

NATIONAL BUREAU OF STANDARDS REPORT

5320

SKELETON TABLES
FOR
MANUAL ON EXPERIMENTAL STATISTICS
FOR ORDNANCE ENGINEERS

A Report to
Office of Ordnance Research
Department of the Army



U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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NATIONAL BUREAU OF STANDARDS REPORT

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Skeleton Tables for

Manual on Experimental Statistics
for Ordnance Engineers

Prepared by

Statistical Engineering Laboratory

A Report to

Office of Ordnance Research
Department of the Army

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N O T I C E

This report presents in skeleton form the tables which will eventually appear in the Manual on Experimental Statistics for Ordnance Engineers, as an aid to evaluation of drafts of various portions of the Manual being circulated for comment.

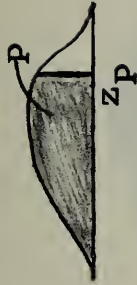
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Table Ia

Cumulative Normal Distribution

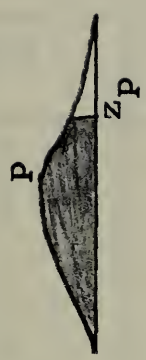


Values of P corresponding to z_p for the normal curve
 z is the standard normal variable.

z_p	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
-3	.0013	.0010	.0007	.0005	.0003	.0002	.0002	.0001	.0001	.0000
-2	.0228	.0179	.0139	.0107	.0082	.0062	.0047	.0035	.0026	.0019
-1	.1587	.1357	.1151	.0968	.0808	.0668	.0548	.0446	.0359	.0287
-0	.5000	.4602	.4207	.3821	.3446	.3085	.2743	.2420	.2119	.1841
+0	.5000	.5398	.5793	.6179	.6554	.6915	.7257	.7580	.7881	.8159
1	.8413	.8643	.8849	.9032	.9192	.9332	.9452	.9554	.9641	.9713
2	.9772	.9821	.9861	.9893	.9918	.9938	.9953	.9965	.9974	.9981
3	.9987	.9990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000

Table Ib

Cumulative Normal Distribution



Values of z_p Corresponding to P for the normal curve.
 z is the standard normal variable

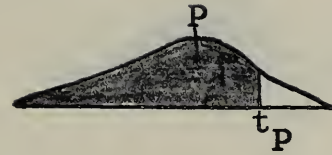
P	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.00	-	-2.33	-2.05	-1.88	-1.75	-1.64	-1.55	-1.48	-1.41	-1.34
.10	-1.28	-1.23	-1.18	-1.13	-1.08	-1.04	-0.99	-0.95	-0.92	-0.88
.20	-0.84	-0.81	-0.77	-0.74	-0.71	-0.67	-0.64	-0.61	-0.58	-0.55
.30	-0.52	-0.50	-0.47	-0.44	-0.41	-0.39	-0.36	-0.33	-0.31	-0.28
.40	-0.25	-0.23	-0.20	-0.18	-0.15	-0.13	-0.10	-0.08	-0.05	-0.03
.50	0.00	0.03	0.05	0.08	0.10	0.13	0.15	0.18	0.20	0.23
.60	0.25	0.28	0.31	0.33	0.36	0.39	0.41	0.44	0.47	0.50
.70	0.52	0.55	0.58	0.61	0.64	0.67	0.71	0.74	0.77	0.81
.80	0.84	0.88	0.92	0.95	0.99	1.04	1.08	1.13	1.18	1.23
.90	1.28	1.34	1.41	1.48	1.55	1.64	1.75	1.88	2.05	2.33

Special values

P	.001	.005	.010	.025	.050	.100
z_p	-3.090	-2.576	-2.326	-1.960	-1.645	-1.282
P	.999	.995	.990	.975	.950	.900
z_p	3.090	2.576	2.326	1.960	1.645	1.282

TABLE II

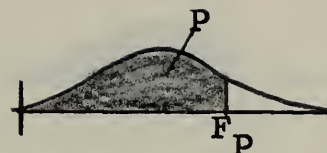
Percentiles of the t Distribution



d.f.	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$
1	3.078	6.314	12.706	31.821	63.657
2					
⋮					
9			2.262	2.821	
⋮					
19		1.729	2.093		
⋮					
120					
∞	1.282	1.645	1.960	2.326	2.576

Use d.f. 1(1)30, 40, 60, 120, ∞. Values taken from Table A-5 Dixon and Massey "Introduction to Statistical Analysis," Second Edition, McGraw-Hill (1957).

Percentiles of the F Distribution



n_1 = degrees of freedom for numerator

n_2 = degrees of freedom for denominator.

1. The first step is to identify the problem or question that needs to be answered. This involves understanding the context and the specific requirements of the task.

2. The second step is to gather relevant information and data. This can be done through research, consultation with experts, or by analyzing existing data sets.

3. The third step is to develop a plan or strategy to address the problem. This involves breaking down the problem into smaller, manageable parts and determining the best approach to solve each part.

4. The fourth step is to implement the plan. This involves carrying out the tasks and actions outlined in the plan, while monitoring progress and making adjustments as needed.

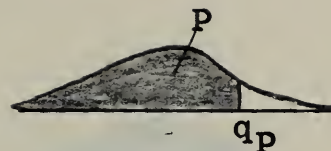
5. The fifth step is to evaluate the results. This involves comparing the outcomes of the implementation against the original goals and objectives, and identifying any areas for improvement.

6. The final step is to communicate the findings and conclusions. This involves sharing the results of the analysis with the relevant stakeholders, and providing recommendations for future action.

Reproduce also $F_{.99}(n_1, n_2)$. This is Table A-7b of the above reference.

TABLE IV

Percentiles of q (Studentized Range)



$q = w/s$. w is the range of t observations, and v is the number of degrees of freedom associated with the standard deviation s .

$q_{.95}$

$v \backslash t$	2(1)20
1 (1) 20	

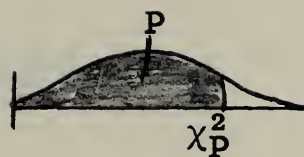
$q_{.99}$

$v \backslash t$	2(1)20
1 (1) 20	

Values for above tables taken from Table A-18, Dixon and Massey, Second Edition (1957).

TABLE V

Percentiles of the χ^2 Distribution



Values of χ^2_p corresponding to P

d.f.	$\chi^2_{.90}$	$\chi^2_{.95}$	$\chi^2_{.975}$	$\chi^2_{.99}$	$\chi^2_{.995}$

For large degrees of freedom,

$$\chi^2_p \doteq (z_p + \sqrt{2(d.f.) - 1})^2 / 2$$

where z_p is given in Table I.

d.f. = 1(1) 16, 18, 20, 24, 30, 40, 60, 120

Values taken from Table A-6a, Dixon and Massey, McGraw-Hill Second Edition (1957).

TABLE VI

Confidence Belts for Proportions

(Change labels,	Ordinate label - P Abscissae label - p)
1st chart	Confidence coefficient .90
2nd chart	Confidence coefficient .95
3rd chart	Confidence coefficient .99

Charts 1,2,3 are reproduced from Dixon and Massey, p. 414, 415, 416, Second Edition, McGraw-Hill (1957).

TABLE VII

Confidence Belts for the Correlation Coefficient
(confidence coefficient .95)

Reproduced from Table A-27, Dixon and Massey, Second Edition,
McGraw-Hill (1957).

TABLE VIII

Weighting Coefficients for Probit Analysis

Y	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.001	0.001	0.001	0.002	0.002	0.003	0.005	0.006	0.008	0.011
2	0.015	0.019	0.025	0.031	0.040	0.050	0.062	0.076	0.092	0.110
3	0.131	0.154	0.180	0.208	0.238	0.269	0.302	0.336	0.370	0.405
4	0.439	0.471	0.503	0.532	0.558	0.581	0.601	0.616	0.627	0.634
5	0.637	0.634	0.627	0.616	0.601	0.581	0.558	0.532	0.503	0.471
6	0.439	0.405	0.370	0.336	0.302	0.269	0.238	0.208	0.180	0.154
7	0.131	0.110	0.092	0.076	0.062	0.050	0.040	0.031	0.025	0.019
8	0.015	0.011	0.008	0.006	0.005	0.003	0.002	0.002	0.001	0.001

Values obtained from page 32, Finney, Cambridge University Press (1952).

TABLE IX

Maximum and Minimum Working Probits and Range

Expected probit Y	Minimum working probit y_0	Range 1/z	Maximum working probit y_{100}	Expected probit Y
1.1	0.8579	5034	9.1421	8.9
1.2	0.9522	3425	9.0478	8.8
1.3	1.0462	2354	8.9538	8.7
1.4	1.1400	1634	8.8600	8.6
1.5	1.2334	1146	8.7666	8.5
1.6	1.3266	811.5	8.6734	8.4
1.7	1.4194	580.5	8.5806	8.3
1.8	1.5118	419.4	8.4882	8.2
1.9	1.6038	306.1	8.3962	8.1
2.0	1.6954	225.6	8.3046	8.0
2.1	1.7866	168.00	8.2134	7.9
2.2	1.8772	126.34	8.1228	7.8
2.3	1.9673	95.96	8.0327	7.7
2.4	2.0568	73.62	7.9432	7.6
2.5	2.1457	57.05	7.8543	7.5
2.6	2.2339	44.654	7.7661	7.4
2.7	2.3214	35.302	7.6786	7.3
2.8	2.4081	28.189	7.5919	7.2
2.9	2.4938	22.736	7.5062	7.1
3.0	2.5786	18.5216	7.4214	7.0
3.1	2.6624	15.2402	7.3376	6.9
3.2	2.7449	12.6662	7.2551	6.8
3.3	2.8261	10.6327	7.1739	6.7
3.4	2.9060	9.0154	7.0940	6.6
3.5	2.9842	7.7210	7.0158	6.5
3.6	3.0606	6.6788	6.9394	6.4
3.7	3.1351	5.8354	6.8649	6.3
3.8	3.2074	5.1497	6.7926	6.2
3.9	3.2773	4.5903	6.7227	6.1
4.0	3.3443	4.1327	6.6557	6.0
4.1	3.4083	3.7582	6.5917	5.9
4.2	3.4687	3.4519	6.5313	5.8
4.3	3.5251	3.2025	6.4749	5.7
4.4	3.5770	3.0010	6.4230	5.6
4.5	3.6236	2.8404	6.3764	5.5
4.6	3.6643	2.7154	6.3357	5.4
4.7	3.6982	2.6220	6.3018	5.3
4.8	3.7241	2.5573	6.2759	5.2
4.9	3.7407	2.5192	6.2593	5.1
5.0	3.7467	2.5066	6.2533	5.0

TABLE X

Tolerance Factors for Normal Distributions

Factors K such that the probability is γ that at least a proportion P of the distribution will be included between $\bar{X} + Ks$, where \bar{X} and s are estimates of the mean and standard deviation computed from a sample of n .

Use format as pp. 102-107, "Techniques of Statistical Analysis", Eisenhart, Hastay and Wallis, McGraw-Hill (1947).
Abridge, using $n = 2(1)20, 25, 30(10)100, 100(100)600,$
 $800, 1000, \infty$.

TABLE XI

Criteria for Rejection of Outlying Observations

Statistic	Number of observations n	Critical values						
		$\alpha =$.30	$\alpha =$.20	$\alpha =$.10	$\alpha =$.05	$\alpha' =$.02	$\alpha' =$.01	$\alpha' =$.005

Reproduced from Table A-8e, Dixon and Massey, McGraw-Hill,
Second Edition (1957).

TABLE XII

Percentiles of T(n) for the "Wilcoxon Signed-ranks Test"

n	T _{.025} (n)	T _{.01} (n)	T _{.005} (n)
6	0	-	-
7	2	0	-
8	4	2	0
9	6	3	2
10	8	5	3
11	11	7	5
12	14	10	7
13	17	13	10
14	21	16	13
15	25	20	16
16	30	24	20
17	35	28	23
18	40	33	28
19	46	38	32
20	52	43	38
21	59	49	43
22	66	56	49
23	73	62	55
24	81	69	61
25	89	77	68

Adapted from Table II, F. Wilcoxon, 1949, "Some rapid approximate statistical procedures", New York:American Cyanamid Company, p. 14.

(See also, Table G, p. 254, Siegel, "Non-parametric Statistics", McGraw-Hill (1956).

For large n,

$$T_p(n) = \frac{n(n+1)}{4} - z_{1-p} \sqrt{\frac{n(n+1)(2n+1)}{24}} \quad \text{approximately}$$

where z is given in Table I.

TABLE XIII

Probabilities Associated with the Mann-Whitney Test

Probabilities associated with values as small as U.

Reproduce tables of Mann and Whitney from Annals of Mathematical Statistics Volume 18, (1947), pp. 52-54.

(Same tables are in Siegel, McGraw-Hill, 1956, Table J).

Eliminate last two columns of last table. Put note above each table:

" n_1 is the smaller of n_A, n_B ,

n_2 is the larger of n_A, n_B ."

TABLE XIV

Percentiles of $U(n_1, n_2)$ for the "Mann Whitney" Test

a) $U_{.001}(n_1, n_2)$ Reproduce Table K_1 p. 274 of *.

b) $U_{.01}(n_1, n_2)$ Reproduce Table K_2 p. 275 of *.

c) $U_{.025}(n_1, n_2)$ Reproduce Table K_3 p. 276 of *.

d) $U_{.05}(n_1, n_2)$ Reproduce Table K_4 p. 277 of *.

Put note above each table: " n_2, n_1 are the larger and smaller respectively of n_A, n_B ."

*) Siegel, McGraw-Hill, 1956.

NOTE: for $n > 20$,

$$U_{\alpha/2} = \frac{n_A n_B}{2} - z_{1-\alpha/2} \sqrt{\frac{n_A n_B (n_A + n_B + 1)}{12}}$$

approximately where z is given in Table I.

TABLE XV

Tables for Distribution-free Tolerance Limits (Two-sided)

Values (r,s) such that we may assert with confidence at least γ that 100P percent of a population lies between the rth smallest and the sth largest of a random sample of n from that population (no assumption of normality required).

n \ p	$\gamma = 0.75$			$\gamma = 0.90$			$\gamma = 0.95$			$\gamma = 0.99$		
	.75	.90	.99	.75	.90	.99	.75	.90	.99	.75	.90	.99
50	5,5	2,1	-	5,4	1,1	-	4,4	1,1	-	3,3	-	-
55	6,6	2,2	1,1	5,5	2,1	-	5,4	1,1	-	4,3	-	-
60	7,6	2,2	1,1	6,5	2,1	-	5,5	1,1	-	4,4	-	-
65	7,7	3,2	1,1	6,6	2,2	-	6,5	2,1	-	5,4	-	-
70	8,7	3,2	1,1	7,6	2,2	-	6,6	2,1	-	5,5	-	-
75	8,8	3,3	1,1	7,7	2,2	-	7,6	2,1	-	5,5	-	-
80	9,8	3,3	2,2	8,7	3,2	-	7,7	2,2	-	6,5	-	-
85	10,9	4,3	2,2	8,8	3,2	-	8,7	2,2	-	6,6	-	-
90	10,10	4,3	2,2	9,8	3,2	-	8,8	3,2	-	7,6	-	-
95	11,10	4,3	2,2	9,9	3,3	-	9,8	3,2	-	7,7	-	-
100	11,11	4,4	2,2	10,10	3,3	-	9,9	3,2	-	8,7	-	-
110	12,12	5,4	2,2	11,11	4,3	-	10,10	3,3	-	9,8	-	-
120	14,13	5,5	2,2	12,12	4,4	-	11,11	4,3	-	10,9	-	-
130	15,14	6,5	3,2	13,13	5,4	-	13,12	4,4	-	11,10	-	-
140	16,15	6,6	3,2	14,14	5,5	-	14,13	4,4	-	12,11	-	-
150	17,17	6,6	3,3	16,15	5,5	-	15,14	5,4	-	13,13	-	-
170	20,19	7,7	4,3	18,17	6,6	-	17,16	6,5	-	15,15	-	-
200	23,23	9,8	4,4	21,21	8,7	-	20,20	7,6	-	18,18	-	-
300	35,35	13,13	6,6	33,32	12,11	-	32,31	11,11	-	29,29	-	-
400	47,47	18,18	9,8	45,44	16,16	1,1	43,43	15,15	1,1	40,40	-	-
500	59,59	23,22	11,11	57,56	21,20	1,1	55,54	20,19	1,1	52,51	-	-
600	72,71	28,27	13,13	68,68	26,25	2,1	67,66	24,24	1,1	63,63	-	-
700	84,83	33,32	16,15	80,80	30,30	2,2	78,78	29,28	1,1	75,74	-	-
800	96,96	37,37	18,18	92,92	35,34	3,2	90,90	33,33	2,2	86,86	-	-
900	108,108	42,42	21,20	104,104	40,39	3,2	102,102	38,37	2,2	98,97	-	-
1000	121,120	47,47	23,22	117,116	44,44	3,3	114,114	43,42	3,2	110,109	-	-

Tables for Distribution-free Tolerance Limits (One-Sided)

Largest values of m such that we may assert with confidence at least γ that 100P percent of a population lies below the m th largest (or above the m th smallest) of a random sample of n from that population (no assumption of normality required).

$\begin{matrix} p \\ n \end{matrix}$		$\gamma = 0.75$				$\gamma = 0.90$				$\gamma = 0.95$				$\gamma = 0.99$			
		.75	.90	.95	.99	.75	.90	.95	.99	.75	.90	.95	.99	.75	.90	.95	.99
50		10	3	1	-	9	2	1	-	8	2	-	-	6	1	-	-
55		12	4	2	-	10	3	1	-	9	2	-	-	7	1	-	-
60		13	4	2	-	11	3	1	-	10	2	1	-	8	1	-	-
65		14	5	2	-	12	4	1	-	11	3	1	-	9	2	-	-
70		15	5	2	-	13	4	1	-	12	3	1	-	10	2	-	-
75		16	6	2	-	14	4	1	-	13	3	1	-	10	2	-	-
80		17	6	3	-	15	5	1	-	14	4	1	-	11	2	-	-
85		19	7	3	-	16	5	2	-	15	4	1	-	12	2	-	-
90		20	7	3	-	17	5	2	-	16	4	1	-	13	3	-	-
95		21	7	3	-	18	6	2	-	17	5	1	-	14	3	1	-
100		22	8	3	-	20	6	2	-	18	5	2	-	15	4	1	-
110		24	9	4	-	22	7	3	-	20	6	2	-	17	4	1	-
120		27	10	4	-	24	8	3	-	22	7	2	-	19	5	1	-
130		29	11	5	-	26	9	3	-	25	8	3	-	21	6	2	-
140		31	12	5	1	28	10	4	-	27	8	3	-	23	6	2	-
150		34	12	6	1	31	10	4	-	29	9	3	-	26	7	2	-
170		39	14	7	1	35	12	5	-	33	11	4	-	30	9	3	-
200		46	17	8	1	42	15	6	-	40	13	5	-	36	11	4	-
300		70	26	12	2	65	23	10	1	63	22	9	1	58	19	7	-
400		94	36	17	3	89	32	15	2	86	30	13	1	80	27	11	-
500		118	45	22	3	113	41	19	2	109	39	17	2	103	35	14	1
600		143	55	26	4	136	51	23	3	133	48	21	2	126	44	18	1
700		167	65	31	5	160	60	28	4	156	57	26	3	149	52	22	2
800		192	74	36	6	184	69	32	5	180	66	30	4	172	61	26	2
900		216	84	41	7	208	79	37	5	204	75	35	4	195	70	30	3
1000		241	94	45	8	233	88	41	6	228	85	39	5	219	79	35	3

TABLE XVII

Confidence Associated with a Tolerance Limit Statement

Confidence γ with which we may assert that 100P percent of the population lies between the largest and smallest of a random sample of n from that population (continuous distribution assumed).

n	P=.75	P=.90	P=.95	P=.99	n	P=.75	P=.90	P=.95	P=.99
3	.16	.03	.01	.00	17	.95	.52	.21	.01
4	.26	.05	.01	.00	18	.96	.55	.23	.01
5	.37	.08	.02	.00	19	.97	.58	.25	.02
6	.47	.11	.03	.00	20	.98	.61	.26	.02
7	.56	.15	.04	.00	25	.99	.73	.36	.03
8	.63	.19	.06	.00	30	1.00-	.82	.45	.04
9	.70	.23	.07	.00	40		.92	.60	.06
10	.76	.26	.09	.00	50		.97	.72	.09
11	.80	.30	.10	.01	60		.99	.81	.12
12	.84	.34	.12	.01	70		.99	.87	.16
13	.87	.38	.14	.01	80		1.00-	.91	.19
14	.90	.42	.15	.01	90			.94	.23
15	.92	.45	.17	.01	100			.96	.26
16	.94	.49	.19	.01					

TABLE XVIIIa

Table of Required Sample Sizes

Sample size required for detecting, with probability $1-\beta$, whether the average m of a new product differs from the standard m_0 (or whether two product averages m_A and m_B differ).

$$d = \frac{m-m_0}{\sigma} \quad (\text{or } d = \frac{m_A-m_B}{\sqrt{\sigma_A^2+\sigma_B^2}} \text{ if we are comparing two products}).$$

The standard deviations are assumed to be known.

$$\alpha = .01$$

$d \backslash 1-\beta$.50	.60	.70	.80	.90	.95	.99
.1	664	801	962	1168	1488	1782	2404
.2	166	201	241	292	372	446	601
.4	42	51	61	73	93	112	151
.6	19	23	27	33	42	50	67
.8	11	13	16	19	24	28	38
1.0	7	9	10	12	15	18	25
1.2	5	6	7	9	11	13	17
1.4	4	5	5	6	8	10	13
1.6	3	4	4	5	6	7	10
1.8	3	3	3	4	5	6	8
2.0	2	3	3	3	4	5	7
3.0	1	1	2	2	2	2	3

If we must estimate σ from our sample, and use Student's t then we should add 4 to the tabulated values to obtain the approximate required sample size. (If we are comparing two product averages, add 2 to the tabulated values, For this case, we must have $\sigma_A = \sigma_B$).

TABLE 1

TABLE 1. SUMMARY OF DATA FOR THE STUDY

The data were collected from 100 subjects who were randomly selected from a population of 1000. The subjects were divided into two groups of 50 each. The first group was the control group and the second group was the experimental group. The data were collected over a period of 12 weeks.

TABLE 1

TABLE 1. SUMMARY OF DATA FOR THE STUDY

TABLE 1

Age	Sex	Height	Weight	Heart Rate	Blood Pressure	Cholesterol	Glucose
20-29	Male	170-180	60-80	60-80	110-130	150-200	80-100
30-39	Male	170-180	60-80	60-80	110-130	150-200	80-100
40-49	Male	170-180	60-80	60-80	110-130	150-200	80-100
50-59	Male	170-180	60-80	60-80	110-130	150-200	80-100
60-69	Male	170-180	60-80	60-80	110-130	150-200	80-100
70-79	Male	170-180	60-80	60-80	110-130	150-200	80-100
80-89	Male	170-180	60-80	60-80	110-130	150-200	80-100
90-99	Male	170-180	60-80	60-80	110-130	150-200	80-100
20-29	Female	150-160	40-60	50-70	90-110	100-150	60-80
30-39	Female	150-160	40-60	50-70	90-110	100-150	60-80
40-49	Female	150-160	40-60	50-70	90-110	100-150	60-80
50-59	Female	150-160	40-60	50-70	90-110	100-150	60-80
60-69	Female	150-160	40-60	50-70	90-110	100-150	60-80
70-79	Female	150-160	40-60	50-70	90-110	100-150	60-80
80-89	Female	150-160	40-60	50-70	90-110	100-150	60-80
90-99	Female	150-160	40-60	50-70	90-110	100-150	60-80

TABLE 1. SUMMARY OF DATA FOR THE STUDY

TABLE XVIIIa (Continued)

$$\alpha = .05$$

$d \backslash 1-\beta$.50	.60	.70	.80	.90	.95	.99
.1	385	490	618	785	1051	1300	1838
.2	97	123	155	197	263	325	460
.4	25	31	39	50	66	82	115
.6	11	14	18	22	30	37	52
.8	7	8	10	13	17	21	29
1.0	4	5	7	8	11	13	19
1.2	3	4	5	6	8	10	13
1.4	2	3	4	5	6	7	10
1.6	2	2	3	4	5	6	8
1.8	2	2	2	3	4	5	6
2.0	1	2	2	2	3	4	5
3.0	1	1	1	1	2	2	3

If we must estimate σ from our sample and use Student's t , then we should add 2 to the tabulated values to obtain the approximate required sample size. (If we are comparing two produce averages, add 1 to the tabulated values).

Table 1

Year	1900	1905	1910	1915	1920	1925	1930
1900	100	100	100	100	100	100	100
1905	100	100	100	100	100	100	100
1910	100	100	100	100	100	100	100
1915	100	100	100	100	100	100	100
1920	100	100	100	100	100	100	100
1925	100	100	100	100	100	100	100
1930	100	100	100	100	100	100	100
1935	100	100	100	100	100	100	100
1940	100	100	100	100	100	100	100
1945	100	100	100	100	100	100	100
1950	100	100	100	100	100	100	100
1955	100	100	100	100	100	100	100
1960	100	100	100	100	100	100	100
1965	100	100	100	100	100	100	100
1970	100	100	100	100	100	100	100
1975	100	100	100	100	100	100	100
1980	100	100	100	100	100	100	100
1985	100	100	100	100	100	100	100
1990	100	100	100	100	100	100	100
1995	100	100	100	100	100	100	100
2000	100	100	100	100	100	100	100

The following table shows the results of the investigation into the effect of the various factors on the rate of the disease. The results are given in the form of percentages. The first column shows the year, the second column shows the rate of the disease, the third column shows the rate of the disease in the absence of the various factors, and the fourth column shows the rate of the disease in the presence of the various factors. The results show that the rate of the disease is generally higher in the presence of the various factors than in the absence of the various factors.

TABLE XVIIIb

Table of Required Sample Sizes

Sample size required for detecting with probability $1-\alpha$ whether

- a) the average m of a new product exceeds that of a standard m_0
- b) the average m of a new product is less than that of a standard m_0
- c) the average of a specified product m_A exceeds the average of another specified product m_B .

The standard deviations are assumed to be known

$$a) \quad d = \frac{m - m_0}{\sigma}$$

$$b) \quad d = \frac{m_0 - m}{\sigma}$$

$$c) \quad d = \frac{m_A - m_B}{\sqrt{\sigma_A^2 + \sigma_B^2}}$$

$$\alpha = .01$$

$d \backslash 1-\beta$.50	.60	.70	.80	.90	.95	.99
.1	542	666	813	1004	1302	1578	2165
.2	136	167	204	251	326	395	542
.4	34	42	51	63	82	99	136
.6	16	19	23	28	37	44	61
.8	9	11	13	16	21	25	34
1.0	6	7	9	11	14	16	22
1.2	4	5	6	7	10	11	16
1.4	3	4	5	6	7	9	12
1.6	3	3	4	4	6	7	9
1.8	2	3	3	4	5	5	7
2.0	2	2	3	3	4	4	6
3.0	1	1	1	2	2	2	3

If we must estimate σ from our sample, and use Student's t , then we should add 3 to the tabulated values to obtain the approximate required sample size. (If we are comparing two product averages, add 2 to the tabulated values. For this case, we must have $\sigma_A = \sigma_B$).

Table XVIIIb (Continued)

$$\alpha = .05$$

$d \backslash 1-\beta$.50	.60	.70	.80	.90	.95	.99
.1	271	361	471	619	857	1083	1578
.2	68	91	118	155	215	271	395
.4	17	23	30	39	54	68	99
.6	8	11	14	18	24	31	44
.8	5	6	8	10	14	17	25
1.0	3	4	5	7	9	11	16
1.2	2	3	4	5	6	8	11
1.4	2	2	3	4	5	6	9
1.6	2	2	2	3	4	5	7
1.8	1	2	2	2	3	4	5
2.0	1	1	2	2	3	3	4
3.0	1	1	1	1	1	2	2

If we must estimate σ from our sample, and use Student's t , then we should add 2 to the tabulated values to obtain the approximate required sample size. (If we are comparing two product averages, add 1 to the tabulated values. For this case, we must have $\sigma_A = \sigma_B$).

Table 1

Year	1900	1901	1902	1903	1904	1905	1906
1	100	100	100	100	100	100	100
2	100	100	100	100	100	100	100
3	100	100	100	100	100	100	100
4	100	100	100	100	100	100	100
5	100	100	100	100	100	100	100
6	100	100	100	100	100	100	100
7	100	100	100	100	100	100	100
8	100	100	100	100	100	100	100
9	100	100	100	100	100	100	100
10	100	100	100	100	100	100	100
11	100	100	100	100	100	100	100
12	100	100	100	100	100	100	100
13	100	100	100	100	100	100	100
14	100	100	100	100	100	100	100
15	100	100	100	100	100	100	100
16	100	100	100	100	100	100	100
17	100	100	100	100	100	100	100
18	100	100	100	100	100	100	100
19	100	100	100	100	100	100	100
20	100	100	100	100	100	100	100
21	100	100	100	100	100	100	100
22	100	100	100	100	100	100	100
23	100	100	100	100	100	100	100
24	100	100	100	100	100	100	100
25	100	100	100	100	100	100	100
26	100	100	100	100	100	100	100
27	100	100	100	100	100	100	100
28	100	100	100	100	100	100	100
29	100	100	100	100	100	100	100
30	100	100	100	100	100	100	100
31	100	100	100	100	100	100	100
32	100	100	100	100	100	100	100
33	100	100	100	100	100	100	100
34	100	100	100	100	100	100	100
35	100	100	100	100	100	100	100
36	100	100	100	100	100	100	100
37	100	100	100	100	100	100	100
38	100	100	100	100	100	100	100
39	100	100	100	100	100	100	100
40	100	100	100	100	100	100	100
41	100	100	100	100	100	100	100
42	100	100	100	100	100	100	100
43	100	100	100	100	100	100	100
44	100	100	100	100	100	100	100
45	100	100	100	100	100	100	100
46	100	100	100	100	100	100	100
47	100	100	100	100	100	100	100
48	100	100	100	100	100	100	100
49	100	100	100	100	100	100	100
50	100	100	100	100	100	100	100
51	100	100	100	100	100	100	100
52	100	100	100	100	100	100	100
53	100	100	100	100	100	100	100
54	100	100	100	100	100	100	100
55	100	100	100	100	100	100	100
56	100	100	100	100	100	100	100
57	100	100	100	100	100	100	100
58	100	100	100	100	100	100	100
59	100	100	100	100	100	100	100
60	100	100	100	100	100	100	100
61	100	100	100	100	100	100	100
62	100	100	100	100	100	100	100
63	100	100	100	100	100	100	100
64	100	100	100	100	100	100	100
65	100	100	100	100	100	100	100
66	100	100	100	100	100	100	100
67	100	100	100	100	100	100	100
68	100	100	100	100	100	100	100
69	100	100	100	100	100	100	100
70	100	100	100	100	100	100	100
71	100	100	100	100	100	100	100
72	100	100	100	100	100	100	100
73	100	100	100	100	100	100	100
74	100	100	100	100	100	100	100
75	100	100	100	100	100	100	100
76	100	100	100	100	100	100	100
77	100	100	100	100	100	100	100
78	100	100	100	100	100	100	100
79	100	100	100	100	100	100	100
80	100	100	100	100	100	100	100
81	100	100	100	100	100	100	100
82	100	100	100	100	100	100	100
83	100	100	100	100	100	100	100
84	100	100	100	100	100	100	100
85	100	100	100	100	100	100	100
86	100	100	100	100	100	100	100
87	100	100	100	100	100	100	100
88	100	100	100	100	100	100	100
89	100	100	100	100	100	100	100
90	100	100	100	100	100	100	100
91	100	100	100	100	100	100	100
92	100	100	100	100	100	100	100
93	100	100	100	100	100	100	100
94	100	100	100	100	100	100	100
95	100	100	100	100	100	100	100
96	100	100	100	100	100	100	100
97	100	100	100	100	100	100	100
98	100	100	100	100	100	100	100
99	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100

The following table shows the results of the experiments conducted during the year 1900-1901. The experiments were conducted in the laboratory of the Department of Agriculture, and the results are given in the following table. The table shows the results of the experiments conducted during the year 1900-1901, and the results are given in the following table. The table shows the results of the experiments conducted during the year 1900-1901, and the results are given in the following table.

TABLE XIX

Percentiles for $\varphi = \frac{\bar{X} - m_0}{w}$

n	$\varphi .95$	$\varphi .975$	$\varphi .99$	$\varphi .995$	$\varphi .999$	$\varphi .9995$
2 (1) 20						

Reproduced from Table A-8c(1), Dixon and Massey,
Second Edition, McGraw-Hill (1957).

TABLE XX

Percentiles for $\varphi' = \frac{\bar{X}_A - \bar{X}_B}{1/2(w_A + w_B)}$

$n=n_A=n_B$	$\varphi'.95$	$\varphi'.975$	$\varphi'.99$	$\varphi'.995$	$\varphi'.999$	$\varphi'.9995$
2						
3						
⋮						
20						

Reproduced from Table A-8c(2), Dixon and Massey, Second Edition, McGraw-Hill (1957).

1914

THE [illegible] OF THE [illegible] IN THE [illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

THE [illegible] OF THE [illegible] IN THE [illegible]

THE [illegible] OF THE [illegible] IN THE [illegible]

Critical values of L for the Link-Wallace Test

 $\alpha = .05$

t = number of groups = number of ranges

n = number in group = number per range

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	3.43	2.37	1.78	1.40	1.16	1.00	.87	.78	.70	.66	.63	.58	.50	.47	.44	.42	.40	.38	.36
3	1.91	1.44	1.13	.94	.80	.70	.62	.56	.51	.47	.43	.40	.38	.36	.33	.32	.30	.29	.27
4	1.63	1.25	1.01	.84	.72	.63	.57	.51	.47	.43	.40	.37	.35	.33	.31	.29	.28	.27	.25
5	1.53	1.19	.96	.81	.70	.61	.55	.50	.45	.42	.39	.36	.34	.32	.30	.29	.27	.26	.25
6	1.50	1.18	.95	.80	.69	.61	.55	.49	.45	.42	.39	.36	.34	.32	.30	.29	.27	.26	.25
7	1.49	1.17	.95	.80	.69	.61	.55	.50	.45	.42	.39	.36	.34	.32	.30	.29	.28	.26	.25
8	1.49	1.17	.96	.81	.70	.62	.55	.50	.46	.42	.39	.37	.35	.33	.31	.29	.28	.27	.25
9	1.50	1.18	.97	.82	.71	.62	.56	.51	.47	.43	.40	.37	.35	.33	.31	.30	.28	.27	.26
10	1.52	1.20	.98	.83	.72	.63	.57	.52	.47	.44	.41	.38	.35	.34	.32	.30	.29	.27	.26
11	1.54	1.21	.99	.84	.73	.64	.58	.52	.48	.44	.41	.38	.36	.34	.32	.31	.29	.28	.27
12	1.56	1.23	1.00	.85	.74	.65	.59	.53	.49	.45	.42	.39	.37	.35	.33	.31	.30	.28	.27
13	1.58	1.25	1.02	.86	.75	.66	.59	.54	.49	.46	.42	.40	.37	.35	.33	.32	.30	.29	.27
14	1.60	1.26	1.03	.87	.76	.67	.60	.55	.50	.46	.43	.40	.38	.36	.34	.32	.31	.29	.28
15	1.62	1.28	1.05	.89	.77	.68	.61	.56	.51	.47	.44	.41	.38	.36	.34	.33	.31	.30	.28
16	1.64	1.30	1.06	.90	.78	.69	.62	.56	.52	.48	.44	.41	.39	.37	.35	.33	.31	.30	.29
17	1.66	1.31	1.08	.91	.79	.70	.63	.57	.52	.48	.45	.42	.39	.37	.35	.33	.32	.30	.29
18	1.68	1.33	1.09	.92	.80	.71	.64	.58	.53	.49	.46	.43	.40	.38	.36	.34	.32	.31	.30
19	1.70	1.34	1.10	.93	.81	.72	.65	.59	.54	.50	.46	.43	.40	.38	.36	.34	.33	.31	.30
20	1.72	1.36	1.11	.95	.82	.73	.65	.59	.54	.50	.47	.44	.41	.39	.37	.35	.33	.32	.30

2 3 4 5 6 7 8 9 10 11 12	1
13 14 15 16 17 18 19 20	2
21 22 23 24 25 26 27 28	3
29 30 31 32 33 34 35 36	4
37 38 39 40 41 42 43 44	5
45 46 47 48 49 50 51 52	6
53 54 55 56 57 58 59 60	7
61 62 63 64 65 66 67 68	8
69 70 71 72 73 74 75 76	9
77 78 79 80 81 82 83 84	10
85 86 87 88 89 90 91 92	11
93 94 95 96 97 98 99 100	12
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109 110 111 112 113 114 115 116	14
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141 142 143 144 145 146 147 148	18
149 150 151 152 153 154 155 156	19
157 158 159 160 161 162 163 164	20
165 166 167 168 169 170 171 172	21
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181 182 183 184 185 186 187 188	23
189 190 191 192 193 194 195 196	24
197 198 199 200 201 202 203 204	25
205 206 207 208 209 210 211 212	26
213 214 215 216 217 218 219 220	27
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309 310 311 312 313 314 315 316	39
317 318 319 320 321 322 323 324	40
325 326 327 328 329 330 331 332	41
333 334 335 336 337 338 339 340	42
341 342 343 344 345 346 347 348	43
349 350 351 352 353 354 355 356	44
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381 382 383 384 385 386 387 388	48
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397 398 399 400 401 402 403 404	50
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453 454 455 456 457 458 459 460	57
461 462 463 464 465 466 467 468	58
469 470 471 472 473 474 475 476	59
477 478 479 480 481 482 483 484	60
485 486 487 488 489 490 491 492	61
493 494 495 496 497 498 499 500	62
501 502 503 504 505 506 507 508	63
509 510 511 512 513 514 515 516	64
517 518 519 520 521 522 523 524	65
525 526 527 528 529 530 531 532	66
533 534 535 536 537 538 539 540	67
541 542 543 544 545 546 547 548	68
549 550 551 552 553 554 555 556	69
557 558 559 560 561 562 563 564	70
565 566 567 568 569 570 571 572	71
573 574 575 576 577 578 579 580	72
581 582 583 584 585 586 587 588	73
589 590 591 592 593 594 595 596	74
597 598 599 600 601 602 603 604	75
605 606 607 608 609 610 611 612	76
613 614 615 616 617 618 619 620	77
621 622 623 624 625 626 627 628	78
629 630 631 632 633 634 635 636	79
637 638 639 640 641 642 643 644	80
645 646 647 648 649 650 651 652	81
653 654 655 656 657 658 659 660	82
661 662 663 664 665 666 667 668	83
669 670 671 672 673 674 675 676	84
677 678 679 680 681 682 683 684	85
685 686 687 688 689 690 691 692	86
693 694 695 696 697 698 699 700	87
701 702 703 704 705 706 707 708	88
709 710 711 712 713 714 715 716	89
717 718 719 720 721 722 723 724	90
725 726 727 728 729 730 731 732	91
733 734 735 736 737 738 739 740	92
741 742 743 744 745 746 747 748	93
749 750 751 752 753 754 755 756	94
757 758 759 760 761 762 763 764	95
765 766 767 768 769 770 771 772	96
773 774 775 776 777 778 779 780	97
781 782 783 784 785 786 787 788	98
789 790 791 792 793 794 795 796	99
797 798 799 800 801 802 803 804	100

Table XXI (Continued)

 $\alpha = .01$

t = number of groups = number of ranges

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	7.92	4.42	2.96	2.06	1.69	1.39	1.20	1.03	.91	.82	.75	.68	.63	.59	.55	.51	.48	.46	.43
3	3.14	2.14	1.57	1.25	1.04	.89	.78	.69	.62	.57	.52	.48	.45	.42	.39	.37	.35	.34	.32
4	2.47	1.74	1.33	1.08	.91	.78	.69	.62	.56	.51	.47	.44	.41	.38	.36	.34	.32	.31	.29
5	2.24	1.60	1.24	1.02	.86	.75	.66	.59	.54	.49	.46	.42	.40	.37	.35	.33	.31	.30	.29
6	2.14	1.55	1.21	.99	.85	.74	.65	.59	.53	.49	.45	.42	.39	.37	.35	.33	.31	.30	.28
7	2.10	1.53	1.21	.99	.84	.74	.65	.59	.53	.49	.45	.42	.40	.37	.35	.33	.32	.30	.29
8	2.08	1.52	1.21	.99	.85	.74	.66	.59	.54	.50	.46	.43	.40	.37	.35	.33	.32	.30	.29
9	2.09	1.53	1.22	1.00	.85	.75	.66	.60	.54	.50	.46	.43	.40	.38	.36	.34	.32	.31	.29
10	2.10	1.55	1.23	1.01	.86	.75	.67	.61	.55	.51	.47	.44	.41	.38	.36	.34	.33	.31	.30
11	2.11	1.56	1.24	1.02	.88	.77	.68	.61	.56	.51	.48	.44	.42	.39	.37	.35	.33	.32	.30
12	2.13	1.58	1.25	1.03	.89	.78	.69	.62	.57	.52	.48	.45	.42	.40	.37	.35	.34	.32	.31
13	2.15	1.60	1.27	1.05	.90	.79	.70	.63	.58	.53	.49	.46	.43	.40	.38	.36	.34	.33	.31
14	2.18	1.62	1.28	1.06	.91	.80	.71	.64	.58	.54	.50	.46	.43	.41	.39	.37	.35	.33	.32
15	2.20	1.64	1.30	1.08	.92	.81	.72	.65	.59	.54	.50	.47	.44	.41	.39	.37	.35	.34	.32
16	2.22	1.65	1.31	1.09	.93	.82	.73	.66	.60	.55	.51	.48	.45	.42	.40	.38	.36	.34	.32
17	2.24	1.67	1.33	1.11	.95	.83	.74	.67	.61	.56	.52	.48	.45	.43	.40	.38	.36	.34	.33
18	2.27	1.69	1.34	1.12	.96	.84	.75	.68	.62	.57	.53	.49	.46	.43	.41	.39	.37	.35	.33
19	2.30	1.71	1.36	1.14	.97	.85	.76	.68	.62	.57	.53	.50	.46	.44	.41	.39	.37	.35	.34
20	2.32	1.73	1.38	1.15	.98	.86	.77	.69	.63	.58	.54	.50	.47	.44	.42	.40	.38	.36	.34

n = number in group = number per range

1875-1876

1876-1877

1877-1878

1878-1879

1879-1880

1880-1881

1881-1882

1882-1883

1883-1884

1884-1885

1885-1886

1886-1887

1887-1888

1888-1889

1889-1890

1890-1891

1891-1892

1892-1893

1893-1894

1894-1895

1895-1896

1896-1897

TABLE XXII

Percentiles of $F' = \frac{w_A}{w_B}$

n_B		n_A								
		2	3	4	5	6	7	8	9	10
2	.005	.0078								
	.01	.0157								
	.025	.039								
	.05	.079								
3	.005									
	.01									
	.025									
	.05									
4	"									
5	"									
6	"									
7	"									
8	"									
9	"									
10	"									

Taken from Table A-8d, Dixon and Massey, Second Edition, (1957).

Tables for Computing Confidence Limits for σ

Degrees of Freedom ν	A .05	A .95	A .025	A .975	A .01	A .99	A .005	A .995
1	.5103	15.947	.4461	31.910	.3882	79.786	.3562	159.576
2	.5778	4.415	.5207	6.285	.4660	9.975	.4344	14.124
3	.6196	2.920	.5665	3.729	.5142	5.111	.4834	6.467
4	.6493	2.372	.5992	2.874	.5489	3.669	.5188	4.396
5	.6721	2.089	.6242	2.453	.5757	3.003	.5464	3.485
6	.6903	1.915	.6444	2.202	.5974	2.623	.5688	2.980
7	.7054	1.797	.6612	2.035	.6155	2.377	.5875	2.660
8	.7183	1.711	.6754	1.916	.6310	2.204	.6037	2.439
9	.7293	1.645	.6878	1.826	.6445	2.076	.6177	2.278
10	.7391	1.593	.6987	1.755	.6564	1.977	.6301	2.154
11	.7477	1.551	.7084	1.698	.6670	1.898	.6412	2.056
12	.7554	1.515	.7171	1.651	.6765	1.833	.6512	1.976
13	.7624	1.485	.7250	1.611	.6852	1.779	.6603	1.909
14	.7688	1.460	.7321	1.577	.6931	1.733	.6686	1.854
15	.7747	1.437	.7387	1.548	.7004	1.694	.6762	1.806
20	.7979	1.358	.7650	1.444	.7297	1.556	.7071	1.640
25	.8149	1.308	.7843	1.380	.7511	1.473	.7299	1.542
30	.8279	1.274	.7991	1.337	.7678	1.416	.7477	1.475
40	.8470	1.228	.8210	1.279	.7925	1.343	.7740	1.390
50	.8606	1.199	.8367	1.243	.8103	1.297	.7931	1.337
60	.8710	1.179	.8487	1.217	.8239	1.265	.8078	1.299
70	.8793	1.163	.8583	1.198	.8349	1.241	.8196	1.272
80	.8861	1.151	.8662	1.183	.8439	1.222	.8293	1.250
90	.8919	1.141	.8728	1.171	.8515	1.207	.8376	1.233
100	.8968	1.133	.8785	1.161	.8581	1.195	.8446	1.219

For large degrees of freedom, the following approximate formula may be used

$$A_p = \sqrt{2\nu/(z_p + \sqrt{2\nu-1})^2}$$

U. S. DEPARTMENT OF COMMERCE

Sinclair Weeks, *Secretary*

NATIONAL BUREAU OF STANDARDS

A. V. Astin, *Director*



THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major field laboratories in Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside front cover of this report.

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Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat and Power. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology and Lubrication. Engine Fuels.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Nuclear Physics. Radioactivity. X-rays. Betatron. Nucleonic Instrumentation. Radiological Equipment. AEC Radiation Instruments.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Gas Chemistry. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

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