

NATIONAL BUREAU OF STANDARDS REPORT

6217

PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION
A Quarterly Report
July through September 1958

FOR OFFICIAL DISTRIBUTION



U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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NBS PROJECT

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July 1 through September 30, 1958

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*Only unclassified projects are included in this report.

Status of Projects

September 30, 1958

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS

Task 1101-12-1104/55-55

Origin: NBS

Authorized 8/29/54

Manager: P. Davis

Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. W. Rheinboldt continued his studies of a general theory of asymptotic convergence and in particular of the applications to linear operators in Hilbert Space.

W. Greub and W. Rheinboldt continued their studies of the applications of functional analysis to numerical analysis. Special attention was given to the method of steepest descent for functional equations in Hilbert Space. A joint paper entitled "On a Generalization of an Inequality of L. V. Kantorovitch" has been submitted to a technical journal.

W. Greub investigated the method of Lototsky for the summation of divergent series. The relation between the Lototsky method and the classical methods of Borel, Euler, and Knopp are obtained in a very natural way if the series transformation is associated with an integral transformation. A manuscript entitled "On the Relations Between Summation Methods and Integral Transforms" has been completed.

F. W. J. Olver has carried out extensive investigations of the uniform asymptotic expansions of parabolic cylinder functions of large order. A manuscript entitled "Uniform Asymptotic Expansions for Weber Parabolic Cylinder Functions of Large Orders" has been completed.

N. Bazley has continued his research in estimating lower bounds for eigenvalues. Considering the radial Schrödinger equation

$$y'' - k^2 y' + \lambda_n \frac{e^{-x}}{x} y = 0, \quad y(0) = y(\infty) = 0,$$

a 704 code has been written which yields upper and lower bounds for the first few eigenvalues λ_n for each fixed k . It is planned to alter the code so that it will work when angular momentum is introduced. Work on lower bound estimates for helium atom eigenvalues is continuing.

E. Brauer has completed a code for the computation of the greatest positive characteristic root of a positive or irreducible non-negative matrix. The code makes use of a method proposed by Alfred Brauer.

E. Haynsworth has prepared a code embodying two variations of the method of A. Brauer. These variations should improve the speed of the convergence.

Publications:

- (1) On the numerical integration of periodic analytic functions. P. J. Davis. To appear in the Proceedings of the Symposium on Numerical Approximation, held at Madison, Wisconsin, April 1958.
- (2) Computation problems concerned with the Hilbert matrix. J. Todd. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.
- (3) Reducible linear differential systems. H. A. Antosiewicz. In manuscript.
- (4) On the convergence of the Rayleigh quotient iteration for the computation of the characteristic roots and vectors, II. A. Ostrowski. Submitted to a technical journal.
- (5) On the bounds of a one-parametric family of matrices. A. Ostrowski. Submitted to a technical journal.
- (6) On permanents of doubly stochastic matrices. M. Marcus and M. Newman. To appear in the Duke Mathematical Journal.
- (7) Field convexity of a linear transformation. M. Marcus and A. J. Goldman. Submitted to a technical journal.
- (8) A continuous poker game. A. J. Goldman and J. J. Stone. Submitted to a technical journal.
- (9) On a determinantal inequality. M. Marcus. To appear in the American Mathematical Monthly.
- (10) On a generalization of an inequality of L. V. Kantorovitch. W. Greub and W. Rheinboldt. To appear in a technical journal.
- (11) On the relations between summation methods and integral transforms. W. Greub. In manuscript.
- (12) Uniform asymptotic expansions for Weber parabolic cylinder functions of large orders. F. W. J. Olver. In manuscript.
- (13) Generalizations of Brauer theorems on stochastic matrices. E. Haynsworth. In manuscript.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO
NUMERICAL ANALYSIS

Task 1101-12-5116/55-56

Origin: NBS

Authorized 8/13/54

Sponsor: Office of Naval Research

Manager: M. Newman

Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. M. Newman is continuing his study of divisibility properties of the coefficients of modular forms. In this connection he has shown that the partition function fills all residue classes infinitely

often modulo 2, 5 and 13. The same fact has been demonstrated for the coefficients of a large class of modular forms and a certain infinite set of moduli. A manuscript is in preparation.

M. Newman is continuing his work on the construction of polynomial bases for classes of automorphic functions.

K. Goldberg has derived a formula for the number of gaps of length m in the sequence of integers less than and relatively prime to n . He was aided in this work by computations made by L. E. Clarenbach.

K. Goldberg has begun a survey of the literature pertaining to incidence matrices. Together with M. R. Kellington he is compiling a complete bibliography of all papers in the field.

E. Brauer completed the computation of the coefficients of the infinite product

$$\prod_{i=1}^t \prod_{n=1}^{\infty} (1 - x^{e_i n r_i})$$

for certain values of e_i, r_i . In all, some 40,000 coefficients were computed representing about ten hours of machine time. These coefficients are of importance in the theory of elliptic modular functions. The actual coefficients computed were suggested by J. C. P. Miller and served to extend and verify tables of Ferguson and Miller.

A. Ostrowski finished a paper on the application of the generalized Rayleigh quotient to a cubically convergent iteration procedure which gives even for nonsymmetric matrices the eigenvalues corresponding to linear elementary divisors. He was assisted in this work by E. Haynsworth.

A. Goldman is preparing an expository paper entitled "Tensor, Grassmann and Clifford algebras."

Publications:

- (1) Some computational problems concerning integral matrices. O. Taussky. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.
- (2) On the minimization of concave and convex functionals. G. B. Dantzig (RAND Corporation), A. J. Hoffman, and W. Hirsch (New York University). In manuscript.
- (3) Abelian groups of unimodular matrices. E. C. Dade. Submitted to a technical journal.
- (4) Incidence algebras. E. C. Dade and K. Goldberg. In manuscript.
- (5) The construction of Hadamard matrices. K. Goldberg and E. C. Dade. Submitted to a technical journal.
- (6) Dense subgraphs and connectivity. R. E. Nettleton (NBS, 3.2), K. Goldberg, and M. S. Green (NBS, 3.2). To appear in the Canadian Journal of Mathematics.
- (7) Some combinatorial lemmas. K. Goldberg. In manuscript.

Status of Projects

- (8) On normal and EPr matrices. M. Pearl. To appear in the Michigan Journal of Mathematics.
- (9) A further extension of Cayley's parameterization. M. Pearl. To appear in the Canadian Journal of Mathematics.
- (10) On a theorem of M. Riesz. M. Pearl. To appear in the Journal of Research, NBS.
- (11) Inclusion theorems for congruence subgroups. M. Newman and I. Reiner (University of Illinois). To appear in Transactions of the American Mathematical Society.
- (12) Further identities and congruences for the coefficients of modular forms. M. Newman. To appear in Canadian Journal of Mathematics.
- (13) Congruences for the coefficients of modular forms and for the coefficients of $j(\tau)$. M. Newman. Proc. Amer. Math. Soc. 9, 609-612(1958).
- (14) Construction and application of a class of modular functions, II. M. Newman. To appear in Proceedings of London Mathematical Society.
- (15) Tensor, Grassmann, and Clifford algebras. A. Goldman. In manuscript.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION

Task 1101-12-5116/56-148

Origin and Sponsor: National Institutes of Health Authorized 9/30/55

Manager: W. Gautschi (11.2)

Full task description: July-Sept 1955 issue, p. 7

Status: INACTIVE.

EVALUATION OF MATRIX COMPUTATION PROGRAM

Task 1101-12-5116/57-200

Origin: NBS

Authorized 12/19/57

Sponsor: David Taylor Model Basin

Manager: M. Newman

Full task description: Oct-Dec 1957 issue, p. 4

Status: COMPLETED. Additional results were incorporated into the publication item below.

Publication:

- (1) The evaluation of matrix inversion programs. M. Newman and J. Todd. To appear in the Journal of the Society for Industrial and Applied Mathematics.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF COULOMB WAVE FUNCTIONS

Task 1102-40-1110/47-2

Origin: NBS

Authorized 7/1/47

Manager: I. A. Stegun

Full task description: Apr-June 1949 issue, p. 45

Status: INACTIVE.

TABLES OF POWER POINTS OF ANALYSIS-OF-VARIANCE TESTS

Task 1102-40-1110/51-8

Origin: Section 11.3, NBS

Authorized 3/26/51

Manager: S. Peavy

Full task description: Apr-June 1951 issue, p. 49

Status: INACTIVE.

REVISION OF MATHEMATICAL TABLES

Task 1102-40-1110/52-7

Origin: NBS

Authorized 8/10/51

Managers: W. F. Cahill, I. Stegun

Full task description: July-Sept 1951 issue, p. 41

Status: INACTIVE.

Status of Projects

SPHEROIDAL WAVE FUNCTIONS

Task 1102-40-1110/52-37

Origin: NBS

Authorized 11/28/51

Manager: D. Liepman

Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE.

SIEVERT'S INTEGRAL

Task 1102-40-1110/52-57

Origin: NBS

Authorized 2/12/52

Managers: M. Paulsen, P. O'Hara

Full task description: Jan-Mar 1952 issue, p. 46

Status: INACTIVE. For status to date, see Jan-Mar 1958 issue, p. 8.

HEAT TRANSFER

Task 1102-40-1110/57-241

(formerly 3711-60-0009/57-241)

Origin: NBS

Authorized 6/30/57

Manager: W. F. Cahill

Full task description: Apr-June 1957 issue, p. 31

Status: COMPLETED.

Publication:

- (1) Heat transfer in laminar flow through a tube. M. Abramowitz, W. F. Cahill, and C. Wade. Submitted to a technical journal.

MATHEMATICAL SUBROUTINES
Task 3711-60-0009/56-160

Origin: NBS

Authorized 9/30/55

Managers: Staff

Full task description: July-Sept 1955 issue, p. 13

Status: CONTINUED. E. Brauer completed the code for finding the largest eigenvalue of a positive matrix (see Apr-June 1958 issue, p.8). E. Haynsworth is now coding a variation of this routine which should speed up the convergence.

A subroutine is being prepared by P. Walsh to solve the general secular equation $|A - \lambda B| = 0$, where A is real, symmetric and B is real, symmetric, and positive definite.

AUTOMATIC CODING
Task 3711-60-1120/55-65

Origin: NBS

Authorized 9/29/54

Manager: J. Wegstein

Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. The CORBIE system was modified to apply with 8192 words of core memory on the 704. A considerable number of experiments were carried out with two routines for minimizing an arbitrary function of up to eight parameters. The minimizers were used for solving simultaneous transcendental equations and curve fitting with non-polynomial functions. However, a satisfactory minimizer has not been found.

The following subroutines were prepared for submission to the SHARE organization:

- (1) 704 Chi Square Distribution Subroutine, by J. Menard and D. Templeton
- (2) 704 Binary Coded Decimal Output Subroutine, by A. Schopf
- (3) 704 Student Distribution Subroutine, by J. Menard and D. Templeton.

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-5113/57-216

Origin and Sponsor: National Science Foundation

Authorized 12/27/56

Manager: I. A. Stegun

Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. The text for Chapter 25, Numerical Interpolation, Differentiation, and Integration, by P. Davis and I. Polonsky, has been revised for editorial review. The accompanying tables are ready in preliminary form.

The text for Chapter 15, Hypergeometric Functions, by F. Oberhettinger, has been typed and is being revised. Planning at the present time does not call for the inclusion of any tables.

Chapter 27, Miscellaneous Functions, has been revised for editorial review, and the accompanying tables are ready in preliminary form.

Drafting of the graphs to be included in chapters 3, 4, 5, 6, 7, 12, 13, and 17 has been completed. Work is currently under way on the graphs for chapters 9, 10, 14, and 22.

Typing and checking of the manuscript of the tabular material has been completed for chapters 5, 7, 13, 14, and 17. Processing of the tabular material is under way for chapters 9, 10, 11, 23, and 25.

3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS

Task 1103-12-1107/51-2

Origin: NBS

Authorized 7/1/50

Manager: C. Eisenhart

Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. N. C. Severo is preparing a manuscript on the convergence of distributions of random variables whose powers are normal. The convergence is studied as the coefficient of variation approaches zero. The paper also presents propagation of error formulas in which the order terms involve the coefficient of variation.

W. S. Connor and N. C. Severo are preparing a manuscript discussing the mathematical formulation and solution of a problem discussed and graphically solved by W. J. Youden in a manuscript entitled "Measurements made by matching with known standards."

Publication:

- (1) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. Joel. Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN

Task 1103-12-1107/53-1

Origin: NBS

Authorized 10/15/52

Manager: W. S. Connor

Full task description: Oct-Dec 1952 issue, p. 60

Status: INACTIVE.

Publication:

- (1) Multi-variable experiments. M. Zelen and W. S. Connor. To appear in Industrial Quality Control.

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES

Task 1103-12-1107/56-170

Origin: NBS

Authorized 12/15/55

Manager: Joan R. Rosenblatt

Full task description: Oct-Dec 1955 issue, p. 14

Status: INACTIVE.

MEASUREMENT OF RELIABILITY

Task 1103-12-1130/56-182

Origin: NBS

Authorized 3/23/56

Manager: M. Zelen

Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. M. Zelen presented a paper entitled "Analysis of two-factor classifications with respect to life tests," at the Institute of Mathematical Statistics meeting, Cambridge, Mass., August 1958.

J. R. Rosenblatt and M. Zelen have been engaged in work on a Defense Department problem pertaining to the reliability of missiles.

Dr. Rosenblatt made a rough translation of "The Calculation of the Reliability of Automatic Electrical Systems", by G. V. Druzhinin (Automatika i Telemekhanika (Moscow) 18, 1136-1138, 1957). She is also editing for publication her paper, "On prediction of system behavior," that was presented at the NYU-Industry Conference on Reliability, Ardsley-on-Hudson, New York, June 1958.

Publications:

- (1) Problems in life testing: Factorial experiments. M. Zelen. To appear in the Proceedings of the Thirteenth Midwest Quality Control Conference, Kansas City, Mo., November 1958.
- (2) Analysis of two-factor classifications with respect to life tests. M. Zelen. In manuscript.

FRACTIONAL FACTORIALS FOR THE MIXED SERIES

Task 1103-12-5148/58-291

Origin and Sponsor: Bureau of Ships

Authorized 9/30/57

Managers: W. S. Connor, M. Zelen

Full task description: July-Sept 1957 issue, p. 43

Status: CONTINUED. About 30 fractional factorial designs of the $2^m 3^n$ series have been constructed and analyzed. It is expected to

construct about 40 designs altogether, viz., one for each of the cases 2^4_3 , ..., 2^9_3 ; $2^3_3^2$, ..., $2^8_3^2$; ... ; $2 \cdot 3^9$. Construction and analysis are being done by hand, rather than by machine. This is feasible because the designs are constructed so that it is easy to estimate the parameters.

Publications:

- (1) Fractional factorial experiments of the $2^m 3^n$ series. W. S. Connor. Proceedings of the Fourteenth Annual Clinic of the Rochester Society for Quality Control, February 18, 1958, pp. 59-70.
- (2) Analysis of fractionally replicated $2^m 3^n$ designs. R. C. Bose and W. S. Connor. To appear in Revue de L'Institut International de Statistique (The Hague).

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS

Task 1104-12-1115/55-57

Origin: NBS

Authorized 9/1/54

Manager: W. H. Pell

Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. C. M. Tchen has completed the manuscript of a paper on turbulent diffusion. In this paper turbulent diffusion is studied on the basis of the statistical mechanics of man-body problems. The structure of the diffusivity has been calculated explicitly in terms of an assumed interaction mechanism. Existing mathematical models of turbulent diffusion have been compared and discussed. In this way, some of the controversies in the phenomenological theories, represented by the Fokker-Planck equation and the Richardson equation, have been clarified by relating them to a common statistical basis. In order to elucidate further the nonlinear interaction, the moment equations have been used. Finally, with the use of Hisenberg's hypothesis of eddy diffusivity and phase considerations a diffusion equation involving the eddy size has been obtained. The solution shows that the turbulent dispersion occurs at a faster rate than the molecular diffusion and depends explicitly on the eddy size.

C. M. Tchen has finished the checking of galley-proofs of "Turbulent Motion". This was written jointly by Dr. Tchen and G. B. Schubauer, and forms Section B, Volume V, of the Princeton series, "High Speed Aerodynamics and Jet Propulsion."

The study of fluctuations in the annual flows of rivers due to annual variations in rainfall by V. M. Yevdjevich is continuing. Seventy stations on rivers in the U. S. are definitely prepared for part I of the statistical computation on the IBM 704. Data on 30 stations from Europe has been collected and are being prepared for statistical analysis with the aid of the 704. Still other stations are to be included in the study. This is believed to be the most comprehensive study of this sort which has been made, because of the large number of stations which is being used and because of their wide distribution over the U. S. and Europe. Concomitant study of the literature on this controversial problem is under way.

A. Ghaffari has now discussed the complete phase-portrait of a nonlinear differential equation $dy/dx = P(x,y)$, where P and Q are known polynomials of degree five (see Jan-Mar 1958 issue, p. 15). He also considered the question of the maximum number of limit cycles

which appear from an equilibrium position of focus type of the above mentioned equation and showed that the Petrovsky inequality, concerning the maximum number of limit cycles, is verified.

Following Lefschetz's approach, Dr. Ghaffari is continuing the discussion concerning the nature of critical points at infinity of the second order nonlinear Rayleigh equation.

A paper has been completed by J. P. Vinti on the "Theory of the Effect of Drag on the Orbital Inclination of an Earth Satellite" and has been submitted for publication.

Publications:

- (1) Diffusion of particles in turbulent flow. C. M. Tchen. To appear in the Proceedings of a Symposium on Atmospheric Diffusion, Oxford, England; August 1958.
- (2) Turbulent motion. C. M. Tchen and G. B. Schubauer (NBS Fluid Mechanics Section). To appear as Section B, Volume V of the Princeton Series, High Speed Aerodynamics and Jet Propulsion.
- (3) Theory of the effect of drag on the orbital inclination of an earth satellite. J. P. Vinti. Submitted to a technical journal.
- (4) The graphical solution of initial value problems. W. H. Pell. Submitted to a technical journal.
- (5) On some mathematical properties of wedge solutions. A. Ghaffari. In manuscript.
- (6) On the domain of regularity of generalized axially symmetric potentials. P. Henrici. To appear in the Proceedings of the American Mathematical Society.
- (7) On the solution of compressible flow past a wedge. A. Ghaffari. In manuscript.
- (8) On the asymptotic behavior of the integral curves of a certain non-linear differential equation. A. Ghaffari. In manuscript.

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS

Task 1104-12-5160/56-154

Origin: NBS

Authorized 9/30/55

Sponsor: Office of Naval Research

Manager: F. Oberhettinger

Full task description: July-Sept 1955 issue, p. 20

Status: INACTIVE.

RESEARCH IN FLUID DYNAMICS OF TWO-PHASE FLOWS

Task 1104-12-5160/56-155

Origin and Sponsor: Office of Naval Research Authorized 9/30/55

Manager: W. H. Pell

Full task description: July-Sept 1955 issue, p. 21

Status: CONTINUED. J. P. Vinti has been developing a theoretical treatment of a hydroduct. This gives the conditions existing in the chamber and at the exit plane, as well as the thrust, as functions of the velocity, depth, and design parameters. Certain conditions must be satisfied, however, viz., 1) that the steam does not become super-saturated or wet before reaching the exit plane, and 2) that one has available a table of the velocity of sound for superheated steam - or that one can describe the steam with simple equations, with the aid of an effective value of γ (the ratio of its specific heats).

RESEARCH IN CONTINUUM MECHANICS

Task 1104-12-5160/55-85

Origin: NBS

Authorized 12/27/54

Sponsor: Office of Scientific Research, ARDC, USAF

Manager: W. H. Pell

Full task description: Oct-Dec 1954 issue, p. 30

Status: CONTINUED. As an extension of the statistical methods given in his manuscript on "Kinetic equation for a plasma with unsteady correlation" (see Apr-June 1958 issue, p. 15), C. M. Tchen has continued the study of the behavior of plasma by including an external electric field. As a result of this, it is found that the collision integral depends on the external electric field, a consequence which cannot be predicted from the Boltzmann equation. The new kinetic equation will be suitable for the formulation of a nonlinear theory of the runaway of electrons, a phenomenon important in explaining the paradox of the high conductivity of plasma, connected with the behavior of the non-Maxwellian tail of the distribution function. The manuscript of C. M. Tchen referred to has been processed and approved for publication in a technical journal.

W. H. Pell and L. E. Payne have continued the study of Stokes flows about axially symmetric bodies, and a manuscript has been written which covers the work on this subject completed to date. In this, the Stokes flow problems for the lens-shaped body and the spindle (two ogives with abutting bases) have been solved. A general relation for the drag valid for axially symmetric Stokes flow about any axially symmetric configuration of bodies lying in the finite plane (meridional) is given in terms of the stream function of the flow. Although not new, as examples of the ease with which their method may be used, solutions

of the Stokes flow problem for the oblate and prolate spheroids and two separated spheres are also given. The authors have now turned their attention to the problem of the torus, which differs fundamentally from the bodies considered in the manuscript mentioned above in that the flow space is doubly-connected. The determination of the appropriate circulation must be made in order to find the drag for this body with the general formula mentioned above, and work has proceeded in this direction. It is hoped that a minimum principle may be used to yield the appropriate circulation.

Publication of the manuscript on this subject is under consideration. Because of its length, it appears that it may be necessary to publish the results in two papers.

A revision of the manuscript of A. Walz on an approximate theory of the compressible boundary layer with heat transfer has been made.

Publications:

- (1) Kinetic equation for a plasma with unsteady correlations. C. M. Tchen. Submitted to a technical journal.
- (2) The Stokes flow problem for a class of axially symmetric bodies. L. E. Payne and W. H. Pell. In manuscript.
- (3) Compressible turbulent boundary layers with heat transfer and pressure gradient in flow direction. Alfred H. Walz. In manuscript.
- (4) Stationary principles for forced vibrations in elasticity and electromagnetism. J. L. Synge. To appear in the Proceedings of the Eighth Symposium in Applied Mathematics of the American Mathematical Society, held at Chicago, Ill., April 1956.
- (5) The vibration of triangular wings. R. F. Dressler. In manuscript. Abstract available in the Proceedings of the Ninth International Congress on Mechanics, Brussels, September 1956.
- (6) On the factorization of a fourth order differential operation occurring in the theory of structures. W. H. Pell. In manuscript.
- (7) Note on the integration of the elastic plate equation with variable flexural rigidity. W. H. Pell. In manuscript.
- (8) Unsteady nonlinear waves in sloping channels. R. F. Dressler. Proc. Roy. Soc.(A) 247, 186-198(1958).
- (9) Bending and stretching of corrugated diaphragms. R. F. Dressler. To appear in the Transactions of the American Society of Mechanical Engineers.

COMPUTATION OF OUTFLOW FROM A BREACHED DAM
Task 1104-12-5160/58-369

Origin and Sponsor: Army Map Service

Authorized 9/30/58

Manager: V. M. Yevdjevich

Objective: (1) To predict the outflow hydrograph from a breached dam. (2) To determine the modification of the hydrograph as it progresses downstream from the dam.

Background: This is an extension of an earlier study by R. F. Dressler (see task 1104-10-5160/54-23, P. & P. Apr-June 1954, p. 21, and July-Sept 1954, p. 27). More specifically, the present study aims at bridging the gap between Dressler's theoretical solution and operational requirements. It is desired to develop a method which can be readily adapted to current use.

Status: NEW. The initial period on the study of outflow from a dam has been spent in the collection and study of literature on the subject. With reference to problem (1) above, the fundamental differential equation for water storage regulation was studied in order to adapt it for the purpose of dam breaching. Consideration of problem (2) has been concentrated on more exact methods of solution, -- the method of finite differences and the method of characteristics.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

1102-40-5126/54-13 AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING

Origin and Sponsor: New York Quartermaster Procurement Agency

Manager: M. Paulsen

Full task description: Oct-Dec 1953 issue, p. 43

Status: Inactive.

3711-60-0009/54-30 SPECTRUM ANALYSIS

Origin: NBS, Division 4

Managers: C. D. Coleman, W. Bozman (4.1)

Full task description: Jan-Mar 1954 issue, p. 46

Status: Continued. A search for new atomic energy levels of thorium I gave two new even levels and five new odd levels. A search for new levels of iodine II gave 22 new even levels. The line list for tantalum has been prepared, and wavenumbers have been calculated for over 3,000 lines. The code for computing an expanded and extended revision of Kayser's "Tabelle der Schwingungszahlen" has been completed, and 170 pages have been computed and printed in a form ready for publication.

3711-60-0009/54-38 EQUATION OF STATE OF REAL GASES

(formerly COMPRESSIBILITY FACTORS OF DRY AIR)

Origin: NBS, Section 3.2

Manager: M. Paulsen

Full task description: Jan-Mar 1954 issue, p. 48

Status: Continued. The code for obtaining the virial coefficients has been expanded to include a variety of calculations involved in the computation of thermodynamic functions of real gases.

The objective of this task has been extended to include the preparation of a compatible ensemble of routines which would tie this program in with existing programs for the calculation of partition functions and for equilibrium properties of gaseous mixtures.

3711-60-0009/55-68 CRYSTAL STRUCTURE CALCULATIONS

Origin: NBS, Division 9

Managers: P. O'Hara, S. Block (9.7)

Full task description: Jan-Mar 1955 issue, p. 18

Status: Continued. A new Fortran program was written by S. Block to compute radial distribution curves. The program was used to calculate curves for solid ammonia and barium borate glass.

Several two-and three-dimensional Fourier maps were calculated for various crystals.

Calculations are being carried out to determine the structure of sodium borate. Normalized structure factors have been computed and phase determination calculations are under way.

3711-60-0009/55-82 THERMOMETER CALIBRATIONS

Origin: NBS, Section 3.1

Manager: S. Prusch

Full task description: Jan-Mar 1955 issue, p. 20

Status: Continued. The ITS constants and tables were computed for approximately 65 thermometers under test. A code for low temperature scales is in preparation.

1102-40-5126/55-88 STRESSES IN A WALL FOUNDATION

Origin and Sponsor: NBS, Section 10.1

Manager: I. Stegun

Full task description: Jan-Mar 1955 issue, p. 22

Status: Continued. Production runs were made on the 704.

1102-40-5126/55-121 ELECTRON PENETRATION

Origin and Sponsor: NBS, Section 4.8

Manager: S. Peavy

Full task description: Apr-June 1955 issue, p. 19

Status: Continued. Modifications were made in the original program so that the spatial moments could be read in rather than computed, and then the distribution was done on these moments.

1102-40-5126/56-162 STRESSES IN A WALL RESTING ON A FOOTING

Origin and Sponsor: NBS, Section 10.1

Manager: I. Stegun

Full task description: Jan-Mar 1956 issue, p. 26

Status: Inactive.

1102-40-5126/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES

Origin and Sponsor: NBS, Section 5.9

Managers: E. Haynsworth, P. Walsh

Full task description: Jan-Mar 1956 issue, p. 27

Status: Continued. Several cases were run using the SCF-1 program developed under task 3711-60-0009/57-223 (see p.19).

1102-40-5126/56-171 COLLISION INTEGRALS USED IN TRANSPORT THEORY

Origin and Sponsor: NBS, Section 3.2

Manager: D. Sumida

Full task description: Oct-Dec 1955 issue, p. 33

Status: Terminated. For further work related to this task, see task 1102-40-5126/58-358, "Reduced Cross-Sections" (Apr-June 1958 issue, p. 30).

1102-40-5126/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS

Origin and Sponsor: NBS, Section 2.5

Manager: S. Prusch

Full task description: July-Sept 1956 issue, p. 33

Status: Continued. Calculations were performed on 11 laboratory sets of gage blocks.

1102-40-5126/57-209 TRAFFIC DISTRIBUTION

Origin and Sponsor: Bureau of Public Roads

Manager: S. Peavy

Full task description: Jan-Mar 1957 issue, p. 32

Status: Inactive.

1102-40-5126/57-221 BESSEL FUNCTIONS FOR COMPLEX ARGUMENTS

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Manager: R. Zucker

Full task description: Oct-Dec 1956 issue, p. 31

Status: Continued. Seventy-six cases were run to evaluate the Bessel and Hankel functions for complex arguments up to order n specified.

1102-40-5126/57-222 ROOTS OF POLYNOMIALS

Origin and Sponsor: Naval Research Laboratory

Manager: J. P. Menard

Full task description: Oct-Dec 1956 issue, p. 32

Status: Inactive.

3711-60-0009/57-223 SELF-CONSISTENT FIELDS

Origin: NBS, Section 3.2

Manager: E. V. Haynsworth

Full task description: Apr-June 1957 issue, p. 28

Status: Continued. The A-matrix routine and the SCF-2 program have been completely checked out. Data is being prepared by the sponsor for production runs.

3711-60-0009/57-229 APPLICATION OF ELECTRONIC DATA PROCESSING MACHINERY
TO PAYROLL OPERATIONS

Origin: NBS, Section 40.0

Managers: M. Paulsen, P. Ruttenberg

Full task description: Jan-Mar 1957 issue, p. 36

Status: Continued. The change routine, a file maintenance routine, was used to generate a Working File composed of 67 employees from Division 42 for pay period 13.

Two output codes were written (see Jan-Mar 1957 issue, p. 36):
(1) Check Issue and Statement of Earnings (card output). These cards are required by the Treasury Department as input for the writing of salary checks. They contain essentially the same information as the current Statement of Earnings and Deductions card that each Bureau employee now receives with his salary check. (2) The Master File (tape output). This contains information found on the current Master card and the Bond Schedule card plus certain additional information not currently included on cards. The annual biweekly and hourly rate are no longer carried with each employee's record. Certain fields are available on the Master File tape that would prove helpful in making certain personnel, budget, labor distribution, cost accounting, and other reports if desired in the future.

The PRCINP (input routine for reading in the Working File), the DPP (IBM Data Processing Package) (see Apr-June issue, p. 20), and the payroll calculation routines were assembled as one code. This code was checked out using as input data the Working File mentioned in the first paragraph above.

The code checking gave results which corresponded to those obtained by the Payroll Section for the 67 employees used in the test.

The Federal Employees Salary Increase Act of 1958 brought about several changes in the method of computing certain rates (effective after pay period 13 for which the code above was checked out). The code for these changes has been written as well as that for another output routine, the Bond Purchase List (see Apr-June 1957 issue, p. 36), but they have not yet been incorporated into the basic code.

The extent to which this project may be extended has not yet been determined.

1102-40-5126/57-234 PERSONNEL SURVEY

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of
the Army

Manager: P. O'Hara

Full task description: Jan-Mar 1957 issue, p. 37

Status: Inactive.

1102-40-5126/57-236 SELF CONSISTENT FIELDS--EIGENVALUES

Origin and Sponsor: NBS, Section 3.6

Manager: E. Haynsworth

Full task description: Apr-June 1957 issue, p. 30

Status: Continued. Production runs were continued.

3711-60-0009/57-247 MECHANICAL IMPEDANCE

Origin: NBS, Section 6.1

Managers: J. P. Menard, M. D. Burkhard (6.1)

Full task description: Apr-June 1957 issue, p. 32

Status: Continued. Production runs involving 19 samples of data were made directly by the sponsor. The program will continue in production under the sponsor's direction.

3711-60-0009/57-248 THE EVALUATION OF A TRIPLE INTEGRAL FOR THE
SOLUTION OF NEGATIVE ION DETACHMENT

Origin: NBS, Section 4.6

Manager: S. Peavy

Full task description: Apr-June 1957 issue, p. 34

Status: Inactive.

3711-60-0009/57-250 AUTOMATIC REDUCTION OF SPECTROPHOTOMETRIC DATA

Origin: NBS, Section 2.1

Manager: W. C. Rheinboldt

Full task description: July-Sept 1957 issue, p. 31

Status: Continued. Only a small number of cases have been run under the immediate supervision of the sponsor. A paper entitled, "Digital Reduction of Speedophotometric Data," was written jointly by K. J. Keegan, J. C. Schleter, D. B. Judd (all NBS 2.1), and W. C. Rheinboldt and J. P. Menard, to be presented at the Forty-third Annual Meeting of the Optical Society of America, Detroit, Michigan, October 9-11.

1102-40-5126/57-251 CURRENT NOISE AND FIXED RESISTORS

Origin and Sponsor: NBS, Section 1.6

Manager: D. Sumida

Full task description: July-Sept 1957 issue, p.32

Status: Inactive.

3711-60-0009/58-254 REPRODUCTION OF COLOR- AND SPECTRAL-ENERGY
DISTRIBUTION OF DAYLIGHT AND OTHER ILLUMINANTS

Origin: NBS, Section 2.3

Manager: W. C. Rheinboldt

Full task description: July-Sept 1957 issue, p. 32

Status: Continued. Approximately 70 cases have been run on the 704 under the immediate direction of the sponsor. The results are used in the

work of the sponsor to design filters which, when combined with a given source of light, simulate light from other sources as accurately as possible in regard to color and spectral energy distribution. In particular, in order to match the light from Plancksian radiation it was necessary to obtain sufficiently accurate values of the energy distribution of Plancksian radiation for any given temperature T. A code has been written to evaluate

$$E_{\text{Planck}} = \frac{C_1}{\lambda^5 (\exp(C_2/\lambda T) - 1)}$$

for $\lambda = 300(10)880 \text{ m } \mu$ and any $T^\circ(\text{K})$. The most recent values of the constants

$$C_1 = 3.74 \times 10^{22} \text{ microwatt} \cdot \text{m } \mu^4 \cdot \text{cm}^{-2}$$

$$C_2 = 1.438 \times 10^7 \text{ m } \mu \cdot \text{degrees}$$

were used.

This code has been completely checked out. The energy distributions for 51 temperature values between $1,600^\circ\text{K}$ and $20,000^\circ\text{K}$ have been computed.

1102-40-5126/58-263 GAS TUBE CHARACTERISTIC

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Manager: W. F. Cahill

Full task description: July-Sept 1957 issue, p. 35

Status: Continued. Runs have been made with the new program. The results have been transmitted to the sponsor.

1102-40-5126/58-264 THEORY OF IONIZATION PROBABILITY

Origin and Sponsor: NBS, Section 4.6

Manager: S. Peavy

Status: Continued. A code has been written and checked out to compute and punch on cards the values of the second integral of the first equation for values of $r' = 1.0(.1)20$, $r = .05(.05)20$, $K_p = 1.0$, and $\eta = 0(1)4$.

3711-60-0009/58-266 DEPOLYMERIZATION, II

Origin and Sponsor: NBS, Section 7.6

Manager: L. S. Joel

Full task description: July-Sept 1957 issue, p. 36

Status: Continued. Code checking on the 704 was continued.

3711-60-0009/58-267 CONVERSION OF THE CIE-CHROMATICITY COORDINATES INTO
THE MUNSELL COLOR SYSTEM

Sponsor: NBS, Section 2.1

Manager: W. C. Rheinboldt

Full task description: July-Sept 1957 issue, p. 37

Status: Continued. Approximately 600 sets of tristimulus data have been transformed into the corresponding Munsell renotations with the help of the existing code. The data were supplied from two outside sources.

The work done under this task together with that done under task 3711-60-0009/57-250 (see p.21) was the subject of an official news release of the Department of Commerce on August 25 under the title, "Electronic Seeing Eye."

A paper entitled "Mechanized Conversion of Colorimetric Data to Munsell Renotations" has been written jointly by W. C. Rheinboldt and J. P. Menard, to be presented at the Forty-third Annual Meeting of the Optical Society of America, Detroit, Mich., October 9-11.

Preliminary investigations have been started to rewrite the existing program such that the code together with the complete set of data can be kept entirely in the 8192-word core memory of the NBS IBM-704 computer.

3711-60-0009/58-268 RESPONSE FUNCTION CALCULATION

Origin: NBS, Section 4.11

Manager: A. Beam

Full task description: July-Sept 1957 issue, p. 37

Status: Completed.

1102-40-5126/58-269 MOLECULAR STRUCTURE, IV

Origin and Sponsor: Naval Research Laboratory, USN

Manager: P. J. O'Hara

Full task description: July-Sept 1957 issue, p. 38

Status: Continued. Initial computations were carried out for the cellobiose crystal which will correct the observed intensities for vibrational motion and place them on an absolute scale. The results of this calculation will be used in phase determination studies by probability methods.

Further studies concerning the structure of the spurrite crystal were undertaken by means of two dimensional least square refinement of atomic position parameters. A three dimensional Fourier map was computed for aureomycin using normalized structure factors as coefficients.

1102-40-5126/58-272 THERMODYNAMIC PROPERTIES OF REAL GASES

Origin and Sponsor: NBS, Section 3.2

Manager: J. P. Menard

Full task description: Oct-Dec 1957 issue, p. 32

Status: Continued. A code has been written and completely checked out for performing the computations described in the Apr-June 1958 issue, page 23. In performing these computations the code produces a magnetic tape inscribed with the following values:

$$\begin{array}{ll}
 1) \quad z = PV/RT & 4) \left[\frac{d(E/RT)}{d \ln} \right]_T \\
 2) \quad E/RT & 5) \left[\frac{dz}{d \ln} \right]_T \\
 3) \quad C_v/R &
 \end{array}$$

The final answers are then computed and inscribed immediately following the above values on the same magnetic tape. These values are:

$$\begin{array}{ll}
 1) \quad C_v/R & 4) \quad \alpha \\
 2) \quad C_p/R & 5) \quad a/a_o \\
 3) \quad \gamma = C_p/C_v &
 \end{array}$$

The code then rewinds the magnetic tape, reads in the final answers, arranges them in the correct orders, and prints them out on the on-line printer in a form suitable for publication.

A second code has been written for the purpose of reading the intermediate values on the magnetic tape, arranging them in the proper order, and printing them on the on-line printer in a form suitable for publication. This code has also been completely checked out.

Production runs have been made on argon free air and on nitrogen for temperatures ranging from 1400°K to 15,000°K. Further runs will be made under the sponsor's direction.

1102-40-5126/58-274 CALCULATIONS FOR d-SPACINGS, II

Origin and Sponsor: NBS, Division 9

Manager: R. Zucker

Full task description: July-Sept 1957 issue, p. 38

Status: Continued. About 26 calculations for d-spacings for ortho-
gonal, hexagonal and orthorhombic crystals were carried out this
quarter, and redetermination of unit cell constants by least squares
fitting to a measured d-spacing was performed for about 32 crystals.

1102-40-5126/58-279 FIRE RESISTANT T-BEAM

Origin and Sponsor: NBS, Section 10.2

Manager: C. Wade

Full task description: Oct-Dec 1957 issue, p.33

Status: Continued. Additional runs have been made to compare corresponding one- and two-dimension cases. Computations have also been made to compare results for several combinations of time interval and grid-block width and three thicknesses of the T-beam. All computations have been for one type of concrete. Some of the results have been transmitted to the sponsor.

1102-40-5126/58-282 MISSILE BOUNDARY LAYER COMPUTATION

Origin and Sponsor: Naval Ordnance Laboratory

Manager: R. Danek (NOL)

Full task description: Jan-Mar 1958 issue, p. 29

Status: Reporting terminated. Advanced phases of the problem involve classified data that cannot be reported here.

1102-40-5126/58-284 EPHEMERIS CALCULATIONS FOR SATELLITES

Origin and Sponsor: Naval Research Laboratory

Managers: W. F. Cahill, J. H. Wegstein

Full task description: Oct-Dec 1957 issue, p. 34

Status: Continued. The program has been used to correct orbit elements (period, angle of perigee, drag coefficients, perigee, latitude and longitude of a fix) for various orbits as requested by sponsor. The results of previous computations were included in papers presented by Dr. R. Jastrow before the IGY Conference in Moscow.

1102-40-5126/58-289 SCATTERING OF ELECTRONS BY HYDROGEN

Origin and Sponsor: NBS, Section 3.6

Manager: R. Zucker

Full task description: Oct-Dec 1957 issue, p. 36

Status: Inactive. For status to date, see Oct-Dec 1957 issue, p. 36.

3711-60-0009/58-294 NUCLEAR SCATTERING OF PHOTONS

Origin: NBS, Section 4.8

Manager: J. P. Menard

Full task description: Oct-Dec 1957 issue, p. 36

Status: Continued. Approximately 10 samples have been run by the sponsor. The program will continue in production under the sponsor's direction.

1102-40-5126/58-297 MANY BODY PROBLEM

Origin and Sponsor: Naval Research Laboratory

Manager: S. Peavy

Full task description: Oct-Dec 1957 issue, p. 38

Status: Completed.

1102-40-5126/58-298 ANALYSIS OF SPECTROCHEMICAL DATA

Origin and Sponsor: NBS, Section 5.10

Managers: S. Peavy, R. N. Varner

Full task description: Oct-Dec 1957 issue, p. 39

Status: Continued. A Fortran code has been written and checked out.

This code converts spectrometer readings of the various elements in standard reference samples certified by the Bureau to concentrations using an experimentally determined calibration curve. From the derived concentrations the minimum, maximum, median, average, standard deviation, and coefficient of variation are determined.

1102-40-5126/58-299 TIME-DEPENDENT SCHROEDINGER EQUATION

Origin and Sponsor: NBS, Section 3.1

Manager: J. Beiman

Full task description: Oct-Dec 1957 issue, p. 39

Status: Continued. Production runs were made, and the results were transmitted to the sponsor.

1102-40-5126/58-300 LAMINAR MIXING IN BOUNDARY LAYERS

Origin: Polytechnic Institute of Brooklyn

Sponsor: Air Force Office of Scientific Research

Manager: W. C. Rheinboldt

Full task description: Oct-Dec 1957 issue, p. 40

Status: Inactive. For status to date, see Apr-June 1958 issue, p. 25.

1102-40-5126/58-304 TRANSPORT PROPERTIES OF AIR AT ELEVATED TEMPERATURES

Origin and Sponsor: NBS, Section 3.2

Manager: P. J. Walsh

Full task description: Oct-Dec 1957 issue, p. 40

Status: Continued. Coding of the modified formulae has been completed and code checking is now in progress.

1102-40-5126/58-306 INTERPOLATION OF COLOR MIXTURE FUNCTIONS

Origin and Sponsor: NBS, Section 2.1

Manager: W. C. Rheinboldt

Full task description: Oct-Dec 1957 issue, p. 42

Status: Continued. A number of production runs have been made under the direction of the sponsor. The results showed that under certain conditions the accuracy of the interpolation can be improved if the

interpolation formula is applied to the logarithm of the function instead of to the function itself. The existing code was augmented to permit the interpolation either of the function directly or of the logarithm of the function. In the latter case the code also computes the antilogarithm of the results.

1102-40-5126/58-307 STUDY OF SURFACE TENSION

Origin and Sponsor: NBS, Section 9.2

Manager: R. Arms

Full task description: Oct-Dec 1957 issue, p. 43

Status: Continued. Tabulation of the integral

$$F(x) = e^{-2x} \int_0^{\infty} \frac{t^{1.5} dt}{\cosh(t-x)}$$

is being considered.

1102-40-5126/58-308 OSCILLATING SPHERE

Origin and Sponsor: NBS, Section 3.4

Manager: S. Prusch

Full task description: Oct-Dec 1957 issue, p. 43

Status: Continued. Several small production runs were performed for the sponsor.

1102-40-5126/58-311 NULL RATE CALCULATIONS

Origin and Sponsor: Naval Research Laboratory

Manager: J. H. Wegstein

Full task description: Oct-Dec 1957 issue, p. 44

Status: Completed. Results were transmitted to the sponsor.

1102-40-5126/58-312 RESPONSE FUNCTION, II

Origin and Sponsor: NBS, Section 4.11

Manager: A. Beam

Full task description: Jan-Mar 1958 issue, p. 33

Status: Continued. The sponsor has made several runs on the 704.

1102-40-5126/58-314 APPROXIMATIONS FOR GAS MIXTURES

Origin and Sponsor: NBS, Section 3.2

Manager: R. Zucker

Full task description: Jan-Mar 1958 issue, p. 33

Status: Inactive. For status to date, see Jan-Mar 1958 issue, p. 33.

1102-40-5126/58-316 INTERSECTION CAPACITY STUDY

Origin and Sponsor: Bureau of Public Roads

Managers: S. Peavy, J. M. Cameron

Full task description: Jan-Mar 1958 issue, p. 33

Status: Continued. The machine code for the transcription of data from punched cards into a form suitable for machine analysis has been written and checked. The code for estimating a nested sequence of prediction equations has been written but not checked.

1102-40-5126/58-321 TABLE OF THERMODYNAMIC FUNCTIONS OF SULFUR

Origin and Sponsor: NBS, Section 3.2

Manager: R. Zucker

Full task description: Jan-Mar 1958 issue, p. 34

Status: Inactive. For status to date, see Jan-Mar 1958 issue, p. 34.

1102-40-5126/58-322 PROPAGATION CONSTANT OF A SOUND WAVE

Origin and Sponsor: NBS, Section 6.1

Manager: R. J. Arms

Full task description: Jan-Mar 1958 issue, p. 34

Status: Continued. Additional production runs were performed.

1102-40-5126/58-324 FITTING A POLYNOMIAL TO A CURVE

Origin and Sponsor: NBS, Section 4.5

Managers: S. Peavy, R. N. Varner

Full task description: Jan-Mar 1958 issue, p. 34

Status: Completed.

1102-40-5126/58-333 CALCIUM HYDROXIDE

Origin and Sponsor: NBS, Section 9.0

Manager: P. O'Hara

Full task description: Jan-Mar 1958 issue, p. 36

Status: Inactive.

1102-40-5126/58-336 HELICAL TRANSFORMS

Origin and Sponsor: National Institutes of Health

Manager: R. N. Varner

Full task description: Jan-Mar 1958 issue, p. 36

Status: Continued. Production runs were made under the direction of the sponsor.

1102-40-5126/58-337 GEORGETOWN LANGUAGE TRANSLATION EXPERIMENTS

Origin and Sponsor: Georgetown University

Manager: R. J. Arms

Full task description: Jan-Mar 1958 issue, p. 37

Status: Continued. A code for sentence searching for certain linguistic information has been partially checked. The sponsor is using a 704 belonging to the Air Force.

1102-40-5126/58-339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS

Origin and Sponsor: NBS, Section 3.4

Manager: H. Oser

Full task description: Jan-Mar 1958 issue, p. 38

Status: Continued. Investigations have been started to obtain the functions F and G as Laplace transforms of F* and G* (as given in full task description). The latter functions have also been given for a more complicated model, which made the analysis even more cumbersome. Approximation methods have to be used for the evaluation of the Laplace integrals. The already written complex arithmetic code will prove to be very useful for these computations.

A table has been computed for F* and G* with a new set of constants.

1102-40-5126/58-343 MINIMIZATION PROBLEM

Origin and Sponsor: Naval Research Laboratory

Manager: S. Peavy

Full task description: Jan-Mar 1958 issue, p. 40

Status: Continued. A code has been written in Fortran for the second order differential equation and is in the process of being checked out.

1102-40-5126/58-344 SEMICONDUCTOR SURFACE MEASUREMENTS

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Managers: W. Hall, J. Beiman

Full task description: Jan-Mar 1958 issue, p. 41

Status: Completed. The program for the second integral has been checked, and the results were transmitted to the sponsor.

1102-40-5126/58-356 MAGNETIC SUSCEPTIBILITY OF IONS

Origin and Sponsor: NBS, Section 3.0

Manager: R. N. Varner

Full task description: Apr-June 1958 issue, p. 29

Status: Completed. Results have been transmitted to the sponsor.

1102-40-5126/58-358 REDUCED CROSS-SECTIONS

Origin and Sponsor: NBS, Section 3.2Manager: W. C. RheinboldtFull task description: Apr-June 1958 issue, p. 30Status: Continued. A small number of runs have been made with the code for the quantum mechanical case. Preparatory investigations have been started for the programming of the classical case.

1102-40-5126/58-359 VELOCITY DISTRIBUTION IN BOUNDARY LAYERS

Origin and Sponsor: NBS, Section 11.4Manager: W. C. RheinboldtFull task description: Apr-June 1958 issue, p. 32Status: Inactive. For status to date, see Apr-June 1958 issue, p. 32.

3711-60-0009/58-360 DIFFUSION COEFFICIENTS

Origin and Sponsor: NBS, Section 5.2Manager: W. C. RheinboldtFull task description: Apr-June 1958 issue, p. 32Status: Continued. The code as outlined in the previous issue has been written and completely checked out. However, the accuracy of the results proved not to be completely satisfactory. This inaccuracy is due to two reasons. The first stems from the fact that experimental data with an accuracy of at most four significant figures have to be numerically differentiated. The second loss of accuracy occurs when these differentiated data are extrapolated to $+\infty$ in order to facilitate the evaluation of the function $\bar{D}(x, t)$.

Due to the unsatisfactory result it was decided that two smaller codes would be written. The object of the first code is to perform the differentiation involved in the main program. The differentiation is performed by fitting a parabola through seven points and then differentiating the resulting quadratic. The method of least squares is used in fitting the parabola. Very satisfactory results have been obtained even though the accuracy is at most two significant figures. The second code evaluates the diffusion coefficient D under the assumption that D is not dependent on the concentration. Under this assumption the general equation may be integrated yielding the solution

$$c(x) = \frac{c_0}{2} \left(1 - \operatorname{erf} \frac{x}{\sqrt{4Dt}} \right).$$

The function $c(x)$ is measured, $c_0 = \max_x c(x)$ being the maximal fringe.

The code then solves the equation

$$\operatorname{erf} y(x) = \left| \frac{2c(x)}{c_0} - 1 \right|$$

for $y(x)$. Averaging the quotients $x/y(x)$ in a certain way gives a value F which in turn yields the diffusion coefficient $D = F^2/4t$, where the time t must be given in seconds.

Both codes have been written and completely checked out. Several production runs have been made under the direct supervision of the sponsor.

1102-40-5126/58-361 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION

Origin and Sponsor: Naval Research Laboratory

Manager: R. J. Arms

Full task description: Apr-June 1958 issue, p. 33

Status: Continued. The code has been checked out. Production runs are being submitted to the sponsor. A second phase of the problem is being coded. Also, an auxiliary code for evaluating the integral of plasma radiation,

$$J(x, kt) = \int_x^{\infty} \frac{du e^{-u} \ln \left(\frac{(\sqrt{x} + \sqrt{u-x})^2}{x} \right)}{\sqrt{u(u-x)} \left(\exp \frac{\alpha}{\sqrt{ktu}} - 1 \right) \left(1 - \exp \frac{-\alpha}{\sqrt{kt(u-x)}} \right)}$$

has been checked out.

1102-40-5126/58-363 CRYSTAL LATTICE COMPUTATIONS

Origin: NBS, Section 3.1

Sponsor: Maryland University

Manager: A. Schopf

Full task description: Apr-June 1958 issue, p. 33

Status: Completed.

1102-40-5126/58-365 ABSOLUTE RECORDING PERFORMANCE OF MAGNETIC TAPE

Origin and Sponsor: NBS, Section 6.1

Manager: J. Beiman

Full task description: Apr-June 1958 issue, p. 34

Status: Completed. The computations were carried out as requested, and the results were transmitted to the sponsor.

1102-40-5126/58-366 RADIATION PATTERNS OF ANTENNAS

Origin and Sponsor: U.S. Information Agency, Department of State

Managers: R. T. Moore, P. J. Walsh

Full task description: Apr-June 1958 issue, p. 35

Status: Continued. The code for tables (a), (b), and (d) as given in part 1 of the full task description in the previous issue has been written and is being checked. The code for computing the values stated in part 2 has been checked out for primary lobes. Code checking is now in progress for the calculations involving secondary lobes.

1102-40-5126/58-368 INTENSITY FUNCTIONS AND CROSS SECTIONS OF LIGHT
SCATTERED BY SPHERICAL PARTICLES

Origin and Sponsor: U. S. Army Signal Research and Development Laboratories,
Atmospheric Physics Branch, Belmar, N. J.

Manager: H. Oser

Objective: To calculate the intensity functions

$$i_1 = \left| \sum_1^{\infty} \frac{2n+1}{(n+1)n} \{ a_n \tau_n + b_n \pi_n \} \right|^2 \quad (1)$$

$$i_2 = \left| \sum_1^{\infty} \frac{2n+1}{(n+1)n} \{ a_n \pi_n + b_n \tau_n \} \right|^2 \quad (2)$$

and the cross sections for absorption, extinction and scattering of light by spherical particles:

$$Q_{\text{ext}} = \frac{2}{x} \sum_{n=1}^{\infty} (2n+1)(-1)^n \{ I(a_n) + I(b_n) \} \quad (3)$$

$$Q_{\text{sca}} = \frac{2}{x} \sum_{n=1}^{\infty} (2n+1) \left\{ |a_n|^2 + |b_n|^2 \right\} \quad (4)$$

$$Q_{\text{abs}} = Q_{\text{ext}} - Q_{\text{sca}}$$

Tables are to be prepared which include a wider range than present tables give, especially with regard to complex refractive indices and large-size parameters.

Background: Mie's theory gives for the intensity of light scattered by a small spherical particle into the angle θ the following relation:

$$I = I_0 f(r, \lambda, m, \theta).$$

For unpolarized incident light we get

$$I = \frac{I_0}{2} \left[i_1(\theta, x, m) + i_2(\theta, x, m) \right]$$

where

$I_0 \equiv$ intensity of incident light

$x \equiv 2\pi r/\lambda =$ size parameter

$r \equiv$ particle radius

$\lambda \equiv$ wave length of incident light

$m \equiv$ complex refractive index

$\theta \equiv$ scattering angle

i_1 and i_2 are the intensity functions which are proportional to the two polarized components of the intensity of scattered light. They are given by (1) and (2) and are due to G. Mie, Annalen der Physik, vol. 25, p. 377. The a_n and b_n can be written in the following way:

$$a_n(x, m) = (-)^{n+\frac{1}{2}} \frac{S'_n(mx) S_n(x) - m S'_n(x) S_n(mx)}{S'_n(mx) \phi_n(x) - m \phi'_n(x) S_n(mx)}$$

$$b_n(x, m) = (-)^{n+\frac{3}{2}} \frac{m S'_n(mx) S_n(x) - S'_n(x) S_n(mx)}{m S'_n(mx) \phi_n(x) - \phi'_n(x) S_n(mx)}$$

where

$$S_n(z) = \left(\frac{\pi z}{2}\right)^{\frac{1}{2}} J_{n+\frac{1}{2}}(z)$$

$$C_n(z) = (-)^n \left(\frac{\pi z}{2}\right)^{\frac{1}{2}} J_{-n-\frac{1}{2}}(z)$$

$$\phi_n(z) = S_n(z) + i C_n(z)$$

$$S'_n(z) = \frac{\partial}{\partial z} (S_n(z)), \quad C'_n(z) = \frac{\partial}{\partial z} (C_n(z))$$

$S_n(z)$ and $C_n(z)$ are the so-called spherical Bessel functions and z takes the values x and mx . π_n and τ_n are given by

$$\pi_n = \frac{d P_n(\cos \theta)}{d (\cos \theta)}, \quad \pi'_n = \frac{d^2 P_n(\cos \theta)}{d (\cos \theta)^2},$$

$$\tau_n = \pi_n \cos \theta - \sin^2 \theta \cdot \pi'_n$$

$P_n(\cos \theta)$ is the Legendre polynomial of n -th degree.

Status: New. Investigations have been made to check the usefulness of recurrence for formulae for the Bessel functions in the present case. It has been found that the loss of accuracy requires calculation with double precision arithmetic. Therefore an interpretive complex double precision arithmetic has been coded for the IBM 704 using an existing double precision code. This program has been checked out and test runs on data used in previous publications of several authors gave very satisfactory results for the scattering coefficients a_n and b_n . The code for the angular functions τ_n and π_n has been written and π_n is being checked out.

1102-40-5126/59-374 END-EFFECT IN THE CYLINDRICAL ROTATIONAL VISCOMETER

Origin and Sponsor: NBS, Section 7.1

Managers: A. Schopf, J. P. Menard

Objective: To calculate the viscosity of a Newtonian liquid contained in a cylindrical cup of radius R and height H wherein another concentric cylinder of radius r is immersed to a depth h and is rotating with constant angular speed and known applied momentum. The influence of the three geometrical parameters r/R , h/H , and R/H (End-effect) has to be investigated.

Background: The problem involves the solution of the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + 3 \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} = 0$$

with $u = 0$ on the outer cup, $u = 1$ on the inner cup, and $\partial u / \partial y = 0$ on the free surface (x being the radial and y the vertical coordinates).

The solution has been expressed by different series of Bessel functions that are valid in different parts of the domain (Syoten Oka, 1956). The coefficients must be found by solving infinite linear systems. This method seems to be unsuited for numerical evaluation, due to instabilities and poor convergences that result from the singularity at the inner cup corner. A new approach is to be made.

This problem was submitted by H. Leaderman (7.1).

Status: New. A direct difference approach has been made, using the minimum principle that belongs to this boundary value problem. The difference solution for one set of the three geometrical parameters yields also a good approximation for neighboring parameters by means of affine transforms.

The resulting symmetric positive definite linear systems of high order (about 80 equations) and bad condition are attacked by a modified version of the c-g-algorithm of Stiefel and Hestenes. The modification is made to improve the stability of this algorithm. That part of the program will be written in a general form to be submitted to SHARE. Coding is under way.

1102-40-5126/59-375 RELAXATION OF POISSON DISTRIBUTION

Origin and Sponsor: NBS, Section 3.1Manager: W. F. CahillObjective: To evaluate

$$x_n(\tau) = \left[\frac{1-e^{-\theta}}{-\tau-e^{-\theta}} \right] \left[\frac{1-e^{-\tau}}{\theta-e^{-\tau}} \right] \exp \left\{ \frac{ae^{-\tau}(e^{-\theta}-1)}{e^{-\tau}-e^{-\theta}} \right\} L_n \left\{ \frac{ae^{-\tau}(e^{-\theta}-1)^2}{(1-e^{-\tau})(e^{-\tau}-e^{-\theta})} \right\}$$

where L_n is the Laguerre polynomial

$$L_n(Y) = \sum_{v=0}^n \binom{n}{n-v} \frac{(-Y)^v}{v!}$$

for $a = 8$, $\theta = 3$, $n = 0(2)16$, $v = 0, \dots, 6, \infty$.Background: The function $x_n(\tau)$ arises in the study of the distribution of vibrating molecules in a shock wave.

The problem was proposed by K. Shuler (3.1).

Status: Completed (New).1102-40-5126/59-376 VARIATIONAL CALCULATION OF SLOW ELECTRON
SCATTERING BY HYDROGEN ATOMSOrigin and Sponsor: NBS, Section 4.6Manager: L. S. JoelObjective: To apply the Kohn variational method to determine the symmetric and antisymmetric scattering phase shifts for s-, p-, and d-wave electrons. The trial function contains, in addition to the phase shift parameter, up to three linear parameters and one screening parameter which enters nonlinearly. The variation of the stationary integral with respect to the nonlinear parameter is to be accomplished by a systematic numerical interpolation method. The dependence of the phase shifts upon the number of linear parameters is also to be investigated to try to get some idea of the degree of convergence of the procedure.Background: This problem arises in the study of phase shifts in elastic scattering of electrons by hydrogen atoms.

The problem was submitted by S. Geltman (4.6).

Status: New. A code for the 704 was written and is being checked for initial values $k = 1$ and $\gamma = 1$.

1102-40-5126/59-381 BOOLEAN FUNCTIONS AND PICTORIAL DATA PROCESSING

Origin and Sponsor: NBS, Section 12.5Manager: B. BenderObjective: To conduct theoretical analyses of certain classes of Boolean algebraic functions in order (a) to determine methods of transforming the functions to their absolutely simplest forms, and (b) to determine minimal representations of the functions under a given probability of error in the function value.

Background: The need for this study occurs in the application of Boolean function techniques to the reduction of the number of bits required in the storage of pictures in a digital computer.

The problem was suggested by R. Ledley (12.5).

Status: New. Several techniques for minimizing truth functions are being examined for possible use in computer programs. A preliminary program is being written.

1102-40-5126/59-383 DESIGN FLOW CAPACITY CURVES

Origin and Sponsor: NBS, Section 10.5

Manager: A. Beam

Objective: To compute design flow capacity curves for drainage pipes under various conditions. The computations are as follows:

$$(1) \text{ To compute } \phi(N) = \frac{(Q_1 + Q_2)_r Q_{stc}}{ad^2 \sqrt{\gamma d}}$$

(2) If $15 \geq \phi(N) \geq 1.95$, compute using Newton's method, the first two positive zeros of

$$N^{-5/8} + N^{3/8} - \phi(N) = 0.$$

The iterative formula reduces to

$$X_{n+1} = \frac{-3\phi(N)e^{5/8 \ln X_n} + 8}{\frac{5}{X_n} \phi(N)e^{5/8 \ln X_n} - 8}$$

(3) For each of the two values of N obtained, to compute

$$Q_2 = \frac{(Q_1 + Q_2)_r Q_{stc}}{1 + N}, \quad Q_1 = N Q_2,$$

$$R = \frac{Q_1 + Q_2}{Q_1}, \quad \text{and} \quad \log_{10} R,$$

where Q_{stc} is the discharge rate of a full pipe under steady flow conditions,

$(Q_1 + Q_2)_r$ is the discharge rate relative to Q_{stc} ,

d is the diameter of the main drain,

γ is the ratio of the vertical distance from the center of gravity of the flow to the center of the pipe and the radius of the pipe,

a is one of three parameters depending on the diameter of the branch pipe.

The curves are obtained by plotting γ versus $\log_{10} R$.

Background: When a main drainage pipe is joined by a branch pipe, then there is a buildup of pressure at this junction depending on the flow, slope of the pipes, roughness of the surface, and pipe diameters. Laboratory experiments at NBS furnished data which made it possible to develop flow equations which yield design flow capacity curves. These curves show the condition under which both the main and branch pipes will flow full.

The problem was submitted by R. S. Wyle (10.5).

Status: Completed (New). The above computations were carried out on the 704 for 2700 sets of $a, \gamma, (Q_1 + Q_2)_r$, and Q_{stc} ; and the results were transmitted to the sponsor.

1102-40-5126/59-391 ION DISTURBANCE AROUND A SATELLITE

Origin and Sponsor: Naval Research Laboratory

Manager: W. F. Cahill

Objective: To find the solution of the systems of differential equations:

$$(1) \quad \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial \psi}{\partial r}) + \frac{1}{r^2} \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial \psi}{\partial \theta}) = e^{\psi} - N(r, \theta)$$

having boundary conditions

$$\begin{aligned} \psi &= \psi_0, & r &= r_0, \\ \psi &= 0, & r &= \infty, \\ \frac{\partial \psi}{\partial \theta} &= 0, & \theta &= 0, \theta = \pi. \end{aligned}$$

$$(2) \quad N(r, \theta) = \frac{N_0}{\sqrt{\dot{r}^2 + r^2 \dot{\theta}^2} [d(r, \theta)]^2}$$

where

$$(3) \quad \begin{aligned} m(\ddot{r} - \rho \dot{\theta}^2) &= - \frac{\partial \psi}{\partial \rho}, \\ m(r \ddot{\theta} + 2\dot{r}\dot{\theta}) &= - \frac{1}{r} \frac{\partial \psi}{\partial \theta} \end{aligned}$$

having initial conditions at $t = 0$,

$$r \sin \theta = C_1, \quad \frac{d}{dt}(r \cos \theta) = 0,$$

$$\frac{d}{dt}(r \cos \theta) = C_2, \quad r = C_3.$$

The C_i are varied to give a uniform dense set of trajectories (r, θ) coming from infinity, and $d(r, \theta)$ is the distance between neighboring trajectories at (r, θ) .

Background: The object of this calculation is to determine the ion density in the vicinity of an earth satellite. Such a satellite is expected to acquire an electrical charge and may thus disturb the partially ionized medium through which it moves. Electromagnetic interactions of this type may affect the drag on the satellite or produce other observable effects. The calculation proceeds in three steps: (1) the potential around the satellite is calculated for an assumed ion density distribution; (2) ion trajectories are determined in this potential field; (3) the change in ion density as a result of this motion is calculated. The process is then repeated until consistency is achieved.

Status: New. The program for solving equation (1) using finite differences and a variable weighted relaxation factor has been checked out.

3711-60-0009/59-393 HEAT TRANSFER IN THE PRESENCE OF MOISTURE

Origin: NBS, Section 10.6

Manager: F. L. Alt

Objective: To study solutions of the system of partial differential equations

$$\frac{\partial u}{\partial t} = \ell \nabla^2 u - gz$$

$$\frac{\partial v}{\partial t} = m \nabla^2 v + z$$

$$\frac{\partial w}{\partial t} = n \nabla^2 w - z$$

where z depends on u and v in a known way. Initial and boundary conditions can be formulated in several ways, the simplest being that u and v are given and the normal derivative of w vanishes on the boundary.

Background: The equations describe the process of drying a moist porous substance by applying heat to its outside. The symbol u represents temperature, w the amount of moisture (liquid), and v the amount of water vapor per unit volume, and z the rate of evaporation; ℓ , m , n are the diffusivities of heat, vapor and water, respectively, and g depends on the heat of evaporation. The equations are nonlinear because the heat transfer coefficient ℓ depends strongly on moisture content w , and because z depends nonlinearly on u and v .

The NBS Heat Transfer Section (10.6) is carrying out experimental determinations of heat transfer in moist materials; the purpose of this study is to account separately for the effects of heat conduction and of the heat carried by the diffusion of vapor. The problem was proposed by H. E. Robinson (10.6).

Status: New. The problem has been studied with some simplifying assumptions about the dependence of z on u and v . Steady state solutions have been derived.

Results obtained so far were reported in a paper, "Heat transfer in the presence of moisture," presented at the International Congress of Mathematicians, Edinburgh, 1958.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING Task 3737-60-0002/51-1

Origin: NBS

Authorized 7/1/50

Managers: W. J. Youden, J. Cameron

Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Spectrometric reference standards: Experimental arrangements for elimination of possible trends in the measurements were worked out for a study of the effect of different makes of electrodes on spectrometer performance and for a study of the homogeneity of a new group of standard samples. Coding of the analysis of the data from regular tests was begun. This code uses the output of a general data conversion code prepared by section 11.2 and described under task 1102-40-5126/58-298. This work is being done for R.E. Michaelis, Section 5.10.

(2) Strength of glass: Methods for fitting the Weibull extreme value distribution were investigated on data on the strength of glass, for M. J. Kerper, Section 9.2.

(3) Standard lamps: Methods of analysis were devised and carried out on data on reduction factors used in the calibration of standard lamps, for H. K. Hammond, Section 2.1.

(4) Transistor aging study: An analysis of data from a large scale transistor aging study was completed, for G. Conrad, Section 1.6.

(5) Machine (704) analysis of data and the preparation of special codes were carried out for: H. B. Kirkpatrick, Section 9.4; J. R. Crandall, Section 9.6; M. J. Kerper, Section 9.2; J. Mandel, Section 7.5; and R. E. Michaelis, Section 5.10.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: CONTINUED. Statistical studies on the relation between transition temperature and several physical and chemical properties of two types of ship steel were continued.

MANUAL ON EXPERIMENTAL STATISTICS
FOR ORDNANCE ENGINEERS
Task 1103-40-5146/55-93

Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: CONTINUED. A draft of Part II, sections 1 through 3, Proportions, is being prepared for distribution. A draft of Part II, section 4, Sensitivity Testing, has been completed. A draft of Part IV, Miscellaneous Topics, has been completed.

Publications:

- (1) A note on the computation of χ^2 . Mary G. Natrella. To appear in the American Statistician.
- (2) The relation between confidence intervals and tests of significance--a teaching aid. Mary G. Natrella. Submitted to a technical journal.

STATISTICAL SERVICES
Task 1103-40-5150/58-346

Origin and Sponsors: Various Agencies Authorized 3/31/58
Manager: J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 45

Status: CONTINUED. Work was done during the quarter for the following agencies:

(1) Veterans Administration Hospital, Perry Point, Maryland: Final results of the analysis of covariance of data on the effect of tranquilizers on mental patients were transmitted to the sponsor.

(2) Bureau of Public Roads: See task 1102-40-5126/58-316, p. 28, for report on this task done jointly with the Computation Laboratory.

APPLICATION OF AUTOMATIC COMPUTER

The record of the use of the IBM 704 for the period July 1 through September 30 is as follows:

<u>Task No.</u>	<u>Title</u>	<u>Code</u>			
		<u>Assembly</u>	<u>Checking</u>	<u>Production</u>	
(M I N U T E S)					
<u>NBS:</u>					
1120/55-65	Automatic coding		1078	187	
0009/56-160	Mathematical subroutines	64	220	24	
0009/54-30	Spectrum analysis	60	207	1454	
0009/54-38	Equation of state of real gases			12	
0009/55-68	Crystal structure calculations	61	198	469	
0009/55-82	Thermometer calibrations	72	151	311	
5126/55-88	Stresses in a wall foundation			90	
5126/55-97	High temperature properties for air ^o	23	163	373	
5126/55-121	Electron penetration		18	61	
0009/56-131	Calculations in optics*	15	43	17	
5126/56-166	SCF-LCAO solution of some hydrides		389	83	
5126/56-171	Collision integrals used in transport theory		58		
5126/57-219	Thermal properties*		28	27	
0009/57-223	Self-consistent fields	40	276		
5126/57-229	Payroll operations	103	130		
5126/57-236	Self-consistent field--eigen- values			94	
5126/57-246	Radiation diffusion**	111	358	3152	
0009/57-247	Mechanical impedance		10	97	
5126/57-248	Negative ion detachment			15	
5126/57-249	Color differences*		10	218	
0009/57-250	Automatic reduction in spectro- photometric data		6		
5126/57-252	Detecting efficiency in a neutral meson experiment**	1	272	280	
0009/58-254	Reproduction of color and spectral- energy distribution of daylight			118	
5126/58-255	Chi functions**		62	604	
5126/58-264	Theory of ionization probability		61	34	
5126/58-266	Depolymerization, II		3		
0009/58-267	Munsell color system conversion		8	213	
5126/58-272	Thermodynamic properties of real gases		376	65	
5126/58-274	Calculations for d-spacings, II			269	
5126/58-279	Fire resistant T-beam		45	124	

<u>Task No.</u>	<u>Title</u>	<u>Code</u>		
		<u>Assembly</u>	<u>Checking</u>	<u>Production</u>
		(M I N U T E S)		
5126/58-294	Nuclear scattering of photons			8
5126/58-298	Analysis of spectrochemical data	38	41	8
5126/58-299	Time-dependent Shroedinger equation		60	347
5126/58-304	Transport properties of air		69	23
5126/58-306	Interpolation of color mixture functions		45	25
5126/58-308	Oscillating sphere		49	60
5126/58-312	Response function, II			108
5126/58-322	Propagation constant of a sound wave			6
5126/58-339	Viscoelasticity properties of materials			37
5126/58-359	Velocity distribution in boundary layers			16
5126/58-360	Diffusion coefficients	29	223	89
5126/58-365	Absolute recording performance of magnetic tape		19	
5126/58-367	Ellipsometer calculations**		21	192
5126/59-374	End-effect in viscometer	17	12	3
5126/59-375	Relaxation of Poisson distribution		6	
5126/59-376	Slow electron scattering by hydrogen atoms		93	
5126/59-383	Design flow capacity curves		37	85
0002/51-1	Statistical engineering	28	100	358
5126	Miscellaneous	530	927	274
	Totals...	1192	5872	10030

OUTSIDE

5116/55-56	Research in mathematical topics applicable to numerical analysis	51	100	657
5116/56-148	Differential equations for nerve excitation			394
5113/57-216	Handbook of mathematical functions		43	25
5126/53-45	Air defense tactics ^o		66	122
5126/55-104	Fuze problem ^o		111	418
5126/57-221	Bessel functions for complex arguments			275
5126/58-263	Gas tube characteristic		38	241
5126/58-269	Molecular structure, IV	81	89	363
5126/58-270	Post Office mechanization**		89	56
5126/58-276	General kinetics, I**		12	1795
5126/58-282	Missile boundary layer computation	239	229	1334
5126/58-284	Ephemeris calculations for satellites	49	249	5353
5126/58-286	Azimuth ^o		10	
5126/58-315	Mechanization of French translation**	18	194	432

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Task No.	Title	Code		
		Assembly	Checking	Production
(M I N U T E S)				
5126/58-316	Intersection capacity study		89	
5126/58-317	Scattering, II		110	123
5126/58-320	Teller emission problem ^o	33	83	515
5126/58-335	List of Bessel functions**		6	43
5126/58-336	Helical transforms*		12	
5126/58-340	M5-17 Fuze Data ^o		74	137
5126/58-343	Minimization problem	136	28	31
5126/58-344	Semi-conductor surface measurements		17	31
5126/58-361	Spectrum of dipole radiation	11	207	173
5126/58-363	Crystal lattice computations			5
5126/58-364	David Taylor Model Basin***			3404
5126/58-366	Radiation patterns of antennas		237	27
5126/58-368	Intensity functions		351	
5126/58-370	Neutrons		101	30
5126/59-391	Ion disturbance around a satellite		31	33
	Classified	106	1099	626
	Miscellaneous	<u>49</u>	<u>235</u>	<u>78</u>
	Totals...	773	3910	16721
	Total time for the period ...	<u>1965</u>	<u>9782</u>	<u>26751</u>

* Problem programmed in the Computation Laboratory; production runs continued under direction of sponsor.

** Problem programmed by sponsor and run under his direction.

*** Machine time provided under contract.

^o Classified task.

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Division Seminar

GREENSTEIN, DAVID S. (University of Michigan) Derivatives, translates, and moments. September 4.

Applied Statistics Seminar

MANDEL, J. (NBS Organic and Fibrous Materials Division). A model for the design and analysis of interlaboratory experiments. August 19.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

CONNOR, W. S. Experiments to study the effects of several variables on one or more response variables. Presented at the Joint Session of the Biometrics Society and the American Institute of Biological Sciences, Bloomington, Indiana, August 25.

EISENHART, C. (1) Some examples of recent work of the Statistical Engineering Laboratory of the National Bureau of Standards. Presented before the Research and Industrial Applications Section of the Royal Statistical Society, St. Salvator's College, St. Andrews, Scotland, August 25. (2) Some cannons of sound experimentation. Presented at the International Statistical Institute, Brussels, Belgium, September 1-8.

- NEWMAN, M. Further identities and congruences for the coefficients of modular forms. Presented at the meeting of the American Mathematical Society, Cambridge, Mass., August 26-29.
- TCHEN, C. M. Structure of turbulent diffusion. Presented at the International Symposium on Atmospheric Diffusion, Oxford, England, August 24-29.
- WEGSTEIN, J. What's next in computer language? Presented at the meeting of the Washington Chapter of the Association for Computing Machinery, August 13.
- YOU DEN, W. J. Testing test procedures. Presented before the American Society for Quality Control, Washington, D.C., September 24.
- ZELEN, M. Factorial analysis of life-tests. Presented before the Institute of Mathematical Statistics, Cambridge, Mass., August 25-28.

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Papers presented at the International Congress of Mathematicians, Edinburgh, Scotland, August 7-14:

- ABRAMOWITZ, M. Heat transfer in laminar flow. (Presented by F. L. Alt.)
- ALT, F. L. Heat transfer in the presence of moisture.
- EISENHART, C. A test for extreme residuals.
- GHAFFARI, A. On the behavior in the large of the integral curves of a non-linear differential equation.
- TCHEN, C. M. Correlation in a system of interacting particles.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.3 Technical Papers

- (1) A survey of Lyapunov's second method. H. Antosiewicz. "Contributions to the theory of nonlinear oscillations," Vol. IV, ed. by S. Lefshetz, pp. 141-166 (Annals of Mathematics Studies No. 41, Princeton University Press, Princeton, N.J., 1958).
- (2) The coefficients in an allocation problem. R. J. Aumann and J. B. Kruskal (University of Wisconsin). Naval Research Logistics Quart. 5, 111-123(1958).
- (3) Fractional factorial experiments of $2^m 3^n$ series. W. S. Connor. Proceedings of the Fourteenth Annual Clinic of the Rochester Society for Quality Control, February 18, 1958, pp. 59-70.
- (4) Additional abscissas and weights for Gaussian quadratures of high order; values for $n = 64, 80, \text{ and } 96$. P. Davis and P. Rabinowitz (Weizmann Institute). J. Research NBS 60, 613-614 (June 1958), RP 2875.
- (5) Note on circular disks containing the eigenvalues of a matrix. Ky Fan. Duke Math. J. 25, 441-446(1958).
- (6) Topological proofs for certain theorems on matrices with non-negative elements. Ky Fan. Monatsch. Math. 62, 219-237(1958).
- (7) Mechanized computation of thermodynamics tables at the National Bureau of Standards. J. Hilsenrath (NBS Thermodynamics Section) and J. Wegstein. Proceedings of the Joint Conference on Thermodynamic and Transport Properties of Fluids, held in London, July 10-12, 1958, pp. 79-90 (Institution of Mechanical Engineers, London, 1958).
- (8) Congruences for the coefficients of modular forms and for the coefficients of $j(\tau)$. M. Newman. Proc. Amer. Math. Soc. 9, 609-612(1958).
- (9) Generation of Bessel Functions on high speed computers. I. Stegun and M. Abramowitz. Math. Tables Aid. Comp. 11, 255-257(1957).

- (10) National physical standards and design of experiment. W. J. Youden. Bull. International Statistical Inst. 35, 191-198(1957).

1.4 Reviews, Notes, and Abstracts

Abstracts of Short Communications and Scientific Programme,
International Congress of Mathematicians, Edinburgh, 1958:

- (1) Heat transfer in laminar flow. M. Abramowitz. p. 135.
- (2) Unsteady nonlinear waves in sloping channels. R. F. Dressler. p. 138.
- (3) On the behavior in the large of the integral curves of a non-linear differential equation. A. Ghaffari. pp. 50-51.

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- (4) The Stokes flow problem for a class of axially symmetric bodies. L. E. Payne and W. H. Pell. Abstract: Notices Amer. Math. Soc. 5, 468(1958).

1.5 Miscellaneous

- (1) Contributions to Handbook of Physics, ed. by E. U. Condon and H. Odishaw (McGraw-Hill Book Co., Inc., New York, 1958):
F. L. Alt, Arithmetic, Chap. 1, pp. 1-4 to 1-9.
M. Abramowitz, Integral equations, Chap. 6, pp. 1-90 to 1-94.
C. Eisenhart and M. Zelen, Elements of probability, Chap. 12, pp. 1-134 to 1-164.
W. J. Youden, Statistical design of experiments, Chap. 13, pp. 1-165 to 1-168.
- (2) "Computers." F. L. Alt. An item in The Encyclopedia of Chemistry Supplement, pp. 91-92 (Reinhold Publishing Corp., New York, 1958).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION SEPTEMBER 30, 1958

2.1 Mathematical Tables

- (1) Tables of the bivariate normal distribution function and related functions. To appear as NBS Applied Mathematics Series 50.
- (2) Fractional factorial experiment designs for factors at three levels. To appear as NBS Applied Mathematics Series 54.
- (3) Tables of osculatory interpolation coefficients. H. E. Salzer. To appear in the NBS Applied Mathematics Series.

2.3 Technical Papers

- (1) Heat transfer in laminar flow through a tube. M. Abramowitz, W. F. Cahill, and C. Wade, Jr. Submitted to a technical journal.
- (2) A note on Hahn's theorem. R. J. Aumann. Submitted to a technical journal.
- (3) A theoretical foundation for the numerical evaluation of worth in subjective allocation problems. R. J. Aumann. Submitted to a technical journal.
- (4) Time phasing in the allocation problem. R. J. Aumann. Submitted to a technical journal.
- (5) Assigning quantitative values to qualitative factors in the Naval electronics problem. R. J. Aumann and J. B. Kruskal (University of Wisconsin). To appear in the Naval Research Logistics Quarterly.
- (6) Analysis of fractionally replicated $2^n 3^m$ designs. R. C. Bose and W. S. Connor. To appear in Revue de L'Institut International de Statistique (The Hague).
- (7) Abelian groups of unimodular matrices. E. C. Dade. Submitted to a technical journal.
- (8) The construction of Hadamard matrices. E. C. Dade and K. Goldberg. Submitted to a technical journal.
- (9) On the numerical integration of periodic analytic functions. P. J. Davis. To appear in the Proceedings of the Symposium on Numerical Approximation, held at the Mathematical Research Center of the U. S. Army, Madison, Wisconsin, April 21-23, 1958.
- (10) Bending and stretching of corrugated diaphragms. R. F. Dressler. Submitted to a technical journal.
- (11) Unsteady nonlinear waves in sloping channels. R. F. Dressler. Submitted to a technical journal.
- (12) Some canons of sound experimentation. C. Eisenhart. To appear in the Proceedings of a Special Session of the International Statistical Institute, Brussels, Belgium, Sept. 1-8, 1958.
- (13) Note on bivariate linear interpolation for analytic functions. W. Gautschi. Submitted to a technical journal.
- (14) Some elementary inequalities relating to a gamma and incomplete gamma function. W. Gautschi. Submitted to a technical journal.

- (15) A continuous poker game. A. J. Goldman and J. J. Stone. Submitted to a technical journal.
- (16) Field convexity of a linear transformation. A. J. Goldman and M. Marcus. Submitted to a technical journal.
- (17) On a generalization of an inequality of L. V. Kantorovich, W. Greub and W. Rheinboldt. Submitted to a technical journal.
- (18) On the domain of regularity of generalized axially symmetric potentials. P. Henrici. To appear in the Proceedings of the American Mathematical Society.
- (19) Mechanized computation of thermodynamics tables at the National Bureau of Standards. II. Equilibrium compositions and thermodynamics properties of dissociated and ionized gaseous systems. J. Hilsenrath, M. Klein (NBS Thermodynamics Section), and D. Y. Sumida. To appear in the Proceedings of a Symposium of the American Society of Mechanical Engineers, to be held at Purdue University, Lafayette, Indiana, February 1959.
- (20) Numerical experiments in potential theory using the Nehari estimates. U. W. Hochstrasser. To appear in Mathematical Tables and Other Aids to Computation.
- (21) Permanents of doubly stochastic processes. M. Marcus and M. Newman. Submitted to a technical journal.
- (22) A note on the computation of χ^2 . M. G. Natrella. To appear in the American Statistician.
- (23) The relation between confidence intervals and tests of significance -- a teaching aid. M. G. Natrella. Submitted to a technical journal.
- (24) Dense subgraphs and connectivity. R. E. Nettleton (NBS, 3.2), K. Goldberg, and S. M. Green (NBS, 3.2). To appear in the Canadian Journal of Mathematics.
- (25) Construction and application of a class of modular functions, II. M. Newman. To appear in the Proceedings of the London Mathematical Society.
- (26) Further identities and congruences for the coefficients of modular forms. M. Newman. To appear in the Canadian Journal of Mathematics.
- (27) Inclusion theorems for congruence subgroups. M. Newman and I. Reiner (University of Illinois). To appear in the Transactions of the American Mathematical Society.

- (28) The evaluation of matrix inversion programs. M. Newman and J. Todd (California Institute of Technology). Submitted to a technical journal.
- (29) On the derivative of Bessel functions with respect to the order. F. Oberhettinger. Submitted to a technical journal.
- (30) On the diffraction and reflection of waves and pulses on wedges and corners. F. Oberhettinger. To appear in the Journal of Research, NBS.
- (31) On Gauss' speeding up device in the theory of single step iteration. A. M. Ostrowski. To appear in Mathematical Tables and Other Aids to Computation.
- (32) On the convergence of the Rayleigh quotient iteration for the computation of the characteristic roots and vectors, II. A. M. Ostrowski. Submitted to a technical journal.
- (33) A further extension of Cayley's parameterization. M. Pearl. To appear in the Canadian Journal of Mathematics.
- (34) A note on commutators. M. Pearl. Submitted to a technical journal.
- (35) On a theorem of M. Riesz. M. Pearl. To appear in the Journal of Research, NBS.
- (36) On normal and EPr matrices. M. Pearl. To appear in the Michigan Journal of Mathematics.
- (37) The graphical solution of initial value problems. W. H. Pell. Submitted to a technical journal.
- (38) The non-central χ^2 as a test statistic. N. C. Severo. Submitted to a technical journal.
- (39) Stationary principles for forced vibrations in elasticity and electromagnetism. J. L. Synge. To appear in the Proceedings of the Eighth Symposium in Applied Mathematics held by the American Mathematical Society, Chicago, Ill., April 1956.
- (40) Some computational problems concerning integral matrices. O. Taussky. To appear in the Proceedings of the 1956 meeting of The Italian Society for the Advancement of Science, held in Sicily.
- (41) Diffusion of particles in turbulent flow. C. M. Tchen. To appear in the Proceedings of a Symposium on Atmospheric Diffusion, Oxford, England, August 1958.

- (42) Turbulent motion. C. M. Tchen and G. B. Schubauer (NBS Fluid Mechanics Section). To appear as Section B, Volume V of the Princeton Series, High Speed Aerodynamics and Jet Propulsion.
- (43) Kinetic equation for a plasma with unsteady correlations. C. M. Tchen. Submitted to a technical journal.
- (44) Computation problems concerned with the Hilbert matrix. J. Todd. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.
- (45) Theory of the effect of drag on the orbital inclination of an earth satellite. J. P. Vinti. Submitted to a technical journal.
- (46) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.
- (47) Statistics -- Engineering viewpoint. W. J. Youden. To appear in the Journal of Engineering Education.
- (48) Problems in life testing: factorial experiments. M. Zelen. To appear in the Proceedings of the Thirteenth Midwest Quality Control Conference, Kansas City, Mo., November 1958.
- (49) Multi-variable experiments. M. Zelen and W. S. Connor. To appear in Industrial Quality Control.
- (50) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. S. Joel. Submitted to a technical journal.

2.4 Reviews and Notes

- (1) A note on algebras. A. J. Goldman. Submitted to a technical journal.



THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major laboratories in Boulder, Colo., is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside front cover.

WASHINGTON, D. C.

Electricity and Electronics. Resistance and Reactance. Electron Devices. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology. Engine Fuels. Free Radicals Research.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Neutron Physics. Radiation Theory. Radioactivity. X-rays. High Energy Radiation. Nucleonic Instrumentation. Radiological Equipment.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Molecular Structure and Properties of Gases. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Concreting Materials. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Air Conditioning, Heating, and Refrigeration. Floor, Roof, and Wall Coverings. Codes and Safety Standards. Heat Transfer.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analog Systems. Application Engineering.

• Office of Basic Instrumentation.

• Office of Weights and Measures.

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships. VHF Research. Ionospheric Communication Systems.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Systems. Navigation Systems. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Radio Systems Application Engineering. Radio-Meteorology.

Radio Standards. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Electronic Calibration Center. Microwave Physics. Microwave Circuit Standards.

