NATIONAL BUREAU OF STANDARDS REPORT

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INDEX TO THE DISTRIBUTIONS OF
MATHEMATICAL STATISTICS

by

Frank A. Haight

NBS

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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INDEX TO THE DISTRIBUTIONS OF MATHEMATICAL STATISTICS

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Statistical Engineering Laboratory

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This index of distribution functions was prepared under the Statistical Engineering Laboratory's program of developing aids for the application of modern statistical methods in the physical sciences. The index is a fairly complete summary of published results on statistical distributions, and should serve to eliminate unnecessary derivation of results already in the literature.

PREFACE

This index was started in April, 1954 with the limited intention of supplying my students at Auckland University College with a small reference pamphlet. In their study of mathematical statistics it appeared that no text book contained a complete treatment of all the distributions which a student might encounter; my index was supposed to facilitate a quick selection of the appropriate book.

Once started, it was not difficult to continue noting information. I even persuaded myself to spend the summer of 1954-55 in a systematic search of journals. By the beginning of 1955 my interest in the project faltered, and simultaneously the supply of statistical journals available in New Zealand ([a] -- [o] of Table 1) failed. I typed the collected results on stencils and published the index in mimeographed form. During the three years following, I sent out several hundred copies of the index in response to requests; finally the stencils wore out.

At the invitation of the National Bureau of Standards, I spent the summer of 1958 at the Statistical Engineering Loboratory supplementing and editing the index for publication. This work has included:

(a) Extending the range of journals covered,

(b) Bringing these up to the end of 1957,

(c) Collecting items from several additional books,

(d) Adding information supplied me by readers of the original version, and

(e) Correcting various mistakes found in the original version.

I wish to thank Dr. Churchill Eisenhart for making possible the invitation to the N. B. S., and Mrs. Lola Deming for helpful advice on the typescript.

Also, I am grateful to Dean L. M. K. Boelter (acting on behalf of the Regents of the University of California) for granting me two months' leave from my work at the Institute of Transportation and Traffic Engineering, even though I was newly employed by them.

Frank A. Haight

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INTRODUCTION

1. Organization.

The material given under each distribution consists of a number of entries, most of which are provided with one or more references. In the case of the normal distribution with mean m and variance v (No. 1.1) the number of entries is fairly large, and therefore the standard order is most easily seen:

I Functions and constants which characterize the distribution

II Derived distributions

(a) of linear quantities

(b) of quadratic quantities

III Estimation

(a) point

(b) interval

IV Testing statistical hypotheses

(a) by linear statistics

(b) by quadratic statistics

V Miscellaneous

VI "See also"

These categories are by no means always used for less important distributions. With the limited information available a complete listing of the headings in such cases would be wasteful since the majority would be empty. Keeping in mind the above order, it should not be difficult to find

the required entry.

Occasionally an entry will be indented; such an entry should be read as a continuation of the preceding one.

2. References.

The references to the literature are of the following types: coded, uncoded, reviews.

I Coded

(a) Journals, e.g. [c]4:17, which refers to the 17th page of the 4th volume of the journal designated as [c] in Table 1.

(b) Books, e.g. [12]53, which refers to the 53rd page of the book designated as [12] in Table 2.

II Uncoded, e.g. Trans. Am. Math. Soc. 17:382, conforming to the usual volume and page reference style.

III Reviews

(a) Mathematical Reviews is designated by MR,

(b) Zentralblatt fur Mathematik is designated by Z.

MR and Z references will in no case offer a review of a paper appearing in coded journals and therefore may be considered to indicate publications in obscure (from the point of view of the present work) sources. Moreover, every effort has been made to avoid a MR or Z references to an uncoded paper quoted and very few duplications of this sort should be found.

The choice between direct (i.e. coded or uncoded) and indirect (i.e. review) references is frequently available. The one given is the one which was actually inspected, with all direct references collected before the search of MR and Z. Consequently each entry corresponding to a direct references is based on the paper, and never its review, and each entry corresponding to an indirect reference is based on the review and never the paper.

Since it is difficult to distinguish priority in a large number of references, a chronological table of the coded and review references is provided. This table also exhibits which volumes have been systematically searched in the preparation of this index.

3. Criteria for Inclusion

I. Distributions. As a general principle, a distribution is included if its density (or probability) function is a known, explicit function. The following exceptions may be noted:

(a) Certain families of distributions are mentioned, e.g. Pearson and Koopman, whose densities are specified only implicitly.

(b) Certain distributions are mentioned in terms of their cumulative probability function or characteristic function.

II. Entries. The general principle governing the selection of entries is this: that it must exhibit a property of the distribution in question. Exceptions to this rule are generally of one of the following forms:

(a) Historical information about well known distributions, although not systematically sought, may in some circumstances be included.

(b) Important applications, such as those which led to the discovery of the distribution are usually supplied.

(c) Bibliographies

Reference to tables has been excluded in almost every case.

It is clear that applications must be severely limited. With a slight exaggeration, several whole branches of statistics may be considered applications of some particular distribution, as exhibited for example in the following table:

DISTRIBUTION	APPLICATION
Binomial	Quality control
Normal	Analysis of variance
Lognormal	Probit analysis
Poisson	Random processes
Deterministic	Applied mathematics

4. Relationship between distributions

I. Mention. In some cases (such as 2.1 and 2.3) the relationship between two distributions is asserted in their designation. In others (such as 5.3 and 5.15) a very close connection is not pointed out. In the majority of cases, however, known relationships are simply listed among the miscellaneous properties of both.

In choosing between these alternatives, an attempt has been made to reproduce current statistical usage and terminology.

II. Inclusion. Very similar principles have been used to decide for or against independent listing. If one distribution is relatively important and its equivalent much less so (for example Chi-square and Erlang) inclusion has been practiced. In other cases independent reputation seems to justify independent categories.

It must certainly be supposed that many of the trivial distributions of Chapter VIII could be included in some larger category, or even combined with each other. The production of such a systematic classification which would exhibit all connections, even if worth doing, is certainly removed from the purpose of this book, and has hardly been attempted.

For example, it is well known that No. 8.1 contains as special cases all the distributions of Chapters I and II;

very likely it also contains dozens of others listed. Nevertheless to indicate this by a system of sub-headings, applied to all entries, would quickly undermine the utility of the whole work, since it is the special cases rather than the general principle which occur in statistical practice.

5. Notation and Terminology

In univariate distributions the stochastic variable is always denoted by x, in bivariate by x and y and in multivariate by $x_1, \dots, x_k$, quite regardless of the domain of definition. This departs from the usage of certain authors in two respects:

(a) The letter n is not used for a discrete variable.

(b) The statistic obeying a particular distribution is not used in the density. For example in Student's "t" distribution we write

$$\left(1 + \frac{x^2}{r}\right)^{-\frac{1}{2}(r+1)}$$

rather than

$$\left(1 + \frac{t^2}{r}\right)^{-\frac{1}{2}(r+1)} .$$

This practice is justified not only by the need for uniformity but by the belief that the alternative is wasteful of the alphabet: t, F, z, D, . . . Similarly we prefer to call distributions by the names of their discoverers (or reputed discoverers) rather than by the symbol used to denote some

statistic found to satisfy them. Of course all known designations will be found in the final index.

In many books the expression $f(x)$ is employed to denote a probability density. However f is commonly used in mathematics for an arbitrary function, and therefore we prefer to adopt something more distinctive for this special function, and have selected $D(x)$ for the purpose.

In the discrete case this replaces the probability distribution, which is often written p_n. $C(x)$ is the cumulative function.

When we come to the characteristic function the situation is a little more complicated. Using t for the variable, statistical works generally have to define several symbols for characteristic functions of various quantities, for example:

$\chi(t)$ = characteristic function of distribution of x

$\varphi(t)$ = characteristic function of distribution of $n\overline{x}$

$\zeta(t)$ = characteristic function of distribution of $\overline{x}$, etc.

Since we will be dealing with many different statistics and possibly their characteristic functions, it is more economical and systematic simply to abbreviate by the following system: $Ch(x)$, $Ch(n\overline{x})$, $Ch(\overline{x})$, etc. Thus it is not necessary to select a new letter to denote the characteristic function of each new statistic.

However, this practice leads to equations like

$$Ch(x) = e^{-\frac{1}{2}vt^2}$$

which may be offensive to some, however clear the meaning. Such readers are advised to interpret the equality sign as an abbreviation for the verb "is".

This interpretation has another important connection with the notation being used. A variety of verbs have been employed to describe the relation between a stochastic variable and its distribution, for example:

x <u>obeys</u> the normal distribution with mean m and variance v,
x <u>follows</u> the normal distribution with mean m and variance v,
x <u>is</u> a normal variable with mean m and variance v.

It seems equally felicitous to assert this relationship by the convenient abbreviation

$$D(x) = N(m,v)$$

which may, if advantageous for any reason, be regarded not as a mathematical equation but as shorthand. In any case it makes possible an unambiguous condensation of the facts.

Similar remarks apply to the expressions MGF(x), FD(p) which are used to mean moment generating function of the distribution of x and fiducial distribution of the parameter p.

Another application of this use of the equality sign relates to the symbols C.-R.(p), MLE(p), MME(p), UMVUE(p), BANE(p), and is exemplified by the following:

$$\text{C.-R.}(\sigma) = \frac{v}{2n}$$

$$\text{MLE}(v) = s^2.$$

For the meaning of these and other abbreviations, the reader is referred to the List of Abbreviations.

LIST OF ABBREVIATIONS

D(x)	Density or probability function of a stochastic variable x
C(x)	Cumulative distribution function of x
Ch(x)	Characteristic function of distribution of x
MGF(x)	Moment generating function of distribution of x
PGF(x)	Probability generating function of x
FD(p)	Fiducial distribution of a parameter p
m	Mean of a population
$\bar{x}$	Mean of a sample
$v = \sigma^2$	Variance of a population
s^2	Variance of a sample
μ_k	k^{th} central moment of a population
α_k	k^{th} moment about the origin for a population
k_k	k^{th} cumulant
r	Correlation coefficient in a sample
ρ	Correlation coefficient in a population
ξ	Median
GM	Geometric mean
HM	Harmonic mean
n	Number of items in a sample
β	Slope of regression line in a population
b	Slope of regression line in a sample
$\sim$	Asymptotic (= large sample)
β_1, β_2	Pearson's betas

C.-R.(p)	Cramer-Rao lower bound for variances of estimates of the parameter p
MLE(p)	Maximum likelihood estimate of the parameter p
MME(p)	Minimax estimate of the parameter p
$M\chi^2E(p)$	Minimum Chi-square estimate of the parameter p
UMVUE(p)	Uniformly minimum variance unbiased estimate of the parameter p
BANE(p)	Best asymptotically normal estimate of the parameter p
LR	Likelihood ratio
L	The likelihood function $\Pi\ D(x_j)$
Seq	Sequential
OC	Operating characteristic
BCR	Best critical region
Q	A quadratic form
E	Expectation

1.1 NORMAL (m,v)

I. Functions and parameters

$D(x) = \frac{1}{\sqrt{2\pi v}} e^{-\frac{(x-m)^2}{2v}}$	[6]108, [5]34, [4]57, [8]91, [9]243
$Ch(x) = \exp(-\frac{1}{2} vt^2 + mit)$	[1]211, [5]62
$MGF(x) = \exp(\frac{1}{2} vt^2 - tm)$	[6]112
Derivatives etc.	[d]2:181
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Obtained from Pearson's differential equation	[4]72
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Mean deviation $E\|x-m\| = (2v/\pi)^{\frac{1}{2}} = .79788\sigma$	[1]258

Probable error = .6745 [4]58

$\alpha_{2k} = (2k - 1)v^k$ [5]XII, [8]98

Quasi-range [d]24:603

II. Derived distributions

$D(\bar{x}) = N(m, vn^{-1})$ [9]270, [6]10.2, [2]243, [4]100, [w]1:93

$D(\bar{x}/s)$ [3]139

$D[(n-1)\ s^{-1}(\bar{x} - m)] =$ Student(n-1) [6]217, [5]98, [2]239, [4]112, [w]1:74

$D[s^{-1}(n-1)^{\frac{1}{2}}(\bar{x} - m_i)]$, where m_i not all equal [d]19:406

$D[(\bar{x} - m)/\text{range}]$ [d]22:469

D (range) etc. MR13:762

$D(\Sigma k_i x_i) = N(\Sigma k_i m_i, \Sigma k_i v_i)$ [4]99, [8]92

$\left(\frac{x-m}{\sigma}\right)^2$ is Chi-square, $\left(\frac{x}{\sigma}\right)^2$ is non-central Chi-square, $(x-m)^2$ is Type III, product of normal variables is Bessel, quotient $\sim$ normal for large m/σ [18]1-150

$$D\left[\frac{m_1-m_2(x_1/x_2)}{\sqrt{v_1+v_2(x_1^2/x_2^2)}}\right] \sim N(0,1),\quad m_2 \gg \sigma_2$$ [2]253

$$D(\bar{x}_1-\bar{x}_2) \sim N(m_1-m_2,\ v_1/n_1 + v_2/n_2)$$ [4]100

$$D\left[\frac{\bar{x}_1-\bar{x}_2}{v\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\right] \sim N(0,1)$$ [6]263

$$D\left[\frac{(\bar{x}_1-\bar{x}_2)\sqrt{n_1+n_2-2}\sqrt{n_1n_2}}{\sqrt{n_1+n_2}\sqrt{n_1s_1^2+n_2s_2^2}}\right] \sim \text{Student}\ (n_1+n_2-2)$$ [4]112, [3]109,112, [5]98, [c]29:350, [c]33:252, MR8:42

$$D\left[(n-2)\,\frac{n_1(\bar{x}_1-m_1)^2+n_2(\bar{x}_2-m_2)^2}{s_1+s_2}\right]$$ [4]132

$\sim$ Snedecor (2, n-2), confidence ellipse

D(k^{th} value from top) [1]374, [d]25:565

D(smallest sample value) $\sim$ No. 8.40 [g]42:408

$D[\frac{1}{2}v^{-1}(x-a)] \sim \text{Gamma}\ (\frac{1}{2})$ [10]150

D(HM) MR4:164

$D(\sqrt{x^2+y^2})$, $D(\sqrt{x^2+y^2+z^2})$ in special circumstances MR16:377

$D(\Sigma x_i^2)$	[1]3:353
$D(\chi^2) = D\left(\Sigma \dfrac{x_i - m_i}{\sigma_i}\right)^2 = \chi^2(n)$	[6]10.3
$D(s^2)$ = Type III $(n/2v, \tfrac{1}{2}n)$	[d]5:281, [6]10.3
$D[(1/n)\ \Sigma\ (x_i - \bar{x})^2]$ = Type III $(\tfrac{1}{2}nv^{-1}, \tfrac{1}{2}(n-1)]$	[6]10.4
$D(s)$ for $n = 2,3$	[c]11:277
$D(s/\bar{x})$, Coefficient of variation	[d]7:129, [a]94:564, [a]95:695
$D(s/\text{range})$	[d]17:366, [c]31:20,
$D[(2s^2)^{-\frac{1}{2}}]$, "precision constant" = Type V, moments, etc.	[d]3:20, [a]97:132
$D(\log s^2)$	[b]8:128
$D(ns^2v^{-1}) = \chi^2(n-1)$, For unequal v, $D(s_1/s_2)$ = generalized Student $(n_2v_1/n_1v_2 + 1, n_1 + n_2 - 1)$	[3]115
$D(s_1^2v_2/s_2^2v_1)$ = Snedecor $(n_1 - 1, n_2 - 1)$	[4]115, [10]197,
testing and confidence intervals	[d]13:371,
power function	[d]17:182

$D(n_1 s_1^2 v^{-1} + n_2 s_2^2 v^{-1}) = \chi^2 \ (n_1 + n_2 - 2)$,

$D\left(\frac{1}{2}\log \frac{n_1(n_2 - 1)s_1^2}{n_2(n_1 - 1)s_2^2}\right) =$ Fisher $(n_1 - 1, n_2 - 1)$ [10]198

$D(v^{-1}\Sigma \ (n_i - 1)s_i^2) = \chi^2$ [4]116

D(variance ratio) [k]11:136

Distribution of various statistics from k normal populations with common variance [e]17:2

[illegible] [illegible]

Ranking variances [g]51:621

Distribution of many quantities in a wide variety of cases J. Soc. Stat. Paris 96:262

D(various Q) MR13:142

$D(\bar{x}, s)$ [2]238, MR8:161

$D(b_2)$ for n=4 is hypergeometric [c]25:411

$D(b_1)$ for n=4 is hypergeometric [c]25:207, [c]33:68

D(midrange) [d]21:100

D(range) [c]17:364, [c]18:173, [c]24:404, [c]39:130

Quasi-range [d]28:179

D(r) [b]15:193

$D(\xi)$	[d]26:114
$D(Q)$	MR22:60
$D(x_1 x_2)$	[d]7:1, [d]18:265
$D(x_1, \ldots, x_n)$	[4]98
$FD(m) = N(\bar{x}, vn^{-1})$	[c]30:401,414, [p]17:231
$FD[n^{\frac{1}{2}} s^{-1} (\bar{x} - m)] =$ Student $(n - 1)$	[3]88
Tests	[3]98, [c]33:173, [n]5:90, [d]9:279, [k]6:395
$FD(m,\sigma)$	[3]89, [k]6:395, [d]10:68
$FD(1/\sigma) =$ Helmert $\left(n-1, \frac{1}{s\sqrt{n}}\right)$	[3]89
A priori distributions of m and $1/\sigma$	MR9:48
Ranking means	[t]7:131

III. Estimation

C.-R.$(m) = vn^{-1}$, i.e. $\bar{x}$ efficient	[1]483, [4]135, [3]20
$\mathrm{Var}(\bar{x}) \leq \mathrm{var}(\xi)$	[4]92

UMVUE(m) = $\bar{x}$	[3]51, [t]4:167
$\bar{x}$ unbiased	[p]7:150
Minimax estimates of m	[d]21:218, [d]22:28
Best "density unbiased" estimate of m	[d]25:399
~ efficiency of ξ is .6366	[1]490, [u]22:706
Estimation of m when it must be integral, etc.	[b]12:192
Mean of k^{th} values from top and bottom has ~ efficiency zero	[1]490
C.-R.(σ) = $v(2n)^{-1}$, hence s is efficient	[2]224
Efficiency of estimates of σ	[c]37:182

In estimating σ,

$$\sqrt{\tfrac{1}{2}n}\ \frac{\Gamma(\frac{1}{2}n)}{\Gamma[\frac{1}{2}(n+1)]}\sqrt{\frac{1}{n}\Sigma(x_i-m)^2}$$ is [1]484

more efficient than

$$\sqrt{\tfrac{1}{2}n}\ \frac{\Gamma[\frac{1}{2}(n-1)]}{\Gamma(\frac{1}{2}n)}\ s$$

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Estimation of σ from percentiles	[i]40:85
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If m constant, s^2 not sufficient	[3]11, [4]136
In estimating v, ~ efficiency of mean deviation = .876	[3]7
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$\bar{x}$, $\frac{n}{n-1}$ s^2 are ~ efficient	[1]494
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The most powerful test of

$$\begin{cases} H: \sigma = \sigma_0, & m \\ Alt: \sigma = \sigma_1^o, & m = m_1 \end{cases} \quad \text{is } \Sigma(x_i - \overline{x})^2 \gtrless c$$

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Seq. ratio test terminates	[e]8:342
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Power functions of tests	[d]17:189
Whether two samples are from the same normal population	[n]7:3, [k]17:302
Tests for normality	[c]28:295,[c]27:310,333 [c]34:209, [o]1:125, [i]20:152, Z19:74

Tests of various composite hypotheses	[d]19:495, [e]9:30, [e]10:29
Decision problems	[b]15:55

V. <u>Miscellaneous</u>

Independence of $\overline{x}$ and s (Student-Fisher Theorem)	[i]19:108, J. Math. Soc. Jap. 1:111, MR1:346
s^2, $\overline{x}$ independent	[4]108, MR14:775
Normality if and only if $\overline{x}$ and s^2 independent	[d]13:91, [d]16:400, [d]13:91, NBS Rep. 2267, J. Math. Soc. Jap. 1:111
Normality if and only if $D(\overline{x}, s) \equiv L.\ sn^{-2}$	[d]14:197
Normality if $D(x)\ D(y) = \phi \sqrt{x^2 + y^2}$	MR10:125
Various characterizations of normality	[e]13:359, [d]28:126, [i]39:59, [d]27:858, [e]14:180, Am. J. Math. 61:726, Math. Z.41:405, Z13:214, 3rd Berkeley Symp. 2:195, MR16:1034
Generalization of Student-Fisher Theorem	[i]20:248

Independence of quadratic forms	[o]1:83, Proc. Roy. Soc. Edin. 60:40, [u]30:178, [c]37:93, [c]14:195, [d]15:427, [d]20:119, MR12:509
Bayes theorem	[n]16-1:113
Cochran's theorem	[4]107,68,
More generally	[d]11:100
History of Normal	[c]16:402
History of distribution of s	[c]23:416
Distributions which converge to normality	[d]10:247
Discrete Analogue	[c]44:365
Regression of x and t, where $m = a - be^{-kt}$	[d]18:596
Comparing percentage points of two normals	[d]19:93
k samples	[3]295
Truncated sample	[d]23:237
Sampling from $N(\Sigma\, a_k m_k, v)$	[4]160

Max Σx_i, min Σx_i [c]40:35

See Also: [d]25:389, [d]1:151, [d]6:197, [d]7:77, [d]10:365, [d]13:235, [d]17:483, [d]20:123, [d]21:362, [d]21:557, [d]22:596, [d]23:43, [d]23:384, [d]23:547, [d]25:16, [ℓ]3:309, [e]9:6, [13]345, [c]10:522, [c]13:287, [c]16:239, [c]24:184, [a]79:455, [f]7:23, [c]31:238, [c]34:61,98, [n]5:3, [n]12-3:65, [c]40:116, [c]32:226,301, [g]48:550, [d]25:636,698, [v]5:337, [g]51:88, Brit. Assoc. Math. Tables (3rd Ed.) V.I p. xxviii, N.B.S. Rep. 2545, C.R. Acad. Sci. Paris 238:444, [o]1:83, App. Sci. Res. (Netherlands) Section A 3:297, J. Franklin Inst. 260:209, N.B.S. Rep. 2267, Am. J. Math. 57:821, Ann. Math. 35:312, Nat. Acad. Sci. 28:297, [y]24:2, Z18:225, MR16:52, MR14:1098, Z5:173, Z19:317, MR17:53, [w]7:193.

1.2 NORMAL: N(0,v)

$D(x) = (2\pi v)^{-\frac{1}{2}} \exp(-\frac{1}{2} x^2 v^{-1})$	[10]50, [c]31:1
$Ch(x) = \exp(-\frac{1}{2} vt^2)$	[2]94
$MGF(x) = \exp(\frac{1}{2} vt^2)$	[2]53
C(x)	[c]25:379
$\alpha_{2k} = \frac{\sigma^{2k}(2k)!}{2^k k!}$	[10]54
2nd cumulant = v, others zero	[2]67
Pearsonian type	[2]141
$D(\bar{x}) = N(0, v/n)$	[2]175, [n]10-3:90
D(x/y) = Cauchy	[18]1-150, [w]1:74
$D(x^2)$ = Type III	[i]26:212
$D(s^2)$	[2]246, [u]28:456, Z15:118
$D(\Sigma x_i^2/2v)$ = Gamma $(\frac{1}{2} n)$	
$D(\Sigma x_i^2)$ etc.	[c]35:47
$D(\Sigma x_i^2)$, $D(Q_1/Q_2)$	[w]1:74
$D(Q/v) = \chi^2(r)$, Q of rank r; $\neq 0$ eigenvalues all +1	[d]9:48, [c]25:122, Z19:357 [c]30:407

$Ch(Q) = [\ \Pi(1-2itvk_i)^{\frac{1}{2}}\ ]^{-1}$	
$D[x\ n^{\frac{1}{2}}\ (\Sigma x_i^2)^{-\frac{1}{2}}\ ]$ = Type II	[i]29:13
D(x) assuming v is Type III	[d]28:510
D(x/s) = Student (testing)	[c]37:65
D(range) for n=3, unbiased critical region	[3]327
FD(v) = Type V[$\Sigma\ x_i^2/n-2,\ \frac{1}{2}(n-2)$]	[p]7:226
C.-R.(v) = $2n^{-1}v^2$	[1]484, [p]7:159
Suppose n obeys No. 3.5	MR14:391
MLE(v) = $n^{-1}\Sigma\ x_i^2$	[4]141
s^2 UMVUE of v, but s not of σ	[3]52,54
Neyman-Pearson on hypothesis testing	[c]20:178
Most powerful test of $\begin{cases} H: \sigma = \sigma_0 \\ Alt: \sigma = \sigma_1 \end{cases}$ is $\bar{x}^2 + s^2 \lessgtr c$	[3]275
Completeness	[e]10:313
Unbiased critical regions	[3]212
Testing s_1^2/s_2^2 etc.	[e]5:157

Testing serial correlation	[i]31:103
Various devices for showing area = +1	[d]5:136
Inference	[b]15:52
As "Maxwell-Bolzmann" distribution	[12]39
Variance of mean deviation is $\sim vn^{-1}(.8068)^2$	[2]217
Mean difference	[c]28:432
Properties of f(x), where x is N(0,v)	[c]17:211

<u>See Also</u>: [d]12:239, [c]3:311, [g]26:178, [c]31:260, [n]13-1:51, [u]30:330, MR9:364, MR3:2, [a]83:127.

1.3 NORMAL: N(m,1)

$D(x) = (2\pi)^{-\frac{1}{2}} \exp[-\frac{1}{2}(x - m)^2]$

$E(x_1^2 + x_2^2) = (\frac{1}{2}\pi)^{\frac{1}{2}}$, $Var(x_1^2 + x_2^2) = 2 - \frac{1}{2}\pi$, $E|x| = (2/\pi)^{\frac{1}{2}}$, $E(e^{ax}) = e^{\frac{1}{2}a^2}$, $var(e^{ax}) = e^{2a^2} - e^{a^2}$ [8]120

$D(\overline{x})$	[3]2
$D(x^2)$, $D(x_1^2 + x_2^2)$, $D(x_1^2 + x_2^2)^{\frac{1}{2}}$	[8]95
$D(xy)$ = Bessel	MR10:200
$FD(m) = N(\overline{x}, n^{-1})$	[3]85
Bayes Distribution (m)	[3]91
$\overline{x}$ sufficient	[3]8, [e]17:211, [p]7:161
$\overline{x}$ is consistent	[3]3,26
$\overline{x}$ is MLE	[4]140, [c]33:125
$\overline{x}$ is minimax	
Efficiency of ξ = .637	[3]6
Confidence intervals for m	[3]63,70, [p]7:222
MLE for χ^2 test	[d]25:580
Remarks on testing	[d]13:62
Most powerful test of $\begin{cases} H: m \leq m_0 \\ Alt: m = m_1 m_0 \end{cases}$ is $\overline{x} > c$	[3]274
UMP test of m = 0	[13]452

Seq test	[c]43:452
Completeness	[e]10:313
Unbiased critical regions	[3]311
Testing equality of several means	[e]8:69
Testing; called "Laplace"	[v]2:251
Peculiar composite hypothesis on m	[3]306
Inference	[b]15:52
Pitman's method	[3]324
Range	[c]38:463

<u>See also</u>: [c]27:466, [c]31:202, [c]36:460, [g]7:95
3rd Berkeley Symposium 1:197.

1.4 NORMAL: N(0,1) (Gaussian)

$D(x) = (2\pi)^{-\frac{1}{2}} \exp(-\frac{1}{2}x^2)$	[7]129
General expose	Acta Math 77:1
C(x) as continued fraction	[2]130
C(x) as a series	[c]19:13
Bounds on C(x)	[c]42:263
Property of C(x)	Math. Zeit 41:405
C(x)	MR10:267
~C(x)	MR16:628
$MGF(x) = \exp(\frac{1}{2}t^2)$	
$Ch(x) = \exp(-\frac{1}{2}t^2)$	[8]164, [1]100
$\alpha_{2k} = \frac{(2k)!}{2^k k!}$	[d]5:32, [d]11:353, [h]1:13,193, [1]208
Absolute moments	Z1:26
Median and quartiles	[c]25:79
$D(n\bar{x})$ = Normal	[c]19:227
$D(s^2)$	[e]5:138

$D(x^2)$	[d]1:340, [e]5:138
$D\left\{\frac{x}{\pi x_i^{2}}\right\}$ etc	[d]17:1
$D(\Sigma x_i^2) = \chi^2(n)$	[2]231, [4]103, [9]331
$D(\Sigma k_i x_i)$	[d]13:17
$C(n^{\frac{1}{2}} s^{-1} \bar{x})$ = Student	[9]336
$D(Q)$, $D(Q_1 / Q_2)$	[e]17:37
$D[\Sigma(x_i - \bar{x})^2] = \chi^2(n-1)$	[9]333
$C(r)$ expressed as an integral	[9]339
$D(x_1 x_2)$ by Mellin Transformation	[d]19:375
$D(x_1/x_2)$ = Cauchy	[d]19:375
$D(\bar{x}, s)$	[o]7:65
D(range) for n=3, $\sim$D(range)	[c]34:111,[c]5:313, [o]8:155, [c]36:142
Moments of sample median	[d]26:600
D(extreme deviate)	[c]35:120
C(range)	[c]32:341
$D(\Sigma x_i^2/\Sigma y_j^2)$ = Snedecor	[d]19:378

Ch fcns of estimates of v	[d]19:257
Generating functions	Z2:200
Estimation of dispersion	[c]36:96
Estimation of mean deviation	[c]33:254, [c]35:304
Variance of median	[c]23:361
Testing N(0,1) against various alternatives	[c]30:139
Multivariate analysis	[3]XXVIII
Limit of binomial	[7]134
Central Limit Theorem	[i]27:139, [i]29:206
K^{th} value from the top	[1]374
Censored samples	[c]41:230
Ordered samples	[e]11:23
Stratified sampling	[d]5:138
Variance in two samples	[n]13-3:49
Ratio of two ranges	[d]21:112
Tetrachloric functions	[c]14:157

Approximations	[d]17:363
Sheppard's tables	[c]2:174
Grouping	[i]32:135
Moments of order statistics	[d]41:200
Occasionally called Laplace-Gauss, or even Laplace	Acta Math. 77:1, R. Acad. Sci. Paris 232:1999

See also: [d]4:109, [d]17:350, [d]22:425, [d]24:133, [d]24:297, [13]63, [c]18:395, [c]24:98, [c]24:280, [c]25:195, [i]6:209, [17]No. 41, [m]6:120, [d]22:418, [y]24:22, [u]29:231, Z4:66, Z8:266, Z20:39,145, MR12:191, Z18:412, MR17:756, C. R. Acad. Sci. Paris 238:444, Philos. Trans. Roy. Soc. London A237:231, MR7:18, Z5:366, [y]4:189.

1.5 TRUNCATED NORMAL

C(x)	[1]248, [6]243
Introduction, estimation, examples	[15]144
$D(\Sigma x_i)$	[b]8:223
Fitting	[c]39:252
Estimating m and v	[g]44:518, [g]47:457, [f]9:489, [o]3:37, MR7:461 [c]40:52

Distribution of estimate of σ	[15]316
Censored sample	[c]42:516
MLE	[i]32:119

See also: [d]9:66, [d]20:458, [g]47:379, Brit. Assoc. Math. Tables (3rd Ed.) v.1 p xxxv, MR2:231.

1.6 GENERALIZED NORMAL (Kapetyn)

$(2\pi v)^{-\frac{1}{2}} \exp[-\frac{1}{2} v^{-1}(f(x) - m)^2]\, df(x)$	[5]93
$C.-R.(m) = v/n$, $C.-R.(v) = 2v^2/n$	[5]139

$MLE(\sigma) = [n^{-1} \Sigma(f(x_i) - m)^2]^{\frac{1}{2}}$,
$MLE(m) = n^{-1}\Sigma f(x_i)$.

See also: [c]5:168.

1.7 NORMALS ADDED

$$D(x) = (1+k)^{-1} \{(2\pi)^{-\frac{1}{2}} \exp(-\tfrac{1}{2}(x + m_1)^2 + kv^{-\frac{1}{2}} (2\pi)^{-\frac{1}{2}} \exp -\tfrac{1}{2} (x - m_2)v^{-1}\}$$ [d]2:340, [h']2:63

$$Var(x) = (1+k)^{-1}(1 + m_1^2 + k(v + m_2^2)]$$

Method for partition with example [s]5:47

Semi-invariants [i]17:1

$D(\overline{x})$ [d]11:219

Three normals added [d]5:237

More generally [d]3:1, [d]5:230, [c]3:85, MR14:485

Sampling theory [n]8-3:67

Many normals added [c]37:429

Called "compound normal" [i]32:180

Bivariates [f]8:328

See also: [c]40:460, [e]14:369, MR11:258, MR17:1102.

1.8 LOGNORMAL (a, m, v)

$D(x) = (x - a)^{-1}(2\pi v)^{-\frac{1}{2}} \exp[-\frac{1}{2} v^{-1} (\log(x-a) - m)^2]$, parameters and moments — [1]258, [c]4:194, [15]160, [5]121, [d]3:45

Graphical determination of parameters — [w]9:102

$\text{Mean} = a + e^{m + \frac{1}{2} v}$, $\text{var} = e^{2m + v}(e^v - 1)$

Another form — [d]4:30, [b]7:155

$$D(x) = \frac{1}{\sqrt{2\pi}\, c(x-a)} \exp\left\{-\frac{1}{2c^2}\left[\log \frac{x-a}{b}\right]^2\right\}$$

$m = be^{\frac{1}{2}c^2} + a$, $\text{mode} = be^{-c^2} + a$, $GM = \xi$ $= a + b$, moments, tables, regression, examples, bibliography

Moments, transformations — [w]7:152, [w]8:83

Complete treatment with bibliography — Aitchison, J. and Brown, J.A.C., The Lognormal Distribution Cambridge, 1957 (MR18:957)

Estimation of m — [e]10:341

MLE — [g]46:206, Intl. Congr. Math (1950) 1:581

Regression — [d]7:196

~ Tests on m — [d]28:1044, [d]27:670

Called "Galton-Macalister"	[c]32:239
Called Gibrat	Kendall and Buckland, A dictionary of Stat. terms
Used to approximate [illegible] Distribution	
Used to approximate Fisher Distribution	[d]12:448
Deduced from hypothesis about errors etc.	[i]28:141
$\frac{x-a}{b-x}$ lognormal	[17]No. 46
Transformation	[c]36:155
Vs. Normal	Geochimica et Cosmochimica Acta 8:53
Discrete lognormal	[c]37:362
Compared with normal by means of Galton-Kapetyn apparatus	[s]4:129
Truncated lognormal	[i]28:150, [c]38:414
Lognormal (0,0,1), $E(x) = e^{\frac{1}{2}}$, $v = e^2 - e$	[8]120,176, [17]No. 45, [c]22:109, [d]4:30

See also: [d]14:120, [l]13:161, [e]12:121, [b]6:174, [b]11:19, [d]15:182, [c]4:179, [c]22:146, [g]34:762, [g]36:493, [f]1:57, [c]36:155, [c]38:427, [g]48:600, Journal of the Franklin Inst., 250:339, 250:419, 251:499, 251:617, [g]50:904, [c]43:404, [a]119:157,185, J. Franklin Inst. 244:471, 250:339, Indus. and Engin. Chem. 40:2289, J. Roy. Soc. (A)216:309, J. Phys. Chem., 56:442, [y]13:29, J. Hygiene 42:328, Z10:173, MR3:4, Nature 156:463.

1.9 WRAPPED-UP NORMAL

$D(x) = k\Sigma e^{-c(x+j)^2}$	Bull. Soc. Math. France 66:32, 67:1, C.R. Soc. Math. France (1938)p 34, Am. Ecole. Norm. Sup. 45:1 [t]55:335, [d]18:589, Handbuch der Physik, Berlin, Springer 3:477

1.10 GRAM-CHARLIER

Two-term $D(x) = (2\pi)^{-\frac{1}{2}} [1-k/6(3x-x^2)] \exp(-\frac{1}{2}x^2)$	[3]103, 137
General Gram-Charlier	[4]77
$D(\bar{x})$	[d]1:199, [d]2:99
$D(x^2)$	[i]26:212
$D(s)$	[d]6:127
t-test	[i]210
$D(x) = f(x) N(m,v)$	[d]23:467, [g]26(P):233
F(various statistics)	[g]4:1
Log Gram-Charlier	[i]28:145
Type B Gram-Charlier	[d]8:183, [d]18:574, Trans. Amer. Math. Soc. 67:206, [d]20.376, [j]5:17,
MGF factorial moments,	Z5:213

<u>See also</u>: [a]88:576, [a]89:129, [c]33:126, [c]36:427, [c]38:58, 87, [c]39:425, [i]7:147, [ℓ]23:283, T.A.M.S. 67:206, Z18:320, Z22:243, Z2:43

1.11 BIVARIATE NORMAL $N\begin{pmatrix} m_1, & v_1, & \rho \\ m_2, & v_2, & \end{pmatrix}$

$D(x,y) = [2\pi\,\sigma_1\sigma_2(1-\rho^2)^{\frac{1}{2}}]^{-1}$ $\exp\left\{-\frac{1}{2}[v_1v_2(1-\rho^2)]^{-1}\right.$ $\left.[(x-m_1)^2v_2-2\rho\sigma_1\sigma_2(x-m_1)(y-m_2)+(y-m_2)^2v_1]\right\}$	[6]165, [5]89, [4]60
Introduction, properties, examples	[15]585
Another form (Koopman-Darmois)	[e)8:322
$Ch(x,y) = \exp\left\{i(m_1s+m_2t) - \frac{1}{2}(v_1s^2 + 2\rho\sigma_1\sigma_2st + v_2t)\right\}$	[1]287
MGF	[6]167
$D(\overline{x}_1,\overline{x}_2)$ = Normal bivariate	[4]101
$D(\overline{x} - \overline{y})$	[c]2:379
$D(y/x)$	[c]24:428
$f(xy)$ and $f(x/y)$	[18]1-151, Am. Math. Monthly 49:26
$Ch(xy)$	[a]42:82
$D(r)$	[4]120
If $\rho=0$, $D(r^2) = B(1, n-2)$	[10]160, 178

If $\rho=0$, $D(b) = B(1, n-1)$	[10]180
D(correlation ratio) = No. 5.3	[10]181
$\sim D(r)$	[c]10:507
$D(rs_1s_2, s_1^2, s_2^2)$	[u]29:264
Distribution of various statistics	[e]17:21
$MLE(m_1, m_2, v_1, v_2, \rho\sigma_1\sigma_2) = \bar{x}, \bar{y}, s_1^2, s_2^2, rs_1s_2$	[3]37
$Var(\bar{x}) = v_1/n$, $var(s_1^2) = v_1/2n$, $var(r) = n^{-1}(1-\rho^2)^2$, $cov(s_1,s_2) = \rho^2\sigma_1\sigma_2/2n$, $cov(r, s_1) = \rho\sigma_1(1-\rho^2)/2n$, $cov(\bar{x},\bar{y}) = \rho\sigma_1\sigma_2/n$	[3]38
MLE from fragmentary information	[d]3:163
$\bar{x}$, $\bar{y}$ are joint efficient in estimating m_1 and m_2; $\bar{x}, \bar{y}, \frac{ns_1^2}{n-1}, \frac{ns_2^2}{n-1}, \frac{n}{n-1} rs_1s_2$ have joint efficiency $(n-1/n)^3$	[1]495-6
$\frac{x-m_1}{\sigma_1} + \frac{y-m_2}{\sigma_2}$ and $\frac{x-m_1}{\sigma_1} - \frac{y-m_2}{\sigma_2}$ are independent and $N(0,2(1+\rho))$, $N(0,2(1-\rho))$ respectively	[6]217

Estimation	[d]17:395, [e]8:322
Estimation, testing	[15]606, [g]50:884
Censored samples	[x]6:83
Dist. ratio standard deviations	[x]6:93
Confidence limits for r	[c]29:157, J. Nat. Inst. Personnel Res. 6:153
Confidence limits for m_1/m_2	[d]13:440, MR13:962
Sufficient statistics	[e]17:212
Comparison of two correlations	[10]203
Tests of seven hypotheses on the parameters	[d]11:410
Testing equality of two r's	[d]12:279
Some tests	[w]7:46
Testing equality of variances	[e]1:13, bibliography [a]109:462
$r_1 - r_2$	[c]25:102
Seq. tests of ρ	[w]8:202
Truncation	MR2:231
Fisher's original work on r and ρ	[n]1-4:1
Sufficient conditions for normal bivariate	[e]6:399, MR15:805

k samples [c]27:145, 227

$x/\sigma_1 + y/\sigma_2$ and $x/\sigma_1 - y/\sigma_2$ are independent normal variables [c]31:9

If $\rho = 0$ then $\dfrac{v_1/v_2}{v_1/v_2 + s_1^2/s_2^2}$ is Beta [c]31:10

Called Bravais distribution [i]19:3

Properties Z15:310, MR1:246

<u>See also</u>: [d]4:196, [d]14:141, [c] 260, [g]26:129, [c]22:1, [c]25:356, [c]25:392, [f]8:328, [n]9-3:90, [c]39:238, [i]27:221, [6]218, [v]5:311, [d]27:1075, [c]44:289, [x]4:85, Harvard Ed. Rev. 1946, p. 52, MR4:280, MR8:283, MR14:1102, MR7:212, [y]1(No. 4) 20.

1.12 BIVARIATE NORMAL $N\begin{pmatrix}0, & v_1, & \\ 0, & v_2, & \rho\end{pmatrix}$

$$D(x,y) = 2\pi M^{\frac{1}{2}})^{-1} \exp[-\tfrac{1}{2} M^{-1}(x^2 v_2 - 2\rho\sigma_1\sigma_2 xy + y^2 v_1)]$$

where $M = \begin{vmatrix} v_1 & \sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & v_2 \end{vmatrix} = v_1 v_2(1-\rho^2)$

[9]308, [2]22,334, [4]60, [8]116, [15]588, [10]95, 106, [c]30:8

$$Ch(x,y) = \exp(-\tfrac{1}{2} n^{-1}(v_1 s^2 + 2\rho\sigma_1\sigma_2 st + t^2 v_2)]$$

$\alpha_{40} = 3v_1^2$, $\alpha_{31} = 3\rho\sigma_1^3\sigma_2$, $\alpha_{22} = (1+2\rho^2)v_1v_2$	[2]80, [4]60
Moments	[o]3:2, [c]12:177
Central moments	[h']4:73
Incomplete moments	[c]13:401, [c]40:22
Product-moments	[c]12:86
As limit of binomial	[a]91:548
If $\rho = 0$, $\sigma_1 = \sigma_2$ called "circular normal", D(r), properties	[c]29:137
C(x,y) with other properties	[c]33:59, [c]38:475
Cumulants	[2]89
$Var(r) = n^{-1}(1-\rho^2)$	[2]336
$Var(b) = n^{-1}\sigma_1/\sigma_2(1-\rho^2)$	[2]337
Marginal and conditional distributions	[4]62
Regression	[3]144
Correlation and regression with generalization	[i]24:1
Bilinear forms	[o]1:103, [d]18:565

Quadratic forms	[d]14:195
$D(x^2, y^2)$	[2]336
D(variance-covariance) = Wishart	[2]340, [1]29.6
$D(r)$	[2]342,
$D(r)$ for $n=4$	[u]26:536
$D(b)$ = a Type VII	[1]402
$D(s_1/s_2)$ = No.8.3 if $\sigma_1 = \sigma_2$	[e]2:65, [c]31:9
$D(e^x, e^y)$	[w]2:155
When, further, $\rho = 0$ generalized Student	[c]30:190
Simple function of x/y is normal	[a]93:442
$D(y/x)$	[c]32:16, [i]20:61
$D(xy)$, $D(x/y)$	MR3:171
Joint distribution of Pearson betas	[c]7:386
$D(s_1^2, s_2^2, rs_1s_2)$ etc.	[d]5:283
$D(s_1^2, s_2^2)$	[e]5:139, [c]25:126
$D\left(\frac{(n-1)^{\frac{1}{2}}(b-\beta)\sigma_1}{\sqrt{v_2-b^2v_1}}\right)$ = Student $(n-1)$	

$D\left(\frac{(n-2)^{\frac{1}{2}}(b_2-\beta)\ s_1}{\sqrt{v_2-bv_1}}\right) = \text{Student}(n-2)$	[9]5.13, [2]348, [3]156
Moments of dist. of covariance from $N\binom{0,\ 1}{0,\ 1,}\rho)$	[3]334
D(radial standard deviation) $= D[n^{-1}\Sigma(x_i - \bar{x})^2 + \Sigma(y_1-\bar{y})\ ]^{\frac{1}{2}}$	[d]15:75
D(sample covariance)	[3]359, [10]138
MLE(ρ)	[3]33
Confidence intervals for ρ	[3]81
MLE(v_1,v_2,ρ)	[2]339
Estimation of ρ when $v_1 = v_2$	[d]9:149
Estimation of ρ by rank correlation	[d]7:40, [c]40:419
Truncation	[d]21:272
and estimation	[e]12:277, MR16:498
Testing v_1/v_2	[3]138
Testing σ_1/σ_2 and ρ	[e]5:151
Test whether two samples are from same population	[3]140

Hotelling's generalized T applied to tolerance limits	[d]14:90
Odd fact for $\rho = 0$	[d]18:442

See also: [o]12:90, [c]2:369, [c]4:498, [c]17:176, [c]20:295, [g]27:254, [a]83:128, [c]29:74, [n]2:040, [n]7:6, [c]32:196, [c]38:371, [i]7:220, [i]24:17, [n]13-1:21, 65, [u]28:457, Z12:267

1.13 TRIVARIATE NORMAL

D(x)	[10]255,260
Moments	[o]4:15, [c]40:23
Correlation	[i]14:158
Partial correlation	[c]10:391
Yielding 2 x 2 x 2 table	[e]10:272
Student test for partial correlation	[10]256
Snedecor test for multiple correlation	[10]257

<u>See Also</u>: [d]8:179, [d]12:94, [n]1-1:151, [u]44:342, MR15:805.

1.14 MULTIVARIATE NORMAL

$D(x) = Ce^{-Q} = [(2\pi)^{\frac{1}{2}n}\sigma_1\ldots\sigma_n R^{\frac{1}{2}}]^{-1}$ $\exp\left\{-\frac{1}{2}\Sigma\Sigma \frac{R_{ij}}{R\sigma_i\sigma_j}(x_i-m_i)(x_j-m_j)\right\}$	[6]177, [2]376, [4]63
where $R = \|r_{ij}\|$	
$C(x)$	[c]40:458, [c]41:351
$Ch(x) = \exp\left\{i\Sigma m_j t_j - \frac{1}{2}\Sigma\Sigma r_{ij}\sigma_i\sigma_j t_i t_j\right\}$	
Moments	[c]40:20, MR5:42
Marginal distributions, conditional distributions, regression	[4]70
Independence of quadratic forms	[o]1:83
Distributions of moments, partial and multiple correlations	[i]24:185, [i]27:235, [i]28:20
D(product moment) = Wishart	[c]20:32, [i]20:218, [u]35:336, [u]29:260, 271, MR10:387, [b]17:82
Cumulants of logarithmic generalized variance	[i]38:17
Independence of distribution of means and second order moments	[4]233

D(multiple correlation)	[d]3:196
Fiducial distribution	[u]34:41
Multiple and partial correlation	[c]19:100
Linear regression theory	[4]245
$D(Q) = \chi^2$	[4]104
D(various Q)	MR13:142
D(vector correlation)	[c]28:353
Sampling	[4]XI, [d]6:202
$MLE(m_1,\ldots,m_n) = (\bar{x}_1,\ldots,\bar{x}_n)$	[6]187
Hotelling's generalized Student test	[4]234, [d]9:240
LR test for variances equal and correlations zero	[d]11:204
LR test for independence of variables	[d]11:17
Characterization	[e]14:367
Hypothesis of equality of means	[4]238
Independence of sets of variables	[4]242
Probability that all n variables are positive	[d]26:484

Tolerance regions	[d]27:174
Testing variance-covariance homogeneity	[c]31:31, [c]34:311
Truncation	[x]5:17
Various tests	[d]17:257, [d]21:293
Quadrivariate	[c]43:206
Bibliography of tests of hypothesis of equality of variances called 'Bipolar' distribution	[e]14:61
Variables separated into two sets	[c]30:295
Many samples	[c]31:221
Seq analysis	[e]12:328
Central limit theorem	[i]28:109
Generalizations of N(m,v) theorems	[e]17:221

<u>See also</u>: [d]8:149, [d]17:344, [d]19:447, [d]21:445, [e]3:273, [e]12:99, [e]6:35, [a]90:136, [c]6:59, [c]15:192, [c]35:58, [o]1:79, [b]18:70, [c]43:212, [x]1:59, [t]6:181, [g]52:200, MR13:366, MR17:278, MR12:345, MR15:141, MR6:159, Z10:406, Z15:220, MR10:312, Trans. Am. Math. Soc. 24:135.

II. TYPE III DISTRIBUTIONS

2.1 TYPE III (p,q)

$D(x) = \frac{p^q}{\Gamma(q)} x^{q-1} e^{-px}$, $(0, \infty)$

$D(x) = \frac{x^b e^{-x/a}}{b!\, a^{b+1}}$ ("gamma") [6]112

$Ch(x) = (1 - it/p)^{-q}$ [2]55, [1]126

$MGF(x) = (1 - at)^{-b-1}$ [6]115, [4]74

$\alpha_1 = q/p$, $\alpha_2 = p^{-2}q(q+1)$, $\alpha_3 = p^{-3}(q+1)(q+2)$ [2]55

$E(x) = a(b+1)$, $v = a^2(b+1)$

r^{th} cumulant $= q(r-1)!p^{-r}$ [2]67

$\mu_2 = qp^{-2}$, $\sigma^2 = q/p^2$, $\mu_3 = 2qp^{-3}$, [2]433
$\mu_4 = 3q(q+2)p^{-4}$

Arithmetic, geometric means Math. Student 13:11

Type III (p/q, p) ("Eulerian") [c]35:6

C(x) [c]25:379

$f(\frac{x}{x+y})$ = Beta [14]41

$\sim$ C(x) MR13:553

Transformations $y = x^n$, $y = e^x$	[d]9:176
Normalizing Transform	Proc. Pak. Stat. Assoc. 5:120
Transformation $y = (x + k)^{\frac{1}{2}}$	[d]14:115
Moments Ch(x), cumulants when x=y-c	[18]1-136, 1-144
Type III (p, p+1)	[e]5:176
$D(\bar{x})$ = Type III (np, nq)	[2]244, [c]18:335 [w]1:73
$D[(nq-1)n^{-1}\bar{x}^{-1}]$ = Reciprocal Type III(p,nq-1)	
$\sim D(\sqrt[3]{x})$	[c]35:297
D(x/y) = Beta of Second Kind (p=q)	[w]1:74
$FD(p^{-1})$ = Type V ($n\bar{x}$/nq-1, nq-1)	[3]87, [c]30:408
Bayes $D(p^{-1})$ = Rectangular	[3]91
D(xy) where x is Type III and y Type V	[i]39:64
D(HM)	MR4:164
D(xy), Ch(xy)	MR16:377
MLE (p) = Moments (p) = $q/\bar{x}$, correcting for bias = $\frac{nq-1}{n\bar{x}}$, sufficient $\sim$efficient, not efficient	[3]26
Var $(\frac{nq-1}{n\bar{x}}) = p^2(nq-2)^{-1}$	[1]505, [u]45:214

$MLE(1/p) = \overline{x}/q$	[3]21
$Var(\overline{x}/q) = p^{-2}n^{-1}q^{-1}$, sufficient	[3]21
Sufficient statistics for p	[e]17:212,219
Ordered LSE is MLE for 1/p	[d]25:315
MLE(p,q), variance-covariance of estimates	[b]14:187 [c]42:22, [r]1:18
$UMVUE(1/p) = \overline{x}/q$, with $Var = p^{-2}n^{-1}q^{-1}$	[3]53
There is a sufficient estimate of q	[3]26
Estimation	[e]8:324
Minimax	[16]64, C.R. [e]14:57
Gauging	[e]15:192
Closest estimate	[u]33:217
Testing n such populations	[3]325
Slippage tests for p	Koninkl. Nederl. Akad.(A)59:329
Testing equality of 1/p	[c]31:205
Confidence intervals for 1/p	[3]74, [e]6:113, MR5:128

Truncated distribution	[g]45:411
Estimation from Truncated Type III	[d]26:659, [d]27:498
Truncated samples	[c]40:52
Relation with Poisson	[o]3:123
Characterized by independence of sum and quotient	[d]26:319
Discrete Type III	[c]44:365
Mills ratio	[d]24:309
Normal limit	Am. Math. Monthly 50:98
Renewal theory	[d]11:448
Trivariate	[d]21:550

See also: [d]7:95, [d]8:17, [d]24:407, [e]2:150, [b]11:101, [c]21:263, [d]25:640, [c]24:300, [v]2:330, [g]50:904, [i]39:171, [q]7:95, Am. Math. Monthly 50:98, MR17:756.

2.2 TYPE III (p,1)

$D(x) = pe^{-px}$	[5]34
Type X	
$\alpha_1 = 1/p$	[5]59, [2]48
$v = p^{-2}$	[5]67
Moments	[2]142, [8]100, [10]18
Characterization	[t]7:60, 3rd Berkeley Symp. 2:195
Cumulants	[2]87, [10]40
C(x)	[c]25:379
Mean difference	[c]28:432
$MGF(x) = (1-x/p)^{-1}$	[10]37
MGF(log x)	[v]7:296
Grouping corrections	[c]39:433
A priori distributions of p	[i]27:36
D(x + y)	[8]95

Examples and applications	[8]29,79,83, [j]20:366, [c]39:168
$D(\overline{x})$	[c]39:168
$D(\xi)$ = No. 8.9, $MGF(\xi)$	[p]7:153
Σx_i where x_i is Type III$(p_i,1)$	MR5:42
Rank variates	[c]24:210,271, [r]4:153
Estimation	[c]35:187, [g]48:493, [s]10:167, [o]8:15 [p]7:152
Censored sample	[c]41:230, [d]23:237
Moments of D(s)	[c]22:53
Variance of mean deviation $\cong \frac{4}{3np^2}$	[2]217
Testing $p = p_0$	[d]9:84
Sequential test	[c]41:252, [d]27:460
Testing against four other possible dist.	[c]43:253
Confidence intervals	[3]84
Estimation from truncated exponential	[d]26:498
Relation with Poisson	[o]2:13

See also: [d]7:19, [d]25:555, [g]48:488, [g]50:904

<u>If $x = y - c$.</u>

Mean $= c + \frac{1}{p}$, var $= \frac{1}{p^2}$, skewness $= 2$, kurtosis $= 6$, Ch(x), cumulants	[18]1-136, 1-144
MLE	[k]8:52
LR test for hypothesis that n such populations are identical, etc.	[3]305
k samples	Z14:269
Best linear estimates of m and σ	[d]25:320
$D\left\{\frac{1}{n-1}\sum_{i=1}^{n-1} \lvert x_{i+1}-x_i \rvert\right\}$	[v]6:133
Life testing	[d]25:373
Original Neyman-Pearson paper on hypothesis testing	[c]20:221
Censored samples	[g]52:58
Estimation by order statistics	[d]26:585
Quasi-range	[d]28:179
LR tests	[d]12:301
Seq. testing	[x]2:86
Confidence intervals	MR5:43

$$\begin{cases} H: p = p_0,\ c = c_0 \\ Alt: p = p_1,\ c = c_1 \end{cases}$$ [3]304

See also: [d]25:409, [d]24:458, [c]30:402,416, [e]24:279

Maxwell-Bolzmann

x^2 is Type III (p, 3/2) [5]39,60

Connection with N(0, v) [12]40

$D(\bar{x})$ [n]10-3:90

See also: [n]17-1:125.

2.3 TYPE III (1,q)

$D(x) = \frac{1}{\Gamma(q)} e^{-x} x^{q-1}$, "Gamma" [10]149

$\alpha_r = \frac{\Gamma(q+r)}{\Gamma(q)}$ [10]150,163

$k_r = q(r-1)!$ [2]96,153

$Ch(x) = (1-ix)^{-q}$ [17]No. 34

Skewness $= q^{-\frac{1}{2}}$ [10]161

HM = q-1	[10]163
$D(n\bar{x})$ = Type III (1,nq)	[c]19:228, [n]10-3:91
$D(\Pi x_i)$	[c]24:474
$D(x_1 - x_2)$	[2]252
$D(x_1 + x_2)$ = Type III $(1, q_1 + q_2)$	[10]151, [p]7:101
D(GM) as a series (with generalization)	[2]251, [d]5:277
$D(\frac{x_1}{x_1 + x_2}) = B(\frac{1}{2} q_1, \frac{1}{2} q_2)$	[10]153
$D(x_1/x_2)$ = Beta of second kind	[10]158, 160, [p]7:102
$D(x_1 - x_2)$ involves a Bessel function if x_1 and x_2 are from two separate distributions, and $D(x_1/x_2)$ is Fisher	[d]7:51
$\sim D(\bar{x})$	[d]25:636
$D(\bar{x}, GM/\bar{x}) = D(\bar{x})\, D(GM/\bar{x})$	[c]30:287
$C.-R.(q) = [n \frac{d^2}{dq^2} \log \Gamma(q)]^{-1}$	
$\bar{x}$ is moments estimate of q, not sufficient	

Closest estimate	[u]33:216
MLE(q) = log GM	[p]7:169
$E(\overline{x}) = q$, var $(\overline{x}) = q/n$, efficiency $(\overline{x}) \rightarrow 0$	[1]504,5
$\sim$D(log GM) = Normal	[1]505
Confidence intervals	[p]7:224
Mellin transform	[d]19:373
Log log $\frac{1}{x}$	[v]8:71
A + B log x	[c]36:165
Multivariate generalization	[d]22:549
Tetrachoric functions	[c]14:161
Fermi-Dirac, x-c is Type III (3/2,const.)	[12]42, [j]8:701.

<u>See also</u>: [d]25:401, [10]161, [d]22:425, [c]24:281, [c]27:409, [c]30:415, [c]36:165, [d]25:784, [e]10:314, [g]51:467, [c]44:265, [d]22:418, Canad. J. Math. 3:140.

2.4 TYPE III (1,1)

$D(x) = e^{-x}$, "exponential"	[6]217
$Ch(x) = \frac{1}{1-it}$	[17]No. 33
$D(\bar{x},s)$	[d]3:128, [d]4:133, [d]4:139,142
$x = y - c$, Type X, confidence intervals	[b]17:90
$D(\Sigma x_i)$ = Type III by convolutions	[d]5:13
$D(\Sigma x_i/i) = D(\max x_i)$	[b]14:43
Doubly truncated, Ch(x)	[n]10:3, [17]No. 32
Ratio of two ranges	[d]21:112
C.-R. theorem false for $x = y - c$	[1]485, [3]47

<u>See also</u>: [d]22:425, [i]36:152, [m]6:120, [d]22:418

2.5 CHI-SQUARE (k)

$D(x) = \frac{1}{2^{\frac{1}{2}k}\Gamma(\frac{1}{2}k)} x^{\frac{1}{2}k-1} e^{-\frac{1}{2}x}$ [Type III $(\frac{1}{2}, \frac{1}{2}k)$]	[5]96, [10]164, [2]17, [4]102, [8]134, MR8:161 [1]3:353, [18]1-161
$\alpha_1 = k$, $v = 2k$	[1]234, [4]103
$\alpha_s = k(k+2)\cdot\ldots\cdot(k+2s-2)$	[1]234
$\mu_2 = 2k$, $\mu_3 = 8k$, $\mu_4 = 48k + 12k^2$, $\mu_5 = 32k(5k+12)$	[2]292
Cumulants	[c]31:216
$Ch(x) = (1 - 2it)^{-\frac{1}{2}k}$	[v]4:8
C(x), relation with Poisson	[1]3:357, [c]37:313
Introduction, properties, examples	[15]253
Obtained as dist. of normal variance	[3]104, Z23:148, [p]7:98
$D(x_1 + x_2)$	[e]7:27
D(log x)	[c]34:170
$D(2\sqrt{xy})$ = Chi square(2n-2) if D(x) = chi-square (n) and D(y) = chi-square (n-1)	

$D(\bar{x},s)$ for $k = 2,3,4$, $n = 3,4$	MR12:345
$D(x_1/x_2)$ = Beta of second kind $(\frac{1}{2}k_1, \frac{1}{2}k_2)$	[10]177
$D\left(\frac{x_1}{x_1+x_2}\right)$ = Beta (k_1,k_2)	[10]177
Obtained as $D(\Sigma\, x_i^2)$	[10]169
$\sim D[(x-k)(2k)^{-\frac{1}{2}}] = N[k,(2k)^{-\frac{1}{2}}]$, $\sim D(2x)^{\frac{1}{2}} = N[(2k)^{\frac{1}{2}}, 1]$	[1]251
$\sim D(-2 \log LR)$	[d]9:60
Reproductive property	[4]105, [10]177
Normal approximation	[d]17:216, [d]27:786
Large parameter	[c]43:92, Proc. A.M.S. 6th Symp. in Appl. Math., p. 251
Convolution for a pair of Chi-square variables	[8]134
Percentage points	[c]41:313, [i]33:168
$\sim$ Significance levels	[d]14:57,93
Elderton's tables	[c]1:155

Minimax estimation	[16]17
Connection between χ^2 test and χ^2 distribution	[c]19:215
Original Neyman-Pearson paper on hypothesis testing	[c]20:263
Queueing	[b]16:82
Called Erlang's Distribution	[d]24:339
Approximation for small sample	[d]9:158

<u>See also</u>: [d]18:89, [g]29:372, [c]22:298, [a]85:87,95, [a]87:442, [c]29:133, [c]29:389, [c]31:346, [c]34:368, [c]32:268, [c]40:421, [p]7:98

2.6 NON-CENTRAL CHI-SQUARE

$$D(x) = e^{-\frac{1}{2}x} e^{-\frac{1}{2}k} 2^{-\frac{1}{2}n} \sum_{j=0}^{\infty} \frac{x^{\frac{1}{2}n+j-1} k^j}{\Gamma(\frac{1}{2}n+j)\, 2^{2j}\, j!}$$ [c]36:204

$$Ch(x) = \frac{\exp[kit/(1-2it)]}{(1-2it)^{n/2}},$$ [18]1-162

$k_r = 2^{r-1}(r-1)!(n+rk), \sim$form

Logarithmic Non-central Chi-square	[o]7:57

<u>See also</u>: [i]34:57, [c]41:538, [i]36 (supplement):18, [d]28:678, [c]44:528

2.7 HELMERT (p,q)

$$D(x) = \frac{(x/q)^{p-1}\, e^{-\frac{1}{2}x^2/q^2}}{q\ \Gamma(\frac{1}{2}p)\ 2^{\frac{1}{2}(p-2)}}$$ [5]94

For $2q^2 = k$, $p=2$, called Rayleigh [12]39

$D(x^2/q^2) =$ Chi-square

Called semi-normal [i]20:61

Refs, Ch(x) [17]No. 42

$$\alpha_1 = \frac{[\frac{1}{2}(p-1)]!}{[\frac{1}{2}(p-2)]!}\, 2^{\frac{1}{2}}\, q, \quad v=pq^2 - \alpha_1^2$$

Non-central [d]23:467

<u>See also</u>: [c]23:418, <u>Electrical Engineering,</u> Nov. 1954, p. 1004, MR8:161, J. Appl. Physics 23:137.

2.8 RECIPROCAL TYPE III (p,q)

$$D(x) = \frac{p^{q+1}\, q^{q+1}}{\Gamma(q+1)}\, x^{-q-2}\, e^{-pq/x}, \quad pq > 0$$

Mean $= p$, var $= \frac{p^2}{q-1}$, Type V

$$\alpha_r = \frac{(pq)^r\, \Gamma(q-r+1)}{\Gamma(q+1)}$$ [2]86, 142

Obtained from Pearson equation	[11]45
Various constants, with an example	[11]78, [d]3:20
Ch(x) in special case	[17]No. 47
If $q = -\frac{1}{2}$, $p = -1$, $Ch(x) = \exp[(-1+i)\sqrt{t}]$	[s]208:318, [17]No. 49
More generally	[d]7:25
MLE	[r]1:19
As dist. of precision const. in normal sample	[a]97:132
C(x)	[c]25:379

See also: [c]26:388, [c]36:165, [o]8:55

2.9 GENERALIZED TYPE III

$D(x) = C(1+x/a)^{pa}\ e^{-px}$, with moments of D(s)	[c]22:52
$D(x) = \dfrac{p^p e^{-p}}{q^p \Gamma(p)} (q + x)^{p-1}\ e^{-px/q}$, with semi-invariants, $D(\overline{x})$ etc.	[c]21:287, [c]24:293
$Ch(x) = e^{-aix}\ (1 - \frac{ix}{p})^{-ap-1}$	[17]No. 35

$$D(x) = \frac{1}{p\,\Gamma(q)} \left(\frac{x-c}{p} \right)^{q-1} \exp \frac{c-x}{p}, \quad c \leq x < \infty$$

MLE(c,p,q)	[3]39
Variance of estimates	[3]42
Tables	[d]1:191
Estimation	[g]48:336
$D(x) = \frac{e^{ba} b^p}{\Gamma(p)} e^{-bx}(x+a)^{p-1}$ from Pearson's equation	[4]74, [2]124, [11]65
$D(\bar{x})$	[n]10-3:91
$D(x) = A(x-c)^{q-1} e^{-p(x-c)}$, $x > c$, $p > 0$, $q > 0$	[1]249
Various constants, with an example	[11]66
One root of quadratic in Pearson equation is ∞	[11]44
$D(x_1/x_2)$ where each is Generalized Type III, or one is Generalized Type III and the other N(m,v)	[2]253
Bayes' Theorem	[n]16-1:114
Counting radioactive particles	[d]18:260

Two Generalized Type III distributions added [n]8-3:76

<u>See also</u>: [d]1:150, [d]1:191, [d]7:18, [c]1:293, [c]3:311, [c]5:173, [c]13:13, [c]16:114, [n]1-3:88, [c]32:294, [u]46:284

2.10 WISHART UNIVARIATE

$$D(x) = \frac{a^{\frac{1}{2}(n-1)}}{\Gamma[\frac{1}{2}(n-1)]} x^{\frac{1}{2}(n-3)} e^{-ax}$$, Type III [1]391

$[a, \frac{1}{2}(n-1)]$

III. BINOMIAL DISTRIBUTIONS

3.1 BINOMIAL (k,p)

$D(x) = \binom{k}{x}p^x(1-p)^{k-x}$, $x=0,1,\ldots,k$, "Bernoulli"	[4]47, [10]46, [5]106, [d]1:118
$\alpha_1 = kp$, $v = kpq$, (where $p+q=1$) $\alpha_3 = kpq(q-p)$, $\alpha_4 = 2k^2p^2q^2 + pq(1-6pq)$	[1]193, [c]5:172, [5]57,66, [10]58, [2]52,117
$\mu_{r+1} = pq(kr\mu_{r-1} + \frac{d\mu_r}{dp})$	[2]118
$\beta_1 = \frac{q-p}{\sqrt{kpq}}$ $\beta_2 = 3 + \frac{1-6pq}{kpq}$	
Factorial moments $\alpha_{[i]} = k^{[i]}p^i$	[1]257, Nature 164:282, [2]87
Long introductory article	[15]23
Moments	[d]6:96, [v]3:325, Z9:28
Cumulants $k_{r+1} = pq\frac{dk_r}{dp}$	[2]135, [c]31:392 [18]1-144
Mean deviation	[c]44:532, MR7:128
Semi-invariants	[d]2:196

Several formulas for moments about the mean	[d]7:191, [c]15:410, Z9:220
C(x) as a Beta integral, recursion formula for moments	[14]33
Moments in general	[d]8:103, [d]11:106, [c]17:165, [c]26:262, MR7:461, [c]30:11, Bull. Am. Math. Soc. April 1934, p. 262, 41:857
Moments and series	[i]14:168
Arcsin transform	[14]210
Another form of D(x)	[d]8:116
$\beta_1 = \frac{(q-p)^2}{kpq}$, $\beta_2 = \frac{1+3pq(k-2)}{kpq}$ etc.	[12]52, [d]4:216
$kp - q \leq \text{mode} \leq kp + p$	[6]57
$Ch(x) = (pe^{it} + 1 - p)^k$	[5]62, [2]55,103
$MGF(x) = (q + pe^t)^k$	[4]48, [10]38
C(x)	[c]38:423
$PGF = (q+px)^n$	[18]1-146
C(x) in terms of incomplete Beta integral	[18]1-152

~ C(x)	[j]8:99
E(1/x)	[g]49:169
$D(n\bar{x})$ = Binomial (nk,p)	[2]243
$D(s^2)$	[c]44:262
FD(p)	[c]37:117
D(x-y) in terms of Legendre functions	MR14:566
Reproductive property by convolutions	[7]216
A convolution with respect to p	[w]6:165
C.-R.(p) = $\frac{pq}{kn}$	[5]141, [15]207, [p]7:160
$\bar{x}$ is sufficient	[p]7:162
$(kn)^{-1}\ \Sigma x_i$ is efficient and unbiased	[5]141, [1]487,
Is MLE	[5]144
~ BCR for k = k_0	[d]18:556
Minimax estimate	[d]21:190
Minimax and Bayes	[d]23:404
Minimax estimation	[16]18
Modified Bayes	MR11:42

BANE(p)	[d]21:402
Biased and unbiased statistics	[t]5:149
MLE(k,p)	[k]18:117
Estimation of p based on runs	MR14:1102
LR comparison of two binomials	[c]37:140
Confidence intervals	[w]2:171,[c]41:275, [c]33:181, [w]5:94, [d]9:174, [y]21:17, [4]129 [w]19:130
Tables	[w]2:171, [c]41:308, [3]81, [c]44:436 [o]8:85
Sequential	[d]17:288,489, [d]18:131, [b]8:98, [c]41:252,
Acceptance inspection	[b]12:301, MR15:727
Chi-square test	[k]7:207
UMP Test	[c]43:465
Sampling inspection	[j]8:626
Multiple sampling	[d]14:363
Analysis of variance	[d]11:335
Order statistics	MR16:729, [s]8:62

Approximate formulas	[8]172, [d]19:592, [n]18 No. 1-2:123
Normal approximation	[9]131, [d]16:319, [c]4:190, [c]29:402, [r]4:47, Proc. Koninkl. Nederl. Akad. (A)57:513, MR10:131
Using generating functions	Proc. 5th Intl. Cong. Math 2:441, [w]1:41
Inequalities for tails	[7]126
Asymptotic behavior	MR15:138
Convergent sequences of binomials	Am. Math. Monthly, 50:96
n binomials	[e]8:11
Normalizing transform $y=k^{\frac{1}{2}} \sin^{-1}(x+a/k)$ and other transforms	[d]14:116, [f]3:52, [c]35:248
$\log \frac{x}{1-x}$, $-2 \tanh^{-1}x$	[v]8:73
Other transforms	[x]2:94
Choosing between several binomials	[j]36:537
If p not constant (called "Lexian")	[k]16:1
Transformations, approximations, applications	[15]668

Chain binomials	[c]40:279
Gambler's ruin	[i]24:52
Connection with Beta distribution	[c]41:304, [n]18:121
Actuarial application	[i]35:11
Binomials added	MR17:862 , [w]5:73
Generalization	[d]20:311
Generalized binomial $Ch(x) = \Pi(q_i + p_i e^{ix})$	[2]122
A modification	[e]15:237,251

<u>See also</u>: [d]6:27, [d]17:13, [d]21:247, [d]22:129, [e]12:248, [b]14:115, [j]8:364, [c]11:269, [c]16:165,202, [f]12:276, [f]13:225, [r]1:15,32, Bull. Am. Math. Soc. 1935, p. 857, 11th Skand. Math-Kongress p. 210, Z5:212, Z18:31, MR6:234, MR11:604, MR12:509, MR9:450, Z3:18, [w]1:9, [g]33:390, [a]83:277, [c]33:222, [g]49:169, [i]7:153, [i]6:77, [i]20:77, [i]22:23, [i]26:22, [i]31:8, [i]32:188, [d]25:770, [c]44:364, J. Proc. Roy. Soc. N.S.W. 81:38, Z19:316, Proc. Ntl. Cong. Math. (1924) 2:801, [y]3:282, [w]8:23.

3.2 BINOMIAL (1,p)

$D(x) = p^x(1-p)^{1-x}$, $x=0,1$

$D(\bar{x})$ = Binomial (n,p)	[6]207, [w]1:73
Chi-square test	[e]13:3
Confidence intervals for p	[6]233
Completeness	[e]10:315

UMVUE(p) = $\bar{x}$, UMVUE(pq) = $\frac{n\bar{x}(1-\bar{x})}{n-1}$

See also: [v]3:324, [w]1:9.

3.3 TRUNCATED BINOMIAL (k,p)

$D(x) = \binom{k}{x} \frac{p^x q^{k-x}}{1-q^k}$, $x=1,\ldots,k$	[6]162
$E(x) = \frac{kp}{1-q^k}$, $v = \frac{kpq}{1-q^k}$,	[d]16:50
moments of x^{-p}	
Estimation	[g]50:877
Tables	[g]49:169,
With an application	[k]14:321

See also: [b]11:2, MR15:969

3.4 NEGATIVE BINOMIAL (k,p)

D(x)	p	q	Mean	Variance	Reference
$\binom{-k}{x} p^x (1-p)^{-k-x}$	p	q	-kp	-kpq	
$(\frac{a}{1+a})^k \binom{-k}{x} \frac{(-1)^x}{(1+a)^x}$	-1/a	$\frac{a+1}{a}$	k/a	$k/a + k/a^2$	[1]259, [a]83:255
coefficient of t^x in $(\frac{a}{1+a})^k (1 - \frac{t}{1+a})^{-k}$	-1/a	$\frac{a+1}{a}$	k/a	$k/a + k/a^2$	[2]125
$p^k \binom{x+k-1}{k-1} (1-p)^x$	$\frac{p-1}{p}$	1/p	qk/p	qk/p^2	[d]17:53, [6]61, [18]1-158, [17]No. 6, [7]218
$\binom{x+k-1}{k-1} \frac{p^x}{(1+p)^{k+x}}$	-p	1+p	p	$p + p^2$	[f]9:176, [4]54
$(\frac{m}{1+bm})^x (1+bm)^{-1/b} \frac{1}{x!} \prod_{j=1}^{x-1} (1+jb)$	-bm	1+bm	m	m(1+bm)	[5]32
$(\frac{n}{n+km})^n \binom{x+n-1}{n-1} (\frac{km}{n+km})^x$	-km/n	$\frac{n+km}{n}$	km	$km + \frac{k^2m^2}{n}$	[c]41:78

If $Qp = 1$, $Q(1-p) = P$ then

$$\beta_1 = \frac{P+Q}{\sqrt{kPQ}}, \quad \beta_2 = 3 + \frac{1+6PQ}{kPQ}$$

Obtained by assuming a Poisson parameter to be Type III	[1]259, [2]125, [c]41:78, [a]110:132, [f]5:162
Some derivations, with interesting properties	[18]1-159, [c]44:530
Moments	Z13:70
If k = h/p, called Polya-Eggenburger	[4]55
Ch(x),	[17]No. 7, extension Mem. Fac. Sci. Kyusku Imp. Univ. (Ser A) 1:178
A "contagious" distribution	[7]83,101, [13]413, [7]128
Skewness, Kurtosis, Cumulants	[18]1-136, 1-144
$C(x) = (p + q)^{-n}$	[c]37:209
$Ch(x) = [1 + bm(1-e^{it})]^{-1/b}$	[5]62
Called compound Poisson	[15]727
Ch(x)	[v]4:9
Limit of contagious	[c]41:269
Limited by Poisson and Pascal	[7]233
PGF	[18]1-146
Problem leading to Negative Binomial, with generalization	[d]17:53

Generalization	MR16:602
Paper by Fisher	[k]11:182
Recurrence formula for cumulants,	Aktuárské Vědy 5:182
Moments	G. Dell Ist. Ital. degli Attuari 6:3
Formulas for tails	[7]237
MLE	[b]8:206, [c]37:114
Mχ^2E	[k]11:109
UMVUE(p)	[e]18:374
Estimation	[d]24:409, Psychometrika 16:107
Sequential	[f]6:59
Sampling	[c]37:358
Fitting	[f]9:176
Truncated	[g]50:877
Moments estimation, MLE	[c]42:58,
Bhattacharyya bounds	[d]27:1182
Transformation $\sinh^{-1}x$	[f]3:52, [c]35:249

Transformations [c]41:315

Called "Pascal", satisfies

$D(x+1) = \frac{(1-p)(x+k)}{(x+1)} D(x)$, etc. [i]14:176

Accident proneness [c]37:24

Telephone traffic [j]35:454

Bibliography [1]437

See also: [7]236, [b]10:260, [f]5:165, [f]7:340,411, [c]35:11, [c]39:178,198, [c]40:203, [c]40:370, [c]44:364, [i]20:78, [i]22:25, [i]31:9, D'Analyse Math 1:331, Psych. Bull., 47:434, [u]45:364, Z6:69, Z18:265, Z13:409, Z14:29, MR17:944, [a]99:733, [w]8:23.

3.5 NEGATIVE BINOMIAL (1, -m)

$D(x) = \frac{1}{1+m} (\frac{m}{1+m})^x$, "Pascal", or "Furry", [5]31,60,66

$\alpha_1 = m$, $v = m^2 + m$,

$Ch(x) = [1 + m(1-e^{it})]^{-1}$

Cumulants [18]1-144

PGF [18]1-146

See also: [c]39:346, [7]59, [v]4:8, [c]36:165, [15]38 [c]44:265

3.6 DISCRETE LEXIAN

$D(x) = \Sigma f(p) \binom{k}{x} p^x (1-p)^{k-x}$, moments etc.	[i]26:34
"Generalized Binomial"	
Poisson-Lexian	[i]26:57
$Ch(x) = [p\phi(t) + q]^k$	[17] No. 69
If <u>a priori</u> distribution of p is Beta	[i]27:39, Bull. Am. Math. Soc. 41:860

3.7 DETERMINISTIC

$D(x) = \begin{cases} 0, & x \neq c \\ 1, & x = c \end{cases}$	[1]192
$Ch(x) = e^{ict}$	[8]209, [5]29,62
For $c = 1$	[2]96
Bibliography	[17]No. 1
Moments, cumulants, Ch(x), PGF	[18]1-136, 1-144, 1-146, [v]3:324

<u>See also</u>: [c]44:366, Z9:363, [w]1:9

3.8 RECIPROCAL TRUNCATED BINOMIAL

D(x)	[n]18N.1-2:77

IV. DISCRETE DISTRIBUTIONS

4.1 POISSON (m)

$D(x) = \frac{e^{-m} m^x}{x!}$, x=0,1,..., Law of small numbers	[6]59, [5]30, [8]X, [7]72,115, [10]47,63 [15]119
$Ch(x) = \exp[m(e^{it} - 1)]$	[1]204, [5]62, [2]66
$MGF(x) = e^{-m} \exp(me^t)$	[6]101, [4]53, [m]2:46
$\alpha_1 = m, \quad \alpha_2 = m^2 + m, \quad v = m$	[6]102, [5]57,66, [4]53
$\alpha_3 = m[(m+1)^2 + m]$, $\alpha_4 = m(m^3 + 6m^2 + 7m + 1)$	[10]59
$\mu_2 = m, \quad \mu_3 = m, \quad \mu_4 = m(1 + 3m)$, $\mu_5 = m(1 + 10m), \quad \mu_6 = m(1 + 25m + 15m^2)$	[d]1:119, [2]86
Skewness, Kurtosis	[18]1-136
PGF	[18]1-146

$\mu_{r+1} = rm\mu_{r-1} + m \frac{d\mu_r}{dm}$	[2]121, Bull. Am. Math. Soc. April 1934, p. 264, 41:857
All cumulants = m	[2]66
Factorial moments $\alpha_{[i]} = m^i$	[1]257
Moments in general	[d]8:103, [i]14:173
Recursion formula for moments, correction with multinomial, C(x) as a Γ integral	[14]36-8
$\beta_1 = 1/m$, $\beta_2 = 3 + 1/m$, $\gamma = 1/m$	[12]52
C(x)	[j]5:604, MR4:194, [c]37:313
Transform $y = \sqrt{x}$	[14]209
$D(\overline{x}) = \frac{e^{-nm}(nm)^{n\overline{x}}}{(n\overline{x})!}$	[1]379, [2]243, [6]208, [15]219
D(x + y)	[10]59
$\sim D(\frac{x-m}{m}) = N(m, m^{\frac{1}{2}})$	[1]250
D(x - y)	[2]251, MR14:566, [a]109:296, [a]100:415, [v]7:175

~ D(x - y)	MR15:138
D(gap between two Poisson events) = exponential	[c]41:251, [g]49:255, MR14:293
$D[\bar{x}^{-1}\Sigma(x_i - \bar{x})]$, i.e., Chi-square test	[b]5:75, Z18:321
Various a <u>priori</u> distributions of m, in particular Type III	[i]27:33
$E(x^2) = E(x + 1)$	[8]119
$E(1/x)$	[g]49:169
Var $(x^{\frac{1}{2}})$	[a]106:143
Reproductive property by convolutions	[7]216
by Ch. functions	[9]279
C.-R.(m) = m/n	[1]487
C.-R.$(m^2) = 4m^3n^{-1}$	
BANE	[d]21:401
Estimation when m must be integral	[b]12:213
Estimation [d]24:406	
Estimation from censored samples	[g]49:158

Estimation of bacteria population	[c]31:170
Estimation of m or 1/m	[g]49:255
$M\chi^2E$	[3]56
Approximation, estimation, application	[15]714
MLE(m) = $\bar{x}$	[4]141, [9]6.22, [5]144, [3]21, [p]7:169
$\bar{x}$ is sufficient	[4]136, [w]22:713
$\bar{x}$ is efficient	[1]487
$\bar{x}$ is unbiased	[3]142
Completeness	[e]10:315
Confidence intervals, tail	[c]41:312,
C.-R.	[16]16
Confidence intervals	[3]71,81, [d]9:173, [c]28:437, [c]44:436, [e]14:25, [p]7:223
Order statistics	MR16:729
Approximate moments of ordered variables	[s]8:78
Testing whether two Poissons are the same	[3]127, [c]37:143,

Two Poissons, etc.	[c]40:447
Whether k Poissons are the same	[d]16:362, Proc. Nat. Inst. Sci. India 3:297
Analysis of variance	[d]11:335, J. Econ. Entom. 37:717
Testing m	[c]31:314, [c]40:354, [g]49:255, [14]205
Testing ratios of means	[o]4:45
Chi-square test	[k]7:207
Testing against contagious	[c]37:59
Sequential testing m	[d]19:400
Small sample tests	[f]12:264
Monograph on Poisson testing and estimation	MR16:383
Obtained from a difference equation analogous to Pearson's differential equation	[f]2:419
Obtained from postulates	Z13:408
Early discussion with numerical examples	[c]10:36

Transform $y = (x + k)^{\frac{1}{2}}$	[d]14:113, [f]3:52, [c]35:247
Transformations	[c]41:312
Domain of attraction	MR3:2
As limit of binomial	MR12:190, [4]52, [7]110, [i]6:78
Generalizations	[c]36:18, Operations Res. 3:198, [c]37:48
A modification	[e]15:237,251
Convergent sequence of Poissons	Am. Math. Monthly 50:97
Connection with hypergeometric	[c]25:300
If m Poisson, called double Poisson	Kendall and Buckland, A Dictionary of Stat. Terms
Normal approximation	[7]146, MR16:1034, MR10:613, [r]4:37
Compounded with binomial	[7]128,221
As approximation to Beta	[j]20:19
Normal approximation	[r]4:37
An approximation	MR18:423

Connection with Gram-Charlier	[2]154
Connection with Type X	[o]2:13
Connection with Type III	[o]3:123
Limiting theorems	[8]148
Characterization	[u]48:206, Proc. A.M.S. 1:813, C. R. Acad. Sci. Paris 239:1114, 3rd Berkeley Symp 2:145
Generalizations	[d]13:410, [d]14:394, MR15:138, MR16:1034, [d]19:414, MR13:258
Possibility of a continuous analogue	[i]14:43
Traffic control	[b]7:65
Accident causation	[b]7:89, [a]90:487
Poisson as a limiting distribution in five different ways, relation with multinomial, exponential	[18]1-156
Accident proneness	[c]37:24
Pedestrian delay	[c]38:383

Insurance risk	[i]40:72
Frequency of war	[a]107:242, [a]112:446
Nomograph for acceptance inspection	[s]4:204
Telephone switchboards	[5]30, [j]6:468
An early treatment, with the famous example of the Prussian horse-kicks	Von. Bortkewjtsch L. Das Gesetz der Kleinen Lahlen, B.G. Teubner, Leipzig 1898
Other applications	[7]119, [j]5:604

See also: [d]14:155, [d]20:523, [d]22:94, [d]22:128, [13]405, [j]7:45, [c]11:267, [c]11:211, [c]26:108, [g]33:390, [g]42:574, [a]83:255, [f]6:17, MR1:246, Z22:243, MR17:53, MR14:485, Z18:31, Z14:138, MR4:20, MR13:633, MR15:541,634, MR7:310, [f]7:340, [c]27:272, [c]30:188, [c]36:250, [n]16-2:285, [k]9:406, [c]38:427, [c]39:346, [i]20:80, [i]22:25, [i]25:158, [i]26:46, [i]31:9, Rio. Ital. di demogr. e Stat 3:219, Proc. First Pakistan Stat. Conference (1950) p. 59, Publ. Math. Debrecen 2:66, Annals of Applied Biology 9:325, Annales de l'Ecole Normalle Superieme 54:321, [v]2:330, [v]3:327, [d]26:147, [c]44:265,365, Brit. Assoc. Math. Tables (3rd Ed.) V.1 pxxxvi, Am. Math. Monthly 50:97, Annals of Math 37:357, Bull. Am. Math. Soc. 1935 p. 861, Am. J. Math 57:827, [u]45:219, Kungl. Lantbruk. Ann. 18:86, Z2:200, MR1:15, MR18:341, MR5:128, MR2:112, Z15:407, Z18:412, MR14:1098, MR13:958, [w]1:9.

4.2 TRUNCATED POISSON

$D(x) = \frac{m^x e^{-m}}{x!} \cdot \frac{1}{1-e^{-m}}$, $x=1,2,\ldots$	[f]8:275, [f]10:402, [f]11:387
Estimation	[c]39:247, [c]40:171, [g]50:906, [i]39:19, [f]9:485.
UMVUE	[e]18:374
Tables	[g]49:169, [11]158, [13]39:247
Servicing machines	[b]13:71
D(1/x)	[n]18 No. 1-2:77
Doubly truncated, D(x) etc.	[g]49:160, Conn. Agric. Exp. Sta. Bull. No. 513.

4.3 COMPOUND POISSON

$D(x) = k_1 \frac{e^{-m} m^x}{x!} + k_2 \frac{e^{-n} n^x}{x!}$	[5]151
double Poisson	
If $k_1 = k_2 = \frac{1}{2}$	[5]150, [g]42:407, [f]8:281

Compound Poisson $D(x) = \frac{1}{x!} \Sigma m_i^x e^{-m_i} k_i$ — [7]237, MR17:862, R. D. Evans, The Atomic Nucleus, p. 766, MR13:633, MR14:770

4.4 UNIFORM

D(x) = 1/k, x=1,...,k	[6]61
Discrete rectangular	
MGF and moments	[d]11:324
Sampling from	[c]21:126
Estimation of range	[g]46:375
Range and quotient of ranges	Int. Congr. Math. (1950) 1:583

See also: MR16:376.

4.5 HYPERGEOMETRIC

$D(x) = \dfrac{\binom{m}{x}\binom{n}{r-x}}{\binom{m+n}{r}}$	[6]61, [7]33, [15]40
D(x) in another form, moments, etc.	[2]126
In the form of a hypergeometric series, asymptotic forms	[c]25:295, [c]26:59,
Various forms	J. Soc. Stat. Paris 96:262
C(x) as a power series	[c]41:317
Ch(x), refs.	[17] No. 8
$E(x) = \dfrac{mr}{m+n}$, $v = \dfrac{mnr\ (m+n-r)}{(m+n)^2(m+n-1)}$	[7]183, [6]98
Difference equation, moments, etc.	[i]14:178
Factorial moments	[2]135, Nature 164:282 [i]6:79
Skewness	[18]1-136,
PGF	[18]1-146
Moments in general	[d]8:103, [d]10:198, [c]16:157, [c]17:57, [c]26:264, [a]89:326 [v]3:326, GANITA 7:1

Binomial and Poisson as limits	[15]690
Binomial as limit	[7]47
Poisson as limit	[7]114, [c]25:300
Normal as limit	[7]146
Normal, Poisson, Binomial approximations	[18]1-155, [d]27:471
Minimax estimation	[d]21:191
Completeness	[e]10:315
"Confidence Limits for the Hypergeometric Distribution"	Chung and DeLury Univ. of Toronto Press 1950, reviewed, [a]115:286
Generalization	[7]39, [c]41:266, see also No. 8.59, [b]18:202
Satisfying difference equation	[w]3:5
Linguistic application	[b]12:27
Truncated hypergeometric, moments	[d]16:59

Double hypergeometric [7]187, Koninkl. Nederl. Akad(A) 60:121

<u>See also</u>: [d]1:113, [d]21:248, [e]11:153, [j]7:39, [j]10:281, [f]8:287, [n]1-4:49, [c]37:140, [h]2:435, [i]22:26, [d]25:762, J. Proc. Roy. Soc. N.S.W. 81:38, N.B.S. Math. Table MT19, MR17:984, Z12:29, Z13:273, MR1:340, U. Calif. Pub. Stat 1, No. 7, MR13:962, MR14:775, [w]1:9.

4.6 CONTAGIOUS

Obtained by considering $\int$ (Poisson) dF(m) MR14:293, [d]14:389 [v]8:13

F step function yields

$$D(x) = \frac{1}{x!} \Sigma\, p_i\, e^{-a_i}\, a_i^x$$

F Type III yields Polya-Eggenberger

$$D(x) = \frac{1}{x!} \frac{\Gamma(x+h/d)}{\Gamma(h/d)} (1+d)^{-d/h} (1+d)^{-x}$$

m itself also Poisson yields

$$D(x) = e^{-k} \frac{c^x}{x!} \Sigma \frac{i^x}{i!} (e^{-c}k)^i$$

Neyman contagious Type A

Fitting [f]11:149

Ch. fcns. [g]49:368

Testing against Poisson [c]37:59

Rutherford contagious [d]25:703

$\sum_j$ (Poisson) $f_j(x)$, Ch(x), sp. cases [17]Nos. 66-8

Bivariate MR15:138

See also: [13]411, [f]9:354, [c]36:450, [c]39:346, [c]40:186, [c]41:268, [d]10:35, J. Econ. Entom. 35:536, MR1:251

4.7 POLLACZEK-GEIRINGER

f(x), Ch(x), [17]No. 9

multiple occurrence of rate events

4.8 BOREL-TANNER

$$f(x) = \frac{e^{-\alpha x}\alpha^{x-r}x^{x-r-1}r}{(x-r)!}, \quad x=r,r+1,\ldots,$$

$0 \leq \alpha \leq 1$, $r = 1,2,\ldots$ [s]214:452, [c]40:58

$$m = \frac{r}{1-\alpha}, \quad v = \frac{r^{\alpha}}{(1-\alpha)^3}$$

4.9 POLYA

$$f(x) = \binom{N}{x} \frac{\prod_{i=0}^{x-1} (m+iR) \prod_{j=0}^{N-x-1} (n-m+jR}{\prod_{k=0}^{N-1} (n+kR)}$$

[7]128, [d]28:1021

Contains Polya-Eggenburger No. 3.4, and Exceedance No. 4.10

4.10 EXCEEDANCE

$$f(x) = \frac{\binom{n}{m} m \binom{N}{x}}{(n+N)\binom{N+n-1}{m+x-1}}, \quad x=0,1,\ldots,N$$

[d]25:762, [w]6:164, [d]28:1021, [d]21:247

Moments — [w]6:165

4.11 INVERSE HYPERGEOMETRIC

$$f(x) = \frac{\binom{n}{m-1}\binom{N-n}{x-m} (n-m+1)}{\binom{N}{x-1}(N-x+1)}$$

, estimation, — Kungl. Lantbuck. Ann. 18:123

truncation

V. DISTRIBUTIONS ON (a,b)

5.1 SERIAL CORRELATION

$D(x) = \frac{\Gamma(\frac{1}{2}k+1)}{\Gamma(\frac{1}{2}k+\frac{1}{2})\Gamma(\frac{1}{2})}(1-x^2)^{\frac{1}{2}(k-1)}(1+r^2-2rx)^{-\frac{1}{2}k}$ [c]41:261

$\alpha_1 = \frac{rk}{k+2}$,

$v = (k+2)^{-1}[1 - r^2k(k+1)(k+2)^{-1}(k+4)^{-1}]$

Moments, called "Leipnik" [c]44:270

See also: [c]35:255,261, [d]13:1,14, [c]43:161,169, [d]18:86, Cowles Commission Papers, New Series No. 42.

5.2 TYPE I

$D(x) = C(x-a)^{p-1}(b-x)^{q-1}$, [1]249, [17]No. 22

$a < x < b, \quad p > 0, \quad q > 0$

Beta for $a=0$, $b=1$ [2]139

normal for $p=q=\frac{1}{2}b^2$, $a=-b$, $b \to \infty$

Type III for $b \to \infty$, $q = \alpha b$ [1]249

Obtained by assuming roots in quadratic of Pearson differential equation real with different sign [4]74, [11]43

Relations of various constants [11]53, [c]16:107

Bayes Theorem [n]16-1:115

Fitting to observations [a]96:306, [c]1:31, [c]1:292, [c]1:408, [c]4:474, [c]7:87

Early volumes of [c] give many examples with $D(x) = C(1+x/a)^m(1-x/b)^n$

See also: [d]1:148, [d]7:20, [d]8:17, [d]10:15, [c]3:311, [c]23:393, [c]26:386, [i]16:53.

5.3 BETA (p,q)

$D(x) = \frac{\Gamma[\frac{1}{2}(p+q)]}{\Gamma(\frac{1}{2}p)\Gamma(\frac{1}{2}q)} x^{\frac{1}{2}p-1}(1-x)^{\frac{1}{2}q-1}$, [2]139, [17]No. 21, [1]243, [10]153

$0<x<1, \quad p< -2, \quad q< -2$

Called Beta distribution of the first kind [d]25:401

$\alpha_1 = \frac{p}{p+q}$, $v = \frac{pq}{(p+q)^2(\frac{1}{2}p+\frac{1}{2}q+1)}$ [2]31,41, [10]153, [14]42

3rd Moments [2]419

Mode $= \frac{p-2}{p+q-4}$, etc. [m]2:128

$\alpha_r = \frac{B(p+r,q)}{B(p,q)}$	[6]117, [b]4:126, [14]42
$HM = \frac{p-2}{p+q-2}$,	[10]163
$GM = \exp\left\{\frac{\partial}{\partial\frac{1}{2}p}\left[\log\Gamma(\frac{1}{2}p) - \log\frac{1}{2}(p+q)\right]\right\}$	
C(x)	[d]20:451
Moments	[3]211
Obtained as a Pearson Type	[4]74
From an example	[10]45
C(x)	[c]19:1, [c]25:379, [c]38:423, [c]37:208, [i]38:192
D($\bar{x}$)	[2]251, [n]8-4:55
When p=q	[c]19:230
Special cases and variants	[2]26
x^2 is Beta	[17]No. 20
Beta (n-k,k)	[1]409
Beta(2n-2,4) connected with tolerance limits	Hoel, Intro. to Math. Stat.
Correlation ratio in samples from uncorrelated bivariate normal is Beta (k-1, n-k)	[2]352, [1]414, [c]21:1, [c]30:290, [10]181,184

D(1-x) = Beta (q,p)	[10]156
If x is Beta (p,q), y is Type III (1, p+q) then xy is Type III (1,p)	
Standardized Beta variable is N(0,1) as $p \to \infty$, $q \to \infty$	[1]252
D(xy)	[e]9:365
No sufficient statistic for ($\frac{1}{2}$p-1)	Proc. Roy. Soc. Lond. (Series A) 154:133
Mellin transform	[d]19:373
more generally	[b]8:136
log log $\frac{1}{x}$, arcsin $\sqrt{x}$,	[v]8:71
Generalized	[c]43:237, [c]44:441
Transform	[c]36:165
Connection with Fisher and Snedecor	[2]419, [c]21:350
Non-central Beta	[d]26:648
Connection with binomial	[c]41:304, [n]18:121
Fitting straight lines	[y]24:23
Approximated by Poisson	[j]20:19

Range of rectangular is [illegible] [4]93, [c]25:417

$D(x) = n(n-1)a^{-n}x^{n-2}(a-x)$

In trivariate normal analysis [3]341

GM $(x_1, \ldots, x_n)$ and GM $(1-x_1, \ldots, 1-x_n)$ are joint sufficient [3]49

In rank correlation [2]418

<u>See also</u>: [d]16:98, [d]17:325, [b]1:214, [c]22:284, [c]22:391, [c]23:143, [c]27:415, [10]154, [c]30:140, [c]33:178, [c]34:368, [c]35:19, [c]32:151,271, [c]36:166, [k]5:75, [c]39:204, [c]37:219, [c]40:281, [g]48:831, [w]1:9

5.4 TYPE II

$$D(x) = \frac{(1-(x-m)^2a^{-2})^p}{aB(\frac{1}{2},p+1)}, \quad m-a \leq x \leq m+a$$ [2]141

Properties MR10:131

Transform to Student [c]28:308

$D(\bar{x})$ [n]10-3:91

$C(x)$ [c]19:12, [c]25:379

~ Dist. of rank correlation coefficient (Pitman) [c]30:259

Called Thompson's distribution, relation with Student's, normal approximation [d]27:784

m location, a scaling, p shape [d]3:86

Likelihood function

C.-R.(m)

Tables

$v = \frac{a^2}{2p+3}$

If m=0 [2]401

Dist. of Spearman's ρ for large n

A numerical example [11]62

Estimation of center [t]4:33

From Pearson system [11]43

See also: [c]4:174, [c]16:114, [c]21:263, [n]12-3:67

5.5 PARTIAL CORRELATION

$D(x) = \pi^{-\frac{1}{2}} \frac{\Gamma[\frac{1}{2}(n-l+1)]}{\Gamma[\frac{1}{2}(n-k)]} (1-x^2)^{\frac{1}{2}(n-k-2)}$, [1]412, [i]24:198

$-1<x<1$

Type II with m=0, a=1, p= $\frac{1}{2}$(n-k-2)

Ch(x)	[17]No. 19
If k=2, transform to Student	[5]99
If corresponding population parameter is zero	[10]256
As No. 5.1 with r=0,	Cowles Commission Papers, New Series No. 10
If population is non-normal	[i]36:16

See also: [d]18:81, [n]2:684, [o]3:45, [a]92:580

5.6 PARABOLIC

$D(x) = \frac{3(a^2-x^2)}{4a^3}$, $-a<x<a$, $v=a^2/5$	[c]39:432
grouping corrections	
Estimation	[d]26:505, [m]6:120

5.7 TYPE IX

$D(x) = \frac{m+1}{a}(1+x/a)^m$, $-a\leq x\leq 0$	[2]142, [17]No. 16

See also: [d]7:26, [c]24:234,240,263.

Type VIII for negative m	[17]No. 23, MR4:21

5.8 TYPE XII

$$D(x) = (p/q)^m \frac{1}{(p+q)B(1+m,1-m)} (1+x/p)^m (1-x/q)^{-m} \qquad [2]143$$

$$|m|<1, \ -p \le x \le q$$

See also: [d]7:27, [17]No. 24.

5.9 CORRELATION DETERMINANT

$$D(x) = \frac{\Gamma[\frac{1}{2}(n-1)]^{k-1} x^{\frac{1}{2}(n-k-2)}}{\pi^{\frac{1}{4}k(k-1)} \Gamma[\frac{1}{2}(n-2)] \cdot \ldots \cdot \Gamma[\frac{1}{2}(n-k)]} \qquad [1]411, [4]120$$

$\alpha_1 = (n-1)^{1-k}(n-2)(n-3) \cdot \ldots \cdot (n-k),$

$v = k(k-1)n^{-2} + 0(n^{-3})$

Downton calls this "Geometric", and mentions the following special cases: [d]25:304

I. $D(x) = px^{p-1}, \ 0 \le x < 1,$

$C(x) = x^p, \ \alpha_1 = p/p-1, \ v = p(p+2)^{-1}(p+1)^{-2}$

II. $D(x) = pb^{-p}(x+a)^{p-1}, \ -a \le x < b-a$

III. $D(x) = pv^{-\frac{1}{2}} b^{-p}(x-m/v^{\frac{1}{2}} + a)^{p-1},$

$m - av^{\frac{1}{2}} \le x < m+(b-a)v^{\frac{1}{2}}, \ \alpha_1 = m,$

$a = p^{\frac{1}{2}}(p+2)^{\frac{1}{2}}, \ b = p^{-\frac{1}{2}}(p+1)^{3/2}, \ p \ge 1$

log log (x) [v]8:70

5.10 TRIANGULAR

$D(x) = 1 - |1 - x|$, $D(\bar{x})$ from rectangular when $n = 2$, D(range) — [c]25:417, [y]24:22, MR 3:171

$D(x) = \dfrac{2x}{2k+1}$, $k \leq x \leq k + 1$ — [3]47, [8]32

Stratified sampling — [c]13:48

$D(x) = (9\sigma)^{-1}[\frac{x-m}{\sigma} + 2\sqrt{2}]$, right triangular — [d]4:256, [d]25:308
$m - 2\sqrt{2}\,\sigma \leq x < m + \sqrt{2}\,\sigma$

$D(x) = 4R^{-2}(\frac{1}{2}R - |x-m|)$, $|x-m| \leq \frac{1}{2}R$, — [d]25:318
best linear estimate of m and σ

Testing — [d]25:695
See also — [d]2:48, [d]28:179

$D(x) = \frac{1}{a}[1 + k - \frac{2k}{a(a-x)}]$, $0 \leq x \leq a$, — [d]4:244
$-1 \leq k \leq 1$, called "linear"

$D(x) = \begin{cases} 1 + x, & -1 \leq x < 0 \\ 1 - x, & 0 \leq x < 1 \end{cases}$ called "Tine" — [d]5:33

$Ch(x) = (2/t^2)(1 - \cos t)$ — [17]No.14

$D(x) = \frac{2}{a^2}(a - x)$, called "semi-triangular" — [c]39:432
$m = a/3$, $v = a^2/18$, grouping corrections

Ch(x) — [17]No.13

Triangular on (a,b). If x and y are extreme values of the sample, then

$$E[\tfrac{1}{2}(x + y)] = \tfrac{1}{2}(a + b)$$

$$Var[\tfrac{1}{2}(x + y)] = \frac{4-\pi}{16n}(b - a)^2 + O(n^{-2})$$

$$E[(x - y)] = [1 - \frac{\sqrt{\pi}}{2n}](b - a) + O(n^{-3/2})$$

$$Var(x - y) = \frac{4-\pi}{4n}(b - a)^2 + O(n^{-2})$$

See Also: [y]16:16

5.11 RECTANGULAR (a-h, a+h)

$D(x) = \frac{1}{2h}$, $m = a$, $v = \frac{h^2}{3}$	[1]244, [5]34
Skewness = 0, kurtosis = -6/5, K_k, Ch(x)	[18]1-136,1-144
$Ch(x) = \frac{\sin ht}{ht} e^{ait}$	[1]259
Special case of Type II	[2]142
Ch(x), bibliography for rectangular over (a,b), (-a, a)	[17]Nos.11,12
$C(x) = \frac{1}{2} + \frac{x}{a}$	[15]93
If x and y are the k^{th} values from the top and bottom of sample,	[1]372

$$E(x) = a + h - \frac{k}{n+1}(2h)$$

$$E[\tfrac{1}{2}(x+y)] = a$$

$$E(x - y) = (1 - \frac{2k}{n+1})(2h)$$

$$\mathrm{Var}(x) = \frac{k(n-k+1)}{(n+1)^2(n+2)}\,(2h)^2$$

$$\mathrm{Var}[\tfrac{1}{2}(x+y)] = \frac{4kh^2}{2(n+1)(n+2)}$$

$$\mathrm{Var}(x-y) = \frac{2k(n-k+1)}{(n+1)^2(n+2)}\,(2h)^2$$

$\sim D(\bar{x}) = N(0, \frac{h^2}{3n})$, $\sim D(c) = \mathrm{Laplace}(0, \frac{h}{n})$, where $c = \frac{1}{2}(\max + \min)$, $n\,\mathrm{var}(c) = 6\,\mathrm{var}(\bar{x})$	[3]48
$D(\xi)$	[d]26:115
Moments of max and min	MR 4:21
$D(\bar{x})$ given incorrectly	[n]10-3:91
$D(\bar{x})$ = No. 5.16	
$D(-2\log \Pi\, x_j)$	[v]4:161
FD(a) = Rectangular(max - h, min + h)	[c]30:402
D(q^{th} ranking item) = Type I	[c]23:390
Testing against simple unimodal distribution	[y]20:111
Grouping corrections	[c]39:430
Transformation to Cauchy	[15]101
C.-R. Theorem may not hold	[1]485
MLE(a-h, a+h) = (max, min)	[6]156, [3]28

Best linear estimate of m and σ	[d]25:308,317
Estimation	[d]17:355
Location and scaling, closest estimate	[u]33:221
Minimax estimate of a	[d]22:37
UMVUE	[14]142
Bayes Theorem	[n]16-1:110
Variance of estimates of a	[g]36:410
Testing a	[d]25:157
Critical regions	[3]280

See also: Archiv. der Math. 3:3, MR 6:235

5.12 RECTANGULAR(0,a)

$D(x) = \frac{1}{a}$, $Ch(x) = \frac{e^{ait} - 1}{ait}$	[2]245
$MGF(x) = \frac{\sinh \frac{1}{2}at}{\frac{1}{2}at}$	[10]38
$\alpha_{2r} = \frac{(\frac{1}{2}a)^{2r}}{2r + 1}$	[10]14
Cumulants	[10]41
$Var(\text{mean deviation}) \simeq \frac{a^2}{45n}$	[2]217

Mean difference	[c]28:432
$Ch(\Sigma x_j)$, $C(\Sigma x_j)$	[9]278
$D(GM) = \frac{n^n x^{n-1}}{a^n \Gamma(n)} (\log a/x)^{n-1}$	[2]246, [d]5:276
$D(range) = n(n-1)\ a^{-n}\ x^{n-2}(a - x)$	[4]92,123, [c]20A:210 [9]241
$D(\sum_{j=1}^{n} \log x_j)$ = Type III	[v]7:296
$D(\bar{x}, s)$ for $n = 2,3$	[d]3:128
D(quotient of ranges)	[g]46:502
$D(max_1/max_2)$ = No. 8.63	[g]50:1136
$FD(a) = k\ x^{-n-1}$	[c]30:408, [d]9:273
$MGF(\log \log \frac{1}{x})$, $MGF(\arcsin \sqrt{x})$	[v]8:69
Completeness	[e]10:314
Estimation by order statistics	[d]26:576
Quasi-range	[d]28:179
Best linear estimate	[d]14:88
$UMVUE(a) = (1 + \frac{1}{n})(max)$, max is sufficient	[14]142

Sufficient statistics	[e]17:214
Confidence intervals for a	[3]83, [b]17:88
Example	[d]11:209
Estimation of dispersion	[c]36:95
UMP test of $a = 1$	[13]

<u>See also</u>: [d]2:48, [d]2:66, [d]4:126, [d]4:139,142,255,[c]23:424.

5.13 RECTANGULAR (0, 1)

$D(x) = 1, \quad 0 \leq x \leq 1$

$D(-\log x) = e^{-x}$, $Ch(-\log x) = (1 - it)^{-1}$ [3]132

$D(-\Sigma \log x_j)$ = Type III (1, k)

$D(\Sigma x_j) = \frac{1}{(n-1)!}[\, x^{n-1} - \binom{n}{1}(x-1)^{n-1} + \binom{n}{2}(x-2)^{n-1} - \ldots]$

"Irwin-Hall" distribution: [d]13:43, [1]245, [w]1:73, [2]240,244, [c]19:234, MR 12:509, MR 15:42, MR 7:311, [c]19:240, [c]41:334

Convolutions	MR 6:88
Sheppard's corrections	[2]88

D(q^{th} value from top of sample) = Type I, $m = 1 - \frac{q}{n+1}$, $v = \frac{q(n-q+1)}{(n+1)^2(n+2)}$	[2]218
$Var(\xi) = \frac{1}{4(n+2)}$	[2]230
$\sim D(\overline{x})$	[d]25:636, MR 9:360
Mellin transform	[d]19:373
Order statistics	[c]24:260, [i]33:214
D(range) = Type I	[c]25:417
$MGF(\log \log \frac{1}{x}) = (1 + t)$ $MGF(\arcsin x) = 2(e^{\frac{1}{2}\pi t} + 1)/(t^2 + 4)$	[v]8:69
Ratio of two ranges	[d]21:112, [x]7:179
Moments of the range	Z 13:30, [c]20:217
C.-R. Theorem may not hold	[1]485
Censored sample	[c]41:230
Stratified sample	[d]13:44
Significance levels for $\overline{x}$	[t]3:172
D(GM) = No. 8.12	[w]1:73

Estimation of center, $D(\bar{x})$, $D(\xi)$ [c]33:126

$D(\max_1 \max_2)$ = No. 8.64 [g]50:1142

Two rectangulars added [n]8-3:74

Hypothesis testing [c]32:321

<u>See Also</u>: [d]23:43, [c]25:203, [m]6:120, [d]22:418, [y]24:21, ME 7:310, Z 11:218, MR 16:602.

5.14 CORRELATION

$$D(x) = \frac{(1-x^2)^{\frac{1}{2}(n-4)}(1-\rho^2)^{\frac{1}{2}(n-1)}\,2^{n-3}}{\pi\,(n-3)!}\sum_{i=0}^{\infty}\frac{(2x\rho)^i}{i!}\Gamma^2\left(\frac{n+i-1}{2}\right)$$

[1]398

$$D(x) = \frac{(1-\rho^2)^{\frac{1}{2}(n-1)}\,(1-x^2)^{\frac{1}{2}(n-4)}}{\pi\,(n-2)!}\,\frac{d^{n-2}}{d(\rho x)^{n-2}}\left[\frac{\cos^{-1}(-\rho x)}{1-\rho^2x^2}\right]$$

[2]342, [10]200

Special cases n = 2,3,4, moments [2]345

n = 4 [u]26:536

$C(x)$ [c]25:71

$\sim D(x)$ [n]1-4:1, [c]38:236

If $\rho = 0$, $D(x)$ = No. 5.5 [6]314, [4]120, [g]26:129

If $\rho = 0$, $D\left(\frac{x}{\sqrt{1 - x^2}}\sqrt{n - 2}\right)$ = Student(n-2) [2]343

Transform $x = \tanh z$, $\rho = \tanh \zeta$ [10]200, [c]21:358

If $\rho = 0$, $D(x^2)$ = Beta $[\frac{1}{4}, \frac{1}{4}(n-2)]$ [10]160,192

Bayes distribution of ρ is No. 5.5(0,1) [3]91, [c]41:278

Moments [n]5:3

Papers dealing with this distribution generally [c]10:507, [c]11:328

Interval estimation [e]7:415

Confidence limits for ρ [c]29:157

Stratified sampling [i]36(Suppl.):87

<u>See also</u>: [b]15:193, [c]21:164, [c]24:383, [o]3:1, Z 21:41

5.15 MULTIPLE CORRELATION

$$D(x) = \frac{\gamma}{B[\frac{1}{2}(n-k), (k-1)]}(1 - R^2)^{\frac{1}{2}(n-1)} x^{\frac{1}{2}(k-3)} (1-x)^{\frac{1}{2}(n-k-2)}$$

where $\gamma = F(\frac{n-1}{2}, \frac{n-1}{2}, \frac{k-1}{2}, R^2x)$ and

$$F(a, b, c, x) = 1 + \frac{ab}{c}x + \frac{a(a+1)b(b+1)}{c(c+1)} \frac{x^2}{2!} + \ldots$$

[14]65, [2]384, [d]3:196, [2]127

If R = 0, $D(x^2)$ = Beta(k, n-k)	[i]30:63, [2]381
Testing	[i]29:25
Another form	[2]387, [3]338
Limiting form when n ∞	[2]387
When R= 0, D(x) = Snedecor	[10]257,262
Mean, variance	[c]22:353
Moments	MR 14:189
More generally	[d]11:6

See also: [e]9:352, [e]10:257, [a]92:445, [n]12-4:67, [i]24:199, [x]1:67,137, [x]4:88, Proc. Roy. Soc. (A) 121:654, [u]46:521

5.16 RECTANGULAR MEAN

$$D(x) = \frac{1}{(n-1)!} \sum_{j=0}^{[x]} (-1)^j \binom{n}{j} (x - j)^{n-1}$$ [1]245

Generalization	[v]3:330
Called Irwin-Hall, cf. No. 5.13	[d]13:43

Compare No. 8.47 and No. 8.70

See Also: London P. O. Res. Rep. 13443, Archiv der Math. 3:3, Proc. Intl. Cong. Math. (1924) 2:795

VI. DISTRIBUTIONS ON $(0, \infty)$

6.1 TYPE VI

$$D(x) = C(x - a)^{p-1}(x - b)^{q-1}, \quad x > b,\ a < b,\ q > 0,\ p + q < 1$$ [1]249

If $b = 0$, $C = \dfrac{a^{1-p-q}}{B(1-p-q,\ p)}$ [2]140

Roots of quadratic in Pearson equation real and same sign [11]45

Truncation [i]39:63, [i]40:18

Various constants and an example [11]83

See also: [d]7:23, [c]23:143, [c]25:379, [17]No. 26, [v]4:167

6.2 SNEDECOR (p,q)

$$D(x) = \frac{(p/q)^{\frac{1}{2}p}\, x^{\frac{1}{2}p-1}(1 + px/q)^{-\frac{1}{2}(p+q)}}{B(\frac{1}{2}p,\ \frac{1}{2}q)}, \quad x > 0$$ [5]100

"F" distribution

$m, v, k_3, k_4, \beta_1, \beta_2$ [p]6:175

Derivation, properties, examples [15]374, [18]1-163

Area unity if p,q both even MR 12:509

Obtained as distribution of ratio of two Chi-square variables [6]10.5, [4]113

$D(\sqrt{x})$ [5]100

$D(\frac{px}{q+px}) = \text{Beta}(\ p - 1,\ \ q - 1)$ [4]115

Therefore called "inverted Beta" [c]33:73

Various properties [d]12:446

$m = \frac{q}{q - 2}$ [10]198

$\alpha_r = \frac{\Gamma(\frac{1}{2}p + r)\ \Gamma(\frac{1}{2}q - r)}{\Gamma(\frac{1}{2}p)\ \Gamma(\frac{1}{2}q)}\ (q/p)^r$ [4]114

$\text{Mode} = \frac{pq - 2q}{pq + 2p}$ [10]197

Approximated by normal distribution [d]13:233

If x, y each Snedecor (n-1,n), then
$D(\sqrt{x/y}) =$ Snedecor(2n-2, 2n-2)

Testing [d]13:371

Used to test multiple correlation coefficient [10]257

<u>See Also</u>: [4]189, [d]6:204, [d]18:89, [c]21:350, [c]37:219, [q]7:96, J. Soc. Statist. Paris 96:262

6.3 BETA OF SECOND KIND(p,q)

$D(x) = \dfrac{x^{p-1}}{B(p,q)\ (1+x)^{p+q}}$, $m = \dfrac{p}{q-1}$, [1]242, [10]156,158,163
[d]25:402

$v = \dfrac{p(p+q-1)}{(q-1)^2(q-2)}$, mode $= \dfrac{p-1}{q+1}$, HM $= \dfrac{p-1}{q}$

if $r < q$, $\alpha_r = \dfrac{p(p+1)\ldots(p+r-1)}{(q-1)(q-2)\ldots(q-r)}$

$D(1/x)$ = No. 6.3(q,p), $D(\frac{1}{1+x})$ = Beta

$p = q$, $x \geq 1$ [d]22:418

Called Fisher's F [w]1:9

C(x) [p]7:102

6.4 HOTELLING

$D(x) = \dfrac{2}{B[\frac{1}{2}(p-q), \frac{1}{2}q](p-1)^{\frac{1}{2}q}} \quad \dfrac{x^{q-1}}{[1 + \frac{x^2}{p-1}]^{\frac{1}{2}p}}$ [4]238

For q=1, this is positive half of Student distribution,
hence called generalized Student [1]409, [d]2:375

$D(x^2)$ [c]32:70

Mellin transform [d]19:373

Percentage points, relation with Chi-square [d]27:1091

<u>See also</u>: [10]207, [c]25:399, [i]30:66, [d]9:235, [c]24:480, [c]4:174, [c] 24:487, [t]7:82

6.5 PARETO

$D(x) = p/q(q/x)^{p+1}$,	[8]120
$\alpha_1 = \frac{p}{p-1} q, \ \xi = 2^{1/p} q$	[1]248, [2]142
More generally	[d]7:26
Testing, location and dispersion	[t]7:115
As Type XI	[c]39:178, [17] No. 25
Ranking	[c]24:234,241,275
Double Pareto	Kendall and Buckland, A Dictionary of Stat. Terms.

See also: [ℓ]19:174, [h']1:149, [g]48:537, [i]8:76, [t]3:77, [y]13:30, MR13:962, Z23:63, C. R. Acad. Sci. Paris 233:1421, [ℓ]25:591, [w]4:147.

6.6 KENDALL

$$D(x) = \frac{re^{-(x-r)/\alpha}(x-r)^{x-1}}{\alpha^x \, \Gamma(x+1)} ,$$

[b]19:211, (cf. Borel-Tanner)

$0 < r < x < \infty , \ 0 \leq \alpha \leq 1,$

$m = \frac{r}{1-\alpha} , \quad v = \frac{r\alpha^2}{(1-\alpha)^3}$

6.7 INVERSE GAUSSIAN

$D(x) = \exp[-\lambda(x-\mu)^2/2\mu^2 x][\lambda/2\pi x^3]^{\frac{1}{2}}$,

Introduction, moments, estimation	[d]28:362,696

VII. DISTRIBUTIONS ON $(-\infty, \infty)$

7.1 TYPE VII

$D(x) = \dfrac{(1+x^2/a^2)^{-m}}{aB(\frac{1}{2}, m-\frac{1}{2})}$, $m > \frac{1}{2}$	[2]142
Estimation	[c]36:412, [t]4:35

See also: [c]15:401, [c]36:412,167

7.2 STUDENT (r)

$D(x) = \dfrac{\Gamma[\frac{1}{2}(r+1)]}{\Gamma(\frac{1}{2}r)(\pi r)}(1+x^2/r)^{-\frac{1}{2}(r+1)}$	[5]97, [2]17, [6]10.6, MR8:161, [14]47, [4]110, [10]186, [1]3:355, [18]1-162, [17]No. 29
Introduction, properties, examples	[15]388
$v = \dfrac{r}{r-2}$	[1]239

Type VII with $m = \frac{1}{2}(r+1)$, $a^2 = r$

"t" distribution

$$\alpha_{2k} = \frac{1.3\cdot\ldots\cdot(2k-1)r^k}{(r-2)(r-4)\cdot\ldots\cdot(r-2k)}$$ [1]239, [10]208

Original paper in which this distribution was discovered — [c]6:1

Ch(x), refs for r=3 — [17]No. 28

Ch(x) — [b]18:212

Distribution of the ratio of a Chi-square variable to a normal variable — [1]387, [4]110, [10]187 [n]5:102

In bivariate normal samples

$$D\left(\frac{\sigma_1(n-1)^{\frac{1}{2}}}{\sigma_2(1-\rho^2)^{\frac{1}{2}}}(b-\beta)\right) = \text{Student } (n-1)$$ [1]29.8

$$D\left(\frac{s_1(n-2)^{\frac{1}{2}}}{s_2(1-r^2)^{\frac{1}{2}}}(b-\beta)\right) = \text{Student } (n-2)$$ [1]29.8

if $\rho=0$

$$D\left[(n-2)^{\frac{1}{2}}\frac{r}{(1-r^2)^{\frac{1}{2}}}\right] = \text{Student } (n-2)$$ [1]29.7

C(x) — [1]3:358, [c]25:389, [n]5:109, [c]37:168

$D(x^2)$ = Snedecor — [6]217, [4]115

Transform to Type II	[c]28:308
D($\bar{x}$)	[n]8-4:92
As $r \rightarrow \infty$, Student $\rightarrow$ N(0,1)	[1]252, [3]101, [a]113:228, [d]27:783, Proc. A.M.S. 6th Symposium in Appl. Math. p. 251
Approximations	[d]7:210, [d]9:87, [d]17:216
D(log x)	[c]34:176
Two Student variables	[c]22:405, [c]23:1
Used to test partial correlation	[10]256
$\sim$ significance levels	[d]14:60
Generalizations	[d]19:406, [d]25:162, [i]34:58

See also: [d]10:265, [d]18:89, [e]11:37, [e]12:89, [j]8:632, [c]33:362, [n]5:90, [c]32:271,300, [c]24:56,296, [i]33:138, [c]44:264, Brit. Assoc. Math. Tables (3rd Ed.) V.1 p xxxiii, J. Soc. Stat. Paris 92:262, MR18:834, Z4:67, [u]21:482,655, [w]1:9

7.3 NORMAL REGRESSION SLOPE

$$D(x) = \frac{[\sigma_1^2\sigma_2^2(1-\rho^2)]^{\frac{1}{2}(n-1)}\Gamma(\frac{1}{2}n)}{\sqrt{\pi}\,\Gamma(\frac{1}{2}(n-1)]\sigma_1^{n-2}(\sigma_2^2 - 2r\sigma_1\sigma_2 x+\sigma_1^2 x^2)^{\frac{1}{2}n}}$$ [1]402

$$v = \frac{1}{n-3}\ \frac{\sigma_2^2}{\sigma_1^2}\ (1-\rho^2)$$ [2]365, [e]1:432

For $\rho=0$, can use Student distribution to test x — [10]194

Stratified sampling — [i]36:96

7.4 CAUCHY (p,q)

$$D(x) = \frac{1}{\pi}\ \frac{p}{p^2 + (x-q)^2}$$ [1]246, [5]35, [18]

$$Ch(x) = \exp[\,qit - p|t|\,]$$ [5]60

q is the mode and median — [5]58

there is no mean, or any moment

Quartiles are $q \pm p$ — [5]67

$\bar{x}$ is not a consistent estimate of q — [5]105

$\bar{x}$ is a "density unbiased" estimate of q — [d]25:400

There are no sufficient estimators	[3]48
C.-R.(q), C.-R.(p), C.-R.(p/q)	[e]8:205
Dist. of t and F statistics	MR13:665
Mean and variance of $\frac{1}{2}$ (x+y), where x and y are respectively the kth values from the top and bottom of the sample	[1]373
$\frac{1}{2}$(x+y) is not a consistent estimate of q	

See also: [d]17:2, [d]21:133

7.5 CAUCHY (1,q)

$D(x) = \frac{1}{\pi} \frac{1}{1+(x-q)^2}$	[6]117
q incorrectly asserted to be the mean	[p]7:165
$E(x)$, $E(x^2)$, $D(x+y)$	[14]43
C.-R.(q) = 2/n	[1]490, [3]24, [p]7:159
$\text{Var}(\xi) \cong \frac{\pi^2}{4n}$	[3]6
$D(\bar{x}) = D(x)$, hence $\bar{x}$ not consistent	[3]2, [1]490, [u]22:702
$D(\xi)$	[3]46
MLE $\neq$ minimax	[16]64

MLE is solution of $\Sigma \frac{2(x_i-q)}{1+(x_i-q)^2} = 0$	[3]24, [p]7:169
Gauging	[e]15:194
There is no sufficient estimator	[9]6.16, [3]27, [p]7:162
There is no UMVUE	[3]51
Information and estimation	[e]8:315
Loss of information	[3]32
Testing $m = m_0$	[d]9:83, [d]13:65
Cauchys added	MR17:863

See also: [b]9:61, [i]20:61, [g]51:641, [d]28:832, [w]1:9.

7.6 CAUCHY (p,0)

$D(x) = \frac{p}{\pi} \frac{1}{p^2 + x^2}$, $Ch(x) = \exp -p\|t\|$	[9]275
Reproductive property	[9]276
Information and estimation	[e]8:316
Completeness	[e]10:314
$D(\overline{x}) = D(x)$	[n]10-3:91

Truncated to (-p,p)	[10]14

7.7 CAUCHY (1,0)

$D(x) = \frac{1}{\pi} \frac{1}{1 + x^2}$	[8]167
$Ch(x) = \exp - \lvert t \rvert$	[2]95, [8]167, [1]246, [9]243, [17]No. 27, [s]213:718, [i]5:133,
Sample median	[d]26:600
$D(\overline{x}) = D(x)$	[2]233,247, [w]1:73
From an example	[8]33, [9]242
Moments	[8]99
As distribution of ratio of two normal variables	[10]159
$C(x) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1} x$	
Censored sample	[c]41:230
Wrapped-up Cauchy	[d]26:245

<u>See also</u>: [d]22:425, [k]7:371, S.D. Poisson (1824) "Sur la Probabilité des résultats moyens des Observations", <u>Connaissance des Tems</u> ou Des Mouvements Célestes à l'usage des Astronomes et des Navigateurs, <u>pour l'an 1827</u>. Le Bureau des Longitudes, Paris. [d]22:418, MR3:232, MR9:235, MR5:124.

7.8 LAPLACE (m,v)

$D(x) = (2v)^{-\frac{1}{2}} \exp - \frac{\lvert x - m \rvert}{\frac{1}{2}(2v)^{\frac{1}{2}}}$	[1]247, [5]35,67 [18]
$Ch(x) = \exp(mit)\cdot(1 + \frac{1}{2}vt^2)^{-1}$	[5]62
Mean and variance of average of greatest and least sample values	[1]375
A priori distributions of m,v	MR9:294
"Best" estimates of m and σ are ξ and $1/n\Sigma \lvert x_i - \xi \rvert$, but ξ not sufficient	[5]147-8
Distribution of smallest sample value	[g]43:408
$D(\xi)$	[d]26:115
Quasi-range	[d]28:179
"Double exponential" distribution	Proc. Roy. Soc. Lond. (Series A) 154:124, [v]7:164
Convolution, estimation, generalization	Sobre la Primera Ley de Errores de Laplace, F. A. Sales Vallés, Thesis, Barcelona 1947.

LAPLACE (0,v), also called "Poisson's first law of error", $D(|\bar{x}|)$, $D(|x| - |y|)$, $D(\log |x| - \log |y|)$, $D(\Sigma x_1^2)$, D(GM), D(HM) [d]6:102

Laplace if $D(x)D(y) = \phi(|x| + |y|)$ MR10:125

See also: [n]10-3:80

LAPLACE (m,1), MLE [3]45

LAPLACE (0,1), [8]120, [9]279, [1]100

Sample Median [d]26:599

$\alpha_{2k} = (2k)!$ [d]5:32

See also: [d]22:425, [17]No. 38, "Laplace (1774), "Memoire sur la Probabilité des causes par les évènemens"

7.9 FISHER (p,q)

$$D(x) = \frac{2p^{\frac{1}{2}p}\, q^{\frac{1}{2}q}\, e^{px}}{B(\frac{1}{2}p,\frac{1}{2}q)(q+pe^{2x})^{\frac{1}{2}(p+q)}}$$ [1]243, [2]249, [14]48, MR8:161

'z' distribution [1]3:355

C(x) as a power series [1]3:360

Cumulants MR9:48,735

Moments, cumulants [y]5:317

Transform to Snedecor by $y = e^{2x}$; $x = \frac{1}{2} \log q\chi_1^2/p\chi_2^2$	[c]23:147
Various properties	[d]12:429, [c]34:173
$Ch(x) = \left(\frac{q}{p}\right)^{\frac{1}{2}it} \frac{\Gamma[\frac{1}{2}(q-it)]\Gamma[\frac{1}{2}(p+it)]}{\Gamma(\frac{1}{2}p)\ \Gamma(\frac{1}{2}q)}$	[3]116
$\sim C(x)$	[d]28:504
Obtained from two Chi-square variables	[2]249, [d]7:52, [g]26:173
Approximate significance levels, transformation to Normal	[d]11:93
Normal limit	Am. Math. Monthly, 50:100
Generalization	[i]34:58
Non-central	[o]7:57

<u>See also</u>: [e]2:423, [b]1:31, [a]94:284, [c]21:350, [c]34:352,359, [c]41:304, Am. Math. Monthly 50:100,382. Proc. Intl. Cong. Math. (1920)805, Current Science 1941, p. 191, [w]5:30, [p]6:183.

7.10 TYPE IV

$D(x) = C(1+x^2/a^2)^{-m} \exp(-p \tan^{-1} x/a)$ [11]69

$(x - h)$ Type IV [3]48

Various constants with an example [11]69

Roots of quadratic in Pearson equation complex [11]44

$\alpha_r = \frac{a}{2m-r-1} [(r-1)a\ \alpha_{r-2} - p\alpha_{r-1}]$ [2]86,140,144

$a=1$ [17]No. 52

See also: [d]7:21, [c]1:39, [c]3:312, [c]6:435, [c]7:74, [c]26:386

VIII. MISCELLANEOUS UNIVARIATE

8.1 PEARSON

$\frac{1}{D(x)} \frac{dD(x)}{dx} = \frac{a_0 + a_1 x}{b_0 + b_1 x + b_2 x^2}$, [d]2:394, [n]11-4:77

types listed with associated parameters

Differential equation for Ch(x) MR8:393

Ch(x) MR10:705

Original paper: Philosophical Trans., 1895.

First seven types treated in [11]

New classification	[d]7:16
$D(\bar{x})$	[d]18:111, [n]8-4:51
Truncation, estimation	[d]22:256, [c]40:50
Bivariate	MR9:363,452
Flexes equidistant from mode, etc.	[d]6:1
Bivariate generalization	[v]3:273
Orthogonal Polynomials	Ann. Soc. Cien. Argentina 155:3
Generalizations	[d]5:124, [c]26:129, (cf No. 8.52), MR10:386, MR17:1095, [d]21:289, J. Gakugei Tokushima Univ. Math. 5:29, [n]12-2:95
Log Pearson distributions	Intl. Cong. Math. (1950)1:580
Romanovsky's generalization	[c]17:106, [c]18:221

See also: [d]8:18, [d]8:206, [d]20:461, [e]6:415, [c]7:127, [c]16:106, [c]16:198, [c]18:264, [c]20:389, [g]26(P):288, [a]85:488, [c]35:113, [c]32:81, [c]36:151, [c]38:4, [i]25:141, MR17:169,272, Z9:314, Z6:268, Z19:73, MR14:755,977, Intl. Cong. Math. (1950) 1:585.

8.2 BESSEL FUNCTION

Mahalanobis' Distribution ("D" distribution)	[e]2:143,385, [e]3:105, [e]4:19,373, 535, [e]8:167, [k]8:379, [14]246, MR4:23
Wilk's distribution of dispersion determinant, etc.	[e]3:26
$D(\overline{x})$	MR5:42
Distribution of vector correlation	[c]28:353
D(x), Ch(x), Moments in a special case	MR14:775
Bivariate Gamma distribution	[e]5:140
Distribution of the range	[d]18:384, [d]21:133
Marginal total of Elfving's distribution	[c]36:142

See also: [e]6:175, [e]8:235, [a]97:125, [a]98:89, [b]16:96, [c]21:168, [c]24:441, [c]24:485,492, [d]18:392, [c]24:39, [c]24:293, [17]Nos. 10,60,61,62,63,64, with Ch(x), refs. [c]21:164, [c]24:293, [c]24:39, Several forms , J. Soc. Stat. Paris 96:262, [u]28:458, MR11:607, MR16:152

8.3 VARIANCE RATIO

$$D(x) = \frac{2(1-\rho^2)^{\frac{1}{2}(n-1)} x^{n-2} (1-4\rho^2 x^2 (1+x^2)^{-2})^{-\frac{1}{2}n}}{B[\frac{1}{2}(n-1), \frac{1}{2}(n-1)](1+x^2)^{n-1}}$$, Bose, [2]365, [e]2:65

For $\rho = 0$, $D[(2n-3)x^2]$ = Hotelling $(2n-2,n-1)$

See also: [p]6:183, [p]7:98, [c]30:190, [c]31:9

8.4 KULLBACH

$$D(x) = \frac{nx^{np-1}}{\Gamma(n)[\Gamma(p)]^2} \sum_{j=0}^{\infty} (-1)^{n+nj+1} \left(\frac{d^{n-1}}{dt^{n-1}} \frac{x^{nt}}{\Gamma(t+1)} \right)_{t=j}$$

Distribution of GM from Type III (1,p) [2]251

8.5 NONCENTRAL STUDENT

D(x)	[c]31:362, [18]1-162, [v]4:173,307
Multivariate	[v]4:331
Application	[c]43:219

See also: [i]36(Suppl.):21, [r]1:28, J. Soc. Stat. Paris 96:262, MR15:46

8.6 CONTINUOUS LEXIAN

$D(x) = \int_0^1 f(p)\binom{n}{x} p^x (1-p)^{n-x} dp$; parameters; [i]31:1, [i]34:197

if f(p) is Beta, D(x) is hypergeometric

8.7 NONCENTRAL SNEDECOR

D(x) [c]36:220, [18]1-163, [e]15:321

See also: [c]38:112, [i]36(Suppl.):33, [r]3:33

8.8 FISHER'S LOGARITHMIC SERIES

$D(x) = \dfrac{k^x}{p \log \frac{1}{1-k}}$, x=1,2,... [c]35:6,

(asserted to be multivariate) Cf. negative binomial (1,-m), No.3.5 [c]37:358

8.9 RANK VARIATE

$D(x) = \frac{N}{\sigma} \; \frac{n!}{(q-1)!(n-q)!} [\exp - \frac{x}{\sigma}(n-q+1)] (1-e^{-x/\sigma})^{q-1}$ [c]24:231, 239, [c]25:79

In special case called Yule's distribution, [c]42:23,425,
MLE, [c]43:248

A Distribution of Type III median [p]7:153

8.10 GENERALIZED PARETO

$D(x) = ax^{-n} \dfrac{1}{e^{b/x}-1}$ [1]6:184

b=1, n=5, a=15/π^4, Planck's radiation function [17]No. 56

8.11 GHOSH

$D(x) = \frac{2}{\Gamma([k] + 1)} x^{[k]} e^{-x^2}$, where $[k]$ is the largest integer $\leq k$.

Furnishes counterexample to theorem on similar regions [e]8:330

8.12 RECTANGULAR GEOMETRIC MEAN

$D(x) = \frac{n^n x^{n-1}}{a^n \Gamma(n)} (\log a/x)^{n-1}$ [2]246, [d]5:276, [w]1:73

8.13 CAUCHY MEDIAN

$D(x) = \frac{(2m + 1)!}{(m!)^2 \pi^{2m+1}} \left(\frac{\pi^2}{4} - [\tan^{-1}(x-k)]^2 \right)^m \frac{1}{1+(x-k)^2}$ [3]46

8.14 SPEARMAN'S RANK CORRELATION

$D(x)$ [c]30:256, [c]34:183, [c]38:131

$\sim D(x)$ = Type II, moments [c]40:409

8.15 CIRCULAR NORMAL CORRELATION

$D(x) = n(n-1) e^{-x^2/v} (1-e^{-x^2/2v})^{n-2} \frac{x}{\sigma}$ [c]39:139, [g]48:496

8.16 KOOPMAN

$D(x) = Q(k)\, R(x) \exp k\, H(x)$, most general distribution admitting a sufficient estimate of k	[3]24, [d]23:403, [c]36:71, Trans Am. Math. Soc. 39:399, [p]7:162

8.17 VON MISES DISTRIBUTION

$f(x) = C \exp k \cos (x-a)$	Physikal. Zeitschr 19:490
'Circular Normal'	[g]48:131, [g]49:53,268, [c]43:344, [d]26:233, [c]43:344

8.18

Family of distributions having all moments equal	[d]11:402

$$D(x) = \frac{1}{6} e^{-x^{\frac{1}{2}}} (1-p \sin x^{\frac{1}{4}}) \quad 0 \leq x < \infty, \; 0 \leq p \leq 1$$

$$\alpha_k = \frac{1}{6} (4k+3)!$$

8.19

Family of distributions having all moments equal	[d]11:402

$$D(x) = e^{-\frac{1}{4}} \pi^{-\frac{1}{2}} x^{-\log x} [1-p \sin(2\pi \log x)]$$

$$\alpha_k = \exp [\tfrac{1}{4} k(k+2)]$$

8.20

"Non-null t^2 distribution", involving a hyper-geometric function [14]48

8.21

$D(x) = \frac{\operatorname{sech}^{k-2} x}{B(1,k-2)}$, the distribution of $\tanh^{-1} r$ in samples from a bivariate normal distribution with zero means and zero correlation [b]15:213, [b]9:61

Ch(x), also special cases and refs [17]Nos. 53-5

$D(\Sigma x_i)$ for $k=3$ Z9:219

8.22

$D(x) = \frac{1}{2a} \operatorname{sech}^2 \left(\frac{x-m}{a}\right)$, connected with lognormal [e]12:122

Called 'logistic', $Ch(x) = \pi x \operatorname{sech} \pi x$ when $m=0$, $a=2$ [v]7:163

Distribution of t and F statistics MR13:665

8.23

Distribution of the correlation ratio, involving series [a]97:121, [c]24:441

8.24

Various distributions of the form exp (- quartic polynomial)	[d]4:1, [d]4:79, [d]19:589
Giving an example where no minimum variance estimator exists	[e]12:43
MLE	[c]31:188, [a]98:114

8.25

$k(1 + x^x)^{-m}$	[2]52
Ch(x)	[2]67

8.26

$D(x) = b \sin 2(a + bx)$	[b]9:61

8.27

Normal multiplied by an eighth degree polynomial	[a]106:361

8.28

$D(x) = (\frac{1}{4} h + \frac{1}{4} h^2 \lvert x \rvert)\, e^{-h\lvert x \rvert}$, $D(\bar{x})$	[n]10-3:90
For h=1, $Ch(x) = (\frac{1}{1+x^2})^2$	[n]10:75, [17]No. 39

8.29

Miscellaneous distributions given in terms of C(x) — [d]13:217, Math. Tables and Other Aids to Computation 5:109, [g]50:209

8.30

$D(x) = \frac{1}{2}k\ (1 + |x|)^{-k-1}$ — [1]225 No. 2,

k negative — [c]33:126

$k = 1+p$ — [c]36:93, [17] No. 17, generalization No. 18

8.31

$D(x) = k\ \exp\ (-ax^p)[1 + q\ \sin\ (bx^p)]$ — [2]106

8.32

Various distributions formed from rational functions of x, rational functions multiplied by $e^{-1/x}$, e^{-x} and $\exp(-\tan^{-1}x)$ — [d]1:137

8.33

$D(x) = (e^2 + |x|)^{-1}[\log\ (e^2 + |x|)]^{-2}$, having a pathologically long tail — [d]17:11

8.34 WEIBULL

$D(x) = ab\, x^{b-1} \exp(-ax^{b})$, $x \geq 0$, $a > 0$, $b > 1$ — [g]43:408, [17]No. 44

Moments of order statistics — [d]26:330

8.35

$D(x) = k \sin^{m}x \cos^{n}x$ — [c]30:182

n=0, value of k, Ch(cos x) — Philos. Magazine Ser. 7, 39:70

8.36

$D(x) = 2h\pi^{-1}(1 + h^2x^2)^{-2}$ — [n]10-3:77

8.37

$D(x) = 2h\pi^{-1}(e^{hx} + e^{-hx})^{-1}$, — [n]10-3:90,
$D(\bar{x})$ — [d]26:153, [v]7:159,
'PERKS' — J. Inst. Actuar. 63:12

8.38

$D(x) = (2\pi)^{-\frac{1}{2}} \frac{x}{k} [\exp(-\frac{1}{2}(x-k)^2) - \exp(-\frac{1}{2}(x+k)^2)]$ — [2]387

8.39

Four distributions formed by multiplying the normal distribution by a polynomial, used to illustrate kurtosis — [g]40:259

8.40 EXTREME VALUE

$D(x) = a \exp[-a(x-m)] \exp[-\exp(-a(x-m))]$ Gumbel Distribution	[d]17:299
Determination of constants	C.R. Acad. Sci. Paris 222:34
Estimation, MLE	[d]24:282,
Bias	[d]27:758
Moments called "Fisher-Tippett Type I", Ch(x), cumulants,	[18]1-144
For m=0, a=1, $Ch(x) = \Gamma(1-ix)$, References	[17]No. 43, [u]24:180,
Connection with No. 2.3	[v]4:8, [g]50:518, [g]42:408
Special cases	[r]1:4
~D(x)	[u]24:180, [g]43:403
With slight modification ~D(x) is distribution of log survival time	J. Hygiene 42:328
Application,	U.S. Dept. Agric. ARS 41:13, Ann. Inst. Henri Poincare 4:115, 5:115, J. de Physique Serie 7 Vol. 8, nos. 8,11, Bull. Am. Meteor. Soc, 23:95, C.R. Acad. Sci. Paris 246:49, 237: 512, Nature 175:270, Cong. Intl. Math.1936, 2:200, [a]99:732, [w]8:97 NBS Appl. Math. Ser. No. 33.

8.41

$D(x) = \frac{2}{\pi} \frac{x^2}{(1+x^2)^2}$, $Ch(x) = e^{-|x|}(1-|x|)$, [n]10:75, [17]No. 30

8.42

D(x+y), where x and y obey various trivial distributions [d]5:16

8.43

$D(x) = C\ x^{-1}(1 + p/x)^{-2}$ [d]6:106

8.44

$D(x) = \frac{a+1}{2a}\ (1-|x|^a)$, $-1<x<1$ [17]No. 15

8.45

$D(x) = \frac{\lambda^a}{\left(1 + \frac{c}{\lambda^b}\right)^n}\ e^{-\lambda x} x^{a-1} \sum_{j=0}^{n} \binom{n}{j} \frac{c^j\ x^{bj}}{\Gamma(bj+a)}$, [17]No. 36

$0<x<\infty$, $\lambda>0$, $c\geq 0$, $b\geq 0$, $a>0$, $n=1,2,\ldots$,

Ch(x), References

8.46

$D(x) = (x-k)x^n\ e^{-ax}$ [g]42:572

8.47 STEVENS-FISHER

$D(x) = \Sigma\ \binom{n}{j} (-1)^j (1-jx)^{n-1}$ [k]9:315, [k]10:14, [4]203

Compare No. 5.16 and No. 8.70

8.48

$D(x) = C \exp(ax^b - cx)$, $0<x<1$ [d]25:641

8.49

$D(x) = C \exp[-a(b - x)^{-c}]$ [d]25:645

8.50

$D(x) = \frac{a}{2\Gamma(1/a)} e^{-|x|^a}$, $-\infty<x<\infty$, $a>0$ [i]5:133, [g]26(Suppl.-H)227
[17]No.40

MLE [u]45:542

8.51

Distribution of non-normal correlation [c]38:224

8.52

$D(x) = C[\frac{p - x^2}{q + x^2}]^{\frac{m}{p+q}}$, $-\sqrt{p} < x < \sqrt{p}$ [17]No.31
Value of C, references

Hansmann's distributions, obtained from a generalized Pearson differential equation [c]26:129

8.53

$D(x) = (k/x) \exp[-ax - \frac{b}{x}]$ [17]No.48

Called "Type Harmonique" C.R. Acad Sci. Paris 213:634

8.54

$D(x) = \frac{a}{\sqrt{\pi}} e^{2a\sqrt{b}} \frac{1}{x^{3/2}} e^{-bx - \frac{a^2}{x}}$ [i]23:101

$Ch(x) = \exp [2a(\sqrt{b} - \sqrt{b-it})]$ [17]No.50

8.55

$D(x) = \frac{a^p |r|}{\Gamma(p)} x^{rp-1} e^{-ax^r}$, $0 < x < \infty$, $a>0$, $p>0$ [17]No.51

8.56

$D(x) = (a - 1)^2 \frac{\log x}{x^a}$, $1 < x < \infty$, $1<a$ [17]No.57

8.57

$D(x) = - [L(b^{1-a}) x^a \log x]^{-1}$, $b < x < \infty$, $b > 1$, $b > 1$, and [17]No.58

$$L(u) = \int_0^u \frac{dv}{\log v}, \quad u \geq 0$$

8.58

$D(x) = -(a + 1)^2 x^a \log x$, $0 < x < 1$, $a>-1$ [17]No.59

8.59

A generalization of the hypergeometric distribution based on the Whittaker function [17]No.65

$$x^{m+\frac{1}{2}} e^{-\frac{1}{2}x} \; {}_1F_1(m+\tfrac{1}{2}-k, 2m+x; x)$$

D(x), Ch(x), references

8.60

$D(x) = C \dfrac{e^{-(x^2/2a^2)}}{b^2 + x^2}$, moments, Ch(x), [v]2:293, [v]3:139

Limiting cases (Cauchy, Normal)

8.61

$D(x) = \frac{1}{\pi} \dfrac{1 - \cos x}{x^2}$ [v]2:328

8.62

D(x) where A + Bf(x) is (a) Normal, or (b) Laplace and f(x) is (i) log x, (ii) $\log\frac{x}{1-x}$, (iii) arcsinh x [c]36:149, [v]5:283

8.63

$$D(x) = \begin{cases} 1 - e^{-\mu NT}, & x = N \\ (1 - e^{-\mu T}) \exp[-\mu T(x-1)], & x = N+1, N+2, \ldots \end{cases}$$

$N = 1, 2, \ldots, \mu > 0, T > 0$

8.64

Garwood's distribution of length of gaps in traffic — [b]7:65, [g]46:117, [c]38:384

8.65 MATCHING

$$D(x) = \frac{1}{x!}\left[1 - \frac{1}{1!} + \frac{1}{2!} - \cdots + \frac{(-1)^{n-x}}{(n-x)!}\right]$$ [a]118:390, MR18:18:346,

Generalization — [4]210

8.66

Cigarette card distribution — [a]118:391

8.67

Cubic polynomial over a finite range, estimation — [g]50:196, [d]26:505,591

8.68

$$D(x) = \begin{cases} \frac{mn}{m+n} x^{m-1}, & 0 < x < 1 \\ \frac{mn}{m+n} x^{-n-1}, & 1 \leq x < \infty \end{cases}$$ [g]50:1137

Moments, etc.

8.69

$$D(x) = \begin{cases} mn\ (m-n)^{-1}\ x^{n-1}\ (1 - x^{m-n}), & m \neq n \\ n^2\ x^{n-1} \log(1/x), & m = n \end{cases}$$ [g]50:1142

Moments

8.70

$$D(x) = \frac{n^n}{(n-1)!} \sum_{j \leq nx} (-1)^j \binom{n}{j} \left(x - \frac{j}{n}\right)^{n-1},$$ [d]26:713

$0 \leq x \leq 1$

Compare No. 5.16 and 8.47

8.71

$$D(x) = \frac{(r-1)!\ (r-x)}{x!\ r^{r-x}}, \quad x = 0,1,\ldots,(r-1)$$ [b]19:339

Moments, approximations

8.72 ARFWEDSON

$$D(x) = \sum_{j=0}^{\infty} (x-j)^n (-1)^j \binom{x}{j}$$ [i]34:121

8.73 STEVENS - CRAIG

$D(x) = C\ n^{(x)}\ \sigma_s^x$, σ_s^x being Stirling's number of second kind [k]8:57, [c]40:173

Generalization [w]7:203.

8.74

$$D(x) = \frac{1}{4} + x^4, \quad -1 < x < 1$$ [m]6:120

8.75 ISING-STEVENS

$$D(x) = \frac{\binom{m-1}{x-1}\binom{m+1}{x}}{\binom{m+n}{m}}$$

Zeit. f. Physik 31:253, [k]9:10, [d]11:370, [t]4:171, [w]8:55,

8.76

$$D(x) = \frac{1}{\pi\sqrt{m^2-x^2}}$$

Am. of Math. 27:18

8.77

$$D(x) = \frac{x^{-2/3}}{3\sqrt{2\pi}}\, e^{-\frac{1}{2}(x^{1/3}-b)^2}$$

Ann. of Math. 27:19

8.78 NEGATIVE HYPERGEOMETRIC

$$D(x) = \binom{n}{x}\frac{B(p+x,q+n-x)}{B(p,q)}$$

[b]10:257, Proc. Intl. Stat. Conf. Rome 1953 paper 71.

Obtained by assuming binomial probability to obey Beta

8.79

$D(x) = k(1 \pm x)^{-2}$ over various ranges

[d]22:425

8.80

$D(x) = (n-1)(1 - \frac{1}{x})^{n-2}\frac{1}{x^2}, \; x \geq 1$

[d]22:425

8.81

$D(x) = \frac{-\log x^2}{\pi^2(1-x^2)}$, $-\infty < x < \infty$ [d]22:425

8.82

$D(x) = \frac{a^{-(a-bx)^2/2cx}}{\sqrt{2\pi cx^3}}$ Called 'inverse Gaussian'   Nature 155:453, [u]43:41, Virginia J. Sci.(New Series) 7:160

Written

$$D(x) = \frac{\exp\left(\frac{-c(x-m)^2}{2m^2x}\right)}{\left(\frac{c}{2\ \ x^3}\right)^{\frac{1}{2}}}$$

[d]28:362,696

$D(x)=k\ x^n e^{-x^2+ax}$, called "Halphen",   Publ. Inst. Statist. Univ. Paris 4:38

8.83

$D(x) = a(\alpha,\beta)/\ [\exp(\alpha^2x^2) - \beta]$ MR16:381

8.84

$D(x) = a \exp\ [-k^2 \log^2 \rho\ ]$ Z10:313

where

$$\rho = \frac{(x-x_0)(x_2-x_1)}{(x_2-x)(x_1-x_0)}$$

8.85

$Ch(x) = \frac{1}{\cosh t}, \frac{t}{\sinh t}, \frac{1}{\cosh^2 t}$ MR11:443

8.86

$D(x) = \frac{2\lambda\ (\lambda+1)}{(x+\lambda-1)(x+\lambda)(x+\lambda+1)}$ [e]18:353

$\lambda > 0$, $x=1,2,\ldots$, $m=1+\lambda$, $v = \infty$

8.87

$D(x) = \frac{N!n!}{\prod_{i=0}^{n} x_i!(i!)^{x_i}}$ [s]5:161

8.88

$D(x) = \frac{ce^{cx}}{(1+e^{cx})^2}$ [s]1:55, [s]3:133

Hyperbolic Error distribution

A discrete distribution from an urn model [d]22:452, [w]7:173

IX. MISCELLANEOUS BIVARIATE

9.1 CAUCHY BIVARIATE

$D(x,y) = \frac{1}{\pi} \frac{1}{(1+x^2+y^2)^2}$ [5]45

$D(x,y) = \frac{1}{2\pi} (1+x^2+y^2)^{-3/2}$ [w]8:235

9.2 STUDENT BIVARIATE

$D(x,y) = (2\pi)^{-1}(1-r^2)^{-\frac{1}{2}}\left(1 + \frac{x^2-2rxy+y^2}{n(1-r^2)}\right)^{-\frac{1}{2}(n+2)}$

Tables [c]22:408, [c]41:154

If x,y independent [3]92

9.3 POISSON BIVARIATE

Discussion	[2]136, [f]7:414, [c]39:196, Psych. Bull. 47:434, Proc. Edin. Math-Soc. IIs 4:18
Special case obtained from binomial	[i]17:98

9.4

Lognormal bivariate	[c]22:130, [d]4:30

9.5

Normal-lognormal	[d]4:30

9.6 BINOMIAL BIVARIATE

$Ch(x,y) = (a \exp(ix+iy) + be^{ix} + ce^{iy} + d)^n$ [2]133

If $a=0$, $D(x,y) = \frac{k!}{x!\ y!\ (k-x-y)!} p^x q^y (1-p-q)^{k-x-y}$, etc.

[i]17:92, [i]19:209

See also: [i]36:74, MR14:995, Z18:154, MR13:665

9.7 GAMMA BIVARIATE

$MGF(x,y) = [\ (1+x)(1-y) - xyr^2\ ]^{-p}$ [e]5:140, [c]25:158, MR3:171

9.8 GAMMA-NORMAL

$MGF(x,y) = (1-x)^{-\frac{1}{2}} \exp[\frac{1}{2} y^2 (1 + \frac{xr^2}{1-x})]$ [e]5:144, [c]25:132

9.9 HYPERGEOMETRIC BIVARIATE

$D(x,y) = \frac{\binom{a}{x}\binom{b}{y}\binom{c}{k-x-y}}{\binom{a+b+c}{k}}$, various properties [i]17:104, [c]16:172, [c]22:140

Moments Ganita 5:97, Koninkl. Nederl. Akad (A) 60:124

9.10 NEGATIVE BINOMIAL BIVARIATE

Various properties [i]17:100

$D(x,y) = \frac{p^p}{(p+2m)^p} \frac{1}{\Gamma(p)} \frac{\Gamma(x+y+p)}{\Gamma(x+1)\Gamma(y+1)} [\frac{m}{p+2m}]^{x+y}$ [c]41:79

correlation $= \frac{m}{p+m}$, regression etc.

Polya-Eggenberger MR11:605

9.11 ELFVING

$D(x,y) = \frac{1}{2} x \exp(-x \cosh y)$, connected with $\sim$D(range) [c]34:111, [c]36:142

9.12

$D(x,y) = C e^{-ax - by} (1-x+y)^p (1+x-y)^q$ [c]14:355

Rhodes surface [c]22:134, [c]41:550

9.13

$$D(x,y) = (1+x/a)^m(1+y/b)^n(1 - \frac{x+y}{c})^q,$$

Filon-Isserlis surface [c]15:222, [c]16:180

9.14

$$D(x,y) = (xy)^k(x-y)[(1-x)(1-y)]^n$$ [c]31:226, [k]9:245

9.15

$$D(x,y) = \frac{x^{n-2}(1-y^2)^{\frac{1}{2}(n-4)}}{(1-2rxy+y^2)^{n-1}}$$ [2]365

9.16

$$D(x,y) = n^{-2} \frac{n!}{[(k-1)!]^2(n-2k)!} (x/n)^{k-1}(y/n)^{k-1}(1-x/n-y/n)^{n-2k}$$

$x > 0, \quad y > 0, \quad x+y<n, \quad 2k < n$

As $n \rightarrow \infty$, $x,y \rightarrow$ independent Type III(1,k)

Special case, k=0 [d]7:149

9.17

$$D(x,y) = C\, x^{\frac{1}{2}(n-3)}(y-x)^{\frac{1}{2}(k-n-2)}\exp -\tfrac{1}{2}(k-1)y,$$

D(x/y) = Beta [b]1:213

9.18

Uniform bivariate, triangular bivariate [c]24:382, [v]5:322

9.19

Gram-Charlier bivariate [c]36:177

9.20

The fifteen constant surface, (quartic polynomial) $\cdot\ e^{-Q}$ [c]17:268

9.21

Pearson's Student-like surfaces [c]15:234, [c]18:229, [c]22:137

9.22

$D(x,y) = x + y$, $D(x + y)$ [8]94

9.23

Normal-negative binomial [k]13:289

9.24

Edgeworth surface [c]38:220, [c]17:314

9.25

Raleigh bivariate: Electrical Engineering November 1954, p. 1004

9.26

Discussion of 'possible' bivariate distributions, Narumi's system [c]15:77,209,222

Generalization [c]22:109

9.27 VON MISES-FISHER DISTRIBUTION

Generalization of No. 8.17 — Proc. Roy. Soc. Lond. Ser. A 217:295, [c]43:344

9.28

Beta Bivariate — [v]2:261

9.29

$$D(x,y) = k \frac{e^{-\frac{1}{2}(ax^2+2bxy+cy^2)}}{m^2 + (ax^2+2bxy+cy^2)}$$ [v]3:153

9.30

$$D(x,y) = k\,[1-a^2x^2 - b^2y^2 + 2abrxy]^n ,$$ [v]3:273

$n+1>0, \quad r^2<1,$

9.31

$$D(x,y) = k \exp[-Q(x,y)] \cdot [h^2+Q(x,y)]^n$$ [v]3:273, [v]5:323

9.32

Defined over (0,0)(0,1)(1,0) from urn model — [v]3:328

9.33

'Correlation by common factor' surface — [c]24:288

9.34

Johnson's system; ten surfaces obtained by Translation — [c]36:297

9.35

Nine surfaces with Pearson or Bessel marginal distributions — MR5:126

9.36

$D(x,y) = C[(x-1)!(h-x)!(y-1)!(k-y)!]^{-1}$ — Hoel, Intro. to Math. Stat. 180

9.37

Type III bivariate, with discussion and calculation of D(r) — [e]7:159

9.38

$D(x,y) = \frac{1}{4}(1+kxy),\ |k| \leq 1,\ -1 \leq x,\ y \leq 1$ — [w]8:234

X. MISCELLANEOUS MULTIVARIATE

10.1 WISHART TRIVARIATE

$$D(x,y,z) = \frac{n^{n-1}(xy-z^2)^{\frac{1}{2}(n-4)}}{4\pi\Gamma(n-2)M^{\frac{1}{2}(n-1)}} \exp\left(-\frac{n}{2M}(v_2x-2\mu z+v_1y)\right),$$

$\mu = \rho\sigma_1\sigma_2,\ M = v_1v_2(1-\rho^2)$ — [1]397, [3]330 [4]226

$$Ch(x,y,z) = \left(\frac{A}{A^*}\right)^{\frac{1}{2}(n-1)}$$

where $A = \begin{vmatrix} \frac{nv_2}{2M} & \frac{\mu n}{2M} \\ -\frac{\mu n}{2M} & \frac{v_1 n}{2M} \end{vmatrix}$ and $A^* = \begin{vmatrix} \frac{nv_2}{2M} - is & \frac{\mu n}{2M} - it \\ -\frac{\mu n}{2M} - it & \frac{v_1 n}{2M} - iu \end{vmatrix}$

Moments and cumulants [3]334

As distribution of normal bivariate variance-covariance [c]10:510, [c]21:164, [c]27:230

<u>See also</u>: [k]9:243, [u]44:295, J. Soc. Stat. Paris 96:262, [u]29:264, [a]92:580

10.2 WISHART MULTIVARIATE

$$D(x_{ij}) = K_{kn} A^{(n-1)} X^{(n-k-2)} \exp(-\Sigma a_{ij} x_{ij}),$$

where $X = |x_{ij}|$, $A = |a_{ij}|$ and

$$K_{kn} = \pi^{\frac{1}{4}k(k-1)} [\Gamma(\tfrac{n-1}{2}) \cdot \ldots \cdot \Gamma(\tfrac{n-k}{2})]^{-1}$$

[4]226, [3]331, [1]391-4, [14]66, [i]30:151, [u]29:260, 271

$$Ch(x_{ij}) = \left(\frac{A}{A^*}\right)^{\frac{1}{2}(n-1)},$$

where $A^* = |a_{ij} - i\epsilon_{ij} t_{ij}|$ and $\epsilon_{ij} = \begin{cases} 1, & i=j \\ \frac{1}{2}, & i \neq j \end{cases}$

Reproductive property	[4]232
Various properties	[c]20:32, [i]24:185
Non-central Wishart	Proc. Roy. Soc. Lond. Ser. A. 229:364

See also: [d]3:197, [d]15:345, [d]17:409, [d]19:262, [e]3:25, [a]97:120, [c]24:476, [c]36:59, [k]9:244, [c]38:470, [i]36:17, [u]35:336, MR10:387, [b]17:79.

10.3 MULTINOMIAL

$D(x_1,\ldots,x_k) = \frac{n!}{\Pi\,(x_i!)}\,\Pi(p_i x_i)$	[6]58, [2]290, [7]124, [18]1-160
$MGF(x_1,\ldots,x_k) = (p_1e^{x_1} + \ldots + p_ke^{x_k})^n$	[4]51
$E(x_i) = np_i$, $Var(x_i) = np_i(1-p_i)$	[4]52, [14]35
Moments	Bull. Amer. Math. Soc. 41:857
Introductory article with applications	[15]36
PGF	[18]1-146
Chi-square test	[e]13:2, [c]36:118, [15]739
Information and estimation	[e]8:325

MLE	[e]18:139
Distinguishing between two multinomials, asymptotic form	[e]7:401
Trivariate	[7]146, [d]21:420
Bivariate multinomial	[d]23:547, Rev. da Fac. de Ciencias de Lisboa 2 Serie (A) 2:197

See also: [d]8:127, [d]21:416, [e]11:367, [d]25:772, [d]28:861, [f]13:451, [t]2:84, Am. Math. Monthly 53:59, Koninkl. Nederl. Akad. (A) 60:121, Z8:122, MR17:56, MR16:839, MR13:665.

10.4 TYPE X MULTIVARIATE

$D(x_1,\ldots,x_n) = C\,e^{-x/b}$ where $x = \Sigma x_i^2$ [c]41:54

10.5

Gamma multivariate [e]11:45

10.6 STUDENT MULTIVARIATE

$$D(x_1,\ldots,x_p) = \frac{A^{\frac{1}{2}}\Gamma[\frac{1}{2}(n+p)]}{(n\pi)^{\frac{1}{2}p}\Gamma(\frac{1}{2}n)}\left[1 + \frac{1}{n}\Sigma a_{ij}x_i x_j\right]^{-\frac{1}{2}(n+p)},$$

Student for p=1 [c]41:153, MR16:602

See also: [w]9:143

10.7 CAUCHY MULTIVARIATE

$D(x_1,\ldots,x_n) = C(a^2+x^2)^{-\frac{1}{2}s-\frac{1}{2}}$, where $x^2 = \Sigma x_i^2$ [c]41:54, MR16:51

$D(x_1,\ldots,x_n) = C_n(1 + \sum_{j=1}^{n} x_y^2)^{-\left(\frac{n+1}{2}\right)}$ [w]8:235

10.8 SPHERICAL

$D(x_1,\ldots,x_n) = (2\pi)^{-\frac{1}{2}n}\, r^{-\frac{1}{2}n+1} \int_0^\infty \rho^{\frac{1}{2}n} J_{\frac{1}{2}n-1}(r\rho)Ch(\rho)d\rho,$

where $r = \sqrt{\Sigma x_i^2}$, $\rho = \sqrt{\Sigma t_i^2}$ [c]41:45

10.9 POISSON MULTIVARIATE

Derivation [e]11:120, [d]28:466, [i]37:1

Without correlation, Multiple Poisson

$D(x_1,\ldots,x_n) = \exp - (k_k+\ldots+k_r) \dfrac{k_1^{x_1}\cdot\ldots\cdot k_n^{x_n}}{x_1!\cdot\ldots\cdot x_n!}$ [7]127, [e]19:210, Z12:113,410

10.10 BINOMIAL MULTIVARIATE

$MGF(x_1,\ldots,x_n) = (1 + \Sigma p_i x_i + \Sigma p_{ij} x_i x_j + \ldots)^N$ [e]11:119, Z12:113, 410

D(x), etc., in special case [i]18:271

10.11

Negative binomial multivariate [i]18:274, [i]19:211, Koninkl. Nederl. Akad. (A)60:121

10.12

Multinomial multivariate	[c]36:47

10.13

Hotelling Multivariate	[k]9:258, [x]7:70

10.14

Multivariate distributions obtained from the Normal multivariate	[i]27:235, [i]28:20

10.15

Generalization of No. 9.14	[k]9:245

10.16

Generalization of No. 8.3	[k]11:136

10.17

Generalization of No. 8.60, No. 9.29	[v]3:153

10.18

Hypergeometric multivariate	[e]15:391, [f]13:488, MR17:634, MR12:722

10.19

Gram-Charlier multivariate	MR14:486

10.20 RUN LENGTH

D(x)	[d]11:367, [4]202,206, [s]5:143

10.21 BETA MULTIVARIATE

Tolerance limits	[4]94

TABLE 1

Journals

[a]	Journal of the Royal Statistical Society, Series A
[b]	Journal of the Royal Statistical Society, Series B
[c]	Biometrika
[d]	Annals of Mathematical Statistics
[e]	Sankhyā
[f]	Biometrics
[g]	Journal of the American Statistical Association
[h]	Nordisk Statistisk Tidskrift
[h']	Nordic Statistical Journal
[i]	Skandinavisk Aktuarietidskrift
[j]	Bell System Technical Journal
[k]	Annals of Eugenics, Annals of Human Genetics
[l]	Econometrica
[m]	Applied Statistics
[n]	Metron
[o]	Annals of the Institute of Statistical Mathematics
[p]	Journal of the Institute of Actuaries Students' Society
[q]	Bulletin of Mathematical Statistics
[r]	Reports of Statistical Application Research, Union of Japanese Scientists and Engineers
[s]	Statistica (Neerlandica)
[t]	Calcutta Statistical Association Bulletin
[u]	Proceedings of the Cambridge Philosophical Society
[v]	Trabajos de Estadistica
[w]	Mitteilungsblatt fur Mathematische Statistik
[x]	Journal of the Indian Society of Agricultural Statistics
[y]	Revue de l'Institut International de Statistique

TABLE 2

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TABLE 3

Chronological

Year	[a]	[b]	[c]	[d]	[e]	[f]	[g]	[h] [h']	[i]
1916	79		11				15		
1917	80		11				15		
1918	81		12				16		1
1919	82		12				16		2
1920	83		13				17		3
1921	84		13				17		4
1922	85		14				18	1	5
1923	86		14-15				18	2	6
1924	87		16				19	3	7
1925	88		17				20	4	8
1926	89		18				21	5	9
1927	90		19				22	6	10
1928	91		20A,B				23	7	11
1929	92		21				24	1	12
1930	93		22	1			25	2	13
1931	94		23	2			26	3	14
1932	95		24	3			27	4	15
1933	96		25	4	1		28		16
1934	97	1	26	5	1		29		17
1935	98	2	27	6	2		30		18
1936	99	3	28	7	2		31		19
1937	100	4	29	8	3		32		20
1938	101	5	30	9	3-4		33		21
1939	102	6	31	10	4		34		22
1940	103	7	32	11	4-5		35		23
1941	104	7	32	12	5		36		24
1942	105		32	13	6		37		25
1943	106		33	14	6		38		26
1944	107		33	15	6		39		27
1945	108		33	16	7	1	40		28
1946	109	8	33	17	7	2	41		29
1947	110	9	34	18	8	3	42		30
1948	111	10	35	19	9	4	43		31
1949	112	11	36	20	9	5	44		32
1950	113	12	37	21	10	6	45		33
1951	114	13	38	22	11	7	46		34
1952	115	14	39	23	12	8	47		35
1953	116	15	40	24	12-13	9	48		36
1954	117	16	41	25	13-14	10	49		37
1955	118	17	42	26	14-16	11	50		38
1956	119	18	43	27	17	12	51		39
1957	120	19	44	28	18	13	52		40

TABLE 3 - Chronological (Continued)

Year	[j]	[k]	[l]	[m]	[n]	[o]	[p]	[q]	[r]
1916									
1917									
1918									
1919									
1920					1				
1921					1				
1922	1				2		2		
1923	2				3				
1924	3				3-4		2		
1925	4	1			4-5				
1926	5	1			6		2		
1927	6	2			7		3		
1928	7	3			7		3		
1929	8				8		3		
1930	9	4			8		3		
1931	10	4			9		3		
1932	11	5			9-10		4		
1933	12	5	1		10-11		4		
1934	13	6	2		11-12		4		
1935	14	6	3		12		4		
1936	15	7	4				4		
1937	16	7-8	5				5		
1938	17	8	6				5		
1939	18	9	7				5		
1940	19	10	8		14				
1941	20	11	9		14				
1942	21	11	10						
1943	22	12	11						
1944	23	12	12						
1945	24	12	13				5		
1946	25	13	14				6		
1947	26	13-14	15				6-7		
1948	27	14	16				7-8		
1949	28	14	17		15	1			
1950	29	15	18			2		4	
1951	30	15-16	19		16	3			1
1952	31	16-17	20	1		4		5	1-2
1953	32	17-18	21	2	17	5		5	2-3
1954	33	18-19	22	3		6			3
1955	34	19-20	23	4		6-7		6	4
1956	35	20-21	24	5		7-8		6-7	4
1957	36	21	25	6	18	8-9		7	4

TABLE 3 - Chronological (Continued)

Year	[s]	[t]	[u]	[v]	[w]	[x]	[y]	MR	[z]
1916			18						
1917									
1918									
1919									
1920			19						
1921			20						
1922									
1923			21						
1924									
1925			22						
1926									
1927			23						
1928			24						
1929			25						
1930			26						
1931			27						1-2
1932			28						2-4
1933			29				1		5-7
1934			30				2		8-10
1935			31				3		10-12
1936			32				4		12-15
1937			33				5		15-17
1938			34				6		17-19
1939			35				7		19-21
1940			36				8	1	21-23
1941			37					2	23-24
1942			38					3	
1943			39					4	
1944			40				12	5	
1945			41				13	6	
1946			42				14	7	
1947		1	43				15	8	
1948		1	44			1	16	9	
1949	3	2	45		1		17	10	
1950	4	2-3	46	1	2	2	18	11	
1951	5	3-4	47	2	3	3	19	12	
1952	6	4	48	3	4	4	20	13	
1953	7	4-5	49	4	5	5	21	14	
1954	8	5	50	5	6	6	22	15	
1955	9	6	51	6	7	7	23	16	
1956	10	6-7	52	7	8	8	24	17	
1957	11	7	53	8	9		25	18	

Index

U. S. DEPARTMENT OF COMMERCE
Sinclair Weeks, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major laboratories in Boulder, Colo., is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside front cover.

WASHINGTON, D. C.

Electricity and Electronics. Resistance and Reactance. Electron Devices. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology. Engine Fuels. Free Radicals Research.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Neutron Physics. Nuclear Physics. Radioactivity. X-rays. Betatron. Nucleonic Instrumentation. Radiological Equipment.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Molecular Structure and Properties of Gases. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

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Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Concreting Materials. Constitution and Microstructure.

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Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analog Systems. Application Engineering.

- Office of Basic Instrumentation.
- Office of Weights and Measures.

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships. VHF Research.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Systems. Navigation Systems. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Radio Systems Application Engineering. Radio Meteorology.

Radio Standards. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Calibration Center. Microwave Physics. Microwave Circuit Standards.

NBS