PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION

A Quarterly Report

January through March 1958
THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major field laboratories in Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside back cover of this report.

WASHINGTON, D. C.


• Office of Basic Instrumentation

BOULDER, COLORADO


NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT

11.0

NBS REPORT

5907

PROJECTS and PUBLICATIONS

of the

APPLIED MATHEMATICS DIVISION

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January 1 through March 31, 1958

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*Only unclassified projects are included in this report.
Status of Projects

March 31, 1958

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-12-1104/55-55

Origin: NBS
Manager: P. Davis
Full task description: July-Sept 1954 issue, p. 1

Authorized 8/29/54

Status: CONTINUED. P. Davis investigated the numerical integration of periodic analytic functions by means of the trapezoidal and Gaussian rules. The following conclusions were reached. For the class of analytic periodic functions which are entire and which are regular in a sufficiently large region of the complex plane which includes the interval of integration in its interior, the trapezoidal rule is (asymptotically) better than the Gaussian rule. For analytic periodic functions which are regular on the interval of integration but which possess isolated singularities of a certain type sufficiently close to the interval, the Gaussian rule is (asymptotically) better than the trapezoidal rule. This latter family of functions includes, among others, the Poisson kernel for sufficiently large values of its parameter. These results will be presented at the Madison, Wisconsin Symposium on Numerical Approximation.

A. J. Goldman is revising the manuscript of his joint paper with J. J. Stone, "Two Continuous Poker Games."

W. Rheinboldt has been investigating the topological foundation of asymptotic convergence and comparing it with the concept of convergence defined by a norm. The problem of a unified theory of general asymptotic expansion is considered, and investigation is underway to determine which operations (defined on the function space in question) leave the connection between a function and its asymptotic series invariant.

N. Bazley has been making calculations of lower bounds for eigenvalues. He has discovered how to apply Aronszajn's theory to symmetric operators with a perturbation term. He is using this method to find lower bounds for ground and simply excited energy levels of He and Li⁺. He is writing a code for the 704 to evaluate the integrals which arise in these calculations. Also, he is applying the Weinstein-Aronszajn methods for lower bound calculations to a clamped square plate with equal loading on two opposite faces.
Status of Projects

Publications:

(5) Reducible linear differential systems. H. A. Antosiewicz. In manuscript.
(13) On the convergence of the Rayleigh quotient iteration for the computation of the characteristic roots and vectors, II. A. M. Ostrowski. Submitted to a technical journal.
(14) On the bounds of a one-parametric family of matrices. A. Ostrowski. In manuscript.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-12-5116/55-56

Origin: NBS
Sponsor: Office of Naval Research
Manager: M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. M. Pearl has been working on maximal non-singular vector spaces in total matrix algebras.

K. Goldberg, D. Mesner, and E. C. Dade have examined ways of identifying incidence algebras generated by latin squares with incidence algebras generated by permutation groups.

A seminar on Hypercomplex Numbers is being conducted by M. Newman.
M. Pearl has delivered three lectures on matrix representations of algebras and algebras over the real numbers; M. Newman two lectures on the number theory of quaternions; and A. Goldman two lectures on linear spaces and tensor algebras. The lectures will be continued through May 1958.

M. Newman has completed a set of double length integer subroutines for the 704. These are written to perform modular arithmetic as well. A code to compute the Hermite normal form of an integral mxn matrix using these subroutines has also been completed.

M. Newman is continuing work on the existence of identities for the coefficients of modular forms. A manuscript has been prepared.

A study of congruence properties of the coefficients of the modular invariant $j(T)$ modulo 13 has been completed and a manuscript prepared. Divisibility properties of the partition function are now being studied.

Publications:
(1) Incidence algebras. E. C. Dade and K. Goldberg. In manuscript.
(3) Abelian groups of unimodular matrices. E. C. Dade. Submitted to a technical journal.
(5) Dense subgraphs and connectivity. R. E. Nettleton (NBS 3.2), K. Goldberg, and M. S. Green (NBS 3.2). Submitted to a technical journal.
(8) A further extension of Cayley's parameterization. M. Pearl. Submitted to a technical journal.
(9) On a converse of a theorem of Pringsheim. P. Davis. Submitted to a technical journal.
Status of Projects

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-12-5116/56-148

Origin and Sponsor: National Institutes of Health  Authorized 9/30/55
Managers: H. A. Antosiewicz, W. Gautschi (11.2)
Full task description: July-Sept 1955 issue, p. 7

Status: CONTINUED. For the one-dimensional case, calculations are continuing and results are being transmitted to the sponsor as they are computed.

The sponsors have submitted a new set of differential equations which refer to a different type of nerve under conditions in which voltage, current and other variables are functions of the distance along the nerve. Characteristic for the nerve under consideration is the presence of nodes which are located at equidistant intervals. It is assumed that the functioning of the nerve is governed by Hodgkin-Huxley's differential equations, to hold at the nodes, and by the usual equations of a passive cable to hold in between the nodes.

The differential equation for the internodes therefore is a parabolic equation of the form

$$\frac{\partial V}{\partial t} = \frac{1}{r^2 c} \frac{\partial^2 V}{\partial x^2} - \frac{V}{r o^c} \quad (r_o, r_2, c \text{ constants}),$$

and the problem consists in finding a continuous solution $V$ subject to certain boundary conditions at the nodes and initial conditions for $t=0$. If the nodes are located at $x=jL$ ($j=0,1,2,...; L > 0$) the boundary condition at the $j$-th node ($j \neq 0$) is of the form

$$C \frac{\partial V}{\partial t} - \frac{1}{r_2} \left\{ \frac{\partial V}{\partial x} \bigg|_{jL+0} - \frac{\partial V}{\partial x} \bigg|_{jL-0} \right\} + G(m,n,h;V) = 0,$$

where $G$ is a certain function, linear in $V$ and nonlinear in $m,n,h$, which already occurred in the previous version of the problem; $m,n$ and $h$ are node functions which are related to $V$ by the same system of differential equations as used previously. A similar boundary condition holds at the first node ($j=0$).

The initial conditions for $t=0$ are

$$V = 0 \quad (0 \leq x < \infty),$$

$$C \frac{\partial V}{\partial t} + G(m_o,n_o,h_o;V) = 0 \quad (x = jL, j \geq 0),$$

where $m_o,n_o,h_o$ are given initial values for $m,n,h$. 
To solve the problem numerically, all differential equations are replaced by explicit finite difference equations. Coding of the problem is in progress.

EVALUATION OF MATRIX COMPUTATION PROGRAM
Task 1101-12-5116/57-200

Origin: NBS
Sponsor: David Taylor Model Basin
Manager: M. Newman
Full task description: Oct-Dec 1957 issue, p. 4

Status: CONTINUED. A program has been planned for determining the characteristic roots of the matrices which were previously inverted; then comparison will be made between the observed results and the theoretical condition number for each matrix. The purpose of the study is to ascertain whether or not the condition number affects the determination of the characteristic roots in the same way that it does the determination of the inverse.
2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF $E_1(z)$, ($z = x+iy$)
Task 1102-40-1110/43-3

Manager: I. Stegun
Full task description: Apr-June 1949 issue, p. 41

Status: CONTINUED. Printing is under way; proofs were received.

Publication:
(1) Table of the exponential integral for complex arguments. Applied Mathematics Series 51. In press.

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-40-1110/47-2

Origin: NBS
Manager: M. Abramowitz
Full task description: Apr-June 1949 issue, p. 45

Status: CONTINUED. A 704 program for computing the regular and irregular solutions for all values of $\gamma$, $\rho$ and $L$ is now being checked.

TABLES OF POWER POINTS OF ANALYSIS-OF-VARIANCE TESTS
Task 1102-40-1110/51-8

Origin: Section 11.3, NBS
Manager: S. Peavy
Full task description: Apr-June 1951 issue, p. 49

Status: INACTIVE.
Status of Projects

REVISION OF MATHEMATICAL TABLES
Task 1102-40-1110/52-7

Origin: NBS
Managers: W. F. Cahill, I. Stegun
Full task description: July-Sept 1951 issue, p. 41

Status: CONTINUED. Printing of the "Table of Natural Logarithms for Arguments from Five to Ten to Sixteen Decimal Places" is under way, for reissue in the Bureau's Applied Mathematics Series. In the past this table was known as Volume IV of "Tables of Natural Logarithms," prepared by the New York Mathematical Tables Project, and was designated as MT12.

Publication:
(1) Table of natural logarithms for arguments from five to ten to sixteen decimal places. Applied Mathematics Series 53. In press.

TABLE OF THE MODIFIED AIRY INTEGRAL
Task 1102-40-1110/52-23

Origin: NBS
Manager: I. Stegun
Full task description: July-Sept 1951 issue, p. 42

Status: CONTINUED. Printing is under way; proofs were received.

Publication:

SPHEROIDAL WAVE FUNCTIONS
Task 1102-40-1110/52-37

Origin: NBS
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE.
Status of Projects

SIEVERT'S INTEGRAL
Task 1102-40-1110/52-57

Origin: NBS
Authorized 2/12/52
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46

Status: CONTINUED. The introduction to the tables is being revised. The tabular material has been prepared for photo-offset.

HEAT TRANSFER
Task 1102-40-1110/57-241
(formerly 3711-60-0009/57-241)

Origin: NBS
Authorized 6/30/57
Managers: M. Abramowitz, W. F. Cahill
Full task description: Apr-June 1957 issue, p. 31

Status: CONTINUED. The coefficients $A_n$, eigenvalues $z_n$, and eigenfunctions $y(r,z_n)$ for the expansion

$$\frac{\theta - \theta_1}{\theta - \theta_1} = \sum_{n=1}^k A_n e^{z_n^2} y(r,z_n)$$

have been computed for $\lambda = 1,10,1000; k = 5, 8$.

The results, where possible, were compared with those given in "Heat Transfer to Hagen-Poiseille Flows," by Knox Millsapps and Karl Pohlhausen, and with those contained in a private communication from S. N. Singh of the Indian Institute of Technology. These comparisons gave excellent agreement.

Publication:
(1) Heat transfer in laminar flow through a tube. M. Abramowitz, W. F. Cahill and C. Wade. In manuscript.

MATHEMATICAL SUBROUTINES
Task 3711-60-0009/56-160

Origin: NBS
Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13
Status: CONTINUED. Alfred Beam has written and submitted to SHARE, the following 704 subroutine: (1) Floating point integration by Gaussian quadrature; (2) Floating point integration by Laguerre quadrature; (3) Floating point integration by Hermite quadrature, and (4) Double precision floating point Arc Tangent subroutine.

The orthonormalization code (see July-Sept 1957 issue, p. 11) is being modified by P. J. Walsh, and a description is being prepared for SHARE.

A subroutine has been written by J. D. Waggoner to compute $XAX' = B$, where $X$ is an arbitrary $m \times n$ matrix and $A$ and $B$ are symmetric matrices stored in lower triangular form (to be submitted to SHARE).

A general purpose control program has been written and is now being checked out by A. Beam. This program is designed for use in problems involving large amounts of data and many sub-programs which will not fit into the core simultaneously and must be used repeatedly. It reads any subprogram from tape into a specified location in the memory, reads in data (a whole item at a time or any part of an item) from tape to a location determined by the control program, allocates working space in core for data to be computed, and when the computation is finished can, if desired, store this data on tape with an identification tag so that it can be called back in later. It also arranges for data or subprograms to be written over if they are not needed at a specified time in the program.

A preliminary study is being made by E. Haynsworth on new methods of finding eigenvalues of matrices, with particular reference to two recent papers by A. Brauer. These methods are to be adapted for use on the computer.

AUTOMATIC CODING
Task 3711-60-1120/55-65

Origin: NBS
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. The following service routines and subroutines were presented to the SHARE organization:
(1) Hermite Quadrature Integration, by A. Beam
(2) Laguerre Quadrature Integration, by A. Beam
(3) Gaussian Quadrature Integration, by A. Beam
(4) Double Precision Arctangent Instruction, by A. Beam
(5) Binary to BCD Conversion of Unrestricted Integers, by V. E. Henriques and Genevie Urban (12.5)
(6) BCD to Binary Conversion of Unrestricted Integers, by V. E. Henriques and Genevie Urban (12.5)

Presentations of the CORBIE system were given by J. H. Wegstein at the Computation Center Seminar, M.I.T; at a Symposium on Recent Advances
in Programming Methods, Ohio State University; at a Seminar on Programming, Westinghouse Co., Baltimore; and at a Seminar for Coders, Aberdeen Proving Ground, Md.

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-5113/57-216

Origin and Sponsor: National Science Foundation
Manager: M. Abramowitz
Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. The following chapters have been completed and submitted to the Bureau's Editorial Committee for review:
Chapter 1. Mathematical constants
3. Powers and roots
4. Elementary transcendental functions
5. The exponential integral and related functions
6. The gamma functions and related functions
7. The error functions and related functions
12. The Struve function and related functions
23. Bernoulli and Euler polynomials

Typing of the tables has been completed for chapters 1 and 3 and is in process for chapters 4, 5 and 6. Work has started on the drafting of the graphs that are included in the above chapters. Final revision of the text of most of the other chapters is in process.

Chapter 5 The Exponential Integral and Related Functions. Tabulation of the integrals \( Ei(x), \ -Ei(-x), \) and \( E_n(x) \) has led to the decision to employ the notation \( E_i(x) \) for \(-Ei(-x)\) and discard the latter notation. Thus the notation \( E_i(x) \) will be consistent with that for \( E_n(x) \). Also, since \( Ei(x) \) is the principal value of the integral

\[
\int_{-\infty}^{x} e^{t-1} dt \quad \text{for} \quad x > 0 \quad \text{and}
\]

\[
Ei(x) = \frac{1}{2} \left[ -E_1(-x+i0) - E_1(-x-i0) \right] \quad \text{for} \quad x > 0,
\]

any ambiguity in using \( Ei(x) \) and \(-Ei(-x)\) is avoided.

A draft of a paper by W. Gautschi, "Evaluation of higher functions by recurrence," has been written. This report which is concerned with the stability of first order recurrence relations indicates how the functions \( E_n(x) = \int_{-\infty}^{x} e^{-xt} t^{-n} dt \) and others may be generated in a recursively stable fashion.

Chapter 13 Orthogonal Polynomials. The text for the chapter, by U. Hochstrasser, has been completed. To emphasize the general properties of orthogonal polynomials the functions have been described according to the various properties satisfied by orthogonal polynomials...
rather than by treating each polynomial separately. All of the classical polynomials are discussed. Tables are given of the coefficients, in addition to graphs and skeleton tabulations, as a function of the order for certain selected arguments.

Chapter 19 Parabolic Cylinder Functions. A preliminary outline of the material to be included has been prepared by J.C.P. Miller.

Chapter 22 Miscellaneous Functions. This chapter of tables and mathematical properties, by I. A. Stegun, covers a number of mathematical functions such as Planck's radiation functions, Sievert's integrals, Clebsch-Gordon coefficients. The functions have been derived from a number of sources. In the case of the Clebsch-Gordon coefficients the explicit formulas are given for the most important cases, as well as a skeleton table for checking purposes in numerical work.

Chapter 23 Bernoulli and Euler Polynomials, by E. V. Haynsworth and K. Goldberg, was completed and submitted for review. The work is characterized by the parallel display of the properties of the Bernoulli and Euler polynomials. In addition, there has been included a section on the Riemann Zeta function with special emphasis on the tabulation of the sums of reciprocal powers. These series are of fundamental importance in increasing the rate of convergence of series.

Chapter 24 Combinatorial Analysis. The text of this chapter, by K. Goldberg, M. Newman and E. V. Haynsworth, has been completed. The aim has been to collect, generalize and condense most of the elementary combinatorial results that have proved most useful. The format is arranged to present the results in a form suitable for ready reference. Some new results have been included. The following section titles indicate the subject matter:

1. Notation
2. Solutions of Counting Problems
3. Binomial Coefficients
4. Stirling Numbers
5. Number Theory
6. Partitions
7. Symmetric Functions
8. Logical Theorems
9. Power Series
10. Aids to Computation
3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-12-1107/51-2

Origin: NBS
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. N. C. Severo has continued the work on the collection of tables and preparation of examples of their use for the Statistical Tables chapter of the Handbook of Mathematical Functions (see task 1102-40-5113/57-216, p. 10).

Publication:
(1) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. Joel. Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-12-1107/53-1

Origin: NBS
Manager: W. S. Connor
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. D. M. Mesner continued his studies of incidence matrices. On 20 February 1958 he presented a paper "Embedding Theorems for 0, 1 Matrices" at the NBS Mathematics Colloquium.

Publications:
(1) Multi-variable experiments. M. Zelen and W. S. Connor. Submitted to a technical journal.
Status of Projects

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-1107/56-170

Origin: NBS
Manager: Joan R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

Status: CONTINUED. J. R. Rosenblatt concluded her in-hours course, "Non-parametric statistical techniques." Four students (NBS and DOFL) completed the course.

MEASUREMENT OF RELIABILITY
Task 1103-12-1130/56-182

Origin: NBS
Manager: M. Zelen
Full task description: Jan-Mar 1956 issue, p. 13


Publication:

FRACTIONAL FACTORIALS FOR THE MIXED SERIES
Task 1103-12-5148/58-291

Origin and Sponsor: Bureau of Ships
Managers: W. S. Connor, M. Zelen
Full task description: July-Sept 1957 issue, p. 43

Status: CONTINUED. W. S. Connor prepared a report on the progress to date for submission to the sponsor. This report presents a method that facilitates the determination of the normal equations, applies the method to a 1/2 replicate of the $2^{33}_2$ factorial, and illustrates the analysis of this design with real data.

Some of the results of this report were given by Dr. Connor at the Statistics Seminar, Virginia Polytechnic Institute, Blacksburg, Virginia, on February 21.
Publication:

(1) Fractional factorial experiments of the $2^m \cdot 3^n$ series. W. S. Connor. To appear in the Transactions of the Rochester Society for Quality Control.
4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-12-1115/55-57

Origin: NBS
Manager: R. F. Dressler
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. The manuscript on compressible fluid flow past a two-dimensional finite wedge, by A. Ghaffari, has been completed. The stream function and velocity potential are found for the steady irrotational ideal compressible flow past a finite wedge for both the supersonic and subsonic regions. Certain analytical properties of the solutions and their derivatives are discussed.

A. Ghaffari has investigated the behavior in the large of the totality of the paths of a dynamical system defined by the nonlinear differential equation

\[
dy/dx = \left[ y(x^2+y^2-2x-3)(x^2+y^2-2x-8) + x \right] \left[ x(x^2+y^2-2x-3)(x^2+y^2-2x-8)-y \right]^{-1}
\]

in the phase plane.

It is found that the origin is the only elementary critical point, which is an unstable focus, and the equator is a limit-cycle.

In polar form the equation transforms into

\[
d\rho/d\theta = \rho (\rho^2-2\rho \cos\theta-3)(\rho^2-2\rho \cos\theta-8),
\]

which is partially discussed by H. Poincaré (Oeuvres, Vol. I, p. 83). One can observe that the circles without contact are defined by \(0 < \rho < 1\) and \(\rho > 4\), and that the circular region \(1 < \rho < 4\) contains only two limit-cycles \(C_1\) and \(C_2\). Thus the totality of the paths of the differential equation consists, aside from the equator and the limit-cycles \(C_1\) and \(C_2\), of a first family of spirals (inside \(C_1\)) spiralling away from the origin, a second family of spirals (between \(C_1\) and \(C_2\)) spiralling away from \(C_1\), and finally a third family of spirals (between \(C_2\) and the equator) approaching asymptotically the limit-cycle \(C_2\) and the equator.

J. Vinti has begun a study of a problem suggested by W. Wildhack, of the Office of Basic Instrumentation, concerning the rotation of the plane of a satellite orbit produced by the "wind" associated with the rotation of the atmosphere of the earth.
The paper on graphical methods of integration by W. H. Pell has now been completed and is ready for publication.

V. M. Yevdjevic has begun a critical study of the techniques which have been used by hydrologists in the analysis of fluctuations of annual rainfalls and runoffs. Two approaches have been used in the analysis of data regarding rainfall and runoff: (1) that in which the oscillations in these quantities are analyzed by specially developed mathematical techniques; normally, this reveals cyclic fluctuations of various periods; (2) that in which a statistical approach is used, and which frequently yields the results that the occurrence of wet and dry years are purely random, and, indeed, that all fluctuations are of this nature.

It is desirable to establish the correctness of one or the other of these conclusions. The object of Dr. Yevdjevic's study is to investigate the relative reliability of the two procedures, i.e., to answer the question as to whether the sequence of annual precipitations and runoffs follow a cyclic, non-cyclic, or semi-cyclic pattern.

Publications:
(2) The graphical solution of initial value problems. W. H. Pell. In manuscript.

RESEARCH IN CONTINUUM MECHANICS
(formerly Mathematical Elasticity)
Task 1104-12-5160/55-85

Origin: NBS
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: R. F. Dressler
Full task description: Oct-Dec 1954 issue, p. 30

Status: CONTINUED. W. H. Pell and L. E. Payne are continuing their study on Stokes flows about certain types of axially symmetric bodies. The differential equation for the stream function is \( L^{-1}_1 \chi = 0 \), where

\[
L^{-1}_1 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r},
\]
x and r being cylindrical coordinates. A. Weinstein [see Oct-Dec 1957 issue, p. 20] and L. E. Payne have given decomposition formulas for the solution of equations involving iterated operators, and these results are being applied to Stokes flow problems. Work has been completed on this problem for a spindle (double ogive) and lens-shaped body, and a manuscript is in preparation.

W. H. Pell has completed the manuscript of the paper on the integration of the elastic plate equation for a plate whose thickness varies linearly in two orthogonal directions in the plane of the plate. This generalizes a result of E. Reissner [Ingenieur-Archiv 8, 80-81 (1937)].

A. Walz has written up the results of his most recent work on an approximate theory for the compressible turbulent boundary layer with heat transfer. This is based on an infinite number of integral conditions (ordinary differential equations) derived from the partial differential equations for the velocity and temperature fields. The properties of this system are discussed. It has been found that the two equations of this system most commonly used in less general boundary layer theories, viz., the momentum and energy integral conditions, are most suitable to describe the boundary layer behavior. To use these two equations in the case of compressible turbulent boundary layers with heat transfer, known semi-empirical laws for wall friction and dissipation in an incompressible flow have been generalized. Approximate formulas derived in this connection have been used for calculating some examples, especially the wall friction coefficient of a flat plate. Comparison with available experimental data shows good agreement. Some possibilities for simplification and improvement of the theory have been outlined.

R. F. Dressler has completed a paper on the nonlinear gravity wave study (see publication (6) below).

Dr. Dressler has also completed a manuscript describing exact stress and displacement behavior of a typical corrugated elastic diaphragm shape with a 10:1 range of the thickness parameter. Interpretation and evaluation of the results for practical design purposes is presented. (Publication (7) below.)

By means of the Bogolubov method of statistical mechanics, C. M. Tchen has obtained the Fokker-Planck equation for a plasma. The coefficients of the Fokker-Planck equation are found to depend on the law of interaction between particles; this implies the screening effect of a plasma cloud and additional friction due to momentum relaxation. Dr. Tchen has also solved the problem of time relaxation by a generalization of the Fokker-Planck equation. This result also gave a statistical foundation of the Debye screening effect. A manuscript is in preparation.

Publications:


(5) Note on the integration of the elastic plate equation with variable flexural rigidity. W. H. Pell. In manuscript.


(8) Compressible turbulent boundary layers with heat transfer and pressure gradient in flow direction. Alfred H. Walz. In manuscript.

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-5160/56-154

Origin: NBS
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: July-Sept 1955 issue, p. 20

Status: CONTINUED. In order to include a supplementary list of the most important pairs of Fourier transforms used by statisticians, a survey of the major statistical journals has been made and a number of pairs most frequently used by statisticians were found and recorded.

RESEARCH IN FLUID DYNAMICS OF TWO-PHASE FLOWS
Task 1104-12-5160/56-155

Origin and Sponsor: Office of Naval Research
Manager: R. F. Dressler
Full task description: July-Sept 1955 issue, p. 21

Status: CONTINUED. J. Vinti has continued his study of the literature on the discharge of steam jets into water and the connection of this problem with that of the propulsion of underwater devices. He has devised a numerical method for reducing measurements of static pressure, stagnation pressure, and stagnation temperature in superheated steam, to obtain a complete description of the state of the steam at a given point of a flow. The method of reduction depends on whether the flow is subsonic or supersonic, and gives the temperature, density, velocity, and Mach number at the point of measurement.

He has also examined the theory of the thrust obtainable by means of a steam jet discharging into water and, utilizing this, has made
a preliminary analysis of the action of some underwater propulsion devices developed by Aerojet Corporation. The aim of this analysis is to find out whether the essential problem that arises in the theory of such devices is one in two-phase flow and cavitation, as has hitherto been supposed, or whether it is really one in gas dynamics.

Publication:
5. MATHEMATICAL AND COMPUTATIONAL SERVICES

1102-40-5126/54-13 AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: New York Quartermaster Procurement Agency
Manager: H. Bremer
Full task description: Oct-Dec 1953 issue, p. 43
Status: Continued. During the past quarter a bid evaluation problem was successfully run on the 704 with the IBM transportation code for a 704 without drums. For the sake of trial and checking, a past problem, previously solved on SEAC, was used. Application of the new code involved overcoming some difficulty resulting from the fact that a restriction—that the cost matrix must have more rows than columns—was not clarified in the code write-up. By using the 704 transportation code, it is now possible to handle problems as large as \( m+n = 700 \) and \( n = 600 \). (On SEAC the size limitation was \( mxn \leq 740 \).)

Consideration and study is being given to overcoming the increase in the overall elapsed time required for the solution of an award problem on the 704. The time is much greater than was experienced for solutions on SEAC. For example, a problem that required three hours on SEAC would presently require four to five days on the 704 even though the total computing time on the 704 would be less than the total computing time on SEAC. This is due to the fact that the present 704 code will solve only "one transportation problem" at a time since it will accept only the matrix to be solved and this in the form of cards, whereas the SEAC transportation code was designed specifically for use in solving bidder award problems with the feature of reading into the memory all possible rows that might be needed to make up any desired matrix.

No bid evaluation problems were submitted by the Army Quartermaster Corps during the quarter, due to a slowdown in the purchasing activity.

3711-60-0009/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Managers: C. D. Coleman, W. Bozman (4.1)
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. A list of 12000 spectral lines of thorium I was put on cards and on tape, wave numbers were computed and listed, and differences between known even levels were computed. Lists of wavenumbers are being prepared for energy level searches.
3711-60-0009/54-38  COMPRESSIBILITY FACTORS OF DRY AIR  
Origin: NBS, Section 3.2  
Manager: M. Paulsen  
Full task description: Jan-Mar 1954 issue, p. 48  
Status: Continued. The 704 code for computing tables of compressibility factors of dry air has been completed. As in the SEAC code the temperature, in degrees Kelvin, is held constant while the pressure, in pounds per square inch, is increased by an increment designated by the sponsor. The format of the printout can be varied quite extensively; this includes a selection of the number of decimal places desired by the sponsor. The calculation of another thermodynamic function, enthalpy, has been added to the code.

3711-60-0009/55-68  CRYSTAL STRUCTURE CALCULATIONS  
Origin: NBS, Division 9  
Managers: P. O'Hara, S. Block (9.7)  
Full task description: Jan-Mar 1955 issue, p. 18  
Status: Continued. A program has been completed for the computation on the 704 of three-dimensional Fourier summations. The program is used to evaluate the electron density throughout a region representing the unit cell of a crystal. The peaks of the electron density function correspond to the locations of individual atoms in the unit cell. The information needed for the calculation consists of several hundred observed structure factors and their associated phase angles. The electron density is given as

\[
\rho(x, y, z) = \frac{1}{V} \sum_{h=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \left| F_{hkl} \right| \exp \left\{ -2\pi i (hx + ky + lz) \right\}
\]

where \( V \) = volume of unit cell,  

\[
\left| F_{hkl} \right| = |A_{hkl}| + i B_{hkl} \quad \text{magnitude of observed structure factor,}
\]

\( h, k, l \) = Miller indices.

The \( x, y, z \) can range from 0 to 1 at intervals of 1/64 with the restriction that the maximum grid for any plane be limited to 2048 points. The expression (1) above can also be stated as

\[
\rho(x, y, z) = \frac{1}{V} \left[ F(000) + 2 \sum_{h=0}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} A_{hkl} \cos 2\pi(hx + ky + lz) 
+ B_{hkl} \sin 2\pi(hx + ky + lz) \right]
\]

\[
A_{hkl} = \frac{F_{hkl}}{ \left| F_{hkl} \right| } \cos \alpha_{hkl}
\]

\[
B_{hkl} = \frac{F_{hkl}}{ \left| F_{hkl} \right| } \sin \alpha_{hkl}
\]

\[
\alpha_{hkl} = \arctan \frac{B_{hkl}}{A_{hkl}}
\]
The Fourier summations are carried out by evaluating one-dimensional summations in three stages and accumulating the sine and cosine terms to obtain a point, line or plane when the index of summation changes. When all the input data have been processed the results for a plane are printed and the entire calculation repeated for the next plane.

The program was checked by computing the electron density distribution for a crystal of known structure.

3711-60-0009/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. Preparation of a code for computing International Temperature Scale constants is in process.

1102-40-5126/55-88 STRESSES IN A WALL FOUNDATION
Origin and Sponsor: NBS, Section 10.1
Full task description: Jan-Mar 1955 issue, p. 22
Status: Inactive.

1102-40-5126/55-117 ATTENUATION OF PRESSURE PULSES OF FINITE AMPLITUDE
Origin and Sponsor: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Apr-June 1955 issue, p. 18
Status: Terminated.

1102-40-5126/55-121 ELECTRON PENETRATION
Origin and Sponsor: NBS, Section 4.8
Manager: S. Peavy
Full task description: Apr-June 1955 issue, p. 19
Status: Continued. At the sponsor's request runs were made on the IBM 704 for two compounds,--air and polystyrene.

1102-40-5126/56-139 STUDY OF INTERNUCLEAR POTENTIAL FOR H₃
Origin and Sponsor: NBS, Section 3.2
Manager: E. Haynsworth
Full task description: July-Sept 1955 issue, p. 36
Status: Terminated.

1102-40-5126/56-162 STRESSES IN A WALL RESTING ON A FOOTING
Origin and Sponsor: NBS, Section 10.1
Manager: I. Stegun
Full task description: Jan-Mar 1956 issue, p. 26
Status: Inactive.
1102-40-5126/56-163 ANGULAR DISTRIBUTIONS AND POLARIZATION EFFECTS IN NUCLEAR SCATTERING
Origin and Sponsor: Naval Research Laboratory
Manager: I. Stegun
Full task description: Oct-Dec 1955 issue, p. 32
Status: Continued. A code for the 704 has been written and checked out for computing the polarization, differential cross section and absorption cross section. As a by-product, a subroutine has been obtained for computing the Legendre functions $P^L_0(\cos \theta)$ and $P^L_1(\cos \theta)$ for varying $L$ and $\theta$.

1102-40-5126/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Managers: E. Haynsworth, P. Walsh
Full task description: Jan-Mar 1956 issue, p. 27
Status: Continued. Roots and vectors for matrices of order up to 18 were calculated. The SCF routine (see task 3711-60-0009/57-223, p. 24) was used to give solutions for several double block problems.

1102-40-5126/56-171 COLLISION INTEGRALS USED IN TRANSPORT THEORY
Origin and Sponsor: NBS, Section 3.2
Manager: D. Sumida
Full task description: Oct-Dec 1955 issue, p. 33
Status: Inactive.

1102-40-5126/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: S. Prusch
Full task description: July-Sept 1956 issue, p. 33
Status: Continued. Calculations were performed on nine laboratory sets of gage blocks; each set consisted of a maximum of 88 blocks.

1102-40-5126/57-209 TRAFFIC DISTRIBUTION
Origin and Sponsor: Bureau of Public Roads
Manager: S. Peavy
Full task description: Jan-Mar 1957 issue, p. 32
Status: Inactive.

1102-40-5126/57-211 METEOROLOGICAL DATA
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: P. O'Hara
Full task description: Oct-Dec 1956 issue, p. 30
Status: Terminated.
Status of Projects

1102-40-5126/57-219  THERMAL PROPERTIES
Origin and Sponsor: NBS, Section 3.2
Manager: D. Sumida
Full task description: Oct-Dec 1956 issue, p. 30
Status: Completed. A routine for the evaluation of the Debye-equations was coded and checked out. Results have been transmitted to the sponsor.

1102-40-5126/57-221  BESSEL FUNCTIONS FOR COMPLEX ARGUMENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Oct-Dec 1956 issue, p. 31
Status: Continued. The code originally written for SEAC was modified for the 704, to be used with the SEAC interpretative code. Twelve cases were run to evaluate the Bessel and Hankel functions for complex arguments up to order n specified.

1102-40-5126/57-222  ROOTS OF POLYNOMIALS
Origin and Sponsor: Naval Research Laboratory
Manager: J. P. Menard
Full task description: Oct-Dec 1956 issue, p. 32
Status: Inactive.

3711-60-0009/57-223  SELF-CONSISTENT FIELDS
Origin: NBS, Section 3.2
Manager: E. V. Haynsworth
Full task description: Apr-June 1957 issue, p. 28
Status: Continued. The code for calculating the A matrix has been completed and checked out. A subroutine to calculate XSX^T (where S is symmetric and stored triangularly) has been added to the SCF program. This subroutine will be used to check the accuracy of the results. The coding of the control program has been completed and code checking is in progress. Modifications are being made in the SCF program to make use of the control program for data manipulation.

1102-40-5126/57-224  TRACK-WHILE-SCAN RADAR PROBLEM
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Managers: E. Haynsworth, P. J. Walsh
Full task description: Oct-Dec 1956 issue, p. 32
Status: Continued. The formulae in the Jan-Mar 1957 issue (p.34) were modified to eliminate integration with respect to Ø. Calculations of the modified M(Ø) were performed using several different parameters and keeping Ø constant. The results have been submitted to the sponsors for analysis.
3711-60-0009/57-229 APPLICATION OF ELECTRONIC DATA PROCESSING MACHINERY TO PAYROLL OPERATIONS

Origin: NBS, Section 40.0
Managers: H. Bremer, P. R. McClenon, M. Paulsen
Full task description: Jan-Mar 1957 issue, p. 36

Status: Continued. During the past quarter programming and code-checking has continued on many of the subroutines that go into producing the payroll program for the 704. Two important subroutines have been written for handling the control of the input and output tapes. Use of these subroutines will reduce the amount of work necessary for producing the tape outputs. If the present rate of progress continues, it should be possible to carry out a test run by the end of the current fiscal year.

1102-40-5126/57-234 PERSONNEL SURVEY

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: P. O'Hara
Full task description: Jan-Mar 1957 issue, p. 37

Status: Inactive.

1102-40-5126/57-236 SELF CONSISTENT FIELDS--EIGENVALUES

Origin and Sponsor: NBS, Section 3.6
Manager: E. Haynsworth
Full task description: Apr-June 1957 issue, p. 30

Status: Continued. The routine for controlling the subroutines has been checked out. Also, results have been obtained for single and double block matrices.

3711-60-0009/57-247 MECHANICAL IMPEDANCE

Origin: NBS, Section 6.1
Managers: J. P. Menard, M. D. Burkhard (6.1)
Full task description: Apr-June 1957 issue, p. 32

Status: Continued. Additional groups of data on the measurements of impedances of the forehead and mastoids of young subjects 18 to 25 years old have been furnished directly by the sponsor for processing on the 704. The computer is used to determine parameters of mass, stiffness and resistance. The computations are used in connection with the development of an artificial mastoid to check bone conduction hearing aids.

3711-60-0009/57-248 THE EVALUATION OF A TRIPLE INTEGRAL FOR THE SOLUTION OF NEGATIVE ION DETACHMENT

Origin: NBS, Section 4.6
Manager: S. Peavy
Full task description: Apr-June 1957 issue, p. 34
Status: Continued. The original problem has been altered to include another integral:

\[ I_4 = \int_0^{1-t^2} ds^2 \int_{-1}^{1} \frac{dq}{(s^2+t^2)^{\frac{3}{2}}} \int_{-1}^{1} d\beta \frac{H^2(K)}{K^2} \]

\[ \cdot [2s\beta-(1+q^2-2q\alpha)^{\frac{3}{2}}]\left[(1-s^2)^2+4s^2\left(\frac{(1-q\alpha)}{(1+q^2-2q\alpha)^{\frac{3}{2}}}\right)^2\right]
\]

The code is now being checked out.

3711-60-0009/57-250 AUTOMATIC REDUCTION OF SPECTROPHOTOMETRIC DATA
Origin: NBS, Section 2.1
Manager: W. C. Rheinboldt
Full task description: July-Sept 1957 issue, p. 31
Status: Continued. Approximately 500 runs were made during the quarter under the direction of the sponsor for the automatic reduction of partially corrected spectrophotometric data to fully corrected spectrophotometric data and into colorimetric terms.

1102-40-5126/57-251 CURRENT NOISE AND FIXED RESISTORS
Origin and Sponsor: NBS, Section 1.6
Manager: D. Sumida
Full task description: July-Sept 1957 issue, p. 32
Status: Inactive.

3711-60-0009/58-254 REPRODUCTION OF COLOR- AND SPECTRAL-ENERGY DISTRIBUTION OF DAYLIGHT AND OTHER ILLUMINANTS
Origin: NBS, Section 2.3
Manager: W. C. Rheinboldt
Full task description: July-Sept 1957 issue, p. 32
Status: Continued. Some additional runs on the 704 have been made under the immediate direction of the sponsor. The computations have facilitated and expedited the development of filters for the laboratory reproduction of daylight and high intensity incandescent sources (photoflood and flash lamps) to be considered by the American Standards Association for use in American standards for sensitometry of black and white and color films. These filters with a calibrated light source produce a very close energy match and a perfect color match with the ideals. This would bring the field of sensitometry up to date for modern films.
1102-40-5126/58-263 GAS TUBE CHARACTERISTIC
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: W. F. Cahill
Full task description: July-Sept 1957 issue, p. 35
Status: Continued. Runs have been made for 25 sets of parameters, and the results have been transmitted to sponsor.

1102-40-5126/58-264 THEORY OF IONIZATION PROBABILITY
Origin and Sponsor: NBS, Section 4.6
Manager: S. Peavy
Status: Inactive.

3711-60-0009/58-266 DEPOLYMERIZATION, II
Origin and Sponsor: NBS, Section 7.6
Manager: L. S. Joel
Full task description: July-Sept 1957 issue, p. 36
Status: Continued. A code for generation of the function for case II, Terminal Initiation, has been written and checked.
A code for Runge-Kutta integration of differential equations using Gill's method (Proc. Cambridge Philos. Soc. 1951, pp. 96-116) has been written and is being checked. The first large system to be solved has 403 equations.

3711-60-0009/58-267 CONVERSION OF THE CIE-CHROMATICITY COORDINATES INTO THE MUNSELL COLOR SYSTEM
Sponsor: NBS, Section 2.1
Manager: W. C. Rheinboldt
Full task description: July-Sept 1957 issue, p. 37
Status: Inactive. For status to date, see Oct-Dec 1957 issue, p. 32.

3711-60-0009/58-268 RESPONSE FUNCTION CALCULATION
Origin: NBS, Section 4.11
Manager: A. Beam
Full task description: July-Sept 1957 issue, p. 37
Status: Continued. The code was checked and turned over to the sponsor. It is used by the sponsor as the need arises.

1102-40-5126/58-269 MOLECULAR STRUCTURE, IV
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. J. O'Hara
Full task description: July-Sept 1957 issue, p. 38
Status: Continued. Phase determination calculations for aureomycin are being carried out at the sponsor's request.
Status of Projects

1102-40-5126/58-272  THERMODYNAMIC PROPERTIES OF REAL GASES
Origin and Sponsor: NBS, Section 3.2
Manager: J. P. Menard
Status: Continued. IBM 704 codes have been written and completely checked out for performing the following differentiations:

\[
\begin{align*}
\left. \frac{d}{d \ln \rho/\rho_0} \frac{PV}{RT} \right|_{T=\text{const}} & = \left. \frac{d}{d \ln \rho/\rho_0} \frac{E}{RT} \right|_{T=\text{const}} \\
\left. \frac{d}{d \ln T} \frac{PV}{RT} \right|_{\rho = \text{const}} & = \left. \frac{d}{d \ln T} \frac{E}{RT} \right|_{\rho = \text{const}}
\end{align*}
\]

where \( PV/RT \equiv \) compressibility factor
\( E/RT \equiv \) internal energy
\( \rho \equiv \) density of given gas.

The codes/set up to handle a variable number of values of the function for equally or unequally spaced values of the argument. Since the table of compressibility factors and internal energies, as submitted by the sponsor, was tabulated with respect to the density for a given temperature, a feature was incorporated in the code for rearranging the values with respect to the temperature for a given density. Furthermore, it was necessary to set up a scheme which would take care of missing values in the data. This was accomplished in two steps. First, a flag word was inserted in the value table wherever a value was missing for a given argument. Then the code was set up to delete both the flag and the corresponding argument value and compress the function to be differentiated. The differentiation was then performed with the assumption that the argument values were unequally spaced.

Production runs have been made, and we are now awaiting further communications from the sponsor for future runs.

1102-40-5126/58-274  CALCULATIONS FOR d-SPACINGS, II
Origin and Sponsor: NBS, Division 9
Manager: R. Zucker
Full task description: July-Sept 1957 issue, p. 38
Status: Continued. About 19 calculations for d-spacings for orthogonal, hexagonal and orthorhombic crystals were carried out this quarter, and redetermination of unit cell constants by least squares fitting to a measured d-spacing was performed for about 26 crystals.
Status of Projects

1102-40-5126/58-279  FIRE RESISTANT T-BEAM
Origin and Sponsor: NBS, Section 10.2
Manager: C. Wade
Full task description: Oct-Dec 1957 issue, p. 33
Status: Continued. Exploratory calculations have been made for one type
of concrete structure in the two-dimensional case for several combinations
of grid block width and time increment.
   For the one-dimensional case a code has been written which
computes temperature on a line through the concrete from the air
surface to the fire surface. The same expression for the gradient as
in the two-dimensional case is used at the air surface. The code is
now being checked out.

1102-40-5126/58-282  MISSILE BOUNDARY LAYER COMPUTATION
Origin and Sponsor: Naval Ordnance Laboratory
Manager: R. Danek (NOL)
Objective: To compute the laminar and turbulent boundary layer growth
about a missile as it traverses its trajectory.
Background: This problem involves the stepwise integration along the
profile of a body shape of quantities specified by a series of inter-
related equations, for each chosen time step in the course of the
trajectory. Although a simple heat transfer relation exists in the
problem at present, a more complete time-dependent treatment of this
phase of the problem is being composed. This computation was initially
coded for the IBM 650 at NOL but due to its time consuming nature and
the desire to extend the original formulation to the time-dependent
scheme, the use of the 704 was deemed advisable.
Status: New. The problem has been analyzed and the original version
has been coded and checked out for 704 using FORTRAN. Programming of
the problem with heat conduction as described above is under way,
although some details require further analysis.

1102-40-5126/58-284  EPHEMERIS CALCULATIONS FOR SATELLITES
Origin and Sponsor: Naval Research Laboratory
Managers: W. F. Cahill, J. H. Wegstein
Full task description: Oct-Dec 1957 issue, p. 34
Status: Continued. Computations on the trajectories of Sputnik I and
Explorer I were made. Results are being analyzed by the sponsor.

1102-40-5126/58-289  SCATTERING OF ELECTRONS BY HYDROGEN
Origin and Sponsor: NBS, Section 3.6
Manager: R. Zucker
Full task description: Oct-Dec 1957 issue, p. 36
Status: Inactive. For status to date, see Oct-Dec 1957 issue, p. 36.
3711-60-0009/58-294 NUCLEAR SCATTERING OF PHOTONS  
Origin: NBS, Section 4.8  
Manager: J. P. Menard  
Full task description: Oct-Dec 1957 issue, p. 36  
Status: Continued. A code has been written and completely checked out. A number of production runs have been made. The program will continue in production under the sponsor's direction.

1102-40-5126/58-297 "MANY BODY" PROBLEM  
Origin and Sponsor: Naval Research Laboratory  
Manager: S. Peavy  
Full task description: Oct-Dec 1957 issue, p. 38  
Status: Inactive.

1102-40-5126/58-298 ANALYSIS OF SPECTROCHEMICAL DATA  
Origin and Sponsor: NBS, Section 5.10  
Managers: S. Peavy, R. Ridinger  
Full task description: Oct-Dec 1957 issue, p. 39  
Status: Continued. The code for the problem has been checked out on the 704, and a few runs have been made at the request of the sponsor.

1102-40-5126/58-299 TIME-DEPENDENT SCHRODINGER EQUATION  
Origin and Sponsor: NBS, Section 3.1  
Manager: J. Beiman  
Full task description: Oct-Dec 1957 issue, p. 39  
Status: Continued. The code has been checked out, and the problem is in production. A number of runs have been made under varying initial conditions.

1102-40-5126/58-300 LAMINAR MIXING IN BOUNDARY LAYERS  
Origin: Polytechnic Institute of Brooklyn  
Sponsor: Air Force Office of Scientific Research  
Manager: W. C. Rheinboldt  
Full task description: Oct-Dec 1957 issue, p. 40  
Status: Continued. In order to solve the boundary value problem in question the following iteration scheme has been outlined:  
\[ c^{(n)} = k + \int_{-\infty}^{\infty} g^{(n)}(\sigma) \, d\sigma \]  
and successively the auxiliary functions,
Status of Projects

\[ h^{(n)}(\sigma) = c^{(n)} + \int_{0}^{\sigma} G^{(n)}(\tau) \, d\tau, \]

\[ K^{(n)}(\sigma) = \exp\left[-\frac{1}{2}(c^{(n)}\sigma)^2 + \int_{0}^{\sigma} (\sigma-\tau)^2 G^{(n)}(\tau) \, d\tau\right], \]

\[ T^{(n)}(\sigma) = \frac{1-k^2}{\int_{-\infty}^{+\infty} K^{(n)}(\tau) \, d\tau} \int_{-\infty}^{\sigma} K^{(n)}(\tau) \, d\tau + k^2, \]

\[ \hat{G}^{(n)}(\sigma) = \beta \int_{0}^{\sigma} \left( (h^{(n)}(\tau))^2 - T(\tau) \right) \frac{K^{(n)}(\sigma)}{K^{(n)}(\tau)} \, d\tau. \]

Then the next approximation \( G^{(n+1)}(\sigma) \) is given by

\[ G^{(n+1)}(\sigma) = \frac{1-k - \int_{-\infty}^{+\infty} \hat{G}^{(n)}(\tau) \, d\tau}{\int_{-\infty}^{+\infty} K^{(n)}(\tau) \, d\tau} K^{(n)}(\sigma) + \hat{G}^{(n)}(\sigma). \]

This iteration scheme has been coded, and the 704 code has been completely checked out.

An initial function,

\[ G^{(0)}(\sigma) = \frac{1-k}{2} \sqrt{\frac{1+k}{\pi}} - \frac{1+k}{2} \frac{\sigma^2}{2} \]

has been used. Furthermore provision has been made to read in a numerically given initial function. The code has been run for 12 different combinations of the parameters \( k \) and \( \beta \). It has been found that the iteration is converging for \(-0.3 \leq \beta \leq 0.4, 0.1 \leq k \leq 0.9\). The resulting functions have been checked with Runge-Kutta method for the solution of the initial value problem of the given system of equations. The result has been shown to be a solution to the system in question and valid to 6 significant figures.

Tests have been made with a different step-width in order to find its influence in accuracy, speed of iteration, etc. While the physical nature of the problem indicates that convergence may fail for \( \beta < -0.3 \), it is still an open question as to why the process fails to give convergence for \( \beta > 0.4 \). Numerical experiments have been started in order to find an answer to this question. However, insufficient funds have made it necessary to discontinue the investigations temporarily.
1102-40-5126/58-304 TRANSPORT PROPERTIES OF AIR AT ELEVATED TEMPERATURES
Origin and Sponsor: NBS, Section 3.2
Manager: P. J. Walsh
Full task description: Oct-Dec 1957 issue, p. 40
Status: Continued. The third phase of the problem has been coded and completely checked out. The formulae involved are similar to those described in the Oct-Dec 1957 issue. The results for several temperatures are now being analyzed by the sponsor.
A set of formulae are to be coded which will generate the $\Omega_{ij}^{(k,l)}$ used in computing the matrix elements. These are the collision integrals which depend upon a set of collision cross-sections calculated under another project.

1102-40-5126/58-306 INTERPOLATION OF COLOR MIXTURE FUNCTIONS
Origin and Sponsor: NBS, Section 2.1
Manager: W. C. Rheinboldt
Full task description: Oct-Dec 1957 issue, p. 42
Status: Continued. The 704 code has been written and completely checked out. The operation of this code has been turned over to the sponsor, and a number of production runs have already been made.

1102-40-5126/58-307 STUDY OF SURFACE TENSION
Origin and Sponsor: NBS, Section 9.2
Manager: R. Arms
Full task description: Oct-Dec 1957 issue, p. 43
Status: Reactivated. Several additional calculations were performed for the sponsor.

1102-40-5126/58-308 OSCILLATING SPHERE
Origin and Sponsor: NBS, Section 3.4
Manager: S. Prusch
Full task description: Oct-Dec 1957 issue, p. 43
Status: Continued. Additional production runs are being analyzed by the sponsor.

1102-40-5126/58-311 NULL RATE CALCULATIONS
Origin and Sponsor: Naval Research Laboratory
Manager: J. H. Wegstein
Full task description: Oct-Dec 1957 issue, p. 44
Status: Continued. Numerous sets of minitrack data were analyzed by this computer program, and the results were transmitted to the sponsor. Several modifications were made in the code for more conveniently handling the data.
1102-40-5126/58-312  RESPONSE FUNCTION, II
Origin and Sponsor: NBS, Section 4.11
Manager: A. Beam
Objective: (1) To invert a 56x56 matrix A. (2) To read in a large group of data and smooth it into a 56-element column vector V. (3) To compute $A^{-1}V = E$.

Background: In the measurement of the energy spectrum of 5 to 115 Mev X-rays, the spectrometer has resolution much poorer than that associated with optical instruments. The response to a Mono energetic x-ray beam is termed the response function. A matrix A, made up of response functions for a family of x-ray energies may be used to convert an energy spectrum vector to a smeared spectrum called a pulse height distribution. Likewise a pulse height distribution vector V may be used to multiply the inverted response function matrix to yield an energy spectrum vector, E.

The problem was communicated by J. Wyckoff (4.11).

Status: New. Computation using the inverse matrix was invalid because of the large magnitude of the elements of the inverse and the nature of the input data. Considerable effort was made to get a valid result by modifying the matrix and vector input, but this effort met with little success. Hence, the code was modified to do multiplication with the original matrix and turned over to the sponsor, who has used it in this form on several occasions.

1102-40-5126/58-314  APPROXIMATIONS FOR GAS MIXTURES
Origin and Sponsor: NBS, Section 3.2
Manager: R. Zucker
Objective: To test approximations for gas mixtures and to examine the relaxation of the thermal conductivity of multicomponent gas mixtures due to thermal diffusion. Binary and ternary systems are considered. Expressions for the elements of determinants of orders 4 and 7 are to be evaluated for given temperatures and components.

Background: The problem arises in gas mixtures and was requested by C. Muckenfuss (3.2).

Status: New. Numerical calculations were carried out, and the results were transmitted to sponsor. They were in good agreement with the experimental work.

1102-40-5126/58-316  INTERSECTION CAPACITY STUDY
Origin and Sponsor: Bureau of Public Roads
Managers: S. Peavy, J. M. Cameron
Objective: To analyze data on flow of vehicular traffic through signalized intersection to determine equations for predicting the flow of such traffic under a variety of conditions.

Background: The procedures for predicting flow through an intersection given in "Highway Capacity Manual," Bureau of Public Roads (U. S. Government Printing Office, Washington, D. C., 1950) are to be applied to data taken since 1950. Some recent studies by the Bureau of Public...
Status of Projects

Roads have indicated that extrapolation from the previous results may be subject to error and it is desired to recompute the predicting equations using the new data to ascertain if the patterns in traffic flow have changed. The problem was communicated by A. A. Carter, Jr.

Status: New.

1102-40-5126/58-321 TABLE OF THERMODYNAMIC FUNCTIONS OF SULFUR

Origin and Sponsor: NBS, Section 3.2
Manager: R. Zucker

Objective: To prepare a table of thermodynamic functions of pure sulfur from smoothing equations, for given values of temperature.

Background: The table is needed for use in connection with a new and more accurate determination of the heat capacity of sulfur from experimental values. It was requested by E. D. West (3.2).

Status: New. The equations for monoclinic sulfur were evaluated for a lower and for an upper range of temperatures. The results were transmitted to the sponsor.

1102-40-5126/58-322 THE PROPAGATION CONSTANT OF A SOUND WAVE

Origin and Sponsor: NBS, Section 6.1
Manager: R. J. Arms

Objective: To calculate the roots of a sixth degree complex polynomial whose coefficients are functions of a single parameter.

Background: Roots of the polynomial give the propagation constant \( K = \alpha + i\beta \), where \( \alpha \) and \( 1/\beta \) are non-dimensional forms, respectively, of the amplitude attenuation per unit length and the wave length for a sound wave in a Maximilian diatomic gas. \( K \) is given as a function of the parameter \( x = w\mu/p \) (\( w = 2\pi \times \text{frequency}, \mu = \text{viscosity}, p = \text{pressure} \)).

The computation was requested by M. Greenspan (6.1).

Status: New. The code has been written and checked out on the 704. The results of several runs have been submitted to the sponsor.

1102-40-5126/58-324 FITTING A POLYNOMIAL TO A CURVE

Origin and Sponsor: NBS, Section 4.5
Managers: S. Peavy, R. Ridinger

Objective: To fit a curve through various known coordinates and then find values of the functions corresponding to other given coordinates. Use is to be made of an orthonormalizing routine to determine the coefficients of the polynomial.

Background: On spectra photos used by the sponsor, measurements were made previously with calibrations at several intervals. In order to save the time of taking many measurements, the curve-fitting procedure using the 704, is being tried. The problem was submitted by E. Horl (4.5).

Status: New. The code has been written and checked. Calculations are being made for each new set of data presented by the sponsor.
1102-40-5126/58-328  MAGNETIC FIELD IN A THIN FERRO-MAGNETIC SHEET DUE TO A LOCAL SOURCE  

**Origin and Sponsor:** NBS, Section 6.1  
**Manager:** S. Prusch  
**Objective:** To evaluate  

\[
y = \frac{2}{\pi} (1-\rho) \left\{ \tan^{-1} \frac{1}{x} + \rho^2 \tan^{-1} \frac{1}{x+2c} + \rho^4 \tan^{-1} \frac{1}{x+4c} + \ldots \right. \\
+ \left. \rho \left[ \tan^{-1} \frac{1}{2c-x} + \rho^2 \tan^{-1} \frac{1}{4c-x} + \rho^4 \tan^{-1} \frac{1}{6c-x} + \ldots \right] \right\}
\]

for \( \rho = 1/5, 1/3, 1/2, 3/5, 5/7, c = 0.25, 0.5(0.5)2.0, \) and \( x = 0(0.2)1.0. \)

**Background:** The relevant field distribution of the source in the absence of the sheet is known. When the sheet is placed near the source, the magnetic field within it can be expressed in infinite series form by using image theory (see Maxwell's Electricity and Magnetism, Vol. 1, p. 443). The problem arose in connection with a magnetic tape recording investigation. The computation was requested by E. Daniel (6.1).  
**Status:** Completed (New). The code was written and checked out on the 704. The computations were carried out and were transmitted to the sponsor.

1102-40-5126/58-330  EPITRON ORBITS  

**Origin and Sponsor:** Naval Research Laboratory, Department of the Navy  
**Manager:** R. J. Arms  
**Objective:** To calculate, by elementary functions, the path of particles in a numerical model of an epitron.  
**Background:** The calculations are for orbits to be expected in an epitron, which is an experimental machine for the testing of magnetic self-focusing. The problem was submitted by J. B. Ehrman (NRL).  
**Status:** New.

1102-40-5126/58-331  COMPUTATION OF HYDRAULIC ELEMENTS  

**Origin and Sponsor:** NBS, Section 10.5  
**Manager:** R. Ridinger  
**Objective:** To compute the values of the following hydraulic elements for a circular pipe section as a function of depth.  

1) \( \frac{R}{R_f} = \left[ 1 + \frac{\sin \theta}{2\pi(1 - \frac{\theta^o}{360})} \right] \)  
2) \( \frac{A}{A_f} = \left[ (1 - \frac{\theta^o}{360}) + \frac{\sin \theta}{2\pi} \right] \)  
3) \( \frac{Q}{Q_f} = \left( \frac{A}{A_f} \right) \left( \frac{R}{R_f} \right)^{2/3} \)  
4) \( \frac{a}{a_f} = (1 - \frac{\theta^o}{360}) \)  
5) \( \frac{V}{V_f} = \left( \frac{R}{R_f} \right)^{2/3} \)  
6) \( \theta = 2 \cos^{-1} (2x-1) \)
Status of Projects

Background: The need arises for computing the volume rate of discharge through a pipe for conditions other than when the pipe flow is full or half full. Rough curves in handbooks show these relationships, but their accuracy is questionable in certain ranges. Thus values were computed using the above equations where x, ranging from 0 to 1 by increments of .01, represents the ratio of the greatest possible depth to the variable depth.

The problem was communicated by R. S. Wyle.

Status: Completed (New). The computations were carried out on the 704, and the results were transmitted to the sponsor.

1102-40-5126/58-333 CALCIUM HYDROXIDE
Origin and Sponsor: NBS, Section 9.0
Manager: P. J. O'Hara
Objective: To calculate structure factors and do a least square fit of atomic position parameters for the calcium hydroxide crystal.
Background: These calculations are an application of task 1102-40-5126/58-269 to the study of the calcium hydroxide crystal (see July-Sept 1957 issue, p. 38). Computations will be carried out using the 704 program NY x R2. The problem was submitted by F. Ordway (9.0).
Status: New.

1102-40-5126/58-334 PRICE COMPUTATION TABLE
Origin and Sponsor: NBS, Section 32.0
Manager: R. Zucker
Objective: To check a price table for weights ranging from 1/4 oz. to 5 lbs. at 1/4 oz. gradations for costs per ounce ranging from 1 to 9 cents at 1 cent gradations
Background: The table is to be used by weight inspectors in checking the accuracy of scales. The computation was communicated by H. Wallin (32.0).
Status: Completed (New). The table was checked, and the results were submitted to the sponsor.

1102-40-5126/58-336 HELICAL TRANSFORMS
Origin and Sponsor: National Institutes of Health
Manager: R. Ridinger
Objective: To calculate $A_n^2 + B_n^2$ for each value of R and L, where L is the layer line and

$$A_n = \sum J_n (2\pi R) \cos \left\{ n \left(\frac{\pi}{2} - \varphi \right) + \frac{2\pi L z}{c} \right\},$$

$$B_n = \sum J_n (2\pi R) \sin \left\{ n \left(\frac{\pi}{2} - \varphi \right) + \frac{2\pi L z}{c} \right\}.$$

Background: Formulae are given for the Fourier transforms of a number of helical structures. These have been used successfully to verify the presence of the $\alpha$-helix in the synthetic polypeptide, poly-r-methyl-L-glutamate.
Since then general predictions have been made concerning the intensities of the x-ray diffraction pattern for other helical structures. These predictions do not necessarily result in correct distribution of layer line intensities. Thus it is desired to calculate the helical intensities in order to compare the observed intensities from model building with the calculated results.

At a point in reciprocal space with cylindrical coordinates \((R, \psi, L/c)\) the Fourier transform is:

\[
F(R, \psi, L/c) = \sum_n \sum_j f_j J_n(2\pi j R) \exp\left\{i \left[ n(\psi - \phi_j + \frac{\pi}{2}) + \frac{2\pi L z_j}{c} \right] \right\}
\]

where \(r_j, \phi_j, z_j\) are the atomic coordinates. This expression can be written:

\[
F(R, \psi, L/c) = \sum_n \left( A_n + B_n \right) \exp in \psi
\]

where \(A_n\) and \(B_n\) are the calculated values.

The problem was communicated by A. Rich (NIH).

**Status**: New. A flow diagram has been drawn, the coding has been completed, and checking is in progress.

1102-40-5126/58-337 GEORGETOWN LANGUAGE TRANSLATION EXPERIMENTS
Origin and Sponsor: Georgetown University
Manager: R. J. Arms
Objective: To investigate usage of the Bureau's computer for Georgetown machine translation experiments, and to offer consulting services about computers to the Georgetown project.
Background: At Georgetown University, Department of Languages and Linguistics, there are about four groups individually developing methods of machine translation. Most of these groups have reached a stage of progress where computer experiments would be of aid in their research. The problem was submitted by L. Dostert (Georgetown University).
Status: New. A small computer experiment has been planned with the group headed by Dr. P. Garvin with supervision of the manager. The coding will be done at Georgetown.

1102-40-5126/58-339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS
Origin and Sponsor: NBS, Section 3.4
Manager: W. Rheinboldt
Objective: To perform computations on the IBM 704 that become necessary in connection with the investigation of the mechanical behavior of certain polymers.
As a first problem, the following complex valued function is to be evaluated:
\[ J^* = \frac{c_2 \cosh(\gamma_1(K+1)) - \frac{L-1}{L+1} \cosh(\gamma_1(K-1))}{\gamma_1 \sinh(\gamma_1(K+1)) - \frac{L-1}{L+1} \sinh(\gamma_1(K-1))} \]

where

\[ c_2 = 3.774 \times 10^{-7}, \quad L = 145.7, \]

\[ K = 7694, \quad r_o = 5.50 \times 10^4, \]

and

\[ \gamma_1 = \sqrt{\omega c_2 r_o} i, \]

with \( \omega \) denoting the (variable) frequency. This function occurs in the investigation of a long line model for the viscoelastic behavior of "standard" polyisobutylene with step-function resistance.

**Background:** The mechanical behavior of many rubberlike polymers can be represented by models which are based on the molecular mechanisms involved in the motion of long chain molecules. Such models are proving valuable in predicting qualitatively and with fair quantitative accuracy many of the mechanical properties of such materials.

These theories frequently involve calculations which become extremely laborious to perform and in some cases are almost impractical to carry out with a desk computer. It is therefore desirable to perform these computations on the IBM 704.

The problem was submitted by R. Marvin (3.4).

**Status:** New. A code has been written and completely checked out that performs the computation of the functions \( J^* \) and \( G^* = 1/J^* \) for 56 frequency values ranging from \( 10^{-8} \) to \( 10^5 \). To facilitate the coding a complex arithmetic abstraction was written which allows the coder to perform complex arithmetic operations directly by use of pseudo operations. The results have been turned over to the sponsor. Discussions about the next computations have been started.

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1102-40-5126/58-341 COSMIC RAYS

**Origin and Sponsor:** Naval Research Laboratory

**Manager:** W. Rheinboldt

**Objective:** 1) To compute a new air density distribution given by

\[ \rho(y) = \rho(y_n) \exp\left( -\frac{1}{d_n} (y-y_n) \right) \quad \text{for} \quad y_n \leq y \leq y_{n+1}, \quad \rho(0) = 1, \]

where the \( d_n \) are the scale heights at the point \( y = y_n \) and are numerically given from experimental data.

2) With this function \( \rho(y) \), to compute the integral

\[ M(h, \theta) = 2.012 \times 10^2 \int_0^{\infty} \rho(-R + \sqrt{(R+h)^2 + x^2 + 2(h+R)x \cos \theta}) \, dx \]
which is connected with the study of absorption of cosmic rays in the earth's atmosphere. The range of parameters is

\[ h = 100(25)200 \quad \text{[miles]} \]
\[ \theta = 0^\circ, 70^\circ, 75^\circ, 80^\circ, 85^\circ, 87.5^\circ \]

and

\[ \theta = 90^\circ(l^\circ) \theta_{\text{max}} \]

where

\[ \theta_{\text{max}} = \frac{\pi}{2} + \arcsin \frac{R}{R+h} \]

and the principal value of the function \( \arcsin \) is taken.

**Background:** High altitude cosmic ray studies necessitate a good knowledge of absorption in the earth's atmosphere at different zenith angles. New density determination based on satellite lifetime measurements make it possible to obtain new results for the problem. The problem was submitted by R. Drachman (NRL).

**Status:** New. A 704 code for the computation of the function \( M(h,\theta) \) has been written and completely checked out. The results were given to the sponsor.

Another 704 code has been written to compute the function

\[ I(y) = \frac{2\pi}{ky} \int_0^{x_m} \frac{x}{1+\exp\left(\frac{x-R}{a}\right)} \left[ \exp(-k|x-y|) - \exp(-k(x+y)) \right] dx \]

for the seven cases:

- \( R_o = 3.3; \ x_m = 6.2; \ y_m = 6.0; \ k = 0.7, 1.0, 1.3; \ a = 0.55 \)
- \( R_o = 4.18; \ x_m = 8.2; \ y_m = 8.0; \ k = 0.7; \ a = 0.55 \)
- \( R_o = 6.5; \ x_m = 11.2; \ y_m = 11.0; \ k = 0.7, 1.0, 1.3; \ a = 0.55 \)

and \( y = 0.2(0.1)y_m \).

This code has also been completely checked out, and the results have been turned over to the sponsor.
Status of Projects

1102-40-5126/58-343 MINIMIZATION PROBLEM

Origin and Sponsor: Naval Research Laboratory
Manager: S. Peavy

Objective: To find the value of $\alpha$ that will give the minimum for each of the following $\epsilon_i$ for each given $\rho$:

\[
\begin{align*}
\epsilon_1 &= T_1 + T_3 + T_4 + W_2 \\
\epsilon_2 &= T_1 + [T_3 + T_4]' + W_2 \\
\epsilon_3 &= \epsilon_1 + T_5 \\
\epsilon_4 &= \epsilon_2 + T_5 \\
\epsilon_5 &= \epsilon_1 + T_6 \\
\epsilon_6 &= \epsilon_3 + T_6 \\
\epsilon_7 &= \epsilon_4 + T_6 \\
\epsilon_8 &= \epsilon_1 - W_2 \\
\epsilon_9 &= \epsilon_7 - W_2
\end{align*}
\]

where

\[
T_1 = \frac{3}{5} k_F^2 + 4\pi \rho a (1 - \frac{1}{p}) + \frac{6\pi \rho}{5p} a(k_F a)^2 + \frac{2\pi \rho}{5p} k_F^2 a^4 \alpha
\]

\[
\frac{4\pi \rho a^2}{h} M[-V_{os}(1/4 - 1/4p + \frac{3}{4(2s + 1)}) - \frac{2}{(2\alpha + \beta_s)^3} - V_{ot}(\frac{3}{4} - \frac{3}{4p} - \frac{3}{4(2s+1)} - \frac{2}{(2\alpha + \beta_t)^3})]
\]

\[
T_3 = \frac{2\pi \rho a^2}{\alpha} (1 - \frac{1}{p}) \alpha
\]

\[
T_4 = \frac{\pi \rho k_F^2 a^2}{\alpha p}
\]

\[
W_2 = -\frac{4\pi \rho a^2 k_F^2 M}{5h} [-V_{os}(\frac{1}{4p} + \frac{3}{4(2s+1)}) (\frac{24}{(2\alpha + \beta_s)^5} + \frac{12a}{(2\alpha + \beta_s)^4} + \frac{2a^2}{(2\alpha + \beta_s)^3})
\]

\[
+ V_{ot}(\frac{3}{4p} + \frac{3}{4(2s+1)} (\frac{24}{(2\alpha + \beta_t)^5} + \frac{12a}{(2\alpha + \beta_t)^4} + \frac{2a^2}{(2\alpha + \beta_t)^3}))]
\]
\[
[ T_3 ^\prime + T_4 ^\prime ] = 4 \pi \rho a^2 k_F \left[ \left( \frac{1}{2} - \frac{52}{35p} \right) \frac{\alpha}{k_F} \right. \\
+ \frac{1}{p} \left\{ \left( \frac{18}{35} + \frac{6 \alpha^2}{5} \right) \tan^{-1} \frac{k_F}{\alpha} + \frac{\alpha^5}{k_F} \left( \frac{3}{5} + \frac{9 \alpha^2}{35 k_F^2} \right) \ln \left( 1 + \frac{k_F^2}{\alpha^2} \right) \\
- \frac{\alpha^3}{k_F^2} \left( \frac{33}{70} + \frac{9 \alpha^2}{35 k_F^2} \right) \right\} \right] \\
T_5 ^\prime = \frac{4 \pi \rho a^3}{p} \alpha^2 \left[ \left( \frac{12}{5} \frac{\alpha}{k_F} + \frac{36}{35} \frac{k_F}{\alpha} \right) \tan^{-1} \frac{k_F}{\alpha} \\
+ \left( \frac{\alpha}{k_F} \right)^4 \left( \frac{12}{5} + \frac{18 \alpha^2}{35 k_F^2} \right) \ln \left( 1 + \frac{k_F^2}{\alpha^2} \right) - \left( \frac{\alpha}{k_F} \right)^2 \left( \frac{18 \alpha^2}{35 k_F^2} + \frac{15}{7} \right) - \frac{48}{35} - \frac{9}{10} \left( \frac{k_F}{\alpha} \right)^2 \right] \\
T_6 ^\prime = \frac{8 \pi \rho a^4}{p} \alpha \left\{ \left[ \frac{3}{5} \frac{\alpha}{k_F} \right]^3 \tan^{-1} \frac{k_F}{\alpha} \\
- \left[ \frac{3}{4} \left( \frac{\alpha}{k_F} \right)^4 + \frac{3}{40} \left( \frac{k_F}{\alpha} \right)^2 + \frac{1}{8} \left( \frac{\alpha}{k_F} \right)^6 + \frac{1}{4} \right] \ln \left( 1 + \frac{k_F^2}{\alpha^2} \right) \\
+ \left( \frac{\alpha}{k_F} \right)^2 \left( \frac{1}{8} \left( \frac{\alpha}{k_F} \right)^2 - \frac{73}{60} \right) + \frac{1}{5} + \frac{119}{800} \left( \frac{k_F}{\alpha} \right)^2 \right\} \\
\right. \\
p = 4 \\
2s + 1 = 2 \\
a = .5 \\
\beta_t = 3.11 \\
\frac{2M}{\hbar} V_{ot} = 16.7 \\
\frac{2M}{\hbar} V_{os} = 9.25
\]

The values of \( \rho \) to be used are .088, .109, .135, .196.

Background: The problem is concerned in finding the minimum energy and density of nuclear matter with the use of the cluster expansion method. The problem was submitted by A. Temkin (NRL).

Status: New. The program has been written for the IBM 704 and checking has been started.

1102-40-5126/58-344 SEMICONDUCTOR SURFACE MEASUREMENTS

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Managers: W. Hall, J. Beiman

Objective: (1) To evaluate

\[
g = \lambda^{\frac{1}{2}} \int_{y}^{0} \frac{(e^{-y}-1) + b \lambda^{-1}(e^{-y}-1)}{[\lambda(e^{-y}-1)+\lambda^{-1}(e^{-4y}-1)+(\lambda-\lambda^{-1})y]^{\frac{1}{2}}} dy.
\]
For $y < 0$, use the positive branch of the square root. 
For $y > 0$, use the negative branch of the square root.
The values of $g$ are proportional to the surface conductivity 
of the space charge layer on a semiconductor.

(2) To obtain data sufficient to plot high accuracy curves of 
$g$ versus the surface potential $Y$ for a wide range of material conductivity 
parameters $\lambda$.

**Background:** In semiconductor surface measurements a known condition exists 
at the minima in the calculated and the experimentally measured curves. 
The charge in the surface states and the surface potential can be found 
by comparison of calculated and experimentally measured surface 
conductivity curves for a material of known bulk conductivity. The 
problem was presented by J. M. Stinchfield.

**Status:** New. The program has been written for $y = -50.0(2)50.0$, and a 
wide range of $\lambda$. The program is in the process of being checked.
6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS
Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Transistor aging studies: An experiment design was devised for evaluating the effect of varying the time between the end of heat treatment and the beginning of testing. This test is being conducted preliminary to a large scale test, and the findings may lead to changes in test procedure. This work is being done for G. Conrad of Section 1.6.

(2) Inventory of spare parts: Work was begun for R. O. Stone and P. Meissner, Section 1.6, on the probabilistic aspects of maintaining an inventory of spare parts for a system with complications introduced by conditions on time of repair.

(3) Standard lamps: A statistical study of the factors affecting measurements of candlepower has been made for H. K. Hammond of Section 2.1. The results of these studies are being used in the design and analysis of current tests.

(4) Statistical analysis of rather large sets of data were carried out for R. E. Michaelis, Section 5.10; M. J. Kerper, Section 9.2; G. Conrad, Section 1.6; R. V. Waterhouse, Section 6.1.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC
Manager: W. J. Youden
Authorized 12/1/51
Full task description: Oct-Dec 1951 issue, p. 58

Status: CONTINUED. Statistical studies on the relation between transition temperature and several physical and chemical properties of two types of ship steel were continued.
Status: CONTINUED. On January 15, C. Eisenhart, J. M. Cameron, W. S. Connor, N. C. Severo and S. Young attended a "Clinic on Statistical Problems in Geology" held by the Committee on Statistics in Geology, Geologic Division, at the U. S. Geological Survey headquarters in Washington. C. Eisenhart outlined some "New developments in statistical treatment of directional properties." As a follow-up on one of the other current problems discussed at the meeting, C. Eisenhart, J. M. Cameron and W. S. Connor visited the laboratory of Lorin Stieff, Nucleonics Group, Geochemistry and Petrology Branch, Geologic Division, USGS, Washington, on February 5th to discuss statistical-design aspects of the scheduling of observations and method of analysis of mass spectrometric data on isotopic abundance.

G. N. Alexander, Water Resources Division, USGS, Washington, conferred at the NBS (a) with W. S. Connor on the use of tables of the non-central t-distribution and (b) with N. C. Severo on the use of the log-normal distribution.

N. C. Severo also was consulted by J. Hack, General Geology Branch, Geologic Division, USGS, on the appropriateness of the Poisson, log-normal, and negative-binomial distributions in studies of gemma-gemma clam distributions.

A memo, "A method for placing confidence limits on preferred directions," by C. Eisenhart and S. Young, based on analysis of variance analogies noted and developed by Sir Ronald Fisher, and by G. S. Watson and E. J. Williams, was sent to F. G. Poole, USGS, Denver, on March 18. A second memo, in preparation as the quarter ended, gives a procedure for determining how many observations are required in order to obtain an estimate of preferred direction having a confidence interval of specified width. The procedure is based on an adaptation to observations, from the von Mises-Fisher family of distributions, of Charles Stein's two-sample procedure (for data from a normal distribution).
Status of Projects

Status: CONTINUED. A preliminary draft of Part I, section 3.1 (Linear Regression) was distributed for comment. A preliminary draft of Part III, sections 3 and 4 (Factorial Experiments and Experiments to Determine Optimum Conditions) is in the process of being reproduced for distribution. Work on preparation of examples for Parts I and III is underway.

STATISTICAL SERVICES
Task 1103-40-5150

Origin and Sponsors: Various Agencies
Manager: J. M. Cameron

Authorized 3/31/58

Objective: To perform statistical services as requested by other agencies of the Federal Government and their contractors.

Background: This project is intended particularly to serve other Government agencies and their contractors whose statistical work is on too small a scale to justify acquisition of a statistical staff, or requires the assistance of specialists not locally available. The experience gained in the performance of such services gives new stimuli to the research staff and benefits the efforts of statisticians in their work on Bureau problems.

Status: NEW. Work was done during the quarter for the following agencies:

(1) Veterans Administration Hospital, Perry Point, Maryland. A covariance analysis with up to nine concommitant variables is being carried out on data on the effect of a number of tranquilizing drugs on mental patients. A special IBM 704 code for analyzing the data on 25 performance measures on each of 800 patients in up to five time periods is being checked out. This work is being done for Dr. M. H. Gordon of the Hospital’s Central NP Research Unit.

(2) General Services Administration. An analysis of data on the effect of time of storage on stockpiled manila rope fiber was done on the IBM 704 for H. LeBovit of the Technical Research and Development Division of the Defense Materials Service.

(3) Bureau of Public Roads. See task 1102-40-5126/58-316, p. 33, for report on this task done jointly with the Computation Laboratory.

(4) Post Office Department. The "Post Office Project" of Division 12, sponsored by the Post Office Department, has as its objectives the performance of operations analysis of mail handling in post offices with the specific aim of recommending and/or developing automatic equipment and techniques for the improvement of the present postal operations. It is hoped that these operations can be improved through (1) speeding the flow of mail, (2) handling more mail without enlarging post office
facilities and/or personnel requirements, and (3) reducing cost of handling the mail.

Various members of the Statistical Engineering Laboratory have been engaged in consulting and advising on the statistical problems encountered by the project for well over two years. Since September 1956 one member of SEL has spent full-time on this work.

Sampling studies have been conducted in the post offices of four cities to determine the statistical nature and various relevant parameters of the flow of letter-mail. This information is being used in the development and design of equipments and systems for improving letter sorting by automation. The following studies have been completed:

(a) Distribution of letter-mail by destination: A sampling plan called the "chain ratio" method, because the formulas involved in the analysis resembled a chain of ratios, was devised for estimating the distribution of mail by destination and was applied in sampling studies conducted in the post offices of San Francisco, Los Angeles and Baltimore. The method proved to be very simple to apply. Data of this type had previously been gathered by costly procedure involving elaborate footage measurements of all of the mail passing through the sorting system during a single fixed time period, such as 24 or 48 hours. On the other hand, chain-ratio sampling was built up from a series of short samples spread over a one week period and involved counting approximately 20,000 letters (about 70 feet of mail) per day compared to measuring about 4000 feet of mail (approximately 1,200,000 letters) per day. Full details of the chain-ratio method and of its applications to the mail of these three cities are given in two NBS Reports by N. C. Severo and A. E. Newman (12.5), entitled "A statistical chain-ratio method for determining the distribution of mail by destination" and "Distribution of mail by destination at the San Francisco, Los Angeles and Baltimore post offices," transmitted to the sponsor in November 1957 and February 1958, respectively. Subsequently, N. C. Severo, M. Zelen and A. E. Newman (12.5) met with H. W. Lieske of the Office of Research and Engineering of the Post Office Department and members of his staff to discuss the meaning and details of the above stated reports.

(b) Letter-mail size and color characteristics: Sampling studies were conducted in San Francisco, Los Angeles, and Washington to determine the distribution of letters by length, height, thickness and color. Control charts were drawn to determine the stability, from sample to sample, of the characteristics studied; and additional statistical tests were then applied to test for differences between cities and between different categories of mail. Statistical tolerance limits were calculated from the data for the sizing characteristics. The results of this study, and of studies (c), (d) and (e) below, are described in detail in an NBS Report by N. C. Severo, A. E. Newman (12.5), S. Young and M. Zelen, entitled "Some applications of statistical sampling methods to outgoing letter mail characteristics," transmitted to the sponsor in February 1958.

(c) Top and bottom clearance space of an addressed envelope: Three studies were conducted to determine top and bottom clearance space of an addressed envelope, i.e., the distances from the top edge of the first line of intelligence of the address to the bottom edge of the envelope.
Separate studies were made for mail from San Francisco, from Los Angeles and from Washington.

(d) Proportions of long and short letters: A study was conducted, using San Francisco and Los Angeles, to determine the ratio of long letters to long plus short letters for a special category of letter mail.

(e) Ratio of hand-cancelled to machine-cancelled mail: A sampling scheme was devised for determining the ratio of hand-cancelled to machine-cancelled letter mail and was applied to Washington mail only.

J. M. Cameron, N. C. Severo and M. Zelen collaborated with members of the Division 12 Post Office Project in devising a method for simulating a sorting machine on the SEAC.

N. C. Severo and M. Zelen attended the periodic meetings of the Division 12 Post Office Project group headed by Mr. I. Rotkin.
The record of the use of the IBM 704 for the period January 1 through March 31 is as follows:

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<td>5126/58-338</td>
<td>Minima by Cox-Prugh</td>
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<td>5126/58-341</td>
<td>Cosmic ray problem</td>
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<td>5126/58-343</td>
<td>Minimization problem</td>
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<td>5126/58-344</td>
<td>Semi-conductor surface measurements</td>
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<td>5126/58-346</td>
<td>GSA data processing</td>
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<td>716</td>
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Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Mathematics Colloquium Series


MESNER, D. M. Embedding theorems for 0,1 matrices. February 7.


Mathematical Statistics Seminar

Papers and Invited Talks
Presented by Members of the Staff at Meetings of Outside Organizations


CONNOR, W. S. (1) Experiments to study the effects of several variables on one or more response variables. Presented at the Wilmington, Delaware Chapter of the American Society for Quality Control, January 9. (2) A fractional factorial experiment of the mixed $2^{m-3}N$ series. Presented before the Rochester Society for Quality Control, Rochester, N. Y., February 18. (3) Use of the direct product of matrices in the analysis of factorials. Presented at a Statistics Seminar, Virginia Polytechnic Institute, Blacksburg, Va., February 21.


MESNER, D. M. An embedding theorem for 0,1 matrices. Presented at a meeting of the American Mathematical Society, Cincinnati, Ohio, January 28-30.

PEARL, M. On Cayley's parameterization, II. Presented at a meeting of the American Mathematical Society, Cincinnati, Ohio, January 28-30.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.3 Technical Papers


Publication Activities

1.4 Reviews and Notes


2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION MARCH 31, 1958

2.1 Mathematical Tables

(1) Tables of the bivariate normal distribution function and related functions. To appear as NBS Applied Mathematics Series 50.

(2) Table of the exponential integral for complex arguments. To appear as NBS Applied Mathematics Series 51.

(3) Integrals of Airy functions. To appear as NBS Applied Mathematics Series 52.

(4) Table of natural logarithms for arguments from five to ten to sixteen decimal places. To appear as NBS Applied Mathematics Series 53.

2.3 Technical Papers


(7) On a converse of a theorem of Pringsheim. P. J. Davis. Submitted to a technical journal.
(8) Additional abscissas and weights for Gaussian quadratures of high order; values for \( n = 64, 80, \) and 96. P. Davis and P. Rabinowitz (Weizmann Institute). Submitted to a technical journal.

(9) Bending and stretching of corrugated diaphragms. R. F. Dressler. Submitted to a technical journal.

(10) Unsteady nonlinear waves in sloping channels. R. F. Dressler. Submitted to a technical journal.


(13) Some elementary inequalities relating to the gamma and incomplete gamma function. W. Gautschi. Submitted to a technical journal.


(24) On the derivative of Bessel functions with respect to the order. F. Oberhettinger. Submitted to a technical journal.


(26) On the convergence of the Rayleigh quotient iteration for the computation of the characteristic roots and vectors, II. A. Ostrowski. Submitted to a technical journal.

(27) A further extension of Cayley's parameterization. M. Pearl. Submitted to a technical journal.


(32) The radial distribution of the center of gravity of random points on a unit circle. F. Scheid. To appear in the Journal of Research, NBS.

(33) The non-central $\chi^2$ as a test statistic. N. C. Severo. Submitted to a technical journal.

(34) Generation of Bessel functions on high speed computers. I. Stegun and M. Abramowitz. Submitted to a technical journal.


(41) Multi-variable experiments. M. Zelen and W. S. Connor. Submitted to a technical journal.

(42) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. S. Joel. Submitted to a technical journal.

2.4 Reviews and Notes

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($0.75), available from the Superintendent of Documents, Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.