NATIONAL BUREAU OF STANDARDS REPORT

6601

PROJECTS and PUBLICATIONS of the APPLIED MATHEMATICS DIVISION A Quarterly Report

July through September 1959

For Official Distribution



U. S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS

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NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT

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of the

APPLIED MATHEMATICS DIVISION

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APPLIED MATHEMATICS DIVISION

July 1 through September 30, 1959

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CONTENTS

Status of Projects* as of September 30, 1959	1
1. Numerical analysis	1
2. Mathematical tables and programming research	8
3. Probability and mathematical statistics	12
4. Mathematical physics	15
5. Mathematical and computational services	19
6. Statistical engineering services	35
Application of automatic computer	37
Lectures and symposia	41
Publication activities	42

*Only unclassified projects are included in this report.

September 30, 1959

1. NUMERICAL ANALYSIS

Note: For administrative reasons the numbers of NBS projects have been slightly modified. The following list will facilitate reference to write-ups on corresponding Numerical Analysis tasks in earlier issues of this Report:

Present	Formerly
1101-12-11110	1101-12-1104
1101-12-11411	1101-12-5116/55-56
1101-12-11414	1101-12-5116/56-148

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS Task 1101-12-11110/55-55

Origin: NBS Manager: P. Davis Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. E. Haynsworth revised the paper on "Matrices with Real Roots" and presented it at the meeting of the American Mathematical Society in Salt Lake City, Utah, September 3, 1959. The manuscript of the paper on "Bounds for Determinants with Positive Diagonals" was completed, and a paper on "Partitioned Matrices" is now in preliminary manuscript form.

R. Silverman continued his investigation of the structure of a metric space proposed in previous work and having combinatorial applications. (Given a class of sets, we may construct a metric over the class by defining the distance between any two sets as the number of elements contained in at least one, but not both of the sets. Various classical combinatorial configurations such as orthogonal Latin squares, projective planes, Hadamard matrices, and balanced incomplete block designs can be represented by appropriate subspaces. As a special case one also obtains the Hamming distance, used in the theory of symmetric binary codes.)

H. F. Weinberger derived the following result in approximation theory. Given the value of u(x) at the n+l equally spaced points i/n, $i=0,\ldots,n$, and the bound

(1)
$$\int_0^1 {u''}^2 dx \leq M^2,$$

Authorized 8/29/54

we find the range of possible values of $u(\xi)$ for a given $0 < \xi < 1$. Let v(x) be the piecewise cubic polynomial having two continuous derivatives and discontinuities in its third derivative only at the points i/n, and such that

(2)
$$v(\frac{i}{n}) = u(\frac{i}{n}),$$

 $v''(0) = v''(1) = 0.$

Then

(3)
$$\left| u(\xi) - v(\xi) \right|^2 \leq c^2 M^2$$

where the constant c is given by

$$c^{2} = \frac{2}{n^{3}} (n\xi - j)(j + 1 - n\xi) [(j + 2 - n\xi)c_{j} + (n\xi + 1 - j)c_{j+1} - \frac{1}{3} (n\xi - j)^{3}(j + 1 - n\xi)^{3}]$$

in terms of the auxiliary constants

$$\begin{aligned} c_{e} &= \frac{1}{6(\tau^{n} - \tau^{-n})} [n\xi (n\xi + 1) (n\xi - 1) (\tau^{e-n} - \tau^{n-e}) - n(1 - \xi) \{ n(1 - \xi) + 1 \} \\ &\cdot \{ n(1 - \xi) + 2 \} (\tau^{e} - \tau^{-e})] - \frac{(n\xi - j)(j + 1 - n\xi)}{6(\tau^{n} - \tau^{-n})(\tau - \tau^{-1})} \\ &\cdot [(j + 2 - n\xi)(\tau^{n-j} - \tau^{j-n}) + (n\xi + 1 - j)(\tau^{n-j-1} - \tau^{-n+j+1})](\tau^{e} - \tau^{-e}), \\ &e = j, j + 1, \quad \tau = -2 + \sqrt{3}, \quad j < n\xi < j + 1. \end{aligned}$$

The error bound is sharp in the sense that there are functions u(x) having the given values u(i/n), satisfying (1) and such that

$$u(\xi) = v(\xi) + cM.$$

N. Bazley and P. Davis continued their investigation of the accuracy of Monte Carlo calculations for Markoff chains. It has been noted that the error in such calculations is bounded by $c \sqrt{n}$, where n is the number of trials and c is a positive constant depending on the statistic involved. These bounds have been shown to be valid in a problem where both an experimental and theoretical solution is known.

An ordinary differential equation which arises in the theory of buckling of beams has been considered by N. Bazley. Upper and lower bounds have been found for the first eigenvalue. The lower bound calculation involved both a simultaneous change of the operator and the boundary conditions.

Publications:

(1) Reduction formulae for partitioned matrices. E. Haynsworth. Submitted to a technical journal.

- (2) Application of a theorem on partitioned matrices. E. Haynsworth. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.
- (3) Regions containing the characteristic roots of a matrix. E. Haynsworth. In manuscript.
- (4) Bounds for determinants with positive diagonals. E. Haynsworth. In manuscript.
- (5) Matrices with real roots. E. Haynsworth. In manuscript.
- (6) Partitioned matrices. E. Haynsworth. In manuscript.
- (7) A metrization for power-sets with applications to combinatorial analysis. R. Silverman. To appear in the Canadian Journal of Mathematics.
- (8) Computation problems concerned with the Hilbert matrix. J. Todd. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.
- (9) On a generalization of an inequality of L. V. Kantorovitch. W. Greub and W. Rheinboldt. Proc. Amer. Math. Soc. 10, 407-415 (1959).
- (10) Uniform asymptotic expansions for Weber parabolic cylinder functions of large orders. F. W. J. Olver. To appear in the Journal of Research NBS, Section B. Mathematics and Mathematical Physics.
- (11) Leonhard Euler's integral: A historical profile of the Gamma function. (In memoriam Milton Abramowitz.) P. J. Davis. To appear in the American Mathematical Monthly.
- (12) Relations between summation methods and integral transforms. W. Greub. J. Research NBS, 63B, 1-13 (1959).
- (13) Linear differential equations of the second order with large parameter. F. W. J. Olver. To appear in the Journal of the Society for Industrial and Applied Mathematics.
- (14) Basic theorems in matrix theory. M. Marcus. To appear as NBS Applied Mathematics Series 57.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS Task 1101-12-11411/55-56

Authorized 8/13/54

Origin: NBS Sponsor: Office of Naval Research Manager: M. Newman Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. M. Newman has completed a manuscript entitled "Subgroups of the modular group and sums of squares." Many theorems on the structure of congruence subgroups of an arbitrary subgroup of the modular group are given, and these are used to prove the existence of linear recurrence relations of length independent of n for the arithmetical function $r_s(n)$, the number of representations of n as the sum of s squares, s any positive integer. A set of notes on the numerical solution of equations and of differential equations has been prepared by M. Newman, in connection with a course in numerical analysis offered by the NBS Graduate School.

K. Goldberg continued his study of the gaps between integers relatively prime to the first n primes. K. Kloss extended the table of very long gaps up to n=36.

K. Goldberg and J. Gager developed a program for testing models for a certain type of human competetive behavior.

K. Goldberg studied group generated incidence spaces and proved the following: If there exists a group G with two subgroups M and N of the same index v, at least one of which has double index two, and $k = [M:M \cap N]$ is less than v, then there exists a v, k, λ design.

K. Goldberg continued his survey of recent work on incidence matrices. He also continued his investigation into the properties of the coefficients in the formal power series for log $e^{x}e^{y}$.

Publications:

- Generating functions for formal power series in non-commuting variables.
 K. Goldberg. To appear in the Proceedings of the American Mathematical Society.
- (2) The minima of cyclic sums. K. Goldberg. To appear in the Journal of the London Mathematical Society.
- (3) The incidence equation $AA^{T} = aA$. K. Goldberg. To appear in the American Mathematical Monthly.
- (4) Principal submatrices of a full-rowed non-negative matrix.K. Goldberg. J. Research NBS 63B, 19-20 (1959).
- (5) The minimum of a certain linear form. K. Goldberg. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.
- (6) Note on a paper by S. Mukhoda and S. Sawaki. K. Goldberg. Submitted to a technical journal.
- (7) Inclusion theorems for congruence subgroups. M. Newman and I. Reiner. Trans. Amer. Math. Soc. 91, 369-379 (1959).
- (8) Some computational problems concerning integral matrices. O. Taussky. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.
- (9) The construction of Hadamard matrices. K. Goldberg and E. C. Dade. To appear in the Michigan Journal of Mathematics.
- (10) Construction and application of a class of modular functions II.
 M. Newman. Proc. London Math. Soc. 9, 373-387 (1959).
- Modular forms whose coefficients possess multiplicative properties.M. Newman. To appear in Annals of Mathematics.
- (12) On the convergence of the Rayleigh quotient iteration for the computation of characteristic roots and vectors: III. Generalized Rayleigh quotient and characteristic roots with linear elementary divisors; and IV. Generalized Rayleigh quotient for nonlinear elementary divisors. A. M. Ostrowski. Arch. Rat. Mech. Anal. <u>3</u>, 325-347 (1959).
 V. Usual Rayleigh quotient for non-Hermitian matrices

and linear elementary divisors, and VI. Usual Rayleigh quotient for nonlinear elementary divisors, to appear in Archive for Rational Mechanics and Analysis.

- (13) Weighted restricted partitions. M. Newman. To appear in Acta Mathematica.
- (14) Über genaherte Determinanten und bedingte Tragheitsindizes. A. M. Ostrowski. To appear in (Wiener) Monatshefte für Mathematik und Physik.

INFORMATION SELECTION SYSTEMS Task 1101-12-11412/60-470

Authorized 9/25/59

Origin: NBS Sponsor: National Science Foundation Managers: K. Goldberg, A. J. Goldman

Objective: To conduct basic mathematical research related to information selection systems.

Background: With the advent of high speed electronic computers it becomes important to utilize these machines for storage, selection and retrieval of information, as in libraries or file systems. This requires a rigorous logical analysis of data and document selection systems. Such an analysis must provide means for the description and treatment of the two major functions of these systems; namely, (1) the <u>storage</u> of rather complete information on a specific topic or topics in such a way as to permit (2) <u>rapid recall</u> of directly or indirectly defined portions of this information.

Status: NEW. K. Goldberg continued his investigation of the problem of classification, especially the problem of "almost partitioning", together with J. Edmonds.

A. J. Goldman resolved some questions left open in an earlier study (with B. K. Bender (11.2)) of combinatorial problems related to a mail sorting device. A paper has been written and accepted for publication.

A. J. Goldman completed the major part of an investigation (with B. K. Bender (11.2) and R. B. Thomas (12.5)) of simplification processes for Boolean functions. A SEAC program requiring remarkably little internal storage and no use of external memory is in operation. Phase I of the program determines the function's core. (For terminology see W. Quine, Amer. Math. Monthly <u>59</u>, (1952), p. 521, and <u>62</u> (1955), p. 627.) Phase II determines the columns of the reduced table of prime implicants. In phase III some selections from these columns are made and some entries are deleted; this effects partial progress toward selecting a simplest disjunctive normal form for the function. The validity of the criteria of phase III (which are believed new) has been established. The remaining topic of study is the invariance of the result of phase III under permutation of the individual steps. A manuscript is in preparation. Publications:

- Zeros of certain polynomials. A. J. Goldman. J. Research NBS <u>63</u>B, 21-22 (1959).
- (2) A continuous poker game. A. J. Goldman and J. J. Stone (Stanford University). To appear in the Duke Mathematical Journal.
- (3) A symmetric continuous poker model. A. J. Goldman and J. J. Stone (Stanford University). To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.
- (4) Analytical comparison of suggested configurations for automatic mailsorting equipment. B. K. Bender and A. J. Goldman. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.
- (5) Capacity requirement of a mail sorting device, II. A. J. Goldman. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.
- (6) A note on algebras. A. J. Goldman. To appear in the American Mathematical Monthly (Math. Notes).

ORTHOGONAL FUNCTIONS IN THE THEORY OF PARTIAL DIFFERENTIAL EQUATIONS Task 1101-12-11413/60-469

Authorized 9/25/59

Origin: NBS Sponsor: Atomic Energy Commission Managers: P. Davis, P. Rabinowitz

Objective: To investigate the method of orthogonal functions in the theory and numerical solution of partial differential equations of elliptic type.

Background: Partial differential equations of elliptic type occur in numerous engineering and scientific problems, such as air flow at subsonic speeds, electromagnetic fields, heat transfer, elastic vibrations. The method of orthogonal functions is one of the two major methods of approach to these problems. In the past, studies of this method have been mainly theoretical rather than numerical. In order to facilitate the application of electronic computing machines to these problems it is of great importance to conduct numerical experiments using the method of orthogonal functions.

Status: NEW. A double precision floating orthonormalization routine was written for use in solution of elliptic partial differential equations by orthogonal functions by P. Rabinowitz.

A double precision floating differencing routine was also written by P. Rabinowitz to check a conjecture by G. Szego concerning the complete monotonicity of the differences of θ_r where $P_n(\cos \theta_r) = 0$. The case n=64 had been checked by Dr. Rabinowitz on WEIZAC and gave verification to the conjecture. It was felt worthwhile to test a case for odd n. When values of $x_r = \cos \theta_r$ became available, the conjecture was tested here and was verified for n=63. The differences of the zeros, x_r , of $P_n(x)$ were also

tested but monotonicity of the differences held only up to differences of order 6 for n=63.

P. Rabinowitz wrote a double precision floating Legendre polynomial subroutine, as well as a double precision floating general quadrature subroutine. These were used to write a double precision floating routine to evaluate

$$L_{m} = \frac{1}{2}(m+1) \int_{-1}^{1} (1-x)^{-1} | P_{m}(x) - P_{m+1}(x) | dx$$

which are the Lebesgue constants of the series of Legendre polynomials at the end point x=1.

In addition a double precision floating routine was written to generate the first 25 basic harmonic polynomials in three variables for use in solution of the Dirichlet problem in three dimensions by means of orthogonal functions.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION Task 1101-12-11414/56-148

Origin and Sponsor: National Institutes of Health Authorized 9/30/55 Manager: P. Rabinowitz Full task description: July-Sept 1955 issue, p. 7

Status: CONTINUED. Computations continued as requested by the sponsor covering, during this quarter, the slightly modified form of the one-dimensional case.

It is interesting to note that this project, which is concerned with the behavior of nerve fibres under electrical stimuli as described by the Hodgkin-Huxley differential equations, is one of the few existing applications of high speed computers to mathematical biology.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

Note: For administrative reasons the numbers of NBS projects have been slightly modified. The following list will facilitate reference to write-ups on corresponding tasks in earlier issues of this Report:

Present	Formerly
1102-12-11120	3711-60-1120
1102-40-11112	1102-40-1110
1102-40-11421	1102-40-5113
3911-61-39952	3711-60-0009

TABLES OF COULOMB WAVE FUNCTIONS Task 1102-40-11112/47-2

Origin: NBS Manager: I. A. Stegun Full task description: Apr-June 1949 issue, p. 45

Status: INACTIVE.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS Task 1102-40-11112/51-8

Origin: Section 11.3, NBS Manager: S. Peavy Full task description: Apr-June 1951 issue, p. 49 Authorized 3/26/51

Authorized 7/1/47

Status: INACTIVE.

REVISION OF MATHEMATICAL TABLES Task 1102-40-11112/52-7

Origin: NBS Manager: I. A. Stegun Full task description: July-Sept 1951 issue, p. 41

Status: INACTIVE.

SPHEROIDAL WAVE FUNCTIONS Task 1102-40-11112/52-37

Origin: NBS Manager: D. Liepman Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE.

SIEVERT'S INTEGRAL Task 1102-40-11112/52-57

Origin: NBS

Authorized 2/12/52

Authorized 11/28/51

Authorized 8/10/51

Managers: M. Paulsen, P. O'Hara Full task description: Jan-Mar 1952 issue, p. 46

Status: INACTIVE.

HANDBOOK OF MATHEMATICAL FUNCTIONS Task 1102-40-11421/57-216

Origin and Sponsor: National Science Foundation Authorized 12/27/56 Manager: I. A. Stegun Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. The textual material for Chapter 26 (Probability functions) is being distributed for comments. The accompanying tables have been punched on cards and are presently being checked before typing.

Chapter 16 (Elliptic functions) and Chapter 15 (Hypergeometric functions) are being revised and prepared for distribution. The tabular material for Chapter 16 has been checked and the manuscript prepared.

Text and tables are being revised for Chapter 8 (Legendre functions), Chapter 20 (Mathieu functions), and Chapter 21 (Spheroidal wave functions).

All chapters are undergoing review for consistency of format and notation. Chapters 4, 7, and 25 are presently being prepared in suitable form for the printer. Revised galley proofs have been received and checked for Chapter 6.

AUTOMATIC CODING Task 1102-12-11120/55-65

Authorized 9/29/54

Origin: NBS Manager: J. Wegstein Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. A guide for using the Black Box Computer was completed and is available to NBS personnel on request. Thirty-minute presentations on this tool are given to small groups of interested persons on request. A differencing operation was added to this computer program and a plotting routine is nearing completion. Service requests to the Black Box Computer are generally completed and returned to the originator within 24 hours.

A new automatic operator system was placed in operation on the 704 computer. This was accomplished through cooperation with the David Taylor Model Basin and consisted of adapting the Bell Laboratories Input-Output and Monitor System (BE SYS2) for NBS use. An instruction manual "NBS Programming with BE SYS2" was prepared by G. M. Galler.

Much 704 computing is now being done with programs written by individuals from outside the Computation Laboratory. This is termed "openshop programming." Because open shop programmers use numerous automatic programming aids, such as FORTRAN and the Bell Systems, the automatic coding tools include on a limited basis answering numerous questions and giving aid to individuals in checking out their programs. MATHEMATICAL SUBROUTINES Task 3911-61-39952/56-160

Origin: NBS Managers: Staff Full task description: July-Sept 1955 issue, p. 13

Status: CONTINUED. The program write-up for the orthonormalization subroutine has been completed, and this write-up and symbolic deck are being put into final form for submission to SHARE.

Since the orthonormalizing code is so general, it was decided to prepare a code for polynomial data fitting only. For this purpose a recursive method of G.E. Forsythe is being adapted by P. Rabinowitz to include the augmentation feature which was employed in the orthonormalization subroutine. This feature provides much of the output obtainable from the present orthonormalizing code. The method is being coded by P. J. Walsh.

Authorized 9/30/55

3. PROBABILITY AND MATHEMATICAL STATISTICS

Note: For administrative reasons the numbers of NBS projects have been slightly modified. The following list will facilitate reference to write-ups on corresponding Statistics tasks in earlier issues of this Report:

Formerly
1103-12-1130
1103-12-1107
1103-12-5148

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS Task 1103-12-11131/51-2

Authorized 7/1/50

Origin: NBS Manager: C. Eisenhart Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. M. Zelen and N. C. Severo have completed their work on the chapter entitled "Probability Functions" which is to appear in the forthcoming N.B.S. Handbook of Mathematical Functions (see p. 9).

M. Zelen and N. C. Severo have written a paper entitled "Graphical computation of bivariate normal probabilities." The methods presented in the paper enable one to quickly calculate the bivariate normal integral with a uniform accuracy of 2D.

M. Zelen and N. C. Severo are preparing a manuscript entitled "Normal approximation to the chi-square and non-central F distributions." The approximations to the chi-square integral give 5S accuracy for degrees of freedom larger than 30. The non-central F approximation gives 2D accuracy, i.e., the same as Patnaik's approximation, but has the advantage that the approximating function is the normal probability function.

Miles Davis prepared a preliminary report on "Distribution of the Cramér-Smirnov statistic in the parametric case."

Publications:

- The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. Joel. To appear in the Annals of Mathematical Statistics.
- Measurements made by matching with known standards. W. J. Youden,
 W. S. Connor and N. C. Severo. Technometrics 1, 101-109 (1959).

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN Task 1103-12-11131/53-1

Authorized 10/15/52

Authorized 12/15/55

Origin: NBS Manager: W. S. Connor Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. M. Zelen and J. M. Cameron are preparing a manuscript dealing with the theory of least squares.

Publications:

(1) Randomization and experimentation. W. J. Youden. To appear in Annals of Mathematical Statistics.

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES Task 1103-12-11131/56-170

Origin: NBS Manager: J. R. Rosenblatt Full task description: Oct-Dec 1955 issue, p. 14

Status: CONTINUED. Shirley Young completed a paper on "Exact and approximate distributions for the Wilcoxon Statistic with ties."

Publication:

(1) On the power of some rank order two-sample tests. J. R. Rosenblatt. To appear in "Contributions to probability and statistics," in press, Stanford University Press (1960).

> MEASUREMENT OF RELIABILITY Task 1103-12-11130/56-182

> > Authorized 3/23/56

Origin: NBS Manager: M. Zelen, J. R. Rosenblatt Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. Miles Davis spoke at a Statistical Engineering Laboratory Reliability Seminar, August 21, on "Stochastic Models for Life Testing." In connection with this talk, he prepared a selected and partially annotated bibliography on models for life testing.

The following translations were prepared by J. R. Rosenblatt: "On mathematical methods of controlling abstract transformations" by

Yu. I. Zhuravlev; selections from the book "Reliability of Electronic Equipment (Collected Papers)" "Soviet Radio" Publishers, Moscow, 1958.

Publications:

- (1) On prediction of system behavior. J. R. Rosenblatt. To appear in the "Proceeding of the NYU-Industry Conference on Reliability Theory," held at Ardsley-on-Hudson, N.Y., June 1958, pp. 39-52.
- (2) Analysis of two-factor classifications with respect to life tests.
 M. Zelen. To appear in "Contributions to probability and statistics," in press, Stanford University Press (1960).
- (3) Factorial experiments in life testing. M. Zelen. Technometrics 1, 269-288 (1959).
- (4) Theoretical problems of reliability measurement and prediction. J. R. Rosenblatt. "Proceedings of the Third Exploratory Conference on Missile Model Design for Reliability Prediction," held at White Sands Missile Range, April 1959, pp. 29-34.

FRACTIONAL FACTORIALS FOR THE MIXED SERIES Task 1103-12-11431/58-291

Authorized 9/30/57

Origin and Sponsor: Bureau of Ships Managers: W. S. Connor, M. Zelen Full task description: July-Sept 1957 issue, p. 43

Status: COMPLETED. A prepublication copy of the catalog of fractional

factorial designs has been circulated to certain specialists in the field for critical comment prior to publication.

Publications:

- (1) Construction of fractional factorial designs of the mixed 2^m3ⁿ series.
 W. S. Connor. To appear in "Contributions to probability and statistics", in press, Stanford University Press (1960).
- (2) Analysis of fractionally replicated 2^m3ⁿ</sup> designs. R. C. Bose and W. S. Connor. To appear in the Proceedings of the 31st Session of the International Statistical Institute, Brussels, 1958.

4. MATHEMATICAL PHYSICS

Note: For administrative reasons the numbers of NBS projects have been slightly modified. The following list will facilitate reference to write-ups on corresponding Mathematical Physics tasks in earlier issues of this Report:

Formerly
1104-12-1115
1104-12-1140
1104-12-5160/59-420
1104-12-5160/56-154

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS Task 1104-12-11141/55-57

Origin: NBS Manager: W. H. Pell Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. A. Ghaffari has been revising the manuscript of his paper on Rayleigh's nonlinear differential equation which is associated with acoustical phenomena and maintained vibrations. His recent result consists of the complete determination of the nature of the higher order critical point P(1,0,0) on Poincaré's unit-sphere; he then deduced the behavior in the large of paths tending to the critical point P.

Taking into account the parity of damping and restoring forces in Rayleigh's equation and their leading coefficients, an analysis of the homogeneous differential equation (see Jan-Mar 1959 issue, p. 13) he shows that the higher order critical point P is a saddle point. Then, it is seen that all paths tend to infinity with a definite asymptotic direction, and they are asymptotic to the x-axis as they tend to the critical point P.

In the third of a series of reports on the Stokes flow problem for a class of axially symmetric bodies W. H. Pell and L. E. Payne consider the case in which the meridian section of the flow region is not simply connected and solve in particular the flow about a torus. The most interesting feature of these problems is the determination of the stream constant on the meridian profile in such a way that the pressure remains single-valued.

Publications:

(1) Turbulent motion. C. M. Tchen and G. B. Schubauer (NBS Fluid Mechanics Section). In press, to appear as Section B, Volume V, of the Princeton Series, High Speed Aerodynamics and Jet Propulsion (Princeton University Press).

Authorized 9/1/54

- (2) Diffusion of particles in turbulent flow. C. M. Tchen. Advances in Geophysics <u>6</u>: Atmospheric Diffusion and Air Pollution, pp. 165-173 (Academic Press, New York and London, 1959).
- (3) Stokes flow problem for a class of axially symmetric bodies. L. E. Payne and W. H. Pell. Submitted to a technical journal.
- (4) Rotational properties of two-dimensional lattices. J. P. Vinti. Submitted to a technical journal.

PLASMA RESEARCH Task 1104-12-11140/59-422

Origin: NBS Manager: C. M. Tchen Full task description: Apr-June 1959 issue, p. 15

Status: CONTINUED. C. M. Tchen has continued his work on dynamics of plasma. Attention was given to the structure of the correlation function and the effects of the magnetic field on the shielding mechanism. The problem of plasma radiation in a magnetic field was also studied. Dr. Tchen is presently attached to the United Kingdom Atomic Energy Research Establishment at Harwell, England, as a Guggenheim Fellow, and will collaborate in the above and other problems of plasma dynamics with his British colleagues.

In August, a talk entitled "Microscopic and Macroscopic Description of Plasma Flow" was given by Dr. Tchen at the General Electric Company, Schenectady, N. Y. The talk gave the derivation of the transfer equations and the calculations of the electric conductivity on the basis of the kinetic equation as derived by Tchen (see Phys. Rev. 114, 394-411, 1959).

Publication:

 Kinetic equation for a plasma with unsteady correlations. C. M. Tchen. Phys. Rev. 114, 394-411 (1959).

> RESEARCH ON SATELLITE ORBITS Task 1104-12-11440/59-420

> > Authorized 12/19/58

Authorized 6/30/59

Origin: NBS Sponsor: Office of Scientific Research, ARDC, USAF Manager: J. P. Vinti Full task description: Oct-Dec 1958 issue, p. 15

Status: CONTINUED. J. P. Vinti spent the month of July 1959 as a participant in the Summer Institute in Dynamical Astronomy at Yale University. While there he gave three hours of lectures on his recent investigations in satellite astronomy, their title being "A New Method of Solution for Unretarded Satellite Orbits."

The main task in completing this research consists in evaluating the six integrals appearing in the kinetic equations of motion and solving the resulting system for the coordinates as explicit functions of time. It appears that the six integrals are all expressible as elliptic integrals, but that to express them as such it is necessary to factor two quartics. Some progress has been made in doing this and also in developing an alternative approach involving series expansions. These series expansions may be converted to series in powers of the oblateness parameter $k = (I_p - I_t)/MR^2$, where I_p , I_t , M, and R are respectively the polar and transverse moments of inertia, the mass, and the equatorial radius of the planet.

Meanwhile it has appeared desirable to try to check directly the outstanding features of the solution that occur when perturbation theory is applied to the problem. These results are that for an orbital inclination $I = 63.4^{\circ}$ there is no motion of the line of apsides relative to the line of nodes and that certain singularities then occur, necessitating a different approach when I \approx 63.4°. To investigate these features before the complete solution of the main problem is at hand, it has proved useful to calculate two frequencies, a spheroidal frequency n_1 and the draconic frequency n_2 . Here $1/n_1$ is the time interval from perigee spheroid to perigee spheroid, and $1/n_2$ is that from ascending node to ascending node. To do so, it is sufficient to calculate the derivatives of the action variables with respect to the Jacobi α 's, the integrals then occurring being somewhat simpler than those in the kinetic equations of motion. It turns out that when $n_1 = n_2$ the spheroidal and draconic periods are both equal to the anomalistic period, the time interval from perigee to perigee. Thus when $n_1 = n_2$, there is no motion of the line of apsides relative to the line of nodes.

Expansion of these frequencies n_1 and n_2 through the first power of k shows that they are equal, to this approximation, if and only if $\cos^2 I = 1/5$, i.e., $I = 63.4^{\circ}$. This result verifies the first outstanding feature of the solution by perturbation theory. To investigate the second feature, i.e., the possibility of singularities, it will be necessary to obtain the expansions for the frequencies through the second power of k. This development is now being carried out for n_1 and n_2 , along with that for the sidereal frequency n_3 .

Publications:

- (1) New approach in the theory of satellite orbits. J. P. Vinti. Phys. Rev. Letters 3, 1(July 1, 1959).
- (2) A new method of solution for unretarded satellite orbits. J. P. Vinti. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS Task 1104-12-11626/56-154

Authorized 9/30/55

Origin: NBS Sponsor: Office of Naval Research Manager: F. Oberhettinger Full task description: July-Sept 1955 issue, p. 20

Status: CONTINUED. Preparation of an inverse table of Fourier transforms has been completed. The manuscript is being prepared in a form suitable for submittal for publication.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

Note: For administrative reasons the numbers of NBS projects have been slightly modified. The following list will facilitate reference to write-ups on corresponding tasks in earlier issues of this Report:

Present	Formerly
1102-40-11645	1102-40-5126
3911-61-39952 .	3711-60-0009

3911-61-39952/54-30 SPECTRUM ANALYSIS Origin: NBS, Division 4 Managers: C. D. Coleman, W. Bozman (4.1) Full task description: Jan-Mar 1954 issue, p. 46 Status: Continued. The wavelength computing codes from Steinhaus were modified to agree with our format and used to compute about 6000 lines of praseo-dymium, 1000 lines of titanium, and 500 lines of holmium. A search for energy levels of iodine II gave about 40 new even and odd levels. Matrices of energy levels of thorium were set up to fit the observations of Zalubas to the theoretically expected levels. A code is being written to calculate observed patterns from interferometry of an atomic beam. 3911-61-39952/54-38 EQUATION OF STATE OF REAL GASES (formerly COMPRESSIBILITY FACTORS OF DRY AIR) Origin: NBS, Section 3.2 Manager: M. Paulsen Full task description: Jan-Mar 1954 issue, p. 48 Status: Continued. The code for punching cards, mentioned in the April-June 1959 issue has been completed and used by the sponsor. This card output can be used as input data for other routines. 3911-61-39952/55-68 CRYSTAL STRUCTURE CALCULATIONS Origin: NBS, Division 9 Managers: P. O'Hara, S. Block (9.7) Full task description: Jan-Mar 1955 issue, p. 18 Status: Continued: A new crystal, aluminum dodecaboride is now being processed by probability methods to determine phase angles and ultimately the crystal structure. Normalized structure factors have been calculated and a modified program to test for rational dependence is being code checked.

3911-61-39952/55-82 THERMOMETER CALIBRATIONS Origin: NBS, Section 3.1 Manager: S. Prusch Full task description: Jan-Mar 1955 issue, p. 20 Status: Continued. ITS constants and tables were computed for 38 thermometers under test. LTS constants and tables were calculated for 36 thermometers. 1102-40-11645/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS Origin and Sponsor: NBS, Section 2.5 Manager: S. Prusch Full task description: July-Sept 1956 issue, p. 33 Status: Continued. Calculations were performed for 12 laboratory sets of gage blocks. 1102-40-11645/57-219 THERMAL PROPERTIES Origin and Sponsor: NBS, Section 3.2 Manager: R. Varner Full task description: Oct-Dec 1956 issue, p. 30 Status: Continued. Production runs were made under the direction of the sponsor. The results have been made available in a report, "Preliminary Report on Thermodynamic Properties of Lithium, Beryllium, Magnesium, Aluminum and Their Compounds with Hydrogen, Nitrogen, Oxygen, Fluorin and Chlorine." 1102-40-11645/57-221 BESSEL FUNCTIONS FOR COMPLEX ARGUMENTS Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army Manager: R. Zucker Full task description: Oct-Dec 1956 issue, p. 31 Status: Inactive. 3911-61-39952/57-223 SELF-CONSISTENT FIELDS Origin: NBS, Section 3.2 Manager: E. V. Haynsworth Full task description: Apr-June 1957 issue, p. 28 Status: Reactivated. Minor changes were made in the code to handle input and output data more conveniently. 3911-61-39952/57-229 APPLICATION OF ELECTRONIC DATA PROCESSING MACHINERY TO PAYROLL OPERATIONS Origin: NBS, Section 40.0 Managers: M. Paulsen, P. Ruttenberg Full task description: Jan-Mar 1957 issue, p. 36 Status: Inactive.

1102-40-11645/57-236 SELF CONSISTENT FIELDS--EIGENVALUES Origin and Sponsor: NBS, Section 3.6 Manager: E. Haynsworth Full task description: Apr-June 1957 issue, p. 30 Status: Inactive.

1102-40-11645/58-263 GAS TUBE CHARACTERISTIC Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army Manager: I. A. Stegun

Full task description: July-Sept 1957 issue, p. 35 Status: Continued. Several reruns have been made for testing convergence and stability. A revised formulation of the problem is being considered.

3911-61-39952/58-266 DEPOLYMERIZATION, II Origin: NBS, Section 7.6 Manager: L. S. Joel Full task description: July-Sept 1957 issue, p. 36 Status: Continued. Plans are underway for polymer chains of lengths 400, 1000, and 2000.

1102-40-11645/58-269 MOLECULAR STRUCTURE, IV Origin and Sponsor: Naval Research Laboratory, USN <u>Manager</u>: P. J. O'Hara

Full task description: July-Sept 1957 issue, p. 38

Status: Continued. Structure determination of two new crystals, lysozyme chloride and n-benzyl dihydronics-tinamide, has been started this past quarter. Intensity data for like reflections of the lysozyme chloride crystal were averaged resulting in a unique deck of about 7500 reflections. Normalized structure factors (E) were computed, and rational dependence tests are proceeding for both crystals.

1102-40-11645/58-270 MATHEMATICAL PROBLEMS RELATED TO POSTAL OPERATIONS Origin: NBS

Sponsor: Post Office Department, Office of Research and Engineering Managers: B. K. Bender, A. J. Goldman

Full task description: Oct-Dec 1958 issue, p. 22

- Status: Continued. The study of the combined incoming-outgoing sort is nearing completion. Work on determining the optimal location for sorting installations is about to begin.
- <u>Publications:</u> (1) Analytic comparison of suggested configurations for automatic mail sorting equipment. B. K. Bender and A. J. Goldman. To appear in the Journal of Research, NBS, Sec. B. (2) Capacity requirement of a mail sorting device, II. A. J. Goldman. To appear in the Journal of Research, NBS, Sec. B.

1102-40-11645/58-272 THERMODYNAMIC PROPERTIES OF REAL GASES Origin and Sponsor: NBS, Section 3.2 Manager: J. P. Menard Full task description: Oct-Dec 1957 issue, p. 32

Status: Continued. A code has been written and is now being checked out which will perform inverse interpolation and subtabulation on a given table of values. The purpose of the code is to retabulate existing tabl of thermodynamic properties versus density at constant temperature to tables of thermodynamic properties versus enthalpy or entropy at constan temperature. All interpolations are performed with Aitken's method of successive approximation. The order of the interpolation may vary with each table.

1102-40-11645/58-274 CALCULATIONS FOR d-SPACINGS, II Origin and Sponsor: NBS, Division 9 Manager: R. Zucker Full task description: July-Sept 1957 issue, p. 38

<u>Status</u>: Continued. About 19 calculations for d-spacings for orthogonal, hexagonal, and orthorhombic crystals were carried out this quarter, and redetermination of unit cell constants by least squares fitting to a measured d-spacing was performed for about 13 crystals.

1102-40-11645/58-299 TIME-DEPENDENT SCHROEDINGER EQUATION Origin and Sponsor: NBS, Section 3.1 Manager: A. Schopf Full task description: Oct-Dec 1957 issue, p. 39 Status: Completed. Results have been transmitted to the sponsor.

1102-40-11645/58-304 TRANSPORT PROPERTIES OF AIR AT ELEVATED TEMPERATURES Origin and Sponsor: NBS, Section 3.2 Manager: P. J. Walsh, J. D. Waggoner

Full task description: Oct-Dec 1957 issue, p. 40

Status: Continued. The supervisory routine for joining three of the subprograms has been checked out. These subprograms calculate η (viscosity), K_o (thermal conductivity due to the transport of translational energies), and C_{ij} (diffusion coefficients). A fourth subprogram has been added which computes K₁ (the Eucken correction to the thermal conductivity due to the transport of rotational and vibrational energy), K₂ (part of the thermal conductivity due to the transport properties, such as the Prandtl numbers.

The code for calculating the potential parameters has been written and is now being checked (see P. and P. Apr-June 1959 issue, pp. 21-22 for a description of these parameters).

Another program has also been written under this project. It determines a set of parameters (called force constants) which are required to correlate the transport properties of gases by means of the Chapman-Euskog kinetic theory. 1102-40-11645/58-307 STUDY OF SURFACE TENSION Origin and Sponsor: NBS, Section 9.2 Manager: R. Arms Full task description: Oct-Dec 1957 issue, p. 43 Status: Inactive.

1102-40-11645/58-308 OSCILLATING SPHERE Origin and Sponsor: NBS, Section 3.4 <u>Manager: S. Prusch</u> Full task description: Oct-Dec 1957 issue, p. 43 Status: Terminated. Production runs will continue under the direction of the sponsor, and the machine time will be reported in the section of this Report entitled, "Application of Automatic Computer."

1102-40-11645/58-316 INTERSECTION CAPACITY STUDY Origin and Sponsor: Bureau of Public Roads Managers: S. Peavy, J. M. Cameron Full task description: Jan-Mar 1958 issue, p. 33 Status: Inactive.

1102-40-11645/58-333 CALCIUM HYDROXIDE Origin and Sponsor: NBS, Section 9.0 Manager: P. O'Hara Full task description: Jan-Mar 1958 issue, p. 36 Status: Completed. Results have been transmitted to the sponsor.

1102-40-11645/58-339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS

Origin and Sponsor: NBS, Section 3.4 Manager: H. Oser

Full task description: Jan-Mar 1958 issue, p. 38

Status: Continued. Expressions for the long-time range down to about 1 sec have been developed for the creep- and stress-relaxation functions by expanding in series of eigenfunctions. The experimental values match excellently with those obtained from this theory. Asymptotic expressions are being developed for the short time range.

1102-40-11645/59-348 RUSSIAN-TO-ENGLISH MACHINE TRANSLATION
Origin: NBS
Sponsor: Office of Ordnance Research, U. S. Army
Manager: I. Rhodes
Full task description: Oct-Dec 1958 issue, p. 26
Status: Continued. During this quarter, under the sponsorship of the
United States Air Force, Mr. Murray Sherry was a Guest Worker at the

NBS. Mr. Sherry is making a survey of mechanical means of translation

and spent the time in association with Mrs. Rhodes to obtain a thorough working knowledge of her proposed method for the mechanical translation of Russian to English.

1102-40-11645/58-358 REDUCED CROSS-SECTIONS <u>Origin and Sponsor</u>: NBS, Section 3.2 <u>Manager</u>: R. J. Arms <u>Full task description</u>: Apr-June 1959 issue, p. 30 <u>Status</u>: Continued. Subroutine for the evaluation of the functions ψ and $\frac{\partial \psi}{\partial r}$, where

$$\psi = g - \varphi_{\alpha}(r) - \frac{g^2 b}{r}$$

were checked out. For examination of stability a code is being prepared for printing ψ , $\partial \psi / \partial r$, φ , $\partial \varphi / \partial r$ over various parameters.

1102-40-11645/58-361 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION

Origin and Sponsor: Naval Research Laboratory

Manager: R. J. Arms

Full task description: Apr-June 1958 issue, p. 33

Status: Continued. The results of several production runs have been turned over to the sponsor. Checkout has been completed on two new programs which facilitate data preparation.

1102-40-11645/58-366 RADIATION PATTERNS OF ANTENNAS

Origin and Sponsor: U. S. Information Agency, Department of State

Managers: P. J. Walsh

Full task description: Apr-June 1958 issue, p. 35

Status: Continued. Part 1. Tables a, b, and c have been printed from tape and submitted to the sponsor. Most of table d has also been printed from tape. A small portion of table d remains to be computed and put on tape for final processing.

Part 2. No additional antennas were analyzed during this quarter.

1102-40-11645/58-368	INTENSITY FUNCTIONS AND CROSS SECTIONS OF LIGHT
	SCATTERED BY SPHERICAL PARTICLES
Origin and Sponsor:	U. S. Army Signal Research and Development
	Laboratories, Atmospheric Physics Branch,
	Belmar, N. J.
Manager: H. Oser	

Full task description: July-Sept 1958 issue, p. 32

Status: Continued. Further production runs have been made for the case of coated spheres, and the results have been turned over to the sponsor.

 $\mathbf{24}$

1102-40-11645/59-374 END-EFFECT IN THE CYLINDRICAL ROTATIONAL VISCOMETER Origin and Sponsor: NBS, Section 7.1 Manager: J. P. Menard Full task description: July-Sept 1958 issue, p. 34 Status: Completed. Results have been transmitted to the sponsor. 1102-40-11645/59-377 LOGICAL DIAGRAM REDUCTION Origin and Sponsor: NBS, Section 12.3 Managers: W. Hall, C. Coleman (4.01) Full task description: Apr-June 1959 issue, p. 25 Status: Continued. Production runs have been made, and more are planned as the original logical design of the PILOT computer is changed. 1102-40-11645/59-388 HEAT PUMP CALCULATIONS Origin and Sponsor: NBS, Section 10.3 Manager: R. Zucker Full task description: Jan-Mar 1959 issue, p. 26 Status: Continued. Several production runs with heating and cooling data were made on the IBM-704. 1102-40-11645/59-389 FREQUENCY ALLOCATION Origin and Sponsor: Civil Aeronautics Administration Manager: L. S. Joel Full task description: Oct-Dec 1958 issue, p. 29 Status: Continued. A run of the code using data will be made as soon as the data file from FAA is completed. Theoretical investigations were continued. 1102-40-11645/59-394 VARIATIONAL CALCULATION OF SLOW ELECTRON SCATTERING BY HYDROGEN ATOMS, II Origin and Sponsor: NBS, Section 4.6 Manager: A. Beam Full task description: Oct-Dec 1958 issue, p. 30 Status: Continued. The final master code for the s, p, and d waves has been completed and checked. The previous s-wave code has been very useful in checking the master code, and because of its greater speed is being used for s-wave production runs. Production runs using the new code were started. 1102-40-11645/59-403 COMPUTATION OF COLOR FADINGS Origin and Sponsor: NBS, Section 2.1 Manager: J. P. Menard Full task description: Oct-Dec 1958 issue, p. 30 Status: Continued. Production runs have been made in which Munsell renotations were obtained for 602 samples, and color differences were obtained for 564 samples. This completes the original test program. The codes will continue to be used by the sponsors and all production runs will be under their direct supervision.

1102-40-11645/59-407 FOURIER COEFFICIENTS

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Manager: R. Zucker

Full task description: Jan-Mar 1959 issue, p. 27

Status: Continued. Additional production runs were made on IBM-704, and answers were submitted to sponsor.

1102-40-11645/59-412 DYNAMICS OF PNEUMATIC PRESSURE REDUCERS

Origin and Sponsor: NBS, Section 3.2

Manager: A. Beam

Full task description: Oct-Dec 1958 issue, p. 31

Status: Continued. Minor changes were made, and production runs were continued.

1102-40-11645/59-414 INFINITE SYSTEMS Origin and Sponsor: NBS, Division 3 Manager: R. Zucker Full task description: Jan-Mar 1959 issue, p. 28

Status: Continued. Additional calculations for t=.2 were carried out in the adiabatically isolated system.

Alternative methods of obtaining the relaxation of x_n were coded and checked out. One method entailed the solution of the system of equations

$$\frac{dx_{i}}{dt} = \sum_{j=0}^{3} a_{ij} x_{j}, \qquad i = 0, 1, 2, 3,$$

where initial conditions were given at t=0. Solutions were obtained and their logarithms were printed out for t= 10^{-n} [n=0(1)3], 5.

Solutions were also checked against the following expression

$$x_{n}(t) = (1-e^{-H})e^{-nH},$$

where

$$H = \ln \frac{e^{-t}(1-e^{-\theta}) - e^{\theta}(1-e^{-\theta})}{e^{-t}(1-e^{-\theta}) - (1-e^{-\theta})}, \quad \theta_0 = 3, \quad \theta = 2$$
$$x_n(\infty) = [1-e^{-\theta}]e^{-n\theta}.$$

1102-40-11645/59-415 COMPLEX LEGENDRE FUNCTIONS Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army Manager: R. Zucker Full task description: Jan-Mar 1959 issue, p. 29 Status: Continued. The code for the double summation F(z) was checked out. F(z) was evaluated for numerous parameters and its expected maxima were obtained. 1102-40-11645/59-418 P-WAVE EQUATION Origin and Sponsor: NBS, Section 4.8 Managers: S. Peavy, R. Varner Full task description: Jan-Mar 1959 issue, p. 29 Status: Continued. A production run was made, and results were transmitted to the sponsor for analysis. 1102-40-11645/59-430 ASYMMETRIC ROTOR EIGENVALUE PROBLEM Origin and Sponsor: NBS, Section 3.2 Managers: E. V. Haynsworth, P. Walsh Objective: To calculate the quantum mechanical energy levels of a rigid asymmetric body rotating in space. Background: In the analysis of molecular spectra a molecule is usually assumed to be a rigid body, in the first approximation, insofar as its rotational properties are concerned. It is thus necessary to compute the quantum-mechanical energy levels of a rigid rotating body to a high accuracy. If the molecule has high symmetry the calculation of energy eigenvalues is trivial. However, for the general case the so-called asymmetric top, a secular equation, often of high degree, must be solved. The problem may be formulated in terms of a single asymmetry parameter which is a function of the moments of inertia of the molecule. A code is needed which will permit calculation of any desired set of energy eigenvalues on the IBM 704 when the value of this asymmetry parameter is specified. The problem was submitted by V. W. Laurie. Status: Completed (New). The calculations were carried out as requested, and the results were transmitted to the sponsor. 1102-40-11645/59-434 PETROLOGICAL COMPUTATION Origin and Sponsor: Geophysical Laboratories, Carnegie Institution of Washington Manager: R. Varner Full task description: Apr-June 1959 issue, p. 28 Status: Continued. Several production runs were made under the sponsor's direction.

1102-40-11645/59-435 ELECTROCARDIOGRAPHIC ANALYSIS Origin: NBS, Division 12.5 Sponsor: Veterans Administration Manager: R. J. Arms Full task description: Apr-June 1959 issue, p. 29 Status: Continued. A code has been checked out for sampling on lists of patients. Plane fitting subroutines are being developed. 1102-40-11645/59-444 HOSPITAL SUBSISTENCE ITEMS SUPPLY Origin and Sponsor: Veterans Administration Manager: L. S. Joel Full task description: Apr-June 1959 issue, p. 29 Status: Continued. Investigation of procedural shortcuts resulting from the nature of the constraints have been made and are being described in a report to the Veterans Administration. 1102-40-11645/59-445 OIL SUPPLY Origin and Sponsor: Military Petroleum Supply Agency, Department of the Navy Manager: L. S. Joel Full task description: Apr-June 1959 issue, p. 30 Status: Continued. Three allocation problems have been run on the IBM 704, and runs continue as data is received. An irregularity in the input code was discovered and corrected by M. Paulsen. 1102-40-11645/60-449 SPECTRAL LINE COLORIMETRY Origin and Sponsor: NBS, Section 2.1 Manager: J. P. Menard Objective: To compute the CIE tristimulus values and chromaticity coordinates of samples illuminated by light sources having spectral lines superimposed on a continuum, such as fluorescent lamps. Background: The Photometry and Colorimetry Section, NBS (Section 2.1) is currently investigating, in cooperation with the research committee of one of the national technical societies, the problem of appearance of objects illuminated by fluorescent lamp sources. The color coordinates of a large number of color specimens, which sample the whole of the color solid, need to be computed to enable the prediction of appearance under a number of illuminants. Problem was communicated by J. C. Schleter (2.1). Status: New. Two programs have been written and completely checked out on

the IBM 704. The first, a Fortran program, is the more elaborate of the two, since it was intended to be run on the IBM 704. The second, a Fortransit program, was written with the intention of running it on the IBM 650. It was written in such a way that it would be compatible to both Fortran and the IBM 704 and Fortransit and the IBM 650. It was first checked out on the IBM 704, and then assembled on the IBM 650 and checked out without difficulty. All codes have been turned over to the sponsors and production runs have been made. Production will continue under the sponsor's direction.

1102-40-11645/60-451 MECHANICAL IMPEDANCE Origin and Sponsor: NBS, Section 6.1

Manager: J. M. Smith

Objective: Given values for the parameters R, M, r, m, k, and 29 of the frequency f ranging from 35 to 10,000, to compute the real and imaginary parts of

$$Z_{x} = i2_{\pi} fm + \frac{(R+i2_{\pi} fM)(r-ik/2_{\pi} f)}{(R+r) + i(2_{\pi} fM-k/2_{\pi} f)}$$

<u>Background</u>: A fairly simple mechanical system has been devised to simulate the mechanical properties of the head and mastoid impedances. (See E. L. R. Corliss, W. Koidan, J. Accoust. Soc. Amer. <u>27</u>, 1164 (1955)). The system is made up of five components, a spring stiffness, two masses, and two damping components. Z_x is the complex impedance of the human head or mastoid.

The problem was requested by K. D. Macknet (6.1). <u>Status</u>: Completed. The program was checked out, and the results of production runs were submitted to the sponsor.

1102-40-11645/60-452 CONSULTING SERVICES FOR PACIFIC MISSILE RANGE Origin and Sponsor: NBS, Section 12.3

Manager: G. W. Reitwiesner

Objective: To assist in providing consulting services and computer test display facilities for the Pacific Missile Range.

Background: These services are to be provided via the NBS Data Processing Division (Div. 12) under Interdepartmental Government Order Naer 01987, 31 August 1959. In particular there applies Item Ia of that order which reads as follows:

"Item I. Consultative services will be provided and assistance given in the general area of the applications of automatic computers and data processing methods to problems of test range instrumentation. This will include consultation and assistance on the following specific tasks, as well as on any additional ones requested by the Bureau of Aeronautics, and within the technical competence of the National Bureau of Standards:

a. Establishment of functional requirements and evaluation of computer methods and equipment for a Real Time Computation System for the Pacific Missile Range. This system will provide on-line computation of missile and satellite trajectories, predicted impact point, vehicle control functions, and target acquisition aids for remote tracking equipment."

<u>Status</u>: New. Thirty-three documents were received from NBS Division 12 for orientation. Twenty-four of these documents ranged over a wide area of subjects: explanations of the operation of digital differential analyzers, formal analyses of range instrumentation problems, brochures from equipment manufacturers, analog-to-digital conversion equipment, etc. 1102-40-11645/60-453 DATA CONVERSION

Origin and Sponsor: Army Map Service

Manager: J. Beiman

<u>Objective</u>: To edit and convert ephemeris data now stored on 704 tapes in floating binary form to fixed point binary coded decimal form suitable for UNIVAC tape.

- Background: An ephemeris of satellite position vector components for given periods has been stored on 704 tapes by Vanguard. The conversion of the format of the data is requested so that it can be copied onto UNIVAC tapes, using a tape-to-tape converter, at the David Taylor Model Basin. The problem was submitted by W. Kahn (Army Map Service).
- Status: New. The problem has been programmed and code checked on the 704. One Vanguard tape has been processed, and the results have been submitted to the sponsor.

1102-40-11645/60-456 FIRING CIRCUIT EQUATIONS Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Manager: R. Zucker

a = 10,100

Objective: To obtain solutions for C_2 in equation (1):

(1)
$$1.15 \times 10^{10} C_2 \left[1 - e^{-1/anRC_2} + \frac{43.45 \times 10^{-12}}{C_2} e^{-1/anRC_2} \right]$$

= $1 - \left[1 - \frac{43.45 \times 10^{-12}}{C_2} \right]^n e^{-1/aRC_2}$

where

 $R = 5 \times 10^{5}, 4 \times 10^{5}, 3 \times 10^{5}, 2 \times 10^{5}, 1 \times 10^{5}, 5 \times 10^{4}$ n = 1770, 1558, 1351, 1149, 948, 749, 551, 358, 170.

With the values of R and the obtained C_2 , solutions for n_1 are to be obtained from equation (2):

(2)
$$1.15 \times 10^{10} C_2 \left[1 - e^{-5/n_1 R C_2} + \frac{43.45 \times 10^{-12}}{C_2} e^{-5/n_1 R C_2} \right]$$

= $1 - \left[1 - \frac{43.45 \times 10^{-12}}{C_2} \right]^{n_1 - 5/R C_2} e^{-5/n_1 R C_2}$

Background: The computations arise as part of an analytical study to optimize the choice of threshold settings in pulse radar altimeters. The optimum condition was to achieve the maximum probability of target detection and the minimum probability of detection of system noise. The problem was proposed by J. W. Seaton (DOFL).

Status: New. The code was checked and calculations were carried out for the above parameters.

Objective: To compile, edit, and prepare in a form suitable for

publication, data relating to the origin, destination, and routing of all domestic airline passengers traveling on certificated carriers.

Background: The Domestic Origin and Destination Airline Passenger Survey has been produced recurrently since 1939. As the volume of passengers in the industry grew the manual method of tabulation was replaced by punched card processing. Further increases in volume have made it essential to explore the use of electronic equipment.

The data given in the Report is of the utmost importance to the Civil Aeronautics Board in its regulatory work. Its other uses throughout the airline industry, as well as by the public in general, supply additional justification for its production on an accurate and current basis.

The reports produced herein will be published by the CAB each quarter and will be summarized for an annual summary report. Status: New.

1102-40-11645/60-459TRANSCENDENTAL EQUATIONSOrigin and Sponsor:NBS, Section 9.01Manager:R. Zucker2

<u>Objective</u>: (1) To determine the roots $\pm \gamma^2$ of a second order determinant whose elements are functions of γ^2 , given parameters u and p, and $\psi(\gamma^2)$, where

$$\psi(\gamma^2) = \frac{-\gamma^2}{2(1+1)} - \frac{\gamma^2}{2(1+2)} - \frac{\gamma^2}{2(1+3)} - \frac{\gamma^2}{2(1+4)} - \frac{\gamma^2}{2(1+5)}$$

(2) To evaluate

$$T = \frac{m^4 (a/e)^4}{4p}$$

where a/e is a solution of the equation $f(\gamma) = f(\gamma')$ with

$$f(\gamma) = E(\gamma) \begin{cases} \tan \frac{\gamma l}{2a} \end{cases}$$

and γ and γ the negative and positive solutions given above.

Background: The calculations arise in the solution of the frequency equations derived from the differential equations for flexural vibration of unrestrained cylinders and disks. [G. Pickett, J. Appl. Phys. <u>16</u>, 820-831 (1945)].

The problem was proposed by W. E. Tefft, 9.01. Status: New. 1102-40-11645/60-460 DESIGN EQUATIONS FOR MAGNETRON INJECTION ELECTRON GUNS

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

<u>Manager</u>: A. Beam Objective: To solve the following set of equations simultaneously:

(1)
$$\ddot{R} - R\dot{\theta}^2 = R[(\dot{\phi}^2 + \dot{\phi})\sin^2\theta + 2\int_0^T g\dot{\theta} dt]$$

(2)
$$2\dot{R}\dot{\theta} + R\ddot{\theta} = R[(\dot{\phi}^2 + \dot{\phi})\sin\theta\cos\theta + g]$$

(3)
$$\dot{g} + g\dot{\theta} \cot \theta + 6\theta \int_{0}^{T} g\dot{\theta} dt = \frac{K}{R \sin \theta}$$

where
$$\dot{\phi} = \frac{1}{2} \left[\frac{\sin^2 \theta_0}{R^2 \sin^2 \theta} - 1 \right]$$
, $K > 0$ is a parameter, $R = \frac{\gamma}{\gamma_0}$.

The initial conditions given for T = 0 are:

$$R(T) = 1$$
, $\theta(T) = \theta_0$, $R(T) = \theta(T) = g(T) = 0$, $0 < \theta < 2_{\pi}$.

<u>Background</u>: We consider the equation of motion of electrons in an axiallysymmetric system, subject to (a) a uniform axial magnetic field, and (b) space-charge-limited emission from an infinite conical cathode. The electric field within the flow is established by the electrons themselves. We assume the potential to be of the form $\phi = \gamma^2 F(\theta)$ (spherical polar coordinates) and obtain the equation of motion, using Newton's second law of motion.

The azimuthal equation of motion is integrable and leads to Busch's well-known result for the angular velocity:

$$\phi = \frac{eB}{2m} \left[\frac{\gamma_0^2 \sin^2 \theta_0}{2 \sin^2 \theta} - 1 \right].$$

Poisson's equation is separable with the assumed potential function; by means not described here, the charge density is expressible as a function of γ and θ .

If we substitute G = dF/dt, the γ and θ equations of motion will involve G and an integral of G θ , and Poisson's equation will involve G, G, θ , γ and two parameters.

Finally, if we take (eB/m)t = T, we obtain the above set of equations. K involves two parameters, one of which is θ_0 .

The problem was submitted by W. E. Waters (DOFL).

Status: New. Coding of the problem was started by R. Herbold.

1102-40-11645/60-461 DATA SYSTEMS LANGUAGES Origin and Sponsor: NBS, Section 11.2 Manager: J. H. Wegstein

- Objective: To participate, as chairman, in a working-committee conducting a fact-finding study of the strengths and weaknesses of existing automatic business compilers, such as FLOWMATIC and AIMACO. The committee is also expected to propose a common computer language suitable for handling business data-processing problems that will be acceptable to most computer manufactures and users. The committee membership consists of representatives of computer manufacturers and some government agencies.
- Background: This committee is one of two committees that resulted from a conference sponsored by the Department of Defense in May 1959. The conference was attended by about 40 representatives of computer manufacturers, by consultants, government representatives, and users of large data processing machines. There was general agreement on the feasibility and need for a common business-data-processing language. It was agreed that the language should be open-ended and capable of continuous change and amendment, that it should be problem-oriented and machine independent, that it should use simple English or pseudo-English and avoid symbolism as far as possible.

Two working committees were formed. The Fact-Finding and Short-Range Language Committee was formed because attendees urged an immediate attempt at defining a language. The current plan is to achieve this objective by December 1, 1959. A second committee, the Intermediate-Range Committee, was also formed, and its objective is to take a longer-range view of the same task. Starting either with the "immediatecommittee" language or independently, it will plan a language that might by implemented by 1961.

Status: New.

1102-40-11645/60-462 CORRELATION OF FUNCTIONS

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of

the Army

Manager: G. W. Reitwiesner <u>Objective</u>: To evaluate the correlation coefficient between two functions F_1 and F_2 , which are defined as

$$F_{i} = \frac{1 + a \cos(p_{i}\alpha)}{1 + a^{2} + 2a \cos(p_{i}\alpha)}, \quad i = 1, 2,$$

for p_1 = 1 and 1 \leq p_2 \leq 2 and for a = 0.5, 0.9, 0.98, where α varies over 0 \leq α \leq $2\pi.$

<u>Background</u>: This computation is of interest in connection with studies of the angular accuracy of Radar Target Tracking.

Status: Completed (New). Solutions were obtained on the 704 for each of 45 combinations of values of p_2 and a for $\alpha = j \cdot 2^{-k} \pi$, $j = 1,3,5, \cdots, (2k-1)$, for $k = 4,5,6,\ldots,13$. Solution for k = 13 sufficed to indicate close to three-figure accuracy for all correlation coefficients.

1102-40-11645/60-464 CORRELATION MATRIX FOR PHS DATA

Origin and Sponsor: Public Health Service

Manager: J. Beiman

Objective: To compute Pearson product moment matrices and sums of cross products.

Background: The Public Health Service is developing a psychological instrument designed to measure an individual's level of anxiety in regard to illness and problems connected with illness. In the course of the development of this instrument it has become important to have a factoranalysis performed in order to gain a clearer idea of the nature of the variables measured by the test. The factor analysis itself will be performed by the Institute of Personality and Ability Testing at the University of Illinois. However, this analysis cannot be performed until a matrix of intercorrelations of the 84-item test is computed. This part will be performed by the National Bureau of Standards.

The problem was submitted by P. R. Robbins (Public Health Service). <u>Status</u>: New. The problem has been programmed and code checked. Results have been transmitted to the sponsor.

6. STATISTICAL ENGINEERING SERVICES

Note: For administrative reasons the numbers of NBS projects have been slightly modified. The following list will facilitate reference to write-ups on corresponding Statistical Service tasks in earlier issues of this Report:

Present	Formerly
1103-40-11430	1103-40-5105
1103-40-11625	1103-40-5150
3911-61-39951	3737-60-0002

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING Task 3911-61-39951/51-1

Origin: NBS Managers: W. J. Youden, J. Cameron Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) <u>Color-matching</u>. J. R. Rosenblatt and M. C. Dannemiller collaborated with I.Nimeroff (2.01) on statistical analyses of a largescale color-matching study, the results of which were presented at a meeting of the Optical Society of America.

(2) Polygons used for angular standards. For the calibration of polygons used for angular standards from closures at two angular separations, designs for minimizing the variance of the determined corrections have been worked out by J. M. Cameron for polygons involving 12 or fewer angles. A simple method for the computations was devised.

(3) <u>Stress-rupture tests of stainless steel</u>. W. J. Youden has collaborated with personnel of the Metallurgy Division on the design of tests to evaluate the uniformity of a stock of material to be used for the preparation of reference specimens for the calibration of stress-rupture testing equipment.

(4) <u>Sampling glass beads</u>. A series of experiments was planned and analyzed investigating the analytical procedures for sampling glass beads. This work was done for C. M. Hunt and R. G. Pike of Section 9.6.

(5) Dr. Youden served on the Commission for the Terminology and Expression of Analytical Results of the Analytical Section of the International Union of Pure and Applied Chemistry.

Authorized 7/1/50

Publications:

- Some canons of sound experimentation. C. Eisenhart. To appear in the Proceedings of the 31st Session of the International Statistical Institute, Brussels, 1958.
- (2) Statistical aspects of the cement testing program. W. J. Youden. To appear in the Proceedings of the American Society for Testing Materials.
- (3) Graphical diagnosis of interlaboratory test results. W. J. Youden. Indust. Quality Control <u>15</u>, 24-28 (1959); also Middle Atlantic Conference Transactions, American Society for Quality Control, 1959, pp. 196-206.
- (4) Problems of the experimenter. W. J. Youden. National Convention Transactions, American Society for Quality Control, 1959, pp. 41-47.
- (5) Statistics--Engineering viewpoint. W. J. Youden. To appear in the Journal of Engineering Education.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC Task 1103-40-11430/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51 Manager: W. J. Youden Full task description: Oct-Dec 1951 issue, p. 58

Status: INACTIVE.

STATISTICAL SERVICES Task 1103-40-11625/58-346

Authorized 3/31/58

Origin and Sponsors: Various Agencies Manager: J. M. Cameron Full task description: Jan-Mar 1958 issue, p. 45

Status: CONTINUED. Work was done during the quarter for the following agencies:

(1) U. S. Geological Survey: Investigations were continued on several mathematical models associated with measurement processes in hydrology.

(2) <u>Bureau of Public Roads</u>: See task 1102-40-11645/58-316 (p. 23) for write-up of this task done jointly with the Computation Laboratory.

(3) <u>Nuclear Materials Management Division, AEC</u>: A first draft of a statistical section for inclusion in an AEC publication entitled "The Use of Uranium Standards in Isotopic Analysis" was prepared by W. J. Youden.

APPLICATION OF AUTOMATIC COMPUTER

The record of the use of the IBM 704 for the period July 1 through September 30 is as follows:

Task No.		Title	Assembly	Checking	Production
			(M	INUT	ES)
NBS:					
11411/55-56	11.1	Research in mathematical			
		topics applicable to			
		Numerical Analysis	79	72	400
11120/55-65	11.2	Automatic coding	277	573	123
39951/56-160	11.2	Mathematical subroutines	114	123	67
39951/51-1	11.3	Statistical engineering	26	23	410
11626/58-369	11.4	Computation of outflow from			
		a breached dam			19
39952/54-30	4.1	Spectrum analysis	56	43	672
39952/54-38	3.2	Equation of state of real ga	ses	2	48
39952/55-68	9.7	Crystal structure calcu-			
		lations	107	130	560
39952/55-82	3.1	Thermometer calibrations	18	48	646
11645/55-97	3.8	High temperature properties			
		for air ^o	33	193	206
39952/56-131	2.2	Calculations in optics*	6	3	13
11645/56-150	4.10	Mathematical expressions*	52		16
11645/56-166	5.9	SCF-LCAO solution of some			
		hydrides*		5	93
11645/56-171	3.2	Collision integrals used in			
		transport theory**	120	224	577
11645/57-219	3.2	Thermal properties*	394	215	466
39952/57-223	3.2	Self-consistent fields	4	75	48
11645/57-246	4.8	Radiation diffusion**	365	153	1977
39952/57-247	6.1	Mechanical impedance*			45
39952/57-250	2.3	Automatic reduction in spect	ro-		
		photometric data*		3	40
11645/57-252	4.11	Detecting efficiency in a			
		neutral meson experiment*	* 10	8	143
39952/58-254	2.3	Reproduction of color- and			
		spectral-energy distri-			
		bution of daylight*	9	4	18
11645/58-255	4.8	Chi functions**			805
11645/58-256	10.6	Composite walls**	231	60	156
11645/58-260	12.5	Prototype accounting**	46	87	466

				Code	
Task No.		Title	Assembly	Checking	Production
			(M	INUTE	S)
11645/58-261	9.0	Temperature induced strain	IS		
		in a slab			16
/58-267	2.1	Munsell color system			20
,		conversion*			4
/58-270	12.5	Mathematical problems			1
,		related to postal			
		operations			13
/58-272	3.2	Thermodynamic properties of	of		10
,		real gases	9	27	172
/58-274	9.7	Calculations for d-spacing	rs		
,		II	4		95
/58-275	7.8	Crystallography**	19	26	63
/58-299	3.1	Time-dependent Shroedinger			00
,	0.12	equation		12	5
/58-304	3.2	Transport properties of ai	r 49	119	159
/58-306	2.1	Interpolation of color			200
,00 000		mixture functions*			16
/58-308	3.4	Oscillating sphere*	43	25	17
/58-314	3.2	Approximations for gas	10	20	±.,
,00 011	0.2	mixtures*	302	161	3425
/58-333	9 0	Calcium hydroxide	27	10	11
/58-339	3.4	Viscoelasticity properties		10	
/00 000	0.1	of materials	87		74
/58-358	32	Reduced cross-sections	11	19	1
/58-360	5.2	Diffusion coefficients*	13	8	110
/59-377	12 3	Logical diagram reduction	181	229	647
/59-378	82 20	Correlation program**	101	16	37
/59-387	30 /	Nuclear reactor design**		13	636
/59-388	10.3	Heat nump calculations		5	13
/59-300	12 5	Flectrocardiogram**		0	73
/59-390	12.5	Slow electron scattering			10
/ 33 334	4.0	by hydrogon atoms	203	356	204
/50-305	77	Adsorption study**	16	300	10
/59-403	21	Computation of color fadin	10	0	29
/59-403	2.1 1 Q	Counter**	65		29
/59-404	12.5	Bank Board**	95	374	397
/59-412	3 2	Dunamics of proumatic	50	574	001
/ 55-412	0.2	by namics of pheumatic	3	240	777
/50-413	6 6	Mass weighing*	5	240	9
/59-413	21	Infinito systems	9	73	78
/59-414	0.I	Spootnum analycic of	5	15	10
/ 59-41 /	2.4	spectrum analysis of			50
/50-418	1 9		15	61	11
/50-410	12 5	Radio intensities**	15	01	14
/50-440	82 10	Manning	171	462	371
/50-440	85 10	Ionospheric data**	10	10	571
/60-440	2 1	Spectral line colonimetry	10	20	50
/00-449	2.I	opectrar time cororinetry	47	44	50

				Code	
Task No.		Title	Assembly	Checking	Production
			(M	INUT	ES)
11645/60-451	6 1	Mochanical impedance	16	1	
/60-457	12 5	Public Housing problem**	14	20	175
/60-450	0 1	Transcondental equations	24	23	18
/60-439	9.1	Plack box computer comices	*	21	10
/60-403	0.J	Migoolloncoug	а О	2	10
/80-900	11.2	MISCEITANEOUS	5	5	15
		Totals (NBS Services)	3308	4372	15849
OUTSIDE					
00151DE:	NTH	Differential equations for			
11414/00-140	NIN	Differential equations for		67	1.06
11645 /52-45	60	Air defense testies ⁹	10	225	190
11645/53-45	SC	Air defense tactics	19	335	294
/58-263	DOFL	Gas tube characteristic	170	/ 50	386
/58-269	NKL	Molecular structure, IV	170	58	321
/58-276	NOL	General kinetics, 1**			4801
/58-282	NOL	Missile boundary layer			
		computation	95	195	230
/58-284	NRL	Sputnik tracking*			2
/58-319	HPBA	Auto tag ^o	8	19	1361
/58-335	DOFL	Roots of Bessel functions**	51	36	273
/58-340	DOFL	M5-17 Fuze Data ^o			72
/58-347	BURR	Computations for war games*	*	65	1962
/58-361	NRL	Spectrum of dipole radiatio	n 67	51	251
/58-370	NRL	Neutron diffusion study ^o			1035
/59-371	NRL	ASWAP ^o	247	146	209
/59-373	DOFL	Rhinitis ^o	4	1	24
/59-408	NASA	NASA**	1509	223	25677
/59-411	HEW	Fitting of exponential			
		curves**	113	127	274
/59-415	DOFL	Complex Legendre functions		23	253
/59-416	DOFL	Analysis of power supply			
		experiments**			6
/59-419	DOFL	Neutrons ^o	78	152	828
/59-423	WB	Weather Bureau**		20	14034
/59-424	NOL	Systems engineering**	40	3	24
/59-425	CU	Molecular orbitals*	14	3	75
/59-434	CIW	Petrological computations			56
/59-435	VA	Electrocardiographic			
		analysis	70	137	224
/59-437	GE	GE Highway studies**	27	10	417
/59-441	GK	Systems engineering**	74	352	124
/59-447	BPRO	Public Roads study**			2076
/59-448	NMR	Vapor transition*	12	15	26
/60-450	ACC	Chemical warfare	25	85	481
/60-453	AMS	Data conversion	6	49	203

					Code	
Task No.		Title		Assembly	Checking	Production
				(1	INUT	E S)
11645/60-456 /60-460	DOFL DOFL	Firing circui Design equati	it equations ion for electro	9 on	19	32
		guns			13	
/60-462	DOFL	Correlation of	of functions	29	13	
		Totals (Outs	side)	2667	2214	55987
Total t	ime for	the quarter	(MINUTES)	. 5975	6586	71836
Total t	ime for	the quarter	(HOURS)	. 99.6	109.8	1197.3

* Problem programmed in the Computation Laboratory; production runs continued under direction of sponsor.

** Problem programmed by sponsor and run under his direction.

^o Classified task.

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Reliability Seminar

DAVIS, Miles. Stochastic models for life testing. August 21.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

- HAYNSWORTH, Emilie V. Matrices with real roots. Presented at a Meeting of the American Mathematical Society, Salt Lake City, Utah, September 3.
- NEWMAN, Morris. Congruence properties of partitions. Presented at Boulder Laboratories, NBS, Boulder, Colo., July 6.
- SEVERO, Norman. Probability and statistics. Presented at the National Science Foundation Summer Institute for High School Teachers of Mathematics, held at the University of Buffalo, N. Y., July 13-17.
- VINTI, John P. A new method of solution for satellite orbits. Presented at the Summer Institute in Dynamical Astronomy, Yale University, New Haven, Conn., July 22, 24.
- WEGSTEIN, Joseph. Artificial language in automatic programming. Presented at the Naval Proving Ground, Dahlgren, Va., July 29.
- YOUDEN, W. J. (1) Making good measurements. Presented at the National Science Foundation Summer Institute for Teachers of Physics, held at the University of Maryland (lecture presented at the NBS), July 22.
 (2) What is scientific evidence? Presented to the NBS Student Trainees, July 28.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.1 Mathematical Tables

- Tables of the bivariate normal distribution function and related functions. NBS Applied Mathematics Series 50. Issued June 15, 1959; price \$3.25, available from U. S. Government Printing Office, Washington 25, D. C.
- 1.2 Technical Notes, Manuals, and Bibliographies
 - (1) Some applications of statistical sampling methods to outgoing letter mail characteristics. Norman C. Severo, Arthur E. Newman, Shirley E. Young, and Marvin Zelen. Technical Note 16. Issued July 1959; price \$2.75. Available from U. S. Dept. of Commerce, Office of Technical Services, Washington 25, D. C.
- 1.3 Technical Papers

The following papers appeared in J. Research NBS 63B (July-Sept 1959):

- Principal submatrices of a full-rowed non-negative matrix.
 K. Goldberg. Pp. 19-20.
- (2) Zeros of certain polynomials. A. J. Goldman. Pp. 21-22.
- (3) Relations between summation methods and integral transformations.W. Greub. Pp. 1-13.
- (4) On a modification of Watson's lemma. F. Oberhettinger. Pp. 15-17.
- (5) Compressible turbulent boundary layers with heat transfer and pressure gradient in flow direction. A. Walz. Pp. 53-70.
- (6) Analytical integration of the differential equation for water storage. V. M. Yevdjevich. Pp. 43-52.

* * * * *

(7) On a generalization of an inequality of L. V. Kantorovich.
 W. Greub and W. Rheinboldt. Proc. Amer. Math. Soc. 10, 407-415 (1959).

- (8) Construction and application of a class of modular functions, II.
 M. Newman. Proc. London Math. Soc. 9, 373-387 (1959).
- (9) Inclusion theorems for congruence subgroups. M. Newman and I. Reiner (University of Illinois). Trans. Amer. Math. Soc. <u>91</u>, 369-379 (1959).
- (10) On the convergence of the Rayleigh quotient iteration for the computation of characteristic roots and vectors, III. (Generalized Rayleigh quotient and characteristic roots with linear elementary divisors); IV. (Generalized Rayleigh quotient for nonlinear elementary divisors). A. Ostrowski. Arch. Rat. Mech. Anal. <u>3</u>, 325-347 (1959).
- (11) On prediction of system behavior. J. Rosenblatt. "Proceedings of the NYU-Industry Conference on Reliability Theory," held at Ardsleyon-Hudson, N. Y., June 1958, pp. 39-52.
- (12) Theoretical problems of reliability measurement and prediction. J. R. Rosenblatt. "Proceedings of the Third Exploratory Conference on Missile Model Design for Reliability Prediction," held at White Sands Missile Range, 1959, pp. 29-34.
- (13) Stationary principles for forced vibrations in elasticity and electromagnetism. J. L. Synge. Appeared in "Proceedings of the Eighth Symposium in Applied Mathematics (Calculus of Variations)", pp. 79-88 (Amer. Math. Soc., Providence, R. I.).
- (14) Diffusion of particles in turbulent flow. C. M. Tchen. Advances in Geophysics 6: Atmospheric Diffusion and Air Pollution. pp. 165-173 (Academic Press, New York and London, 1959).
- (15) Kinetic equation for a plasma with unsteady correlations. C. M. Tchen. Phys. Rev. 114, 394-411 (1959).
- (16) New approach to the theory of satellite orbits. J. P. Vinti. Phys. Rev. Letters 3, S517 (1959).
- Measurements made by matching with known standards. W. J. Youden,
 W. S. Connor, and N. C. Severo. Technometrics 1, 101-109 (1959).
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U.S. DEPARTMENT OF COMMERCE Frederick H. Mueller, Secretary

NATIONAL BUREAU OF STANDARDS

A. V. Astin, Director

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

WASHINGTON, D.C.

Electricity and Electronics. Resistance and Reactance. Electron Devices. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Photographic Technology. Length. Engineering Metrology.

Heat. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology. Molecular Kinetics. Free Radicals Research.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Neutron Physics. Radiation Theory. Radioactivity. X-rays. High Energy Radiation. Nucleonic Instrumentation. Radiological Equipment.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Molecular Structure and Properties of Gases. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Air Conditioning, Heating, and Refrigeration. Floor, Roof, and Wall Coverings. Codes and Safety Standards. Heat Transfer. Concreting Materials.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analog Systems. Application Engineering.

• Office of Basic Instrumentation.

• Office of Weights and Measures.

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Lique-faction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships. VHF Research. Radio Warning Services. Airglow and Aurora. Radio Astronomy and Arctic Propagation.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Research. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation Obstacles Engineering. Radio-Meteorology.⁺ Lower Atmosphere Physics.

Radio Standards. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Electronic Calibration Center. Microwave Physics. Microwave Circuit Standards.

Radio Communication and Systems. Low Frequency and Very Low Frequency Research. Iligh Frequency and Very High Frequency Research. Ultra High Frequency and Super High Frequency Research. Modulation Research. Antenna Research. Navigation Systems. Systems Analysis. Field Operations.

NBS