NATIONAL BUREAU OF STANDARDS REPORT

6316

PROJECTS and PUBLICATIONS

of the

APPLIED MATHEMATICS DIVISION

A Quarterly Report

October through December 1958

⬦NBS⬦

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the back cover.

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Inquiries regarding the Bureau's reports should be addressed to the Office of Technical Information, National Bureau of Standards, Washington 25, D. C.

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NBS PROJECT

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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

October 1 through December 31, 1958

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	Andreas Schopf	Y. M. Yevdjevich, D.Sc.

*On leave of absence   **Part time

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*Only unclassified projects are included in this report.
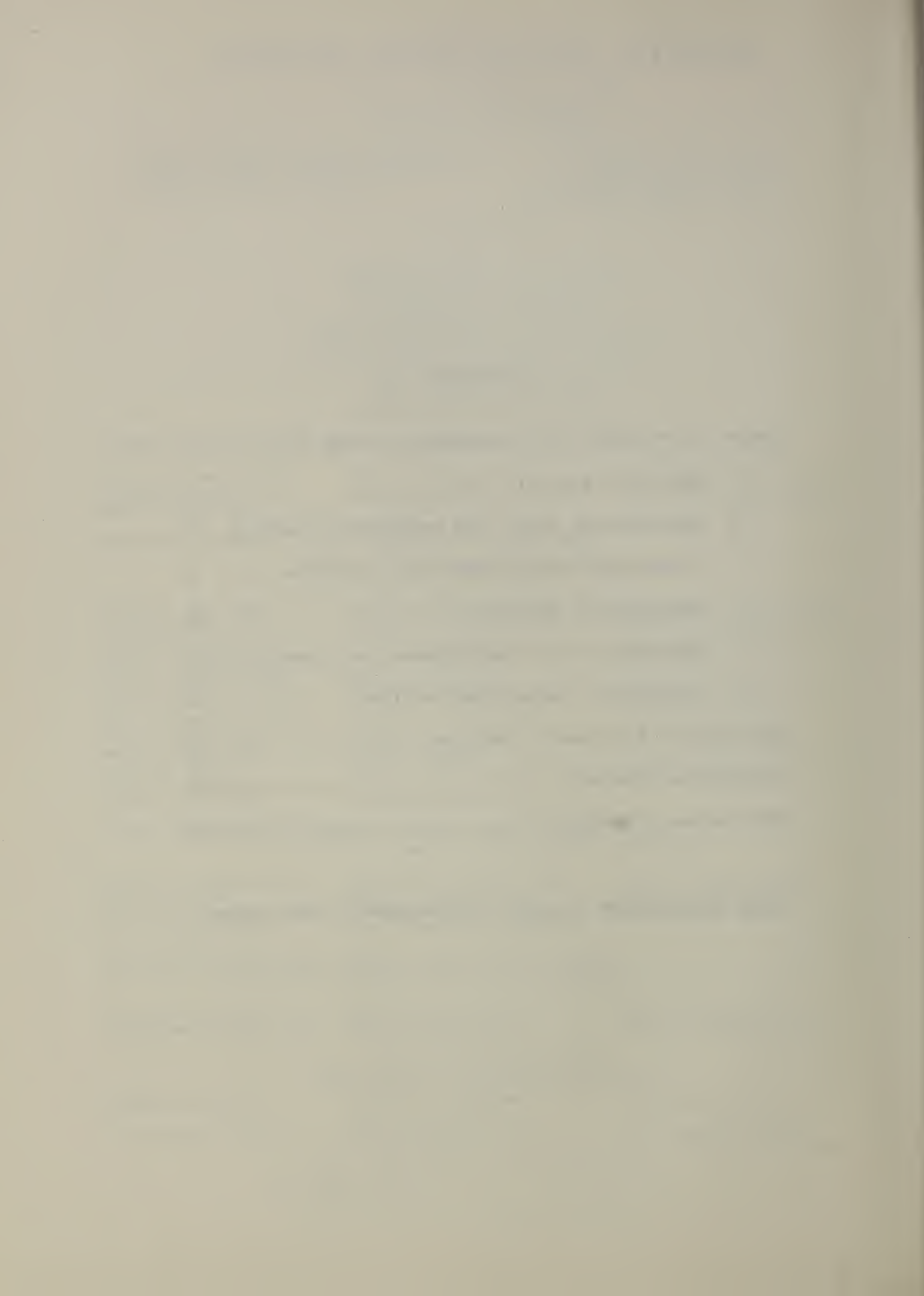
Status of Projects

December 31, 1958

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-12-1104/55-55

Origin: NBS   Authorized 8/29/54
Manager: P. Davis
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. W. Rheinboldt continued his studies of the
application of functional analysis to numerical analysis. Dr. Rheinboldt,
W. Gautschi, and J. Arms jointly presented a series of seminar lectures
on functional analysis to members of the division staff. In this seminar
special attention was given to Newton's method for operator equations in
Banach spaces.

N. Bazley has obtained numerical estimates of a lower bound
for the second eigenvalue of helium from his extension of Aronszajn's
method, and this has been combined with Temple's formula to find a
satisfactory and rigorous lower bound for the first eigenvalue. An
account of the method, together with its combination with Weinstein's
method, has been partially written up and will be completed soon.

N. Bazley has also been investigating applications of Monte
Carlo methods to the numerical calculation of Markov chains.

E. Haynsworth has found that the theorems in her paper
"Generalizations of Brauer theorems on stochastic matrices" (see July-
Sept 1958 issue, p. 2) were special cases of a more general theorem on
partitioned matrices. For this reason the paper has been rewritten under
the title "Reduction formulae for partitioned matrices."

The code for finding the largest positive eigenvalue of a
positive matrix using a modified Brauer method has been completed and
checked out. It works especially well for matrices which are nearly
singular, e.g., the Hilbert matrix.

In a paper entitled "Optimal approximations and error bounds"
(Proceedings, Conference on Numerical Approximation, Mathematics Research
Center, U. S. Army 1958, to appear), M. Golomb and H. F. Weinberger
showed how to find a best linear approximation to a certain linear
functional in terms of given data, and a bound for the error in this
approximation. If the data are the values of a function at N equally
spaced points and a bound on the square integral of the kth derivative,
the above process involves the inversion of a matrix of order $N + k$.
Since in practice N may be large, although k is in general fairly small,

such matrix inversion presents numerical problems. ·H. F. Weinberger has used special features of the matrix to reduce the problem of inverting the above matrix to that of finding the zeros of a polynomial of degree 2k and inverting a matrix of order k. This work is now in progress and will be applied to various particular problems.

Publications:
(1) On the numerical integration of periodic analytic functions. P. J. Davis. To appear in the Proceedings of the Symposium on Numerical Approximation, held at Madison, Wisconsin, April 1958.
(2) Computation problems concerned with the Hilbert matrix. J. Todd. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.
(3) On the minimum of the permanent of a doubly stochastic matrix. M. Marcus and M. Newman. To appear in the Duke Mathematical Journal.
(4) Field convexity of a linear transformation. M. Marcus and A. J. Goldman. Submitted to a technical journal.
(5) A continuous poker game. A. J. Goldman and J. J. Stone. Submitted to a technical journal.
(6) On a determinantal inequality. M. Marcus. To appear in the American Mathematical Monthly.
(7) On a generalization of an inequality of L. V. Kantorovitch. W. Greub and W. Rheinboldt. To appear in Proceedings of the American Mathematical Society.
(8) On the relations between summation methods and integral transforms. W. Greub. In manuscript.
(9) Uniform asymptotic expansions for Weber parabolic cylinder functions of large orders. F. W. J. Olver. In manuscript.
(10) Reduction formulae for partitioned matrices. E. Haynsworth. In manuscript.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO
NUMERICAL ANALYSIS
Task 1101-12-5116/55-56

Origin: NBS   Authorized 8/13/54
Sponsor: Office of Naval Research
Manager: M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. M. Newman has completed two manuscripts entitled respectively "Modular forms whose coefficients possess Euler products", and "Weighted restricted partitions". In the first all modular forms of a certain class are determined whose coefficients are multiplicative and for which the associated Dirichlet series possess Euler products. In the second, linear recurrence formulas of fixed length are determined for weighted partitions into parts not divisible by a fixed integer.

A seminar on Vinogradov's methods in number theory is being conducted by M. Newman. Five lectures covering introductory material have been given.

K. Goldberg has continued his work on algebras with an incidence matrix basis, together with E. C. Dade.

Dr. Goldberg has shown that a conjectured result on the gaps in the sequence of integers relatively prime to a fixed integer n implies that there is always a prime between x^2 and $(x+1)^2$. The conjecture on gaps has been verified for those n which have 15 or fewer distinct prime factors.

Dr. Goldberg has also investigated certain general types of combinatorial generation and has derived a necessary and sufficient condition that an incidence matrix have a permutation matrix for one of its principal submatrices.

A. Ostrowski prepared a paper on the application of the classical Rayleigh quotient to nonsymmetric matrices. Although the resultant convergence is only quadratic, the iteration procedure offers from the computational point of view an advantage as compared with the generalized Rayleigh quotient iteration.

Dr. Ostrowski also finished a paper on the qualitative sharpening of Sylvester's law of inertia in which he derives bounds for the factors multiplying the eigenvalues of a symmetric matrix as it is transformed by a linear transformation. He prepared a paper on the determination of inertia characters and a quadratic form considered on linear manifold. Another note was completed developing the results of a previous note on univalent transformations in the n-dimensional space.

Preparation of the expository manuscript by A. J. Goldman, "Tensor, Grassmann, and Clifford algebras," continued.

Publications:

(1) Some computational problems concerning integral matrices. O. Taussky. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.

(2) Abelian groups of unimodular matrices. E. C. Dade. Submitted to a technical journal.

(3) Incidence algebras. E. C. Dade and K. Goldberg. In manuscript.

(4) The construction of Hadamard matrices. K. Goldberg and E. C. Dade. To appear in the Michigan Journal of Mathematics.

(5) Dense subgraphs and connectivity. R. E. Nettleton (NBS, 3.2), K. Goldberg, and M. S. Green (NBS, 3.2). To appear in the Canadian Journal of Mathematics.

(6) Some combinatorial lemmas. K. Goldberg. In manuscript.

(7) On normal and EPr matrices. M. Pearl. To appear in the Michigan Journal of Mathematics.

(8) A further extension of Cayley's parameterization. M. Pearl. To appear in the Canadian Journal of Mathematics.

(9) On a theorem of M. Riesz. M. Pearl. To appear in the Journal of Research, NBS.

(10) Inclusion theorems for congruence subgroups. M. Newman and I. Reiner (University of Illinois). To appear in Transactions of the American Mathematical Society.

(11) Further identities and congruences for the coefficients of modular forms. M. Newman. Canadian J. Math. <u>10</u>, 577-586 (1958).

(12) Construction and application of a class of modular functions, II. M. Newman. To appear in Proceedings of London Mathematical Society.

(13) On the bounds of a one-parametric family of matrices. A. Ostrowski. To appear in the Journal für die Reine und Angewandte Mathematik.

(14) Un nouveau critere d'univalence des transformations dans un R^n. A. Ostrowski. To appear in Comptes Rendus de l'Academie des Sciences Paris.

(15) On the convergence of the Rayleigh quotient iteration for the computation of the characteristic roots and vectors, II. A. Ostrowski. To appear in Archive for Rational Mechanics and Analysis.

(16) On the convergence of the Rayleigh quotient iteration for the computation of characteristic roots and vectors, III. (Generalized Rayleigh quotient and characteristic roots with linear elementary divisors). A. Ostrowski. In manuscript.

(17) On the convergence of the Rayleigh quotient iteration for the computation of characteristic roots and vectors, IV. (Generalized Rayleigh quotient for nonlinear elementary divisors.) A. Ostrowski. In manuscript.

(18) A quantitative formulation of Sylvester's law of inertia. A. Ostrowski. In manuscript.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-12-5116/56-148

Origin and Sponsor: National Institutes of Health   Authorized 9/30/55
Manager: W. Gautschi (11.2)
Full task description: July-Sept 1955 issue, p. 7

Status: INACTIVE.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-40-1110/47-2

Origin: NBS   Authorized 7/1/47
Manager: I. A. Stegun
Full task description: Apr-June 1949 issue, p. 45

Status: INACTIVE.

TABLES OF POWER POINTS OF ANALYSIS-OF-VARIANCE TESTS
Task 1102-40-1110/51-8

Origin: Section 11.3, NBS   Authorized 3/26/51
Manager: S. Peavy
Full task description: Apr-June 1951 issue, p. 49

Status: INACTIVE.

REVISION OF MATHEMATICAL TABLES
Task 1102-40-1110/52-7

Origin: NBS   Authorized 8/10/51
Managers: W. F. Cahill, I. Stegun
Full task description: July-Sept 1951 issue, p. 41

Status: INACTIVE.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-40-1110/52-37

Origin: NBS   Authorized 11/28/51
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE.

SIEVERT'S INTEGRAL
Task 1102-40-1110/52-57

Origin: NBS   Authorized 2/12/52
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46

      Status: INACTIVE. For status to date, see Jan-Mar 1958 issue, p. 8.

MATHEMATICAL SUBROUTINES
Task 3711-60-0009/56-160

Origin: NBS   Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

      Status: CONTINUED. The Corbato eigenvalue routine was modified by R. Zucker to print the eigenvalues (in descending magnitude) and their corresponding eigenvectors in column form, five columns at a time. Previously, the eigenvalues were printed in random order and the eigenvectors were printed contiguously.
      A code has been written and checked out by C. Wade, which incorporates a new matrix inversion routine designed for the solution of large systems of equations.

AUTOMATIC CODING
Task 3711-60-1120/55-65

Origin: NBS   Authorized 9/29/54
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

      Status: CONTINUED. A code for performing the computations associated with mass weighings was placed in service as the first of a series of Black Box Computer codes. This code can be called into service by merely sending its number and the data to be used to the computer. This is a special purpose code and is useful to only one laboratory. Other general purpose-code packages are being added to the Black Box collection which will interpolate data, do least-squares, minimizations, etc. These can be called into action by anyone who supplies the proper calling number along with the data to be used.

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-5113/57-216

Origin and Sponsor: National Science Foundation   Authorized 12/27/56
Manager: I. A. Stegun
Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. Revision and editing of the textual material
for chapters 9, 10, 11 (Bessel Functions of Integer Order, Bessel
Functions of Fractional Order, and Integrals of Bessel Functions) was
in process.

Typing of the textual material was in progress for chapter 19
(Parabolic Cylinder Functions). The accompanying tables are ready in
preliminary form.

Work is currently under way on the preparation and checking of
the graphs for chapters 9, 10, 14, 19, 22. Typing and checking of the
manuscript of the tabular material for chapter 11 (Integrals of Bessel
Functions) was completed and is in progress on chapter 9 (Bessel Functions
of Integral Order). Revisions of chapters 3, 13, 23, 24 continued.

3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-12-1107/51-2

Origin: NBS   Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

     Status: CONTINUED. N. C. Severo continued his work on a manuscript on the convergence of distributions of random variables whose powers are normal.
     W. S. Connor, N. C. Severo and W. J. Youden completed a draft of a manuscript entitled "Measurements made by matching with known standards," giving the probabilities associated with positioning of unknowns with respect to a series of standards.

     Publication:
(1) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. Joel. Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-12-1107/53-1

Origin: NBS   Authorized 10/15/52
Manager: W. S. Connor
Full task description: Oct-Dec 1952 issue, p. 60

     Status: INACTIVE.

     Publication:
(1) Multi-variable experiments. M. Zelen and W. S. Connor. To appear in Industrial Quality Control.

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-1107/56-170

Origin: NBS   Authorized 12/15/55
Manager: Joan R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

     Status: INACTIVE.

MEASUREMENT OF RELIABILITY
Task 1103-12-1130/56-182

Origin: NBS   Authorized 3/23/56
Manager: M. Zelen
Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. M. Zelen and C. Dannemiller have completed
sampling experiments on the IBM 704 to test the robustness of procedures
for analyzing life tests having an underlying exponential distribution.
The results of the experimental sampling strongly indicate that these
procedures depend heavily on the exponential assumption.

J. R. Rosenblatt and M. Zelen have completed work on a
Department of Defense problem pertaining to the reliability of missiles.

J. R. Rosenblatt made a rough translation of "Redundant systems
with contact elements" by G. V. Druzhinin (Izvestiya Akademii Nauk SSSR,
Otdelenie Tekhnicheskikh Nauk, No. 6, pp. 96-98, June 1958).

M. Zelen presented a paper entitled "Problems in life testing:
factorial experiments" before the Thirteenth Midwest Quality Control
Conference, Kansas City, Missouri, November 1958.

M. Zelen presented a paper entitled "Factorial experiments in
life testing" before the annual meeting of the American Statistical
Association, Chicago, Illinois, December 1958.

Publications:
(1) Problems in life testing: Factorial experiments. M. Zelen. To
    appear in the Transactions of the Thirteenth Midwest Quality Control
    Conference, Kansas City, Mo., November 1958, pp. 21-33.
(2) Analysis of two-factor classifications with respect to life tests.
    M. Zelen. In manuscript.

FRACTIONAL FACTORIALS FOR THE MIXED SERIES
Task 1103-12-5148/58-291

Origin and Sponsor: Bureau of Ships   Authorized 9/30/57
Managers: W. S. Connor, M. Zelen
Full task description: July-Sept 1957 issue, p. 43

Status: CONTINUED. Thirty-nine fractional factorial designs of
the $2^m 3^n$ series have been constructed and analyzed. There is one for each
of the cases $2^4 3, \ldots, 2^9 3; 2^3 3^2, \ldots, 2^8 3^2; \ldots; 2^1 3^9$. The designs are such
that the grand average, main effects, and interaction effects can easily
be estimated.

W. S. Connor presented a paper entitled "Some recent work on
mixed fractional factorial designs" before the Annual Meeting of the Amer-
ican Statistical Association in Chicago, Illinois, December 1958.

Publication:
(1) Analysis of fractionally replicated $2^m 3^n$ designs. R. C. Bose and
    W. S. Connor. To appear in Revue de L'Institut International de
    Statistique (The Hague).

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-12-1115/55-57

Origin: NBS   Authorized 9/1/54
Manager: W. H. Pell
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. C. M. Tchen has continued work on the statistical method initiated in his paper on "Diffusion of particles in turbulent flow" (see July-Sept 1958 issue, p. 12). As an extension, Tchen is studying the trapping of particles by an inhomogeneous and unsteady magnetic field. This serves as a statistical model of the Fermi mechanism of acceleration to explain the origin of cosmic rays. The checking of galley proofs of "Diffusion of particles in turbulent flow", to appear in the Proceedings of the Symposium on Atmospheric Diffusion, Oxford University, has been finished.

A. Ghaffari has studied the critical points at infinity of Rayleigh's nonlinear equation

$$(1) \qquad \frac{d^2x}{dt^2} + \mu[-\frac{dx}{dt} + \frac{1}{3}(\frac{dx}{dt})^3] + x = 0, \qquad 0 < \mu < 2,$$

which is associated with acoustical phenomena and steady vibrations, and has analyzed the full phase-portrait. Noting that (1) may be replaced by the equivalent system

$$(2) \qquad \frac{dx}{dt} = y, \qquad \frac{dy}{dt} = -x - \mu(-y + \frac{1}{3}y^3),$$

and following the analysis of Lefshetz [see Differential Equations: Geometric Theory (Interscience, 1958)], he has obtained the following results. The unit sphere $x^2 + y^2 + z^2 = 1$ has four critical points on its equator $z = 0$ located at its intersections with the lines $x = 0$, $y = 0$ (in the plane $z = 0$). There are also critical points at the poles $(0,0,\pm1)$ of the unit sphere, which correspond to the critical point of (1) at the origin (unstable focus). The arcs of the equator $z = 0$ between consecutive critical points on it are paths of (1).

The analysis by V. M. Yevdjevich of the fluctuations in the flows of rivers due to annual variations in rainfall continues. Following is the status to date:

(a) Analysis of the statistics of annual flows has been computed for 65 river stations using the IBM 704. (This part is principally completed.)

(b) The data on annual flows for another 60 river stations from many parts of the world have been gathered, studied, corrected, and prepared for processing on the 704.

(c) The data on monthly river flows for about 45 stations has been collected, studied, and forwarded for processing on the 704. (This is for the second part of the project.)

(d) The study of the influence of the errors in the river flow data and of the inconsistence (nonhomogeneity) of flow time series upon the fluctuations in annual flows was initiated.

(e) The study of the very abundant literature has been continued.

(f) The first part of the report on the fluctuation of annual flow has been written.

J. P. Vinti has prepared a paper, primarily of didactic interest, deducing the rotations that leave a two-dimensional lattice invariant and the forms of the corresponding lattices. Unlike previous treatments, the method does not involve the use of group theory or matrix algebra, but makes use only of the algebra of complex numbers and of the integer solutions of certain quadratic equations. It should serve as a simple and rigorous introduction to the theory of two-dimensional crystals.

Publications:

(1) Rotational properties of two-dimensional lattices. J. P. Vinti. In manuscript.

(2) Diffusion of particles in turbulent flow. C. M. Tchen. To appear in the Proceedings of a Symposium on Atmospheric Diffusion, Oxford, England, August 1958.

(3) Turbulent motion. C. M. Tchen and G. B. Schubauer (NBS Fluid Mechanics Section). To appear as Section B, Volume V of the Princeton Series, High Speed Aerodynamics and Jet Propulsion.

(4) Theory of the effect of drag on the orbital inclination of an earth satellite. J. P. Vinti. To appear in the Journal of Research of the National Bureau of Standards.

(5) The graphical solution of initial value problems. W. H. Pell. Submitted to a technical journal.

(6) On some mathematical properties of wedge solution. A. Ghaffari. In manuscript.

(7) On the domain of regularity of generalized axially symmetric potentials. P. Henrici. To appear in the Proceedings of the American Mathematical Society.

(8) On the solution of compressible flow past a wedge. A. Ghaffari. In manuscript.

(9) On the asymptotic behavior of the integral curves of a certain nonlinear differential equation. A. Ghaffari. In manuscript.

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-5160/56-154

Origin: NBS   Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: July-Sept 1955 issue, p. 20

Status: CONTINUED. Publication is delayed pending the prepa-
ration of a section on inverse tables of Fourier transforms to be included.

RESEARCH IN FLUID DYNAMICS OF TWO-PHASE FLOWS
Task 1104-12-5160/56-155

Origin and Sponsor: Office of Naval Research   Authorized 9/30/55
Manager: W. H. Pell
Full task description: July-Sept 1955 issue, p. 21

Status: TERMINATED. J. P. Vinti is preparing a summary report
for the sponsor on the theory of a hydroduct.

RESEARCH IN CONTINUUM MECHANICS
Task 1104-12-5160/55-85

Origin: NBS   Authorized 12/27/54
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: W. H. Pell
Full task description: Oct-Dec 1954 issue, p. 30

Status: CONTINUED. C. M. Tchen has continued his study of
statistical plasma dynamics. Emphasis has been placed on the phenomenon
of degeneration of high order correlation functions into lower order ones.
Application of this result to the study of the runaway of electrons is
being made.
C. M. Tchen presented a paper "Structure of Correlation in
Plasma with an External Field" at the Divisional Meeting of the Fluid
Dynamics Division of the American Physical Society, San Diego, Cal.,
November 26. Dr. Tchen visited General Atomics, San Diego, and discussed
problems of plasma dynamics with Drs. M. Rosenbluth and N. Rostoker, and
also discussed magnetohydrodynamics with Prof. H. Liepmann of California
Institute of Technology and with Drs. Wegener and Davies at the Jet
Propulsion Laboratory, Pasadena, Cal.
W. H. Pell presented a paper "Bending and Stretching of
Corrugated Diaphragms" at the American Society of Mechanical Engineers

meeting in New York, December 3, in the absence of its author, R. F. Dressler.

Work on the problem of Stokes flows about axially symmetric bodies has been continued by W. H. Pell. The body currently under consideration is the torus [see July-Sept 1958 issue, p. 14]. Finding the appropriate stream function involves the determination of the circulation about the torus. This has now been shown to be determined uniquely by the condition that the pressure in the fluid be single-valued.

A paper is under preparation for publication which will contain the general theory as well as the Stokes flow solution for the lens-shaped body contained in the initial manuscript by L. E. Payne and W. H. Pell [see July-Sept 1958 issue, p. 15]. It is planned to include a table of Stokes drag for various bodies. The remainder of the first manuscript will form a second paper.

A limited number of prepublication copies of the following manuscripts are available:

a) Kinetic equation for a plasma with unsteady correlations. C. M. Tchen.

b) Compressible turbulent boundary layers with heat transfer and pressure gradient in flow direction. Alfred H. Walz.

Publications:

(1) Kinetic equation for a plasma with unsteady correlations. C. M. Tchen. To appear in Physical Review.
(2) The Stokes flow problem for a class of axially symmetric bodies. L. E. Payne and W. H. Pell. In manuscript.
(3) Compressible turbulent boundary layers with heat transfer and pressure gradient in flow direction. Alfred H. Walz. In manuscript.
(4) Stationary principles for forced vibrations in elasticity and electro-magnetism. J. L. Synge. To appear in the Proceedings of the Eighth Symposium in Applied Mathematics of the American Mathematical Society, held at Chicago, Ill., April 1956.
(5) The vibration of triangular wings. R. F. Dressler. In manuscript. Abstract available in the Proceedings of the Ninth International Congress on Applied Mechanics, Brussels, September 1956.
(6) On the factorization of a fourth order differential operation occuring in the theory of structures. W. H. Pell. In manuscript.
(7) Note on the integration of the elastic plate equation with variable flexural rigidity. W. H. Pell. In manuscript.
(8) Bending and stretching of corrugated diaphragms. R. F. Dressler. To appear in the Transactions of the American Society of Mechanical Engineers.

COMPUTATION OF OUTFLOW FROM A BREACHED DAM
Task 1104-12-5160/58-369

Origin and Sponsor: Army Map Service   Authorized 9/30/58
Manager: V. M. Yevdjevich
Full task description: July-Sept 1958 issue, p. 16

   Status: CONTINUED. V. M. Yevdjevich has continued (1) the study of the outflow hydrograph from a breached dam, and (2) the analysis of the modification of the hydrograph as it progresses downstream from the dam.
   A report on the first of these problems entitled "The Analytical Integration of the Differential Equation for Water Storage," which treats the outflow hydrograph from a breached dam, has been finished, and submitted to the sponsor. A second paper on this problem entitled "The Influence of the Negative Wave and the Effect of Flow Resistance on the Free Outflow Hydrograph of Rapid Openings in Bodies of Water" has been started.
   A study of possible devices for obtaining the modification of the hydrograph as it passes downstream is in progress; in particular, a review of existing methods and procedures is under way.

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RESEARCH ON SATELLITE ORBITS
Task 1104-12-5160/59-420

Origin: NBS   Authorized 12/19/58
Sponsor: Office of Scientific Research, ARDC, USAF.
Manager: J. P. Vinti

   Objective: (a) To examine the physical conditions for the existence of "quasi-elliptic", "quasi-circular" and "ballistic" stages of satellite motion, (b) to analyse the motion of a spherical satellite under the action of the inverse-square component of the earth's gravitational field and an assumed aerodynamic drag force, applying direct perturbation methods that may be feasible.

   Background: The lifetime of a satellite orbit, except for meteoric collisions, depends on the drag exerted on the satellite by the tenuous atmosphere through which it moves. Calculations of this effect have been carried out by several authors, using various methods, including a method which amounts to averaging quantities over each revolution and then using difference methods for tracing the effects of drag through successive revolutions.
   The possibility clearly exists, however, that the disappearance of orbits having apsides may be followed by a number of revolutions, perhaps hundreds or thousands, during which the satellite moves in a spiral of gradually diminishing radius, before it finally plunges to earth in a "ballistic" orbit. In general, one might expect: first, a "quasi-

elliptic" stage, characterized by the existence of apsides; second, a
"quasi-circular" stage, characterized by the absence of apsides and
possibly also by the approximate validity of the relation $rv^2 = \mu$
satisfied by circular orbits; and last a "ballistic" stage, without apsides,
with rv^2 markedly different from μ, and with large drag.

       It is of interest to try to establish the conditions for the
existence of all three stages, to estimate the number of revolutions in
each stage, and to find the factors that govern the approximate validity
of $rv^2 = \mu$ in the second, quasi-circular stage. Any information about
these matters would contribute greatly to our knowledge of the fundamental
principles of satellite motion. These questions have already arisen
in considerations about the effect of drag on the inclination of a
satellite orbit.

       Status: NEW. J. P. Vinti has set up a model for the theoretical
study of the effect of drag on satellite orbits. On the assumptions that
the drag is proportional to the product of the projected area, the density,
the square of the speed v, and that the density varies exponentially with
altitude, the differential equations of the orbit of a spherical satellite
are expressed in terms of the radius r and the quantity $\mu = rv^2$.

       Restriction to the case of an initially small eccentricity
of the orbit enables one to express the problem in terms of a simple
system of two nonlinear differential equations. A perturbation theory
is then appropriate. The theory of first-order perturbations then
results in a single nonlinear differential equation of third order.
Numerical solution is planned to find how the orbit varies with changes
in the critical distance occurring in the density function and with
changes in the initial altitude and in the initial eccentricity.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

1102-40-5126/54-13   AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: New York Quartermaster Procurement Agency
Manager: M. Paulsen
Full task description: Oct-Dec 1953 issue, p. 43
Status: Inactive.

3711-60-0009/54-30   SPECTRUM ANALYSIS
Origin: NBS, Division 4
Managers: C. D. Coleman, W. Bozman (4.1)
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. A search for new atomic energy levels of Th I gave
  three new even levels and 15 new odd levels. Wavenumbers were computed
  for about 4000 unclassified lines of Hf I. Then a search for new odd
  levels was made, disclosing the important lowest known odd level at
  10508.88 wavenumbers.
        Work continued on the revision of Kayser's "Table of Wavenumbers."
  Eight hundred pages have been calculated, and the first volume, consisting
  of 500 pages, has been printed in a form suitable for publication.

3711-60-0009/54-38   EQUATION OF STATE OF REAL GASES
                        (formerly COMPRESSIBILITY FACTORS OF DRY AIR)
Origin: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Continued. An editing and tape writing code has been prepared
  and is being checked out which will either create a library tape or
  update this tape. This enables the output of the partition functions
  program to be put on the above mentioned library tape, which in turn
  is used as input by the program which calculates the equilibrium
  properties of gaseous mixtures.
        A polynomial substitution was written using Fortran. It has
  been checked out and is being used by the sponsor. Although it is a
  specific program, it can be used to the following extent: (a) Number
  (No) of sets of coefficients (Ai) can vary from 1 to 6; (b) The number
  of X's (M) can vary from 1 to 100; (c) The number of coefficients per
  set (N) can vary from 1 to 20; (d) A table of X's is calculated for
  $X = 10(10)400$; (e) The special heading may easily be changed. The y's
  ($y = 1 + A_1X + A_2X^2 + \ldots$) are printed eight to a line for each X
  given for each set of coefficients, with a heading for each set; finally,
  a table with (No + 1) columns is printed with the argument (X) varying
  from 10(1)400 and with a corresponding value of y as calculated using
  the (No) sets of coefficients.

3711-60-0009/55-68   CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Division 9
Managers: P. O'Hara, S. Block (9.7)
Full task description: Jan-Mar 1955 issue, p. 18
Status: Continued. Additional computations were undertaken to determine
   the structure of several crystals. Extensive use was made of the
   existing least squares and Fourier synthesis programs.
          The programming of an improved version of the Fourier code is
   now under way.

3711-60-0009/55-82   THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. ITS constants and tables were computed for approxi-
   mately 21 thermometers under test. A code for computing low temperature
   constants and tables was written and is being checked out.

1102-40-5126/55-88   STRESSES IN A WALL FOUNDATION
Origin and Sponsor: NBS, Section 10.1
Manager: I. Stegun
Full task description: Jan-Mar 1955 issue, p. 22
Status: Terminated.

1102-40-5126/55-121   ELECTRON PENETRATION
Origin and Sponsor: NBS, Section 4.8
Manager: S. Peavy
Full task description: Apr-June 1955 issue, p. 19
Status: Inactive

1102-40-5126/56-162   STRESSES IN A WALL RESTING ON A FOOTING
Origin and Sponsor: NBS, Section 10.1
Manager: I. Stegun
Full task description: Jan-Mar 1956 issue, p. 26
Status: Inactive.

1102-40-5126/56-166   SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Managers: E. Haynsworth, P. Walsh
Full task description: Jan-Mar 1956 issue, p. 27
Status: Continued. The sponsor is using several of the codes prepared
   under this task for production runs.

1102-40-5126/56-186   MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: S. Prusch
Full task description: July-Sept 1956 issue, p. 33
Status: Continued. Calculations were performed for eight laboratory sets
    of gage blocks.

1102-40-5126/57-209   TRAFFIC DISTRIBUTION
Origin and Sponsor: Bureau of Public Roads
Manager: S. Peavy
Full task description: Jan-Mar 1957 issue, p. 32
Status: Completed. Results were transmitted to the sponsor.

1102-40-5126/57-221   BESSEL FUNCTIONS FOR COMPLEX ARGUMENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of
                     the Army
Manager: R. Zucker
Full task description: Oct-Dec 1956 issue, p. 31
Status: Inactive.

1102-40-5126/57-222   ROOTS OF POLYNOMIALS
Origin and Sponsor: Naval Research Laboratory
Manager: J. P. Menard
Full task description: Oct-Dec 1956 issue, p. 32
Status: Inactive.

3711-60-0009/57-223   SELF-CONSISTENT FIELDS
Origin: NBS, Section 3.2
Manager: E. V. Haynsworth
Full task description: Apr-June 1957 issue, p. 28
Status: Continued. Data is being prepared by the sponsor for production
    runs.

3711-60-0009/57-229   APPLICATION OF ELECTRONIC DATA PROCESSING
                         MACHINERY TO PAYROLL OPERATIONS
Origin: NBS, Section 40.0
Managers: M. Paulsen, P. Ruttenberg
Full task description: Jan-Mar 1957 issue, p. 36
Status: Inactive.

1102-40-5126/57-234   PERSONNEL SURVEY
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of
                     the Army
Manager: P. O'Hara
Full task description: Jan-Mar 1957 issue, p. 37
Status: Terminated.

1102-40-5126/57-236   SELF CONSISTENT FIELDS--EIGNEVALUES
Origin and Sponsor: NBS, Section 3.6
Manager: E. Haynsworth
Full task description: Apr-June 1957 issue, p. 30
Status: Continued. Data is being prepared by the sponsor for production
   runs.

3711-60-0009/57-247   MECHANICAL IMPEDANCE
Origin: NBS, Section 6.1
Managers: J. P. Menard, M. D. Burkhard (6.1)
Full task description: Apr-June 1957 issue, p. 32
Status: Continued. Production runs involving 24 samples of data were
   made directly by the sponsor. The program will continue in production
   under the sponsor's direction.

3711-60-0009/57-248   THE EVALUATION OF A TRIPLE INTEGRAL FOR THE
                       SOLUTION OF NEGATIVE ION DETACHMENT
Origin: NBS, Section 4.6
Manager: S. Peavy
Full task description: Apr-June 1957 issue, p. 34
Status: Inactive.

3711-60-0009/57-250   AUTOMATIC REDUCTION OF SPECTROPHOTOMETRIC DATA
Origin: NBS, Section 2.1
Manager: W. C. Rheinboldt
Full task description: July-Sept 1957 issue, p. 31
Status: Continued. Production runs involving about 100 samples of data
   were made directly by the sponsor. The paper entitled, "Digital
   Reduction of Speedophotometric Data,"   written jointly by K. J.
   Keegan, J. C. Schleter, D.B. Judd (all NBS 2.1), and W. C. Rheinboldt
   and J. P. Menard, was presented at the Forty-third Annual Meeting of
   the Optical Society of America, Detroit, Michigan, October 9.

1102-40-5126/57-251   CURRENT NOISE AND FIXED RESISTORS
Origin and Sponsor: NBS, Section 1.6
Manager: D. Sumida
Full task description: July-Sept 1957 issue, p. 32
Status: Inactive.

3711-60-0009/58-254   REPRODUCTION OF COLOR- AND SPECTRAL-ENERGY
                       DISTRIBUTION OF DAYLIGHT AND OTHER ILLUMINANTS
Origin: NBS, Section 2.3
Manager: W. C. Rheinboldt
Full task description: July-Sept 1957 issue, p. 32
Status: Continued. The code for the computation of the energy distribution
   of a Planckian radiator has been extended to facilitate also the

evaluation of the trilinear color coordinates from this energy distri-
bution using three different sets of extinction data. Approximately
80 cases have been run with this code, corresponding to temperature
values between 1,600°K and 20,000°K. In addition, about 25 cases
have been run with the original code under the immediate direction of
the sponsor. The program will continue in production under the sponsor's
direction.

1102-40-5126/58-263   GAS TUBE CHARACTERISTIC
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of
                        the Army
Manager: W. F. Cahill
Full task description: July-Sept 1957 issue, p. 35
Status: Continued. The computer program has been generalized to
  distinguish between molecular gases and atomic gases. Runs are being
  made for neon, a neon-argon mixture, and hydrogen. Some of the results
  have been transmitted to the sponsor.

1102-40-5126/58-264   THEORY OF IONIZATION PROBABILITY

Manager: S. Peavy
Full task description: Oct-Dec 1957 issue, p. 30
Status: Continued. A code is being written to compute the first
  integral of the first equation (see full task write-up) for values
  of $r = 0(1)4$ and $\lambda = 0(1)4$.

3711-60-0009/58-266   DEPOLYMERIZATION, II
Origin and Sponsor: NBS, Section 7.6
Manager: L. S. Joel
Full task description: July-Sept 1957 issue, p. 36
Status: Inactive.

3711-60-0009/58-267   CONVERSION OF THE CIE-CHROMATICITY COORDINATES
                        INTO THE MUNSELL COLOR SYSTEM
Sponsor: NBS, Section 2.1
Manager: W. C. Rheinboldt
Full task description: July-Sept 1957 issue, p. 37
Status: Continued. Production runs involving about 500 samples of data
  were made directly by the sponsor. A paper entitled, "Mechanized
  Conversional Colorimetric Data to Munsell Renotations" has been written
  jointly by W. C. Rheinboldt and J. P. Menard and was presented by
  W. C. Rheinboldt at the Forty-third Annual Meeting of the Optical
  Society of America, Detroit, Michigan, October 9.

1102-40-5126/58-269   MOLECULAR STRUCTURE, IV
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. J. O'Hara
Full task description: July-Sept 1957 issue, p. 38
Status: Continued. Several two-dimensional Fourier projections and least
   square refinements were computed for the Spurite crystal.
          A program was written to compute any of the several types of
   correction terms which must be applied to raw crystallographic data.
   These corrections are a function of the physical characteristics of the
   measuring equipment and also of the dimensions and configuration of the
   unit cell of the crystal. The observations from three sets of data
   were corrected and corresponding values from different sets were scaled
   and averaged. The results are now ready for use in various structure
   determination calculations.

1102-40-5126/57-270   MATHEMATICAL PROBLEMS RELATED TO POSTAL OPERATIONS
Origin: NBS
Sponsor: Post Office Department, Office of Research and Engineering
Managers: B. Bender, A. J. Goldman
Objective: To investigate those "operations research" problems en-
   countered by the Post Office Department which are appropriate for
   mathematical analysis. In particular, (a) to determine how automatic
   mail sorting devices can be most effectively arranged and employed, and
   (b) to develop methods for studying the problem of optimal location of
   mail-processing organizations.
Background: This task is being undertaken as part of a broader study of
   Post Office mechanization, involving the Data Processing Systems Division
   and Electricity and Electronics Division of the Bureau.
Status: Analytical methods have been developed for the comparison of
   proposed configurations for automatic mail sorting equipment. In a
   specific numerical case, these methods yielded a sorting system which
   was proved to be within 4 percent of optimum. A report on this material
   is being prepared for the sponsor. In addition, the capacity requirement
   for an idealized version of a certain sorting device has been determined
   and found to be far lower than had been estimated. A paper on this
   subject will appear in the NBS Journal of Research.

Publication:
          Capacity requirement of mail sorting device. B. Bender and
   A. J. Goldman. To appear in the NBS Journal of Research.

1102-40-5126/58-272   THERMODYNAMIC PROPERTIES OF REAL GASES
Origin and Sponsor: NBS, Section 3.2
Manager: J. P. Menard
Full task description: Oct-Dec 1957 issue, p. 32
Status: Inactive.

1102-40-5126/58-274   CALCULATIONS FOR d-SPACINGS, II
Origin and Sponsor: NBS, Division 9
Manager: R. Zucker
Full task description. July-Sept 1957 issue, p. 38
Status: Continued. About 100 calculations for d-spacings for orthogonal,
   hexagonal and orthorhombic crystals were carried out this quarter, and
   redetermination of unit cell constants by least squares fitting to a
   measured d-spacing was performed for about 12 crystals.

1102-40-5126/58-279   FIRE RESISTANT T-BEAM
Origin and Sponsor: NBS, Section 10.2
Manager: C. Wade
Full task description: Oct-Dec 1957 issue, p. 33
Status: Completed. Runs were made for an additional thickness of the
   T-beam for several combinations of time intervals and grid-block width.
   As before, the one- and two-dimensional cases were compared. The
   results have been given to the sponsor.

1102-40-5126/58-284   EPHEMERIS CALCULATIONS FOR SATELLITES
Origin and Sponsor: Naval Research Laboratory
Managers: W. F. Cahill, J. H. Wegstein
Full task description: Oct-Dec 1957 issue, p. 34
Status: Completed. Further work using programs developed under this
   task is being carried on under the direction of the National Aeronautics
   and Space Administration.

1102-40-5126/58-289   SCATTERING OF ELECTRONS BY HYDROGEN
Origin and Sponsor: NBS, Section 3.6
Manager: R. Zucker
Full task description: Oct-Dec 1957 issue, p. 36
Status: Completed. Results were submitted to the sponsor.

3711-60-0009/58-294   NUCLEAR SCATTERING OF PHOTONS
Origin: NBS, Section 4.8
Manager: J. P. Menard
Full task description: Oct-Dec 1957 issue, p. 36
Status: Continued. The existing code has been modified to print the
   value of the dispersion integral I(E) where

$$I(E) = P \int_{0}^{\alpha} \frac{G_a(E')}{E'^2 - E^2} \, dE'.$$

P denotes that the principal value of the integral is to be taken. The
program will continue in production under the sponsor's direction.

1102-40-5126/58-298   ANALYSIS OF SPECTROCHEMICAL DATA
Origin and Sponsor: NBS, Section 5.10
Managers: S. Peavy, R. N. Varner
Full task description: Oct-Dec 1957 issue, p. 39
Status: Continued. An addition to change the final output has been made
  in the Fortran code. The same addition has been made to the two previous
  codes. The first two codes have been checked out, and checking of the
  Fortran code is under way. The sponsor has made several runs using the
  initial code.

1102-40-5126/58-299   TIME-DEPENDENT SCHROEDINGER EQUATION
Origin and Sponsor: NBS, Section 3.1
Manager: A. Schopf
Full task description: Oct-Dec 1957 issue, p. 39
Status: Continued. A new general code has been written and checked out.
  A first production run has been made and compared with earlier results.
  The differential equation has been solved by step-wise Taylor expansions.
  The solution obtained agrees well with the asymptotic series and also
  with the earlier Runge-Kutta solutions of Ostrowski.

1102-40-5126/58-300   LAMINAR MIXING IN BOUNDARY LAYERS
Origin: Polytechnic Institute of Brooklyn
Sponsor: Air Force Office of Scientific Research
Manager: W. C. Rheinboldt
Full task description: Oct-Dec 1957 issue, p. 40
Status: Inactive.

1102-40-5126/58-304   TRANSPORT PROPERTIES OF AIR AT ELEVATED TEMPERATURES
Origin and Sponsor: NBS, Section 3.2
Manager: P. J. Walsh
Full task description: Oct-Dec 1957 issue, p. 40
Status: Continued. The data have been assembled for production runs. One
  production run was made and the results were transmitted to the sponsor.

1102-40-5126/58-306   INTERPOLATION OF COLOR MIXTURE FUNCTIONS
Origin and Sponsor: NBS, Section 2.1
Manager: W. C. Rheinboldt
Full task description: Oct-Dec 1957 issue, p. 42
Status: Continued. Approximately 15 production runs have been made
  directly by the sponsor. The program will continue in production under
  his direction.

1102-40-5126/58-307   STUDY OF SURFACE TENSION
Origin and Sponsor: NBS, Section 9.2
Manager: R. Arms
Full task description: Oct-Dec 1957 issue, p. 43
Status: Inactive

1102-40-5126/58-308   OSCILLATING SPHERE
Origin and Sponsor: NBS, Section 3.4
Manager: S. Prusch
Full task description: Oct-Dec 1957 issue, p. 43
Status: Continued. Small production runs were performed for the sponsor.

1102-40-5126/58-312   RESPONSE FUNCTION, II
Origin and Sponsor: NBS, Section 4.11
Manager: A. Beam
Full task description: Jan-Mar 1958 issue, p. 33
Status: Continued. The sponsor has made several runs on the 704.

1102-40-5126/58-314   APPROXIMATIONS FOR GAS MIXTURES
Origin and Sponsor: NBS, Section 3.2
Manager: R. Zucker
Full task description: Jan-Mar 1958 issue, p. 33
Status: Completed. The results have been submitted to the sponsor.

1102-40-5126/58-316   INTERSECTION CAPACITY STUDY
Origin and Sponsor: Bureau of Public Roads
Manager: S. Peavy, J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 33
Status: Continued. An auxiliary code has been written and checked out for
   the analysis of data related to the study of highway accident experience
   as related to control of access. Results have been transmitted to the
   sponsor. Work is being continued on the intersection capacity study.

1102-40-5126/58-321   TABLE OF THERMODYNAMIC FUNCTIONS OF SULFUR
Origin and Sponsor: NBS, Section 3.2
Manager: R. Zucker
Full task description: Jan-Mar 1958 issue, p. 34
Status: Completed. Results have been submitted to the sponsor.

1102-40-5126/58-322   PROPAGATION CONSTANT OF A SOUND WAVE
Origin and Sponsor: NBS, Section 6.1
Manager: R. J. Arms
Full task description: Jan-Mar 1958 issue, p. 34
Status: Inactive.

1102-40-5126/58-333   CALCIUM HYDROXIDE
Origin and Sponsor: NBS, Section 9.0
Manager: P. O'Hara
Full task description: Jan-Mar 1958 issue, p. 36
Status: Inactive.

1102-40-5126/58-336   HELICAL TRANSFORMS
Origin and Sponsor: National Institutes of Health
Manager: R. N. Varner
Full task description: Jan-Mar 1958 issue, p. 36
Status: Completed. Results have been transmitted to the sponsor.

1102-40-5126/58-337   GEORGETOWN LANGUAGE TRANSLATION EXPERIMENTS
Origin and Sponsor: Georgetown University
Manager: R. J. Arms
Full task description: Jan-Mar 1958 issue, p. 37
Status: Inactive.

1102-40-5126/58-339   COMPUTATION OF VISCOELASTICITY PROPERTIES OF
                                    MATERIALS
Origin and Sponsor: NBS, Section 3.4
Manager: H. Oser
Full task description: Jan-Mar 1958 issue, p. 38
Status: Continued. Investigations have been carried out in close
  cooperation with the sponsor.

1102-40-5126/58-343   MINIMIZATION PROBLEM
Origin and Sponsor: Naval Research Laboratory
Manager: S. Peavy
Full task description: Jan-Mar 1958 issue, p. 40
Status: Continued. The second order differential equation, due to its
  complexity, has been converted to include only portions of the original
  equation. A Fortran code has been written to evaluate this equation,
  and code checking and analysis of the results are now in progress.

1102-40-5126/59-348   RUSSIAN-TO-ENGLISH MACHINE TRANSLATION
Origin: NBS
Sponsor: Office of Ordnance Research, U. S. Army
Manager: I. Rhodes
Objective: To develop possible techniques for machine translation of
  technical material from Russian to English. To facilitate the
  development, attention is to be confined in the beginning to obtaining
  crude translation.
Background: Because of the limited number of linguists in this country
  familiar with the Russian language, a tremendous backlog of untranslated
  Russian technical literature has accumulated. The knowledge represented

is inaccessible to our engineers and scientists because of their inability
to read the articles in the original language. It is in the national
interest to provide translation means quickly.

Large-scale electronic digital computers have tremendous potential
for use as mechanical translation devices. The proposed program is set
up to develop the potentialities.

Status: The problem falls into two parts: (1) glossary lookup of individual
source words in a sentence, and (2) organization of the separate target
words into a meaningful sentence which renders correctly the intention
of the original. The first part has been completed and is running on
the IBM 704. The plan for the second part of the program has been
completed, but coding has not yet been started.

1102-40-5126/58-358   REDUCED CROSS-SECTIONS
Origin and Sponsor: NBS, Section 3.2
Manager: R. J. Arms
Full task description: Apr-June 1958 issue, p. 30
Status: Terminated.

1102-40-5126/58-359   VELOCITY DISTRIBUTION IN BOUNDARY LAYERS
Origin and Sponsor: NBS, Section 11.4
Manager: W. C. Rheinboldt
Full task description: Apr-June 1958 issue, p. 32
Status: Inactive.

3711-60-0009/58-360   DIFFUSION COEFFICIENTS
Origin: NBS, Section 5.2
Manager: W. C. Rheinboldt
Full task description: Apr-June 1958 issue, p. 32
Status: Continued. A Fortran code has been written for the solution of
the nonlinear diffusion problem

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D(c) \frac{\partial c}{\partial x}\right)$$
$$(-\infty < x < +\infty)$$
$$c(0,x) = c_O(x)$$

where $D(c)$ is given numerically. The problem is solved by employing a
suitable difference method after approximating $D(c)$ with a polynomial or
a stepfunction. $c_O(x)$ can be either a stepfunction, a polynomial or a
numerically given function. The code has been completely checked out
and gives very satisfactory results in the predicted stability region
of the difference method. Approximately 75 production runs have been
made by the sponsor. The program will continue under the direct
supervision of the sponsor.

1102-40-5126/58-361   CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION
Origin and Sponsor: Naval Research Laboratory
Manager: R. J. Arms
Full task description: Apr-June 1958 issue, p. 33
Status: Continued. Tests are being conducted on the feasibility of
  estimating a multifold integral by random variable generation.

1102-40-5126/58-366   RADIATION PATTERNS OF ANTENNAS
Origin and Sponsor: U. S. Information Agency, Department of State
Managers: R. T. Moore, P. J. Walsh
Full task description: Apr-June 1958 issue, p. 35
Status: Continued. Part (1) (see full write-up): The code for table (a)
  has been completely checked out. Code checking on tables (b) and (d) is
  now in progress. The code for table (c) has been written and code
  checking will begin soon.
          Part (2): Results for primary lobes for certain antennae have
  been calculated and submitted to the sponsor. Results for secondary
  lobes on a test case have been obtained from the machine and checking
  by hand calculation is now in progress. The parameters for 82 antennae
  have been submitted by the sponsor and have been prepared for production
  runs.

1102-40-5126/58-368   INTENSITY FUNCTIONS AND CROSS SECTIONS OF LIGHT
                        SCATTERED BY SPHERICAL PARTICLES
Origin and Sponsor: U. S. Army Signal Research and Development Laboratories,
                     Atmospheric Physics Branch, Belmar, N. J.
Manager: H. Oser
Full task description: July-Sept 1958 issue, p. 32
Status: Continued. A misprint occurred in equation (3), which should read

$$Q_{ext} = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1)(-1)^n \left\{ I(an) - I(bn) \right\}.$$

First production runs have been made, and the results have been submitted
to the sponsor.

1102-40-5126/59-374   END-EFFECT IN THE CYLINDRICAL ROTATIONAL VISCOMETER
Origin and Sponsor: NBS, Section 7.1
Managers: A. Schopf, J. P. Menard
Full task description: July-Sept 1958 issue, p. 34
Status: Inactive.

1102-40-5126/59-376   VARIATIONAL CALCULATION OF SLOW ELECTRON
                       SCATTERING BY HYDROGEN ATOMS
Origin and Sponsor: NBS, Section 4.6
Manager: L. S. Joel
Full task description: July-Sept 1958 issue, p. 35
Status: Terminated. See task 1102-40-5126/59-394, p. 30.

1102-40-5126/59-381   BOOLEAN FUNCTIONS AND PICTORIAL DATA PROCESSING
Origin and Sponsor: NBS, Section 12.5
Manager: B. Bender
Full task description: July-Sept 1958 issue, p. 35
Status: Continued. A method has been devised and two programs written
   for SEAC to aid in simplifying truth functions of up to 14 variables.
   The first of the programs obtains the core of truth function, i.e.,
   lists all the prime implicants the alternation of which must appear
   in any simplest normal equivalent, and lists all clauses not subsumed
   by at least one prime implicant in the core. The second program finds
   for each of these clauses all the prime implicants which subsume it.
   (For a definition of unfamiliar terms, see W. V. Quine, "The Problem
   of Simplifying Truth Functions," Amer. Math. Month., Oct. 1952.)
        A projected modification in the second program would enable a
   selection to be made among some of the prime implicants not in the
   core to obtain part of one (not necessarily unique) simplest normal
   equivalent, and a projected third phase would attempt to complete the
   process.

1102-40-5126/59-389   FREQUENCY ALLOCATION
Origin and Sponsor: Civil Aeronautics Administration
Manager: L. S. Joel
Objective: To find if possible a systematic algorithm for optimizing
   frequency assignment (in the sense of making fewest changes). An
   interim objective is to write a program to "automate" the present method
   of assignment.
Background: The Civil Aeronautics Administration maintains a network of
   radio transmitters emiting signals at fixed frequencies which delineate
   lanes as navigational aids for aircraft making point-to-point flights.
        A similar network is used for making "instrument landings" at
   airfields.
        Frequencies for the transmitters are selected from a fixed set
   subject to a set of restrictions determined by the range of the trans-
   mitters and the selectivity of the receiving devices available in
   airplanes. As new stations are added to the network frequencies are
   assigned to them, and if necessary existing frequencies are changed to
   accomodate the new stations.
        The problem was transmitted by E. Estes (CAA).
Status: New. Two preliminary bookkeeping programs have been written and
   checked with sample data.

1102-40-5126/59-391   ION DISTURBANCE AROUND A SATELLITE
Origin and Sponsor: Naval Research Laboratory
Manager: W. F. Cahill
Full task description: July-Sept 1958 issue, p.37
Status: Completed. The results were transmitted to the sponsor.

3711-60-0009/59-393   HEAT TRANSFER IN THE PRESENCE OF MOISTURE
Origin: NBS, Section 10.6
Manager: F. L.Alt
Full task description: July-Sept 1958 issue, p.38
Status: Inactive. For status to date, see July-Sept 1958 issue.

1102-40-5126/59-394   VARIATIONAL CALCULATION OF SLOW ELECTRON SCATTERING
                       BY HYDROGEN ATOMS,II
Origin and Sponsor: NBS, Section 4.6
Manager: A. Beam
Objective: To apply the Kohn variational method to determine the symmetric
  and antisymmetric scattering phase shifts for s-, p-, and d-wave electrons.
  The trial function contains, in addition to the phase shift parameter,
  up to three linear parameters and one screening parameter which enters
  nonlinearly. The variation of the stationary integral with respect to
  the nonlinear parameter is to be accomplished by a systematic numerical
  interpolation method. The dependence of the phase shifts upon the
  number of linear parameters is also to be investigated to try to get
  some idea of the degree of convergence of the procedure.
Background: This problem arises in the study of phase shifts in elastic
  scattering of electrons by hydrogen atoms. While the prime objective is
  still that of task 1102-40-5126/59-376, a more general approach in the
  study of phase shifts, etc. is planned, allowing for changes in initial
  values. The problem was proposed by S. Geltman (4.6).
Status: New. A Fortran code for the 704 was written for the s-wave, and
  all desired results for this part of the problem have been obtained. The
  code for the p-wave has been written and is being checked out.

1102-40-5126/59-403   COMPUTATION OF COLOR FADINGS
Origin and Sponsor: NBS, Section 2.1
Manager: W. C. Rheinboldt, J. P. Menard
Objective: (1) To write a revised program for the conversion of CIE-
  chromaticity coordinates into terms of the Munsell renotation system,
  which will store all necessary data directly in the core of the 704.
  (2) To write a code for the computation of color difference.
Background: The Photometry and Colorimetry Section (2.1) is presently
  engaged in a test-program for a commercial company. This program in-
  volves the reduction of a considerable amount of spectrophotometric data
  into terms of the Munsell renotation system and additionally the compu-
  tations of color differences for the evaluation of the amount of fading
  of given color samples.
        The original program for the conversion of CIE-chromaticity
  coordinates was written under task 1102-40-5126/57-267 and required that

the data of the Munsell Renotation System be stored on magnetic tape.
For the present needs this code is too slow. The revised program will
store all necessary data directly in the core memory and will incorporate
other improvements in the method of computation including more generalized
input and output.

      The problem has been transmitted by H. Keegan (2.1)

Status: New. The revised code has been written and is in the process of
being checked out.

1102-40-5126/59-412   DYNAMICS OF PNEUMATIC PRESSURE REDUCERS
Origin and Sponsor: NBS, Section 3.2
Manager: W. Gautschi
Objective: To solve numerically a set of nonlinear systems of six
differential equations under given initial conditions and additional
constraint conditions.
Background: The problem relates to the study of the dynamic behavior of
pneumatic pressure reducers with respect to various design parameters
and initial operating conditions. Of particular interest are the natural
frequencies, damping, and the regions of stability. The dynamics of the
reducers is governed by a system of six differential equations.

      The dynamics of pneumatic pressure reducers (such as those used
in the control systems of aircraft, rockets, etc.) is being studied in
the NBS Thermodynamics Section (3.2) under the sponsorship of the Bureau
of Aeronautics. The problem was transmitted by D. H. Tsai (3.2).
Status: New. A code for 704 has been written which integrates the
simultaneous differential equations numerically taking into account the
constraint conditions. The code is being checked with a set of parameters
furnished by the sponsor.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS
RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS   Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

  Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

  (1) <u>Interlaboratory Transistor Measurement Experiment</u>: Methods of "editing" data were developed to mitigate the effect of test-set bias discovered during the early stages of the experiment. Work on this project is being done for G. Conrad, Section 1.6.

  (2) <u>Measurement of Gage Blocks</u>: Procedures for machine computation of measures of precision and for testing for possible systematic errors were worked out in collaboration with J. Beers, Section 2.4.

  (3) <u>Machine (704) analyses</u> of data and/or the preparation of special codes were carried out for:

     H. B. Kirkpatrick, 9.4
     J. R. Crandall, 9.6
     M. J. Kerper, 9.2
     J. Mandel, 7.5
     R. E. Michaelis, 5.10
     F. M. Reinhart, 8.4
     H. C. Allen, 4.2
     D. R. Lide, 3.2

  (4) An NBS in-hours course, "Design of Multi-factor Experiments," is being presented by W. S. Connor and M. Zelen. Twenty-three have enrolled for the course.

  (5) <u>Uniformity of Titanium</u>: As a member of the NAS Materials Advisory Board Sampling Subpanel, W. J. Youden devised a method for selecting specimens from sheets of rolled titanium alloy to give valid estimates of the effect of nine factors on the mechanical properties of the metal. The schedule calls for a subset of 1/9th of the complete set of 7776 tests called for in the full factorial design using the nine factors.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC   ˙Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

      Status: CONTINUED. Statistical studies on the relation between
nil-ductility transition temperature and several physical and chemical
properties of two types of ship steel were begun.

MANUAL ON EXPERIMENTAL STATISTICS
FOR ORDNANCE ENGINEERS
Task 1103-40-5146/55-93

Origin and Sponsor: Office of Ordnance Research   Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

      Status: CONTINUED. The major portions of Part II (Qualitative
Data) and Part IV (Miscellaneous Topics) are being readied for distribution
for final comments. Work proceeded on preparing examples for the sections:
Regression, and Design of Experiments.

      Publications:
(1) A note on the computation of χ^2. Mary G. Natrella. To appear in
    The American Statistician.
(2) The relation between confidence intervals and tests of significance--a
    teaching aid. Mary G. Natrella. Submitted to a technical journal.

STATISTICAL SERVICES
Task 1103-40-5150/58-346

Origin and Sponsors: Various Agencies   Authorized 3/31/58
Manager: J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 45

    ˙ Status: CONTINUED. Work was done during the quarter for the
following agencies:
      (1) <u>Veterans Administration Hospital, Perry Point, Maryland</u>:
A "data preparation" code for the analysis of a new set of data on mental
patients was written for use with the previously prepared analysis of
covariance code.
      (2) <u>Bureau of Public Roads</u>: See task 1102-40-5126/58-316, p.25,
for report on this task done jointly with the Computation Laboratory.
      (3) <u>Chemical Corps</u>: Analysis of a problem arising in the work of
the Chemical Corps was begun in collaboration with A. J. Goldman of the
Numerical Analysis Section (11.1).

The record of the use of the IBM 704 for the period October 1 through December 31 is as follows:

Task No.	Title	Assembly	Code Checking	Production
		(M I N U T E S)		
NBS:				
1104/55-55	Research in numerical analysis and related fields	6	37	10
1107/51-2	Studies in probability and statistics	31	81	303
1115/55-57	Research in mathematical physics	15	55	56
0002/51-1	Statistical engineering	34	46	195
1120/55-65	Automatic coding	5	964	24
0009/56-160	Mathematical subroutines	60	211	165
0009/54-30	Spectrum analysis	9	108	731
0009/54-38	Equation of state of real gases	54	7	44
0009/55-68	Crystal structure calculations	78		813
0009/55-82	Thermometer calibrations			79
5126/55-88	Stresses in a wall foundation	15		5
5126/55-97	High temperature properties for air°		268	126
0009/56-131	Calculations in optics*	55	46	99
5126/56-150	Mathematical expressions*	2	4	120
5126/56-166	SCF-LCAO solution of some hydrides		231	239
5126/56-181	Coulomb wave function		8	9
5126/57-219	Thermal properties*			2
0009/57-223	Self-consistent fields		11	
5126/57-236	Self-consistent field--eigenvalues			21
5126/57-246	Radiation diffusion**	48	63	2168
0009/57-247	Mechanical impedance			81
0009/57-250	Automatic reduction in spectro-photometric data*		5	46
5126/57-252	Detecting efficiency in a neutral meson experiment**		255	204
0009/58-254	Reproduction of color-and spectral-energy distribution of daylight*	4	18	28
5126/58-255	Chi functions**		210	831
5126/58-256	Composite walls**		107	13
5126/58-260	Prototype accounting**	101	142	41
5126/58-264	Theory of ionization probability	17	72	20
0009/58-267	Munsell color system conversion*	20	1	103
5126/58-272	Thermodynamic properties of real gases**		23	
5126/58-274	Calculations for d-spacing,II*			250

Task No.	Title	Assembly	Checking	Production
		(M I N U T E S)		
5126/58-279	Fire resistant T-beam			105
5126/58-281	Psi function**	6		
5126/58-294	Nuclear scattering of photons*		12	48
5126/58-298	Analysis of spectrochemical data	11	53	52
5126/58-299	Time-dependent Shroedinger equation	21	6	6
5126/58-304	Transport properties of air		101	
5126/58-306	Interpolation of color mixture functions*			4
5126/58-308	Oscillating sphere			38
5126/58-312	Response function, II*			18
5126/58-314	Approximations for gas mixtures			4
5126/58-339	Viscoelasticity properties of materials		192	120
5126/58-353	Ship steel**		27	
5126/58-357	Eigenvalues***	61	110	146
5126/58-358	Reduced cross-sections**	51	306	20
5126/58-360	Diffusion coefficients*	112	22	324
5126/58-367	Ellipsometer calculations**			7
5126/59-375	Relaxation of Poisson distribution	6		2
5126/59-376	Slow electron scattering by hydrogen atoms		10	
5126/59-377	Logical diagram reduction**	181	333	216
5126/59-378	Correlation program **	9	5	28
5126/59-379	Transcendental equations**	13	18	13
5126/59-382	Phase shift**		36	136
5126/59-384	Least squares fit*			10
5126/59-387	Nuclear reactor design***		31	124
5126/59-388	Air conditioning	10	9	19
5126/59-390	Electrocardiogram**	10	41	
5126/59-394	Slow electron scattering by hydrogen atoms	75	147	248
5126/59-395	Adsorption study**	13	41	8
5126/59-396	Bessel integral evaluation	17	53	44
5126/59-398	Machine design**	55	170	388
5126/59-401	Rates**		33	
5126/59-403	Color fadings	50	33	
5126/59-404	Counter***		19	17
5126/59-409	Bank Board**		16	5
5126/59-413	Mass weighing***			18
	Totals (NBS)...	1,255	4,797	8,994
OUTSIDE				
5116/55-56	Research in mathematical topics applicable to numerical analysis		101	40
5126/53-45	Air defense tactics°		172	93
5126/58-263	Gas tube characteristic		24	545

Task No.	Title	Assembly	Code Checking	Production
		(M I N U T E S)		
5126/58-269	Molecular structure, IV	136	271	741
5126/58-270	Post Office mechanization**		44	64
5126/58-276	General kinetics, I**	1315	34	2439
5126/58-282	Missile boundary layer computation °		177	2422
5126/58-284	Ephemeris calculations for satellites		45	955
5126/58-286	Azimuth°		12	
5126/58-315	Mechanization of French translation		32	174
5126/58-316	Intersection capacity study		134	33
5126/58-319	Auto tag°	5	570	366
5126/58-320	Teller emission problem°	52	116	155
5126/58-335	List of Bessel functions**	4	12	445
5126/58-338	Minima by Cox Prugh**		12	
5126/58-340	M5-17 Fuze Data°	18	65	107
5126/58-343	Minimization problem	49	43	52
5126/58-348	Russian-to-English machine translation	126	216	
5126/58-361	Spectrum of dipole radiation	29	200	507
5126/58-366	Radiation patterns of antennas		217	19
5126/58-368	Intensity functions		324	155
5126/58-370	Neutron diffusion study°	18	389	9
5126/59-371	ASWAP°	15	13	109
5126/59-373	Rhinitis°	27	28	10
5126/59-389	Frequency allocation		7	
5126/59-391	Ion disturbance around a satellite		237	111
5126/59-397	Thermoequations°			36
5126/59-407	Fourier series	5	16	
5126/59-408	NASA**	68	102	2111
5126/59-411	Fitting of exponential curves***			8
	Totals (Outside)....	1,867	3,613	11,706
	Total time for the quarter	3,122	8,410	20,700

*Problem programmed in the Computation Laboratory; production runs continued under direction of sponsor.

**Problem programmed by sponsor and run under his direction.

***Machine time provided under contract.

°Classified task.

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are
not available from the National Bureau of Standards. If and when a
paper is to be published, it will be listed in the section of this report
on Publication Activities.

Applied Mathematics Division Seminars

HERMANN, ROBERT (Harvard University). The ideas of Elie Cartan in
differential geometry. October 28.

ROTA, GIAN-CARLO (Harvard University). Spectral theory of differential
operators. November 18.

Applied Statistics Seminar

YOUDEN, W. J. Interpreting interlaboratory tests. December 3.

Papers and Invited Talks
Presented by Members of the Staff
at Meetings of Outside Organizations

ALT, F. L. Mathematical methods in simulation. Presented before the
Institute of Management Sciences, held at Philadelphia, Pa.,
October 17.

EISENHART, C. (1) The meaning of 'least' in Least Squares. Presented at a
Joint Statistics Seminar of Birkbeck College and Imperial College of
Science and Technology, London, November 5. (2) The anatomy of
measurement. Presented at a Statistics Seminar, London School
of Economics, London, England, November 20. (3) Measurement-
taking in the presence of trend or drift. Presented at a
Statistics Seminar, University of Cambridge, England, November 24.
(4) Some examples of the effective use of simple statistical
techniques. Presented before the Industrial Applications Section,

Royal Statistical Society (South Wales Group), Newport, South
Wales, December 8.   (5) Some antecedents of modern experiment
design.   Presented before the Study Section of the London Group,
Royal Statistical Society, London, December 11.

HAYNSWORTH, E. V. (1) Generalizations of Brauer theorems on stochastic
matrices.   Presented at a meeting of the American Mathematical
Society, Princeton, N.J., October 25.   (2) Similarity trans-
formation for partitioned matrices.   Presented at a Mathematics
Colloquium, University of North Carolina, Chapel Hill, N.C.,
November 13.

OLVER, F. W. J.   Linear differential equations of the second order with
large parameter.   Presented at a Symposium on Asymptotic
Expansions, held at Pomona College, Claremont, Calif., November
21.

OSTROWSKI, A.   On Sylvester's law of inertia.   Presented before the Mathe-
matics Department, Westinghouse Corporation, Pittsburgh, Pa.,
in December.

PELL, W. H. (1) Theoretical research in continuum mechanics.   Presented
at the Air Research Development Command-Office of Scientific
Research, Research Contractor's Conference, held at Midwest
Institute, Kansas City, Mo., October 23-24.   (2) Bending and
stretching of corrugated diaphragms, by R. F. Dressler.   Pre-
sented by Dr. Pell at the 1958 Annual Meeting of the American
Society of Mechanical Engineers, New York, N. Y., December 1-4.

RHEINBOLDT, W. C., and J. P. MENARD Mechanized conversion of colorimetric
data to Munsell renotations.   Presented at a meeting of the
Optical Society of America, Detroit, Mich., October 9-11.

TCHEN, C. M.   Structure of correlation in plasma with an external field.
Presented at a meeting of the Division of Fluid Dynamics,
American Physical Society, San Diego, Calif., November 26.

YOUDEN, W. J.   (1) Problems of the experimenter.   Presented before the
Central Massachusetts Section, American Chemical Society,
Worcester, Mass., October 6; before the Western Vermont Section,
American Chemical Society, Middlebury, Vt., October 8; and
before the Connecticut Chapter, American Statistical Association,
New Haven, Conn., October 9.
(2) Interpretation of chemical data.   Presented before the American
Chemical Society: before the Maine Section Brunswick, Me.,
October 4; and before the Rhode Island Section, Providence, R.I.,
October 10.
(3) Graphical diagnosis of interlaboratory test results.
Presented before the Statistics Department, Harvard University,
Cambridge, Mass., October 7; and before the Connecticut Valley
Section, American Chemical Society, Springfield, Mass., October 11.

(4) Applications of statistics in experiments at the National
Bureau of Standards. Presented before the Boston Chapter,
American Statistical Association, Boston, Mass., October 7.
(5) Interpreting interlaboratory tests. Presented before the
Statistics Department, Virginia Polytechnic Institute,
Blacksburg, Va., October 31. (6) The reliability of estimates
of physical properties. Presented before the Society for
Experimental Stress Analysis, Albany, N.Y., November 12.
(7) Experimental designs. Presented at the Maryland Institute
of Metals, Baltimore, Md., December 2. (8) Statistics. Presented
at the Annual Princeton Conference of the American Society for
Quality Control and the SPES of the American Statistical Assoc-
iation, Princeton, N. J., December 5.

ZELEN, M. Problems in life testing: factorial experiments. Presented
        before the 13th Midwest American Society for Quality Control
        Conference, Kansas City, Mo., November 6.

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Papers presented at the joint meeting of the American Statistical
Association and the Biometric Society, Chicago, Ill., December 27-30.

CONNOR, W. S. Some recent work on mixed fractional factorial designs.

YOUDEN, W. J. Measurements made by matching with known standards.

ZELEN, M. Factorial experiments in life testing.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.3 Technical Papers

    (1) Unsteady nonlinear waves in sloping channels. R. F. Dressler.
        Proc. Roy. Soc. {A} $\underline{247}$, 186-198 (1958).

    (2) Further identities and congruences for the coefficients of modular
        forms. M. Newman. Canadian J. Math. $\underline{10}$, 577-586 (1958).

    (3) On the diffraction and reflection of waves and pulses on wedges
        and corners. F. Oberhettinger. J. Research, NBS, $\underline{61}$, 343-365
        (1958), RP2906.

    (4) On the bounds of a one-parametric family of matrices. A. M.
        Ostrowski. J. Reine Angew. Math. $\underline{200}$, 190-199 (1958).

    (5) Problems in life testing: factorial experiments. M. Zelen.
        To appear in the Transactions of the Thirteenth Midwest
        Quality Control Conference, Kansas City, Mo., November 1958,pp.21-33.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION DECEMBER 31, 1958

2.1 Mathematical Tables

    (1) Tables of the bivariate normal distribution function and related
        functions. To appear as NBS Applied Mathematics Series 50.

    (2) Tables of osculatory interpolation coefficients. H. E. Salzer.
        To appear in the NBS Applied Mathematics Series.

2.2 Manuals, Bibliographies, and Indices

    (1) Fractional factorial experiment designs for factors at three
        levels. To appear as NBS Applied Mathematics Series 54.

2.3 Technical Papers

    (1) Heat transfer in laminar flow through a tube. M. Abramowitz,
        W. F. Cahill, and C. Wade, Jr. To appear in the Journal of
        Research, NBS.

(2) A note on Hahn's theorem. R. J. Aumann. Submitted to a technical journal.

(3) A theoretical foundation for the numerical evaluation of worth in subjective allocation problems. R. J. Aumann. Submitted to a technical journal.

(4) Time phasing in the allocation problem. R. J. Aumann. Submitted to a technical journal.

(5) Assigning quantitative values to qualitative factors in the Naval electronics problem. R. J. Aumann and J. B. Kruskal (University of Wisconsin). To appear in the Naval Research Logistics Quarterly.

(6) Analysis of fractionally replicated $2^n 3^m$ designs. R. C. Bose and W. S. Connor. To appear in Revue de L'Institut International de Statistique (The Hague).

(7) Abelian groups of unimodular matrices. E. C. Dade. Submitted to a technical journal.

(8) The construction of Hadamard matrices. E. C. Dade and K. Goldberg. To appear in the Michigan Journal of Mathematics.

(9) On the numerical integration of periodic analytic functions. P. J. Davis. To appear in the Proceedings of the Symposium on Numerical Approximation, held at the Mathematical Research Center of the U. S. Army, Madison, Wisconsin, April 21-23,1958.

(10) Bending and stretching of corrugated diaphragms. R. F. Dressler. To appear in the Transactions of the American Society of Mechanical Engineers.

(11) Some canons of sound experimentation. C. Eisenhart. To appear in the Proceedings of a Special Session of the International Statistical Institute, Brussels, Belgium, Sept. 1-8, 1958.

(12) Note on bivariate linear interpolation for analytic functions. W. Gautschi. Submitted to a technical journal.

(13) Some elementary inequalities relating to a gamma and incomplete gamma function. W. Gautschi. To appear in the Journal of Mathematics and Physics.

(14) The exponential integral $\int_1^\infty e^{-xt} t^{-n} dt$ for large values of n. W. Gautschi. To appear in the Journal of Research, NBS.

(15) A continuous poker game. A. J. Goldman and J. J. Stone. Submitted to a technical journal.

(16) Field convexity of a linear transformation. A. J. Goldman and
     M. Marcus. Submitted to a technical journal.

(17) On a generalization of an inequality of L. V. Kantorovich.
     W. Greub and W. Rheinboldt. To appear in the Proceedings of the
     American Mathematical Society.

(18) On the domain of regularity of generalized axially symmetric
     potentials. P. Henrici. To appear in the Proceedings of the
     American Mathematical Society.

(19) Mechanized computation of thermodynamics tables at the National
     Bureau of Standards. II. Equilibrium compositions and thermo-
     dynamics properties of dissociated and ionized gaseous systems.
     J. Hilsenrath, M. Klein (NBS Thermodynamics Section), and
     D. Y. Sumida. To appear in the Proceedings of a Symposium of the
     American Society of Mechanical Engineers, to be held at Purdue
     University, Lafayette, Indiana, February 1959.

(20) Numerical experiments in potential theory using the Nehari
     estimates. U. W. Hochstrasser. To appear in Mathematical
     Tables and Other Aids to Computation.

(21) On the minimum of the permanent of a doubly stochastic matrix.
     M. Marcus and M. Newman. To appear in the Duke Mathematical
     Journal.

(22) A note on the computation of χ^2. M. G. Natrella. To appear
     in the American Statistician.

(23) The relation between confidence intervals and tests of
     significance--a teaching aid. M. G. Natrella. Submitted to
     a technical journal.

(24) Dense subgraphs and connectivity. R. E. Nettleton (NBS, 3.2),
     K. Goldberg and S. M. Green (NBS,3.2). To appear in the
     Canadian Journal of Mathematics.

(25) Construction and application of a class of modular functions, II.
     M. Newman. To appear in the Proceedings of the London Mathe-
     matical Society.

(26) Inclusion theorems for congruence subgroups. M. Newman and
     I. Reiner (University of Illinois). To appear in the Transactions
     of the American Mathematical Society.

(27) The evaluation of matrix inversion programs. M. Newman and
     J. Todd (California Institute of Technology). To appear in the
     Journal of the Society for Industrial and Applied Mathematics.

(28) On the derivative of Bessel functions with respect to the order.
     F. Oberhettinger. Submitted to a technical journal.

(29) Linear differential equations of the second order with a large
     parameter. F. W. J. Olver. Submitted to a technical journal.

(30) On Gauss' speeding up device in the theory of single step
     iteration. A. M. Ostrowski. To appear in Mathematical Tables
     and Other Aids to Computation.

(31) On the convergence of the Rayleigh quotient iteration for the
     computation of the characteristic roots and vectors, II.
     A. M. Ostrowski. To appear in Archive for Rational Mechanics
     and Analysis.

(32) Un nouveau critere d'univalence des transformations dans un R^n.
     A. M. Ostrowski. Submitted to a technical journal.

(33) A further extension of Cayley's parameterization. M. Pearl.
     To appear in the Canadian Journal of Mathematics.

(34) A note on commutators. M. Pearl. Submitted to a technical journal.

(35) On a theorem of M. Riesz. M. Pearl. To appear in the Journal of
     Research, NBS.

(36) On normal and EPr matrices. M. Pearl. To appear in the Michigan
     Journal of Mathematics.

(37) The graphical solution of initial value problems. W. H. Pell.
     Submitted to a technical journal.

(38) The non-central χ^2 as a test statistic. N. C. Severo. Sub-
     mitted to a technical journal.

(39) Stationary principles for forced vibrations in elasticity and
     electromagnetism. J. L. Synge. To appear in the Proceedings of
     the Eighth Symposium in Applied Mathematics held by the American
     Mathematical Society, Chicago, Ill., April 1956.

(40) Some computational problems concerning integral matrices.
     O. Taussky. To appear in the Proceedings of the 1956 meeting of
     The Italian Society for the Advancement of Science, held in
     Sicily.

(41) Diffusion of particles in turbulent flow. C. M. Tchen. To
     appear in the Proceedings of a Symposium on Atmospheric
     Diffusion, Oxford, England, August 1958.

(42) Kinetic equation for a plasma with unsteady correlations. C. M.
     Tchen. To appear in Physical Review.

(43) Turbulent motion. C. M. Tchen and G. B. Schubauer (NBS Fluid
     Mechanics Section). To appear as Section B, Volume V of the
     Princeton Series, High Speed Aerodynamics and Jet Propulsion.

(44) Computation problems concerned with the Hilbert matrix. J. Todd.
     To appear in the Proceedings of the 1956 meeting of the Italian
     Society for the Advancement of Science, held in Sicily.

(45) Theory of the effect of drag on the orbital inclination of an
     earth satellite. J. P. Vinti. To appear in the Journal of
     Research, NBS.

(46) Graphical diagnosis of interlaboratory test results. W. J. Youden.
     To appear in the Proceedings of the Midatlantic Conference of the
     American Society for Quality Control to be held in Atlantic City,
     February 6-7, 1959.

(47) Randomization and experimentation. W. J. Youden. To appear in
     Annals of Mathematical Statistics.

(48) Statistics--Engineering viewpoint. W. J. Youden. To appear in
     the Journal of Engineering Education.

(49) Multi-variable experiments. M. Zelen and W. S. Connor. To
     appear in Industrial Quality Control.

(50) The weighted compounding of two probabilities from independent
     significance tests. M. Zelen and L. S. Joel. Submitted to a
     technical journal.

2.4 Reviews and Notes

(1) A note on algebras. A. J. Goldman. To appear in the American
    Mathematical Monthly (Math. Notes).

(2) Review of "Cours de Geometrie Differentielle Locale" by J. Favard
    (Paris, Gauthier-Villare, 1957). A. J. Goldman. To appear in
    Scripta Mathematica.

U. S. DEPARTMENT OF COMMERCE
Lewis L. Strauss, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major laboratories in Boulder, Colo., is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside front cover.

WASHINGTON, D. C.

Electricity and Electronics. Resistance and Reactance. Electron Devices. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology. Engine Fuels. Free Radicals Research.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Neutron Physics. Radiation Theory. Radioactivity. X-rays. High Energy Radiation. Nucleonic Instrumentation. Radiological Equipment.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Molecular Structure and Properties of Gases. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Concreting Materials. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Air Conditioning, Heating, and Refrigeration. Floor, Roof, and Wall Coverings. Codes and Safety Standards. Heat Transfer.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Anolog Systems. Application Engineering.

• Office of Basic Instrumentation.   • Office of Weights and Measures.

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships. VHF Research. Ionospheric Communication Systems.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Systems. Navigation Systems. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Radio Systems Application Engineering. Radio-Meteorology.

Radio Standards. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Electronic Calibration Center. Microwave Physics. Microwave Circuit Standards.