NATIONAL BUREAU OF STANDARDS REPORT

5433

PROJECTS and PUBLICATIONS of the APPLIED MATHEMATICS DIVISION

A Quarterly Report

April through June 1957

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($0.75), available from the Superintendent of Documents, Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.

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NBS PROJECT	NBS REPORT
11.0	5433

PROJECTS and PUBLICATIONS of the APPLIED MATHEMATICS DIVISION

April through June 1957

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

April 1 through June 30, 1957

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CONTENTS

*Only unclassified projects are included in this report.

Status of Projects

June 30, 1957

1. NUMERICAL ANALYSIS

RESEARCH IN THEORIES OF DISCRETE STRUCTURES

Task 1100-11-5170/56-159

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: O. Taussky-Todd
Full task description: July-Sept 1955 issue, p. 1

Status: CONTINUED. O. Taussky-Todd noticed that the incidence matrix of a finite projective geometry has the property that a sufficiently large power has positive elements only. Alternative proofs were subsequently communicated by H. J. Ryser (Ohio State) and M. Newman.

O. Taussky-Todd continued work on the coding of a problem concerning the 3-class group of imaginary quadratic number fields. E. C. Dade is collaborating with her on this problem, using a new approach of his own with a new code.

O. Taussky-Todd collaborated with A. Lax (New York University) and with M. Gerstenhaber (University of Pennsylvania) on problems both connected with pairs of matrices with property L.

E. C. Dade prepared a manuscript, "Abelian groups of unimodular matrices." This paper gives a very strong generalization of the theorem of K. Goldberg (J. Washington Acad. Sci. 46 (1956) 337-338), which states that any two commutative unimodular 2 x 2 matrices with rational integral elements are powers of another matrix of this type. An alternative proof of this was obtained by J. Todd and O. Taussky-Todd. They also generalized the theorem to matrices with coefficients in an imaginary quadratic field and found that certain complications occur. Analyzing these complications, Dade studied the general case of commutative n x n matrices with elements in the ring of integers of an arbitrary algebraic number field and the determinant a unit in this field. His method consists in imbedding the matrices into a commutative algebra formed by n x n matrices with elements in the ring of integers of an algebraic number field. The structure of the group of units of this algebra is then investigated.

Alternative proofs for Goldberg's original result have been obtained in the meantime by abstract group theory using the fact that the modular group in two dimensions is a free product (by A. Karrass and D.Solitar)

and also by non-Euclidean geometry (by H.S.M. Coxeter).

Publications:

(1) Incidence algebras. E. C. Dade and K. Goldberg. In manuscript.
(2) Commutativity in finite matrices. O. Taussky. Amer. Math. Monthly 64, 229-235 (1957).
(3) On matrix classes corresponding to an ideal and its inverse. O. Taussky. Illinois J. Math. 1, 108-113 (1957).
(4) The number of representations of a quadratic form as a sum of four squares. G. Pall (Illinois Institute of Technology) and O. Taussky. To appear in the Proceedings of the Royal Irish Academy.
(5) Pairs of matrices of order two which generate free groups. K. Goldberg and M. Newman. To appear in the Illinois Journal of Mathematics.
(6) A method for computing eigenvectors. K. Goldberg. In manuscript.
(7) A computation of cyclic cubic units. H. Cohn and S. Gorn (Moore School of Electrical Engineering). To appear in the Journal of Research, NBS.
(8) A numerical study of Dedekind's cubic class number formula. H. Cohn. To appear in the Journal of Research, NBS.
(9) Some computational problems concerning integral matrices. O. Taussky. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.
(10) Abelian groups of unimodular matrices. E. C. Dade. In manuscript.

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-12-1104/55-55

Origin: NBS — Authorized 8/13/54
Managers: J. Todd, P. Davis — Revised 8/29/54
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. H. A. Antosiewicz is preparing a manuscript, "Reducible linear differential systems, I." It presents the results he has obtained so far concerning the reducibility of systems $\dot{x}=[C+B(t)]x$ to $\dot{y} = Cy$ by use of differentiable matrices $Z(t)$ that are bounded together with $Z^{-1}(t)$ on $[0, \infty)$.

J. Todd discussed the condition of the matrices associated with certain discretizations of the two-dimensional Laplacian operator, in particular, two nine-point approximations. These have condition numbers of the same order $O(n^2)$ as the usual five-point approximations, so that decisions on which are most suitable to use in practice will depend rather on the corresponding truncation errors: these do not appear to be known in the nine-point cases.

E. Brauer and J. Gager are collaborating with J. Todd in connection with experiments to determine the radius of univalence of certain functions. They are also checking some tables of high order Legendre polynomials using the IBM 704.

C. Spector is working on a predicative theory of denumerable sets, hyperarithmetical predicates, constructive ordinals, and permanents of doubly stochastic matrices.

M. Pearl is studying the characterizations of normal matrices over arbitrary fields.

M. Marcus and M. Newman are considering the following problem suggested by H. Ryser: Let p(X) denote the permanent of the square matrix X. What is $\min_{X \in S_n} p(X)$, where S_n is the totality of all n-square doubly stochastic matrices? It is conjectured that $J_n = (1/n)$ is the unique minimizing matrix with min $p = n!/n^n$. Thus far the following preliminary results have been established:

(i) There is at most one interior relative extreme value for p on S_n, and this is a relative minimum at J_n. Hence, if the minimum of p(X) occurs in the interior of S_n it must be at J_n.

(ii) The result is true for n equal to 2 and 3. Under an inductive hypothesis, any minimizing matrix X must have all permanental cofactors corresponding to non-zero elements of X equal, and all corresponding to zero elements equal.

(iii) $$\frac{1}{n^{2n}} \leq \operatorname{Min}_{X \in S_n} p(X) \leq 1.$$

Publications:

(1) Complete sequences and approximations in normed linear spaces. P. Davis and K. Fan. To appear in the Duke Mathematical Journal.
(2) Note on bounds for certain determinants. E. V. Haynsworth. To appear in the Duke Mathematical Journal.
(3) A survey of Lyapunov's second method. H. A. Antosiewicz. To appear in Annals of Mathematics Studies.
(4) Convex functions of quadratic forms. M. Marcus. To appear in Duke Mathematical Journal.
(5) On subdeterminants of doubly stochastic matrices. M. Marcus. To appear in the Illinois Journal of Mathematics.
(6) On a determinantal inequality. M. Marcus. To appear in American Mathematical Monthly.
(7) Some extreme value results for indefinite Hermitian matrices. M. Marcus; B. N. Moyles and R. Westwick (University of British Columbia). To appear in the Illinois Journal of Mathematics.
(8) Extreme value properties of Hermitian matrices. M. Marcus. To appear in Journal of the London Mathematical Society.
(9) Computational problems concerned with the Hilbert matrix. J. Todd. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.
(10) On the condition of matrices, III. J. Todd. To appear in Journal of Research, NBS.
(11) On normal and EPr matrices. M. Pearl. In manuscript.
(12) A determinantal inequality of H. P. Robertson, II. M. Marcus. To appear in Journal of the Washington Academy of Science.

(13) Reducible linear differential systems, I. H. A. Antosiewicz. In manuscript.
(14) Field convexity of a linear transformation. M. Marcus and A. J. Goldman, To appear in American Mathematical Monthly.
(15) A note on values of a quadratic form. M. Marcus. J. Wash. Acad. Sci. 47, 97-99 (1957).
(16) On doubly stochastic transforms of a vector. M. Marcus. Submitted to a technical journal.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-12-5116/55-56

Origin: NBS Authorized 8/13/54
Sponsor: Office of Naval Research
Managers: J. Todd, M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. M. Newman and J. Todd are continuing with experiments in connection with their report on the evaluation of matrix inversion programs [see Oct-Dec 1956 issue, pp. 2,3], which were postponed because of the Training Program. Various norms for the errors obtained in the inversion of twelve representative matrices by the method programmed by M. Newman for SEAC have been computed. The same norms will be computed for a typical inversion program on the 704. The matrices are of order 28, the largest that can be handled internally on SEAC. They range from an orthogonal matrix (P-condition) to the Hilbert matrix (P-condition experimentally infinite).

K. Fan completed a study of Existence theorems and extreme solutions for inequalities concerning convex functions or linear transformations. This deals with consistency conditions and extreme solutions for relations of the following two types:

(i) $f_\nu(x) \leq 0$ ($\nu \in I$), where each f_ν is a lower semi-continuous convex function defined on a compact convex set in a topological vector space.

(ii) $Ax - \lambda x - y_o \in C$, where A is a completely continuous linear operator in a Banach space X, C is a convex set (not necessarily a cone) in X containing 0, $\lambda \neq 0$ and $y_o \in X$ are given.

K. Fan solved the following problem: Given a square matrix $B = (b_{ij})$ of order n with non-negative elements and with $b_{ii} = 0$, determine all ordered n-tuples of positive numbers $\rho_1, \rho_2, \ldots, \rho_n$ with the following property: For any square matrix $A = (a_{ij})$ of order n with complex elements and satisfying $a_{ij} \leq b_{ij}$ ($i \neq j$), every eigenvalue of A lies in at least one of the n circular disks $|z - a_{ii}| \leq \rho_i$ ($1 \leq i \leq n$).

A proof given by Alexandroff-Hopf for the Perron-Frobenius theorem is well-known and makes use of the Brouwer fixed point theorem.

K. Fan found that several theorems on matrices with non-negative elements can be proved by using those elementary topological facts which are closely related to the Brouwer fixed point theorem.

M. Newman has derived some new congruences for the coefficients of modular forms. Two consequences of these are the following congruences modulo 13 for the coefficients c(n) of the complete modular invariant $12^3 J(\tau)$:

$$c(91n) = 0 \pmod{13}, \quad (n,7) = 1$$
$$c(143n) = 0 \pmod{13}, \quad (n,11) = 1.$$

Publications:

(1) On the minimization of concave and convex functionals. G. B. Dantzig (RAND Corporation), A. J. Hoffman and W. Hirsch (New York University). In manuscript.
(2) Estimation of the frequencies of thin elastic plates with free edges. H. Fujita (University of Tokyo), T. Kato, Y. Nakata (University of Tokyo) and M. Newman. To appear in the Journal of Research, NBS.
(3) Systems of inequalities involving convex functions. Ky Fan (Notre Dame University), I. Glicksberg (RAND Corporation) and A. J. Hoffman. To appear in the Proceedings of the American Mathematical Society.
(4) Construction and application of a class of modular functions. M. Newman. To appear in the Proceedings of the London Mathematical Society.
(5) Congruences for the coefficients of modular forms and some new congruences for the partition function. M. Newman. To appear in Canadian Journal of Mathematics.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-12-5116/56-148

Origin and Sponsor: National Institutes of Health, Bethesda, Md. Authorized 9/30/55
Manager: H. A. Antosiewicz
Full task description: July-Sept 1955 issue, p. 7

Status: CONTINUED. Work on a new code of the entire problem for use on IBM 704 has begun. The proposed code will incorporate all decision processes, such as choice of integration interval, search for acceptable parameter values, and computation of initial conditions for each run which previously had to be made off the computer.

TRAINING PROGRAM IN NUMERICAL ANALYSIS
Task 1101-40-5114/57-237

Origin and Sponsor: National Science Foundation Authorized 3/27/57
Manager: J. Todd
Full task description: Jan-Mar 1957 issue, p. 5

Status: COMPLETED. The training program was completed as planned. The following main courses of lectures were presented during this quarter:

Partial differential equations	(D. M. Young (R. D. Richtmeyer
Recursive functions and Turing machines	S. C. Kleene
Integral equations	H. F. Buckner
Linear programming and related topics	C. B. Tompkins
Algebraic equations	U. Hochstrasser
Statistical computations	M. Zelen, J.M. Cameron
Discrete variable problems	M. Hall, M. Newman, O. T. Todd
Advanced topics	H. Cohn, P. Lax, S. M. Ulam
Sources of problems	F. L. Alt

An introduction to programming for the IBM 704 was given by M. Newman. In addition, the courses in supporting mathematics were continued as follows:

H. A. Antosiewicz and K. Fan continued the series on Functional Analysis begun by H. F. Weinberger.

M. Marcus and K. Fan carried on in the series on Bounds for Eigenvalues of Matrices begun by O. Taussky-Todd.

P. Davis and J. Todd presented a series of talks on Approximation Theory.

M. R. Hestenes presented special lecture, and L. W. Cohen devoted an hour to answering questions raised by the program. S. N. Alexander spoke of the activities of the Data Processing Systems Division. G. Porter (Chief, Personnel Division, NBS) spoke about the needs of NBS for scientists and the opportunities for scientists at NBS.

The question of preparing some record of the lectures is under consideration. Several speakers have submitted manuscripts.

Among the computations completed by the participants were the following:

D. C. Lewis, Jr.: SEAC code for drawing a unicursal curve through a topological map given by an incidence matrix.

F. R. Olsen: Table of coefficients for the Bernoulli numbers $B^{(n)}$ expressed as polynomials in n of degree ν; the cases n = 13(1)19 were covered.

C. L. Seebeck, Jr.: Various financial tables, using double precision fixed-point arithmetic.

H. C. Griffith: Double precision codes for elliptic functions. There is now available a table of values of $K(k^2)$, good to 21D for k^2=0(.01)1.00, and tables of $sn(pk,k^2)$, $cn(pk,k^2)$, $dn(pk,k^2)$ for the same values of k^2 and for p = 0(.01)1.00.

F. Scheid: Monte Carlo calculations of the radial distribution of the center of gravity of n random points on the unit circle. The results for n = 6(1)24 have been obtained by J. A. Greenwood and D. Durand. The cases n = 3, 4, 5 were obtained on SEAC, to about 3D.

All other participants had completed exploratory calculations on major problems when use of SEAC was discontinued in favor of the IBM 704; it is expected that the problems will be completed on the respective local machines.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF $E_1(z)$, $(z=x+iy)$

Task 1102-40-1110/43-3

Manager: I. Stegun — Authorized 7/1/47
Full task description: Apr-June 1949 issue, p. 41

Status: CONTINUED. Checking of the final manuscript continued.

TABLES OF COULOMB WAVE FUNCTIONS

Task 1102-40-1110/47-2

Origin: NBS — Authorized 7/1/47
Manager: M. Abramowitz
Full task description: Apr-June 1949 issue, p. 45

Status: CONTINUED. Checking of the tables continued.

TABLES OF POWER POINTS OF ANALYSIS-OF-VARIANCE TESTS

Task 1102-40-1110/51-8

Origin: Section 11.3, NBS — Authorized 3/26/51
Manager: S. Peavy
Full task description: Apr-June 1951 issue, p. 49

Status: CONTINUED. Calculations for the tables have been completed.

REVISION OF MATHEMATICAL TABLES
Task 1102-40-1110/52-7

Origin: NBS Authorized 8/10/51
Managers: W. F. Cahill, I. Stegun
Full task description: July-Sept 1951 issue, p. 41

Status: INACTIVE. For status to date, see Jan-Mar 1957 issue, p. 9.

TABLE OF THE MODIFIED AIRY INTEGRAL
Task 1102-40-1110/52-23

Origin: NBS Authorized 10/4/51
Manager: I. Stegun
Full task description: July-Sept 1951 issue, p. 42

Status: INACTIVE. For status to date, see Jan-Mar 1957 issue, p. 9.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-40-1110/52-37

Origin: NBS Authorized 11/28/51
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE.

SIEVERT'S INTEGRAL
Task 1102-40-1110/52-57

Origin: NBS Authorized 2/12/52
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46

Status: CONTINUED. Checking of the tabular values and preparation of the introductory material continued.

L-SHELL CONVERSION COEFFICIENTS
Task 1102-40-1110/53-52

Origin: Oak Ridge National Laboratory Authorized 5/20/53
Manager: W. Hall
Full task description: Apr-June 1953 issue, p. 45

Status: CONTINUED. The finite nucleus computations were completed this quarter for selected portions of the original tables. For L_I and L_{II} shells z = 55, 65, 75, 85, 95; k = .05, .1, .2, .6, 1.0. For K shell z = 55, 65; k = 0.5, .1, .2, .6, 1.0.

AUTOMATIC CODING
Task 3711-60-1120/55-65

Origin: NBS Authorized 9/29/54
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. The CORBIE tape system for using the IBM 704 as described in the Jan-Mar 1957 issue, p. 10, was placed in operation. Considerable debugging was required before it functioned as intended. It is now working satisfactorily and appears to be used in more than half of the codes being run on the 704. The assumption that codes can be checked out using symbolic notation entirely has been found to be valid. Statistics on the performance of the CORBIE system are being obtained.

A routine was written and checked out by F. L. Alt and W. F. Cahill which permits the 704 to simulate the SEAC. Some SEAC codes have been run on the 704 using this simulator.

Sections 1, 2, 3, 4, 14, 15, 16, and 18 of a programmer's guide for the 704 were prepared by J. Wegstein. These include an introduction to the 704, the tape library, the CORBIE system, and the BCS (Binary Card System) for the 704.

MATHEMATICAL SUBROUTINES
Task 3711-60-0009/56-160

Origin: NBS Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

Status: CONTINUED. A subroutine was written which computes the function

$$w(z) = e^{-z^2} \left(1 + \frac{2i}{\pi} \int_0^z e^{t^2}\, dt\right)$$

in floating point arithmetic for any complex z. The same code will be used to provide subroutines for evaluating the Fresnel integrals and the error function for complex arguments. A subroutine for the evaluation of the repeated error integral is in preparation.

A subroutine was written to evaluate the error function

$$F(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

using a rational approximation which should give accuracy to $\pm$ 3 in the seventh place. The argument x is real and $0 \leq x \leq 3.855$. If $x > 3.855$ the value of the error function is set equal to 1.

Tests were made on the matrix subroutines for the addition, subtraction and multiplication of real matrices. The determinant and inverse of a Hilbert matrix (order 6) were also evaluated. The results were accurate to one significant digit in the determinant and inverse of this matrix.

A subroutine was written to multiply matrix A by the transpose of matrix B, where A and B are real and stored row-wise. A subroutine was also written to multiply the transpose of matrix A by matrix B where A and B are real and stored row-wise. Both of these subroutines may be used when matrix B is the same as matrix A and here only matrix A need be stored.

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-5113/57-216

Origin and Sponsor: National Science Foundation Authorized 12/27/56
Manager: M. Abramowitz
Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. A meeting with the Advisory Committee has produced some further minor modifications in the proposed table of contents. The rational approximations to be given are to be included in each chapter. Floating numbers are to be given in the form ($\pm$ a) $\pm$ b, where a is the exponent corresponding to the number $b = b_1 \cdot b_2\ b_3 \cdots$. In printing tabular values digits are to be given in groups of five; a number containing six digits will be given as one group. When using auxiliary arguments, like 1/x, the functions are to be tabulated in increasing order of x. When using an auxiliary function, the original function is to be defined on the tabular page. In the case where ambiguous definitions exist, this is to be

stated on the tabular page.

The chapter on Powers, Real and Complex Roots will contain the following tables:

Table 3.1: $x^2, x^3, x^4, x^{10}, x^{17}, x^{24}$,

Prime factors, or $\log_{10} x$ if x prime, x = 1(1)1000, 10S.

$x^{1/2}, x^{1/3}, x^{1/4}, x^{1/5}$,

Table 3.2: $\sqrt{1 + x^2}$, $\sqrt{1 - x^2}$, x = 0(.01)1, 10D.

Table 3.3: $(1 + iy)^{1/n}$, n = 2(1)5, y = 0(.01)1, 10D.

The chapter on Struve functions will contain the following tables:

Table 12:1:

$H_o(x)$, $H_1(x)$, x = 0(.1)5.0, 7D.

$$\left.\begin{array}{l} \displaystyle\int_o^x H_o(t)\,dt, \\ I_o(x) - L_o(x),\ I_1(x) - L_1(x), \\ \displaystyle\int_o^x [I_o(t) - L_o(t)]\,dt, \\ \displaystyle\frac{2}{\pi}\int_x^{\infty} \frac{H_o(t)}{t}\,dt, \end{array}\right\} x = 0(.1)5.0,\ 6D.$$

Table 12.2:

$$\left.\begin{array}{l} H_o(x) - Y_o(x),\ H_1(x) - Y_1(x), \\ f_o = \displaystyle\int_o^x [H_o(t) - Y_o(t)]\,dt - \frac{2}{\pi}\ln x, \\ I_o(x) - L_o(x),\ I_1(x) - L_1(x), \\ g_o = \displaystyle\int_o^x [I_1(t) - L_o(t)]\,dt - \frac{2}{\pi}\ln x, \\ \displaystyle\int_x^{\infty} [H_o(t) - Y_o(t)]\,\frac{dt}{t}, \end{array}\right\} 1/x = 0(.01).20,\ 6D.$$

Chapter 27 on Binary Tables will contain the following:

Table 27.1: 2^x, x = .001(.001).009(.01).9(.1).9, 10S

27.2: 2^x, $\pm$ x = 0(1)50, exact

27.3: Addition and Multiplication Tables for Octal System

27.4: $\log_2 x$, x = 0(.001)1.2, 2(1)100(10)1000(100)10,000, 8D

27.5: Mathematical Constants in Octal Notation

3. PROBABILITY AND MATHEMATICAL STATISTICS

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-12-1107/49-1a

Origin: NBS Authorized 1/9/49
Manager: L. S. Deming
Full task description: Apr-June 1949 issue, p. 75

Status: TERMINATED. For status to date, see Jan-Mar 1957 issue, page 14. Beginning next quarter work on this task will be reported under task 1103-12-1107/51-2, "Miscellaneous Studies in Probability and Statistics."

TABLES TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-12-1107/51-1

Origin: NBS Authorized 7/1/50
Managers: C. Eisenhart, L. S. Deming
Full task description: July-Sept 1950 issue, p. 57

Status: TERMINATED. For status to date, see July-Sept 1952 issue, page 64. Beginning next quarter work on this task will be reported under task 1103-12-1107/51-2, "Miscellaneous Studies in Probability and Statistics."

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-12-1107/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. C. Eisenhart, N. C. Severo, and V. Martinez continued the study of the effects of data transformations on correlation analysis.

N. C. Severo prepared a short paper on "The non-central chi-square as a test statistic," to be submitted to a technical journal.

M. Zelen has prepared a manuscript entitled "Linear estimation and related topics," which summarizes his lectures presented at the NBS in May 1957 as part of the National Bureau of Standards-National Science Foundation Training Program in Numerical Analysis. This report discusses (i) the Gauss-Markov theorem and extensions, (ii) distribution of quadratic forms, and (iii) likelihood ratio tests of the general linear hypothesis.

Publication:

(1) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. Joel. Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-12-1107/53-1

Origin: NBS Authorized 10/15/52
Manager: W. S. Connor
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. W. S. Connor studied a problem concerning the triangular association scheme. The triangular association scheme is an array of n rows and n columns with the following properties: (1) The positions in the principal diagonal are blank. (2) The $n(n-1)/2$ positions above the principal diagonal are filled by the numbers $1,2,\ldots,n(n-1)/2$ corresponding to treatments. (3) The array is symmetric about the principal diagonal. (4) The first associates of any treatment θ are the treatments which lie in the same row and column as θ, and the remaining treatments are second associates of θ. As is well known, from this definition it follows that (a) the number of treatments which are first associates of any treatment θ is $2n-4$, and (b) withrespect to any two treatments θ_1 and θ_2 which are i-th associates $(i=1,2)$ of each other, there are $2(n-4)-i(n-6)$ other treatments which are first associates of both θ_1 and θ_2. The problem studied was whether the conditions (a) and (b) can be satisfied by an association scheme different from the triangular association scheme. It was proved for $n \geq 9$ that this is impossible. This work is contained in a paper referenced below.

J. M. Cameron continued his investigation of weighing designs for use in calibration work (see the Jan-Mar 1957 issue, p. 16). A number of these designs have been constructed which are such that with respect to any two objects θ_1 and θ_2, there are λ_1 weighings which have θ_1 and θ_2 in the same pan, and λ_2 weighings which have θ_1 and θ_2 in opposite pans.

Publications:

(1) Fractional factorial experiment designs for factors at two levels. NBS Applied Mathematics Series 48. Appeared April 1957. (Available from U. S. Government Printing Office, Washington, D. C.)
(2) On the identity relationship for fractional replicates of the 2^n series. R. C. Burton and W. S. Connor. To appear in the Annals of Mathematical Statistics.
(3) Experiences with incomplete block designs: Examples. W. S. Connor. To appear in the Proceedings of the Symposium on the Design of Industrial Experiments, Raleigh, N. C., November 1956.
(4) The principle of randomization in the design of experiments. C. Eisenhart. Synopsis to appear in the Proceedings of the 1955 Conference on the Design of Experiments in Army Research, Development and Testing, Washington, D. C., October 1955.
(5) Design of experiments in research and development. W J. Youden. To appear in the Proceedings of the 1955 Conference on the Design of Experiments in Army Research, Development and Testing, Washington, D.C., October 1955.
(6) The analysis of covariance for incomplete block designs. M. Zelen. To appear in Biometrics.
(7) The analysis of incomplete block designs. M. Zelen. To appear in the June issue of the Journal of the American Statistical Association.
(8) The use of incomplete block designs for asymmetrical factorial arrangements. M. Zelen. Submitted to a technical journal.
(9) Experiments with many factors. M. Zelen. To appear in the Proceedings of the Third Annual Statistical Engineering Symposium, U. S. Army Chemical Corps, Edgewood, Maryland, May 1957.
(10) The uniqueness of the triangular association scheme. W. S. Connor. Submitted to a technical journal.

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-1107/56-170

Origin: NBS Authorized 12/15/55
Manager: Joan R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

Status: INACTIVE. For status to date, see July-Sept 1956 issue, page 15.

Publication:

(1) Table of the first moment of ranked extremes. J. Lieblein and H. E. Salzer. To appear in the Journal of Research, NBS.

MEASUREMENT OF RELIABILITY
Task 1103-12-1130/56-182

Origin: NBS Authorized 3/23/56
Manager: M. Zelen
Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. J. R. Rosenblatt presented a paper, "On some aspects of prediction in the study of complex systems," at the NYU-RCA Working Conference on Theory of Reliability, Ardsley-on-Hudson, New York, April 18.

J. R. Rosenblatt and J. M. Smith are calculating additional numerical examples to illustrate the effect of interdependence of components on system performance.

H. Gumbel (Naval Air Missile Test Center, Point Mugu, California) addressed Reliability Seminar V on the subject, "What is a failure?" on April 15.

Publications:

(1) On prediction of system performance from information on component performance. J. R. Rosenblatt. Proceedings of the 1957 Western Joint Computer Conference, Los Angeles, California, February 1957, pp. 85-94.
(2) On some aspects of prediction in the study of complex systems. J. R. Rosenblatt. To appear in the Proceedings of the NYU-RCA Working Conference on Theory of Reliability, Ardsley-on-Hudson, New York, April 1957.

CATALOG OF FRACTIONAL REPLICATION DESIGNS
Task 1103-12-5147/57-213

Origin and Sponsor: Bureau of Ships Authorized 8/30/56
Managers: W. S. Connor, M. Zelen
Full task description: July-Sept 1956 issue, p. 37

Status: COMPLETED. The catalog of fractional replication designs for the 3^n series has been completed and transmitted to the sponsor. It is also being readied for publication in the NBS Applied Mathematics Series. The catalog contains designs for the fractions 1/3, 1/9, 1/27, 1/81, 1/243 for n ranging from 4 to 10 factors. These experiment plans have been constructed with the treatment combinations grouped in blocks of equal size so that the experimenter may take advantage of any homogeneous grouping of the experimental material to increase the precision of comparisons among the various factors.

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-12-1115/55-57

Origin: NBS Authorized 9/1/54
Manager: R. F. Dressler
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. W. Pell has developed iterative methods for the graphical integration of certain types of ordinary differential equations, both linear and non-linear. Among these are the important equations

$$g_2(y)\,\ddot{y} + g_1(y)\,\dot{y} + y = f(x), \qquad (\dot{y} = dy/dx)$$

$$f_2(x)\,\ddot{y} + f_1(x)\,\dot{y} + y = f(x).$$

To illustrate the procedure and accuracy of these methods they were applied to certain special cases, and it appears that one iteration of the procedure will give results sufficiently accurate for engineering purposes for integrals of x of reasonable length. In one period both amplitude and period of the solution of $\ddot{y} + y = 0$, $y(0) = 0$, $\dot{y}(0) = 1$, are accurate to within 0.5%. Such accuracy, of course, may not obtain in other cases. A report was begun which will incorporate some examples with an exposition of the methods used.

A. Ghaffari continued a study concerning the analytic properties of the solution of compressible flow past a wedge. It is shown that the solution for the stream function around the wedge as well as its first derivative are both continuous in the main stream. It is also found that the solution for the stream function is bounded when the compressible flow velocity around the wedge reaches its maximum.

Publications:

(1) On the representation of a certain integral involving Bessel functions of hypergeometric series. P. Henrici. To appearin the Journal of Mathematics and Physics.
(2) On the domain of regularity of generalized axially symmetric potentials. P. Henrici. To appear in the Proceedings of the American Mathematical Society.
(3) The quotient-difference algorithm. P. Henrici. To appear in NBS Applied Mathematics Series 49, "Further contributions to the solution of simultaneous linear equations and the determination of eigenvalues."
(4) Graphical solution of the single-degree-of-freedom vibration problem with arbitrary damping and restoring forces. W. H. Pell. Trans. ASME, Appl. Mech. 24, 311-312 (1957).

(5) The transmission of Rayleigh waves across an ocean floor with two surface layers. Part I. R. Stoneley. Bull. Seismological Soc. Amer. 47, 7-12 (1957).
(6) On the propagation of turbidity currents. R. Stoneley. To appear in the Vening Meinesz Jubilee Volume.
(7) The attenuation of Rayleigh waves with depth in a medium with two surface layers. R. Stoneley and U. Hochstrasser. Monthly Notices of Royal Astronomical Society, Geophysical Supplement 7, 279-288.
(8) On the solution of compressible flow past a wedge. A. Ghaffari. In manuscript. Abstract to appear in the Bulletin of the American Mathematical Society.
(9) On some expansions for Bessel integral functions. F. Oberhettinger. To appear in Journal of Research, NBS.

RESEARCH IN CONTINUUM MECHANICS
(formerly Mathematical Elasticity)
Task 1104-12-5160/55-85

Origin: NBS Authorized 12/27/54
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: R. F. Dressler
Full task description: Oct-Dec 1954 issue, p. 30

Status: CONTINUED. The problem of the flow about a ring wing in the presence of an axially symmetric body is still under consideration. As a first phase of the problem it was deemed advisable to investigate the mutual interference effects of the simplest cases of this problem viz., the ring vortex in the presence of a doublet and the ring vortex in the presence of a sphere lying on its axis of symmetry. These simple cases are under study.

R. F. Dressler is investigating the edge effect on bent plates for non-zero correction at the edge, and the analogous problem for generalized plane stress. He plans to attend the NATO-AGARD Wind Tunnel Panel at Schweringen, Holland. Also he will serve as a temporary consultant for the European Office of the Air Research Development Command.

Publications:
(1) Stationary principles for forced vibrations in elasticity and electromagnetism. J. L. Synge. To appear in the Proceedings of the Eighth Symposium in Applied Mathematics of the American Mathematical Society, held at Chicago, Ill., April 1956.
(2) Elastic waves in anisotropic media. J. L. Synge. To appear in the Journal of Mathematics and Physics.

(3) The torsion of a hollow square. J. L. Synge and W. F. Cahill. To appear in the Quarterly of Applied Mathematics.
(4) The vibration of triangular wings. R. F. Dressler. In manuscript. Abstract available in the Proceedings of the Ninth International Congress on Mechanics, Brussels, September 1956.
(5) The elastic problem for a ring of uniform force in an infinite body. W. H. Pell. In manuscript.

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-5160/56-154

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: July-Sept 1955 issue, p. 20

Status: CONTINUED. The tables of formulas have been prepared for publication. An introduction is being written.

RESEARCH IN FLUID DYNAMICS OF TWO-PHASE FLOWS
Task 1104-12-5160/56-155

Origin and Sponsor: Office of Naval Research Authorized 9/30/55
Manager: R. F. Dressler
Full task description: July-Sept 1955 issue, p. 21

Status: CONTINUED. In order to study the condensation process in a mixing chamber, J. M. Burgers and A. Ghaffari studied the growth of a water droplet surrounded by condensing steam. The condensation upon a plane surface and upon spherical drops of given radius was investigated. The amount of heat to be taken up by the water in both cases was found and compared. The equations of motion for a mixture of steam and water droplets were given as affording an idea about what happens in the mixing process and as a means for finding out something about the shape to be given to the mixing chamber. A number of examples were studied using specific values for entrance needs and temperatures of water and steam and for exit pressure.

The results were written up in an NBS report, "On the application of steam-driven water jets for the propulsion of an underwater vessel."

Publication:
(1) Condensation interfaces in two-phase flows. P. Chiarulli and R. F. Dressler. To appear in the Journal of Applied Physics. Abstract

appeared in the Proceedings of the Ninth International Congress on Mechanics, Brussels, September 1956.

RESEARCH IN RADIATION THEORY
Task 1104-12-5160/56-175

Origin: NBS Authorized 12/28/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: Oct-Dec 1955 issue, p. 18

Status: COMPLETED.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

1102-40-5126/51-37 MOLECULAR STRUCTURE, III
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. O'Hara
Full task description: July-Sept 1951 issue, p. 50
Status: Continued. The problem is being studied from the standpoint of adapting it for solution with existing IBM 704 codes.

1102-40-5126/52-44 CALCULATIONS FOR d-SPACINGS
Origin and Sponsor: NBS, Division 9
Full task description: Oct-Dec 1951 issue, p. 47
Manager: R. Zucker
Status: Continued. About 16 calculations for d-spacings for tetragonal, hexagonal, orthorhombic and monoclinic crystals were performed. Also redetermination of unit cell constants by least squares fitting to a measured d-spacing were carried out for about 16 crystals.

Work is in progress for writing a general d-spacing code for the 704.

1102-40-5126/53-51 RADIATION DIFFUSION, III
Origin: NBS, Section 4.8
Sponsor: Atomic Energy Commission
Manager: J. Doggett (4.8)
Full task description: Apr-June 1953 issue, p. 57 (Neutron Diffusion, III)
Status: Completed.

1102-40-5126/54-13 AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: New York Quartermaster Procurement Agency
Manager: H. Bremer
Full task description: Oct-Dec 1953 issue, p. 43
Status: Continued. Three procurement contracts were awarded during this quarter.

Test runs on the 704 with a transportation code furnished by the IBM Corporation were started. This work is being done in order to become familiar with the 704 code and with a view toward standardizing procedures for handling all future contract award problems on the 704.

3711-60-0009/54-22 ENERGY DISTRIBUTIONS ON OPTICAL IMAGE
Origin: NBS, Section 2.2
Manager: L. S. Joel
Full task description: Jan-Mar 1954 issue, p. 43
Status: Inactive. For status to date, see Jan-Mar 1956 issue, p. 19.

3711-60-0009/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Manager: S. Prusch
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. Preparation of new spectrum analysis programs for the IBM 704 have been started. The program for computing vacuum wave-numbers from wave-lengths has been written and has been used to compute 2000 new wave-numbers of iodine II. Further programs to evaluate and compute differences between known energy levels, to search for line pairs with these differences, to predict new levels, and to check for reoccurrence of predicted levels and evaluate the predictions, have been partially written, and code checking is proceeding.

3711-60-0009/54-38 COMPRESSIBILITY FACTORS OF DRY AIR
Origin: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Inactive.

3711-60-0009/55-68 CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Division 9
Manager: P. O'Hara, S. Block (9.7)
Full task description: Jan-Mar 1955 issue, p. 18
Status: Continued. Existing codes have been modified for solution of the problem on the 704, and code checking is in progress.

3711-60-0009/55-75 PARAMETER OF THE DISPERSION EQUATION FOR OPTICAL GLASS
Origin: NBS, Section 2.2
Manager: R. Zucker
Full task description: Jan-Mar 1955 issue, p. 20
Status: Inactive.

3711-60-0009/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. Preparation of a code for the calculations to be performed on the IBM 704 computer was started.

1102-40-5126/55-88 STRESSES IN A WALL FOUNDATION
Origin and Sponsor: NBS, Section 10.1
Manager: I. Stegun
Full task description: Jan-Mar 1955 issue, p. 22
Status: Inactive.

1102-40-5126/55-113 REACTOR DESIGN
Origin: Westinghouse Atomic Power Division
Sponsor: Atomic Energy Commission
Manager: U. Hochstrasser
Full task description: Jan-Mar 1955 issue, p. 28
Status: Completed. Results have been transmitted to the sponsor.

1102-40-5126/55-117 ATTENUATION OF PRESSURE PULSES OF FINITE AMPLITUDE
Origin: NBS, Section 3.2
Sponsor: Bureau of Aeronautics, U. S. Navy
Manager: M. Paulsen
Full task description: Apr-June 1955 issue, p. 18
Status: Inactive.

1102-40-5126/55-121 ELECTRON PENETRATION
Origin: NBS, Section 4.8
Sponsor: Atomic Energy Commission
Manager: S. Peavy
Full task description: Apr-June 1955 issue, p. 19
Status: Continued. The problem has been recoded for the IBM 704 machine. The routine for calculation of the moments has been completed and checked out. The remainder of the program is in the process of being checked out.

1102-40-5126/55-127 VIBRATIONS OF CIRCULAR DISC
Origin and Sponsor: Evans Signal Laboratory
Manager: U. Hochstrasser
Full task description: July-Sept 1955 issue, p. 32
Status: Completed. The results have been transmitted to the sponsor.

1102-40-5126/56-136 CALCULATION OF WAVE FUNCTIONS BY HARTREE METHOD
Origin and Sponsor: Naval Research Laboratory
Manager: S. Peavy
Full task description: July-Sept 1955 issue, p. 34
Status: Inactive.

1102-40-5126/56-140 MULTIPLE CORRELATION ROUTINES
Origin and Sponsor: Agricultural Economics Division, Department of Agriculture
Managers: H. Bremer, M. Paulsen
Full task description: Oct-Dec 1955 issue, p. 26
Status: Completed. Seven problems were run during the quarter. A final report on the problem has been prepared.

1102-40-5126/56-162 STRESSES IN A WALL RESTING ON A FOOTING
Origin and Sponsor: NBS, Section 10.1
Manager: I. Stegun
Full task description: Jan-Mar 1956 issue, p. 26
Status: Inactive.

1102-40-5126/56-163 ANGULAR DISTRIBUTIONS AND POLARIZATION EFFECTS IN NUCLEAR SCATTERING
Origin and Sponsor: Naval Research Laboratory
Manager: I. Stegun
Full task description: Oct-Dec 1955 issue, p. 32
Status: Continued. A code for the solution of the Coulomb wave equation is being prepared for the IBM 704 computer.

1102-40-5126/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Manager: E. Haynsworth
Full task description: Jan-Mar 1956 issue, p. 27
Status: Continued. Several additional cases were computed.

1102-40-5126/56-171 COLLISION INTEGRALS USED IN TRANSPORT THEORY
Origin and Sponsor: NBS, Section 3.2
Manager: J. Cooper
Full task description: Oct-Dec 1955 issue, p. 33
Status: Inactive.

1102-40-5126/56-172 NUMERICAL EVALUATION OF SPECIAL INTEGRAL EXPRESSIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: L. Joel
Full task description: Jan-Mar 1956 issue, p. 29
Status: Inactive.

1102-40-5126/56-179 NORMAL PROPAGATION CONSTANT
Origin and Sponsor: NBS, Section 82.10
Manager: H. H. Howe (82.10)
Full task description: Apr-June 1956 issue, p. 30
Status: Continued. Very extensive computations were made during this quarter. These involved several different formulas as follows: (1) The basic formula (1) given in the Apr-June 1956 issue, page 30, extending to much higher values of G (ground resistivity) and to different values of K (dielectric constant of ground). (2) The same ionospheric model, but including the first spherical correction described on page 31 of the report for Apr-June 1956. (3) The same model,but a third spherical correction, namely one in which the right side of equation (2) on page 31 is multiplied by $\sqrt{c^2}/(1.01\ c^2 - .01)$. (4) The two-layered model described in the Jan-Mar 1957 issue, page 30, for the basic formula. (5) The two-layered model, with the first spherical correction included.
Some additional curves showing Im(n) in the c-plane were computed for use in a projected report.

1102-40-5126/56-184 GAIN CALCULATIONS OF AN ITERATED TRANSISTOR AMPLIFIER
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: E. Haynsworth
Full task description: Apr-June 1956 issue, p. 32
Status: Inactive. For status to date, see July-Sept 1956 issue, p. 32.

1102-40-5126/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: S. Prusch
Full task description: July-Sept 1956 issue, p. 33
Status: Inactive.

1102-40-5126/56-192 NOISE MEASUREMENT
Origin and Sponsor: NBS, Section 6.1
Manager: W. F. Cahill
Full task description: Oct-Dec 1956 issue, p. 28
Status: Completed. All results have been transmitted to the sponsor.

1102-40-5126/57-209 TRAFFIC DISTRIBUTION
Origin and Sponsor: Bureau of Public Roads
Manager: S. T. Peavy
Full task description: Jan-Mar 1957 issue, p. 32
Status: Continued. The program has been completed and checked out on the IBM 705 (at the Treasury Department).

3711-60-0009/57-210 SOUND VELOCITY
Origin: NBS, Section 3.2
Manager: U. Hochstrasser
Full task description: July-Sept 1956 issue, p. 34
Status: Inactive. For status to date, see Jan-Mar 1957 issue, p. 32.

1102-40-5126/57-211 METEOROLOGICAL DATA
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: P. O'Hara
Full task description: Oct-Dec 1956 issue, p. 30
Status: Inactive.

1102-40-5126/57-215 COMPUTATION OF INTEGRALS INVOLVING BESSEL FUNCTIONS
Origin and Sponsor: NBS, Section 6.1
Manager: U. Hochstrasser
Full task description: July-Sept 1956 issue, p. 35
Status: Completed. The results have been transmitted to the sponsor.

1102-40-5126/57-219 THERMAL PROPERTIES
Origin and Sponsor: NBS, Section 3.2
Managers: J. Cooper, D. Sumida
Full task description: Oct-Dec 1956 issue, p. 30
Status: Continued. Coding of the problem for the IBM 704 computer is in progress.

1102-40-5126/57-221 BESSEL FUNCTIONS FOR COMPLEX ARGUMENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Oct-Dec 1956 issue, p. 31
Status: Continued. Several cases were run on SEAC to evaluate the Bessel and Hankel functions for given complex arguments up to the order n specified. The functions X_n, Y_n and Z_n were also evaluated. The results were submitted to the sponsor.

1102-40-5126/57-222 ROOTS OF POLYNOMIALS
Origin and Sponsor: Naval Research Laboratory
Manager: U. Hochstrasser
Full task description: Oct-Dec 1956 issue, p. 32
Status: Inactive.

3711-60-0009/57-223 SELF-CONSISTENT FIELDS
Origin: NBS, Section 3.2
Manager: E. V. Haynsworth
Objective: To prepare a very generalized form of the original code for the 704 for solving self-consistent field problems. The new code will incorporate a number of additional calculations as well as a master program and a control program to make it as flexible as possible. It is intended that this code should operate with efficiency for any given combination of core, drum, and tape storage, as it is to be used at several different installations.
Background: The original code was prepared in connection with task 1102-40-5126/56-139 (see July-Sept 1955 issue, p. 36). The decision to rewrite the code in a generalized form resulted from consultations with R. A. Nesbet of Boston University, Cambridge, Mass. E. J. Ransil (3.2) transmitted the request.
Status: New.

1102-40-5126/57-224 TRACK-WHILE-SCAN RADAR PROBLEM
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Managers: E. Haynsworth, P. J. Walsh
Full task description: Oct-Dec 1956 issue, p. 32
Status: Continued. Modifications were made in several of the factors to be computed. The code has been rewritten for the 704 computer, and code checking is now in progress.

1102-40-5126/57-225 DAMAGE ASSESSMENT PROBLEM, II
Origin and Sponsor: Corps of Engineers, U. S. Army
Managers: H. Bremer, W. G. Hall, L. S. Joel
Full task description: Oct-Dec 1956 issue, p. 33
Status: Inactive.

1102-40-5126/57-228 MICROWAVE PROPERTIES OF FERRITES
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: B. Walter
Full task description: Jan-Mar 1957 issue, p. 35
Status: Completed. The results were transmitted to the sponsor.
The calculations of $F(H_a)$ under the assumption of Bloch-Bloembergen damping showed no significant differences from previous calculations assuming Landau-Lifschitz damping. The same general results occurred.

3711-60-0009/57-229 APPLICATION OF ELECTRONIC DATA PROCESSING MACHINERY TO PAYROLL OPERATIONS

Origin: NBS, Section 40.0
Managers: H. Bremer, P. R. McClenon, M. Paulsen, J. B. Tallerico
Full task description: Jan-Mar 1957 issue, p. 36
Status: Continued. During the past quarter item layouts, card layouts and print layouts were drawn up and agreed upon. Two sets of flow charts were drawn up indicating the logical flow of data and the flow of the calculations of individual pay and summarization of this pay.

The problem has been broken up into many small sections to make it possible to assign a section to each member of the Accounting Division for actual coding. It is hoped that in this manner the Accounting Division will be able to eventually handle all of the coding for its own work. One section which concerns breaking the input data into separate items has been coded and is being code checked now.

1102-40-5126/57-234 PERSONNEL SURVEY

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: P. O'Hara
Full task description: Jan-Mar 1957 issue, p. 37
Status: Inactive.

3711-60-0009/57-232 POLYNOMIAL EVALUATION

Origin: NBS, Section 3.1
Manager: R. Durrah
Objective: To evaluate

$$X_n^k(J) = \sum_{j=o}^{k} \frac{p_n(\mu_j)\, e^{-\mu_j J}}{\sum_{m=o}^{k} p_m^2(\mu_j)\, e^{m\theta}}$$

for $k = 3,4,5$,

$J = .01,\ 0.1,\ 1,3,10$,

$n = 0,1,2,3,\ldots,k,\ (n \leq k)$,

$\theta = 1$.

The $p_n(\mu)$ are Gottlier polynomials (see Amer. J. Math. 60, 455(1937)).

The μ_j are the roots of the equation: $P_{r+1}(\mu) = 0$.

Background: The problem arises in the application of the theory of stochastic processes to chemical kinetics and deals particularly with the calculation of the rate of dissociation of diatomic molecules at high temperatures such as are found in shock waves. The problem was transmitted by K. E. Shuler (3.1).

Status: New. The code was written for the 704 and is in the process of bring checked out.

3711-60-0009/57-235 TRIPLE INTEGRALS--ENTROPY CALCULATIONS
Origin: NBS, Section 3.2
Managers: W. Gautschi, A. Beam
Objective: To evaluate integrals of the form

$$I = \int_R f_\lambda(x)\, f_\lambda(y)\, f_\lambda(z)\, dx\, dy\, dz$$

with the region R defined by the inequalities $x+y-z \geq 0$, $y+z-x \geq 0$ $z+x-y \geq 0$, $x \geq 0$, $y \geq 0$, $z \geq 0$. For various values of the parameter λ, the functions $f_\lambda(x)$ are given in tabular form for $x \geq 1$ and are defined by $f_\lambda(x) = -x$ for $0 \leq x < 1$. They have a discontinuity at $x = 1$.
Background: The integral arises in the computation of entropy densities for monotomic gases. The problem was communicated by R. E. Nettleton of Section 3.2.
Status: New. By taking into account all the symmetries of the problem, we found that

$$I_\lambda = 4 \int_0^{\infty} f_\lambda(x)dx \int_{x/2}^{x} f_\lambda(y)dy \int_{x-y}^{y} f_\lambda(z)dz.$$

The last integral was computed by numerical quadrature, after splitting apart the integral over the unit cube. A code was written for the IBM 704 and is in the process of being checked.

3711-60-0009/57-236 SELF-CONSISTENT FIELD— EIGENVALUES
Origin: NBS, Section 3.6
Manager: E. V. Haynsworth
Objective: To solve a self-consistent field problem using a code already written for previous similar problems. It is necessary to adjust the code for the present problem which is more complicated.
Background: This problem is similar to those described in tasks 1102-40-5126/56-139 (see July-Sept 1955 issue, p. 36) and 1102-40-5126/56-166 (see Jan-Mar 1956 issue, p. 27). It is requested by M. Boyd (3.6).
Status: Completed (New). Results have been submitted to the sponsor.

3711-60-0009/57-238 MODIFIED LANDAU FUNCTION
Origin: NBS, Section 4.8
Manager S. Peavy, R. Allsbrook
Objective: To evaluate

$$L_B(\lambda) = \frac{1}{2\pi}\int_o^{oo} \exp\left\{ i\lambda y + iy \ln y - \frac{\pi}{2} y + i\frac{\pi}{2}B + B \ln y \right\} dy$$

$$+ \frac{1}{2\pi}\int_o^{oo} \exp\left\{ -i\lambda y - iy \ln y - \frac{\pi}{2}y - i\frac{\pi}{2}B + B \ln y \right\} dy$$

for λ = -6(.2)10 and B = -15(.2)4.
Background: The modified Landau function arises in the problem of electron range straggling. Its importance is as a weight function for range distributions. It describes the small energy loss distribution with very little modification. This distribution includes effects due to radiative as well as non-radiative interactions.
The problem was submitted by L. Spencer (4.8).
Status: New.

3711-60-0009/57-241 HEAT TRANSFER
Origin: NBS, Section 11.2
Managers: M. Abramowitz, W. F. Cahill
Objective: To solve for various values of λ the partial differential equation

$$(1-r^2)\frac{\partial\theta}{\partial\xi} = \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{1}{\lambda^2}\frac{\partial^2\theta}{\partial\xi^2}$$

under the boundary conditions

$$\theta = \theta_o \quad \text{for } \xi = 0,$$
$$\theta \text{ finite for } \xi = oo,$$
$$\theta = \theta_1 \text{ for } r = 1.$$

Background: The above differential equation describes the heat transfer in a tube of infinite extent due to a fluid moving with parabolic velocity distribution.
Status: New. Assuming a solution in the form

$$\frac{\theta - \theta_1}{\theta_o - \theta_1} = \sum_{n=1}^{oo} A_n e^{z_n\xi} \cdot y(r,z_n),$$

there exists for each value of λ a set of eigenfunctions $y(r,z_n)$ and corresponding eigenvalues z_n which satisfy the differential system

$$y'' + \frac{1}{r} y' + \left[\frac{z_n^2}{\lambda^2} - z_n(1 - r^2)\right] y = 0,$$

$$y(r, z_n) = 0 \quad \text{for } r = 1.$$

The code to compute these eigenvalues and eigenfunctions on the 704 was written and checked out.

3711-60-0009/57-247 MECHANICAL IMPEDANCE
Origin: NBS, Section 6.1
Manager: J. P. Menard
Objective: (I) To compute calibration constants K(f), equal to the complex conjugate of $\bar{K}(f)$, for 23 values of the frequency f ranging from 40 to 10,000.

$$\bar{K}(f) = 1/N \sum_{n=1}^{N} [F_n/D_n - F_o/D_o] [1/M_n (2\pi f)^2]$$

(II) To compute the complex impedance Z_x from measured values F_x and D_x of forces and displacements, for the frequencies and corresponding values of K(f) from (I).

$$Z_x = [1/i (2\pi f) K(f)][F_x/D_x - F_o/D_o]$$

(III) Given Z_x from (II), to calculate by approximate methods parameters for the following equation:

$$Z_x = R_x + iX_x = i(2\pi f)m + \frac{[R + i(2\pi f)M][r - ik/(2\pi f)]}{[R + r] + i[(2\pi f)M - k/(2\pi f)]}$$

The following parameters are to be obtained:

1) $k = \frac{1}{N_1} \sum_f [- (2\pi f)X_x], \quad 400 \leq f \leq 750;$

N_1 is the number of frequency points in the range between 400 and 750.

2) $r = \frac{1}{N_2} \sum_f R_x, \quad 400 \leq f \leq 4000;$

N_2 is the number of frequency points in the range between 400 and 4000.

3) $m = \frac{1}{N_3} \sum_f \frac{X_x}{2\pi f} + \frac{k}{(2\pi f)^2}, \quad 1500 \leq f \leq 4000;$

N_3 is the number of frequency points in the range between 1500 and 4000.

4) $$Q = \frac{f(\text{zero})}{f(\text{min}) - f(\text{max})}, \quad f < 250.$$

f(zero) is the frequency at which X is nearest zero.

5) $$M = \frac{k}{[2\pi f(\text{zero})]^2 (1 + 1/Q^2)}$$

6) $$R = \frac{-r + \sqrt{r^2 + 4\,[2\pi f(\text{zero})]^2 M^2/Q^2}}{2}$$

Background: A method has been developed for determining the resistive and reactive components of the impedance of the human head and mastoid, or of other high mechanical impedances, using a special direct-recording measuring system that records both force and motion. (See E.L.R. Corliss, W. Koidan, J. Accoust. Soc. Amer. 27, 1164 (1955)). For a given frequency f one or more known masses are used for calibration, and the voltages (expressed in complex form) from the force pickup and displacement measuring device are recorded. At certain frequencies measurements are also made with no load for correction purposes. In the above expressions,

M_n is the value of the individual mass,

N is the number of masses,

F and D are measured values of force and displacement expressed in a complex form,

Z_x is the complex impedance of the human head or mastoid,

the subscript n refers to a mass M_n,

the subscript 0 designates no load,

the subscript x refers to the head or mastoid.

The problem was requested by M. D. Burkhard (6.1)

Status: New.

3711-60-0009/57-248 THE EVALUATION OF A TRIPLE INTEGRAL FOR THE SOLUTION OF NEGATIVE ION DETACHMENT

Origin: NBS, Section 4.6
Manager: S. Peavy
Objective: The following quantity is to be computed:

$$\sigma = \frac{4}{\pi}\frac{t}{k_o} N_o^2 \left[I_1(t^2) + I_1'(t^2) - I_2(t^2)\right]$$

where

$$I_1(t^2) = \int_o^{1-t^2} ds^2 \; sq \int_{-1}^{1} \frac{d\alpha}{(1+q^2-2q\alpha)^2} \int_{-1}^{1} d\beta \, \frac{H^2(K)}{K^2}$$

$$I_1'(t^2) = \int_o^{1-t^2} ds^2 \; sq \int_{-1}^{1} \frac{d\alpha}{(1+s^2-2s\alpha)^2} \int_{-1}^{1} d\beta \, \frac{H^2(K')}{(K')^2}$$

$$I_2(t^2) = \int_o^{1-t^2} ds^2 \; sq \int_{-1}^{1} \frac{d\alpha}{1+q-2q\alpha} \int_{-1}^{1} d\beta \, \frac{H^2(K)}{K^2} \, T$$

for

$$T = \left[\left[(1-s^2)^2 + 4s^2\left\{\beta - s\frac{(1-q\alpha)}{(1+q^2-2q\alpha)^{\frac{1}{2}}}\right\}\left\{\beta - \frac{1}{s}\frac{(1-q\alpha)}{(1+q^2-2q\alpha)^{\frac{1}{2}}}\right\}\right]\right]^{-\frac{1}{2}}$$

$$K^2 = 2\frac{k_o^2}{t^2}\left[1 - \frac{t^2}{2} - q\alpha - s\beta\,(1+q^2-2q\alpha)^{\frac{1}{2}}\right]$$

$$(K')^2 = 2\frac{k_o^2}{t^2}\left[1 - \frac{t^2}{2} - s\alpha - q\beta\,(1+s^2-2s\alpha)^{\frac{1}{2}}\right]$$

$$H(K) = \int_o^{\infty} dr\, r\, \eta_o(r) \sin Kr; \quad \eta_o(r) = \begin{cases} e^{-b_o r}\, {}_1F_1\left(1-\frac{1}{b_o};2;2b_o r\right), & \text{for } r \le r_o \\ A_o \frac{e^{-k_o r}}{r}, & \text{for } r \ge r_o \end{cases}$$

$$s^2 + q^2 + 1 - t^2, \; s \text{ and } q \ge 0$$

The results are needed for the following values of t^2:
.5, .25, .15, .10, .075, .05, .025, .015, .0125, .01, .0075, .005
.004, .003, .002;
and for the following two sets of constants:
Case 1: $k_o^2 = .05512$, $b_o = 1.057$, $A_o = .5186$, $r_o = 1.882$, $N_o^2 = 2.292$

Case 2: $k_o^2 = .02825$, $b_o = .5060$, $A_o = -1.840$, $r_o = 8.778$, $N_o^2 = .4164$

Background: The problem arises in the process of an electron striking a negative ion and detaching one of its bounded electrons. The computations were requested by S. Geltman (4.6).
Status: New.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Plastic panels: A fractional replication of a $4^3 \times 5$ design was constructed for use in an interlaboratory study of the properties of panels used in the maritime industry, for F. W. Reinhart, Section 7.7.

(2) Transistor aging study: Analysis of extensive data on the behavior of transistors with time was begun for G. Conrad, Section 1.6.

(3) Spectrogaphic analysis: An analysis of data from an interlaboratory study of the within-day, between-day and between-laboratory variation in a proposed standard method for spectrographic analysis of metal alloys was carried out for R. Alvarez, Section 5.10.

The two-semester in-hours course on "Selected Techniques of Statistical Analysis" was completed. Drs. Connor, Eisenhart, Rosenblatt and Zelen served as instructors.

Work was begun on the preparation of a battery of programs for statistical analysis of data on the new computer.

Publications:

(1) National physical standards and design of experiments. W. J. Youden. To appear in Revue de L'Institut International de Statistique (The Hague).
(2) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.
(3) Statistics: engineering viewpoint. W. J. Youden. To appear in the Journal of Engineering Education.
(4) Methods of testing small fire extinguishers. H. Shoub, T. G. Lee (Section 10.2); and J. M. Cameron. NBS Building Materials and Structures Report 150 (U.S. Government Printing Office, Washington, D. C., June 14, 1957).

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: CONTINUED. A brief report was prepared, summarizing the lengthy statistical analysis of 9 properties on 12 plates and giving recommendations for sampling future heats.

Analysis of data on 6 plates from each of 2 new steels was begun.

STATISTICAL ANALYSIS AND DESIGN OF EXPERIMENTS FOR THE U. S. GEOLOGICAL SURVEY
Task 1103-40-5140/54-1

Origin and Sponsor: U. S. Geological Survey, Department of Interior Authorized 10/9/53
Managers: C. Eisenhart, W. J. Youden
Full task description: Oct-Dec 1953 issue, p. 50

Status: CONTINUED. Comments of C. Eisenhart, W. J. Youden, M. Zelen and others on a manuscript entitled "The meaning and importance of 'semi-quantitative' analysis," by E. M. Shoemaker (U.S.G.S., Colorado Plateau District, Grand Junction) were transmitted to the author.

Full details of the multiple regression analysis of the dependence of the size of uranium ore deposits on chemical composition of the ore (see Jan-Mar 1957 issue, p. 40) were transmitted to A. T. Miesch (Northwestern University) and E. M. Shoemaker (U.S.G.S., Colorado Plateau District, Grand Junction) in a memorandum by J. M. Cameron and W. S. Connor, with additional comments by C. Eisenhart.

C. Eisenhart and W. S. Connor completed a preliminary study of whether and to what extent success in locating ore and mineralized deposits by indicator plants depends on the depth of the deposits below the ground surface. Data provided by H. L. Cannon (Geochemical Exploration Section, Denver) was used. The results were transmitted to Mrs. Cannon in memorandum form.

On April 30th, J. M. Cameron, C. Eisenhart, M. G. Natrella and J. R. Rosenblatt participated in a Washington meeting of the Committee on Statistics in Geology, of the Geologic Division, U.S.G.S. W. L. Newman (Colorado Plateau District, Grand Junction) commented on the difficulties met thus far by the Denver and Grand Junction groups in their attempts to gain acceptance of their geologic work involving 'semi-quantitative' spectrochemical analysis. C. Wahshaftig (U.S.G.S. Center, Menlo Park, California) discussed some of the statistical problems arising in the work of the Menlo Park Center.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-5146/55-93

Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: CONTINUED. The planned contents of the Manual is as follows:

Preface
Introduction
Some Basic Statistical Concepts
Part I. Some Standard Techniques for Quantitative Data
1. Performance of an item
2. Statistical tests concerning averages and dispersions
3. Description, prediction and correlation
Part II. Some Standard Techniques for Qualitative Data
1. Proportions and percentages
2. Sensitivity testing
Part III. The Planning and Analysis of Comparative Experiments
1. General considerations
2. Comparing the performance of several items, products or processes
3. Experiments in which several factors are studied simultaneously (factorial experiments)
4. Experiments to determine optimum conditions or levels
Part IV. Miscellaneous
1. Rejection of observations
Tables

A revised draft of Part I, sections 1 and 2 (Estimation and Tests) is in the process of being reproduced for circulation. The current draft was prepared with consideration to the coordinated report of comments received on a previous draft, 16 August 1956. Major additions in the new version include some short-cut tests and distribution-free tests.

Part I, section 3 (Regression) is essentially complete, except for the addition of some figures and examples. It is planned to circulate the test of this portion for comment.

Parts II, III, and IV, have been drafted, but examples have to be added. The selection and preparation of tables is completed.

APPLICATION OF ELECTRONIC COMPUTERS

During this quarter the installation in the Computation Laboratory of a new IBM type 704 electronic computer was completed, thus modernizing the Bureau's computing facilities by making available one of the newest and fastest machines. Problems previously solved on the SEAC are being recoded for solution on the 704. After this quarter SEAC will no longer be regularly maintained as a Bureau service facility. It will be used by the Data Processing Systems Division as an applications research facility and an engineering tool for studies of components and systems.

The record of SEAC operations for tasks of the Applied Mathematics Division for the period April 1 through June 30 is as follows:

Task No.	Title	Code Checking	Productive Operation
NBS:		(H O U R S)	
1104/55-55	Research in numerical analysis		2
5114/57-237	Numerical analysis training program	8	55
5116/56-148	Study of differential equations for nerve fiber excitation		2
1110/53-52	L-shell conversion coefficients		74
1110/51-8	Tables of power points of analysis-of-variance tests		18
1110/52-37	Spheroidal wave functions		1
5126/52-44	Calculations for d-spacings		4
5126/54-30	Spectrometer analysis		43
5126/55-68	Crystal structure calculations	2	6
5126/55-97	High temperature properties of air		15
5126/55-121	Electron penetration		3
0009/56-160	Mathematical subroutines	7	1
5126/56-162	Stresses in a wall resting on a footing		4
5126/56-166	SCF-LCAO Solution of some hydrides		5
5126/56-179	Normal propagation constant	35	73
5126/56-192	Noise measurement		2
5126/57-219	Thermal properties	6	
5126/57-223	Self-consistent fields		52
0009/57-236	Self-consistent field--eigenvalues		11
0002/51-1	Statistical engineering	2	12
1115/55-57	Research in mathematical physics and related fields	1	4
	Miscellaneous items	25	71
	Totals:	86	458

OTHER:		Code Checking	Productive Operation
5113/57-216	Handbook of mathematical functions	3	14
5126/54-13	Award of procurement contracts by linear programming		10
5126/55-113	Reactor design		10
5126/55-127	Vibration of a circular disc		9
5126/56-140	Multiple correlation routines		2
5126/56-163	Nuclear scattering		8
5126/56-184	Transistor amplifier		11
5126/57-224	Track-while-scan radar problem	4	143
	Classified tasks	16	106
	Totals:	23	311

The record of the use of the IBM 704 for the period May 14 through June 30 is as follows:

Task No.	Title	Assembly	Code Checking	Production
		(M I N U T E S)		
5116/55-56	Research in mathematical topics applicable to numerical analysis	34	2126	
5170/56-159	Research in theories of discrete structures	11	2	
5114/57-237	Numerical analysis training program	42		
5116/56-148	Nerve fiber excitation	19	73	30
5113/57-216	Handbook of mathematical tables	43	164	
1120/55-65	Automatic coding	14	2330	
0009/56-160	Staff subroutines	244	368	
5126/52-44	Calculations of d-spacings	10	9	
5126/54-30	Spectrometer analysis	85	462	102
5126/55-68	Crystal structure calculations		139	
5126/55-97	High temperature properties of air	12	144	
5126/55-121	Electron penetration	24	179	
5126/56-192	Noise measurement	4		
0009/57-229	Payroll operations	21	21	
0009/57-232	Polynomial evaluation	33	136	
0009/57-235	Triple integrals	5	47	
0009/57-238	Modified Landau functions	23	120	21
0009/57-241	Heat transfer	55	162	

Task No.	Title	Assembly	Code Checking	Production
		(M I N U T E S)		
*5126/57-243	Weather Bureau	52	443	1459
*5126/57-246	Nuclear Physics Section	42	248	178
*5126/57-252	Betatron Laboratory	12	54	
*0002/51-1	Statistical engineering	54	51	
*5710/--	Data Processing--Post Office Problem			28
5126/--	Classified	20	648	68
	Totals:	859	7926	1886

*Managed by the requesting agency.

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Colloquium Series

KAC, M. (Cornell University).Properties of continuous processes derived from discrete observations. May 29.

Applied Statistics Seminar

SIEGEL, S. (Pennsylvania State University). Decision making and measurement: Human behavior in two-choice situations. April 8.

GUMBEL, H. (U.S. Naval Air Missile Test Center, Point Mugu, California). Reliability Seminar, V: What is a failure? April 15.

Statistical Theory Seminar

McFADDEN, J. A. (Naval Ordnance Laboratory). The zero-crossing intervals of random functions. May 29.

MESNER, D. M. (Purdue University). Incidence matrices: Algebraic and combinatorial structure. June 4.

Mathematical Physics Section Seminar

CABANNES, H. (University of Marseille). Theories of the detached shock wave. April 18.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

ALT, F.L. (1) Problems for future computers. Presented at a Seminar on Automatic Computers and their Capabilities, held at the Moore School of Electrical Engineering, University of Pennsylvania, Philadelphia, Pa., April 10. (2) Mathematical methods in management and data processing problems. Presented before the Data Processing Advisory Committee, U. S. Navy, Washington, D. C., April 29.

CANNON, E. W. Some outstanding developments in the technical program of the National Bureau of Standards over the past year. Presented at a meeting of the Division of Mathematics of the National Academy of Sciences-National Research Council, Washington, D. C., May 18.

GHAFFARI, A. The value of science. A comment presented before the Discussion Panel on Professor Robert Oppenheimer's lecture, "Science, Values and Human Community," at the Fulbright Conference on Higher Education, held at Bronxville, N. Y., June 12-16.

ROSENBLATT, J.R. On some aspects of prediction in the study of complex systems. Presented at the New York University-RCA Working Conference on Theory of Reliability, held at Ardsley-on-Hudson, N. Y., April 18.

TAUSSKY-TODD, O. Commutativity in finite matrices. Presented before the Mathematics Department, Ohio State University, Columbus, Ohio, May 16.

WEGSTEIN, J. Giant computing machines. Presented at the Mt. Rainier Junior High School, Mt. Rainier, Md., May 22.

YOUDEN, W. J. (1) Dice, data and deductions. Presented at a meeting of the Detroit Section of the American Chemical Society, held at the University of Michigan, Ann Arbor, Mich., May 21; also presented at the University of Notre Dame, Notre Dame, Indiana, May 24. (2) Problems of the experimenter. Presented at a meeting of the American Association of Physics Teachers, Washington, D. C., April 6; also presented at a Seminar at the E. I. duPont de Nemours Co., Newburgh, N.Y., April 17. (3) Interpretation of chemical data. Presented at a meeting of the Toledo Section of the American Chemical Society, held at Bowling Green University, Bowling Green, Ohio, May 20; also presented before the Michigan State University Section of the American Chemical Society, East Lansing, Mich., May 23. (4) Control of experimental data by

statistical design. Presented at the University of Michigan, Ann Arbor, Mich., May 22.

ZELEN, M. Experiments with many factors. Presented at the Third Annual Statistical Engineering Symposium of the U. S. Army Chemical Corps Engineering Command, held at Edgewood, Md., May 2.

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Papers presented at the meeting of the American Mathematical Society held at New York University, New York, N. Y., April 5-6:

GHAFFARI, A. On the solution of compressible flow past a wedge.

MARCUS, M. On subdeterminants of doubly stochastic matrices.

NEWMAN, M. Some theorems about $p_r(n)$.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.2 Manuals, Bibliographies, and Indices

(1) Fractional factorial experiment designs for factors at two levels. NBS Applied Mathematics Series 48. Appeared April 1957. Available from the U. S. Government Printing Office, Washington 25, D. C., 50 cents.

(2) Methods of testing small fire extinguishers. H. Shoub, T. G. Lee (Section 10.2); and J. M. Cameron. NBS Building Materials and Structures Report 150. Available from U. S. Government Printing Office, Washington, D. C., 15 cents.

1.3 Technical Papers

(1) A note on values of a quadratic form. M. Marcus. J. Wash. Acad. Sci. 47, 97-99 (1957).

(2) On prediction of system performance from information on component performance. J. R. Rosenblatt. Proceedings of the Western Joint Computer Conference, Los Angeles, California, February 1957, pp. 85-94.

(3) Pitfalls in computation. I. A. Stegun and M. Abramowitz. J. Soc. Indust. Appl. Math. 4, 207-219 (1956).

(4) The transmission of Rayleigh waves across an ocean floor with two surface layers. R. Stoneley. Bull. Seismological Soc. Amer. 47, 7-12 (1957).

(5) The attenuation of Rayleigh waves with depth in a medium with two surface layers. R. Stoneley and U. Hochstrasser. Monthly Notices of Royal Astronomical Society, Geophysical Supplement 7, 279-288 (1957).

(6) Commutativity in finite matrices. O. Taussky. Amer. Math. Month. 64, 229-235 (1957).

(7) On matrix classes corresponding to an ideal and its inverse. O. Taussky. Illinois J. Math. 1, 108-113 (1957).

1.4 Reviews and Notes

(1) Graphical solution of the single-degree-of-freedom vibration problem with arbitrary damping and restoring forces. W. H. Pell. Trans. ASME, J. Appl. Mech. 24, 311-312 (1957).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION JUNE 30, 1957

2.1 Mathematical Tables

(1) Tables of the bivariate normal distribution function and related functions. To appear as NBS Applied Mathematics Series 50.

(2) Table of the first moment of ranked extremes. J. Lieblein and H. E. Salzer. To appear in the Journal of Research, NBS.

2.2 Manuals, Bibliographies, and Indices

(1) Analyzing straight line data. F. S. Acton. To appear as a book in the Applied Statistics Series of the Wiley Publications in Statistics.

2.3 Technical Papers

(1) A survey of Lyapunov's second method. H. Antosiewicz. To appear in Annals of Mathematics Studies.

(2) On the identity relationship for fractional replicates of the 2^n series. R. C. Burton and W. S. Connor. To appear in Annals of Mathematical Statistics.

(3) Some examples of the use of high speed computers in statistics. J. Cameron. To appear in the Proceedings of the Office of Ordnance Research Conference on Designs of Experiments in Army Research, Development and Testing, Washington, D. C., October 1955.

(4) Use of computers in statistical calculations. J. Cameron. To appear in the Proceedings of the Fourth Annual High-Speed Computer Conference, 1957, held at the University of Louisiana, Baton Rouge, La., March 1957.

(5) Condensation interfaces in two-phase flows. P. Chiarulli and R. F. Dressler. To appear in the Journal of Applied Physics.

(6) A numerical study of Dedekind's cubic class number formula. H. Cohn. To appear in the Journal of Research, NBS.

(7) A computation of cyclic cubic units. H. Cohn and S. Gorn (Moore School of Electrical Engineering). To appear in the Journal of Research, NBS.

(8) Experiences with incomplete block designs: Examples. W. S. Connor. To appear in the Proceedings of the Symposium on the Design of Industrial Experiments, held at the University of North Carolina, Raleigh, N. C., November 1956.

(9) The uniqueness of the triangular association scheme. W. S. Connor. Submitted to a technical journal.

(10) Uniqueness theory for asymptotic expansions in general regions. P. Davis. To appear in the Pacific Journal of Mathematics.

(11) Complete sequences and approximations in normed linear spaces. P. Davis and K. Fan. To appear in the Duke Mathematical Journal.

(12) Existence theorems and extreme solutions for inequalities concerning convex functions or linear transforms. K. Fan. Submitted to a technical journal.

(13) Systems of inequalities involving convex functions. Ky Fan, I. Glicksburg (University of Notre Dame), and A. J. Hoffman. To appear in the Proceedings of the American Mathematical Society.

(14) Pairs of matrices of order two which generate free groups. K. Goldberg and M. Newman. To appear in the Illinois Journal of Mathematics.

(15) A matrix minimization problem. A. J. Goldman. Submitted to a technical journal.

(16) Games with random payoff matrices. A. J. Goldman. Summary only to appear in the Proceedings of the Third Conference on Games, held by the Office of Naval Research Logistics Project at Princeton University, Princeton, N. J., March 1957.

(17) Note on bounds for certain determinants. E. Haynsworth. To appear in the Duke Journal of Mathematics.

(18) On the domain of regularity of generalized axially symmetric potentials. P. Henrici. To appear in the Proceedings of the American Mathematical Society.

(19) On the representation of a certain integral involving Bessel functions by hypergeometric series. P. Henrici. To appear in the Journal of Mathematics and Physics.

(20) Mechanized computation of thermodynamics tables at the National Bureau of Standards. J. Hilsenrath (NBS Thermodynamics Section) and J. Wegstein. To appear in the Proceedings of the Joint Conference on Thermodynamic Transport Properties of Fluids,

sponsored by the Institute of Mechanical Engineers, London, 1957.

(21) Numerical experiments in potential theory using the Nehari estimates. U. W. Hochstrasser. Submitted to a technical journal.

(22) Estimation of the frequencies of thin elastic plates with free edges. T. Kato, H. Fujita, Y. Nakata (University of Tokyo); and M. Newman. To appear in the Journal of Research, NBS.

(23) A determinantal inequality of H. P. Robertson, II. M. Marcus. To appear in the Journal of the Washington Academy of Sciences.

(24) On a determinantal inequality. M. Marcus. To appear in the American Mathematical Monthly.

(25) On doubly stochastic transforms of a vector. M. Marcus. Submitted to a technical journal.

(26) On subdeterminants of doubly stochastic matrices. M. Marcus. To appear in the Illinois Journal of Mathematics.

(27) Some extreme value results for indefinite Hermitian matrices. M. Marcus, B. N. Moyles and R. Westwick (University of British Columbia). To appear in the Illinois Journal of Mathematics.

(28) Congruences for the coefficients of modular forms and some new congruences for the partition function. M. Newman. To appear in the Canadian Journal of Mathematics.

(29) Construction and application of a class of modular functions. M. Newman. To appear in the Proceedings of the London Mathematical Society.

(30) On some expansions for Bessel integral functions. F. Oberhettinger. To appear in the Journal of Research, NBS.

(31) The number of representations of a quadratic form as a sum of four squares. G. Pall (Illinois Institute of Technology) and O. Taussky. To appear in the Proceedings of the Royal Irish Academy.

(32) On some aspects of prediction in the study of complex systems. J. R. Rosenblatt. To appear in the Proceedings of the NYU-RCA Working Conference on Theory of Reliability, held in New York, N.Y., April 1957.

(33) Generation of Bessel functions on high speed computers. I. Stegun and M. Abramowitz. Submitted to a technical journal.

(34) Elastic waves in anisotropic media. J. L. Synge. To appear in the Journal of Mathematics and Physics.

(35) Stationary principles for forced vibrations in elasticity and electromagnetism. J. L. Synge. To appear in the Proceedings of the Eighth Symposium in Applied Mathematics held by the American Mathematical Society, Chicago, Ill., April 1956.

(36) The torsion of a hollow square. J. L. Synge and W. F. Cahill. To appear in the Quarterly of Applied Mathematics.

(37) A determinantal inequality of H. P. Robertson, I. O. Taussky. Submitted to a technical journal.

(38) Some computationsl problems concerning integral matrices. O. Taussky. To appear in the 1956 Meeting of the Italian Society for the Advancement of Science, held in Sicily.

(39) Computational problems concerned with the Hilbert matrix. J. Todd. To appear in the Proceedings of the 1956 Meeting of the Italian Society for the Advancement of Science, held in Sicily.

(40) The condition of certain matrices, III. J. Todd. To appear in the Journal of Research, NBS.

(41) Automatic coding principles. J. Wegstein. To appear in the Proceedings of a Symposium on Automatic Programming, held by the Office of Naval Research, Washington, D. C., June 1956.

(42) Design of experiments in research and development. W. J. Youden. To appear in the Proceedings of the Office of Ordnance Research Conference on Design of Experiments in Army Research, Development, and Testing, Washington, D. C., October 1955.

(43) National physical standards and design of experiment. W. J. Youden. To appear in Revue de L'Institut International de Statistique (The Hague).

(44) Discussion of the paper by W. C. Otto, "A statistical study on flexural strength of concrete beams." W. J. Youden. To appear in the Proceedings of the Annual Meeting of the American Society for Testing Materials for 1956.

(45) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.

(46) Statistics--Engineering viewpoint. W. J. Youden. To appear in the Journal of Engineering Education.

(47) Experiments with many factors. M. Zelen. To appear in the Proceedings of the Third Annual Statistical Engineering Symposium of the U. S. Army Chemical Corps Engineering Command, Army Chemical Center, Md., May 1957.

(48) The analysis of covariance for incomplete block designs. M. Zelen. To appear in Biometrics.

(49) The analysis of incomplete block designs, M. Zelen. To appear in the Journal of the American Statistical Association.

(50) The use of incomplete block designs for asymmetrical factorial arrangements. M. Zelen. Submitted to a technical journal.

(51) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. S. Joel. Submitted to a technical journal.

2.4 Reviews and Notes

(1) Essential similarity: a counter-example. A. J. Goldman. Submitted to a technical journal.

(2) The probability of a saddle point. A. J. Goldman. Submitted to a technical journal.

(3) Convexity of the field of a linear transformation. A. J. Goldman and M. Marcus. To appear in the American Mathematical Monthly (Math. Notes).

2.5 Miscellaneous

(1) Further contributions to the solution of simultaneous linear equations and the determination of eigenvalues. To appear as Applied Mathematics Series 49.

U. S. DEPARTMENT OF COMMERCE
Sinclair Weeks, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major field laboratories in Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside front cover of this report.

WASHINGTON, D. C.

Electricity and Electronics. Resistance and Reactance. Electron Tubes. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat and Power. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology and Lubrication. Engine Fuels.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Nuclear Physics. Radioactivity. X-rays. Betatron. Nucleonic Instrumentation. Radiological Equipment. AEC Radiation Instruments.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Gas Chemistry. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Organic Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Concreting Materials. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Heating and Air Conditioning. Floor, Roof, and Wall Coverings. Codes and Specifications.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analogue Systems. Application Engineering.

• Office of Basic Instrumentation

• Office of Weights and Measures

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Systems. Navigation Systems. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Radio Systems Application Engineering.

Radio Standards. Radio Frequencies. Microwave Frequencies. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Calibration Center. Microwave Physics. Microwave Circuit Standards.


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