NATIONAL BUREAU OF STANDARDS REPORT

5656

PROJECTS and PUBLICATIONS of the APPLIED MATHEMATICS DIVISION

A Quarterly Report

July through September 1957

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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NBS PROJECT

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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

July 1 through September 30, 1957

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CONTENTS

*Only unclassified projects are included in this report.

Status of Projects

September 30, 1957

1. NUMERICAL ANALYSIS

RESEARCH IN THEORIES OF DISCRETE STRUCTURES
Task 1100-11-5170/56-159

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: O. Taussky-Todd
Full task description: July-Sept 1955 issue, p. 1

Status: TERMINATED. O. Taussky-Todd studied unimodular group matrices with rational integral elements belonging to the symmetric group with three elements. Several matrices of finite and infinite order were found. Their characteristic roots and the groups generated by them were studied.

E. C. Dade and O. Taussky-Todd continued work on the 3-class group of imaginary quadratic fields. If $D < 0$ is a square free integer $\equiv 5(8)$, then it is known that the ideal class number of the field generated by $\sqrt{D}$ differs from the number of form classes with discriminant 3 by a factor 3. E. C. Dade prepared a program (for the I.B.M. 704) which allows the structure of the group of form or ideal classes to be determined. In this way it is easy to determine whether a new base class is added by going from ideals to forms or whether the period of an existing base element is increased. A table is being prepared which shows which of these two cases happens for a special D. E. C. Dade has also prepared a code to determine the structure of the class group for $D \equiv 1,2,3,6(8)$. The computation of class numbers and generators of class groups has now been carried far beyond the range of previously existing tables.

Relevant to a conjecture of P. Erdos, E. C. Dade has proved the following number-theoretic identity:

Let $n = p_1^{a_1} p_2^{a_2} \ldots p_k^{a_k}$ be an integer with prime factors $p_1, p_2, \ldots, p_k$ satisfying $p_i > k$. Let $a_1, a_2, \ldots, a_s$ be the integers less than and prime to n. Then

$$\sum_{i=1}^{s-1} (a_{i+1} - a_i)^2 = 2n \prod_{i=1}^{k} (1 + p_i^{-1}) - n - 4.$$

E. C Dade and K. Goldberg have studied incidence algebras with applications of Hadamard matrices and problems in graph theory. A manuscript is in preparation.

Future reports on topics under this task will be included in the status for task 1101-12-5116/55-56, "Research in mathematical topics applicable to numerical analysis."

Publications:

(1) Incidence algebras. E. C. Dade and K. Goldberg. In manuscript.
(2) Pairs of matrices of order two which generate free groups. K. Goldberg and M. Newman. Illinois J. Math. 1, 446-448 (1957).
(3) A method for computing eigenvectors. K. Goldberg. In manuscript.
(4) A computation of cyclic cubic units. H. Cohn and S. Gorn (Moore School of Electrical Engineering). J. Res. NBS 59, 155-168 (1957), RP2783.
(5) A numerical study of Dedekind's cubic class number formula. H. Cohn. To appear in the Journal of Research, NBS.
(6) Some computational problems concerning integral matrices. O. Taussky. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.
(7) Abelian groups of unimodular matrices. E. C. Dade. In manuscript.
(8) The number of representations of a quadratic form as a sum of four squares. G. Pall (Illinois Institute of Technology) and O. Taussky. Proc. Roy. Irish Acad. Sect. A., 58, 23-28 (1957).

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-12-1104/55-55

Origin: NBS Authorized 8/29/54
Managers: J. Todd, P. Davis
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. P. Davis has prepared a manuscript which summarizes his experience to date on the numerical solution of problems in potential theory using orthonormalization techniques.

M. Newman and J. Todd have completed a report entitled "The evaluation of matrix inversion programs." A set of 13 test matrices whose inverses are known and whose condition numbers range from 1 to e^{cn} is described, and the results obtained by inverting them on SEAC, the I.B.M. 704, the ORACLE, etc. are analyzed. It was found that the results obtained using floating arithmetic were significantly better in general than those obtained with fixed point arithmetic, and that all the results obtained were in substantial agreement with the error estimates of von Neumann and Goldstine. A report was presented at the Wayne Matrix Conference and a manuscript has been prepared.

M. Marcus and M. Newman have submitted a manuscript to a technical journal on permanents of doubly stochastic matrices. If p(S)

denotes the permanent of the doubly stochastic matrix S, some results are obtained on a conjecture of van der Waerden on the minimum value of p(S).

M. Pearl has found an upper bound for the index of a nilpotent commutator. A manuscript describing this result is being submitted to a technical journal.

M. Pearl has been working with M. Reisz (Institute for Fluid Dynamics and Applied Mathematics, University of Maryland) on problems concerning the characterization of matrices belonging to classical groups.

M. Pearl has extended his earlier results (to appear in the Canadian Journal of Mathematics) concerning Cayley's parametrization of automorphs of symmetric matrices to arbitrary fields.

E. Brauer and J. Gager have written a code which computes the Legendre polynomials $P_n(\cos\theta)$ up to n = 80 using the following formula:

$$P_n(\cos\theta) = \frac{2n!}{2^{2n}n!n!}\,[2(a_o \cos n\theta + a_1 \cos(n-2)\theta + \ldots + a_{(n-2)/2}\cos 2\theta) + a_{n/2}\cos\theta]$$

where $a_{r+1} = a_r \cdot \frac{2r+1}{r+1} \cdot \frac{n-r}{2n-2r-1}$, $a_o = 1$. The Legendre polynomials were computed for the purpose of checking a table of Legendre polynomials by Clark and Churchill, "Tables of Legendre Polynomials" (Engineering Research Institute, University of Michigan, 1957). The values found in the table are correctly rounded to six decimal places. The values $P_n(\cos\theta)$ computed by this code are believed to be correct to seven decimal places.

There has been interest recently in the radius of univalence of erf z (Nehari, Umezawa, Reade), and various estimates have been obtained, e.g., $\sqrt{\frac{\pi}{2}} = 1.25\ldots$ This quantity can clearly be estimated experimentally. Accordingly, J. Gager and E. Brauer have written a code which evaluates $\operatorname{erf} z = \int_o^z e^{-t^2}\,dt$ for $\theta = 0(1°)90°$ and r = .1(.1)2.0, where $z = x + iy$, $x = r\cos\theta$, $y = r\sin\theta$. The behavior of the maps of $|z| = r$ was observed and it appears that the radius of univalence is the first value of r for which the map touches the axis. This appears to be 1.5748376. The final results, which were spot-checked against the tables of Karpov and Faddeeva and Terentiev are believed to be accurate to seven decimal places. All operations are performed in fixed point; the power-series for erf z was computed by the usual Horner-Newton recurrence; sine and cosine subroutines (which should give nine correct decimal digits) were written as a training problem and were incorporated into the final code. Theoretical studies on the problem are being continued by J. Todd.

Publications:

(1) Complete sequences and approximations in normed linear spaces. P. Davis and K. Fan. Duke Math. J. 24, 183-192 (1957).

(2) Note on bounds for certain determinants. E. V. Haynsworth. Duke Math. J. 24, 313-320 (1957).
(3) A survey of Lyapunov's second method. H. A. Antosiewicz. To appear in Annals of Mathematics Studies.
(4) Convex functions of quadratic forms. M. Marcus. To appear in the Duke Mathematical Journal.
(5) On subdeterminants of doubly stochastic matrices. M. Marcus. To appear in the Illinois Journal of Mathematics.
(6) On a determinantal inequality. M. Marcus. To appear in the American Mathematical Monthly.
(7) Some extreme value results for indefinite Hermitian matrices. M. Marcus, B. N. Moyles and R. Westwick (University of British Columbia). To appear in the Illinois Journal of Mathematics.
(8) Extreme value properties of Hermitian matrices. M. Marcus. To appear in the Journal of the London Mathematical Society.
(9) Computational problems concerned with the Hilbert matrix. J. Todd. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.
(10) On the condition of matrices, III. J. Todd. To appear in the Journal of Research, NBS.
(11) On normal and EPr matrices. M. Pearl. Submitted to a technical journal.
(12) A determinantal inequality of H. P. Robertson, II. M. Marcus. To appear in the Journal of the Washington Academy of Sciences.
(13) Reducible linear differential systems. H. A. Antosiewicz. In manuscript.
(14) Field convexity of a linear transformation. M. Marcus and A.J.Goldman. To appear in the American Mathematical Monthly (Math. Notes).
(15) On doubly stochastic transforms of a vector. M. Marcus. Submitted to a technical journal.
(16) On permanents of doubly stochastic matrices. M. Marcus and M. Newman. Submitted to a technical journal.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-12-5116/55-56

Origin: NBS Authorized 8/13/54
Sponsor: Office of Naval Research
Managers: J. Todd, M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. M. Newman has derived further identities for the coefficients $p_r(n)$ defined by $\prod(1-x^n)^r = \sum p_r(n)x^n$ and has extended previous work to include the cases $p = 2$, $p = 3$. It was shown for example that

$$p_{15}(27n+23) - 1836p_{15}(3n+2) = -3^{13}p_{15}(\frac{n-1}{3})$$

and since $p_{15}(53) = 0$, this implies that

$$p_{15}(\frac{429\cdot 9^{t}-5}{8}) = 0.$$

A manuscript has been prepared.

P. Davis has proved a converse of a theorem of Pringsheim relating to the singularities of power series. Assuming that a power series has a singularity of a certain type at z = 1, a conclusion is made as to the distribution of its coefficients. A manuscript has been prepared.

Publications:

(1) On the minimization of concave and convex functionals. G. B. Dantzig (RAND Corporation), A. J. Hoffman and W. Hirsch (New York University). In manuscript.
(2) Estimation of the frequencies of thin elastic plates with free edges. T. Kato, H. Fujita, Y. Nakata (University of Tokyo); and M. Newman. J. Res. NBS 59, 169-186 (1957), RP2784.
(3) Systems of inequalities involving convex functions. K. Fan (Notre Dame University), I. Glicksberg (RAND Corporation) and A. J. Hoffman. Proc. Amer. Math. Soc. 8, 617-622 (1957).
(4) Construction and application of a class of modular functions. M. Newman. Proc. London Math. Soc. (3) 7, 334-350 (1957).
(5) Congruences for the coefficients of modular forms and some new congruences for the partition function. M. Newman. To appear in Canadian Journal of Mathematics.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-12-5116/56-148

Origin and Sponsor: National Institutes of Health, Bethesda, Md. Authorized 9/30/55
Manager: H. A Antosiewicz
Full task description: July-Sept 1955 issue, p. 7

Status: CONTINUED. The I.B.M. 704 code for the entire problem has been completed and checked out. Runs have been made for seven sets of data, and the results were transmitted to the sponsor.

TRAINING PROGRAM IN NUMERICAL ANALYSIS
Task 1101-40-5114/57-237

Origin and Sponsor: National Science Foundation Authorized 3/27/57
Manager: J. Todd
Full task description: Jan-Mar 1957 issue, p. 5

Status: COMPLETED.

Publications:

(1) The radial distribution of the center of gravity of random points on a unit circle. F. Scheid. To appear in the Journal of Research, NBS.
(2) A class of triple-diagonal matrices for test purposes. P. Clement. In manuscript.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF $E_1(z)$, $(z=x+iy)$
Task 1102-40-1110/43-3

Manager: I. Stegun — Authorized 7/1/47
Full task description: Apr-June 1949 issue, p. 41

Status: CONTINUED. Checking of the final manuscript was completed, and the table was submitted for publication in the Applied Mathematics Series.

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-40-1110/47-2

Origin: NBS — Authorized 7/1/47
Manager: M. Abramowitz
Full task description: Apr-June 1949 issue, p. 45

Status: CONTINUED. A manuscript of the tables has been prepared, and a short introduction has been drafted in preliminary form.

TABLES OF POWER POINTS OF ANALYSIS-OF-VARIANCE TESTS
Task 1102-40-1110/51-8

Origin: Section 11.3, NBS — Authorized 3/26/51
Manager: S. Peavy
Full task description: Apr-June 1951 issue, p. 49

Status: INACTIVE.

REVISION OF MATHEMATICAL TABLES
Task 1102-40-1110/52-7

Origin: NBS Authorized 8/10/51
Managers: W. F. Cahill, I. Stegun
Full task description: July-Sept 1951 issue, p. 41

Status: CONTINUED. The "Table of Natural Logarithms for Arguments from 5 to 10 to Sixteen Decimal Places" has been prepared for reissue in the Applied Mathematics Series. In the past this table was known as Volume IV of "Tables of Natural Logarithms," prepared by the New York Mathematical Tables Project, and was designated as MT12.

TABLE OF THE MODIFIED AIRY INTEGRAL
Task 1102-40-1110/52-23

Origin: NBS Authorized 10/4/51
Manager: I. Stegun
Full task description: July-Sept 1951 issue, p. 42

Status: CONTINUED. Checking of the final manuscript was completed, and the table has been submitted for publication in the Applied Mathematics Series.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-40-1110/52-37

Origin: NBS Authorized 11/28/51
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE.

SIEVERT'S INTEGRAL
Task 1102-40-1110/52-57

Origin: NBS Authorized 2/12/52
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46

Status: INACTIVE.

L-SHELL CONVERSION COEFFICIENTS
Task 1102-40-1110/53-52

Origin: Oak Ridge National Laboratory Authorized 5/20/53
Manager: W. Hall
Full task description: Apr-June 1953 issue, p. 45

Status: COMPLETED. The original project, computation of internal conversion coefficients for K, L_I, L_{II}, and L_{III} shells for Z = 15(10)95, was completed in the summer of 1956. For these computations the nucleus was considered as a point mass at the origin.

It was found that better accuracy could be attained, particularly for the higher values of Z, if the nucleus were considered to be a sphere of finite radius. Since the programs for computing coefficients for each shell were quite long (K, L_I, and L_{II} programs are each about 6,000 instructions; L_{III}, about 10,000) and the remaining time during which SEAC was to be available was small, it was desirable to make minimal changes in the existing programs.

The lower limit of integration was changed from zero to R, the radius of the nucleus; and increments computed by the following formulae were added to the eigenvalues:

$$\Delta w_K = \frac{6(1+\gamma)(\alpha Z)^2(2\alpha ZR)^{2\gamma}}{\gamma\Gamma(2\gamma+4)} \cdot \left[1 - \frac{\gamma(2\gamma+3)}{(\gamma+1)(\gamma+2)}\alpha ZR\right]$$

$$\Delta w_{L_I} = \frac{3(\alpha Z)^2(1+w)(2w-1)}{\gamma\Gamma(2\gamma+4)} \left(\frac{\alpha ZR}{w}\right)^{2\gamma}\left[1 - \frac{\gamma(2\gamma+3)\,\alpha ZR}{(2w-1)(\gamma+1)(\gamma+2)}\right]$$

$$\Delta w_{L_{II}} = \frac{3(\alpha Z)^2(1-w)(2w+1)}{\gamma\Gamma(2\gamma+4)} \left(\frac{\alpha ZR}{w}\right)^{2\gamma}\left[1 - \frac{\gamma(2\gamma+3)\,\alpha ZR}{(2w+1)(\gamma+1)(\gamma+2)}\right]$$

In all cases in the above, $\gamma = \sqrt{1 - \alpha^2 Z^2}$, and as always $\alpha = 1/137.0$.

It was then necessary to replace the starting values $f_{\kappa}^{(o)}$, $g_{\kappa}^{(o)}$, $F_K^{(o)}$, and $G_K^{(o)}$ by $f_{\kappa}^{(R)}$, $g_{\kappa}^{(R)}$, $F_K^{(R)}$, and $G_K^{(R)}$. The formulae for new starting values are:

$$b_+ = R(w+1) + \frac{3\alpha Z}{Z}, \qquad b_- = b_+ - 2R.$$

At r = R for $\kappa < 0$:

$$f_n(R) = \sum_{n=0}^{\infty} \nu_n, \qquad g_n^{(R)} = \sum_{n=0}^{\infty} \gamma_n,$$

where

$$\nu_n = \frac{1}{2n+2\kappa+1}\left[\frac{\alpha Z}{2}\gamma_{n-1} - b_-\gamma_n\right],$$

$$\gamma_{n+1} = \frac{1}{2(n+1)} \left[b_+ \nu_n - \frac{\alpha Z}{Z} \nu_{n-1} \right],$$

$$\gamma_0 = 1; \quad \gamma_{-1} = 0; \quad \nu_n = 0 \text{ for } n < 0.$$

For $\mathcal{H} > 0$:

$$f_{\mathcal{H}}^{(R)} = \sum_{n=0}^{\infty} \delta_n, \quad g_{\mathcal{H}}^{(R)} = \sum_{n=0}^{\infty} \epsilon_n$$

where

$$\delta_{n+1} = \frac{1}{2(n+1)} \left[\frac{\alpha Z}{2} \epsilon_{n-1} - b_- \epsilon_n \right],$$

$$\epsilon_n = \frac{1}{2n+2\mathcal{H}+1} \left[b_+ \delta_n - \frac{\alpha Z}{2} \delta_{n-1} \right],$$

$$\delta_o = 1; \quad \delta_n = 0 \text{ for } n < 0; \quad \epsilon_n = 0 \text{ for } n < 0,$$

$F_K(R)$ and $G_K(R)$ are computed the same way, with W substituted for w, and k for $\mathcal{H}$. (K = -1 for K and L_I shells, and K = 1 for L_{II} shells.)

All starting values were then normalized so that $f_{\mathcal{H}}(R) = f_{\mathcal{H}}(0)$ and $F_K(R) = F_K(0)$ in order to use existing programs without changing scaling at any point. This was permissible since the ratio of $f_{\mathcal{H}}$ to $g_{\mathcal{H}}$ is important, rather than values of $f_{\mathcal{H}}$ and $g_{\mathcal{H}}$.

The following cases were run using the new routines for k = .05, .1, .2, .6, 1.0:

K shell: Z = 55,

L_I shell: Z = 55, 65, 75, 85, 95,

L_{II} shell: Z = 55, 65, 75, 85, 95.

AUTOMATIC CODING
Task 3711-60-1120/55-65

Origin: NBS Authorized 9/29/54
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. Several latent errors were removed from the CORBIE system, and several new features were added. About 75 per cent of those who program for the 704 use the CORBIE system, and this system is used on about half of the problems being run on the 704 computer.

The average code checking run on the 704 takes about three minutes with the CORBIE system.

Publication:

(1) Automatic coding principles. J. Wegstein. Symposium on Advanced Programming Methods for Digital Computers, Washington, D. C., pp. 3-6 (Office of Naval Research, Department of the Navy, Washington, D.C.).

MATHEMATICAL SUBROUTINES
Task 3711-60-0009/56-160

Origin: NBS Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

Status: CONTINUED. A routine was written by J. P. Menard to compute the real roots of $P_n(x) = \sum_{i=0}^{n} a_i x^{n-i}$, where the a_i are real numbers $10^{-35} \leq |a_i| \leq 10^{35}$ and $2 \leq n \leq 100$. The routine requires that the polynomial shall have all real roots. The correction Δ for an approximation x_o of a root is given by the formula (derived by Laguerre):

$$\Delta = n \Big/ \left\{ s_1 \pm \sqrt{(n-1)(n\, s_2 - s_1^2)} \right\}$$

where

$$s_1 = -P_n'(x_o)/P_n(x_o), \quad P_n'(x_o) = dP_n/dx,$$

$$s_2 = [P_n'(x_o)/P_n(x_o)] - [P_n''(x_o)/P_n(x_o)].$$

Then $x_1 = x_o + \Delta$ is a better approximation for the nearest root.

A general purpose orthonormalizing code has been written by P.J.Walsh which employs the basic methods of Davis-Rabinowitz [Journal ACM, 1 (1954) 183-191]. The routine contains all the features of the Bram orthonormalizing code written for SEAC. In addition, the code employs a self-correcting technique suggested by P. Davis. Thus, at each step of the orthonormalizing procedure, greater accuracy is achieved, especially when the number of vectors is large. Two other features have been added to allow more flexibility in preliminary weighting and augmentation of the vectors. Matrix multiplication subroutines were introduced to facilitate coding.

Subroutines have been written by E. Haynsworth (1) to replace a symmetric matrix s which is stored in lower triangular form by the lower triangular matrix M which has the property that $M\, S\, M^T = I$, (2) to replace a real symmetric matrix H, stored in the lower triangular form, by its

transform $\bar{H}$ where $\bar{H} = M H M^T$, and (3) to transform the eigenvectors Y of $\bar{H}$ into corresponding vectors X for H, X = MY.

These three subroutines have been written as separate units but when linked together they provide a means of transforming the eigenvalue problem $(H - \lambda S) X = 0$ into the simpler problem $(\bar{H} - \lambda I) Y = 0$.

A series of tests proposed by M. Newman and J. Todd were carried out by P. J. Walsh to determine the accuracy of the matrix inverse routines written for the 704 and also to compare the results with tests previously conducted on SEAC. A routine was written to compute

$$n^{-1}\left\{\sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}^2\right\}^{\frac{1}{2}}, \qquad n^{-2}\left\{\sum_{i=1}^{n}\sum_{j=1}^{n} |r_{ij}|\right\},$$

for

$$R_1 = A A^{-1} - I, \quad R_2 = A^{-1} A - I, \quad R_3 = (A^{-1})^{-1} - A.$$

The matrix $A = (a_{ij})$ was generated internally; then the above norms were computed.

Subroutines are now available to generate the test matrices of order n and may be used to avoid the tedious and time-consuming task of data preparation. They will be submitted to SHARE. The following matrices were used in the tests:

(1) Triple diagonal matrix: $a_{ij} = -2$ if $i = j$,
$= 1$ if $j = i \pm 1$,
$= 0$ otherwise.

A is real symmetric and negative definite.

(2) Quintuple diagonal matrix: $A = \bar{A}^2$ where $\bar{A}$ is A as defined in (1) above. A is real symmetric and positive definite.

(3) Lehmer matrix 1: Let $n = p-1$ where p is a prime,

$a_{ij} = 0$ if $i + j = p$,
$= -1$ if $i+j$ is a perfect square,
$= 1$ otherwise.

(4) Lehmer matrix 2: $a_{ij} = i/j$ if $i \leq j$,
$= j/i$ if $i \geq j$.

(5) Givens matrix: $a_{ij} = 2 \min(i,j) - 1$.

(6) Hilbert matrix: $a_{ij} = (1+j+1)^{-1}$.

(7) Orthogonal matrix: $a_{ij} = \sqrt{2/(n+1)} \sin(ij\pi/(n+1))$.

(8) Rutishauser matrix: $a_{ij} = (1+j)!/i!j!$.

(9) Circulant matrix 1: The first row is (1,2,3,...,n).

(10) Circulant matrix 2: $A = n\,I + P$, where $(P_{ij}) = 1$, $(i,j)=1,2,3,\ldots,n)$.

(11) Circulant matrix 3: The first row is $(n^2, 1,2,\ldots,n-1,n,n-1,2,1)$.

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-5113/57-216

Origin and Sponsor: National Science Foundation Authorized 12/27/56
Manager: M. Abramowitz
Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. Work is progressing steadily on the various chapters, and preliminary drafts are available for many of them. The methods of computation included in the individual chapters are a significant feature in the use and extension of the tables. Particular attention is being given to the application of these methods to large scale computers as well as to their use with desk computers.

P. Davis and I. Polonsky have prepared a manuscript of chapter 25, tentatively entitled, "Interpolation, Differentiation and Integration," which includes those formulas and tables which have been found most useful in the Computation Laboratory for routine numerical analysis. The tables of chapter 6, "The Gamma Function and Related Functions," were checked by differencing, in addition to applying various functional checks.

The chapter on "Exponential, Logarithmic, Circular and Hyperbolic Functions" will contain the following tables:

Table 4.1. e^x, $\pm x = 0(.001)1$, 18D; $x = 1(.1)5$, 15D; $x = 5(.1)10$, 12D; $x = 1(1)100$, 19S; $-x = 1(.1)2.5$, 18D; $-x = 2.5(.1)10$, 20D; $-x = 1(1)100$, 19S.

4.2. e^x, e^{-x}, $x = 10^{-n}(10^{-n})10^{-n+1}$, $n = 10(-1)1$, 25D.

4.3. $\log_{10} x$, $x = 100(1)1200$, 10D.

4.4. $\ln x$, $x = 0(.001)2.000$, 16D.

4.5. $\ln x$, x prime, $1 < x < 1000$, 25D.

4.6. $\ln(1+x), -\ln(1-x)$, $x = 10^{-n}(10^{-n})10^{-n+1}$, $n = 13(-1)1$, 25D.

4.7. $-\ln(-\ln x)$, $x = 0(.001)1.000$, 5D;
$-\ln(\ln x)$, $x = 1(.1)10(1)20$, 5D.

Table 4.8. $\sin x$, $\cos x$, $x = 0(.001)1.6$, 23D.

4.9. $\sin x$, $\cos x$, $x = 10^{-n}(10^{-n})10^{-n+1}$, $n = 10(-1)4$, 25D.

4.10. $\sin x$, $\cos x$, $x = 1(1)100$, 23D; $x = 100(1)1000$, 8D.

4.11. $\tan x$, $\cot x$, $\sec x$, $\csc x$, $x = 0(.01)1.60$, 8S.

4.12. $\frac{1}{x} - \cot x$, $\csc x - \frac{1}{x}$, $x = 0(.01).50$, 8D.

4.13. $\sin \theta$, $\cos \theta$, $\theta = 0(.1°)45°$, 15D.

4.14. $\tan \theta$, $\cot \theta$, $\theta = 0(.5°)45°$, 15D.
$\sec \theta$, $\csc \theta$, $\theta = 0(.5°)45°$, 9S.

4.15. $\sin \frac{\pi}{2}x$, $\cos \frac{\pi}{2}x$, $\tan \frac{\pi}{2}x$, $\cot \frac{\pi}{2}x$, $\sec \frac{\pi}{2}x$, $\csc \frac{\pi}{2}x$:

$x = 0(.01).50$, 20D.

4.16. Harmonic Analysis: $\sin \frac{2\pi}{s}$, $\cos \frac{2\pi}{s}$, $r = 1(1)[s/2]$, s even;

$r = 1(1)[(s-1)/2]$, s odd; $s = 3(1)25$; 10D.

4.17. arc sin x, arc tan x, $x = 0(.001)1.000$, 12D.

4.18. $f(x) = (2x)^{-\frac{1}{2}}[\pi/2 - \text{arc sin } (1-x)]$, $x = 0(.001).050$, 12D.

4.19. $\sinh x$, $\cosh x$, $x = 0(.01)2.00$, 9D; $x = 2(.1)10$, 9D.
$\tanh x$, $\coth x$, $x = 0(.01)2.00$, 8S; $x = 2(.1)10$, 10D.

4.20. $e^{\pm \pi x}$, $\sinh \pi x$, $\cosh \pi x$, $\tanh \pi x$, $x = 0(.01)1.00$, 10D.

4.21. arc sinh x, arc tanh x, $x = 0(.01)1.00$, 9D.

4.22. arc tanh $x + \frac{1}{2} \ln(1-x)$, $x = .50(.01)1.00$, 8D.

4.23. arc sinh x, (arc cosh x)$/\sqrt{x^2-1}$, $x = 1.00(.01)2.00$, 8D, 9D.

4.24. arc sinh $x - \ln x$, arc cosh $x - \ln x$, $1/x = 0(.01).50$, 10D.

4.25. Roots x_n of $\tan x_n = \lambda x_n$, $1/\lambda = -1.00(.05)1.00$, $-\lambda = 0(.05)1.00$, } n=1(1)9, 5D.

$x_1 / \sqrt{3(\lambda-1)}$, $1/\lambda = .5(.05)1.00$, 5D.

4.26. Roots x_n of $\cot x_n = \lambda x_n$, $1/\lambda = 0(.05)1.00$, $-\lambda = 0(.05)1.00$, } n=1(1)9, 5D.

$x_1 \cdot \sqrt{1/\lambda}$, $1/\lambda = 0(.05)1.00$, 5D.

Table 4.27. Roots x_n of $\cos x_n \cosh x_n = \pm 1$, $n = 1(1)5, 7D, 6D$.

HEAT TRANSFER
Task 1102-40-1110/57-241
(formerly 3711-60-0009/57-241)

Origin: NBS, Section 11.2 Authorized 6/30/57
Managers: M. Abramowitz, W. F. Cahill
Full task description: Apr-June 1957 issue, p. 31

Status: CONTINUED. The code to compute

$$\alpha_{ij} = \int_0^1 r(1-r^2)y(r,z_i)y(r,z_j)dr,$$
$$\beta_i = \int_0^1 r(1-r^2)y(r,y_i)dr,$$

which form the coefficients of the linear system satisfied by the coefficients in the characteristic function expansion has been written and checked out. The first ten characteristic functions for $\lambda = 1$ have been computed.

3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-12-1107/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. M. Zelen continued work on his chapter for the Handbook of Mathematical Tables (see p.13; also Oct-Dec 1956 issue, p. 10). Portions dealing with the normal probability integral, bivariate normal, and the chi-square and incomplete gamma functions were revised and checked. The last section to be completed is the incomplete beta function. Upon completion of the last section, the tables will be collected and examples of computations using the tables will be shown.

M. Zelen has completed in manuscript form a paper entitled, "Linear estimation and related topics." This contains the material recently given before the NSF-NBS Training Program in Numerical Analysis (see Apr-June 1957 issue, p. 6).

Publication:

(1) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. Joel. Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-12-1107/53-1

Origin: NBS Authorized 10/15/52
Manager: W. S. Connor
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. D. M. Mesner prepared a draft of a paper, "On linear associative algebras corresponding to association schemes of partially balanced designs," to be published jointly with R. C. Bose. Dr. Mesner also began a study of an algebra which is associated with the binomial coefficient C_r^v.

M. Zelen and W. S. Connor prepared in manuscript form an expository paper, "Multi-variable experiments."

Publications:

(1) Multi-variable experiments. M. Zelen and W. S. Connor. In manuscript.

(2) On the identity relationship for fractional replicates of the 2^n series. R. C. Burton and W. S. Connor. Ann. Math. Stat. 28, 762-767 (1957).

(3) Experiences with incomplete block designs: Examples. W. S. Connor. To appear in the Proceedings of the Symposium on the Design of Industrial Experiments, held at the University of North Carolina, Raleigh, N. C., November 1956.

(4) The principle of randomization in the design of experiments. C. Eisenhart. Synopsis appeared in the Proceedings of the Conference on the Design of Experiments in Army Research, Development and Testing, Office of Ordnance Research Report 57-1, June 1957, Durham, N. C., pp. 15-16.

(5) Design of experiments in research and development. W. J. Youden. Proceedings of the Conference on the Design of Experiments in Army Research, Development and Testing, Office of Ordnance Research Report 57-1, June 1957, Durham, N. C., pp. 9-14.

(6) The analysis of covariance for incomplete block designs. M. Zelen. To appear in Biometrics.

(7) The analysis of incomplete block designs. M. Zelen. J. Amer. Stat. Assoc. 57, 204-217 (1957).

(8) The use of incomplete block designs for asymmetrical factorial arrangements. M. Zelen. To appear in the Annals of Mathematical Statistics.

(9) Experiments with many factors. M. Zelen. To appear in the Proceedings of the Third Annual Statistical Engineering Symposium, U. S. Army Chemical Corps, Edgewood, Maryland, 1957.

(10) The uniqueness of the triangular association scheme. W. S. Connor. Submitted to a technical journal.

(11) On linear associative algebras corresponding to association schemes of partially balanced designs. D. M. Mesner and R. C. Bose. In manuscript.

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-1107/56-170

Origin: NBS Authorized 12/15/55
Manager: Joan R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

Status: CONTINUED. J. R. Rosenblatt is teaching an in-hours course, "Non-Parametric Statistical Techniques," in the NBS Graduate School program. The first semester of classes began in September. Initial registration included 15 students from the NBS, the Diamond Ordnance Fuze Laboratories, the U. S. Geological Survey, and other agencies.

Publication:

(1) Table of the first moment of ranked extremes. J. Lieblein and H. E. Salzer. J. Res. NBS 59, 203-206 (1957), RP 2787.

MEASUREMENT OF RELIABILITY
Task 1103-12-1130/56-182

Origin: NBS Authorized 3/23/56
Manager: M. Zelen
Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. M. Zelen prepared a paper entitled, "Multi-factor experiments for evaluating reliability", for presentation at the Joint Military-Industry Guided Missile Reliability Symposium, November 5-7, at Point Mugu, California. The paper considers various aspects of the planning and analysis of factorial experiments when the type of experiment is one where failure data are being taken. Methods are discussed for evaluating the failure data with respect to different environmental conditions of operation. A mathematical model is developed which permits: (1) the relative importance of each factor to be evaluated with respect to its influence in causing failure of the items under test; (2) requirements to be made as to when accelerated life testing at some extreme environmental condition can be made, so as to permit meaningful extrapolation to normal operating conditions; (3) the experimenter to plan tests so as to minimize the number of different combinations of the factors and still have meaningful results.

J. R. Rosenblatt initiated work on a class of estimation problems suggested by the need to formalize an intuitively appealing procedure for presenting the results of aging studies. Preliminary work on these problems was discussed in an address entitled, "Models for the analysis of component aging experiments", presented before the Statistical Summer Session at the Massachusetts Institute of Technology, August 8-9.

J. M. Smith carried out some numerical calculations to illustrate possible effects of time-dependent inter-relations among components in a system. J. R. Rosenblatt is preparing a report describing the simple Markov process models used.

Publication:

(1) On some aspects of prediction in the study of complex systems. J. R. Rosenblatt. To appear in the Proceedings of the NYU-RCA Working Conference on Theory of Reliability, Ardsley-on-Hudson, New York, April 1957.

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-12-1115/55-57

Origin: NBS Authorized 9/1/54
Manager: R. F. Dressler
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. W. H. Pell has completed a paper on graphical methods of integration for ordinary differential equations of certain types. Included among these are equations of importance in the theory of mechanical vibrations and oscillations in electrical circuits. Methods are developed which are applicable to non-linear as well as linear equations. Figures are now being drawn which illustrate the methods employed.

F. Chilton completed the writing of a paper on some theoretical aspects of the Cartesian Diver balance, and the manuscript has been submitted to a technical journal.

A theoretical study was undertaken by F. Chilton concerning the spinning baseball problem, in conjunction with some experimental work being done at NBS by L. J. Briggs. The study included an analysis of experimental data and theoretical examination and evaluation of the experimental method used and various suggested alternatives. Calculations for the problem of collision of baseball and launching projectile and the subsequent motion are in progress.

R. F. Dressler investigated and integrated a differential equation, at the request of the Chemistry Division, which describes the rate at which a super-cooled liquid in which crystal initiation exists will approach the equilibrium temperature.

A. Ghaffari obtained the solutions for the stream function of a steady irrotational compressible flow past a wedge for both subsonic and supersonic regions. It is shown that the stream function for subsonic regions is a positive monotonically increasing function of τ, and also the stream function for supersonic regions is a positive decreasing function of τ. The dimensionless independent variable τ which is of practical significance ranges between zero when the air speed is zero and unity when the air speed reaches its maximum possible value as the pressure is reduced to zero. This paper was presented at the American Mathematical Society meeting at Pennsylvania State University, August 27-30.

Publications:

(1) On the representation of a certain integral involving Bessel functions of hypergeometric series. P. Henrici. To appear in the Journal of Mathematics and Physics.

(2) On the domain of regularity of generalized axially symmetric potentials. P. Henrici. To appear in the Proceedings of the American Mathematical Society.
(3) The quotient-difference algorithm. P. Henrici. To appear in NBS Applied Mathematics Series 49, "Further contributions to the solution of simultaneous linear equations and the determination of eigenvalues."
(4) Graphical solution of initial value problems. W. H. Pell. In manuscript.
(5) On the propagation of turbidity currents. R. Stoneley. To appear in the Vening Meinesz Jubilee Volume.
(6) On the solution of compressible flow past a wedge. A. Ghaffari. In manuscript. Abstract only: Bull.Amer. Math. Soc. 63, 244-245 (1957).
(7) On some expansions for Bessel integral functions. F. Oberhettinger. J. Res. NBS 59, 197-201 (1957), RP2786.

RESEARCH IN CONTINUUM MECHANICS
(formerly Mathematical Elasticity)
Task 1104-12-5160/55-85

Origin: NBS Authorized 12/27/54
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: R. F. Dressler
Full task description: Oct-Dec 1954 issue, p. 30

Status: CONTINUED. C. M. Tchen has begun a study of the fundamental equations of magnetohydrodynamics. The main difficulty lies in the energy equation; this has been solved by considering macroscopically the electromagnetic effect both as body force and as stresses. These equations have the same limitations as the classical hydrodynamical equations. In order to extend them and to study the structure of the physical coefficients, their statistical basis has been investigated. For this purpose, a non-linear transition probability depending upon the distribution function was introduced to study the "master equation" of the transport phenomena. It is hoped that a more generalized transport equation than the Boltzmann equation can be found in this way, which will serve as a basis of the fundamental equations of magnetohydrodynamics.

W. H. Pell has completed an investigation and written a paper on the factorization of the differential operator which appears in the equation

$$\frac{1}{g(x)} \frac{d^2}{dx^2} \left(f(x) \frac{d^2 w}{dx^2}\right) + k^2 w = 0,$$

which occurs in problems in the bending and vibration of beams and

cylindrical shells with variable bending stiffness. The factorization permits one to give the solutions, hitherto unknown, of a class of problems occurring in these disciplines.

R. F. Dressler attended the AGARD Wind Tunnel Panel on Hypersonics at Scheveningen, Holland, in July and the Anglo-American Aeronautical Conference, in London, September, and served as Consultant for Aerodynamics and Elasticity for AFOSR in Europe during August and September.

Publications:

(1) Stationary principles for forced vibrations in elasticity and electromagnetism. J. L. Synge. To appear in the Proceedings of the Eighth Symposium in Applied Mathematics of the American Mathematical Society, held at Chicago, Ill., April 1956.
(2) Elastic waves in anisotropic media. J. L. Synge. J. Math. Phys. 35, 323-334 (1957).
(3) The torsion of a hollow square. J. L. Synge and W. F. Cahill. To appear in the Quarterly of Applied Mathematics.
(4) The vibration of triangular wings. R. F. Dressler. In manuscript. Abstract available in the Proceedings of the Ninth International Congress on Mechanics, Brussels, September 1956.
(5) The elastic problem for a ring of uniform force in an infinite body. W. H. Pell. To appear in the Journal of Research, NBS.
(6) On the factorization of a fourth order differential operator occurring in the theory of structures. W. H. Pell. In manuscript.

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-5160/56-154

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: July-Sept 1955 issue, p. 20

Status: INACTIVE.

RESEARCH IN FLUID DYNAMICS OF TWO-PHASE FLOWS
Task 1104-12-5160/56-155

Origin and Sponsor: Office of Naval Research Authorized 9/30/55
Manager: R. F. Dressler
Full task description: July-Sept 1955 issue, p. 21

Status: CONTINUED. J. M Burgers and A. Ghaffari extended their

study to the particularities of the condensation process in the mixing chamber. The question of the appearance of a shock wave either in the nozzle delivering the high speed steam or in the mixing chamber is being considered. The application of constant pressure in the mixing chamber will prevent the occurrence of a condensation shock, on the ground that such shocks do not seem to appear in steam injectors. A paper has been completed on one phase of the investigation.

Publications:

(1) Condensation interfaces in two-phase flows. P. Chiarulli and R. F. Dressler. J. Appl. Phys. 28, 990-997 (1957). Abstract appeared in the Proceedings of the Ninth International Congress on Mechanics, Brussels, September 1956.
(2) On the application of steam driven water jets for propulsion purposes. J. M. Burgers and A. Ghaffari. To appear in the Journal of Research, NBS.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

1102-40-5126/51-37 MOLECULAR STRUCTURE, III
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. O'Hara
Full task description: July-Sept 1951 issue, p. 50
Status: Terminated. This problem is continued under task 1102-40-5126/58-269, see page38.

1102-40-5126/52-44 CALCULATIONS FOR d-SPACINGS
Origin and Sponsor: NBS, Division 9
Full task description: Oct-Dec 1951 issue, p. 47
Manager: R. Zucker
Status: Terminated. This problem is continued under task 1102-40-5126/58-274, see page 38.

1102-40-5126/54-13 AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: New York Quartermaster Procurement Agency
Manager: H. Bremer
Full task description: Oct-Dec 1953 issue, p. 43
Status: Continued. The 704 code for the transportation problem, as written by IBM and modified for the particular 704 in our installation, is being checked out by testing it on some of the earlier problems that were done on SEAC. No further problems were submitted by the sponsor.

3711-60-0009/54-22 ENERGY DISTRIBUTIONS ON OPTICAL IMAGE
Origin: NBS, Section 2.2
Manager: L. S. Joel
Full task description: Jan-Mar 1954 issue, p. 43
Status: Inactive. For status to date, see Jan-Mar 1956 issue, p. 19.

3711-60-0009/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Managers: C. D. Coleman, W. Bozman (4.1)
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. The 704 codes have been written for evaluating and computing differences between known energy levels, for searching for line pairs with these differences, for predicting new levels, and for checking for reoccurrence of predicted levels. The final phase of the code for evaluating predictions is being code checked.

These codes have been used to compute new odd levels in rhenium II. Data are being prepared for calculation of new even levels of rhenium II and of technetium I.

3711-60-0009/54-38 COMPRESSIBILITY FACTORS OF DRY AIR
Origin: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Inactive.

3711-60-0009/55-68 CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Division 9
Manager: P. O'Hara, S. Block (9.7)
Full task description: Jan-Mar 1955 issue, p. 18
Status: Continued. Structure factor calculations and least square refinements of atomic position parameters were carried out for the following crystals: azulene, suanite, calcium apatite, and triclinic magnesium borate.

The share program NYXRI was used to carry out the calculations on the IBM 704.

3711-60-0009/55-75 PARAMETER OF THE DISPERSION EQUATION FOR OPTICAL GLASS
Origin: NBS, Section 2.2
Manager: R. Zucker
Full task description: Jan-Mar 1955 issue, p. 20
Status: Inactive.

3711-60-0009/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. The code for applying bridge corrections to thermometer data and averaging the data was prepared; the code for calculating related thermometer tables was written. Code checking is in progress.

1102-40-5126/55-88 STRESSES IN A WALL FOUNDATION
Origin and Sponsor: NBS, Section 10.1
Manager: I. Stegun
Full task description: Jan-Mar 1955 issue, p. 22
Status: Inactive.

1102-40-5126/55-117 ATTENUATION OF PRESSURE PULSES OF FINITE AMPLITUDE
Origin: NBS, Section 3.2
Sponsor: Bureau of Aeronautics, U.S.Navy
Manager: M. Paulsen
Full task description: Apr-June 1955 issue, p. 18
Status: Inactive.

1102-40-5126/55-121 ELECTRON PENETRATION
Origin: NBS, Section 4.8
Sponsor: Atomic Energy Commission
Manager: S. Peavy
Full task description: Apr-June 1955 issue, p. 19
Status: Continued. The code for carrying out the solution on the 704 was completely checked out. Results have been calculated for z = 6, 13, 29, 50, 82, for e = .025, .05, .1, .2,.4, .7, 1,2,4 Mev.

1102-40-5126/56-136 CALCULATION OF WAVE FUNCTIONS BY HARTREE METHOD
Origin and Sponsor: Naval Research Laboratory
Manager: S. Peavy
Full task description: July-Sept 1955 issue, p. 34
Status: Inactive.

1102-40-5126/56-139 STUDY OF INTERNUCLEAR POTENTIAL FOR H_3
Origin and Sponsor: NBS, Section 3.2
Manager: E. Haynsworth
Full task description: July-Sept 1955 issue, p. 36
Status: Reactivated. This task has been reactivated to provide better values of molecular energies and wave functions than heretofore obtained. The data provided earlier serves as a basis for the additional calculations.

1102-40-5126/56-162 STRESSES IN A WALL RESTING ON A FOOTING
Origin and Sponsor: NBS, Section 10.1
Manager: I. Stegun
Full task description: Jan-Mar 1956 issue, p. 26
Status: Inactive.

1102-40-5126/56-163 ANGULAR DISTRIBUTIONS AND POLARIZATION EFFECTS IN NUCLEAR SCATTERING
Origin and Sponsor: Naval Research Laboratory
Manager: I. Stegun
Full task description: Oct-Dec 1955 issue, p. 32
Status: Continued. Various time-saving modifications are being made in the code for the solution of the Coulomb wave equation.

1102-40-5126/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Managers: E. Haynsworth, P. Walsh
Full task description: Jan-Mar 1956 issue, p. 27
Status: Continued. Various calculations, all related to the original problem, were required:

(1) The inverses of several matrices were computed and submitted to the sponsor.

(2) A routine was written to compute AWA^{-1} and $(AWA^{-1})'$ where A is an nxn matrix, and W is an nx1 column vector. The code has been checked out and several cases for n = 18 have been computed.

(3) The simulated SEAC code for computing eigenvalues and their corresponding eigenvectors was also used on matrices of order 9,10, and 18.

1102-40-5126/56-171 COLLISION INTEGRALS USED IN TRANSPORT THEORY
Origin and Sponsor: NBS, Section 3.2
Manager: D. Sumida
Full task description: Oct-Dec 1955 issue, p. 33
Status: Inactive.

1102-40-5126/56-172 NUMERICAL EVALUATION OF SPECIAL INTEGRAL EXPRESSIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: L. Joel
Full task description: Jan-Mar 1956 issue, p. 29
Status: Inactive.

1102-40-5126/56-179 NORMAL PROPAGATION CONSTANT
Origin and Sponsor: NBS, Section 82.10
Manager: H.H. Howe (82.10)
Full task description: Apr-June 1956 issue, p. 30
Status: Continued. A few computations were made to complete the work described on page 26 of the Apr-June 1957 report. The main work of the quarter consisted of transferring the results of that work from magnetic wire to paper tape to punched cards, and sorting the cards. Some 60,000 cards are now on hand.

In addition, computations were made showing the ionospheric reflection coefficient R_s, given by the formulas on page 30 of the report for Jan-Mar 1957, for various values of the parameters L, D/L, L/L_o, and c, using real values of c.

1102-40-5126/56-184 GAIN CALCULATIONS OF AN ITERATED TRANSISTOR AMPLIFIER
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: E. Haynsworth
Full task description: Apr-June 1956 issue, p. 32
Status: Inactive. For status to date, see July-Sept 1956 issue, p. 32.

1102-40-5126/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: S. Prusch
Full task description: July-Sept 1956 issue, p. 33
Status: Continued. Calculation of variance was run for 15 laboratory test sets of master gage blocks--where each set consists of a maximum of 88 blocks.

1102-40-5126/57-209 TRAFFIC DISTRIBUTION
Origin and Sponsor: Bureau of Public Roads
Manager: S. Peavy
Full task description: Jan-Mar 1957 issue, p. 32
Status: Continued. Checking out on the IBM 705 (at the Treasury Department) has been completed, and several runs have been made. The resulting data are being analyzed by the sponsor.

3711-60-0009/57-210 SOUND VELOCITY
Origin: NBS, Section 3.2
Manager: J. P. Menard
Full task description: July-Sept 1956 issue, p. 34
Status: Inactive. For status to date, see Jan-Mar 1957 issue, p. 32.

1102-40-5126/57-211 METEOROLOGICAL DATA
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: P. O'Hara
Full task description: Oct-Dec 1956 issue, p. 30
Status: Inactive.

1102-40-5126/57-219 THERMAL PROPERTIES
Origin and Sponsor: NBS, Section 3.2
Manager: D. Sumida
Full task description: Oct-Dec 1956 issue, p. 30
Status: Inactive. For status to date, see Apr-June 1957 issue, p. 27.

1102-40-5126/57-221 BESSEL FUNCTIONS FOR COMPLEX ARGUMENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Oct-Dec 1956 issue, p. 31
Status: Inactive.

1102-40-5126/57-222 ROOTS OF POLYNOMIALS
Origin and Sponsor: Naval Research Laboratory
Manager: J. P. Menard
Full task description: Oct-Dec 1956 issue, p. 32
Status: Inactive.

3711-60-0009/57-223 SELF-CONSISTENT FIELDS
Origin: NBS, Section 3.2
Manager: E. V. Haynsworth
Full task description: Apr-June 1957 issue, p. 28
Status: Continued. All the subroutines for the main SCF routine on the 704 have been written and checked out separately. To run the complete routine a floating point subroutine for finding eigenvalues and eigenvectors is needed. Such a routine has been submitted to SHARE and should be available soon, so that the routine may be checked out in its entirety.

Work has been started on the auxiliary routine to calculate the A matrix which is a function of the direct and exchange integrals determined by a preliminary program being coded elsewhere.

Also, preliminary flow charts and plans have been made for the control program and master program which will call in data and subroutines from tape and drum as they are needed and put them out again on tape and drum when a specific part of the computation is completed. As it is anticipated that this SCF program will be used at a number of different installations with different amounts of core, tape and drum space available, the master program will be coded to use these amounts as parameters with which to compute the necessary variations in the control program.

1102-40-5126/57-224 TRACK-WHILE-SCAN RADAR PROBLEM
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Managers: E. Haynsworth, P.J.Walsh
Full task description: Oct-Dec 1956 issue, p. 32
Status: Continued. Slight modifications were introduced in some of the factors of the original formulae. The code has been completely checked and several successful runs for various parameters have been submitted to the sponsor.

1102-40-5126/57-225 DAMAGE ASSESSMENT PROBLEM, II
Origin and Sponsor: Corps of Engineers, U. S. Army
Managers: H. Bremer, W. G. Hall, L. S. Joel
Full task description: Oct-Dec 1956 issue, p. 33
Status: Completed.

3711-60-0009/57-229 APPLICATION OF ELECTRONIC DATA PROCESSING MACHINERY TO PAYROLL OPERATIONS
Origin: NBS, Section 40.0
Managers: H. Bremer, P. R. McClenon, M. Paulsen
Full task description: Jan-Mar 1957 issue, p. 36
Status: Continued. The payroll task has been separated into five separate parts: (1) To create a master tape; (2) To provide for file maintenance of the master tape (this is to include both personnel actions and time reports); (3) To calculate gross pay and all appropriate payroll deductions; (4) To calculate labor cost distribution; (5) To provide for various output reports.

Creation of the master tape is almost completed. Detailed machine instructions have been written and checked. The file maintenance job has been started. Payroll calculation has been represented on 11 individual flow charts, of which about half have been reduced to specific machine instructions. A considerable amount of planning has been done in connection with the required outputs.

3711-60-0009/57-232 POLYNOMIAL EVALUATION
Origin: NBS, Section 3.1
Manager: W. F. Cahill
Full task description: Apr-June 1957 issue, p. 29
Status: Completed. The summation was evaluated on the 704 as requested, and the results were transmitted to the sponsor.

1102-40-5126/57-234 PERSONNEL SURVEY
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: P. O'Hara
Full task description: Jan-Mar 1957 issue, p. 37
Status: Inactive.

3711-60-0009/57-235 TRIPLE INTEGRALS—ENTROPY CALCULATIONS
Origin: NBS, Section 3.2
Managers: W. Gautschi, A. Beam
Full task description: Apr-June 1957 issue, p. 30
Status: Completed. The integrals were evaluated on the 704 as requested and the results were transmitted to the sponsor.

3711-60-0009/57-238 MODIFIED LANDAU FUNCTION
Origin: NBS, Section 4.8
Managers: S. Peavy, R. Allsbrook
Full task description: Apr-June 1957 issue, p. 31
Status: Completed. The function has been evaluated on the IBM 704 as requested, and the results have been transmitted to the sponsor.

1102-40-5126/57-244 EXCHANGE INTEGRAL ANALYSIS
Origin: NBS, Section 3.2
Sponsors: NBS, Section 3.2; National Science Foundation
Manager: E. V. Haynsworth
Objective: To test the dependence of the electronic properties of molecules on exchange integrals, possibly providing direction as to when estimation of exchange integrals is permissible and when it is not.
Background: Knowledge of this type is invaluable to those involved in such calculations because of the very real difficulties still encountered in obtaining reliable values of exchange integrals, despite the revolution introduced by electronic computers. The importance of using exact values of exchange integrals in molecular computations is at best a moot question. A few computations, notably one on CO_2, yielded quite respectable results employing estimated values of these quantities, while other computations fared not so well. Discussions of the latter cases include statements concerning the need for using exact values together with the comment that the former good results arise from fortuitous cancelling out of error.

In an effort to establish something more definitive than opinion and hearsay on this point, two molecules, for which exact values of all integrals to six decimal places are available, have been chosen and errors are to be methodically introduced into the exchange integrals singly and in groups. Total electronic energies, orbital energies (which correspond to molecular ionization potentials) and wave functions are then to be computed using an SCF routine that was developed under task 1102-40-5126/56-166 (see p. 26). The problem was communicated by B. J. Ransil, 3.2.
Status: New.

3711-60-0009/57-247 MECHANICAL IMPEDANCE
Origin: NBS, Section 6.1
Managers: J. P. Menard, M. D. Burkhard (6.1)
Full task description: Apr-June 1957 issue, p. 32
Status: Continued. A program has been written and checked for each of the three parts of the problem. The operation of the completed programs has been turned over to the sponsor, and a number of production runs have already been made. Production runs on the program will continue under the sponsor's direction.

3711-60-0009/57-248 THE EVALUATION OF A TRIPLE INTEGRAL FOR THE SOLUTION OF NEGATIVE ION DETACHMENT

Origin: NBS, Section 4.6
Manager: S. Peavy
Full task description: Apr-June 1957 issue, p. 34
Status: Continued. The code, for the 704, is in preparation.

1102-40-5126/57-249 COLOR DIFFERENCES

Origin and Sponsor: NBS, Section 9.4
Manager: D. Sumida
Objective: To obtain statistics from gloss and color measurements of various specimens of enamels exhibiting effects of weathering. The following quantities are to be evaluated on the 704:

(1) Per cent gloss retained $= \frac{G_f}{G_i} \times 100$

(2) Per cent gloss change $= \frac{G_i - G_f}{G_i} \times 100$

(3) $L = 10\sqrt{Rd}$

(4) $\Delta L = L_i - L_f$

(5) $\Delta a = a_i - a_f$

(6) $\Delta b = b_i - b_f$

(7) $\Delta E = \sqrt{a^2 + b^2 + L^2}$

where G_i is the initial gloss reading; G_f, the final gloss reading; a_i, b_i and Rd_i, the initial color readings; a_f, b_f and Rd_f, the final color readings; and ΔE, the color difference.

Background: Increased use of porcelain enamel for architectural panels has caused manufacturers to take a greater interest in the causes and effects of weathering. The ultimate objective of the investigation is to develop an accelerated test to determine weather resistance. By the use of the accelerated test, fabricators will be able to determine if enamels will be good or poor when exposed to weathering. Information obtained from weathered specimens will give a basis for developing this test.
Status: Completed (New). The calculations were carried out as requested, and the results were transmitted to the sponsor.

3711-60-0009/57-250 AUTOMATIC REDUCTION OF SPECTROPHOTOMETRIC DATA

Origin: NBS, Section 2.1
Manager: W. C. Rheinboldt
Objective: To convert given spectrophotometric data into the 1931 CIE - Standard-Observer and Coordinate System for any given color temperature.

The spectrophotometric data are either corrected or uncorrected for photometric scale error; in the latter case the program is to perform the correction process. Also two transformations of the standard system are to be incorporated into the code,—namely, the transformation into the Judd Uniform Chromaticity Scale (UCS), and the transformation into the Breckenridge-Schaub Rectangular Uniform Chromaticity Scale (RUCS). Furthermore, the cone-to-rod ratio is to be computed.

Background: A large amount of spectrophotometric data has to be reduced in the course of the work of the Photometry and Colorimetry Section (2.1). Until now this was done by hand computation. This situation gave rise to an investigation of the possibility of reducing the data on the IBM 704. It was found that this was possible and that various kinds of problems could be incorporated in one 704 program. The problem was transmitted by H. Keegan.

Status: New. The code has been written and checked out on the IBM 704. Computations have been started.

1102-40-5126/57-251 CURRENT NOISE AND FIXED RESISTORS

Origin: NBS, Section 1.6

Manager: D. Sumida

Objective: Starting from measurements taken on groups of electrical resistors, to obtain the conversion gain of each resistor and certain statistical data for each group of resistors. The statistical data include the mean, 95 per cent confidence limits, variance, and minimum and maximum values of conversion gain of a group of resistors.

Background: The purpose of the project is to measure the noise developed by dc current flowing through fixed resistors and to analyze the results obtained. Current noise measurements have been made on approximately 3,000 resistors, which were usually measured in groups of ten. The measurements on a group of resistors consist of a calibration figure which is kept constant for the group and individual resistor measurements of set noise, total noise, and the dc loading power. Computations must be performed, using these measurements, to obtain a noise index figure called conversion gain. Statistical data using conversion gain within the group must also be computed to aid in the analysis of results. Because of the large volume of measurements, computation on a high speed digital machine is advantageous. The problem was submitted by G. T. Conrad, Jr. (1.6).

Status: New. The code has been prepared for the 704 and checked out. Production runs have been scheduled.

3711-60-0009/58-254 REPRODUCTION OF COLOR- AND SPECTRAL-ENERGY DISTRIBUTION OF DAYLIGHT AND OTHER ILLUMINANTS

Origin: NBS, Section 2.3

Manager: W. C. Rheinboldt

Objective: (1) To compute the spectral energy distribution of light transmitted through a filter which consists of a two-compartment cell

filled with a given mixture of three chemical solutions. (2) To compute the trilinear color coordinates from this energy distribution using three different sets of excitation data, one of which is that adopted by the Optical Society of America. Provision is made to use the second part of the program separately and to compute these trilinear coordinates for any given energy distribution.

Background: Combinations of a given source of light with filters of the described type have been extensively used to simulate light from other sources as accurately as possible in regard to color and spectral energy distribution. For instance such filter-source combinations have been designed by the sponsor to match mean-noon-sunlight and-daylight and these filters have become International Standards. In continuation of this work especially in order to simulate daylight and other illuminants and to develop American Standard - Illuminants for photographic sensitometry a large amount of data has to be reduced. It was therefore desirable to mechanize these calculations and to reduce the data automatically on the IBM 704. The problem was submitted by R. Davis (2.3).

Status: New. The problem has been analyzed, coded for the 704, and checked out. Production runs have been started.

1102-40-5126/58-258 NOISE MEASUREMENT II

Origin and Sponsor: NBS, Section 6.1

Manager: W. F. Cahill

Objective: To compute the distribution of kinetic energy near the boundaries of a reverberation chamber excited by a sound field. Also, to compute the radiation resistance seen by a simple source radiating a finite band width of frequencies and situated midway between two plane parallel walls which are imperfect reflectors.

Background: These computations are of importance in using a reverberation chamber. The problem was transmitted by R. V. Waterhouse (6.1).

Status: New. The output of a simple source in a spherical enclosure and the kinetic energy distributions near a two-wall edge and near a two-wall corner have been computed; the results have been transmitted to the sponsor. The problems of kinetic energy distributions for a three-wall corner and along a line of symmetry near a corner are being formulated.

3711-60-0009/58-261 TEMPERATURE-INDUCED STRAINS IN A SLAB

Origin: NBS, Division 9.0

Manager: J. P. Menard

Objective: To compute tables showing the ratio ft(q,x), "fractional temperature excess", of the internal temperature to the surface temperature for a slab, a cylinderical rod, and a sphere, after a sudden change in the surface temperature. Also, the average fractional temperature excess, aft(q), for the slab and a "generalized" temperature average for the cylinder and the sphere are to be computed.

The functions to be computed in each case are as follows:

1. For a slab:

$$\underline{ft}(q,x) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{Q_n} \exp\left(-\frac{Q_n^2}{4q^2}\right)\cos\left(Q_n \frac{X}{a}\right)$$

$$\underline{aft}(q) = \int_o^1 \underline{ft}(q,x)\,d\,(x/a)$$

$$= 2\sum_{n=1}^{\infty} \frac{\exp\left(-\frac{Q_n^2}{4q^2}\right)}{Q_n^2}$$

where $Q_n = (2n-1)\pi/2$, $q = a/2\sqrt{kt}$,

$q = 0.3(0.02)3.0(0.1)10.0$,

$x/a = \quad 0(0.1)1.0.$

2. For a cylindrical rod:

$$\underline{ft}(q,x) = 2\sum_{n=1}^{\infty} \exp\left(-\frac{R_n^2}{4q^2}\right)\frac{J_o\left(R_n \frac{x}{a}\right)}{R_n J_1(R_n)}$$

$$\frac{1}{(x/a)^2}\int_o^{x/a} \frac{x}{a}\underline{ft}(q,x)\,d(x/a) = \frac{2}{x/a}\sum_{n=1}^{\infty} \frac{1}{R_n^2} \exp\left(-\frac{R_n^2}{4q^2}\right)\frac{J_1\left(R_n \frac{x}{a}\right)}{J_1(R_n)}$$

where R_n is the n^{th} positive root of $J_o(z) = 0$,

$q = 0.4(0.02)3.0(0.1)10.0$,

$x/a = \quad 0(0.1)1.0.$

3. For a sphere:

$$\underline{ft}(q,x) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{S_n} \exp\left(-\frac{S_n^2}{4q^2}\right)\frac{a}{x} \sin\left(\frac{S_n x}{a}\right)$$

$$\frac{1}{(x/a)^3}\int_o^{x/a}(x/a)^2\,\underline{ft}(q,x)d(x/a)$$

$$=\frac{2}{(x/a)^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{S_n^3}\exp\left(-\frac{S_n^2}{4q^2}\right)\left[\frac{a}{x}\sin\frac{S_n x}{a}-S_n\cos\frac{S_n x}{a}\right]$$

where $S_n = n\pi$,

$q = 0.5(0.02)3.0(0.1)10.0$,

$x/a = 0(0.1)1.0$.

Background: The problem arises in the study of the strains in glass when submitted to a sudden change in temperature and was transmitted by L. H. Adams (30.50).

Status: New. The code has been written for the case of the slab and has been checked out on the IBM 704. The results for this part of the problem have been turned over to the sponsor.

1102-40-5126/58-263 GAS TUBE CHARACTERISTIC

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Manager: W. F. Cahill

Objective: To compute voltage-current static-characteristics for cold cathode gas tubes from a knowledge of the basic properties of the gas and the electrode materials, and the tube geometry. The problem involves solving the system of differential equations

$$\frac{dy_1}{dt}=\frac{G}{y_1}[H\,y_2-M]$$

$$\frac{dy_2}{dt}=N\,y_2\exp(Q/y_1^{\frac{1}{2}})$$

under the boundary conditions

$$y_2(0)=R,\quad y_2(d)=M;$$

and forming the integrals

$$\int_o^d y_1(t)dt \text{ and } \int_o^d \frac{p\,dt}{M+y_2(t)}$$

for various values of the parameters.

Background: There has been good agreement between theory and experimental measurements in the very low current, positive slope region of gas tube characteristics, and also in the glow region, after the tube has broken down. However, there has been no successful theory in the critical negative slope region which determines the tube's breakdown characteristics. Preliminary calculations show that the effect of space charge upon the electric field in the gas can explain the negative slope region. New experimental data is being taken to compare with computations. This will allow tubes to be designed for optimum performance without "endless" experimental testing.

The problem was transmitted by A. L. Ward (DOFL).

Status: New. The problem has been coded, using the Runge-Kutta method of integration. The code has been checked out, and results have been computed for some values of the parameters. However, a difficulty has arisen in choosing $y_1(0)$ for other values of the parameters; this is now being investigated.

3711-60-0009/58-266 DEPOLYMERIZATION, II

Origin and Sponsor: NBS, Section 7.6

Manager: L. S. Joel

Objective: To obtain numerical solutions for the system of differential equations

$$\frac{dQ_{N-1}}{dT} = -(1+\sigma)(N-4)Q_{N-1}$$

$$\frac{dQ_n}{dT} = -(1+\sigma)(n-3)Q_n + 2\left(\frac{K_0}{K_1} - \frac{1}{4}\sigma\right)Q_{n+2} + 2\sum_{i=1}^{\frac{N-n-3}{2}} \frac{K_i}{k_1} Q_{n+2i+2}, \quad 4 \le n \le N-3,$$

with boundary conditions at $T = 0$: $\begin{cases} Q_{N-1} = 1, \\ Q_N = 0, \end{cases}$

where K_i is defined in terms of the parameters σ and ϵ by the equation

$$\frac{K_i}{k_1} = (2+\sigma)\left[1-\left(1-\frac{3}{4}\epsilon\right)(1-\epsilon)^i\right] + \sigma.$$

Background: This system arises in the theory of chain reactions which describes the depolymerization of long polymer molecules as developed by R. Simha, L. A. Wall, and P. M. Blatz (see J. Polymer Science, 1950). (N-1) represents the initial number of units in a chain, Q_n the fraction of molecules containing n units, and T is a dimensionless variable related to time. $1/\epsilon$ is the average kinetic chain length of reaction and σ the ratio between the probabilities of transfer and initiation of a free radical chain.

This task is essentially a reactivation of task 3711-60-0009/54-17 (see Oct-Dec 1953 issue, p. 44) for extended values of the parameters and with the problem coded for the IBM 704.

Status: New. Analysis of the problem for computation on the 704 has been started. The parameters ϵ and σ, which were constants in the SEAC computation will be functions of R, the radical concentration, in solving the system with the 704.

3711-60-0009/58-267 CONVERSION OF THE CIE - CHROMATICITY COORDINATES INTO THE MUNSELL COLOR SYSTEM

Sponsor: NBS, Section 2.1

Manager: W. C. Rheinboldt

Objective: To write a program for the conversion of CIE - chromaticity coordinates into terms of the Munsell renotation system. Mathematically this is equivalent to a three-dimensional coordinate transformation, where the transformation is only numerically given, i.e., for a grid of discrete points. The use of approximations for hue or chroma lines in the different Munsell value charts is explicitly excluded.

Background: Since the problem arises very often in the work of the Photometry and Colorimetry Section (2.1), much time was spent in hand computation. It is therefore desirable to automatize the process with the help of the IBM 704 computer. The background theory is given in the paper, "Final report of the O.S.A. Subcommittee on the Spacing of the Munsell Colors," by Sidney M. Newhall, Dorothy Nickerson and Deane B. Judd (Optical Soc. Amer. 33, 385-418, July 1943). The problem was submitted by H. Keegan.

Status: New. The problem has been analyzed. A method has been devised to locate for any given point in the Y,x,z coordinate system the neighboring points for which the Munsell coordinates V, H, C are known. Then the Munsell coordinates for the given point can be evaluated by interpolation.

Work is in progress for writing the code for the 704.

3711-60-0009/58-268 RESPONSE FUNCTION CALCULATION

Origin: NBS, Section 4.11

Manager: A. Beam

Objective: To compute the matrix products

(1) $AB = C$

(2) $CK = D$

where A is an nxn inverse Bremsstrahlung matrix, and B and K are nxl column vectors which have been obtained experimentally. Then to print B, C, K, and D. Also, to provide for printing A at any desired time.

Background: The evaluation of the response of instruments measuring the energy spectrum of high energy Xrays is complicated by the lack of sources of mono-energetic Xrays. One way to measure the response of a sodium

iodide total absorption Xray spectrometer is to use the broad Bremsstrahlung Xray spectrum from a synchrotron. As the maximum energy of such a spectrum is moved up in steps, the signal from a channel of the spectrometer is measured. The response of any one channel is a folding of the instrument response with the spectrum of Xrays sent into the instrument. A multiplication by the inverse Bremsstrahlung matrix is required in the unfolding of the instrument response from the known spectrum of Xrays sent in.

The problem was transmitted by J. Wyckoff (4.11).

Status: New. The code was written and checked out on the 704. It has been turned over to the sponsor.

1102-40-5126/58-269 MOLECULAR STRUCTURE, IV
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. J. O'Hara
Objective: To determine the prominent maxima of

$$\phi(x,y,z) = \sum_{h,k,\ell} C_{h,k,\ell} \cos 2\pi(hx+ky+\ell z)$$

for preassigned values of h, k, ℓ. To make auxiliary computations necessary for determining the coefficients $C_{h,k,\ell}$. Incidental to the calculations will be the evaluation of a large number of third order determinants.

Background: This task is a continuation of the work under 1102-40-5126/51-37, using the IBM 704 computer (see p.23). The problem arises in the determination of molecular structures of crystals. It was transmitted by J. Karle and H. Hauptman (NRL).

Status: New. The phase determination problem was recoded for the IBM 704 and code checking has been started.

1102-40-5126/58-274 CALCULATIONS FOR d-SPACINGS, II
Origin and Sponsor: NBS, Division 9
Manager: R. Zucker
Objective: To perform calculations for d-spacings from unit cell measurements.

Background: This is a continuation of task 1102-40-5126/52-44 with the problem to be set up for computation on the IBM 704. The problem was transmitted by H. E. Swanson (9.7).

Status: New. A general program that will calculate d-spacings for all crystallographic space groups has been written for the 704. The code evaluates d-spacings, sorts them in descending order and prints them and their corresponding indices. This code replaces the four separate codes for SEAC (tetragonal, hexagonal, orthorhombic and monoclinic). Test cases have been run and the code has been completely checked out. Ten orthorhombic cases, one monoclinic, one hexagonal case have been run.

A general code for redetermination of unit cell constants by least squares fitting to measured d-spacing has been written for the 704. The following expression is being evaluated:

$$Q = \frac{1}{d^2} = h^2 a^{*2} + k^2 b^{*2} + \ell^2 c^{*2} + 2k\ell b^* c^* \cos \alpha^* + 2\ell h c^* a^* \cos \beta^* + 2hk a^* b^* \cos \gamma^*$$

where

$$a^* = \frac{bc \sin \alpha}{D}, \qquad \cos \alpha^* = \frac{\cos \beta \cdot \cos \gamma - \cos \alpha}{\sin \beta \sin \gamma},$$

$$b^* = \frac{ca \sin \beta}{D}, \qquad \cos \beta^* = \frac{\cos \gamma \cos \alpha - \cos \beta}{\sin \gamma \sin \alpha},$$

$$c^* = \frac{ab \sin \gamma}{D}, \qquad \cos \gamma^* = \frac{\cos \alpha \cos \beta - \cos \gamma}{\sin \alpha \sin \beta},$$

and

$$D = abc \sqrt{1 + 2 \cos \alpha \cos \beta \cos \gamma - \cos^2 \alpha - \cos^2 \beta - \cos^2 \gamma}.$$

Symmetry conditions for least squares are imposed at the beginning before solving for the starred quantities. The code covers the following crystallographic space groups:

Cubic	$a = b = c$,	$\alpha = \beta = \gamma = 90°$
Tetragonal	$a = b \neq c$,	$\alpha = \beta = \gamma = 90°$
Orthorhombic	$a \neq b \neq c$,	$\alpha = \beta = \gamma = 90°$
Hexagonal	$a = b \neq c$,	$\alpha = \beta = 90°$, $\gamma = 120°$
Rhombic	$a = b = c$,	$\alpha = \beta = \gamma \neq 90°$
Monoclinic	$a \neq b \neq c$,	$\alpha = \gamma = 90°$, $\beta \neq 90°$
Triclinic	$a \neq b \neq c$,	$\alpha, \beta, \gamma \neq 90°$ (generally)

The unit cell constants a, b, c, cos α, cos β, cos γ are obtained from the starred quantities by the following relations:

$$v^* = a^* b^* c^* \sqrt{1 - \cos^2 \alpha^* - \cos^2 \beta^* - \cos^2 \gamma^* + 2 \cos \alpha^* \cos \beta^* \cos \gamma^*}$$

$$a = \frac{b^* c^* \sin \alpha^*}{v^*}, \qquad \cos \alpha = \frac{\cos \beta^* \cos \gamma^* - \cos \alpha^*}{\sin \beta^* \sin \gamma^*}$$

$$b = \frac{a^* c^* \sin \beta^*}{v^*}, \qquad \cos \beta = \frac{\cos \alpha^* \cos \gamma^* - \cos \beta^*}{\sin \alpha^* \sin \gamma^*}$$

$$c = \frac{a^* b^* \sin \gamma^*}{v^*}, \qquad \cos \gamma = \frac{\cos \alpha^* \cos \beta^* - \cos \gamma^*}{\sin \alpha^* \sin \beta^*}$$

With the redetermination of these unit cell constants, the code then evaluates d-spacings and the differences between the observed and computed d-spacings. Test cases have been run and the code checked.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Stress Rupture Standards: W. J. Youden prepared for Division 8 a detailed program for selecting 27 stress rupture test specimens from four ingots and the assignment of these specimens to three pieces of test equipment. The assignment of the test specimens to three pieces of test equipment follows a pattern so that any comparisons of interest always involve the same number of specimens from each piece of test equipment providing thereby an automatic compensation for any differences that might exist between the different test equipments. The three specimens used in each equipment are completely balanced as to origin—both as to ingot and position in the bar.

(2) Spectrochemistry: A computer program for the statistical analysis of data automatically recorded on punched cards has been checked out. The data are from experiments designed to test the homogeneity of standard reference samples of metal rods, for R. E. Michaelis, Section 5.10.

(3) Transistor aging study: A preliminary analysis of shelf-aging experiments with transistors was completed for G. Conrad, Section 1.6. Also, a possible plan for an inter-laboratory experiment on transistor measurement to ascertain differences in measurement techniques was discussed with a BuShips Task Group.

(4) Analysis of tolerances for screw gauges: An investigation of the relationship between two systems (U.S. and English) of the tolerances for screw gauges was conducted for I. H. Fullmer, 2.5.

(5) Statistical analyses including weighted least square estimation of constants, fitting of polynomials, multiple regression were performed on the IBM 704 for personnel of twelve other NBS laboratories.

Publications:

(1) National physical standards and design of experiments. W. J. Youden. To appear in Revue de L'Institut International de Statistique (The Hague).
(2) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.
(3) Statistics: Engineering viewpoint. W. J. Youden. To appear in the Journal of Engineering Education.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: CONTINUED. A multiple regression analysis was made to represent the relation between transition temperature and several physical and chemical properties of two types of ship steel.

STATISTICAL ANALYSIS AND DESIGN OF EXPERIMENTS FOR THE U. S. GEOLOGICAL SURVEY
Task 1103-40-5140/54-1

Origin and Sponsor: U. S. Geological Survey, Department of Interior Authorized 10/9/53
Managers: C. Eisenhart, W. J. Youden
Full task description: Oct-Dec 1953 issue, p. 50

Status: CONTINUED. Correspondence was exchanged with the staffs of the Grand Junction and Denver Centers of the Geological Survey on problems in the statistical analysis of data on ore deposits.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-5146/55-93

Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: CONTINUED. A revised draft of part I, sections 1 and 2 (Estimation and Tests) was distributed for comment to a limited number of statisticians and engineers.

A report presenting in skeleton form the tables which will eventually appear in the Manual was prepared and given limited distribution.

The section on regression was revised and is nearly ready to be reproduced.

FRACTIONAL FACTORIALS FOR THE MIXED SERIES
Task 1103-12-5148/58-291

Origin and Sponsor: Bureau of Ships Authorized 9/30/57
Managers: W. S. Connor, M. Zelen

Objective: To conduct fundamental research on methods of construction and analysis for fractional replications of the mixed factorial designs which will ultimately lead to a catalog of these designs.

Background: The use of factorial designs has now become widely accepted as an efficient way for carrying out experiments involving many different factors. However, one of the main difficulties with factorial designs is that the number of measurements required may be large and in some cases prohibitive. Another disadvantage is that in many experimental situations it is not practical to plan an entire experimental program in advance, but to make a few smaller experiments which serve as a guide to further work. This latter condition is especially true when measurements are made singly or in small groups, such that the experimental results become known sequentially as they are taken. There are now prepared catalogs of the 2^n series and the 3^n series. For the 2^m3^n mixed series, i.e., m factors each at two levels and n factors each at three levels, there is no theory for the construction and analysis of fractional replication of this family. The work under this project is intended to extend the theory of fractional designs to these mixed series, thus generalizing the work done under task 1103-12-5147/57-213 (see July-Sept 1956 issue, p. 37).

Status: NEW. R. C. Bose of the Department of Statistics, University of North Carolina, was a guest worker at the National Bureau of Standards during the month of August 1957. He conducted the background work on the construction of the mixed 2^m3^n fractional factorial designs by preparing a set of notes which show how to consider factorial designs from the regression standpoint.

Professor Bose spoke on the topic, "Some mathematical problems arising in the construction of fractionally replicated designs and designs for the study of response surfaces", on September 5.

W. S. Connor studied the formation of the normal equations for fractions of the mixed factorial 2^m3^n series and developed a simplifying procedure which utilizes the internal structures of the designs formed by the factors at two levels and three levels considered separately. The procedure applies to any fractional factorial, whether of a mixed or pure series.

APPLICATION OF AUTOMATIC COMPUTER

The record of the use of the IBM 704 for the period July 1 through September 30 is as follows:

Task No.	Title	Assembly	Code Checking	Production
NBS:			(M I N U T E S)	
1104/55-55	Research in numerical analysis and related fields	92	343	54
5116/56-148	Study of diffraction equations for nerve excitation	19	50	1267
5170/56-159	Research in theories of discrete structures	108	293	644
1110/52-37	Spheroidal wave functions		112	
5113/57-216	Handbook of mathematical functions	118	1130	122
1120/55-65	Automatic coding	12	764	19
0009/56-160	Mathematical subroutines	134	833	6
0009/54-30	Spectrum analysis	104	1056	40
0009/55-68	Crystal structure calculations	8	329	30
0009/55-82	Thermometer calculations		59	
5126/55-97	High temperature properties for air	28	220	
5126/55-121	Electron penetration		578	33
0009/56-131	Calculations in optics	128	124	27
5126/56-166	SCF-LCAO solution of some hydrides	19	69	2
5126/56-169	Crystal field effects for atoms			376
5126/56-192	Noise measurement	7	25	
0009/57-223	Self-consistent fields	109	146	
0009/57-229	Payroll operations	19	176	
0009/57-232	Polynomial evaluation	16	168	
0009/57-235	Triple-integrals--entropy calculations		45	
0009/57-238	Modified Landau functions	4	412	62
1110/57-241	Heat transfer		32	91
5126/57-246	Radiation diffusion	125	285	3919
0009/57-247	Mechanical impedance		284	44
5126/57-249	Color differences	17	83	
0009/57-250	Automatic reduction in spectrophotometric data	120	169	
5126/57-251	Current noise and fixed resistor	15	380	2
0009/57-252	Detection efficiency in a neutral meson experiment	15	230	187

Task No.	Title	Assembly	Code Checking	Production
0009/58-254	Reproduction of color - and spectral-energy distribution of daylight	83	223	
5126/58-255	Chi function calculations		329	
5126/58-256	Composite walls		200	
5126/58-258	Noise measurement, II		248	
5126/58-260	Photo-type accounting	14	2	
0009/58-261	Temperature-induced strains in a slab		140	11
0009/58-262	Calibration of pyrometric lamps		33	15
0009/58-267	Munsell color system conversion		54	
0009/58-268	Response function calculation		40	
	Totals...	1314	9664	6951
OUTSIDE:				
5126/53-45	Air defense tactics	5	386	324
5126/55-104	Fuze problem		167	1312
5116/57-200	Evaluation of matrix inversion program	19	94	
5126/57-221	Bessel functions for complex arguments			33
5126/57-222	Roots of polynomials			4
5126/57-224	Track-while-scan radar problem		881	
5126/57-243	(Weather Bureau)			2643
5126/57-245	Missile Trajectory, III	176	465	75
5126/57-253	Airplane wing and structure stresses			10
5126/57-259	Kill probabilities			40
5126/58-263	Gas tube characteristic		144	
5126/58-269	Molecular structure	35	57	
5126/58-270	Post Office mechanization		28	8
	Totals...	235	2222	4449

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Mathematical Statistics Seminar

BOSE, R. C. (University of North Carolina). Some mathematical problems arising in the construction of fractionally replicated designs and designs for the study of response surfaces. September 5.

Mathematical Physics Section Seminar

BULLEN, K. (University of Sydney, Australia). Seismology and the earth's interior. September 27.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

EISENHART, C. Karl Pearson: An appreciation of one aspect of his work. Presented at the meeting of the American Statistical Association, Atlantic City, N. J., September 11.

GHAFFARI, A. On some mathematical properties of wedge solutions. Presented at the meeting of the American Mathematical Society, University Park, Penn., August 26-30.

HILSENRATH, J. (NBS Thermodynamics Section), J. H. Wegstein and J. W. Cooper. Mechanized computation of thermodynamic data. Presented before the Joint Conference on Thermodynamic and Transport Properties of Fluids, London, England, July 10-12.

PEARL, M. On normal matrices. Presented at the meeting of the American Mathematical Society, University Park, Penn., August 26-30.

ROSENBLATT, J.R. On models for the analysis of component aging experiments. Presented at the Bell Telephone Laboratories Statistical Summer Session, MIT, Dedham, Mass., August 8.

SEVERO, N.C. A comparison of tests on the mean of a logarithmic normal distribution with known variance. Presented at a Symposium on Statistical Methods in Radio Wave Propagation, held at the University of California at Los Angeles, August, 1957.

TODD, J., and NEWMAN, M. The evaluation of matrix inversion programs. Presented at the Conference on Matrix Computations, held at Wayne State University, Detroit, Michigan, September 3-6, 1957.

WEGSTEIN, J.H. The system for using the 704 at the National Bureau of Standards. Presented before the IBM Progress Report on Automatic Programming Systems, Washington, D. C., September 12.

YOUDEN, W. J. Problems of the experimenter. Presented at the American Institute of Chemical Engineers, East Tennessee Section, Kingsport, Tennessee. September 26.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Table of the first moment of ranked extremes. J. Lieblein and H. E. Salzer. J. Res. NBS 59, 203-206 (1957).

1.3 Technical Papers

(1) On the identity relationship for fractional replicates of the 2^n series. R. C. Burton and W. S. Connor. Ann. Math. Stat. 28, 762-767 (1957).

(2) Some examples of the use of high speed computers in statistics. J. Cameron. Proceedings of the Conference on Designs of Experiments in Army Research, Development and Testing, Office of Ordnance Research Report 57-1, Durham, N.C., June 1957, pp. 129-135.

(3) Condensation interfaces in two-phase flows. P. Chiarulli and R. F. Dressler. J. Appl. Phys. 28, 990-997 (1957).

(4) A computation of cyclic cubic units. H. Cohn and S. Gorn (Moore School of Electrical Engineering). J. Res. NBS 59, 155-168 (1957), RP2783.

(5) Complete sequences and approximations in normed linear spaces. P. Davis and K. Fan. Duke Math. J. 24, 183-192 (1957).

(6) Uniqueness theory for asymptotic expansions in general regions. P. Davis. Pac. J. Math. 7, 849-859 (1957).

(7) The principle of randomization in the design of experiments. C. Eisenhart. Synopsis appeared in Proceedings of the Conference on the Design of Experiments in Army Research, Development and Testing, Office of Ordnance Research Report 57-1, Durham, N.C., June 1957, pp. 15-16.

(8) Systems of inequalities involving convex functions. Ky Fan, I. Glicksburg (University of Notre Dame), and A. J. Hoffman. Proc. Amer. Math. Soc. 8, 617-622 (1957).

(9) Pairs of matrices of order two which generate free groups. K. Goldberg and M. Newman. Ill. J. Math. 1, 446-448 (1957).

(10) Games with random payoff matrices. A. J. Goldman. Summary only: Report of the Third Conference on Games, held by the Office of Naval Research Logistics Project at Princeton University, Princeton, N. J., March 11, 12, 1957.

(11) Note on bounds for certain determinants. E. Haynsworth. Duke Math. J. 24, 313-320 (1957).

(12) Estimation of the frequencies of thin elastic plates with free edges. T. Kato, H. Fujita, Y. Nakata (University of Tokyo); and M. Newman. J. Res. NBS 59, 169-186 (1957), RP2784.

(13) Construction and application of a class of modular functions. M. Newman. Proc. London Math. Soc. (3) 7, 334-350 (1957).

(14) On some expansions for Bessel integral functions. F. Oberhettinger. J. Res. NBS 59, 197-201 (1957), RP2786.

(15) The number of representations of a quadratic form as a sum of four squares. G. Pall (Illinois Institute of Technology) and O. Taussky. Proc. Roy. Irish Acad. Sect. A., No. 3, 23-28 (1957).

(16) Elastic waves in anisotropic media. J. L. Synge. J. Math. Phys. 35, 323-334 (1957).

(17) Automatic coding principles. J. Wegstein. Symposium on Advanced Programming Methods for Digital Computers, Washington, D. C., pp. 3-6 (Office of Naval Research, Department of the Navy, Washington, D. C.

(18) Design of experiments in research and development. W. J. Youden. Proceedings of the Conference on Design of Experiments in Army Research, Development, and Testing, Office of Ordnance Research Report 57-1, Durham, N.C., June 1957, pp. 9-14.

(19) The analysis of incomplete block designs. M. Zelen. J. Amer. Stat. Assoc. 52, 204-217 (1957).

2. MANUSCRIPTS IN THE PROCESS OF PUELICATION SEPTEMBER 30, 1957

2.1 Mathematical Tables

(1) Tables of the bivariate normal distribution function and related functions. To appear as NBS Applied Mathematics Series 50.

(2) Table of the exponential integral for complex arguments. To appear in the NBS Applied Mathematics Series.

(3) Integrals of Airy functions. To appear in the NBS Applied Mathematics Series.

(4) Table of natural logarithms for arguments from five to ten to sixteen decimal places. To appear in the NBS Applied Mathematics Series.

2.2 Manuals, Bibliographies, and Indices

(1) Analyzing straight line data. F. S. Acton. To appear as a book in the Applied Statistics Series of the Wiley Publications in Statistics.

2.3 Technical Papers

(1) A survey of Lyapunov's second method. H. Antosiewicz. To appear in Annals of Mathematics Studies.

(2) On the application of steam-driven water jets for propulsion purposes. J. M. Burgers and A. Ghaffari. To appear in the Journal of Research, NBS.

(3) Use of computers in statistical calculations. J. Cameron. To appear in the Proceedings of the Fourth Annual High-Speed Computer Conference, 1957, held at the University of Louisiana, Baton Rouge, La., March 1957.

(4) A numerical study of Dedekind's cubic class number formula. H. Cohn. To appear in the Journal of Research, NBS.

(5) Experiences with incomplete block designs: Examples. W. S. Connor. To appear in the Proceedings of the Symposium on the Design of Industrial Experiments, held at the University of North Carolina, Raleigh, N. C., November 1956.

(6) The uniqueness of the triangular association scheme. W. S. Connor. Submitted to a technical journal.

(7) Abelian groups of unimodular matrices. E. C. Dade. Submitted to a technical journal.

(8) Existence theorems and extreme solutions for inequalities concerning convex functions or linear transforms. K. Fan. To appear in Mathematische Zeitschrift.

(9) Note on circular disks containing the eigenvalues of a matrix. Ky Fan. Submitted to a technical journal.

(10) Topological proofs for certain theorems on matrices with non-negative elements. Ky Fan. Submitted to a technical journal.

(11) A matrix minimization problem. A. J. Goldman. Submitted to a technical journal.

(12) On the domain of regularity of generalized axially symmetric potentials. P. Henrici. To appear in the Proceedings of the American Mathematical Society.

(13) On the representation of a certain integral involving Bessel functions by hypergeometric series. P. Henrici. To appear in the Journal of Mathematics and Physics.

(14) Mechanized computation of thermodynamics tables at the National Bureau of Standards. J. Hilsenrath (NBS Thermodynamics Section) and J. Wegstein. To appear in the Proceedings of the Joint Conference on Thermodynamic Transport Properties of Fluids sponsored by the Institute of Mechanical Engineers, London, 1957.

(15) Numerical experiments in potential theory using the Nehari estimates. U. W. Hochstrasser. Submitted to a technical journal.

(16) A determinantal inequality of H. P. Robertson, II. M. Marcus. To appear in the Journal of the Washington Academy of Sciences.

(17) On a determinantal inequality. M. Marcus. To appear in the American Mathematical Monthly.

(18) On doubly stochastic transforms of a vector. M. Marcus. Submitted to a technical journal.

(19) On subdeterminants of doubly stochastic matrices. M. Marcus. To appear in the Illinois Journal of Mathematics.

(20) Some extreme value results for indefinite Hermitian matrices. M. Marcus, B. N. Moyles and R. Westwick (University of British Columbia). To appear in the Illinois Journal of Mathematics.

(21) Permanents of doubly stochastic processes. M. Marcus and M. Newman. Submitted to a technical journal.

(22) Congruences for the coefficients of modular forms and some new congruences for the partition function. M. Newman. To appear in the Canadian Journal of Mathematics.

(23) A method of speeding up iterations with super-linear convergence. A. M. Ostrowski. Submitted to a technical journal.

(24) On the derivative of Bessel functions with respect to the order. F. Oberhettinger. Submitted to a technical journal.

(25) On normal and EPr matrices. M. Pearl. Submitted to a technical journal.

(26) The elastic problem for a ring of uniform force in an infinite body. W. H. Pell. To appear in the Journal of Research, NBS.

(27) On some aspects of prediction in the study of complex systems. J. R. Rosenblatt. To appear in the Proceedings of the NYU-RCA Working Conference on Theory of Reliability, held in New York, N.Y., April 1957.

(28) The radial distribution of the center of gravity of random points on a unit circle. F. Scheid. To appear in the Journal of Research, NBS.

(29) The non-central χ^2 as a test statistic. N. C. Severo. Submitted to a technical journal.

(30) Generation of Bessel functions on high speed computers. I. Stegun and M. Abramowitz. Submitted to a technical journal.

(31) Stationary principles for forced vibrations in elasticity and electromagnetism. J. L. Synge. To appear in the Proceedings of the Eighth Symposium in Applied Mathematics held by the American Mathematical Society, Chicago, Ill., April 1956.

(32) The torsion of a hollow square. J. L. Synge and W. F. Cahill. To appear in the Quarterly of Applied Mathematics.

(33) A determinantal inequality of H. P. Robertson, I. O. Taussky. To appear in the Journal of the Washington Academy of Sciences.

(34) Some computational problems concerning integral matrices. O. Taussky. To appear in the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.

(35) Computational problems concerned with the Hilbert matrix. J. Todd. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.

(36) The condition of certain matrices, III. J. Todd. To appear in the Journal of Research, NBS.

(37) National physical standards and design of experiment. W. J. Youden. To appear in Revue de L'Institut International de Statistique (The Hague).

(38) Discussion of the paper by W. C. Otto, "A statistical study on flexural strength of concrete beams." W. J. Youden. To appear in the Proceedings of the Annual Meeting of the American Society for Testing Materials for 1956.

(39) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.

(40) Statistics--Engineering viewpoint. W. J. Youden. To appear in the Journal of Engineering Education.

(41) Experiments with many factors. M. Zelen. To appear in the Proceedings of the Third Annual Statistical Engineering Symposium of the U. S. Army Chemical Corps Engineering Command, Army Chemical Center, Md., May 1957.

(42) Multi-factor experiments for evaluating reliability. M. Zelen. To appear in the 1957 Joint Military-Industry Guided Missile Reliability Symposium held at Point Mugu,California, November, 1957.

(43) The analysis of covariance for incomplete block designs. M. Zelen. To appear in Biometrics.

(44) The use of incomplete block designs for asymmetrical factorial arrangements. M. Zelen. To appear in the Annals of Mathematical Statistics.

(45) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. S. Joel. Submitted to a technical journal.

2.4 Reviews and Notes

(1) Essential similarity: a counter-example. A. J. Goldman. To appear in the American Mathematical Monthly (Math. Notes).

(2) The probability of a saddle point. A. J. Goldman. To appear in the American Mathematical Monthly (Math. Notes).

(3) Field convexity of a linear transformation. A. J. Goldman and M. Marcus. To appear in the American Mathematical Monthly (Math. Notes).

2.5 Miscellaneous

(1) Further contributions to the solution of simultaneous linear equations and the determination of eigenvalues. To appear as Applied Mathematics Series 49.

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U. S. DEPARTMENT OF COMMERCE
Sinclair Weeks, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major field laboratories in Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside back cover of this report.

WASHINGTON, D. C.

Electricity and Electronics. Resistance and Reactance. Electron Tubes. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat and Power. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology and Lubrication. Engine Fuels.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Nuclear Physics. Radioactivity. X-rays. Betatron. Nucleonic Instrumentation. Radiological Equipment. AEC Radiation Instruments.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Gas Chemistry. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Organic Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metals Physics.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Concreting Materials. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Heating and Air Conditioning. Floor, Roof, and Wall Coverings. Codes and Specifications.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analogue Systems. Application Engineering.

- Office of Basic Instrumentation
- Office of Weights and Measures

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Systems. Navigation Systems. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Radio Systems Application Engineering.

Radio Standards. Radio Frequencies. Microwave Frequencies. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Calibration Center. Microwave Physics. Microwave Circuit Standards.


NBS