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NATIONAL BUREAU OF STANDARDS REPORT

6273

Second Draft of

Part II (Some Standard Statistical Techniques for Qualitative Data)

for

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS

A Report to

Office of Ordnance Research
Department of the Army

NBS

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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Prepared by

Statistical Engineering Laboratory

A Report to
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NOTICE

This report is a draft of Part II, sections 1, 2 and 3, covering techniques for estimating and comparing proportions. The final draft of Part II will include an additional section on Sensitivity Testing which is not included here.

Table references are to Tables listed in NBS Report 5320, or to the additional Tables provided at the end of this report.

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Tables

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II. Some Standard Statistical Techniques for Qualitative Data

What is Qualitative Data?

There exist physical and visual tests for which it is impossible to obtain and record actual measurements. The only possible observation will consist of a notation as to whether the test item passes or fails a shock test, fires or does not fire, is acceptable or unacceptable with respect to a standard color chip. In some other tests, the required expense or inconvenience may make it undesirable from a practical point of view to obtain the exact measurement. For example, a dimension can always be measured, but in large-scale production excessive time may be required and go-no-go gauges are used instead.

Wherever it is possible to obtain actual measurements, these measurements do provide more information than do counts. In planning tests this must be kept in mind and all factors considered. Time, money, availability of experienced personnel, all contribute to the decision of whether to measure or gauge. If measurements are taken, the methods of Part I apply and not the methods of this Part.

For whatever reason, whether necessity or economic choice, the observations now consist not of measurements, but of a series of responses (e.g., hit-miss, pass-fail, larger than-less than, yes-no). This is what is called qualitative data.

The majority of practical applications will involve only two mutually exclusive categories of classification such as those mentioned. The two-category case may be thought of as a YES-NO classification, and in fact it often is exactly that (pass-fail). The most extensive treatment will be given to this case. (sections 1 and 2.)

Methods are also available for handling results which fall into three or more categories. An example of a three category case might be ACCEPT-REJECT-REWORK such as commonly found in screening inspection. The methods used for three categories apply for any larger number of categories and no purpose is served by further distinctions.

The raw data will always consist of <u>counts</u> of the number of test items or test runs which fall into each of the categories of the classification system. For the purpose of processing the data, the counts will be expressed as the proportion of the total number of items examined. In general the equivalent percentages are not used in analysis, even though final presentation of this kind of result is often made in percentages.

The organization of this part of the manual will parallel that of Part I as much as possible. Instead of estimating the true average of a lot with respect to some property we estimate the true proportion of items in the lot which have the pertinent characteristic. (For single estimates, see section 1.1. For interval estimates, see section 1.2.) Comparisons can be made between new and standard products, or between any two products, with regard to the percentage of individual items exhibiting the chosen characteristic. The methods for comparison are given in 2.1 and 2.2 for 2-category classifications (the pass-fail type) and in 3.1 and 3.2 for several-category classifications. A brief section dealing with the case where an item is classified by two different criteria, each of which has several categories, is included (see 3.3) because the technique of analysis is similar to that of 3.2.

1. Estimating the true proportion or percentage of items which have a given quality characteristic (two categories of classification).

Given:

A sample of n items selected at random from a much larger group. Upon examination or test, r of the n items show the presence of the given characteristic.

Example:

Ten fuzes are taken at random from a production line and tested under a specified set of conditions. Four fail to function.

Questions: The general question is "what can be said about the larger group with regard to the proportion of defective items contained therein?" —specifically,

(1) What proportion, p, of the fuzes produced would be expected to fail under the prescribed conditions?

(2) Can we give an interval estimate of the proportion of defective fuzes, and state a confidence coefficient associated with this interval—i.e., a measure of our assurance that the interval will bracket the true proportion?

1.1 Best single estimate.

The best estimate of the true proportion of items having a given characteristic is equal to the number of sample items which have the characteristic divided by the total number of items in the sample.

The best estimate of the actual proportion of fuzes that will fail is equal to the number of defective fuzes in the sample, divided by the total number of fuzes in the sample.

We assume that the samples are chosen by an unbiased method.

Procedure:	Example:
Compute the estimated proportion p, as follows:	
$p = \frac{r}{n}$	$p = 4/10 = .4$

1.2 Interval estimates of the true proportion or percentage.

1.2.1 Two-sided confidence intervals (For fuller explanation of confidence intervals read the Introduction.)

Although the best single estimate of the true proportion of items having a given characteristic is the proportion of such items in the sample, an interval estimate may be preferred.

A confidence coefficient, i.e., a measure of assurance that the stated interval does contain the true proportion, can be given for the interval estimate. These interval estimates are obtained as follows: For $n > 30$, use Charts VIa, VIb or VIc for 90%, 95%, and 99% confidence intervals respectively. (For $n \leq 30$, use Table XXIV.) On the charts there are 2 curves for each of a number of values of $\underline{n}$. The upper and lower curve for a particular $\underline{n}$ constitute a confidence belt for the true proportion. First locate the observed proportion, r/n, on the horizontal scale. From this point travel up to the curves for the sample n and read off upper and lower limits for the population proportion P. For example, in a sample of $n = 100$, where observed proportion is .4, 95% confidence limits for the true proportion are .31 to .51.

The three charts give $(1-\alpha)$ confidence interval estimates for $\alpha = .10, .05, .01$. If we use these charts a large number of times to make interval estimates of P, the true proportion, we can expect $100(1-\alpha)\%$ of these intervals to contain P.

Approximate method.

An alternative approximate method of obtaining confidence intervals is useful when $.1<P<.9$, and nP and $n(1-P)$ are both greater than 5.

1) Choose the desired confidence level $1-\alpha$.

2) Look up $z_{1-\alpha/2}$ in Table Ib.

3) Compute $p_1 = p + z_{1-\alpha/2} \sqrt{\frac{p(1-p)}{n}}$

$$p_2 = p - z_{1-\alpha/2} \sqrt{\frac{p(1-p)}{n}}$$

The interval from p_1 to p_2 is the $1-\alpha$ confidence interval estimate of P.

1.2.2 One-sided confidence intervals

A one-sided interval estimate will state that P is less than a proportion p_1', or alternatively that it is larger than p_2', and the statement carries given confidence level in the manner discussed above. Use Charts V1a, V1b, V1c to obtain .95, .975, and .995 one-sided confidence intervals respectively, by using only the top portion, or only the bottom portion of the belt for a given sample size. For $n \leq 30$, Table XXV should be used.

Approximate method (useful when $.1 < P < .9$, and nP and $n(1-P)$ are both greater than 5). A $(1-\alpha)$ one-sided confidence interval can be calculated by computing:

$$p_1' = p + z_{1-\alpha} \sqrt{\frac{p(1-p)}{n}}$$, or alternatively

$$p_2' = p - z_{1-\alpha} \sqrt{\frac{p(1-p)}{n}} .$$

The interval below p_1', or the interval above p_2' is a one-sided confidence interval estimate of P.

1.3 Sample size required to estimate the true proportion.

We shall discuss two problems:

Problem 1.3.1 Specified error in either direction permitted (i.e., to estimate P within $\pm d$).

Problem 1.3.2 Specified error in only one direction permitted (i.e., to estimate P within +d, or within -d).

In 1.3.1 we are indifferent as to whether our estimate is too high or too low. In 1.3.2 we wish to protect ourselves against an overestimate, but do not worry about an underestimate (or vice versa).

Problem 1.3.1 Sample size required to estimate the true proportion within a stated amount ($\pm d$).

Graphical method.

The problem could be restated as follows: we wish to make a two-sided confidence interval estimate for P and the width of the interval should be not greater than 2d.

It is therefore possible to use the charts (Table VI) in reverse, that is to find the sample size belt whose maximum width (vertical distance on the charts) is 2d. The maximum width of confidence interval for a particular *n* will occur when the observed proportion is equal to 0.5. If past records on

the particular process indicate that the observed proportion has always been within some range (e.g., always less than 0.1), use the widths of the intervals at this point rather than the maximum widths.

Numerical method.

The basic formula for sample size is

$$n = \frac{z^2_{1-\alpha/2} \, P'(1-P')}{d^2}$$

A sample of size n guarantees a probability not greater than α that our estimate of P is in error by more than d.

Since the true proportion P is unknown, what should be used as a value for P'? The rules are as follows:

1) If there is no prior information about P available, or if P is believed to be in the neighborhood of 0.5, use P' = 0.5. The formula then simplifies to

$$n = \frac{z^2_{1-\alpha/2}}{4d^2}$$

2) If the true proportion P can safely be assumed less than 0.5, let P' be the largest reasonable guess for P.

3) If the true proportion P can safely be assumed to be greater than 0.5, let P' be the <u>smallest</u> reasonable guess for P.

It is obvious that the largest sample size will be required when the true P is 0.5, and the purpose of these three rules is to be as conservative as possible.

Problem 1.3.2 Sample size required to estimate the true proportion with error in specified direction not to exceed d (i.e., $+d$, or $-d$).

Sometimes we do not care if we underestimate P, the true proportion, but wish to protect ourselves against overestimating P. The risk of overestimating (underestimating) P by more than d is to be not greater than d. The error in estimate is to be only in the direction that we choose.

The basic formula for sample size is

$$n = \frac{z^2_{1-\alpha}\, P'(1-P')}{d^2}$$

Since P is unknown, we use the same rules as in 1.2.1 for substituting a value for P'.

1) If there is no prior information about P available, or if P is believed to be in the neighborhood of 0.5, use $P' = 0.5$. The formula then simplifies to:

$$n = \frac{z^2_{1-\alpha}}{4d^2}$$

2) If the true P can safely be assumed to be less than 0.5, let P' be the largest reasonable guess for P.

3) If the true P can safely be assumed to be greater than 0.5, let P' be the <u>smallest</u> reasonable guess for P.

The largest sample size will be required when $P = 0.5$, and the purpose of the rules is to be as conservative as possible.

2. Comparisons between proportions (two categories of classification).

In addition to estimating proportions, there are cases where we want to compare proportions. For a given kind of ammunition, we may have a specification which prescribes the maximum percentage of duds. Production lots of this ammunition will not be acceptable if they exceed this specified percent defective. Where we are comparing an observed proportion with a specification or standard, the methods of section 2.1 apply.

A different kind of comparison is involved where we have no standard value, but wish to compare two observed proportions. For example, there are two methods of manufacture for a certain component. Method I is cheaper and would obviously be preferable unless it produces a higher percentage of defectives than Method II.

As in Part I, the procedure used will lead to one of two conclusions. If we ask the question "Do these two lots differ with regard to proportion defective?", our procedure will give either (a) or (b) as the answer:

(a) The two lots differ with regard to proportion defective.

(b) There is no reason to believe that the two lots differ in this respect.

In the case of comparison with a standard, the comparison will be made by computing a confidence interval for the observed proportion. The procedures in this section, therefore, look somewhat different than the procedures in Part I, section 2.2, but they are not essentially different. (For amplification of this relationship, read the section on "Relationship between confidence intervals and tests of significance" in the Introduction.)

This section will give solutions for the following problems:

2.1 Comparison of an observed proportion with a standard.

2.2 Comparison of two observed proportions.

2.1 Comparison of an observed proportion with a standard.

<u>Given</u>:

A sample of *n* items selected at random from a much larger group. Upon examination, *r* of the *n* items show the presence of the pertinent characteristic. $p = r/n$ = observed proportion. P_0 = the known proportion of individual items in the standard product that exhibit the pertinent characteristic.

<u>Example</u>:

<u>Problem 2.1.1</u> Does the new product differ from the standard with regard to the proportion which exhibits the pertinent characteristic? (Does P differ from P_0?)

Solution for $n \leq 30$

<u>Procedure:</u> <u>Example:</u>

i) Choose confidence level, $1-\alpha$. Tables are provided for confidence levels .90, .95, .99.

ii) Enter Table XXIV with n and r. Select column equal to chosen confidence level.

iii) If the tabled limits do not include P_0, conclude that P differs from P_0.

iv) If the tabled limits do include P_0, there is no reason to believe that P is different from P_0.

Solution for $n > 30$

<u>Procedure:</u>

<u>Example:</u>

i) Choose confidence level, $1-\alpha$. Charts are provided for confidence levels .90, .95, .99.

ii) Go to Chart V1a, V1b, or V1c, as determined by choice of confidence level. Locate observed r on horizontal scale. Locate curves for proper n.

iii) Read off upper and lower limits for P. If these limits do not include P_0, conclude that P differs from P_0.

iv) If the limits do include P_0, there is no reason to believe that P differs from P_0.

Problem 2.1.2 Does the characteristic proportion for the new product exceed that for the standard? (Is $P > P_0$?)

Solution for $n \leq 30$

Procedure:

i) Choose confidence level, $1-\alpha$. Tables are provided for confidence levels .90, .95, .99.

ii) Go to Table XXV. Find n and chosen confidence level. Enter Table in row n-r. Subtract tabled value from 1. This is a lower one-sided limit for observed P.

iii) If limit obtained in ii) exceeds P_0, conclude that the characteristic proportion for the new product exceeds that for the standard.

Example:

iv) If limit obtained in ii) is not larger than P_0, there is no reason to believe that the proportion for the new product exceeds that for the standard.

Solution for $n > 30$

Procedure:	Example:
i) Choose confidence level $1-\alpha$. Tables VIa, VIb, VIc, will provide confidence levels .95, .99, .995 for one-sided tests.	
ii) Go to Chart VIa, VIb, or VIc as determined by choice of confidence level. Locate observed r on horizontal scale. Locate curves for proper n.	
iii) Read off lower confidence limit for P. If P_0 is smaller than this limit, conclude that the proportion for the new product exceeds that for the standard product.	

iv) If P_0 is larger than the limit and is therefore included in the one-sided confidence interval for P, there is no reason to believe that P is larger than P_0.

Problem 2.1.3 Is the characteristic proportion for the new product less than that for the standard? (Is $P < P_0$?)

Solution for $n \leq 30$

Procedure:

i) Choose confidence level $1-\alpha$. Tables are provided for confidence levels .90, .95, .99.

ii) Go to Table XXV. Enter table with n, r, and chosen confidence level. Table entry is an upper one-sided limit for observed p.

iii) If tabled limit is less than P_0, conclude that the characteristic proportion for the new product is less than that for the standard.

Example:

iv) If the tabled limit is larger than P_0, there is no reason to believe that the proportion for the new product is less than the standard.

Solution for $n > 30$

<u>Procedure:</u> <u>Example:</u>

i) Choose confidence level $1-\alpha$. Use of Charts Vla, Vlb, Vlc will provide confidence levels .95, .975, .995 for one-sided tests.

ii) Go to Chart Vla, Vlb, or Vlc as determined by choice of confidence level. Locate observed r on horizontal scale. Locate curves for proper n.

iii) Read off upper confidence limit for P. If P_0 is larger than this limit, conclude that the proportion for the new product is less than that for the standard product.

iv) If P_0 is less than this limit and is therefore included in the one-sided confidence limit for P, there is no reason to believe that P is less than P_0.

Problem 2.1.4 Sample size required to detect a difference from a standard proportion without regard to the sign of the difference.

Given:

P_0 = the known proportion of the population of standard items which exhibit the pertinent characteristic. P_0 may be given by specification or standard.

To be specified for this problem:

α = the significance level, or risk of announcing a difference when in fact there is none.

β = the risk of failing to detect a difference when in fact the true proportion for the new product differs from the standard by an amount δ (i.e., $|P - P_0| = \delta$).

δ = the amount of difference which is considered important to detect.

Charts and tables to be used:

Table XXVIIa gives the required sample size for a number of values of P_0 and P for $\alpha = .05$ and $1-\beta = .50, .80, .90, .95$, and $.99$. The table is given largely for illustration, to demonstrate how the required sample size is affected by the magnitude of the P_0 and δ involved, and by different choices of β. For desired values of α and β which are not included in

Table XXVIIa, use Table XXVI, a table to convert the difference between the proportions into the necessary form for use with Table XVIIIa.

Procedure: Example:

i) Specify δ, the amount of difference considered important to detect.

ii) Choose α and β.

iii) For $\alpha = .05$, $1-\beta = .50$, .80, .90, .95, and .99, go to Table XXVIIa.

iv) Let $P = P_0 + \delta$ or $P = P_0 - \delta$, whichever makes P closer to 0.5.

v) If either P or P_0 is less than 0.5, enter Table XXVIIa with P and P_0. If neither P nor P_0 is less than 0.5, enter Table XXVIIa with 1-P and $1-P_0$. In either case,

the smaller of the two proportions determines the column and the larger determines the row. Read off n directly. n is the required sample size for the new product.

vi) For values of α, β, and P which are not included in Table XXVIIa, go to Table XXVI. Look up

$\theta_0 = \theta$ corresponding to P_0

$\theta = \theta$ corresponding to P

vii) Compute $d = |\theta - \theta_0|$

viii) Enter Table XVIIIa with chosen α, $1-\beta$, and d (from step vii). The tabled n is the required sample size for the new product. (Footnote to table should be ignored.)

Problem 2.1.5 Sample size required to detect a difference from a standard proportion with regard to the sign of the difference.

Given:

P_0 = the known proportion of the population of standard items which exhibit the pertinent characteristic. P_0 may be given by specification or standard.

To be specified for this problem:

δ = the amount of difference which is considered important to detect.

α = the significance level, or risk of announcing a difference when in fact there is none.

β = the risk of failing to detect a difference when in fact the true proportion for the new product is $P = P_0 + \delta$ or $P_0 - \delta$, as specified.

Charts and tables to be used:

Table XXVIIb gives the required sample size for a number of values of P_0 and P for $\alpha = .05$ and $1-\beta = .50, .80, .90, .95$ and .99. The table is given largely for illustration, to demonstrate how the required sample size is affected by the magnitude of the P_0 and δ involved, and by different choices of β. For desired values of α and β which are not included in Table XXVIIb, use: Table XXVI, a table to convert the difference between the proportions into the necessary form for use with Table XVIIIb.

Procedure: Example:

i) Specify $+\delta$ or $-\delta$, the signed difference from the standard proportion that is considered important to detect. Then $P = P_0 + \delta$ or $P = P_0 - \delta$, as specified.

ii) Choose α and β.

iii) For $\alpha = .05$, $1-\beta = .50$, .80, .90, .95, and .99, go to Table XXVIIb.

iv) Let $P = P_0 + \delta$ or $P = P_0 - \delta$, as specified.

v) If either P or P_0 is less than 0.5, enter Table XXVIIb with P and P_0. If neither P nor P_0 is less than 0.5, enter Table XXVIIb with 1-P and $1-P_0$. In either case, the smaller of

the two proportions
determines the column and
the larger determines the
row. Read off n directly.
n is the required sample
size for the new product.

vi) For values of α, β, and P not included in Table XXVIIb, go to Table XXVI. Look up
$\Theta_0 = \Theta$ corresponding to P_0
$\Theta = \Theta$ corresponding to P.

vii) Compute $|d| = |\Theta_0 - \Theta|$

viii) Enter Tables XVIIIb with chosen α, $1-\beta$, and d (from step vii). The tabled n is the required sample size for the new product.

2.2 Comparison of two observed proportions.

Two problems will be discussed:

Problem a. To test whether the proportion having a given characteristic is different for two materials, products, or processes. There is no particular concern about which proportion is larger.

Problem b. To test whether the proportion having a given characteristic is larger in Product A than the corresponding proportion in Product B.

It is again important to decide which problem is appropriate before taking the observations. If this is not done and if the choice of the problem is influenced by the observations, the significance level of the test and the operating characteristics of the test may differ considerably from their nominal values.

In the following it is assumed that the appropriate problem has been selected and that n_A and n_B items are taken from Products A and B respectively. In Product A, there are r_A items which are classified in Class I; in Product B, r_B items fall in Class I.

The items will be classified and the observations recorded in a 2x2 table as in Table 2.2:

Table 2.2

	Class I	Class II	Total
Sample from A	r_A	s_A	$n_A = r_A + s_A$
Sample from B	r_B	s_B	$n_B = r_B + s_B$
TOTAL	r	s	n

The rows in the table are the two samples and the columns are the two classes into which the observed items have been classified. The classes may be success-failure or any other two-category classification. Entries in the table are counts. If Class I is the class of interest, the observed proportions are $p_A = r_A/n_A$ and $p_B = r_B/n_B$.

The solutions to the two problems will be discussed separately for three cases:

Case 1. Equal samples ($n_A = n_B$) [Section 2.2.1]

Case 2. Small unequal samples ($n_A \neq n_B$, both less than 20) [Section 2.2.2]

Case 3. Large unequal samples [Section 2.2.3]

2.2.1 Comparing two proportions based on equal samples.

Tables to be used: Table XXVIIIa and XXVIIIb are available for equal sample sizes by intervals of one up to 20 and $n_A = n_B = 30, 40, 50, 60, 70, 80, 90, 100, 150, 200, 300, 400, 500$.

Use Table XXVIIIa for $\alpha = .05$ in Problem a
for $\alpha = .025$ in Problem b.

Use Table XXVIIIb for $\alpha = .01$ in Problem a
for $\alpha = .005$ in Problem b.

Problem 2.2.1a Is P_A different from P_B? (equal sample sizes).

Steps in solution:

The solution involves two steps:

Step (1). Pick out the proper ordered pair (as described below) from the 4 entries in the data table (Table 2.2). This pair of entries is called the "observed contrast" pair.

Step (2). Compare the "observed contrast" pair with the minimum contrast pair given in Table XXVIII, and judge whether or not the observed contrast is significant.

With a little practice, both steps can be done quickly by eye. After recording the data as in Table 2.2, the detailed procedure is:

Step (1) Find the ordered pair (a_1, a_2) where

a_1 = smallest entry of all 4

a_2 = entry in <u>same</u> <u>class</u> as a_1 from the other sample.

(If a_1 should equal a_2, proceed no further. Data gives no reason to believe that the two proportions differ.)

For example, consider the following table of observed counts:

	Class I	Class II	Total
Product A	$r_A = 15$	$s_A = 2$	$n_A = 17$
Product B	$r_B = 7$	$s_B = 10$	$n_B = 17$
TOTAL	$r = 22$	$s = 12$	$n = 34$

According to the rule, $a_1 = 2$ and $a_2 = 10$ and we use the pair (2, 10) for comparison with Table XXVIII.

Step (2) Compare the observed contrast pair with the minimum contrast pair listed in Table XXVIII.

Table XXVIII shows the "least different" pairs of entries in a 2x2 table which are significant at the chosen level.

A "more different" pair is of course significant also. For example, look at the entries in Table XXVIIIa for $n_A = n_B = 17$.

The "minimum contrast required" are (0,5), (1,7), (2,9), (3,10), etc. Since (0,5) is significant, so also is (0,6), (0,7), etc. Since (1,7) is significant, so also is (1,8), (1,9), etc.

In the example shown in Step 1, the observed contrast pair (a_1, a_2) is (2,10). If the chosen significance level is $\alpha = .05$, we go to Table XXVIIIa. For $n_A = n_B = 17$, we find an entry (2,9) which is significant. Since (2,9) is significant, (2,10) is also significant and we conclude that the two products differ with regard to the characteristic proportion.

<u>Summary of procedure:</u> Is P_A different from P_B? (equal sample sizes)

<u>Example:</u>

i) Choose α, the significance level of the test.

ii) Use Table XXVIIIa for $\alpha = .05$ or Table XXVIIIb for $\alpha = .01$.

iii) Obtain data contrast pair as in Step 1 above. Call this pair (a_1, a_2).

iv) Enter table with sample size (n_A or n_B) for each group.

v) Call table pairs (A_1, A_2). Find the table pair where $A_1 = a_1$.

vi) If a_2 is equal to or larger than A_2, the observed contrast is significant at the chosen level. Conclude that the two products differ with regard to the characteristic proportion considered. Otherwise, there is no reason to believe that the two proportions differ.

Problem 2.2.1b   Is P_A larger than P_B? (equal sample sizes)

The solution to this problem is the same as that of Problem 2.2.1a, with two important exceptions:

(1) First, compare the observed proportion for A (i.e., p_A) with the observed proportion for B (i.e., p_B). If p_A is not larger than p_B, proceed no further. There is no reason to believe that the characteristic proportion for Product A exceeds that for Product B.

(2) If p_A is larger than p_B, obtain the observed contrast pair exactly as in 2.2.1a and compare with the minimum contrast pair in Table XXVIII. With regard to the question asked here, the significance level of Table XXVIIIa is .025 and of Table XXVIIIb is .005.

Summary of procedure: Is P_A larger than P_B? (equal sample sizes)

Example:

i) Choose α, the significance level of the test.

ii) Use Table XXVIIIa for $\alpha = .025$ and Table XXVIIIb for $\alpha = .005$.

iii) Compute the observed proportion for Product A and the observed proportion for Product B. (See Table 2.2). If Class I is the class considered $p_A = r_A/n_A$ and $p_B = r_B/n_B$.

iv) If p_A is not larger than p_B, proceed no further. There is no reason to believe that the true proportion P_A is larger than P_B.

v) If p_A is larger than p_B, obtain data contrast pair as in Step 1 of 2.2.1a. Call this pair (a_1, a_2).

vi) Enter table with sample size (n_A or n_B) for each group.

vii) Call table pairs (A_1, A_2). Find the table pair where $A_1 = a_1$.

viii) If a_2 is equal to or larger than A_2, the observed contrast is significant at the chosen level. Conclude that the proportion for Product A exceeds that for Product B. Otherwise there is no reason to believe that the two proportions differ.

2.2.2 Comparing two proportions—samples of 20 or less and unequal in size.

Table XXIX is to be used for this case.

<u>Problem 2.2.2a</u> Is P_A different from P_B?

<u>Steps in solution</u>:

(1) The data table (Table 2.2) should be rearranged as follows:

Arrange rows so that the larger sample is in the first row. Identify the samples as to source.

	Class I	Class II	Total
n_1= larger sample	r_1	s_1	n_1
n_2= smaller sample	r_2	s_2	n_2
	r	s	n

(2) Focus on:

Class I if $r_1/n_1 \geq r_2/n_2$; Class II if $s_1/n_1 \geq s_2/n_2$. The observed contrast pair to be called (a_1, a_2) is equal to:

(r_1, r_2) if $r_1/n_1 \geq r_2/n_2$

(s_1, s_2) if $s_1/n_1 \geq s_2/n_2$.

(3) Enter Table XXIX in the section for n_1 and n_2 and the line for a_1. The observed a_2 must then be equal to or smaller than the bold face number in the body of the table for significance.

Example:

Consider the following data

	Class I	Class II	Total
Product A	$r_A = 4$	$s_A = 2$	$n_A = 6$
Product B	$r_B = 0$	$s_B = 10$	$n_B = 10$
Total	$r = 4$	$s = 12$	$n = 16$

Step 1. The rows are rearranged so that the larger sample is in the first row:

	Class I	Class II	Total
n_1	$r_1 = 0$	$s_1 = 10$	$n_1 = 10$
n_2	$r_2 = 4$	$s_2 = 2$	$n_2 = 6$
	$r = 4$	$s = 12$	$n = 16$

Step 2. Since $s_1/n_1 > s_2/n_2$, focus on Class II. The observed contrast pair (a_1,a_2) is equal to $(s_1,s_2) = (10,2)$.

Step 3. If the significance level is to be 0.05, we find in Table XXIX for $n_1 = 10$, $n_2 = 6$, and $a_1 = 10$, that a_2 must be 2 or less for significance. Therefore we conclude that P_A does differ from P_B.

<u>Problem 2.2.2b</u> Is P_A larger than P_B?

Consider the original data table (Table 2.2).

(1) Focus on the class of interest. If this is Class I, compute $p_A = r_A/n_A$ and $p_B = r_B/n_B$. If p_A is not larger than p_B, proceed no further. The data give no reason to believe that the true proportion P_A is larger than P_B.

(2) If p_A is larger than p_B proceed exactly as in 2.2.2a — i.e., rearrange rows to have the larger sample first and pick out the class which shows the larger proportion.

<u>Example</u>:

Consider the example of Problem 2.2.2a, but suppose that the only meaningful question to be asked is "Is P_A larger than P_B?

Step 1. Focus on the class of interest. If this is Class I, compute $p_A = r_A/n_A$ and $p_B = r_B/n_B$. Since p_A is larger than p_B, proceed to Step 2. (If p_A were not larger than p_B, no further steps would have been necessary.)

Step 2. Rearrange rows to put the larger sample in the first row:

	Class I	Class II	Total
n_1	$r_1 = 0$	$s_1 = 10$	$n_1 = 10$
n_2	$r_2 = 4$	$s_2 = 2$	$n_2 = 6$
Total	$r = 4$	$s = 12$	$n = 16$

Since $s_1/n_1 > s_2/n_2$, focus on Class II. The observed contrast pair (a_1, a_2) is equal to $(s_1, s_2) = 10, 2$. If the chosen significance level is to be 0.5, we find in Table XXIX for $n_1 = 10$, $n_2 = 6$, and $a_1 = 10$ that a_2 must be 3 or less for significance. We therefore conclude that P_A is larger than P_B.

2.2.3 Comparing two proportions—approximate method for large samples.

Problem 2.2.3a Is P_A different from P_B?

Procedure:

Example:

i) Choose α, the significance level of the test.

ii) Look up $\chi^2_{1-\alpha}$ for one degree of freedom in Table V.

iii) Compute:

$$\chi^2 = \frac{n\left[\,|r_A s_B - r_B s_A| - \frac{n}{2}\,\right]^2}{n_A\ r\ n_B\ s}$$

See NOTE below.

iv) If $\chi^2 \geq \chi^2_{1-\alpha}$, decide that the two products differ with regard to the proportion having the given characteristic. Otherwise, there is no reason to believe that the products differ in this respect.

<u>NOTE</u>: The computation of χ^2 is most conveniently done in terms of the actual counts in the table, as given in Step (iii) above. The formula can be expressed in terms of the observed proportions as follows:

$$\chi^2 = \frac{(n'|p_A - p_B| - \frac{1}{2})^2}{n' p(1-p)}$$

where

$$p_A = r_A/n_A$$

$$p_B = r_B/n_B$$

$$p = \frac{r_A + r_B}{n_A + n_B}$$

and

$$n' = \frac{n_A n_B}{n_A + n_B}$$

The two formulas are algebraically equivalent, but use of the form given in this note requires extra arithmetic and rounding. In spite of the fact that the question is put in terms of the difference between proportions, the answer is obtained more easily using observed <u>counts</u>. Furthermore, using the formula in terms of counts highlights the fact that one <u>cannot</u> judge the difference between two proportions without knowing the sample sizes involved.

Problem 2.2.3b Is P_A larger than P_B?

Procedure: | Example:

i) Choose α, the significance level of the test.

ii) Look up $\chi^2_{1-2\alpha}$ for one degree of freedom in Table V.

iii) Compute:

$$\chi^2 = \frac{n\left[\,|r_A s_B - r_B s_A| - \frac{n}{2}\right]^2}{n_A \; r \; n_B \; s}$$

See NOTE below.

iv) If $\chi^2 \geq \chi^2_{1-2\alpha}$ and r_A/n_A is larger than r_B/n_B, decide that the proportion in Class I for Product A exceeds the proportion in Class I for Product B. Otherwise there is no reason to believe the proportions differ.

__NOTE:__ The computation of χ^2 is most conveniently done in terms of the actual counts in the table, as given in Step (iii) above. The formula can be expressed in terms of the observed proportions as follows:

$$\chi^2 = \frac{\left(n' \left| p_A - p_B \right| - \frac{1}{2}\right)^2}{n' \, p(1-p)}$$

where

$$p_A = r_A/n_A$$

$$p_B = r_B/n_B$$

$$p = \frac{r_A + r_B}{n_A + n_B}$$

$$n' = \frac{n_A n_B}{n_A + n_B}$$

The two formulas are algebraically equivalent, but use of the form given in this note requires extra arithmetic and rounding. In spite of the fact that the question is put in terms of the difference between proportions, the answer is obtained more easily using observed __counts__. Furthermore, using the formula in terms of counts highlights the fact that one cannot judge the difference between proportions without knowing the sample sizes involved.

2.2.4 Sample size required to detect a difference between two proportions.

2.2.4a Sample size required to detect a difference between two proportions without regard to the sign of the difference.

Unfortunately, the sample size required depends on the true but unknown values of the two proportions involved. Very often the experimenter has some idea of the magnitude of (or an upper bound for) one of these values, and then must specify the size of the difference which the experiment should be designed to detect. For a fixed difference to be detected, the largest sample sizes will be required if the true proportions are in the neighborhood of 0.5. A look at Table XXVII, however, will show that over-conservatism does not pay. Suppose, for example, that one of the proportions can safely be assumed to be less than 0.4. The most conservative assumption would be that it is equal to 0.4 (this being the closest reasonable guess to 0.5). Attempting to be over-cautious by placing it at 0.45 will extract a heavy price in the number of tests to be run.

Given:

For this problem there is nothing given, but—

Assumed:

P' = an estimate of one of the two proportions.

To be conservative, make this estimate as close to 0.5 as is reasonable.

To be specified for this problem:

α = the significance level, or risk of announcing a difference when in fact there is none.

β = the risk of failing to detect a difference when in fact the true proportions differ by an amount δ (i.e., $|P' - P''| = \delta$).

δ = the amount of difference which is considered important to detect.

Tables and charts to be used:

Table XXVIIa can be used for α = .05 and 1-β = .50, .80, .90, .95, and .99, and certain values for the proportions. The entry in Table XXVIIa must be doubled to give n'. n' is the required sample size to be taken from each product.

For other desired values of α and β use:

Table XXVI, a table to convert the difference between the proportions into the necessary form for use with Table XVIIIa.

Procedure: Example:

i) Specify δ, the amount of difference considered important to detect.

ii) Choose α and β.

iii) For $\alpha = .05$ and $1-\beta = .50$, .80, .90, .95, or .99, go to Table XXVIIa.

iv) P' = an estimate of one of the proportions. Let $P'' = P' + \delta$ or $P' - \delta$, whichever makes P'' closer to 0.5.

v) If either P' or P'' is less than 0.5, enter Table XXVIIa with P' or P''. If neither P' nor P'' is less than 0.5, enter Table XXVIIa with $1-P'$ and $1-P''$. In either case the smaller of the two proportions determines the

column and the larger of the two determines the row. Read off n and double it to obtain n'. n' is the required sample size to be taken from each product.

vi) For other values of α and β, go to Table XXVI. Look up

$\theta' = \theta$ corresponding to P'

$\theta'' = \theta$ corresponding to P''.

vii) Compute $d = |\theta' - \theta''|$.

viii) Enter Table XVIIIa with α, β and d (from Step vii). Read off n and double it to obtain n'. Then n' is the required sample size to be taken from each product.

2.2.4b Sample size required to detect a difference between two proportions with regard to the sign of the difference.

Read first paragraph of 2.2.4a.

Given:

For this problem there is nothing given, but—

Assumed:

P' = an estimate of one of the two proportions.

To be conservative, make this estimate as close to 0.5 as is reasonable.

To be specified for this problem:

α = the significance level, or risk of announcing a difference when in fact there is none.

β = the risk of failing to detect a difference when in fact the true proportion for the other product is $P'' = P' + \delta$ or $P'' = P' - \delta$, as specified.

δ = the amount of difference considered important to detect.

Tables and charts to be used:

Table XXVIIb can be used for $\alpha = .05$; $1-\beta = .50, .80, .90, .95$, and $.99$; and certain values for the proportions.

For other desired values of α and β, use Table XXVI, a table to convert the difference between the proportions into the necees-sary form for use with Table XVIIIb.

Question to be answered:

Is P_A larger than P_B?

Estimate available:

Either P'_A, an estimate of P_A;

or P'_B, an estimate of P_B.

Procedure: | Example:

i) Specify δ, the amount of difference considered important to detect. If P'_A is available, then $P'' = P'_A - \delta$. If P'_B is available, then $P'' = P'_B + \delta$.

ii) Choose α and β.

iii) For $\alpha = .05$, $1-\beta = .50$, .80, .90, .95 or .99, go to Table XXVIIb.

iv) If either P' or P'' is less than 0.5, enter Table XXVIIb with P' and P''. If neither P' nor P'' is less

than 0.5, enter Table XXVIIb with 1-P' and 1-P". In either case, the smaller of the two proportions determines the column and the larger determines the row.

v) Read off n and double it to obtain n'. n' is the required sample size to be taken from each product.

vi) For other values of α and β, go to Table XXVI. Look up $\theta' = \theta$ corresponding to P' $\theta'' = \theta$ corresponding to P".

vii) Compute $d = |\theta' - \theta''|$.

viii) Enter Table XVIIIb with α, β, and d (from Step vii). Read off n and double it to obtain n'. n' is the required sample size to be taken from each product.

3. Comparisons of sets of proportions (three or more categories of classification).

In some inspection and testing procedures, two categories of classification (e.g., good-bad) will not be sufficient. For example, we might wish to classify an item into 3 categories (1) acceptable as is, (2) unacceptable but reworkable, and (3) unusable. In classifications by size, color, or structure it may be necessary to distinguish more than 2 categories in some comparisons. For classifications of this sort, we cannot use the methods given in preceding sections.

3.1 Comparison of an item, product or process with a standard, when a characteristic is classified into 3 or more categories.

Given:

A sample of n items selected at random from a much larger population. The items are classified into k categories, according to some criterion. A_i of the items are observed to be in the i^{th} category ($A_1 + A_2 + \ldots + A_k = n$). P_i = the known percentage of standard items which fall in category i. ("Standard item" may be a theoretical standard or specification standard.)

We shall make one of two decisions on the basis of analysis of data:

(1) The new item, product, or process differs from the

standard with regard to proportions in each category.

(2) There is no reason to believe that the new item, product, or process differs from the standard with regard to proportions in each category.

Solution:

The solution is approximate, but if $nP_i \geq 5.0$, the approximation is ordinarily very good. If $nP_i < 5$ for several categories, these categories may be pooled to obtain a theoretical frequency of at least 5 for the combined cells.

Procedure: Example:

i) Choose α, the significance level of the test.

ii) Look up $\chi^2_{1-\alpha}$ for k-1 degrees of freedom in Table V.

iii) Compute nP_i, the theoretical value for each category.

iv) Compute:

$$\chi^2 = \sum_{i=1}^{k} (A_i^2/nP_i) - n \quad .$$

v) If $\chi^2 \geq \chi^2_{1-\alpha}$, conclude that the item, product, or process differs from the standard with regard to proportions in each category. Otherwise, there is no reason to believe that they differ.

3.2 Comparison of two or more items, products or processes when each has several categories of classification.

Symbols to be used:

m = number of items, products, or processes to be compared.

k = number of categories of classification.

n_i = size of sample for the i^{th} item, product, or process.

x_{ij} = number of items of the i^{th} kind which are classified in the j^{th} category.

C_j = total number in the j^{th} category.

The data will be tabulated in the following form:

Item, Product, or Process	Category 1	2	...	k	Total
1	x_{11}	x_{12}	...	x_{1k}	n_1
2	x_{21}	x_{22}	...	x_{2k}	n_2
⋮	⋮	⋮		⋮	⋮
m	x_{m1}	x_{m2}	...	x_{mk}	n_m
Total	C_1	C_2	...	C_k	n

After analysis of the data, we shall make one of the following decisions:

1) The items, products, or processes differ with respect to the proportions in each category.

2) There is no reason to believe that the items, products, or processes differ in this regard.

Solution:

The solution is approximate, but should be quite accurate if the smallest $n_i C_j / n \geq 5$.

Procedure:

Example:

i) Choose α, the significance level of the test.

ii) Look up $\chi^2_{1-\alpha}$ for $(k-1)(m-1)$ degrees of freedom in Table V.

iii) Compute:

$$\chi^2 = n\left(\sum_{i=1}^{m}\sum_{j=1}^{k} \frac{x_{ij}^2}{n_i C_j} - 1\right)$$

iv) If $\chi^2 \geq \chi^2_{1-\alpha}$, decide that the items, products, or processes differ with regard to the proportion in each category. Otherwise, there is no reason to believe that they differ in this regard.

Simplified computation for the special case m=2

In this case the tabulation would consist of only the first two rows of the table given above, and

$$\chi^2 = \sum_{j=1}^{k} \left[\frac{n_1 n_2}{x_{1j} + x_{2j}} \left(\frac{x_{1j}}{n_1} - \frac{x_{2j}}{n_2} \right)^2 \right]$$

The degrees of freedom for χ^2 is k-1.

This form is convenient if the data are given in terms of proportions.

Further simplification for m=2 when $n_1 = n_2$

$$\chi^2 = \sum_{j=1}^{k} \frac{(x_{1j} - x_{2j})^2}{x_{1j} + x_{2j}}$$

with degrees of freedom = k-1.

NOTE: This short-cut has an analog for m=3 when $n_1 = n_2 = n_3$. For each category take all three possible differences, sum the squares of the three differences, and divide by the sum of the three observations. Finally, sum this quantity over all categories to obtain χ^2.

3.3 A test of association between two methods of classification.

There are situations in which individuals are classified into categories by means of two different criteria. For example, in a study of tire wear*, records of scrapping of tires were kept and tires were classified as front and rear, left and right. In another study of the cause of failure of vacuum tubes**, two criteria of classification were position in shell and type of failure. In each study the question was: Is there any association or relation between the criteria of classification?

This problem is a different one than the problem of section 3.2, but it is placed here because of the similarity in analysis.

We shall assume we have n individual items, classified by criteria A and B into k and m categories respectively. Let x_{ij} be the number of individuals in the i^{th} category of A and the j^{th} category of B. Let R_i and C_j be the total number of individuals classified in the i^{th} category of A and the j^{th} category of B respectively.

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* A. W. Swan, "The χ^2 Significance Test - Expected vs. Observed Results", The Engineer, December 31, 1948, p. 679.

** Besse B. Day, "Application of Statistical Methods to Research and Development in Engineering", Review of the International Statistical Institute, 1949, Nos. 3 and 4.

We would tabulate as follows:

		Criterion B				
		1	2	...	k	Total
Criterion A	1	x_{11}	x_{12}	...	x_{1k}	R_1
	2	x_{21}	x_{22}	...	x_{2k}	R_2
		⋮	⋮		⋮	⋮
	m	x_{m1}	x_{m2}	...	x_{mk}	R_m
Total		C_1	C_2	...	C_k	n

We will make, as a result of the analysis of the data, one of two decisions:

1) There is some relation or association between the two criteria of classification.

2) There is no reason to believe that such an association exists.

Solution: The solution is approximate, but should be quite accurate if the smallest of $R_i C_j / n \geq 5.0$.

Procedure: Example:

i) Choose α, the level of significance of the test.

ii) Look up $\chi^2_{1-\alpha}$ for $(k-1)(m-1)$ degrees of freedom in Table V.

iii) Compute:

$$\chi^2 = n\left(\sum_{i=1}^{m}\sum_{j=1}^{k} \frac{x_{ij}^2}{R_i C_j} - 1\right)$$

iv) If $\chi^2 \geq \chi^2_{1-\alpha}$, conclude that there is an association between the two criteria of classification. Otherwise, there is no reason to believe that such an association exists.

Table XXIV

Confidence Limits for a Proportion
(Two-Sided)

n=1				n=2			
r	90%	95%	99%	r	90%	95%	99%
0				0			
1				1			
				2			

n = 1,2,3,...,30

r = 0,1,2,3,...,n

Upper limits are in bold face. The observed proportion in a random sample is r/n.

To be reproduced from "Statistics Manual", NAVORD REPORT 3369, NOTS 948, by E. L. Crow, F. A. Davis and M. W. Maxfield, U. S. Naval Ordnance Test Station, China Lake, California, 1955.

Table XXV

Confidence Limits for a Proportion
(One-Sided)

n=2				n=3			
r	90%	95%	99%	r	90%	95%	99%
0				0			
1				1			
				2			

n = 1,2,3,...,30

r = 0,1,2,3,...,n

If the observed proportion is r/n, enter the table with **n** and **r** for an upper one-sided limit. For a lower one-sided limit, enter the table with **n** and **n-r** and subtract the table entry from 1.

To be reproduced from "Statistics Manual", NAVORD REPORT 3369, NOTS 948, by E. L. Crow, F. A. Davis and M. W. Maxfield, U. S. Naval Ordnance Test Station, China Lake, California, 1955.

Table XXVI

$\theta = 2 \text{ arc sin } \sqrt{P}$

P	θ	P	θ	P	θ	P	θ
.00	.00	.25	1.05	.50	1.57	.75	2.09
.01	.20	.26	1.07	.51	1.59	.76	2.12
.02	.28	.27	1.09	.52	1.61	.77	2.14
.03	.35	.28	1.12	.53	1.63	.78	2.17
.04	.40	.29	1.14	.54	1.65	.79	2.19
.05	.45	.30	1.16	.55	1.67	.80	2.21
.06	.49	.31	1.18	.56	1.69	.81	2.24
.07	.54	.32	1.20	.57	1.71	.82	2.27
.08	.57	.33	1.22	.58	1.73	.83	2.29
.09	.61	.34	1.25	.59	1.75	.84	2.32
.10	.64	.35	1.27	.60	1.77	.85	2.35
.11	.68	.36	1.29	.61	1.79	.86	2.37
.12	.71	.37	1.31	.62	1.81	.87	2.40
.13	.74	.38	1.33	.63	1.83	.88	2.43
.14	.77	.39	1.35	.64	1.85	.89	2.47
.15	.80	.40	1.37	.65	1.88	.90	2.50
.16	.82	.41	1.39	.66	1.90	.91	2.53
.17	.85	.42	1.41	.67	1.92	.92	2.57
.18	.88	.43	1.43	.68	1.94	.93	2.61
.19	.90	.44	1.45	.69	1.96	.94	2.65
.20	.93	.45	1.47	.70	1.98	.95	2.69
.21	.95	.46	1.49	.71	2.00	.96	2.74
.22	.98	.47	1.51	.72	2.03	.97	2.79
.23	1.00	.48	1.53	.73	2.05	.98	2.86
.24	1.02	.49	1.55	.74	2.07	.99	2.94
						1.00	3.14

Table XXVII

Sample Size Required for Comparing a Proportion with a Standard Proportion

(Consists of two parts — Table XXVIIa and Table XXVIIb)

The use of Table XXVII (or the equivalent use of Table XXVI and Table XVIII) is based on the inverse-sine transformation of the binomial to an approximately normal distribution.

Exact determination of required sample size could be made from tables of the binomial distribution, so far as the tables are available. (See "Tables of the Cumulative Normal Probability Distribution", Staff, Computation Laboratory, Harvard University, Harvard University Press, 1955, Introduction on Applications.)

The entries computed for the table were rounded to 3 significant figures and the rounding was always upward.

The table may also be used to determine the sample size required for comparing two proportions (see section 2.2.4).

Table XXVIIa

Sample Size Required to Detect a Difference
from a Standard Proportion
(without regard to the sign of the difference)

$\alpha = .05, \quad 1-\beta = .50$

Larger Proportion	Smaller Proportion											
	.001	.002	.005	.01	.02	.05	.10	.20	.30	.40	.45	.50
.01	205	313	1120									
.02	80	102	190	551								
.05	26	30	41	62	138							
.10	12	13	16	20	30	104						
.20	6	6	7	8	10	17	48					
.30	4	4	4	5	6	8	15	72				
.40	3	3	3	3	4	5	8	20	88			
.45	2	3	3	3	3	4	6	14	40	376		
.50	2	2	2	3	3	4	5	10	23	95	383	
.55	2	2	2	2	2	3	4	7	15	43	96	383
.60	2	2	2	2	2	3	4	6	11	24	43	95
.70	2	2	2	2	2	2	3	4	6	11	15	23
.80	1	1	1	1	2	2	2	3	4	6	7	10
.90	1	1	1	1	1	1	2	2	3	4	4	5
1.00	1	1	1	1	1	1	1	1	1	2	2	2

continued

Table XXVIIa

$\alpha = .05, \quad 1-\beta = .80$

Larger Proportion	Smaller Proportion .001	.002	.005	.01	.02	.05	.10	.20	.30	.40	.45	.50
.01	419	640	2280									
.02	162	208	388	1130								
.05	53	61	82	125	281							
.10	24	26	32	40	61	212						
.20	11	12	13	15	19	35	98					
.30	7	7	8	9	11	16	30	146				
.40	5	5	6	6	7	10	15	41	178			
.45	4	5	5	5	6	8	12	27	82	767		
.50	4	4	4	5	5	7	10	19	47	194	782	
.55	4	4	4	4	5	6	8	15	30	87	196	782
.60	3	3	3	4	4	5	7	11	21	49	87	194
.70	3	3	3	3	3	4	5	8	12	21	30	47
.80	2	2	2	2	3	3	4	5	8	11	15	19
.90	2	2	2	2	2	2	3	4	5	7	8	10
1.00	1	1	1	1	1	2	2	2	2	3	3	4

continued

Table XXVIIa

$\alpha = .05, \quad 1-\beta = .90$

Larger Proportion	Smaller Proportion .001	.002	.005	.01	.02	.05	.10	.20	.30	.40	.45	.50
.01	560	857	3040									
.02	217	279	520	1510								
.05	70	81	110	168	376							
.10	32	35	42	54	82	284						
.20	15	15	18	20	26	47	131					
.30	9	10	11	12	14	21	40	196				
.40	7	7	7	8	9	13	20	54	239			
.45	6	6	6	7	8	11	16	36	109	1030		
.50	5	5	6	6	7	9	13	26	63	260	1050	
.55	5	5	5	5	6	8	10	20	41	116	262	1050
.60	4	4	4	5	5	7	9	15	28	65	116	260
.70	3	3	4	4	4	5	6	10	16	28	41	63
.80	3	3	3	3	3	4	5	7	10	15	20	26
.90	2	2	2	2	3	3	4	5	6	9	10	13
1.00	2	2	2	2	2	2	2	3	3	4	4	5

continued

Table XXVIIa

$\alpha = .05, \quad 1-\beta = .95$

Larger Proportion	Smaller Proportion .001	.002	.005	.01	.02	.05	.10	.20	.30	.40	.45	.50
.01	693	1060	3760									
.02	268	345	642	1870								
.05	87	100	136	207	465							
.10	39	43	52	67	101	351						
.20	18	19	22	25	32	58	162					
.30	11	12	13	15	17	26	49	242				
.40	8	8	9	10	12	16	25	67	295			
.45	7	7	8	9	10	13	19	45	135	1270		
.50	6	6	7	7	8	11	16	32	77	321	1300	
.55	6	6	6	7	7	9	13	24	50	143	324	1300
.60	5	5	5	6	6	8	11	19	35	81	143	321
.70	4	4	4	5	5	6	8	12	20	35	50	77
.80	3	3	4	4	4	5	6	8	12	19	24	32
.90	3	3	3	3	3	4	4	6	8	11	13	16
1.00	2	2	2	2	2	2	3	3	4	5	5	6

continued

Table XXVIIa

$\alpha = .05, \quad 1-\beta = .99$

Larger Proportion	Smaller Proportion .001	.002	.005	.01	.02	.05	.10	.20	.30	.40	.45	.50
.01	979	1500	5320									
.02	378	487	908	2640								
.05	123	141	192	293	658							
.10	55	60	73	94	142	496						
.20	25	27	30	35	45	81	229					
.30	16	17	18	20	24	37	70	342				
.40	11	12	13	14	16	22	35	94	417			
.45	10	10	11	12	14	18	27	63	190	1800		
.50	9	9	9	10	12	15	22	45	109	453	1830	
.55	8	8	8	9	10	13	18	34	71	202	458	1830
.60	7	7	7	8	9	11	15	26	49	114	202	453
.70	5	6	6	6	7	8	11	17	28	49	71	109
.80	4	5	5	5	5	6	8	12	17	26	34	45
.90	4	4	4	4	4	5	6	8	11	15	18	22
1.00	2	2	3	3	3	3	3	4	5	6	7	8

Table XXVIIb

Sample Size Required to Detect a Difference from a Standard Proportion (with regard to the sign of the difference)

$\alpha = .05, \quad 1-\beta = .50$

Larger Proportion	Smaller Proportion											
	.001	.002	.005	.01	.02	.05	.10	.20	.30	.40	.45	.50
.01	145	221	783									
.02	56	72	134	389								
.05	19	21	29	44	97							
.10	9	9	11	14	21	74						
.20	4	4	5	6	7	12	34					
.30	3	3	3	3	4	6	11	51				
.40	2	2	2	2	3	4	6	14	62			
.45	2	2	2	2	2	3	4	10	28	265		
.50	2	2	2	2	2	3	4	7	16	67	270	
.55	2	2	2	2	2	2	3	5	11	30	68	270
.60	1	1	2	2	2	2	3	4	8	17	30	67
.70	1	1	1	1	1	2	2	3	4	8	11	16
.80	1	1	1	1	1	1	2	2	3	4	5	7
.90	1	1	1	1	1	1	1	2	2	3	3	4
1.00	1	1	1	1	1	1	1	1	1	1	1	2

continued

Table XXVIIb

$\alpha = .05, \quad 1-\beta = .80$

Larger Proportion	Smaller Proportion .001	.002	.005	.01	.02	.05	.10	.20	.30	.40	.45	.50
.01	330	504	1790									
.02	128	164	306	888								
.05	42	48	65	99	222							
.10	19	21	25	32	48	167						
.20	9	9	11	12	15	28	77					
.30	6	6	6	7	9	13	24	115				
.40	4	4	5	5	6	8	12	32	141			
.45	4	4	4	4	5	6	10	21	64	604		
.50	3	3	4	4	4	5	8	15	37	153	617	
.55	3	3	3	3	4	5	6	12	24	68	155	617
.60	3	3	3	3	3	4	5	9	17	39	68	153
.70	2	2	2	2	3	3	4	6	10	17	24	37
.80	2	2	2	2	2	2	3	4	6	9	12	15
.90	2	2	2	2	2	2	2	3	4	5	6	8
1.00	1	1	1	1	1	1	1	2	2	2	3	3

continued

Table XXVIIb

$\alpha = .05,\ 1-\beta = .90$

Larger Proportion	Smaller Proportion .001	.002	.005	.01	.02	.05	.10	.20	.30	.40	.45	.50
.01	457	698	2480									
.02	177	227	424	1230								
.05	57	66	90	137	307							
.10	26	28	34	44	67	232						
.20	12	13	14	17	21	38	107					
.30	8	8	9	10	12	18	33	160				
.40	6	6	6	7	8	11	17	44	195			
.45	5	5	5	6	7	9	13	30	89	837		
.50	4	4	5	5	6	7	10	21	51	212	854	
.55	4	4	4	4	5	6	9	16	33	95	214	854
.60	3	4	4	4	4	5	7	13	23	53	95	212
.70	3	3	3	3	3	4	5	8	13	23	33	51
.80	2	2	2	3	3	3	4	6	8	13	16	21
.90	2	2	2	2	2	3	3	4	5	7	9	10
1.00	1	1	1	1	2	2	2	2	3	3	4	4

continued

Table XXVIIb

$\alpha = .05, \quad 1-\beta = .95$

Larger Proportion	Smaller Proportion .001	.002	.005	.01	.02	.05	.10	.20	.30	.40	.45	.50
.01	577	882	3140									
.02	223	287	535	1560								
.05	73	83	113	173	388							
.10	33	36	43	56	84	293						
.20	15	16	18	21	27	48	135					
.30	10	10	11	12	15	22	41	202				
.40	7	7	8	8	10	13	21	56	246			
.45	6	6	7	7	8	11	16	37	112	1060		
.50	5	5	6	6	7	9	13	27	64	267	1080	
.55	5	5	5	6	6	8	11	20	42	119	270	1080
.60	4	4	5	5	5	7	9	16	29	67	119	267
.70	3	4	4	4	4	5	7	10	16	29	42	64
.80	3	3	3	3	3	4	5	7	10	16	20	27
.90	2	2	2	3	3	3	4	5	7	9	11	13
1.00	2	2	2	2	2	2	2	3	3	4	4	5

continued

Table XXVIIb

α = .05, 1-β = .99

Larger Proportion	Smaller Proportion											
	.001	.002	.005	.01	.02	.05	.10	.20	.30	.40	.45	.50
.01	841	1290	4570									
.02	325	418	779	2270								
.05	105	121	165	251	565							
.10	47	52	63	81	122	426						
.20	22	23	26	30	39	70	196					
.30	14	14	16	18	21	32	60	293				
.40	10	10	11	12	14	19	30	81	358			
.45	8	9	9	10	12	16	24	54	163	1540		
.50	7	8	8	9	10	13	19	39	94	389	1580	
.55	7	7	7	8	9	11	15	29	61	174	393	1580
.60	6	6	6	7	8	10	13	23	42	98	174	389
.70	5	5	5	5	6	7	9	15	24	42	61	94
.80	4	4	4	4	5	6	7	10	15	23	29	39
.90	3	3	3	3	4	4	5	7	9	13	15	19
1.00	2	2	2	2	2	3	3	4	5	6	6	7

Table XXVIII

Minimum Contrasts Required for Significance in 2x2 Tables with Equal Samples[1/]

(Consists of two parts — Table XXVIIIa and Table XXVIIIb)

Directions for filling in significant contrasts which have been omitted from Tables XXVIII a and b:

In some sample sizes, some entries have been omitted, but only where they are easy to supply. For example, see $n_A = n_B = 80$. There is an entry (16,29) followed by an entry (23,36). The difference between the first numbers of these pairs is the same as the difference between the second numbers of the pairs. Thus contrast pairs (17,30), (18,31), (19,32), etc., are also significant contrasts and are omitted only to save space.

Procedure for values of n which are not tabulated:

In many cases, Table XXVIII can be used to give a good idea of the significance of an observed contrast for values of n intermediate to those tabulated. For example, consider two samples of n = 320 items each:

	Class I	Class II	Total
Sample A	92	228	320
Sample B	117	203	320

Looking in the table for n = 300, we find that a significant contrast would be (92,116), and for n = 400, a significant contrast would be (92,118). We therefore know that the observed contrast (92,117) is approximately significant at the 5% level.

If this method is not considered sufficient in a particular case, use the χ^2 method described in section 2.2.3. (The χ^2 method is an approximation which gives good results for cases not covered by the table.)

1/ Adapted, with permission, from Tables I and II of D. Mainland, L. Herrera and M. Sutcliffe "Tables for Use with Binomial Samples", Department of Medical Statistics, New York University College of Medicine, 1956.

Table XXVIIIa

Minimum Contrasts Required in 2x2 Tables
with Equal Samples for Significance at the:

5% Level - Two-Sided ("Is P_A different from P_B?")
2.5% Level - One-Sided ("Is P_A larger than P_B?")

Sample Size $n_A = n_B$	A_1, A_2								
4	0,4								
5	0,4								
6	0,5								
7	0,5	1,6							
8	0,5	1,6							
9	0,5	1,6							
10	0,5	1,7	2,8						
11	0,5	1,7	2,8						
12	0,5	1,7	2,8	3,9					
13	0,5	1,7	2,8	3,9					
14	0,5	1,7	2,8	3,10					
15	0,5	1,7	2,9	3,10	4,11				
16	0,5	1,7	2,9	3,10	4,11				
17	0,5	1,7	2,9	3,10	4,11	5,12			
18	0,5	1,7	2,9	3,10	4,11	5,12			
19	0,5	1,7	2,9	3,10	4,11	5,12			
20	0,5	1,7	2,9	3,10	4,11	5,13	6,14		
30	0,6	1,8	2,9	3,11	4,12	5,13	6,15	7,16	8,17
	9,18	10,19							
40	0,6	1,8	2,9	3,11	4,12	5,14	6,15	7,16	8,18
	9,19	10,20	15,25						

Table XXVIIIa

(continued)

Sample Size $n_A = n_B$	A_1, A_2
50	0,6 1,8 2,10 3,11 4,13 5,14 6,15 7,17 8,18 9,19 10,20 11,22 19,30
60	0,6 1,8 2,10 3,11 4,13 5,14 6,16 7,17 8,18 9,20 10,21 11,22 12,23 13,24 14,26 24,36
70	0,6 1,8 2,10 3,11 4,13 5,14 6,16 7,17 8,18 9,20 10,21 11,22 12,23 13,25 18,30 19,32 20,33 28,41
80	0,6 1,8 2,10 3,11 4,13 5,14 6,16 7,17 8,19 9,20 10,21 11,22 12,24 13,25 14,26 15,27 16,29 23,36 24,38 33,47
90	0,6 1,8 2,10 3,11 4,13 5,14 6,16 7,17 8,19 9,20 10,21 11,23 12,24 13,25 14,26 15,28 20,33 21,35 31,45 32,47 37,52
100	0,6 1,8 2,10 3,11 4,13 5,15 6,16 7,17 8,19 9,20 10,21 11,23 12,24 13,25 14,27 18,31 19,33 25,39 26,41 42,57
150	0,6 1,8 2,10 3,12 4,13 5,15 6,16 7,18 8,19 9,20 10,22 11,23 12,24 13,26 14,27 15,28 16,30 19,33 20,35 25,40 26,42 32,48 33,50 41,58 42,60 66,84
200	0,6 1,8 2,10 3,12 4,13 5,15 6,16 7,18 8,19 9,21 10,22 11,23 12,25 13,26 14,27 15,29 18,32 19,34 22,37 23,39 27,43 28,45 33,50 34,52 41,59 42,61 51,70 52,72 65,85 66,87 89,110

Table XXVIIIa

(continued)

Sample Size $n_A = n_B$	A_1, A_2
300	0,6 1,8 2,10 3,12 4,13 5,15 6,16 7,18 8,19 9,21 10,22 11,24 12,25 13,26 14,28 15,29 16,30 17,31 18,33 19,34 20,35 21,37 24,40 25,42 29,46 30,48 35,53 36,55 41,60 42,62 48,68 49,70 56,77 57,79 66,88 67,90 78,101 79,103 95,119 96,121 137,162
400	0,6 1,8 2,10 3,12 4,13 5,15 6,17 7,18 8,19 9,21 10,22 11,24 12,25 13,26 14,28 15,29 16,30 17,32 20,35 21,37 24,40 25,42 28,45 29,47 33,51 34,53 38,57 39,59 44,64 45,66 51,72 52,74 58,80 59,82 67,90 68,92 76,100 77,102 87,112 88,114 100,126 101,128 117,144 118,146 141,169 142,171 185,214
500	0,6 1,8 2,10 3,12 4,13 5,15 6,17 7,18 8,19 9,21 10,22 11,24 12,25 13,26 14,28 15,29 16,30 17,32 18,33 19,34 20,36 23,39 24,41 27,44 28,46 32,50 33,52 37,56 38,58 42,62 43,64 48,69 49,71 55,77 56,79 62,85 63,87 70,94 71,96 79,104 80,106 89,115 90,117 100,127 101,129 113,141 114,143 128,157 129,159 147,177 148,179 172,203 173,205 234,266

Table XXVIIIb

Minimum Contrasts Required in 2x2 Tables
with Equal Samples for Significance at the:

1% Level - Two-Sided ("Is P_A different from P_B?")
0.5% Level - One-Sided ("Is P_A larger than P_B?")

Sample Size $n_A = n_B$	A_1, A_2
5	0,5
6	0,6
7	0,6
8	0,6
9	0,6 1,8
10	0,7 1,8
11	0,7 1,8 2,9
12	0,7 1,8 2,10
13	0,7 1,9 2,10
14	0,7 1,9 2,10 3,11
15	0,7 1,9 2,10 3,11
16	0,7 1,9 2,10 3,12
17	0,7 1,9 2,11 3,12 4,13
18	0,7 1,9 2,11 3,12 4,13
19	0,7 1,9 2,11 3,12 4,13 5,14
20	0,7 1,9 2,11 4,13 5,15
30	0,8 1,10 2,12 3,13 4,15 9,20
40	0,8 1,10 2,12 3,14 4,15 5,17 8,20 9,22 13,26
50	0,8 1,10 2,12 3,14 4,15 5,17 6,18 7,20 9,22 10,24 18,32

Table XXVIIIb

(continued)

Sample Size $n_A = n_B$	A_1, A_2
60	0,8 1,10 2,12 3,14 4,16 5,17 6,19 8,21 9,23 11,25 12,27 19,34 20,36 22,38
70	0,8 1,10 2,12 3,14 4,16 5,17 6,19 7,20 8,22 10,24 11,26 14,29 15,31 21,37 22,39 26,43
80	0,8 1,10 2,12 3,14 4,16 5,18 6,19 7,21 9,23 10,25 12,27 13,29 16,32 17,34 24,41 25,43 31,49
90	0,8 1,10 2,12 3,14 4,16 5,18 6,19 7,21 8,22 9,24 11,26 12,28 15,31 16,33 19,36 20,38 28,46 29,48 35,54
100	0,8 1,10 2,13 3,14 4,16 5,18 6,19 7,21 8,22 9,24 10,25 11,27 14,30 15,32 18,35 19,37 23,41 24,43 33,52 34,54 40,60
150	0,8 1,11 2,13 3,15 4,16 5,18 6,20 7,21 8,23 9,24 10,26 11,27 12,29 14,31 15,33 17,35 18,37 21,40 22,42 26,46 27,48 31,52 32,54 39,61 40,63 51,74 52,76 63,87
200	0,8 1,11 2,13 3,15 4,16 5,18 6,20 7,21 8,23 9,24 10,26 11,27 12,29 13,30 14,32 16,34 17,36 19,38 20,40 23,43 24,45 26,47 27,49 31,53 32,55 36,59 37,61 43,67 44,69 51,76 52,78 63,89 64,91 86,113
300	0,8 1,11 2,13 3,15 4,17 5,18 6,20 7,22 8,23 9,25 10,26 11,28 12,29 13,31 15,33 16,35 17,36 18,38 20,40 21,42 23,44 24,46 27,49 28,51 31,54 32,56 35,59 36,61 40,65 41,67

Table XXVIIIb

(continued)

Sample Size $n_A = n_B$	A_1, A_2
300 (cont'd)	45,71 46,73 51,78 52,80 58,86 59,88 66,95 67,97 76,106 77,108 88,119 89,121 107,139 108,141 133,166
400	0,8 1,11 2,13 3,15 4,17 5,18 6,20 7,22 8,23 9,25 10,26 11,28 12,29 13,31 14,32 15,34 17,36 18,38 19,39 20,41 22,43 23,45 26,48 27,50 29,52 30,54 33,57 34,59 37,62 38,64 41,67 42,69 46,73 47,75 52,80 53,82 57,86 58,88 64,94 65,96 71,102 72,104 79,111 80,113 88,121 89,123 98,132 99,134 111,146 112,148 127,163 128,165 152,189 153,191 181,219
500	0,8 1,11 2,13 3,15 4,17 5,18 6,20 7,22 8,24 9,25 10,27 11,28 12,30 14,32 15,34 16,35 17,37 19,39 20,41 22,43 23,45 25,47 26,49 28,51 29,53 32,56 33,58 35,60 36,62 40,66 41,68 44,71 45,73 49,77 50,79 54,83 55,85 59,89 60,91 65,96 66,98 72,104 73,106 79,112 80,114 86,120 87,122 95,130 96,132 104,140 105,142 115,152 116,154 127,165 128,167 141,180 142,182 159,199 160,201 184,225 185,227 229,271

Table XXIX

Tables for Testing Significance in 2x2 Tables with Unequal Samples

	a_1	Significance Level			
		0.05 (0.10)	0.025 (0.05)	0.01 (.02)	0.005 (.10)
$n_1 = 3$ $n_2 = 3$	3				
$n_1 = 4$ $n_2 = 4$	4				
3	4				
$n_1 = 5$ $n_2 = 5$	5 4				
4	5 4				
3	5				
2	5				
⋮					
$n_1 = 20$ $n_2 = 20, 19, 18, \ldots, 2$					

The table shows (1) in bold type for given a_1, n_1, and n_2, the value of a_2 which is just significant at the probability level quoted in parentheses for a two-sided test and without parentheses for a one-sided test, (2) in small type, for given n_1, n_2 and $a_1 + a_2$, the exact probability (if there is independence) that a_2 is equal to or less than the integer shown in bold type.

To be reproduced from D. J. Finney, Table 38, pp 188-193, Biometrika Tables for Statisticians I, edited by E. S. Pearson and H. O. Hartley, Cambridge University Press (1954) and R. Latscha, "Tests of Significance in a 2x2 Contingency Table: Extension of Finney's Table", Biometrika 40, Parts 1 and 2 (June 1953).

U. S. DEPARTMENT OF COMMERCE
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Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships. VHF Research. Ionospheric Communication Systems.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Systems. Navigation Systems. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Radio Systems Application Engineering. Radio-Meteorology.

Radio Standards. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Electronic Calibration Center. Microwave Physics. Microwave Circuit Standards.

NBS