# NATIONAL BUREAU OF STANDARDS REPORT

5199

## FRACTIONAL FACTORIAL EXPERIMENT DESIGNS FOR FACTORS AT THREE LEVELS

By the Statistical Engineering Laboratory

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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**NBS PROJECT** **NBS REPORT**

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## FRACTIONAL FACTORIAL EXPERIMENT DESIGNS FOR FACTORS AT THREE LEVELS

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# P R E F A C E

The experiment plans presented here make available a collection of fractional factorial designs in which each factor is at three levels. It is hoped that the publication of this volume will accelerate the application of fractional factorial designs in experimental work. This project was carried out in the Statistical Engineering Laboratory of the National Bureau of Standards under contract with the Bureau of Ships, Department of the Navy. The work was performed under the direction of W. S. Connor and M. Zelen. The experiment plans were constructed by R. C. Burton, Lola S. Deming, F. L. Miller, Jr., and H. M. Pettigrew. Lola S. Deming supervised the preparation of the manuscript in its final form. All of these experiment plans were checked by J. M. Cameron and M. Carroll Dannemiller on the National Bureau of Standard's electronic computer SEAC.

# FRACTIONAL FACTORIAL EXPERIMENT DESIGNS FOR FACTORS AT THREE LEVELS

1. Introduction. One of the most difficult types of experiments to carry out is that which requires the experimenter to jointly evaluate the characteristics of several factors with respect to some material or process. Often the number of factors is so large that it is neither economical nor feasible to study all combinations of them. For these cases, fractional factorial experiment plans can be used to enable the experimenter to choose a fraction of the possible factorial combinations. Study of this smaller number of combinations may, in many instances, contain enough information to fulfill the original experimental objectives. In still other experimental situations it may not be practical to plan an entire experimental program in advance, but rather to conduct a sequence of smaller experiments. For these cases, particularly efficient use can be made of fractional factorial experiment plans. This monograph contains a collection of fractional factorial experiment designs, sometimes termed fractional replicates, for the case when all factors are at three levels. This catalogue can be regarded as a sequel to Fractional Factorial Experiment Designs for Factors at Two Levels [1].

The technique for analyzing fractional factorial plans is discussed in the books by Davies [2] and Kempthorne [3]. The relation of fractional factorials to the theory of groups is discussed by Finney [5] and Kempthorne [4]; their relation to orthogonal arrays and hypercubes is given by Rao [6]. Other tabulations of fractional factorials for the $3^n$ series can be found in Davies [2], and Kitagawa and Mitome [7].

2. Description of Experiment Plans. A factorial experiment involving n factors, each at three levels, results in $3^n$ different factorial combinations. A fractional factorial requiring only $1/3^p \times 3^n = 3^{n-p}$, $n > p$, of these combinations is termed a $1/3^p$ fractional factorial design. The fractional replicates catalogued in this monograph are for the fractions 1/3, 1/9, 1/27, 1/81, 1/243 and for n ranging from 4 to 10 factors. These experiment plans have been constructed so that the treatment combinations are grouped into blocks. Use of the blocks permits the experimenter to take advantage of any homogeneous grouping of the experimental material, and will often result in greater precision of the comparisons among the various factors.

Each plan has a designation r.n.k. which denotes that the plan is a 1/r replicate ($r=3^p$, $p=1,2,3,4,5$) of the $3^n$ factorial system having $b = 3^n/rk = 3^s$ blocks of $k=3^{n-p-s}$ treatments each. For example, plan 3.4.9 refers to a 1/3 replicate of a $3^4$ factorial design having 3 blocks of 9 units each.

In all plans, the $n$ factors are denoted by capital letters A,B,C,..., except for I which is not used to denote a factor. The factorial treatment combinations are designated by lower case letters having the exponent 0, 1, or 2, i.e., $a^{x_1}b^{x_2}c^{x_3}\ldots$ $(x_i = 0,1,2)$, where the exponent corresponds to one of the three different levels associated with that particular factor. For example with respect to factor A, the three levels are given by 1, a, $a^2$, where $a^0=1$ is simply used as a unity element. Thus, for an experiment involving the factors A,B,C,D, the treatment combination $a^2cd$ indicates A at level 2, B at level 0, and C and D at level 1. The factorial combination where all factors are at the $0^{th}$ level is denoted by (1).

The experiment plans in this monograph may be regarded as grouped in families where all designs belonging to a particular family have the same fraction and the same number of factors, but differ only with respect to the number of treatments within a block. The experimental layout for the plan having the smallest number of units per block is always written out in full. Generally, the other members of the family can be obtained from this initial design by re-grouping the blocks of the initial design. Occasionally there is a second experiment plan, other than that with the smallest block size, which has the complete experimental layout written out

in full. This situation arises when it is not possible to obtain an "optimum" design from the plan having the smallest block size. For example, the family of 9 factors having a 1/81 replicate has the treatment combinations written out in full for Plan 81.9.9 and also for Plan 81.9.27. Although a plan for 9 blocks of 27 treatments each can be obtained from Plan 81.9.9, a plan resulting in more information is given by Plan 81.9.27.

In addition to using the capital letters to represent the various factors, the capital letters are also used to represent the various main effects and interactions associated with the respective factors. Main effects having 2 degrees of freedom are designated by capital letters alone. Two factor interactions will have 4 degrees of freedom and can be split into two orthogonal sums of squares, each carrying 2 degrees of freedom. With respect to factors A and B, the two mutually orthogonal parts of the two factor interaction are denoted by AB and $AB^2$. In general if $F_1, F_2, \ldots, F_s$ represents an appropriate selection of the factors A,B,C,..., then the interaction between the s factors will have $2^s$ degrees of freedom and can be split into $2^{s-1}$ mutually orthogonal sums of squares each containing 2 degrees of freedom. These are

designated by

$$F_1 F_2^{\lambda_2} F_3^{\lambda_3} \cdots F_s^{\lambda_s}$$

where the exponents take the value 1 or 2, and the $2^{s-1}$ different sets are generated by considering all combinations of $(\lambda_2, \lambda_3, \ldots, \lambda_s)$ as exponents. Note that the first letter in every interaction always appears with an exponent of unity.

3. Loss of information. The reduction in the number of observations is accomplished at the expense of "losing" information on the main effects and interactions. This loss of information results in all main effects and interactions being entangled or aliased with other main effects or interactions. That is, a particular estimate will serve as the estimate of several of the parameters in the underlying mathematical model describing the experimental situation. However, since in many experiments, interactions involving three or more factors (second or higher-order interactions) can be considered negligible, the fractional designs in this catalogue have been constructed so as (1) to have all main effects aliased with three-factor and higher-order interactions and (2) to have as many two-factor interactions as possible confounded (aliased) with three-factor and higher-order interactions. Two-factor interactions which are only

aliased with higher order interactions are termed measurable.*

Listed with every experimental plan is information describing the measurable two-factor interactions. If the fractional design is used as a completely randomized design (without regard to blocking) the measurable two-factor interactions are given under the heading "Completely randomized." This information is given (for a particular $r$ and $n$) with the design having minimum block size. For example, in a 1/3 replicate of the $3^4$ factorial design the information is found with Plan 3.4.3.

When treatments are grouped into blocks, this usually entails loss of additional information on interactions. The additional loss of information on interactions results in certain interactions and their aliases being confounded with the blocks of the experiment. For the case of blocking, the information on measurable two-factor interactions is found under the heading "With blocks."

Associated with the $1/3^p$ fractional design is a fundamental identity which consists of the symbol I and $(3^p-1)/2$ groups of letters connected by equal signs, i.e.,

$$I = A^{a_1}B^{b_1}C^{c_1} \ldots = A^{a_2}B^{b_2}C^{c_2} \ldots = \ldots = A^{a_t}B^{b_t}C^{c_t} \ldots$$

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* Although considerable effort was made to find solutions which have the maximum number of two-factor interactions confounded with three-factor and higher-order interactions, other solutions may exist having a larger number of measurable two-factor interactions.

where $t = (3^p-1)/2$ and the $a_i, b_j, c_k, \ldots$ take on values 0,1, or 2. It will be convenient to call a group of such letters a "word". The words in the fundamental identity are such that the first letter of every word always has unity as an exponent, and a letter having a zero exponent is omitted from the word, or can be regarded as being a unity element, i.e., $A^0 = B^0 = C^0 = \ldots 1$.

The fundamental identity for a $1/3^p$ fraction will contain p words which are underlined. These are referred to as the generators of the fundamental identity; they are used to generate all the words in the fundamental identity in the following way. Let $G_1, G_2, \ldots, G_p$ denote the generators and let $(\lambda_1, \lambda_2, \ldots, \lambda_p)$ be a vector of p elements such that each element takes on the value 0,1, or 2, ($\lambda_i = 0,1,2$). Two such vectors $(\lambda_1, \lambda_2, \ldots, \lambda_p)$ and $(\lambda_1', \lambda_2', \ldots, \lambda_p')$ are said to be equivalent if $(\lambda_1, \lambda_2, \ldots, \lambda_p) = (c\lambda_1', c\lambda_2', \ldots, c\lambda_p')$ where c is an integer and the $c\lambda_i'$ are reduced modulo 3. Then all words in the fundamental identity are obtained from

$$G_1^{\lambda_1} G_2^{\lambda_2} \ldots G_p^{\lambda_p}$$

by considering all possible vectors $(\lambda_1, \lambda_2, \ldots, \lambda_p)$, but omitting both the null vector $(0,0,\ldots,0)$ and those vectors which are equivalent. Finally all exponents associated with each letter are reduced modulo 3 such that the first letter

in every word has an exponent of unity. For example consider a $1/3^2$ replicate of a $3^6$ experiment. Here p=2 and the generators* are given by $G_1 = ACDE$, $G_2 = BC^2DE^2F$. The different vectors $(\lambda_1,\lambda_2)$ making up the fundamental identity are (1,0),(0,1),(1,1), (1,2). The equivalent vectors which are not considered are (2,0), (0,2), (2,2), and (2,1) respectively. Thus the words of the fundamental identity are

$$I = G_1 = G_2 = G_1G_2 = G_1G_2^2$$

$$= \underline{ACDE} = \underline{BC^2DE^2F} = ABD^2F = AB^2C^2E^2F^2$$

The fundamental identity serves two purposes: (1) it is used to select the appropriate sub-set of treatments from the full factorial and, (2) it determines the manner in which the various main effects and interactions are aliased with one another as a consequence of taking only a sub-set of measurements from the full factorial. If the p generators of a $1/3^p$ fractional replicate are denoted by

$$G_i = A^{a_i} B^{b_i} C^{c_i} \ldots \qquad i=1,2,\ldots,p$$

then the factorial combinations $a^{x_1}b^{x_2}c^{x_3}\ldots$ selected for the $1/3^p$ fraction satisfy simultaneously the p equations

* The generators listed for the fundamental identities in this catalogue are not unique. In this example, for instance, another set of generators is $G_1G_2$ and $G_1G_2^2$ .

$$a_i x_1 + b_i x_2 + c_i x_3 + \ldots = 0 \pmod 3 \qquad i=1,2,\ldots,p.$$

Now if X represents a main effect or part of the four degrees of freedom associated with a two-factor interaction, then the quantities aliased with X are obtained by multiplying X with each word and with the square of each word in the fundamental identity. Thus, X will be aliased with $3^p-1$ other interactions. Carrying out the formal operation of multiplication results in

$$X = XA^{a_1}B^{b_1}C^{c_1}\ldots = XA^{a_2}B^{b_2}C^{c_2}\ldots = \ldots = XA^{a_t}B^{b_t}C^{c_t}\ldots$$

$$= XA^{2a_1}B^{2b_1}C^{2c_1}\ldots = XA^{2a_2}B^{2b_2}C^{2c_2}\ldots = \ldots = XA^{2a_t}B^{2b_t}C^{2c_t}\ldots$$

where all exponents are reduced modulo 3 and the leading letter of every word has an exponent of unity. Then X is said to be aliased with those main effects and interactions set equal to X, i.e.,

$$XA^{a_1}B^{b_1}C^{c_1}\ldots,\ XA^{a_2}B^{b_2}C^{c_2}\ldots,\ \ldots,\ XA^{2a_t}B^{2b_t}C^{2c_t}\ldots$$

In order to arrange the $3^{n-p}$ treatment combinations into $3^s$ blocks of $3^{n-p-s}$ treatments each, it is necessary to confound $(3^s-1)/2$ additional interaction components and their aliases with the blocks. With respect to any plan, the $(3^s-1)/2$ interactions used to arrange the treatments into blocks are

found under the heading "Block confounding". Note that s of these interactions are underlined. These are the generators of the block confounding interactions from which all other $(3^s-1)/2$ block confounding interactions are obtained. These are generated in the same way as are the words in the fundamental identity. Thus, if $W_1, W_2, \ldots, W_s$ represent the s block generators of a plan having $3^s$ blocks, the $(3^s-1)/2$ block confounding interactions are generated by considering all combinations of $(\lambda_1, \lambda_2, \ldots, \lambda_s)$ with respect to $W_1^{\lambda_1} W_2^{\lambda_2} \ldots W_s^{\lambda_s}$, but omitting both the equivalent and null vectors and reducing all exponents modulo 3.

Now if a block generator is of the form

$$W_i = A^{a_i} B^{b_i} C^{c_i} \ldots \qquad i=1,2,\ldots,s$$

the $3^{n-p}$ treatments satisfying the fundamental identity are arranged in s blocks such that the treatments in the $g^{th}$ block satisfy the s simultaneous equations

$$\begin{aligned}
a_1x_1 + b_1x_2 + c_1x_3 + \ldots &= \alpha_{1g} \pmod 3 \\
a_2x_1 + b_2x_2 + c_2x_3 + \ldots &= \alpha_{2g} \pmod 3 \\
&\vdots \qquad\qquad \vdots \qquad\quad \vdots \\
a_sx_1 + b_sx_2 + c_sx_3 + \ldots &= \alpha_{sg} \pmod 3
\end{aligned}$$

where the $\alpha_{ig} = 0, 1, \text{ or } 2$ for $i = 1,2,\ldots,s$. The $3^s$ blocks are obtained by considering all possible combinations of $(\alpha_{1g},\alpha_{2g},\ldots,\alpha_{sg})$ over the numbers 0,1,2.

To illustrate the use of the fundamental identity and the block confounding relationships, consider Plan 9.6.3 which is a 1/9 replicate of a $3^6$ factorial design in 27 blocks of 3 treatments each. The full factorial requires $3^6 = 729$ treatments for one complete replication. These are given in Table 1. The fundamental identity is

$$I = \underline{ACDE} = \underline{BC^2DE^2F} = ABD^2F = AB^2C^2E^2F^2 \ .$$

The treatments in parentheses in Table 1 are the 81 treatments which satisfy the two simultaneous equations

$$x_1 + x_3 + x_4 + x_5 = 0 \qquad \text{(mod 3)}$$

$$x_2 + 2x_3 + x_4 + 2x_5 + x_6 = 0 \qquad \text{(mod 3)} \ .$$

In order to determine the aliases of any main effect or interaction, one must multiply all the words and the square of the words in the fundamental identity by that particular main effect or interaction. Table 2 lists the aliases of all the main effects and interactions such that the leading exponent of every word is unity and all exponents are reduced modulo 3.

Table 1

THE COMPLETE REPLICATION OF A $3^6$ FACTORIAL DESIGN

Treatments of the 1/9 replicate are shown in parentheses

$A_o$

| | | $B_o$ | | | $B_1$ | | | $B_2$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $C_o$ | $C_1$ | $C_2$ | $C_o$ | $C_1$ | $C_2$ | $C_o$ | $C_1$ | $C_2$ |
| $D_o$ | $E_o$ $F_o$ | (1) | c | $c^2$ | b | bc | $bc^2$ | $b^2$ | $b^2c$ | $b^2c^2$ |
| | $E_o$ $F_1$ | f | cf | $c^2f$ | bf | bcf | $bc^2f$ | $(b^2f)$ | $b^2cf$ | $b^2c^2f$ |
| | $E_o$ $F_2$ | $f^2$ | $cf^2$ | $c^2f^2$ | $(bf^2)$ | $bcf^2$ | $bc^2f^2$ | $b^2f^2$ | $b^2cf^2$ | $b^2c^2f^2$ |
| | $E_1$ $F_o$ | e | ce | $(c^2e)$ | be | bce | $bc^2e$ | $b^2e$ | $b^2ce$ | $b^2c^2e$ |
| | $E_1$ $F_1$ | ef | cef | $c^2ef$ | bef | bcef | $bc^2ef$ | $b^2ef$ | $b^2cef$ | $(b^2c^2ef)$ |
| | $E_1$ $F_2$ | $ef^2$ | $cef^2$ | $c^2ef^2$ | $bef^2$ | $bcef^2$ | $(bc^2ef^2)$ | $b^2ef^2$ | $b^2cef^2$ | $b^2c^2ef^2$ |
| | $E_2$ $F_o$ | $e^2$ | $(ce^2)$ | $c^2e^2$ | $be^2$ | $bce^2$ | $bc^2e^2$ | $b^2e^2$ | $b^2ce^2$ | $b^2c^2e^2$ |
| | $E_2$ $F_1$ | $e^2f$ | $ce^2f$ | $c^2e^2f$ | $be^2f$ | $bce^2f$ | $bc^2e^2f$ | $b^2e^2f$ | $(b^2ce^2f)$ | $b^2c^2e^2f$ |
| | $E_2$ $F_2$ | $e^2f^2$ | $ce^2f^2$ | $c^2e^2f^2$ | $be^2f^2$ | $(bce^2f^2)$ | $bc^2e^2f^2$ | $b^2e^2f^2$ | $b^2ce^2f^2$ | $b^2c^2e^2f^2$ |
| $D_1$ | $E_o$ $F_o$ | d | cd | $c^2d$ | bd | bcd | $(bc^2d)$ | $b^2d$ | $b^2cd$ | $b^2c^2d$ |
| | $E_o$ $F_1$ | df | cdf | $(c^2df)$ | bdf | bcdf | $bc^2df$ | $b^2df$ | $b^2cdf$ | $b^2c^2df$ |
| | $E_o$ $F_2$ | $df^2$ | $cdf^2$ | $c^2df^2$ | $bdf^2$ | $bcdf^2$ | $bc^2df^2$ | $b^2df^2$ | $b^2cdf^2$ | $(b^2c^2df^2)$ |
| | $E_1$ $F_o$ | de | cde | $c^2de$ | bde | (bcde) | $bc^2de$ | $b^2de$ | $b^2cde$ | $b^2c^2de$ |
| | $E_1$ $F_1$ | def | (cdef) | $c^2def$ | bdef | bcdef | $bc^2def$ | $b^2def$ | $b^2cdef$ | $b^2c^2def$ |
| | $E_1$ $F_2$ | $def^2$ | $cdef^2$ | $c^2def^2$ | $bdef^2$ | $bcdef^2$ | $bc^2def^2$ | $b^2def^2$ | $(b^2cdef^2)$ | $b^2c^2def^2$ |
| | $E_2$ $F_o$ | $de^2$ | $cde^2$ | $c^2de^2$ | $(bde^2)$ | $bcde^2$ | $bc^2de^2$ | $b^2de^2$ | $b^2cde^2$ | $b^2c^2de^2$ |
| | $E_2$ $F_1$ | $(de^2f)$ | $cde^2f$ | $c^2de^2f$ | $bde^2f$ | $bcde^2f$ | $bc^2de^2f$ | $b^2de^2f$ | $b^2cde^2f$ | $b^2c^2de^2f$ |
| | $E_2$ $F_2$ | $de^2f^2$ | $cde^2f^2$ | $c^2de^2f^2$ | $bde^2f^2$ | $bcde^2f^2$ | $bc^2de^2f^2$ | $(b^2de^2f^2)$ | $b^2cde^2f^2$ | $b^2c^2de^2f^2$ |
| $D_2$ | $E_o$ $F_o$ | $d^2$ | $cd^2$ | $c^2d^2$ | $bd^2$ | $bcd^2$ | $bc^2d^2$ | $b^2d^2$ | $(b^2cd^2)$ | $b^2c^2d^2$ |
| | $E_o$ $F_1$ | $d^2f$ | $cd^2f$ | $c^2d^2f$ | $bd^2f$ | $(bcd^2f)$ | $bc^2d^2f$ | $b^2d^2f$ | $b^2cd^2f$ | $b^2c^2d^2f$ |
| | $E_o$ $F_2$ | $d^2f^2$ | $(cd^2f^2)$ | $c^2d^2f^2$ | $bd^2f^2$ | $bcd^2f^2$ | $bc^2d^2f^2$ | $b^2d^2f^2$ | $b^2cd^2f^2$ | $b^2c^2d^2f^2$ |
| | $E_1$ $F_o$ | $d^2e$ | $cd^2e$ | $c^2d^2e$ | $bd^2e$ | $bcd^2e$ | $bc^2d^2e$ | $(b^2d^2e)$ | $b^2cd^2e$ | $b^2c^2d^2e$ |
| | $E_1$ $F_1$ | $d^2ef$ | $cd^2ef$ | $c^2d^2ef$ | $(bd^2ef)$ | $bcd^2ef$ | $bc^2d^2ef$ | $b^2d^2ef$ | $b^2cd^2ef$ | $b^2c^2d^2ef$ |
| | $E_1$ $F_2$ | $(d^2ef^2)$ | $cd^2ef^2$ | $c^2d^2ef^2$ | $bd^2ef^2$ | $bcd^2ef^2$ | $bc^2d^2ef^2$ | $b^2d^2ef^2$ | $b^2cd^2ef^2$ | $b^2c^2d^2ef^2$ |
| | $E_2$ $F_o$ | $d^2e^2$ | $cd^2e^2$ | $c^2d^2e^2$ | $bd^2e^2$ | $bcd^2e^2$ | $bc^2d^2e^2$ | $b^2d^2e^2$ | $b^2cd^2e^2$ | $(b^2c^2d^2e^2)$ |
| | $E_2$ $F_1$ | $d^2e^2f$ | $cd^2e^2f$ | $c^2d^2e^2f$ | $bd^2e^2f$ | $bcd^2e^2f$ | $(bc^2d^2e^2f)$ | $b^2d^2e^2f$ | $b^2cd^2e^2f$ | $b^2c^2d^2e^2f$ |
| | $E_2$ $F_2$ | $d^2e^2f^2$ | $cd^2e^2f^2$ | $(c^2d^2e^2f^2)$ | $bd^2e^2f^2$ | $bcd^2e^2f^2$ | $bc^2d^2e^2f^2$ | $b^2d^2e^2f^2$ | $b^2cd^2e^2f^2$ | $b^2c^2d^2e^2f^2$ |

$A_1$

| | | $B_o$ | | | $B_1$ | | | $B_2$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $C_o$ | $C_1$ | $C_2$ | $C_o$ | $C_1$ | $C_2$ | $C_o$ | $C_1$ | $C_2$ |
| $D_o$ | $E_o$ $F_o$ | a | ac | $ac^2$ | ab | abc | $abc^2$ | $ab^2$ | $ab^2c$ | $(ab^2c^2)$ |
| | $E_o$ $F_1$ | af | acf | $ac^2f$ | abf | abcf | $(abc^2f)$ | $ab^2f$ | $ab^2cf$ | $ab^2c^2f$ |
| | $E_o$ $F_2$ | $af^2$ | $acf^2$ | $(ac^2f^2)$ | $abf^2$ | $abcf^2$ | $abc^2f^2$ | $ab^2f^2$ | $ab^2cf^2$ | $ab^2c^2f^2$ |
| | $E_1$ $F_o$ | ae | ace | $ac^2e$ | abe | abce | $abc^2e$ | $ab^2e$ | $(ab^2ce)$ | $ab^2c^2e$ |
| | $E_1$ $F_1$ | aef | acef | $ac^2ef$ | abef | (abcef) | $abc^2ef$ | $ab^2ef$ | $ab^2cef$ | $ab^2c^2ef$ |
| | $E_1$ $F_2$ | $aef^2$ | $(acef^2)$ | $ac^2ef^2$ | $abef^2$ | $abcef^2$ | $abc^2ef^2$ | $ab^2ef^2$ | $ab^2cef^2$ | $ab^2c^2ef^2$ |
| | $E_2$ $F_o$ | $ae^2$ | $ace^2$ | $ac^2e^2$ | $abe^2$ | $abce^2$ | $abc^2e^2$ | $(ab^2e^2)$ | $ab^2ce^2$ | $ab^2c^2e^2$ |
| | $E_2$ $F_1$ | $ae^2f$ | $ace^2f$ | $ac^2e^2f$ | $(abe^2f)$ | $abce^2f$ | $abc^2e^2f$ | $ab^2e^2f$ | $ab^2ce^2f$ | $ab^2c^2e^2f$ |
| | $E_2$ $F_2$ | $(ae^2f^2)$ | $ace^2f^2$ | $ac^2e^2f^2$ | $abe^2f^2$ | $abce^2f^2$ | $abc^2e^2f^2$ | $ab^2e^2f^2$ | $ab^2ce^2f^2$ | $ab^2c^2e^2f^2$ |
| $D_1$ | $E_o$ $F_o$ | ad | (acd) | $ac^2d$ | abd | abcd | $abc^2d$ | $ab^2d$ | $ab^2cd$ | $ab^2c^2d$ |
| | $E_o$ $F_1$ | adf | acdf | $ac^2df$ | abdf | abcdf | $abc^2df$ | $ab^2df$ | $(ab^2cdf)$ | $ab^2c^2df$ |
| | $E_o$ $F_2$ | $adf^2$ | $acdf^2$ | $ac^2df^2$ | $abdf^2$ | $(abcdf^2)$ | $abc^2df^2$ | $ab^2df^2$ | $ab^2cdf^2$ | $ab^2c^2df^2$ |
| | $E_1$ $F_o$ | (ade) | acde | $ac^2de$ | abde | abcde | $abc^2de$ | $ab^2de$ | $ab^2cde$ | $ab^2c^2de$ |
| | $E_1$ $F_1$ | adef | acdef | $ac^2def$ | abdef | abcdef | $abc^2def$ | $(ab^2def)$ | $ab^2cdef$ | $ab^2c^2def$ |
| | $E_1$ $F_2$ | $adef^2$ | $acdef^2$ | $ac^2def^2$ | $(abdef^2)$ | $abcdef^2$ | $abc^2def^2$ | $ab^2def^2$ | $ab^2cdef^2$ | $ab^2c^2def^2$ |
| | $E_2$ $F_o$ | $ade^2$ | $acde^2$ | $(ac^2de^2)$ | $abde^2$ | $abcde^2$ | $abc^2de^2$ | $ab^2de^2$ | $ab^2cde^2$ | $ab^2c^2de^2$ |
| | $E_2$ $F_1$ | $ade^2f$ | $acde^2f$ | $ac^2de^2f$ | $abde^2f$ | $abcde^2f$ | $abc^2de^2f$ | $ab^2de^2f$ | $ab^2cde^2f$ | $(ab^2c^2de^2f)$ |
| | $E_2$ $F_2$ | $ade^2f^2$ | $acde^2f^2$ | $ac^2de^2f^2$ | $abde^2f^2$ | $abcde^2f^2$ | $(abc^2de^2f^2)$ | $ab^2de^2f^2$ | $ab^2cde^2f^2$ | $ab^2c^2de^2f^2$ |
| $D_2$ | $E_o$ $F_o$ | $ad^2$ | $acd^2$ | $ac^2d^2$ | $abd^2$ | $abcd^2$ | $abc^2d^2$ | $ab^2d^2$ | $ab^2cd^2$ | $ab^2c^2d^2$ |
| | $E_o$ $F_1$ | $(ad^2f)$ | $acd^2f$ | $ac^2d^2f$ | $abd^2f$ | $abcd^2f$ | $abc^2d^2f$ | $ab^2d^2f$ | $ab^2cd^2f$ | $ab^2c^2d^2f$ |
| | $E_o$ $F_2$ | $ad^2f^2$ | $acd^2f^2$ | $ac^2d^2f^2$ | $abd^2f^2$ | $abcd^2f^2$ | $abc^2d^2f^2$ | $(ab^2d^2f^2)$ | $ab^2cd^2f^2$ | $ab^2c^2d^2f^2$ |
| | $E_1$ $F_o$ | $ad^2e$ | $acd^2e$ | $ac^2d^2e$ | $abd^2e$ | $abcd^2e$ | $(abc^2d^2e)$ | $ab^2d^2e$ | $ab^2cd^2e$ | $ab^2c^2d^2e$ |
| | $E_1$ $F_1$ | $ad^2ef$ | $acd^2ef$ | $(ac^2d^2ef)$ | $abd^2ef$ | $abcd^2ef$ | $abc^2d^2ef$ | $ab^2d^2ef$ | $ab^2cd^2ef$ | $ab^2c^2d^2ef$ |
| | $E_1$ $F_2$ | $ad^2ef^2$ | $acd^2ef^2$ | $ac^2d^2ef^2$ | $abd^2ef^2$ | $abcd^2ef^2$ | $abc^2d^2ef^2$ | $ab^2d^2ef^2$ | $ab^2cd^2ef^2$ | $(ab^2c^2d^2ef^2)$ |
| | $E_2$ $F_o$ | $ad^2e^2$ | $acd^2e^2$ | $ac^2d^2e^2$ | $abd^2e^2$ | $(abcd^2e^2)$ | $abc^2d^2e^2$ | $ab^2d^2e^2$ | $ab^2cd^2e^2$ | $ab^2c^2d^2e^2$ |
| | $E_2$ $F_1$ | $ad^2e^2f$ | $(acd^2e^2f)$ | $ac^2d^2e^2f$ | $abd^2e^2f$ | $abcd^2e^2f$ | $abc^2d^2e^2f$ | $ab^2d^2e^2f$ | $(ab^2cd^2e^2f^2)$ | $ab^2c^2d^2e^2f$ |
| | $E_2$ $F_2$ | $ad^2e^2f^2$ | $acd^2e^2f^2$ | $ac^2d^2e^2f^2$ | $abd^2e^2f^2$ | $abcd^2e^2f^2$ | $abc^2d^2e^2f^2$ | $ab^2d^2e^2f^2$ | $ab^2cd^2e^2f^2$ | $ab^2c^2d^2e^2f^2$ |

$A_2$

| | | $B_o$ | | | $B_1$ | | | $B_2$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $C_o$ | $C_1$ | $C_2$ | $C_o$ | $C_1$ | $C_2$ | $C_o$ | $C_1$ | $C_2$ |
| $D_o$ | $E_o$ $F_o$ | $a^2$ | $a^2c$ | $a^2c^2$ | $a^2b$ | $(a^2bc)$ | $a^2bc^2$ | $a^2b^2$ | $a^2b^2c$ | $a^2b^2c^2$ |
| | $E_o$ $F_1$ | $a^2f$ | $(a^2cf)$ | $a^2c^2f$ | $a^2bf$ | $a^2bcf$ | $a^2bc^2f$ | $a^2b^2f$ | $a^2b^2cf$ | $a^2b^2c^2f$ |
| | $E_o$ $F_2$ | $a^2f^2$ | $a^2cf^2$ | $a^2c^2f^2$ | $a^2bf^2$ | $a^2bcf^2$ | $a^2bc^2f^2$ | $a^2b^2f^2$ | $(a^2b^2cf^2)$ | $a^2b^2c^2f^2$ |
| | $E_1$ $F_o$ | $a^2e$ | $a^2ce$ | $a^2c^2e$ | $(a^2be)$ | $a^2bce$ | $a^2bc^2e$ | $a^2b^2e$ | $a^2b^2ce$ | $a^2b^2c^2e$ |
| | $E_1$ $F_1$ | $(a^2ef)$ | $a^2cef$ | $a^2c^2ef$ | $a^2bef$ | $a^2bcef$ | $a^2bc^2ef$ | $a^2b^2ef$ | $a^2b^2cef$ | $a^2b^2c^2ef$ |
| | $E_1$ $F_2$ | $a^2ef^2$ | $a^2cef^2$ | $a^2c^2ef^2$ | $a^2bef^2$ | $a^2bcef^2$ | $a^2bc^2ef^2$ | $(a^2b^2ef^2)$ | $a^2b^2cef^2$ | $a^2b^2c^2ef^2$ |
| | $E_2$ $F_o$ | $a^2e^2$ | $a^2ce^2$ | $a^2c^2e^2$ | $a^2be^2$ | $a^2bce^2$ | $(a^2bc^2e^2)$ | $a^2b^2e^2$ | $a^2b^2ce^2$ | $a^2b^2c^2e^2$ |
| | $E_2$ $F_1$ | $a^2e^2f$ | $a^2ce^2f$ | $(a^2c^2e^2f)$ | $a^2be^2f$ | $a^2bce^2f$ | $a^2bc^2e^2f$ | $a^2b^2e^2f$ | $a^2b^2ce^2f$ | $a^2b^2c^2e^2f$ |
| | $E_2$ $F_2$ | $a^2e^2f^2$ | $a^2ce^2f^2$ | $a^2c^2e^2f^2$ | $a^2be^2f^2$ | $a^2bce^2f^2$ | $a^2bc^2e^2f^2$ | $a^2b^2e^2f^2$ | $a^2b^2ce^2f^2$ | $(a^2b^2c^2e^2f^2)$ |
| $D_1$ | $E_o$ $F_o$ | $a^2d$ | $a^2cd$ | $a^2c^2d$ | $a^2bd$ | $a^2bcd$ | $a^2bc^2d$ | $(a^2b^2d)$ | $a^2b^2cd$ | $a^2b^2c^2d$ |
| | $E_o$ $F_1$ | $a^2df$ | $a^2cdf$ | $a^2c^2df$ | $(a^2bdf)$ | $a^2bcdf$ | $a^2bc^2df$ | $a^2b^2df$ | $a^2b^2cdf$ | $a^2b^2c^2df$ |
| | $E_o$ $F_2$ | $(a^2df^2)$ | $a^2cdf^2$ | $a^2c^2df^2$ | $a^2bdf^2$ | $a^2bcdf^2$ | $a^2bc^2df^2$ | $a^2b^2df^2$ | $a^2b^2cdf^2$ | $a^2b^2c^2df^2$ |
| | $E_1$ $F_o$ | $a^2de$ | $a^2cde$ | $a^2c^2de$ | $a^2bde$ | $a^2bcde$ | $a^2bc^2de$ | $a^2b^2de$ | $a^2b^2cde$ | $(a^2b^2c^2de)$ |
| | $E_1$ $F_1$ | $a^2def$ | $a^2cdef$ | $a^2c^2def$ | $a^2bdef$ | $a^2bcdef$ | $(a^2bc^2def)$ | $a^2b^2def$ | $a^2b^2cdef$ | $a^2b^2c^2def$ |
| | $E_1$ $F_2$ | $a^2def^2$ | $a^2cdef^2$ | $(a^2c^2def^2)$ | $a^2bdef^2$ | $a^2bcdef^2$ | $a^2bc^2def^2$ | $a^2b^2def^2$ | $a^2b^2cdef^2$ | $a^2b^2c^2def^2$ |
| | $E_2$ $F_o$ | $a^2de^2$ | $a^2cde^2$ | $a^2c^2de^2$ | $a^2bde^2$ | $a^2bcde^2$ | $a^2bc^2de^2$ | $a^2b^2de^2$ | $a^2b^2cde^2$ | $a^2b^2c^2de^2$ |
| | $E_2$ $F_1$ | $a^2de^2f$ | $a^2cde^2f$ | $a^2c^2de^2f$ | $a^2bde^2f$ | $(a^2bcde^2f)$ | $a^2bc^2de^2f$ | $a^2b^2de^2f$ | $a^2b^2cde^2f$ | $a^2b^2c^2de^2f$ |
| | $E_2$ $F_2$ | $a^2de^2f^2$ | $(a^2cde^2f^2)$ | $a^2c^2de^2f^2$ | $a^2bde^2f^2$ | $a^2bcde^2f^2$ | $a^2bc^2de^2f^2$ | $a^2b^2de^2f^2$ | $a^2b^2cde^2f^2$ | $a^2b^2c^2de^2f^2$ |
| $D_2$ | $E_o$ $F_o$ | $a^2d^2$ | $a^2cd^2$ | $(a^2c^2d^2)$ | $a^2bd^2$ | $a^2bcd^2$ | $a^2bc^2d^2$ | $a^2b^2d^2$ | $a^2b^2cd^2$ | $a^2b^2c^2d^2$ |
| | $E_o$ $F_1$ | $a^2d^2f$ | $a^2cd^2f$ | $a^2c^2d^2f$ | $a^2bd^2f$ | $a^2bcd^2f$ | $a^2bc^2d^2f$ | $a^2b^2d^2f$ | $a^2b^2cd^2f$ | $(a^2b^2c^2d^2f)$ |
| | $E_o$ $F_2$ | $a^2d^2f^2$ | $a^2cd^2f^2$ | $a^2c^2d^2f^2$ | $a^2bd^2f^2$ | $a^2bcd^2f^2$ | $(a^2bc^2d^2f^2)$ | $a^2b^2d^2f^2$ | $a^2b^2cd^2f^2$ | $a^2b^2c^2d^2f^2$ |
| | $E_1$ $F_o$ | $a^2d^2e$ | $(a^2cd^2e)$ | $a^2c^2d^2e$ | $a^2bd^2e$ | $a^2bcd^2e$ | $a^2bc^2d^2e$ | $a^2b^2d^2e$ | $a^2b^2cd^2e$ | $a^2b^2c^2d^2e$ |
| | $E_1$ $F_1$ | $a^2d^2ef$ | $a^2cd^2ef$ | $a^2c^2d^2ef$ | $a^2bd^2ef$ | $a^2bcd^2ef$ | $a^2bc^2d^2ef$ | $a^2b^2d^2ef$ | $(a^2b^2cd^2ef)$ | $a^2b^2c^2d^2ef$ |
| | $E_1$ $F_2$ | $a^2d^2ef^2$ | $a^2cd^2ef^2$ | $a^2c^2d^2ef^2$ | $a^2bd^2ef^2$ | $(a^2bcd^2ef^2)$ | $a^2bc^2d^2ef^2$ | $a^2b^2d^2ef^2$ | $a^2b^2cd^2ef^2$ | $a^2b^2c^2d^2ef^2$ |
| | $E_2$ $F_o$ | $(a^2d^2e^2)$ | $a^2cd^2e^2$ | $a^2c^2d^2e^2$ | $a^2bd^2e^2$ | $a^2bcd^2e^2$ | $a^2bc^2d^2e^2$ | $a^2b^2d^2e^2$ | $a^2b^2cd^2e^2$ | $a^2b^2c^2d^2e^2$ |
| | $E_2$ $F_1$ | $a^2d^2e^2f$ | $a^2cd^2e^2f$ | $a^2c^2d^2e^2f$ | $a^2bd^2e^2f$ | $a^2bcd^2e^2f$ | $a^2bc^2d^2e^2f$ | $(a^2b^2d^2e^2f)$ | $a^2b^2cd^2e^2f$ | $a^2b^2c^2d^2e^2f$ |
| | $E_2$ $F_2$ | $a^2d^2e^2f^2$ | $a^2cd^2e^2f^2$ | $a^2c^2d^2e^2f^2$ | $(a^2bd^2e^2f^2)$ | $a^2bcd^2e^2f^2$ | $a^2bc^2d^2e^2f^2$ | $a^2b^2d^2e^2f^2$ | $a^2b^2cd^2e^2f^2$ | $a^2b^2c^2d^2e^2f^2$ |

Table 2

Aliases

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $A = AC^2D^2E^2 = ABC^2DE^2F = AB^2DF^2 = ABCEF = CDE = AB^2CD^2EF^2 = BD^2F = BCEF$ | | | | | | | |
| $B = ABCDE = BCD^2EF^2 = AB^2D^2F = AC^2E^2F^2 = AB^2CDE = CD^2EF^2 = AD^2F = ABC^2E^2F^2$ | | | | | | | |
| $C = AC^2DE = BDE^2F = ABCD^2F = AB^2E^2F^2 = ADE = BCDE^2F = ABC^2D^2F = AB^2CE^2F^2$ | | | | | | | |
| $D = ACD^2E = BC^2D^2E^2F = ABF = AB^2C^2DE^2F^2 = ACE = BC^2E^2F = ABDF = AB^2C^2D^2E^2F^2$ | | | | | | | |
| $E = ACDE^2 = BC^2DF = ABD^2EF = AB^2C^2F^2 = ACD = BC^2DEF = ABD^2E^2F = AB^2C^2EF^2$ | | | | | | | |
| $F = ACDEF = BC^2DE^2F^2 = ABD^2F^2 = AB^2C^2E^2 = ACDEF^2 = BC^2DE^2 = ABD^2 = AB^2C^2E^2F$ | | | | | | | |
| $AB = AB^2C^2D^2E^2 = AB^2C^2DE^2F = ABDF^2 = ACEF = BC^2D^2E^2 = ACD^2EF^2 = DF^2 = BC^2E^2F^2$ | | | | | | | |
| $AB^2 = ABC^2D^2E^2 = AC^2DE^2F = ADF^2 = AB^2CEF = BCDE = ABCD^2EF^2 = BDF^2 = CEF$ | | | | | | | |
| $AC = ACD^2E^2 = ABDE^2F = AB^2C^2DF^2 = ABEF = DE = AB^2C^2D^2EF^2 = BC^2D^2F = BC^2EF$ | | | | | | | |
| $AC^2 = AD^2E^2 = ABCDE^2F = AB^2CDF^2 = ABC^2EF = CD^2E^2 = AB^2D^2EF^2 = BCD^2F = BEF$ | | | | | | | |
| $AD = AC^2DE^2 = ABC^2D^2E^2F = AB^2F^2 = ABCD^2EF = CE = AB^2CEF^2 = BDF = BCDEF$ | | | | | | | |
| $AD^2 = AC^2E^2 = ABC^2E^2F = AB^2D^2F^2 = ABCDEF = CD^2E = AB^2CDEF^2 = BF = BCD^2EF$ | | | | | | | |
| $AE = AC^2D^2E = ABC^2DF = AB^2DE^2F^2 = ABCF = CD = AB^2CD^2E^2F^2 = BD^2E^2F = BCE^2F$ | | | | | | | |
| $AE^2 = AC^2D^2 = ABC^2DEF = AB^2DEF^2 = ABCE^2F = CDE^2 = AB^2CD^2F^2 = BD^2EF = BCF$ | | | | | | | |
| $AF = AC^2D^2E^2F^2 = ABC^2DE^2F^2 = AB^2DF = ABCE = CDEF^2 = AB^2CD^2E = BD^2 = BCEF^2$ | | | | | | | |
| $AF^2 = AC^2D^2E^2F = ABC^2DE^2 = AB^2D = ABCEF^2 = CDEF = AB^2CD^2EF = BD^2F^2 = BCE$ | | | | | | | |
| $BC = ABC^2DE = BD^2EF^2 = AB^2CD^2F = AE^2F^2 = AB^2DE = CDE^2F = AC^2D^2F = ABCE^2F^2$ | | | | | | | |
| $BC^2 = ABDE = BC^2D^2EF^2 = AB^2C^2D^2F = ACE^2F^2 = AB^2C^2DE = DE^2F = ACD^2F = ABE^2F^2$ | | | | | | | |
| $BD = ABCD^2E = BCDEF^2 = AB^2F = AC^2DE^2F^2 = AB^2CE = CEF^2 = ADF = ABC^2D^2E^2F^2$ | | | | | | | |
| $BE = ABCDE^2 = BCD^2F^2 = AB^2D^2EF = AC^2F^2 = AB^2CD = CD^2E^2F^2 = AD^2E^2F = ABC^2EF^2$ | | | | | | | |
| $BE^2 = ABCD = BCD^2E^2F^2 = AB^2D^2E^2F = AC^2EF^2 = AB^2CDE^2 = CD^2F^2 = AD^2EF = ABC^2F^2$ | | | | | | | |
| $BF^2 = ABCDEF^2 = BCD^2E = AB^2D^2 = AC^2E^2F = AB^2CDEF = CD^2EF = AD^2F^2 = ABC^2E^2$ | | | | | | | |
| $CD^2 = AC^2E = BE^2F = ABCDF = AB^2D^2E^2F^2 = AD^2E = BCD^2E^2F = ABC^2F = AB^2CDE^2F^2$ | | | | | | | |
| $CE^2 = AC^2D = BDEF = ABCD^2E^2F = AB^2EF^2 = ADE^2 = BCDF = ABC^2D^2EF = AB^2CF^2$ | | | | | | | |
| $CF = AC^2DEF = BDE^2F^2 = ABCD^2F^2 = AB^2E^2 = ADEF^2 = BCDE^2 = ABC^2D^2 = AB^2CE^2F$ | | | | | | | |
| $CF^2 = AC^2DEF^2 = BDE^2 = ABCD^2 = AB^2E^2F = ADEF = BCDE^2F^2 = ABC^2D^2F^2 = AB^2CE^2$ | | | | | | | |
| $DE^2 = ACD^2 = BC^2D^2EF = ABE^2F = AB^2C^2DEF^2 = ACE^2 = BC^2F = ABDEF = AB^2C^2D^2F^2$ | | | | | | | |
| $DF = ACD^2EF = BC^2D^2E^2F^2 = ABF^2 = AB^2C^2DE^2 = ACEF^2 = BC^2E^2 = ABD = AB^2C^2D^2E^2F$ | | | | | | | |
| $EF = ACDE^2F = BC^2DF^2 = ABD^2EF^2 = AB^2C^2 = ACDF^2 = BC^2DE = ABD^2E^2 = AB^2C^2EF$ | | | | | | | |
| $EF^2 = ACDE^2F^2 = BC^2D = ABD^2E = AB^2C^2F = ACDF = BC^2DEF^2 = ABD^2E^2F^2 = AB^2C^2E$ | | | | | | | |
| $ABC = AB^2CD^2E^2 = AB^2DE^2F = ABC^2DF^2 = AEF = BD^2E^2 = AC^2D^2EF^2 = CDF^2 = BCE^2F^2$ | | | | | | | |
| $ABE = AB^2C^2D^2E = AB^2C^2DF = ABDE^2F^2 = ACF = BC^2D^2 = ACD^2E^2F^2 = DEF^2 = BC^2EF^2$ | | | | | | | |
| $BCD = ABC^2D^2E = BDEF^2 = AB^2CF = ADE^2F^2 = AB^2E = CE^2F = AC^2DF = ABCD^2E^2F^2$ | | | | | | | |
| $BDE = ABCD^2E^2 = BCDF^2 = AB^2EF = AC^2DF^2 = AB^2C = CE^2F^2 = ADE^2F = ABC^2D^2EF^2$ | | | | | | | |
| $CDF = AC^2D^2EF = BD^2E^2F^2 = ABCF^2 = AB^2DE^2 = AEF^2 = BCE^2 = ABC^2D = AB^2CD^2E^2F$ | | | | | | | |
| $DEF = ACD^2E^2F = BC^2D^2F^2 = ABEF^2 = AB^2C^2D = ACF^2 = BC^2E = ABDE^2 = AB^2C^2D^2EF$ | | | | | | | |
| $ABE^2 = AB^2C^2D^2 = AB^2C^2DEF = ABDEF^2 = ACE^2F = BC^2D^2E = ACD^2F^2 = DE^2F^2 = BC^2F^2$ | | | | | | | |
| $AC^2F = AD^2E^2F^2 = ABCDE^2F^2 = AB^2CDF = ABC^2E = CD^2E^2F = AB^2D^2E = BCD^2 = BEF^2$ | | | | | | | |
| $AE^2F = AC^2D^2F^2 = ABC^2DEF^2 = AB^2DEF = ABCE^2 = CDE^2F^2 = AB^2CD^2 = BD^2E = BCF^2$ | | | | | | | |
| $BE^2F^2 = ABCDF^2 = BCD^2E^2 = AB^2D^2E^2 = AC^2EF = AB^2CDE^2F = CD^2F = AD^2EF^2 = ABC^2$ | | | | | | | |

For example the aliases of the main effect A are

$$A = AC^2D^2E^2 = ABC^2DE^2F = AB^2DF^2 = ABCEF = CDE = AB^2CD^2EF^2$$
$$= BD^2F = BCEF.$$

Thus, all main effects have interactions composed of at least three factors as aliases, and there are 12 two-factor interaction components that are not measurable. These are AB, AC, AD, $AD^2$, AE, AF, $BD^2$, BF, CD, CE, DE, and $DF^2$, and represent the information one would lose if the fractional replicate is used as a completely randomized design. The interactions AC, BC, $ABC^2$, $AB^2$, BF, ABCF, $BC^2F^2$, $AB^2C^2F$, AF, $AB^2CF^2$, $CF^2$, $AC^2F^2$ and $ABF^2$ are used to arrange the 81 treatments into 27 blocks of 3 units each. These interactions and their aliases are then confounded with the blocks. This results in leaving only 11 two-factor interaction components measurable, i.e., $AC^2$, $AE^2$, $AF^2$, $BC^2$, BD, $BE^2$, $BF^2$, $CD^2$, CF, $DE^2$, and EF. The 3 equations which serve as a basis for placing the 81 treatments into blocks are

$$x_1 + x_3 = \alpha_{1g} \pmod 3$$
$$x_2 + x_3 = \alpha_{2g} \pmod 3 \qquad g=1,2,\ldots,27$$
$$x_2 + x_6 = \alpha_{3g} \pmod 3$$

Thus, $(\alpha_{1g},\alpha_{2g},\alpha_{3g})$ is equal to (0,0,0) for block 1, (0,0,1) for block 2, (0,0,2) for block 3, (0,1,0) for block 4, ..., (2,2,2) for block 27.

## 4. The Analysis of Fractional Factorial Plans.

Let the mathematical model underlying a $3^n$ factorial experiment be

observation = true value + block effect + random error.

This model can be written as

$$(a^{x_1}b^{x_2}c^{x_3}\ldots) = (A^{x_1}B^{x_2}C^{x_3}) + b_j + \epsilon_{x_{123}\ldots}$$

where $(a^{x_1}b^{x_2}c^{x_3}\ldots)$ represents the actual observation of the factorial combination, $(A^{x_1}B^{x_2}C^{x_3}\ldots)$ represents the true value of the observation, $b_j$ $(j=1,2,\ldots,b)$ represents the block effect associated with the block in which the measurement is made, and $\{\epsilon_{x_{123}\ldots}\}$ denotes a sequence of uncorrelated random variables having a mean zero and (unknown) variance $\sigma^2$.

Following Kempthorne [3], define:

$$\mu = (1/3^n) \sum (A^{x_1}B^{x_2}C^{x_3}\ldots) \qquad \text{(Summation is over all } 3^n \text{ factorial combinations)}$$

$$(A)_i = (1/3^{n-1}) \sum_{x_1=i} (A^{x_1}B^{x_2}C^{x_3}\ldots) - \mu \qquad i=0,1,2$$

$$(B)_i = (1/3^{n-1}) \sum_{x_2=i} (A^{x_1}B^{x_2}C^{x_3}\ldots) - \mu \qquad i=0,1,2$$

$$\vdots \qquad\qquad \vdots$$

$$(AB)_i = (1/3^{n-1}) \sum_{x_1+x_2=i \,(\mathrm{mod}\ 3)} (A^{x_1}B^{x_2}C^{x_3}\ldots) - \mu \qquad i=0,1,2$$

$$(AB^2)_i = (1/3^{n-1}) \sum_{x_1+2x_2=i \,(\mathrm{mod}\ 3)} (A^{x_1}B^{x_2}C^{x_3}\ldots) - \mu \qquad i=0,1,2$$

$$(AB^2C^2\ldots)_i = (1/3^{n-1}) \sum_{x_1+2x_2+2x_3\ldots=i \,(\mathrm{mod}\ 3)} (A^{x_1}B^{x_2}C^{x_3}\ldots) - \mu \qquad i=0,1,2$$

where $\sum_{x_1=i}$ sums over all $3^{n-1}$ factorial combinations holding

$x_1=i$ fixed, $\sum_{x_2=i}$ sums over all $3^{n-1}$ factorial combinations

holding $x_2=i$ fixed, ..., $\sum_{x_1+2x_2+2x_3+\ldots=i}$ sums over all $3^{n-1}$ factorial combinations such that $x_1+2x_2+2x_3+\ldots=i$ is fixed. The quantities $(A)_i$, $(B)_i$, $(C)_i$,... are parameters associated with the main effects of factors A, B, C, ..., respectively. The quantities $(AB)_i$, $(AB^2)_i$, $(AC)_i$, $(AC^2)_i$, $(BC)_i$, $(BC^2)_i$, ... are parameters associated with the two factor interactions AB, $AB^2$, AC, $AC^2$, BC, $BC^2$, ..., etc. With the above definitions, the true value of a treatment combination, $(A^{x_1}B^{x_2}C^{x_3}\ldots)$, can be written as a linear function of these parameters, i.e.,

$$(A^{x_1}B^{x_2}C^{x_3}\ldots) = \mu + (A)_{x_1} + (B)_{x_2} + (C)_{x_3} + \ldots$$
$$+ (AB)_{x_1+x_2} + (AB^2)_{x_1+2x_2} + \ldots + (BC)_{x_2+x_3}$$
$$+ (BC^2)_{x_2+2x_3} + \ldots + (ABC)_{x_1+x_2+x_3}$$
$$+ \ldots + (AB^2C^2\ldots)_{x_1+2x_2+2x_3+\ldots}$$

where all indices are reduced modulo 3. For example, if n=3, then

$$(A^{x_1}B^{x_2}C^{x_3}) = \mu + (A)_{x_1} + (B)_{x_2} + (C)_{x_3} + (AB)_{x_1+x_2}$$
$$+ (AB^2)_{x_1+2x_2} + (AC)_{x_1+x_3} + (AC^2)_{x_1+2x_3}$$
$$+ (BC)_{x_2+x_3} + (BC^2)_{x_2+2x_3} + (ABC)_{x_1+x_2+x_3}$$
$$+ (AB^2C)_{x_1+2x_2+x_3} + (ABC^2)_{x_1+x_2+2x_3} + (AB^2C^2)_{x_1+2x_2+2x_3}.$$

Further if $x_1=0$, $x_2=1$, $x_3=2$

$$(BC^2) = \mu + (A)_0 + (B)_1 + (C)_2 + (AB)_1 + (AB^2)_2$$
$$+ (AC)_2 + (AC^2)_1 + (BC)_0 + (BC^2)_2$$
$$+ (ABC)_0 + (AB^2C)_1 + (ABC^2)_2 + (AB^2C^2)_0.$$

Now if the experimental plan is a $1/3^p$ fractional replicate of the $3^n$ factorial experiment, the estimates of the above parameters are

$$\hat{\mu} = 1/3^{n-p} \sum (a^{x_1}b^{x_2}c^{x_3}\ldots) \qquad \text{(Sum over entire } 3^{n-p} \text{ combinations)}$$

$$(\hat{A})_i = 1/3^{n-p-1} \sum_{x_1=i} (a^{x_1}b^{x_2}c^{x_3}\ldots) - \hat{\mu} \qquad (i=0,1,2)$$

$$(\hat{B})_i = 1/3^{n-p-1} \sum_{x_2=i} (a^{x_1}b^{x_2}c^{x_3}\ldots) - \hat{\mu} \qquad (i=0,1,2)$$

$$\vdots \qquad\qquad \vdots$$

$$(\hat{AB})_i = 1/3^{n-p-1} \sum_{x_1+x_2=i} (a^{x_1}b^{x_2}c^{x_3}\ldots) - \hat{\mu} \qquad (i=0,1,2)$$

$$(\hat{AB^2})_i = 1/3^{n-p-1} \sum_{x_1+2x_2=i} (a^{x_1}b^{x_2}c^{x_3}\ldots) - \hat{\mu} \qquad (i=0,1,2)$$

$$\vdots \qquad\qquad \vdots$$

where all summations refer only to those $3^{n-p}$ measurements selected for the $1/3^p$ fractional replicate.

Every estimate will be biased, and the manner in which an estimate is biased depends on all of its aliases. For a $1/3^p$ fractional replicate, let the $(3^p-1)/2$ words in the fundamental identity be denoted by

$$U_1, U_2, \ldots, U_t \quad \text{where} \quad U_j = A^{a_j} B^{b_j} C^{c_j} \ldots \qquad j=1,2,\ldots,t.$$

Let $X = A^{\alpha}B^{\beta}C^{\gamma}\ldots$ be a particular main effect or interaction and let its $(3^p-1)$ aliases be denoted by $XU_1, XU_2, \ldots, XU_t$, $XU_1^2, XU_2^2, \ldots, XU_t^2$ where

$$XU_j = A^{a'_j} B^{b'_j} C^{c'_j} \ldots$$

$$XU_j^2 = A^{a''_j} B^{b''_j} C^{c''_j} \ldots \quad . \qquad (j=1,2,\ldots,t)$$

In all of the above, it is always assumed that the first non-zero exponent is unity. Then the expected value of the estimate of $(X)_i$ is

$$E(\hat{X})_i = (X)_i + \sum_{j=1}^{t} (XU_j)_{\lambda_j} + \sum_{j=1}^{t} (XU_j^2)_{\rho_j} + \sum_{j=1}^{t} (U_j)_0$$

where $\gamma_j$ and $\rho_j$ take on the values 0,1, or 2. It will always be possible to write

$$a'_j x_1 + b'_j x_2 + c'_j x_3 + \ldots = \Theta'_j \, [a_j x_1 + b_j x_2 + c_j x_3 + \ldots]$$

$$+ \phi'_j \, [\alpha x_1 + \beta x_2 + \gamma x_3 + \ldots]$$

where $\Theta'_j$, $\phi'_j$ are equal to 1 or 2, and all coefficients are reduced modulo 3. For particular values of $(x_1, x_2, \ldots)$ $i = \alpha x_1 + \beta x_2 + \gamma x_3 + \ldots \pmod 3$ and $\lambda_j$ is given by

$$\lambda_j = i\phi'_j \pmod 3.$$

Similarly for $XU_j^2$, it is always possible to write

$$a''_j x_1 + b''_j x_2 + c''_j x_3 + \ldots = \Theta''_j \, [a_j x_1 + b_j x_2 + c_j x_3 + \ldots]$$

$$+ \phi''_j \, [\alpha x_1 + \beta x_2 + \gamma x_3 + \ldots]$$

and $\rho_j$ is given by $\rho_j = i\phi''_j$ where $i = (\alpha x_1 + \beta x_2 + \gamma x_3 + \ldots) \pmod 3$.

In general, the expected value of $\hat{\mu}$ is $E(\hat{\mu}) = \mu + \sum_{j=1}^{t} (U_j)_0$.

Also if $x_1=0$, $x_2=0$, $x_3=0$, ..., then $i = \alpha x_1 + \beta x_2 + \gamma x_3 + \ldots = 0$ and $\lambda_j = \rho_j = 0$ for all $j$. Hence

$$E(\hat{X})_0 = \sum_{j=1}^{t} (XU_j)_0 + \sum_{j=1}^{t} (XU_j^2)_0 + \sum_{j=1}^{t} (U_j)_0 .$$

Referring back to our example, i.e., 1/9 replicate of the $3^6$ factorial, consider the aliases of the two-factor interaction X=AB. Here $\alpha$=1, $\beta$=1 and the expected values of $(\hat{AB})_0$, $(\hat{AB})_1$, $(\hat{AB})_2$ are

$$E(\hat{AB})_0 = (AB)_0 + (AB^2C^2D^2E^2)_0 + (AB^2C^2DE^2F)_0 + (ABDF^2)_0$$
$$+ (ACEF)_0 + (BC^2D^2E^2)_0 + (ACD^2EF^2)_0 + (DF^2)_0$$
$$+ (BC^2E^2F^2)_0 + (ACDE)_0 + (BC^2DE^2F)_0$$
$$+ (ABD^2F)_0 + (AB^2C^2E^2F^2)_0$$

$$E(\hat{AB})_1 = (AB)_1 + (AB^2C^2D^2E^2)_2 + (AB^2C^2DE^2F)_1 + (ABDF^2)_2$$
$$+ (ACEF)_2 + (BC^2D^2E^2)_1 + (ACD^2EF^2)_1 + (DF^2)_1$$
$$+ (BC^2E^2F^2)_2 + (ACDE)_0 + (BC^2DE^2F)_0$$
$$+ (ABD^2F)_0 + (AB^2C^2E^2F^2)$$

$$E(\hat{AB})_2 = (AB)_2 + (AB^2C^2D^2E^2)_1 + (AB^2C^2DE^2F)_2 + (ABDF^2)_1$$
$$+ (ACEF)_1 + (BC^2D^2E^2)_2 + (ACD^2EF^2)_2 + (DF^2)_2$$
$$+ (BC^2E^2F^2)_1 + (ACDE)_0 + (BC^2DE^2F)_0$$
$$+ (ABD^2F)_0 + (AB^2C^2E^2F^2)_0 .$$

To show explicitly how one of these indices is obtained, consider $XU_1 = AB^2C^2D^2E^2$ which comes from multiplying $X=AB$ by $U_1=ACDE$. Thus, $x_1+2x_2+2x_3+2x_4+2x_5 = 2(x_1+x_3+x_4+x_5) + 2(x_1+x_2)$, from which $\theta_1'=2$, $\phi_1'=2$. Therefore, $\lambda_1=2(x_1+x_2)$, and is equal to 1 or 2 depending on whether $X_1+X_2=2$ or 1, respectively.

Analysis of Variance. The analysis of variance follows in a straightforward manner, once the estimates of the various main effects or interactions are obtained. If $(\hat{X})_i$ represents the estimate of the parameter $(A^\alpha B^\beta C^\gamma \ldots)_i$ associated with the measurable interaction $A^\alpha B^\beta C^\gamma \ldots$, the appropriate sum of squares having 2 degrees of freedom is given by

$$3^{n-p-1} \sum_{i=0}^{2} (\hat{X})_i^2 \quad .$$

Thus, the sum of squares associated with the main effect of (say) A is given by

$$3^{n-p-1} \sum_{i=0}^{2} (\hat{A})_i^2 \quad ;$$

the sums of squares associated with (say) AB and $AB^2$, each having two degrees of freedom, is

$$3^{n-p-1} \sum_{i=0}^{2} (\hat{AB})_i^2$$

and

$$3^{n-p-1} \sum_{i=0}^{2} (\hat{AB^2})_i^2$$

respectively, etc.

One can make the usual F- or variance-ratio tests on the main effects and measurable two factor interactions, if it is possible to assume (1) that three-factor and higher-order interactions are negligible, and (2) the variance-ratio tests can be justified from randomization principles, cf. Kempthorne [3], or that the $\{\epsilon_{x_{123\ldots}}\}$ follow a normal distribution. Note however, that for a $1/3^p$ fractional replicate there will be $(3^p-1)$ alias terms. Thus even though the higher-order terms may be considered negligible, the large number of such terms may still bias the particular treatment mean square.

Referring to our example, the analysis of variance for the 1/9 replicate of the $3^6$ factorial experiment used as a completely randomized design is shown in Table 3.

Table 3

| Source | Degrees of freedom |
|---|---|
| Main effects | 12 |
| Measurable two-factor interactions | 36 |
| Non-measurable two-factor interactions | 12 |
| Three-factor interactions | 20 |
| Total | 80 |

On the other hand, if the same plan is used with 27 blocks, the analysis of variance breakdown results in a smaller number of measurable two-factor interactions. Table 4 summarizes the appropriate analysis of variance.

Table 4

| Source | Degrees of freedom |
|---|---|
| Blocks | 26 |
| Main effects | 12 |
| Measurable two-factor interactions | 22 |
| Three-factor interactions | 20 |
| Total | 80 |

The estimate of the residual error is supplied in both cases by the mean-square associated with the three-factor interactions having 20 degrees of freedom.

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Plan 3.4.3. 1/3 replication of 4 factors in 9 blocks of 3 units each.

Factors: A, B, C, D.

$I = \underline{ABCD}$.

Completely randomized: The following two-factor interactions are measurable: $AB^2$, $AC^2$, $AD^2$, $BC^2$, $BD^2$, $CD^2$.

Block confounding: $\underline{AB}$, $\underline{AC^2}$, $AB^2C$, $BC$.

With blocking: The following two-factor interactions are measurable: $AB^2$, $AD^2$, $BC^2$, $CD^2$.

Blocks

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| (1) | $c^2d$ | $cd^2$ | $bd^2$ | $bc^2$ |
| $ab^2cd^2$ | $ab^2$ | $ab^2c^2d$ | $acd$ | $ad^2$ |
| $a^2bc^2d$ | $a^2bcd^2$ | $a^2b$ | $a^2b^2c^2$ | $a^2b^2cd$ |

| 6 | 7 | 8 | 9 |
|---|---|---|---|
| $bcd$ | $b^2d$ | $b^2c^2d^2$ | $b^2c$ |
| $ac^2$ | $abc$ | $abd$ | $abc^2d^2$ |
| $a^2b^2d^2$ | $a^2c^2d^2$ | $a^2c$ | $a^2d$ |

Plan 3.4.9. 1/3 replication of 4 factors in 3 blocks of 9 units each.

Factors: A, B, C, D.

$I = \underline{ABCD}$.

Block confounding: $\underline{AB}$.

With blocking: The following two-factor interactions are measurable: $AB^2$, $AC^2$, $AD^2$, $BC^2$, $BD^2$, $CD^2$.

Blocks

Combine blocks of Plan 3.4.3 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1, 2, and 3 | 4, 5, and 6 | 7, 8, and 9 |

Plan 3.5.3. 1/3 replication of 5 factors in 27 blocks of 3 units each.

Factors: A, B, C, D, E.

$I = \underline{ABCDE}$.

Completely randomized: All two-factor interactions are measurable.

Block confounding: $\underline{ABC^2E}$, $\underline{BCDE^2}$, $AB^2D$, $ACD^2E^2$, $\underline{ACD}$, $AB^2D^2E^2$ $ABC^2D^2E^2$, $ABC^2D$, $ACE$, $BCD^2E$, $AB^2E$, $BC$, $DE^2$.

With blocking: All two-factor interactions except $AB$, $AC^2$, $AD^2$, $AE^2$, $BC$, $BD$, $BE$, $CD^2$, $CE^2$, and $DE^2$ are measurable.

Blocks

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| (1) | abe | $a^2b^2e^2$ | $b^2e$ | $ae^2$ | $a^2b$ |
| $a^2bc^2d^2e^2$ | $b^2c^2d^2$ | $ac^2d^2e$ | $a^2c^2d^2$ | $bc^2d^2e$ | $ab^2c^2d^2e^2$ |
| $ab^2cde$ | $a^2cde^2$ | $bcd$ | $abcde^2$ | $a^2b^2cd$ | $cde$ |
| **7** | **8** | **9** | **10** | **11** | **12** |
| $be^2$ | $ab^2$ | $a^2e$ | $c^2d$ | $abc^2de$ | $a^2b^2c^2de^2$ |
| $a^2b^2c^2d^2e$ | $c^2d^2e^2$ | $abc^2d^2$ | $a^2bce^2$ | $b^2c$ | $ace$ |
| $acd$ | $a^2bcde$ | $b^2cde^2$ | $ab^2d^2e$ | $a^2d^2e^2$ | $bd^2$ |
| **13** | **14** | **15** | **16** | **17** | **18** |
| $b^2c^2de$ | $ac^2de^2$ | $a^2bc^2d$ | $bc^2de^2$ | $ab^2c^2d$ | $a^2c^2de$ |
| $a^2c$ | $bce$ | $ab^2ce^2$ | $a^2b^2ce$ | $ce^2$ | $abc$ |
| $abd^2e^2$ | $a^2b^2d^2$ | $d^2e$ | $ad^2$ | $a^2bd^2e$ | $b^2d^2e^2$ |
| **19** | **20** | **21** | **22** | **23** | **24** |
| $cd^2$ | $abcd^2e$ | $a^2b^2cd^2e^2$ | $b^2cd^2e$ | $acd^2e^2$ | $a^2bcd^2$ |
| $a^2bde^2$ | $b^2d$ | $ade$ | $a^2d$ | $bde$ | $ab^2de^2$ |
| $ab^2c^2e$ | $a^2c^2e^2$ | $bc^2$ | $abc^2e^2$ | $a^2b^2c^2$ | $c^2e$ |

| 25 | 26 | 27 |
|---|---|---|
| $bcd^2e^2$ | $ab^2cd^2$ | $a^2cd^2e$ |
| $a^2b^2de$ | $de^2$ | $abd$ |
| $ac^2$ | $a^2bc^2e$ | $b^2c^2e^2$ |

Plan 3.5.9. 1/3 replication of 5 factors in 9 blocks of 9 units each.

Factors: A, B, C, D, E.

I = ABCDE.

Block confounding: $AB^2C^2D$, $AB^2E^2$, $AB^2CD^2E$, $CD^2E^2$.

With blocking: All two-factor interactions except BD are measurable.

Blocks

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| (1) | $a^2b^2e^2$ | abe | $a^2b^2c^2$ | $abc^2e^2$ |
| ace | $b^2c$ | $a^2bce^2$ | $b^2e$ | $a^2b$ |
| $a^2b^2cd$ | $abcde^2$ | cde | abd | $de^2$ |
| $b^2c^2de$ | $a^2bc^2d$ | $ac^2de^2$ | $a^2bcde$ | acd |
| $a^2bd^2e$ | $ad^2$ | $b^2d^2e^2$ | $ac^2d^2e$ | $b^2c^2d^2$ |
| $a^2c^2e^2$ | $ab^2c^2e$ | $bc^2$ | $ab^2ce^2$ | bce |
| $abc^2d^2$ | $c^2d^2e^2$ | $a^2b^2c^2d^2e$ | $cd^2$ | $a^2b^2cd^2e^2$ |
| $bcd^2e^2$ | $a^2cd^2e$ | $ab^2cd^2$ | $a^2d^2e^2$ | $ab^2d^2e$ |
| $ab^2de^2$ | bde | $a^2d$ | $bc^2de^2$ | $a^2c^2de$ |

| 6 | 7 | 8 | 9 |
|---|---|---|---|
| $c^2e$ | abc | $ce^2$ | $a^2b^2ce$ |
| $ae^2$ | $a^2bc^2e$ | $ac^2$ | $b^2c^2e^2$ |
| $a^2b^2de$ | $c^2d$ | $a^2b^2c^2de^2$ | $abc^2de$ |
| $b^2cde^2$ | ade | $b^2d$ | $a^2bde^2$ |
| $a^2bc^2d^2e^2$ | $b^2cd^2e$ | $a^2bcd^2$ | $acd^2e^2$ |
| $a^2c$ | $be^2$ | $a^2e$ | $ab^2$ |
| $abcd^2e$ | $a^2b^2d^2$ | $abd^2e^2$ | $d^2e$ |
| $bd^2$ | $ab^2c^2d^2e^2$ | $bc^2d^2e$ | $a^2c^2d^2$ |
| $ab^2c^2d$ | $a^2cde^2$ | $ab^2cde$ | bcd |

Plan 3.5.27. 1/3 replication of 5 factors in 3 blocks of 27 units each.

Factors: A, B, C, D, E.

$I = \underline{ABCDE}$.

Block confounding: $\underline{AB^2C^2D}$.

With blocking: All two-factor interactions are measurable.

Blocks

Combine blocks of Plan 3.5.9 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1, 2, and 3 | 4, 5, and 6 | 7, 8, and 9 |

Plan 3.6.9. 1/3 replication of 6 factors in 27 blocks of 9 units each.

Factors: A, B, C, D, E, F.

$I = \underline{AB^2CDE^2F}$.

Completely randomized: All two-factor interactions are measurable.

Block confounding: $\underline{AC^2D^2E^2F}$, $\underline{AB^2D^2E^2}$, $ABCD^2E^2F^2$, $BC^2F$, $\underline{ACD^2E}$, $AD^2F^2$, $ABC^2D^2$, $BEF^2$, $ABD^2EF$, $CEF$, $BCE^2$ $AB^2CD^2F$, $AB^2C^2D^2EF^2$.

With blocking: All two-factor interactions except $AB^2$ and DE are measurable.

Blocks

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| (1) | $a^2e^2$ | ae | $bc^2e$ | $a^2bc^2$ |
| $abcde^2$ | bcde | $a^2bcd$ | $ab^2d$ | $b^2de^2$ |
| $abd^2ef^2$ | $bd^2f^2$ | $a^2bd^2e^2f^2$ | $ab^2c^2d^2e^2f^2$ | $b^2c^2d^2ef^2$ |
| $a^2b^2cf^2$ | $ab^2ce^2f^2$ | $b^2cef^2$ | $a^2ef^2$ | $af^2$ |
| $cd^2ef$ | $a^2cd^2f$ | $acd^2e^2f$ | $bd^2e^2f$ | $a^2bd^2ef$ |
| $a^2b^2c^2d^2e$ | $ab^2c^2d^2$ | $b^2c^2d^2e^2$ | $a^2cd^2e^2$ | $acd^2e$ |
| $a^2b^2de^2f$ | $ab^2def$ | $b^2df$ | $a^2c^2df$ | $ac^2de^2f$ |
| $abc^2f$ | $bc^2e^2f$ | $a^2bc^2ef$ | $ab^2cef$ | $b^2cf$ |
| $c^2de^2f^2$ | $a^2c^2def^2$ | $ac^2df^2$ | $bcdf^2$ | $a^2bcde^2f^2$ |

| 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|
| $abc^2e^2$ | $b^2ce^2$ | $a^2b^2ce$ | $ab^2c$ | adf |
| $a^2b^2de$ | $ac^2de$ | $c^2d$ | $a^2c^2de^2$ | $a^2bcd^2e^2f$ |
| $a^2b^2c^2d^2f^2$ | $acd^2f^2$ | $cd^2e^2f^2$ | $a^2cd^2ef^2$ | $a^2be$ |
| $e^2f^2$ | $a^2bc^2e^2f^2$ | $abc^2ef^2$ | $bc^2f^2$ | $b^2cd$ |
| $abd^2f$ | $b^2c^2d^2f$ | $a^2b^2c^2d^2e^2f$ | $ab^2c^2d^2ef$ | $acef^2$ |
| $cd^2$ | $a^2bd^2$ | $abd^2e^2$ | $bd^2e$ | $b^2c^2ef$ |
| $c^2def$ | $a^2bcdef$ | abcdf | $bcde^2f$ | $b^2d^2e^2f^2$ |
| $a^2b^2ce^2f$ | $ae^2f$ | ef | $a^2f$ | $a^2bc^2df^2$ |
| $abcdef^2$ | $b^2def^2$ | $a^2b^2df^2$ | $ab^2de^2f^2$ | $ac^2d^2e^2$ |

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Plan 3.6.9. (Continued)

| 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|
| $de^2f$ | $a^2def$ | $abc^2def$ | $bc^2df$ | $a^2bc^2de^2f$ |
| $abcd^2ef$ | $bcd^2f$ | $a^2b^2d^2f$ | $ab^2d^2e^2f$ | $b^2d^2ef$ |
| $ab$ | $be^2$ | $a^2b^2c^2e^2$ | $ab^2c^2e$ | $b^2c^2$ |
| $a^2b^2cde^2$ | $ab^2cde$ | $de$ | $a^2d$ | $ade^2$ |
| $cf^2$ | $a^2ce^2f^2$ | $abe^2f^2$ | $bef^2$ | $a^2bf^2$ |
| $a^2b^2c^2f$ | $ab^2c^2e^2f$ | $ce^2f$ | $a^2cef$ | $acf$ |
| $a^2b^2d^2ef^2$ | $ab^2d^2f^2$ | $c^2d^2f^2$ | $a^2c^2d^2e^2f^2$ | $ac^2d^2ef^2$ |
| $abc^2de^2f^2$ | $bc^2def^2$ | $a^2b^2cdef^2$ | $ab^2cdf^2$ | $b^2cde^2f^2$ |
| $c^2d^2e$ | $a^2c^2d^2$ | $abcd^2$ | $bcd^2e^2$ | $a^2bcd^2e$ |

| 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|
| $ab^2cde^2f$ | $b^2cdef$ | $a^2b^2cdf$ | $a^2d^2f^2$ | $ad^2e^2f^2$ |
| $a^2c^2d^2ef$ | $ac^2d^2f$ | $c^2d^2e^2f$ | $bce^2f^2$ | $a^2bcef^2$ |
| $a^2c$ | $ace^2$ | $ce$ | $bdef$ | $a^2bdf$ |
| $bc^2de^2$ | $a^2bc^2de$ | $abc^2d$ | $ab^2cd^2f$ | $b^2cd^2e^2f$ |
| $ab^2c^2f^2$ | $b^2c^2e^2f^2$ | $a^2b^2c^2ef^2$ | $a^2cde$ | $acd$ |
| $bf$ | $a^2be^2f$ | $abef$ | $ab^2c^2def^2$ | $b^2c^2df^2$ |
| $bcd^2ef^2$ | $a^2bcd^2f^2$ | $abcd^2e^2f^2$ | $ab^2e^2$ | $b^2e$ |
| $a^2de^2f^2$ | $adef^2$ | $df^2$ | $bc^2d^2$ | $a^2bc^2d^2e^2$ |
| $ab^2d^2e$ | $b^2d^2$ | $a^2b^2d^2e^2$ | $a^2c^2e^2f$ | $ac^2ef$ |

| 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|
| $d^2ef^2$ | $a^2bc^2d^2ef^2$ | $abc^2d^2f^2$ | $bc^2d^2e^2f^2$ | $a^2b^2cd^2e^2f^2$ |
| $abcf^2$ | $b^2f^2$ | $a^2b^2e^2f^2$ | $ab^2ef^2$ | $c^2ef^2$ |
| $abde^2f$ | $b^2c^2de^2f$ | $a^2b^2c^2def$ | $ab^2c^2df$ | $cdf$ |
| $a^2b^2cd^2ef$ | $ad^2ef$ | $d^2f$ | $a^2d^2e^2f$ | $abc^2d^2e^2f$ |
| $cde^2$ | $a^2bde^2$ | $abde$ | $bd$ | $a^2b^2c^2d$ |
| $a^2b^2c^2de^2f^2$ | $acde^2f^2$ | $cdef^2$ | $a^2cdf^2$ | $abdf^2$ |
| $a^2b^2$ | $ac^2$ | $c^2e^2$ | $a^2c^2e$ | $abce$ |
| $abc^2d^2e$ | $b^2cd^2e$ | $a^2b^2cd^2$ | $ab^2cd^2e^2$ | $d^2e^2$ |
| $c^2f$ | $a^2bcf$ | $abce^2f$ | $bcef$ | $a^2b^2ef$ |

| 26 | 27 |
|---|---|
| $ab^2cd^2ef^2$ | $b^2cd^2f^2$ |
| $a^2c^2f^2$ | $ac^2e^2f^2$ |
| $a^2cde^2f$ | $acdef$ |
| $bc^2d^2ef$ | $a^2bc^2d^2f$ |
| $ab^2c^2de^2$ | $b^2c^2de$ |
| $bde^2f^2$ | $a^2bdef^2$ |
| $bc$ | $a^2bce^2$ |
| $a^2d^2e$ | $ad^2$ |
| $ab^2f$ | $b^2e^2f$ |

Plan 3.6.27. 1/3 replication of 6 factors in 9 blocks of 27 units each.

Factors: A, B, C, D, E, F.

$I = \underline{AB^2CDE^2F}$.

Block confounding: $\underline{AC^2D^2E^2F}$, $\underline{AB^2D^2E^2}$, $ABCD^2E^2F^2$, $BC^2F$.

With blocking: All two-factor interactions are measurable.

Blocks

Combine blocks of Plan 3.6.9 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1, 2, and 3 | 4, 5, and 6 | 7, 8, and 9 |
| 4 | 5 | 6 |
| 10, 11, and 12 | 13, 14 and 15 | 16, 17 and 18 |
| 7 | 8 | 9 |
| 19, 20, and 21 | 22, 23, and 24 | 25, 26, and 27 |

Plan 3.6.81. 1/3 replication of 6 factors in 3 blocks of 81 units each.

Factors: A, B, C, D, E, F.

$I = \underline{AB^2CDE^2F}$.

Block confounding: $AC^2D^2E^2F$.

With blocking: All two-factor interactions are measurable.

Blocks

Combine blocks of Plan 3.6.9 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1 through 9 | 10 through 18 | 19 through 27 |

Plan 3.7.27, 1/3 replication of 7 factors in 27 blocks of 27 units each.

Factors: A, B, C, D, E, F, G.

$I = \underline{AB^2CDE^2FG}$.

Completely randomized: All two-factor interactions are measurable.

Block confounding: $\underline{AC^2D^2E^2F}$, $\underline{AB^2D^2E^2}$, $ABCD^2E^2F^2$, $BC^2F$, $\underline{ACD^2E}$, $AD^2F^2$, $ABC^2D^2$, $BEF^2$, $ABD^2EF$, $CEF$, $BCE^2$, $AB^2CD^2F$, $AB^2C^2D^2EF^2$.

With blocking: All two-factor interactions are measurable.

Blocks

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| (1) | $a^2e^2$ | $ae$ | $bc^2e$ |
| $abcde^2$ | $bcde$ | $a^2bcd$ | $ab^2d$ |
| $abd^2ef^2$ | $bd^2f^2$ | $a^2bd^2e^2f^2$ | $ab^2c^2d^2e^2f^2$ |
| $a^2b^2cf^2$ | $ab^2ce^2f^2$ | $b^2cef^2$ | $a^2ef^2$ |
| $cd^2ef$ | $a^2cd^2f$ | $acd^2e^2f$ | $bd^2e^2f$ |
| $a^3b^2c^2d^2e$ | $ab^2c^2d^2$ | $b^2c^2d^2e^2$ | $a^2cd^2e^2$ |
| $a^3b^2de^2f$ | $ab^2def$ | $b^2df$ | $a^2c^2df$ |
| $abc^2f$ | $bc^2e^2f$ | $a^2bc^2ef$ | $ab^2cef$ |
| $c^2de^2f^2$ | $a^2c^2def^2$ | $ac^2df^2$ | $bcdf^2$ |
| $adg$ | $de^2g$ | $a^2deg$ | $abc^2deg$ |
| $a^2bcd^2e^2g$ | $abcd^2eg$ | $bcd^2g$ | $a^2b^2d^2g$ |
| $a^2bef^2g$ | $abf^2g$ | $be^2f^2g$ | $a^2b^2c^2e^2f^2g$ |
| $b^2cdf^2g$ | $a^2b^2cde^2f^2g$ | $ab^2cdef^2g$ | $def^2g$ |
| $acefg$ | $cfg$ | $a^2ce^2fg$ | $abe^2fg$ |
| $b^2c^2eg$ | $a^2b^2c^2g$ | $ab^2c^2e^2g$ | $ce^2g$ |
| $b^2d^2e^2fg$ | $a^2b^2d^2efg$ | $ab^2d^2fg$ | $c^2d^2fg$ |
| $a^2bc^2dfg$ | $abc^2de^2fg$ | $bc^2defg$ | $a^2b^2cdefg$ |
| $ac^2d^2e^2f^2g$ | $c^2d^2ef^2g$ | $a^2c^2d^2f^2g$ | $abcd^2f^2g$ |
| $a^2d^2g^2$ | $ad^2e^2g^2$ | $d^2eg^2$ | $a^2bc^2d^2eg^2$ |
| $bce^2g^2$ | $a^2bceg^2$ | $abcg^2$ | $b^2g^2$ |
| $bdef^2g^2$ | $a^2bdf^2g^2$ | $abde^2f^2g^2$ | $b^2c^2de^2f^2g^2$ |
| $ab^2cd^2f^2g^2$ | $b^2cd^2e^2f^2g^2$ | $a^2b^2cd^2ef^2g^2$ | $ad^2ef^2g^2$ |
| $a^2cdefg^2$ | $acdfg^2$ | $cde^2fg^2$ | $a^2bde^2fg^2$ |
| $ab^2c^2deg^2$ | $b^2c^2dg^2$ | $a^2b^2c^2de^2g^2$ | $acde^2g^2$ |
| $ab^2e^2fg^2$ | $b^2efg^2$ | $a^2b^2fg^2$ | $ac^2fg^2$ |
| $bc^2d^2fg^2$ | $a^2bc^2d^2e^2fg^2$ | $abc^2d^2efg^2$ | $b^2cd^2efg^2$ |
| $a^2c^2e^2f^2g^2$ | $ac^2ef^2g^2$ | $c^2f^2g^2$ | $a^2bcf^2g^2$ |

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Plan 3.7.27. (Continued)

| 5 | 6 | 7 | 8 |
|---|---|---|---|
| $a^2bc^2$ | $abc^2e^2$ | $b^2ce^2$ | $a^2b^2ce$ |
| $b^2de^2$ | $a^2b^2de$ | $ac^2de$ | $c^2d$ |
| $b^2c^2d^2ef^2$ | $a^2b^2c^2d^2f^2$ | $acd^2f^2$ | $cd^2e^2f^2$ |
| $af^2$ | $e^2f^2$ | $a^2bc^2e^2f^2$ | $abc^2ef^2$ |
| $a^2bd^2ef$ | $abd^2f$ | $b^2c^2d^2f$ | $a^2b^2c^2d^2e^2f$ |
| $acd^2e$ | $cd^2$ | $a^2bd^2$ | $abd^2e^2$ |
| $ac^2de^2f$ | $c^2def$ | $a^2bcdef$ | $abcdf$ |
| $b^2cf$ | $a^2b^2ce^2f$ | $ae^2f$ | $ef$ |
| $a^2bcde^2f^2$ | $abcdef^2$ | $b^2def^2$ | $a^2b^2df^2$ |
| $bc^2dg$ | $a^2bc^2de^2g$ | $ab^2cde^2g$ | $b^2cdeg$ |
| $ab^2d^2e^2g$ | $b^2d^2eg$ | $a^2c^2d^2eg$ | $ac^2d^2g$ |
| $ab^2c^2ef^2g$ | $b^2c^2f^2g$ | $a^2cf^2g$ | $ace^2f^2g$ |
| $a^2df^2g$ | $ade^2f^2g$ | $bc^2de^2f^2g$ | $a^2bc^2def^2g$ |
| $befg$ | $a^2bfg$ | $ab^2c^2fg$ | $b^2c^2e^2fg$ |
| $a^2ceg$ | $acg$ | $bg$ | $a^2be^2g$ |
| $a^2c^2d^2e^2fg$ | $ac^2d^2efg$ | $bcd^2efg$ | $a^2bcd^2fg$ |
| $ab^2cdfg$ | $b^2cde^2fg$ | $a^2de^2fg$ | $adefg$ |
| $bcd^2e^2f^2g$ | $a^2bcd^2ef^2g$ | $ab^2d^2ef^2g$ | $b^2d^2f^2g$ |
| $abc^2d^2g^2$ | $bc^2d^2e^2g^2$ | $a^2b^2cd^2e^2g^2$ | $ab^2cd^2eg^2$ |
| $a^2b^2e^2g^2$ | $ab^2eg^2$ | $c^2eg^2$ | $a^2c^2g^2$ |
| $a^2b^2c^2def^2g^2$ | $ab^2c^2df^2g^2$ | $cdf^2g^2$ | $a^2cde^2f^2g^2$ |
| $d^2f^2g^2$ | $a^2d^2e^2f^2g^2$ | $abc^2d^2e^2f^2g^2$ | $bc^2d^2ef^2g^2$ |
| $abdefg^2$ | $bdfg^2$ | $a^2b^2c^2dfg^2$ | $ab^2c^2de^2fg^2$ |
| $cdeg^2$ | $a^2cdg^2$ | $abdg^2$ | $bde^2g^2$ |
| $c^2e^2fg^2$ | $a^2c^2efg^2$ | $abcefg^2$ | $bcfg^2$ |
| $a^2b^2cd^2fg^2$ | $ab^2cd^2e^2fg^2$ | $d^2e^2fg^2$ | $a^2d^2efg^2$ |
| $abce^2f^2g^2$ | $bcef^2g^2$ | $a^2b^2ef^2g^2$ | $ab^2f^2g^2$ |

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Plan 3.7.27. (Continued)

| 9 | 10 | 11 | 12 |
|---|---|---|---|
| $ab^2c$ | $adf$ | $de^2f$ | $a^2def$ |
| $a^2c^2de^2$ | $a^2bcd^2e^2f$ | $abcd^2ef$ | $bcd^2f$ |
| $a^2cd^2ef^2$ | $a^2be$ | $ab$ | $be^2$ |
| $bc^2f^2$ | $b^2cd$ | $a^2b^2cde^2$ | $ab^2cde$ |
| $ab^2c^2d^2ef$ | $acef^2$ | $cf^2$ | $a^2ce^2f^2$ |
| $bd^2e$ | $b^2c^2ef$ | $a^2b^2c^2f$ | $ab^2c^2e^2f$ |
| $bcde^2f$ | $b^2d^2e^2f^2$ | $a^2b^2d^2ef^2$ | $ab^2d^2f^2$ |
| $a^2f$ | $a^2bc^2df^2$ | $abc^2de^2f^2$ | $bc^2def^2$ |
| $ab^2de^2f^2$ | $ac^2d^2e^2$ | $c^2d^2e$ | $a^2c^2d^2$ |
| $a^2b^2cdg$ | $a^2d^2fg$ | $ad^2e^2fg$ | $d^2efg$ |
| $c^2d^2e^2g$ | $bce^2fg$ | $a^2bcefg$ | $abcfg$ |
| $cef^2g$ | $bdeg$ | $a^2bdg$ | $abde^2g$ |
| $abc^2df^2g$ | $ab^2cd^2g$ | $b^2cd^2e^2g$ | $a^2b^2cd^2eg$ |
| $a^2b^2c^2efg$ | $a^2cdef^2g$ | $acdf^2g$ | $cde^2f^2g$ |
| $abeg$ | $ab^2c^2defg$ | $b^2c^2dfg$ | $a^2b^2c^2de^2fg$ |
| $abcd^2e^2fg$ | $ab^2e^2f^2g$ | $b^2ef^2g$ | $a^2b^2f^2g$ |
| $dfg$ | $bc^2d^2f^2g$ | $a^2bc^2d^2e^2f^2g$ | $abc^2d^2ef^2g$ |
| $a^2b^2d^2e^2f^2g$ | $a^2c^2e^2g$ | $ac^2eg$ | $c^2g$ |
| $b^2cd^2g^2$ | $fg^2$ | $a^2e^2fg^2$ | $aefg^2$ |
| $ac^2e^2g^2$ | $abcde^2fg^2$ | $bcdefg^2$ | $a^2bcdfg^2$ |
| $acdef^2g^2$ | $abd^2eg^2$ | $bd^2g^2$ | $a^2bd^2e^2g^2$ |
| $a^2bc^2d^2f^2g^2$ | $a^2b^2cg^2$ | $ab^2ce^2g^2$ | $b^2ceg^2$ |
| $b^2c^2defg^2$ | $cd^2ef^2g^2$ | $a^2cd^2f^2g^2$ | $acd^2e^2f^2g^2$ |
| $a^2bdeg^2$ | $a^2b^2c^2d^2efg^2$ | $ab^2c^2d^2fg^2$ | $b^2c^2d^2e^2fg^2$ |
| $a^2bce^2fg^2$ | $a^2b^2de^2f^2g^2$ | $ab^2def^2g^2$ | $b^2df^2g^2$ |
| $ad^2fg^2$ | $abc^2f^2g^2$ | $bc^2e^2f^2g^2$ | $a^2bc^2ef^2g^2$ |
| $b^2e^2f^2g^2$ | $c^2de^2g^2$ | $a^2c^2deg^2$ | $ac^2dg^2$ |

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Plan 3.7.27. (Continued)

| 13 | 14 | 15 | 16 |
|---|---|---|---|
| $abc^2def$ | $bc^2df$ | $a^2bc^2de^2f$ | $ab^2cde^2f$ |
| $a^2b^2d^2f$ | $ab^2d^2e^2f$ | $b^2d^2ef$ | $a^2c^2d^2ef$ |
| $a^2b^2c^2e^2$ | $ab^2c^2e$ | $b^2c^2$ | $a^2c$ |
| $de$ | $a^2d$ | $ade^2$ | $bc^2de^2$ |
| $abe^2f^2$ | $bef^2$ | $a^2bf^2$ | $ab^2c^2f^2$ |
| $ce^2f$ | $a^2cef$ | $acf$ | $bf$ |
| $c^2d^2f^2$ | $a^2c^2d^2e^2f^2$ | $ac^2d^2ef^2$ | $bcd^2ef^2$ |
| $a^2b^2cdef^2$ | $ab^2cdf^2$ | $b^2cde^2f^2$ | $a^2de^2f^2$ |
| $abcd^2$ | $bcd^2e^2$ | $a^2bcd^2e$ | $ab^2d^2e$ |
| $a^2bc^2d^2efg$ | $abc^2d^2fg$ | $bc^2d^2e^2fg$ | $a^2b^2cd^2e^2fg$ |
| $b^2fg$ | $a^2b^2e^2fg$ | $ab^2efg$ | $c^2efg$ |
| $b^2c^2de^2g$ | $a^2b^2c^2deg$ | $ab^2c^2dg$ | $cdg$ |
| $ad^2eg$ | $d^2g$ | $a^2d^2e^2g$ | $abc^2d^2e^2g$ |
| $a^2bde^2f^2g$ | $abdef^2g$ | $bdf^2g$ | $a^2b^2c^2df^2g$ |
| $acde^2fg$ | $cdefg$ | $a^2cdfg$ | $abdfg$ |
| $ac^2f^2g$ | $c^2e^2f^2g$ | $a^2c^2ef^2g$ | $abcef^2g$ |
| $b^2cd^2ef^2g$ | $a^2b^2cd^2f^2g$ | $ab^2cd^2e^2f^2g$ | $d^2e^2f^2g$ |
| $a^2bcg$ | $abce^2g$ | $bceg$ | $a^2b^2eg$ |
| $bc^2efg^2$ | $a^2bc^2fg^2$ | $abc^2e^2fg^2$ | $b^2ce^2fg^2$ |
| $ab^2dfg^2$ | $b^2de^2fg^2$ | $a^2b^2defg^2$ | $ac^2defg^2$ |
| $ab^2c^2d^2e^2g^2$ | $b^2c^2d^2eg^2$ | $a^2b^2c^2d^2g^2$ | $acd^2g^2$ |
| $a^2eg^2$ | $ag^2$ | $e^2g^2$ | $a^2bc^2e^2g^2$ |
| $bd^2e^2f^2g^2$ | $a^2bd^2ef^2g^2$ | $abd^2f^2g^2$ | $b^2c^2d^2f^2g^2$ |
| $a^2cd^2e^2fg^2$ | $acd^2efg^2$ | $cd^2fg^2$ | $a^2bd^2fg^2$ |
| $a^2c^2df^2g^2$ | $ac^2de^2f^2g^2$ | $c^2def^2g^2$ | $a^2bcdef^2g^2$ |
| $ab^2cef^2g^2$ | $b^2cf^2g^2$ | $a^2b^2ce^2f^2g^2$ | $ae^2f^2g^2$ |
| $bcdg^2$ | $a^2bcde^2g^2$ | $abcdeg^2$ | $b^2deg^2$ |

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Plan 3.7.27. (Continued)

| 17 | 18 | 19 | 20 |
|---|---|---|---|
| $b^2cdef$ | $a^2b^2cdf$ | $a^2d^2f^2$ | $ad^2e^2f^2$ |
| $ac^2d^2f$ | $c^2d^2e^2f$ | $bce^2f^2$ | $a^2bcef^2$ |
| $ace^2$ | $ce$ | $bdef$ | $a^2bdf$ |
| $a^2bc^2de$ | $abc^2d$ | $ab^2cd^2f$ | $b^2cd^2e^2f$ |
| $b^2c^2e^2f^2$ | $a^2b^2c^2ef^2$ | $a^2cde$ | $acd$ |
| $a^2be^2f$ | $abef$ | $ab^2c^2def^2$ | $b^2c^2df^2$ |
| $a^2bcd^2f^2$ | $abcd^2e^2f^2$ | $ab^2e^2$ | $b^2e$ |
| $adef^2$ | $df^2$ | $bc^2d^2$ | $a^2bc^2d^2e^2$ |
| $b^2d^2$ | $a^2b^2d^2e^2$ | $a^2c^2e^2f$ | $ac^2ef$ |
| $ab^2cd^2efg$ | $b^2cd^2fg$ | $f^2g$ | $a^2e^2f^2g$ |
| $a^2c^2fg$ | $ac^2e^2fg$ | $abcde^2f^2g$ | $bcdef^2g$ |
| $a^2cde^2g$ | $acdeg$ | $abd^2efg$ | $bd^2fg$ |
| $bc^2d^2eg$ | $a^2bc^2d^2g$ | $a^2b^2cfg$ | $ab^2ce^2fg$ |
| $ab^2c^2de^2f^2g$ | $b^2c^2def^2g$ | $cd^2eg$ | $a^2cd^2g$ |
| $bde^2fg$ | $a^2bdefg$ | $a^2b^2c^2d^2ef^2g$ | $ab^2c^2d^2f^2g$ |
| $bcf^2g$ | $a^2bce^2f^2g$ | $a^2b^2de^2g$ | $ab^2deg$ |
| $a^2d^2ef^2g$ | $ad^2f^2g$ | $abc^2g$ | $bc^2e^2g$ |
| $ab^2g$ | $b^2e^2g$ | $c^2de^2fg$ | $a^2c^2defg$ |
| $a^2b^2cefg^2$ | $ab^2cfg^2$ | $adf^2g^2$ | $de^2f^2g^2$ |
| $c^2dfg^2$ | $a^2c^2de^2fg^2$ | $a^2bcd^2e^2f^2g^2$ | $abcd^2ef^2g^2$ |
| $cd^2e^2g^2$ | $a^2cd^2eg^2$ | $a^2befg^2$ | $abfg^2$ |
| $abc^2eg^2$ | $bc^2g^2$ | $b^2cdfg^2$ | $a^2b^2cde^2fg^2$ |
| $a^2b^2c^2d^2e^2f^2g^2$ | $ab^2c^2d^2ef^2g^2$ | $aceg^2$ | $cg^2$ |
| $abd^2e^2fg^2$ | $bd^2efg^2$ | $b^2c^2ef^2g^2$ | $a^2b^2c^2f^2g^2$ |
| $abcdf^2g^2$ | $bcde^2f^2g^2$ | $b^2d^2e^2g^2$ | $a^2b^2d^2eg^2$ |
| $ef^2g^2$ | $a^2f^2g^2$ | $a^2bc^2dg^2$ | $abc^2de^2g^2$ |
| $a^2b^2dg^2$ | $ab^2de^2g^2$ | $ac^2d^2e^2fg^2$ | $c^2d^2efg^2$ |

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Plan 3.7.27. (Continued)

| 21 | 22 | 23 | 24 |
|---|---|---|---|
| $d^2ef^2$ | $a^2bc^2d^2ef^2$ | $abc^2d^2f^2$ | $bc^2d^2e^2f^2$ |
| $abcf^2$ | $b^2f^2$ | $a^2b^2e^2f^2$ | $ab^2ef^2$ |
| $abde^2f$ | $b^2c^2de^2f$ | $a^2b^2c^2def$ | $ab^2c^2df$ |
| $a^2b^2cd^2ef$ | $ad^2ef$ | $d^2f$ | $a^2d^2e^2f$ |
| $cde^2$ | $a^2bde^2$ | $abde$ | $bd$ |
| $a^2b^2c^2de^2f^2$ | $acde^2f^2$ | $cdef^2$ | $a^2cdf^2$ |
| $a^2b^2$ | $ac^2$ | $c^2e^2$ | $a^2c^2e$ |
| $abc^2d^2e$ | $b^2cd^2e$ | $a^2b^2cd^2$ | $ab^2cd^2e^2$ |
| $c^2f$ | $a^2bcf$ | $abce^2f$ | $bcef$ |
| $aef^2g$ | $bc^2ef^2g$ | $a^2bc^2f^2g$ | $abc^2e^2f^2g$ |
| $a^2bcdf^2g$ | $ab^2df^2g$ | $b^2de^2f^2g$ | $a^2b^2def^2g$ |
| $a^2bd^2e^2fg$ | $ab^2c^2d^2e^2fg$ | $b^2c^2d^2efg$ | $a^2b^2c^2d^2fg$ |
| $b^2cefg$ | $a^2efg$ | $afg$ | $e^2fg$ |
| $acd^2e^2g$ | $bd^2e^2g$ | $a^2bd^2eg$ | $abd^2g$ |
| $b^2c^2d^2e^2f^2g$ | $a^2cd^2e^2f^2g$ | $acd^2ef^2g$ | $cd^2f^2g$ |
| $b^2dg$ | $a^2c^2dg$ | $ac^2de^2g$ | $c^2deg$ |
| $a^2bc^2eg$ | $ab^2ceg$ | $b^2cg$ | $a^2b^2ce^2g$ |
| $ac^2dfg$ | $bcdfg$ | $a^2bcde^2fg$ | $abcdefg$ |
| $a^2def^2g^2$ | $abc^2def^2g^2$ | $bc^2df^2g^2$ | $a^2bc^2de^2f^2g^2$ |
| $bcd^2f^2g^2$ | $a^2b^2d^2f^2g^2$ | $ab^2d^2e^2f^2g^2$ | $b^2d^2ef^2g^2$ |
| $be^2fg^2$ | $a^2b^2c^2e^2fg^2$ | $ab^2c^2efg^2$ | $b^2c^2fg^2$ |
| $ab^2cdefg^2$ | $defg^2$ | $a^2dfg^2$ | $ade^2fg^2$ |
| $a^2ce^2g^2$ | $abe^2g^2$ | $beg^2$ | $a^2bg^2$ |
| $ab^2c^2e^2f^2g^2$ | $ce^2f^2g^2$ | $a^2cef^2g^2$ | $acf^2g^2$ |
| $ab^2d^2g^2$ | $c^2d^2g^2$ | $a^2c^2d^2e^2g^2$ | $ac^2d^2eg^2$ |
| $bc^2deg^2$ | $a^2b^2cdeg^2$ | $ab^2cdg^2$ | $b^2cde^2g^2$ |
| $a^2c^2d^2fg^2$ | $abcd^2fg^2$ | $bcd^2e^2fg^2$ | $a^2bcd^2efg^2$ |

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Plan 3.7.27. (Continued)

| 25 | 26 | 27 |
|---|---|---|
| $a^2b^2cd^2e^2f^2$ | $ab^2cd^2ef^2$ | $b^2cd^2f^2$ |
| $c^2ef^2$ | $a^2c^2f^2$ | $ac^2e^2f^2$ |
| $cdf$ | $a^2cde^2f$ | $acdef$ |
| $abc^2d^2e^2f$ | $bc^2d^2ef$ | $a^2bc^2d^2f$ |
| $a^2b^2c^2d$ | $ab^2c^2de^2$ | $b^2c^2de$ |
| $abdf^2$ | $bde^2f^2$ | $a^2bdef^2$ |
| $abce$ | $bc$ | $a^2bce^2$ |
| $d^2e^2$ | $a^2d^2e$ | $ad^2$ |
| $a^2b^2ef$ | $ab^2f$ | $b^2e^2f$ |
| $b^2ce^2f^2g$ | $a^2b^2cef^2g$ | $ab^2cf^2g$ |
| $ac^2def^2g$ | $c^2df^2g$ | $a^2c^2de^2f^2g$ |
| $acd^2fg$ | $cd^2e^2fg$ | $a^2cd^2efg$ |
| $a^2bc^2e^2fg$ | $abc^2efg$ | $bc^2fg$ |
| $b^2c^2d^2g$ | $a^2b^2c^2d^2e^2g$ | $ab^2c^2d^2eg$ |
| $a^2bd^2f^2g$ | $abd^2e^2f^2g$ | $bd^2ef^2g$ |
| $a^2bcdeg$ | $abcdg$ | $bcde^2g$ |
| $ae^2g$ | $eg$ | $a^2g$ |
| $b^2defg$ | $a^2b^2dfg$ | $ab^2de^2fg$ |
| $ab^2cde^2f^2g^2$ | $b^2cdef^2g^2$ | $a^2b^2cdf^2g^2$ |
| $a^2c^2d^2ef^2g^2$ | $ac^2d^2f^2g^2$ | $c^2d^2e^2f^2g^2$ |
| $a^2cfg^2$ | $ace^2fg^2$ | $cefg^2$ |
| $bc^2de^2fg^2$ | $a^2bc^2defg^2$ | $abc^2dfg^2$ |
| $ab^2c^2g^2$ | $b^2c^2e^2g^2$ | $a^2b^2c^2eg^2$ |
| $bf^2g^2$ | $a^2be^2f^2g^2$ | $abef^2g^2$ |
| $bcd^2eg^2$ | $a^2bcd^2g^2$ | $abcd^2e^2g^2$ |
| $a^2de^2g^2$ | $adeg^2$ | $dg^2$ |
| $ab^2d^2efg^2$ | $b^2d^2fg^2$ | $a^2b^2d^2e^2fg^2$ |

Plan 3.7.81. 1/3 replication of 7 factors in 9 blocks of 81 units each.

Factors: A, B, C, D, E, F, G.

$I = \underline{AB^2CDE^2FG}$.

Block confounding: $\underline{AC^2D^2E^2F}$, $\underline{AB^2D^2E^2}$, $ABCD^2E^2F^2$, $BC^2F$.

With blocking: All two-factor interactions are measurable.

Blocks

Combine blocks of Plan 3.7.27 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1, 2, and 3 | 4, 5, and 6 | 7, 8, and 9 |
| 4 | 5 | 6 |
| 10, 11, and 12 | 13, 14, and 15 | 16, 17, and 18 |
| 7 | 8 | 9 |
| 19, 20, and 21 | 22, 23, and 24 | 25, 26, and 27 |

Plan 3.7.243. 1/3 replication of 7 factors in 3 blocks of 243 units each.

Factors: A, B, C, D, E, F, G.

$I = \underline{AB^2CDE^2FG}$.

Block confounding: $\underline{AC^2D^2E^2F}$.

With blocking: All two-factor interactions are measurable.

Blocks

Combine blocks of Plan 3.7.27 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1 through 9 | 10 through 18 | 19 through 27 |

Plan 9.6.3. 1/9 replication of 6 factors in 27 blocks of 3 units each.

Factors: A, B, C, D, E, F.

$I = \underline{ACDE} = \underline{BC^2DE^2F} = ABD^2F = AB^2C^2E^2F^2$.

Completely randomized: All two-factor interactions except the following are measurable: AB, AC, AD, $AD^2$, AE, AF, $BD^2$, BF, CD, CE, DE, and $DF^2$.

Block confounding: AC, BC, $ABC^2$, $AB^2$, BF, ABCF, $BC^2F^2$, $AB^2C^2F$, AF, $AB^2CF^2$, $CF^2$, $AC^2F^2$, $ABF^2$.

With blocking: The following two-factor interactions are measurable: $AC^2$, $AE^2$, $AF^2$, $BC^2$, BD, $BE^2$, $BF^2$, $CD^2$, CF, $DE^2$ and EF.

Blocks

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| (1) | $de^2f$ | $d^2ef^2$ | $a^2cd^2e$ | $a^2cf$ | $a^2cde^2f^2$ |
| $abc^2de^2f^2$ | $abc^2d^2e$ | $abc^2f$ | $bf^2$ | $bde^2$ | $bd^2ef$ |
| $a^2b^2cd^2ef$ | $a^2b^2cf^2$ | $a^2b^2cde^2$ | $ab^2c^2de^2f$ | $ab^2c^2d^2ef^2$ | $ab^2c^2$ |
| **7** | **8** | **9** | **10** | **11** | **12** |
| $ac^2de^2$ | $ac^2d^2ef$ | $ac^2f^2$ | ade | $ad^2f$ | $ae^2f^2$ |
| $a^2bcd^2ef^2$ | $a^2bc$ | $a^2bcde^2f$ | $a^2bc^2d^2f^2$ | $a^2bc^2e^2$ | $a^2bc^2def$ |
| $b^2f$ | $b^2de^2f^2$ | $b^2d^2e$ | $b^2ce^2f$ | $b^2cdef^2$ | $b^2cd^2$ |
| **13** | **14** | **15** | **16** | **17** | **18** |
| $ce^2$ | cdef | $cd^2f^2$ | $a^2c^2d^2$ | $a^2c^2e^2f$ | $a^2c^2def^2$ |
| $abdef^2$ | $abd^2$ | $abe^2f$ | $bce^2f^2$ | bcde | $bcd^2f$ |
| $a^2b^2c^2d^2f$ | $a^2b^2c^2e^2f^2$ | $a^2b^2c^2de$ | $ab^2def$ | $ab^2d^2f^2$ | $ab^2e^2$ |

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Plan 9.6.3. (Continued)

| 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|
| $a^2d^2e^2$ | $a^2ef$ | $a^2df^2$ | $acd$ | $acd^2e^2f$ | $acef^2$ |
| $bc^2ef^2$ | $bc^2d$ | $bc^2d^2e^2f$ | $a^2bd^2e^2f^2$ | $a^2be$ | $a^2bdf$ |
| $ab^2cdf$ | $ab^2cd^2e^2f^2$ | $ab^2ce$ | $b^2c^2ef$ | $b^2c^2df^2$ | $b^2c^2d^2e^2$ |

| 25 | 26 | 27 |
|---|---|---|
| $c^3e$ | $c^2df$ | $c^2d^2e^2f^2$ |
| $abcdf^2$ | $abcd^2e^2$ | $abcef$ |
| $a^2b^2d^2e^2f$ | $a^2b^2ef^2$ | $a^2b^2d$ |

Plan 9.6.9. 1/9 replication of 6 factors in 9 blocks of 9 units each.

Factors: A, B, C, D, E, F.

$I = \underline{ACDE} = \underline{BC^2DE^2F} = ABD^2F = AB^2C^2E^2F^1$.

Block confounding: $\underline{ABC^2}$, $\underline{AD}$, $AB^2CD^2$, $BC^2D^2$.

With blocking: All two-factor interactions except the following are measurable: AB, AC, AD, $AD^2$, AE, AF, $BD^2$, BF, CD, CE, DE, and $DF^2$.

Blocks

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| (1) | $a^2b^2cd^2ef$ | $abc^2de^2f^2$ | $bf^2$ | $a^2cd^2e$ |
| $a^2c^2def^2$ | $ab^2e^2$ | $bcd^2f$ | $a^2bc^2def$ | $ae^2f^2$ |
| $bce^2f^2$ | $a^2c^2d^2$ | $ab^2def$ | $b^2ce^2f$ | $a^2bc^2d^2f^2$ |
| $a^2bdf$ | $acef^2$ | $b^2c^2d^2e^2$ | $a^2b^2d$ | $abcef$ |
| $a^2b^2cde^2$ | $abc^2f$ | $d^2ef^2$ | $a^2cde^2f^2$ | $ab^2c^2$ |
| $acd^2e^2f$ | $b^2c^2df^2$ | $a^2be$ | $abcd^2e^2$ | $c^2df$ |
| $b^2c^2ef$ | $a^2bd^2e^2f^2$ | $acd$ | $c^2e$ | $a^2b^2d^2e^2f$ |
| $ab^2d^2f^2$ | $bcde$ | $a^2c^2e^2f$ | $ad^2f$ | $b^2cdef^2$ |
| $abc^2d^2e$ | $de^2f$ | $a^2b^2cf^2$ | $ab^2c^2d^2ef^2$ | $bde^2$ |

| 6 | 7 | 8 | 9 |
|---|---|---|---|
| $ab^2c^2de^2f$ | $b^2f$ | $a^2bcd^2ef^2$ | $ac^2de^2$ |
| $b^2cd^2$ | $a^2b^2c^2de$ | $abe^2f$ | $cd^2f^2$ |
| $ade$ | $ce^2$ | $a^2b^2c^2d^2f$ | $abdef^2$ |
| $c^2d^2e^2f^2$ | $a^2df^2$ | $ab^2ce$ | $bc^2d^2e^2f$ |
| $bd^2ef$ | $a^2bcde^2f$ | $ac^2f^2$ | $b^2d^2e$ |
| $a^2b^2ef^2$ | $ab^2cd^2e^2f^2$ | $bc^2d$ | $a^2ef$ |
| $abcdf^2$ | $bc^2ef^2$ | $a^2d^2e^2$ | $ab^2cdf$ |
| $a^2bc^2e^2$ | $abd^2$ | $cdef$ | $a^2b^2c^2e^2f^2$ |
| $a^2cf$ | $ac^2d^2ef$ | $b^2de^2f^2$ | $a^2bc$ |

Plan 9.6.27. 1/9 replication of 6 factors in 3 blocks of 27 units each.

Factors: A, B, C, D, E, F.

$I = \underline{ACDE} = \underline{BC^2DE^2F} = ABD^2F = AB^2C^2E^2F^2$.

Block confounding: $\underline{AC}$.

With blocking: All two-factor interactions except the following are measurable: AB, AC, AD, $AD^2$, AE, AF, $BD^2$, BF, CD, CE, DE, and $DF^2$.

Blocks

Combine blocks of Plan 9.6.3 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1 through 9 | 10 through 18 | 19 through 27 |

Plan 9.7.9. 1/9 replication of 7 factors in 27 blocks of 9 units each.

Factors: A, B, C, D, E, F, G.

$I = \underline{ABCDE} = \underline{CD^2EF^2G^2} = ABC^2E^2F^2G^2 = ABD^2FG.$

Completely randomized: All two-factor interactions are measurable.

Block confounding: $\underline{AB^2C}$, $\underline{BCG}$, $AC^2G$, $ABG^2$, $\underline{BF^2G^2}$, $ACF^2G^2$, $BC^2F$, $ABC^2F^2$, $AB^2F^2G$, $ABCFG$, $CFG^2$, $AB^2C^2FG^2$, $AF$.

With blocking: All two-factor interactions except the following are measurable: AF, CD, and EG.

Blocks

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| (1) | $a^2cde^2g^2$ | $ac^2d^2eg$ | $f^2g$ | $a^2cde^2f^2$ |
| $ab^2cd^2f^2$ | $b^2c^2e^2f^2g^2$ | $a^2b^2def^2g$ | $ab^2cd^2fg$ | $b^2c^2e^2f$ |
| $a^2b^2e^2fg$ | $ab^2cdef$ | $b^2c^2d^2fg^2$ | $a^2b^2e^2g^2$ | $ab^2cdeg$ |
| $bcd^2e^2g$ | $a^2bc^2e$ | $abdg^2$ | $bcd^2e^2f^2g^2$ | $a^2bc^2ef^2g$ |
| $a^2cd^2efg^2$ | $ac^2fg$ | $de^2f$ | $a^2cd^2e$ | $ac^2g^2$ |
| $a^2bc^2df$ | $abd^2e^2fg^2$ | $bcefg$ | $a^2bc^2dg$ | $abd^2e^2$ |
| $abef^2g^2$ | $bcdf^2g$ | $a^2bc^2d^2e^2f^2$ | $abef$ | $bcdfg^2$ |
| $b^2c^2deg^2$ | $a^2b^2d^2g$ | $ab^2ce^2$ | $b^2c^2def^2$ | $a^2b^2d^2f^2g^2$ |
| $ac^2de^2f^2g$ | $d^2ef^2$ | $a^2cf^2g^2$ | $ac^2de^2fg^2$ | $d^2efg$ |

| 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|
| $ac^2d^2ef^2g^2$ | $fg^2$ | $a^2cde^2fg$ | $ac^2d^2ef$ | $bc^2d^2ef$ |
| $a^2b^2defg^2$ | $ab^2cd^2g^2$ | $b^2c^2e^2g$ | $a^2b^2de$ | $ade$ |
| $b^2c^2d^2$ | $a^2b^2e^2f^2$ | $ab^2cdef^2g^2$ | $b^2c^2d^2f^2g$ | $a^2c^2d^2f^2g$ |
| $abdf^2$ | $bcd^2e^2f$ | $a^2bc^2efg^2$ | $abdfg$ | $b^2dfg$ |
| $de^2g$ | $a^2cd^2ef^2g$ | $ac^2f^2$ | $de^2f^2g^2$ | $a^2bde^2f^2g^2$ |
| $bceg^2$ | $a^2bc^2df^2g^2$ | $abd^2e^2f^2g$ | $bcef^2$ | $a^2b^2cef^2$ |
| $a^2bc^2d^2e^2fg$ | $abeg$ | $bcd$ | $a^2bc^2d^2e^2g^2$ | $ab^2c^2d^2e^2g^2$ |
| $ab^2ce^2f^2g$ | $b^2c^2defg$ | $a^2b^2d^2f$ | $ab^2ce^2fg^2$ | $ce^2fg^2$ |
| $a^2cf$ | $ac^2de^2$ | $d^2eg^2$ | $a^2cg$ | $abcg$ |

(Continued next page)

Plan 9.7.9. (Continued)

11

$a^2bfg^2$
$cd^2g^2$
$ae^2f^2$
$a^2b^2cd^2e^2f$
$abcd^2ef^2g$
$ab^2c^2df^2g^2$
$b^2eg$
$a^2c^2defg$
$bc^2de^2$

12

$abcde^2fg$
$a^2c^2e^2g$
$cdef^2g^2$
$ab^2c^2efg^2$
$bc^2f^2$
$b^2d^2e^2f^2g$
$a^2b^2cd$
$ad^2f$
$a^2bd^2eg^2$

13

$bc^2d^2eg$
$adef^2g$
$a^2c^2d^2fg^2$
$b^2dg^2$
$a^2bde^2f$
$a^2b^2cefg$
$ab^2c^2d^2e^2f^2$
$ce^2$
$abcf^2g^2$

14

$a^2b$
$cd^2f^2$
$ae^2fg$
$a^2b^2cd^2e^2g$
$abcd^2efg^2$
$ab^2c^2df$
$b^2ef^2g^2$
$a^2c^2deg^2$
$bc^2de^2f^2g$

15

$abcde^2g^2$
$a^2c^2e^2f^2g^2$
$cdef$
$ab^2c^2e$
$bc^2fg$
$b^2d^2e^2fg^2$
$a^2b^2cdf^2g$
$ad^2g$
$a^2bd^2ef^2$

16

$bc^2d^2ef^2g^2$
$adefg^2$
$a^2c^2d^2$
$b^2df^2$
$a^2bde^2g$
$a^2b^2ceg^2$
$ab^2c^2d^2e^2fg$
$ce^2f^2g$
$abcf$

17

$a^2bf^2g$
$cd^2fg$
$ae^2g^2$
$a^2b^2cd^2e^2f^2g^2$
$abcd^2e$
$ab^2c^2dg$
$b^2ef$
$a^2c^2def^2$
$bc^2de^2fg^2$

18

$abcde^2f^2$
$a^2c^2e^2f$
$cdeg$
$ab^2c^2ef^2g$
$bc^2g^2$
$b^2d^2e^2$
$a^2b^2cdfg^2$
$ad^2f^2g^2$
$a^2bd^2efg$

19

$b^2cde^2f^2$
$abc^2e^2f$
$a^2bcdeg$
$c^2ef^2g$
$a^2b^2c^2g^2$
$a^2d^2e^2$
$acdfg^2$
$bd^2f^2g^2$
$ab^2d^2efg$

20

$a^2b^2c^2d^2ef^2g^2$
$bdefg^2$
$abc^2d^2$
$a^2df^2$
$ab^2de^2g$
$aceg^2$
$c^2d^2e^2fg$
$a^3bce^2f^2g$
$b^2cf$

21

$ab^2f^2g$
$a^2bcd^2fg$
$be^2g^2$
$acd^2e^2f^2g^2$
$b^2cd^2e$
$c^2dg$
$a^2ef$
$abc^2def^2$
$a^2b^2c^2de^2fg^2$

22

$b^2cde^2fg$
$abc^2e^2g$
$a^2bcdef^2g^2$
$c^2efg^2$
$a^2b^2c^2f^2$
$a^2d^2e^2f^2g$
$acd$
$bd^2f$
$ab^2d^2eg^2$

23

$a^2b^2c^2d^2ef$
$bde$
$abc^2d^2f^2g$
$a^2dfg$
$ab^2de^2f^2g^2$
$acef^2$
$c^2d^2e^2g^2$
$a^2bce^2fg^2$
$b^2cg$

24

$ab^2fg^2$
$a^2bcd^2g^2$
$be^2f^2$
$acd^2e^2f$
$b^2cd^2ef^2g$
$c^2df^2g^2$
$a^2eg$
$abc^2defg$
$a^2b^2c^2de^2$

25

$b^2cde^2g^2$
$abc^2e^2f^2g^2$
$a^2bcdef$
$c^2e$
$a^2b^2c^2fg$
$a^2d^2e^2fg^2$
$acdf^2g$
$bd^2g$
$ab^2d^2ef^2$

26

$a^2b^2c^2d^2eg$
$bdef^2g$
$abc^2d^2fg^2$
$a^2dg^2$
$ab^2de^2f$
$acefg$
$c^2d^2e^2f^2$
$a^2bce^2$
$b^2cf^2g^2$

27

$ab^2$
$a^2bcd^2f^2$
$be^2fg$
$acd^2e^2g$
$b^2cd^2efg^2$
$c^2df$
$a^2ef^2g^2$
$abc^2deg^2$
$a^2b^2c^2de^2f^2g$

Plan 9.7.27. 1/9 replication of 7 factors in 9 blocks of 27 units each.

Factors: A, B, C, D, E, F, G.

$I = \underline{ABCDE} = \underline{CD^2EF^2G^2} = ABC^2E^2F^2G^2 = ABD^2FG.$

Block confounding: $\underline{AB^2C}$, $\underline{BCG}$, $AC^2G$, $ABG^2$.

With blocking: All two-factor interactions are measurable.

Blocks

Combine blocks of Plan 9.7.9 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1, 2, and 3 | 4, 5, and 6 | 7, 8, and 9 |
| 4 | 5 | 6 |
| 10, 11, and 12 | 13, 14, and 15 | 16, 17, and 18 |
| 7 | 8 | 9 |
| 19, 20, and 21 | 22, 23, and 24 | 25, 26, and 27 |

Plan 9.7.81. 1/9 replication of 7 factors in 3 blocks of 81 units each.

Factors: A, B, C, D, E, F, G.

$I = \underline{ABCDE} = \underline{CD^2EF^2G^2} = ABC^2E^2F^2G^2 = ABD^2FG.$

Block confounding: $\underline{AB^2C}$.

With blocking: All two-factor interactions are measurable.

Blocks

Combine blocks of Plan 9.7.9 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1 through 9 | 10 through 18 | 19 through 27 |

Plan 9.8.27. 1/9 replication of 8 factors in 27 blocks of 27 units each.

Factors: A, B, C, D, E, F, G, H.

$I = \underline{ABCDEH^2} = \underline{CD^2EF^2G^2} = ABC^2E^2F^2G^2H^2 = ABD^2FGH^2$.

Completely randomized: All two-factor interactions are measurable.

Block confounding: $\underline{AB^2CH}$, $\underline{BCGH^2}$, $AC^2G$, $ABG^2H^2$, $\underline{BF^2G^2H}$, $ACF^2G^2H^2$, $BC^2F$, $ABC^2F^2H$, $AB^2F^2G$, $ABCFG$, $CFG^2H$, $AB^2C^2FG^2H^2$, $AFH$.

With blocking: All two-factor interactions are measurable.

Blocks

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| (1) | $a^2cde^2g^2$ | $ac^2d^2eg$ | $f^2g$ |
| $ab^2cd^2f^2$ | $b^2c^2e^2f^2g^2$ | $a^2b^2def^2g$ | $ab^2cd^2fg$ |
| $a^2b^2e^2fg$ | $ab^2cdef$ | $b^2c^2d^2fg^2$ | $a^2b^2e^2g^2$ |
| $bcd^2e^2g$ | $a^2bc^2e$ | $abdg^2$ | $bcd^2e^2f^2g^2$ |
| $a^2cd^2efg^2$ | $ac^2fg$ | $de^2f$ | $a^2cd^2e$ |
| $a^2bc^2df$ | $abd^2e^2fg^2$ | $bcefg$ | $a^2bc^2dg$ |
| $abef^2g^2$ | $bcdf^2g$ | $a^2bc^2d^2e^2f^2$ | $abef$ |
| $b^2c^2deg^2$ | $a^2b^2d^2g$ | $ab^2ce^2$ | $b^2c^2def^2$ |
| $ac^2de^2f^2g$ | $d^2ef^2$ | $a^2cf^2g^2$ | $ac^2de^2fg^2$ |
| $a^2bcde^2g^2h$ | $abc^2d^2egh$ | $bh$ | $a^2bcde^2f^2h$ |
| $c^2e^2f^2g^2h$ | $a^2def^2gh$ | $acd^2f^2h$ | $c^2e^2fh$ |
| $acdefh$ | $c^2d^2fg^2h$ | $a^2e^2fgh$ | $acdegh$ |
| $a^2b^2c^2eh$ | $ab^2dg^2h$ | $b^2cd^2e^2gh$ | $a^2b^2c^2ef^2gh$ |
| $abc^2fgh$ | $bde^2fh$ | $a^2bcd^2efg^2h$ | $abc^2g^2h$ |
| $ab^2d^2e^2fg^2h$ | $b^2cefgh$ | $a^2b^2c^2dfh$ | $ab^2d^2e^2h$ |
| $b^2cdf^2gh$ | $a^2b^2c^2d^2e^2f^2h$ | $ab^2ef^2g^2h$ | $b^2cdfg^2h$ |
| $a^2d^2gh$ | $ace^2h$ | $c^2deg^2h$ | $a^2d^2f^2g^2h$ |
| $bd^2ef^2h$ | $a^2bcf^2g^2h$ | $abc^2de^2f^2gh$ | $bd^2efgh$ |
| $ab^2c^2d^2egh^2$ | $b^2h^2$ | $a^2b^2cde^2g^2h^2$ | $ab^2c^2d^2ef^2g^2h^2$ |
| $a^2bdef^2gh^2$ | $abcd^2f^2h^2$ | $bc^2e^2f^2g^2h^2$ | $a^2bdefg^2h^2$ |
| $bc^2d^2fg^2h^2$ | $a^2be^2fgh^2$ | $abcdefh^2$ | $bc^2d^2h^2$ |
| $adg^2h^2$ | $cd^2e^2gh^2$ | $a^2c^2eh^2$ | $adf^2h^2$ |
| $b^2de^2fh^2$ | $a^2b^2cd^2efg^2h^2$ | $ab^2c^2fgh^2$ | $b^2de^2gh^2$ |
| $cefgh^2$ | $a^2c^2dfh^2$ | $ad^2e^2fg^2h^2$ | $ceg^2h^2$ |
| $a^2c^2d^2e^2f^2h^2$ | $aef^2g^2h^2$ | $cdf^2gh^2$ | $a^2c^2d^2e^2fgh^2$ |
| $abce^2h^2$ | $bc^2deg^2h^2$ | $a^2bd^2gh^2$ | $abce^2f^2gh^2$ |
| $a^2b^2cf^2g^2h^2$ | $ab^2c^2de^2f^2gh^2$ | $b^2d^2ef^2h^2$ | $a^2b^2cfh^2$ |

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Plan 9.8.27. (Continued)

| 5 | 6 | 7 | 8 |
|---|---|---|---|
| $a^2cde^2f^2$ | $ac^2d^2ef^2g^2$ | $fg^2$ | $a^2cde^2fg$ |
| $b^2c^2e^2f$ | $a^2b^2defg^2$ | $ab^2cd^2g^2$ | $b^2c^2e^2g$ |
| $ab^2cdeg$ | $b^2c^2d^2$ | $a^2b^2e^2f^2$ | $ab^2cdef^2g^2$ |
| $a^2bc^2ef^2g$ | $abdf^2$ | $bcd^2e^2f$ | $a^2bc^2efg^2$ |
| $ac^2g^2$ | $de^2g$ | $a^2cd^2ef^2g$ | $ac^2f^2$ |
| $abd^2e^2$ | $bceg^2$ | $a^2bc^2df^2g^2$ | $abd^2e^2f^2g$ |
| $bcdfg^2$ | $a^2bc^2d^2e^2fg$ | $abeg$ | $bcd$ |
| $a^2b^2d^2f^2g^2$ | $ab^2ce^2f^2g$ | $b^2c^2defg$ | $a^2b^2d^2f$ |
| $d^2efg$ | $a^2cf$ | $ac^2de^2$ | $d^2eg^2$ |
| $abc^2d^2ef^2g^2h$ | $bf^2gh$ | $a^2bcde^2fgh$ | $abc^2d^2efh$ |
| $a^2defg^2h$ | $acd^2fgh$ | $c^2e^2gh$ | $a^2deh$ |
| $c^2d^2h$ | $a^2e^2g^2h$ | $acdef^2g^2h$ | $c^2d^2f^2gh$ |
| $ab^2df^2h$ | $b^2cd^2e^2f^2g^2h$ | $a^2b^2c^2efg^2h$ | $ab^2dfgh$ |
| $bde^2gh$ | $a^2bcd^2eh$ | $abc^2f^2h$ | $bde^2f^2g^2h$ |
| $b^2ceg^2h$ | $a^2b^2c^2dgh$ | $ab^2d^2e^2f^2gh$ | $b^2cef^2h$ |
| $a^2b^2c^2d^2e^2fgh$ | $ab^2efh$ | $b^2cdh$ | $a^2b^2c^2d^2e^2g^2h$ |
| $ace^2f^2gh$ | $c^2def^2h$ | $a^2d^2fh$ | $ace^2fg^2h$ |
| $a^2bcfh$ | $abc^2de^2fg^2h$ | $bd^2eg^2h$ | $a^2bcgh$ |
| $b^2f^2gh^2$ | $a^2b^2cde^2f^2h^2$ | $ab^2c^2d^2efh^2$ | $b^2fg^2h^2$ |
| $abcd^2fgh^2$ | $bc^2e^2fh^2$ | $a^2bdeh^2$ | $abcd^2g^2h^2$ |
| $a^2be^2g^2h^2$ | $abcdegh^2$ | $bc^2d^2f^2gh^2$ | $a^2be^2f^2h^2$ |
| $cd^2e^2f^2g^2h^2$ | $a^2c^2ef^2gh^2$ | $adfgh^2$ | $cd^2e^2fh^2$ |
| $a^2b^2cd^2eh^2$ | $ab^2c^2g^2h^2$ | $b^2de^2f^2g^2h^2$ | $a^2b^2cd^2ef^2gh^2$ |
| $a^2c^2dgh^2$ | $ad^2e^2h^2$ | $cef^2h^2$ | $a^2c^2df^2g^2h^2$ |
| $aefh^2$ | $cdfg^2h^2$ | $a^2c^2d^2e^2g^2h^2$ | $aegh^2$ |
| $bc^2def^2h^2$ | $a^2bd^2f^2g^2h^2$ | $abce^2fg^2h^2$ | $bc^2defgh^2$ |
| $ab^2c^2de^2fg^2h^2$ | $b^2d^2efgh^2$ | $a^2b^2cgh^2$ | $ab^2c^2de^2h^2$ |

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Plan 9.8.27. (Continued)

| 9 | 10 | 11 | 12 |
|---|---|---|---|
| $ac^2d^2ef$ | $bc^2d^2ef$ | $a^2bfg^2$ | $abcde^2fg$ |
| $a^2b^2de$ | $ade$ | $cd^2g^2$ | $a^2c^2e^2g$ |
| $b^2c^2d^2f^2g$ | $a^2c^2d^2f^2g$ | $ae^2f^2$ | $cdef^2g^2$ |
| $abdfg$ | $b^2dfg$ | $a^2b^2cd^2e^2f$ | $ab^2c^2efg^2$ |
| $de^2f^2g^2$ | $a^2bde^2f^2g^2$ | $abcd^2ef^2g$ | $bc^2f^2$ |
| $bcef^2$ | $a^2b^2cef^2$ | $ab^2c^2df^2g^2$ | $b^2d^2e^2f^2g$ |
| $a^2bc^2d^2e^2g^2$ | $ab^2c^2d^2e^2g^2$ | $b^2eg$ | $a^2b^2cd$ |
| $ab^2ce^2fg^2$ | $ce^2fg^2$ | $a^2c^2defg$ | $ad^2f$ |
| $a^2cg$ | $abcg$ | $bc^2de^2$ | $a^2bd^2eg^2$ |
| $bfg^2h$ | $a^2b^2fg^2h$ | $ab^2cde^2fgh$ | $b^2c^2d^2efh$ |
| $acd^2g^2h$ | $bcd^2g^2h$ | $a^2bc^2e^2gh$ | $abdeh$ |
| $a^2e^2f^2h$ | $abe^2f^2h$ | $bcdef^2g^2h$ | $a^2bc^2d^2f^2gh$ |
| $b^2cd^2e^2fh$ | $a^2cd^2e^2fh$ | $ac^2efg^2h$ | $dfgh$ |
| $a^2bcd^2ef^2gh$ | $ab^2cd^2ef^2gh$ | $b^2c^2f^2h$ | $a^2b^2de^2f^2g^2h$ |
| $a^2b^2c^2df^2g^2h$ | $ac^2df^2g^2h$ | $d^2e^2f^2gh$ | $a^2cef^2h$ |
| $ab^2egh$ | $egh$ | $a^2cdh$ | $ac^2d^2e^2g^2h$ |
| $c^2defgh$ | $a^2bc^2defgh$ | $abd^2fh$ | $bce^2fg^2h$ |
| $abc^2de^2h$ | $b^2c^2de^2h$ | $a^2b^2d^2eg^2h$ | $ab^2cgh$ |
| $a^2b^2cde^2fgh^2$ | $acde^2fgh^2$ | $c^2d^2efh^2$ | $a^2fg^2h^2$ |
| $bc^2e^2gh^2$ | $a^2b^2c^2e^2gh^2$ | $ab^2deh^2$ | $b^2cd^2g^2h^2$ |
| $abcdef^2g^2h^2$ | $b^2cdef^2g^2h^2$ | $a^2b^2c^2d^2f^2gh^2$ | $ab^2e^2f^2h^2$ |
| $a^2c^2efg^2h^2$ | $abc^2efg^2h^2$ | $bdfgh^2$ | $a^2bcd^2e^2fh^2$ |
| $ab^2c^2f^2h^2$ | $c^2f^2h^2$ | $a^2de^2f^2g^2h^2$ | $acd^2ef^2gh^2$ |
| $ad^2e^2f^2gh^2$ | $bd^2e^2f^2gh^2$ | $a^2bcef^2h^2$ | $abc^2df^2g^2h^2$ |
| $cdh^2$ | $a^2bcdh^2$ | $abc^2d^2e^2g^2h^2$ | $begh^2$ |
| $a^2bd^2fh^2$ | $ab^2d^2fh^2$ | $b^2ce^2fg^2h^2$ | $a^2b^2c^2defgh^2$ |
| $b^2d^2eg^2h^2$ | $a^2d^2eg^2h^2$ | $acgh^2$ | $c^2de^2h^2$ |

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Plan 9.8.27. (Continued)

| 13 | 14 | 15 | 16 |
|---|---|---|---|
| $bc^2d^2eg$ | $a^2b$ | $abcde^2g^2$ | $bc^2d^2ef^2g^2$ |
| $adef^2g$ | $cd^2f^2$ | $a^2c^2e^2f^2g^2$ | $adefg^2$ |
| $a^2c^2d^2fg^2$ | $ae^2fg$ | $cdef$ | $a^2c^2d^2$ |
| $b^2dg^2$ | $a^2b^2cd^2e^2g$ | $ab^2c^2e$ | $b^2df^2$ |
| $a^2bde^2f$ | $abcd^2efg^2$ | $bc^2fg$ | $a^2bde^2g$ |
| $a^2b^2cefg$ | $ab^2c^2df$ | $b^2d^2e^2fg^2$ | $a^2b^2ceg^2$ |
| $ab^2c^2d^2e^2f^2$ | $b^2ef^2g^2$ | $a^2b^2cdf^2g$ | $ab^2c^2d^2e^2fg$ |
| $ce^2$ | $a^2c^2deg^2$ | $ad^2g$ | $ce^2f^2g$ |
| $abcf^2g^2$ | $bc^2de^2f^2g$ | $a^2bd^2ef^2$ | $abcf$ |
| $a^2b^2h$ | $ab^2cde^2g^2h$ | $b^2c^2d^2egh$ | $a^2b^2f^2gh$ |
| $bcd^2f^2h$ | $a^2bc^2e^2f^2g^2h$ | $abdef^2gh$ | $bcd^2fgh$ |
| $abe^2fgh$ | $bcdefh$ | $a^2bc^2d^2fg^2h$ | $abe^2g^2h$ |
| $a^2cd^2e^2gh$ | $ac^2eh$ | $dg^2h$ | $a^2cd^2e^2f^2g^2h$ |
| $ab^2cd^2efg^2h$ | $b^2c^2fgh$ | $a^2b^2de^2fh$ | $ab^2cd^2eh$ |
| $ac^2dfh$ | $d^2e^2fg^2h$ | $a^2cefgh$ | $ac^2dgh$ |
| $ef^2g^2h$ | $a^2cdf^2gh$ | $ac^2d^2e^2f^2h$ | $efh$ |
| $a^2bc^2deg^2h$ | $abd^2gh$ | $bce^2h$ | $a^2bc^2def^2h$ |
| $b^2c^2de^2f^2gh$ | $a^2b^2d^2ef^2h$ | $ab^2cf^2g^2h$ | $b^2c^2de^2fg^2h$ |
| $acde^2g^2h^2$ | $c^2d^2egh^2$ | $a^2h^2$ | $acde^2f^2h^2$ |
| $a^2b^2c^2e^2f^2g^2h^2$ | $ab^2def^2gh^2$ | $b^2cd^2f^2h^2$ | $a^2b^2c^2e^2fh^2$ |
| $b^2cdefh^2$ | $a^2b^2c^2d^2fg^2h^2$ | $ab^2e^2fgh^2$ | $b^2cdegh^2$ |
| $abc^2eh^2$ | $bdg^2h^2$ | $a^2bcd^2e^2gh^2$ | $abc^2ef^2gh^2$ |
| $c^2fgh^2$ | $a^2de^2fh^2$ | $acd^2efg^2h^2$ | $c^2g^2h^2$ |
| $bd^2e^2fg^2h^2$ | $a^2bcefgh^2$ | $abc^2dfh^2$ | $bd^2e^2h^2$ |
| $a^2bcdf^2gh^2$ | $abc^2d^2e^2f^2h^2$ | $bef^2g^2h^2$ | $a^2bcdfg^2h^2$ |
| $ab^2d^2gh^2$ | $b^2ce^2h^2$ | $a^2b^2c^2deg^2h^2$ | $ab^2d^2f^2g^2h^2$ |
| $a^2d^2ef^2h^2$ | $acf^2g^2h^2$ | $c^2de^2f^2gh^2$ | $a^2d^2efgh^2$ |

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Plan 9.8.27. (Continued)

| 17 | 18 | 19 | 20 |
|---|---|---|---|
| $a^2bf^2g$ | $abcde^2f^2$ | $b^2cde^2f^2$ | $a^2b^2c^2d^2ef^2g^2$ |
| $cd^2fg$ | $a^2c^2e^2f$ | $abc^2e^2f$ | $bdefg^2$ |
| $ae^2g^2$ | $cdeg$ | $a^2bcdeg$ | $abc^2d^2$ |
| $a^2b^2cd^2e^2f^2g^2$ | $ab^2c^2ef^2g$ | $c^2ef^2g$ | $a^2df^2$ |
| $abcd^2e$ | $bc^2g^2$ | $a^2b^2c^2g^2$ | $ab^2de^2g$ |
| $ab^2c^2dg$ | $b^2d^2e^2$ | $a^2d^2e^2$ | $aceg^2$ |
| $b^2ef$ | $a^2b^2cdfg^2$ | $acdfg^2$ | $c^2d^2e^2fg$ |
| $a^2c^2def^2$ | $ad^2f^2g^2$ | $bd^2f^2g^2$ | $a^2bce^2f^2g$ |
| $bc^2de^2fg^2$ | $a^2bd^2efg$ | $ab^2d^2efg$ | $b^2cf$ |
| $ab^2cde^2f^2h$ | $b^2c^2d^2ef^2g^2h$ | $a^2c^2d^2ef^2g^2h$ | $af^2gh$ |
| $a^2bc^2e^2fh$ | $abdefg^2h$ | $b^2defg^2h$ | $a^2b^2cd^2fgh$ |
| $bcdegh$ | $a^2bc^2d^2h$ | $ab^2c^2d^2h$ | $b^2e^2g^2h$ |
| $ac^2ef^2gh$ | $df^2h$ | $a^2bdf^2h$ | $abcd^2e^2f^2g^2h$ |
| $b^2c^2g^2h$ | $a^2b^2de^2gh$ | $ade^2gh$ | $cd^2eh$ |
| $d^2e^2h$ | $a^2ceg^2h$ | $abceg^2h$ | $bc^2dgh$ |
| $a^2cdfg^2h$ | $ac^2d^2e^2fgh$ | $bc^2d^2e^2fgh$ | $a^2befh$ |
| $abd^2f^2g^2h$ | $bce^2f^2gh$ | $a^2b^2ce^2f^2gh$ | $ab^2c^2def^2h$ |
| $a^2b^2d^2efgh$ | $ab^2cfh$ | $cfh$ | $a^2c^2de^2fg^2h$ |
| $c^2d^2ef^2g^2h^2$ | $a^2f^2gh^2$ | $abf^2gh^2$ | $bcde^2f^2h^2$ |
| $ab^2defg^2h^2$ | $b^2cd^2fgh^2$ | $a^2cd^2fgh^2$ | $ac^2e^2fh^2$ |
| $a^2b^2c^2d^2h^2$ | $ab^2e^2g^2h^2$ | $e^2g^2h^2$ | $a^2cdegh^2$ |
| $bdf^2h^2$ | $a^2bcd^2e^2f^2g^2h^2$ | $ab^2cd^2e^2f^2g^2h^2$ | $b^2c^2ef^2gh^2$ |
| $a^2de^2gh^2$ | $acd^2eh^2$ | $bcd^2eh^2$ | $a^2bc^2g^2h^2$ |
| $a^2bceg^2h^2$ | $abc^2dgh^2$ | $b^2c^2dgh^2$ | $a^2b^2d^2e^2h^2$ |
| $abc^2d^2e^2fgh^2$ | $befh^2$ | $a^2b^2efh^2$ | $ab^2cdfg^2h^2$ |
| $b^2ce^2f^2gh^2$ | $a^2b^2c^2def^2h^2$ | $ac^2def^2h^2$ | $d^2f^2g^2h^2$ |
| $acfh^2$ | $c^2de^2fg^2h^2$ | $a^2bc^2de^2fg^2h^2$ | $abd^2efgh^2$ |

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Plan 9.8.27. (Continued)

| 21 | 22 | 23 | 24 |
|---|---|---|---|
| $ab^2f^2g$ | $b^2cde^2fg$ | $a^2b^2c^2d^2ef$ | $ab^2fg^2$ |
| $a^2bcd^2fg$ | $abc^2e^2g$ | $bde$ | $a^2bcd^2g^2$ |
| $be^2g^2$ | $a^2bcdef^2g^2$ | $abc^2d^2f^2g$ | $be^2f^2$ |
| $acd^2e^2f^2g^2$ | $c^2efg^2$ | $a^2dfg$ | $acd^2e^2f$ |
| $b^2cd^2e$ | $a^2b^2c^2f^2$ | $ab^2de^2f^2g^2$ | $b^2cd^2ef^2g$ |
| $c^2dg$ | $a^2d^2e^2f^2g$ | $acef^2$ | $c^2df^2g^2$ |
| $a^2ef$ | $acd$ | $c^2d^2e^2g^2$ | $a^2eg$ |
| $abc^2def^2$ | $bd^2f$ | $a^2bce^2fg^2$ | $abc^2defg$ |
| $a^2b^2c^2de^2fg^2$ | $ab^2d^2eg^2$ | $b^2cg$ | $a^2b^2c^2de^2$ |
| $cde^2f^2h$ | $a^2c^2d^2efh$ | $afg^2h$ | $cde^2fgh$ |
| $ab^2c^2e^2fh$ | $b^2deh$ | $a^2b^2cd^2g^2h$ | $ab^2c^2e^2gh$ |
| $a^2b^2cdegh$ | $ab^2c^2d^2f^2gh$ | $b^2e^2f^2h$ | $a^2b^2cdef^2g^2h$ |
| $bc^2ef^2gh$ | $a^2bdfgh$ | $abcd^2e^2fh$ | $bc^2efg^2h$ |
| $a^2c^2g^2h$ | $ade^2f^2g^2h$ | $cd^2ef^2gh$ | $a^2c^2f^2h$ |
| $a^2bd^2e^2h$ | $abcef^2h$ | $bc^2df^2g^2h$ | $a^2bd^2e^2f^2gh$ |
| $abcdfg^2h$ | $bc^2d^2e^2g^2h$ | $a^2begh$ | $abcdh$ |
| $b^2d^2f^2g^2h$ | $a^2b^2ce^2fg^2h$ | $ab^2c^2defgh$ | $b^2d^2fh$ |
| $ad^2efgh$ | $cgh$ | $a^2c^2de^2h$ | $ad^2eg^2h$ |
| $a^2bc^2d^2ef^2g^2h^2$ | $abfg^2h^2$ | $bcde^2fgh^2$ | $a^2bc^2d^2efh^2$ |
| $defg^2h^2$ | $a^2cd^2g^2h^2$ | $ac^2e^2gh^2$ | $deh^2$ |
| $ac^2d^2h^2$ | $e^2f^2h^2$ | $a^2cdef^2g^2h^2$ | $ac^2d^2f^2gh^2$ |
| $a^2b^2df^2h^2$ | $ab^2cd^2e^2fh^2$ | $b^2c^2efg^2h^2$ | $a^2b^2dfgh^2$ |
| $abde^2gh^2$ | $bcd^2ef^2gh^2$ | $a^2bc^2f^2h^2$ | $abde^2f^2g^2h^2$ |
| $ab^2ceg^2h^2$ | $b^2c^2df^2g^2h^2$ | $a^2b^2d^2e^2f^2gh^2$ | $ab^2cef^2h^2$ |
| $b^2c^2d^2e^2fgh^2$ | $a^2b^2egh$ | $ab^2cdh^2$ | $b^2c^2d^2e^2g^2h^2$ |
| $a^2ce^2f^2gh^2$ | $ac^2defgh^2$ | $d^2fh^2$ | $a^2ce^2fg^2h^2$ |
| $bcfh^2$ | $a^2bc^2de^2h^2$ | $abd^2eg^2h^2$ | $bcgh^2$ |

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Plan 9.8.27. (Continued)

| 25 | 26 | 27 |
|---|---|---|
| $b^2cde^2g^2$ | $a^2b^2c^2d^2eg$ | $ab^2$ |
| $abc^2e^2f^2g^2$ | $bdef^2g$ | $a^2bcd^2f^2$ |
| $a^2bcdef$ | $abc^2d^2fg^2$ | $be^2fg$ |
| $c^2e$ | $a^2dg^2$ | $acd^2e^2g$ |
| $a^2b^2c^2fg$ | $ab^2de^2f$ | $b^2cd^2efg^2$ |
| $a^2d^2e^2fg^2$ | $acefg$ | $c^2df$ |
| $acdf^2g$ | $c^2d^2e^2f^2$ | $a^2ef^2g^2$ |
| $bd^2g$ | $a^2bce^2$ | $abc^2deg^2$ |
| $ab^2d^2ef^2$ | $b^2cf^2g^2$ | $a^2b^2c^2de^2f^2g$ |
| $a^2c^2d^2egh$ | $ah$ | $cde^2g^2h$ |
| $b^2def^2gh$ | $a^2b^2cd^2f^2h$ | $ab^2c^2e^2f^2g^2h$ |
| $ab^2c^2d^2fg^2h$ | $b^2e^2fgh$ | $a^2b^2cdefh$ |
| $a^2bdg^2h$ | $abcd^2e^2gh$ | $bc^2eh$ |
| $ade^2fh$ | $cd^2efg^2h$ | $a^2c^2fgh$ |
| $abcefgh$ | $bc^2dfh$ | $a^2bd^2e^2fg^2h$ |
| $bc^2d^2e^2f^2h$ | $a^2bef^2g^2h$ | $abcdf^2gh$ |
| $a^2b^2ce^2h$ | $ab^2c^2deg^2h$ | $b^2d^2gh$ |
| $cf^2g^2h$ | $a^2c^2de^2f^2gh$ | $ad^2ef^2h$ |
| $abh^2$ | $bcde^2g^2h^2$ | $a^2bc^2d^2egh^2$ |
| $a^2cd^2f^2h^2$ | $ac^2e^2f^2g^2h^2$ | $def^2gh^2$ |
| $e^2fgh^2$ | $a^2cdefh^2$ | $ac^2d^2fg^2h^2$ |
| $ab^2cd^2e^2gh^2$ | $b^2c^2eh^2$ | $a^2b^2dg^2h^2$ |
| $bcd^2efg^2h^2$ | $a^2bc^2fgh^2$ | $abde^2fh^2$ |
| $b^2c^2dfh^2$ | $a^2b^2d^2e^2fg^2h^2$ | $ab^2cefgh^2$ |
| $a^2b^2ef^2g^2h^2$ | $ab^2cdf^2gh^2$ | $b^2c^2d^2e^2f^2h^2$ |
| $ac^2deg^2h^2$ | $d^2gh^2$ | $a^2ce^2h^2$ |
| $a^2bc^2de^2f^2gh^2$ | $abd^2ef^2h^2$ | $bcf^2g^2h^2$ |

Plan 9.8.81. 1/9 replication of 8 factors in 9 blocks of 81 units each.

Factors: A, B, C, D, E, F, G, H.

$I = \underline{ABCDEH^2} = \underline{CD^2EF^2G^2} = ABC^2E^2F^2G^2H^2 = ABD^2FGH^2$.

Block confounding: $\underline{AB^2CH}$, $\underline{BCGH^2}$, $AC^2G$, $ABG^2H^2$.

With blocking: All two-factor interactions are measurable.

Blocks

Combine blocks of Plan 9.8.27 as follows:

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 1, 2, and 3 | 4, 5, and 6 | 7, 8, and 9 |
| 4 | 5 | 6 |
| 10, 11, and 12 | 13, 14, and 15 | 16, 17, and 18 |
| 7 | 8 | 9 |
| 19, 20, and 21 | 22, 23, and 24 | 25, 26, and 27 |

Plan 9.8.243. 1/9 replication of 8 factors in 3 blocks of 243 units each.

Factors: A, B, C, D, E, F, G, H.

$I = \underline{ABCDEH^2} = \underline{CD^2EF^2G^2} = ABC^2E^2F^2G^2H^2 = ABD^2FGH^2$.

Block confounding: $\underline{AB^2CH}$.

With blocking: All two-factor interactions are measurable.

Blocks

Combine blocks of Plan 9.8.27 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1 through 9 | 10 through 18 | 19 through 27 |

Plan 27.7.3. 1/27 replication of 7 factors in 27 blocks of 3 units each.

Factors: A, B, C, D, E, F, G.

$I = \underline{ACDEF^2G} = \underline{BC^2EF^2G} = ABDE^2FG^2 = AB^2C^2D = \underline{ABCEG^2} = AB^2CD^2EF = AB^2E^2F^2 = ABC^2D^2F^2G^2 = AD^2E^2G = BD^2FG = AC^2FG = CD^2E^2F^2 = BCDE^2G.$

Completely randomized: The following two-factor interactions are measurable: AB, AC, AE, $AG^2$, $BC^2$, BD, $BE^2$, $BF^2$, $BG^2$, CD, CE, CF, CG, $DE^2$, DG, $EF^2$, EG, $FG^2$.

Block confounding: $\underline{AF}$, $\underline{DF}$, $ADF^2$, $AD^2$, $\underline{AC^2DF}$, $ACD^2F$, $AC^2D^2F^2$, ACD, $ACF^2$, $CD^2$, $AC^2$, CF, $CDF^2$.

With blocking: The following two-factor interactions are measurable: AB, AC, AE, BD, $BF^2$, CD, CE, CG, DG, $EF^2$, EG, $FG^2$.

## Blocks

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| (1) | $ab^2df^2$ | $a^2bd^2f$ | cdg |
| $abcdef^2g$ | $a^2cd^2efg$ | $b^2ceg$ | $abc^2d^2ef^2g^2$ |
| $a^2b^2c^2d^2e^2fg^2$ | $bc^2e^2g^2$ | $ac^2de^2f^2g^2$ | $a^2b^2e^2f$ |
| **5** | **6** | **7** | **8** |
| $ab^2cd^2f^2g$ | $a^2bcfg$ | $c^2d^2g^2$ | $ab^2c^2f^2g^2$ |
| $a^2c^2efg^2$ | $b^2c^2deg^2$ | $abef^2$ | $a^2def$ |
| $bde^2$ | $ad^2e^2f^2$ | $a^2b^2cde^2fg$ | $bcd^2e^2g$ |
| **9** | **10** | **11** | **12** |
| $a^2bc^2dfg^2$ | ace | $a^2b^2cdef^2$ | $bcd^2ef$ |
| $b^2d^2e$ | $a^2bc^2de^2f^2g$ | $c^2d^2e^2fg$ | $ab^2c^2e^2g$ |
| $ace^2f^2g$ | $b^2d^2fg^2$ | $abg^2$ | $a^2df^2g^2$ |

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Plan 27.7.3. (Continued)

| 13 | 14 | 15 | 16 |
|---|---|---|---|
| $b^2cf$ | $abcd$ | $a^2cd^2f^2$ | $b^2c^2dfg$ |
| $ac^2deg$ | $a^2b^2c^2d^2ef^2g$ | $bc^2efg$ | $ad^2eg^2$ |
| $a^2bd^2e^2f^2g^2$ | $e^2fg^2$ | $ab^2de^2g^2$ | $a^2bce^2f^2$ |

| 17 | 18 | 19 | 20 |
|---|---|---|---|
| $abc^2d^2g$ | $a^2c^2f^2g$ | $a^2c^2e^2$ | $b^2c^2de^2f^2$ |
| $a^2b^2ef^2g^2$ | $bdefg^2$ | $bdf^2g$ | $ad^2fg$ |
| $cde^2f$ | $ab^2cd^2e^2$ | $ab^2cd^2efg^2$ | $a^2bceg^2$ |

| 21 | 22 | 23 | 24 |
|---|---|---|---|
| $a^2b^2g$ | $a^2de^2g$ | $b^2d^2e^2f^2g$ | $abe^2fg$ |
| $cdef^2g^2$ | $bcd^2f^2g^2$ | $acfg^2$ | $a^2b^2cdg^2$ |
| $abc^2d^2e^2f$ | $ab^2c^2ef$ | $a^2bc^2de$ | $c^2d^2ef^2$ |

| 25 | 26 | 27 |
|---|---|---|
| $bc^2f^2$ | $ac^2df$ | $a^2b^2c^2d^2$ |
| $ab^2defg$ | $a^2bd^2eg$ | $ef^2g$ |
| $a^2cd^2e^2g^2$ | $b^2ce^2f^2g^2$ | $abcde^2fg^2$ |

Plan 27.7.9. 1/27 replication of 7 factors in 9 blocks of 9 units each.

Factors: A, B, C, D, E, F, G.

I = Same as Plan 27.7.3

Block confounding: AF, DF, $ADF^2$, $AD^2$.

With blocking: The following two-factor interactions are measurable: AB, AC, AE, $AG^2$, $BC^2$, BD, $BE^2$, $BF^2$, $BG^2$, CD, CE, CF, CG, $DE^2$, DG, $EF^2$, EG, $FG^2$.

## Blocks

Combine blocks of Plan 27.7.3 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1, 2, and 3 | 4, 5, and 6 | 7, 8, and 9 |
| 4 | 5 | 6 |
| 10, 11, and 12 | 13, 14, and 15 | 16, 17, and 18 |
| 7 | 8 | 9 |
| 19, 20, and 21 | 22, 23, and 24 | 25, 26, and 27 |

Plan 27.7.27. 1/27 replication of 7 factors in 3 blocks of 27 units each.

Factors: A, B, C, D, E, F, G.

I = Same as Plan 27.7.3.

Block confounding: AF.

With blocking: The following two-factor interactions are measurable: AB, AC, AE, $AG^2$, $BC^2$, BD, $BE^2$, $BF^2$, $BG^2$, CD, CE, CF, CG, $DE^2$, DG, $EF^2$, EG, $FG^2$.

Blocks

Combine blocks of Plan 27.7.3 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1 through 9 | 10 through 18 | 19 through 27 |

Plan 27.8.9. 1/27 replication of 8 factors in 27 blocks of 9 units each.

Factors: A, B, C, D, E, F, G, H.

$I = \underline{BCDEFG} = \underline{ACDE^2F^2H} = ABC^2D^2GH = AB^2EFG^2H = \underline{ABD^2E^2F} = AB^2CF^2G = AB^2C^2E^2H^2 = ABCD^2EF^2G^2H^2 = ADFGH^2 = AC^2DEG^2 = BC^2DF^2H^2 = CE^2FG^2H^2 = BDE^2G^2H.$

Completely randomized: All two-factor interactions are measurable.

Block confounding: $\underline{ABC^2EF^2}$, $\underline{AB^2C^2DF}$, $AC^2D^2E^2$, $BDE^2F^2$, $\underline{AB^2CD^2F}$, $ADE^2$, $AB^2F$, $ABD^2EF^2$, $ACE^2$, $BC^2D^2E^2F^2$, $CD^2$, $BCE^2F^2$, $ABCDEF^2$.

With blocking: All two-factor interactions except $AH^2$, $EF^2$, $CD^2$, and $BG^2$ are measurable.

## Blocks

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| (1) | $ac^2e^2fg$ | $a^2cef^2g^2$ | $b^2cfg^2$ |
| $bcdefg$ | $abdf^2g^2$ | $a^2bc^2de^2$ | $c^2def^2$ |
| $acde^2f^2h$ | $a^2degh$ | $c^2dfg^2h$ | $ab^2c^2de^2g^2h$ |
| $abc^2d^2gh$ | $a^2bcd^2e^2fg^2h$ | $bd^2ef^2h$ | $ad^2fh$ |
| $a^2be^2f^2gh^2$ | $bc^2eg^2h^2$ | $abcfh^2$ | $a^2ce^2h^2$ |
| $b^2c^2d^2e^2f^2g^2$ | $ab^2cd^2e$ | $a^2b^2d^2fg$ | $bd^2e^2g$ |
| $a^2c^2d^2efh^2$ | $cd^2f^2gh^2$ | $ad^2e^2g^2h^2$ | $a^2b^2d^2ef^2g^2h^2$ |
| $a^2b^2cdg^2h^2$ | $b^2de^2fh^2$ | $ab^2c^2def^2gh^2$ | $a^2bc^2dfgh^2$ |
| $ab^2efg^2h$ | $a^2b^2c^2f^2h$ | $b^2ce^2gh$ | $abcef^2gh$ |

| 5 | 6 | 7 | 8 |
|---|---|---|---|
| $ab^2e^2f^2$ | $a^2b^2c^2eg$ | $bc^2f^2g$ | $abce^2g^2$ |
| $acdg$ | $a^2de^2fg^2$ | $b^2deg^2$ | $ab^2c^2df$ |
| $a^2b^2cdefh$ | $b^2df^2gh$ | $abde^2fgh$ | $a^2bc^2def^2g^2h$ |
| $a^2c^2d^2e^2f^2gh$ | $cd^2eg^2h$ | $ab^2cd^2f^2g^2h$ | $a^2b^2d^2e^2h$ |
| $efgh^2$ | $ac^2f^2g^2h^2$ | $a^2b^2c^2e^2fg^2h^2$ | $b^2cef^2h^2$ |
| $abc^2d^2efg^2$ | $a^2bcd^2f^2$ | $cd^2e^2f$ | $ad^2ef^2g$ |
| $b^2c^2d^2h^2$ | $ab^2cd^2e^2fgh^2$ | $a^2bcd^2egh^2$ | $bd^2fg^2h^2$ |
| $bcde^2f^2g^2h^2$ | $abdeh^2$ | $a^2df^2h^2$ | $c^2de^2gh^2$ |
| $a^2bg^2h$ | $bc^2e^2fh$ | $ac^2eh$ | $a^2cfgh$ |

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Plan 27.8.9. (Continued)

| 9 | 10 | 11 | 12 |
|---|---|---|---|
| $a^2bef$ | $b^2ef^2g$ | $ab^2c^2g^2$ | $a^2b^2ce^2f$ |
| $a^2b^2cde^2f^2g$ | $cde^2g^2$ | $adef$ | $a^2c^2df^2g$ |
| $bcdh$ | $ab^2cdfgh$ | $a^2b^2de^2f^2g^2h$ | $b^2c^2deh$ |
| $b^2c^2d^2efgh$ | $ac^2d^2ef^2g^2h$ | $a^2cd^2h$ | $d^2e^2fgh$ |
| $ab^2gh^2$ | $a^2fg^2h^2$ | $c^2e^2f^2h^2$ | $acegh^2$ |
| $a^2c^2d^2g^2$ | $bc^2d^2f$ | $abcd^2e^2f^2g$ | $a^2bd^2eg^2$ |
| $abc^2d^2e^2f^2h^2$ | $a^2b^2c^2d^2e^2gh^2$ | $b^2cd^2efg^2h^2$ | $ab^2d^2f^2h^2$ |
| $acdefg^2h^2$ | $a^2bcdef^2h^2$ | $bdgh^2$ | $abc^2de^2fg^2h^2$ |
| $e^2f^2g^2h$ | $abe^2h$ | $a^2bc^2efgh$ | $bcf^2g^2h$ |

| 13 | 14 | 15 | 16 |
|---|---|---|---|
| $bce$ | $abfg$ | $a^2bc^2e^2f^2g^2$ | $c^2efg^2$ |
| $b^2c^2de^2fg$ | $ab^2cdef^2g^2$ | $a^2b^2d$ | $bde^2f^2$ |
| $abc^2df^2h$ | $a^2bcde^2gh$ | $bdefg^2h$ | $adg^2h$ |
| $ab^2d^2egh$ | $a^2b^2c^2d^2fg^2h$ | $b^2cd^2e^2f^2h$ | $abcd^2efh$ |
| $a^2b^2cf^2gh^2$ | $b^2e^2g^2h^2$ | $ab^2c^2efh^2$ | $a^2bc^2h^2$ |
| $d^2f^2g^2$ | $ac^2d^2e^2$ | $a^2cd^2efg$ | $b^2cd^2g$ |
| $a^2bd^2e^2fh^2$ | $bc^2d^2ef^2gh^2$ | $abcd^2g^2h^2$ | $a^2cd^2e^2f^2g^2h^2$ |
| $a^2c^2deg^2h^2$ | $cdfh^2$ | $ade^2f^2gh^2$ | $a^2b^2defgh^2$ |
| $ace^2fg^2h$ | $a^2ef^2h$ | $c^2gh$ | $ab^2c^2e^2f^2gh$ |

| 17 | 18 | 19 | 20 |
|---|---|---|---|
| $acf^2$ | $a^2e^2g$ | $be^2fg^2$ | $abc^2ef^2$ |
| $abc^2deg$ | $a^2bcdfg^2$ | $b^2cdf^2$ | $ab^2de^2g$ |
| $a^2c^2de^2fh$ | $cdef^2gh$ | $abcdeg^2h$ | $a^2bdfh$ |
| $a^2bd^2f^2gh$ | $bc^2d^2e^2g^2h$ | $ab^2c^2d^2e^2fh$ | $a^2b^2cd^2ef^2gh$ |
| $bce^2fgh^2$ | $abef^2g^2h^2$ | $a^2b^2eh^2$ | $b^2c^2fgh^2$ |
| $ab^2d^2e^2fg^2$ | $a^2b^2c^2d^2ef^2$ | $c^2d^2eg$ | $acd^2fg^2$ |
| $d^2eh^2$ | $ac^2d^2fgh^2$ | $a^2bc^2d^2f^2g^2h^2$ | $bcd^2e^2h^2$ |
| $b^2c^2df^2g^2h^2$ | $ab^2cde^2h^2$ | $a^2cde^2fgh^2$ | $def^2g^2h^2$ |
| $a^2b^2ceg^2h$ | $b^2fh$ | $af^2gh$ | $a^2c^2e^2g^2h$ |

| 21 | 22 | 23 | 24 |
|---|---|---|---|
| $a^2bcg$ | $ce^2f^2g$ | $aeg^2$ | $a^2c^2f$ |
| $a^2b^2c^2defg^2$ | $bc^2dg^2$ | $abcde^2f$ | $a^2bdef^2g$ |
| $bc^2de^2f^2gh$ | $ac^2defgh$ | $a^2cdf^2g^2h$ | $de^2h$ |
| $b^2d^2g^2h$ | $abd^2e^2f^2g^2h$ | $a^2bc^2d^2eh$ | $bcd^2fgh$ |
| $ab^2ce^2f^2g^2h^2$ | $a^2bcefg^2h^2$ | $bf^2h^2$ | $abc^2e^2gh^2$ |
| $a^2d^2e^2f^2$ | $b^2d^2ef$ | $ab^2c^2d^2f^2g$ | $a^2b^2cd^2e^2g^2$ |
| $abd^2efgh^2$ | $a^2d^2gh^2$ | $c^2d^2e^2fg^2h^2$ | $acd^2ef^2h^2$ |
| $ac^2dh^2$ | $a^2b^2c^2de^2f^2h^2$ | $b^2cdegh^2$ | $ab^2dfg^2h^2$ |
| $cefh$ | $ab^2ch$ | $a^2b^2e^2fgh$ | $b^2c^2ef^2g^2h$ |

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Plan 27.8.9. (Continued)

| 25 | 26 | 27 |
|---|---|---|
| $b^2c^2e^2$ | $ab^2cefg$ | $a^2b^2f^2g^2$ |
| $dfg$ | $ac^2de^2f^2g^2$ | $a^2cde$ |
| $ab^2def^2h$ | $a^2b^2c^2dgh$ | $b^2cde^2fg^2h$ |
| $acd^2e^2gh$ | $a^2d^2efg^2h$ | $c^2d^2f^2h$ |
| $a^2c^2ef^2gh^2$ | $cg^2h^2$ | $ae^2fh^2$ |
| $bcd^2ef^2g^2$ | $abd^2$ | $a^2bc^2d^2e^2fg$ |
| $a^2b^2cd^2fh^2$ | $b^2d^2e^2f^2gh^2$ | $ab^2c^2d^2eg^2h^2$ |
| $a^2bde^2g^2h^2$ | $bc^2defh^2$ | $abcdf^2gh^2$ |
| $abc^2fg^2h$ | $a^2bce^2f^2h$ | $begh$ |

Plan 27.8.27. 1/27 replication of 8 factors in 9 blocks of 27 units each.

Factors: A, B, C, D, E, F, G, H.

I = Same as Plan 27.8.9.

Block confounding: $ABC^2EF^2$, $AB^2C^2DF$, $AC^2D^2E^2$, $BDE^2F^2$.

With blocking: All two-factor interactions are measurable.

Blocks

Combine blocks of Plan 27.8.9 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1, 2, and 3 | 4, 5, and 6 | 7, 8, and 9 |
| 4 | 5 | 6 |
| 10, 11, and 12 | 13, 14, and 15 | 16, 17, and 18 |
| 7 | 8 | 9 |
| 19, 20, and 21 | 22, 23, and 24 | 25, 26, and 27 |

Plan 27.8.81. 1/27 replication of 8 factors in 3 blocks of 81 units each.

Factors: A, B, C, D, E, F, G, H.

I = Same as Plan 27.8.9.

Block confounding: $ABC^2EF^2$.

With blocking: All two-factor interactions are measurable.

Blocks

Combine blocks of Plan 27.8.9 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1 through 9 | 10 through 18 | 19 through 27 |

Plan 27.9.27. 1/27 replication of 9 factors in 27 blocks of 27 units each.

Factors: A, B, C, D, E, F, G, H, J.

$I = \underline{BCDEFG} = \underline{ACDE^2F^2H} = ABC^2D^2GH = AB^2EFG^2H = \underline{ABD^2E^2FJ} = AB^2CF^2GJ = AB^2C^2E^2H^2J^2 = ABCD^2EF^2G^2H^2J^2 = ADFGH^2J^2 = AC^2DEG^2J = BC^2DF^2H^2J = CE^2FG^2H^2J = BDE^2G^2HJ^2.$

Completely randomized: All two-factor interactions are measurable.

Block confounding: $\underline{ABC^2EF^2}$, $\underline{AB^2C^2DF}$, $AC^2D^2E^2$, $BDE^2F^2$, $\underline{AB^2CD^2E}$, $ADEF$, $AB^2E^2F^2$, $ABD^2$, $ACF^2$, $BC^2D^2F$, $CD^2E^2F$, $BCE$, $ABCDE^2F$.

With blocking: All two-factor interactions are measurable.

Blocks

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| (1) | $bcd^2g^2j$ | $a^2bc^2d^2ef^2gj$ | $c^2d^2fgj$ |
| $abd^2e^2fj$ | $ab^2cde^2fg^2j^2$ | $b^2c^2dgj^2$ | $abc^2de^2f^2gj^2$ |
| $ab^2cf^2gj$ | $ac^2d^2f^2j^2$ | $d^2efg^2j^2$ | $ab^2d^2g^2j^2$ |
| $a^2cd^2e^2gj^2$ | $a^2bc^2de^2$ | $abdf^2g^2$ | $a^2de^2fg^2$ |
| $b^2c^2d^2e^2f^2g^2$ | $de^2f^2gj$ | $a^2cdfj$ | $b^2cde^2j$ |
| $a^2bcehj$ | $a^2b^2c^2d^2eg^2hj^2$ | $ab^2d^2e^2f^2ghj^2$ | $a^2bd^2efghj^2$ |
| $b^2cd^2fhj^2$ | $c^2dfg^2h$ | $a^2degh$ | $b^2df^2gh$ |
| $c^2ef^2ghj^2$ | $bd^2ef^2h$ | $a^2bcd^2e^2fg^2h$ | $cd^2eg^2h$ |
| $abc^2d^2gh$ | $ab^2dhj$ | $b^2cdef^2g^2hj$ | $abcdfg^2hj$ |
| $a^2d^2f^2g^2hj$ | $a^2bcdf^2ghj^2$ | $abc^2defhj^2$ | $a^2c^2dhj^2$ |
| $acde^2f^2h$ | $abc^2e^2f^2g^2hj$ | $bfghj$ | $ae^2ghj$ |
| $ab^2efg^2h$ | $acd^2efghj$ | $c^2d^2e^2hj$ | $ab^2c^2d^2ef^2hj$ |
| $bde^2g^2hj^2$ | $b^2ce^2gh$ | $a^2b^2c^2f^2h$ | $bc^2e^2fh$ |
| $a^2b^2c^2de^2fghj$ | $a^2e^2fhj^2$ | $acg^2hj^2$ | $a^2b^2ce^2f^2g^2hj^2$ |
| $a^2b^2def^2j^2$ | $a^2cef^2g^2$ | $ac^2e^2fg$ | $a^2b^2c^2eg$ |
| $a^2bc^2fg^2j^2$ | $a^2b^2d^2fg$ | $ab^2cd^2e$ | $a^2bcd^2f^2$ |
| $ac^2deg^2j$ | $abegj^2$ | $bce^2f^2j^2$ | $acefj^2$ |
| $bcdefg$ | $b^2c^2efj$ | $a^2b^2e^2g^2j$ | $bef^2g^2j$ |
| $ab^2c^2e^2h^2j^2$ | $ad^2e^2g^2h^2$ | $cd^2f^2gh^2$ | $ab^2cd^2e^2fgh^2$ |
| $bc^2df^2h^2j$ | $b^2f^2g^2h^2j^2$ | $a^2b^2cefgh^2j^2$ | $bcgh^2j^2$ |
| $ce^2fg^2h^2j$ | $bc^2d^2e^2fgh^2j^2$ | $a^2bd^2h^2j^2$ | $d^2e^2f^2h^2j^2$ |
| $a^2b^2cdg^2h^2$ | $a^2c^2gh^2j$ | $aef^2h^2j$ | $a^2b^2fh^2j$ |
| $adfgh^2j^2$ | $abcfh^2$ | $bc^2eg^2h^2$ | $ac^2f^2g^2h^2$ |
| $a^2c^2d^2efh^2$ | $a^2bdefg^2h^2j$ | $abcde^2gh^2j$ | $a^2cdef^2gh^2j$ |
| $a^2be^2f^2gh^2$ | $a^2b^2cd^2e^2f^2h^2j$ | $ab^2c^2d^2fg^2h^2j$ | $a^2bc^2d^2e^2g^2h^2j$ |
| $b^2d^2egh^2j$ | $cdeh^2j^2$ | $a^2c^2de^2f^2g^2h^2j^2$ | $b^2c^2defg^2h^2j^2$ |
| $abcd^2ef^2g^2h^2j^2$ | $ab^2c^2def^2gh^2$ | $b^2de^2fh^2$ | $abdeh^2$ |

Plan 27.9.27. (Continued)

| 5 | 6 | 7 | 8 |
|---|---|---|---|
| $a^2d^2ej$ | $acd^2e^2f^2g^2j$ | $a^2b^2cd^2efg^2j$ | $ab^2c^2d^2e^2gj$ |
| $bdfj^2$ | $a^2bcdeg^2j^2$ | $cdf^2g^2j^2$ | $a^2c^2defgj^2$ |
| $b^2cd^2ef^2gj^2$ | $a^2b^2c^2d^2e^2fj^2$ | $bc^2d^2ej^2$ | $a^2bd^2e^2f^2g^2j^2$ |
| $acdg$ | $c^2def^2$ | $ab^2c^2df$ | $b^2deg^2$ |
| $a^2b^2c^2df^2g^2j$ | $ab^2defgj$ | $a^2bdgj$ | $abcdef^2j$ |
| $abcd^2e^2hj^2$ | $bc^2d^2f^2g^2hj^2$ | $ac^2d^2e^2fg^2hj^2$ | $d^2ghj^2$ |
| $a^2b^2cdefh$ | $ab^2c^2de^2g^2h$ | $a^2bc^2def^2g^2h$ | $abde^2fgh$ |
| $a^2c^2d^2e^2f^2gh$ | $ad^2fh$ | $a^2b^2d^2e^2h$ | $ab^2cd^2f^2g^2h$ |
| $bc^2deghj$ | $a^2bde^2f^2hj$ | $defhj$ | $a^2cde^2g^2hj$ |
| $adef^2g^2hj^2$ | $cde^2fghj^2$ | $ab^2cdeghj^2$ | $b^2c^2de^2f^2hj^2$ |
| $cf^2hj$ | $a^2c^2efg^2hj$ | $b^2c^2g^2hj$ | $a^2b^2ef^2ghj$ |
| $b^2d^2e^2fg^2hj$ | $a^2b^2cd^2ghj$ | $bcd^2e^2f^2ghj$ | $a^2bc^2d^2fhj$ |
| $a^2bg^2h$ | $abcef^2gh$ | $a^2cfgh$ | $ac^2eh$ |
| $ab^2c^2fghj^2$ | $b^2ehj^2$ | $abf^2hj^2$ | $bcefg^2hj^2$ |
| $ab^2e^2f^2$ | $b^2cfg^2$ | $abce^2g^2$ | $bc^2f^2g$ |
| $abc^2d^2efg^2$ | $bd^2e^2g$ | $ad^2ef^2g$ | $cd^2e^2f$ |
| $c^2e^2g^2j^2$ | $a^2f^2gj^2$ | $b^2e^2fgj^2$ | $a^2b^2cj^2$ |
| $a^2bce^2fgj$ | $abc^2j$ | $a^2c^2e^2f^2j$ | $afg^2j$ |
| $b^2c^2d^2h^2$ | $a^2b^2d^2ef^2g^2h^2$ | $bd^2fg^2h^2$ | $a^2bcd^2egh^2$ |
| $a^2bc^2ef^2h^2j^2$ | $abe^2fg^2h^2j^2$ | $a^2eg^2h^2j^2$ | $ace^2f^2gh^2j^2$ |
| $a^2cd^2fg^2h^2j^2$ | $ac^2d^2egh^2j^2$ | $a^2b^2c^2d^2f^2gh^2j^2$ | $ab^2d^2efh^2j^2$ |
| $ab^2ceg^2h^2j$ | $b^2c^2e^2f^2gh^2j$ | $abc^2efgh^2j$ | $be^2h^2j$ |
| $efgh^2$ | $a^2ce^2h^2$ | $b^2cef^2h^2$ | $a^2b^2c^2e^2fg^2h^2$ |
| $ac^2de^2fh^2j$ | $dg^2h^2j$ | $ab^2de^2f^2g^2h^2j$ | $b^2cdfgh^2j$ |
| $abd^2f^2gh^2j$ | $bcd^2efh^2j$ | $acd^2h^2j$ | $c^2d^2ef^2g^2h^2j$ |
| $a^2b^2de^2gh^2j^2$ | $ab^2cdf^2h^2j^2$ | $a^2bcde^2fh^2j^2$ | $abc^2dg^2h^2j^2$ |
| $bcde^2f^2g^2h^2$ | $a^2bc^2dfgh^2$ | $c^2de^2gh^2$ | $a^2df^2h^2$ |

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Plan 27.9.27. (Continued)

| 9 | 10 | 11 | 12 |
|---|---|---|---|
| $b^2d^2f^2j$ | $cd^2ef^2j$ | $a^2c^2d^2e^2fg^2j$ | $ad^2gj$ |
| $ade^2j^2$ | $abcdj^2$ | $bc^2def^2g^2j^2$ | $a^2bde^2fgj^2$ |
| $abcd^2fgj^2$ | $ab^2c^2d^2efgj^2$ | $b^2d^2e^2j^2$ | $a^2b^2cd^2f^2g^2j^2$ |
| $a^2b^2cde^2f^2g$ | $a^2c^2df^2g$ | $adef$ | $cde^2g^2$ |
| $bc^2de^2fg^2j$ | $b^2dfg^2j$ | $a^2b^2cdegj$ | $ab^2c^2de^2f^2j$ |
| $a^2cd^2ef^2hj^2$ | $a^2bc^2d^2e^2f^2hj^2$ | $abd^2fg^2hj^2$ | $bcd^2eghj^2$ |
| $bcdh$ | $b^2c^2deh$ | $a^2b^2de^2f^2g^2h$ | $ab^2cdfgh$ |
| $b^2c^2d^2efgh$ | $d^2e^2fgh$ | $a^2cd^2h$ | $ac^2d^2ef^2g^2h$ |
| $ac^2df^2ghj$ | $abdef^2ghj$ | $bcde^2fhj$ | $a^2bc^2dg^2hj$ |
| $a^2b^2dfg^2hj^2$ | $a^2cdefg^2hj^2$ | $ac^2de^2ghj^2$ | $df^2hj^2$ |
| $ab^2ce^2fhj$ | $ac^2fhj$ | $eg^2hj$ | $a^2ce^2f^2ghj$ |
| $abd^2eg^2hj$ | $ab^2cd^2e^2g^2hj$ | $b^2c^2d^2f^2ghj$ | $a^2b^2d^2efhj$ |
| $e^2f^2g^2h$ | $bcf^2g^2h$ | $a^2bc^2efgh$ | $abe^2h$ |
| $a^2bc^2e^2ghj^2$ | $a^2b^2ghj^2$ | $ab^2cef^2hj^2$ | $b^2c^2e^2fg^2hj^2$ |
| $a^2bef$ | $a^2b^2ce^2f$ | $ab^2c^2g^2$ | $b^2ef^2g$ |
| $a^2c^2d^2g^2$ | $a^2bd^2eg^2$ | $abcd^2e^2f^2g$ | $bc^2d^2f$ |
| $ab^2c^2ef^2g^2j^2$ | $ae^2f^2g^2j^2$ | $cfgj^2$ | $a^2c^2ej^2$ |
| $cegj$ | $bc^2e^2gj$ | $a^2bf^2j$ | $abcefg^2j$ |
| $abc^2d^2e^2f^2h^2$ | $ab^2d^2f^2h^2$ | $b^2cd^2efg^2h^2$ | $a^2b^2c^2d^2e^2gh^2$ |
| $c^2fh^2j^2$ | $befh^2j^2$ | $a^2bce^2g^2h^2j^2$ | $abc^2f^2gh^2j^2$ |
| $b^2cd^2e^2g^2h^2j^2$ | $c^2d^2g^2h^2j^2$ | $a^2d^2ef^2gh^2j^2$ | $acd^2e^2fh^2j^2$ |
| $a^2bcf^2g^2h^2j$ | $a^2b^2c^2ef^2g^2h^2j$ | $ab^2e^2fgh^2j$ | $b^2ch^2j$ |
| $ab^2gh^2$ | $acegh^2$ | $c^2e^2f^2h^2$ | $a^2fg^2h^2$ |
| $a^2b^2c^2deh^2j$ | $a^2de^2h^2j$ | $acdf^2g^2h^2j$ | $c^2defgh^2j$ |
| $a^2d^2e^2fgh^2j$ | $a^2bcd^2fgh^2j$ | $abc^2d^2eh^2j$ | $bd^2e^2f^2g^2h^2j$ |
| $bdef^2gh^2j^2$ | $b^2cde^2f^2gh^2j^2$ | $a^2b^2c^2dfh^2j^2$ | $ab^2deg^2h^2j^2$ |
| $acdefg^2h^2$ | $abc^2de^2fg^2h^2$ | $bdgh^2$ | $a^2bcdef^2h^2$ |

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Plan 27.9.27. (Continued)

| 13 | 14 | 15 | 16 |
|---|---|---|---|
| $a^2b^2d^2e^2f^2gj$ | $ab^2cd^2fj$ | $b^2c^2d^2eg^2j$ | $abc^2d^2f^2g^2j$ |
| $degj^2$ | $a^2cde^2f^2j^2$ | $ac^2dfg^2j^2$ | $a^2b^2c^2de^2g^2j^2$ |
| $bcd^2e^2fg^2j^2$ | $a^2bc^2d^2gj^2$ | $abd^2ef^2j^2$ | $a^2d^2fj^2$ |
| $ab^2cdef^2g^2$ | $b^2c^2de^2fg$ | $a^2b^2d$ | $bde^2f^2$ |
| $a^2bc^2defj$ | $abde^2g^2j$ | $bcdf^2gj$ | $acde^2fgj$ |
| $acd^2f^2ghj^2$ | $c^2d^2efhj^2$ | $a^2d^2e^2g^2hj^2$ | $b^2d^2ef^2g^2hj^2$ |
| $a^2bcde^2gh$ | $abc^2df^2h$ | $bdefg^2h$ | $adg^2h$ |
| $a^2b^2c^2d^2fg^2h$ | $ab^2d^2egh$ | $b^2cd^2e^2f^2h$ | $abcd^2efh$ |
| $c^2de^2f^2g^2hj$ | $a^2dfghj$ | $acdehj$ | $a^2b^2cdf^2hj$ |
| $ab^2de^2fhj^2$ | $b^2cdg^2hj^2$ | $a^2b^2c^2def^2ghj^2$ | $bc^2dfghj^2$ |
| $b^2cefghj$ | $a^2b^2c^2e^2hj$ | $ab^2f^2g^2hj$ | $a^2be^2fg^2hj$ |
| $bd^2hj$ | $a^2bcd^2ef^2g^2hj$ | $abc^2d^2e^2fghj$ | $a^2c^2d^2eghj$ |
| $a^2ef^2h$ | $ace^2fg^2h$ | $c^2gh$ | $ab^2c^2e^2f^2gh$ |
| $abc^2eg^2hj^2$ | $be^2f^2ghj^2$ | $a^2bcfhj^2$ | $ce^2hj^2$ |
| $abfg$ | $bce$ | $a^2bc^2e^2f^2g^2$ | $c^2efg^2$ |
| $ac^2d^2e^2$ | $d^2f^2g^2$ | $a^2cd^2efg$ | $b^2cd^2g$ |
| $b^2c^2f^2j^2$ | $a^2b^2efg^2j^2$ | $ab^2ce^2gj^2$ | $a^2bcef^2gj^2$ |
| $a^2cg^2j$ | $ac^2ef^2gj$ | $e^2fj$ | $ab^2ej$ |
| $bc^2d^2ef^2gh^2$ | $a^2bd^2e^2fh^2$ | $abcd^2g^2h^2$ | $a^2cd^2e^2f^2g^2h^2$ |
| $a^2c^2e^2fgh^2j^2$ | $ah^2j^2$ | $cef^2g^2h^2j^2$ | $ab^2cfg^2h^2j^2$ |
| $a^2b^2cd^2eh^2j^2$ | $ab^2c^2d^2e^2f^2g^2h^2j^2$ | $b^2d^2fgh^2j^2$ | $abd^2e^2gh^2j^2$ |
| $abce^2f^2h^2j$ | $bc^2fg^2h^2j$ | $a^2begh^2j$ | $f^2gh^2j$ |
| $b^2e^2g^2h^2$ | $a^2b^2cf^2gh^2$ | $ab^2c^2efh^2$ | $a^2bc^2h^2$ |
| $ab^2c^2dgh^2j$ | $b^2def^2h^2j$ | $a^2b^2cde^2fg^2h^2j$ | $bcdeg^2h^2j$ |
| $ad^2efg^2h^2j$ | $cd^2e^2gh^2j$ | $a^2c^2d^2f^2h^2j$ | $b^2c^2d^2e^2fh^2j$ |
| $a^2bdf^2g^2h^2j^2$ | $abcdefgh^2j^2$ | $bc^2de^2h^2j^2$ | $ac^2def^2h^2j^2$ |
| $cdfh^2$ | $a^2c^2deg^2h^2$ | $ade^2f^2gh^2$ | $a^2b^2defgh^2$ |

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Plan 27.9.27. (Continued)

| 17 | 18 | 19 | 20 |
|---|---|---|---|
| $bd^2efgj$ | $a^2bcd^2e^2j$ | $a^2b^2c^2d^2j$ | $ab^2d^2ef^2g^2j$ |
| $ab^2df^2gj^2$ | $b^2cdefj^2$ | $c^2de^2fj^2$ | $a^2dg^2j^2$ |
| $acd^2eg^2j^2$ | $c^2d^2e^2f^2gj^2$ | $bd^2f^2gj^2$ | $a^2bcd^2efj^2$ |
| $a^2bcdfg^2$ | $abc^2deg$ | $ab^2de^2g$ | $b^2cdf^2$ |
| $c^2dj$ | $a^2def^2g^2j$ | $a^2bcde^2f^2g^2j$ | $abc^2dfgj$ |
| $a^2b^2cd^2e^2fghj^2$ | $ab^2c^2d^2hj^2$ | $ad^2ehj^2$ | $cd^2e^2f^2g^2hj^2$ |
| $cdef^2gh$ | $a^2c^2de^2fh$ | $a^2bdfh$ | $abcdeg^2h$ |
| $bc^2d^2e^2g^2h$ | $a^2bd^2f^2gh$ | $a^2b^2cd^2ef^2gh$ | $ab^2c^2d^2e^2fh$ |
| $ab^2c^2defg^2hj$ | $b^2de^2ghj$ | $cdghj$ | $a^2c^2def^2hj$ |
| $a^2bdehj^2$ | $abcde^2f^2g^2hj^2$ | $ab^2c^2df^2g^2hj^2$ | $b^2defghj^2$ |
| $abcghj$ | $bc^2ef^2hj$ | $b^2e^2f^2hj$ | $a^2b^2cfg^2hj$ |
| $ad^2e^2f^2hj$ | $cd^2fg^2hj$ | $bc^2d^2efg^2hj$ | $a^2bd^2e^2ghj$ |
| $b^2fh$ | $a^2b^2ceg^2h$ | $a^2c^2e^2g^2h$ | $af^2gh$ |
| $a^2c^2f^2g^2hj^2$ | $aefghj^2$ | $abce^2fghj^2$ | $bc^2hj^2$ |
| $a^2e^2g$ | $acf^2$ | $abc^2ef^2$ | $be^2fg^2$ |
| $a^2b^2c^2d^2ef^2$ | $ab^2d^2e^2fg^2$ | $acd^2fg^2$ | $c^2d^2eg$ |
| $abc^2e^2fj^2$ | $bg^2j^2$ | $b^2ceg^2j^2$ | $a^2b^2c^2e^2f^2gj^2$ |
| $b^2ce^2f^2g^2j$ | $a^2b^2c^2fgj$ | $a^2efgj$ | $ace^2j$ |
| $ac^2d^2fgh^2$ | $d^2eh^2$ | $bcd^2e^2h^2$ | $a^2bc^2d^2f^2g^2h^2$ |
| $b^2c^2egh^2j^2$ | $a^2b^2e^2f^2h^2j^2$ | $a^2cf^2h^2j^2$ | $ac^2efg^2h^2j^2$ |
| $bcd^2f^2h^2j^2$ | $a^2bc^2d^2efg^2h^2j^2$ | $a^2b^2d^2e^2fg^2h^2j^2$ | $ab^2cd^2gh^2j^2$ |
| $a^2cefh^2j$ | $ac^2e^2g^2h^2j$ | $abg^2h^2j$ | $bcef^2gh^2j$ |
| $abef^2g^2h^2$ | $bce^2fgh^2$ | $b^2c^2fgh^2$ | $a^2b^2eh^2$ |
| $a^2bc^2de^2f^2gh^2j$ | $abdfh^2j$ | $ab^2cdefh^2j$ | $b^2c^2de^2g^2h^2j$ |
| $a^2b^2d^2g^2h^2j$ | $ab^2cd^2ef^2gh^2j$ | $ac^2d^2e^2f^2gh^2j$ | $d^2fh^2j$ |
| $de^2fg^2h^2j^2$ | $a^2cdgh^2j^2$ | $a^2bc^2degh^2j^2$ | $abde^2f^2h^2j^2$ |
| $ab^2cde^2h^2$ | $b^2c^2df^2g^2h^2$ | $def^2g^2h^2$ | $a^2cde^2fgh^2$ |

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Plan 27.9.27. (Continued)

| 21 | 22 | 23 | 24 |
|---|---|---|---|
| $b^2cd^2e^2fgj$ | $abcd^2egj$ | $bc^2d^2e^2f^2j$ | $a^2bd^2fg^2j$ |
| $acdef^2gj^2$ | $a^2b^2cdfgj^2$ | $ab^2c^2dej^2$ | $b^2de^2f^2g^2j^2$ |
| $abc^2d^2e^2g^2j^2$ | $a^2c^2d^2ef^2g^2j^2$ | $ad^2e^2fgj^2$ | $cd^2j^2$ |
| $a^2b^2c^2defg^2$ | $bc^2dg^2$ | $a^2bdef^2g$ | $abcde^2f$ |
| $bdej$ | $adf^2j$ | $cdefg^2j$ | $a^2c^2de^2gj$ |
| $a^2c^2d^2fghj^2$ | $b^2c^2d^2e^2ghj^2$ | $a^2b^2d^2f^2hj^2$ | $ab^2cd^2efg^2hj^2$ |
| $bc^2de^2f^2gh$ | $ac^2defgh$ | $de^2h$ | $a^2cdf^2g^2h$ |
| $b^2d^2g^2h$ | $abd^2e^2f^2g^2h$ | $bcd^2fgh$ | $a^2bc^2d^2eh$ |
| $ade^2fg^2hj$ | $a^2b^2deg^2hj$ | $ab^2cde^2f^2ghj$ | $b^2c^2dfhj$ |
| $a^2b^2cde^2hj^2$ | $bcdef^2hj^2$ | $a^2bc^2de^2fg^2hj^2$ | $abdghj^2$ |
| $ab^2c^2eghj$ | $a^2bc^2f^2ghj$ | $abefhj$ | $bce^2g^2hj$ |
| $abcd^2f^2hj$ | $a^2cd^2e^2fhj$ | $ac^2d^2g^2hj$ | $d^2ef^2ghj$ |
| $cefh$ | $ab^2ch$ | $b^2c^2ef^2g^2h$ | $a^2b^2e^2fgh$ |
| $a^2bef^2g^2hj^2$ | $fg^2hj^2$ | $a^2ceghj^2$ | $ac^2e^2f^2hj^2$ |
| $a^2bcg$ | $ce^2f^2g$ | $a^2c^2f$ | $aeg^2$ |
| $a^2d^2e^2f^2$ | $b^2d^2ef$ | $a^2b^2cd^2e^2g^2$ | $ab^2c^2d^2f^2g$ |
| $ab^2fj^2$ | $a^2be^2j^2$ | $abcf^2g^2j^2$ | $bc^2efgj^2$ |
| $c^2f^2g^2j$ | $ab^2c^2e^2fg^2j$ | $b^2gj$ | $a^2b^2cef^2j$ |
| $abd^2efgh^2$ | $a^2d^2gh^2$ | $acd^2ef^2h^2$ | $c^2d^2e^2fg^2h^2$ |
| $e^2gh^2j^2$ | $ab^2ef^2gh^2j^2$ | $b^2ce^2fh^2j^2$ | $a^2b^2c^2g^2h^2j^2$ |
| $b^2c^2d^2ef^2h^2j^2$ | $abc^2d^2fh^2j^2$ | $bd^2eg^2h^2j^2$ | $a^2bcd^2e^2f^2gh^2j^2$ |
| $a^2bc^2e^2fh^2j$ | $c^2eh^2j$ | $a^2e^2f^2g^2h^2j$ | $acfgh^2j$ |
| $ab^2ce^2f^2g^2h^2$ | $a^2bcefg^2h^2$ | $abc^2e^2gh^2$ | $bf^2h^2$ |
| $a^2b^2df^2gh^2j$ | $bde^2fgh^2j$ | $a^2bcdh^2j$ | $abc^2def^2g^2h^2j$ |
| $a^2cd^2eg^2h^2j$ | $b^2cd^2f^2g^2h^2j$ | $a^2b^2c^2d^2efgh^2j$ | $ab^2d^2e^2h^2j$ |
| $bcdfg^2h^2j^2$ | $acde^2g^2h^2j^2$ | $c^2df^2gh^2j^2$ | $a^2defh^2j^2$ |
| $ac^2dh^2$ | $a^2b^2c^2de^2f^2h^2$ | $ab^2dfg^2h^2$ | $b^2cdegh^2$ |

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Plan 27.9.27. (Continued)

| 25 | 26 | 27 |
|---|---|---|
| $d^2e^2g^2j$ | $a^2cd^2f^2gj$ | $ac^2d^2efj$ |
| $abdefg^2j^2$ | $bcde^2gj^2$ | $a^2bc^2df^2j^2$ |
| $ab^2cd^2e^2f^2j^2$ | $b^2c^2d^2fg^2j^2$ | $a^2b^2d^2egj^2$ |
| $a^2cde$ | $ac^2de^2f^2g^2$ | $dfg$ |
| $b^2c^2def^2gj$ | $a^2b^2de^2fj$ | $ab^2cdg^2j$ |
| $a^2bcd^2g^2hj^2$ | $abc^2d^2ef^2ghj^2$ | $bd^2e^2fhj^2$ |
| $b^2cde^2fg^2h$ | $a^2b^2c^2dgh$ | $ab^2def^2h$ |
| $c^2d^2f^2h$ | $a^2d^2efg^2h$ | $acd^2e^2gh$ |
| $abc^2de^2hj$ | $bdf^2g^2hj$ | $a^2bcdefghj$ |
| $a^2de^2f^2ghj^2$ | $acdfhj^2$ | $c^2deg^2hj^2$ |
| $acef^2g^2hj$ | $c^2e^2fghj$ | $a^2hj$ |
| $ab^2d^2fghj$ | $b^2cd^2ehj$ | $a^2b^2c^2d^2e^2f^2g^2hj$ |
| $begh$ | $a^2bce^2f^2h$ | $abc^2fg^2h$ |
| $a^2b^2c^2efhj^2$ | $ab^2e^2g^2hj^2$ | $b^2cf^2ghj^2$ |
| $a^2b^2f^2g^2$ | $ab^2cefg$ | $b^2c^2e^2$ |
| $a^2bc^2d^2e^2fg$ | $abd^2$ | $bcd^2ef^2g^2$ |
| $ac^2gj^2$ | $ef^2j^2$ | $a^2ce^2fg^2j^2$ |
| $bcfj$ | $a^2bc^2eg^2j$ | $abe^2f^2gj$ |
| $ab^2c^2d^2eg^2h^2$ | $b^2d^2e^2f^2gh^2$ | $a^2b^2cd^2fh^2$ |
| $bc^2e^2f^2g^2h^2j^2$ | $a^2bfgh^2j^2$ | $abceh^2j^2$ |
| $cd^2efgh^2j^2$ | $a^2c^2d^2e^2h^2j^2$ | $ad^2f^2g^2h^2j^2$ |
| $a^2b^2ce^2gh^2j$ | $ab^2c^2f^2h^2j$ | $b^2efg^2h^2j$ |
| $ae^2fh^2$ | $cg^2h^2$ | $a^2c^2ef^2gh^2$ |
| $a^2c^2dfg^2h^2j$ | $adegh^2j$ | $cde^2f^2h^2j$ |
| $a^2bd^2ef^2h^2j$ | $abcd^2e^2fg^2h^2j$ | $bc^2d^2gh^2j$ |
| $b^2dh^2j^2$ | $a^2b^2cdef^2g^2h^2j^2$ | $ab^2c^2de^2fgh^2j^2$ |
| $abcdf^2gh^2$ | $bc^2defh^2$ | $a^2bde^2g^2h^2$ |

Plan 27.9.81. 1/27 replication of 9 factors in 9 blocks of 81 units each.

Factors: A, B, C, D, E, F, G, H, J.

I = Same as Plan 27.9.27.

Block confounding: $ABC^2EF^2$, $AB^2C^2DF$, $AC^2D^2E^2$, $BDE^2F^2$.

With blocking: All two-factor interactions are measurable.

Blocks

Combine blocks of Plan 27.9.27 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1, 2, and 3 | 4, 5, and 6 | 7, 8, and 9 |
| 4 | 5 | 6 |
| 10, 11, and 12 | 13, 14, and 15 | 16, 17, and 18 |
| 7 | 8 | 9 |
| 19, 20, and 21 | 22, 23, and 24 | 25, 26, and 27 |

Plan 27.9.243. 1/27 replication of 9 factors in 3 blocks of 243 units each.

Factors: A, B, C, D, E, F, G, H, J.

I = Same as Plan 27.9.27.

Block confounding: $ABC^2EF^2$.

With blocking: All the two-factor interactions are measurable.

Blocks

Combine blocks of Plan 27.9.27 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1 through 9 | 10 through 18 | 19 through 27 |

Plan 81.8.3. 1/81 replication of 8 factors in 27 blocks of 3 units each.

Factors: A, B, C, D, E, F, G, H.

$I = \underline{ACDEF^2G} = \underline{BC^2EF^2G} = ABDE^2FG^2 = AB^2C^2D = \underline{ABCEG^2} = AB^2CD^2EF = AB^2E^2F^2 = ABC^2D^2F^2G^2 = AD^2E^2G = BD^2FG = AC^2FG = CD^2E^2F^2 = BCDE^2G = \underline{AB^2CD^2E^2F^2G^2H^2} = ABCF^2H = AD^2FH^2 = AC^2E^2G^2H = AB^2EFGH = ACDFG^2H = EFG^2H^2 = AB^2C^2DE^2F^2GH = BC^2E^2H^2 = BC^2FG^2H = ABCE^2FGH^2 = BCDEF^2G^2H = AB^2G^2H^2 = AC^2EF^2H^2 = BD^2E^2G^2H = ABC^2D^2EGH^2 = BCDFH^2 = CD^2EFGH = BD^2EF^2H^2 = AB^2CD^2GH = CD^2G^2H^2 = AD^2EF^2G^2H = ABC^2D^2E^2FH = ACDE^2H^2 = ABDEH = AB^2C^2DEFG^2H^2 = ABDF^2GH^2.$

Completely randomized: The following two-factor interactions are measurable: AB, AC, AE, AH, BD, $BF^2$, $BG^2$, CD, CF, CG, $DE^2$, $DH^2$, $EF^2$, EG, FH, $GH^2$.

Block confounding: $\underline{AB^2}$, $\underline{BH^2}$, $AH^2$, ABH, $\underline{ABCD}$, $AC^2D^2$, $AB^2CDH^2$, $AB^2C^2D^2H$, $ABC^2D^2H^2$, $BC^2D^2$, ACDH, BCDH, $CDH^2$.

With blocking: The following two-factor interactions are measurable: AB, AE, AH, $BF^2$, $BG^2$, CD, CF, CG, $DE^2$, $DH^2$, $EF^2$, $GH^2$.

Blocks

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| (1) | $a^2b^2gh^2$ | $abg^2h$ | $ef^2gh^2$ |
| $a^2b^2cde^2fgh^2$ | $abcde^2fg^2h$ | $cde^2f$ | $a^2b^2cdg^2h$ |
| $abc^2d^2ef^2g^2h$ | $c^2d^2ef^2$ | $a^2b^2c^2d^2ef^2gh^2$ | $abc^2d^2e^2f$ |

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Plan 81.8.3. (Continued)

| 5 | 6 | 7 | 8 |
|---|---|---|---|
| $abcd$ | $cdgh^2$ | $e^2fg^2h$ | $c^2d^2g^2h$ |
| $c^2d^2e^2fgh^2$ | $a^2b^2c^2d^2e^2fg^2h$ | $a^2b^2cdef^2$ | $a^2b^2e^2f$ |
| $a^2b^2ef^2g^2h$ | $abef^2$ | $abc^2d^2gh^2$ | $abcdef^2gh^2$ |

| 9 | 10 | 11 | 12 |
|---|---|---|---|
| $a^2b^2c^2d^2$ | $ad^2fg$ | $ac^2deg$ | $a^2bd^2fh$ |
| $abe^2fgh^2$ | $b^2ce^2f^2g^2h^2$ | $b^2d^2fg^2h^2$ | $ace^2f^2g$ |
| $cdef^2g^2h$ | $a^2bc^2deh$ | $a^2bce^2f^2h$ | $b^2c^2deg^2h^2$ |

| 13 | 14 | 15 | 16 |
|---|---|---|---|
| $b^2cfh$ | $a^2bcfg$ | $a^2bd^2eg$ | $b^2ceg$ |
| $a^2bc^2de^2f^2g$ | $ac^2de^2f^2g^2h^2$ | $acfg^2h^2$ | $a^2bc^2dfg^2h^2$ |
| $ad^2eg^2h^2$ | $b^2d^2eh$ | $b^2c^2de^2f^2h$ | $ad^2e^2f^2h$ |

| 17 | 18 | 19 | 20 |
|---|---|---|---|
| $ac^2dfh$ | $aceh$ | $a^2df^2g^2$ | $ab^2df^2h^2$ |
| $b^2d^2e^2f^2g$ | $b^2c^2dfg$ | $ab^2cd^2e^2h^2$ | $bcd^2e^2gh$ |
| $a^2bceg^2h^2$ | $a^2bd^2e^2f^2g^2h^2$ | $bc^2efgh$ | $a^2c^2efg^2$ |

| 21 | 22 | 23 | 24 |
|---|---|---|---|
| $bdf^2gh$ | $bc^2e^2g^2$ | $a^2c^2e^2h^2$ | $a^2cd^2f^2h^2$ |
| $a^2cd^2e^2g^2$ | $a^2defh^2$ | $ab^2defgh$ | $ab^2c^2e^2gh$ |
| $ab^2c^2efh^2$ | $ab^2cd^2f^2gh$ | $bcd^2f^2g^2$ | $bdefg^2$ |

| 25 | 26 | 27 |
|---|---|---|
| $bc^2f^2h^2$ | $ab^2de^2g^2$ | $bde^2h^2$ |
| $a^2de^2gh$ | $bcd^2efh^2$ | $a^2cd^2efgh$ |
| $ab^2cd^2efg^2$ | $a^2c^2f^2gh$ | $ab^2c^2f^2g^2$ |

Plan 81.8.9. 1/81 replication of 8 factors in 9 blocks of 9 units each.

Factors: A, B, C, D, E, F, G, H.

I = Same as Plan 81.8.3.

Completely randomized: The following two-factor interactions are measurable: AB, AC, AE, AH, BD, $BF^2$, $BG^2$, CD, CF, CG, $DE^2$, $DH^2$, $EF^2$, EG, FH, $GH^2$.

Block confounding: $AB^2$, $BH^2$, $AH^2$, ABH.

With blocking: The following two-factor interactions are measurable: AB, AC, AE, AH, BD, $BF^2$, $BG^2$, CD, CF, CG, $DE^2$, $DH^2$, $EF^2$, EG, FH, $GH^2$.

Blocks

| 1 | 2 | 3 |
|---|---|---|
| 1, 2, and 3 | 4, 5, and 6 | 7, 8, and 9 |
| 4 | 5 | 6 |
| 10, 11, and 12 | 13, 14, and 15 | 16, 17, and 18 |
| 7 | 8 | 9 |
| 19, 20, and 21 | 22, 23, and 24 | 25, 26, and 27 |

Plan 81.8.27. 1/81 replication of 8 factors in 3 blocks of 27 units each.

Factors: A, B, C, D, E, F, G, H.

I = Same as Plan 81.8.3.

Block confounding: $AB^2$.

With blocking: The following two-factor interactions are measurable: AB, AC, AE, AH, BD, $BF^2$, $BG^2$, CD, CF, CG, $DE^2$, $DH^2$, $EF^2$, EG, FH, $GH^2$.

Blocks

Combine blocks of Plan 81.8.3 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1 through 9 | 10 through 18 | 19 through 27 |

Plan 81.9.9. 1/81 replication of 9 factors in 27 blocks of 9 units each.

Factors: A, B, C, D, E, F, G, H, J.

$I = \underline{BCDEFG} = \underline{ACDE^2F^2H} = ABC^2D^2GH = AB^2EFG^2H = \underline{ABD^2E^2FJ} = AB^2CF^2GJ = AB^2C^2E^2H^2J^2 = ABCD^2EF^2G^2H^2J^2 = BDE^2G^2HJ^2 = AC^2DEG^2J = BC^2DF^2H^2J = CE^2FG^2H^2J = ADFGH^2J^2 = \underline{ABC^2EF^2} = AB^2DE^2G = AB^2D^2F^2H^2 = ABC^2DE^2FG^2H^2 = BCF^2G^2H = ABCDJ^2 = AE^2F^2G^2J^2 = BEFH^2J^2 = CDGHJ = AC^2D^2E^2F^2GH^2J = BD^2F^2GJ^2 = AD^2EHJ^2 = ABCE^2FGHJ^2 = AB^2CD^2E^2G^2HJ = ACD^2FG^2 = BCD^2E^2H^2 = DEF^2G^2H^2 = ACEGH^2 = CD^2EF^2J = BC^2E^2GJ = AC^2FHJ = ABDEF^2GHJ = AB^2C^2DF^2G^2HJ^2 = AB^2C^2D^2EFGJ^2 = AB^2CDEFH^2J = ABG^2H^2J = BC^2D^2EFG^2HJ.$

Completely randomized: All two-factor interactions are measurable.

Block confounding: $\underline{AG^2}$, $\underline{FGH}$, $AFH$, $AF^2GH^2$, $\underline{FG^2J^2}$, $AFGJ^2$, $FH^2J$, $AF^2G^2HJ^2$, $AH^2J^2$, $AF^2J$, $GH^2J^2$, $AGHJ$, $AFG^2H^2J$.

With blocking: All two-factor interactions except the following are measurable: $AG^2$, $BF^2$, $CJ^2$, $DE$, $DH^2$, $EH$.

Blocks

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| (1) | $bc^2d^2ej^2$ | $b^2cde^2j$ | $bcdh$ |
| $a^2cde^2g^2hj$ | $a^2bg^2h$ | $a^2b^2c^2d^2eg^2hj^2$ | $a^2bc^2d^2e^2g^2h^2j$ |
| $abde^2fgh$ | $ab^2c^2fghj^2$ | $acd^2efghj$ | $ab^2cd^2e^2fgh^2$ |
| $bcd^2efh^2j$ | $b^2de^2fh^2$ | $c^2fh^2j^2$ | $b^2c^2efj$ |
| $ab^2cf^2gj$ | $ad^2ef^2g$ | $abc^2de^2f^2gj^2$ | $ac^2df^2ghj$ |
| $ac^2d^2egh^2j^2$ | $abcde^2gh^2j$ | $ab^2gh^2$ | $abegj^2$ |
| $a^2b^2d^2ef^2g^2h^2$ | $a^2c^2de^2f^2g^2h^2j^2$ | $a^2bcf^2g^2h^2j$ | $a^2cef^2g^2$ |
| $b^2c^2de^2f^2hj^2$ | $cf^2hj$ | $bd^2ef^2h$ | $d^2e^2f^2h^2j^2$ |
| $a^2bc^2fg^2j^2$ | $a^2b^2cd^2efg^2j$ | $a^2de^2fg^2$ | $a^2b^2dfg^2hj^2$ |

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Plan 81.9.9. (Continued)

| 5 | 6 | 7 | 8 |
|---|---|---|---|
| $b^2ehj^2$ | $c^2d^2e^2hj$ | $b^2c^2d^2h^2$ | $cdeh^2j^2$ |
| $a^2b^2cdg^2h^2$ | $a^2eg^2h^2j^2$ | $a^2b^2e^2g^2j$ | $a^2c^2d^2g^2$ |
| $adfgh^2j^2$ | $abc^2efgh^2j$ | $ac^2e^2fg$ | $abcd^2fgj^2$ |
| $cd^2e^2f$ | $bdfj^2$ | $defhj$ | $bc^2e^2fh$ |
| $abcef^2gh$ | $ab^2d^2e^2f^2ghj^2$ | $abd^2f^2gh^2j$ | $ab^2c^2def^2gh^2$ |
| $ab^2c^2d^2e^2gj$ | $acdg$ | $ab^2cdeghj^2$ | $ae^2ghj$ |
| $a^2bd^2e^2f^2g^2j^2$ | $a^2b^2c^2df^2g^2j$ | $a^2bc^2def^2g^2h$ | $a^2b^2ce^2f^2g^2hj^2$ |
| $bc^2df^2h^2j$ | $b^2cef^2h^2$ | $bce^2f^2j^2$ | $b^2d^2f^2j$ |
| $a^2c^2efg^2hj$ | $a^2bcd^2e^2fg^2h$ | $a^2cd^2fg^2h^2j^2$ | $a^2bdefg^2h^2j$ |

| 9 | 10 | 11 | 12 |
|---|---|---|---|
| $be^2h^2j$ | $ab^2cd^2e$ | $ade^2j^2$ | $abc^2j$ |
| $a^2bcdeg^2j^2$ | $b^2c^2g^2hj$ | $cd^2eg^2h$ | $bde^2g^2hj^2$ |
| $ab^2defgj$ | $a^2cfgh$ | $a^2bd^2efghj^2$ | $a^2b^2c^2de^2fghj$ |
| $b^2cd^2fhj^2$ | $ac^2de^2fh^2j$ | $abcfh^2$ | $ab^2d^2efh^2j^2$ |
| $ace^2f^2gh^2j^2$ | $a^2bc^2d^2ef^2gj$ | $a^2b^2cde^2f^2g$ | $a^2f^2gj^2$ |
| $abc^2d^2gh$ | $a^2b^2de^2gh^2j^2$ | $a^2c^2gh^2j$ | $a^2bcd^2egh^2$ |
| $a^2d^2f^2g^2hj$ | $bcde^2f^2g^2h^2$ | $b^2f^2g^2h^2j^2$ | $c^2d^2ef^2g^2h^2j$ |
| $c^2def^2$ | $abf^2hj^2$ | $ab^2c^2d^2ef^2hj$ | $acde^2f^2h$ |
| $a^2b^2c^2e^2fg^2h^2$ | $d^2efg^2j^2$ | $bc^2de^2fg^2j$ | $b^2cfg^2$ |

| 13 | 14 | 15 | 16 |
|---|---|---|---|
| $ac^2eh$ | $abcd^2e^2hj^2$ | $ab^2dhj$ | $abdeh^2$ |
| $dg^2h^2j$ | $bc^2eg^2h^2$ | $b^2cd^2e^2g^2h^2j^2$ | $bcd^2g^2j$ |
| $a^2bc^2dfgh^2$ | $a^2b^2cefgh^2j^2$ | $a^2d^2e^2fgh^2j$ | $a^2b^2d^2fg$ |
| $abd^2e^2fj$ | $ab^2c^2df$ | $acefj^2$ | $ab^2ce^2fhj$ |
| $a^2b^2ef^2ghj$ | $a^2c^2d^2e^2f^2gh$ | $a^2bcdf^2ghj^2$ | $a^2cdef^2gh^2j$ |
| $a^2cd^2e^2gj^2$ | $a^2bdgj$ | $a^2b^2c^2eg$ | $a^2bc^2e^2ghj^2$ |
| $b^2c^2d^2e^2f^2g^2$ | $cdf^2g^2j^2$ | $bef^2g^2j$ | $e^2f^2g^2h$ |
| $ab^2cdf^2h^2j^2$ | $aef^2h^2j$ | $abc^2d^2e^2f^2h^2$ | $ac^2d^2f^2j^2$ |
| $bcefg^2hj^2$ | $b^2d^2e^2fg^2hj$ | $c^2dfg^2h$ | $b^2c^2defg^2h^2j^2$ |

| 17 | 18 | 19 | 20 |
|---|---|---|---|
| $ab^2c^2e^2h^2j^2$ | $acd^2h^2j$ | $a^2bc^2de^2$ | $a^2b^2cj^2$ |
| $b^2deg^2$ | $c^2e^2g^2j^2$ | $abd^2eg^2hj$ | $ab^2c^2de^2g^2h$ |
| $a^2c^2defgj^2$ | $a^2bce^2fgj$ | $b^2c^2d^2efgh$ | $cde^2fghj^2$ |
| $ad^2fh$ | $abc^2defhj^2$ | $a^2b^2fh^2j$ | $a^2c^2d^2efh^2$ |
| $a^2be^2f^2gh^2$ | $a^2b^2c^2d^2f^2gh^2j^2$ | $de^2f^2gj$ | $bc^2f^2g$ |
| $a^2b^2cd^2ghj$ | $a^2degh$ | $bcgh^2j^2$ | $b^2d^2egh^2j$ |
| $bc^2d^2f^2g^2hj^2$ | $b^2cdef^2g^2hj$ | $ac^2f^2g^2h^2$ | $abcd^2ef^2g^2h^2j^2$ |
| $abcdef^2j$ | $ab^2e^2f^2$ | $a^2cd^2ef^2hj^2$ | $a^2bde^2f^2hj$ |
| $ce^2fg^2h^2j$ | $bd^2fg^2h^2$ | $ab^2cde^2fg^2j^2$ | $afg^2j$ |

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Plan 81.9.9. (Continued)

| 21 | 22 | 23 | 24 |
|---|---|---|---|
| $a^2d^2ej$ | $a^2b^2d^2e^2h$ | $a^2c^2dhj^2$ | $a^2bcehj$ |
| $acg^2hj^2$ | $ab^2ceg^2h^2j$ | $ad^2e^2g^2h^2$ | $abc^2dg^2h^2j^2$ |
| $bfghj$ | $efgh^2$ | $bc^2d^2e^2fgh^2j^2$ | $b^2cdfgh^2j$ |
| $a^2bcde^2fh^2j^2$ | $a^2cdfj$ | $a^2bef$ | $a^2b^2c^2d^2e^2fj^2$ |
| $b^2cd^2ef^2gj^2$ | $bcd^2e^2f^2ghj$ | $b^2df^2gh$ | $c^2ef^2ghj^2$ |
| $c^2de^2gh^2$ | $b^2c^2dgj^2$ | $cegj$ | $bd^2e^2g$ |
| $ab^2de^2f^2g^2h^2j$ | $abdf^2g^2$ | $ab^2c^2ef^2g^2j^2$ | $acd^2e^2f^2g^2j$ |
| $a^2b^2c^2f^2h$ | $a^2bc^2ef^2h^2j^2$ | $a^2b^2cd^2e^2f^2h^2j$ | $a^2df^2h^2$ |
| $abc^2d^2efg^2$ | $ac^2d^2e^2fg^2hj^2$ | $abcdfg^2hj$ | $ab^2efg^2h$ |

| 25 | 26 | 27 |
|---|---|---|
| $a^2ce^2h^2$ | $a^2bd^2h^2j^2$ | $a^2b^2c^2deh^2j$ |
| $ac^2deg^2j$ | $abce^2g^2$ | $ab^2d^2g^2j^2$ |
| $bcdefg$ | $b^2e^2fgj^2$ | $c^2d^2fgj$ |
| $a^2bc^2d^2fhj$ | $a^2b^2cdefh$ | $a^2e^2fhj^2$ |
| $b^2c^2e^2f^2gh^2j$ | $cd^2f^2gh^2$ | $bdef^2gh^2j^2$ |
| $d^2ghj^2$ | $bc^2deghj$ | $b^2ce^2gh$ |
| $ab^2cd^2f^2g^2h$ | $adef^2g^2hj^2$ | $abc^2e^2f^2g^2hj$ |
| $a^2b^2def^2j^2$ | $a^2c^2e^2f^2j$ | $a^2bcd^2f^2$ |
| $abe^2fg^2h^2j^2$ | $ab^2c^2d^2fg^2h^2j$ | $acdefg^2h^2$ |

Plan 81.9.27. 1/81 replication of 9 factors in 9 blocks of 27 units each.

Factors: A, B, C, D, E, F, G, H, J.

I = Same as Plan 81.9.9.

Block confounding: $\underline{AB^2C^2DF}$, $\underline{AB^2CD^2E}$, $AB^2E^2F^2$, $CD^2E^2F$.

With blocking: All two-factor interactions are measurable.

Blocks

| 1 | 2 | 3 |
|---|---|---|
| (1) | $a^2bc^2de^2$ | $ab^2cd^2e$ |
| $abc^2d^2gh$ | $b^2ce^2gh$ | $a^2degh$ |
| $bcdefg$ | $a^2b^2d^2fg$ | $ac^2e^2fg$ |
| $ab^2efg^2h$ | $c^2dfg^2h$ | $a^2bcd^2e^2fg^2h$ |
| $acde^2f^2h$ | $bd^2ef^2h$ | $a^2b^2c^2f^2h$ |
| $bde^2g^2hj^2$ | $a^2b^2c^2d^2eg^2hj^2$ | $acg^2hj^2$ |
| $ab^2c^2e^2h^2j^2$ | $cdeh^2j^2$ | $a^2bd^2h^2j^2$ |
| $b^2cd^2fhj^2$ | $a^2e^2fhj^2$ | $abc^2defhj^2$ |
| $adfgh^2j^2$ | $bc^2d^2e^2fgh^2j^2$ | $a^2b^2cefgh^2j^2$ |
| $abcd^2ef^2g^2h^2j^2$ | $b^2f^2g^2h^2j^2$ | $a^2c^2de^2f^2g^2h^2j^2$ |
| $ac^2deg^2j$ | $bcd^2g^2j$ | $a^2b^2e^2g^2j$ |
| $ce^2fg^2h^2j$ | $a^2bdefg^2h^2j$ | $ab^2c^2d^2fg^2h^2j$ |
| $abd^2e^2fj$ | $b^2c^2efj$ | $a^2cdfj$ |
| $ab^2cf^2gj$ | $de^2f^2gj$ | $a^2bc^2d^2ef^2gj$ |
| $a^2b^2cdg^2h^2$ | $ad^2e^2g^2h^2$ | $bc^2eg^2h^2$ |
| $b^2c^2d^2e^2f^2g^2$ | $a^2cef^2g^2$ | $abdf^2g^2$ |
| $a^2be^2f^2gh^2$ | $ab^2c^2def^2gh^2$ | $cd^2f^2gh^2$ |
| $a^2c^2d^2efh^2$ | $abcfh^2$ | $b^2de^2fh^2$ |
| $b^2d^2egh^2j$ | $a^2c^2gh^2j$ | $abcde^2gh^2j$ |
| $a^2bcehj$ | $ab^2dhj$ | $c^2d^2e^2hj$ |
| $bc^2df^2h^2j$ | $a^2b^2cd^2e^2f^2h^2j$ | $aef^2h^2j$ |
| $a^2d^2f^2g^2hj$ | $abc^2e^2f^2g^2hj$ | $b^2cdef^2g^2hj$ |
| $a^2b^2c^2de^2fghj$ | $acd^2efghj$ | $bfghj$ |
| $a^2cd^2e^2gj^2$ | $abegj^2$ | $b^2c^2dgj^2$ |
| $c^2ef^2ghj^2$ | $a^2bcdf^2ghj^2$ | $ab^2d^2e^2f^2ghj^2$ |
| $a^2b^2def^2j^2$ | $ac^2d^2f^2j^2$ | $bce^2f^2j^2$ |
| $a^2bc^2fg^2j^2$ | $ab^2cde^2fg^2j^2$ | $d^2efg^2j^2$ |

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Plan 81.9.27. (Continued)

| 4 | 5 | 6 |
|---|---|---|
| $a^2b^2c^2eg$ | $acdg$ | $bd^2e^2g$ |
| $cd^2eg^2h$ | $a^2bg^2h$ | $ab^2c^2de^2g^2h$ |
| $a^2de^2fg^2$ | $abc^2d^2efg^2$ | $b^2cfg^2$ |
| $bc^2e^2fh$ | $a^2b^2cdefh$ | $ad^2fh$ |
| $b^2df^2gh$ | $a^2c^2d^2e^2f^2gh$ | $abcef^2gh$ |
| $a^2c^2dhj^2$ | $abcd^2e^2hj^2$ | $b^2ehj^2$ |
| $bcgh^2j^2$ | $a^2b^2de^2gh^2j^2$ | $ac^2d^2egh^2j^2$ |
| $a^2bd^2efghj^2$ | $ab^2c^2fghj^2$ | $cde^2fghj^2$ |
| $b^2c^2defg^2h^2j^2$ | $a^2cd^2fg^2h^2j^2$ | $abe^2fg^2h^2j^2$ |
| $d^2e^2f^2h^2j^2$ | $a^2bc^2ef^2h^2j^2$ | $ab^2cdf^2h^2j^2$ |
| $b^2cde^2j$ | $a^2d^2ej$ | $abc^2j$ |
| $a^2b^2fh^2j$ | $ac^2de^2fh^2j$ | $bcd^2efh^2j$ |
| $c^2d^2fgj$ | $a^2bce^2fgj$ | $ab^2defgj$ |
| $bef^2g^2j$ | $a^2b^2c^2df^2g^2j$ | $acd^2e^2f^2g^2j$ |
| $abdeh^2$ | $b^2c^2d^2h^2$ | $a^2ce^2h^2$ |
| $a^2bcd^2f^2$ | $ab^2e^2f^2$ | $c^2def^2$ |
| $ac^2f^2g^2h^2$ | $bcde^2f^2g^2h^2$ | $a^2b^2d^2ef^2g^2h^2$ |
| $ab^2cd^2e^2fgh^2$ | $efgh^2$ | $a^2bc^2dfgh^2$ |
| $a^2bc^2d^2e^2g^2h^2j$ | $ab^2ceg^2h^2j$ | $dg^2h^2j$ |
| $ae^2ghj$ | $bc^2deghj$ | $a^2b^2cd^2ghj$ |
| $a^2cdef^2gh^2j$ | $abd^2f^2gh^2j$ | $b^2c^2e^2f^2gh^2j$ |
| $ab^2c^2d^2ef^2hj$ | $cf^2hj$ | $a^2bde^2f^2hj$ |
| $abcdfg^2hj$ | $b^2d^2e^2fg^2hj$ | $a^2c^2efg^2hj$ |
| $ab^2d^2g^2j^2$ | $c^2e^2g^2j^2$ | $a^2bcdeg^2j^2$ |
| $a^2b^2ce^2f^2g^2hj^2$ | $adef^2g^2hj^2$ | $bc^2d^2f^2g^2hj^2$ |
| $abc^2de^2f^2gj^2$ | $b^2cd^2ef^2gj^2$ | $a^2f^2gj^2$ |
| $acefj^2$ | $bdfj^2$ | $a^2b^2c^2d^2e^2fj^2$ |

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Plan 81.9.27. (Continued)

| 7 | 8 | 9 |
|---|---|---|
| $abce^2g^2$ | $b^2deg^2$ | $a^2c^2d^2g^2$ |
| $a^2b^2d^2e^2h$ | $ac^2eh$ | $bcdh$ |
| $ab^2c^2df$ | $cd^2e^2f$ | $a^2bef$ |
| $a^2cfgh$ | $abde^2fgh$ | $b^2c^2d^2efgh$ |
| $a^2bc^2def^2g^2h$ | $ab^2cd^2f^2g^2h$ | $e^2f^2g^2h$ |
| $ab^2cdeghj^2$ | $d^2ghj^2$ | $a^2bc^2e^2ghj^2$ |
| $a^2eg^2h^2j^2$ | $abc^2dg^2h^2j^2$ | $b^2cd^2e^2g^2h^2j^2$ |
| $ac^2d^2e^2fg^2hj^2$ | $bcefg^2hj^2$ | $a^2b^2dfg^2hj^2$ |
| $a^2bcde^2fh^2j^2$ | $ab^2d^2efh^2j^2$ | $c^2fh^2j^2$ |
| $a^2b^2c^2d^2f^2gh^2j^2$ | $ace^2f^2gh^2j^2$ | $bdef^2gh^2j^2$ |
| $a^2bdgj$ | $ab^2c^2d^2e^2gj$ | $cegj$ |
| $abc^2efgh^2j$ | $b^2cdfgh^2j$ | $a^2d^2e^2fgh^2j$ |
| $a^2b^2cd^2efg^2j$ | $afg^2j$ | $bc^2de^2fg^2j$ |
| $a^2c^2e^2f^2j$ | $abcdef^2j$ | $b^2d^2f^2j$ |
| $c^2de^2gh^2$ | $a^2bcd^2egh^2$ | $ab^2gh^2$ |
| $ad^2ef^2g$ | $bc^2f^2g$ | $a^2b^2cde^2f^2g$ |
| $b^2cef^2h^2$ | $a^2df^2h^2$ | $abc^2d^2e^2f^2h^2$ |
| $bd^2fg^2h^2$ | $a^2b^2c^2e^2fg^2h^2$ | $acdefg^2h^2$ |
| $acd^2h^2j$ | $be^2h^2j$ | $a^2b^2c^2deh^2j$ |
| $b^2c^2g^2hj$ | $a^2cde^2g^2hj$ | $abd^2eg^2hj$ |
| $ab^2de^2f^2g^2h^2j$ | $c^2d^2ef^2g^2h^2j$ | $a^2bcf^2g^2h^2j$ |
| $bcd^2e^2f^2ghj$ | $a^2b^2ef^2ghj$ | $ac^2df^2ghj$ |
| $defhj$ | $a^2bc^2d^2fhj$ | $ab^2ce^2fhj$ |
| $bc^2d^2ej^2$ | $a^2b^2cj^2$ | $ade^2j^2$ |
| $abf^2hj^2$ | $b^2c^2de^2f^2hj^2$ | $a^2cd^2ef^2hj^2$ |
| $cdf^2g^2j^2$ | $a^2bd^2e^2f^2g^2j^2$ | $ab^2c^2ef^2g^2j^2$ |
| $b^2e^2fgj^2$ | $a^2c^2defgj^2$ | $abcd^2fgj^2$ |

Plan 81.9.81. 1/81 replication of 9 factors in 3 blocks of 81 units each.

Factors: A, B, C, D, E, F, G, H, J.

I = Same as Plan 81.9.9.

Block confounding: $AB^2C^2DF$.

With blocking: All two-factor interactions are measurable.

Blocks

Combine blocks of Plan 81.9.27 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1, 2, and 3 | 4, 5, and 6 | 7, 8, and 9 |

Plan 243.9.3. 1/243 replication of 9 factors in 27 blocks of 3 units each.

Factors: A, B, C, D, E, F, G, H, J.

$I = BCDEFG = ACDE^2F^2H = ABC^2D^2GH = AB^2EFG^2H = ABD^2E^2FJ =$
$AB^2CF^2GJ = AB^2C^2E^2H^2J^2 = ABCD^2EF^2G^2H^2J^2 = BDE^2G^2HJ^2 =$
$AC^2DEG^2J = BC^2DF^2H^2J = CE^2FG^2H^2J = ADFGH^2J^2 =$
$ABC^2EF^2 = AB^2DE^2G = AB^2D^2F^2H^2 = ABC^2DE^2FG^2H^2 = BCF^2G^2H =$
$ABCDJ^2 = AE^2F^2G^2J^2 = BEFH^2J^2 = CDGHJ = AC^2D^2E^2F^2GH^2J =$
$BD^2F^2GJ^2 = AD^2EHJ^2 = ABCE^2FGHJ^2 = AB^2CD^2E^2G^2HJ = ACD^2FG^2 =$
$BCD^2E^2H^2 = DEF^2G^2H^2 = ACEGH^2 = CD^2EF^2J = BC^2E^2GJ = AC^2FHJ =$
$ABDEF^2GHJ = AB^2C^2DF^2G^2HJ^2 = AB^2C^2D^2EFGJ^2 = AB^2CDEFH^2J =$
$ABG^2H^2J = BC^2D^2EFG^2HJ = AB^2C^2DF = AD^2EF^2G = ABDEH^2 =$
$AC^2F^2G^2H^2 = CD^2EG^2H = ACEFJ^2 = AB^2D^2G^2J^2 = DEFHJ =$
$BCD^2E^2F^2GHJ = ABC^2EFGH^2J = BEF^2G^2J = ABF^2HJ^2 = AB^2CDEGHJ^2 =$
$ABCDFG^2HJ = AC^2D^2E^2 = AB^2CDEF^2G^2 = CDFH^2 = BEGH = ABCDF^2GH^2 =$
$BCFJ = BCD^2E^2FG^2J^2 = ABC^2DE^2HJ = AB^2D^2FGHJ = ABC^2EG^2HJ^2 =$
$AC^2GJ^2 = ABCE^2F^2H^2J = AD^2EFG^2H^2J = CD^2EFGH^2J^2 = BC^2DG^2 =$
$ABCD^2EFH = AB^2C^2E^2F^2GH = ABD^2E^2F^2G^2H = AB^2EJ = ACDE^2FGJ =$
$ABC^2D^2FH^2J^2 = ACDE^2G^2H^2J^2 = FG^2HJ^2 = AB^2C^2E^2FG^2J = CE^2HJ^2 =$
$BC^2DFGHJ^2 = ABD^2E^2GH^2J^2 = ABCE^2G^2 = BC^2E^2FH = BD^2FG^2H^2 =$
$AB^2CD^2E^2FGH^2 = BC^2D^2EJ^2 = CDF^2G^2J^2 = AB^2C^2D^2EF^2HJ = AE^2GHJ =$
$AC^2D^2E^2FG^2HJ^2 = ABC^2DE^2F^2GJ^2 = ACD^2H^2J = AB^2DE^2F^2G^2H^2J =$
$BCGH^2J^2 = BDE^2F^2 = CE^2F^2G = AB^2CH = AC^2DEFGH = ADG^2H =$
$ADF^2J = ABCD^2EGJ = AC^2DEF^2H^2J^2 = AB^2CFG^2H^2J^2 = BDE^2FGH^2J =$
$ABC^2D^2F^2G^2J = BCDEF^2HJ^2 = BCDEG^2H^2J = AB^2EF^2GH^2J^2 = ABFG =$
$AE^2FH^2 = AB^2C^2D^2EG^2H^2 = BC^2D^2EF^2GH^2 = AB^2CD^2E^2F^2J^3 = ABDEFG^2J^3 =$
$BD^2HJ = BC^2E^2F^2G^2H^2J^2 = AB^2C^2DGH^2J = DEGJ^2 = AB^2DE^2FHJ^2 =$
$ACD^2F^2GHJ^2 = ACEF^2G^2HJ.$

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Plan 243.9.3. (Continued)

Completely randomized: No two-factor interactions are measurable.

Block confounding: $\underline{ABC^2}$, $\underline{BC}$, $AB^2$, AC, $\underline{GH}$, $ABC^2GH$, BCGH, $AB^2GH$, ACGH, $ABC^2G^2H^2$, $BCG^2H^2$, $AB^2G^2H^2$, $ACG^2H^2$.

With blocking: No two-factor interactions are measurable.

Blocks

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| (1) | $de^2f^2gj$ | $d^2efg^2j^2$ | $b^2c^2efj$ |
| $abc^2e^2f^2g^2hj$ | $abc^2defhj^2$ | $abc^2d^2gh$ | $acg^2hj^2$ |
| $a^2b^2cefgh^2j^2$ | $a^2b^2cdg^2h^2$ | $a^2b^2cd^2e^2f^2h^2j$ | $a^2be^2f^2gh^2$ |
| **5** | **6** | **7** | **8** |
| $b^2c^2dgj^2$ | $b^2c^2d^2e^2f^2g^2$ | $bce^2f^2j^2$ | bcdefg |
| $acde^2f^2h$ | $acd^2efghj$ | $ab^2efg^2h$ | $ab^2dhj$ |
| $a^2bdefg^2h^2j$ | $a^2bd^2h^2j^2$ | $a^2c^2gh^2j$ | $a^2c^2de^2f^2g^2h^2j^2$ |
| **9** | **10** | **11** | **12** |
| $bcd^2g^2j$ | $adfgh^2j^2$ | $ad^2e^2g^2h^2$ | $aef^2h^2j$ |
| $ab^2d^2e^2f^2ghj^2$ | $a^2bc^2de^2$ | $a^2bc^2d^2ef^2gj$ | $a^2bc^2fg^2j^2$ |
| $a^2c^2d^2efh^2$ | $b^2cdef^2g^2hj$ | $b^2cd^2fhj^2$ | $b^2ce^2gh$ |
| **13** | **14** | **15** | **16** |
| $a^2cdfj$ | $bd^2ef^2h$ | bfghj | $c^2dfg^2h$ |
| $bde^2g^2hj^2$ | $ab^2c^2d^2fg^2h^2j$ | $ab^2c^2e^2h^2j^2$ | $abcde^2gh^2j$ |
| $ab^2c^2def^2gh^2$ | $a^2cd^2e^2gj^2$ | $a^2cef^2g^2$ | $a^2b^2def^2j^2$ |

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Plan 243.9.3. (Continued)

| 17 | 18 | 19 | 20 |
|---|---|---|---|
| $a^2b^2d^2fg$ | $abcfh^2$ | $ab^2cd^2e$ | $bc^2eg^2h^2$ |
| $c^2d^2e^2hj$ | $a^2b^2e^2g^2j$ | $a^2d^2f^2g^2hj$ | $ab^2cf^2gj$ |
| $abcd^2ef^2g^2h^2j^2$ | $c^2ef^2ghj^2$ | $bc^2d^2e^2fgh^2j^2$ | $a^2e^2fhj^2$ |

| 21 | 22 | 23 | 24 |
|---|---|---|---|
| $a^2degh$ | $cd^2f^2gh^2$ | $abegj^2$ | $cdeh^2j^2$ |
| $bc^2df^2h^2j$ | $abd^2e^2fj$ | $a^2b^2c^2f^2h$ | $abdf^2g^2$ |
| $ab^2cde^2fg^2j^2$ | $a^2b^2c^2d^2eg^2hj^2$ | $ce^2fg^2h^2j$ | $a^2b^2c^2de^2fghj$ |

| 25 | 26 | 27 |
|---|---|---|
| $b^2d^2egh^2j$ | $a^2bcehj$ | $b^2de^2fh^2$ |
| $ac^2d^2f^2j^2$ | $b^2f^2g^2h^2j^2$ | $ac^2deg^2j$ |
| $a^2bcd^2e^2fg^2h$ | $ac^2e^2fg$ | $a^2bcdf^2ghj^2$ |

Plan 243.9.9. 1/243 replication of 9 factors in 9 blocks of 9 units each.

Factors: A, B, C, D, E, F, G, H, J.

I = Same as Plan 243.9.3.

Block confounding: $\underline{ABC^2}$, $\underline{BC}$, $AB^2$, AC.

With blocking: No two-factor interactions are measurable.

Blocks

Combine blocks of Plan 243.9.3 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1, 2, and 3 | 4, 5, and 6 | 7, 8, and 9 |
| 4 | 5 | 6 |
| 10, 11, and 12 | 13, 14, and 15 | 16, 17, and 18 |
| 7 | 8 | 9 |
| 19, 20, and 21 | 22, 23, and 24 | 25, 26, and 27 |

Plan 243.9.27. 1/243 replication of 9 factors in 3 blocks of 27 units each.

Factors: A, B, C, D, E, F, G, H, J.

I = Same as Plan 243.9.3.

Block confounding: $ABC^2$.

With blocking: No two-factor interactions are measurable.

Blocks

Combine blocks of Plan 243.9.3 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1 through 9 | 10 through 18 | 19 through 27 |

Plan 243.10.9. 1/243 replication of 10 factors in 27 blocks of 9 units each.

Factors: A, B, C, D, E, F, G, H, J, K.

$I = \underline{BCDEFG} = \underline{ACDE^2F^2H} = ABC^2D^2GH = AB^2EFG^2H = \underline{ABD^2E^2FJ} =$
$AB^2CF^2GJ = AB^2C^2E^2H^2J^2 = ABCD^2EF^2G^2H^2J^2 = BDE^2G^2HJ^2 =$
$AC^2DEG^2J = BC^2DF^2H^2J = CE^2FG^2H^2J = ADFGH^2J^2 = \underline{ABC^2EF^2} =$
$AB^2DE^2G = AB^2D^2F^2H^2 = ABC^2DE^2FG^2H^2 = BCF^2G^2H = ABCDJ^2 =$
$AE^2F^2G^2J^2 = BEFH^2J^2 = CDGHJ = AC^2D^2E^2F^2GH^2J = BD^2F^2GJ^2 =$
$AD^2EHJ^2 = ABCE^2FGHJ^2 = AB^2CD^2E^2G^2HJ = ACD^2FG^2 = BCD^2E^2H^2 =$
$DEF^2G^2H^2 = ACEGH^2 = CD^2EF^2J = BC^2E^2GJ = AC^2FHJ = ABDEF^2GHJ =$
$AB^2C^2DF^2G^2HJ^2 = AB^2C^2D^2EFGJ^2 = AB^2CDEFH^2J = ABG^2H^2J =$
$BC^2D^2EFG^2HJ = \underline{AB^2C^2DFK} = AD^2EF^2GK = ABDEH^2K^2 = AC^2F^2G^2H^2K^2 =$
$CD^2EG^2HK^2 = ACEFJ^2K^2 = AB^2D^2G^2J^2K^2 = DEFHJK = BCD^2E^2F^2GHJK =$
$ABC^2EFGH^2JK = BEF^2G^2JK^2 = ABF^2HJ^2K = AB^2CDEGHJ^2K =$
$ABCDFG^2HJK^2 = AC^2D^2E^2K^2 = AB^2CDEF^2G^2K^2 = CDFH^2K^2 = BEGHK =$
$ABCDF^2GH^2K = BCFJK = BCD^2E^2FG^2J^2K^2 = ABC^2DE^2HJK = AB^2D^2FGHJK =$
$ABC^2EG^2HJ^2K^2 = AC^2GJ^2K = ABCE^2F^2H^2JK^2 = AD^2EFG^2H^2JK^2 =$
$CD^3EFGH^2J^2K = BC^2DG^2K^2 = ABCD^2EFHK = AB^2C^2E^2F^2GHK =$
$ABD^2E^2F^2G^2HK^2 = AB^2EJK = ACDE^2FGJK = ABC^2D^2FH^2J^2K^2 =$
$ACDE^2G^2H^2J^2K^2 = FG^2HJ^2K^2 = AB^2C^2E^2FG^2JK^2 = CE^2HJ^2K =$
$BC^2DFGHJ^2K = ABD^2E^2GH^2J^2K = ABCE^2G^2K = BC^2E^2FHK^2 = BD^2FG^2H^2K =$
$AB^2CD^2E^2FGH^2K^2 = BC^2D^2EJ^2K = CDF^2G^2J^2K = AB^2C^2D^2EF^2HJK^2 =$
$AE^2GHJK^2 = AC^2D^2E^2FG^2HJ^2K = ABC^2DE^2F^2GJ^2K^2 = ACD^2H^2JK =$
$AB^2DE^2F^2G^2H^2JK = BCGH^2J^2K^2 = BDE^2F^2K = CE^2F^2GK^2 = AB^2CHK^2 =$
$AC^2DEFGHK^2 = ADG^2HK = ADF^2JK^2 = ABCD^2EGJK^2 = AC^2DEF^2H^2J^2K =$
$AB^2CFG^2H^2J^2K = BDE^2FGH^2JK^2 = ABC^2D^2F^2G^2JK = BCDEF^2HJ^2K^2 =$
$BCDEG^2H^2JK = AB^2EF^2GH^2J^2K^2 = ABFGK^2 = AE^2FH^2K =$
$AB^2C^2D^2EG^2H^2K = BC^2D^2EF^2GH^2K^2 = AB^2CD^2E^2F^2J^2K = ABDEFG^2J^3K =$
$BD^2HJK^2 = BC^2E^2F^2G^2H^2J^2K = AB^2C^2DGH^2JK^2 = DEGJ^2K^2 =$
$AB^2DE^2FHJ^2K^2 = ACD^2F^2GHJ^2K^2 = ACEF^2G^2HJK.$

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Plan 243.10.9. (Continued)

Completely randomized: All two-factor interactions are measurable.

Block confounding: $\underline{FH^2J}$, $\underline{FGH}$, $FG^2J^2$, $GH^2J^2$, $\underline{AG^2}$, $AFG^2H^2J$, $AFH$, $AF^2J$, $AGHJ$, $AF^2G^2HJ^2$, $AF^2GH^2$, $AFGJ^2$, $AH^2J^2$.

With blocking: All two-factor interactions except the following are measurable: $AG^2$, $BF^2$, $CJ^2$, $DE$, $DH^2$, $DK^2$, $EH$, $EK$, $HK^2$.

Blocks

| 1 | 2 | 3 |
|---|---|---|
| (1) | $d^2efg^2j^2$ | $de^2f^2gj$ |
| $a^2bc^2fg^2j^2$ | $a^2bc^2d^2ef^2gj$ | $a^2bc^2de^2$ |
| $bcd^2efh^2jk^2$ | $bcde^2f^2g^2h^2k^2$ | $bcgh^2j^2k^2$ |
| $a^2b^2d^2ef^2g^2h^2k^2$ | $a^2b^2de^2gh^2j^2k^2$ | $a^2b^2fh^2jk^2$ |
| $a^2cde^2g^2hjk$ | $a^2cfghk$ | $a^2cd^2ef^2hj^2k$ |
| $ab^2cf^2gj$ | $ab^2cd^2e$ | $ab^2cde^2fg^2j^2$ |
| $b^2c^2de^2f^2hj^2k$ | $b^2c^2g^2hjk$ | $b^2c^2d^2efghk$ |
| $abde^2fghk$ | $abf^2hj^2k$ | $abd^2eg^2hjk$ |
| $ac^2d^2egh^2j^2k^2$ | $ac^2de^2fh^2jk^2$ | $ac^2f^2g^2h^2k^2$ |

| 4 | 5 | 6 |
|---|---|---|
| $bdfj^2k^2$ | $bef^2g^2jk^2$ | $bd^2e^2gk^2$ |
| $a^2b^2c^2df^2g^2jk^2$ | $a^2b^2c^2egk^2$ | $a^2b^2c^2d^2e^2fj^2k^2$ |
| $b^2cef^2h^2k$ | $b^2cd^2e^2g^2h^2j^2k$ | $b^2cdfgh^2jk$ |
| $a^2eg^2h^2j^2k$ | $a^2d^2e^2fgh^2jk$ | $a^2df^2h^2k$ |
| $a^2bcd^2e^2fg^2h$ | $a^2bcdf^2ghj^2$ | $a^2bcehj$ |
| $acdgk^2$ | $acefj^2k^2$ | $acd^2e^2f^2g^2jk^2$ |
| $c^2d^2e^2hj$ | $c^2dfg^2h$ | $c^2ef^2ghj^2$ |
| $ab^2d^2e^2f^2ghj^2$ | $ab^2dhj$ | $ab^2efg^2h$ |
| $abc^2efgh^2jk$ | $abc^2d^2e^2f^2h^2k$ | $abc^2dg^2h^2j^2k$ |

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Plan 243.10.9. (Continued)

| 7 | 8 | 9 |
|---|---|---|
| $b^2d^2f^2jk$ | $b^2deg^2k$ | $b^2e^2fgj^2k$ |
| $a^2c^2d^2g^2k$ | $a^2c^2defgj^2k$ | $a^2c^2e^2f^2jk$ |
| $cdeh^2j^2$ | $ce^2fg^2h^2j$ | $cd^2f^2gh^2$ |
| $a^2bdefg^2h^2j$ | $a^2be^2f^2gh^2$ | $a^2bd^2h^2j^2$ |
| $a^2b^2ce^2f^2g^2hj^2k^2$ | $a^2b^2cd^2ghjk^2$ | $a^2b^2cdefhk^2$ |
| $abcd^2fgj^2k$ | $abcdef^2jk$ | $abce^2g^2k$ |
| $bc^2e^2fhk^2$ | $bc^2d^2f^2g^2hj^2k^2$ | $bc^2deghjk^2$ |
| $ae^2ghjk^2$ | $ad^2fhk^2$ | $adef^2g^2hj^2k^2$ |
| $ab^2c^2def^2gh^2$ | $ab^2c^2e^2h^2j^2$ | $ab^2c^2d^2fg^2h^2j$ |

| 10 | 11 | 12 |
|---|---|---|
| $bd^2ef^2h$ | $bde^2g^2hj^2$ | $bfghj$ |
| $a^2b^2c^2d^2eg^2hj^2$ | $a^2b^2c^2de^2fghj$ | $a^2b^2c^2f^2h$ |
| $b^2cde^2jk^2$ | $b^2cfg^2k^2$ | $b^2cd^2ef^2gj^2k^2$ |
| $a^2de^2fg^2k^2$ | $a^2f^2gj^2k^2$ | $a^2d^2ejk^2$ |
| $a^2bcf^2g^2h^2jk$ | $a^2bcd^2egh^2k$ | $a^2bcde^2fh^2j^2k$ |
| $acd^2efghj$ | $acde^2f^2h$ | $acg^2hj^2$ |
| $c^2fh^2j^2k$ | $c^2d^2ef^2g^2h^2jk$ | $c^2de^2gh^2k$ |
| $ab^2gh^2k$ | $ab^2d^2efh^2j^2k$ | $ab^2de^2f^2g^2h^2jk$ |
| $abc^2de^2f^2gj^2k^2$ | $abc^2jk^2$ | $abc^2d^2efg^2k^2$ |

| 13 | 14 | 15 |
|---|---|---|
| $b^2ehj^2k^2$ | $b^2d^2e^2fg^2hjk^2$ | $b^2df^2ghk^2$ |
| $a^2c^2efg^2hjk^2$ | $a^2c^2d^2e^2f^2ghk^2$ | $a^2c^2dhj^2k^2$ |
| $cd^2e^2fk$ | $cdf^2g^2j^2k$ | $cegjk$ |
| $a^2bd^2e^2f^2g^2j^2k$ | $a^2bdgjk$ | $a^2befk$ |
| $a^2b^2cdg^2h^2$ | $a^2b^2cefgh^2j^2$ | $a^2b^2cd^2e^2f^2h^2j$ |
| $abcef^2ghk^2$ | $abcd^2e^2hj^2k^2$ | $abcdfg^2hjk^2$ |
| $bc^2df^2h^2j$ | $bc^2eg^2h^2$ | $bc^2d^2e^2fgh^2j^2$ |
| $adfgh^2j^2$ | $aef^2h^2j$ | $ad^2e^2g^2h^2$ |
| $ab^2c^2d^2e^2gjk$ | $ab^2c^2dfk$ | $ab^2c^2ef^2g^2j^2k$ |

| 16 | 17 | 18 |
|---|---|---|
| $defhjk$ | $e^2f^2g^2hk$ | $d^2ghj^2k$ |
| $a^2bc^2def^2g^2hk$ | $a^2bc^2e^2ghj^2k$ | $a^2bc^2d^2fhjk$ |
| $bce^2f^2j^2$ | $bcd^2g^2j$ | $bcdefg$ |
| $a^2b^2e^2g^2j$ | $a^2b^2d^2fg$ | $a^2b^2def^2j^2$ |
| $a^2cd^2fg^2h^2j^2k^2$ | $a^2cdef^2gh^2jk^2$ | $a^2ce^2h^2k^2$ |
| $ab^2cdeghj^2k$ | $ab^2ce^2fhjk$ | $ab^2cd^2f^2g^2hk$ |
| $b^2c^2d^2h^2k^2$ | $b^2c^2defg^2h^2j^2k^2$ | $b^2c^2e^2f^2gh^2jk^2$ |
| $abd^2f^2gh^2jk^2$ | $abdeh^2k^2$ | $abe^2fg^2h^2j^2k^2$ |
| $ac^2e^2fg$ | $ac^2d^2f^2j^2$ | $ac^2deg^2j$ |

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Plan 243.10.9. (Continued)

| 19 | 20 | 21 |
|---|---|---|
| $b^2de^2fh^2$ | $b^2f^2g^2h^2j^2$ | $b^2d^2egh^2j$ |
| $a^2c^2de^2f^2g^2h^2j^2$ | $a^2c^2gh^2j$ | $a^2c^2d^2efh^2$ |
| $cf^2hjk^2$ | $cd^2eg^2hk^2$ | $cde^2fghj^2k^2$ |
| $a^2bg^2hk^2$ | $a^2bd^2efghj^2k^2$ | $a^2bde^2f^2hjk^2$ |
| $a^2b^2cd^2efg^2jk$ | $a^2b^2cde^2f^2gk$ | $a^2b^2cj^2k$ |
| $abcde^2gh^2j$ | $abcfh^2$ | $abcd^2ef^2g^2h^2j^2$ |
| $bc^2d^2ej^2k$ | $bc^2de^2fg^2jk$ | $bc^2f^2gk$ |
| $ad^2ef^2gk$ | $ade^2j^2k$ | $afg^2jk$ |
| $ab^2c^2fghj^2k^2$ | $ab^2c^2d^2ef^2hjk^2$ | $ab^2c^2de^2g^2hk^2$ |

| 22 | 23 | 24 |
|---|---|---|
| $d^2e^2f^2h^2j^2k^2$ | $dg^2h^2jk^2$ | $efgh^2k^2$ |
| $a^2bc^2d^2e^2g^2h^2jk^2$ | $a^2bc^2dfgh^2k^2$ | $a^2bc^2ef^2h^2j^2k^2$ |
| $bcdhk$ | $bcefg^2hj^2k$ | $bcd^2e^2f^2ghjk$ |
| $a^2b^2dfg^2hj^2k$ | $a^2b^2ef^2ghjk$ | $a^2b^2d^2e^2hk$ |
| $a^2cef^2g^2$ | $a^2cd^2e^2gj^2$ | $a^2cdfj$ |
| $ab^2cd^2e^2fgh^2k^2$ | $ab^2cdf^2h^2j^2k^2$ | $ab^2ceg^2h^2jk^2$ |
| $b^2c^2efj$ | $b^2c^2d^2e^2f^2g^2$ | $b^2c^2dgj^2$ |
| $abegj^2$ | $abd^2e^2fj$ | $abdf^2g^2$ |
| $ac^2df^2ghjk$ | $ac^2ehk$ | $ac^2d^2e^2fg^2hj^2k$ |

| 25 | 26 | 27 |
|---|---|---|
| $be^2h^2jk$ | $bd^2fg^2h^2k$ | $bdef^2gh^2j^2k$ |
| $a^2b^2c^2e^2fg^2h^2k$ | $a^2b^2c^2d^2f^2gh^2j^2k$ | $a^2b^2c^2deh^2jk$ |
| $b^2cd^2fhj^2$ | $b^2cdef^2g^2hj$ | $b^2ce^2gh$ |
| $a^2d^2f^2g^2hj$ | $a^2degh$ | $a^2e^2fhj^2$ |
| $a^2bcdeg^2j^2k^2$ | $a^2bce^2fgjk^2$ | $a^2bcd^2f^2k^2$ |
| $ace^2f^2gh^2j^2k$ | $acd^2h^2jk$ | $acdefg^2h^2k$ |
| $c^2def^2k^2$ | $c^2e^2g^2j^2k^2$ | $c^2d^2fgjk^2$ |
| $ab^2defgjk^2$ | $ab^2e^2f^2k^2$ | $ab^2d^2g^2j^2k^2$ |
| $abc^2d^2gh$ | $abc^2defhj^2$ | $abc^2e^2f^2g^2hj$ |

Plan 243.10.27. 1/243 replication of 10 factors in 9 blocks of 27 units each.

Factors: A, B, C, D, E, F, G, H, J, K.

I = Same as Plan 243.10.9.

Block confounding: $FH^2J$, FGH, $FG^2J^2$, $GH^2J^2$.

With blocking: All two-factor interactions except the following are measurable: DE, $HK^2$.

## Blocks

Combine blocks of Plan 243.10.9 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1, 2, and 3 | 4, 5, and 6 | 7, 8, and 9 |
| 4 | 5 | 6 |
| 10, 11, and 12 | 13, 14, and 15 | 16, 17, and 18 |
| 7 | 8 | 9 |
| 19, 20, and 21 | 22, 23, and 24 | 25, 26, and 27 |

Plan 243.10.81. 1/243 replication of 10 factors in 3 blocks of 81 units each.

Factors: A, B, C, D, E, F, G, H, J, K.

I = Same as Plan 243.10.9.

Block confounding: $FH^2J$.

With blocking: All two-factor interactions are measurable.

Blocks

Combine blocks of Plan 243.10.9 as follows:

| 1 | 2 | 3 |
|---|---|---|
| 1 through 9 | 10 through 18 | 19 through 27 |

U. S. DEPARTMENT OF COMMERCE
Sinclair Weeks, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

# THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major field laboratories in Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside front cover of this report.

## WASHINGTON, D. C.

**Electricity and Electronics.** Resistance and Reactance. Electron Tubes. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

**Optics and Metrology.** Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

**Heat and Power.** Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology and Lubrication. Engine Fuels.

**Atomic and Radiation Physics.** Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Nuclear Physics. Radioactivity. X-rays. Betatron. Nucleonic Instrumentation. Radiological Equipment. AEC Radiation Instruments.

**Chemistry.** Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Gas Chemistry. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

**Mechanics.** Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

**Organic and Fibrous Materials.** Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Organic Plastics. Dental Research.

**Metallurgy.** Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

**Mineral Products.** Engineering Ceramics. Glass. Refractories. Enameled Metals. Concreting Materials. Constitution and Microstructure.

**Building Technology.** Structural Engineering. Fire Protection. Heating and Air Conditioning. Floor, Roof, and Wall Coverings. Codes and Specifications.

**Applied Mathematics.** Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

**Data Processing Systems.** SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analogue Systems. Application Engineering.

- Office of Basic Instrumentation
- Office of Weights and Measures

## BOULDER, COLORADO

**Cryogenic Engineering.** Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

**Radio Propagation Physics.** Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships.

**Radio Propagation Engineering.** Data Reduction Instrumentation. Modulation Systems. Navigation Systems. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Radio Systems Application Engineering.

**Radio Standards.** Radio Frequencies. Microwave Frequencies. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Calibration Center. Microwave Physics. Microwave Circuit Standards.


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