NATIONAL BUREAU OF STANDARDS REPORT

5297

PROJECTS and PUBLICATIONS of the APPLIED MATHEMATICS DIVISION

A Quarterly Report

January through March 1957

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the back cover.

Reports and Publications

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Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($0.75), available from the Superintendent of Documents, Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.

NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT	NBS REPORT
11.0	5297

PROJECTS and PUBLICATIONS of the APPLIED MATHEMATICS DIVISION

January through March 1957

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

January 1 through March 31, 1957

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under sponsorship of the National Science Foundation

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*On leave of absence.

CONTENTS

*Only unclassified projects are included in this report.

Status of Projects

March 31, 1957

1. NUMERICAL ANALYSIS

RESEARCH IN THEORIES OF DISCRETE STRUCTURES

Task 1100-11-5170/56-159

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: O. Taussky-Todd
Full task description: July-Sept 1955 issue, p. 1

Status: CONTINUED. O. Taussky-Todd studied a determinantal inequality obtained by H. P. Robertson in connection with applications to physics. The inequality concerns the real and complex parts of a positive definite Hermitian matrix. Several further inequalities were obtained, one by application of a theorem by H. Weyl.

O. Taussky-Todd continued work on a coding problem concerning the three-class group in a complex quadratic field.

O. Taussky-Todd is preparing a manuscript containing a report of the lectures she gave in the Training Program.

H. Cohn (now at Washington University, St. Louis) has completed the paper describing the work he carried out on SEAC in the summer of 1956.

Publications:

(1) On the Hilbert matrix. T. Kato. Proc. Amer. Math. Soc. 8, 73-81 (1957).
(2) Classes of positive definite unimodular circulants. M. Newman and O. Taussky. Canadian J. Math. 9, 71-73 (1957).
(3) Incidence algebras. E. C. Dade and K. Goldberg. In manuscript.
(4) Commutativity in finite matrices. O. Taussky. To appear in the American Mathematical Monthly.
(5) Classes of matrices. O. Taussky. To appear in the Illinois Journal of Mathematics.
(6) The number of representations of a quadratic form as a sum of four squares. G. Pall (Illinois Institute of Technology) and O. Taussky. To appear in the Proceedings of the Royal Irish Academy.
(7) Commuting bilinear transformations and matrices. O. Taussky and J. Todd. J. Washington Acad. Sci. 46, 373-375 (1956).
(8) Pairs of matrices of order two which generate free groups. K. Goldberg

and M. Newman. To appear in the Illinois Journal of Mathematics.
(9) A method for computing eigenvectors. K. Goldberg. In manuscript.
(10) A computation of cyclic cubic units. H. Cohn and S. Gorn (Moore School of Engineering). To appear in the Journal of Research, NBS.
(11) A numerical study of Dedekind's cubic class number formula. H. Cohn. To appear in the Journal of Research, NBS.
(12) Some computational problems concerning integral matrices. O. Taussky. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-10-1104/55-55

Origin: NBS — Authorized 8/13/54
Managers: J. Todd, P. Davis — Revised 8/29/54
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. H. A. Antosiewicz continued his study of the reducibility of systems $\dot{x} = [C + B(t)]x$ to $\dot{y} = Cy$ by use of differentiable matrices $Z(t)$ that are bounded together with $Z^{-1}(t)$ on $[0,\infty)$. One of the sufficient conditions for reducibility he obtained may be formulated thus: If the eigenvalues of C have distinct real parts and the elements $b_{ij}(t)$ of $B(t)$ satisfy $b_{ii}(t) \in L(0,\infty)$, $b_{ij}(t) \in L^p(0,\infty)$, $i \neq j$, where $1 \leq p \leq 2$, then there exists a $Z(t)$ such that $Z(t) \to I$ as $t \to \infty$ and $y = Z^{-1}(t)x$ satisfies $\dot{y} = Cy$.

G. Weiss is investigating the runs properties of pseudo-random numbers and a special form of restricted random walk. A manuscript describing the results is in preparation.

M. Marcus has completed the following manuscripts: (1) Field convexity of a linear transformation (with A. J. Goldman). The field F(A) of a matrix A is the set of complex numbers (Ax,x) where x is a unit vector. It is a classical theorem of Hausdorff that F(A) is convex. This is now proved by an elementary induction on the size of A which reduces the essential computation to that for a 2 x 2 matrix. This work was suggested by lectures of O. Taussky-Todd in the Training Program. (2) On an inequality of H. P. Robertson, II. The result here is the following: Let $1 \leq k \leq n$ and assume A is positive definite Hermitian. Then

(a) $$\prod_{j=1}^{k} \lambda_j(A+\bar{A}) \geq 2^k \prod_{j=1}^{k} \lambda_j(A);$$

(b) If $A - \bar{A}$ is non-singular and $k = 2s$, then

$$\prod_{j=1}^{k} \lambda_{n-j+1}^{2/k}(A+\bar{A}) \geq \prod_{j=1}^{s} \lambda_{n-j+1}^{4/k}(A-\bar{A}) + 4 \prod_{j=1}^{k} \lambda_j^{2/k}(A);$$

(c) if $A - \bar{A}$ is non-singular and $k = 2s + 1$, then

$$\prod_{j=1}^{k} \lambda_{n-j+1}^{2/k}(A+\bar{A}) \geq \lambda_{n-s}^{2/k}(A-\bar{A}) \prod_{j=1}^{k} \lambda_{n-j+1}^{4/k}(A-\bar{A}) + 4 \prod_{j=1}^{k} \lambda_j^{2/k}(A).$$

Here $\lambda_j(X) \leq \lambda_{j+1}(X)$ when X is a matrix with real eigenvalues. This is an extension of some of the results recently obtained by O. Taussky-Todd (see task 1100-11-5170/56-159, p. 1).

At present M. Marcus is working on problems related to those considered by O. Taussky-Todd and T. Motzkin on commutativity of finite linear transformations. In particular, let $\alpha A + A^*$ have eigenvalues $S(\alpha\lambda + \lambda)$ where $\lambda = (\lambda_1, \ldots, \lambda_n)$ are the eigenvalues of A and S is a real orthogonal matrix. Then the question considered is: When is A normal?

A paper "On a converse of a theorem of Pringshein," by P. Davis was completed.

Publications:

(1) Complete sequences and approximations in normed linear spaces. P. Davis and K. Fan. To appear in the Duke Journal of Mathematics.
(2) Note on bounds for certain determinants. E. V. Haynsworth. To appear in the Duke Journal of Mathematics.
(3) A survey of Lyapunov's second method. H. A. Antosiewicz. To appear in Annals of Mathematics Studies.
(4) Convex functions of quadratic forms. M. Marcus. To appear in the Duke Journal of Mathematics.
(5) Some inequalities for quadratic forms and eigenvalues. M. Marcus. Coalesced with (4).
(6) On subdeterminants of doubly stochastic matrices. M. Marcus. To appear in the Illinois Journal of Mathematics.
(7) A note on symmetric functions of eigenvalues. M. Marcus and R. Thompson (University of British Columbia). Duke Math. J. 24, 43-46 (1957).
(8) On a determinantal inequality. M. Marcus. Submitted to a technical journal.
(9) Some extreme value results for indefinite Hermitian matrices. M. Marcus, B. N. Moyles and R. Westwick (University of British Columbia). To appear in the Illinois Journal of Mathematics.
(10) Extreme value properties of Hermitian matrices. M. Marcus. To appear in Journal of London Mathematical Society.
(11) Computational problems concerned with the Hilbert matrix. J. Todd. To appear in the Proceedings of the 1956 meeting of the Italian Society for the Advancement of Science, held in Sicily.
(12) Field convexity of a linear transformation. M. Marcus and A. J. Goldman. In manuscript.

(13) A determinantal inequality of H. P. Robertson, II. M. Marcus. Submitted to a technical journal.
(14) Numerical computation of the transfinite diameter of two collinear segments. P. Davis. J. Research, NBS. 58, 155-156 (1957).

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-12-5116/55-56

Origin: NBS Authorized 8/13/54
Sponsor: Office of Naval Research
Managers: J. Todd, M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. M. Newman has completed a manuscript entitled "Congruences for the coefficients of modular forms and some new congruences for the partition function." It is shown, for example, that for $n \equiv 6 \pmod{13}$, $p(13^2n-7) \equiv 6p(n) \pmod{13}$. This leads to a variety of congruences for $p(n)$ modulo 13, of which the simplest is $p(155\,\Delta_n + 84) \equiv 0 \pmod{13}$, $\Delta_n \equiv (13/24)(13^{2n}-1)$.

T. Kato has completed a manuscript on an elementary proof for a theorem estimating eigenvalues of linear transformations of the form T*T, which was used elsewhere in calculating the fundamental frequency of a square plate with free edges. This work was started when Dr. Kato was a guest worker at the Bureau.

Publications:
(1) On dominant eigenvalues of positive matrices. T. Kato. In manuscript.
(2) On the minimization of concave and convex functionals. G. B. Dantzig. (RAND Corporation), A. J. Hoffman, W. Hirsch (New York University). In manuscript.
(3) Estimation of the frequencies of thin elastic plates with free edges. H. Fujita (University of Tokyo), T. Kato, Y. Nakata (University of Tokyo), and M. Newman. To appear in the Journal of Research, NBS.
(4) An inclusion theorem for modular groups. M. Newman. Amer. Math. Soc. 8, 125-127 (1957).
(5) Systems of inequalities involving convex functions. Ky Fan (Notre Dame University), I. Glicksberg (RAND Corporation) and A. J. Hoffman. To appear in the Proceedings of the American Mathematical Society.
(6) Some theorems on $P_r(n)$. M. Newman. Canadian J. Math. 9, 68-70 (1957).
(7) Construction and application of a class of modular functions. M. Newman. To appear in the Proceedings of the London Mathematical Society.
(8) Congruences for the coefficients of modular forms and some new congruences for the partition function. M. Newman. Submitted to a technical journal.

(9) On a theorem estimating eigenvalues. T. Kato. In manuscript.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-12-5116/56-148

Origin and Sponsor: National Institutes of Health, Bethesda, Md. Authorized 9/30/55
Manager: H. A. Antosiewicz
Full task description: July-Sept 1955 issue, p. 7

Status: CONTINUED. A number of cases have been computed as a third approximation between the previously obtained two best approximations to the desired solution with prescribed behavior in the large. Certain difficulties that arose due to indeterminancies in the differential equations have been minimized by a change in the computing procedure.

NUMERICAL EXPERIMENTS ON POTENTIAL THEORY USING THE NEHARI ESTIMATES
Task 1101-12-5116/56-189

Origin: NBS Authorized 7/10/56
Sponsor: Air Research and Development Command, USAF
Managers: P. Davis, U. Hochstrasser
Full task description: Apr-June 1956 issue, p. 6

Status: COMPLETED. A final report was distributed.

Publication:
(1) Numerical experiments on potential theory using the Nehari estimates. U. Hochstrasser. Submitted to a technical journal.

TRAINING PROGRAM IN NUMERICAL ANALYSIS
Task 1101-40-5114/57-237

Origin and Sponsor: National Science Foundation Authorized 3/27/57
Manager: J. Todd

Objective: To provide an intensive training program in numerical analysis to university teachers who are experts in related mathematical fields and who aspire to teach and conduct research in numerical analysis

and to direct university computer centers.

Background: The present national shortage of mathematicians competent in programming for high speed automatic digital computer machines and in carrying out the required numerical analysis has motivated the presentation of a concentrated course in the subject. Few universities have staffs with sufficient experience in the field to qualify them to give appropriate instruction and direction of research. Also, the high speed computing equipment presently being acquired by many institutions may be misused unless trained staffs are available. The present program is aimed at helping to fill these needs.

Status: NEW. The first two weeks of the program are devoted to an introduction to numerical analysis and to programming for automatic computation. Although the training program is not a coding course, all participants have prepared and run simple test problems on SEAC. The basis of the formal teaching, which was given by M. Newman and J. Todd, was a discussion of a complex of problems which illustrated various topics in numerical analysis and programming. This set of problems could readily be adapted for solution on any type of digital computer.

The second phase of the program will be devoted to surveys of particular chapters in numerical analysis. It will continue for about thirteen weeks. To date the following topics have been covered:

Classical Numerical analysis	J. Todd
Linear Equations and Matrix Inversion.	U. Hochstrasser M. Newman J. Todd
Quadrature and Ordinary Differential Equations.	H. A. Antosiewicz M. Abramowitz
Characteristic Values of Matrices.	J.W. Givens (Wayne State University), C. Donald LaBudde (New York University, Inst. of Math. Sci.)
Partial Differential Equations	P. Davis

In addition to the formal lectures, in which the practical aspects as well as theoretical ones are emphasized, there were demonstrations of current SEAC techniques by W. F. Cahill, I. A. Stegun, and W. Gautschi, and also by the instructors.

Each participant has chosen a significant computation problem and is in process of preparing a code for it. A list of the participants, their institution and a short title of their chosen problem is given below:

P. A. Clement, State College of Washington, Bessel functions $K_{in}(x)$ of pure imaginary index

R. F. Gabriel, Rutgers University, Conformal mapping of a triangle

H. C. Griffith, Florida State University, Table of Jacobian elliptic functions

D. C. Lewis, Johns Hopkins University, Unicursal curves and maps

Mary Lister, Pennsylvania State College, Elliptic partial differential equations with curved boundaries

F. R. Olson, University of Buffalo, Bernoulli polynomials

O. W. Rechard, Washington State College (part-time participant)

R. F. Reeves, Ohio State University (part-time participant)

F. Scheid, Boston University, Monte Carlo problems and Coulomb wave functions

C. L. Seebeck, Jr., University of Alabama, Compound interest tables

C. Spector, Ohio State University, Compiler

V. J. Varineau, University of Wyoming, Eigenvalues of Toeplitz matrices

In addition to the above, H. W. Wicke, a guest worker from Sandia Corporation, is attending the course and is making a table of "semi-conductor" integrals.

A series of lectures on supporting mathematics has been arranged. So far, the following have participated:

F. Oberhettinger, Asymptotics, especially of integral transforms.

O. Todd, M. Marcus, and A. Brauer (University of North Carolina), Bounds for characteristic values of matrices.

H. F. Weinberger (University of Maryland), Functional analysis.

Advantage has been taken of the presence in Washington of various experts in numerical analysis and some have addressed the group. In particular, there was one lecture from A. S. Householder (ORNL and Army Mathematics Center) and one from J. Barkley Rosser (Cornell), and two from E. Stiefel (Zurich).

No attempt has been made to hide the present unsatisfactory state of numerical analysis, and the discrepancy between results which are mathematically secure and those which are currently produced has been emphasized. In this connection, various research problems have been uncovered and some seem amenable to solution. Progress on these will be reported in detail in future reports.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF $E_1(z)$, $(z=x+iy)$

Task 1102-40-1110/43-3

Manager: I. Stegun — Authorized 7/1/47
Full task description: Apr-June 1949 issue, p. 41

Status: CONTINUED. Checking of the final manuscript continued.

TABLES OF COULOMB WAVE FUNCTIONS

Task 1102-40-1110/47-2

Origin: NBS — Authorized 7/1/47
Manager: M. Abramowitz
Full task description: Apr-June 1949 issue, p. 45

Status: CONTINUED. Computation of F_o, F'_o, G_o and G'_o for $\eta = 0(.5)25$, $\rho = 0(.5)40$ has been completed. Checking of the computations is in progress.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS

Task 1102-40-1110/51-8

Origin: Section 11.3, NBS — Authorized 3/26/51
Manager: S. Peavy
Full task description: Apr-June 1951 issue, p. 49

Status: INACTIVE. For status to date, see Oct-Dec 1955 issue, p. 8.

REVISION OF MATHEMATICAL TABLES
Task 1102-40-1110/52-7

Origin: NBS Authorized 8/10/51
Managers: W. F. Cahill, I. Stegun
Full task description: July-Sept 1951 issue, p. 41

Status: CONTINUED. Revision of "Tables of Natural Logarithms," vol. IV, Mathematical Table MT12 (1941) has been submitted for reissue in the Applied Mathematics Series.

TABLE OF THE MODIFIED AIRY INTEGRAL
Task 1102-40-1110/52-23

Origin: NBS Authorized 10/4/51
Manager: I. Stegun
Full task description: July-Sept 1951 issue, p. 42

Status: CONTINUED. The final manuscript is receiving a "last" review prior to submittal for publication.

TABLE OF ERROR FUNCTION FOR COMPLEX ARGUMENTS
Task 1102-40-1110/52-25

Origin: NBS Authorized 10/5/51
Manager: W. Hall
Full task description: July-Sept 1951 issue, p. 42

Status: TERMINATED. The preparation of the table has been discontinued because a more comprehensive table of the function was recently published elsewhere.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-40-1110/52-37

Origin: NBS Authorized 11/28/51
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE

SIEVERT'S INTEGRAL
Task 1102-40-1110/52-57

Origin: NBS Authorized 2/12/52
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46

Status: CONTINUED. Checking of the tabular values and preparation of the introductory material continued.

L-SHELL CONVERSION COEFFICIENTS
Task 1102-40-1110/53-52

Origin: Oak Ridge National Laboratory Authorized 5/20/53
Manager: W. Hall
Full task description: Apr-June 1953 issue, p. 45

Status: CONTINUED. At the request of the sponsor, certain additional exploratory calculations were made and transmitted to him for evaluation.

AUTOMATIC CODING
Task 3711-60-1120/55-65

Origin: NBS Authorized 9/29/54
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. An experimental system for using the 704 computer at the NBS has been developed by J. Wegstein and J. Cooper. This system, called CORBIE, is expected to facilitate rapid and easy code checking as well as problem solution. It does not require the presence of a drum, or more than 4096 words of core memory.

Codes are to be initially read into the computer in symbolic form from cards. They are then stored on magnetic tape and thereafter are always loaded into the computer from the magnetic tape. If some lines of a code are to be changed, only the changes are read into the computer from symbolic cards. The old code is read into the computer from magnetic tape and the new modified code is written back on magnetic tape.

Code checking is to be done entirely in symbolic form, and several code checking routines are automatically available. During code checking the progress through the code is recorded by the monitor on-line printer, which prints information at various pre-determined break points.

The system has provision for the later addition of a symbolic trace or automonitoring routine. The codes of numerous programmers are stored on the same tape and each code is identified by a call number. Up to four such tapes may be on the line at the same time. If someone wishes to read a certain code into the computer he drops control cards into the card hopper,which contain the code identification number. The computer automatically finds this code on any of the tapes currently on-line and reads it into the computer. The activity of the computer is controlled by control cards, most of which originate with the programmer.

An important feature of this system is a technique of compressing the symbolic code before it is stored on magnetic tape. The bulk of the code of this system is concerned with automatically up-dating the magnetic tape files of codes and data. It also includes the SHARE assembly program (SAP) as an integral part.

The code for this system has been written and is about two-thirds checked out. The checking out has been accomplished on the IBM 704 in New York. About 2.79 hours of 704 time have been used to date on this project.

Additional seminars on programming for the IBM 704 were held.

MATHEMATICAL SUBROUTINES
Task 3711-60-0009/56-160

Origin: NBS Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

Status: CONTINUED. The code that computes the characteristic polynomial of a real matrix (using a modified version of Voetters method) has been completed. An additional code was prepared for testing the method systematically. Numerical experiments are being started.

Two subroutines for operation on symmetric matrices stored triangularly have been written for the 704. The first obtains the product AA', where A is rectangular and stores the result triangularly. The second obtains the trace of AB, where A and B are both symmetric and stored triangularly.

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-5113/57-216

Origin and Sponsor: National Science Foundation Authorized 12/27/56
Manager: M. Abramowitz
Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. Assignments for most of the chapters have been made. It is intended that each chapter shall be self-contained, having its own table of contents and page numbers. Conventions have been adopted concerning the notation to be employed, and formulas will be numbered employing the decimal system. For each function the important mathematical properties, methods of computation, bibliography and mathematical tables will be given. The methods of computation will describe interpolation techniques peculiar to the functions in the particular chapter, in addition to procedures for extending the tables. Many of the elementary functions will be given to 10D or 10S. In the case of basic functions, such as the exponential function, eighteen places or more will be given. There will be an introduction giving interpolation and computational techniques. A chapter will be included giving a comprehensive list of polynomial and rational approximations.

A table of mathematical constants has been completed by D. S. Liepman. It lists approximately 250 values to an accuracy varying from 15 to 25 significant figures.

The text for the chapter on the Gamma function and related functions has been prepared by P. Davis. The tables have been compiled and are being checked. The table of contents for the chapter is:

I. Mathematical properties
  1) The Gamma Function
  2) The Beta Function
  3) The Psi Function
  4) The Polygamma Function
  5) The Incomplete Gamma Function
  6) The Incomplete Beta Function

II. Methods of Computation
  7) Use and Extension of the Table
  8) Summation of Rational Series by Means of Polygamma Functions

III. Bibliography

IV. Tables of Functions

Table 6.1: $\Gamma(x)$, $\ln \Gamma(x)$, $\psi(x)$, $\psi'(x)$, $x = 1(.005)2$, 10D.

Table 6.2: $\psi''(x)$, $\psi^{(3)}(x)$, $x = 1(.01)2$, 10D.

Table 6.3: $\Gamma(n)$, $n = 1(1)101$, 11S; $[\Gamma(n)]^{-1}$, $n = 1(1)101$, 9S; $\ln \Gamma(n)$, $\ln \Gamma(n+1/3)$, $\ln \Gamma(n+\frac{1}{2})$, $\ln \Gamma(n+2/3)$, $n = 1(1)101$, 8S.

Table 6.4: $\psi(n)$, $n = 1(1)101$, 10D;

$$f_1(n) = \frac{\Gamma(n)}{n^{n-\frac{1}{2}} e^{-n}\sqrt{2\pi}},$$

$$f_2(n) = \ln \Gamma(n) - (n-\tfrac{1}{2}) \ln n + n,$$

$$f_3(n) = n - \psi(n),$$

$n = 1(1)101$, 8D.

Table 6.5: $f_1(n)$, $f_2(n)$, $f_3(n)$, $1/n = 0(.001).015$.

Table 6.6: $n!, n = 100(100)1000$, 20S.

Table 6.7: $\ln\Gamma(x+iy)$, $x = 1(.1)2$, $y = 0(.1)10$, 12D.

Table 6.8: $\psi(x+iy)$, $x = 1(.1)2$, $y = 0(.1).0$, 5D; $R\,\psi(1+iy)$ is given to 10D.

Table 6.9: $f_4(y) = R\psi(1+iy) - \ln y$, $1/y = 0(.01).11$, 8D.

Methods are given for obtaining these functions for all values of the argument. This is in accordance with one of the aims of the volume,--namely, to give tables or procedures for making use of simple auxiliary functions to obtain the function values over the entire range of tabular arguments.

3. PROBABILITY AND MATHEMATICAL STATISTICS

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-12-1107/49-1a

Origin: NBS Authorized 1/9/49
Manager: L. S. Deming
Full task description: Apr-June 1949 issue, p. 75

Status: CONTINUED. A card file of abstracts taken from the two main reviewing services of mathematical and statistical publications, namely, Zentralblatt für Mathematik and Mathematical Reviews, is maintained. Abstracts from Zentralblatt cover the years 1931 through 1941; those from Mathematical Reviews begin with 1939 and continue with current issues.

As the current issues of Mathematical Reviews arrive, the abstracts of papers on probability and mathematical statistics are selected, mounted on 5 x 8 cards, arranged alphabetically by author, and held in a separate file until the annual subject-index issue arrives. A code number is then written on each abstract card which identifies it by subject according to the classification followed in Mathematical Reviews. When the abstracts for each volume (i.e., all monthly issues for the year) have been classified by subject, they are added to the combined Zentralblatt and Mathematical Reviews file that covers the period from 1939 to the current reviews.

The subject-index volume for 1955 was received late in 1956. Hence the abstracts have been classified by subject through 1955 and have been added to the combined file. The 1956 abstracts have been prepared on cards but await the receipt of their index for coding by subject.

TABLES TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-12-1107/51-1

Origin: NBS Authorized 7/1/50
Managers: C. Eisenhart, L. S. Deming
Full task description: July-Sept 1950 issue, p. 57

Status: INACTIVE. For status to date, see July-Sept 1952 issue, p. 64.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-12-1107/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. C. Eisenhart, N. C. Severo, and V. Martinez are undertaking a study in correlation analysis. They are investigating the likely consequences of "large" and "small" samples (a) of failure to recognize that it is NOT the directly observed quantities X and Y that have a bivariate normal distribution BUT RATHER their transforms U = g(X) and V = h(Y), and (b) of erroneous transformation to U and V before analysis, when in fact it is X and Y (and not U and V) that have a bivariate normal distribution. To date consideration has been given primarily to cases where U and V are either logarithmic or simple exponential functions of X and Y. "Large sample" properties are investigated analytically via the usual asymptotic formulae for functions of moments, and "small sample" properties are investigated by empirical sampling.

N. C. Severo is also investigating the following: If (1) X be a random variable such that Y = g(X) is normally distributed, (2) μ_x is the mean value of X, and (3) μ_y and σ_y^2 are the mean and variance of Y respectively, then application of the most powerful test of the hypothesis $\mu_x = {}_0\mu_x$ against the simple alternative $\mu_x = {}_1\mu_x$ requires knowledge of the values u_y and σ_y^2. In general, the values u_y and σ_y^2 may be difficult to evaluate directly. Now, if the approximate values indicated by the so-called "propagation of error" formulae are substituted for μ_y and σ_y^2, what is the effect on the operating characteristic function of the test?

W. C. Connor presented an invited address on the "Techniques and rationale of multiple comparisons" at the February 18th meeting of the Washington Statistical Society, a chapter of the American Statistical Association.

N. C. Severo presented an invited paper on "Tests concerning the means of certain distributions" at the Institute of Mathematical Statistics Eastern Regional Meeting in Washington, D. C., March 8th.

Publications:

(1) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. Joel. Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-12-1107/53-1

Origin: NBS Authorized 10/15/52
Manager: W. S. Connor
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. W. S. Connor constructed three fractional factorial designs for the mixed 2^n 3^m series. Two constructions were given for a 1/2 replicate of the 2^3 3^2 design, one following Kempthorne, Design and Analysis of Experiments, pp. 419-420, and the other following Morrison, "Fractional replication for mixed series," Biometrics 12 (1956), pp. 1-19. One construction was given for a 1/3 replicate of the 2^3 3^2 design.

An investigation of modified weighing designs was begun by J. M. Cameron. In calibration work involving sets of nearly identical objects, measurements can be made only on the differences between groups having an equal number of objects. The problem is to construct designs in which n objects are partitioned into b blocks of size 2k so that when measurements are made on the difference between the two groups of size k within a block, it is possible to estimate differences among all n objects.

W. S. Connor prepared the final draft of a paper "Experiences with incomplete block designs: Examples" (see the complete reference below).

R. C. Bose and R. C. Burton presented a contributed paper, "On a problem in Abelian groups and the construction of fractionally replicated designs," and M. Zelen contributed a paper on "The use of incomplete block designs for asymmetrical factorial arrangements," to the Eastern Regional Meeting of the Institute of Mathematical Statistics, Catholic University, Washington, D. C., March 7-8.

Publications:

(1) Fractional factorial experiment designs for factors at two levels. To appear as NBS Applied Mathematics Series 48.
(2) On the identity relationship for fractional replicates of the 2^n series. R. C. Burton and W. S. Connor. To appear in the Annals of Mathematical Statistics.
(3) Experiences with incomplete block designs: Examples. W. S. Connor. To appear in the Proceedings of the Symposium on the Design of Industrial Experiments, held at Raleigh, N. C., November 1956.
(4) The principle of randomization in the design of experiments. C. Eisenhart. To appear in the Proceedings of the 1955 Conference on the Design of Experiments in Army Research, Development and Testing, Washington, D. C., October 1955.
(5) Design of experiments in research and development. W. J. Youden. To appear in the Proceedings of the 1955 Conference on the Design of Experiments in Army Research, Development and Testing, Washington, D. C., October 1955.
(6) The analysis of covariance for incomplete block designs. M. Zelen. To appear in Biometrics.

(7) The analysis of incomplete block designs. (Formerly entitled: Exact tests of significance for combining inter- and intra-block information). M. Zelen. To appear in the Journal of the American Statistical Association.
(8) The use of incomplete block designs for asymmetrical factorial arrangements. M. Zelen. Submitted to a technical journal.

RESEARCH ON MATHEMATICAL ASPECTS OF ORDER STATISTICS METHODS
Task 1103-12-1107/55-110

Origin: NBS Authorized 3/3/55
Manager: C. Eisenhart
Full task description: Jan-Mar 1955 issue, p. 31

Status: TERMINATED. Activities of the type heretofore reported under this task will be reported in the future as activities of the Study of Non-Parametric Statistical Techniques (see task 1103-10-1107/56-170).

Publication:
(1) Table of the first moment of ranked extremes. J. Lieblein and H. E. Salzer. To appear in the Journal of Research,NBS.

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-1107/56-170

Origin: NBS Authorized 12/15/55
Manager: Joan R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

Status: INACTIVE. For status to date, see July-Sept 1956 issue, page 15.

Publication:
(1) On the independence of tests of randomness and other hypotheses. I. R. Savage. Appeared: J. Amer. Stat. Assoc. 52, 53-57 (1957).

MEASUREMENT OF RELIABILITY
Task 1103-12-1130/56-182

Origin: NBS Authorized 3/23/56
Manager: M. Zelen
Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. J. R. Rosenblatt presented a paper, "On prediction of system performance from information on component performance," at the 1957 Western Joint Computer Conference, "Techniques for Reliability," Los Angeles, California, February 27. This paper was also presented (1) at an NBS Reliability Seminar, February 21; (2) at a Statistics Department Seminar, University of California, Berkeley, March 1; and (3) at a Diamond Ordnance Fuze Laboratories Staff Meeting, March 26.

M. Zelen is continuing his investigations of non-exponential distributions for describing the distribution of failures. Current problems being worked on are: (a) To find estimates of the population parameters using order statistics; (b) to find suitable tests of significance using the order statistics estimates, and (c) to determine optimum replacement procedures.

Discussions have been initiated to determine what types of simple circuits would be suitable for testing mathematical models associated with the failure of assemblies.

M. Zelen and J. R. Rosenblatt attended meetings of the 1957 IRE-ASQC Symposium on Electronic Reliability.

Publication:

(1) On prediction of system performance from information on component performance. J. R. Rosenblatt. To appear in the Proceedings of the 1957 Western Joint Computer Conference, Los Angeles, California, February 1957.

CATALOGUE OF FRACTIONAL REPLICATION DESIGNS
Task 1103-12-5147/57-213

Origin and Sponsor: Bureau of Ships Authorized 8/30/56
Managers: W. S Connor, M. Zelen
Full task description: July-Sept 1956 issue, p. 37

Status: CONTINUED. All of the 3^n fractional factorial experiment plans were checked on SEAC by C. Dannemiller and J. M. Cameron. Before this could be done, some preliminary checks had to be made and certain information had to be extracted from the plans and coded so that SEAC could perform its operations. This phase of the work was carried out by L. S. Deming.

A first draft of the introduction to the catalogue that will present the analysis of these fractional designs with particular emphasis on the interpretation of the "aliases" has been written by M. Zelen.

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-12-1115/55-57

Origin: NBS Authorized 9/1/54
Manager: R. F. Dressler
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. U. Hochstrasser is continuing the geophysical study for the static deformation of the earth under gravitational forces. The case for earth tides, which had been computed previously by Takeuchi (Trans. Amer. Geophys. Union 31, 1950, p. 651-689), has been analyzed with the SEAC code described in previous issues. Although newer data for the density and rigidity distribution have been used, the results are in close agreement with the ones obtained by Takeuchi.

A manuscript describing results of a theoretical analysis of the application of a Cartesian diver as a delicate balance is being prepared by F. Chilton and P. Chiarulli.

A. Ghaffari is continuing a study of certain non-linear ordinary differential equations. In particular the second order Rayleigh equation is being investigated for self-excited vibrations. Using Poincaré's approach, the nature of the critical points of this equation at infinity has been studied, and also the full phase-portrait is being analyzed.

W. Pell has completed a second study of certain new graphical methods of solution for several types of ordinary differential equations, both linear and non-linear. These methods apply particularly well to the second-order equations for rather general vibration problems. A manuscript describing this second phase of the study is now in preparation.

F. Oberhettinger has completed investigations concerning the behavior of Bessel integral functions and related functions, especially for small and large values of the variables. A manuscript presenting the results has been prepared and accepted for publication. F. Oberhettinger is also preparing the chapter on the Gauss hypergeometric function for the NBS Handbook of Mathematical Tables.

In applying the hodograph method to the determination of the steady, irrotational flow of compressible fluid past a wedge, A. Ghaffari has obtained the solution for the region in which the main stream speed is greater than the fluid speed around the wedge. In this method he starts with the incompressible flow around the wedge and transforms this into the corresponding compressible flow for the limiting case of high velocities by using various properties of hypergeometric functions. A manuscript on this has been prepared.

Publications:

(1) A discussion of the paper "Design of corrugated diaphragms," by J. A. Haringx. R. F. Dressler. Trans. ASME 79, 61-62 (Jan. 1957).
(2) On the representation of a certain integral involving Bessel functions of hypergeometric series. P. Henrici. To appear in the Journal of Mathematics and Physics.
(3) On the domain of regularity of generalized axially symmetric potentials. P. Henrici. To appear in the Proceedings of the American Mathematical Society.
(4) The quotient-difference algorithm. P. Henrici. To appear in NBS Applied Mathematics Series 49, "Further contributions to the solution of simultaneous linear equations and the determination of eigenvalues."
(5) Graphical solution of the single degree of freedom vibration problem with arbitrary damping and restoring forces. W. H. Pell. To appear in the Journal of Applied Mechanics.
(6) The transmission of Rayleigh waves across an ocean floor with two surface layers. Part I. R. Stoneley. To appear in the Bulletin of the Seismological Society of America.
(7) On the propagation of turbidity currents. R. Stoneley. To appear in the Vening Meinesz Jubilee Volume.
(8) The attenuation of Rayleigh waves with depth in a medium with two surface layers. R. Stoneley and U. Hochstrasser. To appear in the Geophysical Supplement, Monthly Notices, Royal Astronomical Society.
(9) On the solution of compressible flow past a wedge. A. Ghaffari. In manuscript. Abstract to appear in the Bulletin of the American Mathematical Society.
(10) On some expansions for Bessel integral functions. F. Oberhettinger. To appear in Journal of Research, NBS.

RESEARCH IN CONTINUUM MECHANICS
(formerly Mathematical Elasticity)
Task 1104-12-5160/55-85

Origin: NBS Authorized 12/27/54
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: R. F. Dressler
Full task description: Oct-Dec 1954 issue, p. 30

Status: CONTINUED. W. H. Pell has completed the manuscript of a paper in which the Kelvin integral solutions of the Navier equations have been employed to obtain the displacements in an infinite elastic body due to a constant concentrated force applied along,and normal to,a circle lying within the body. These results should be useful in the ultimate solution for displacements and stresses in the elastic cone due to an axially symmetric distribution of force on its surface, since the present results represent the first of the two solutions needed to

solve this problem using the superposition method. This method is also applicable to a force whose magnitude is variable along the circle but the calculations in such a case become excessive.

A study of the effect of thermal stresses in shells has been initiated. Except for the case of cylindrical shells, little work has been done in this field. It appears from preliminary studies that in certain simple axially symmetric shapes with axially symmetric temperature distributions the thermal stresses may be found; the work is proceeding along these lines.

Another problem now under consideration is that of the flow about a ring wing (annular wing) in the presence of an axially symmetric body so situated that the axes of symmetry of both bodies coincide. The ring wing alone has been studied by Dickmann, Stewart, and Weissinger, while the flow about the axially symmetric body is considered in the standard texts. Using the results for these bodies, the problem is being considered from the point of view of the interference of each body on the flow about the other.

All the curves exhibiting the rapid stress variations for all stress components in a sequence of corrugated elastic diaphragms have been drawn, and R. F. Dressler is continuing the preparation of a text of a manuscript to evaluate these results.

R. F. Dressler has resumed an investigation of the elastic edge layer in plates subjected to a generalized plane stress system of forces. A modification of the stretching method already used to analyze the rapid variations in the edge layer for bent plates is being employed. The interior lowest order asymptotic approximation yields the classical plane stress equations, which are being interrelated with the lowest order effect in the edge layer, where the applied stresses at the edge are permitted a rapid variation across the plate thickness.

Using the non-linear analogy between unsteady compressible flow and open channel hydraulic waves, R. F. Dressler has undertaken an investigation of the classic problem for unsteady reservoir discharge down an inclined channel with arbitrary slope. The non-homogeneous equations for any slope have been transformed into a homogeneous system by a suitable transformation of dependent and independent variables thus permitting exact solution by hodograph inversion. The problem is now being solved, following the Riemann method of integration, through use of the suitable hypergeometric auxiliary function. In the hydraulic case this can be transformed conveniently into a complete elliptic integral of the second kind.

Publications:

(1) Investigations of the properties of corrugated diaphragms. W. A. Wildhack (NBS OBI), R. F. Dressler, and E. C. Lloyd (NBS Mechanics Division). Trans. ASME 79, 65-82 (1957).

(2) Stationary principles for forced vibrations in elasticity and electromagnetism. J. L. Synge. To appear in the Proceedings of the Eighth Symposium in Applied Mathematics of the American Mathematical Society, held at Chicago, Ill., April 1956.

(3) Elastic waves in anisotropic media. J. L. Synge. Submitted to a technical journal.
(4) The torsion of a hollow square. J. L. Synge and W. F. Cahill. Submitted to a technical journal.
(5) The vibration of triangular wings. R. F. Dressler. In manuscript. Abstract available in the Proceedings of the Ninth International Congress on Mechanics, Brussels, September 1956.
(6) The elastic problem for a ring of uniform force in an infinite body. W. H. Pell. In manuscript.

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-5160/56-154

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: July-Sept 1955 issue, p. 20

Status: CONTINUED. The normalization factors have been calculated, and a few further results have been added to the computation. The tables of formulas are now being prepared for printing.

RESEARCH IN FLUID DYNAMICS OF TWO-PHASE FLOWS
Task 1104-12-5160/56-155

Origin and Sponsor: Office of Naval Research Authorized 9/30/55
Manager: R. F. Dressler
Full task description: July-Sept 1955 issue, p. 21

Status: CONTINUED. An attempt is being made by J. M. Burgers and A. Ghaffari to analyze a propulsion problem based on one of the cases considered by P. Chiarulli and R. F. Dressler in "Condensation Interfaces in Two-Phase Flows." The relative advantages of a steam jet and a steam-driven water jet are being considered. For a water jet which can serve as a propulsion mechanism, the basic relations referring to the transfer of momentum from a jet of superheated steam to water are being studied. Following a typical calculated example it is shown how the equations of continuity, momentum, and of enthalpy can be applied to find velocities and the temperature of the mixture, and to obtain the thrust, the power delivered, and the jet efficiency.

In analyzing the various possibilities it is shown that from certain standpoints propulsion should be obtained preferably by means of a jet of water that receives its momentum from a jet of steam driven

into it in such a way that the steam completely condenses. This analysis involves the study of the growth of a water droplet surrounded by condensing steam.

Publication:

(1) Condensation interfaces in two-phase flows. P. Chiarulli and R. F. Dressler. Submitted to a technical journal. Abstract appeared in the Proceedings of the Ninth International Congress on Mechanics, Brussels, September 1956.

RESEARCH IN RADIATION THEORY
Task 1104-12-5160/56-175

Origin: NBS Authorized 12/28/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: Oct-Dec 1955 issue, p. 18

Status: INACTIVE.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

1102-40-5126/51-37 MOLECULAR STRUCTURE, III
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. O'Hara
Full task description: July-Sept 1951 issue, p. 50
Status: Continued. Probability methods were used to compute parameters for phase determination formulas. These formulas yielded phase angles well within the desired tolerance when applied to a crystal of space group P_1.

1102-40-5126/52-44 CALCULATIONS FOR d-SPACINGS
Origin and Sponsor: NBS, Division 9
Full task description: Oct-Dec 1951 issue, p. 47
Manager: R. Zucker
Status: Continued. About 18 calculations for d-spacings for tetragonal, hexagonal, orthorhombic and monoclinic crystals were performed. Also redetermination of unit cell constants by least squares fitting to a measured d-spacing were carried out for about 40 crystals.

1102-40-5126/53-51 RADIATION DIFFUSION, III
Origin: NBS, Section 4.8
Sponsor: Atomic Energy Commission
Manager: J. Doggett (4.8)
Full task description: Apr-June 1953 issue, p. 57 (Neutron Diffusion III)
Status: Inactive.

1102-40-5126/54-13 AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: New York Quartermaster Procurement Agency
Manager: H. Bremer
Full task description: Oct-Dec 1953 issue, p. 43
Status: Continued. During the quarter determination of procurement contracts by using the "Transportation Problem" as coded for SEAC continued. In the period five awards were solved. In all cases they represented routine considerations. Some of the items contracted for were combat boots, shoes and jackets.

3711-60-0009/54-22 ENERGY DISTRIBUTIONS ON OPTICAL IMAGE
Origin: NBS, Section 2.2
Manager: L. S Joel
Full task description: Jan-Mar 1954 issue, p. 43
Status: Inactive. For status to date, see Jan-Mar 1956 issue, p. 19.

3711-60-0009/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Manager: S. Prusch
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. The spectrum of tantalum I has been searched with 50 additional levels of even parity. The results have been compiled and are awaiting analysis. A second search for even levels in hafnium I has been compiled. The search was carried out with 150 odd level differences. A previous search utilizing 50 even level differences yielded 45 new odd levels. A total of about 600 hafnium I lines have thereby been classified.

3711-60-0009/54-38 COMPRESSIBILITY FACTORS OF DRY AIR
Origin: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Continued. Using data received, a small table of compressibility factors was computed for T = 300°K, and P(in lbs/in.^{2}) = 10(10)100 (20)1000(50)4000 (100)5000 was calculated. The results were submitted to the sponsor.

3711-60-0009/55-68 CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Division 9
Manager: R. Prosen (Div. 9), P. O'Hara
Full task description: Jan-Mar 1955 issue, p. 18
Status: Continued. Structure factor calculations were carried out for the $Mg_2B_2O_5$ and $z_nB_2O_4$ crystals using several sets of new position parameters.

3711-60-0009/55-75 PARAMETER OF THE DISPERSION EQUATION FOR OPTICAL GLASS
Origin: NBS, Section 2.2
Manager: R. Zucker
Full task description: Jan-Mar 1955 issue, p. 20
Status: Inactive

1102-40-5126/55-81 COMBINING TESTS FOR SIGNIFICANCE
Origin: NBS, Section 11.3
Manager: L. S. Joel
Full task description: Oct-Dec 1954 issue, p. 23
Status: Completed. Results have been transmitted to the sponsor.

3711-60-0009/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. Calibration tables were computed for five thermometers under test.

1102-40-5126/55-88 STRESSES IN A WALL FOUNDATION
Origin and Sponsor: NBS, Section 10.1
Manager: I. Stegun
Full task description: Jan-Mar 1955 issue, p. 22
Status: Inactive.

1102-40-5126/55-113 REACTOR DESIGN
Origin: Westinghouse Atomic Power Division
Sponsor: Atomic Energy Commission
Manager: U. Hochstrasser
Full task description: Jan-Mar 1955 issue, p. 28
Status: Continued. The code for the bare reactor problem has been checked out. Some runs have been made to check the theory and to investigate some features of the convergence of the procedure used.

1102-40-5126/55-117 ATTENUATION OF PRESSURE PULSES OF FINITE AMPLITUDE
Origin: NBS, Section 3.2
Sponsor: Bureau of Aeronautics, U.S. Navy
Manager: M. Paulsen
Full task description: Apr-June 1955 issue, p. 18
Status: Inactive.

1102-40-5126/55-118 THERMOMETER CALIBRATION, II
Origin and Sponsor: NBS, Section 3.1
Manager: B. G. Urban
Full task description: Apr-June 1955 issue, p. 18
Status: TERMINATED.

1102-40-5126/55-121 ELECTRON PENETRATION
Origin: NBS, Section 4.8
Sponsor: Atomic Energy Commission
Manager: S. Peavy
Full task description: Apr-June 1955 issue, p. 19
Status: Continued. The main problem is being programmed for the 704. Additional computations are being done on SEAC.

1102-40-5126/55-123 TEMPERATURE DISTRIBUTION
Origin and Sponsor: NBS, Section 6.4
Manager: W. F. Cahill
Full task description: Apr-June 1955 issue, p. 19
Status: Completed. All results have been transmitted to the sponsor.

1102-40-5126/55-127 VIBRATIONS OF CIRCULAR DISC
Origin and Sponsor: Evans Signal Laboratory
Manager: U. Hochstrasser
Full task description: July-Sept 1955 issue, p. 32
Status: Continued. The table of ϵ vs. d/h mentioned in the July-Sept 1955 issue, p. 27, has been completed and differenced in order to find any missing values. Further, a table of the solutions for $.9 \leq \epsilon \leq 1$ has been computed in the range of $0 < d/h < 200$. (The equation in this case is different from the one given in the original task description since β is imaginary). Some missing values are being computed.

1102-40-5126/56-136 CALCULATION OF WAVE FUNCTIONS BY HARTREE METHOD
Origin: Naval Research Laboratory
Sponsor: " "
Manager: S. Peavy
Full task description: July-Sept 1955 issue, p. 34
Status: Continued. A few new runs and some continuous runs were made at the request of the sponsor.

1102-40-5126/56-140 MULTIPLE CORRELATION ROUTINES
Origin and Sponsor: Agricultural Economics Division, Department of Agriculture
Managers: H. Bremer, M. Paulsen
Full task description: Oct-Dec 1955 issue, p. 26
Status: Continued. This routine is in the nature of a "canned routine" and as such is available for use on any problem that requires the computation of multiple correlations, regression coefficients and standard deviations as long as the number of variables is less than or equal to nine, and the number of observations less than 90 for a four-variable problem and less than 37 for a nine-variable problem. In general, the sponsor sets up his data to conform with the input format as specified by the SEAC routine; then we furnish a programmer who

checks the correctness of the data format and operates SEAC. For an average problem, with five variables and 30 observations, machine time is approximately 20 minutes.

The Department of Agriculture sponsored the original programming of this routine. During this quarter about 20 problems were run off using the routine.

1102-40-5126/56-162 STRESSES IN A WALL RESTING ON A FOOTING
Origin and Sponsor: NBS, Section 10.1
Manager: I. Stegun
Full task description: Jan-Mar 1956 issue, p. 26
Status: Continued. The data for the solution of 100 simultaneous equations has been checked and several iterations have been computed.

1102-40-5126/56-163 ANGULAR DISTRIBUTIONS AND POLARIZATION EFFECTS IN NUCLEAR SCATTERING
Origin and Sponsor: Naval Research Laboratory
Manager: I. Stegun
Full task description: Oct-Dec 1955 issue, p. 32.
Status: Continued. The solutions of approximately 25 problems have been obtained.

1102-40-5126/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Manager: E. Haynsworth
Full task description: Jan-Mar 1956 issue, p. 27
Status: Inactive.

1102-40-5126/56-169 CRYSTAL FIELD EFFECTS FOR ATOMS
Origin and Sponsor: NBS, Division 3
Manager: I. Stegun
Full task description: Jan-Mar 1956 issue, p. 28
Status: Completed. Results have been transmitted to the sponsor.

1102-40-5126/56-171 COLLISION INTEGRALS USED IN TRANSPORT THEORY
Origin and Sponsor: NBS, Section 3.2
Manager: J. Cooper
Full task description: Oct-Dec 1955 issue, p. 33
Status: Inactive.

1102-40-5126/56-172 NUMERICAL EVALUATION OF SPECIAL INTEGRAL EXPRESSIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Department of the Army
Manager: L. Joel
Full task description: Jan-Mar 1956 issue, p. 29
Status: Inactive

1102-40-5126/56-176 MODEL GOVERNMENT PAYROLL ON HIGH SPEED COMPUTERS
Origin and Sponsor: NBS, Section 12.5
Manager: G. H. Urban
Full task description: Jan-Mar 1956 issue, p. 30
Status: Terminated.

1102-40-5126/56-179 NORMAL PROPAGATION CONSTANT
Origin and Sponsor: NBS, Section 82.10
Manager: H. H. Howe (82.10)
Full task description: Apr-June 1956 issue, p. 30
Status: Continued. The problem was tackled for a different model, which should be somewhat closer to reality. Previously, the ionosphere was considered as a homogeneous medium with a sharp lower boundary. Now, a two-layered model is used, with two sharp boundaries. H is now the height of the lower boundary, measured in wave-lengths. D is a new parameter, namely the distance between the two boundaries, measured in wave-lengths. As before, G measures the resistivity of the ground. L now measures the resistivity of the layer of ionosphere between the boundaries. A new parameter, L_o, measures the resistivity of the ionosphere assumed to extend indefinitely above the upper boundary. Although for the most part we consider $L_o < L$, the case $L_o > L$ is also possible, although it is not likely that this would be applicable to the low frequencies for which the mode theory is useful.

The formulas given on pages 30-31 of the Jan-Mar 1956 issue are modified as follows: R_s contains an additional factor Q, in the following ways:

$$R_s = \frac{(L-i)c - Q\sqrt{c^2L^2 - Li}}{(L-i)c + Q\sqrt{c^2L^2 - Li}}$$

where Q is given by

$$Q = \frac{(L-i)\sqrt{c^2L_o^2 - L_oi} + (L_o-i)\sqrt{c^2L^2 - Li}\tanh P}{(L_o-i)\sqrt{c^2L^2 - Li} + (L-i)\sqrt{c^2L_o^2 - L_oi}\tanh P},$$

and where

$$P = 2\pi i (D/L)\sqrt{c^2L^2 - Li}\ .$$

It will be seen that this degenerates into the previous case whenever Q = 1. This happens, for example, when L = L_o, for then the upper boundary disappears. It also happens if D becomes infinite (i.e., the lower layer becomes the entire ionosphere), for then tanh P becomes unity.

During the report quarter, this job was coded for SEAC, and considerable progress was made in debugging the code.

1102-40-5126/56-184 GAIN CALCULATIONS OF AN ITERATED TRANSISTOR AMPLIFIER
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: E. Haynsworth
Full task description: Apr-June 1956 issue, p. 32
Status: Inactive. For status to date, see July-Sept 1956 issue, p. 32

1102-40-5126/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: S. Prusch
Full task description: July-Sept 1956 issue, p. 33
Status: Inactive.

1102-40-5126/56-192 NOISE MEASUREMENT
Origin and Sponsor: NBS, Section 6.1
Manager: W. F. Cahill
Full task description: Oct-Dec 1956 issue, p. 28
Status: Continued. The function f(r,z) has been tabulated for r=0(.01)2.0, z = 0(.05)1.50.

1102-40-5126/57-206 EXPECTED VALUES OF ORDER STATISTICS
Origin and Sponsor: NBS, Section 11.3
Manager: R. Durrah
Full task description: July-Sept 1956 issue, p. 34
Status: Completed. Results have been transmitted to the sponsor.

The title of this task should have read more accurately, "Moments of Order Statistics," and the objective more specifically stated as being the computation of the r-th order moments of order statistics defined by the formulas given in the July-September 1956 issue (see p. 34). (In that write-up the formulas for $P_r(x)$ should have $k \cdot C_{n,k}$ in place of $C_{n,k}$ as given, and the denominator of the second fraction in the formula for $x_r(n,k)$ should have read (k-r)! instead of (k-r+i)! as given.)

Linear functions of order statistics arise in a large number of decision problems and since the computation of the exact distribution is usually impracticable one has to depend on a knowledge of the moments.

By a simple reduction formula, the expected values and the moments of the 2nd, 3rd, and 4th order of order statistics can be obtained. The basis for this work is contained in the paper, "Moments of Order Statistics from a Normal Population," by R. C. Bose and S. Gupta (Institute of Statistics Mimeo Series No. 154, University of North Carolina, July 1956).

1102-40-5126/57-209 TRAFFIC DISTRIBUTION
Origin and Sponsor: Bureau of Public Roads
Manager: S. T. Peavy
Objective: To evaluate the relative effectiveness of four methods of predicting traffic volume distribution. If the results warrant, distributions for future periods will be calculated.
Background: A home interview type of traffic survey was made in the Washington metropolitan area in 1948 and again in 1955. Four generally accepted methods of predicting traffic volume distribution are to be applied to the data received in the 1948 survey to obtain an estimated distribution for 1955. Then, using the data obtained in the 1955 survey as a control, the error will be determined.

In order to provide more adequate roads and highways in the most effective routing patterns, it is essential to study methods for predicting traffic volume distribution and if possible to determine the most accurate one.

The problem was transmitted by W. L. Mertz.
Status: New. The problem has been coded for the IBM 705 at the Treasury Department, and code checking is in progress.

3711-60-0009/57-210 SOUND VELOCITY
Origin and Sponsor: NBS, Section 3.2
Manager: U. Hochstrasser
Full task description: July-Sept 1956 issue, p. 34
Status: Continued. The problem has been discussed with the sponsor. It has been found that a reformulation of it will simplify and improve the computation and the results.

1102-40-5126/57-211 METEOROLOGICAL DATA
Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Department of the Army
Manager: P. O'Hara
Full task description: Oct-Dec 1956 issue, p. 30
Status: Cont. Temperature and density distribution patterns are being computed using data gathered at the Oklahoma City weather station. Two additional pressure surfaces have been added to the five surfaces originally under consideration.

1102-40-5126/57-215 COMPUTATION OF INTEGRALS INVOLVING BESSEL FUNCTIONS
Origin and Sponsor: NBS, Section 6.1
Manager: U. Hochstrasser
Full task description: July-Sept 1956 issue, p. 35
Status: Continued. New tables for $P(x,\theta)$ for large values of x have been computed using a more accurate formula. The table for $Q(x,\theta)$ has been completed and transmitted to the sponsor.

3711-60-0009/57-219 THERMAL PROPERTIES
Origin and Sponsor: NBS, Section 3.2
Managers: J. Cooper, D. Sumida
Full task description: Oct-Dec 1956 issue, p. 30
Status: Continued. Code-checking is now in progress. Portions of the code have been checked out satisfactorily on several trial runs.

1102-40-5126/57-221 BESSEL FUNCTIONS FOR COMPLEX ARGUMENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Department of the Army
Manager: R. Zucker
Full task description: Oct-Dec 1956 issue, p. 31
Status: Continued. The subroutine to evaluate

$$\sum_{n=1}^{\infty} z_n = \frac{2n+1}{n(n+1)} \left[\frac{i}{\frac{\hat{j}_n'(B_c)}{\hat{j}_n(B_c)} - \frac{1}{k^*} \frac{\hat{H}_n'(B_{oc})}{\hat{H}_n(B_{oc})}} \right]$$

was written, checked and incorporated into the main routine. Numerous cases were computed on SEAC.

1102-40-5126/57-222 ROOTS OF POLYNOMIALS
Origin and Sponsor: Naval Research Laboratory
Manager: U. Hochstrasser
Full task description: Oct-Dec 1956 issue, p. 32
Status: Continued. Since all the polynomials should have real roots and the degree of the polynomials is not higher than 18, the polynomial routine has been modified accordingly. Thus the roots are obtained at least twice as fast. (An 18th degree polynomial took 10 minutes). The roots of 54 polynomials have been found and transmitted to the sponsor.

1102-40-5126/57-224 TRACK-WHILE-SCAN RADAR PROBLEM

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Managers: E. Haynsworth, P. J. Walsh

Full task description: Oct-Dec 1956 issue, p. 32

Status: Continued. The sponsor requested computation of the multiple integral

$$\overline{M(\bar{\phi})} = \frac{1}{(\Delta\phi)^2} \int_o^\infty \frac{e^{-x/x_o}}{x_o} \int_{-\infty}^{\infty} \frac{e^{-\phi^2/2\sigma^2}}{\sqrt{2}\cdot\sigma\sqrt{\pi}} \int_{-\Delta\phi/2}^{\Delta\phi/2} M(\bar{\phi},\phi_o,\phi,x)\,d\phi_o\,d\phi dx$$

where

$$M(\bar{\phi},\phi_o,\phi,x) = \frac{\sqrt{1-\rho^2}}{2\pi a^2 \sigma_\tau \sigma_T} e^{-c^2/1-\rho^2} + \frac{b}{4\sqrt{\pi}a^3\sigma_\tau\sigma_T} e^{\frac{1}{1-\rho^2}[(b^2/4a^2)-c^2]}$$

$$\cdot\left[\operatorname{erf}\left(\frac{b}{2a\sqrt{1-\rho^2}}\right)\right]$$

and

$$a^2 = \frac{1}{2\sigma_T^2} - \frac{\rho\bar{R}}{\sigma_\tau\sigma_T} + \frac{\bar{R}^2}{2\sigma_\tau^2}, \qquad b = \frac{\bar{T}}{\sigma_T^2} - \frac{\rho(\bar{R}\,\bar{T}+\bar{\tau})}{\sigma_\tau\sigma_T} + \frac{\bar{R}\,\bar{\tau}}{\sigma_\tau^2}$$

$$c^2 = \frac{\bar{\tau}^2}{2\sigma_\tau^2} - \frac{\rho\,\bar{\tau}\,\bar{T}}{\sigma_T\sigma_\tau} + \frac{\bar{T}^2}{2\sigma_T^2}, \qquad \bar{R} = \frac{1}{\Delta\phi}(\bar{\phi} - \phi_o)$$

$$\tau = \sum_{i=1}^{n} \bar{\tau}_i, \quad \sigma_\tau^2 = \sum_{i=1}^{n} \sigma_{\tau_i}^2, \quad \bar{\tau}_i = (k+i)(A\,f_{k+i}\,x + N_o)$$

$$\bar{T} = \sum_{i=1}^{n} \bar{T}_{k+i}, \quad \sigma_T^2 = \sum_{i=1}^{n} \sigma_{T_{k+i}}^2, \quad \bar{T}_{k+i} = A\,f_{k+1}\,x + N_o$$

$$\sigma_{T_{k+i}}^2 = (A\,f_{k+i}\,x + 3N_o)^2 - 6N_o^2, \quad \sigma_{\tau_i}^2 = (k+i)^2[(A\,f_{k+i}x + 3N_o)^2 - 6N_o^2$$

$$\rho = \frac{\sum_{i=1}^{n}(k+i)[(A\,f_{k+i}x + 3N_o)^2 - 6N_o^2]}{\sigma_\tau\,\sigma_T}, \quad f_{k+i} = \frac{\sin^4\frac{\pi\alpha}{\lambda}\Phi}{\left(\frac{\pi\alpha}{\lambda}\Phi\right)^4}$$

where $\Phi = (\phi - \phi_o) + (k+i)\Delta\phi$, and A, N_o, x_o, α, λ, N, k, $\sqrt{2}\,\sigma$, $\Delta\phi$, and $\bar{\phi}$ are input parameters. M is the probability density for the ratio of two random variables. $M(\bar{\phi})$ is the probability distribution of the measured position of a fixed target in a single measurement. The distribution indicates the amount of uncertainity introduced by the noise.

The code has been written and checked out on SEAC. Running of the code for various parameters is now in progress.

1102-40-5126/57-225 DAMAGE ASSESSMENT PROBLEM II
Origin and Sponsor: Corps of Engineers, U.S. Army
Managers: H. Bremer, W. G. Hall, L. S. Joel
Full task description: Oct-Dec 1956 issue, p. 33
Status: Continued. Consultation services continued as described in Oct-Dec 1956 issue, p. 33. Towards the end of the current quarter this service had been reduced to answering only specific questions as related to Damage Assessment Problem I as it was coded for UNIVAC.

1102-40-5126/57-228 MICROWAVE PROPERTIES OF FERRITES
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: B. Walter
Objective: To calculate size effect for a small ferrite sphere

$$F(H_a) = K \frac{(\mu-1)[\gamma R \cosh \gamma R - \sinh \gamma R]}{(\mu-1)[\gamma R \cosh \gamma R - \sinh \gamma R] + (\gamma R)^2 \sinh \gamma R}$$

$F(H_a)$ gives apparent permeability of a sphere provided the radius is very small compared with wavelength measured in air;
μ is the intrinsic microwave permeability; it is a function of the applied steady magnetic field H_a, of the saturation magnetization for the single crystal under consideration, and of the damping parameter;
R is the radius of the sphere: R = .02816cm.;
$\gamma = 6.2682 \sqrt{u}$, $2.10 \times 10^3 \leq H_a \leq 6.0 \times 10^3$ oersteds.
Background: The function arises in the study of microwave properties of polycrystalline and single-crystal ferrites. The components of the microwave intrinsic permeability tensor of ferrites can be measured by observing the proportional change of resonant frequency ($\delta f/f$) of a cavity when a very small sphere or cylinder of ferrite is placed in the cavity. The function $F(H_a)$ shows how $\delta f/f$ varies as a function of the applied field H_a.

The problem was transmitted by J. E. Tompkins (DOFL 44.3).
Status: New. The calculations of $F(H_a)$ have been completed for a single-crystal sample of ferrite under the assumption of Landau-Lifschitz damping. Excellent agreement with experimental results has been obtained.

Calculations under the assumption of Bloch-Bloembergen damping have been started.

3711-60-0009/57-229 APPLICATION OF ELECTRONIC DATA PROCESSING MACHINERY TO PAYROLL OPERATIONS

Origin: NBS, Section 40.0

Managers: H. Bremer, P.R.McClenon, M. Paulsen, J.B.Tallerico

Objective: To investigate the feasibility of computing and assembling in final form the NBS payroll and other personal services reports using electronic data processing machinery. It is planned to duplicate with the IBM 704 all the output currently obtained with punched-card equipment and to compare the efficiency and flexibility of the two systems.

Background: In January 1956 the Bureau adopted a mechanical payroll system using punched-card equipment to compute and assemble in finished form the payroll for its nearly 3000 employees and various personal services reports. This system has proved to be superior in efficiency and flexibility to the former hand operation; however, certain phases of the operation are somewhat time-consuming. It is now planned to explore the possibility of achieving greater flexibility and efficiency in the overall operation by utilizing the IBM 704. The minimum reports that are required in the Bureau's payroll operation and that will be produced on the 704 are outlined below to give some indication of the magnitude of the task.

(1) Master Pay List, giving for each employee salary rates (annual, biweekly, hourly), deductions to be made from earnings, and other information needed to calculate the biweekly pay. This is a complete file of employees on the Bureau rolls.

(2) Payroll for Personal Services, listing for each employee by name and number details of biweekly gross earnings, deductions, and net pay. The Bureau is required by regulation to submit a certified copy of the biweekly payroll to the Treasury Department, which serves as their authority to issue salary checks.

(3) Check Issue and Statement of Earnings Cards, giving on punched cards essentially the same information for each employee as is given in the Payroll for Personal Services. The cards accompany the payroll listing to the Treasury Department where they are actually used as input for the writing of salary checks. The earnings cards are ultimately issued to the employees with the salary checks.

(4) Time Cards, giving for each employee, punched and printed on the cards, name, number, division, section, pay-period number, and tenure code. This information is reproduced from the Master File after the payroll has been calculated. In this way, considerable keypunching is eliminated and delays in payroll processing that might be caused by incorrect employee numbers are minimized.

(5) Division Total Payroll Control, summarizing in one line for each division the total gross earnings (base pay, overtime, holiday, etc.), deductions from gross pay (federal and state taxes, FICA, retirement, etc.), and net pay. This is used as a source document to determine the amount to charge each division for its share of the Government's contributions to the FICA and Government Life Insurance programs.

(6) Bond Schedule, giving for approximately 1100 subscribers

each subscriber's bond account number, bond purchase price, and bond account balance. This schedule is referred to when making changes to a subscriber's bond allotment or when it is necessary to refund a bond account balance to an employee when he resigns from the Bureau or cancels his bond allotment.

(7) Bond Purchase List, containing names of subscribers who have paid for bonds, the prices of the bonds purchased, subscribers' addressoplate numbers which identify the plates used by the Treasury to inscribe the bonds, and the total number of bonds to be issued for each denomination, and the total purchase price. The list is forwarded to the Treasury Department biweekly.

(8) Employee Cumulative Earnings Report, giving for each employee the year to date total for gross earnings and for deductions from gross (taxes, retirement, etc.). This information is used to control the earnings of employees subject to a maximum earnings limitation and also to certify wages to various states for unemployment compensation purposes.

(9) Labor Distribution Report, a biweekly statement reporting the distribution of labor costs to approximately 1100 active Bureau projects. The statement is prepared according to division and project order number, and shows for each employee the number of labor hours charged to the project and the labor cost.

(10) Annual Individual Pay Record, summarizing by payperiod each employee's detailed earnings and deductions for the year. Regulations require that this record be retained indefinitely.

(11) Withholding Tax Statement (W-2), giving for each employee, annual gross earnings, Federal, state and Social Security taxes withheld during the year.

The results obtained with the 704 will be evaluated and compared with the operating requirements of the present system insofar as time, cost and flexibility factors are concerned to determine whether it will be practical to prepare the above reports using the 704. It is expected that this study will also be of interest to other Government agencies and private organizations.

Status: New. Work sheets were prepared outlining the calculations required to obtain base pay, overtime and holiday pay, night differential, various allowances such asforeign post differentials and cost of living allowances, and deductions from gross pay such as federal and state taxes, insurance, rent, retirement, FICA, etc. Input and output requirements were identified, and a preliminary flow diagram of the problem was prepared.

1102-40-5126/57-234 PERSONNEL SURVEY

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Manager: P. O'Hara

Objective: To use operations research methods to predict one major aspect of the future personnel status of a medium sized research-

engineering organization. A succession of interdependent linear processes is used to approximate what is essentially a non-linear development process.

Background: The problem was communicated by L. M. Court (DOFL).

Status: New.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Standard reference samples: Analysis of data from study of homogeneity of metal rods, for R. E. Michaelis, Section 5.10.

(2) Thickness measurements of electroplated metal: Analysis of data from previously designed inter-laboratory tests, for F. Ogburn, Section 5.6.

(3) Calibration of 12-sided polygon: Least squares estimation of correction to angles from closures at 30°, 60°, and 120°, for C. G. Haven, Section 2.5.

(4) Radio propagation: Design of experiment for use in study of scattering of signals by meteors, for G. R. Ochs and R. J. Carpenter, Section 82.20.

(5) Participation in the Child Safety Project: Assistance was furnished in the planning of the study, and in the analysis and interpretation of the data collected. This project was concerned with the reaction of children to release devices used on refrigerators, and with the behavior of children in a small, dark enclosure. The nature of the program and some of its findings are discussed in a report, "Behavior of young children under simulated refrigerator entrapment," prepared for release to the press on April 16.

Publications:

(1) National physical standards and design of experiments. W. J. Youden. To appear in Revue de L'Institut International de Statistique (The Hague).
(2) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.
(3) Statistics: Engineering viewpoint. W. J. Youden. To appear in the Journal of Engineering Education.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: CONTINUED. The statistical analysis on SEAC of results from nine physical and chemical tests on ship plate was completed.

The use of logarithmic transformations on data from the Charpy V-Notch test was investigated to see if its use would improve the efficiency of transition temperature estimates.

STATISTICAL ANALYSIS AND DESIGN OF EXPERIMENTS FOR THE U. S. GEOLOGICAL SURVEY
Task 1103-40-5140/54-1

Origin and Sponsor: U. S. Geological Survey Authorized 10/9/53
Department of Interior
Managers: C. Eisenhart, W. J. Youden
Full task description: Oct-Dec 1953 issue, p. 50

Status: CONTINUED. C. Eisenhart, N. Severo, and V. Martinez continued an empirical investigation of methods for deriving confidence limits for "preferred directions," using data provided by F.G. Poole (Colorado Plateau District). A final report on this work was completed in preliminary draft.

J. M. Cameron, W. S. Connor, M. T. Carson, and M. C. Dannemiller carried out a multiple regression analysis of the dependence of size of uranium ore deposits on the percentages of eight chemical elements found in the ore. Data on 75 uranium ore deposits furnished by A.T. Miesch (Colorado Plateau District) was used. The value of the coefficient of multiple correlation was found to be 0.67, which is statistically significant at the 0.5% level. A more detailed breakdown revealed that the dependence of size of deposit on the percentages present of two particular elements, considered jointly, accounted for about three-fourths of the reduction in over-all variation ascribable to dependence on these eight elements and that the residual dependence on the remaining six elements, while statistically significant, was of negligible utility from a prediction viewpoint.

C. Eisenhart and W. S. Connor began a study of whether success in locating ore and mineralized deposits by indicator plants depends on the depth of the deposits below the ground surface. The data used was provided by H. L. Cannon (Geochemical Exploration Section, Denver).

On January 17th, W. S Connor and M. G. Natrella participated in a Washington meeting of the Committee on Statistics in Geology, of

the Geologic Division, USGS. The discussion centered on some statistical problems met by the Chief Geologist, W. H. Bradley, in his studies of certain clam colonies in tidal flats.

J. R. Rosenblatt reviewed and commented on a revised manuscript on "Semiquantitative spectrographic analysis and rank correlation in geochemistry," by F. J. Flanagan (Geochemistry and Petrology Branch, Washington). Comments of C. Eisenhart, W. J. Youden, M. Zelen, and others on a manuscript, "The meaning and importance of 'semi-quantitative' analysis", by E. M. Shoemaker (Colorado Plateau District) were assembled for transmittal to the author.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-5146/55-93

Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: CONTINUED. Drafts have now been completed for almost all of the sections of the manual. The four major parts are--

I. Some Standard Statistical Techniques for Quantitative Data.
II. Some Standard Statistical Techniques for Qualitative Data
III. Data Collection and Analysis (The Design of Experiments)
IV. Miscellaneous

A beginning has been made on the revision of Part I.

APPLICATION

OF

NATIONAL BUREAU OF STANDARDS AUTOMATIC COMPUTER (SEAC)

The record of SEAC operations for tasks of the Applied Mathematics Division for the period January 1 through March 31 is as follows:

Task No. NBS:	Title	Code Checking	Productive Operation
1104/55-56	Research in mathematical topics applicable to numerical analysis	1	
1110/47-2	Tables of Coulomb wave functions	25	67
1110/53-51	Radiation diffusion		7
1110/53-52	L-shell conversion coefficients	10	2
1115/55-57	Research in mathematical physics and related fields	5	37
5114/57-237	Numerical analysis training program	2	25
5116/56-148	Study of differential equations for nerve fiber excitation	1	22
5126/52-44	Calculations for d-spacings		9
5126/54-30	Spectrometer analysis	2	71
5126/55-68	Crystal structure calculations	5	2
5126/55-97	High temperature properties of air	3	191
5126/55-121	Electron penetration	4	10
5126/55-123	Temperature distribution		20
5126/56-144	Auto-correlation		6
0009/56-160	Mathematical subroutines	11	2
5126/56-162	Stresses in a wall resting on a footing		53
5126/56-192	Noise measurement	3	12
5126/57-206	Expected values of order statistics		9
5126/57-210	Sound velocity	3	
5126/57-215	Computation of integrals involving Bessel functions	2	3
5126/57-219	Thermal properties	13	
0002/52-1	Statistical engineering	6	36
	Training (Computation Laboratory)	16	17
	Miscellaneous items	7	10
	Totals:	119	611
OTHER:			
5113/57-216	Handbook of mathematical tables	12	16
5126/54-13	Award of procurement contracts by linear programming	1	36
5126/55-113	Reactor design	3	24
5126/55-127	Vibration of a circular disc	5	65

	Title	Code Checking	Productive Operation
5126/56-136	Calculation of wave functions		7
5126/56-140	Multiple correlation routines	1	13
5126/56-163	Nuclear scattering	1	55
5126/56-184	Transistor amplifier		2
5126/57-221	Bessel function for complex arguments		2
5126/57-222	Roots of polynomials	3	16
5126/57-224	Track-while-scan radar problem	30	57
5126/57-231	Parameter evaluation	4	9
	Classified tasks	30	136
	Miscellaneous items		2
	Totals:	90	440

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Colloquium Series

KNESER, M. (University of Heidelberg, and Institute of Mathematical Sciences, New York University). Orthogonal groups. January 28.

Reliability Seminar

ROSENBLATT, J. R. On prediction of system performance from information on component performance. February 21.

Mathematical Physics Section Seminar

OLSZAK, W. (Polish Academy of Sciences). Plastic non-homogeneity. March 22.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

ANTOSIEWICZ, H. A. (1) Reducible linear systems. Presented before the Department of Mathematics, Catholic University, Washington, D. C., March 29. (2) Some stability problems in nonlinear ordinary differential equations. Presented at a Mathematics Colloquium, Stevens Institute of Technology, Hoboken, N. J., February 27.

CAMERON, J. M. Use of computers in statistical calculations. Presented at the High-Speed Computer Conference, 1957, held at the Louisiana State University, Baton Rouge, La., March 5-8.

CANNON, E. W., and I. R. Rhodes. The early history of electronic digital computer development. Presented before the Local Chapter of the Association for Computing Machinery, San Diego, California, February 21.

CONNOR, W. S. Techniques and rationale of multiple comparisons. Presented at a meeting of the Washington Statistical Society, Washington, D. C., February 18.

EISENHART, C. E. Recognition of basic elements of a career program for scientific and technical personnel. Presented at the Fourth Annual Meeting of the Associated Boards for Research and Development Personnel, held at the U. S. Naval Ordnance Laboratory, White Oak, Md., March 22.

GHAFFARI, A. Rayleigh equation in the projective plane. Presented at a Mathematics Seminar, Princeton University, Princeton, N. J., March 6.

GOLDMAN, A. J. On games with random payoff matrices. Presented at the Third Conference on Games, held by the Office of Naval Research Logistics Project, Princeton University, Princeton, N. J., March 11.

HOCHSTRASSER, U. W. Numerical experiments in potential theory using Nehari estimates. Presented to the Mathematics Department, University of Kansas, Lawrence, Kansas, January 25.

ROSENBLATT, J. R. On prediction of system performance from information on component performance. Presented at the 1957 Western Joint Computer Conference, Los Angeles, California, February 26-28; at a Statistics Department Seminar, University of California, Berkeley, March 1; and also presented at a Diamond Ordnance Fuze Laboratories Staff Meeting, Washington, D. C., March 26.

SEVERO, N. C. Analysis of variance. Two lectures presented at a meeting of the American Society for Quality Control, Washington, D. C., January 29 and February 1.

TAUSSKY-TODD, O. Commutativity of matrices. Presented at the California Institute of Technology, Pasadena, California, January 14.

TODD, J. Computational problems concerning the Hilbert matrix. Presented at the California Institute of Technology, Pasadena, California, January 15.

WEGSTEIN, J. H. Mathematics and computers. Presented at a meeting of the Washington Junior Academy of Sciences, Washington, D. C., February 16.

YOUDEN, W. J. (1) Statistics in engineering research. Presented at a meeting of the Washington Society for Engineers, Cosmos Club, Washington, D C., January 2. (2) Randomization and experimentation. Presented to the Department of Bio-Statistics, The Johns Hopkins University, Baltimore, Md., January 10. (3) Statistics for skeptics. Presented to the Milwaukee Section, American Society for Quality Control, Milwaukee, Wis., January 21.

(3) Design of experiments in the physical sciences. Two lectures presented at a meeting of the American Society for Quality Control, Washington, D. C. February 12, 19. (5) General introduction to Operations Research. Presented at the Naval Gun Factory, Washington, D. C., February 14. (6) Applications of mathematics in statistics and probability. Presented to the Washington Junior Academy of Sciences, Washington, D. C., February 16.

(7) Statistics for skeptics. Presented at a meeting of the Pittsburgh Section of the American Chemical Society, Pittsburgh, Pa., February 21. (8) Problems of an experimenter. Presented at Smith, Kline and French Laboratories, Research Development Division Seminar, Philadelphia, Pa., March 15; and also at a General Chemistry Seminar, Cornell University, Ithaca, New York, March 21. (9) Statistics-engineering viewpoint. Presented at an Industrial Engineering Seminar, Cornell University. March 22.

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Papers presented at the Eastern Regional Meeting of the Institute of Mathematical Statistics, Washington, D. C , March 8:

SEVERO, N. C. Tests concerning the means of certain distributions.

ZELEN, M. The use of incomplete block designs for asymmetrical factorial arrangements.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.3 Technical Papers

(1) Numerical computation of the transfinite diameter of two collinear line segments. P. Davis. J. Res. NBS 58, 155-156(1957).

(2) A discussion of "Design of corrugated diaphragms" by J. A. Haringx. R. F. Dressler. Trans. Am. Soc. Mech. Eng. 79, 61-62 (1957).

(3) Systems of distinct representatives and linear programming. A. J. Hoffman and H. W. Kuhn (Bryn Mawr College). Amer. Math. Mo. 63, 455-460 (1956).

(4) On the Hilbert matrix. T. Kato. Proc. Amer. Math. Soc. 8, 73-81 (1957).

(5) Solvability and consistency for linear equations and inequalities. H. W. Kuhn (Bryn Mawr College). Amer. Math. Mo. 63, 217-232 (1956).

(6) A note on symmetric functions of eigenvalues. M. Marcus and R. Thompson (University of British Columbia). Duke Math. J. 24, 43-46 (1957).

(7) An inclusion theorem for modular groups. M. Newman. Proc. Amer. Math. Soc. 8, 125-127 (1957).

(8) Some theorems about $P_r(n)$. M. Newman. Canadian J. Math. 9, 68-70 (1957).

(9) Classes of positive definite unimodular circulants. M. Newman and O. Taussky. Canadian J. Math. 9, 71-73 (1957).

(10) On the independence of tests of randomness and other hypotheses. I. R. Savage. J. Amer. Stat. Assoc. 52, 53-57 (1957).

(11) Commuting bilinear transformations and matrices. O. Taussky and J. Todd. J. Wash. Acad. Sci. 46, 373-375 (1956).

(12) A direct approach to the problem of stability in the numerical solution of partial differential equations. J. Todd. Comm. Pure

Appl. Math. (New York University) 9, 597-612 (1956).

(13) Investigations of the properties of corrugated diaphragms. W. A. Wildhack (NBS Office of Basic Instrumentation), R.F. Dressler and E. C. Lloyd (NBS Mechanics Division). Amer. Soc. Mech. Eng. 79, 65-82 (1957).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION MARCH 31, 1957

2.1 Mathematical Tables

(1) Tables of the bivariate normal distribution function and related functions. To appear as NBS Applied Mathematics Series 50.

(2) Table of the first moment of ranked extremes. J. Lieblein and H. E. Salzer. To appear in the Journal of Research, NBS.

2.2 Manuals, Bibliographies, and Indices

(1) Analyzing straight line data. F. S. Acton. To appear as a book in the Applied Statistics Series of the Wiley Publications in Statistics.

(2) Fractional factorial experimental designs for factors at two levels. To appear as NBS Applied Mathematics Series 48.

2.3 Technical Papers

(1) A survey of Lyapunov's second method. H. Antosiewicz. To appear in Annals of Mathematics Studies.

(2) On the identity relationship for fractional replicates of the 2^n series. R. C. Burton and W. S. Connor. To appear in Annals of Mathematical Statistics.

(3) Some examples of the use of high speed computers in statistics. J. Cameron. To appear in the Proceedings of the Office of Ordnance Research Conference on Designs of Experiments in Army Research, Development and Testing, Washington, D. C., October 1955.

(4) Use of computers in statistical calculations. J. Cameron. To appear in the Proceedings of the Fourth Annual High-Speed Computer Conference, 1957, held at the University of Louisiana, Baton Rouge, La., March 5-8, 1957.

(5) Condensation interfaces in two-phase flows. P. Chiarulli and R. F. Dressler. Submitted to a technical journal.

(6) A numerical study of Dedekind's cubic class number formula. H. Cohn. To appear in the Journal of Research, NBS.

(7) A computation of cyclic cubic units. H. Cohn and S. Gorn (Moore School of Engineering). To appear in the Journal of Research, NBS.

(8) Experiences with incomplete block designs: Examples. W. S. Connor. To appear in the Proceedings of the Symposium on the Design of Industrial Experiments, held at the University of North Carolina, Raleigh, N. C.

(9) Uniqueness theory for asymptotic expansions in general regions. P. Davis. To appear in the Pacific Journal of Mathematics.

(10) Complete sequences and approximations in normed linear spaces. P. Davis and K. Fan. To appear in the Duke Mathematical Journal.

(11) Systems of inequalities involving convex functions. Ky Fan, I. Glicksburg (University of Notre Dame), and A. J. Hoffman. To appear in the Proceedings of the American Mathematical Society.

(12) Estimation of the frequencies of thin elastic plates with free edges. H. Fujita, T. Kato, Y. Nakata (University of Tokyo), and M. Newman. To appear in the Journal of Research, NBS.

(13) Pairs of matrices of order two which generate free groups. K. Goldberg and M. Newman. To appear in the Illinois Journal of Mathematics.

(14) Note on bounds for certain determinants. E. Haynsworth. To appear in the Duke Journal of Mathematics.

(15) On the domain of regularity of generalized axially symmetric potentials. P. Henrici. To appear in the Proceedings of the American Mathematical Society.

(16) On the representation of a certain integral involving Bessel functions by hypergeometric series. P. Henrici. To appear in the Journal of Mathematics and Physics.

(17) Mechanized computation of thermodynamics tables at the National Bureau of Standards. J. Hilsenrath (NBS Thermodynamics Section) and J. Wegstein. To appear in the Proceedings of the Joint Conference on Thermodynamic Transport Properties of Fluids, sponsored by the Institute of Mechanical Engineers, London, 1957.

(18) Numerical experiments in potential theory using the Nehari estimates. U. W. Hochstrasser. Submitted to a technical journal.

(19) Estimation of the frequencies of thin elastic plates with free edges. T. Kato, H. Fujita, Y. Nakata, and M. Newman. To appear in the Journal of Research, NBS.

(20) A determintal inequality of H. P. Robertson,II. M. Marcus. Submitted to a technical journal.

(21) A note on values of a quadratic form. M. Marcus. Submitted to a technical journal.

(22) On a determinantal inequality. M. Marcus. Submitted to a technical journal.

(23) On subdeterminants of doubly stochastic matrices. M. Marcus. To appear in the Illinois Journal of Mathematics.

(24) Some extreme value results for indefinite Hermitian matrices. M. Marcus, B. N. Moyles and R. Westwick (University of British Columbia). To appear in the Illinois Journal of Mathematics.

(25) Congruences for the coefficients of modular forms and some new congruences for the partition function. M. Newman. Submitted to a technical journal.

(26) Construction and application of a class of modular functions. M. Newman. To appear in the Proceedings of the London Mathematical Society.

(27) On some expansions for Bessel integral functions. F. Oberhettinger. To appear in the Journal of Research, NBS.

(28) The number of representations of a quadratic form as a sum of four squares. G. Pall (Illinois Institute of Technology) and O. Taussky. To appear in the Proceedings of the Royal Irish Academy.

(29) Graphical solution of the single degree of freedom vibration problem with arbitrary damping and restoring forces. W. H. Pell. To appear in the Journal of Applied Mechanics.

(30) On prediction of system performance from information on component performance. J. R. Rosenblatt. To appear in the Proceedings of the 1957 Western Joint Computer Conference, Los Angeles, California.

(31) Generation of Bessel functions on high speed computers. I. Stegun. and M. Abramowitz. Submitted to a technical journal.

(32) Pitfalls in computation. I. A. Stegun and M. Abramowitz. To appear in the Journal of the Society for Industrial and Applied Mathematics.

(33) The transmission of Rayleigh waves across an ocean floor with two surface layers. R. Stoneley. To appear in the Bulletin of the

Seismological Society of America.

(34) The attenuation of Rayleigh waves with depth in a medium with two surface layers. R. Stoneley and U. Hochstrasser. To appear in the Geophysical Supplement, Monthly Notices, Royal Astronomical Society.

(35) Elastic waves in anisotropic media. J. L. Synge. Submitted to a technical journal.

(36) Stationary principles for forced vibrations in elasticity and electromagnetism. J. L. Synge. To appear in the Proceedings of the Eighth Symposium in Applied Mathematics held by the American Mathematical Society, Chicago, Ill., April 1956.

(37) The torsion of a hollow square. J. L. Synge and W. F. Cahill. Submitted to a technical journal.

(38) A determinantal inequality of H. P. Robertson, I. O. Taussky. Submitted to a technical journal.

(39) Classes of matrices. O. Taussky. To appear in the Illinois Journal of Mathematics.

(40) Commutativity in finite matrices. O. Taussky. To appear in the American Mathematical Monthly.

(41) Some computational problems concerning integral matrices. O. Taussky. To appear in the 1956 Meeting of the Italian Society for the Advancement of Science, held in Sicily.

(42) Computational problems concerned with the Hilbert matrix. J. Todd. To appear in the Proceedings of the 1956 Meeting of the Italian Society for the Advancement of Science, held in Sicily.

(43) Automatic coding principles. J. Wegstein. To appear in the Proceedings of a Symposium on Automatic Programming, held by the Office of Naval Research, Washington, D. C., June 1956.

(44) Design of experiments in research and development. W. J. Youden. To appear in the Proceedings of the Office of Ordnance Research Conference on Design of Experiments in Army Research, Development, and Testing, Washington, D. C., October 1955.

(45) National physical standards and design of experiment. W. J. Youden. To appear in Revue de L'Institut International de Statistique (The Hague).

(46) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.

(47) Statistics--Engineering viewpoint. W J. Youden. To appear in the Journal of Engineering Education.

(48) The analysis of incomplete block designs (formerly entitled: Exact tests of significance for combining intra- and inter-block information). M. Zelen. To appear in the Journal of the American Statistical Association.

(49) The analysis of covariance for incomplete block designs. M. Zelen. To appear in Biometrics.

(50) The use of incomplete block designs for asymmetrical factorial arrangements. M. Zelen. Submitted to a technical journal.

(51) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. S. Joel. Submitted to a technical journal.

2.5 Miscellaneous

(1) Further contributions to the solution of simultaneous linear equations and the determination of eigenvalues. To appear as Applied Mathematics Series 49.

U. S. DEPARTMENT OF COMMERCE
Sinclair Weeks, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major field laboratories in Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside front cover of this report.

WASHINGTON, D. C.

Electricity and Electronics. Resistance and Reactance. Electron Tubes. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat and Power. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology and Lubrication. Engine Fuels.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Nuclear Physics. Radioactivity. X-rays. Betatron. Nucleonic Instrumentation. Radiological Equipment. AEC Radiation Instruments.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Gas Chemistry. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Organic Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metals Physics.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Concreting Materials. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Heating and Air Conditioning. Floor, Roof, and Wall Coverings. Codes and Specifications.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analogue Systems. Application Engineering.

- Office of Basic Instrumentation
- Office of Weights and Measures

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Systems. Navigation Systems. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Radio Systems Application Engineering.

Radio Standards. Radio Frequencies. Microwave Frequencies. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Calibration Center. Microwave Physics. Microwave Circuit Standards.


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