NATIONAL BUREAU OF STANDARDS REPORT

6493

PROJECTS and PUBLICATIONS

of the

APPLIED MATHEMATICS DIVISION

A Quarterly Report

April through June 1959

FOR OFFICIAL DISTRIBUTION

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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NBS PROJECT
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APPLIED MATHEMATICS DIVISION

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*On leave of absence

**Part time

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*Only unclassified projects are included in this report.

Status of Projects

June 30, 1959

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-12-1104/55-55

Origin: NBS Authorized 8/29/54
Manager: P. Davis
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. N. Bazley studied the computational advantages of the use of Bessel's inequality in estimating quadratic functionals. A calculation of upper and lower bounds for the torsional rigidity of a hollow square is in progress.

N. Bazley and P. Davis continued their study of Monte Carlo methods. A certain game has been investigated both theoretically and experimentally. The average length of the game as computed from Markov chain theory compared favorably with that computed by playing the game 10^4 times.

E. Haynsworth continued investigations on bounds for determinants and charactoristic roots of a matrix, on matrices with real roots, and on methods of reducing partitioned matrices of a special type which arise in problems concerning lattices of two or more dimensions. Papers on these subjects are now in manuscript form.

A survey of basic theorems in matrix theory has been prepared for publication in the Applied Mathematics Series of the NBS. The survey was made by Marvin Marcus incidental to his studies in the theory of eigenvalue bounds during his stay at the NBS under the NBS-National Research Council Postdoctoral Fellowship Program in 1956-7. It covers basic identities and inequalities of matrix theory, giving results dealing with elementary properties, canonical forms, invariance, congruence, commutativity, orthogonalization, eigenvalues, determinants, submatrices, rank, determinant and rank inequalities, numerical methods for inversion and eigenvalues, and condition numbers.

Publications:

(1) Computation problems concerned with the Hilbert matrix. J. Todd. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.

(2) Convexity of the field of a linear transformation. A. J. Goldman and M. Marcus. Canadian Math. Bull. 2, 15-18(1959).

(3) A continous poker game. A. J. Goldman and J. J. Stone. Submitted to a technical journal.
(4) On a generalization of an inequality of L. V. Kantorovitch. W. Greub and W. Rheinboldt. To appear in Proceedings of the American Mathematical Society.
(5) On the relations between summation methods and integral transforms. W. Greub. In manuscript.
(6) Uniform asymptotic expansions for Weber parabolic cylinder functions of large orders. F. W. J. Olver. To appear in the Journal of Research NBS, Section B. Mathematics and Mathematical Physics.
(7) Reduction formulae for partitioned matrices. E. Haynsworth. Submitted to a technical journal.
(8) Lower bounds for eigenvalues with applications to the helium atom. N. Bazley. Proc. Nat. Acad. Sci. 45, 850-853(1959).
(9) Leonhard Euler's integral: A historical profile of the Gamma function. (In memoriam Milton Abramowitz.) P. J. Davis. To appear in the American Mathematical Monthly.
(10) The zeros of certain polynomials. A. J. Goldman. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.
(11) Linear differential equations of the second order with large parameter. F. W. J. Olver. Submitted to a technical journal.
(12) Applications of a theorem on partitioned matrices. E. V. Haynsworth. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.
(13) Basic theorems in matrix theory. M. Marcus. To appear in the NBS Applied Mathematics Series.
(14) Bounds for determinants with positive diagonals. E. Haynsworth. In manuscript.
(15) Matrices with real roots. E. Haynsworth. In manuscript.
(16) Regions containing the characteristic roots of a matrix. E. Haynsworth. In manuscript.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-12-5116/55-56

Origin: NBS Authorized 8/13/54
Sponsor: Office of Naval Research
Manager: M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. K. Goldberg and M. R. Kellington continued work on a survey of the literature pertaining to incidence matrices. Dr. Goldberg is preparing an introduction.

Publications:

(1) Some computational problems concerning integral matrices. O. Taussky. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.

(2) The construction of Hadamard matrices. K. Goldberg and E. C. Dade. To appear in the Michigan Journal of Mathematics.

(3) Inclusion theorems for congruence subgroups. M. Newman and I. Reiner (University of Illinois). To appear in Transactions of the American Mathematical Society.

(4) Construction and application of a class of modular functions, II. M. Newman. To appear in Proceedings of London Mathematical Society.

(5) On the convergence of the Rayleigh quotient iteration for the computation of characteristic roots and vectors: III. Generalized Rayleigh quotient and characteristic roots with linear elementary divisors; IV. Generalized Rayleigh quotient for nonlinear elementary divisors; V. Usual Rayleigh quotient for non-Hermitian matrices and linear elementary divisors; VI. Usual Rayleigh quotient for nonlinear elementary divisors. A. M. Ostrowski. To appear in Archive for Rational Mechanics and Analysis.

(6) A quantitative formulation of Sylvester's law of inertia. A. Ostrowski. Proc. Nat. Acad. Sci. 45, 740-744(1959).

(7) Generating functions for formal power series in noncommuting variables. K. Goldberg. To appear in the Proceedings of the American Mathematical Society.

(8) Note on a paper by S. Mukhoda and S. Sawaki. K. Goldberg. Submitted to a technical journal.

(9) The incidence equation AA^T-aA. K. Goldberg. To appear in the American Mathematical Monthly.

(10) The minima of cyclic sums. K. Goldberg. To appear in the Journal of the London Mathematical Society.

(11) The minimum of a certain linear form. K. Goldberg. Submitted to a technical journal.

(12) Principal submatrices of a full-rowed non-negative matrix. K. Goldberg. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(13) Weighted restricted partitions. M. Newman. To appear in Acta Mathematica.

(14) Über genäherte Determinanten und bedingte Tragheitsindizes. A. M. Ostrowski. To appear in (Wiener) Monatshefte für Mathematik und Physik.

(15) Modular forms whose coefficients possess Euler products. M. Newman. Submitted to a technical journal.

TRAINING PROGRAM IN NUMERICAL ANALYSIS, II
Task 1101-40-5114/57-237

Origin and Sponsor: National Science Foundation Authorized 3/27/57
Manager: P. Davis

Status: COMPLETED. The second NBS-NSF Training Program in Numerical Analysis was held at the National Bureau of Standards from February 9 through June 5, 1959. The program was under the direction of P. J. Davis, and he was assisted by M. Newman. There were nine participants.

During this quarter survey lectures of particular chapters in numerical analysis continued as follows:

Parabolic and Hyperbolic Partial Differential Equations	J. Douglas
Linear Programming	A. Charnes
Inequalities	K. Fan
Permutations and Matrices	K. Goldberg
Integral Equations	H. Bueckner
Statistical Computations	N. Severo
Orthonormalizing Codes in Numerical Analysis	P. Davis
Eigenvalue Computations	M. Marcus
Finite Differences and Maximum Principles	A. Schopf

Individual or shorter courses of lectures were given by I. Stakgold (ONR), R. Jackson (University of Delaware), H. Polacheck (DTMB), A. J. Goldman, J. Horowitz (Princeton University), P. J. Morse (M.I.T.), I. Schoenberg (University of Pennsylvania), H. Shapiro (New York University), N. Bazley, H. Weinberger (University of Maryland), D. Shanks (DTMB), T. A. Jeeves (Westinghouse), J. D. Wilkes (ONR).

As a direct outcome of the program, several papers on particular problems that were investigated are in preparation for submittal to technical journals.

The lectures of the first Training Program (1957), edited by John Todd of the California Institute of Technology, have been accepted for publication by a commercial publisher. The material to be published in this volume has been augmented by a number of lectures which were given during the 1959 program.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-12-5116/56-148

Origin and Sponsor: National Institutes of Health Authorized 9/30/55
Manager: W. Gautschi (11.2)
Full task description: July-Sept 1955 issue, p. 7

Status: CONTINUED. Computations continued as requested by the sponsor for both the one- and the two-dimensional cases-the former in a slightly modified form.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-40-1110/47-2

Origin: NBS Authorized 7/1/47
Manager: I. A. Stegun
Full task description: Apr -June 1949 issue, p. 45

Status: INACTIVE.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Task 1102-40-1110/51-8

Origin: Section 11.3, NBS Authorized 3/26/51
Manager: S. Peavy
Full task description: Apr-June 1951 issue, p. 49

Status: INACTIVE.

REVISION OF MATHEMATICAL TABLES
Task 1102-40-1110/52-7

Origin: NBS Authorized 8/10/51
Managers: I. A. Stegun
Full task description: July-Sept 1951 issue, p. 41

Status: INACTIVE.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-40-1110/52-37

Origin: NBS Authorized 11/28/51
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE.

SIEVERT'S INTEGRAL
Task 1102-40-1110/52-57

Origin: NBS Authorized 2/12/52
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46

Status: INACTIVE.

MATHEMATICAL SUBROUTINES
Task 3711-60-0009/56-160

Origin: NBS Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

Status: CONTINUED. The orthonormalizing routine has been prepared as a subroutine to be submitted to SHARE. Some new features have been incorporated which give additional information. For example, the Fourier coefficients, the sums of squares of residuals and the Gram determinant are now preserved during the calculation and are available to the programmer. Augmentation of the vectors is now completely automatic unless the coder specifies that special augmentation is required.

The orthonormalizing code to be used at the NBS laboratory will require decimal input only. The programmer supplies a list of key words (zeros or ones) which specify the information to be computed and printed.

A main use of the orthonormalizing code has been the least-squares fitting of the form

$$y_i = a_1 f_1(X_i) + a_2 f_2(X_i) + \ldots + a_n f_n(X_i).$$

Recent experiments suggested by P. Davis and conducted by E. Haynsworth and P. Walsh extend the class of functions which may be fitted to the data to a variety of algebraic curves, such as circles, ellipses and rational functions with numerators and denominators up to cubic degree.

AUTOMATIC CODING
Task 3711-60-1120/55-65

Origin: NBS Authorized 9/29/54
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. The Black Box Computer was readied for general use by the addition of a program package called "Tablemaker." Tablemaker is a general purpose program for reading, computing, and printing tables of numbers. The computations are determined by a list of instructions from the problem author. These instructions are punched almost verbatim on IBM cards and read into the computer along with cards containing input data.

Space in the memory is provided for eleven tables of numbers (vectors). Each table can contain up to 200 numbers. The general procedure is to read from one to three tables, to compute from one to eight tables, and to print from one to eight columns of numbers where each column is one of the tables.

The problem author expresses each value in a computed table by means of a formula that may utilize any previously computed number. These formulas can be any "freshman algebra" type of formula and can include functions like log, sin, cos, exp, square root, arctan, and tanh. In addition to these formulas, the problem author can also utilize special routines from a list which includes integration, interpolation, and curve fitting. In the case of curve fitting, the author indicates that the curve fitting routine is to be used, then gives the arbitrary function which is to be fitted to the data. This promises to be a very useful tool. As the need arises, other special routines can be readily added to the repertoire.

The bulk of the Tablemaker code is kept in binary form on cards or magnetic tape. When a computation is to be performed, the statements from the author are punched almost verbatim on cards that are inserted into a small skeletal Fortran program. This program is then compiled and automatically read into the computer along with the bulk of the Tablemaker code in binary form. Computation then begins immediately.

A guide for using the Black Box Computer is in preparation and will be distributed throughout the NBS.

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-5113/57-216

Origin and Sponsor: National Science Foundation Authorized 12/27/56
Manager: I. A. Stegun
Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. The textual material for Chapter 22 (Orthogonal Polynomials), Chapter 27 (Miscellaneous Functions), and Chapter 28 (Scales of Notation) has been distributed for comments. The manuscripts for the tabular material have been prepared on the card-controlled typewriter.

Typing of the textual material is under way for Chapter 16 (Elliptic Functions) and Chapter 26 (Mathematical Statistics). The tables and graphs for Chapter 16 have already been prepared; those for Chapter 26 are being compiled.

Tabular and graphic material is being compiled for Chapter 20 (Mathieu Functions), Chapter 21 (Spheroidal Wave Functions), Chapter 8 (Legendre Functions), and Chapter 18 (Weierstrass Elliptic Functions).

Various other chapters have been undergoing revision based on comments and suggestions of the Advisory Committee.

3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-12-1107/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. In connection with work on the Handbook of Mathematical Functions, M. Zelen has found new approximations to the distributions of Student's t, non-central F, and the ratio of two non-central χ^2 variates.

In connection with research on transformations, N. C. Severo prepared a draft of a manuscript, "Convergence of the 'Square-root Normal' distribution as the coefficient of variation becomes small".

A listing of some 1200 references covering ergodic theory, Markov chains and stochastic processes, and time series was made available to Prof. Herman Wold of Uppsala University, Sweden, who is directing the preparation of a bibliography on time series and stochastic processes as part of the "Teaching Aids" program of the International Statistical Institute under the sponsorship of UNESCO.

Publications:

(1) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. Joel. To appear in the Annals of Mathematical Statistics.
(2) Measurements made by matching with known standards. W. J. Youden, W. S. Connor and N. C. Severo. To appear in Technometrics.
(3) The non-central χ^2 as a test statistic. N. C. Severo. Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-12-1107/53-1

Origin: NBS Authorized 10/15/52
Manager: W. S. Connor
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. M. Zelen presented a seminar entitled, "Recent Advances in Experiment Design", before the Department of Agriculture (May 1959).

Publications:

(1) Fractional factorial experiment designs for factors at three levels. W. S. Connor and M. Zelen. National Bureau of Standards Applied Mathematical Series 54 (U.S.Government Printing Office, Washington,D.C., 1959, 30 cents).

(2) Randomization and experimentation. W. J. Youden. To appear in Annals of Mathematical Satistics.

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-1107/56-170

Origin: NBS Authorized 12/15/55
Manager: J. R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

Status: INACTIVE.

Publication:

(1) On the power of some rank order two-sample tests. J. R. Rosenblatt. To appear in "Contributions to probability and statistics," in press, Stanford University Press (1960).

MEASUREMENT OF RELIABILITY
Task 1103-12-1130/56-182

Origin: NBS Authorized 3/23/56
Manager: M. Zelen, J. R. Rosenblatt
Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. J. R. Rosenblatt gave a talk on "Theoretical Problems of Reliability Measurement and Prediction" at the Third Exploratory Conference on Missile Model Design for Reliability Prediction, at White Sands Missile Range, April 1959. A preliminarylisting of "USSR Bibliography on Reliability Theory" was prepared for distribution at the Conference.

M. Zelen presented the following seminars: (1) Multifactor experiments in life testing, Department of Statistics, Virginia Polytechnic Institute, April 1959; and (2) Lifetesting, Department of Statistics, Michigan State University, April 1959.

Publications:

(1) On prediction of system behavior. J. R. Rosenblatt. To appear in the Proceedings of the NYU-Industry Conference on Reliability Theory, held at Ardsley-on-Hudson, N.Y., June 1958.

(2) Analysis of two-factor classifications with respect to life tests. M. Zelen. To appear in "Contributions to probability and statistics," in press, Stanford University Press (1960).
(3) Factorial experiments in life testing. M. Zelen. To appear in Technometrics.
(4) Theoretical problems of reliability measurement and prediction. J. R. Rosenblatt. To appear in the Proceedings of the Third Exploratory Conference on Missile Model Design for Reliability Prediction, held at White Sands Missile Range, April 1959.

FRACTIONAL FACTORIALS FOR THE MIXED SERIES
Task 1103-12-5148/58-291

Origin and Sponsor: Bureau of Ships Authorized 9/30/57
Managers: W. S. Connor, M. Zelen
Full task description: July-Sept 1957 issue, p. 43

Status: CONTINUED. The catalog of fractional factorial designs has been completed, checked, and typed. An introduction which explains the method of construction and use of the designs has been prepared.

Publications:
(1) Construction of fractional factorial designs of the mixed 2^m3^n series. W. S. Connor. To appear in "Contributions to probability and statistics", in press, Stanford University Press (1960).
(2) Analysis of fractionally replicated 2^m3^n designs. R. C. Bose and W. S. Connor. To appear in the Proceedings of a Special Session of the International Statistical Institute, Brussels, 1958.

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-12-1115/55-57

Origin: NBS Authorized 9/1/54
Manager: W. H. Pell
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. The study of plasma gases by the techniques of statistical mechanics was continued by C. M. Tchen as one of the topics of the plasma program of NBS. This work has been concentrated on the study of the Fokker-Planck equation in external fields. The kinetic equation for a plasma in the form of the Fokker-Planck equation is derived by solving a system of "hierachy" equations. The latter system governs the singlet distribution function and the double correlation function, while the triple correlation function is degenerated into lower order functions. An anisotropy is produced by the movements of the particles and by the presence of the external fields. The related problem of the structure of the correlation function has been studied under a separate project. A manuscript covering the problem of the Fokker-Planck equation and the problem of the structure of the correlation function in external fields is in preparation. Other problems described in the plasma program (see Jan-Mar 1959 issue, p. 12) will be studied during the forthcoming year at the Atomic Energy Research Establishment, Harwell, England, and the Max-Planck Institute for Physics and Astrophysics, Munich, West Germany, under a Guggenheim Fellowship which has been awarded to Dr. Tchen.

The statistical analysis on the IBM 704 of the annual and monthly flows of rivers has been completed and the major part of the report referred to in the Jan-Mar 1959 issue (p. 14) has been finished.

The study of the errors of a computed mean discharge, an application of statistics to a problem in hydrology, is finished. This has been a joint project of V. Yevdjevich, working under The American University contract, and N. Matalas, Hydraulic Engineer, Research Section (SW), Water Resources Division, U. S. Geological Survey. A report is being prepared.

A. Ghaffari has completed the investigation on Rayleigh's nonlinear differential equation (see Oct-Dec 1958 issue, p. 11, and Jan-Mar 1959 issue, p. 12). The notion of critical points at infinity due to H. Poincaré and extended by S. Lefschetz is used throughout. Instead of Rayleigh's differential equation the corresponding phase-portrait system is considered, and the relation of its free oscillation to the nature of its critical points at infinity is studied. Analysis of the critical points at infinity of Rayleigh's equation gives the asymptotic behavior of the paths. By application of the existence criterion, given by Levinson and Smith, the existence and uniqueness of a periodic solution is deduced.

The complete system of the paths on the Poincaré sphere, including the equator and its critical points, is treated. A manuscript has been prepared.

J. P. Vinti and R. F. Dressler have written a report on the functional representation of curves which, when ploted against a set of variables x_i on Cartesian coordinate paper, another set y_i on semi-log paper, and a third set z_k on log paper, are all rectilinear. The most general function exhibiting such behavior is found.

A first report on Stokes flow problems by L. E. Payne and W. H. Pell has been issued and has been submitted to an outside journal for publication. A second report has been finished, and work on a third is continuing.

Publications:

(1) Diffusion of particles in turbulent flow. C. M Tchen. Appeared in Advances in Geophysics 6: "Atmospheric Diffusion and Air Pollution," pp. 165-174 (Academic Press, New York, 1959).
(2) Rotational properties of two-dimensional lattices. J. P. Vinti. Submitted to a technical journal.
(3) Turbulent motion. C. M. Tchen and G. B. Schubauer (NBS Fluid Mechanics Section). To appear as Section B, Volume V, of the Princeton Series, High Speed Aerodynamics and Jet Propulsion (Princeton University Press).
(4) The graphical solution of initial value problems. W. H. Pell. Submitted to a technical journal.
(5) Stokes flow problem for a class of axially symmetric bodies. L. E. Payne and W. H. Pell. Submitted to a technical journal.

RESEARCH IN CONTINUUM MECHANICS
Task 1104-12-5160/55-85

Origin: NBS Authorized 12/27/54
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: W. H. Pell
Full task description: Oct-Dec 1954 issue, p. 30

Status: TERMINATED. Future reports on the progress of studies started here will appear in this section under the tasks, "Research on Satellite Orbits," "Plasma Research," and "Research in Mathematical Physics and Related Fields."

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-5160/56-154

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: July-Sept 1955 issue, p. 20

Status: CONTINUED. Preparation of an introduction continued. Also preparation of an inverse table of Fourier transforms was started for inclusion in the publication.

COMPUTATION OF OUTFLOW FROM A BREACHED DAM
Task 1104-12-5160/58-369

Origin and Sponsor: Army Map Service Authorized 9/30/58
Manager: V. M. Yevdjevich
Full task description: July-Sept 1958 issue, p. 16

Status: COMPLETED. V. M. Yevdjevich has continued his analysis of the outflow hydrograph from a breached dam and has completed a study entitled, "Effect of the Sudden Water Release on the Reservoir Outflow Hydrographs." This has been combined with the paper listed in the Jan-Mar 1959 issue (p. 15) and certain prefatory remarks to constitute a report to the sponsor covering all of the activity under this project.

Publications:
(1) Analytical integration of the differential equation for water storage. V. Yevdjevich. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.
(2) Effect of sudden water release on the reservoir outflow hydrographs. V. Yevdjevich. In manuscript.

RESEARCH ON SATELLITE ORBITS
Task 1104-12-5160/59-420

Origin: NBS Authorized 12/19/58
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: J. P. Vinti
Full task description: Oct-Dec 1958 issue, p. 15

Status: CONTINUED. J. P. Vinti is continuing his investigation of satellite motion in the gravitational field obtained by using the axially symmetric solution of Laplace's equation in spheroidal coordinates

described in the Jan-Mar 1959 issue (p. 15). This potential, which makes the Hamilton-Jacobi equation for a satellite orbit separable, has an expansion in zonal harmonics in which the amplitudes of the zeroth and second harmonics can be adjusted to agree exactly with the values for any axially symmetric planet and a fourth harmonic which then agrees approximately with the latest value for that of the earth. The net result is therefore a reduction of the problem of satellite motion to quadratures, for a potential that is much closer to the empirically accepted one for the earth than any heretofore used as the starting point of a calculation. The method can take into account a first harmonic in the potential, in case observations are reduced to a center which does not coincide with the center of mass of the planet.

Work is being continued on theevaluation of the integrals that occur in the kinetic equations of motion which follow from the formal solution of the Hamilton-Jacobi equation.

Publications:

(1) A new approach in the theory of satellite orbits. J. P. Vinti. To appear in Physical Review Letters.

(2) A new method of solution for unretarded satellite orbits. J. P. Vinti. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

PLASMA RESEARCH
Task 1104-12-1140/59-422

Origin: NBS Authorized 6/30/59
Manager: C. M. Tchen

Objective: To conduct mathematical research in magnetohydrodynamics and in problems of statistical mechanics related to the physics of plasma gases.

Background: Plasma gases are used in the production of nuclear energy by processes of controlled fusion. Because of the extreme thermodynamic conditions prevailing in plasma, the classical theories of fluid dynamics are not applicable and entirely new theoretical approaches must be found.

Status: NEW. C. M. Tchen has continued his study of statistical plasma dynamics. Emphasis is centered on the correlation function governing the cooperative phenomena between a system of charged particles in external fields. The external fields are found to play two main roles: One is to produce a convection of the singlet distribution function, and a second is to modify the shielding by introducing an anisotropy. The related problem of the kinetic equation in external fields has been studied and described under a separate project. A manuscript covering the

problem of the correlation function and the related problem of the kinetic equation is in preparation. Dr. Tchen has been granted a Guggenheim Fellowship award (1959-1960) for further studies of plasma physics and magnetohydrodynamics to be carried out at the Atomic Energy Research Establishment, Harwell, England, and at the Max Planck Institute for Physics and Astrophysics, Munich, W. Germany.

Publications:

(1) Kinetic equation for a plasma with unsteady correlations. C. M. Tchen. To appear in Physical Review.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

3711-60-0009/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Managers: C. D. Coleman, W. Bozman (4.1)
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. A search for even levels of hafnium I gave 12 new levels. Matrix calculations are being made to determine the probable electron configurations of the levels.

Computation of the revision of Kayser's "Table of Wavenumbers" has been completed and the master pages have all been submitted for publication.

Two codes from Steinhaus of Los Alamos are being tested for use in calculating wavelengths from comparator settings. The first part (Roger D.P.) is a double precision least squares curve fitting code for the standard lines, and the second part (Spectra 1) computes wavelengths from the computed curve.

3711-60-0009/54-38 EQUATION OF STATE OF REAL GASES
(formerly COMPRESSIBILITY FACTORS OF DRY AIR)
Origin: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Continued. The subtabulation routine mentioned in the Jan-Mar 1959 issue, p. 16, has been used by the sponsor and others. The feature for writing on tape is now available.

Fortran code mentioned in the Jan-Mar 1959 issue has been completed and is being used by the sponsor. Provision has been made to handle up to 10 functions. However, the heading for this table is only 72 characters. A second version of this code, one that will punch cards, will be available soon.

3711-60-0009/55-68 CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Section 9.7
Managers: P. O'Hara, S. Block (9.7)
Full task description: Jan-Mar 1955 issue, p. 18
Status: Continued. The phase determination program was rewritten to take advantage of the 32K memory with a resultant 50 percent reduction in computing time.

Two and three dimensional Fourier maps were calculated for several crystals.

The $NaBO_2 \cdot 4H_2O$ crystal is being tested for rational dependence.

Status of Projects

3711-60-0009/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. ITS constants and tables were computed for 48 thermometers under test.

1102-40-5126/55-121 ELECTRON PENETRATION
Origin and Sponsor: NBS Section 4.8
Manager: S. Peavy
Full task description: Apr-June 1955 issue, p. 19
Status: Terminated. Production runs are continuing under the direction of the sponsor. (In future reports, production time will be reported in the section, "Application of Automatic Computer.")

1102-40-5126/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Managers: E. Haynsworth, P. Walsh
Full task description: Jan-Mar 1956 issue, p. 27
Status: Terminated. The codes are now being used directly by the sponsor for production runs. (In the future reports, production time will be reported in the section, "Application of Automatic Computer.")

1102-40-5126/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: S. Prusch
Full task description: July-Sept 1956 issue, p. 33
Status: Continued. Calculations were performed for 15 laboratory sets of gage blocks.

1102-40-5126/57-219 THERMAL PROPERTIES
Origin and Sponsor: NBS, Section 3.2
Manager: R. Varner
Full task description: Oct-Dec 1956 issue, p. 30
Status: Continued. The original code has been revised to handle mixed data. A Fortran code has been written to evaluate the following:

$$Q = \sum_{i=1}^{5} a_i T^i - \sum_{i=1}^{3} b_i / T^i .$$

This code has been checked and used for production runs. A Fortran integration code was written by the sponsor.

1102-40-5126/57-221 BESSEL FUNCTIONS FOR COMPLEX ARGUMENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Oct-Dec 1956 issue, p. 31
Status: Inactive.

3711-60-0009/57-223 SELF-CONSISTENT FIELDS
Origin: NBS, Section 3.2
Manager: E. V. Haynsworth
Full task description: Apr-June 1957 issue, p. 28
Status: Terminated. The codes are now being used directly by the sponsor for production runs. Detailed instructions on the A-matrix program and the Self-Consistent Field Program have been prepared so that these codes may be used at other installations. (In future reports, production time will be reported in the section, "Application of Automatic Computer.")

3711-60-0009/57-229 APPLICATION OF ELECTRONIC DATA PROCESSING MACHINERY TO PAYROLL OPERATIONS
Origin: NBS, Section 40.0
Managers: M. Paulsen, P. Ruttenberg
Full task description: Jan-Mar 1957 issue, p. 36
Status: Inactive.

1102-40-5126/57-236 SELF CONSISTENT FIELDS--EIGENVALUES
Origin and Sponsor: NBS, Section 3.6
Manager: E. Haynsworth
Full Task Description: Apr-June 1957 issue, p. 30
Status: Continued. Data is being prepared for production runs.

3711-60-0009/57-248 THE EVALUATION OF A TRIPLE INTEGRAL FOR THE SOLUTION OF NEGATIVE ION DETACHMENT
Origin: NBS, Section 4.6
Manager: S. Peavy
Full task description: Apr-June 1957 issue, p. 34
Status: Completed. Results have been transmitted to the sponsor.

1102-40-5126/58-263 GAS TUBE CHARACTERISTIC
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: I.A. Stegun
Full task description: July-Sept 1957 issue, p. 35
Status: Continued. Tests were made for argon, neon, krypton.

Status of Projects

3711-60-0009/58-266 DEPOLYMERIZATION, II
Origin and Sponsor: NBS, Section 7.6
Manager: L. S. Joel
Full task description: July-Sept 1957 issue, p. 36
Status: Continued. Code checking continued. The program has been modified to include exact formulation of some variables which were approximated in the SEAC code.

1102-40-5126/58-269 MOLECULAR STRUCTURE, IV
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. J. O'Hara
Full task description: July-Sept 1957 issue, p. 38
Status: Continued. A program was written to test for rational dependence of atoms and thereby locate subgroups whose average differs significantly from the overall average. New phases were computed for the spurite crystal and in nearly all cases the results were improved.

Several two-dimensional Fourier maps were calculated for the arginnine and spurite crystal. Structure factors were computed for aureomycin.

1102-40-5126/57-270 MATHEMATICAL PROBLEMS RELATED TO POSTAL OPERATIONS
Origin: NBS
Sponsor: Post Office Department, Office of Research and Engineering
Managers: B. Bender, A. J. Goldman
Full task description: Oct-Dec 1958 issue, p. 22
Status: Continued. The work on optimal configurations of sorting equipment for outgoing mail was transmitted to the sponsor, and a study of the combined incoming-outgoing sort is in progress.
Publication: Capacity requirement of a mail sorting device. B. Bender and A. J. Goldman, J. Research NBS, 62, 171-173 (1959), RP2948.

1102-40-5126/58-272 THERMODYNAMIC PROPERTIES OF REAL GASES
Origin and Sponsor: NBS, Section 3.2
Manager: J. P. Menard
Full task description: Oct-Dec 1957 issue, p. 32
Status: Continued. A code has been written and checked out for printing certain thermodynamic functions from a binary tape. The printing may be on-line or off-line. The functions are printed first according to density with the temperature held constant and then according to temperature with the density held constant. Production runs have been made with this code.

Status of Projects

1102-40-5126/58-274 CALCULATIONS FOR d-SPACINGS, II
Origin and Sponsor: NBS, Section 9.7
Manager: R. Zucker
Full task description: July-Sept 1957 issue, p. 38
Status: Continued. About 30 calculations for d-spacing for orthogonal, hexagonal and orthorhombic crystals were carried out this quarter, and redetermination of unit cell constants by least squares fitting to a measured d-spacing was performed for about 15 crystals.

3711-60-0009/58-294 NUCLEAR SCATTERING OF PHOTONS
Origin: NBS, Section 4.8
Manager: J. P. Menard
Full task description: Oct-Dec 1957 issue, p. 36
Status: Terminated. The program will continue in production under the sponsor's direction. (Production time will be reported hereafter in the section, "Application of Automatic Computer.")

1102-40-5126/58-298 ANALYSIS OF SPECTROCHEMICAL DATA
Origin and Sponsor: NBS, Section 5.10
Managers: S. Peavy, R. N. Varner
Full task description: Oct-Dec 1957 issue, p. 39
Status: Terminated. All the codes requested by the sponsor have been written and checked out. Production runs are being made under the supervision of the sponsor. (Production time will be reported hereafter in the section, "Application of Automatic Computer.")

1102-40-5126/58-299 TIME-DEPENDENT SCHROEDINGER EQUATION
Origin and Sponsor: NBS, Section 3.1
Manager: A. Schopf
Full task description: Oct-Dec 1957 issue, p. 39
Status: Continued. The existing codes for the various parts of the problem have been rewritten in a single automatic and self-reassembling binary code, allowing changes of meshlength and various other parameters. The deck has been checked out and transmitted to the sponsor.

1102-40-5126/58-304 TRANSPORT PROPERTIES OF AIR AT ELEVATED TEMPERATURES
Origin and Sponsor: NBS, Section 3.2
Manager: P. J. Walsh
Full task description: Oct-Dec 1957 issue, p. 40
Status: Continued. The theory for the prediction of the transport properties of gases requires certain quantities $\Omega_{ij}^{(\ell,s)}$ as input. These are generally called collision integrals. They include information concerning the potential of interaction between a particle of species i and one of species j. As described in Hirschfeld, Curtiss, and Bird, Molecular Theory of Gases and Liquids (John Wiley and Sons, New York), these integrals are obtained from certain general integrals $\Omega^{(\ell,s)*}$ which

are independent of i and j. One obtains the $\Omega_{ij}^{(\ell,s)}$ from the $\Omega^{(\ell,s)*}$ by substituting parameters characteristic of the pair i,j into a table of the latter integrals. These parameters are in turn obtained by means of comparison of the experimental transport properties for the mixture of i,j molecules and the transport properties predicted in general for the potential using the $\Omega^{(\ell,s)*}$ A code is being written which obtains these parameters from the data. This code will also provide a method for comparing the potentials with each other with respect to the predictions of transport properties.

Code checking of the supervisory routine for joining the codes previously written is now in progress. Some calculations have been made and the results have been submitted to the sponsor. A code has been written which compares the potentials, and also one which computes potential parameters from experimental data.

1102-40-5126/58-307 STUDY OF SURFACE TENSION
Origin and Sponsor: NBS, Section 9.2
Manager: R. Arms
Full task description: Oct-Dec 1957 issue, p. 43
Status: Inactive.

1102-40-5126/58-308 OSCILLATING SPHERE
Origin and Sponsor: NBS, Section 3.4
Manager: S. Prusch
Full task description: Oct-Dec 1957 issue, p. 43
Status: Inactive.

1102-40-5126/58-312 RESPONSE FUNCTION, II
Origin and Sponsor: NBS, Section 4.11
Manager: A. Beam
Full task description: Jan-Mar 1958 issue, p. 33
Status: Completed. Results have been transmitted to the sponsor.

1102-40-5126/58-316 INTERSECTION CAPACITY STUDY
Origin and Sponsor: Bureau of Public Roads
Manager: S. Peavy, J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 33
Status: Continued. The code has been written and code checking is nearly completed.

1102-40-5126/58-333 CALCIUM HYDROXIDE
Origin and Sponsor: NBS, Section 9.0
Manager: P. O'Hara
Full task description: Jan-Mar 1958 issue, p. 36
Status: Inactive.

Status of Projects

1102-40-5126/58-337 GEORGETOWN LANGUAGE TRANSLATION EXPERIMENTS
Origin and Sponsor: Georgetown University
Manager: R. J. Arms
Full task description: Jan-Mar 1958 issue, p. 37
Status: Terminated.

1102-40-5126/58-339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS
Origin and Sponsor: NBS, Section 3.4
Manager: H. Oser
Full task description: Jan-Mar 1958 issue, p. 38
Status: Continued. Tabulation of the functions in question was continued. Further theoretical results are asymptotic expressions for small and large time parameters. In order to cut down the amount of machine time for further computations more theoretical efforts are necessary. Investigations have been started to get analytic expressions for intermediate ranges of the time variable.

1102-40-5126/58-343 MINIMIZATION PROBLEM
Origin and Sponsor: Naval Research Laboratory
Manager: S. Peavy
Full task description: Jan-Mar 1958 issue, p. 40
Status: Completed. Results have been transmitted to the sponsor.

1102-40-5126/59-348 RUSSIAN-TO-ENGLISH MACHINE TRANSLATION
Origin: NBS
Sponsor: Office of Ordnance Research, U. S. Army
Manager: I. Rhodes
Full task description: Oct-Dec 1958 issue, p. 26
Status: Continued. Mrs. Rhodes attended The International Conference on Information Processing in Paris, June 13-22. At a symposium of the Conference on Machine Translation, she gave an informal talk on the progress of her language translation technique.

1102-40-5126/58-358 REDUCED CROSS-SECTIONS
Origin and Sponsor: NBS, Section 3.2
Manager: R. J. Arms
Full task description: Apr-June 1959 issue, p. 30
Status: Reactivated. Preliminary test codes are being written.

1102-40-5126/58-359 VELOCITY DISTRIBUTION IN BOUNDARY LAYERS
Origin and Sponsor: NBS, Section 11.4
Manager: W. C. Rheinboldt
Full task description: Apr-June 1958 issue, p. 32
Status: Terminated. The program will continue in production under the sponsor's direction. (Production time will be reported herafter in the section, "Application of Automatic Computer.")

3711-60-0009/58-360 DIFFUSION COEFFICIENTS
Origin: NBS, Section 5.2
Manager: W. C. Rheinboldt
Full task description: Apr-June 1958 issue, p. 32
Status: Terminated. The program will continue in production under the sponsor's direction. (Production time will be reported hereafter in the section, "Application of Automatic Computer.")

1102-40-5126/58-361 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION
Origin and Sponsor: Naval Research Laboratory
Manager: R. J. Arms
Full task description: Apr-June 1958 issue, p. 33
Status: Continued. A code for input generation has been checked out.

1102-40-5126/58-366 RADIATION PATTERNS OF ANTENNAS
Origin and Sponsor: U. S. Information Agency, Department of State
Managers: R. T. Moore, P. J. Walsh
Full task description: Apr-June 1958 issue, p. 35
Status: Continued. Part (1): Most of table (d) has been computed and printed on tape.

Part (2): Approximately 20 more antennas were analyzed, and results were submitted to the sponsor.

1102-40-5126/58-368 INTENSITY FUNCTIONS AND CROSS SECTIONS OF LIGHT SCATTERED BY SPHERICAL PARTICLES
Origin and Sponsor: U. S. Army Signal Research and Development Laboratories, Atmospheric Physics Branch, Belmar, N. J.
Manager: H. Oser
Full task description: July-Sept 1958 issue, p. 32
Status: Continued. Production runs have been made for the single sphere problem. Comparison of the results with the experiments showed excellent agreement.

As an extension of the original problem a code has been written by A. Beam to compute scattering functions of coated spheres. To this end the wave equation has to be solved in three different regions and the solutions have to be matched at the boundaries of these regions. The problem can be solved exactly in terms of spherical Bessel and Hankel functions.

The code works properly, and production has started.

1102-40-5126/59-374 END-EFFECT IN THE CYLINDRICAL ROTATIONAL VISCOMETER
Origin and Sponsor: NBS, Section 7.1
Managers: A. Schopf, J. P. Menard
Full task description: July-Sept 1958 issue, p. 34
Status: Continued. Several iterative codes for solving linear equations by slight modifications of the conjugate gradient method have been written and tested for accuracy and numerical stability with various test matrices. A method that gives upper and lower bounds for the end-effect coefficient has been developed.

1102-40-5126/59-377 LOGICAL DIAGRAM REDUCTION
Origin and Sponsor: NBS, Section 12.3
Managers: W. Hall, C. Coleman (4.01)
Objective: (a) To check a previously prepared list of "stages" of an electronic network for permissibility according to a set of criteria furnished by the sponsor. Certain of these criteria are arbitrary such as permissibility of stage name, but most are functions of the basic machine design; for example, the number, type, and phase of inputs to an AND-gate and an OR-gate for both single and double package states. Other criteria include numbers of AND-gates and OR-gates per stage, phase and type of signal destination, signal drive, and signal load.

(b) To form lists of those stages in which the output of a given stage appears as input. This is accomplished largely by sorting and merging of data obtained incidentally to computation of permissibility.

(c) To establish the needed wiring connections between stages and select from them a tree (in the topological sense) of minimal length.

(d) To check trees so obtained for certain additional criteria, such as required maximum or minimum length of wires, maximum of connections to one node.

(e) To record the result in the form of wiring tables.

Background: The computations are in support of a project in the Digital Systems Section (12.3) for study of the use of digital computers as an aid to the design of electronic circuits (including other digital computers). One of the possible applications is to the Bureau's future Pilot computer.
Status: New. The coding has been completed and checked. Production running is pending until changes in machine design are considered.

1102-40-5126/59-388 HEAT PUMP CALCULATIONS
Origin and Sponsor: NBS, Section 10.3
Manager: R. Zucker
Full task description: Jan-Mar 1959 issue, p. 26
Status: Continued. Additional cases were run on the IBM 704 for psychrometric and flowmeter cooling.

Status of Projects

1102-40-5126/59-389 FREQUENCY ALLOCATION
Origin and Sponsor: Civil Aeronautics Administration
Manager: L. S. Joel
Full task description: Oct-Dec 1958 issue, p. 29
Status: Continued. A linear program solution of the problem suggested by Dr. A. J. Hoffman, of General Electric, was found to be unfeasible due to the large number of variables. An approach using subsystems is being investigated.

1102-40-5126/59-394 VARIATIONAL CALCULATION OF SLOW ELECTRON SCATTERING BY HYDROGEN ATOMS, II
Origin and Sponsor: NBS, Section 4.6
Manager: A. Beam
Full task description: Oct-Dec 1958 issue, p. 30
Status: Continued. To date, the code for the s-wave has been completed and it has been used to obtain all the desired results.

An error in the formulation of the problem for the p-wave has invalidated the presently completed p-wave code. It was found that in the preparation of a revised code, all scattering waves of interest (s, p, and d) could be combined in a single code if the integrals which enter are evaluated by quadrature methods.

The already obtained s-wave results and parts of the old p-wave calculation will be useful in checking out the new code for the general case.

1102-40-5126/59-403 COMPUTATION OF COLOR FADINGS
Origin and Sponsor: NBS, Section 2.1
Managers: W. C. Rheinboldt, J. P. Menard
Full task description: Oct-Dec 1958 issue, p. 30
Status: Continued. A production run consisting of 300 samples has been made, and the results were turned over to the sponsor.

1102-40-5126/59-407 FOURIER COEFFICIENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Jan-Mar 1959 issue, p. 27
Status: Continued. Additional cases were run on the IBM 704.

1102-40-5126/59-412 DYNAMICS OF PNEUMATIC PRESSURE REDUCERS
Origin and Sponsor: NBS, Section 3.2
Manager: W. Gautschi
Full task description: Oct-Dec 1958 issue, p. 31
Status: Continued. Production runs have been continued on the basis of the supplementary code (see Jan-Mar 1959 issue) and of a new auxiliary code which facilitates the determination of the dependence of the stability characteristics from the parameters.

1102-40-5126/59-414 INFINITE SYSTEMS
Origin and Sponsor: NBS, Division 3
Manager: R. Zucker
Full task description: Jan-Mar 1959 issue, p. 28
Status: Continued. The code was extended to include the following: —
δ-function reaction:

$$x_n(\tau) = \sum_{j=0}^{N} \frac{\ell_m(\mu_j)\ell_n(\mu_j)e^{m\theta}}{\sum_{s=o}^{N} \ell_s^2(\mu_j)e^{s\theta}} e^{-\mu_j\tau}$$

where μ_j are the roots of $\ell_{N+1}(\mu)=0$, and

$$\ell_n(\mu) = e^{-n\theta} \sum_{\nu=o}^{n} (1-e^{\theta}) \binom{n}{\nu} \binom{\mu}{\nu} ;$$

δ-function relaxation:

$$x_n(\tau) = \frac{(1-e^{\theta})e^{m\theta}\beta^m}{(e^{-\tau}-e^{\theta})} \sum_{i=o}^{[\min n,m]} \frac{m!n!}{(m-i)!(n-i)!(i!)^2} \beta^{n-i}(\beta-\alpha)^i$$

$$\beta = \frac{e^{-\tau}-1}{e^{-\tau}-e^{\theta}}, \qquad \alpha = \frac{e^{-\tau}e^{-\theta}}{e^{-\tau}-1}$$

The δ-function relaxation was modified to include the evaluation of the expression

$$H(\tau) = \sum_{n=0}^{5} x_n(\tau) \ln x_n(\tau)$$

for the adiabatically isolated system.

The code has been checked out and production runs were made.

The δ-function reaction and relaxation were evaluated for m=3, N=4, θ=1, n=0(1)5 and τ =.01, .5, 2, 5. The zeros of $\ell_n(\mu)$ for n=0(1)11 for θ=.1, 1, 5 were computed.

For the adiabatically isolated system computations were carried out for m=3, θ=.2877, n=0(1)5, τ =.01, .05, .1, .5, 1.0, 2.0.

1102-40-5126/59-415 COMPLEX LEGENDRE FUNCTIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R Zucker
Full task description: Jan-Mar 1959 issue, p. 29
Status: Continued. The code for the double summation was written and is in the process of being checked out.

Status of Projects

1102-40-5126/59-418 P-WAVE EQUATION
Origin and Sponsor: NBS, Section 4.8
Managers: S. Peavy, R. Varner
Full task description: Jan-Mar 1959 issue, p. 29
Status: Continued. A code, using a SHARE subroutine to integrate the second order differential equation in the unperturbed and non-exchange equations, was written by the sponsor. A Fortran code using the SHARE subroutine to evaluate the exchange equation has been written and is being checked out.

1102-40-5126/59-427 MAGNETIC SCATTERING OF NEUTRONS BY PARAMAGNETIC MnF_2
Origin and Sponsor: Naval Research Laboratory
Manager: W. Hall
Full task description: Jan-Mar 1959 issue, p. 31
Status: Completed. Results have been transmitted to the sponsor.

1102-40-5126/59-429 WATCH JEWEL PERFORMANCE
Origin and Sponsor: NBS, Section 6.6
Manager: J. Beiman
Objective: To reduce raw data on watch jewel performances into watch rates.
Background: The NBS was asked to determine whether the Federal Government should specify 7 jewel or 21 jewel stop watches for laboratory use. In connection with the study, 80 initial chronometer observations, 2400 watch final observations were made on the effect of jeweling on the performance of laboratory stop watches. The problem was requested by H. Bowman (6.6).
Status: Completed (New). The problem has been coded and checked out. Production results have been submitted to the sponsor for analysis.

1102-40-5126/59-434 PETROLOGICAL COMPUTATION
Origin and Sponsor: Geophysical Laboratories, Carnegie Institution of Washington
Manager: R. Varner
Objective: To compute the diffraction transforms for two dimensional analogs of layered minerals with arbitrary lengths and levels of short range ordering.
Background: This problem arises in experimental petrology and, because of the extensive calculations involved, the use of a large scale computer is required. These computations were requested by Dr. Felix Chayes of the Geophysical Laboratories.
Status: New. In collaboration with Dr. Chayes, a program for the calculation of individual intensities was prepared and a number of cases have been run.

1102-40-5126/59-435 ELECTROCARDIOGRAPHIC ANALYSIS
Origin: NBS, Division 12.5
Sponsor: Veterans Administration
Manager: R. J. Arms
Objective: Statistical analysis of large samples of vectorcardiographic heart signals.
Background: With the use of analog devices, vectorcardiographic readings have been the subject of extensive study. A facility for converting heart signals to digital readings (at intervals of one millisecond) on 704 magnetic tape has been designed by Division 12. Analysis of the data by digital methods, having the natural advantage of speed, permits examination of large samples. Improvement of criteria for normal versus abnormal heart signals is the desired result.

The problem was submitted by Dr. H. V. Pipberger (V. A.).

Status: New. Input and output codes have been prepared and are in the process of being checked out.

1102-40-5126/59-443 MICROWAVE SCATTERING
Origin and Sponsor: NBS, Division 5.7
Manager: H. Oser
Objective: To solve a linear nth order system of differential equations for a given set of initial values.
Background: Study of vibrational modes of chain molecules leads to an nth order system of differential equations for the amplitude functions. The solution leads to Rutherford types of matrices, whose determinants can be expressed in trigonometric form. The initial values are such that the solutions can be carried through exactly and can be represented in closed form.

The results are to be published in a chemical journal as part of a paper dealing with the absorption of microwaves in substances which are characterized by the presence of long chain molecules.

The problem was requested by E. Lustig (5.7).

Status: Completed (New). Results have been transmitted to the sponsor.

1102-40-5126/59-444 HOSPITAL SUBSISTENCE ITEMS SUPPLY
Origin and Sponsor: Veterans Administration
Manager: L. S. Joel
Objective: To adapt the recent results of Markowitz and Manne[1] and R. Gomory[2] in order to outline construction of a method of solution (for an IBM 650 computer if possible or for an IBM 704) to the problem of the minimization of

$$(1)\quad F(x,y) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\,x_{ij} + \sum_{j=1}^{n} d_j\left(\sum_{i=1}^{m} x_{ij}\right) + \sum_{j=1}^{n}\sum_{k=1}^{p} f_{jk}\,y_{jk},$$

subject to the constraints

$$(2)\quad \sum_{i} x_{ij} \geq \sum_{k} y_{jk}, \qquad j = 1,2,\ldots,n,$$

(3) $\sum_j y_{jk} \geq b_k$, $k = 1,2,\ldots,p$,

(4) $x_{ij} \geq 0$, $y_{jk} \geq 0$, $i = 1,2,\ldots,m$; $j = 1,2,\ldots,n$; $k = 1,2,\ldots,p$,

where x_{ij} = number of units shipped from origin i to depot j,

y_{jk} = number of units shipped from depot j to hospital k,

b_k = number of units required at the k^{th} hospital,

c_{ij} = unit shipping cost from origin i to depot j,

d_j = unit storage cost in depot j,

f_{jk} = unit shipping cost from depot j to hospital k,

c_{ij}, d_j and f_{jk} are piecewise linear functions of their coefficients in (1).

Background: The present study is undertaken because conventional linear programming techniques are not applicable to the above minimization problem as economies of scale (i.e., decreasing unit shipping and storage costs for volume shipments) impose the "wrong"* kind of nonlinearity on the objective function. (*A cost function which is piecewise linear and strictly concave can be treated by the simplex method with very few simple modifications.)

References: [1]. Markowitz and Mann, On the solution of discrete programming problems, Econometrica, Vol. 25, No. 1 (1957). [2]. R. Gomory, An algorithm for integer solutions to linear programs, Princeton-IBM Math. Res. Project, Technical Report No. 1, 1958.

The problem was submitted by R. Dean (V. A.).

Status: New.

1102-40-5126/59-445 OIL SUPPLY

Origin and Sponsor: Military Petroleum Supply Agency, Department of the Navy

Manager: L. S. Joel

Objective: To solve a transportation problem using a standard 704 simplex-transportation code.

Background: The problem concerns the purchase and allocation to depots of jet fuels by the MPSA. The problem was transmitted by G. Suzuki (MPSA).

Status: New.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Gage blocks. A computing machine program was prepared for the analysis of calibration data on gage blocks. In addition to the calibration values a statistical analysis is made to assess the precision and accuracy of mechanical and interferometric measurements and included as part of the print-out for each set of data. This work was done for John Beers, Section 2.4.

(2) Standard voltage cells. An analysis of data on the precision of measurements on standard voltage cells was undertaken to ascertain the degree of dependence of the precision upon type and year of manufacture, for Catherine Law, Section 1.6.

(3) Transistor measurement procedures. Design and analysis were done for an experiment to investigate and compare the performance of two transistor types with respect to two different methods of test to verify a certain aging specification, for G. Conrad, 1.6.

(4) Color-matching. Methods were developed for the analysis, using multivariate techniques, of individual observer data from a large-scale color-matching investigation. This work is carried on in collaboration with I. Nimeroff, Section 2.01.

(5) An in-hours course, "Design of Multi-factor Experiments" presented by W. S. Connor and M. Zelen was concluded.

(6) Use of sampling methods in studies of Post Office operation. N. C. Severo and A. E. Newman have prepared a manuscript entitled, "A statistical chain-ratio method for determining the distribution of mail by destination".

Publications:

(1) Some canons of sound experimentation. C. Eisenhart. To appear in the Proceedings of a Special Session of the International Statistical Institute, Burssels, 1958.

(2) Graphical diagnosis of interlaboratory test results. W. J. Youden. To appear in the Proceedings of the Midatlantic Conference of the American Society for Quality Control to be held in Atlantic City, February 1959.
(3) Problems of the experimenter. W. J. Youden. Submitted to a technical journal.
(4) Statistics--Engineering viewpoint. W. J. Youden. To appear in the Journal of Engineering Education.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: CONTINUED. A study of the effect of location of the plate on the variability of transition temperature results was conducted on the available data on ship plate.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-5146/55-93

Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: COMPLETED. Drafts of all sections (see Apr-June 1957 issue, p. 38) have been sent to the sponsor.

Publication:
(1) The relation between confidence intervals and test of significance--a teaching aid. Mary G. Natrella. Submitted to a technical journal.

STATISTICAL SERVICES
Task 1103-40-5150/58-346

Origin and Sponsors: Various Agencies Authorized 3/31/58
Manager: J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 45

Status: CONTINUED. Work was done during the quarter for the

following agencies:

(1) U. S. Geological Survey: Investigations were begun on several mathematical models associated with measurement processes of hydrology, in particular concerning the construction of "synthetic" stream discharge records from records on stages of the stream. Another investigation involves study of the properties of a mathematical model relating discharge to rainfall.

(2) Naval Ordnance Laboratory: Regression analysis was done to relate fatigue data for certain aluminum and magnesium alloys to maximum stress and degree of defect (porosity or microshrinkage, respectively).

(3) Army Chemical Corps: A study of the use of Monte Carlo methods in a problem of the Chemical Corps was conducted by A. J. Goldman and J. M. Cameron.

(4) Bureau of Public Roads: See task 1102-40-5126/58-316, p. 23 , for write-up of this task done jointly with the Computation Laboratory.

APPLICATION OF AUTOMATIC COMPUTER

The record of the use of the IBM 704 for the period April 1 through June 30 is as follows:

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
NBS:					
5114/57-237	11.1	Numerical analysis training program	145	279	492
0002/51-1	11.3	Statistical engineering	40	57	613
5160/58-369	11.4	Computation of outflow from a breached dam		14	249
1120/55-65	11.2	Automatic coding	377	184	146
0009/56-160	11.2	Mathematical subroutines	16	55	65
0009/54-30	4.1	Spectrum analysis	21	224	694
0009/54-38	3.2	Equation of state of real gases	37	29	61
0009/55-68	9.7	Crystal structure calculations	87	117	632
0009/55-82	3.1	Thermometer calibrations	14	289	123
5126/55-97	3.8	High temperature properties for air°	38	250	111
0009/56-131	2.2	Calculations in optics*	41	36	173
5126/56-150	4.10	Mathematical expressions*	2	15	135
5126/56-166	5.9	SCF-LCAO solution of some hydrides*			129
5126/56-171	3.2	Collision integrals used in transport theory**	128	239	185
5126/57-219	3.2	Thermal properties*	148	158	1171
0009/57-223	3.2	Self-consistent fields	27	123	
5126/57-229	40.0	Payroll operations	2		
5126/57-236	3.6	Self-consistent field--eigenvalues		3	103
5126/57-246	4.8	Radiation diffusion**	515	174	1674
0009/57-247	6.1	Mechanical impedance*			113
0009/57-250	2.3	Automatic reduction in spectrophotometric data*	7	9	118
5126/57-252	4.11	Detecting efficiency in a neutral meson experiment**		15	17
0009/58-254	2.3	Reproduction of color- and spectral-energy distribution of daylight*		6	83
5126/58-255	4.8	Chi functions**	71	459	983

Task No.		Title	Code Assembly	Code Checking	Code Production
			(M I N U T E S)		
5126/58-256	10.6	Composite walls**	293	42	141
5126/58-258	6.1	Noise measurement, II*		14	7
5126/58-260	12.5	Prototype accounting**	13	162	467
5126/58-267	2.1	Munsell color system conversion*			30
5126/58-272	3.2	Thermodynamic properties of real gases**		156	68
5126/58-274	9.7	Calculations for d-spacings, II			177
5126/58-275	7.8	Crystallography**	15		176
5126/58-294	4.8	Nuclear scattering of photons*			20
5126/58-296	10.2	Lump network**		7	
5126/58-298	5.10	Analysis of spectrochemical data			13
5126/58-299	3.1	Time-dependent Shroedinger equation	39	54	18
5126/58-304	3.2	Transport properties of air	60	307	115
5126/58-306	2.1	Interpolation of color mixture functions*			16
5126/58-308	3.4	Oscillating sphere	28	24	2
5126/58-313	11.3	Fractional factorials	9	9	17
5126/58-314	3.2	Approximations for gas mixtures*	381	156	573
5126/58-339	3.4	Viscoelasticity properties of materials		27	115
5126/58-357	3.3	Eigenvalues**			5
5126/58-360	5.2	Diffusion coefficients*			7
5126/58-367	8.4	Ellipsometer calculations**			18
5126/59-374	7.1	End-effect in the cylindrical rotational viscometer	45		22
5126/59-377	12.3	Logical diagram reduction	76	616	1335
5126/59-378	82.20	Correlation program**			18
5126/59-382	4.6	Phase shift**		19	30
5126/59-383	10.5	Design flow capacity*			1
5126/59-387	30.4	Nuclear reactor design**			134
5126/59-388	10.3	Air conditioning			20
5126/59-390	12.5	Electrocardiogram**	99	141	71
5126/59-394	4.6	Slow electron scattering by hydrogen atoms	84	97	128
5126/59-395	7.7	Adsorption study**	15	47	28
5126/59-403	2.1	Computation of color fadings			60
5126/59-404	4.9	Counter**	4	30	75
5126/59-409	12.5	Bank Board**	47	50	253
5126/59-412	3.2	Dynamics of pneumatic pressure reducers	31	138	158
5126/59-413	6.6	Mass weighing*		2	89

Task No.		Title	Code Assembly	Checking	Production
			(M I N U T E S)		
5126/59-414	3.1	Infinite systems	2	11	202
5126/59-418	4.8	P-wave equation	43	103	12
5126/59-428	12.5	Radio intensities**	21	20	9
5126/59-429	6.6	Watch jewel performance			20
5126/59-430	3.2	Asymmetric rotor eigenvalue problem	6	10	37
5126/59-433	2.3	Color of signals*			61
5126/59-440	82.10	Mapping**	82	97	57
5126/59-446	85.10	Ionospheric data**	31	11	13
		Totals(NBS).....	3140	5085	12888
OUTSIDE:					
5116/55-56	ONR	Research in mathematical topics applicable to numerical analysis	4	37	184
5116/56-148	NIH	Differential equations for nerve excitation	7	11	991
5126/53-45	SC	Air defense tactics°	62	56	34
5126/58-263	DOFL	Gas tube characteristic		205	310
5126/58-269	NRL	Molecular structure,IV	174	79	761
5126/58-273	DOFL	Administration problem**		6	
5126/58-276	NOL	General kinetics,I**			5483
5126/58-282	NOL	Missile boundary layer computation°	53	325	987
5126/58-284	NRL	Sputnik tracking*		1	13
5126/58-315	DOFL	Mechanization of French translation**		433	282
5126/58-316	BPR	Intersection capacity study	43	61	
5126/58-319	HPBA	Auto tag°		32	22
5126/58-320	NRL	Teller emission problem°	152	62	1141
5126/58-335	DOFL	Roots of Bessel functions**			31
5126/58-340	DOFL	M5-17 Fuze Data°	10		165
5126/58-347	BURR	Computations for war games**	174	402	6601
5126/58-348	OOR	Russian-to-English machine translation	52	84	45
5126/58-361	NRL	Spectrum of dipole radiation	109	32	191
5126/58-366	USIA	Radiation patterns of antennas		7	140
5126/58-368	SC	Intensity functions	17	117	174
5126/58-370	NRL	Neutron diffusion study°		126	1219
5126/59-371	NRL	ASWAP°	85	22	19
5126/59-407	DOFL	Fourier series			100
5126/59-408	NASA	NASA**	1650	432	20367
5126/59-410	GA	Gallaudet training program**	21	36	9

Task No.		Title	Code Assembly	Code Checking	Code Production
			(M I N U T E S)		
5126/59-411	HEW	Fitting of exponential curves***	71	15	26
5126/59-415	DOFL	Complex Legendre functions	2	114	106
5126/59-416	DOFL	Analysis of power supply experiments**	27	40	206
5126/59-419	DOFL	Neutrons °	24	114	1805
5126/59-423	WB	Weather Bureau**	4	35	19923
5126/59-424	NOL	Systems engineering**	153	27	56
5126/59-425	CU	Molecular orbitals*			4
5126/59-427	NRL	Magnetic scattering of neutrons	124	49	227
5126/59-431	BUSH	Factorials*	15	16	814
5126/59-434	CIW	Petrological computations	68	138	242
5126/59-435	VA	Electrocardiographic analysis	18	67	43
5126/59-445	NPSA	Oil Supply		27	566
5126/59-447	BPRO	Public Roads**(General Electric)			610
5126/59-448	NMR	Vapor transition*	10	1	
		Totals (Outside)	3129	3209	63897
		Total time for the quarter(MINUTES).....	6269	8294	76785
		Total time for the quarter (HOURS)	104.5	138.2	1279.8

* Problem programmed in the Computation Laboratory; production runs continued under direction of sponsor.

** Problem programmed by sponsor and run under his direction.

*** Machine time provided under contract.

° Classified task.

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Mathematical Statistics Seminar

SHRIKHANDE, S. S. (University of North Carolina) The construction of orthogonal Latin squares and falsity of Euler's conjecture. April 27.

CONNOR, W. S. Fractional factorials of the $2^m 3^n$ series. May 1.

MENGER, E. (Illinois Institute of Technology) Probabilistic geometry. May 21.

BOSE, R. C. (University of North Carolina) Two-error correcting binary codes. May 27.

Applied Statistics Seminar

MATALAS, N. C. (U.S. Geological Survey) The use of statistical methods for the analysis of droughts. May 6.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

ALT, F. L. Electronic computers for research. Presented at the Fourth Institute on Research and Development Administration of The American University, Washington, D.C., April 24.

BAZLEY, N. (1) Lower bounds for eigenvalues of self-adjoint operators. Presented at a meeting of the American Mathematical Society, New York, N.Y., April 23. (2) Lower bounds to helium atom energies. Presented at a meeting of the American Physical Society, Washington, D.C., May 1.

EISENHART, C. The background and evolution of the method of least squares. Presented before the British Society for the History of Science, London, England, May 4.

GOLDBERG, K. Group generated incidence matrices. Presented at a meeting of the American Mathematical Society, New York, N.Y., April 25.

NEWMAN, M. Number theory and computers. Presented at a meeting of the Mathematical Association of America, Baltimore, Md., May 2.

ROSENBLATT, J. R. Theoretical problems of reliability measurement and prediction. Presented at the Third Session of the Exploratory Conference on Missile Model Design for Reliability Prediction, held at White Sands Missile Range, New Mexico, April 20-23.

SHERLIN, M. B., with Sherlin, G. C. (6.3), Preparations for successful science projects. Presented before the Mathematics and Science Department of Brooks Junior-Senior High and Calvert County Junior-Senior High Schools, Prince Frederick, Md., May 25.

YEVDJEVICH, V. M. Effect of sudden water release on reservoir outflow hydrographs. Presented before the Hydrodynamics Laboratory of the Massachusetts Institute of Technology, Cambridge, Mass., April 22.

YOUDEN, W. J. (1) Graphical diagnosis of interlaboratory test results. Presented before the American Society for Quality Control, Oshkosh, Wis., April 13; and before the American Society for Quality Control, Chicago, Ill., April 28. (2) What is scientific evidence?. Presented before a meeting of Sigma Xi, Chicago, Ill., May 13. (3) Randomization and experimentation. Given at a meeting of the American Statistical Association, Chicago, Ill., May 20. (4) Problems of the experimenter. Delivered at the 13th Annual Convention, American Society for Quality Control, Cleveland, Ohio, May 25. (5) Seminar on problems of the experimenter. Presented before the Chemistry Department, University of Chicago, Chicago, Ill., June 1. (6) A new graphic method for statistical treatment and evaluation of interlaboratory tests. Presented before a meeting of the American Society for Testing Materials, Atlantic City, N. J., June 25.

ZELEN, M. (1) Multi-factor experiments in life testing. Presented before the Department of Statistics, Virginia Polytechnic Institute, Blacksburg, Va., April 17. (2) Life-testing. Presented before the Department of Statistics, Michigan State University, Lansing, Mich., April 28. (3) Recent development in the design of experiments. Presented before the Committee on Statistical Design and Analysis, U.S. Department of Agriculture, Washington, D.C., May 21.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Tables of osculatory interpolation coefficients. H. E. Salzer. National Bureau of Standards Applied Mathematics Series 56. Issued May 8, 1959; price 30 cents. Available from U. S. Government Printing Office, Washington 25, D.C.

1.2 Manuals, Bibliographies, and Indices.

(1) Fractional factorial experiment designs for factors at three levels. W. S. Connor and M. Zelen. National Bureau of Standards Applied Mathematics Series 54. Issued May 1, 1959; price 30 cents. Available from U. S. Government Printing Office, Washington 25, D.C.

(2) Analysis of straight line data. F. S. Acton. (J. Wiley and Sons, Inc., New York, 1959).

1.3 Technical Papers

(1) Assigning quantitative values to qualitative factors in the Naval electronics problem. R. J. Aumann and J. B. Kruskal (University of Wisconsin). Naval Res. Log. Quart. 6, 1-16 (1959).

(2) Lower bounds for eigenvalues with applications to the helium atom. N. Bazley. Proc. Nat. Acad. Sci. 45, 850-853 (1959).

(3) Capacity requirement of a mail sorting device. B. Bender and A. J. Goldman. J. Research NBS 62, 171-173 (1959), RP2948.

(4) Bivariate linear interpolation for analytic functions. W. Gautschi. Math. Tables Aids Comp. 13, 91-96(1959).

(5) Some elementary inequalities relating to a gamma and incomplete gamma function. W. Gautschi. J. Math. Phys. 38, 77-81(1959).

(6) Convexity of the field of a linear transformation. A. J. Goldman and M. Marcus. Canadian Math. Bull. 2, 15-18 (1959).

(7) On the derivative of Bessel functions with respect to the order. F. Oberhettinger. J. Math. Phys. 37, 75-78 (1958).

(8) A quantitative formulation of Sylvester's law of inertia. A. M. Ostrowski. Proc. Nat. Acad. Sci. 45, 740-744 (1959).

(9) Diffusion of particles in turbulent flow. C. M. Tchen. Appeared in Advances in Geophysics 6: Atmospheric Diffusion and Air Pollution, pp. 165-173 (Academic Press, New York and London, 1959).

(10) Graphical diagnosis of interlaboratory test results. W. J. Youden. Indust. Quality Control 15, 24-28 (1959); also Middle Atlantic Conference Transactions, American Society for Quality Control,1959, pp. 196-206.

(11) Problems of the experimenter. W. J. Youden. National Convention Transactions, American Society for Quality Control, 1959, pp. 41-47.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATIONS JUNE 30,1959.

2.1 Mathematical Tables

(1) Tables of the bivariate normal distribution function and related functions. To appear as NBS Applied Mathematics Series 50.

2.2 Manuals, Bibliographies, and Indices

(1) Handbook of Mathematical Functions. To appear in the NBS Applied Mathematics Series.

(2) Basic theorems in matrix theory. M. Marcus. To appear in the NBS Applied Mathematics Series.

2.3 Technical Papers

(1) Analytic comparison of suggested configurations for automatic mail sorting equipment. B. Bender and A. J. Goldman. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(2) Analysis of fractionally replicated 2^n3^m designs. R. C. Bose and W. S. Connor. To appear in Proceedings of a Special Session of the International Statistical Institute, Brussels, 1958.

(3) Construction of fractional factorial designs of the mixed 2^m3^n series. W. S. Connor. To appear in "Contributions to probability and statistics," in press, Stanford University Press (1960).

(4) The construction of Hadamard matrices. E. C. Dade and K. Goldberg. To appear in the Michigan Journal of Mathematics.

(5) Leonhard Euler's integral: A historical profile of the Gamma function. (In memoriam: Milton Abramowitz). P. J. Davis. To appear in the American Mathematical Monthly.

(6) Bending and stretching of corrugated diaphragms. R. F. Dressler. To appear in the Transactions of the American Society of Mechanical Engineers.

(7) Some canons of sound experimentation. C. Eisenhart. To appear in the Proceedings of a Special Session of the International Statistical Institute, Brussels, 1958.

(8) Absorption of radiation by a cylindrical sample of a strong absorber. P. H. Fang (Mineral Products Division) and I. A. Stegun. Submitted to a technical journal.

(9) Generating functions for formal power series in noncommuting variables. K. Goldberg. To appear in the Proceedings of the American Mathematical Society.

(10) Note on a paper by S. Mukhoda and S. Sawaki. K. Goldberg. Submitted to a technical journal.

(11) The incidence equation $AA^T = aA$. K. Goldberg. To appear in the American Mathematical Monthly.

(12) The minima of cyclic sums. K. Goldberg. To appear in the Journal of the London Mathematical Society.

(13) The minimum of a certain linear form. K. Goldberg. Submitted to a technical journal.

(14) Principal submatrices of a full-rowed non-negative matrix. K. Goldberg. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(15) The zeros of certain polynomials. A. J. Goldman. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(16) A continuous poker game. A. J. Goldman and J. J. Stone. Submitted to a technical journal.

(17) Relations between summation methods and integral transformations. W. Greub. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(18) On a generalization of an inequality of L. V. Kantorovich. W. Greub and W. Rheinboldt. To appear in the Proceedings of the American Mathematical Society.

(19) Applications of a theorem on partitioned matrices. E. V. Haynsworth. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(20) Reduction formulas for partitioned matrices. E. V. Haynsworth. Submitted to a technical journal.

(21) The relation between confidence intervals and tests of significance--a teaching aid. M. G. Natrella. Submitted to a technical journal.

(22) Construction and application of a class of modular functions,II. M. Newman. To appear in the Proceedings of the London Mathematical Society.

(23) Modular forms whose coefficients posses Euler products. M. Newman. Submitted to a technical journal.

(24) Weighted restricted partitions. M. Newman. To appear in Acta Arithmetica.

(25) Inclusion theorems for congruence subgroups. M. Newman and I. Reiner (University of Illinois). To appear in the Transactions of the American Mathematical Society.

(26) Linear differential equations of the second order with a large parameter. F. W. J. Olver. Submitted to a technical journal.

(27) Uniform asymptotic expansions for Weber parabolic cylinder functions of large orders. F. W. J. Olver. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(28) On the convergence of Gauss' alternating procedure in the method of least squares, I. A. M. Ostrowski. To appear in Annali di Matematica Pura ed Applicata.

(29) On the convergence of the Rayleigh quotient iteration for the computation of characteristic roots and vectors: III. Generalized Rayleight quotient and characteristic roots with linear elementary divisors; IV. Generalized Rayleigh quotient for nonlinear elementary divisors; V. Usual Rayleigh quotient for non-Hermitian matrices and linear elementary divisors; VI. Usual Rayleigh quotient for nonlinear elementary divisors. A. Ostrowski. To appear in Archive for Rational Mechanics and Analysis.

(30) Über genäherte Determinanten und bedingte Tragheitsindizes. A. M. Ostrowski. To appear in (Wiener) Monatshefte für Mathematik und Physik.

(31) Stokes flow problem for a class of axially symmetric bodies. L. E. Payne and W. H. Pell. Submitted to a technical journal.

(32) The graphical solution of initial value problems. W. H. Pell. Submitted to a technical journal.

(33) On prediction of system behavior. J. Rosenblatt. To appear in the Proceedings of the NYU-Industry Conference on Reliability, held at Ardsley-on-Hudson, N. Y., June 1958.

(34) On the power of some rank order two-sample tests. J. R. Rosenblatt. To appear in "Contributions to probability and statistics", in press, Stanford University Press (1960).

(35) Theoretical problems of reliability measurement and prediction. J. R. Rosenblatt. To appear in the Proceedings of the Third Exploratory Conference on Missile Model Design for Reliability Prediction, held at White Sands Missile Range, 1959.

(36) The non-central χ^2 as a test statistic. N. C. Severo. Submitted to a technical journal.

(37) Stationary principles for forced vibrations in elasticity and electromagnetism. J. L. Synge. To appear in the Proceedings of the Eighth Symposium in Applied Mathematics held by the American Mathematical Society, Chicago, Ill., April 1956.

(38) Some computational problems concerning integral matrices. O. Taussky. To appear in the Proceedings of the 1956 meeting of The Italian Society for the Advancement of Science, held in Sicily.

(39) Kinetic equation for a plasma with unsteady correlations. C. M. Tchen. To appear in Physical Review.

(40) Turbulent motion. C. M. Tchen and G. B. Schubauer (NBS Fluid Mechanics Section). To appear as Section B, Volume V of the Princeton Series, High Speed Aerodynamics and Jet Propulsion.

(41) Computation problems concerned with the Hilbert matrix. J. Todd. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.

(42) A new approach to the theory of satellite orbits. J. P. Vinti. To appear in Physical Review Letters.

(43) A new method of solution for unretarded satellite orbits. J. P. Vinti. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(44) Rotational properties of two-dimensional lattices. J. P. Vinti. Submitted to a technical journal.

(45) Compressible turbulent boundary layers with heat transfer and pressure gradient in flow direction. A. Walz. To appear in Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(46) Analytical integration of the differential equation for water storage. V. M. Yevdjevich. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(47) Discussion of random balance. W. J. Youden. Submitted to a technical journal.

(48) Randomization and experimentation. W. J. Youden. To appear in Annals of Mathematical Statistics.

(49) Statistics--Engineering viewpoint. W. J. Youden. To appear in the Journal of Engineering Education.

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(51) Factorial experiments in life testing. M. Zelen. To appear in Technometrics.

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(53) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. S. Joel. To appear in the Annals of Mathematical Statistics.

2.4 Reviews and Notes

(1) A note on algebras. A. J. Goldman. To appear in the American Mathematical Monthly (Math. Notes).

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U.S. DEPARTMENT OF COMMERCE
Frederick H. Mueller, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

WASHINGTON, D.C.

Electricity and Electronics. Resistance and Reactance. Electron Devices. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Photographic Technology. Length. Engineering Metrology.

Heat. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology. Molecular Kinetics. Free Radicals Research.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Neutron Physics. Radiation Theory. Radioactivity. X-rays. High Energy Radiation. Nucleonic Instrumentation. Radiological Equipment.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Molecular Structure and Properties of Gases. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Air Conditioning, Heating, and Refrigeration. Floor, Roof, and Wall Coverings. Codes and Safety Standards. Heat Transfer. Concreting Materials.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analog Systems. Application Engineering.

- Office of Basic Instrumentation.
- Office of Weights and Measures.

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships. VHF Research. Radio Warning Services. Airglow and Aurora. Radio Astronomy and Arctic Propagation.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Research. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation Obstacles Engineering. Radio-Meteorology. Lower Atmosphere Physics.

Radio Standards. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Electronic Calibration Center. Microwave Physics. Microwave Circuit Standards.

Radio Communication and Systems. Low Frequency and Very Low Frequency Research. High Frequency and Very High Frequency Research. Ultra High Frequency and Super High Frequency Research. Modulation Research. Antenna Research. Navigation Systems. Systems Analysis. Field Operations.


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