NATIONAL BUREAU OF STANDARDS REPORT

6431

PROJECTS and PUBLICATIONS

of the

APPLIED MATHEMATICS DIVISION

A Quarterly Report

January through March 1959

⟨NBS⟩

U. S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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Inquiries regarding the Bureau's reports should be addressed to the Office of Technical Information, National Bureau of Standards, Washington 25, D. C.

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NBS PROJECT

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IMPORTANT NOTICE

⟨NBS⟩

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

January 1 through March 31, 1959

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under contract with The American University

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	Andreas Schopf	Y. M. Yevdjevich, D.Sc.

PARTICIPANTS IN NUMERICAL ANALYSIS TRAINING PROGRAM
under the sponsorship of the National Science Foundation

Richard V. Andree, Ph.D.	Robert T. Gregory, Ph.D.	E. P. Miles, Jr., Ph.D.
Truman A. Botts, Ph.D.	Ralph E. Lee, Ph.D.	Bill C. Moore, Ph.D.
G. Cleaves Byers, Ph.D.	Leroy F. Meyers, Ph.D.	Jack D. Munn, M.A.

*On leave of absence   **Part time

CONTENTS

*Only unclassified projects are included in this report.

Status of Projects

March 31, 1959

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-12-1104/55-55

Origin: NBS   Authorized 8/29/54
Manager: P. Davis
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. W. Rheinboldt continued his comparative studies of the different iteration methods in functional analysis and their application to numerical analysis. He also continued the series of seminar lectures on functional analysis to members of the division staff. In these lectures the general theory of Newton's method for operator equations in Banach spaces was presented.

A. J. Goldman has determined the range of the real parameter P for which all zeros of $z^{n+1} - z^n + P$ are interior to the unit circle. The solution is $0 < P < 2 \sin(\pi/(4n+2))$. The problem arose in examining the radius of convergence of a generating function.

N. Bazley has completed a note describing the method of intermediate problems for finding lower bounds to the eigenvalues of self-adjoint operators. This note will appear in a technical publication. Numerical bounds are given for the $E(1^1 S)$ and $E(2^1 S)$ energy levels of the helium atom. Convergence of the lower bounds to the true eigenvalues has been established when the inverse operators are completely continuous.

Numerical calculations of Markov chains with a large number of states have been carried out by P. Davis and N. Bazley. It should be emphasized that even if the matrix of transition probabilities has large order numerical solution via matrix powering is feasible when most of the matrix elements vanish. A "sparse" Markov chain of order 100 has been programmed for the 704 and a number of its parameters obtained. A Monte Carlo solution will be programmed and compared with these results.

E. Haynsworth has completed the paper "Reduction Formulae for Partitioned Matrices" and has submitted it to a technical journal. A second paper, "Application of a Theorem on Partitioned Matrices," is in manuscript form.

H. F. Weinberger solved the problem of bounding a function in terms of its values at N equally spaced points and the square integral of its kth derivative was solved by interpolation with a function which is piecewise a polynomial of degree 2k-1 with discontinuities in its (2k-1)th

derivative. This interpolation problem is reduced to the solution of a
Dirichlet boundary value problem for a finite difference equation of order
2k. In this way the bounds can be obtained in terms of N by inverting only
k x k matrices. This is important when k is relatively small while N is
large.

        Publications:
(1) On the numerical integration of periodic analytic functions. P. J.
     Davis. Appeared in "On Numerical Approximation," ed. by R. E. Langer,
     pp. 45-59 (The University of Wisconsin Press, Madison, Wis., 1959).
(2) Computation problems concerned with the Hilbert matrix. J. Todd.
     To appear in the Proceedings of the 1956 meeting of the Italian Society
     for the Advancement of Science, held in Sicily.
(3) On the minimum of the permanent of a doubly stochastic matrix. M.
     Marcus and M. Newman. Duke Math. J. 26, 61-72(1959).
(4) Field convexity of a linear transformation. M. Marcus and A. J.
     Goldman. Submitted to a technical journal.
(5) A continuous poker game. A. J. Goldman and J. J. Stone. Submitted to
     a technical journal.
(6) On a generalization of an inequality of L. V. Kantorovitch. W. Greub
     and W. Rheinboldt. To appear in Proceedings of the American Mathe-
     matical Society.
(7) On the relations between summation methods and integral transforms.
     W. Greub. In manuscript.
(8) Uniform asymptotic expansions for Weber parabolic cylinder functions
     of large orders. F. W. J. Olver. To appear in the Journal of Research
     NBS, Section B. Mathematics and Mathematical Physics.
(9) Reduction formulae for partitioned matrices. E. Haynsworth. Submitted
     to a technical journal.
(10) Lower bounds for eigenvalues with applications to the helium atom.
     N. Bazley. To appear in the Proceedings of the National Academy of
     Sciences.
(11) Leonhard Euler's integral: A historical profile of the Gamma function.
     (In memoriam Milton Abramowitz.) P. J. Davis. To appear in the Amer-
     ican Mathematical Monthly.
(12) The zeros of certain polynomials. A. J. Goldman. Submitted to a
     technical journal.
(13) Linear differential equations of the second order with large parameter.
     F. W. J. Olver. Submitted to a technical journal.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO
NUMERICAL ANALYSIS
Task 1101-12-5116/55-56

Origin: NBS   Authorized 8/13/54
Sponsor: Office of Naval Research
Manager: M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. K. Goldberg has developed a formula for the expected value of the nth block in a flow of random data, with a block defined as a montonically increasing sequence. In the limit, it is a polynomial of degree n in e = 2.71828... which closely approximates 2.

K. Goldberg has continued his investigation of gaps between values of a given function which are relatively prime to the first n primes. If the function is $x^2 + 1$ and $n \leq 19$, the maximum gap is less than the nth prime. If this is generally true, then $x^2 + 1$ represents an infinity of primes.

K. Goldberg has continued his investigation of algebras with an incidence matrix basis, with E. C. Dade.

M. Newman is preparing a manuscript on congruence properties of the coefficients of modular forms. Work on polynomial bases for classes of automorphic forms is continuing. Certain questions of structure of modular subgroups are being investigated.

In connection with the training course, a set of notes on matrix computation has been prepared by M. Newman. These will ultimately appear in the book on the lectures presented in Numerical Analysis Program I (1957) under preparation by J. Todd. The seminar on Vinogradov's methods is continuing; the introductory matter was concluded.

Publications:
(1) Some computational problems concerning integral matrices. O. Taussky. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.
(2) Abelian groups of unimodular matrices. E. C. Dade. Illinois J. Math. 3, 11-27(1959).
(3) Incidence algebras. E. C. Dade and K. Goldberg. Submitted to a technical journal.
(4) The construction of Hadamard matrices. K. Goldberg and E. C. Dade. To appear in the Michigan Journal of Mathematics.
(5) Dense subgraphs and connectivity. R. E. Nettleton (NBS,3.2), K. Goldberg, and M. S. Green (NBS,3.2). Canadian J. Math. 11,262-268(1959).
(6) Some combinatorial lemmas. K. Goldberg. In manuscript.
(7) On normal and EPr matrices. M. Pearl. Michigan J. Math. 6, No. 1, 1-5(1959).
(8) A further extension of Cayley's parameterization. M. Pearl. Canadian J. Math. 11, No. 1,48-50(1959).
(9) On a theorem of M. Riesz. M. Pearl. J. Research NBS, 62, 89-94(1959), RP 2935.

(10) Inclusion theorems for congruence subgroups. M. Newman and I. Reiner
     (University of Illinois). To appear in Transactions of the American
     Mathematical Society.
(11) Construction and application of a class of modular functions, II. M.
     Newman. To appear in Proceedings of London Mathematical Society.
(12) Un nouveau critere d'univalence des transformations dans un R^n.
     A. Ostrowski. Compt. Rend. Acad. Sci. Paris $\underline{248}$, 348-350(1959).
(13) On the convergence of the Rayleigh quotient iteration for the computation
     of the characteristic roots and vectors, II. A. Ostrowski. Arch.
     Rat. Mech. Anal. $\underline{2}$, 423-428(1959).
(14) On the convergence of the Rayleigh quotient iteration for the computation
     of characteristic roots and vectors; III. Generalized Rayleigh quotient
     and characteristic roots with linear elementary divisors; IV.
     Generalized Rayleigh quotient for nonlinear elementary divisors; V.
     Usual Rayleigh quotient for non-Hermitian matrices and linear elemen-
     tary divisors; VI. Usual Rayleigh quotient for nonlinear elementary
     divisors. A. M. Ostrowski. To appear in Archive for Rational
     Mechanics and Analysis.
(15) A quantitative formulation of Sylvester's law of inertia. A. Ostrowski.
     Submitted to a technical journal.
(16) Generating functions for formal power series in noncommuting variables.
     K. Goldberg. Submitted to a technical journal.
(17) Note on a paper by S. Mukhoda and S. Sawaki. K. Goldberg. Submitted
     to a technical journal.
(18) The incidence equation $AA^T = aA$. K. Goldberg. To appear in the
     American Mathematical Monthly.
(19) The minima of cyclic sums. K. Goldberg. Submitted to a technical
     journal.
(20) The minimum of a certain linear form. K. Goldberg. Submitted to a
     technical journal.
(21) Principal submatrices of a full-rowed non-negative matrix. K. Goldberg.
     To appear in the Journal of Research, NBS, Section B. Mathematics
     and Mathematical Physics.
(22) Weighted restricted partitions. M. Newman. Submitted to a technical
     journal.
(23) Über genäherte Determinanten und bedingte Tragheitsindizes. A. M.
     Ostrowski. To appear in (Wiener) Monatshefte für Mathematik und
     Physik.
(24) Modular forms whose coefficients possess Euler products. M. Newman.
     Submitted to a technical journal.
(25) On the bounds of a one-parametric family of matrices. A. M. Ostrowski.
     J. Reine Angew. Math. $\underline{200}$, 190-199(1958).

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-12-5116/56-148

Origin and Sponsor: National Institutes of Health   Authorized 9/30/55
Manager: W. Gautschi (11.2)
Full task description: July-Sept 1955 issue, p. 7

    Status: CONTINUED. Computations have been resumed for both the one-
and the two-dimensional cases,--the former in a slightly modified form.
Results are being transmitted to the sponsor.

TRAINING PROGRAM IN NUMERICAL ANALYSIS, II
Task 1101-40-5114/57-237

Origin and Sponsor: National Science Foundation   Authorized 3/27/57
Manager: P. Davis
Full task description: Jan-Mar 1957 issue, p. 5

    Status: REACTIVATED. The first two weeks of the program were devoted
to an introduction to programming for automatic computation. Although the
training program is not primarily a coding course, all participants have
prepared and run simple test problems on the 704. The basis of the formal
teaching, given by M. Newman, was a discussion of a complex of problems
which illustrated various topics in numerical analysis and programming.
This set of problems could readily be adapted for solution on any type of
digital computer.
    The second phase of the program will be devoted to surveys of
particular chapters in numerical analysis. It will continue for about
thirteen weeks. To date the following topics have been covered:

Linear Equations	E. Haynsworth
Nonlinear Equations	U. Hochstrasser
Matrix Computation	M. Newman
Interpolation and Approximation	P. Davis
Ordinary Differential Equations	W. Gautschi
Elliptic Partial Differential Equations	
	D. Young,Jr.(U. of Tex.)
Monte Carlo Methods	J. Todd (Cal. Tech.)
Bounds for Eigenvalues	O. T. Todd (Cal. Tech.)

    In addition to these surveys, individual or shorter courses of
lectures have been given by E. W. Cannon, I. Stegun, R. J. Arms, J. Wegstein,
S. Gorn (Univ. of Pa.), A. Brauer (U. of N.C.), M. Hall (Ohio State), F. Alt,
A. Grad (N.S.F.), and J. Pasta (A.E.C.).
    Each participant has chosen a significant computation problem and
is in process of preparing a code for it. The participants, and their
institutions, are:
    R. V. Andree, The University of Oklahoma
    T. A. Botts, University of Virginia
    G. Cleaves Byers, Michigan College of Mining and Technology

R. T. Gregory, University of Texas
R. E. Lee, The University of Missouri
L. F. Meyers, The Ohio State University
E. P. Miles, Jr., The Florida State University
B. C. Moore, The Agriculture and Mechanical College of Texas
J. D. Munn, Mississippi Southern College

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-40-1110/47-2

Origin: NBS   Authorized 7/1/47
Manager: I. A. Stegun
Full task description: Apr-June 1949 issue, p. 45

    Status: INACTIVE.

TABLES OF POWER POINTS OF ANALYSIS-OF-VARIANCE TESTS
Task 1102-40-1110/51-8

Origin: Section 11.3, NBS   Authorized 3/26/51
Manager: S. Peavy
Full task description: Apr-June 1951 issue, p. 49

    Status: INACTIVE.

REVISION OF MATHEMATICAL TABLES
Task 1102-40-1110/52-7

Origin: NBS   Authorized 8/10/51
Manager: I. A. Stegun
Full task description: July-Sept 1951 issue, p. 41

    Status: INACTIVE.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-40-1110/52-37

Origin: NBS   Authorized 11/28/51
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

    Status: INACTIVE.

SIEVERT'S INTEGRAL
Task 1102-40-1110/52-57

Origin: NBS   Authorized 2/12/52
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46

       Status: INACTIVE. For status to date, see Jan-Mar 1958 issue, p.8.

AUTOMATIC CODING
Task 3711-60-1120/55-65

Origin: NBS   Authorized 9/29/54
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

       Status: CONTINUED. A program, BS LIST, was prepared by G. Galler
for distribution to the SHARE organization. This program causes the 704
computer to read absolute or relocatable binary program cards and print
them on-line or off-line in symbolic language.
       Experiments were begun on a general purpose table generating
program for inclusion in the Black Box Computer. A side product was
the preparation of tables from data obtained in the analysis of watch
jewel performance.

MATHEMATICAL SUBROUTINES
Task 3711-60-0009/56-160

Origin: NBS   Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

       Status: INACTIVE.

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-5113/57-216

Origin and Sponsor: National Science Foundation   Authorized 12/27/56
Manager: I. A. Stegun
Full task description: Oct-Dec 1956 issue, p. 10

       Status: CONTINUED. The textual material for Chapters 9,10,11
(Bessel Functions of Integral Order, Bessel Functions of Fractional Order,
Integrals of Bessel Functions), Chapter 25 (Numerical Interpolation,
Differentiation, and Integration), and Chapter 19 (Parabolic Cylinder
Functions) has been distributed for comments. The mansucripts of the tables
for Chapters 9,10 and 11 have been completed and are under way for Chapters
25 and 19. Chapter 15 (Hypergeometric Functions), Chapter 22(Orthogonal
Polynomials), Chapter 27(Miscellaneous Functions), and Chapter 28(Scales of
Notation) are being prepared for distribution.

3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-12-1107/51-2

Origin: NBS   Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

     Status: CONTINUED. N. C. Severo presented a paper entitled "Mathematical problems associated with measurements made by matching with known standards" at the Eastern Regional meetings of the Institute of Mathematical Statistics held in Pittsburgh March 19-21. The paper, which is a joint effort by W. S. Connor and N. C. Severo, concerned the evaluation of several integrals by means of appropriate use of the convolution formula.
     References and certain other material from the abstract cards for the Bibliography of Statistical Literature (see Jan-Mar 1957 issue, p.14, for description) have been entered on punched cards and listings were made by subject classification. Addition of titles and other pertinent information is in progress.

     Publications:
(1) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. Joel. To appear in the Annals of Mathematical Statistics.
(2) Measurements made by matching with known standards. W. J. Youden, W. S. Connor, and N. C. Severo. To appear in Technometrics.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-12-1107/53-1

Origin: NBS   Authorized 10/15/52
Manager: W. S. Connor
Full task description: Oct-Dec 1952 issue, p. 60

     Status: INACTIVE.

     Publication:
(1) Multi-variable experiments. M. Zelen and W. S. Connor. Industrial Quality Control 15, 14-17(1959).

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-1107/56-170

Origin: NBS   Authorized 12/15/55
Manager: M. Zelen, Joan R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

    Status: CONTINUED. J. R. Rosenblatt completed the manuscript of a
paper, "On the power of some rank order two-sample tests."

    Publication:
(1) On the power of some rank order two-sample tests. J. R. Rosenblatt.
     Submitted for publication.

MEASUREMENT OF RELIABILITY
Task 1103-12-1130/56-182

Origin: NBS   Authorized 3/23/56
Manager: M. Zelen
Full task description: Jan-Mar 1956 issue, p. 13

    Status: CONTINUED. J. R. Rosenblatt made a rough translation of
"Dependence of the effectiveness of redundancy on operating time of the
system", by G. V. Druzhinin (Izvestiya Akademii Nauk SSSR, Otdelenie
Tekhnicheskikh Nauk, No. 11, pp. 83-86, Nov. 1958). She continued com-
piling a bibliography of USSR publications on problems of reliability
theory.
    M. Zelen completed a manuscript entitled, "Factorial experi-
ments in life testing". This paper deals, in part, with an investigation
of the robustness of proposed procedures for analyzing life test data.

    Publications:
(1) On prediction of system behavior. J. R. Rosenblatt. To appear in
     Proceedings of the NYU-Industry Conference on Reliability Theory,
     held at Ardsley-on-Hudson, N.Y., June 1958.
(2) Analysis of two-factor classifications with respect to life tests.
     M. Zelen. Submitted for publication.
(3) Factorial experiments in life testing. M. Zelen. Submitted to a
     technical journal.

FRACTIONAL FACTORIALS FOR THE MIXED SERIES
Task 1103-12-5148/58-291

Origin and Sponsor: Bureau of Ships   Authorized 9/30/57
Managers: W. S. Connor, M. Zelen
Full task description: July-Sept 1957 issue, p. 43

Status: CONTINUED. Four activities are in progress:
(1) the electronic computer is being used to generate fractional repli-
cates of the 3^n which will be suitably adjoined to fractional replicates
of the 2^m to produce the mixed fractionals;
(2) these fractional replicates of the 2^m are being generated manually;
(3) certain small matrices needed for the analysis of the mixed fractionals
are being inverted manually; and
(4) a code is being written which will enable the electronic computer to
produce the normal equations for the designs, to be used to check equations
already derived.

Publications:
(1) Construction of fractional factorial designs of the mixed $2^m 3^n$ series.
     W. S. Connor. Submitted for publication.
(2) Analysis of fractionally replicated $2^m 3^n$ designs. R. C. Bose and
     W. S. Connor. To appear in the Proceedings of a Special Session of
     the International Statistical Institute, Brussels, 1958.

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-12-1115/55-57

Status: CONTINUED. C. M. Tchen has continued his work on the statistical approach to plasma dynamics. Emphasis is given to the relation between the microscopic and macroscopic description of a plasma. In connection with an NBS-wide proposed program for research in plasma dynamics, Dr. Tchen has prepared a survey and an outline of specific problems in plasma dynamics and magnetohydrodynamics as a plan of research for such a program. Proposed for investigation are some aspects of the general foundations of plasma dynamics, including the derivation of general kinetic equations for a plasma. Also to be investigated are transport properties and collective behavior of plasmas, such as conductivity and diffusion, radiation phenomena, various types of plasma waves, and other magnetohydrodynamical and plasma problems encountered in astrophysics, geophysics, and thermonuclear research.

The checking of page proofs of the paper "Diffusion of Particles in Turbulent Flow", by C. M. Tchen, to appear in the Proceedings of the Symposium on Atmospheric Diffusion has been completed.

The statistical analysis of the annual flows of rivers for Y. Yevdjevich's study of the fluctuation of river flows is essentially finished. The similar analysis for monthly flows is in progress. A paper entitled "Fluctuation of Wet and Dry Years of River Flow" is in preparation. A report entitled "The Error in the Computed Mean River Flow: Applied Statistics in Hydrology" is also being written. This study was considered necessary as a result of the analysis of the fluctuations referred to above. It is being prepared by Y. Yevdjevich in collaboration with Dr. Nicholas Matelas, Hydraulic Engineer, U. S. Geologic Survey, Water Resources Division. Some computations will be done on the Datatron in the Geologic Survey.

A. Ghaffari has continued his investigation of Rayleigh's equation (see Oct-Dec 1958 issue, p. 11). To describe the complete system of paths on the unit sphere it was first necessary to determine the nature of the critical points, and it was found that there are four, in two antipodal pairs, $(0,\pm 1,0)$, $(\pm 1,0,0)$ on the equator $z = 0$. The behavior of the paths on the unit sphere is governed by the homogeneous equation

$$z[\mu(-yz^2 + \tfrac{1}{3}y^3) + xz^2]dx + yz^3 dy$$

$$= [y^2 z^2 + x\{xz^2 + \mu(-yz^2 + \tfrac{1}{3}y^3)\}]dz.$$

It is found that the point (0,1,0) is the simplest type of node which is unstable, i.e., paths emanate from the node in every direction; similarly for its antipodal associate. In the phase plane the curves tending to infinity in the direction of the y-axis will have vertical asymptotes. It is shown similarly that the point (1,0,0) and its associate are critical points of higher order which behave like an ordinary saddle point: paths tending to the saddle point are tangent to one another.

    Publications:
(1) Rotational properties of two-dimensional lattices. J. P. Vinti.
     Submitted to a technical journal.
(2) Diffusion of particles in turbulent flow. C. M. Tchen. To appear
     in the Proceedings of a Symposium on Atmospheric Diffusion, Oxford,
     England, August 1958.
(3) Turbulent motion. C. M. Tchen and G. B. Schubauer (NBS Fluid
     Mechanics Section). To appear as Section B, Volume V of the
     Princeton Series, High Speed Aerodynamics and Jet Propulsion.
(4) Theory of the effect of drag on the orbital inclination of an earth
     satellite. J. P. Vinti. J. Research NBS, 62, 79-88(1959),RP2934.
(5) The graphical solution of initial value problems. W. H. Pell. Sub-
     mitted to a technical journal.

RESEARCH IN CONTINUUM MECHANICS
Task 1104-12-5160/55-85

Origin: NBS   Authorized 12/27/54
Sponsor: Office of Scientific Research, ARDC, USAF.
Manager: W. H. Pell
Full task description: Oct-Dec 1954 issue, p. 30

     Status: CONTINUED. C. M. Tchen has continued his study of statis-
tical plasma dynamics with the analysis centering on the kinetic equation.
In particular, the structure of the correlation function is under scrutiny.
Since the kinetic equation determines the distribution function of a
single particle, a similar approach, based on the hierarchy equations, can
lead to the determination of the correlation function for a plasma in
nonequilibrium.
     C. M. Tchen presented an invited lecture on "Recent Development of
Satistical Plasma Dynamics" at the Magnetohydrodynamics Seminar at the
Rensselaer Polytechnic Institute, Troy, N.Y.

        The paper on Stokes flow containing the general theory, the
solution for the lens-shaped body including the drag for the hemispherical
cup as well as a table of the drags for other configurations, has been
completed by W. H. Pell.

        Publications:
(1) Kinetic equation for a plasma with unsteady correlations. C. M. Tchen.
    To appear in Physical Review.
(2) The Stokes flow problem for a class of axially symmetric bodies.
    L. E. Payne and W. H. Pell. In manuscript.
(3) Compressible turbulent boundary layers with heat transfer and pressure
    gradient in flow direction. Alfred H. Walz. To appear in Journal of
    Research, NBS, Section B. Mathematics and Mathematical Physics.
(4) Stationary principles for forced vibrations in elasticity and electro-
    magnetism. J. L. Synge. To appear in the Proceedings of the Eighth
    Symposium in Applied Mathematics of the American Mathematical Society,
    held at Chicago, Ill., April 1956.
(5) Note on the integration of the elastic plate equation with variable
    flexural rigidity. W. H. Pell. In manuscript.
(6) Bending and stretching of corrugated diaphragms. R. F. Dressler.
    To appear in the Transactions of the American Society of Mechanical
    Engineers.

            FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
                        Task 1104-12-5160/56-154

Origin: NBS   Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: July-Sept 1955 issue, p. 20

        Status: INACTIVE.

                COMPUTATION OF OUTFLOW FROM A BREACHED DAM
                        Task 1104-12-5160/58-369

Origin and Sponsor: Army Map Service   Authorized 9/30/58
Manager: V. M. Yevdjevich
Full task description: July-Sept 1958 issue, p. 16

        Status: CONTINUED. The analysis of the outflow hydrograph from a
breached dam has been continued, and the report on the influence of the
negative wave and the effect of flow resistance on the free outflow
hydrograph of rapid openings in bodies of water is about 80% finished.

It has been decided to change the title of this paper from that reported
in the Oct-Dec 1958 issue to "The Effect of Sudden Water Release on the
Reservoir Outflow Hydrograph". This, together with the earlier paper
"The Analytical Integration of the Differential Equation for Water Storage"
will form a portion of an interim report to the sponsor which will cover
the year's activity on the project. This interim report has been started.

       Publication:
(1)   The analytical integration of the differential equation for water
       storage. V. Yevdjevich. To appear in the Journal of Research of the
       National Bureau of Standards, Section B. Mathematics and Mathematical
       Physics.

RESEARCH ON SATELLITE ORBITS
Task 1104-12-5160/59-420

Origin: NBS   Authorized 12/19/58
Sponsor: Office of Scientific Research, ARDC, USAF.
Manager: J. P. Vinti
Full task description: Oct-Dec 1958 issue, p. 15

       Status: CONTINUED. Since the previous report it has appeared
advisable to make a detour into the theory of the effect of oblateness
on satellite motion, without drag.
       By solving the Laplace equation in a system of orthogonal
curvilinear coordinates, J. P. Vinti has found an axially symmetric
solution for the potential that everywhere represents exactly the $1/r$
term and the second harmonic and takes into account about half of the
fourth harmonic, with correct sign. This potential makes the Hamilton-
Jacobi equation separable, so that it permits exact solution of the
latter. This method, when worked out, should thus make any perturbation
very small.
       The kinetic equations of motion have been set up formally in
terms of certain integrals which are now being investigated.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

1102-40-5126/54-13   AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: New York Quartermaster Procurement Agency
Manager: M. Paulsen
Full task description: Oct-Dec 1953 issue, p. 43
Status: Terminated. The sponsor has assumed the recoding and the
  continued running of the problem on his IBM 650.

3711-60-0009/54-30   SPECTRUM ANALYSIS
Origin: NBS, Division 4
Managers: C. D. Coleman, W. Bozman (4.1)
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. The number of known atomic energy levels of thorium I
  has been increased from 5 even and 23 odd levels to 20 even and 170 odd
  levels. The hafnium I analysis is nearly completed. After a search for
  new even levels has been made, the list will be prepared for publication.
       Work continued on the revision of Kayser's "Table of Wave
  Numbers." Four hundred pages of Volume II have been calculated and printed
  in a form suitable for publication.
       A paper covering work done on this project was presented to the
  Optical Society Meeting in New York on April 4.

3711-60-0009/54-38   EQUATION OF STATE OF REAL GASES
                     (formerly COMPRESSIBILITY FACTORS OF DRY AIR)
Origin: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Continued. A routine was written and checked out using an Nth
  order Floating Point Interpolation Subroutine (Share-Distribution No.265).
  This routine enables one to subtabulate an existing table(reads in
  regular DEC cards as the existing table), rearrange and/or omit any
  number of columns of the existing table (and at same time subtabulate if
  desired). The resulting table can be printed on-line and/or punched on
  cards with the heading and format chosen by the user. Writing the
  resulting table on tape is possible, but the code has not been checked
  out.
       Another routine is being written, using Fortran, which will
  evaluate
$$f_i(T) = AT^{-2} + BT^{-1} + C + DT^1 + ET^2 + FT^3 + GT^{-1}L\emptyset G_{10}T + HTL\emptyset G_{10}T + IT^2 L\emptyset G_{10}T,$$
  and print a table with the T argument and the corresponding values of $f_i(T)$,
  where i can vary from 1 to 5.

3711-60-0009/55-68   CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Division 9
Managers: P. O'Hara, S. Block (9.7)
Full task description: Jan-Mar 1955 issue, p. 18
Status: Continued. A final structure was determined for the monoclinic
   form of the $Na_4P_4O_{12} \cdot 4H_2O$ crystal. Structure factors computed using the
   present atomic coordinates agree very well with observed structure
   factors and the structure is chemically reasonable.
        The new version of the Fourier synthesis routine has been
   completed, and several three-dimensional maps have been computed.
        A new and more powerful least square refinement program has been
   used successfully in the past quarter. This program was written by Dr.
   William Busing of the Oak Ridge National Laboratory, and although it
   requires more time per refinement cycle the total number of cycles is
   considerably reduced.

3711-60-0009/55-82   THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. ITS constants and tables were computed for approxi-
   mately 23 thermometers under test. The code for computing low temperature
   constants and tables is being tested.

1102-40-5126/55-121   ELECTRON PENETRATION
Origin and Sponsor: NBS, Section 4.8
Manager: S. Peavy
Full task description: Apr-June 1955 issue, p. 19
Status: Continued. The sponsor is using the code to make runs as necessary.

1102-40-5126/56-162   STRESSES IN A WALL RESTING ON A FOOTING
Origin and Sponsor: NBS, Section 10.1
Manager: I. Stegun
Full task description: Jan-Mar 1956 issue, p. 26
Status: Completed. Solutions have been obtained for a 100x100 system of
   equations.

1102-40-5126/56-166   SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Managers: E. Haynsworth, P. Walsh
Full task description: Jan-Mar 1956 issue, p. 27
Status: Continued. The codes are now being used by the sponsor for
   production runs.

1102-40-5126/56-186   MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: S. Prusch
Full task description: July-Sept 1956 issue, p. 33
Status: Continued.   Calculations were performed on seven laboratory sets
  of gage blocks.

1102-40-5126/57-219   THERMAL PROPERTIES
Origin and Sponsor: NBS, Section 3.2
Manager: R. Varner
Full task description: Oct-Dec 1956 issue, p. 30
Status: Reactivated.   Additional production runs have been made at the
  request of the sponsor.

1102-40-5126/57-221   BESSEL FUNCTIONS FOR COMPLEX ARGUMENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of
                        the Army
Manager: R. Zucker
Full task description: Oct-Dec 1956 issue, p. 31
Status: Continued.   Sixteen cases were run to evaluate the Bessel and
  Hankel functions for complex arguments up to order n specified.

1102-40-5126/57-222   ROOTS OF POLYNOMIALS
Origin and Sponsor: Naval Research Laboratory
Manager: J. P. Menard
Full task description: Oct-Dec 1956 issue, p. 32
Status: Completed.   Results have been transmitted to the sponsor.

3711-60-0009/57-223   SELF-CONSISTENT FIELDS
Origin: NBS, Section 3.2
Manager: E. V. Haynsworth
Full task description: Apr-June 1957 issue, p. 28
Status: Continued.   Some minor modifications were made in the A-matrix
  and SCF programs.   Several production runs have been made.

3711-60-0009/57-229   APPLICATION OF ELECTRONIC DATA PROCESSING MACHINERY
                        TO PAYROLL OPERATIONS
Origin: NBS, Section 40.0
Managers: M. Paulsen, P. Ruttenberg
Full task description: Jan-Mar 1957 issue, p. 36
Status: Inactive.

1102-40-5126/57-236   SELF CONSISTENT FIELDS--EIGENVALUES
Origin and Sponsor: NBS, Section 3.6
Manager: E. Haynsworth
Full task description: Apr-June 1957 issue, p. 30
Status: Continued. A few cases have been prepared and run. In one case
   the process did not converge to a solution, and further investigation
   may be necessary to determine why oscillation occurred.

3711-60-0009/57-247   MECHANICAL IMPEDANCE
Origin: NBS, Section 6.1
Managers: J. P. Menard, M. D. Burkhard (6.1)
Full task description: Apr-June 1957 issue, p. 32
Status: Terminated. Several production runs have been made directly by
   the sponsor. The program will continue in production under the sponsor's
   direction. (In future reports production time will be reported in the
   section, "Application of Automatic Computer.")

3711-60-0009/57-248   THE EVALUATION OF A TRIPLE INTEGRAL FOR THE
                      SOLUTION OF NEGATIVE ION DETACHMENT
Origin: NBS, Section 4.6
Manager: S. Peavy
Full task description: Apr-June 1957 issue, p. 34
Status: Inactive.

3711-60-0009/57-250   AUTOMATIC REDUCTION OF SPECTROPHOTOMETRIC DATA
Origin: NBS, Section 2.1
Manager: W. C. Rheinboldt
Full task description: July-Sept 1957 issue, p. 31
Status: Terminated. Several production runs have been made directly by the
   sponsor. The program will continue in production under the sponsor's
   direction.   In future reports production time will be reported in the
   section, "Application of Automatic Computer."

1102-40-5126/57-251   CURRENT NOISE AND FIXED RESISTORS
Origin and Sponsor: NBS, Section 1.6
Manager: D. Sumida
Full task description: July-Sept 1957 issue, p. 32
Status: Completed.

3711-60-0009/58-254   REPRODUCTION OF COLOR- AND SPECTRAL-ENERGY
                      DISTRIBUTION OF DAYLIGHT AND OTHER ILLUMINANTS
Origin: NBS, Section 2.3
Manager: W. C. Rheinboldt
Full task description: July-Sept 1957 issue, p. 32
Status: Terminated. Several production runs have been made directly by
   the sponsor. The program will continue under the sponsor's direction.
    In future reports production time will be reported in the section,
   "Application of Automatic Computer."

1102-40-5126/58-263   GAS TUBE CHARACTERISTIC
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department
                              of the Army
Manager: I. A. Stegun
Full task description: July-Sept 1957 issue, p. 35
Status: Continued. Four runs were made, and the results were transmitted
   to the sponsor.

1102-40-5126/58-264   THEORY OF IONIZATION PROBABILITY
Origin and Sponsor: NBS, Section 4.6
Manager: S. Peavy
Full task description: Oct-Dec 1957 issue, p. 30
Status: Completed. Requested production runs were made. Results were
   transmitted to the sponsor.

3711-60-0009/58-266   DEPOLYMERIZATION, II
Origin: NBS, Section 7.6
Manager: L. S. Joel
Full task description: July-Sept 1957 issue, p. 36
Status: Continued. A new code for terminal initiation has been written
   using Fortran. It is being checked on a system of 1,000 equations.

3711-60-0009/58-267   CONVERSION OF THE CIE-CHROMATICITY COORDINATES INTO
                         THE MUNSELL COLOR SYSTEM
Sponsor: NBS, Section 2.1
Manager: W. C. Rheinboldt
Full task description: July-Sept 1957 issue, p. 37
Status: Terminated. Several production runs have been made directly by
   the sponsor. The program will continue in production under the sponsor's
   direction. (Production time will be reported hereafter in the section,
   "Application of Automatic Computer.")

1102-40-5126/58-269   MOLECULAR STRUCTURE, IV
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. J. O'Hara
Full task description: July-Sept 1957 issue, p.38
Status: Continued. In the past quarter a final structure was determined
   for the spurite crystal. Least square refinement of position and
   temperature parameters was made by means of the NYXR1 program.
        The structure determination of this crystal was complicated by a
new effect in that there exists a rational dependence of atoms. This
necessitates a minor modification in the phase determination procedures
and creates a need for a program to detect rational dependence.
        Phase determination calculations were started for the arginine
crystal, which also exhibits rational dependence of atoms. A means of
correcting for this effect is under consideration.

1102-40-5126/58-270   MATHEMATICAL PROBLEMS RELATED TO POSTAL OPERATIONS
Origin: NBS
Sponsor: Post Office Department, Office of Research and Engineering
Managers: B. K. Bender, A. J. Goldman
Full task description: Oct-Dec 1958 issue, p. 22
Status: Continued. The report on the "Analytic Comparison of Suggested
  Configurations for Automatic Mail Sorting Equipment" has been completed.
  A report is being prepared on the extent to which incoming mail should
  be mechanically sorted for postal carriers.
Publication: Capacity requirement for a mail sorting device. B. K. Bender
  and A. J. Goldman. To appear in the Journal of Research of the NBS,
  Section B. Mathematics and Mathematical Physics.

1102-40-5126/58-272   THERMODYNAMIC PROPERTIES OF REAL GASES
Origin and Sponsor: NBS, Section 3.2
Manager: J. P.Menard
Full task description: Oct-Dec 1957 issue, p. 32
Status: Continued. An editing code is being written for printing out
  previously computed data already stored on magnetic tape in a suitable
  form for publication.

1102-40-5126/58-274   CALCULATIONS FOR d-SPACINGS, II
Origin and Sponsor: NBS, Division 9
Manager: R. Zucker
Full task description: July-Sept 1957 issue, p. 38
Status: Continued. About 100 calculations for d-spacings for orthogonal,
  hexagonal and orthorhombic crystals were carried out this quarter, and
  redetermination of unit cell constants by least squares fitting to a
  measured d-spacing was   performed for about 20 crystals.

3711-60-0009/58-294   NUCLEAR SCATTERING OF PHOTONS
Origin: NBS, Section 4.8
Manager: J. P. Menard
Full task description: Oct-Dec 1957 issue, p. 36
Status: Continued. A misprint has been found in the equation for the
  dispersion integral as defined in the Oct-Dec 1958 status report (p.23).
  The equation should read

$$I(E) = P \int_{o}^{\infty} \frac{G_o(E')}{E'- E^2} \, dE',$$

where P denotes that the principal value of the integral is to be taken.
Production runs continued under the sponsor's direction.

1102-40-5126/58-298   ANALYSIS OF SPECTROCHEMICAL DATA
Origin and Sponsor: NBS, Section 5.10
Managers: S. Peavy, R. N. Varner
Full task description: Oct-Dec 1957 issue, p. 39
Status: Continued. The Fortran code for the third phase of the problem
  has been checked out. Several runs using the code for the second phase
  of the problem have been made by the sponsor.

1102-40-5126/58-299   TIME-DEPENDENT SCHRÖDINGER EQUATION
Origin and Sponsor: NBS, Section 3.1
Manager: A. Schopf
Full task description: Oct-Dec 1957 issue, p. 39
Status: Continued. Five more production runs have been made for various
  domains, time steps, and parameters σ . The results agree well with
  previous results and have been transmitted to the sponsor.

1102-40-5126/58-300   LAMINAR MIXING IN BOUNDARY LAYERS
Origin: Polytechnic Institute of Brooklyn
Sponsor: Air Force Office of Scientific Research
Manager: W. C. Rheinboldt
Full task description: Oct-Dec 1957 issue, p. 40
Status: Completed.

1102-40-5126/58-304   TRANSPORT PROPERTIES OF AIR AT ELEVATED TEMPERATURES
Origin and Sponsor: NBS, Section 3.2
Managers: P. J. Walsh, J. D. Waggoner
Full task description: Oct-Dec 1957 issue, p. 40
Status: Continued. Revisions have been made in the code for calculating
  the transport properties. The input data is now available from tape, and
  the results of the calculations are also processed onto tape. Codes
  have been written to rearrange and print the final results in table
  form.
        The codes for computing the transport properties K, η and C_{ij}
  will be joined together by a supervisory routine which will determine
  the calculations of these transport properties for all pressures at a
  specified temperature. The amount of read-in time and data handling
  will thus be considerably reduced.

1102-40-5126/58-306   INTERPOLATION OF COLOR MIXTURE FUNCTIONS
Origin and Sponsor: NBS, Section 2.1
Manager: W. C. Rheinboldt
Full task description: Oct-Dec 1957 issue, p. 42
Status: Terminated. A number of production runs have been made directly
  by the sponsor. The program will continue in production under his
  direction. (Production time will be reported hereafter in the section,
  "Application of Automatic Computer.")

1102-40-5126/58-307   STUDY OF SURFACE TENSION
Origin and Sponsor: NBS, Section 9.2
Manager: R. Arms
Full task description: Oct-Dec 1957 issue, p. 43
Status: Inactive.

1102-40-5126/58-308   OSCILLATING SPHERE
Origin and Sponsor: NBS, Section 3.4
Manager: S. Prusch
Full task description: Oct-Dec 1957 issue, p. 43
Status: Inactive.

1102-40-5126/58-312   RESPONSE FUNCTION, II
Origin and Sponsor: NBS, Section 4.11
Manager: A. Beam
Full task description: Jan-Mar 1958 issue, p. 33
Status: Continued.   The sponsor has made several runs on the 704.

1102-40-5126/58-316   INTERSECTION CAPACITY STUDY
Origin and Sponsor: Bureau of Public Roads
Managers: S. Peavy, J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 33
Status: Continued. A code has been written for the analysis of the
   intersection capacity study.   Checking of the code is under way.

1102-40-5126/58-322   PROPAGATION CONSTANT OF A SOUND WAVE
Origin and Sponsor: NBS, Section 6.1
Manager: R. J. Arms
Full task description: Jan-Mar 1958 issue, p. 34
Status: Completed.

1102-40-5126/58-333   CALCIUM HYDROXIDE
Origin and Sponsor: NBS, Section 9.0
Manager: P. O'Hara
Full task description: Jan-Mar 1958 issue, p. 36
Status: Inactive.

1102-40-5126/58-337   GEORGETOWN LANGUAGE TRANSLATION EXPERIMENTS
Origin and Sponsor: Georgetown University
Manager: R. J. Arms
Full task description: Jan-Mar 1958 issue, p. 37
Status: Inactive.

1102-40-5126/58-339   COMPUTATION OF VISCOELASTICITY PROPERTIES OF
                                   MATERIALS
Origin and Sponsor: NBS, Section 3.4
Manager: H. Oser
Full task description: Jan-Mar 1958 issue, p. 38
Status: Continued. Production runs have been made and the results were
  turned over to the sponsor. A report on these computations is in
  preparation and will be submitted to the Office of Naval Research,
  Washington, D.C.

1102-40-5126/58-343   MINIMIZATION PROBLEM
Origin and Sponsor: Naval Research Laboratory
Manager: S. Peavy
Full task description: Jan-Mar 1958 issue, p. 40
Status: Inactive. For status to date, see Oct-Dec 1958 issue, p. 26.

1102-40-5126/59-348   RUSSIAN-TO-ENGLISH MACHINE TRANSLATION
Origin: NBS
Sponsor: Office of Ordnance Research, U. S. Army
Manager: I. Rhodes
Full task description: Oct-Dec 1958 issue, p. 26
Status: Continued. As reported previously, programming and coding for the
  glossary lookup of individual source words in a sentence has been
  completed and is in use. Coding has been started for the organization
  of the separate words into a meaningful sentence that correctly renders
  the intention of the original.

1102-40-5126/58-359   VELOCITY DISTRIBUTION IN BOUNDARY LAYERS
Origin and Sponsor: NBS, Section 11.4
Manager: W. C. Rheinboldt
Full task description: Apr-June 1958 issue, p. 32
Status: Inactive.

3711-60-0009/58-360 DIFFUSION COEFFICIENTS
Origin: NBS, Section 5.2
Manager: W. C. Rheinboldt
Full task description: Apr-June 1958 issue, p.32
Status: Continued. Several production runs have been made directly by the
  sponsor. The program will continue in production under the direct
  supervision of the sponsor.

1102-40-5126/58-361   CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION
Origin and Sponsor: Naval Research Laboratory
Manager: R. J. Arms
Full task description: Apr-June 1958 issue, p. 33
Status: Continued. By agreement with the sponsor, the random vector
   integration has been discontinued. In its place, a new and comparatively
   simple program has been started. Several production runs on the dipole
   radiation code have been turned over to the sponsor.

1102-40-5126/58-366   RADIATION PATTERNS OF ANTENNAS
Origin and Sponsor: U. S. Information Agency, Department of State
Managers: R. T. Moore, P. J. Walsh
Full task description: Apr-June 1958 issue, p. 35
Status: Continued. Part (1) (see full write-up): The codes for all four
   tables have been written and checked out. Tables (a), (b) and (c) have
   already been produced, and copies have been transmitted to sponsor.
         Part (2). The code has been checked out and results for
   approximately 80 antennae have been submitted to sponsor. In some cases
   both primary and secondary lobe information was obtained.

1102-40-5126/58-368   INTENSITY FUNCTIONS AND CROSS SECTIONS OF LIGHT
                      SCATTERED BY SPHERICAL PARTICLES
Origin and Sponsor: U. S. Army Signal Research and Development Laboratories,
                      Atmospheric Physics Branch, Belmar, N. J.
Manager: H. Oser
Full task description: July-Sept 1958 issue, p. 32
Status: Continued. Production continued with the computation of tables
   of moduli and intensities. Results have been submitted to the sponsor.

1102-40-5126/59-374   END-EFFECT IN THE CYLINDRICAL ROTATIONAL VISCOMETER
Origin and Sponsor: NBS, Section 7.1
Managers: A. Schopf, J. P. Menard
Full task description: July-Sept 1958 issue, p. 34
Status: Inactive.

1102-40-5126/59-381   BOOLEAN FUNCTIONS AND PICTORIAL DATA PROCESSING
Origin and Sponsor: NBS, Section 12.5
Manager: B. Bender
Full task description: July-Sept 1958 issue, p.35
Status: Completed. Two further programs were written for SEAC to aid in
   obtaining one simplest normal equivalent of any truth function of up to
   14 variables. A paper describing the process is being prepared.

1102-40-5126/59-388   HEAT PUMP CALCULATIONS
Origin and Sponsor: NBS, Section 10.3
Manager: R. Zucker
Objective: To perform general computations on the IBM 704 that become
  necessary in connection with the investigation of the heat pump used for
  heating and cooling of homes and in a research program directed towards
  design data on heat pumps.
Background: During the early testing of samples, calculations using desk
  computers became too laborious. By the use of psychrometric tables and
  the adapting of known equations, it is possible to carry out the
  computations on the IBM 704 with a considerable saving of time.
          The problem was proposed by J. C. Davis(10.3).
Status: New. The code has been prepared and checked, and runs were made
  for many sets of data. The results of early test runs were checked
  against corresponding results obtained by desk calculation methods and
  close agreement was found.

1102-40-5126/59-389   FREQUENCY ALLOCATION
Origin and Sponsor: Civil Aeronautics Administration
Manager: L. S. Joel
Full task description: Oct-Dec 1958 issue, p. 29
Status: Continued. A code for chains of substitutions of length of arc,
  and a set of data file updating and editing codes, have been written
  and checked. Investigations continue for the existence of a complete
  solution in closed form.

3711-60-0009/59-393   HEAT TRANSFER IN THE PRESENCE OF MOISTURE
Origin: NBS, Section 10.6
Manager: F. L. Alt
Full task description: July-Sept 1958 issue, p. 38
Status: Completed.

1102-40-5126/59-394   VARIATIONAL CALCULATION OF SLOW ELECTRON SCATTERING
                      BY HYDROGEN ATOMS
Origin and Sponsor: NBS, Section 4.6
Manager: A. Beam
Full task description: Oct-Dec 1958 issue, p. 30
Status: Continued. The p-wave computation was found to be unsatisfactory
  at very low energies due to defects in the boundary condition of the
  trial function. A new condition has been introduced and the changes are
  being made in the Fortran code for the p-wave.

1102-40-5126/59-403   COMPUTATION OF COLOR FADINGS
Origin and Sponsor: NBS, Section 2.1
Managers: W. C. Rheinboldt, J. P. Menard
Full task description: Oct-Dec 1958 issue, p. 30
Status: Continued. The revised code for the conversion of CIE-chromaticity
   coordinates into terms of the Munsell renotation system has been written
   and completely checked out. The program performs this conversion at
   the speed of approximately 3.5 seconds per sample as compared to 24 seconds
   per sample for the previous code.
        A code has been written and completely checked out for computing
   color differences with Godlove's formula,

$$\Delta I = 5 \left\{ 2c_1 \, c_2 \left[\, 1-\cos\left(2\pi \, \frac{H_1-H_2}{100}\right) \right] + (c_1-c_2)^2 + 16(v_1-v_2)^2 \right\}$$

[in N.B.S. units] where H_1, v_1, c_1 and H_2, v_2, c_2 are the Munsell
renotations of the given samples.
        Both codes have been written such that their inputs and outputs
are compatible.
        NBS Section 2.1 has used the first code for evaluating the Munsell
renotations for approximately 1700 samples and the second code for obtain-
ing color differences on approximately 600 pairs of samples. These
samples were obtained as part of a test-program for a commercial company.
The test-program will be continued under the direct supervision of NBS
Section 2.1.

1102-40-5126/59-412   DYNAMICS OF PNEUMATIC PRESSURE REDUCERS
Origin and Sponsor: NBS, Section 3.2
Manager: W. Gautschi
Full task description: Oct-Dec 1958 issue, p. 31
Status: Continued. A particular system has been solved and the results
   transmitted to the sponsor. A supplementary code has been written to
   determine approximations to the stability characteristics of the problem.
   Production runs are under way.

1102-40-5126/59-407   FOURIER COEFFICIENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of
                         the Army
Manager: R. Zucker
Objective: To calculate (a) Fourier coefficients A_n and B_n for n equally
   spaced abscissas, (b) the amplitude of the frequency component, and the
   frequency, and (c) the integral of the series.
Background: The data is the result of measurements of an electromagnetic
   field using a loop antenna, the output of which is the derivative of the
   field. In the telemetering circuitry in some cases electrical integraters
   were used, and in other cases the output of the antenna was recorded
   directly. It is desired to obtain the actual frequency versus amplitude
   spectrum of the electromagnetic field.
        The problem was proposed by R. R. Puttcamp (DOFL).

<u>Status</u>:   New.   The code has been prepared and checked, and production runs
were started.

1102-40-5126/59-414   INFINITE SYSTEMS
<u>Origin and Sponsor</u>:   NBS, Division 3
<u>Manager</u>:   R. Zucker
<u>Objective</u>:   To solve the set of simultaneous equations:

$$\sum_n H_{f,n}[\gamma\sqrt{\epsilon+m-n}] \, e^{\frac{\gamma^2}{4}(n-m)} \, B_n = -H^*_{f,m}[\gamma\sqrt{\epsilon}\,]$$

where $H^*_{f,m}[\gamma\sqrt{\epsilon}]$ is the complex conjugate of $H_{f,m}[\gamma\sqrt{\epsilon}\,]$
for the unknowns B_n, and

$$H_{f,n}(\alpha) = \frac{1}{\sqrt{f!\ n!}} \sum_{r=0}^{\min[f,n]} \binom{f}{r}\binom{n}{r} r! \left(\frac{\alpha i}{\sqrt{2}}\right)^{f+n-2r}$$

Sets of values of B_n are desired as a function of ϵ where ϵ is given and
lies in the region $0 < \epsilon < 10$, for $\gamma = 1/4$, 1, 4, and m = 0,1,2,3,4,5,
for select cases. Also, to evaluate for each solution the normalization
condition:

$$\sum_{n=0}^{[n]\,\le\,\epsilon+m} \left[\frac{\epsilon-n+m}{\epsilon}\right]^{\frac{1}{2}} \left| B_n \right|^2 = 1.$$

<u>Background</u>:   These equations arise in the quantum-mechanical calculation
of the probabilities of transfer of translational to vibrational energy
in collisions between atoms and diatomic molecules.   The particular form
of the equations is peculiar to a hard sphere interaction potential
between the atom and the diatomic molecule for the case in which the atom
approaches the molecule along the axis of the molecule.   The equations
appear in the literature: Castellan and Hulburt, J. Chem. Phys. <u>18</u>,
312(1950).

The problem was requested by R. Rubin and K. E. Shuler (Div. 3).
<u>Status</u>:   New.   The code was checked, and calculations were carried out
for the following sets of parameters: $\gamma = 1$, m = 0,1,2, for various
values of ϵ in the range $0 < \epsilon \le 5$.

1102-40-5126/59-415   COMPLEX LEGENDRE FUNCTIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of
                                  the Army
Manager: R. Zucker
Objective: To evaluate a double summation of the form

$$\sum_{n=0}^{\infty} \sum_{m=-n}^{n} F(z)$$

where F(z) involves expressions of the following types:

$$A_{mnp} = \int_{0}^{1} A \cdot B \, dt \quad \text{and} \quad E_{mp} = C \cdot D.$$

Here A, B, C, and D may be P_p^m or $P_p^{m'}$ (associated Legendre functions)
for complex z or its conjugate.
Background: The computations arise in studies of the absorption spectra
   of ferides.   The problem was proposed by O. R. Cruzan (DOFL).
Status: New.   The code for evaluating the Legendre function and its
   derivative and the associated integrals was completed and checked out.
   Certain expected maxima of E_{mp} have been obtained.

1102-40-5126/59-418   P-WAVE EQUATION
Origin and Sponsor: NBS, Section 4.8
Managers: S. Peavy, R. Varner
Objective: To find the solution of the following second order differential
   equations and evaluate   the integrals:
   1. Unperturbed Equation

$$\left(-\frac{d^2}{dr^2} - \frac{2}{r^2} + K^2 \right) u_p(r) = 0; \quad \int_{r}^{\infty} e^{-r_1} \frac{u_p(r_1)}{r} \, dr_1$$

   2. Non-exchange Equation

$$\left(\frac{d^2}{dr^2} - \frac{2}{r^2} + 2e^{-2r} (1 + \frac{1}{r}) + K^2 \right) u_p(r) = 0;$$

$$\int_{r}^{\infty} \frac{e^{-r_1} u_p(r_1)}{r_1} \, dr_1$$

3. Exchange Equation

$$\left[-\frac{d^2}{dr^2} + \frac{2}{r^2} - K^2 - 2e^{-2r} \left\{ 1 + \frac{1}{r} \right\} \right] u_p(r)$$

$$\underset{=}{\pm} \left[\frac{8e^{-5}}{3} \left\{ \frac{1}{r} \int_o^r r_1^2 e^{-r_1} u_p(r_1)\, dr_1 + r^2 \int_o^\infty \frac{e^{-r_1} u_p(r_1)}{r_1}\, dr_1 \right. \right.$$

$$\left. \left. -r^2 \int_o^r \frac{e^{-r_1} u_p(r_1)}{r_1}\, dr_1 \right\} \right] = 0$$

The boundary conditions are:

$$u_p(0) = 0$$

$$u_p'(0) = 0$$

$$u_p''(0) = 2K^2/3$$

$$u_p(r+\Delta r) = 2u_p(r) - u_p(r-\Delta r) + (\Delta r)^2 u_p''(r)$$

$$u_p(\Delta r) = (\Delta r)^2 K^2/3$$

$$u_p'(r) = \frac{1}{\Delta r} \left[u_p(r+\Delta r) - u_p(r) - \frac{u_p''(r)(\Delta r)^2}{2} \right]$$

While solving the second order differential equation δ will approach a limit where

$$\delta = \xi_e + \varphi_e$$

$$\xi_e = f(K,r)$$

$$\varphi_e = f(K, r, u_p(r), u_p'(r))$$

Background: The problem is concerned with the scattering of electrons from hydrogen atoms and the computation of a phase shift which is related to the solution of a differential equation. Various differential equations are given corresponding to degrees of approximation in the description of the above process.

Status: New. Fortran programs have been written for the three equations. Results for the non-exchange and unperturbed equations have been submitted to the sponsor.

1102-40-5126/59-427   MAGNETIC SCATTERING OF NEUTRONS BY PARAMAGNETIC MnF_2

Origin and Sponsor: Naval Research Laboratory

Manager: W. Hall

Objective: To evaluate the differential cross section and the second moment of neutron energy transfer for neutrons in the 1A range incident on MnF_2 at temperatures T such that $T \gg \theta$, θ being the Curie-Weiss constant.   Theoretical studies of the energy spectrum of neutrons scattered magnetically by exchange coupled lattices in the paramagnetic domain are to be employed to interpret the corresponding experimental results in terms of the coupling constants of these compounds.

Background: The following exchange-coupling model for MnF_2 was chosen for these computations: the 8 Mn^{+2} ions at the nearest-neighbor sites are coupled antiferromagnetically with the central Mn^{+2} ion which is also coupled with the 2 nearest neighbors along the c-axis.   The latter coupling can be either ferromagnetic or antiferromagnetic.   The results will serve to illustrate a general theory of the magnetic scattering of slow neutrons by paramagnetic substances and to evaluate the results of current experiments on MnF_2 at the NRL reactor.

The problem was submitted by A. W. Saenz.

Status: New.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS
RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS   Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

     Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

     (1) <u>Transistor Measurement Experiment</u>: A design of a large scale transistor experiment involving automatic data recording was worked out. Specifications for data processing on high speed computers were suggested. Work on this project is being done for G. Conrad, Section 1.6.

     (2) <u>Color Matching</u>: Methods were developed for the analysis, using multivariate techniques, of individual observer data from a large scale color matching investigation. This work was carried on in collaboration with I. Nimeroff, Section 2.01.

     (3) <u>Machine (704) Analyses</u> of data and/or the preparation of special codes were under way for:

               H. B. Kirkpatrick, 9.4
               M. J. Kerper, 9.2
               J. Mandel, 7.5
               F. M. Reinhart, 8.4
               H. C. Allen, 4.2
               J. B. Wachtman, 9.1

     (4) An in-hours course, "Design of Multi-factor Experiments" is being presented by W. S. Connor and M. Zelen.

     (5) <u>Use of Sampling Methods in Studies of Post Office Operation</u>: N. Severo continued his collaboration with B. M. Levin and A. Newman of Section 12.5 on the use of sampling methods for estimation of parameters of distributions arising in their study of Post Office operations. Dr. Severo presented a paper "A statistician and the Post Office: a case history in operations research" before the American Society for Quality Control on March 5, giving a summary of the use of the chain-ratio method in studies of mail distribution.

Publication:
(1) A Satistician and the Post Office: A Case History in Operations Re-
     search. N. C. Severo. Transactions of the 1959 Conference of the
     Administrative Applications Division, A.S.Q.C., pp. 97-101.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC   Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

     Status: CONTINUED. W. J. Youden participated in a meeting of the
NRC-NSF Ship Structure Committee on planning the balance of the program
for exploring uniformity of steel plates from current production. Statis-
tical studies on the relation between nil-ductility transition temperature
and several physical and chemical properties of two types of ship steel
were completed.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-5146/55-93

Origin and Sponsor: Office of Ordnance Research   Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

     Status: CONTINUED. Part IV (Miscellaneous Topics) was circulated
for comment. Three major sections of Part II (Qualitative Data) were in-
cluded in another report for limited distribution for comment. A draft of
a section on Multiple Regression and a draft of a section on Sensitivity
Testing were completed. Work proceeded on preparing examples for all
sections.

     Publications:
(1) A note on the computation of χ^2. Mary G. Natrella. Amer. Stat. 13,
     20-21 (1959).
(2) The relation between confidence intervals and tests of significance--
     a teaching aid. Mary G. Natrella. Submitted to a technical journal.

STATISTICAL SERVICES
Task 1103-40-5150/58-346

Origin and Sponsors: Various Agencies   Authorized 3/31/58
Manager: J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 45

      Status: CONTINUED. Work was done during the quarter for the
following agencies:
      (1) Veterans Administration Hospital, Perry Point, Maryland:
The analysis of a set of data on mental patients was completed, and the
results were transmitted to the sponsor.
      (2) Bureau of Public Roads: See task 1102-40-5126/58-316, p. 23,
for report on this task done jointly with the Computation Laboratory.
      (3) Chemical Corps: Analysis of a problem arising in the work of
the Chemical Corps was continued in collaboration with A. J. Goldman of
the Numerical Analysis Section (11.1).

APPLICATION OF AUTOMATIC COMPUTER

The record of the use of the IBM 704 for the period January 1 through March 31 is as follows:

Task No.	Title	Assembly	Code Checking	Production
			(M I N U T E S)	
NBS:				
5114/57-237	Numerical analysis training program	88	334	191
0002/51-1	Statistical engineering	81	126	619
5160/58-369	Computation of outflow from a breached dam			60
1120/55-65	Automatic coding	49	148	9
0009/56-160	Mathematical subroutines	32	55	107
0009/54-30	Spectrum analysis	38	105	980
0009/54-38	Equation of state of real gases	9	35	102
0009/55-68	Crystal structure calculations	103	49	839
0009/55-82	Thermometer calibrations		30	163
5126/55-97	High temperature properties for air°	19	217	102
5126/55-121	Electron penetration*		10	29
0009/56-131	Calculations in optics*	3	2	294
5126/56-150	Mathematical expressions*			40
5126/56-166	SCF-LCAO solution of some hydrides*		12	81
5126/56-171	Collision integrals used in transport theory**	34	224	25
5126/57-219	Thermal properties*		6	42
0009/57-223	Self-consistent fields	41	66	62
5126/57-233	Loran computations°	12		22
5126/57-236	Self-consistent field--eigenvalues			101
5126/57-246	Radiation diffusion**	34	60	1563
0009/57-247	Mechanical impedance*			134
5126/57-249	Color differences*			155
0009/57-250	Automatic reduction in spectrophotometric data*			110
5126/57-252	Detecting efficiency in a neutral meson experiment**	183	22	262
0009/58-254	Reproduction of color-and spectral-energy distribution of daylight*		13	63
5126/58-255	Chi functions**		156	807
5126/58-256	Composite walls**	87	9	85
5126/58-260	Prototype accounting**	95	176	258
5126/58-264	Theory of ionization probability	32	39	51

Task No.	Title	Assembly	Code Checking	Production
		(M I N U T E S)		
0009/58-267	Munsell color system conversion*			34
5126/58-272	Thermodynamic properties of real gases**		2	11
5126/58-274	Calculations for d-spacing, II*			151
5126/58-294	Nuclear scattering of photons*	41	23	8
5126/58-299	Time-dependent Shroedinger equation	20	86	810
5126/58-304	Transport properties of air	36	219	27
5126/58-306	Interpolation of color mixture functions*		3	34
5126/58-308	Oscillating sphere			2
5126/58-312	Response function, II*			28
5126/58-314	Approximations for gas mixtures*		60	201
5126/58-339	Viscoelasticity properties of materials		135	276
5126/58-357	Eigenvalues***			48
5126/58-360	Diffusion coefficients*			93
5126/59-377	Logical diagram reduction**	637	873	990
5126/59-382	Phase shift**		8	136
5126/59-383	Design flow capacity	7	8	33
5126/59-387	Nuclear reactor design***		8	85
5126/59-388	Air conditioning			10
5126/59-390	Electrocardiogram**	19	47	12
5126/59-394	Slow electron scattering by hydrogen atoms	38	65	327
5126/59-395	Adsorption study**	22	5	4
5126/59-401	Rates**	22	23	22
5126/59-403	Computation of color fadings	32	95	310
5126/59-404	Counter***		14	89
5126/59-409	Bank Board**	84	60	193
5126/59-412	Dynamics of pneumatic pressure reducers	25	45	83
5126/59-413	Mass weighing*			90
5126/59-414	Infinite systems	28	116	193·
0009/59-417	Spectrum analysis of ruthenium			120
5126/59-418	P-wave equation	20	91	21
0009/59-421	Traffic assimilation**		12	
5126/59-428	Radio intensities**	28	34	
5126/59-429	Watch jewel performance	4	7	
	Total (NBS)	2,003	3,933	11,797

Task No.	Title	Assembly	Code Checking	Production
		(M I N U T E S)		
OUTSIDE:				
5116/55-56	Research in mathematical topics applicable to numerical analysis		23	102
5116/56-148	Differential equations for nerve excitation		83	105
5126/53-45	Air defense tactics°	122	56	84
5126/57-221	Bessel functions for complex arguments			78
5126/58-263	Gas tube characteristic			251
5126/58-269	Molecular structure, IV	69	47	677
5126/58-276	General kinetics, I**	49	9	4770
5126/58-278	Polaris computations**	30	18	34
5126/58-282	Missile boundary layer computation°	1074	522	817
5126/58-315	Mechanization of French translation**	1	51	124
5126/58-316	Intersection capacity study		118	
5126/58-318	Scattering, III**	4		
5126/58-319	Auto tag°	5	232	229
5126/58-320	Teller emission problem°	124	71	176
5126/58-335	Roots of Bessel functions**			56
5126/58-338	Minima by Cox Prugh**	8	24	11
5126/58-340	M5-17 Fuze Data°	21	56	146
5126/58-343	Minimization problem	3	8	10
5126/58-347	Computations for war games**	73	83	16
5126/58-348	Russian-to-English machine translation	68	122	12
5126/58-361	Spectrum of dipole radiation			590
5126/58-366	Radiation patterns of antennas		152	137
5126/58-368	Intensity functions			41
5126/58-370	Neutron diffusion study°	40	322	278
5126/59-373	Rhinitis°			5
5126/59-389	Frequency allocation	13	471	61
5126/59-407	Fourier series	42	21	69
5126/59-408	NASA**	775	322	8255
5126/59-411	Fitting of exponential curves***			15
5126/59-415	Complex Legendre functions	21	46	104
5126/59-416	Analysis of power supply experiments**	23	53	24
5126/59-423	Weather Bureau**			10666
5126/59-424	Systems engineering**	30		18
5126/59-425	Molecular orbitals			5
5126/59-427	Belvoir analysis data conversion		23	321
	Totals (Outside)...	2,595	2,933	28,287

Total time for the quarter (MINUTES).. 4,598   6,866   40,084

Total time for the quarter (HOURS) ...   76.6   114.4   668.1

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   *Problem programmed in the Computation Laboratory; production runs
     continued under direction of sponsor.
 **Problem programmed by sponsor and run under his direction.
***Machine time provided under contract.
  °Classified task.

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Statistics Seminar

SEVERO, N. C. Mathematical problems associated with measurements made by matching known standards. March 17.

Papers and Invited Talks
Presented by Members of the Staff
at Meetings of Outside Organizations

CANNON, E. W. The evolution of programming methods. Presented before the Central Ohio Association for Computing Machinery, Columbus, Ohio, March 21.

DAVIS, P. J. On the use of orthogonalizing codes in numerical analysis. Presented at a Computing Center Seminar of the Institute of Mathematical Sciences, New York University, New York, January 9.

EISENHART, C. (1) The treatment of outlying operations. Presented at a Statistics Seminar of the London School of Economics, London, England, February 26. (2) A decade of statistical engineering at the National Bureau of Standards. Presented at a meeting of the Royal Statistical Society, London, England, March 18.

GAUTSCHI, W. (1) On some questions of stability arising in the use of recurrence relations. Presented at the Westinghouse Research Laboratories, Pittsburgh, Pa., February 6. (2) Examples of numerical instability. Presented before the Applied Mathematics Panel, Oak Ridge National Laboratory, Oak Ridge, Tenn., March 16.

GHAFFARI, A. The behavior of Rayleigh's equation at infinity. Presented at the Annual Meeting of the American Mathematical Society, Philadelphia, Pa., January 20-22.

OSTROWSKI, A. M. Quadratic equation--A study of the regula falsi and difference equations. Presented at a Joint Mathematics Colloquium

of Catholic University-National Bureau of Standards-University of Maryland, Washington, D.C., January 9.

SEVERO, N. C. (1) A statistician and the Post Office: A case history in operations research. Presented before the Administration Applications Division of the American Society for Quality Control, Washington, D.C., March 5. (2) Mathematical problems associated with measurements made by matching with known standards. Presented before the Eastern Regional Meeting of the Institute of Mathematical Statistics, Pittsburgh, Pa., March 19-21.

TCHEN, C. M. (1) Some magnetohydrodynamical phenomena in cosmical medium. Presented at a meeting of the Physics Honor Society, Sigma Pi Sigma, Howard University, Washington, D.C., January 16. (2) Recent development in statistical plasma dynamics. Presented at the Magneto-hydrodynamics Seminar, Rensselaer Polytechnic Institute, Troy, N.Y., March 12.

WEGSTEIN, W. UNCOL--Useful avenue or blind alley. Presented before the Delaware Valley Chapter of the Association for Computing Machinery, Philadelphia, Penn., March 19.

YOUDEN, W. J. (1) Why design experiments. Presented at a meeting of the North Jersey Section, American Chemical Society, Summit, N. J., February 3, and also before a meeting of Committee D-13 on Textile Materials of the American Society for Testing Materials, New York, March 19. (2) What is scientific evidence? Presented before a meeting of the D. C. Council of Engineering and Architectural Societies, Washington, D.C., February 4; also at the Frontiers of Science Lectures held by the Joint Board on Science Education of the Washington Academy of Sciences, Washington, D.C., March 7. (3) Graphical diagnosis of interlaboratory tests. Presented before the American Society for Quality Control, Atlantic City, N.J., February 6. (4) Statistical techniques for ASTM Committee D-20. Presented at a meeting of Committee D-20 of the American Society for Testing Materials, Washington, D.C., February 18.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.3 Technical Papers

(1) Heat transfer in laminar flow through a tube. M. Abramowitz, W. F. Cahill, and C. Wade, Jr. J. Research, NBS, $\underline{62}$, 101-105 (Mar. 1959), RP2937.

(2) Abelian groups of unimodular matrices. E. C. Dade. Illinois J. Math. $\underline{3}$, 11-27(1959).

(3) On the numerical integration of periodic analytic functions. P. J. Davis. Appeared in "On Numerical Approximation," ed. by R. E. Langer, pp. 45-49 (The University of Wisconsin Press, Madison, Wisc., 1959).

(4) The exponential integral $\int_1^\infty e^{-xt} t^{-n} dt$ for large values of n. W. Gautschi. J. Research, NBS, $\underline{62}$, 123-125(1959), RP2941.

(5) Numerical experiments in potential theory using the Nehari estimates. U. W. Hochstrasser. Math. Tables Aids Comp. $\underline{12}$, 26-33(1958).

(6) Mechanized computation of thermodynamics tables at the National Bureau of Standards. II. Equilibrium compositions and thermo-dynamics properties of dissociated and ionized gaseous systems. J. Hilsenrath, M. Klein (NBS Thermodynamics Section), and D. Y. Sumida. Appeared in "Thermodynamic and Transport Properties of Gases, Liquids and Solids," pp. 416-437 (The Amer. Soc. Mech. Eng., New York).

(7) Preparation of new solution standards of radium. W. B. Mann, L. L. Stockman, W. J. Youden (SEL), A. Schwebel, P. A. Mullen, S. B. Garfinkel. J. Research NBS, $\underline{62}$, 21-26(1959), RP2924 (A publication primarily of the Radiation and Atomic Physics Division, NBS).

(8) On the minimum of the permanent of a doubly stochastic matrix. M. Marcus and M. Newman. Duke Math. J. $\underline{26}$, 61-72 (1959).

(9) A note on the computation of χ^2. M. G. Natrella. Amer. Stat. $\underline{13}$, 20-21 (1959).

(10) Dense subgraphs and connectivity. R. E. Nettleton (NBS, 3.2), K. Goldberg and S. M. Green (NBS, 3.2). Canadian J. Math. $\underline{11}$, 262-268 (1959).

(11) The evaluation of matrix inversion programs. M. Newman and J. Todd (California Institute of Technology). J. Soc. Indust. Appl. Math. $\underline{6}$, 466-476 (1958).

(12) On Gauss' speeding up device in the theory of single step iteration. A. M. Ostrowski. Math. Tables Other Aids Comp. $\underline{12}$, 116-132 (1958).

(13) On the convergence of the Rayleigh quotient iteration for the computation of the characteristic roots and vectors, II. A. M. Ostrowski. Arch. Rat. Mech. Anal. $\underline{2}$, 423-428 (1959).

(14) Un nouveau critere d'univalence des transformations dans un R^n. A. M. Ostrowski. Compt. Rend. Acad. Sci. Paris $\underline{248}$, 348-350 (1959).

(15) A further extension of Cayley's parameterization. M. Pearl. Canadian J. Math. $\underline{11}$, No. 1, 48-50 (1959).

(16) On a theorem of M. Riesz. M. Pearl. J. Research NBS $\underline{62}$, 89-94 (1959), RP2935.

(17) On normal and EPr matrices. M. Pearl. Michigan J. Math. $\underline{6}$, No. 1, 1-5 (1959).

(18) A statistician and the Post Office: A case history in Operations Research. N. C. Severo. Transactions of the 1959 Conference of the Administrative Applications Division of the ASQC, pp. 97-101.

(19) Theory of the effect of drag on the orbital inclination of an earth satellite. J. P. Vinti. J. Research NBS, $\underline{62}$, 79-88 (1959), RP2934.

(20) From formulas to computer oriented language. J. Wegstein. Communications ACM $\underline{2}$, 6-8 (1959).

(21) Multi-variable experiments. M. Zelen and W. S. Connor. Indust. Qual. Control $\underline{15}$, 14-17 (1959).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION MARCH 31, 1959

2.1 Mathematical Tables

(1) Tables of the bivariate normal distribution function and related functions. To appear as NBS Applied Mathematics Series 50.

(2) Tables of osculatory interpolation coefficients. H. E. Salzer. To appear in the NBS Applied Mathematics Series.

2.2 Manuals, Bibliographies, and Indices

(1) Fractional factorial experiment designs for factors at three levels. To appear as NBS Applied Mathematics Series 54.

2.3 Technical Papers

(1) A note on Hahn's theorem. R. J. Aumann. Submitted to a technical journal.

(2) A theoretical foundation for the numerical evaluation of worth in subjective allocation problems. R. J. Aumann. Submitted to a technical journal.

(3) Time phasing in the allocation problem. R. J. Aumann. Submitted to a technical journal.

(4) Assigning quantitative values to qualitative factors in the Naval electronics problem. R. J. Aumann and J. B. Kruskal (University of Wisconsin). To appear in the Naval Research Logistics Quarterly.

(5) Lower bounds for eigenvalues with applications to the helium atom. N. Bazley. To appear in the Proceedings of the National Academy of Sciences.

(6) Capacity requirement of a mail sorting device. B. Bender and A. J. Goldman. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(7) Analysis of fractionally replicated $2^n 3^m$ designs. R. C. Bose and W. S. Connor. To appear in Proceedings of a Special Session of the International Statistical Institute, Brussels, 1958.

(8) Construction of fractional factorial designs of the mixed $2^m 3^n$ series. W. S. Connor. Submitted for publication.

(9) The construction of Hadamard matrices. E. C. Dade and K. Goldberg. To appear in the Michigan Journal of Mathematics.

(10) Leonhard Euler's integral: A historical profile of the Gamma function. (In memoriam: Milton Abramowitz) P. J. Davis. To appear in the American Mathematical Monthly.

(11) Bending and stretching of corrugated diaphragms. R. F. Dressler. To appear in the Transactions of the American Society of Mechanical Engineers.

(12) Some canons of sound experimentation. C. Eisenhart. To appear in the Proceedings of a Special Session of the International Satistical Institute, Brussels, 1958.

(13) Note on bivariate linear interpolation for analytic functions. W. Gautschi. To appear in Mathematical Tables and Other Aids to Computation.

(14) Some elementary inequalities relating to a gamma and incomplete gamma function. W. Gautschi. To appear in the Journal of Mathematics and Physics.

(15) Generating functions for formal power series in noncommuting variables. K. Goldberg. Submitted to a technical journal.

(16) Note on a paper by S. Mukhoda and S. Sawaki. K. Goldberg. Submitted to a technical journal.

(17) The incidence equation $AA^T = aA$. K. Goldberg. To appear in the American Mathematical Monthly.

(18) The minima of cyclic sums. K. Goldberg. Submitted to a technical journal.

(19) The minimum of a certain linear form. K. Goldberg. Submitted to a technical journal.

(20) Principal submatrices of a full-rowed non-negative matrix. K. Goldberg. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(21) The zeros of certain polynomials. A. J. Goldman. Submitted to a technical journal.

(22) A continuous poker game. A. J. Goldman and J. J. Stone. Submitted to a technical journal.

(23) Field convexity of a linear transformation. A. J. Goldman and M. Marcus. Submitted to a technical journal.

(24) On a generalization of an inequality of L. V. Kantorovich. W. Greub and W. Rheinboldt. To appear in the Proceedings of the American Mathematical Society.

(25) Reduction formulas for partitioned matrices. E. V. Haynsworth. Submitted to a technical journal.

(26) The relation between confidence intervals and tests of significance--a teaching aid. M. G. Natrella. Submitted to a technical journal.

(27) Construction and application of a class of modular functions, II. M. Newman. To appear in the Proceedings of the London Mathematical Society.

(28) Weighted restricted partitions. M. Newman. To appear in Acta Arithmetica.

(29) Inclusion theorems for congruence subgroups. M. Newman and I. Reiner (University of Illinois). To appear in the Transactions of the American Mathematical Society.

(30) On the derivative of Bessel functions with respect to the order. F. Oberhettinger. Submitted to a technical journal.

(31) Linear differential equations of the second order with a large parameter. F. W. J. Olver. Submitted to a technical journal.

(32) Uniform asymptotic expansions for Weber parabolic cylinder functions of large orders. F. W. J. Olver. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(33) A quantitative formulation of Sylvester's law of inertia. A. M. Ostrowski. To appear in the Proceedings of the National Academy of Sciences.

(34) On the covergence of Gauss' alternating procedure in the method of least squares, I. A. M. Ostrowski. To appear in Annali di Matematica Pura ed Applicata.

(35) On the convergence of the Rayleigh quotient iteration for the computation of characteristic roots and vectors: III. Generalized Rayleigh quotient and characteristic roots with linear elementary divisors; IV. Generalized Rayleigh quotient for nonlinear elementary divisors; V. Usual Rayleigh quotient for non-Hermitian matrices and linear elementary divisors; VI. Usual Rayleigh quotient for nonlinear elementary divisors. A. Ostrowski. To appear in Archive for Rational Mechanics and Analysis.

(36) Über geranderte Determinanten und bedingte Tragheitsindizes. A. M. Ostrowski. To appear in (Wiener) Monatshefte für Mathematik und Physik.

(37) The graphical solution of initial value problems. W. H. Pell. Submitted to a technical journal.

(38) On prediction of system behavior. J. Rosenblatt. To appear in the Proceedings of the NYU-Industry Conference on Reliability, held at Ardsley-on-Hudson, N.Y., June 1958.

(39) On the power of some rank order two-sample test. J. Rosenblatt. Submitted for publication.

(40) The non-central χ^2 as a test statistic. N. C. Severo. Submitted to a technical journal.

(41) Stationary principles for forced vibrations in elasticity and electro-
     magnetism. J. L. Synge. To appear in the Proceedings of the
     Eighth Symposium in Applied Mathematics held by the American Mathe-
     matical Society, Chicago, Ill., April 1956.

(42) Some computational problems concerning integral matrices. O. Taussky.
     To appear in the Proceedings of the 1956 meeting of the Italian
     Society for the Advancement of Science, held in Sicily.

(43) Diffusion of particles in turbulent flow. C. M. Tchen. To appear
     in the Proceedings of a Symposium on Atmospheric Diffusion, Oxford,
     England, August 1958.

(44) Kinetic equation for a plasma with unsteady correlations. C. M.
     Tchen. To appear in Physical Review.

(45) Turbulent motion. C. M. Tchen and G. B. Schubauer (NBS Fluid
     Mechanics Section). To appear as Section B, Volume V of the Prince-
     ton Series, High Speed Aerodynamics and Jet Propulsion.

(46) Computation problems concerned with the Hilbert Matrix. J. Todd.
     To appear in the Proceedings of the 1956 meeting of the Italian
     Society for the Advancement of Science, held in Sicily.

(47) Rotational properties of two-dimensional lattices. J. P. Vinti.
     Submitted to a technical journal.

(48) Compressible turbulent boundary layers with heat transfer and
     pressure gradient in flow direction. A. Walz. To appear in
     Journal of Research, NBS, Section B. Mathematics and Mathematical
     Physics.

(49) Analytical integration of the differential equation for water storage.
     V. M. Yevdjevich. To appear in the Journal of Research, NBS, Section
     B. Mathematics and Mathematical Physics.

(50) Graphical diagnosis of interlaboratory test results. W. J. Youden.
     To appear in the Proceedings of the Midatlantic Conference of the
     American Society for Quality Control to be held in Atlantic City,
     February 6-7,1959.

(51) Problems of the experimenter. W. J. Youden. Submitted to a
     technical journal.

(52) Randomization and experimentation. W. J. Youden. To appear in
     Annals of Mathematical Statistics.

(53) Statistics--Engineering viewpoint. W. J. Youden. To appear in
     the Journal of Engineering Education.

(54) Measurements made by matching with known standards. W. J. Youden,
     W. S. Connor, and N. C. Severo. To appear in Technometrics.

(55) Analysis of two-factor classifications with respect to life tests. M. Zelen. Submitted for publication.

(56) Factorial experiments in life testing. M. Zelen. Submitted to a technical journal.

(57) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. S. Joel. To appear in the Annals of Mathematical Statistics.

2.4 Reviews and Notes

(1) A note on algebras. A. J. Goldman. To appear in the American Mathematical Monthly (Math. Notes).

(2) Review of "Cours de Geometrie Differentielle Locale" by J. Favard (Paris, Gauthier-Villare, 1957). A. J. Goldman. To appear in Scripta Mathematica.

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major laboratories in Boulder, Colo., is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside front cover.

WASHINGTON, D. C.

Electricity and Electronics. Resistance and Reactance. Electron Devices. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology. Engine Fuels. Free Radicals Research.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Neutron Physics. Radiation Theory. Radioactivity. X-rays. High Energy Radiation. Nucleonic Instrumentation. Radiological Equipment.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Molecular Structure and Properties of Gases. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Concreting Materials. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Air Conditioning, Heating, and Refrigeration. Floor, Roof, and Wall Coverings. Codes and Safety Standards. Heat Transfer.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Anolog Systems. Application Engineering.

• Office of Basic Instrumentation. • Office of Weights and Measures.

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships. VHF Research. Ionospheric Communication Systems.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Systems. Navigation Systems. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Radio Systems Application Engineering. Radio-Meteorology.

Radio Standards. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Electronic Calibration Center. Microwave Physics. Microwave Circuit Standards.