NATIONAL BUREAU OF STANDARDS REPORT

5907

PROJECTS and PUBLICATIONS of the APPLIED MATHEMATICS DIVISION

A Quarterly Report

January through March 1958

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

U. S. DEPARTMENT OF COMMERCE
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The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major field laboratories in Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside back cover of this report.

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Electricity and Electronics. Resistance and Reactance. Electron Tubes. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

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Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analogue Systems. Application Engineering.

- Office of Basic Instrumentation
- Office of Weights and Measures

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Systems. Navigation Systems. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Radio Systems Application Engineering.

Radio Standards. Radio Frequencies. Microwave Frequencies. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Calibration Center. Microwave Physics. Microwave Circuit Standards.

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NBS PROJECT	NBS REPORT
11.0	5907

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A Quarterly Report

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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

January 1 through March 31, 1958

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*Only unclassified projects are included in this report.

Status of Projects

March 31, 1958

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-12-1104/55-55

Origin: NBS Authorized 8/29/54
Manager: P. Davis
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. P. Davis investigated the numerical integration of periodic analytic functions by means of the trapezoidal and Gaussian rules. The following conclusions were reached. For the class of analytic periodic functions which are entire and which are regular in a sufficiently large region of the complex plane which includes the interval of integration in its interior, the trapezoidal rule is (asymptotically) better than the Gaussian rule. For analytic periodic functions which are regular on the interval of integration but which possess isolated singularities of a certain type sufficiently close to the interval, the Gaussian rule is (asymptotically) better than the trapezoidal rule. This latter family of functions includes, among others, the Poisson kernel for sufficiently large values of its parameter. These results will be presented at the Madison, Wisconsin Symposium on Numerical Approximation.

A. J. Goldman is revising the manuscript of his joint paper with J. J. Stone, "Two Continuous Poker Games."

W. Rheinboldt has been investigating the topological foundation of asymptotic convergence and comparing it with the concept of convergence defined by a norm. The problem of a unified theory of general asymptotic expansion is considered, and investigation is underway to determine which operations (defined on the function space in question) leave the connection between a function and its asymptotic series invariant.

N. Bazley has been making calculations of lower bounds for eigenvalues. He has discovered how to apply Aronszajn's theory to symmetric operators with a perturbation term. He is using this method to find lower bounds for ground and simply excited energy levels of He and Li^+. He is writing a code for the 704 to evaluate the integrals which arise in these calculations. Also, he is applying the Weinstein-Aronszajn methods for lower bound calculations to a clamped square plate with equal loading on two opposite faces.

Publications:

(1) The condition of matrices, III. J. Todd. J. Res. NBS 60, 1-7 (1957), RP2815.
(2) On a determinantal inequality. M. Marcus. To appear in the American Mathematical Monthly.
(3) Computation problems concerned with the Hilbert matrix. J. Todd. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.
(4) Some extreme value results for indefinite Hermitian matrices. M. Marcus, B. N. Moyles and R. Westwick (University of British Columbia). Illinois J. Math. 1, 449-457 (1957).
(5) Reducible linear differential systems. H. A. Antosiewicz. In manuscript.
(6) Field convexity of a linear transformation. M. Marcus and A. J. Goldman. To appear in the American Mathematical Monthly (Math. Notes).
(7) On doubly stochastic transforms of a vector. M. Marcus. Oxford Quart. J. Math. 9, 74-80 (1958).
(8) On permanents of doubly stochastic matrices. M. Marcus and M. Newman. Submitted to a technical journal.
(9) Essential similarity: A counterexample. A. J. Goldman. Amer. Math. Monthly 65, 546 (1958).
(10) A matrix minimization problem. A. J. Goldman. J. Washington Acad. Sci. 47, 405-406 (1957).
(11) A continuous poker game. A. J. Goldman and J. J. Stone. Submitted to a technical journal.
(12) A method of speeding up iterations with super-linear convergence. A. M. Ostrowski. J. Math. Mech. 7, 117-120 (1958).
(13) On the convergence of the Rayleigh quotient iteration for the computation of the characteristic roots and vectors, II. A. M. Ostrowski. Submitted to a technical journal.
(14) On the bounds of a one-parametric family of matrices. A. Ostrowski. In manuscript.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-12-5116/55-56

Origin: NBS Authorized 8/13/54
Sponsor: Office of Naval Research
Manager: M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. M. Pearl has been working on maximal non-singular vector spaces in total matrix algebras.

K. Goldberg, D. Mesner, and E. C. Dade have examined ways of identifying incidence algebras generated by latin squares with incidence algebras generated by permutation groups.

A seminar on Hypercomplex Numbers is being conducted by M.Newman.

M. Pearl has delivered three lectures on matrix representations of algebras and algebras over the real numbers; M. Newman two lectures on the number theory of quaternions; and A. Goldman two lectures on linear spaces and tensor algebras. The lectures will be continued through May 1958.

M. Newman has completed a set of double length integer subroutines for the 704. These are written to perform modular arithmetic as well. A code to compute the Hermite normal form of an integral mxn matrix using these subroutines has also been completed.

M. Newman is continuing work on the existence of identities for the coefficients of modular forms. A manuscript has been prepared.

A study of congruence properties of the coefficients of the modular invariant $j(\tau)$ modulo 13 has been completed and a manuscript prepared. Divisibility properties of the partition function are now being studied.

Publications:

(1) Incidence algebras. E. C. Dade and K. Goldberg. In manuscript.
(2) Some computational problems concerning integral matrices. O. Taussky. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.
(3) Abelian groups of unimodular matrices. E. C. Dade. Submitted to a technical journal.
(4) Random notes on matrices. K. Goldberg. To appear in the Journal of Research, NBS.
(5) Dense subgraphs and connectivity. R. E. Nettleton (NBS 3.2), K.Goldberg, and M. S. Green (NBS 3.2). Submitted to a technical journal.
(6) The construction of Hadamard matrices. K. Goldberg and E. C. Dade. Submitted to a technical journal.
(7) On normal and EPr matrices. M. Pearl. To appear in the Michigan Journal of Mathematics.
(8) A further extension of Cayley's parameterization. M. Pearl. Submitted to a technical journal.
(9) On a converse of a theorem of Pringsheim. P. Davis. Submitted to a technical journal.
(10) On the minimization of concave and convex functionals. G. B. Dantzig (RAND Corporation), A. J. Hoffman and W. Hirsch (New York University). In manuscript.
(11) Inclusion theorems for congruence subgroups. M. Newman and I. Reiner. (University of Illinois). To appear in Transactions of the American Mathematical Society.
(12) Further identities and congruences for the coefficients of modular forms. M. Newman. To appear in Canadian Journal of Mathematics.
(13) Congruences for the coefficients of modular forms and for the coefficients of $j(\tau)$. M. Newman. To appear in the Proceedings of the American Mathematical Society.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-12-5116/56-148

Origin and Sponsor: National Institutes of Health Authorized 9/30/55
Managers:H. A. Antosiewicz, W. Gautschi (11.2)
Full task description: July-Sept 1955 issue, p. 7

Status: CONTINUED. For the one-dimensional case, calculations are continuing and results are being transmitted to the sponsor as they are computed.

The sponsors have submitted a new set of differential equations which refer to a different type of nerve under conditions in which voltage, current and other variables are functions of the distance along the nerve. Characteristic for the nerve under consideration is the presence of nodes which are located at equidistant intervals. It is assumed that the functioning of the nerve is governed by Hodgkin-Huxley's differential equations, to hold at the nodes, and by the usual equations of a passive cable to hold in between the nodes.

The differential equation for the internodes therefore is a parabolic equation of the form

$$\frac{\partial V}{\partial t} = \frac{1}{r_2 c}\frac{\partial^2 V}{\partial x^2} - \frac{V}{r_o c} \qquad (r_o,\ r_2,\ c \text{ constants}),$$

and the problem consists in finding a continuous solution V subject to certain boundary conditions at the nodes and initial conditions for t=0. If the nodes are located at x=jL (j=0,1,2,...; L > 0) the boundary condition at the j-th node ($j \neq 0$) is of the form

$$C\frac{\partial V}{\partial t} - \frac{1}{r_2}\left\{\frac{\partial V}{\partial x}_{jL+0} - \frac{\partial V}{\partial x}\Big|_{jL-0}\right\} + G(m,n,h;V) = 0,$$

where G is a certain function, linear in V and nonlinear in m,n,h, which already occurred in the previous version of the problem; m,n and h are node functions which are related to V by the same system of differential equations as used previously. A similar boundary condition holds at the first node (j=0).

The initial conditions for t=0 are

$$V = 0 \qquad (0 \leq x < \infty),$$

$$C\frac{\partial V}{\partial t} + G(m_o,n_o,h_o;V) = 0 \qquad (x = jL,\ j \geq 0),$$

where m_o,n_o,h_o are given initial values for m,n,h.

To solve the problem numerically, all differential equations are replaced by explicit finite difference equations. Coding of the problem is in progress.

EVALUATION OF MATRIX COMPUTATION PROGRAM
Task 1101-12-5116/57-200

Origin: NBS Authorized 12/19/57
Sponsor: David Taylor Model Basin
Manager: M. Newman
Full task description: Oct-Dec 1957 issue, p. 4

Status: CONTINUED. A program has been planned for determining the characteristic roots of the matrices which were previously inverted; then comparison will be made between the observed results and the theoretical condition number for each matrix. The purpose of the study is to ascertain whether or not the condition number affects the determination of the characteristic roots in the same way that it does the determination of the inverse.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF $E_1(z)$, $(z = x+iy)$
Task 1102-40-1110/43-3

Manager: I. Stegun — Authorized 7/1/47
Full task description: Apr-June 1949 issue, p. 41

Status: CONTINUED. Printing is under way; proofs were received.

Publication:
(1) Table of the exponential integral for complex arguments. Applied Mathematics Series 51. In press.

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-40-1110/47-2

Origin: NBS — Authorized 7/1/47
Manager: M. Abramowitz
Full task description: Apr-June 1949 issue, p. 45

Status: CONTINUED. A 704 program for computing the regular and irregular solutions for all values of η, ρ and L is now being checked.

TABLES OF POWER POINTS OF ANALYSIS-OF-VARIANCE TESTS
Task 1102-40-1110/51-8

Origin: Section 11.3, NBS — Authorized 3/26/51
Manager: S. Peavy
Full task description: Apr-June 1951 issue, p. 49

Status: INACTIVE.

REVISION OF MATHEMATICAL TABLES
Task 1102-40-1110/52-7

Origin: NBS Authorized 8/10/51
Managers: W. F. Cahill, I. Stegun
Full task description: July-Sept 1951 issue, p. 41

Status: CONTINUED. Printing of the "Table of Natural Logarithms for Arguments from Five to Ten to Sixteen Decimal Places" is under way, for reissue in the Bureau's Applied Mathematics Series. In the past this table was known as Volume IV of "Tables of Natural Logarithms," prepared by the New York Mathematical Tables Project, and was designated as MT12.

Publication:
(1) Table of natural logarithms for arguments from five to ten to sixteen decimal places. Applied Mathematics Series 53. In press.

TABLE OF THE MODIFIED AIRY INTEGRAL
Task 1102-40-1110/52-23

Origin: NBS Authorized 10/4/51
Manager: I. Stegun
Full task description: July-Sept 1951 issue, p. 42

Status: CONTINUED. Printing is under way; proofs were received.

Publication:
(1) Integrals of Airy functions. Applied Mathematics Series 52. In press.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-40-1110/52-37

Origin: NBS Authorized 11/28/51
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE.

SIEVERT'S INTEGRAL
Task 1102-40-1110/52-57

Origin: NBS Authorized 2/12/52
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46

Status: CONTINUED. The introduction to the tables is being revised. The tabular material has been prepared for photo-offset.

HEAT TRANSFER
Task 1102-40-1110/57-241
(formerly 3711-60-0009/57-241)

Origin: NBS Authorized 6/30/57
Managers: M. Abramowitz, W. F. Cahill
Full task description: Apr-June 1957 issue, p. 31

Status: CONTINUED. The coefficients A_n, eigenvalues z_n and eigenfunctions $y(r,z_n)$ for the expansion

$$\frac{\theta - \theta_1}{\theta_o - \theta_1} = \sum_{n=1}^{k} A_n e^{z_n \xi} y(r,z_n)$$

have been computed for $\lambda = 1,10,1000; k = 5, 8$.

The results, where possible, were compared with those given in "Heat Transfer to Hagen-Poiseille Flows," by Knox Millsapps and Karl Pohlhausen, and with those contained in a private communication from S. N. Singh of the Indian Institute of Technology. These comparisons gave excellent agreement.

Publication:
(1) Heat transfer in laminar flow through a tube. M. Abramowitz, W. F. Cahill and C. Wade. In manuscript.

MATHEMATICAL SUBROUTINES
Task 3711-60-0009/56-160

Origin: NBS Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

Status: CONTINUED. Alfred Beam has written and submitted to SHARE, the following 704 subroutine: (1) Floating point integration by Gaussian quadrature; (2) Floating point integration by Laguerre quadrature; (3) Floating point integration by Hermite quadrature, and (4) Double precision floating point Arc Tangent subroutine.

The orthonormalization code (see July-Sept 1957 issue, p. 11) is being modified by P. J. Walsh, and a description is being prepared for SHARE.

A subroutine has been written by J. D. Waggoner to compute XAX' = B, where X is an arbitrary m x n matrix and A and B are symmetric matrices stored in lower triangular form (to be submitted to SHARE).

A general purpose control program has been written and is now being checked out by A. Beam. This program is designed for use in problems involving large amounts of data and many sub-programs which will not fit into the core simultaneously and must be used repeatedly. It reads any subprogram from tape into a specified location in the memory, reads in data (a whole item at a time or any part of an item) from tape to a location determined by the control program, allocates working space in core for data to be computed, and when the computation is finished can, if desired, store this data on tape with an identification tag so that it can be called back in later. It also arranges for data or subprograms to be written over if they are not needed at a specified time in the program.

A preliminary study is being made by E. Haynsworth on new methods of finding eigenvalues of matrices, with particular reference to two recent papers by A. Brauer. These methods are to be adapted for use on the computer.

AUTOMATIC CODING
Task 3711-60-1120/55-65

Origin: NBS Authorized 9/29/54
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. The following service routines and subroutines were presented to the SHARE organization:

(1) Hermite Quadrature Integration, by A. Beam
(2) Laguerre Quadrature Integration, by A. Beam
(3) Gaussian Quadrature Integration, by A. Beam
(4) Double Precision Arctangent Instruction, by A. Beam
(5) Binary to BCD Conversion of Unrestricted Integers, by V. E. Henriques and Genevie Urban (12.5)
(6) BCD to Binary Conversion of Unrestricted Integers, by V. E. Henriques and Genevie Urban (12.5)

Presentations of the CORBIE system were given by J. H. Wegstein at the Computation Center Seminar, M.I.T; at a Symposium on Recent Advances

in Programming Methods, Ohio State University; at a Seminar on Programming, Westinghouse Co., Baltimore; and at a Seminar for Coders, Aberdeen Proving Ground, Md.

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-5113/57-216

Origin and Sponsor: National Science Foundation Authorized 12/27/56
Manager: M. Abramowitz
Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. The following chapters have been completed and submitted to the Bureau's Editorial Committee for review:

Chapter 1. Mathematical constants
3. Powers and roots
4. Elementary transcendental functions
5. The exponential integral and related functions
6. The gamma functions and related functions
7. The error functions and related functions
12. The Struve function and related functions
23. Bernoulli and Euler polynomials

Typing of the tables has been completed for chapters 1 and 3 and is in process for chapters 4, 5 and 6. Work has started on the drafting of the graphs that are included in the above chapters. Final revision of the text of most of the other chapters is in process.

Chapter 5 The Exponential Integral and Related Functions. Tabulation of the integrals $Ei(x)$, $-Ei(-x)$, and $E_n(x)$ has led to the decision to employ the notation $E_1(x)$ for $-Ei(-x)$ and discard the latter notation. Thus the notation $E_1(x)$ will be consistent with that for $E_n(x)$. Also, since $Ei(x)$ is the principal value of the integral $\int_{-\infty}^{x} e^t t^{-1} dt$ for $x > 0$ and

$$Ei(x) = \tfrac{1}{2} [-E_1(-x+i0) - E_1(-x-i0)] \quad \text{for } x > 0,$$

any ambiguity in using $Ei(x)$ and $-Ei(-x)$ is avoided.

A draft of a paper by W. Gautschi, "Evaluation of higher functions by recurrence," has been written. This report which is concerned with the stability of first order recurrence relations indicates how the functions $E_n(x) = \int_1^{\infty} e^{-xt} t^{-n} dt$ and others may be generated in a recursively stable fashion.

Chapter 13 Orthogonal Polynomials. The text for the chapter, by U. Hochstrasser, has been completed. To emphasize the general properties of orthogonal polynomials the functions have been described according to the various properties satisfied by orthogonal polynomials

rather than by treating each polynomial separately. All of the classical polynomials are discussed. Tables are given of the coefficients, in addition to graphs and skeleton tabulations, as a function of the order for certain selected arguments.

Chapter 19 Parabolic Cylinder Functions. A preliminary outline of the material to be included has been prepared by J.C.P. Miller.

Chapter 22 Miscellaneous Functions. This chapter of tables and mathematical properties, by I. A. Stegun, covers a number of mathematical functions such as Planck's radiation functions, Sievert's integrals, Clebsch-Gordon coefficients. The functions have been derived from a number of sources. In the case of the Clebsch-Gordon coefficients the explicit formulas are given for the most important cases, as well as a skeleton table for checking purposes in numerical work.

Chapter 23 Bernoulli and Euler Polynomials, by E. V. Haynsworth and K. Goldberg, was completed and submitted for review. The work is characterized by the parallel display of the properties of the Bernoulli and Euler polynomials. In addition, there has been included a section on the Riemann Zeta function with special emphasis on the tabulation of the sums of reciprocal powers. These series are of fundamental importance in increasing the rate of convergence of series.

Chapter 24 Combinatorial Analysis. The text of this chapter, by K. Goldberg, M. Newman and E. V. Haynsworth, has been completed. The aim has been to collect, generalize and condense most of the elementary combinatorial results that have proved most useful. The format is arranged to present the results in a form suitable for ready reference. Some new results have been included. The following section titles indicate the subject matter:

1. Notation
2. Solutions of Counting Problems
3. Binomial Coefficients
4. Stirling Numbers
5. Number Theory
6. Partitions
7. Symmetric Functions
8. Logical Theorems
9. Power Series
10. Aids to Computation

3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-12-1107/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. N. C. Severo has continued the work on the collection of tables and preparation of examples of their use for the Statistical Tables chapter of the Handbook of Mathematical Functions (see task 1102-40-5113/57-216, p. 10).

Publication:

(1) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. Joel. Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-12-1107/53-1

Origin: NBS Authorized 10/15/52
Manager: W. S. Connor
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. D. M. Mesner continued his studies of incidence matrices. On 20 February 1958 he presented a paper "Embedding Theorems for 0, 1 Matrices" at the NBS Mathematics Colloquium.

Publications:

(1) Multi-variable experiments. M. Zelen and W. S. Connor. Submitted to a technical journal.
(2) The use of group divisible designs for confounded asymmetrical factorial arrangements. M. Zelen. Ann. Math. Stat. 29, 22-40 (1958).
(3) Experiments with many factors. M. Zelen. Proceedings of the Third Annual Statistical Engineering Symposium, Army Chemical Center, Maryland, 2-3 May 1957, pp. 1-12.
(4) The uniqueness of the triangular association scheme. W. S. Connor. Ann. Math. Stat. 29, 262-266 (1958).

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-1107/56-170

Origin: NBS Authorized 12/15/55
Manager: Joan R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

Status: CONTINUED. J. R. Rosenblatt concluded her in-hours course, "Non-parametric statistical techniques." Four students (NBS and DOFL) completed the course.

MEASUREMENT OF RELIABILITY
Task 1103-12-1130/56-182

Origin: NBS Authorized 3/23/56
Manager: M. Zelen
Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. J. R. Rosenblatt attended sessions of the Fourth National Symposium on Reliability and Quality Control in Electronics, Washington, D. C., January 6-8.

Publication:
(1) On some aspects of prediction in the study of complex systems. J. R. Rosenblatt. To appear in the Proceedings of the NYU-RCA Working Conference on Theory of Reliability, held at Ardsley-on-Hudson, New York, April 1957.

FRACTIONAL FACTORIALS FOR THE MIXED SERIES
Task 1103-12-5148/58-291

Origin and Sponsor: Bureau of Ships Authorized 9/30/57
Managers: W. S. Connor, M. Zelen
Full task description: July-Sept 1957 issue, p. 43

Status: CONTINUED. W. S. Connor prepared a report on the progress to date for submission to the sponsor. This report presents a method that facilitates the determination of the normal equations, applies the method to a 1/2 replicate of the $2^3 3^2$ factorial, and illustrates the analysis of this design with real data.

Some of the results of this report were given by Dr. Connor at the Statistics Seminar, Virginia Polytechnic Institute, Blacksburg, Virginia, on February 21.

Publication:

(1) Fractional factorial experiments of the $2^m 3^n$ series. W. S. Connor. To appear in the Transactions of the Rochester Society for Quality Control.

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-12-1115/55-57

Origin: NBS Authorized 9/1/54
Manager: R. F. Dressler
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. The manuscript on compressible fluid flow past a two-dimensional finite wedge, by A. Ghaffari, has been completed. The stream function and velocity potential are found for the steady irrotational ideal compressible flow past a finite wedge for both the supersonic and subsonic regions. Certain analytical properties of the solutions and their derivatives are discussed.

A. Ghaffari has investigated the behavior in the large of the totality of the paths of a dynamical system defined by the nonlinear differential equation

$$dy/dx = [y(x^2+y^2-2x-3)(x^2+y^2-2x-8) + x][x(x^2+y^2-2x-3)(x^2+y^2-2x-8)-y]^{-1}$$

in the phase plane.

It is found that the origin is the only elementary critical point, which is an unstable focus, and the equator is a limit-cycle.

In polar form the equation transforms into

$$d\rho/d\theta = \rho\,(\rho^2-2\rho\,\cos\theta-3)(\rho^2-2\rho\,\cos\theta-8),$$

which is partially discussed by H. Poincaré (Oeuvres, Vol. I, p. 83). One can observe that the circles without contact are defined by $0 < \rho < 1$ and $\rho > 4$, and that the circular region $1 < \rho < 4$ contains only two limit-cycles C_1 and C_2. Thus the totality of the paths of the differential equation consists, aside from the equator and the limit-cycles C_1 and C_2, of a first family of spirals (inside C_1) spiralling away from the origin, a second family of spirals (between C_1 and C_2) spiralling away from C_1, and finally a third family of spirals (between C_2 and the equator) approaching asymptotically the limit-cycle C_2 and the equator.

J. Vinti has begun a study of a problem suggested by W.Wildhack, of the Office of Basic Instrumentation, concerning the rotation of the plane of a satellite orbit produced by the "wind" associated with the rotation of the atmosphere of the earth.

The paper on graphical methods of integration by W. H. Pell has now been completed and is ready for publication.

V. M. Yevdjevic has begun a critical study of the techniques which have been used by hydrologists in the analysis of fluctuations of annual rainfalls and runoffs. Two approaches have been used in the analysis of data regarding rainfall and runoff: (1) that in which the oscillations in these quantities are analyzed by specially developed mathematical techniques; normally, this reveals cyclic fluctuations of various periods; (2) that in which a statistical approach is used, and which frequently yields the results that the occurrence of wet and dry years are purely random, and, indeed, that all fluctuations are of this nature.

It is desirable to establish the correctness of one or the other of these conclusions. The object of Dr. Yevdjevic's study is to investigate the relative reliability of the two procedures, i.e., to answer the question as to whether the sequence of annual precipitations and runoffs follow a cyclic, non-cyclic, or semi-cyclic pattern.

Publications:

(1) On some mathematical properties of wedge solutions. A Ghaffari. In manuscript. Abstract only: Bull. Amer. Math. Soc. 63, 388-389 (1957).
(2) The graphical solution of initial value problems. W. H. Pell. In manuscript.
(3) On the domain of regularity of generalized axially symmetric potentials. P. Henrici. To appear in the Proceedings of the American Mathematical Society.
(4) On the solution of compressible flow past a wedge. A. Ghaffari. In manuscript. Abstract only: Bull. Amer. Math. Soc. 63, 244-245,(1957).

RESEARCH IN CONTINUUM MECHANICS
(formerly Mathematical Elasticity)
Task 1104-12-5160/55-85

Origin: NBS Authorized 12/27/54
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: R. F. Dressler
Full task description: Oct-Dec 1954 issue, p. 30

Status: CONTINUED. W. H. Pell and L. E. Payne are continuing their study on Stokes flows about certain types of axially symmetric bodies. The differential equation for the stream function is $L_{-1}^2 \chi = 0$, where

$$L_{-1} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} ,$$

x and r being cylindrical coordinates. A. Weinstein [see Oct-Dec 1957 issue, p. 20] and L. E. Payne have given decomposition formulas for the solution of equations involving iterated operators, and these results are being applied to Stokes flow problems. Work has been completed on this problem for a spindle (double ogive) and lens-shaped body, and a manuscript is in preparation.

W. H. Pell has completed the manuscript of the paper on the integration of the elastic plate equation for a plate whose thickness varies linearly in two orthogonal directions in the plane of the plate. This generalizes a result of E. Reissner [Ingenieur-Archiv 8, 80-81 (1937)].

A. Walz has written up the results of his most recent work on an approximate theory for the compressible turbulent boundary layer with heat transfer. This is based on an infinite number of integral conditions (ordinary differential equations) derived from the partial differential equations for the velocity and temperature fields. The properties of this system are discussed. It has been found that the two equations of this system most commonly used in less general boundary layer theories, viz., the momentum and energy integral conditions, are most suitable to describe the boundary layer behavior. To use these two equations in the case of compressible turbulent boundary layers with heat transfer, known semi-empirical laws for wall friction and dissipation in an incompressible flow have been generalized. Approximate formulas derived in this connection have been used for calculating some examples, especially the wall friction coefficient of a flat plate. Comparison with available experimental data shows good agreement. Some possibilities for simplification and improvement of the theory have been outlined.

R. F. Dressler has completed a paper on the nonlinear gravity wave study (see publication (6) below).

Dr. Dressler has also completed a manuscript describing exact stress and displacement behavior of a typical corrugated elastic diaphragm shape with a 10:1 range of the thickness parameter. Interpretation and evaluation of the results for practical design purposes is presented. (Publication (7) below.)

By means of the Bogolubov method of statistical mechanics, C. M.Tchen has obtained the Fokker-Planck equation for a plasma. The coefficients of the Fokker-Planck equation are found to depend on the law of interaction between particles; this implies the screening effect of a plasma cloud and additional friction due to momentum relaxation. Dr. Tchen has also solved the problem of time relaxation by a generalization of the Fokker-Planck equation. This result also gave a statistical foundation of the Debye screening effect. A manuscript is in preparation.

Publications:

(1) Stationary principles for forced vibrations in elasticity and electromagnetism. J. L. Synge. To appear in the Proceedings of the Eighth Symposium in Applied Mathematics of the American Mathematical Society, held at Chicago, Ill., April 1956.

(2) The vibration of triangular wings. R. F. Dressler. In manuscript. Abstract available in the Proceedings of the Ninth International Congress on Mechanics, Brussels, September 1956.

(3) The elastic problem for a ring of uniform force in an infinite body. W. H. Pell. To appear in the Journal of Research, NBS.
(4) On the factorization of a fourth order differential operator occurring in the theory of structures. W. H. Pell. In manuscript.
(5) Note on the integration of the elastic plate equation with variable flexural rigidity. W. H. Pell. In manuscript.
(6) Unsteady nonlinear waves in sloping channels. R. F. Dressler. Submitted to a technical journal.
(7) Bending and stretching of corrugated diaphragms. R. F. Dressler. Submitted to a technical journal.
(8) Compressible turbulent boundary layers with heat transfer and pressure gradient in flow direction. Alfred H. Walz. In manuscript.

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-5160/56-154

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: July-Sept 1955 issue, p. 20

Status: CONTINUED. In order to include a supplementary list of the most important pairs of Fourier transforms used by statisticians, a survey of the major statistical journals has been made and a number of pairs most frequently used by statisticians were found and recorded.

RESEARCH IN FLUID DYNAMICS OF TWO-PHASE FLOWS
Task 1104-12-5160/56-155

Origin and Sponsor: Office of Naval Research Authorized 9/30/55
Manager: R. F. Dressler
Full task description: July-Sept 1955 issue, p. 21

Status: CONTINUED. J. Vinti has continued his study of the literature on the discharge of steam jets into water and the connection of this problem with that of the propulsion of underwater devices. He has devised a numerical method for reducing measurements of static pressure, stagnation pressure, and stagnation temperature in superheated steam, to obtain a complete description of the state of the steam at a given point of a flow. The method of reduction depends on whether the flow is subsonic or supersonic, and gives the temperature, density, velocity, and Mach number at the point of measurement.

He has also examined the theory of the thrust obtainable by means of a steam jet discharging into water and, utilizing this, has made

a preliminary analysis of the action of some underwater propulsion devices developed by Aerojet Corporation. The aim of this analysis is to find out whether the essential problem that arises in the theory of such devices is one in two-phase flow and cavitation, as has hitherto been supposed, or whether it is really one in gas dynamics.

Publication:

(1) On the application of steam-driven water jets for propulsion purposes. J. M. Burgers and A. Ghaffari. J. Research NBS, 60, 137-141 (1958), RP2830.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

1102-40-5126/54-13 AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: New York Quartermaster Procurement Agency
Manager: H. Bremer
Full task description: Oct-Dec 1953 issue, p. 43
Status: Continued. During the past quarter a bid evaluation problem was successfully run on the 704 with the IBM transportation code for a 704 without drums. For the sake of trial and checking, a past problem, previously solved on SEAC, was used. Application of the new code involved overcoming some difficulty resulting from the fact that a restriction--that the cost matrix must have more rows than columns--was not clarified in the code write-up. By using the 704 transportation code, it is now possible to handle problems as large as $m+n = 700$ and $n = 600$. (On SEAC the size limitation was $mxn \leq 740$.)

Consideration and study is being given to overcoming the increase in the overall elapsed time required for the solution of an award problem on the 704. The time is much greater than was experienced for solutions on SEAC. For example, a problem that required three hours on SEAC would presently require four to five days on the 704 even though the total computing time on the 704 would be less than the total computing time on SEAC. This is due to the fact that the present 704 code will solve only "one transportation problem" at a time since it will accept only the matrix to be solved and this in the form of cards, whereas the SEAC transportation code was designed specifically for use in solving bidder award problems with the feature of reading into the memory all possible rows that might be needed to make up any desired matrix.

No bid evaluation problems were submitted by the Army Quartermaster Corps during the quarter, due to a slowdown in the purchasing activity.

3711-60-0009/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Managers: C. D. Coleman, W. Bozman (4.1)
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. A list of 12000 spectral lines of thorium I was put on cards and on tape, wave numbers were computed and listed, and differences between known even levels were computed. Lists of wavenumbers are being prepared for energy level searches.

3711-60-0009/54-38 COMPRESSIBILITY FACTORS OF DRY AIR
Origin: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Continued. The 704 code for computing tables of compressibility factors of dry air has been completed. As in the SEAC code the temperature, in degrees Kelvin, is held constant while the pressure, in pounds per square inch, is increased by an increment designated by the sponsor. The format of the printout can be varied quite extensively; this includes a selection of the number of decimal places desired by the sponsor. The calculation of another thermodynamic function, enthalpy, has been added to the code.

3711-60-0009/55-68 CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Division 9
Managers: P. O'Hara, S. Block (9.7)
Full task description: Jan-Mar 1955 issue, p. 18
Status: Continued. A program has been completed for the computation on the 704 of three-dimensional Fourier summations. The program is used to evaluate the electron density throughout a region representing the unit cell of a crystal. The peaks of the electron density function correspond to the locations of individual atoms in the unit cell. The information needed for the calculation consists of several hundred observed structure factors and their associated phase angles.

The electron density is given as

$$(1) \qquad \rho(x,y,z) = \frac{1}{V} \sum_{h=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} |F_{hkl}| \exp\left\{-2\pi i(hx+ky+lz)\right\}$$

where V = volume of unit cell,

$|F_{hkl}| = |A_{hkl} + i\, B_{hkl}|$ = magnitude of observed structure factor,

h,k,l = Miller indices.

The x,y,z can range from 0 to 1 at intervals of 1/64 with the restriction that the maximum grid for any plane be limited to 2048 points. The expression (1) above can also be stated as

$$\rho(x,y,z) = \frac{1}{V}\left[F_{(000)} + 2 \sum_{h=0}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} A_{hkl} \cos 2\pi(hx + ky + lz) + B_{hkl} \sin 2\pi(hx + ky + lz)\right]$$

$$A_{hkl} = |F_{hkl}| \cos \alpha_{hkl}$$

$$B_{hkl} = |F_{hkl}| \sin \alpha_{hkl}$$

$$\alpha_{hkl} = \arctan \frac{B_{hkl}}{A_{hkl}}$$

The Fourier summations are carried out by evaluating one-dimensional summations in three stages and accumulating the sine and cosine terms to obtain a point, line or plane when the index of summation changes. When all the input data have been processed the results for a plane are printed and the entire calculation repeated for the next plane.

The program was checked by computing the electron density distribution for a crystal of known structure.

3711-60-0009/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. Preparation of a code for computing International Temperature Scale constants is in process.

1102-40-5126/55-88 STRESSES IN A WALL FOUNDATION
Origin and Sponsor: NBS, Section 10.1
Full task description: Jan-Mar 1955 issue, p. 22
Status: Inactive.

1102-40-5126/55-117 ATTENUATION OF PRESSURE PULSES OF FINITE AMPLITUDE
Origin and Sponsor: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Apr-June 1955 issue, p. 18
Status: Terminated.

1102-40-5126/55-121 ELECTRON PENETRATION
Origin and Sponsor: NBS Section 4.8
Manager: S. Peavy
Full task description: Apr-June 1955 issue, p. 19
Status: Continued. At the sponsor's request runs were made on the IBM 704 for two compounds,--air and polystyrene.

1102-40-5126/56-139 STUDY OF INTERNUCLEAR POTENTIAL FOR H_3
Origin and Sponsor: NBS, Section 3.2
Manager: E. Haynsworth
Full task description: July-Sept 1955 issue, p. 36
Status: Terminated.

1102-40-5126/56-162 STRESSES IN A WALL RESTING ON A FOOTING
Origin and Sponsor: NBS, Section 10.1
Manager: I. Stegun
Full task description: Jan-Mar 1956 issue, p. 26
Status: Inactive.

1102-40-5126/56-163 ANGULAR DISTRIBUTIONS AND POLARIZATION EFFECTS IN NUCLEAR SCATTERING
Origin and Sponsor: Naval Research Laboratory
Manager: I. Stegun
Full task description: Oct-Dec 1955 issue, p. 32
Status: Continued. A code for the 704 has been written and checked out for computing the polarization, differential cross section and absorption cross section. As a by-product, a subroutine has been obtained for computing the Legendre functions $P^{o}_{\lambda}(\cos\theta)$ and $P^{1}_{\lambda}(\cos\theta)$ for varying λ and θ.

1102-40-5126/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Managers: E. Haynsworth, P. Walsh
Full task description: Jan-Mar 1956 issue, p. 27
Status: Continued. Roots and vectors for matrices of order up to 18 were calculated. The SCF routine (see task 3711-60-0009/57-223, p. 24) was used to give solutions for several double block problems.

1102-40-5126/56-171 COLLISION INTEGRALS USED IN TRANSPORT THEORY
Origin and Sponsor: NBS, Section 3.2
Manager: D. Sumida
Full task description: Oct-Dec 1955 issue, p. 33
Status: Inactive.

1102-40-5126/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: S. Prusch
Full task description: July-Sept 1956 issue, p. 33
Status: Continued. Calculations were performed on nine laboratory sets of gage blocks; each set consisted of a maximum of 88 blocks.

1102-40-5126/57-209 TRAFFIC DISTRIBUTION
Origin and Sponsor: Bureau of Public Roads
Manager: S. Peavy
Full task description: Jan-Mar 1957 issue, p. 32
Status: Inactive.

1102-40-5126/57-211 METEOROLOGICAL DATA
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: P. O'Hara
Full task description: Oct-Dec 1956 issue, p. 30
Status: Terminated.

1102-40-5126/57-219 THERMAL PROPERTIES
Origin and Sponsor: NBS, Section 3.2
Manager: D. Sumida
Full task description: Oct-Dec 1956 issue, p. 30
Status: Completed. A routine for the evaluation of the Debye-equations was coded and checked out. Results have been transmitted to the sponsor.

1102-40-5126/57-221 BESSEL FUNCTIONS FOR COMPLEX ARGUMENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Oct-Dec 1956 issue, p. 31
Status: Continued. The code originally written for SEAC was modified for the 704, to be used with the SEAC interpretative code. Twelve cases were run to evaluate the Bessel and Hankel functions for complex arguments up to order n specified.

1102-40-5126/57-222 ROOTS OF POLYNOMIALS
Origin and Sponsor: Naval Research Laboratory
Manager: J. P. Menard
Full task description: Oct-Dec 1956 issue, p. 32
Status: Inactive.

3711-60-0009/57-223 SELF-CONSISTENT FIELDS
Origin: NBS, Section 3.2
Manager: E. V. Haynsworth
Full task description: Apr-June 1957 issue, p. 28
Status: Continued. The code for calculating the A matrix has been completed and checked out. A subroutine to calculate XSX^T (where S is symmetric and stored triangularly) has been added to the SCF program. This subroutine will be used to check the accuracy of the results. The coding of the control program has been completed and code checking is in progress. Modifications are being made in the SCF program to make use of the control program for data manipulation.

1102-40-5126/57-224 TRACK-WHILE-SCAN RADAR PROBLEM
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Managers: E. Haynsworth, P. J. Walsh
Full task description: Oct-Dec 1956 issue, p. 32
Status: Continued. The formulae in the Jan-Mar 1957 issue (p.34) were modified to eliminate integration with respect to Ø. Calculations of the modified $M(\bar{\emptyset})$ were performed using several different parameters and keeping Ø constant. The results have been submitted to the sponsors for analysis.

3711-60-0009/57-229 APPLICATION OF ELECTRONIC DATA PROCESSING MACHINERY TO PAYROLL OPERATIONS

Origin: NBS, Section 40.0
Managers: H. Bremer, P. R. McClenon, M. Paulsen
Full task description: Jan-Mar 1957 issue, p. 36
Status: Continued. During the past quarter programming and code-checking has continued on many of the subroutines that go into producing the payroll program for the 704. Two important subroutines have been written for handling the control of the input and output tapes. Use of these subroutines will reduce the amount of work necessary for producing the tape outputs. If the present rate of progress continues, it should be possible to carry out a test run by the end of the current fiscal year.

1102-40-5126/57-234 PERSONNEL SURVEY

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: P. O'Hara
Full task description: Jan-Mar 1957 issue, p. 37
Status: Inactive.

1102-40-5126/57-236 SELF CONSISTENT FIELDS--EIGENVALUES

Origin and Sponsor: NBS, Section 3.6
Manager: E. Haynsworth
Full task description: Apr-June 1957 issue, p. 30
Status: Continued. The routine for controlling the subroutines has been checked out. Also, results have been obtained for single and double block matrices.

3711-60-0009/57-247 MECHANICAL IMPEDANCE

Origin: NBS, Section 6.1
Managers: J. P. Menard, M. D. Burkhard (6.1)
Full task description: Apr-June 1957 issue, p. 32
Status: Continued. Additional groups of data on the measurements of impedances of the forehead and mastoids of young subjects 18 to 25 years old have been furnished directly by the sponsor for processing on the 704. The computer is used to determine parameters of mass, stiffness and resistance. The computations are used in connection with the development of an artificial mastoid to check bone conduction hearing aids.

3711-60-0009/57-248 THE EVALUATION OF A TRIPLE INTEGRAL FOR THE SOLUTION OF NEGATIVE ION DETACHMENT

Origin: NBS, Section 4.6
Manager: S. Peavy
Full task description: Apr-June 1957 issue, p. 34

Status: Continued. The original problem has been altered to include another integral:

$$I_4 = \int_0^{1-t^2} ds^2 \frac{sq}{(s^2+t^2)} \int_{-1}^{1} \frac{d\alpha}{(1+q^2-2q\alpha)^{\frac{1}{2}}} \int_{-1}^{1} d\beta \frac{H^2(K)}{K^2}$$

$$\cdot[2s\beta-(1+q^2-2q\alpha)^{\frac{1}{2}}]\left[(1-s^2)^2+4s^2\left\{\beta-s\frac{(1-q\alpha)}{(1+q^2-2q\alpha)^{\frac{1}{2}}}\right\}\left\{\beta-\frac{1}{s}\frac{(1-q\alpha)}{(1+q^2-2q\alpha)^{\frac{1}{2}}}\right\}\right]^{-\frac{1}{2}}$$

The code is now being checked out.

3711-60-0009/57-250 AUTOMATIC REDUCTION OF SPECTROPHOTOMETRIC DATA
Origin: NBS, Section 2.1
Manager: W. C. Rheinboldt
Full task description: July-Sept 1957 issue, p. 31
Status: Continued. Approximately 500 runs were made during the quarter under the direction of the sponsor for the automatic reduction of partially corrected spectrophotometric data to fully corrected spectrophotometric data and into colorimetric terms.

1102-40-5126/57-251 CURRENT NOISE AND FIXED RESISTORS
Origin and Sponsor: NBS, Section 1.6
Manager: D. Sumida
Full task description: July-Sept 1957 issue, p. 32
Status: Inactive.

3711-60-0009/58-254 REPRODUCTION OF COLOR- AND SPECTRAL-ENERGY DISTRIBUTION OF DAYLIGHT AND OTHER ILLUMINANTS
Origin: NBS, Section 2.3
Manager: W. C. Rheinboldt
Full task description: July-Sept 1957 issue, p. 32
Status: Continued. Some additional runs on the 704 have been made under the immediate direction of the sponsor. The computations have facilitated and expedited the development of filters for the laboratory reproduction of daylight and high intensity incandescent sources (photoflood and flash lamps) to be considered by the American Standards Association for use in American standards for sensitometry of black and white and color films. These filters with a calibrated light source produce a very close energy match and a perfect color match with the ideals. This would bring the field of sensitometry up to date for modern films.

1102-40-5126/58-263 GAS TUBE CHARACTERISTIC
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: W. F. Cahill
Full task description: July-Sept 1957 issue, p. 35
Status: Continued. Runs have been made for 25 sets of parameters, and the results have been transmitted to sponsor.

1102-40-5126/58-264 THEORY OF IONIZATION PROBABILITY
Origin and Sponsor: NBS, Section 4.6
Manager: S. Peavy
Status: Inactive.

3711-60-0009/58-266 DEPOLYMERIZATION, II
Origin and Sponsor: NBS, Section 7.6
Manager: L. S. Joel
Full task description: July-Sept 1957 issue, p. 36
Status: Continued. A code for generation of the function for case II, Terminal Initiation, has been written and checked.

A code for Runge-Kutta integration of differential equations using Gill's method (Proc. Cambridge Philos. Soc. 1951, pp. 96-116) has been written and is being checked. The first large system to be solved has 403 equations.

3711-60-0009/58-267 CONVERSION OF THE CIE-CHROMATICITY COORDINATES INTO THE MUNSELL COLOR SYSTEM
Sponsor: NBS, Section 2.1
Manager: W. C. Rheinboldt
Full task description: July-Sept 1957 issue, p. 37
Status: Inactive. For status to date, see Oct-Dec 1957 issue, p. 32.

3711-60-0009/58-268 RESPONSE FUNCTION CALCULATION
Origin: NBS, Section 4.11
Manager: A. Beam
Full task description: July-Sept 1957 issue, p. 37
Status: Continued. The code was checked and turned over to the sponsor. It is used by the sponsor as the need arises.

1102-40-5126/58-269 MOLECULAR STRUCTURE, IV
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. J. O'Hara
Full task description: July-Sept 1957 issue, p. 38
Status: Continued. Phase determination calculations for aureomycin are being carried out at the sponsor's request.

1102-40-5126/58-272 THERMODYNAMIC PROPERTIES OF REAL GASES
Origin and Sponsor: NBS, Section 3.2
Manager: J. P. Menard
Status: Continued. IBM 704 codes have been written and completely checked out for performing the following differentiations:

$$\frac{d}{d \ln \rho/\rho_o} \left.\frac{PV}{RT}\right|_{T=\text{const}}, \qquad \frac{d}{d \ln \rho/\rho_o} \left.\frac{E}{RT}\right|_{T=\text{const}},$$

$$\frac{d}{d \ln T} \left.\frac{PV}{RT}\right|_{\rho=\text{const}}, \qquad \frac{d}{d \ln T} \left.\frac{E}{RT}\right|_{\rho=\text{const}},$$

where PV/RT ≡ compressibility factor
E/RT ≡ internal energy
ρ ≡ density of given gas.

The codes are set up to handle a variable number of values of the function for equally or unequally spaced values of the argument. Since the table of compressibility factors and internal energies, as submitted by the sponsor, was tabulated with respect to the density for a given temperature, a feature was incorporated in the code for rearranging the values with respect to the temperature for a given density. Furthermore, it was necessary to set up a scheme which would take care of missing values in the data. This was accomplished in two steps. First, a flag word was inserted in the value table wherever a value was missing for a given argument. Then the code was set up to delete both the flag and the corresponding argument value and compress the function to be differentiated. The differentiation was then performed with the assumption that the argument values were unequally spaced.

Production runs have been made, and we are now awaiting further communications from the sponsor for future runs.

1102-40-5126/58-274 CALCULATIONS FOR d-SPACINGS, II
Origin and Sponsor: NBS, Division 9
Manager: R. Zucker
Full task description: July-Sept 1957 issue, p. 38
Status: Continued. About 19 calculations for d-spacings for orthogonal, hexagonal and orthorhombic crystals were carried out this quarter, and redetermination of unit cell constants by least squares fitting to a measured d-spacing was performed for about 26 crystals.

1102-40-5126/58-279 FIRE RESISTANT T-BEAM
Origin and Sponsor: NBS, Section 10.2
Manager: C. Wade
Full task description: Oct-Dec 1957 issue, p. 33
Status: Continued. Exploratory calculations have been made for one type of concrete structure in the two-dimensional case for several combinations of grid block width and time increment.

For the one-dimensional case a code has been written which computes temperature on a line through the concrete from the air surface to the fire surface. The same expression for the gradient as in the two-dimensional case is used at the air surface. The code is now being checked out.

1102-40-5126/58-282 MISSILE BOUNDARY LAYER COMPUTATION
Origin and Sponsor: Naval Ordnance Laboratory
Manager: R. Danek (NOL)
Objective: To compute the laminar and turbulent boundary layer growth about a missile as it traverses its trajectory.
Background: This problem involves the stepwise integration along the profile of a body shape of quantities specified by a series of inter-related equations, for each chosen time step in the course of the trajectory. Although a simple heat transfer relation exists in the problem at present, a more complete time-dependent treatment of this phase of the problem is being composed. This computation was initially coded for the IBM 650 at NOL but due to its time consuming nature and the desire to extend the original formulation to the time-dependent scheme, the use of the 704 was deemed advisable.
Status: New. The problem has been analyzed and the original version has been coded and checked out for 704 using FORTRAN. Programming of the problem with heat conduction as described above is under way, although some details require further analysis.

1102-40-5126/58-284 EPHEMERIS CALCULATIONS FOR SATELLITES
Origin and Sponsor: Naval Research Laboratory
Managers: W. F. Cahill, J. H. Wegstein
Full task description: Oct-Dec 1957 issue, p. 34
Status: Continued. Computations on the trajectories of Sputnik I and Explorer I were made. Results are being analyzed by the sponsor.

1102-40-5126/58-289 SCATTERING OF ELECTRONS BY HYDROGEN
Origin and Sponsor: NBS, Section 3.6
Manager: R. Zucker
Full task description: Oct-Dec 1957 issue, p. 36
Status: Inactive. For status to date, see Oct-Dec 1957 issue, p. 36.

3711-60-0009/58-294 NUCLEAR SCATTERING OF PHOTONS
Origin: NBS, Section 4.8
Manager: J. P. Menard
Full task description: Oct-Dec 1957 issue, p. 36
Status: Continued. A code has been written and completely checked out. A number of production runs have been made. The program will continue in production under the sponsor's direction.

1102-40-5126/58-297 "MANY BODY" PROBLEM
Origin and Sponsor: Naval Research Laboratory
Manager: S. Peavy
Full task description: Oct-Dec 1957 issue, p. 38
Status: Inactive.

1102-40-5126/58-298 ANALYSIS OF SPECTROCHEMICAL DATA
Origin and Sponsor: NBS, Section 5.10
Managers: S. Peavy, R. Ridinger
Full task description: Oct-Dec 1957 issue, p. 39
Status: Continued. The code for the problem has been checked out on the 704, and a few runs have been made at the request of the sponsor.

1102-40-5126/58-299 TIME-DEPENDENT SCHRODINGER EQUATION
Origin and Sponsor: NBS, Section 3.1
Manager: J. Beiman
Full task description: Oct-Dec 1957 issue, p. 39
Status: Continued. The code has been checked out, and the problem is in production. A number of runs have been made under varying initial conditions.

1102-40-5126/58-300 LAMINAR MIXING IN BOUNDARY LAYERS
Origin: Polytechnic Institute of Brooklyn
Sponsor: Air Force Office of Scientific Research
Manager: W. C. Rheinboldt
Full task description: Oct-Dec 1957 issue, p. 40
Status: Continued. In order to solve the boundary value problem in question the following iteration scheme has been outlined:

Given $G^{(n)}(\sigma)$ as the n-th approximation of $g''(\sigma)$ to evaluate the auxiliary constant,

$$c^{(n)} = k + \int_{-\infty}^{o} G^{(n)}(\sigma)\, d\sigma$$

and successively the auxiliary functions,

$$h^{(n)}(\sigma) = c^{(n)} + \int_0^{\sigma} G^{(n)}(\tau)\, d\tau,$$

$$K^{(n)}(\sigma) = \exp[-\tfrac{1}{2}(c^{(n)}\sigma^2 + \int_0^{\sigma} (\sigma-\tau)^2\, G^{(n)}(\tau)\, d\tau)],$$

$$T^{(n)}(\sigma) = \frac{1-k^2}{\int_{-\infty}^{+\infty} K^{(n)}(\tau)d\tau} \int_{-\infty}^{\sigma} K^{(n)}(\tau)d\tau + k^2,$$

$$\hat{G}^{(n)}(\sigma) = \beta \int_0^{\sigma} [(h^{(n)}(\tau))^2 - T(\tau)]\, \frac{K^{(n)}(\sigma)}{K^{(n)}(\tau)}\, d\tau .$$

Then the next approximation $G^{(n+1)}(\sigma)$ is given by

$$G^{(n+1)}(\sigma) = \frac{1 - k - \int_{-\infty}^{+\infty} \hat{G}^{(n)}(\tau)\, d\tau}{\int_{-\infty}^{+\infty} K^{(n)}(\tau)d\tau} K^{(n)}(\sigma) + \hat{G}^{(n)}(\sigma).$$

This iteration scheme has been coded, and the 704 code has been completely checked out.

An initial function,

$$G^{(o)}(\sigma) = \frac{1-k}{2}\sqrt{\frac{1+k}{\pi}}\, e^{-\frac{1+k}{2}\frac{\sigma^2}{2}}$$

has been used. Furthermore provision has been made to read in a numerically given initial function. The code has been run for 12 different combinations of the parameters k and β. It has been found that the iteration is converging for $-0.3 \leq \beta \leq 0.4$, $0.1 \leq k \leq 0.9$. The resulting functions have been checked with Runge-Kutta method for the solution of the initial value problem of the given system of equations. The result has been shown to be a solution to the system in question and valid to 6 significant figures.

Tests have been made with a different step-width in order to find its influence in accuracy, speed of iteration, etc. While the physical nature of the problem indicates that convergence may fail for $\beta < -0.3$, it is still an open question as to why the process fails to give convergence for $\beta > 0.4$. Numerical experiments have been started in order to find an answer to this question. However, insufficient funds have made it necessary to discontinue the investigations temporarily.

1102-40-5126/58-304 TRANSPORT PROPERTIES OF AIR AT ELEVATED TEMPERATURES
Origin and Sponsor: NBS, Section 3.2
Manager: P. J. Walsh
Full task description: Oct-Dec 1957 issue, p. 40
Status: Continued. The third phase of the problem has been coded and completely checked out. The formulae involved are similar to those described in the Oct-Dec 1957 issue. The results for several temperatures are now being analyzed by the sponsor.

A set of formulae are to be coded which will generate the $\Omega_{ij}^{(k,l)}$ used in computing the matrix elements. These are the collision integrals which depend upon a set of collision cross-sections calculated under another project.

1102-40-5126/58-306 INTERPOLATION OF COLOR MIXTURE FUNCTIONS
Origin and Sponsor: NBS, Section 2.1
Manager: W. C. Rheinboldt
Full task description: Oct-Dec 1957 issue, p. 42
Status: Continued. The 704 code has been written and completely checked out. The operation of this code has been turned over to the sponsor, and a number of production runs have already been made.

1102-40-5126/58-307 STUDY OF SURFACE TENSION
Origin and Sponsor: NBS, Section 9.2
Manager: R. Arms
Full task description: Oct-Dec 1957 issue, p. 43
Status: Reactivated. Several additional calculations were performed for the sponsor.

1102-40-5126/58-308 OSCILLATING SPHERE
Origin and Sponsor: NBS, Section 3.4
Manager: S. Prusch
Full task description: Oct-Dec 1957 issue, p. 43
Status: Continued. Additional production runs are being analyzed by the sponsor.

1102-40-5126/58-311 NULL RATE CALCULATIONS
Origin and Sponsor: Naval Research Laboratory
Manager: J. H. Wegstein
Full task description: Oct-Dec 1957 issue, p. 44
Status: Continued. Numerous sets of minitrack data were analyzed by this computer program, and the results were transmitted to the sponsor. Several modifications were made in the code for more conveniently handling the data.

1102-40-5126/58-312 RESPONSE FUNCTION, II
Origin and Sponsor: NBS, Section 4.11
Manager: A. Beam
Objective: (1) To invert a 56x56 matrix A. (2) To read in a large group of data and smooth it into a 56-element column vector V. (3) To compute $A^{-1}V = E$.
Background: In the measurement of the energy spectrum of 5 to 115 Mev X-rays, the spectrometer has resolution much poorer than that associated with optical instruments. The response to a Mono energetic x-ray beam is termed the response function. A matrix A, made up of response functions for a family of x-ray energies may be used to convert an energy spectrum vector to a smeared spectrum called a pulse height distribution. Likewise a pulse height distribution vector V may be used to multiply the inverted response function matrix to yield an energy spectrum vector, E.

The problem was communicated by J. Wyckoff (4.11).

Status: New. Computation using the inverse matrix was invalid because of the large magnitude of the elements of the inverse and the nature of the input data. Considerable effort was made to get a valid result by modifying the matrix and vector input, but this effort met with little success. Hence, the code was modified to do multiplication with the original matrix and turned over to the sponsor, who has used it in this form on several occasions.

1102-40-5126/58-314 APPROXIMATIONS FOR GAS MIXTURES
Origin and Sponsor: NBS, Section 3.2
Manager: R. Zucker
Objective: To test approximations for gas mixtures and to examine the relaxation of the thermal conductivity of multicomponent gas mixtures due to thermal diffusion. Binary and ternary systems are considered. Expressions for the elements of determinants of orders 4 and 7 are to be evaluated for given temperatures and components.
Background: The problem arises in gas mixtures and was requested by C. Muckenfuss (3.2).
Status: New. Numerical calculations were carried out, and the results were transmitted to sponsor. They were in good agreement with the experimental work.

1102-40-5126/58-316 INTERSECTION CAPACITY STUDY
Origin and Sponsor: Bureau of Public Roads
Managers: S. Peavy, J. M. Cameron
Objective: To analyze data on flow of vehicular traffic through signalized intersection to determine equations for predicting the flow of such traffic under a variety of conditions.
Background: The procedures for predicting flow through an intersection given in "Highway Capacity Manual," Bureau of Public Roads(U. S. Government Printing Office, Washington, D. C., 1950) are to be applied to data taken since 1950. Some recent studies by the Bureau of Public

Roads have indicated that extrapolation from the previous results may be subject to error and it is desired to recompute the predicting equations using the new data to ascertain if the patterns in traffic flow have changed. The problem was communicated by A. A. Carter, Jr.
Status: New.

1102-40-5126/58-321 TABLE OF THERMODYNAMIC FUNCTIONS OF SULFUR
Origin and Sponsor: NBS, Section 3.2
Manager: R. Zucker
Objective: To prepare a table of thermodynamic functions of pure sulfur from smoothing equations, for given values of temperature.
Background: The table is needed for use in connection with a new and more accurate determination of the heat capacity of sulfur from experimental values. It was requested by E. D. West (3.2).
Status: New. The equations for monoclinic sulfur were evaluated for a lower and for an upper range of temperatures. The results were transmitted to the sponsor.

1102-40-5126/58-322 THE PROPAGATION CONSTANT OF A SOUND WAVE
Origin and Sponsor: NBS, Section 6.1
Manager: R. J. Arms
Objective: To calculate the roots of a sixth degree complex polynomial whose coefficients are functions of a single parameter.
Background: Roots of the polynomial give the propagation constant $K=\alpha+i\beta$, where α and $1/\beta$ are non-dimensional forms, respectively, of the amplitude attenuation per unit length and the wave length for a sound wave in a Maximillian diatomic gas. K is given as a function of the parameter $x = w\mu/p$ ($w = 2\pi \times$ frequency, μ = viscosity, p = pressure). The computation was requested by M. Greenspan (6.1).
Status: New. The code has been written and checked out on the 704. The results of several runs have been submitted to the sponsor.

1102-40-5126/58-324 FITTING A POLYNOMIAL TO A CURVE
Origin and Sponsor: NBS, Section 4.5
Managers: S. Peavy, R. Ridinger
Objective: To fit a curve through various known coordinates and then find values of the functions corresponding to other given coordinates. Use is to be made of an orthonormalizing routine to determine the coefficients of the polynomial.
Background: On spectra photos used by the sponsor, measurements were made previously with calibrations at several intervals. In order to save the time of taking many measurements, the curve-fitting procedure using the 704, is being tried. The problem was submitted by E. Horl (4.5).
Status: New. The code has been written and checked. Calculations are being made for each new set of data presented by the sponsor.

1102-40-5126/58-328 MAGNETIC FIELD IN A THIN FERRO-MAGNETIC SHEET DUE TO A LOCAL SOURCE

Origin and Sponsor: NBS, Section 6.1

Manager: S. Prusch

Objective: To evaluate

$$y = \frac{2}{\pi}(1-\rho)\left\{ \tan^{-1}\frac{1}{x} + \rho^2 \tan^{-1}\frac{1}{x+2c} + \rho^4 \tan^{-1}\frac{1}{x+4c} + \ldots \right.$$

$$\left. + \rho\left[\tan^{-1}\frac{1}{2c-x} + \rho^2 \tan^{-1}\frac{1}{4c-x} + \rho^4 \tan^{-1}\frac{1}{6c-x} + \ldots\right]\right\}$$

for ρ = 1/5, 1/3, 1/2, 3/5, 5/7, c = 0.25, 0.5(.5)2.0, and x = 0(.2)1.0.

Background: The relevant field distribution of the source in the absence of the sheet is known. When the sheet is placed near the source, the magnetic field within it can be expressed in infinite series form by using image theory (see Maxwell's Electricity and Magnetism, Vol. 1, p. 443). The problem arose in connection with a magnetic tape recording investigation. The computation was requested by E. Daniel (6.1).

Status: Completed (New). The code was written and checked out on the 704. The computations were carried out and were transmitted to the sponsor.

1102-40-5126/58-330 EPITRON ORBITS

Origin and Sponsor: Naval Research Laboratory, Department of the Navy

Manager: R. J. Arms

Objective: To calculate, by elementary functions, the path of particles in a numerical model of an epitron.

Background: The calculations are for orbits to be expected in an epitron, which is an experimental machine for the testing of magnetic self-focusing. The problem was submitted by J. B. Ehrman (NRL).

Status: New.

1102-40-5126/58-331 COMPUTATION OF HYDRAULIC ELEMENTS

Origin and Sponsor: NBS, Section 10.5

Manager: R. Ridinger

Objective: To compute the values of the following hydraulic elements for a circular pipe section as a function of depth.

1) $\frac{R}{R_f} = \left[1 + \frac{\sin\theta}{2\pi(1 - \frac{\theta^o}{360})}\right]$

2) $\frac{A}{A_f} = \left[(1 - \frac{\theta^o}{360}) + \frac{\sin\theta}{2\pi}\right]$

3) $\frac{Q}{Q_f} = (\frac{A}{A_f})(\frac{R}{R_f})^{2/3}$

4) $\frac{a}{a_f} = (1 - \frac{\theta^o}{360})$

5) $\frac{V}{V_f} = (\frac{R}{R_f})^{2/3}$

6) $\theta = 2\cos^{-1}(2x-1)$

Background: The need arises for computing the volume rate of discharge through a pipe for conditions other than when the pipe flow is full or half full. Rough curves in handbooks show these relationships, but their accuracy is questionable in certain ranges. Thus values were computed using the above equations where x, ranging from 0 to 1 by increments of .01, represents the ratio of the greatest possible depth to the variable depth.

The problem was communicated by R. S. Wyle.

Status: Completed (New). The computations were carried out on the 704, and the results were transmitted to the sponsor.

1102-40-5126/58-333 CALCIUM HYDROXIDE

Origin and Sponsor: NBS, Section 9.0

Manager: P. J. O'Hara

Objective: To calculate structure factors and do a least square fit of atomic position parameters for the calcium hydroxide crystal.

Background: These calculations are an application of task 1102-40-5126/58-269 to the study of the calcium hydroxide crystal (see July-Sept 1957 issue, p. 38). Computations will be carried out using the 704 program NY x R2. The problem was submitted by F. Ordway (9.0).

Status: New.

1102-40-5126/58-334 PRICE COMPUTATION TABLE

Origin and Sponsor: NBS, Section 32.0

Manager: R. Zucker

Objective: To check a price table for weights ranging from 1/4 oz. to 5 lbs. at 1/4 oz. gradations for costs per ounce ranging from 1 to 9 cents at 1 cent gradations

Background: The table is to be used by weight inspectors in checking the accuracy of scales. The computation was communicated by H. Wallin (32.0).

Status: Completed (New). The table was checked, and the results were submitted to the sponsor.

1102-40-5126/58-336 HELICAL TRANSFORMS

Origin and Sponsor: National Institutes of Health

Manager: R. Ridinger

Objective: To calculate $A_n^2 + B_n^2$ for each value of R and L, where L is the layer line and

$$A_n = \sum_j f_j J_n(2\pi r_j R) \cos\left\{ n\left(\frac{\pi}{2} - \phi_j\right) + \frac{2\pi L z_j}{c} \right\},$$

$$B_n = \sum_j f_j J_n(2\pi r_j R) \sin\left\{ n\left(\frac{\pi}{2} - \phi_j\right) + \frac{2\pi L z_j}{c} \right\}.$$

Background: Formulae are given for the Fourier transforms of a number of helical structures. These have been used successfully to verify the presence of the α-helix in the synthetic polypeptide, poly-r-methyl-L-glutamate.

Since then general predictions have been made concerning the intensities of the x-ray diffraction pattern for other helical structures. These predictions do not necessarily result in correct distribution of layer line intensities. Thus it is desired to calculate the helical intensities in order to compare the observed intensities from model building with the calculated results.

At a point in reciprocal space with cylindrical coordinates $(R,\psi, L/c)$ the Fourier transform is:

$$F(R,\psi, L/c) = \sum_n \sum_j^{\bullet} f_j J_n(2\pi r_j R) \exp\left[i \left\{ n(\psi-\phi_j+\frac{\pi}{2}) + \frac{2\pi L z_j}{c} \right\}\right]$$

where r_j, ϕ_j, z_j are the atomic coordinates. This expression can be written

$$F(R,\psi,L/c) = \sum_n (A_n + B_n)\exp \text{ in } \psi$$

where A_n and B_n are the calculated values.

The problem was communicated by A. Rich (NIH).

Status: New. A flow diagram has been drawn, the coding has been completed, and checking is in progress.

1102-40-5126/58-337 GEORGETOWN LANGUAGE TRANSLATION EXPERIMENTS

Origin and Sponsor: Georgetown University

Manager: R. J. Arms

Objective: To investigate usage of the Bureau's computer for Georgetown machine translation experiments, and to offer consulting services about computers to the Georgetown project.

Background: At Georgetown University, Department of Languages and Linguistics, there are about four groups individually developing methods of machine translation. Most of these groups have reached a stage of progress where computer experiments would be of aid in their research. The problem was submitted by L. Dostert (Georgetown University).

Status: New. A small computer experiment has been planned with the group headed by Dr. P. Garvin with supervision of the manager. The coding will be done at Georgetown.

1102-40-5126/58-339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS

Origin and Sponsor: NBS, Section 3.4

Manager: W. Rheinboldt

Objective: To perform computations on the IBM 704 that become necessary in connection with the investigation of the mechanical behavior of certain polymers.

As a first problem, the following complex valued function is to be evaluated:

$$J^* = \frac{c_2}{\gamma^1} \frac{\cosh(\gamma_1(K+1)) - \frac{L-1}{L+1}\cosh(\gamma_1(K-1))}{\sinh(\gamma_1(K+1)) - \frac{L-1}{L+1}\sinh(\gamma_1(K-1))}$$

where

$c_2 = 3.774 \times 10^{-7}$, $L = 145.7$,

$K = 7694$, $r_o = 5.50 \times 10^4$,

and $\gamma_1 = \sqrt{\omega c_2 r_o i}$,

with ω denoting the (variable) frequency. This function occurs in the investigation of a long line model for the viscoelastic behavior of "standard" polyisobutylene with step-function resistance.

Background: The mechanical behavior of many rubberlike polymers can be represented by models which are based on the molecular mechanisms involved in the motion of long chain molecules. Such models are proving valuable in predicting qualitatively and with fair quantitative accuracy many of the mechanical properties of such materials.

These theories frequently involve calculations which become extremely laborious to perform and in some cases are almost impractical to carry out with a desk computer. It is therefore desirable to perform these computations on the IBM 704.

The problem was submitted by R. Marvin (3.4).

Status: New. A code has been written and completely checked out that performs the computation of the functions J* and G* = 1/J* for 56 frequency values ranging from 10^{-8} to 10^5. To facilitate the coding a complex arithmetic abstraction was written which allows the coder to perform complex arithmetic operations directly by use of pseudo operations. The results have been turned over to the sponsor. Discussions about the next computations have been started.

1102-40-5126/58-341 COSMIC RAYS

Origin and Sponsor: Naval Research Laboratory

Manager: W. Rheinboldt

Objective: 1) To compute a new air density distribution given by

$$\rho(y) = \rho(y_n) \exp(-\frac{1}{d_n}(y-y_n)) \quad \text{for } y_n \le y \le y_{n+1}, \quad \rho(0) = 1,$$

where the d_n are the scale heights at the point $y=y_n$ and are numerically given from experimental data.

2) With this function $\rho(y)$, to compute the integral

$$M(h,\theta) = 2.012 \times 10^2 \int_o^\infty \rho(-R + \sqrt{(R+h)^2 + x^2 + 2(h+R)x\cos\theta})\, dx$$

which is connected with the study of absorption of cosmic rays in the earth's atmosphere. The range of parameters is

$$h = 100(25)200 \quad \text{[miles]}$$

$$\theta = 0^{\circ}, 70^{\circ}, 75^{\circ}, 80^{\circ}, 85^{\circ}, 87.5^{\circ}$$

and

$$\theta = 90^{\circ}(1^{\circ})\ \theta_{max}$$

where $$\theta_{max} = \frac{\pi}{2} + \arcsin \frac{R}{R+h}$$

and the principal value of the function arcsin is taken.

Background: High altitude cosmic ray studies necessitate a good knowledge of absorption in the earth's atmosphere at different zenith angles. New density determination based on satellite lifetime measurements make it possible to obtain new results for the problem. The problem was submitted by R. Drachman (NRL).

Status: New. A 704 code for the computation of the function $M(h,\theta)$ has been written and completely checked out. The results were given to the sponsor.

Another 704 code has been written to compute the function

$$I(y) = \frac{2\pi}{ky} \int_0^{x_m} \frac{x}{1+\exp\left(\frac{x-R_o}{a}\right)} \left[\exp\left(-k|x-y|\right) - \exp(-k(x+y))\right] dx$$

for the seven cases:

$R_o = 3.3$; $x_m = 6.2$; $y_m = 6.0$; $k = 0.7, 1.0, 1.3$; $a = 0.55$

$R_o = 4.18$; $x_m = 8.2$; $y_m = 8.0$; $k = 0.7$; $a = 0.55$

$R_o = 6.5$; $x_m = 11.2$; $y_m = 11.0$; $k = 0.7, 1.0, 1.3$; $a = 0.55$

and $y = 0.2(0.1)y_m$.

This code has also been completely checked out, and the results have been turned over to the sponsor.

1102-40-5126/58-343 MINIMIZATION PROBLEM
Origin and Sponsor: Naval Research Laboratory
Manager: S. Peavy
Objective: To find the value of α that will give the minimum for each of the following ϵ_i for each given ρ:

$$\epsilon_1 = T_1 + T_3 + T_4 + W_2$$

$$\epsilon_2 = T_1 + [T_3 + T_4]' + W_2$$

$$\epsilon_3 = \epsilon_1 + T_5'$$

$$\epsilon_4 = \epsilon_2 + T_5'$$

$$\epsilon_5 = \epsilon_1 + T_6'$$

$$\epsilon_6 = \epsilon_3 + T_6'$$

$$\epsilon_7 = \epsilon_4 + T_6'$$

$$\epsilon_8 = \epsilon_1 - W_2$$

$$\epsilon_9 = \epsilon_7 - W_2$$

where

$$T_1 = \frac{3}{5} k_F^2 + 4\pi\rho a(1 - \frac{1}{p}) + \frac{6\pi\rho}{5p} a(k_F a)^2 + \frac{2\pi\rho k_F^2 a^4 \alpha}{5p} +$$

$$\frac{4\pi\rho a^2}{\hbar} M[-V_{os}(1/4 - 1/4p + \frac{3}{4(2s+1)}) \frac{2}{(2\alpha+\beta_s)^3} - V_{ot}(\frac{3}{4} - \frac{3}{4p} - \frac{3}{4(2s+1)}) \frac{2}{(2\alpha+\beta_t)^3}]$$

$$T_3 = 2\pi\rho a^2 (1 - \frac{1}{p}) \alpha$$

$$T_4 = \frac{\pi\rho k_F^2 a^2}{\alpha p}$$

$$W_2 = -\frac{4\pi\rho a^2 k_F^2 M}{5\hbar} [-V_{os}(-\frac{1}{4p} + \frac{3}{4(2s+1)})(\frac{24}{(2\alpha+\beta_s)^5} + \frac{12a}{(2\alpha+\beta_s)^4} + \frac{2a^2}{(2\alpha+\beta_s)^3})$$

$$+ V_{ot}(\frac{3}{4p} + \frac{3}{4(2s+1)})(\frac{24}{(2\alpha+\beta_t)^5} + \frac{12a}{(2\alpha+\beta_t)^4} + \frac{2a^2}{(2\alpha+\beta_t)^3})]$$

$$[T_3+T_4]' = 4\pi\rho a^2 k_F\Big[\Big(\tfrac{1}{2} - \frac{52}{35p}\Big)\frac{\alpha}{k_F} + \frac{1}{p}\Big\{\Big(\frac{18}{35} + \frac{6}{5}\frac{\alpha^2}{k_F^2}\Big)\tan^{-1}\frac{k_F}{\alpha} + \frac{\alpha^5}{k_F^5}\Big(\frac{3}{5} + \frac{9}{35}\frac{\alpha^2}{k_F^2}\Big)\ln\Big(1 + \frac{k_F^2}{\alpha^2}\Big) - \frac{\alpha^3}{k_F^3}\Big(\frac{33}{70} + \frac{9}{35}\frac{\alpha^2}{k_F^2}\Big)\Big\}\Big]$$

$$T_5' = \frac{4\pi\rho a^3}{p}\alpha^2\Big[\Big(\frac{12}{5}\frac{\alpha}{k_F} + \frac{36}{35}\frac{k_F}{\alpha}\Big)\tan^{-1}\frac{k_F}{\alpha} + \Big(\frac{\alpha}{k_F}\Big)^4\Big(\frac{12}{5} + \frac{18}{35}\frac{\alpha^2}{k_F^2}\Big)\ln\Big(1 + \frac{k_F^2}{\alpha^2}\Big) - \Big(\frac{\alpha}{k_F}\Big)^2\Big(\frac{18}{35}\frac{\alpha^2}{k_F^2} + \frac{15}{7}\Big) - \frac{48}{35} - \frac{9}{10}\Big(\frac{k_F}{\alpha}\Big)^2\Big]$$

$$T_6' = \frac{8\pi\rho a^4\alpha^3}{p}\Big\{\frac{8}{5}\Big(\frac{\alpha}{k_F}\Big)^3\tan^{-1}\frac{k_F}{\alpha} - \Big[\frac{3}{4}\Big(\frac{\alpha}{k_F}\Big)^4 + \frac{3}{40}\Big(\frac{k_F}{\alpha}\Big)^2 + \frac{1}{8}\Big(\frac{\alpha}{k_F}\Big)^6 + \frac{1}{4}\Big]\ln\Big(1 + \frac{k_F^2}{\alpha^2}\Big) + \Big(\frac{\alpha}{k_F}\Big)^2\Big(\frac{1}{8}\Big(\frac{\alpha}{k_F}\Big)^2 - \frac{73}{60}\Big) + \frac{1}{5} + \frac{119}{800}\Big(\frac{k_F}{\alpha}\Big)^2\Big\}$$

$p = 4$ $\qquad \beta_s = 2.58$

$2s + 1 = 2$ $\qquad \frac{2M}{h^2} V_{ot} = 16.7$

$a = .5$

$\beta_t = 3.11$ $\qquad \frac{2M}{h^2} V_{os} = 9.25$

The values of ρ to be used are .088, .109, .135, .196.

Background: The problem is concerned in finding the minimum energy and density of nuclear matter with the use of the cluster expansion method. The problem was submitted by A. Temkin (NRL).

Status: New. The program has been written for the IBM 704 and checking has been started.

1102-40-5126/58-344 SEMICONDUCTOR SURFACE MEASUREMENTS

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Managers: W. Hall, J. Beiman

Objective: (1) To evaluate

$$g = \lambda^{\frac{1}{2}}\int_y^o \frac{(e^{-y}-1) + b\,\lambda^{-1}(e^y-1)}{-[\lambda(e^{-y}-1) + \lambda^{-1}(e^4-1) + (\lambda - \lambda^{-1})y]^{\frac{1}{2}}}\,dy.$$

For $y < 0$, use the positive branch of the square root.
For $y > 0$, use the negative branch of the square root.

The values of g are proportional to the surface conductivity of the space charge layer on a semiconductor.

(2) To obtain data sufficient to plot high accuracy curves of g versus the surface potential Y for a wide range of material conductivity parameters λ.

Background: In semiconductor surface measurements a known condition exists at the minima in the calculated and the experimentally measured curves. The charge in the surface states and the surface potential can be found by comparison of calculated and experimentally measured surface conductivity curves for a material of known bulk conductivity. The problem was presented by J. M. Stinchfield.

Status: New. The program has been written for y = -50.0(2)50.0, and a wide range of λ. The program is in the process of being checked.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Transistor aging studies: An experiment design was devised for evaluating the effect of varying the time between the end of heat treatment and the beginning of testing. This test is being conducted preliminary to a large scale test, and the findings may lead to changes in test procedure. This work is being done for G. Conrad of Section 1.6.

(2) Inventory of spare parts: Work was begun for R. O. Stone and P. Meissner, Section 1.6, on the probabilistic aspects of maintaining an inventory of spare parts for a system with complications introduced by conditions on time of repair.

(3) Standard lamps: A statistical study of the factors affecting measurements of candlepower has been made for H. K. Hammond of Section 2.1. The results of these studies are being used in the design and analysis of current tests.

(4) Statistical analysis of rather large sets of data were carried out for R. E. Michaelis, Section 5.10; M. J. Kerper, Section 9.2; G. Conrad, Section 1.6; R. V. Waterhouse, Section 6.1.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: CONTINUED. Statistical studies on the relation between transition temperature and several physical and chemical properties of two types of ship steel were continued.

STATISTICAL ANALYSIS AND DESIGN OF EXPERIMENTS FOR THE U. S. GEOLOGICAL SURVEY
Task 1103-40-5140/54-1

Origin and Sponsor: U. S. Geological Survey, Department of Interior — Authorized 10/9/53
Managers: C. Eisenhart, W. J. Youden
Full task description: Oct-Dec 1953 issue, p. 50

Status: CONTINUED. On January 15, C. Eisenhart, J. M. Cameron, W. S. Connor, N. C. Severo and S. Young attended a "Clinic on Statistical Problems in Geology" held by the Committee on Statistics in Geology, Geologic Division, at the U. S. Geological Survey headquarters in Washington. C. Eisenhart outlined some "New developments in statistical treatment of directional properties." As a follow-up on one of the other current problems discussed at the meeting, C. Eisenhart, J. M. Cameron and W. S. Connor visited the laboratory of Lorin Stieff, Nucleonics Group, Geochemistry and Petrology Branch, Geologic Division, USGS, Washington, on February 5th to discuss statistical-design aspects of the scheduling of observations and method of analysis of mass spectrometric data on isotopic abundance.

G. N. Alexander, Water Resources Division, USGS, Washington, conferred at the NBS (a) with W. S. Connor on the use of tables of the non-central t-distribution and (b) with N. C. Severo on the use of the log-normal distribution.

N. C. Severo also was consulted by J. Hack, General Geology Branch, Geologic Division, USGS, on the appropriateness of the Poisson, log-normal, and negative-binomial distributions in studies of gemma-gemma clam distributions.

A memo, "A method for placing confidence limits on preferred directions," by C. Eisenhart and S. Young, based on analysis of variance analogies noted and developed by Sir Ronald Fisher, and by G. S. Watson and E. J. Williams, was sent to F. G. Poole, USGS, Denver, on March 18. A second memo, in preparation as the quarter ended, gives a procedure for determining how many observations are required in order to obtain an estimate of preferred direction having a confidence interval of specified width. The procedure is based on an adaptation to observations, from the von Mises-Fisher family of distributions, of Charles Stein's two-sample procedure (for data from a normal distribution).

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-5146/55-93

Origin and Sponsor: Office of Ordnance Research — Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: CONTINUED. A preliminary draft of Part I, section 3.1 (Linear Regression) was distributed for comment. A preliminary draft of Part III, sections 3 and 4 (Factorial Experiments and Experiments to Determine Optimum Conditions) is in the process of being reproduced for distribution. Work on preparation of examples for Parts I and III is underway.

STATISTICAL SERVICES
Task 1103-40-5150

Origin and Sponsors: Various Agencies Authorized 3/31/58
Manager: J. M. Cameron

Objective: To perform statistical services as requested by other agencies of the Federal Government and their contractors.

Background: This project is intended particularly to serve other Government agencies and their contractors whose statistical work either is on too small a scale to justify acquisition of a statistical staff, or requires the assistance of specialists not locally available. The experience gained in the performance of such services gives new stimuli to the research staff and benefits the efforts of statisticians in their work on Bureau problems.

Status: NEW. Work was done during the quarter for the following agencies:

(1) Veterans Administration Hospital, Perry Point, Maryland. A covariance analysis with up to nine concommitant variablesis being carried out on data on the effect of a number of tranquilizing drugs on mental patients. A special IBM 704 code for analyzing the data on 25 performance measures on each of 800 patients in up to five time periods is being checked out. This work is being done for Dr. M. H. Gordon of the Hospital's Central NP Research Unit.

(2) General Services Administration. An analysis of data on the effect of time of storage on stockpiled manila rope fiber was done on the IBM 704 for H. LeBovit of the Technical Research and Development Division of the Defense Materials Service.

(3) Bureau of Public Roads. See task 1102-40-5126/58-316, p. 33, for report on this task done jointly with the Computation Laboratory.

(4) Post Office Department. The "Post Office Project" of Division 12, sponsored by the Post Office Department, has as its objectives the performance of operations analysis of mail handling in post offices with the specific aim of recommending and/or developing automatic equipment and techniques for the improvement of the present postal operations. It is hoped that these operations can be improved through (1) speeding the flow of mail, (2) handling more mail without enlarging post office

facilities and/or personnel requirements, and (3) reducing cost of handling the mail.

Various members of the Statistical Engineering Laboratory have been engaged in consulting and advising on the statistical problems encountered by the project for well over two years. Since September 1956 one member of SEL has spent full-time on this work.

Sampling studies have been conducted in the post offices of four cities to determine the statistical nature and various relevant parameters of the flow of letter-mail. This information is being used in the development and design of equipments and systems for improving letter sorting by automation. The following studies have been completed:

(a) Distribution of letter-mail by destination: A sampling plan called the "chain ratio" method, because the formulas involved in the analysis resembled a chain of ratios, was devised for estimating the distribution of mail by destination and was applied in sampling studies conducted in the post offices of San Francisco, Los Angeles and Baltimore. The method proved to be very simple to apply. Data of this type had previously been gathered by costly procedure involving elaborate footage measurements of all of the mail passing through the sorting system during a single fixed time period, such as 24 or 48 hours. On the other hand, chain-ratio sampling was built up from a series of short samples spread over a one week period and involved counting approximately 20,000 letters (about 70 feet of mail) per day compared to measuring about 4000 feet of mail (approximately 1,200,000 letters) per day. Full details of the chain-ratio method and of its applications to the mail of these three cities are given in two NBS Reports by N. C. Severo and A. E. Newman (12.5), entitled "A statistical chain-ratio method for determining the distribution of mail by destination" and "Distribution of mail by destination at the San Francisco, Los Angeles and Baltimore post offices," transmitted to the sponsor in November 1957 and February 1958, respectively. Subsequently, N. C. Severo, M. Zelen and A. E. Newman (12.5) met with H. W. Lieske of the Office of Research and Engineering of the Post Office Department and members of his staff to discuss the meaning and details of the above stated reports.

(b) Letter-mail size and color characteristics: Sampling studies were conducted in San Francisco, Los Angeles, and Washington to determine the distribution of letters by length, height, thickness and color. Control charts were drawn to determine the stability, from sample to sample, of the characteristics studied; and additional statistical tests were then applied to test for differences between cities and between different categories of mail. Statistical tolerance limits were calculated from the data for the sizing characteristics. The results of this study, and of studies (c), (d) and (e) below, are described in detail in an NBS Report by N. C. Severo, A. E. Newman (12.5), S. Young and M. Zelen, entitled "Some applications of statistical sampling methods to outgoing letter mail characteristics," transmitted to the sponsor in February 1958.

(c) Top and bottom clearance space of an addressed envelope: Three studies were conducted to determine top and bottom clearance space of an addressed envelope, i.e., the distances from the top edge of the first line of intelligence of the address to the bottom edge of the envelope.

Separate studies were made for mail from San Francisco, from Los Angeles and from Washington.

(d) Proportions of long and short letters: A study was conducted, using San Francisco and Los Angeles, to determine the ratio of long letters to long plus short letters for a special category of letter mail.

(e) Ratio of hand-cancelled to machine-cancelled mail: A sampling scheme was devised for determining the ratio of hand-cancelled to machine-cancelled letter mail and was applied to Washington mail only.

J. M. Cameron, N. C. Severo and M. Zelen collaborated with members of the Division 12 Post Office Project in devising a method for simulating a sorting machine on the SEAC.

N. C. Severo and M. Zelen attended the periodic meetings of the Division 12 Post Office Project group headed by Mr. I. Rotkin.

APPLICATION OF AUTOMATIC COMPUTER

The record of the use of the IBM 704 for the period January 1 through March 31 is as follows:

Task No.	Title	Assembly	Code Checking	Production
		(M I N U T E S)		
NBS:				
5116/55-55	Research in numerical analysis	16	17	2
5116/55-56	Research in mathematical topics applicable to numerical analysis	20	24	
5116/56-148	Study of diffraction equations for nerve excitation			199
1110/57-241	Heat transfer		4	14
1110/58-292	Planck's radiation function		88	
1120/55-65	Automatic coding	1	635	
0009/56-160	Mathematical subroutines	35	248	185
5113/57-216	Handbook of mathematical functions	10	62	212
0009/54-30	Spectrum analysis	47	265	89
0009/54-38	Compressibility factors of dry air		62	
0009/55-68	Crystal structure calculations	24	327	21
0009/55-82	Thermometer calibrations		12	
5126/55-97	High temperature properties for air		230	
5126/55-121	Electron penetration		192	
0009/56-131	Calculations in optics			64
5126/56-166	SCF-LCAO solution of some hydrides		35	94
5126/57-219	Thermal properties		64	49
0009/57-223	Self-consistent fields	2	107	
5126/57-229	Payroll operations	17	62	
5126/57-232	Polynomial evaluation		72	
5126/57-233	Inverse Loran		134	
5126/57-236	Self-consistent field--eigenvalues		211	
5126/57-246	Radiation diffusion	66	1092	460
0009/57-247	Mechanical impedance		12	72
5126/57-248	Negative ion detachment		65	
5126/57-249	Color differences		48	
0009/57-250	Automatic reduction in spectrophotometric data		7	4
5126/57-252	Detecting efficiency in a neutral meson experiment	8	261	11

Task No.	Title	Assembly	Code Checking	Production
			(M I N U T E S)	
0009/58-254	Reproduction of color and spectral-energy distribution of daylight			25
5126/58-255	Chi function calculations	39	408	11
5126/56-256	Composite walls		199	
5126/58-258	Noise measurement, II		35	
5126/58-260	Proto-type accounting	51	64	
5126/58-264	Theory of ionization probability		21	
0009/58-267	Munsell color system conversion		21	
0009/58-268	Response function calculation		15	
5126/58-270	Post office mechanization	53	315	68
5126/58-272	Thermodynamic properties of real gases		101	
5126/58-274	Calculation for d-spacings			76
5126/58-279	Fire resistance T-beam		142	
5126/58-281	Psi evaluation		7	
5126/58-294	Nuclear scattering of photons		57	4
5126/58-296	Lump network		126	
5126/58-298	Analysis of spectrochemical data		210	
5126/58-299	Time-dependent Shroedinger equation		444	73
5126/58-301	Spectrochemistry	4	8	7
5126/58-303	Determination of bulk modulus		3	11
5126/58-304	Transport properties of air		170	
5126/58-306	Interpolation of color mixture functions		84	
5126/58-308	Oscillating sphere		79	
5126/58-312	Response function, II	27	63	17
5126/58-321	Table of thermodynamic functions of sulfur		21	5
5126/58-322	Propagation constant of a sound wave		33	
5126/58-324	Fitting a polynomial to a curve	9	22	43
5126/58-327	Heat of combustion			29
5126/58-328	Magnetic field	7	27	
5126/58-329	Collagen model		109	
5126/58-331	Computation of hydraulic elements		23	

Task No.	Title	Assembly	Code Checking	Production
			(M I N U T E S)	
5126/58-334	Price computation table		2	
5126/58-339	Computation of viscoelasticity properties	4	46	
5126/58-347	War games	4	9	
5126/58-349	Machine simulation		13	
0002/51-1	Statistical engineering	17	41	54
5148/58-291	Fractional factorials for the mixed series		14	56
	Totals...	461	7268	1955
OUTSIDE:				
5116/57-200	Evaluation of matrix computation		36	17
5126/53-45	Air defense tactics	53	707	
5126/54-13	Procurement contracts	11	31	
5126/56-163	Nuclear scattering	84	182	13
5126/57-221	Bessel functions for complex arguments			31
5126/57-224	Track-while-scan radar problem		991	25
5126/57-245	Missile trajectory,III		7	
5126/58-263	Gas tube characteristic		720	845
5126/58-269	Molecular structure,IV	69	218	1433
5126/58-273	Administration for DOFL			11
5126/58-276	General kinetics,I	3	84	2511
5126/58-278	Polaris		14	
5126/58-282	Missile boundary layer computation	79	238	1995
5126/58-283	General kinetics,II	160	222	19
5126/58-284	Ephemeris calculations for satellites	35		6512
5126/58-286	Azimuth	5	432	220
5126/58-300	Laminar mixing in boundary layers		612	
5126/58-311	Null rate calculations		786	215
5126/58-315	Mechanization of French translation	112	373	5
5126/58-317	Scattering,II		461	
5126/58-318	Scattering,III	9	25	
5126/58-320	Teller emission problem	2	292	
5126/58-325	Covariance analysis	29	25	
5126/58-326	Technical operations		158	21
5126/58-330	Epitron orbits		2	

Task No.	Title	Assembly	Code Checking	Production
			(M I N U T E S)	
5126/58-332	Mechanization of Russian translation		3	
5126/58-335	List of Bessel functions	30	11	30
5126/58-336	Helical transforms		60	
5126/58-338	Minima by Cox-Prugh	21	10	
5126/58-341	Cosmic ray problem		113	
5126/58-343	Minimization problem	8		
5126/58-344	Semi-conductor surface measurements		37	
5126/58-346	GSA data processing	6	2	
	Classified		260	
	Totals	716	7112	13903

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Mathematics Colloquium Series

GOLDBERG, K. Incidence algebras and applications to graph theory and Hadamard matrices. January 9.

RHEINBOLDT, W. C. On non-selfadjoint boundary-value problems in ordinary differential equations. January 16.

OLVER, F.W.J. Modern table making. January 23.

ARMS, R. J. On the asymptotic behavior of the method of steepest descent. January 30.

LAASONEN, P. (University of California) On the simultaneous determination of several eigen-solutions of a self-adjoint system of ordinary differential equations. February 6.

NEHARI, Z. (Carnegie Institute of Technology) Some properties of solutions of certain classes of second-order non-linear differential equations. February 13.

MESNER, D. M. Embedding theorems for 0,1 matrices. February 7.

SCHOPF, A. Difference methods and the variational principle. March 13.

PUCCI, C. (University of Maryland) Cauchy problem for elliptic equations. March 27.

Mathematical Statistics Seminar

EISENHART, C. The meaning of "least" in Least Squares. January 7.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

CAMERON, J. and C. RAVITSKY (DOFL). Objectives in selecting and evaluating scientists and engineers. Presented at the Fifth Conference of the Associated Boards of U. S. Civil Service Examiners for Scientific and Technical Personnel, Washington, D. C., March 20.

CONNOR, W. S. (1) Experiments to study the effects of several variables on one or more response variables. Presented at the Wilmington, Delaware Chapter of the American Society for Quality Control, January 9. (2) A fractional factorial experiment of the mixed 2^m3^n series. Presented before the Rochester Society for Quality Control, Rochester, N. Y., February 18. (3) Use of the direct product of matrices in the analysis of factorials. Presented at a Statistics Seminar, Virginia Polytechnic Institute, Blacksburg, Va., February 21.

EISENHART, C. The meaning of "least" in Least Squares. Presented before the Mathematics Club of George Washington University, Washington, D. C., January 8.

GOLDMAN, A. J. Continuous poker and von Neumann's theory of games. Presented at a Mathematics Seminar, Catholic University, Washington, D. C., March 21.

MESNER, D. M. An embedding theorem for 0,1 matrices. Presented at a meeting of the American Mathematical Society, Cincinnati, Ohio, January 28-30.

PEARL, M. On Cayley's parameterization, II. Presented at a meeting of the American Mathematical Society, Cincinnati, Ohio, January 28-30.

YOUDEN, W. J. (1) Design of experiments. Presented at a meeting of the American Society for Quality Control, Baltimore, Md., January 14. (2) Problems of the experimenter. Presented before the American Society for Quality Control, Stevens Institute of Technology, Hoboken, N. J., January 25; before the Bureau of Ships Naval Reserve Unit 5-2, Washington, D. C., January 31; and at a Seminar at the Boyce Thompson Institute for Plant Research, Inc. Yonkers, N. Y., March 28. (3) Some modern developments in planning experiments. Presented at the Technical Staff Meeting, Diamond Ordnance Fuze Laboratories, Washington, D. C., February 18. (4) Some problems in testing. Presented at a meeting of the Technical Association of the Pulp and Paper Industry, New York, N. Y., February 19. (5) Statistics in engineering research. Presented at a Bureau of Mines Seminar, College Park, Md., March 6.

(6) Tactics and strategy of experimentation. Presented at a meeting of the Executive Development Program, Fort Belvoir, Va., March 21.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.3 Technical Papers

(1) On the application of steam-driven water jets for propulsion purposes. J. M. Burgers and A. Ghaffari. J. Research NBS 60, 137-141 (Feb. 1958), RP2830.

(2) The uniqueness of the triangular association scheme. W. S. Connor. Ann. Math. Stat. 29, 262-266 (1958).

(3) Existence theorems and extreme solutions for inequalities concerning convex functions or linear transforms. K. Fan. Math. Zeit. 68, 205-216 (1957).

(4) A matrix minimization problem. A. J. Goldman. J. Washington Acad. Sci. 47, 405-406 (1957).

(5) On doubly stochastic transforms of a vector. M. Marcus. Oxford Quart. J. Math. 9, 74-80 (1958).

(6) Some extreme value results for indefinite Hermitian matrices. M. Marcus, B. N. Moyles and R. Westwick (University of British Columbia). Illinois J. Math. 1, 449-457 (1957).

(7) A method of speeding up iterations with super-linear convergence. A. M. Ostrowski. J. Math. Mech. 7, 117-120 (1958).

(8) Asymptotic behavior of tests on the mean of a logarithmico-normal distribution with known variances. N. C. Severo. Ann. Math. Stat. 28, 1044-6 (1957).

(9) The condition of certain matrices, III. J. Todd. J. Research NBS 60, 1-7 (1957), RP2815.

(10) Experiments with many factors. M. Zelen. Proceedings of the Third Annual Statistical Engineering Symposium, held at the Chemical Center, Md., May 1957, pp. 1-12.

(11) The use of group divisible designs for asymmetrical factorial arrangements. M. Zelen. Ann. Math. Stat. 29, 22-40 (1958).

1.4 Reviews and Notes

(1) Essential similarity: a counter-example. A. J. Goldman. Amer. Math. Monthly (Math. Notes) 65, 546 (1958).

(2) On a determinantal inequality. M. Marcus. Amer. Math. Monthly (Math. Notes) 65, 266-268 (1958).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION MARCH 31, 1958

2.1 Mathematical Tables

(1) Tables of the bivariate normal distribution function and related functions. To appear as NBS Applied Mathematics Series 50.

(2) Table of the exponential integral for complex arguments. To appear as NBS Applied Mathematics Series 51.

(3) Integrals of Airy functions. To appear as NBS Applied Mathematics Series 52.

(4) Table of natural logarithms for arguments from five to ten to sixteen decimal places. To appear as NBS Applied Mathematics Series 53.

2.3 Technical Papers

(1) A survey of Lyapunov's second method. H. Antosiewicz. To appear in Annals of Mathematics Studies.

(2) Assigning quantitative values to qualitative factors in the Naval electronics problem. R. J. Aumann and J. B. Kruskal (University of Wisconsin). Submitted to a technical journal.

(3) The coefficients in an allocation problem. R. J. Aumann and J. B. Kruskal (University of Wisconsin). Submitted to a technical journal.

(4) Fractional factorial experiments of $2^m 3^n$ series. W. S. Connor. To appear in the Transactions of the Rochester Society for Quality Control.

(5) Abelian groups of unimodular matrices. E. C. Dade. Submitted to a technical journal.

(6) The construction of Hadamard matrices. E. C. Dade and K. Goldberg. Submitted to a technical journal.

(7) On a converse of a theorem of Pringsheim. P. J. Davis. Submitted to a technical journal.

(8) Additional abscissas and weights for Gaussian quadratures of high order; values for n = 64, 80, and 96. P. Davis and P. Rabinowitz (Weizmann Institute). Submitted to a technical journal.

(9) Bending and stretching of corrugated diaphragms. R. F. Dressler. Submitted to a technical journal.

(10) Unsteady nonlinear waves in sloping channels. R. F. Dressler. Submitted to a technical journal.

(11) Note on circular disks containing the eigenvalues of a matrix. Ky Fan. To appear in the Duke Mathematical Journal.

(12) Topological proofs for certain theorems on matrices with non-negative elements. Ky Fan. To appear in Monatschefte fur Mathematik.

(13) Some elementary inequalities relating to the gamma and incomplete gamma function. W. Gautschi. Submitted to a technical journal.

(14) Random notes on matrices. K. Goldberg. To appear in the Journal of Research, NBS.

(15) A continuous poker game. A. J. Goldman and J. J. Stone. Submitted to a technical journal.

(16) On the domain of regularity of generalized axially symmetric potentials. P. Henrici. To appear in the Proceedings of the American Mathematical Society.

(17) Mechanized computation of thermodynamics tables at the National Bureau of Standards. J. Hilsenrath (NBS Thermodynamics Section) and J. Wegstein. To appear in the Proceedings of the Joint Conference on Thermodynamic Transport Properties of Fluids sponsored by the Institute of Mechanical Engineers, London, 1957.

(18) Numerical experiments in potential theory using the Nehari estimates. U. W. Hochstrasser. To appear in Mathematical Tables and Other Aids to Computation.

(19) Permanents of doubly stochastic processes. M. Marcus and M. Newman. Submitted to a technical journal.

(20) Dense subgraphs and connectivity. R. E. Nettleton (NBS 3.2), K. Goldberg, and S. M. Green (NBS 3.2). Submitted to a technical journal.

(21) Congruences for the coefficients of modular forms and for the coefficients of $j(\tau)$. M. Newman. To appear in the Proceedings of the American Mathematical Society.

(22) Further identities and congruences for the coefficients of modular forms. M. Newman. To appear in the Canadian Journal of Mathematics.

(23) Inclusion theorems for congruence subgroups. M. Newman and I. Reiner (University of Illinois). To appear in the Transactions of the American Mathematical Society.

(24) On the derivative of Bessel functions with respect to the order. F. Oberhettinger. Submitted to a technical journal.

(25) On the diffraction and reflection of waves and pulses on wedges and corners. F. Oberhettinger. To appear in the Journal of Research, NBS.

(26) On the convergence of the Rayleigh quotient iteration for the computation of the characteristic roots and vectors, II. A. Ostrowski. Submitted to a technical journal.

(27) A further extension of Cayley's parameterization. M. Pearl. Submitted to a technical journal.

(28) A note on commutators. M. Pearl. Submitted to a technical journal.

(29) On normal and EPr matrices. M. Pearl. To appear in the Michigan Journal of Mathematics.

(30) The elastic problem for a ring of uniform force in an infinite body. W. H. Pell. To appear in the Journal of Research, NBS.

(31) On some aspects of prediction in the study of complex systems. J. R. Rosenblatt. To appear in the Proceedings of the NYU-RCA Working Conference on Theory of Reliability, held at Ardsley-on-Hudson, N. Y., April 1957.

(32) The radial distribution of the center of gravity of random points on a unit circle. F. Scheid. To appear in the Journal of Research, NBS.

(33) The non-central χ^2 as a test statistic. N. C. Severo. Submitted to a technical journal.

(34) Generation of Bessel functions on high speed computers. I. Stegun and M. Abramowitz. Submitted to a technical journal.

(35) Stationary principles for forced vibrations in elasticity and electromagnetism. J. L. Synge. To appear in the Proceedings of the Eighth Symposium in Applied Mathematics held by the American Mathematical Society, Chicago, Ill., April 1956.

(36) Some computational problems concerning integral matrices. O. Taussky. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.

(37) Computation problems concerned with the Hilbert matrix. J. Todd. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.

(38) National physical standards and design of experiment. W. J. Youden. To appear in Revue de L'Institut International de Statistique (The Hague).

(39) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.

(40) Statistics--Engineering viewpoint. W. J. Youden. To appear in in the Journal of Engineering Education.

(41) Multi-variable experiments. M. Zelen and W. S. Connor. Submitted to a technical journal.

(42) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. S. Joel. Submitted to a technical journal.

2.4 Reviews and Notes

(1) Field convexity of a linear transformation. A. J. Goldman and M. Marcus. To appear in the American Mathematical Monthly (Math. Notes).

USCOMM--DC

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($0.75), available from the Superintendent of Documents, Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.


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