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GRAPHS AND TABLES OF THE SIGNIFICANCE LEVELS $F(\nu_1, \nu_2, p)$ FOR THE FISHER-SNEDECOR VARIANCE RATIO

by

Lewis E. Vogler and Kenneth A. Norton

NBS

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS
BOULDER LABORATORIES
Boulder, Colorado

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1. Definition of the Fisher-Snedecor Variance Ratio $F(\nu_1, \nu_2)$

The Fisher-Snedecor[1/] [2/] variance ratio is the ratio of two independent variables each distributed as χ^2 and normalized by their corresponding numbers of degrees of freedom. Thus, let u be a random variable distributed as $\chi^2(\nu_1)$ with ν_1 degrees of freedom, while v is another random variable distributed independently of u as $\chi^2(\nu_2)$ with ν_2 degrees of freedom. The ratio

$$F(\nu_1, \nu_2) \equiv \frac{u/\nu_1}{v/\nu_2} \tag{1}$$

is, by definition, the Fisher-Snedecor variance ratio.

As a first example suppose that the $\nu_1 + \nu_2$ random variables $x_1, \ldots, x_i, \ldots, x_{\nu_1}, y_1, \ldots, y_j, \ldots, y_{\nu_2}$ are independent and normal with population means μ_1 and μ_2, respectively, and with standard deviations σ_1 and σ_2, respectively. Now define:

1/ R. A. Fisher, "On a distribution yielding the error functions of several well-known statistics," Proceedings of the International Mathematical Conference, p. 805, Toronto, 1924.

2/ George W. Snedecor, "Analysis of Variance and Covariance," Collegiate Press, Inc., Ames, Iowa, 1934.

$$u = \frac{1}{\sigma_1^2} \sum_{i=1}^{\nu_1} (x_i - \mu_1)^2 \qquad (2)$$

$$v = \frac{1}{\sigma_2^2} \sum_{j=1}^{\nu_2} (y_j - \mu_2)^2 \qquad (3)$$

Since the ν_1 normalized deviations $(x_i - \mu_1)/\sigma_1$ are independent and normal with zero mean and unit standard deviation, it can be shown[3/] that u is distributed as $\chi^2(\nu_1)$ with ν_1 degrees of freedom; similarly it can be shown that v is distributed as $\chi^2(\nu_2)$ with ν_2 degrees of freedom. Since the deviations $(x_i - \mu_1)$ and $(y_j - \mu_2)$ are independent, u and v will be independent and it follows that their normalized ratio

$$F(\nu_1, \nu_2) = \frac{\dfrac{1}{\nu_1 \sigma_1^2} \displaystyle\sum_{i=1}^{\nu_1} (x_i - \mu_1)^2}{\dfrac{1}{\nu_2 \sigma_2^2} \displaystyle\sum_{j=1}^{\nu_2} (y_j - \mu_2)^2} \qquad (4)$$

will be distributed as the Fisher-Snedecor variance ratio.

As a second example, suppose that m + n random variables $x_1, \ldots, x_i, \ldots, x_m, y_1, \ldots, y_j, \ldots, y_n$ are independent and normal with possibly different mean values μ_1 and μ_2 and possibly different

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[3/] Harold Cramèr, "Mathematical Methods of Statistics," Princeton University Press, 1946, Chapters 18, 29, 36.

standard deviations σ_1 and σ_2. Now define:

$$u = \frac{1}{\sigma_1^2} \sum_{i=1}^{m} (x_i - \bar{x})^2 \tag{5}$$

$$v = \frac{1}{\sigma_2^2} \sum_{j=1}^{n} (y_j - \bar{y})^2 \tag{6}$$

$$\bar{x} = \frac{1}{m} \sum_{i=1}^{m} x_i \tag{7}$$

$$\bar{y} = \frac{1}{n} \sum_{j=1}^{n} y_j \tag{8}$$

It can be shown[3/] that u is distributed as $\chi^2(\nu_1)$ with $\nu_1 = (m - 1)$ degrees of freedom, while v is distributed as $\chi^2(\nu_2)$ with $\nu_2 = (n - 1)$ degrees of freedom. Since the deviations $(x_i - \bar{x})$ and $(y_j - \bar{y})$ are independent, it follows that u and v are independent and thus their normalized ratio

$$F(\nu_1, \nu_2) = \frac{\dfrac{1}{(m-1)\,\sigma_1^2} \displaystyle\sum_{i=1}^{m} (x_i - \bar{x})^2}{\dfrac{1}{(n-1)\,\sigma_2^2} \displaystyle\sum_{j=1}^{n} (y_j - \bar{y})^2} \equiv \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} \tag{9}$$

will be distributed as the Fisher-Snedecor variance ratio. In most applications the x_i and y_j are assumed to be from normal populations with the same standard deviation so that $\sigma_1 = \sigma_2$. In this case we see by (9) that $F(\nu_1, \nu_2)$ is free of the population parameters; in any other applications the ratio σ_1/σ_2 must be assumed to be known.

Random variables u and v as defined above have the following χ^2 frequency distribution:

$$f(\chi^2)\,d(\chi^2) = \frac{1}{2\Gamma(\nu/2)} (\chi^2/2)^{(\nu/2)-1} \exp(-\chi^2/2)\,d(\chi^2) \qquad (0 \leq \chi^2 < \infty) \tag{10}$$

The simultaneous or joint distribution of the two independent χ^2 variables u and v is then:

$$f(u, v)\,du\,dv = \frac{1}{4\Gamma(\nu_1/2)\,\Gamma(\nu_2/2)} (u/2)^{(\nu_1/2)-1} (v/2)^{(\nu_2/2)-1} \exp\{-(u+v)/2\}\,du\,dv \tag{11}$$

If we substitute $u = \nu_1 F v/\nu_2$ and $v = v$ in (11), we obtain:

$$f(F, v)\,dF\,dv = \frac{(\nu_1/\nu_2)^{\nu_1/2} \quad F^{(\nu_1/2)-1}}{2\Gamma(\nu_1/2)\,\Gamma(\nu_2/2)} (v/2)^{\frac{\nu_1+\nu_2-2}{2}} \exp\left\{- \frac{v(1+\frac{\nu_1}{\nu_2}F)}{2}\right\} dF\,dv \tag{12}$$

The frequency distribution of $F(\nu_1, \nu_2)$ may now be determined by integration of (12) with respect to v from 0 to ∞:

$$f(F)\,dF = \frac{(\nu_1/\nu_2)^{\nu_1/2}}{B(\nu_1/2, \nu_2/2)} F^{(\nu_1/2)-1} \left(1 + \frac{\nu_1}{\nu_2} F\right)^{-(\nu_1+\nu_2)/2} dF \tag{13}$$

The significance levels $F(\nu_1, \nu_2, p)$ are here defined as

$$p = \int_{F(\nu_1, \nu_2, p)}^{\infty} f(F)\,dF \tag{14}$$

It is the purpose of this paper to present graphs and tables of these significance levels $F(\nu_1, \nu_2, p)$ of the random variable $F(\nu_1, \nu_2)$ for several values of ν_1 and ν_2 ranging from 1 to ∞ and for probabilities p from 0.0001 to 0.9999; p is the probability of observing, in random sampling from normal populations with ν_1 and ν_2 degrees of freedom, a value of $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$.

The mean μ_F and variance σ_F^2 of $F(\nu_1, \nu_2)$ are:

$$\mu_F = \frac{\nu_2}{\nu_2 - 2} \qquad (\nu_2 > 2) \qquad (15)$$

$$\sigma_F^2 = \frac{2\nu_2^2(\nu_1 + \nu_2 - 2)}{\nu_1(\nu_2 - 2)^2(\nu_2 - 4)} \qquad (\nu_2 > 4) \qquad (16)$$

The tables and graphs give the significance levels $F(\nu_1, \nu_2, p)$ for a wide range of ν_1, ν_2, and p; the tabulated values are believed to be correct to four significant figures throughout, and to five significant figures in most cases. In view of the relation $F(\nu_2, \nu_1, 1 - p) = 1/F(\nu_1, \nu_2, p)$ the tables and graphs need only have been extended from 0.0001 to 0.5, but are extended instead to 0.9999 for greater convenience to the reader. Our tables and graphs are based on the values published by Merrington and Thompson[4/] for p = 0.005, 0.01, 0.025, 0.05, 0.1, 0.25, and 0.5; new values were computed for p = 0.0001 and 0.001. Our values for p = 0.001 were compared with those published by Fisher and Yates[5/] and by Pearson

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[4/] M. Merrington and C. M. Thompson, "Tables of Percentage Points of the Inverted Beta (F) Distribution," Biometrika, vol. 33, pp. 73-88, 1943.

[5/] R. A. Fisher and F. Yates, "Statistical Tables for Use in Biological, Agricultural and Medical Research," 1942, Oliver and Boyd, Edinburgh.

and Hartley[6] and those values which were significantly different are listed in Appendix I. Since the relation $F(\nu_2, \nu_1, 1-p) = 1/F(\nu_1, \nu_2, p)$ was used in conjunction with the five significant figure tables of Merrington and Thompson[4] to obtain the values for p = 0.75, 0.9, 0.95, 0.975, 0.99, 0.995, some of these values may be accurate, because of rounding, to only four significant figures.

The distribution of F is one of the most useful now available in the literature for testing statistical hypotheses concerning data from normal populations.* The only other requirement for the application of the F distribution, aside from the assumption that the observations are from normal populations, is that the individual squared deviations in the χ^2 variables u and v be statistically independent and that the numbers, ν_1 and ν_2, of independent deviations in u and v, respectively, be known. Some other distributions derivable from that of $F(\nu_1, \nu_2)$ are described briefly in following sections of this paper and these illustrate a few of the applications of the $F(\nu_1, \nu_2)$ distribution.

2. Methods of Interpolation

Interpolation within the tables, either ν_1-wise or ν_2-wise may be accomplished by use of the function $120/\nu$. Thus if F' and F" are the tabulated values between which the required value F lies, then

$$F = \delta F' + (1 - \delta) F'' \tag{17}$$

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[6] E. S. Pearson and H. O. Hartley, "Biometrika Tables for Statisticians," vol. I, Cambridge University Press, 1954.

* The F distribution is also useful for testing non-normal data, but in such cases the conclusions reached are only approximate.

where $$\delta = \frac{\frac{120}{\nu''} - \frac{120}{\nu}}{\frac{120}{\nu''} - \frac{120}{\nu'}} = \frac{\nu'(\nu'' - \nu)}{\nu(\nu'' - \nu')} \tag{18}$$

For $p < 0.5$, p-wise interpolation by the following formula should give at least three figure accuracy:

$$F = \frac{\nu_2}{\nu_1}\left\{(p)^{-2/\nu_2}[a_o(p')^{2/\nu_2}(1 + \frac{\nu_1}{\nu_2}F') + a_1(p'')^{2/\nu_2}(1 + \frac{\nu_1}{\nu_2}F'')] - 1\right\}, \tag{19}$$

$$a_o = \frac{p^{2/\nu_2} - p''^{2/\nu_2}}{p'^{2/\nu_2} - p''^{2/\nu_2}}, \quad a_1 = (1 - a_o). \tag{20}$$

For interpolation formulas giving greater accuracy, reference may be made to a discussion by Hartley in a paper by Thompson.[7] When ν_1 and ν_2 are both very large, say greater than 120, the following approximation is useful:

$$F(\nu_1, \nu_2, p) \cong \left\{1 - \frac{2}{9\nu_1} + \frac{2}{9\nu_2} + X(p)\sqrt{\frac{2}{9\nu_1} + \frac{2}{9\nu_2} + \frac{8}{\nu_1\nu_2}}\right\}^3 \tag{21}$$

In the above $X(p)$ is the standardized normal deviate, i.e. $X(p) = +\sqrt{F(1, \infty, 2p)} = t(\infty, 2p)$ for $p < 0.5$ and $X(p) = -\sqrt{F(1, \infty, 2 - 2p)}$ $= -t(\infty, 2 - 2p)$ for $p > 0.5$. The significance levels $t(\infty, p)$ are given later in tables and graphs. The above formula reduces to the Wilson-Hilferty approximation[8] to $\chi^2(\nu, p)/\nu$ when ν_2 is allowed to increase without limit. Appendix II gives a more accurate formula for large ν_1 and ν_2.

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[7] Catherine M. Thompson, "Tables of Percentage Points of the Incomplete Beta-Function," Biometrika, vol. 32, pp. 151-181, 1941-1942; see especially the discussion by H. O. Hartley on "Methods of Interpolation," pp. 161-167.

[8] E. B. Wilson and M. M. Hilferty, "The distribution of chi-square," Proc. Nat. Acad., vol. 17, p. 694, 1931.

3. The χ^2 Distribution

The frequency distribution of a χ^2 variable is given by (10). The significance levels $\chi^2(\nu, p)$ are here defined as

$$p = \int_{\chi^2(\nu,\, p)}^{\infty} f(\chi^2)\, d\chi^2 \tag{22}$$

These significance levels may be obtained from the significance levels $F(\nu_1, \infty, p)$ as follows. If we let ν_2 increase without limit in (3), then v/ν_2 approaches the constant value 1 and (1) may be expressed $u = \nu_1 F(\nu_1, \infty)$; thus we see that the variable $u \equiv \chi^2(\nu)$ is distributed exactly the same as $\nu F(\nu, \infty)$ with $\nu_1 = \nu$ and $\nu_2 = \infty$ degrees of freedom. Tables and graphs are given of the significance levels $\chi^2(\nu, p)$ for several values of ν and for probabilities p from 0.0001 to 0.9999; p is the probability of observing a value of $\chi^2(\nu) > \chi^2(\nu, p)$ in random sampling from normal populations. For $\nu > 120$ we may determine χ^2 by means of (21).

4. Student's t Distribution

If we let $u = n(\bar{y} - \mu_2)^2/\sigma_2^2$ and $v = \sum_{i=1}^{n} (y_i - \bar{y})^2/\sigma_2^2$, then it may be shown that u is distributed independently of v as χ^2 with one degree of freedom and (1) becomes

$$F(1, n-1) = \frac{n(\bar{y} - \mu_2)^2}{\frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2} \equiv t^2 \equiv \frac{n(\bar{y} - \mu_2)^2}{s_2^2} \tag{23}$$

If we assume that $\sigma_1 = \sigma_2 = \sigma$ and let $u = m(\bar{x} - \mu_1)^2/\sigma^2$, then u is distributed independently of v as χ^2 with one degree of freedom and (1) becomes:

$$F(1,\ n-1) = \frac{m(\bar{x} - \mu_1)^2}{\frac{1}{n-1}\sum_{i=1}^{n} (y_i - \bar{y})^2} \equiv t^2 \equiv \frac{m(\bar{x} - \mu_1)^2}{s_2^2} \tag{24}$$

Thus we see that the variable t^2 in (23) or in (24) is distributed exactly like $F(1,\ n-1)$. Tables and graphs are also given of the significance levels $t(\nu,\ p) \equiv +\sqrt{F(1,\ \nu,\ p)}$ for several values of ν and for probabilities p from 0.0001 to 0.9999; p is the probability of observing a value of $|t| > t(\nu,\ p)$ in random sampling from normal populations. Note that $\bar{x}$ in (24) is the mean of m observations ($m \geq 1$), independent of the n observations y_i used for obtaining the estimate, s_2^2, of σ^2 and

$$t \equiv \frac{\sqrt{m}\ (\bar{x} - \mu_1)}{s_2} \tag{25}$$

Note that t may range from $-\infty$ to $+\infty$, positive and negative values exceeding a given magnitude being equally likely. It follows that

$$\begin{aligned} p'[t > t(\nu,\ p)] &= 0.5\,p[\,|t| > t(\nu,\ p)] \\ p'[t > -t(\nu,\ p)] &= 1 - 0.5\,p[\,|t| > t(\nu,\ p)] \\ p'[t < -t(\nu,\ p)] &= 0.5\,p[\,|t| > t(\nu,\ p)] \end{aligned} \tag{26}$$

If we let $m = 1$, then $\bar{x} = x_1$, i.e., a single observation, independent of the n observations y_i used for obtaining the estimate s_2^2. Note that (23) represents Student's definition of t which provides a test for the significance of a mean value while the definition (24) makes possible the prediction of a confidence band for the expected mean $\bar{x}$ (measured relative to a proposed mean μ_1) of a future set of m observations ($m \geq 1$) based on the prior knowledge of the variance obtained from a set of n earlier observations from this population.

The t distribution may also be used for testing the significance of the difference between two sample mean values on the assumption that the population variances of the samples of m and n, respectively, have the same value σ^2. The argument leading to this application is as follows. Since $(\bar{x} - \mu_1)$ is a random variable normally distributed about zero with variance σ^2/m and $(\bar{y} - \mu_2)$ is a random variable normally distributed about zero with variance σ^2/n, it follows that the difference $(\bar{x} - \bar{y} - \mu_1 + \mu_2)$ is a random variable normally distributed about zero with variance $\left(\frac{1}{m} + \frac{1}{n}\right)\sigma^2$. Thus it follows that

$$u' = \frac{(\bar{x} - \bar{y} - \mu_1 + \mu_2)^2}{\sigma^2\left(\frac{1}{m} + \frac{1}{n}\right)} \tag{27}$$

is distributed as χ^2 with one degree of freedom. Since the sum of two independent χ^2 variables is a χ^2 variable with degrees of freedom equal to the sum of the degrees of freedom of the two variables, it follows that

$$v' = [(m - 1) s_1^2 + (n - 1) s_2^2]/\sigma^2 \tag{28}$$

is distributed as χ^2 with $(m + n - 2)$ degrees of freedom. It can be shown[3/] that v' is statistically independent of u'; thus we conclude that:

$$F(1, m + n - 2) = t^2 = \frac{(\bar{x} - \bar{y} - \mu_1 + \mu_2)^2 (m + n - 2)}{[(m - 1) s_1^2 + (n - 1) s_2^2]\left(\frac{1}{m} + \frac{1}{n}\right)} \tag{29}$$

is distributed as t^2 with $(m + n - 2)$ degrees of freedom.

The above may be used for testing the significance of the difference between two mean values on the assumption that the population variances of the samples of m and n, respectively, are the same. When the population variances may not be assumed to be equal, reference may be made to papers by Welch and Aspin.[9/] [10/] [11/]

Finally, with $v' = (n - 1)\, s_2^2/\sigma^2$ and $\mu_1 = \mu_2$ in (27), we obtain the following expression for predicting a confidence band for the expected mean $\bar{x}$ of m future observations ($m \geq 1$) measured relative to the observed mean $\bar{y}$ of n prior observations on the assumption that the future and prior observations are from the same population:

$$F(1,\ n - 1) = t^2 = \frac{(\bar{x} - \bar{y})^2}{s_2^2\left(\frac{1}{m} + \frac{1}{n}\right)} \tag{30}$$

5. Thompson's τ Distribution

Consider the random variable τ

$$\tau = \frac{\bar{x}_k - \bar{x}}{\left\{\frac{1}{m}\sum_{i=1}^{m}(x_i - \bar{x})^2\right\}^{1/2}} \tag{31}$$

$$\bar{x}_k = \frac{1}{k}\sum_{i=1}^{k} x_i \qquad (1 \leq k < m) \tag{32}$$

9/ B. L. Welch, "Further Note on Mrs. Aspin's Tables and on Certain Approximations to the Tabled Function," Biometrika, vol. 36, 1949, pp. 293-296.

10/ Alice A. Aspin, "Tables for Use in Comparisons Whose Accuracy Involves Two Variances, Separately Estimated," Biometrika, vol. 36, 1949, pp. 290-296.

11/ B. L. Welch, "Note on some criticisms made by Sir Ronald Fisher," Jour. Royal Statistical Society," B, vol. 18, 297-302, 1956.

It may be shown[3/] [12/] that the random variable

$$F(1, m-2) = \frac{\tau^2 k(m-2)}{(m-k-k\tau^2)} \qquad (m > 2) \qquad (33)$$

is distributed as the Fisher-Snedecor variance ratio F with $\nu_1 = 1$ and $\nu_2 = m - 2$ degrees of freedom. This result may be used when $k > 1$ for testing the significance of the difference between a mean of a random sub-group and the general mean. When $k = 1$, the τ distribution may be used for the rejection of outlying observations; other suitable methods for this purpose are given in reference 6, paragraphs 11, 12, 13 and 14.

Solving (33) for τ^2, we obtain:

$$\tau^2 = \frac{(m-k)F(1, m-2)}{k\{m-2+F(1, m-2)\}} = \frac{(m-k)}{k\{1+(m-2)/F(1, m-2)\}} \qquad (34)$$

Note that τ is distributed about a mean of zero over the finite range from $-\sqrt{(m-k)/k}$ to $+\sqrt{(m-k)/k}$. Since positive and negative values of τ exceeding a given magnitude are equally likely, it follows that the probability p' of observing a value of τ greater than $\pm\{(m-k)F(1, m-2, p)/k(m-2+F(1, m-2, p))\}^{1/2}$ is

$$p' = \frac{0.5p}{1-0.5p} \qquad (35)$$

6. Hotelling's Generalized T Distribution

Consider a k dimensional normal distribution and let y_{ji}, $j = 1$ to k and $i = 1$ to n denote a sample of n independent points in

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[12/] W. R. Thompson, "On a criterion for the rejection of observations and the distribution of the ratio of deviation to sample standard deviation," Annals of Math. Stat., vol. 6, Dec., 1935, pp. 214-219.

the k dimensional space. Let $L = |m_{jh}|$ denote the value of the determinant of the moment matrix describing the sample of n points.

$$m_{jh} = r_{jh} s_j s_h \equiv \frac{1}{(n-1)} \sum_{i=1}^{n} (y_{ji} - \bar{y}_j)(y_{hi} - \bar{y}_h). \tag{36}$$

Now let m^{jh} denote the corresponding elements of the reciprocal matrix. Hotelling's invariant form T^2 may now be expressed:

$$T^2 = n \sum_{j=1}^{k} \sum_{h=1}^{k} m^{jh} (\bar{y}_j - \mu_j)(\bar{y}_h - \mu_h) \tag{37}$$

where μ_j(j = 1 to k) denotes the population mean of the distribution.

It may be shown[3/] that the variable

$$F(k, n-k) = T^2(n-k)/k(n-1) \qquad (n > k) \tag{38}$$

is distributed as the Fisher-Snedecor variance ratio F(k, n - k) with $\nu_1 = k$ and $\nu_2 = (n - k)$ degrees of freedom. For k = 1 this yields Student's t distribution. For k = 2 we have

$$T^2 = \frac{n}{1 - r_{12}^2} \left\{ \frac{(\bar{y}_1 - \mu_1)^2}{s_1^2} - \frac{2r_{12}(\bar{y}_1 - \mu_1)(\bar{y}_2 - \mu_2)}{s_1 s_2} + \frac{(\bar{y}_2 - \mu_2)^2}{s_2^2} \right\} \tag{39}$$

$$p = \left\{ 1 + \frac{T^2(2, n-2, p)}{n-1} \right\}^{-(n-2)/2} \tag{40}$$

and in the limit as n approaches infinity:

$$p = \exp\{-T^2(2, \infty, p)/2\} \tag{41}$$

Here p is the probability, in random sampling from bivariate normal distributions, of observing a value of $T^2 > T^2(2, n-2, p)$.

In view of the relation (38) it follows that:

$$p = \left\{ 1 + \frac{2F(2, \nu_2, p)}{\nu_2} \right\}^{-\nu_2/2} \tag{42}$$

or

$$F(2, \nu_2, p) = \frac{\nu_2}{2} \{(1/p)^{2/\nu_2} - 1\} \tag{43}$$

and in the limit as ν_2 approaches infinity:

$$F(2, \infty, p) = \ln(1/p) \tag{44}$$

Finally we note that the random variable $F(2, \infty)$ is Rayleigh distributed[13/] [14/]; such distributions have played a prominent role in many physical investigations. Thus, we may identify $F(2, \infty)$ with the ratio, $E_s^2/(\overline{E_s^2})$, of the square of the instantaneous Rayleigh distributed vector amplitude, E_s, to the mean square amplitude, $(\overline{E_s^2})$, and find by (44) that

$$p(E_s > z) = \exp[-z^2/(\overline{E_s^2})] \tag{45}$$

[13/] Lord Rayleigh, (a) "On the resultant of a large number of vibrations of the same pitch and of arbitrary phase," Phil. Mag., vol. 10, pp. 73-78; August, 1880; and vol. 27, pp. 460-469; June, 1889. (b) "Theory of Sound," 2nd ed., par. 42a; MacMillan and Co., Ltd., London; 1896. Same edition republished by Dover Publications, Inc.; 1945. (c) "On the problem of random vibrations and of random flights in 1, 2, or 3 dimensions," Scientific Papers, Cambridge Univ. Press, Cambridge, England, vol. 1, p. 491; 1899. (d) Phil. Mag., vol. 37, pp. 321-347; April, 1919.

[14/] K. A. Norton, L. E. Vogler, W. V. Mansfield and P. J. Short, "The Probability Distribution of the Amplitude of a Constant Vector Plus a Rayleigh-Distributed Vector," Proc. IRE, vol. 43, no. 10, pp. 1354-1361, October, 1955.

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Appendix I

Comparison of the values of $F(\nu_1, \nu_2, 0.001)$ which differ from those in the tables of Fisher and Yates[5/] and of Pearson and Hartley.[6/]

$F(\nu_1, \nu_2, 0.001)$	F. and Y.	P. and H.	V. and N.
F(2, 60, 0.001)	7.76	7.76	7.7678
F(3, 40, 0.001)	6.60	6.60	6.5945
F(3, 120, 0.001)	5.79	5.79	5.7814
F(4, 7, 0.001)	17.19	17.19	17.198
F(5, 6, 0.001)	20.81	20.81	20.803
F(6, 5, 0.001)	28.84	28.84	28.834
F(6, 10, 0.001)	9.92	9.92	9.9256
F(8, 5, 0.001)	27.64	27.64	27.649
F(8, 8, 0.001)	12.04	12.04	12.046
F(8, 60, 0.001)	3.87	3.87	3.8648
F(12, 60, 0.001)	3.31	3.31	3.3153
F(24, 5, 0.001)	25.14	25.14	25.133
F(24, 6, 0.001)	16.89	16.89	16.897
F(120, 6, 0.001)	- -	15.99	15.981
F(120, 120, 0.001)	- -	1.76	1.7667

Appendix II

Formulas for $F(\nu_1, \nu_2, p)$ for Large ν_1 and ν_2

Equation (21) may be used when ν_1 and ν_2 are both large; it is principally useful, however, only for calculating $\chi^2(\nu, p)/\nu$ in the limiting case of $\nu_2 = \infty$ which is the Wilson-Hilferty approximation.[8/] Table II-2 shows how the values obtained from (21) compare with our tabulated values for $p = 0.0001$, 0.05, and 0.5.

A much more dependable formula for large ν_1 and ν_2 has been developed by Carter;[13/] thus

$$F(\nu_1, \nu_2, p) = \exp(2z) \tag{45}$$

where

$$z = X(p)\sqrt{h + \lambda}/h + \left[\frac{1}{\nu_2 - 1} - \frac{1}{\nu_1 - 1}\right]\left[\frac{5}{6} + \lambda - \frac{1}{3}\left(\frac{1}{\nu_2 - 1} + \frac{1}{\nu_1 - 1}\right)\right],$$

$$h = \frac{2}{\dfrac{1}{\nu_2 - 1} + \dfrac{1}{\nu_1 - 1}}, \qquad \lambda = \frac{1}{6}\left[X^2(p) - 3\right],$$

$$X(p) = \begin{cases} +\, t(\infty, 2p) & , \quad p < 0.5 \\ -\, t(\infty.\ 2 - 2p) & , \quad p > 0.5 \end{cases}$$

The values of the standard normal deviate $X(p)$ and of λ are given for several values of p in Table II-1.

[13/] A. H. Carter, "Approximation to Percentage Points of the z-Distribution", Biometrika, vol. 34, pp. 352-358, 1947.

Table II-1

p	X(p)	λ
0.0001	3.719016	1.805181
0.001	3.090232	1.091589
0.005	2.575829	0.605816
0.01	2.326348	0.401982
0.025	1.959964	0.140243
0.05	1.644854	-0.049076
0.1	1.281552	-0.226271
0.25	0.674490	-0.424177
0.5	0	-0.500000
0.75	-0.674490	-0.424177
0.9	-1.281552	-0.226271
0.95	-1.644854	-0.049076
0.975	-1.959964	0.140243
0.99	-2.326348	0.401982
0.995	-2.575829	0.605816
0.999	-3.090232	1.091589
0.9999	-3.719016	1.805181

We see by Table II-2 that (45) gives at least four significant figure accuracy when ν_1 and ν_2 are both greater than 120, and it is recommended for use in this case.

Table II-2

$F(\nu_1, \nu_2, p)$	Tabulated Value	Carter F	Carter Δ	Equation (21) F	Equation (21) Δ
F(∞, 120, 0.0001)	1.6966	1.6960	-0.0006	1.5686	-0.1280
F(∞, 60, 0.0001)	2.1821	2.1797	-0.0024	1.8610	-0.3211
F(∞, 30, 0.0001)	3.2404	3.2314	-0.0090	2.3393	-0.9011
F(120, ∞. 0.0001)	1.5527	1.5522	-0.0005	1.5536*	+0.0009
F(120, 120, 0.0001)	1.9877	1.9877	0.0000	1.9192	-0.0685
F(120, 60, 0.0001)	2.4405	2.4398	-0.0007	2.2250	-0.2155
F(120, 30, 0.0001)	3.4852	3.4797	-0.0055	2.7679	-0.7173
F(60, ∞. 0.0001)	1.8250	1.8234	-0.0016	1.8276*	+0.0026
F(60, 120, 0.0001)	2.2301	2.2300	-0.0001	2.2062	-0.0239
F(60, 60, 0.0001)	2.6723	2.6726	+0.0003	2.5430	-0.1293
F(60, 30, 0.0001)	3.7163	3.7139	-0.0024	3.1606	-0.5557
F(30, ∞. 0.0001)	2.2544	2.2492	-0.0052	2.2619*	+0.0075
F(30, 120, 0.0001)	2.6480	2.6464	-0.0016	2.7027	+0.0547
F(30, 60, 0.0001)	3.0894	3.0902	+0.0008	3.1130	+0.0236
F(30, 30, 0.0001)	4.1492	4.1518	+0.0026	3.1722	-0.9770
F(∞, 120, 0.05)	1.2539	1.2540	+0.0001	1.2341	-0.0198
F(∞, 60, 0.05)	1.3893	1.3898	+0.0005	1.3449	-0.0444
F(∞, 30, 0.05)	1.6223	1.6243	+0.0020	1.5168	-0.1055
F(120, ∞, 0.05)	1.2214	1.2215	+0.0001	1.2214*	0.0000
F(120, 120, 0.05)	1.3519	1.3519	0.0000	1.3579	+0.0060
F(120, 60, 0.05)	1.4673	1.4675	+0.0002	1.4666	-0.0007
F(120, 30, 0.05)	1.6835	1.6848	+0.0013	1.6506	-0.0329
F(60, ∞, 0.05)	1.3180	1.3184	+0.0004	1.3180*	0.0000
F(60, 120, 0.05)	1.4290	1.4291	+0.0001	1.4523	+0.0233
F(60, 60, 0.05)	1.5343	1.5343	0.0000	1.5666	+0.0323
F(60, 30, 0.05)	1.7396	1.7404	+0.0008	1.7665	+0.0269
F(30, ∞, 0.05)	1.4591	1.4601	+0.0010	1.4589*	-0.0002
F(30, 120, 0.05)	1.5543	1.5547	+0.0004	1.6045	+0.0502
F(30, 60, 0.05)	1.6491	1.6492	+0.0001	1.7343	+0.0852
F(30, 30, 0.05)	1.8409	1.8411	+0.0002	1.7609	-0.0800
F(∞, 120, 0.5)	1.0056	1.0056	0.0000	1.0056	0.0000
F(∞, 60, 0.5)	1.0112	1.0112	0.0000	1.0112	0.0000
F(∞, 30, 0.5)	1.0226	1.0224	-0.0002	1.0224	-0.0002
F(120, ∞, 0.5)	0.99445	0.99446	+0.00001	0.99445*	0.00000
F(120, 120, 0.5)	1.0000	1.0000	0.00000	1.0000	0.00000
F(120, 60, 0.5)	1.0056	1.0056	0.0000	1.0056	0.0000
F(120, 30, 0.5)	1.0170	1.0168	-0.0002	1.0168	-0.0002
F(60, ∞, 0.5)	0.98891	0.98895	+0.00004	0.98893*	+0.00002
F(60, 120, 0.5)	0.99443	0.99446	+0.00003	0.99445	+0.00002
F(60, 60, 0.5)	1.0000	1.0000	0.00000	1.0000	0.00000
F(60, 30, 0.5)	1.0113	1.0111	-0.0002	1.0112	-0.0001
F(30, ∞, 0.5)	0.97787	0.97805	+0.00018	0.97794*	+0.00007
F(30, 120, 0.5)	0.98333	0.98350	+0.00017	0.98343	+0.00010
F(30, 60, 0.5)	0.98884	0.98897	+0.00013	0.98893	+0.00009
F(30, 30, 0.5)	1.0000	1.0000	0.00000	1.0000	0.00000

* These values also represent the Wilson-Hilferty [8/] approximation to $\chi^2(\nu, p)/\nu$.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 1$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5	p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
1	(+7) 4.0528	(+5) 4.0528	(+4) 1.6211	(+3) 4.0522	(+2) 6.4779	(+2) 1.6145	(+1) 3.9864	5.8285	1.0000	(-1) 1.7157	(-2) 2.5085	(-3) 6.1939	(-3) 1.5437	(-4) 2.4678	(-5) 6.1687	(-6) 2.4674	(-8) 2.4674	1
1.2	(6) 2.3821	(4) 5.1319	(3) 3.5094	(3) 1.1048	(2) 2.3927	(1) 7.4802	(1) 2.3000	4.3669	(-1) 8.7158	(-1) 1.5815	(-2) 2.3424	(-3) 5.7938	(-3) 1.4446	(-4) 2.3097	(-5) 5.7736	(-6) 2.3094	(-8) 2.3094	1.2
1.5	(5) 1.4790	(3) 6.8637	(2) 8.0184	(2) 3.1756	(1) 9.2839	(1) 3.6200	(1) 1.3728	3.3235	(-1) 7.6142	(-1) 1.4540	(-2) 2.1794	(-3) 5.3994	(-3) 1.3468	(-4) 2.1536	(-5) 5.3834	(-6) 2.1533	(-8) 2.1533	1.5
2	(+3) 9.9985	(+2) 9.9850	(+2) 1.9850	(+1) 9.8503	(+1) 3.8506	(+1) 1.8513	8.5263	2.5714	(-1) 6.6667	(-1) 1.3333	(-2) 2.0202	(-3) 5.0125	(-3) 1.2508	(-4) 2.0002	(-5) 5.0000	(-6) 2.0000	(-8) 2.0000	2
3	(+2) 7.8401	(+2) 1.6703	(+1) 5.5552	(+1) 3.4116	(+1) 1.7443	(+1) 1.0128	5.5383	2.0239	(-1) 5.8506	(-1) 1.2195	(-2) 1.8659	(-3) 4.6359	(-3) 1.1572	(-4) 1.8507	(-5) 4.6264	(-6) 1.8506	(-8) 1.8505	3
4	(+2) 2.4162	(+1) 7.4137	(+1) 3.1333	(+1) 2.1198	(+1) 1.2218	7.7086	4.5448	1.8074	(-1) 5.4863	(-1) 1.1654	(-2) 1.7911	(-3) 4.4528	(-3) 1.1116	(-4) 1.7779	(-5) 4.4444	(-6) 1.7778	(-8) 1.7778	4
5	(+2) 1.2494	(+1) 4.7181	(+1) 2.2785	(+1) 1.6258	(+1) 1.0007	6.6079	4.0604	1.6925	(-1) 5.2807	(-1) 1.1338	(-2) 1.7470	(-3) 4.3448	(-3) 1.0848	(-4) 1.7350	(-5) 4.3373	(-6) 1.7349	(-8) 1.7349	5
6	(+1) 8.2489	(+1) 3.5507	(+1) 1.8635	(+1) 1.3745	8.8131	5.9874	3.7760	1.6214	(-1) 5.1489	(-1) 1.1132	(-2) 1.7181	(-3) 4.2737	(-3) 1.0671	(-4) 1.7068	(-5) 4.2668	(-6) 1.7067	(-8) 1.7067	6
7	(+1) 6.2167	(+1) 2.9245	(+1) 1.6236	(+1) 1.2246	8.0727	5.5914	3.5894	1.5732	(-1) 5.0572	(-1) 1.0986	(-2) 1.6976	(-3) 4.2235	(-3) 1.0546	(-4) 1.6868	(-5) 4.2167	(-6) 1.6867	(-8) 1.6867	7
8	(+1) 5.0694	(+1) 2.5415	(+1) 1.4688	(+1) 1.1259	7.5709	5.3177	3.4579	1.5384	(-1) 4.9898	(-1) 1.0879	(-2) 1.6824	(-3) 4.1862	(-3) 1.0453	(-4) 1.6718	(-5) 4.1797	(-6) 1.6718	(-8) 1.6718	8
9	(+1) 4.3477	(+1) 2.2857	(+1) 1.3614	(+1) 1.0561	7.2093	5.1174	3.3603	1.5121	(-1) 4.9382	(-1) 1.0796	(-2) 1.6706	(-3) 4.1573	(-3) 1.0381	(-4) 1.6604	(-5) 4.1509	(-6) 1.6603	(-8) 1.6603	9
10	(+1) 3.8577	(+1) 2.1040	(+1) 1.2826	(+1) 1.0044	6.9367	4.9646	3.2850	1.4915	(-1) 4.8973	(-1) 1.0729	(-2) 1.6613	(-3) 4.1343	(-3) 1.0324	(-4) 1.6513	(-5) 4.1281	(-6) 1.6512	(-8) 1.6512	10
12	(+1) 3.2427	(+1) 1.8643	(+1) 1.1754	9.3302	6.5538	4.7472	3.1765	1.4613	(-1) 4.8369	(-1) 1.0631	(-2) 1.6473	(-3) 4.0999	(-3) 1.0238	(-4) 1.6377	(-5) 4.0940	(-6) 1.6376	(-8) 1.6376	12
15	(+1) 2.7448	(+1) 1.6587	(+1) 1.0798	8.6831	6.1995	4.5431	3.0732	1.4321	(-1) 4.7775	(-1) 1.0534	(-2) 1.6335	(-3) 4.0659	(-3) 1.0154	(-4) 1.6241	(-5) 4.0601	(-6) 1.6240	(-8) 1.6240	15
20	(+1) 2.3399	(+1) 1.4819	9.9439	8.0960	5.8715	4.3513	2.9747	1.4037	(-1) 4.7192	(-1) 1.0437	(-2) 1.6197	(-3) 4.0321	(-3) 1.0069	(-4) 1.6106	(-5) 4.0264	(-6) 1.6105	(-8) 1.6105	20
24	(+1) 2.1663	(+1) 1.4028	9.5513	7.8229	5.7167	4.2597	2.9271	1.3898	(-1) 4.6902	(-1) 1.0389	(-2) 1.6129	(-3) 4.0153	(-3) 1.0028	(-4) 1.6040	(-5) 4.0096	(-6) 1.6039	(-8) 1.6039	24
30	(+1) 2.0092	(+1) 1.3293	9.1797	7.5625	5.5675	4.1709	2.8807	1.3761	(-1) 4.6616	(-1) 1.0341	(-2) 1.6060	(-3) 3.9986	(-4) 9.9860	(-4) 1.5973	(-5) 3.9930	(-6) 1.5972	(-8) 1.5972	30
40	(+1) 1.8668	(+1) 1.2609	8.8278	7.3141	5.4239	4.0848	2.8354	1.3626	(-1) 4.6330	(-1) 1.0294	(-2) 1.5993	(-3) 3.9818	(-4) 9.9443	(-4) 1.5906	(-5) 3.9765	(-6) 1.5906	(-8) 1.5906	40
60	(+1) 1.7377	(+1) 1.1973	8.4946	7.0771	5.2857	4.0012	2.7914	1.3493	(-1) 4.6053	(-1) 1.0247	(-2) 1.5925	(-3) 3.9651	(-4) 9.9030	(-4) 1.5840	(-5) 3.9599	(-6) 1.5839	(-8) 1.5839	60
120	(+1) 1.6204	(+1) 1.1380	8.1790	6.8510	5.1524	3.9201	2.7478	1.3362	(-1) 4.5774	(-1) 1.0200	(-2) 1.5858	(-3) 3.9487	(-4) 9.8619	(-4) 1.5774	(-5) 3.9434	(-6) 1.5774	(-8) 1.5774	120
∞	(+1) 1.5137	(+1) 1.0828	7.8794	6.6349	5.0239	3.8415	2.7055	1.3233	(-1) 4.5494	(-1) 1.0153	(-2) 1.5791	(-3) 3.9321	(-4) 9.8203	(-4) 1.5708	(-5) 3.9270	(-6) 1.5708	(-8) 1.5708	∞

$\nu_1 = 1$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 2$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5	p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
1	(+7) 5.0000	(+5) 5.0000	(+4) 2.0000	(+3) 4.9995	(+2) 7.9950	(+2) 1.9950	(+1) 4.9500	7.5000	1.5000	(-1) 3.8889	(-1) 1.1728	(-2) 5.4016	(-2) 2.5970	(-2) 1.0152	(-3) 5.0378	(-3) 1.0015	(-4) 1.0002	1
1.2	(6) 2.7850	(4) 5.9999	(3) 4.1033	(3) 1.2921	(2) 2.8011	(1) 8.7817	(1) 2.7250	5.4476	1.3049	(-1) 3.6913	(-1) 1.1518	(-2) 5.3550	(-2) 2.5860	(-2) 1.0135	(-3) 5.0335	(-3) 1.0013	(-4) 1.0001	1.2
1.5	(5) 1.6158	(3) 7.4992	(2) 8.7646	(2) 3.4737	(2) 1.0185	(1) 3.9966	(1) 1.5408	4.0122	1.1399	(-1) 3.5064	(-1) 1.1312	(-2) 5.3088	(-2) 2.5750	(-2) 1.0118	(-3) 5.0293	(-3) 1.0012	(-4) 1.0001	1.5
2	(+3) 9.9990	(+2) 9.9900	(+2) 1.9900	(+1) 9.9000	(+1) 3.9000	(+1) 1.9000	9.0000	3.0000	1.0000	(-1) 3.3333	(-1) 1.1111	(-2) 5.2632	(-2) 2.5641	(-2) 1.0101	(-3) 5.0251	(-3) 1.0010	(-4) 1.0001	2
3	(+2) 6.9474	(+2) 1.4850	(+1) 4.9799	(+1) 3.0817	(+1) 1.6044	9.5521	5.4624	2.2798	(-1) 8.8110	(-1) 3.1712	(-1) 1.0915	(-2) 5.2181	(-2) 2.5533	(-2) 1.0084	(-3) 5.0208	(-3) 1.0008	(-4) 1.0001	3
4	(+2) 1.9800	(+1) 6.1246	(+1) 2.6284	(+1) 1.8000	(+1) 1.0649	6.9443	4.3246	2.0000	(-1) 8.2843	(-1) 3.0941	(-1) 1.0819	(-2) 5.1956	(-2) 2.5479	(-2) 1.0076	(-3) 5.0188	(-3) 1.0008	(-4) 1.0001	4
5	(+1) 9.7027	(+1) 3.7122	(+1) 1.8314	(+1) 1.3274	8.4336	5.7861	3.7797	1.8528	(-1) 7.9877	(-1) 3.0489	(-1) 1.0761	(-2) 5.1824	(-2) 2.5447	(-2) 1.0071	(-3) 5.0176	(-3) 1.0007	(-4) 1.0001	5
6	(+1) 6.1633	(+1) 2.7000	(+1) 1.4544	(+1) 1.0925	7.2598	5.1433	3.4633	1.7622	(-1) 7.7976	(-1) 3.0192	(-1) 1.0723	(-2) 5.1733	(-2) 2.5425	(-2) 1.0067	(-3) 5.0168	(-3) 1.0007	(-4) 1.0001	6
7	(+1) 4.5132	(+1) 2.1689	(+1) 1.2404	9.5466	6.5415	4.7374	3.2574	1.7010	(-1) 7.6655	(-1) 2.9983	(-1) 1.0696	(-2) 5.1672	(-2) 2.5410	(-2) 1.0065	(-3) 5.0161	(-3) 1.0006	(-4) 1.0001	7
8	(+1) 3.6000	(+1) 1.8494	(+1) 1.1042	8.6491	6.0595	4.4590	3.1131	1.6569	(-1) 7.5683	(-1) 2.9828	(-1) 1.0676	(-2) 5.1624	(-2) 2.5398	(-2) 1.0063	(-3) 5.0158	(-3) 1.0006	(-4) 1.0001	8
9	(+1) 3.0342	(+1) 1.6387	(+1) 1.0107	8.0215	5.7147	4.2565	3.0065	1.6236	(-1) 7.4938	(-1) 2.9708	(-1) 1.0660	(-2) 5.1586	(-2) 2.5389	(-2) 1.0062	(-3) 5.0153	(-3) 1.0006	(-4) 1.0001	9
10	(+1) 2.6548	(+1) 1.4905	9.4270	7.5594	5.4564	4.1028	2.9245	1.5975	(-1) 7.4349	(-1) 2.9612	(-1) 1.0648	(-2) 5.1557	(-2) 2.5382	(-2) 1.0060	(-3) 5.0150	(-3) 1.0006	(-4) 1.0001	10
12	(+1) 2.1850	(+1) 1.2974	8.5096	6.9266	5.0959	3.8853	2.8068	1.5595	(-1) 7.3477	(-1) 2.9469	(-1) 1.0629	(-2) 5.1512	(-2) 2.5371	(-2) 1.0059	(-3) 5.0145	(-3) 1.0006	(-4) 1.0001	12
15	(+1) 1.8109	(+1) 1.1339	7.7008	6.3589	4.7650	3.6823	2.6952	1.5227	(-1) 7.2619	(-1) 2.9327	(-1) 1.0610	(-2) 5.1469	(-2) 2.5361	(-2) 1.0057	(-3) 5.0143	(-3) 1.0006	(-4) 1.0001	15
20	(+1) 1.5119	9.9526	6.9865	5.8489	4.4613	3.4928	2.5893	1.4870	(-1) 7.1773	(-1) 2.9186	(-1) 1.0592	(-2) 5.1424	(-2) 2.5350	(-2) 1.0055	(-3) 5.0138	(-3) 1.0006	(-4) 1.0001	20
24	(+1) 1.3853	9.3394	6.6610	5.6136	4.3187	3.4028	2.5383	1.4695	(-1) 7.1356	(-1) 2.9116	(-1) 1.0582	(-2) 5.1403	(-2) 2.5345	(-2) 1.0054	(-3) 5.0135	(-3) 1.0005	(-4) 1.0001	24
30	(+1) 1.2718	8.7734	6.3547	5.3904	4.1821	3.3158	2.4887	1.4524	(-1) 7.0941	(-1) 2.9046	(-1) 1.0573	(-2) 5.1382	(-2) 2.5339	(-2) 1.0054	(-3) 5.0133	(-3) 1.0005	(-4) 1.0001	30
40	(+1) 1.1698	8.2508	6.0664	5.1785	4.0510	3.2317	2.4404	1.4355	(-1) 7.0531	(-1) 2.8976	(-1) 1.0564	(-2) 5.1358	(-2) 2.5334	(-2) 1.0053	(-3) 5.0133	(-3) 1.0005	(-4) 1.0001	40
60	(+1) 1.0781	7.7678	5.7950	4.9774	3.9253	3.1504	2.3932	1.4188	(-1) 7.0122	(-1) 2.8907	(-1) 1.0555	(-2) 5.1337	(-2) 2.5329	(-2) 1.0052	(-3) 5.0130	(-3) 1.0005	(-4) 1.0001	60
120	9.9549	7.3211	5.5393	4.7865	3.8046	3.0718	2.3473	1.4024	(-1) 6.9717	(-1) 2.8838	(-1) 1.0545	(-2) 5.1316	(-2) 2.5323	(-2) 1.0051	(-3) 5.0128	(-3) 1.0005	(-4) 1.0001	120
∞	9.2103	6.9078	5.2983	4.6052	3.6889	2.9957	2.3026	1.3863	(-1) 6.9315	(-1) 2.8768	(-1) 1.0536	(-2) 5.1293	(-2) 2.5318	(-2) 1.0050	(-3) 5.0123	(-3) 1.0005	(-4) 1.0001	∞

$\nu_1 = 2$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 3$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5	p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
1	(+7) 5.4038	(+5) 5.4038	(+4) 2.1615	(+3) 5.4033	(+2) 8.6416	(+2) 2.1571	(+1) 5.3593	8.1999	1.7092	(-1) 4.9410	(-1) 1.8056	(-2) 9.8736	(-2) 5.7330	(-2) 2.9312	(-2) 1.8001	(-3) 5.9868	(-3) 1.2755	1
1.2	(6) 2.9549	(4) 6.3660	(3) 4.3538	(3) 1.3710	(2) 2.9731	(1) 9.3286	(1) 2.9023	5.8883	1.4842	(-1) 4.7352	(-1) 1.8079	(-1) 1.0030	(-2) 5.8709	(-2) 3.0192	(-2) 1.8586	(-3) 6.1978	(-3) 1.3218	1.2
1.5	(5) 1.6727	(3) 7.7635	(2) 9.0745	(2) 3.5973	(2) 1.0557	(1) 4.1506	(1) 1.6083	4.2806	1.2947	(-1) 4.5502	(-1) 1.8158	(-1) 1.0225	(-2) 6.0349	(-2) 3.1222	(-2) 1.9269	(-3) 6.4432	(-3) 1.3756	1.5
2	(+3) 9.9992	(+2) 9.9917	(+2) 1.9917	(+1) 9.9166	(+1) 3.9165	(+1) 1.9164	9.1618	3.1534	1.1349	(-1) 4.3863	(-1) 1.8307	(-1) 1.0469	(-2) 6.2329	(-2) 3.2450	(-2) 2.0081	(-3) 6.7340	(-3) 1.4394	2
3	(+2) 6.5934	(+2) 1.4111	(+1) 4.7467	(+1) 2.9457	(+1) 1.5439	9.2766	5.3908	2.3555	1.0000	(-1) 4.2454	(-1) 1.8550	(-1) 1.0780	(-2) 6.4771	(-2) 3.3948	(-2) 2.1067	(-3) 7.0868	(-3) 1.5167	3
4	(+2) 1.8102	(+1) 5.6177	(+1) 2.4259	(+1) 1.6694	9.9792	6.5914	4.1908	2.0467	(-1) 9.4054	(-1) 4.1839	(-1) 1.8717	(-1) 1.0968	(-2) 6.6221	(-2) 3.4831	(-2) 2.1647	(-3) 7.2939	(-3) 1.5620	4
5	(+1) 8.6292	(+1) 3.3202	(+1) 1.6530	(+1) 1.2060	7.7636	5.4095	3.6195	1.8843	(-1) 9.0715	(-1) 4.1502	(-1) 1.8835	(-1) 1.1094	(-2) 6.7182	(-2) 3.5415	(-2) 2.2030	(-3) 7.4305	(-3) 1.5919	5
6	(+1) 5.3680	(+1) 2.3703	(+1) 1.2917	9.7795	6.5988	4.7571	3.2888	1.7844	(-1) 8.8578	(-1) 4.1292	(-1) 1.8923	(-1) 1.1185	(-2) 6.7866	(-2) 3.5828	(-2) 2.2303	(-3) 7.5275	(-3) 1.6132	6
7	(+1) 3.8676	(+1) 1.8772	(+1) 1.0882	8.4513	5.8898	4.3468	3.0741	1.7169	(-1) 8.7095	(-1) 4.1149	(-1) 1.8989	(-1) 1.1253	(-2) 6.8381	(-2) 3.6138	(-2) 2.2505	(-3) 7.5998	(-3) 1.6290	7
8	(+1) 3.0456	(+1) 1.5829	9.5965	7.5910	5.4160	4.0662	2.9238	1.6683	(-1) 8.6004	(-1) 4.1044	(-1) 1.9041	(-1) 1.1306	(-2) 6.8776	(-2) 3.6378	(-2) 2.2662	(-3) 7.6559	(-3) 1.6413	8
9	(+1) 2.5404	(+1) 1.3902	8.7171	6.9919	5.0781	3.8626	2.8129	1.6315	(-1) 8.5168	(-1) 4.0967	(-1) 1.9084	(-1) 1.1348	(-2) 6.9094	(-2) 3.6570	(-2) 2.2788	(-3) 7.7006	(-3) 1.6511	9
10	(+1) 2.2038	(+1) 1.2553	8.0807	6.5523	4.8256	3.7083	2.7277	1.6028	(-1) 8.4508	(-1) 4.0905	(-1) 1.9119	(-1) 1.1382	(-2) 6.9353	(-2) 3.6726	(-2) 2.2891	(-3) 7.7371	(-3) 1.6590	10
12	(+1) 1.7899	(+1) 1.0804	7.2258	5.9526	4.4742	3.4903	2.6055	1.5609	(-1) 8.3530	(-1) 4.0816	(-1) 1.9173	(-1) 1.1436	(-2) 6.9750	(-2) 3.6966	(-2) 2.3048	(-3) 7.7933	(-3) 1.6713	12
15	(+1) 1.4635	9.3353	6.4760	5.4170	4.1528	3.2874	2.4898	1.5202	(-1) 8.2569	(-1) 4.0730	(-1) 1.9230	(-1) 1.1490	(-2) 7.0161	(-2) 3.7213	(-2) 2.3210	(-3) 7.8509	(-3) 1.6839	15
20	(+1) 1.2050	8.0984	5.8177	4.9382	3.8587	3.0984	2.3801	1.4808	(-1) 8.1621	(-1) 4.0647	(-1) 1.9288	(-1) 1.1547	(-2) 7.0587	(-2) 3.7467	(-2) 2.3377	(-3) 7.9103	(-3) 1.6969	20
24	(+1) 1.0964	7.5545	5.5190	4.7181	3.7211	3.0088	2.3274	1.4615	(-1) 8.1153	(-1) 4.0607	(-1) 1.9318	(-1) 1.1576	(-2) 7.0801	(-2) 3.7597	(-2) 2.3462	(-3) 7.9406	(-3) 1.7036	24
30	9.9942	7.0545	5.2388	4.5097	3.5894	2.9223	2.2761	1.4426	(-1) 8.0689	(-1) 4.0568	(-1) 1.9349	(-1) 1.1606	(-2) 7.1018	(-2) 3.7729	(-2) 2.3548	(-3) 7.9714	(-3) 1.7103	30
40	9.1278	6.5945	4.9759	4.3126	3.4633	2.8387	2.2261	1.4239	(-1) 8.0228	(-1) 4.0528	(-1) 1.9381	(-1) 1.1635	(-2) 7.1240	(-2) 3.7863	(-2) 2.3636	(-3) 8.0026	(-3) 1.7171	40
60	8.3526	6.1712	4.7290	4.1259	3.3425	2.7581	2.1774	1.4055	(-1) 7.9770	(-1) 4.0491	(-1) 1.9413	(-1) 1.1666	(-2) 7.1469	(-2) 3.8000	(-2) 2.3725	(-3) 8.0343	(-3) 1.7241	60
120	7.6584	5.7814	4.4973	3.9493	3.2270	2.6802	2.1300	1.3873	(-1) 7.9314	(-1) 4.0453	(-1) 1.9446	(-1) 1.1697	(-2) 7.1700	(-2) 3.8137	(-2) 2.3816	(-3) 8.0665	(-3) 1.7311	120
∞	7.0359	5.4221	4.2794	3.7816	3.1161	2.6049	2.0838	1.3694	(-1) 7.8866	(-1) 4.0417	(-1) 1.9479	(-1) 1.1728	(-2) 7.1932	(-2) 3.8278	(-2) 2.3907	(-3) 8.0992	(-3) 1.7383	∞

$\nu_1 = 3$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 4$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5	p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
1	(+7) 5.6250	(+5) 5.6250	(+4) 2.2500	(+3) 5.6246	(+2) 8.9958	(+2) 2.2458	(+1) 5.5833	8.5810	1.8227	(-1) 5.5328	(-1) 2.2003	(-1) 1.2973	(-2) 8.1846	(-2) 4.7174	(-2) 3.1915	(-2) 1.3488	(-3) 4.1387	1
1.2	(6) 3.0478	(4) 6.5663	(3) 4.4908	(3) 1.4142	(2) 3.0671	(1) 9.6274	(1) 2.9990	6.1265	1.5810	(-1) 5.3271	(-1) 2.2249	(-1) 1.3347	(-2) 8.5083	(-2) 4.9438	(-2) 3.3572	(-2) 1.4256	(-3) 4.3849	1.2
1.5	(5) 1.7038	(3) 7.9077	(2) 9.2437	(2) 3.6648	(2) 1.0759	(1) 4.2343	(1) 1.6446	4.4235	1.3780	(-1) 5.1487	(-1) 2.2608	(-1) 1.3811	(-2) 8.9012	(-2) 5.2170	(-2) 3.5571	(-2) 1.5181	(-3) 4.6821	1.5
2	(+3) 9.9992	(+2) 9.9925	(+2) 1.9925	(+1) 9.9249	(+1) 3.9248	(+1) 1.9247	9.2434	3.2320	1.2071	(-1) 5.0000	(-1) 2.3124	(-1) 1.4400	(-2) 9.3906	(-2) 5.5555	(-2) 3.8046	(-2) 1.6328	(-3) 5.0505	2
3	(+2) 6.4019	(+2) 1.3710	(+1) 4.6195	(+1) 2.8710	(+1) 1.5101	9.1172	5.3427	2.3901	1.0632	(-1) 4.8859	(-1) 2.3862	(-1) 1.5171	(-1) 1.0021	(-2) 5.9902	(-2) 4.1222	(-2) 1.7801	(-3) 5.5243	3
4	(+2) 1.7187	(+1) 5.3436	(+1) 2.3155	(+1) 1.5977	9.6045	6.3883	4.1073	2.0642	1.0000	(-1) 4.8445	(-1) 2.4347	(-1) 1.5654	(-1) 1.0412	(-2) 6.2590	(-2) 4.3187	(-2) 1.8714	(-3) 5.8183	4
5	(+1) 8.0527	(+1) 3.1085	(+1) 1.5556	(+1) 1.1392	7.3879	5.1922	3.5202	1.8927	(-1) 9.6456	(-1) 4.8256	(-1) 2.4688	(-1) 1.5985	(-1) 1.0679	(-2) 6.4425	(-2) 4.4532	(-2) 1.9338	(-3) 6.0193	5
6	(+1) 4.9419	(+1) 2.1924	(+1) 1.2028	9.1483	6.2272	4.5337	3.1808	1.7872	(-1) 9.4191	(-1) 4.8156	(-1) 2.4939	(-1) 1.6226	(-1) 1.0873	(-2) 6.5759	(-2) 4.5506	(-2) 1.9792	(-3) 6.1657	6
7	(+1) 3.5222	(+1) 1.7198	(+1) 1.0050	7.8467	5.5226	4.1203	2.9605	1.7157	(-1) 9.2619	(-1) 4.8100	(-1) 2.5132	(-1) 1.6409	(-1) 1.1020	(-2) 6.6774	(-2) 4.6249	(-2) 2.0138	(-3) 6.2771	7
8	(+1) 2.7493	(+1) 1.4392	8.8051	7.0060	5.0526	3.8378	2.8064	1.6642	(-1) 9.1464	(-1) 4.8065	(-1) 2.5285	(-1) 1.6554	(-1) 1.1136	(-2) 6.7572	(-2) 4.6834	(-2) 2.0410	(-3) 6.3648	8
9	(+1) 2.2766	(+1) 1.2560	7.9559	6.4221	4.7181	3.6331	2.6927	1.6253	(-1) 9.0580	(-1) 4.8045	(-1) 2.5408	(-1) 1.6670	(-1) 1.1230	(-2) 6.8217	(-2) 4.7306	(-2) 2.0629	(-3) 6.4357	9
10	(+1) 1.9630	(+1) 1.1283	7.3428	5.9943	4.4683	3.4780	2.6053	1.5949	(-1) 8.9882	(-1) 4.8031	(-1) 2.5511	(-1) 1.6766	(-1) 1.1307	(-2) 6.8747	(-2) 4.7694	(-2) 2.0811	(-3) 6.4941	10
12	(+1) 1.5793	9.6327	6.5211	5.4119	4.1212	3.2592	2.4801	1.5503	(-1) 8.8848	(-1) 4.8017	(-1) 2.5671	(-1) 1.6916	(-1) 1.1427	(-2) 6.9570	(-2) 4.8298	(-2) 2.1092	(-3) 6.5849	12
15	(+1) 1.2783	8.2527	5.8029	4.8932	3.8043	3.0556	2.3614	1.5071	(-1) 8.7830	(-1) 4.8010	(-1) 2.5847	(-1) 1.7071	(-1) 1.1552	(-2) 7.0432	(-2) 4.8928	(-2) 2.1385	(-3) 6.6796	15
20	(+1) 1.0415	7.0960	5.1743	4.4307	3.5147	2.8661	2.2489	1.4652	(-1) 8.6830	(-1) 4.8012	(-1) 2.6013	(-1) 1.7234	(-1) 1.1682	(-2) 7.1327	(-2) 4.9586	(-2) 2.1692	(-3) 6.7786	20
24	9.4246	6.5892	4.8898	4.2184	3.3794	2.7763	2.1949	1.4447	(-1) 8.6335	(-1) 4.8015	(-1) 2.6103	(-1) 1.7318	(-1) 1.1750	(-2) 7.1793	(-2) 4.9925	(-2) 2.1850	(-3) 6.8298	24
30	8.5437	6.1245	4.6233	4.0179	3.2499	2.6896	2.1422	1.4244	(-1) 8.5844	(-1) 4.8019	(-1) 2.6196	(-1) 1.7404	(-1) 1.1819	(-2) 7.2265	(-2) 5.0271	(-2) 2.2013	(-3) 6.8822	30
40	7.7592	5.6981	4.3738	3.8283	3.1261	2.6060	2.0909	1.4045	(-1) 8.5357	(-1) 4.8028	(-1) 2.6291	(-1) 1.7492	(-1) 1.1889	(-2) 7.2754	(-2) 5.0628	(-2) 2.2179	(-3) 6.9358	40
60	7.0599	5.3067	4.1399	3.6491	3.0077	2.5252	2.0410	1.3848	(-1) 8.4873	(-1) 4.8038	(-1) 2.6388	(-1) 1.7581	(-1) 1.1961	(-2) 7.3249	(-2) 5.0992	(-2) 2.2349	(-3) 6.9907	60
120	6.4357	4.9472	3.9207	3.4796	2.8943	2.4472	1.9923	1.3654	(-1) 8.4392	(-1) 4.8049	(-1) 2.6488	(-1) 1.7674	(-1) 1.2035	(-2) 7.3757	(-2) 5.1366	(-2) 2.2523	(-3) 7.0470	120
∞	5.8782	4.6167	3.7151	3.3192	2.7858	2.3719	1.9449	1.3463	(-1) 8.3918	(-1) 4.8063	(-1) 2.6591	(-1) 1.7768	(-1) 1.2110	(-2) 7.4278	(-2) 5.1746	(-2) 2.2701	(-3) 7.1046	∞

$\nu_1 = 4$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 5$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5	p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
1	(+7) 5.7640	(+5) 5.7640	(+4) 2.3056	(+3) 5.7637	(+2) 9.2185	(+2) 2.3016	(+1) 5.7241	8.8198	1.8937	(-1) 5.9084	(-1) 2.4628	(-1) 1.5133	(-2) 9.9930	(-2) 6.1508	(-2) 4.3889	(-2) 2.1195	(-3) 8.0038	1
1.2	(6) 3.1062	(4) 6.6921	(3) 4.5769	(3) 1.4413	(2) 3.1262	(1) 9.8152	(1) 3.0596	6.2753	1.6415	(-1) 5.7046	(-1) 2.5048	(-1) 1.5689	(-1) 1.0483	(-2) 6.5149	(-2) 4.6708	(-2) 2.2701	(-3) 8.6059	1.2
1.5	(5) 1.7233	(3) 7.9984	(2) 9.3500	(2) 3.7072	(2) 1.0887	(1) 4.2867	(1) 1.6673	4.5121	1.4298	(-1) 5.5327	(-1) 2.5636	(-1) 1.6385	(-1) 1.1087	(-2) 6.9633	(-2) 5.0181	(-2) 2.4561	(-3) 9.3515	1.5
2	(+3) 9.9993	(+2) 9.9930	(+2) 1.9930	(+1) 9.9299	(+1) 3.9298	(+1) 1.9296	9.2926	3.2799	1.2519	(-1) 5.3972	(-1) 2.6457	(-1) 1.7283	(-1) 1.1857	(-2) 7.5335	(-2) 5.4603	(-2) 2.6938	(-2) 1.0306	2
3	(+2) 6.2817	(+2) 1.3458	(+1) 4.5392	(+1) 2.8237	(+1) 1.4885	9.0135	5.3092	2.4095	1.1024	(-1) 5.3070	(-1) 2.7628	(-1) 1.8486	(-1) 1.2881	(-2) 8.2919	(-2) 6.0496	(-2) 3.0118	(-2) 1.1589	3
4	(+2) 1.6613	(+1) 5.1712	(+1) 2.2456	(+1) 1.5522	9.3645	6.2560	4.0506	2.0723	1.0367	(-1) 5.2835	(-1) 2.8407	(-1) 1.9260	(-1) 1.3536	(-2) 8.7781	(-2) 6.4284	(-2) 3.2170	(-2) 1.2418	4
5	(+1) 7.6911	(+1) 2.9752	(+1) 1.4940	(+1) 1.0967	7.1464	5.0503	3.4530	1.8947	1.0000	(-1) 5.2779	(-1) 2.8960	(-1) 1.9801	(-1) 1.3993	(-2) 9.1183	(-2) 6.6934	(-2) 3.3611	(-2) 1.3002	5
6	(+1) 4.6747	(+1) 2.0803	(+1) 1.1464	8.7459	5.9876	4.3874	3.1075	1.7852	(-1) 9.7654	(-1) 5.2784	(-1) 2.9373	(-1) 2.0201	(-1) 1.4331	(-2) 9.3703	(-2) 6.8904	(-2) 3.4681	(-2) 1.3436	6
7	(+1) 3.3056	(+1) 1.6206	9.5221	7.4604	5.2852	3.9715	2.8833	1.7111	(-1) 9.6026	(-1) 5.2812	(-1) 2.9692	(-1) 2.0509	(-1) 1.4592	(-2) 9.5639	(-2) 7.0423	(-2) 3.5508	(-2) 1.3772	7
8	(+1) 2.5635	(+1) 1.3485	8.3018	6.6318	4.8173	3.6875	2.7265	1.6575	(-1) 9.4831	(-1) 5.2846	(-1) 2.9946	(-1) 2.0754	(-1) 1.4799	(-2) 9.7191	(-2) 7.1628	(-2) 3.6167	(-2) 1.4040	8
9	(+1) 2.1112	(+1) 1.1714	7.4711	6.0569	4.4844	3.4817	2.6106	1.6170	(-1) 9.3916	(-1) 5.2879	(-1) 3.0154	(-1) 2.0953	(-1) 1.4968	(-2) 9.8445	(-2) 7.2611	(-2) 3.6705	(-2) 1.4259	9
10	(+1) 1.8120	(+1) 1.0481	6.8723	5.6363	4.2361	3.3258	2.5216	1.5853	(-1) 9.3193	(-1) 5.2913	(-1) 3.0327	(-1) 2.1119	(-1) 1.5108	(-2) 9.9493	(-2) 7.3432	(-2) 3.7152	(-2) 1.4440	10
12	(+1) 1.4471	8.8921	6.0711	5.0643	3.8911	3.1059	2.3940	1.5389	(-1) 9.2124	(-1) 5.2975	(-1) 3.0598	(-1) 2.1378	(-1) 1.5327	(-1) 1.0113	(-2) 7.4716	(-2) 3.7853	(-2) 1.4726	12
15	(+1) 1.1621	7.5674	5.3721	4.5556	3.5764	2.9013	2.2730	1.4938	(-1) 9.1073	(-1) 5.3048	(-1) 3.0883	(-1) 2.1651	(-1) 1.5558	(-1) 1.0286	(-2) 7.6069	(-2) 3.8594	(-2) 1.5028	15
20	9.3880	6.4606	4.7616	4.1027	3.2891	2.7109	2.1582	1.4500	(-1) 9.0038	(-1) 5.3135	(-1) 3.1185	(-1) 2.1939	(-1) 1.5802	(-1) 1.0468	(-2) 7.7501	(-2) 3.9378	(-2) 1.5348	20
24	8.4578	5.9768	4.4857	3.8951	3.1548	2.6207	2.1030	1.4285	(-1) 8.9527	(-1) 5.3186	(-1) 3.1343	(-1) 2.2089	(-1) 1.5929	(-1) 1.0564	(-2) 7.8247	(-2) 3.9788	(-2) 1.5515	24
30	7.6322	5.5339	4.2276	3.6990	3.0265	2.5336	2.0492	1.4073	(-1) 8.9019	(-1) 5.3237	(-1) 3.1505	(-1) 2.2243	(-1) 1.6059	(-1) 1.0662	(-2) 7.9014	(-2) 4.0211	(-2) 1.5687	30
40	6.8987	5.1283	3.9860	3.5138	2.9037	2.4495	1.9968	1.3863	(-1) 8.8516	(-1) 5.3296	(-1) 3.1673	(-1) 2.2402	(-1) 1.6194	(-1) 1.0763	(-2) 7.9808	(-2) 4.0647	(-2) 1.5865	40
60	6.2465	4.7565	3.7600	3.3389	2.7863	2.3683	1.9457	1.3657	(-1) 8.8017	(-1) 5.3356	(-1) 3.1845	(-1) 2.2566	(-1) 1.6333	(-1) 1.0867	(-2) 8.0632	(-2) 4.1097	(-2) 1.6049	60
120	5.6661	4.4157	3.5482	3.1735	2.6740	2.2900	1.8959	1.3453	(-1) 8.7521	(-1) 5.3422	(-1) 3.2023	(-1) 2.2736	(-1) 1.6476	(-1) 1.0975	(-2) 8.1473	(-2) 4.1562	(-2) 1.6239	120
∞	5.1490	4.1030	3.3499	3.0173	2.5665	2.2141	1.8473	1.3251	(-1) 8.7029	(-1) 5.3493	(-1) 3.2206	(-1) 2.2910	(-1) 1.6624	(-1) 1.1086	(02) 8.2345	(-2) 4.2043	(-2) 1.6435	∞

$\nu_1 = 5$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 6$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5	p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
1	(+7) 5.8594	(+5) 5.8594	(+4) 2.3437	(+3) 5.8590	(+2) 9.3711	(+2) 2.3399	(+1) 5.8204	8.9833	1.9422	(-1) 6.1675	(-1) 2.6483	(-1) 1.6702	(-1) 1.1347	(-2) 7.2754	(-2) 5.3662	(-2) 2.8163	(-2) 1.2123	1
1.2	(6) 3.1463	(4) 6.7785	(3) 4.6360	(3) 1.4599	(2) 3.1668	(1) 9.9439	(1) 3.1012	6.3770	1.6828	(-1) 5.9653	(-1) 2.7040	(-1) 1.7403	(-1) 1.1975	(-2) 7.7613	(-2) 5.7558	(-2) 3.0441	(-2) 1.3171	1.2
1.5	(5) 1.7367	(3) 8.0606	(2) 9.4230	(2) 3.7363	(2) 1.0974	(1) 4.3226	(1) 1.6829	4.5724	1.4652	(-1) 5.7991	(-1) 2.7806	(-1) 1.8288	(-1) 1.2760	(-2) 8.3679	(-2) 6.2430	(-2) 3.3302	(-2) 1.4492	1.5
2	(+3) 9.9993	(+2) 9.9933	(+2) 1.9933	(+1) 9.9332	(+1) 3.9331	(+1) 1.9330	9.3255	3.3121	1.2824	(-1) 5.6747	(-1) 2.8874	(-1) 1.9443	(-1) 1.3774	(-2) 9.1533	(-2) 6.8757	(-2) 3.7037	(-2) 1.6225	2
3	(+2) 6.1991	(+2) 1.3285	(+1) 4.4838	(+1) 2.7911	(+1) 1.4735	8.9406	5.2847	2.4218	1.1289	(-1) 5.6041	(-1) 3.0406	(-1) 2.1021	(-1) 1.5154	(-1) 1.0225	(-2) 7.7417	(-2) 4.2188	(-2) 1.8629	3
4	(+2) 1.6219	(+1) 5.0525	(+1) 2.1975	(+1) 1.5207	9.1973	6.1631	4.0098	2.0766	1.0617	(-1) 5.5953	(-1) 3.1439	(-1) 2.2057	(-1) 1.6059	(-1) 1.0931	(-2) 8.3139	(-2) 4.5613	(-2) 2.0235	4
5	(+1) 7.4426	(+1) 2.8834	(+1) 1.4513	(+1) 1.0672	6.9777	4.9503	3.4045	1.8945	1.0240	(-1) 5.6016	(-1) 3.2180	(-1) 2.2793	(-1) 1.6701	(-1) 1.1434	(-2) 8.7230	(-2) 4.8071	(-2) 2.1392	5
6	(+1) 4.4909	(+1) 2.0030	(+1) 1.1073	8.4661	5.8197	4.2839	3.0546	1.7821	1.0000	(-1) 5.6114	(-1) 3.2738	(-1) 2.3343	(-1) 1.7183	(-1) 1.1812	(-2) 9.0310	(-2) 4.9926	(-2) 2.2267	6
7	(+1) 3.1567	(+1) 1.5521	9.1554	7.1914	5.1186	3.8660	2.8274	1.7059	(-1) 9.8334	(-1) 5.6215	(-1) 3.3173	(-1) 2.3772	(-1) 1.7558	(-1) 1.2107	(-2) 9.2713	(-2) 5.1378	(-2) 2.2954	7
8	(+1) 2.4357	(+1) 1.2858	7.9520	6.3707	4.6517	3.5806	2.6683	1.6508	(-1) 9.7111	(-1) 5.6306	(-1) 3.3523	(-1) 2.4115	(-1) 1.7858	(-1) 1.2343	(-2) 9.4643	(-2) 5.2548	(-2) 2.3507	8
9	(+1) 1.9974	(+1) 1.1128	7.1338	5.8018	4.3197	3.3738	2.5509	1.6091	(-1) 9.6175	(-1) 5.6392	(-1) 3.3810	(-1) 2.4396	(-1) 1.8105	(-1) 1.2537	(-2) 9.6237	(-2) 5.3510	(-2) 2.3963	9
10	(+1) 1.7081	9.9256	6.5446	5.3858	4.0721	3.2172	2.4606	1.5765	(-1) 9.5436	(-1) 5.6472	(-1) 3.4050	(-1) 2.4631	(-1) 1.8311	(-1) 1.2700	(-2) 9.7561	(-2) 5.4316	(-2) 2.4345	10
12	(+1) 1.3560	8.3788	5.7570	4.8206	3.7283	2.9961	2.3310	1.5286	(-1) 9.4342	(-1) 5.6600	(-1) 3.4427	(-1) 2.5000	(-1) 1.8635	(-1) 1.2956	(-2) 9.9661	(-2) 5.5590	(-2) 2.4950	12
15	(+1) 1.0819	7.0917	5.0708	4.3183	3.4147	2.7905	2.2081	1.4820	(-1) 9.3267	(-1) 5.6750	(-1) 3.4829	(-1) 2.5393	(-1) 1.8980	(-1) 1.3229	(-1) 1.0190	(-2) 5.6952	(-2) 2.5597	15
20	8.6789	6.0186	4.4721	3.8714	3.1283	2.5990	2.0913	1.4366	(-1) 9.2210	(-1) 5.6918	(-1) 3.5257	(-1) 2.5812	(-1) 1.9348	(-1) 1.3521	(-1) 1.0429	(-2) 5.8411	(-2) 2.6291	20
24	7.7896	5.5504	4.2019	3.6667	2.9946	2.5082	2.0351	1.4143	(-1) 9.1687	(-1) 5.7013	(-1) 3.5482	(-1) 2.6031	(-1) 1.9542	(-1) 1.3675	(-1) 1.0555	(-2) 5.9181	(-2) 2.6658	24
30	7.0017	5.1223	3.9492	3.4735	2.8667	2.4205	1.9803	1.3923	(-1) 9.1169	(-1) 5.7110	(-1) 3.5714	(-1) 2.6259	(-1) 1.9743	(-1) 1.3834	(-1) 1.0686	(-2) 5.9980	(-2) 2.7039	30
40	6.3031	4.7306	3.7129	3.2910	2.7444	2.3359	1.9269	1.3706	(-1) 9.0654	(-1) 5.7218	(-1) 3.5956	(-1) 2.6495	(-1) 1.9950	(-1) 1.3999	(-1) 1.0822	(-2) 6.0811	(-2) 2.7435	40
60	5.6830	4.3721	3.4918	3.1187	2.6274	2.2540	1.8747	1.3491	(-1) 9.0144	(-1) 5.7330	(-1) 3.6206	(-1) 2.6739	(-1) 2.0166	(-1) 1.4171	(-1) 1.0963	(-2) 6.1674	(-2) 2.7847	60
120	5.1323	4.0437	3.2849	2.9559	2.5154	2.1750	1.8238	1.3278	(-1) 8.9637	(-1) 5.7448	(-1) 3.6466	(-1) 2.6993	(-1) 2.0389	(-1) 1.4349	(-1) 1.1109	(-2) 6.2574	(-2) 2.8277	120
∞	4.6427	3.7430	3.0913	2.8020	2.4082	2.0986	1.7741	1.3068	(-1) 8.9135	(-1) 5.7577	(-1) 3.6735	(-1) 2.7257	(-1) 2.0622	(-1) 1.4535	(-1) 1.1262	(-2) 6.3511	(-2) 2.8725	∞

$\nu_1 = 6$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 7$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5
1	(+7) 5.9287	(+5) 5.9287	(+4) 2.3715	(+3) 5.9283	(+2) 9.4822	(+2) 2.3677	(+1) 5.8906	9.1021	1.9774
1.2	(6) 3.1755	(4) 6.8413	(3) 4.6790	(3) 1.4735	(2) 3.1963	(2) 1.0038	(1) 3.1314	6.4509	1.7128
1.5	(5) 1.7465	(3) 8.1059	(2) 9.4761	(2) 3.7574	(2) 1.1037	(1) 4.3487	(1) 1.6941	4.6161	1.4908
2	(+3) 9.9994	(+2) 9.9936	(+2) 1.9936	(+1) 9.9356	(+1) 3.9355	(+1) 1.9353	9.3491	3.3352	1.3045
3	(+2) 6.1388	(+2) 1.3158	(+1) 4.4434	(+1) 2.7672	(+1) 1.4624	8.8868	5.2662	2.4302	1.1482
4	(+2) 1.5931	(+1) 4.9658	(+1) 2.1622	(+1) 1.4976	9.0741	6.0942	3.9790	2.0790	1.0797
5	(+1) 7.2611	(+1) 2.8163	(+1) 1.4200	(+1) 1.0456	6.8531	4.8759	3.3679	1.8935	1.0414
6	(+1) 4.3566	(+1) 1.9463	(+1) 1.0786	8.2600	5.6955	4.2066	3.0145	1.7789	1.0169
7	(+1) 3.0477	(+1) 1.5019	8.8854	6.9928	4.9949	3.7870	2.7849	1.7011	1.0000
8	(+1) 2.3421	(+1) 1.2398	7.6942	6.1776	4.5286	3.5005	2.6241	1.6448	(-1) 9.8757
9	(+1) 1.9140	(+1) 1.0698	6.8849	5.6129	4.1971	3.2927	2.5053	1.6022	(-1) 9.7805
10	(+1) 1.6319	9.5175	6.3025	5.2001	3.9498	3.1355	2.4140	1.5688	(-1) 9.7054
12	(+1) 1.2892	8.0009	5.5245	4.6395	3.6065	2.9134	2.2828	1.5197	(-1) 9.5943
15	(+1) 1.0231	6.7408	4.8473	4.1415	3.2934	2.7066	2.1582	1.4718	(-1) 9.4850
20	8.1577	5.6920	4.2569	3.6987	3.0074	2.5140	2.0397	1.4252	(-1) 9.3776
24	7.2980	5.2349	3.9905	3.4959	2.8738	2.4226	1.9826	1.4022	(-1) 9.3245
30	6.5375	4.8173	3.7416	3.3045	2.7460	2.3343	1.9269	1.3795	(-1) 9.2719
40	5.8640	4.4355	3.5088	3.1238	2.6238	2.2490	1.8725	1.3571	(-1) 9.2197
60	5.2672	4.0864	3.2911	2.9530	2.5068	2.1665	1.8194	1.3349	(-1) 9.1679
120	4.7380	3.7670	3.0874	2.7918	2.3948	2.0867	1.7675	1.3128	(-1) 9.1164
∞	4.2682	3.4746	2.8968	2.6393	2.2875	2.0096	1.7167	1.2910	(-1) 9.0654

p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
(-1) 6.3565	(-1) 2.7860	(-1) 1.7885	(-1) 1.2387	(-2) 8.1659	(-2) 6.1592	(-2) 3.4194	(-2) 1.6086	1
(-1) 6.1560	(-1) 2.8523	(-1) 1.8702	(-1) 1.3130	(-2) 8.7554	(-2) 6.6435	(-2) 3.7203	(-2) 1.7609	1.2
(-1) 5.9944	(-1) 2.9432	(-1) 1.9741	(-1) 1.4064	(-2) 9.4990	(-2) 7.2556	(-2) 4.1029	(-2) 1.9557	1.5
(-1) 5.8789	(-1) 3.0699	(-1) 2.1109	(-1) 1.5287	(-1) 1.0475	(-2) 8.0619	(-2) 4.6106	(-2) 2.2157	2
(-1) 5.8245	(-1) 3.2530	(-1) 2.3005	(-1) 1.6979	(-1) 1.1832	(-2) 9.1895	(-2) 5.3270	(-2) 2.5856	3
(-1) 5.8285	(-1) 3.3778	(-1) 2.4270	(-1) 1.8107	(-1) 1.2744	(-2) 9.9502	(-2) 5.8146	(-2) 2.8391	4
(-1) 5.8442	(-1) 3.4682	(-1) 2.5179	(-1) 1.8921	(-1) 1.3404	(-1) 1.0502	(-2) 6.1706	(-2) 3.0251	5
(-1) 5.8620	(-1) 3.5368	(-1) 2.5867	(-1) 1.9537	(-1) 1.3905	(-1) 1.0923	(-2) 6.4430	(-2) 3.1679	6
(-1) 5.8785	(-1) 3.5908	(-1) 2.6406	(-1) 2.0020	(-1) 1.4300	(-1) 1.1254	(-2) 6.6584	(-2) 3.2812	7
(-1) 5.8931	(-1) 3.6342	(-1) 2.6841	(-1) 2.0411	(-1) 1.4620	(-1) 1.1523	(-2) 6.8334	(-2) 3.3733	8
(-1) 5.9063	(-1) 3.6701	(-1) 2.7198	(-1) 2.0733	(-1) 1.4884	(-1) 1.1746	(-2) 6.9784	(-2) 3.4498	9
(-1) 5.9179	(-1) 3.7003	(-1) 2.7499	(-1) 2.1004	(-1) 1.5106	(-1) 1.1933	(-2) 7.1006	(-2) 3.5143	10
(-1) 5.9372	(-1) 3.7480	(-1) 2.7974	(-1) 2.1433	(-1) 1.5458	(-1) 1.2230	(-2) 7.2954	(-2) 3.6174	12
(-1) 5.9591	(-1) 3.7991	(-1) 2.8484	(-1) 2.1892	(-1) 1.5837	(-1) 1.2551	(-2) 7.5054	(-2) 3.7287	15
(-1) 5.9837	(-1) 3.8540	(-1) 2.9032	(-1) 2.2388	(-1) 1.6246	(-1) 1.2897	(-2) 7.7330	(-2) 3.8495	20
(-1) 5.9970	(-1) 3.8830	(-1) 2.9321	(-1) 2.2650	(-1) 1.6463	(-1) 1.3080	(-2) 7.8541	(-2) 3.9139	24
(-1) 6.0114	(-1) 3.9131	(-1) 2.9623	(-1) 2.2923	(-1) 1.6689	(-1) 1.3272	(-2) 7.9806	(-2) 3.9813	30
(-1) 6.0266	(-1) 3.9446	(-1) 2.9937	(-1) 2.3208	(-1) 1.6925	(-1) 1.3473	(-2) 8.1129	(-2) 4.0518	40
(-1) 6.0430	(-1) 3.9774	(-1) 3.0264	(-1) 2.3505	(-1) 1.7172	(-1) 1.3682	(-2) 8.2516	(-2) 4.1258	60
(-1) 6.0599	(-1) 4.0116	(-1) 3.0605	(-1) 2.3816	(-1) 1.7430	(-1) 1.3902	(-2) 8.3970	(-2) 4.2035	120
(-1) 6.0783	(-1) 4.0473	(-1) 3.0962	(-1) 2.4141	(-1) 1.7701	(-1) 1.4132	(-2) 8.5499	(-2) 4.2852	∞

$\nu_1 = 7$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 8$

ν_2	p=0.0001	p=0.001	p=0.005	p=0.01	p=0.025	p=0.05	p=0.1	p=0.25	p=0.5	p=0.75	p=0.9	p=0.95	p=0.975	p=0.99	p=0.995	p=0.999	p=0.9999	ν_2
1	(+7) 5.9814	(+5) 5.9814	(+4) 2.3925	(+3) 5.9816	(+2) 9.5666	(+2) 2.3888	(+1) 5.9439	9.1922	2.0041	(-1) 6.5003	(-1) 2.8919	(-1) 1.8805	(-1) 1.3208	(-2) 8.8818	(-2) 6.8083	(-2) 3.9347	(-2) 1.9726	1
1.2	(6) 3.1977	(4) 6.8891	(3) 4.7117	(3) 1.4838	(2) 3.2187	(2) 1.0109	(1) 3.1544	6.5069	1.7355	(-1) 6.3014	(-1) 2.9668	(-1) 1.9717	(-1) 1.4045	(-2) 9.5596	(-2) 7.3741	(-2) 4.3021	(-2) 2.1719	1.2
1.5	(5) 1.7539	(3) 8.1403	(2) 9.5165	(2) 3.7736	(2) 1.1086	(1) 4.3685	(1) 1.7027	4.6492	1.5103	(-1) 6.1436	(-1) 3.0693	(-1) 2.0882	(-1) 1.5104	(-1) 1.0420	(-2) 8.0954	(-2) 4.7737	(-2) 2.4293	1.5
2	(+3) 9.9994	(+2) 9.9937	(+2) 1.9937	(+1) 9.9374	(+1) 3.9373	(+1) 1.9371	9.3668	3.3526	1.3213	(-1) 6.0354	(-1) 3.2122	(-1) 2.2427	(-1) 1.6503	(-1) 1.1562	(-2) 9.0563	(-2) 5.4072	(-2) 2.7778	2
3	(+2) 6.0929	(+2) 1.3062	(+1) 4.4126	(+1) 2.7489	(+1) 1.4540	8.8452	5.2517	2.4364	1.1627	(-1) 5.9941	(-1) 3.4202	(-1) 2.4593	(-1) 1.8464	(-1) 1.3173	(-1) 1.0420	(-2) 6.3173	(-2) 3.2834	3
4	(+2) 1.5711	(+1) 4.8996	(+1) 2.1352	(+1) 1.4799	8.9796	6.0410	3.9549	2.0805	1.0933	(-1) 6.0089	(-1) 3.5633	(-1) 2.6057	(-1) 1.9792	(-1) 1.4273	(-1) 1.1357	(-2) 6.9485	(-2) 3.6373	4
5	(+1) 7.1226	(+1) 2.7649	(+1) 1.3961	(+1) 1.0289	6.7572	4.8183	3.3393	1.8923	1.0545	(-1) 6.0332	(-1) 3.6677	(-1) 2.7119	(-1) 2.0759	(-1) 1.5079	(-1) 1.2046	(-2) 7.4158	(-2) 3.9009	5
6	(+1) 4.2541	(+1) 1.9030	(+1) 1.0566	8.1016	5.5996	4.1468	2.9830	1.7760	1.0298	(-1) 6.0577	(-1) 3.7477	(-1) 2.7928	(-1) 2.1498	(-1) 1.5697	(-1) 1.2575	(-2) 7.7773	(-2) 4.1057	6
7	(+1) 2.9644	(+1) 1.4634	8.6781	6.8401	4.8994	3.7257	2.7516	1.6969	1.0126	(-1) 6.0798	(-1) 3.8108	(-1) 2.8567	(-1) 2.2082	(-1) 1.6188	(-1) 1.2997	(-2) 8.0658	(-2) 4.2697	7
8	(+1) 2.2706	(+1) 1.2046	7.4960	6.0289	4.4332	3.4381	2.5893	1.6396	1.0000	(-1) 6.0990	(-1) 3.8620	(-1) 2.9086	(-1) 2.2557	(-1) 1.6587	(-1) 1.3340	(-2) 8.3019	(-2) 4.4042	8
9	(+1) 1.8503	(+1) 1.0368	6.6933	5.4671	4.1020	3.2296	2.4694	1.5961	(-1) 9.9037	(-1) 6.1162	(-1) 3.9044	(-1) 2.9515	(-1) 2.2951	(-1) 1.6919	(-1) 1.3627	(-2) 8.4987	(-2) 4.5166	9
10	(+1) 1.5736	9.2042	6.1159	5.0567	3.8549	3.0717	2.3772	1.5621	(-1) 9.8276	(-1) 6.1312	(-1) 3.9401	(-1) 2.9876	(-1) 2.3282	(-1) 1.7199	(-1) 1.3868	(-2) 8.6654	(-2) 4.6120	10
12	(+1) 1.2381	7.7104	5.3451	4.4994	3.5118	2.8486	2.2446	1.5120	(-1) 9.7152	(-1) 6.1561	(-1) 3.9968	(-1) 3.0451	(-1) 2.3811	(-1) 1.7647	(-1) 1.4255	(-2) 8.9330	(-2) 4.7653	12
15	9.7796	6.4707	4.6743	4.0045	3.1987	2.6408	2.1185	1.4631	(-1) 9.6046	(-1) 6.1843	(-1) 4.0581	(-1) 3.1071	(-1) 2.4383	(-1) 1.8132	(-1) 1.4675	(-2) 9.2240	(-2) 4.9325	15
20	7.7573	5.4400	4.0900	3.5644	2.9128	2.4471	1.9985	1.4153	(-1) 9.4959	(-1) 6.2158	(-1) 4.1244	(-1) 3.1743	(-1) 2.5003	(-1) 1.8660	(-1) 1.5133	(-2) 9.5423	(-2) 5.1159	20
24	6.9201	4.9912	3.8264	3.3629	2.7791	2.3551	1.9407	1.3918	(-1) 9.4422	(-1) 6.2332	(-1) 4.1596	(-1) 3.2101	(-1) 2.5334	(-1) 1.8942	(-1) 1.5378	(-2) 9.7131	(-2) 5.2145	24
30	6.1802	4.5814	3.5801	3.1726	2.6513	2.2662	1.8841	1.3685	(-1) 9.3889	(-1) 6.2516	(-1) 4.1964	(-1) 3.2474	(-1) 2.5681	(-1) 1.9238	(-1) 1.5635	(-2) 9.8925	(-2) 5.3182	30
40	5.5257	4.2070	3.3498	2.9930	2.5289	2.1802	1.8289	1.3455	(-1) 9.3361	(-1) 6.2716	(-1) 4.2348	(-1) 3.2864	(-1) 2.6043	(-1) 1.9548	(-1) 1.5905	(-1) 1.0081	(-2) 5.4275	40
60	4.9465	3.8648	3.1344	2.8233	2.4117	2.0970	1.7748	1.3226	(-1) 9.2838	(-1) 6.2925	(-1) 4.2751	(-1) 3.3275	(-1) 2.6424	(-1) 1.9874	(-1) 1.6189	(-1) 1.0280	(-2) 5.5430	60
120	4.4333	3.5519	2.9330	2.6629	2.2994	2.0164	1.7220	1.2999	(-1) 9.2318	(-1) 6.3147	(-1) 4.3174	(-1) 3.3705	(-1) 2.6825	(-1) 2.0218	(-1) 1.6488	(-1) 1.0491	(-2) 5.6652	120
∞	3.9785	3.2656	2.7444	2.5113	2.1918	1.9384	1.6702	1.2774	(-1) 9.1802	(-1) 6.3383	(-1) 4.3619	(-1) 3.4158	(-1) 2.7246	(-1) 2.0581	(-1) 1.6805	(-1) 1.0714	(-2) 5.7947	∞

$\nu_1 = 8$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 9$

ν_2	p=0.0001	p=0.001	p=0.005	p=0.01	p=0.025	p=0.05	p=0.1	p=0.25	p=0.5	p=0.75	p=0.9	p=0.95	p=0.975	p=0.99	p=0.995	p=0.999	p=0.9999	ν_2
1	(+7) 6.0228	(+5) 6.0228	(+4) 2.4091	(+3) 6.0225	(+2) 9.6328	(+2) 2.4054	(+1) 5.9858	9.2631	2.0250	(-1) 6.6133	(-1) 2.9759	(-1) 1.9541	(-1) 1.3871	(-2) 9.4688	(-2) 7.3454	(-2) 4.3750	(-2) 2.3001	1
1.2	(6) 3.2151	(4) 6.9266	(3) 4.7374	(3) 1.4919	(2) 3.2363	(2) 1.0165	(1) 3.1725	6.5509	1.7533	(-1) 6.4159	(-1) 3.0579	(-1) 2.0531	(-1) 1.4785	(-1) 1.0221	(-2) 7.9818	(-2) 4.8018	(-2) 2.5438	1.2
1.5	(5) 1.7597	(3) 8.1674	(2) 9.5482	(2) 3.7862	(2) 1.1123	(1) 4.3841	(1) 1.7094	4.6752	1.5255	(-1) 6.2613	(-1) 3.1697	(-1) 2.1801	(-1) 1.5950	(-1) 1.1182	(-2) 8.7980	(-2) 5.3537	(-2) 2.8612	1.5
2	(+3) 9.9994	(+2) 9.9939	(+2) 1.9939	(+1) 9.9388	(+1) 3.9387	(+1) 1.9385	9.3805	3.3661	1.3344	(-1) 6.1592	(-1) 3.3261	(-1) 2.3493	(-1) 1.7499	(-1) 1.2466	(-2) 9.8941	(-2) 6.1024	(-2) 3.2958	2
3	(+2) 6.0567	(+2) 1.2986	(+1) 4.3882	(+1) 2.7345	(+1) 1.4473	8.8123	5.2400	2.4410	1.1741	(-1) 6.1293	(-1) 3.5550	(-1) 2.5889	(-1) 1.9692	(-1) 1.4302	(-1) 1.1472	(-2) 7.1933	(-2) 3.9364	3
4	(+2) 1.5538	(+1) 4.8475	(+1) 2.1139	(+1) 1.4659	8.9047	5.9988	3.9357	2.0814	1.1040	(-1) 6.1527	(-1) 3.7137	(-1) 2.7525	(-1) 2.1195	(-1) 1.5571	(-1) 1.2569	(-2) 7.9616	(-2) 4.3925	4
5	(+1) 7.0133	(+1) 2.7244	(+1) 1.3772	(+1) 1.0158	6.6810	4.7725	3.3163	1.8911	1.0648	(-1) 6.1843	(-1) 3.8305	(-1) 2.8722	(-1) 2.2300	(-1) 1.6510	(-1) 1.3385	(-2) 8.5370	(-2) 4.7366	5
6	(+1) 4.1732	(+1) 1.8688	(+1) 1.0391	7.9761	5.5234	4.0990	2.9577	1.7733	1.0398	(-1) 6.2147	(-1) 3.9202	(-1) 2.9640	(-1) 2.3150	(-1) 1.7236	(-1) 1.4018	(-2) 8.9863	(-2) 5.0065	6
7	(+1) 2.8987	(+1) 1.4330	8.5138	6.7188	4.8232	3.6767	2.7247	1.6931	1.0224	(-1) 6.2414	(-1) 3.9915	(-1) 3.0370	(-1) 2.3826	(-1) 1.7816	(-1) 1.4525	(-2) 9.3476	(-2) 5.2246	7
8	(+1) 2.2141	(+1) 1.1767	7.3386	5.9106	4.3572	3.3881	2.5612	1.6350	1.0097	(-1) 6.2653	(-1) 4.0496	(-1) 3.0964	(-1) 2.4378	(-1) 1.8291	(-1) 1.4940	(-2) 9.6451	(-2) 5.4046	8
9	(+1) 1.7999	(+1) 1.0107	6.5411	5.3511	4.0260	3.1789	2.4403	1.5909	1.0000	(-1) 6.2858	(-1) 4.0979	(-1) 3.1457	(-1) 2.4839	(-1) 1.8688	(-1) 1.5288	(-2) 9.8945	(-2) 5.5560	9
10	(+1) 1.5275	8.9558	5.9676	4.9424	3.7790	3.0204	2.3473	1.5563	(-1) 9.9232	(-1) 6.3040	(-1) 4.1386	(-1) 3.1875	(-1) 2.5228	(-1) 1.9024	(-1) 1.5583	(-1) 1.0107	(-2) 5.6851	10
12	(+1) 1.1976	7.4797	5.2021	4.3875	3.4358	2.7964	2.2135	1.5054	(-1) 9.8097	(-1) 6.3339	(-1) 4.2036	(-1) 3.2543	(-1) 2.5852	(-1) 1.9564	(-1) 1.6058	(-1) 1.0449	(-2) 5.8939	12
15	9.4218	6.2559	4.5364	3.8948	3.1227	2.5876	2.0862	1.4556	(-1) 9.6981	(-1) 6.3674	(-1) 4.2742	(-1) 3.3266	(-1) 2.6529	(-1) 2.0153	(-1) 1.6577	(-1) 1.0825	(-2) 6.1235	15
20	7.4394	5.2392	3.9564	3.4567	2.8365	2.3928	1.9649	1.4069	(-1) 9.5884	(-1) 6.4057	(-1) 4.3510	(-1) 3.4054	(-1) 2.7271	(-1) 2.0799	(-1) 1.7147	(-1) 1.1239	(-2) 6.3776	20
24	6.6197	4.7968	3.6949	3.2560	2.7027	2.3002	1.9063	1.3828	(-1) 9.5342	(-1) 6.4267	(-1) 4.3921	(-1) 3.4477	(-1) 2.7669	(-1) 2.1146	(-1) 1.7454	(-1) 1.1463	(-2) 6.5153	24
30	5.8960	4.3930	3.4505	3.0665	2.5746	2.2107	1.8490	1.3590	(-1) 9.4805	(-1) 6.4491	(-1) 4.4352	(-1) 3.4920	(-1) 2.8087	(-1) 2.1512	(-1) 1.7778	(-1) 1.1699	(-2) 6.6609	30
40	5.2564	4.0243	3.2220	2.8876	2.4519	2.1240	1.7929	1.3354	(-1) 9.4272	(-1) 6.4725	(-1) 4.4803	(-1) 3.5387	(-1) 2.8527	(-1) 2.1898	(-1) 1.8121	(-1) 1.1950	(-2) 6.8155	40
60	4.6907	3.6873	3.0083	2.7185	2.3344	2.0401	1.7380	1.3119	(-1) 9.3743	(-1) 6.4981	(-1) 4.5280	(-1) 3.5878	(-1) 2.8991	(-1) 2.2306	(-1) 1.8483	(-1) 1.2215	(-2) 6.9798	60
120	4.1901	3.3793	2.8083	2.5586	2.2217	1.9588	1.6843	1.2886	(-1) 9.3218	(-1) 6.5253	(-1) 4.5781	(-1) 3.6397	(-1) 2.9483	(-1) 2.2739	(-1) 1.8868	(-1) 1.2498	(-2) 7.1550	120
∞	3.7467	3.0975	2.6210	2.4073	2.1136	1.8799	1.6315	1.2654	(-1) 9.2698	(-1) 6.5544	(-1) 4.6313	(-1) 3.6945	(-1) 3.0004	(-1) 2.3199	(-1) 1.9277	(-1) 1.2799	(-2) 7.3423	∞

$\nu_1 = 9$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 10$

ν_2	p=0.0001	p=0.001	p=0.005	p=0.01	p=0.025	p=0.05	p=0.1	p=0.25	p=0.5	p=0.75	p=0.9	p=0.95	p=0.975	p=0.99	p=0.995	p=0.999	p=0.9999	ν_2
1	(+7) 6.0562	(+5) 6.0562	(+4) 2.4224	(+3) 6.0558	(+2) 9.6863	(+2) 2.4188	(+1) 6.0195	9.3202	2.0419	(-1) 6.7047	(-1) 3.0441	(-1) 2.0143	(-1) 1.4416	(-2) 9.9562	(-2) 7.7967	(-2) 4.7529	(-2) 2.5922	1
1.2	(6) 3.2291	(5) 6.9569	(3) 4.7581	(3) 1.4984	(2) 3.2506	(2) 1.0210	(1) 3.1870	6.5863	1.7677	(-1) 6.5084	(-1) 3.1319	(-1) 2.1198	(-1) 1.5397	(-1) 1.0772	(-2) 8.4934	(-2) 5.2325	(-2) 2.8772	1.2
1.5	(5) 1.7644	(3) 8.1893	(2) 9.5739	(2) 3.7964	(2) 1.1154	(1) 4.3967	(1) 1.7148	4.6961	1.5378	(-1) 6.3565	(-1) 3.2516	(-1) 2.2555	(-1) 1.6651	(-1) 1.1819	(-2) 9.3921	(-2) 5.8562	(-2) 3.2508	1.5
2	(+3) 9.9994	(+2) 9.9940	(+2) 1.9940	(+1) 9.9399	(+1) 3.9398	(+1) 1.9396	9.3916	3.3770	1.3450	(-1) 6.2598	(-1) 3.4194	(-1) 2.4374	(-1) 1.8327	(-1) 1.3229	(-1) 1.0608	(-2) 6.7090	(-2) 3.7668	2
3	(+2) 6.0275	(+2) 1.2925	(+1) 4.3686	(+1) 2.7229	(+1) 1.4419	8.7855	5.2304	2.4447	1.1833	(-1) 6.2391	(-1) 3.6661	(-1) 2.6967	(-1) 2.0723	(-1) 1.5262	(-1) 1.2375	(-2) 7.9664	(-2) 4.5377	3
4	(+2) 1.5398	(+1) 4.8053	(+1) 2.0967	(+1) 1.4546	8.8439	5.9644	3.9199	2.0820	1.1126	(-1) 6.2700	(-1) 3.8383	(-1) 2.8752	(-1) 2.2380	(-1) 1.6683	(-1) 1.3619	(-2) 8.8630	(-2) 5.0942	4
5	(+1) 6.9250	(+1) 2.6917	(+1) 1.3618	(+1) 1.0051	6.6192	4.7351	3.2974	1.8899	1.0730	(-1) 6.3080	(-1) 3.9657	(-1) 3.0068	(-1) 2.3607	(-1) 1.7742	(-1) 1.4551	(-1) 9.5413	(-2) 5.5187	5
6	(+1) 4.1077	(+1) 1.8411	(+1) 1.0250	7.8741	5.4613	4.0600	2.9369	1.7708	1.0478	(-1) 6.3432	(-1) 4.0640	(-1) 3.1083	(-1) 2.4557	(-1) 1.8567	(-1) 1.5280	(-1) 1.0075	(-2) 5.8546	6
7	(+1)		8.3803	6.6201	4.7611	3.6365	2.7025	1.6898	1.0304	(-1) 6.3743	(-1) 4.1425	(-1) 3.1893	(-1) 2.5318	(-1) 1.9230	(-1) 1.5867	(-1) 1.0507	(-2) 6.1279	7
8	(+1) 2.1683	(+1) 1.1540	7.2107	5.8143	4.2951	3.3472	2.5380	1.6310	1.0175	(-1) 6.4016	(-1) 4.2066	(-1) 3.2555	(-1) 2.5941	(-1) 1.9776	(-1) 1.6351	(-1) 1.0865	(-2) 6.3549	8
9	(+1) 1.7590	9.8943	6.4171	5.2565	3.9639	3.1373	2.4163	1.5863	1.0077	(-1) 6.4255	(-1) 4.2602	(-1) 3.3108	(-1) 2.6462	(-1) 2.0233	(-1) 1.6757	(-1) 1.1166	(-2) 6.5467	9
10	(+1) 1.4901	8.7539	5.8467	4.8492	3.7168	2.9782	2.3226	1.5513	1.0000	(-1) 6.4462	(-1) 4.3055	(-1) 3.3577	(-1) 2.6905	(-1) 2.0622	(-1) 1.7104	(-1) 1.1424	(-2) 6.7111	10
12	(+1) 1.1647	7.2920	5.0855	4.2961	3.3736	2.7534	2.1878	1.4996	(-1) 9.8856	(-1) 6.4809	(-1) 4.3781	(-1) 3.4329	(-1) 2.7617	(-1) 2.1250	(-1) 1.7664	(-1) 1.1841	(-2) 6.9783	12
15	9.1309	6.0808	4.4236	3.8049	3.0602	2.5437	2.0593	1.4491	(-1) 9.7732	(-1) 6.5198	(-1) 4.4573	(-1) 3.5149	(-1) 2.8395	(-1) 2.1938	(-1) 1.8279	(-1) 1.2302	(-2) 7.2744	15
20	7.1805	5.0752	3.8470	3.3682	2.7737	2.3479	1.9367	1.3995	(-1) 9.6626	(-1) 6.5638	(-1) 4.5440	(-1) 3.6049	(-1) 2.9252	(-1) 2.2699	(-1) 1.8961	(-1) 1.2814	(-2) 7.6049	20
24	6.3750	4.6379	3.5870	3.1681	2.6396	2.2547	1.8775	1.3750	(-1) 9.6081	(-1) 6.5880	(-1) 4.5905	(-1) 3.6534	(-1) 2.9714	(-1) 2.3111	(-1) 1.9330	(-1) 1.3093	(-2) 7.7852	24
30	5.6641	4.2388	3.3440	2.9791	2.5112	2.1646	1.8195	1.3507	(-1) 9.5540	(-1) 6.6142	(-1) 4.6395	(-1) 3.7043	(-1) 3.0202	(-1) 2.3547	(-1) 1.9722	(-1) 1.3389	(-2) 7.9770	30
40	5.0363	3.8744	3.1167	2.8005	2.3882	2.0772	1.7627	1.3266	(-1) 9.5003	(-1) 6.6419	(-1) 4.6911	(-1) 3.7581	(-1) 3.0718	(-1) 2.4008	(-1) 2.0137	(-1) 1.3704	(-2) 8.1816	40
60	4.4815	3.5415	2.9042	2.6318	2.2702	1.9926	1.7070	1.3026	(-1) 9.4471	(-1) 6.6711	(-1) 4.7456	(-1) 3.8152	(-1) 3.1266	(-1) 2.4498	(-1) 2.0580	(-1) 1.4040	(-2) 8.4007	60
120	3.9907	3.2372	2.7052	2.4721	2.1570	1.9105	1.6524	1.2787	(-1) 9.3943	(-1) 6.7029	(-1) 4.8035	(-1) 3.8758	(-1) 3.1848	(-1) 2.5022	(-1) 2.1052	(-1) 1.4400	(-2) 8.6358	120
∞	3.5564	2.9588	2.5188	2.3209	2.0483	1.8307	1.5987	1.2549	(-1) 9.3418	(-1) 6.7372	(-1) 4.8652	(-1) 3.9403	(-1) 3.2470	(-1) 2.5582	(-1) 2.1559	(-1) 1.4787	(-2) 8.8890	∞

$\nu_1 = 10$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 12$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5	p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
1	(+7) 6.1067	(+5) 6.1067	(+4) 2.4426	(+3) 6.1063	(+2) 9.7671	(+2) 2.4391	(+1) 6.0705	9.4064	2.0674	(-1) 6.8432	(-1) 3.1481	(-1) 2.1065	(-1) 1.5258	(-1) 1.0718	(-2) 8.5077	(-2) 5.3638	(-2) 3.0838	1
1.2	(6) 3.2504	(4) 7.0027	(3) 4.7894	(3) 1.5083	(2) 3.2721	(2) 1.0278	(1) 3.2090	6.6399	1.7894	(-1) 6.6485	(-1) 3.2449	(-1) 2.2223	(-1) 1.6344	(-1) 1.1636	(-2) 9.3036	(-2) 5.9318	(-2) 3.4412	1.2
1.5	(5) 1.7715	(3) 8.2223	(2) 9.6126	(2) 3.8119	(2) 1.1200	(1) 4.4157	(1) 1.7230	4.7276	1.5563	(-1) 6.5009	(-1) 3.3771	(-1) 2.3719	(-1) 1.7741	(-1) 1.2824	(-1) 1.0338	(-2) 6.6769	(-2) 3.9143	1.5
2	(+3) 9.9994	(+2) 9.9942	(+2) 1.9942	(+1) 9.9416	(+1) 3.9415	(+1) 1.9413	9.4081	3.3934	1.3610	(-1) 6.4123	(-1) 3.5628	(-1) 2.5738	(-1) 1.9624	(-1) 1.4437	(-1) 1.1751	(-2) 7.7080	(-2) 4.5768	2
3	(+2) 5.9833	(+2) 1.2832	(+1) 4.3387	(+1) 2.7052	(+1) 1.4337	8.7446	5.2156	2.4500	1.1972	(-1) 6.4066	(-1) 3.8380	(-1) 2.8651	(-1) 2.2350	(-1) 1.6799	(-1) 1.3839	(-2) 9.2557	(-2) 5.5867	3
4	(+2) 1.5186	(+1) 4.7412	(+1) 2.0705	(+1) 1.4374	8.7512	5.9117	3.8955	2.0826	1.1255	(-1) 6.4504	(-1) 4.0321	(-1) 3.0682	(-1) 2.4265	(-1) 1.8478	(-1) 1.5335	(-1) 1.0381	(-2) 6.3321	4
5	(+1) 6.7908	(+1) 2.6418	(+1) 1.3384	9.8883	6.5246	4.6777	3.2682	1.8877	1.0855	(-1) 6.4981	(-1) 4.1771	(-1) 3.2197	(-1) 2.5700	(-1) 1.9746	(-1) 1.6471	(-2) 1.1246	(-2) 6.9104	5
6	(+1) 4.0081	(+1) 1.7989	(+1) 1.0034	7.7183	5.3662	3.9999	2.9047	1.7668	1.0600	(-1) 6.5419	(-1) 4.2900	(-1) 3.3377	(-1) 2.6822	(-1) 2.0744	(-1) 1.7370	(-1) 1.1935	(-2) 7.3745	6
7	(+1) 2.7644	(+1) 1.3707	8.1764	6.4691	4.6658	3.5747	2.6681	1.6843	1.0423	(-1) 6.5802	(-1) 4.3806	(-1) 3.4324	(-1) 2.7728	(-1) 2.1554	(-1) 1.8101	(-1) 1.2499	(-2) 7.7566	7
8	(+1) 2.0985	(+1) 1.1194	7.0149	5.6668	4.1997	3.2840	2.5020	1.6244	1.0293	(-1) 6.6138	(-1) 4.4551	(-1) 3.5105	(-1) 2.8475	(-1) 2.2225	(-1) 1.8709	(-1) 1.2970	(-2) 8.0771	8
9	(+1) 1.6967	9.5700	6.2274	5.1114	3.8682	3.0729	2.3789	1.5788	1.0194	(-1) 6.6428	(-1) 4.5177	(-1) 3.5760	(-1) 2.9105	(-1) 2.2792	(-1) 1.9223	(-1) 1.3369	(-2) 8.3503	9
10	(+1) 1.4330	8.4452	5.6613	4.7059	3.6209	2.9130	2.2841	1.5430	1.0116	(-1) 6.6684	(-1) 4.5708	(-1) 3.6319	(-1) 2.9642	(-1) 2.3277	(-1) 1.9664	(-1) 1.3714	(-2) 8.5861	10
12	(+1) 1.1144	7.0046	4.9063	4.1553	3.2773	2.6866	2.1474	1.4902	1.0000	(-1) 6.7105	(-1) 4.6568	(-1) 3.7222	(-1) 3.0513	(-1) 2.4066	(-1) 2.0382	(-1) 1.4276	(-2) 8.9732	12
15	8.6859	5.8121	4.2498	3.6662	2.9633	2.4753	2.0171	1.4383	(-1) 9.8863	(-1) 6.7586	(-1) 4.7508	(-1) 3.8213	(-1) 3.1474	(-1) 2.4940	(-1) 2.1180	(-1) 1.4905	(-2) 9.4075	15
20	6.7837	4.8229	3.6779	3.2311	2.6758	2.2776	1.8924	1.3873	(-1) 9.7746	(-1) 6.8129	(-1) 4.8551	(-1) 3.9314	(-1) 3.2544	(-1) 2.5917	(-1) 2.2076	(-1) 1.5613	(-2) 9.8996	20
24	5.9992	4.3929	3.4199	3.0316	2.5412	2.1834	1.8319	1.3621	(-1) 9.7194	(-1) 6.8432	(-1) 4.9116	(-1) 3.9912	(-1) 3.3127	(-1) 2.6452	(-1) 2.2566	(-1) 1.6003	(-1) 1.0171	24
30	5.3075	4.0006	3.1787	2.8431	2.4120	2.0921	1.7727	1.3369	(-1) 9.6647	(-1) 6.8757	(-1) 4.9714	(-1) 4.0547	(-1) 3.3746	(-1) 2.7021	(-1) 2.3090	(-1) 1.6421	(-1) 1.0464	30
40	4.6973	3.6425	2.9531	2.6648	2.2882	2.0035	1.7146	1.3119	(-1) 9.6104	(-1) 6.9104	(-1) 5.0350	(-1) 4.1222	(-1) 3.4408	(-1) 2.7630	(-1) 2.3651	(-1) 1.6870	(-1) 1.0778	40
60	4.1585	3.3153	2.7419	2.4961	2.1692	1.9174	1.6574	1.2870	(-1) 9.5566	(-1) 6.9478	(-1) 5.1028	(-1) 4.1943	(-1) 3.5115	(-1) 2.8285	(-1) 2.4255	(-1) 1.7354	(-1) 1.1119	60
120	3.6823	3.0161	2.5439	2.3363	2.0548	1.8337	1.6012	1.2621	(-1) 9.5032	(-1) 6.9881	(-1) 5.1752	(-1) 4.2717	(-1) 3.5876	(-1) 2.8991	(-1) 2.4907	(-1) 1.7879	(-1) 1.1490	120
∞	3.2612	2.7425	2.3583	2.1848	1.9447	1.7522	1.5458	1.2371	(-1) 9.4503	(-1) 7.0319	(-1) 5.2532	(-1) 4.3550	(-1) 3.6699	(-1) 2.9755	(-1) 2.5615	(-1) 1.8452	(-1) 1.1896	∞

$\nu_1 = 12$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 15$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5
1	(+7) 6.1576	(+5) 6.1576	(+4) 2.4630	(+3) 6.1573	(+2) 9.8487	(+2) 2.4595	(+1) 6.1220	9.4934	2.0931
1.2	(6) 3.2718	(4) 7.0489	(3) 4.8210	(3) 1.5183	(2) 3.2938	(2) 1.0347	(1) 3.2312	6.6939	1.8112
1.5	(5) 1.7787	(3) 8.2557	(2) 9.6518	(2) 3.8275	(2) 1.1247	(1) 4.4349	(1) 1.7313	4.7594	1.5750
2	(+3) 9.9994	(+2) 9.9943	(+2) 1.9943	(+1) 9.9432	(+1) 3.9431	(+1) 1.9429	9.4247	3.4098	1.3771
3	(+2) 5.9384	(+2) 1.2737	(+1) 4.3085	(+1) 2.6872	(+1) 1.4253	8.7029	5.2003	2.4552	1.2111
4	(+2) 1.4971	(+1) 4.6761	(+1) 2.0438	(+1) 1.4198	8.6565	5.8578	3.8689	2.0829	1.1386
5	(+1) 6.6544	(+1) 2.5911	(+1) 1.3146	9.7222	6.4277	4.6188	3.2380	1.8851	1.0980
6	(+1) 3.9068	(+1) 1.7559	9.8140	7.5590	5.2687	3.9381	2.8712	1.7621	1.0722
7	(+1) 2.6819	(+1) 1.3324	7.9678	6.3143	4.5678	3.5108	2.6322	1.6781	1.0543
8	(+1) 2.0274	(+1) 1.0841	6.8143	5.5151	4.1012	3.2184	2.4642	1.6170	1.0412
9	(+1) 1.6331	9.2381	6.0325	4.9621	3.7694	3.0061	2.3396	1.5705	1.0311
10	(+1) 1.3747	8.1288	5.4707	4.5582	3.5217	2.8450	2.2435	1.5338	1.0232
12	(+1) 1.0630	6.7092	4.7214	4.0096	3.1772	2.6169	2.1049	1.4796	1.0115
15	8.2290	5.5351	4.0698	3.5222	2.8621	2.4035	1.9722	1.4263	1.0000
20	6.3748	4.5618	3.5020	3.0880	2.5731	2.2033	1.8449	1.3736	(-1) 9.8870
24	5.6112	4.1387	3.2456	2.8887	2.4374	2.1077	1.7831	1.3474	(-1) 9.8312
30	4.9385	3.7527	3.0057	2.7002	2.3072	2.0148	1.7223	1.3213	(-1) 9.7759
40	4.3455	3.4003	2.7811	2.5216	2.1819	1.9245	1.6624	1.2952	(-1) 9.7211
60	3.8221	3.0781	2.5705	2.3523	2.0613	1.8364	1.6034	1.2691	(-1) 9.6667
120	3.3600	2.7833	2.3727	2.1915	1.9450	1.7505	1.5450	1.2428	(-1) 9.6128
∞	2.9509	2.5132	2.1868	2.0385	1.8326	1.6664	1.4871	1.2163	(-1) 9.5593

p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
(-1) 6.9828	(-1) 3.2539	(-1) 2.2011	(-1) 1.6130	(-1) 1.1517	(-2) 9.2610	(-2) 6.0287	(-2) 3.6432	1
(-1) 6.7903	(-1) 3.3603	(-1) 2.3278	(-1) 1.7328	(-1) 1.2546	(-1) 1.0166	(-2) 6.6968	(-2) 4.0868	1.2
(-1) 6.6473	(-1) 3.5055	(-1) 2.4922	(-1) 1.8878	(-1) 1.3887	(-1) 1.1350	(-2) 7.5809	(-2) 4.6800	1.5
(-1) 6.5673	(-1) 3.7103	(-1) 2.7157	(-1) 2.0986	(-1) 1.5726	(-1) 1.2986	(-2) 8.8190	(-2) 5.5221	2
(-1) 6.5781	(-1) 4.0164	(-1) 3.0419	(-1) 2.4080	(-1) 1.8460	(-1) 1.5442	(-1) 1.0712	(-2) 6.8329	3
(-1) 6.6353	(-1) 4.2348	(-1) 3.2727	(-1) 2.6286	(-1) 2.0437	(-1) 1.7233	(-1) 1.2117	(-2) 7.8228	4
(-1) 6.6943	(-1) 4.3995	(-1) 3.4467	(-1) 2.7961	(-1) 2.1951	(-1) 1.8615	(-1) 1.3215	(-2) 8.6050	5
(-1) 6.7476	(-1) 4.5288	(-1) 3.5836	(-1) 2.9285	(-1) 2.3157	(-1) 1.9721	(-1) 1.4101	(-2) 9.2427	6
(-1) 6.7944	(-1) 4.6335	(-1) 3.6947	(-1) 3.0364	(-1) 2.4146	(-1) 2.0630	(-1) 1.4835	(-2) 9.7743	7
(-1) 6.8348	(-1) 4.7203	(-1) 3.7867	(-1) 3.1263	(-1) 2.4972	(-2) 2.1394	(-1) 1.5454	(-1) 1.0225	8
(-1) 6.8700	(-1) 4.7934	(-1) 3.8646	(-1) 3.2024	(-1) 2.5675	(-1) 2.2044	(-1) 1.5985	(-1) 1.0614	9
(-1) 6.9008	(-1) 4.8560	(-1) 3.9313	(-1) 3.2678	(-1) 2.6282	(-1) 2.2606	(-1) 1.6445	(-1) 1.0952	10
(-1) 6.9527	(-1) 4.9576	(-1) 4.0399	(-1) 3.3746	(-1) 2.7276	(-1) 2.3531	(-1) 1.7206	(-1) 1.1513	12
(-1) 7.0111	(-1) 5.0705	(-1) 4.1606	(-1) 3.4939	(-1) 2.8391	(-1) 2.4571	(-1) 1.8067	(-1) 1.2152	15
(-1) 7.0786	(-1) 5.1967	(-1) 4.2965	(-1) 3.6286	(-1) 2.9657	(-1) 2.5756	(-1) 1.9053	(-1) 1.2890	20
(-1) 7.1164	(-1) 5.2659	(-1) 4.3710	(-1) 3.7029	(-1) 3.0358	(-1) 2.6414	(-1) 1.9604	(-1) 1.3304	24
(-1) 7.1567	(-1) 5.3396	(-1) 4.4508	(-1) 3.7826	(-1) 3.1113	(-1) 2.7125	(-1) 2.0201	(-1) 1.3754	30
(-1) 7.2005	(-1) 5.4189	(-1) 4.5366	(-1) 3.8685	(-1) 3.1929	(-1) 2.7894	(-1) 2.0851	(-1) 1.4246	40
(-1) 7.2485	(-1) 5.5042	(-1) 4.6294	(-1) 3.9617	(-1) 3.2818	(-1) 2.8733	(-1) 2.1562	(-1) 1.4787	60
(-1) 7.3003	(-1) 5.5969	(-1) 4.7301	(-1) 4.0632	(-1) 3.3789	(-1) 2.9654	(-1) 2.2347	(-1) 1.5386	120
(-1) 7.3578	(-1) 5.6977	(-1) 4.8407	(-1) 4.1748	(-1) 3.4863	(-1) 3.0673	(-1) 2.3218	(-1) 1.6055	∞

$\nu_1 = 15$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 20$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5	p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
1	(+7) 6.2091	(+5) 6.2091	(+4) 2.4836	(+3) 6.2087	(+2) 9.9310	(+2) 2.4801	(+1) 6.1740	9.5813	2.1190	(-1) 7.1240	(-1) 3.3617	(-1) 2.2982	(-1) 1.7031	(-1) 1.2352	(-1) 1.0056	(-2) 6.7482	(-2) 4.2736	1
1.2	(6) 3.2935	(4) 7.0956	(3) 4.8530	(3) 1.5283	(2) 3.3157	(2) 1.0417	(1) 3.2536	6.7484	1.8333	(-1) 6.9338	(-1) 3.4780	(-1) 2.4362	(-1) 1.8348	(-1) 1.3502	(-1) 1.1080	(-2) 7.5289	(-2) 4.8187	1.2
1.5	(5) 1.7860	(3) 8.2895	(2) 9.6914	(2) 3.8433	(2) 1.1294	(1) 4.4543	(1) 1.7396	4.7914	1.5939	(-1) 6.7958	(-1) 3.6370	(-1) 2.6164	(-1) 2.0063	(-1) 1.5009	(-1) 1.2429	(-2) 8.5708	(-2) 5.5549	1.5
2	(+3) 9.9995	(+2) 9.9945	(+2) 1.9945	(+1) 9.9449	(+1) 3.9448	(+1) 1.9446	9.4413	3.4263	1.3933	(-1) 6.7249	(-1) 3.8620	(-1) 2.8630	(-1) 2.2415	(-1) 1.7097	(-1) 1.4313	(-1) 1.0048	(-2) 6.6142	2
3	(+2) 5.8930	(+2) 1.2642	(+1) 4.2778	(+1) 2.6690	(+1) 1.4167	8.6602	5.1845	2.4602	1.2252	(-1) 6.7531	(-1) 4.2015	(-1) 3.2275	(-1) 2.5915	(-1) 2.0250	(-1) 1.7189	(-1) 1.2348	(-2) 8.2988	3
4	(+2) 1.4752	(+1) 4.6100	(+1) 2.0167	(+1) 1.4020	8.5599	5.8025	3.8443	2.0828	1.1517	(-1) 6.8250	(-1) 4.4466	(-1) 3.4891	(-1) 2.8452	(-1) 2.2570	(-1) 1.9326	(-1) 1.4092	(-2) 9.6018	4
5	(+1) 6.5157	(+1) 2.5395	(+1) 1.2903	9.5527	6.3285	4.5581	3.2067	1.8820	1.1106	(-1) 6.8966	(-1) 4.6335	(-1) 3.6888	(-1) 3.0403	(-1) 2.4374	(-1) 2.1001	(-1) 1.5479	(-1) 1.0652	5
6	(+1) 3.8036	(+1) 1.7120	9.5888	7.3958	5.1684	3.8742	2.8363	1.7569	1.0845	(-1) 6.9609	(-1) 4.7817	(-1) 3.8476	(-1) 3.1966	(-1) 2.5830	(-1) 2.2361	(-1) 1.6615	(-1) 1.1522	6
7	(+1) 2.5977	(+1) 1.2932	7.7540	6.1554	4.4667	3.4445	2.5947	1.6712	1.0664	(-1) 7.0166	(-1) 4.9027	(-1) 3.9777	(-1) 3.3251	(-1) 2.7037	(-1) 2.3491	(-1) 1.7569	(-1) 1.2258	7
8	(+1) 1.9547	(+1) 1.0480	6.6082	5.3591	3.9995	3.1503	2.4246	1.6088	1.0531	(-1) 7.0656	(-1) 5.0038	(-1) 4.0865	(-1) 3.4331	(-1) 2.8055	(-1) 2.4450	(-1) 1.8382	(-1) 1.2891	8
9	(+1) 1.5680	8.8976	5.8318	4.8080	3.6669	2.9365	2.2983	1.5611	1.0429	(-1) 7.1078	(-1) 5.0893	(-1) 4.1792	(-1) 3.5255	(-1) 2.8929	(-1) 2.5276	(-1) 1.9087	(-1) 1.3442	9
10	(+1) 1.3150	7.8037	5.2740	4.4054	3.4186	2.7740	2.2007	1.5235	1.0349	(-1) 7.1454	(-1) 5.1634	(-1) 4.2591	(-1) 3.6053	(-1) 2.9689	(-1) 2.5994	(-1) 1.9703	(-1) 1.3927	10
12	(+1) 1.0101	6.4048	4.5299	3.8584	3.0728	2.5436	2.0597	1.4678	1.0231	(-1) 7.2082	(-1) 5.2843	(-1) 4.3906	(-1) 3.7372	(-1) 3.0949	(-1) 2.7189	(-1) 2.0734	(-1) 1.4741	12
15	7.7582	5.2484	3.8826	3.3719	2.7559	2.3275	1.9243	1.4127	1.0114	(-1) 7.2801	(-1) 5.4203	(-1) 4.5386	(-1) 3.8864	(-1) 3.2383	(-1) 2.8555	(-1) 2.1921	(-1) 1.5687	15
20	5.9516	4.2900	3.3178	2.9377	2.4645	2.1242	1.7938	1.3580	1.0000	(-1) 7.3638	(-1) 5.5748	(-1) 4.7077	(-1) 4.0576	(-1) 3.4040	(-1) 3.0140	(-1) 2.3310	(-1) 1.6802	20
24	5.2084	3.8732	3.0624	2.7380	2.3273	2.0267	1.7302	1.3307	(-1) 9.9436	(-1) 7.4107	(-1) 5.6603	(-1) 4.8019	(-1) 4.1535	(-1) 3.4972	(-1) 3.1037	(-1) 2.4100	(-1) 1.7441	24
30	4.5540	3.4928	2.8230	2.5487	2.1952	1.9317	1.6673	1.3033	(-1) 9.8877	(-1) 7.4621	(-1) 5.7531	(-1) 4.9041	(-1) 4.2579	(-1) 3.5991	(-1) 3.2016	(-1) 2.4969	(-1) 1.8147	30
40	3.9772	3.1450	2.5984	2.3689	2.0677	1.8389	1.6052	1.2758	(-1) 9.8323	(-1) 7.5182	(-1) 5.8538	(-1) 5.0155	(-1) 4.3720	(-1) 3.7110	(-1) 3.3096	(-1) 2.5931	(-1) 1.8933	40
60	3.4681	2.8266	2.3872	2.1978	1.9445	1.7480	1.5435	1.2481	(-1) 9.7773	(-1) 7.5798	(-1) 5.9637	(-1) 5.1377	(-1) 4.4976	(-1) 3.8348	(-1) 3.4295	(-1) 2.7005	(-1) 1.9817	60
120	3.0180	2.5344	2.1881	2.0346	1.8249	1.6587	1.4821	1.2200	(-1) 9.7228	(-1) 7.6488	(-1) 6.0853	(-1) 5.2734	(-1) 4.6378	(-1) 3.9733	(-1) 3.5640	(-1) 2.8218	(-1) 2.0820	120
∞	2.6193	2.2657	1.9998	1.8783	1.7085	1.5705	1.4206	1.1914	(-1) 9.6687	(-1) 7.7262	(-1) 6.2212	(-1) 5.4253	(-1) 4.7955	(-1) 4.1302	(-1) 3.7169	(-1) 2.9605	(-1) 2.1976	∞

$\nu_1 = 20$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 24$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5	p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
1	(+7) 6.2350	(+5) 6.2350	(+4) 2.4940	(+3) 6.2346	(+2) 9.9725	(+2) 2.4905	(+1) 6.2002	9.6255	2.1321	(-1) 7.1953	(-1) 3.4164	(-1) 2.3476	(-1) 1.7493	(-1) 1.2783	(-1) 1.0470	(-2) 7.1286	(-2) 4.6161	1
1.2	(6) 3.3044	(4) 7.1192	(3) 4.8691	(3) 1.5334	(2) 3.3267	(2) 1.0452	(1) 3.2649	6.7759	1.8444	(-1) 7.0062	(-1) 3.5378	(-1) 2.4916	(-1) 1.8872	(-1) 1.3996	(-1) 1.1556	(-2) 7.9704	(-2) 5.2180	1.2
1.5	(5) 1.7897	(3) 8.3065	(2) 9.7113	(2) 3.8512	(2) 1.1318	(1) 4.4641	(1) 1.7438	4.8075	1.6034	(-1) 6.8707	(-1) 3.7039	(-1) 2.6799	(-1) 2.0673	(-1) 1.5592	(-1) 1.2994	(-2) 9.0987	(-2) 6.0349	1.5
2	(+3) 9.9995	(+2) 9.9946	(+2) 1.9946	(+1) 9.9458	(+1) 3.9456	(+1) 1.9454	9.4496	3.4345	1.4014	(-1) 6.8050	(-1) 3.9396	(-1) 2.9388	(-1) 2.3155	(-1) 1.7814	(-1) 1.5013	(-1) 1.0707	(-2) 7.2185	2
3	(+2) 5.8700	(+2) 1.2593	(+1) 4.2622	(+1) 2.6598	(+1) 1.4124	8.6385	5.1764	2.4626	1.2322	(-1) 6.8423	(-1) 4.2966	(-1) 3.3236	(-1) 2.6874	(-1) 2.1195	(-1) 1.8119	(-1) 1.3237	(-2) 9.1208	3
4	(+2) 1.4642	(+1) 4.5766	(+1) 2.0030	(+1) 1.3929	8.5109	5.7744	3.8310	2.0827	1.1583	(-1) 6.9219	(-1) 4.5560	(-1) 3.6019	(-1) 2.9591	(-1) 2.3706	(-1) 2.0451	(-1) 1.5176	(-1) 1.0611	4
5	(+1) 6.4455	(+1) 2.5133	(+1) 1.2780	9.4665	6.2780	4.5272	3.1905	1.8802	1.1170	(-1) 7.0004	(-1) 4.7551	(-1) 3.8158	(-1) 3.1698	(-1) 2.5673	(-1) 2.2293	(-1) 1.6732	(-1) 1.1823	5
6	(+1) 3.7512	(+1) 1.6897	9.4741	7.3127	5.1172	3.8415	2.8183	1.7540	1.0907	(-1) 7.0706	(-1) 4.9138	(-1) 3.9869	(-1) 3.3393	(-1) 2.7272	(-1) 2.3799	(-1) 1.8017	(-1) 1.2838	6
7	(+1) 2.5550	(+1) 1.2732	7.6450	6.0743	4.4150	3.4105	2.5753	1.6675	1.0724	(-1) 7.1317	(-1) 5.0439	(-1) 4.1278	(-1) 3.4797	(-1) 2.8605	(-1) 2.5060	(-1) 1.9103	(-1) 1.3702	7
8	(+1) 1.9177	(+1) 1.0295	6.5029	5.2793	3.9472	3.1152	2.4041	1.6043	1.0591	(-1) 7.1849	(-1) 5.1528	(-1) 4.2461	(-1) 3.5983	(-1) 2.9736	(-1) 2.6134	(-1) 2.0035	(-1) 1.4451	8
9	(+1) 1.5349	8.7239	5.7292	4.7290	3.6142	2.9005	2.2768	1.5560	1.0489	(-1) 7.2317	(-1) 5.2458	(-1) 4.3474	(-1) 3.7000	(-1) 3.0713	(-1) 2.7064	(-1) 2.0847	(-1) 1.5106	9
10	(+1) 1.2845	7.6376	5.1732	4.3269	3.3654	2.7372	2.1784	1.5179	1.0408	(-1) 7.2727	(-1) 5.3262	(-1) 4.4352	(-1) 3.7885	(-1) 3.1565	(-1) 2.7878	(-1) 2.1561	(-1) 1.5686	10
12	9.8314	6.2488	4.4315	3.7805	3.0187	2.5055	2.0360	1.4613	1.0289	(-1) 7.3416	(-1) 5.4588	(-1) 4.5800	(-1) 3.9351	(-1) 3.2986	(-1) 2.9241	(-1) 2.2764	(-1) 1.6669	12
15	7.5168	5.1009	3.7859	3.2940	2.7006	2.2878	1.8990	1.4052	1.0172	(-1) 7.4217	(-1) 5.6082	(-1) 4.7445	(-1) 4.1027	(-1) 3.4618	(-1) 3.0811	(-1) 2.4162	(-1) 1.7821	15
20	5.7336	4.1493	3.2220	2.8594	2.4076	2.0825	1.7667	1.3494	1.0057	(-1) 7.5148	(-1) 5.7797	(-1) 4.9341	(-1) 4.2968	(-1) 3.6523	(-1) 3.2654	(-1) 2.5818	(-1) 1.9200	20
24	5.0002	3.7354	2.9667	2.6591	2.2693	1.9838	1.7019	1.3214	1.0000	(-1) 7.5677	(-1) 5.8758	(-1) 5.0408	(-1) 4.4066	(-1) 3.7607	(-1) 3.3707	(-1) 2.6771	(-1) 1.9999	24
30	4.3545	3.3572	2.7272	2.4689	2.1359	1.8874	1.6377	1.2933	(-1) 9.9438	(-1) 7.6260	(-1) 5.9805	(-1) 5.1573	(-1) 4.5269	(-1) 3.8800	(-1) 3.4869	(-1) 2.7828	(-1) 2.0891	30
40	3.7852	3.0111	2.5020	2.2880	2.0069	1.7929	1.5741	1.2649	(-1) 9.8880	(-1) 7.6899	(-1) 6.0950	(-1) 5.2854	(-1) 4.6598	(-1) 4.0124	(-1) 3.6161	(-1) 2.9012	(-1) 2.1897	40
60	3.2825	2.6938	2.2898	2.1154	1.8817	1.7001	1.5107	1.2361	(-1) 9.8328	(-1) 7.7610	(-1) 6.2216	(-1) 5.4277	(-1) 4.8079	(-1) 4.1606	(-1) 3.7615	(-1) 3.0352	(-1) 2.3043	60
120	2.8373	2.4019	2.0890	1.9500	1.7597	1.6084	1.4472	1.2068	(-1) 9.7780	(-1) 7.8407	(-1) 6.3633	(-1) 5.5875	(-1) 4.9754	(-1) 4.3292	(-1) 3.9273	(-1) 3.1890	(-1) 2.4368	120
∞	2.4422	2.1325	1.8983	1.7908	1.6402	1.5173	1.3832	1.1767	(-1) 9.7236	(-1) 7.9321	(-1) 6.5244	(-1) 5.7700	(-1) 5.1672	(-1) 4.5235	(-1) 4.1193	(-1) 3.3687	(-1) 2.5929	∞

$\nu_1 = 24$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 30$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5
1	(+7) 6.2610	(+5) 6.2610	(+4) 2.5044	(+3) 6.2607	(+3) 1.0014	(+2) 2.5009	(+1) 6.2265	9.6698	2.1452
1.2	(6) 3.3154	(4) 7.1428	(3) 4.8852	(3) 1.5385	(2) 3.3378	(2) 1.0487	(1) 3.2763	6.8034	1.8555
1.5	(5) 1.7934	(3) 8.3236	(2) 9.7314	(2) 3.8592	(2) 1.1342	(1) 4.4739	(1) 1.7480	4.8237	1.6129
2	(+3) 9.9995	(+2) 9.9947	(+2) 1.9947	(+1) 9.9466	(+1) 3.9465	(+1) 1.9462	9.4579	3.4428	1.4096
3	(+2) 5.8469	(+2) 1.2545	(+1) 4.2466	(+1) 2.6505	(+1) 1.4081	8.6166	5.1681	2.4650	1.2393
4	(+2) 1.4530	(+1) 4.5429	(+1) 1.9892	(+1) 1.3838	8.4613	5.7459	3.8174	2.0825	1.1649
5	(+1) 6.3746	(+1) 2.4869	(+1) 1.2656	9.3793	6.2269	4.4957	3.1741	1.8784	1.1234
6	(+1) 3.6984	(+1) 1.6672	9.3583	7.2285	5.0652	3.8082	2.8000	1.7510	1.0969
7	(+1) 2.5118	(+1) 1.2530	7.5345	5.9921	4.3624	3.3758	2.5555	1.6635	1.0785
8	(+1) 1.8803	(+1) 1.0109	6.3961	5.1981	3.8940	3.0794	2.3830	1.5996	1.0651
9	(+1) 1.5013	8.5476	5.6248	4.6486	3.5604	2.8637	2.2547	1.5506	1.0548
10	(+1) 1.2536	7.4688	5.0705	4.2469	3.3110	2.6996	2.1554	1.5119	1.0467
12	9.5570	6.0898	4.3309	3.7008	2.9633	2.4663	2.0115	1.4544	1.0347
15	7.2707	4.9502	3.6867	3.2141	2.6437	2.2468	1.8728	1.3973	1.0229
20	5.5105	4.0050	3.1234	2.7785	2.3486	2.0391	1.7382	1.3401	1.0114
24	4.7867	3.5935	2.8679	2.5773	2.2090	1.9390	1.6721	1.3113	1.0057
30	4.1492	3.2171	2.6278	2.3860	2.0739	1.8409	1.6065	1.2823	1.0000
40	3.5868	2.8721	2.4015	2.2034	1.9429	1.7444	1.5411	1.2529	(-1) 9.9440
60	3.0894	2.5549	2.1874	2.0285	1.8152	1.6491	1.4755	1.2229	(-1) 9.8884
120	2.6480	2.2621	1.9839	1.8600	1.6899	1.5543	1.4094	1.1921	(-1) 9.8333
∞	2.2544	1.9901	1.7891	1.6964	1.5660	1.4591	1.3419	1.1600	(-1) 9.7787

p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
(-1) 7.2669	(-1) 3.4714	(-1) 2.3976	(-1) 1.7961	(-1) 1.3223	(-1) 1.0894	(-2) 7.5228	(-2) 4.9771	1
(-1) 7.0790	(-1) 3.5981	(-1) 2.5476	(-1) 1.9405	(-1) 1.4502	(-1) 1.2046	(-2) 8.4290	(-2) 5.6401	1.2
(-1) 6.9461	(-1) 3.7715	(-1) 2.7444	(-1) 2.1295	(-1) 1.6190	(-1) 1.3577	(-2) 9.6487	(-2) 6.5442	1.5
(-1) 6.8852	(-1) 4.0182	(-1) 3.0159	(-1) 2.3911	(-1) 1.8551	(-1) 1.5736	(-1) 1.1398	(-2) 7.8631	2
(-1) 6.9319	(-1) 4.3935	(-1) 3.4220	(-1) 2.7860	(-1) 2.2174	(-1) 1.9088	(-1) 1.4175	(-1) 1.0006	3
(-1) 7.0205	(-1) 4.6681	(-1) 3.7180	(-1) 3.0770	(-1) 2.4889	(-1) 2.1630	(-1) 1.6328	(-1) 1.1705	4
(-1) 7.1058	(-1) 4.8800	(-1) 3.9470	(-1) 3.3041	(-1) 2.7034	(-1) 2.3654	(-1) 1.8070	(-1) 1.3102	5
(-1) 7.1824	(-1) 5.0497	(-1) 4.1314	(-1) 3.4883	(-1) 2.8789	(-1) 2.5322	(-1) 1.9523	(-1) 1.4282	6
(-1) 7.2490	(-1) 5.1897	(-1) 4.2839	(-1) 3.6417	(-1) 3.0262	(-1) 2.6727	(-1) 2.0759	(-1) 1.5296	7
(-1) 7.3073	(-1) 5.3076	(-1) 4.4127	(-1) 3.7717	(-1) 3.1520	(-1) 2.7932	(-1) 2.1827	(-1) 1.6181	8
(-1) 7.3584	(-1) 5.4083	(-1) 4.5235	(-1) 3.8841	(-1) 3.2610	(-1) 2.8981	(-1) 2.2763	(-1) 1.6961	9
(-1) 7.4036	(-1) 5.4960	(-1) 4.6198	(-1) 3.9822	(-1) 3.3567	(-1) 2.9904	(-1) 2.3592	(-1) 1.7655	10
(-1) 7.4800	(-1) 5.6411	(-1) 4.7799	(-1) 4.1459	(-1) 3.5173	(-1) 3.1459	(-1) 2.4996	(-1) 1.8841	12
(-1) 7.5683	(-1) 5.8062	(-1) 4.9633	(-1) 4.3343	(-1) 3.7034	(-1) 3.3270	(-1) 2.6647	(-1) 2.0249	15
(-1) 7.6728	(-1) 5.9977	(-1) 5.1768	(-1) 4.5554	(-1) 3.9236	(-1) 3.5423	(-1) 2.8630	(-1) 2.1959	20
(-1) 7.7322	(-1) 6.1061	(-1) 5.2983	(-1) 4.6819	(-1) 4.0504	(-1) 3.6668	(-1) 2.9787	(-1) 2.2965	24
(-1) 7.7985	(-1) 6.2247	(-1) 5.4321	(-1) 4.8218	(-1) 4.1911	(-1) 3.8055	(-1) 3.1084	(-1) 2.4101	30
(-1) 7.8722	(-1) 6.3565	(-1) 5.5810	(-1) 4.9778	(-1) 4.3493	(-1) 3.9618	(-1) 3.2556	(-1) 2.5400	40
(-1) 7.9548	(-1) 6.5036	(-1) 5.7484	(-1) 5.1546	(-1) 4.5292	(-1) 4.1406	(-1) 3.4251	(-1) 2.6908	60
(-1) 8.0489	(-1) 6.6716	(-1) 5.9400	(-1) 5.3579	(-1) 4.7378	(-1) 4.3484	(-1) 3.6238	(-1) 2.8693	120
(-1) 8.1593	(-1) 6.8662	(-1) 6.1641	(-1) 5.5969	(-1) 4.9845	(-1) 4.5956	(-1) 3.8626	(-1) 3.0860	∞

$\nu_1 = 30$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 40$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5	p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
1	(+7) 6.2871	(+5) 6.2871	(+4) 2.5148	(+3) 6.2868	(+3) 1.0056	(+2) 2.5114	(+1) 6.2529	9.7144	2.1584	(-1) 7.3389	(-1) 3.5268	(-1) 2.4481	(-1) 1.8437	(-1) 1.3672	(-1) 1.1328	(-2) 7.9306	(-2) 5.3566	1
1.2	(6) 3.3264	(4) 7.1666	(3) 4.9015	(3) 1.5436	(2) 3.3490	(2) 1.0522	(1) 3.2877	6.8310	1.8677	(-1) 7.1522	(-1) 3.6589	(-1) 2.6044	(-1) 1.9946	(-1) 1.5019	(-1) 1.2548	(-2) 8.9046	(-2) 6.0850	1.2
1.5	(5) 1.7971	(3) 8.3408	(2) 9.7515	(2) 3.8672	(2) 1.1366	(1) 4.4837	(1) 1.7523	4.8399	1.6225	(-1) 7.0221	(-1) 3.8399	(-1) 2.8099	(-1) 2.1929	(-1) 1.6803	(-1) 1.4176	(-1) 1.0221	(-2) 7.0830	1.5
2	(+3) 9.9995	(+2) 9.9948	(+2) 1.9947	(+1) 9.9474	(+1) 3.9473	(+1) 1.9471	9.4663	3.4511	1.4178	(-1) 6.9662	(-1) 4.0977	(-1) 3.0943	(-1) 2.4685	(-1) 1.9311	(-1) 1.6484	(-1) 1.2120	(-2) 8.5485	2
3	(+2) 5.8236	(+2) 1.2496	(+1) 4.2308	(+1) 2.6411	(+1) 1.4037	8.5944	5.1597	2.4674	1.2464	(-1) 7.0230	(-1) 4.4922	(-1) 3.5227	(-1) 2.8874	(-1) 2.3188	(-1) 2.0097	(-1) 1.5164	(-1) 1.0956	3
4	(+2) 1.4418	(+1) 4.5089	(+1) 1.9752	(+1) 1.3745	8.4111	5.7170	3.8036	2.0821	1.1716	(-1) 7.1200	(-1) 4.7826	(-1) 3.8373	(-1) 3.1989	(-1) 2.6121	(-1) 2.2863	(-1) 1.7550	(-1) 1.2888	4
5	(+1) 6.3031	(+1) 2.4602	(+1) 1.2530	9.2912	6.1751	4.4638	3.1573	1.8763	1.1297	(-1) 7.2134	(-1) 5.0080	(-1) 4.0825	(-1) 3.4439	(-1) 2.8459	(-1) 2.5088	(-1) 1.9500	(-1) 1.4495	5
6	(+1) 3.6450	(+1) 1.6445	9.2408	7.1432	5.0125	3.7743	2.7812	1.7477	1.1031	(-1) 7.2961	(-1) 5.1897	(-1) 4.2810	(-1) 3.6438	(-1) 3.0386	(-1) 2.6933	(-1) 2.1139	(-1) 1.5865	6
7	(+1) 2.4680	(+1) 1.2326	7.4225	5.9084	4.3089	3.3404	2.5351	1.6593	1.0846	(-1) 7.3687	(-1) 5.3405	(-1) 4.4464	(-1) 3.8113	(-1) 3.2012	(-1) 2.8500	(-1) 2.2545	(-1) 1.7053	7
8	(+1) 1.8425	9.9194	6.2875	5.1156	3.8398	3.0428	2.3614	1.5945	1.0711	(-1) 7.4322	(-1) 5.4678	(-1) 4.5867	(-1) 3.9543	(-1) 3.3411	(-1) 2.9853	(-1) 2.3770	(-1) 1.8097	8
9	(+1) 1.4672	8.3685	5.5186	4.5667	3.5055	2.8259	2.2320	1.5450	1.0608	(-1) 7.4884	(-1) 5.5776	(-1) 4.7081	(-1) 4.0785	(-1) 3.4631	(-1) 3.1037	(-1) 2.4849	(-1) 1.9025	9
10	(+1) 1.2222	7.2971	4.9659	4.1653	3.2554	2.6609	2.1317	1.5056	1.0526	(-1) 7.5381	(-1) 5.6731	(-1) 4.8142	(-1) 4.1873	(-1) 3.5708	(-1) 3.2085	(-1) 2.5811	(-1) 1.9856	10
12	9.2778	5.9278	4.2282	3.6192	2.9063	2.4259	1.9861	1.4471	1.0405	(-1) 7.6225	(-1) 5.8323	(-1) 4.9913	(-1) 4.3702	(-1) 3.7526	(-1) 3.3863	(-1) 2.7454	(-1) 2.1289	12
15	7.0197	4.7959	3.5850	3.1319	2.5850	2.2043	1.8454	1.3888	1.0287	(-1) 7.7208	(-1) 6.0154	(-1) 5.1962	(-1) 4.5832	(-1) 3.9657	(-1) 3.5957	(-1) 2.9409	(-1) 2.3012	15
20	5.2817	3.8564	3.0215	2.6947	2.2873	1.9938	1.7083	1.3301	1.0171	(-1) 7.8382	(-1) 6.2298	(-1) 5.4380	(-1) 4.8363	(-1) 4.2214	(-1) 3.8485	(-1) 3.1797	(-1) 2.5143	20
24	4.5669	3.4468	2.7654	2.4923	2.1460	1.8920	1.6407	1.3004	1.0113	(-1) 7.9058	(-1) 6.3528	(-1) 5.5776	(-1) 4.9828	(-1) 4.3706	(-1) 3.9968	(-1) 3.3210	(-1) 2.6418	24
30	3.9370	3.0716	2.5241	2.2992	2.0089	1.7918	1.5732	1.2703	1.0056	(-1) 7.9815	(-1) 6.4889	(-1) 5.7326	(-1) 5.1469	(-1) 4.5384	(-1) 4.1641	(-1) 3.4818	(-1) 2.7880	30
40	3.3804	2.7268	2.2958	2.1142	1.8752	1.6928	1.5056	1.2397	1.0000	(-1) 8.0665	(-1) 6.6419	(-1) 5.9074	(-1) 5.3328	(-1) 4.7299	(-1) 4.3558	(-1) 3.6673	(-1) 2.9582	40
60	2.8870	2.4086	2.0789	1.9360	1.7440	1.5943	1.4373	1.2081	(-1) 9.9441	(-1) 8.1639	(-1) 6.8157	(-1) 6.1076	(-1) 5.5469	(-1) 4.9520	(-1) 4.5792	(-1) 3.8855	(-1) 3.1604	60
120	2.4471	2.1128	1.8709	1.7628	1.6141	1.4952	1.3676	1.1752	(-1) 9.8887	(-1) 8.2781	(-1) 7.0185	(-1) 6.3428	(-1) 5.7998	(-1) 5.2159	(-1) 4.8461	(-1) 4.1489	(-1) 3.4073	120
∞	2.0516	1.8350	1.6691	1.5923	1.4835	1.3940	1.2951	1.1404	(-1) 9.8339	(-1) 8.4154	(-1) 7.2627	(-1) 6.6273	(-1) 6.1084	(-1) 5.5411	(-1) 5.1765	(-1) 4.4791	(-1) 3.7208	∞

$\nu_1 = 40$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 60$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5	p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
1	(+7) 6.3134	(+5) 6.3134	(+4) 2.5253	(+3) 6.3130	(+3) 1.0098	(+2) 2.5220	(+1) 6.2794	9.7591	2.1716	(-1) 7.4113	(-1) 3.5824	(-1) 2.4993	(-1) 1.8919	(-1) 1.4130	(-1) 1.1772	(-2) 8.3521	(-2) 5.7548	1
1.2	(6) 3.3375	(4) 7.1904	(3) 4.9178	(3) 1.5488	(2) 3.3602	(2) 1.0558	(1) 3.2991	6.8588	1.8779	(-1) 7.2258	(-1) 3.7204	(-1) 2.6619	(-1) 2.0497	(-1) 1.5547	(-1) 1.3064	(-2) 9.3973	(-2) 6.5529	1.2
1.5	(5) 1.8008	(3) 8.3581	(2) 9.7718	(2) 3.8753	(2) 1.1390	(1) 4.4937	(1) 1.7565	4.8562	1.6321	(-1) 7.0985	(-1) 3.9090	(-1) 2.8763	(-1) 2.2575	(-1) 1.7430	(-1) 1.4792	(-1) 1.0815	(-2) 7.6516	1.5
2	(+3) 9.9995	(+2) 9.9948	(+2) 1.9948	(+1) 9.9483	(+1) 3.9481	(+1) 1.9479	9.4746	3.4594	1.4261	(-1) 7.0482	(-1) 4.1785	(-1) 3.1742	(-1) 2.5476	(-1) 2.0091	(-1) 1.7256	(-1) 1.2874	(-2) 9.2758	2
3	(+2) 5.8002	(+2) 1.2447	(+1) 4.2149	(+1) 2.6316	(+1) 1.3992	8.5720	5.1512	2.4697	1.2536	(-1) 7.1149	(-1) 4.5926	(-1) 3.6257	(-1) 2.9918	(-1) 2.4237	(-1) 2.1146	(-1) 1.6204	(-1) 1.1972	3
4	(+2) 1.4305	(+1) 4.4746	(+1) 1.9611	(+1) 1.3652	8.3604	5.6878	3.7896	2.0817	1.1782	(-1) 7.2213	(-1) 4.8996	(-1) 3.9601	(-1) 3.3248	(-1) 2.7404	(-1) 2.4155	(-1) 1.8844	(-1) 1.4165	4
5	(+1) 6.2309	(+1) 2.4333	(+1) 1.2402	9.2020	6.1225	4.4314	3.1402	1.8742	1.1361	(-1) 7.3223	(-1) 5.1395	(-1) 4.2224	(-1) 3.5890	(-1) 2.9950	(-1) 2.6596	(-1) 2.1024	(-1) 1.6009	5
6	(+1) 3.5910	(+1) 1.6214	9.1219	7.0568	4.9589	3.7398	2.7620	1.7443	1.1093	(-1) 7.4123	(-1) 5.3342	(-1) 4.4366	(-1) 3.8060	(-1) 3.2065	(-1) 2.8639	(-1) 2.2873	(-1) 1.7596	6
7	(+1) 2.4238	(+1) 1.2119	7.3088	5.8236	4.2544	3.3043	2.5142	1.6548	1.0908	(-1) 7.4912	(-1) 5.4963	(-1) 4.6157	(-1) 3.9891	(-1) 3.3864	(-1) 3.0385	(-1) 2.4471	(-1) 1.8985	7
8	(+1) 1.8041	9.7272	6.1772	5.0316	3.7844	3.0053	2.3391	1.5892	1.0771	(-1) 7.5609	(-1) 5.6344	(-1) 4.7687	(-1) 4.1465	(-1) 3.5420	(-1) 3.1904	(-1) 2.5874	(-1) 2.0216	8
9	(+1) 1.4327	8.1865	5.4104	4.4831	3.4493	2.7872	2.2085	1.5389	1.0667	(-1) 7.6225	(-1) 5.7537	(-1) 4.9017	(-1) 4.2838	(-1) 3.6785	(-1) 3.3241	(-1) 2.7120	(-1) 2.1319	9
10	(+1) 1.1904	7.1224	4.8592	4.0819	3.1984	2.6211	2.1072	1.4990	1.0585	(-1) 7.6770	(-1) 5.8582	(-1) 5.0186	(-1) 4.4049	(-1) 3.7997	(-1) 3.4433	(-1) 2.8237	(-1) 2.2314	10
12	8.9933	5.7623	4.1229	3.5355	2.8478	2.3842	1.9597	1.4393	1.0464	(-1) 7.7700	(-1) 6.0335	(-1) 5.2154	(-1) 4.6100	(-1) 4.0062	(-1) 3.6471	(-1) 3.0163	(-1) 2.4047	12
15	6.7628	4.6377	3.4803	3.0471	2.5242	2.1601	1.8168	1.3796	1.0345	(-1) 7.8796	(-1) 6.2367	(-1) 5.4454	(-1) 4.8513	(-1) 4.2512	(-1) 3.8903	(-1) 3.2488	(-1) 2.6164	15
20	5.0463	3.7030	2.9159	2.6077	2.2234	1.9464	1.6768	1.3193	1.0228	(-1) 8.0122	(-1) 6.4788	(-1) 5.7208	(-1) 5.1427	(-1) 4.5500	(-1) 4.1890	(-1) 3.5378	(-1) 2.8834	20
24	4.3397	3.2946	2.6585	2.4035	2.0799	1.8424	1.6073	1.2885	1.0170	(-1) 8.0900	(-1) 6.6194	(-1) 5.8820	(-1) 5.3143	(-1) 4.7272	(-1) 4.3672	(-1) 3.7122	(-1) 3.0465	24
30	3.7163	2.9196	2.4151	2.2079	1.9400	1.7396	1.5376	1.2571	1.0113	(-1) 8.1773	(-1) 6.7774	(-1) 6.0639	(-1) 5.5090	(-1) 4.9298	(-1) 4.5716	(-1) 3.9140	(-1) 3.2369	30
40	3.1642	2.5737	2.1838	2.0194	1.8028	1.6373	1.4672	1.2249	1.0056	(-1) 8.2775	(-1) 6.9575	(-1) 6.2723	(-1) 5.7339	(-1) 5.1653	(-1) 4.8102	(-1) 4.1518	(-1) 3.4638	40
60	2.6723	2.2523	1.9622	1.8363	1.6668	1.5343	1.3952	1.1912	1.0000	(-1) 8.3949	(-1) 7.1674	(-1) 6.5176	(-1) 5.9995	(-1) 5.4457	(-1) 5.0963	(-1) 4.4400	(-1) 3.7420	60
120	2.2301	1.9502	1.7469	1.6557	1.5299	1.4290	1.3203	1.1555	(-1) 9.9443	(-1) 8.5361	(-1) 7.4206	(-1) 6.8152	(-1) 6.3251	(-1) 5.7927	(-1) 5.4523	(-1) 4.8028	(-1) 4.0975	120
∞	1.8250	1.6601	1.5325	1.4730	1.3883	1.3180	1.2400	1.1164	(-1) 9.8891	(-1) 8.7154	(-1) 7.7429	(-1) 7.1979	(-1) 6.7467	(-1) 6.2477	(-1) 5.9224	(-1) 5.2897	(-1) 4.5828	∞

$\nu_1 = 60$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

The Probability Distribution of Fisher's Variance Ratio F

$\nu_1 = 120$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5
1	(+7) 6.3397	(+5) 6.3397	(+4) 2.5359	(+3) 6.3394	(+3) 1.0140	(+2) 2.5325	(+1) 6.3061	9.8041	2.1848
1.2	(6) 3.3486	(4) 7.2144	(3) 4.9342	(3) 1.5539	(2) 3.3714	(2) 1.0593	(1) 3.3106	6.8867	1.8892
1.5	(5) 1.8045	(3) 8.3755	(2) 9.7922	(2) 3.8834	(2) 1.1415	(1) 4.5036	(1) 1.7608	4.8725	1.6417
2	(+3) 9.9995	(+2) 9.9949	(+2) 1.9949	(+1) 9.9491	(+1) 3.9490	(+1) 1.9487	9.4829	3.4677	1.4344
3	(+2) 5.7766	(+2) 1.2397	(+1) 4.1989	(+1) 2.6221	(+1) 1.3947	8.5494	5.1425	2.4720	1.2608
4	(+2) 1.4190	(+1) 4.4400	(+1) 1.9468	(+1) 1.3558	8.3092	5.6581	3.7753	2.0812	1.1849
5	(+1) 6.1580	(+1) 2.4060	(+1) 1.2274	9.1118	6.0693	4.3984	3.1228	1.8719	1.1426
6	(+1) 3.5364	(+1) 1.5981	9.0015	6.9690	4.9045	3.7047	2.7423	1.7407	1.1156
7	(+1) 2.3790	(+1) 1.1909	7.1933	5.7372	4.1989	3.2674	2.4928	1.6502	1.0969
8	(+1) 1.7652	9.5321	6.0649	4.9460	3.7279	2.9669	2.3162	1.5836	1.0832
9	(+1) 1.3976	8.0014	5.3001	4.3978	3.3918	2.7475	2.1843	1.5325	1.0727
10	(+1) 1.1580	6.9443	4.7501	3.9965	3.1399	2.5801	2.0818	1.4919	1.0645
12	8.7031	5.5931	4.0149	3.4494	2.7874	2.3410	1.9323	1.4310	1.0523
15	6.4995	4.4750	3.3722	2.9595	2.4611	2.1141	1.7867	1.3698	1.0403
20	4.8031	3.5438	2.8058	2.5168	2.1562	1.8963	1.6433	1.3074	1.0285
24	4.1037	3.1357	2.5463	2.3099	2.0099	1.7897	1.5715	1.2754	1.0227
30	3.4852	2.7595	2.2997	2.1107	1.8664	1.6835	1.4989	1.2424	1.0170
40	2.9349	2.4103	2.0635	1.9172	1.7242	1.5766	1.4248	1.2080	1.0113
60	2.4405	2.0821	1.8341	1.7263	1.5810	1.4673	1.3476	1.1715	1.0056
120	1.9877	1.7667	1.6055	1.5330	1.4327	1.3519	1.2646	1.1314	1.0000
∞	1.5527	1.4468	1.3637	1.3246	1.2684	1.2214	1.1686	1.0838	(-1) 9.9445

p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
(-1) 7.4839	(-1) 3.6393	(-1) 2.5510	(-1) 1.9408	(-1) 1.4596	(-1) 1.2226	(-2) 8.7873	(-2) 6.1714	1
(-1) 7.2998	(-1) 3.7824	(-1) 2.7201	(-1) 2.1056	(-1) 1.6086	(-1) 1.3592	(-2) 9.9068	(-2) 7.0438	1.2
(-1) 7.1754	(-1) 3.9788	(-1) 2.9436	(-1) 2.3232	(-1) 1.8072	(-1) 1.5425	(-1) 1.1432	(-2) 8.2502	1.5
(-1) 7.1306	(-1) 4.2602	(-1) 3.2554	(-1) 2.6284	(-1) 2.0892	(-1) 1.8053	(-1) 1.3659	(-1) 1.0045	2
(-1) 7.2082	(-1) 4.6948	(-1) 3.7311	(-1) 3.0989	(-1) 2.5321	(-1) 2.2236	(-1) 1.7297	(-1) 1.3058	3
(-1) 7.3239	(-1) 5.0193	(-1) 4.0863	(-1) 3.4551	(-1) 2.8739	(-1) 2.5506	(-1) 2.0214	(-1) 1.5538	4
(-1) 7.4333	(-1) 5.2745	(-1) 4.3668	(-1) 3.7397	(-1) 3.1511	(-1) 2.8163	(-1) 2.2647	(-1) 1.7649	5
(-1) 7.5313	(-1) 5.4831	(-1) 4.5977	(-1) 3.9755	(-1) 3.3831	(-1) 3.0442	(-1) 2.4730	(-1) 1.9484	6
(-1) 7.6173	(-1) 5.6577	(-1) 4.7923	(-1) 4.1757	(-1) 3.5819	(-1) 3.2390	(-1) 2.6546	(-1) 2.1106	7
(-1) 7.6929	(-1) 5.8072	(-1) 4.9593	(-1) 4.3490	(-1) 3.7553	(-1) 3.4095	(-1) 2.8154	(-1) 2.2556	8
(-1) 7.7604	(-1) 5.9372	(-1) 5.1052	(-1) 4.5011	(-1) 3.9084	(-1) 3.5609	(-1) 2.9592	(-1) 2.3866	9
(-1) 7.8204	(-1) 6.0518	(-1) 5.2342	(-1) 4.6361	(-1) 4.0451	(-1) 3.6966	(-1) 3.0891	(-1) 2.5058	10
(-1) 7.9233	(-1) 6.2453	(-1) 5.4535	(-1) 4.8667	(-1) 4.2803	(-1) 3.9310	(-1) 3.3155	(-1) 2.7157	12
(-1) 8.0463	(-1) 6.4725	(-1) 5.7127	(-1) 5.1414	(-1) 4.5631	(-1) 4.2146	(-1) 3.5929	(-1) 2.9762	15
(-1) 8.1967	(-1) 6.7472	(-1) 6.0288	(-1) 5.4798	(-1) 4.9150	(-1) 4.5702	(-1) 3.9457	(-1) 3.3135	20
(-1) 8.2864	(-1) 6.9099	(-1) 6.2174	(-1) 5.6828	(-1) 5.1282	(-1) 4.7870	(-1) 4.1634	(-1) 3.5244	24
(-1) 8.3886	(-1) 7.0952	(-1) 6.4338	(-1) 5.9175	(-1) 5.3763	(-1) 5.0406	(-1) 4.4206	(-1) 3.7764	30
(-1) 8.5092	(-1) 7.3121	(-1) 6.6881	(-1) 6.1954	(-1) 5.6728	(-1) 5.3450	(-1) 4.7330	(-1) 4.0864	40
(-1) 8.6543	(-1) 7.5740	(-1) 6.9979	(-1) 6.5364	(-1) 6.0397	(-1) 5.7244	(-1) 5.1277	(-1) 4.4842	60
(-1) 8.8386	(-1) 7.9076	(-1) 7.3970	(-1) 6.9798	(-1) 6.5232	(-1) 6.2286	(-1) 5.6601	(-1) 5.0309	120
(-1) 9.1017	(-1) 8.3850	(-1) 7.9751	(-1) 7.6313	(-1) 7.2438	(-1) 6.9876	(-1) 6.4792	(-1) 5.8940	∞

$\nu_1 = 120$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

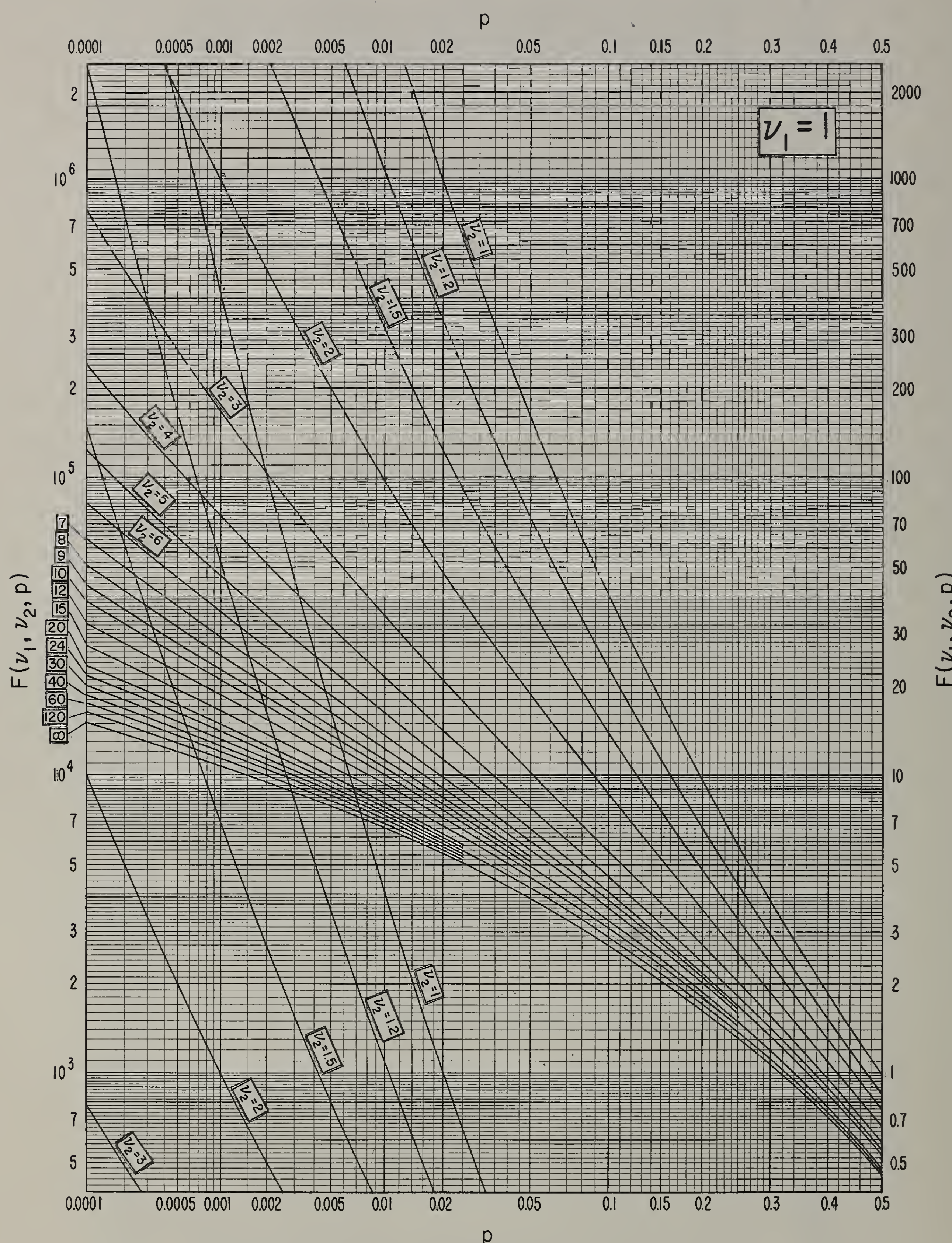
The Probability Distribution of Fisher's Variance Ratio F

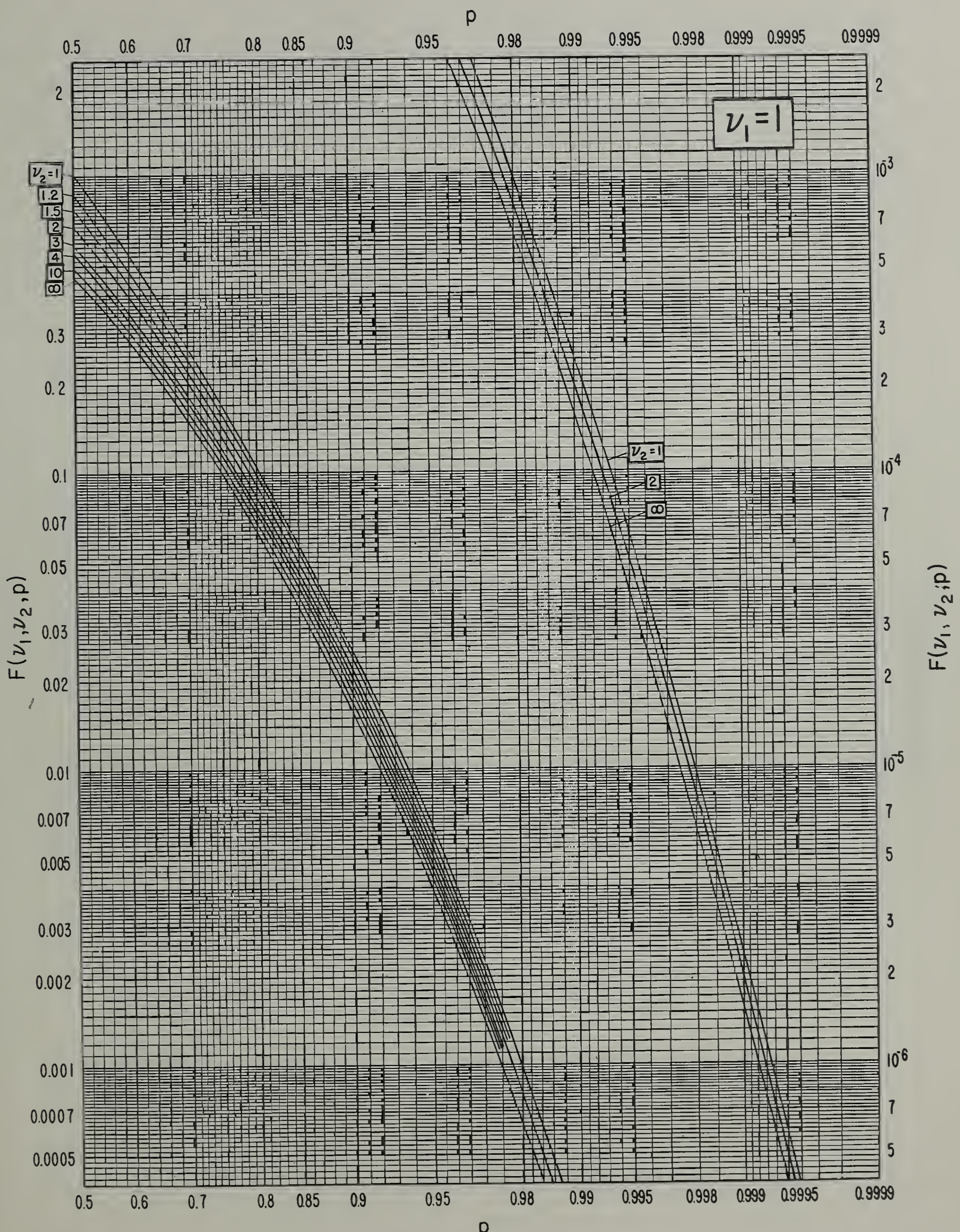
$\nu_1 = \infty$

ν_2	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5	p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν_2
1	(+7) 6.3662	(+5) 6.3662	(+4) 2.5465	(+3) 6.3660	(+3) 1.0183	(+2) 2.5432	(+1) 6.3328	9.8492	2.1981	(-1) 7.5569	(-1) 3.6962	(-1) 2.6031	(-1) 1.9905	(-1) 1.5072	(-1) 1.2691	(-2) 9.2357	(-2) 6.6065	1
1.2	(6) 3.3598	(4) 7.2384	(3) 4.9507	(3) 1.5591	(2) 3.3827	(2) 1.0629	(1) 3.3222	6.9147	1.9905	(-1) 7.3741	(-1) 3.8449	(-1) 2.7790	(-1) 2.1623	(-1) 1.6636	(-1) 1.4133	(-1) 1.0437	(-2) 7.6055	1.2
1.5	(5) 1.8083	(3) 8.3929	(2) 9.8126	(2) 3.8916	(2) 1.1439	(1) 4.5136	(1) 1.7651	4.8889	1.6514	(-1) 7.2527	(-1) 4.0494	(-1) 3.0119	(-1) 2.3901	(-1) 1.8729	(-1) 1.6076	(-1) 1.2076	(-2) 8.9520	1.5
2	(+3) 9.9995	(+2) 9.9950	(+2) 1.9951	(+1) 9.9501	(+1) 3.9498	(+1) 1.9496	9.4913	3.4761	1.4427	(-1) 7.2134	(-1) 4.3429	(-1) 3.3381	(-1) 2.7108	(-1) 2.1715	(-1) 1.8874	(-1) 1.4476	(-1) 1.0857	2
3	(+2) 5.7528	(+2) 1.2347	(+1) 4.1829	(+1) 2.6125	(+1) 1.3902	8.5265	5.1337	2.4742	1.2680	(-1) 7.3025	(-1) 4.7989	(-1) 3.8389	(-1) 3.2091	(-1) 2.6444	(-1) 2.3368	(-1) 1.8443	(-1) 1.4213	3
4	(+2) 1.4075	(+1) 4.4051	(+1) 1.9325	(+1) 1.3463	8.2573	5.6281	3.7607	2.0806	1.1916	(-1) 7.4278	(-1) 5.1417	(-1) 4.2160	(-1) 3.5896	(-1) 3.0128	(-1) 2.6917	(-1) 2.1660	(-1) 1.7012	4
5	(+1) 6.0844	(+1) 2.3785	(+1) 1.2144	9.0204	6.0153	4.3650	3.1050	1.8694	1.1490	(-1) 7.5466	(-1) 5.4133	(-1) 4.5165	(-1) 3.8964	(-1) 3.3142	(-1) 2.9852	(-1) 2.4372	(-1) 1.9421	5
6	(+1) 3.4812	(+1) 1.5745	8.8793	6.8801	4.8491	3.6688	2.7222	1.7368	1.1219	(-1) 7.6523	(-1) 5.6367	(-1) 4.7651	(-1) 4.1525	(-1) 3.5689	(-1) 3.2349	(-1) 2.6717	(-1) 2.1539	6
7	(+1) 2.3336	(+1) 1.1696	7.0760	5.6495	4.1423	3.2298	2.4708	1.6452	1.1031	(-1) 7.7459	(-1) 5.8251	(-1) 4.9761	(-1) 4.3716	(-1) 3.7889	(-1) 3.4521	(-1) 2.8781	(-1) 2.3429	7
8	(+1) 1.7257	9.3337	5.9505	4.8588	3.6702	2.9276	2.2926	1.5777	1.0893	(-1) 7.8284	(-1) 5.9873	(-1) 5.1589	(-1) 4.5625	(-1) 3.9820	(-1) 3.6438	(-1) 3.0623	(-1) 2.5135	8
9	(+1) 1.3620	7.8128	5.1875	4.3105	3.3329	2.7067	2.1592	1.5257	1.0788	(-1) 7.9026	(-1) 6.1293	(-1) 5.3194	(-1) 4.7313	(-1) 4.1540	(-1) 3.8153	(-1) 3.2285	(-1) 2.6690	9
10	(+1) 1.1250	6.7625	4.6385	3.9090	3.0798	2.5379	2.0554	1.4843	1.0705	(-1) 7.9688	(-1) 6.2551	(-1) 5.4624	(-1) 4.8821	(-1) 4.3087	(-1) 3.9701	(-1) 3.3797	(-1) 2.8118	10
12	8.4063	5.4195	3.9039	3.3608	2.7249	2.2962	1.9036	1.4221	1.0582	(-1) 8.0834	(-1) 6.4691	(-1) 5.7071	(-1) 5.1422	(-1) 4.5771	(-1) 4.2403	(-1) 3.6464	(-1) 3.0664	12
15	6.2287	4.3070	3.2602	2.8684	2.3953	2.0658	1.7551	1.3591	1.0461	(-1) 8.2217	(-1) 6.7245	(-1) 6.0010	(-1) 5.4567	(-1) 4.9056	(-1) 4.5729	(-1) 3.9791	(-1) 3.3888	15
20	4.5503	3.3778	2.6904	2.4212	2.0853	1.8432	1.6074	1.2943	1.0343	(-1) 8.3935	(-1) 7.0393	(-1) 6.3674	(-1) 5.8531	(-1) 5.3240	(-1) 5.0005	(-1) 4.4136	(-1) 3.8178	20
24	3.8566	2.9685	2.4276	2.2107	1.9353	1.7331	1.5327	1.2607	1.0284	(-1) 8.4983	(-1) 7.2296	(-1) 6.5907	(-1) 6.0968	(-1) 5.5841	(-1) 5.2679	(-1) 4.6894	(-1) 4.0947	24
30	3.2404	2.5889	2.1760	2.0062	1.7867	1.6223	1.4564	1.2256	1.0226	(-1) 8.6207	(-1) 7.4521	(-1) 6.8535	(-1) 6.3857	(-1) 5.8948	(-1) 5.5894	(-1) 5.0249	(-1) 4.4357	30
40	2.6876	2.2326	1.9318	1.8047	1.6371	1.5089	1.3769	1.1883	1.0169	(-1) 8.7689	(-1) 7.7214	(-1) 7.1736	(-1) 6.7408	(-1) 6.2802	(-1) 5.9913	(-1) 5.4494	(-1) 4.8743	40
60	2.1821	1.8905	1.6885	1.6006	1.4822	1.3893	1.2915	1.1474	1.0112	(-1) 8.9574	(-1) 8.0645	(-1) 7.5873	(-1) 7.2031	(-1) 6.7889	(-1) 6.5253	(-1) 6.0237	(-1) 5.4793	60
120	1.6966	1.5434	1.4311	1.3805	1.3104	1.2539	1.1926	1.0987	1.0056	(-1) 9.2268	(-1) 8.5572	(-1) 8.1873	(-1) 7.8839	(-1) 7.5494	(-1) 7.3330	(-1) 6.9077	(-1) 6.4403	120
∞	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	∞

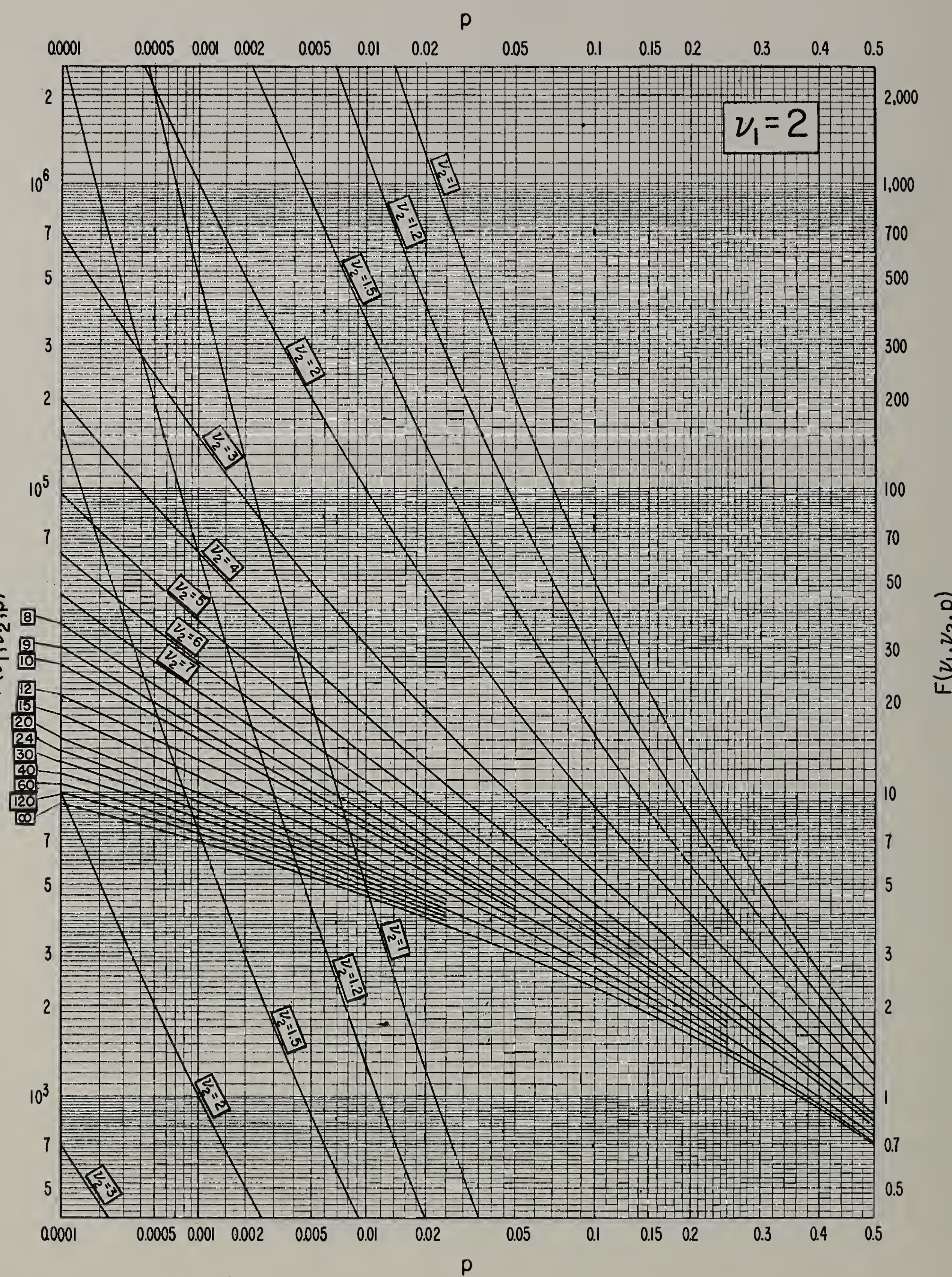
$\nu_1 = \infty$

Interpolation should be carried out using the reciprocals of the degrees of freedom; the function $120/\nu$ is convenient for this purpose. Fisher's variance ratio $F(\nu_1, \nu_2) > F(\nu_1, \nu_2, p)$ with probability p. $F(\nu_1, \nu_2) = \{u/\nu_1\}/\{v/\nu_2\}$ where u and v are random variables independently distributed as χ^2 with ν_1 and ν_2 degrees of freedom, respectively. In particular s_1^2/s_2^2 is distributed as $F(\nu_1, \nu_2)$ where s_1^2 and s_2^2 are independent mean squares from normally distributed populations estimating a common variance σ^2 and based on ν_1 and ν_2 degrees of freedom, respectively. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.

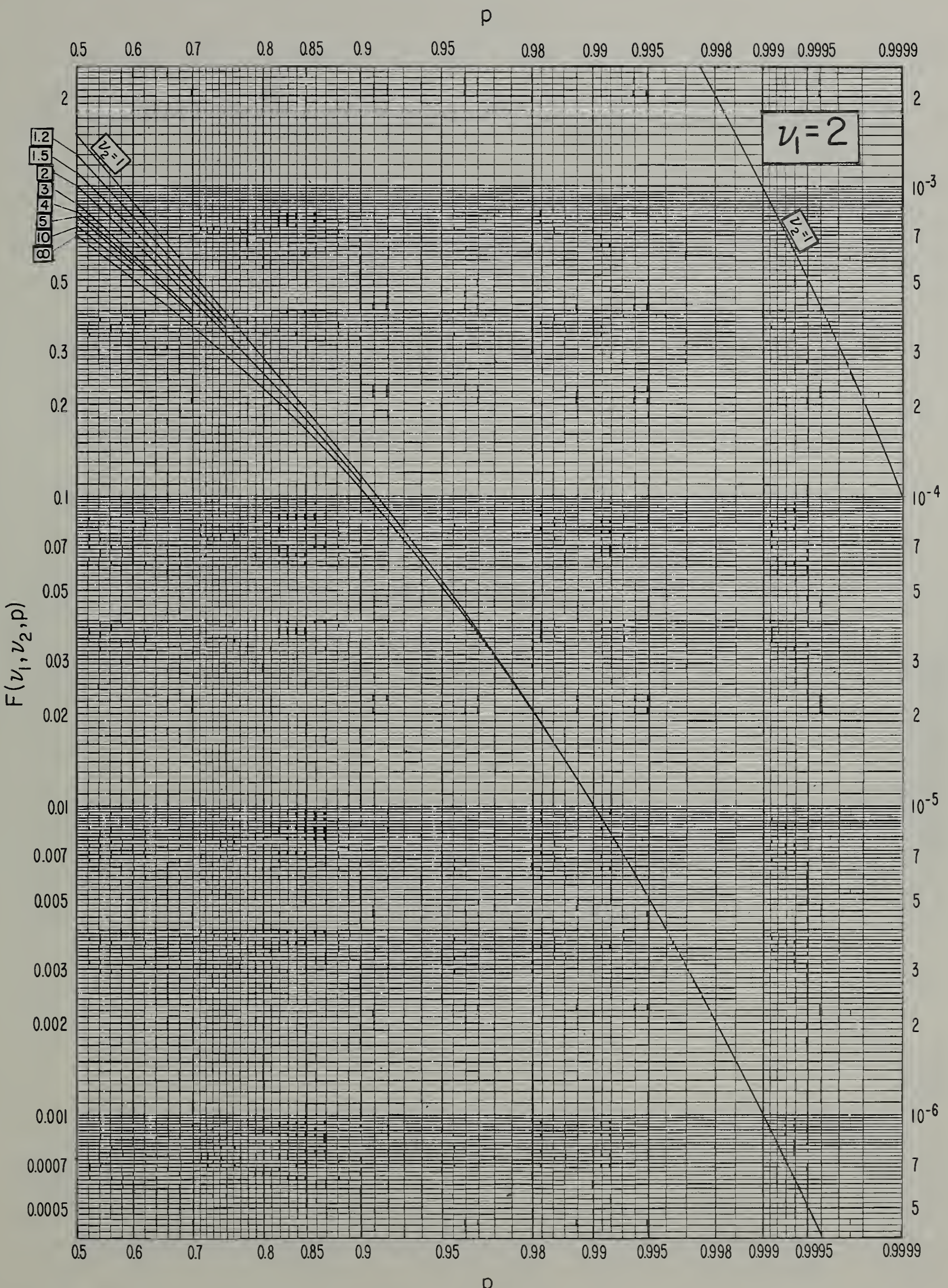

p
0.0001
0.0005
0.001
0.002
0.005
0.01
0.02
0.05
0.1
0.15
0.2
0.3
0.4
0.5
$\nu_1 = 1$
$\nu_2=1$
$\nu_2=1.2$
$\nu_2=1.5$
$\nu_2=2$
$\nu_2=3$
$\nu_2=4$
$\nu_2=5$
$\nu_2=6$
7
8
9
10
12
15
20
24
30
40
60
120
∞
$F(\nu_1, \nu_2, p)$
10^6
10^5
10^4
10^3
2000
1000
700
500
300
200
100
70
50
30
20
10
7
5
3
2
1
0.7
0.5

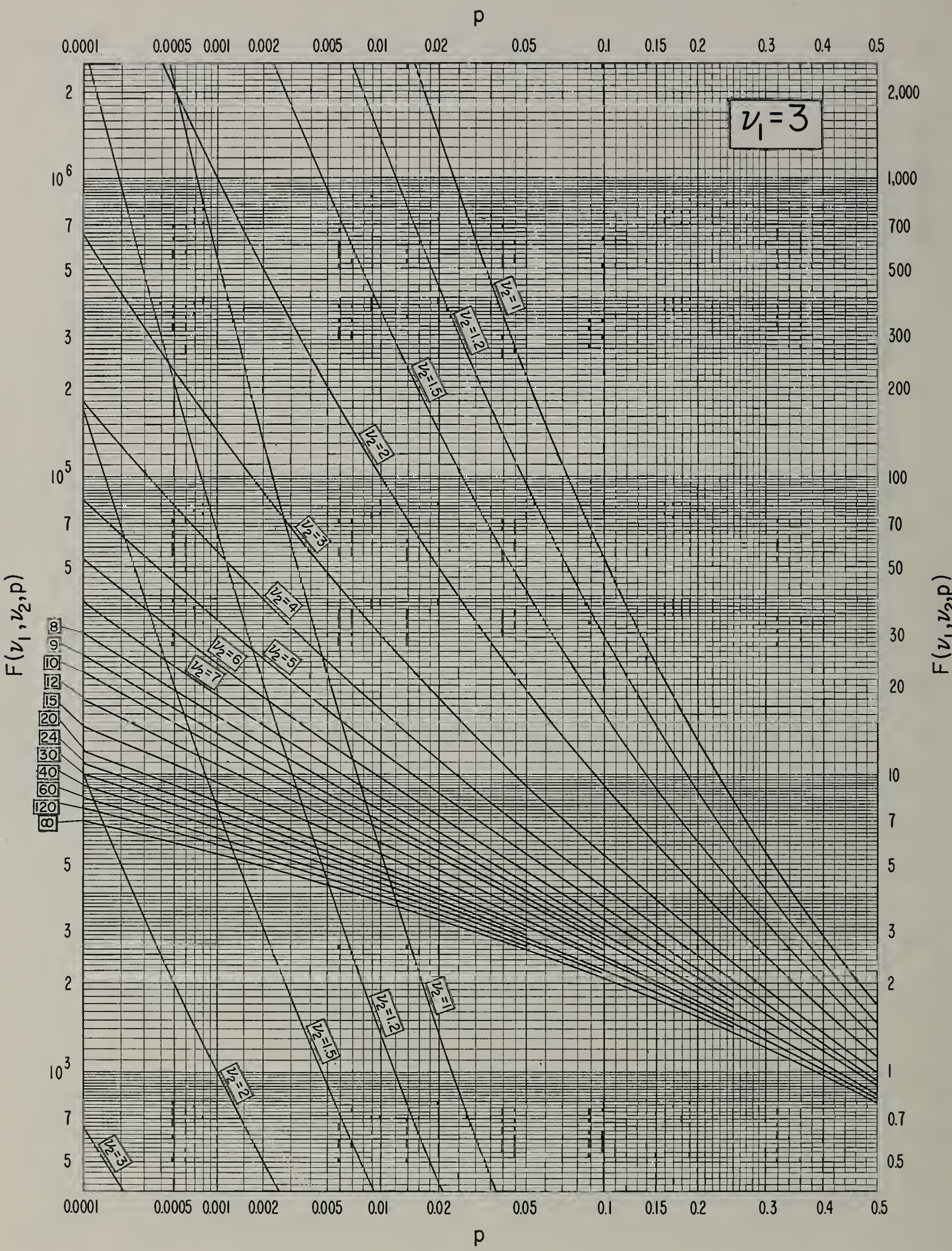


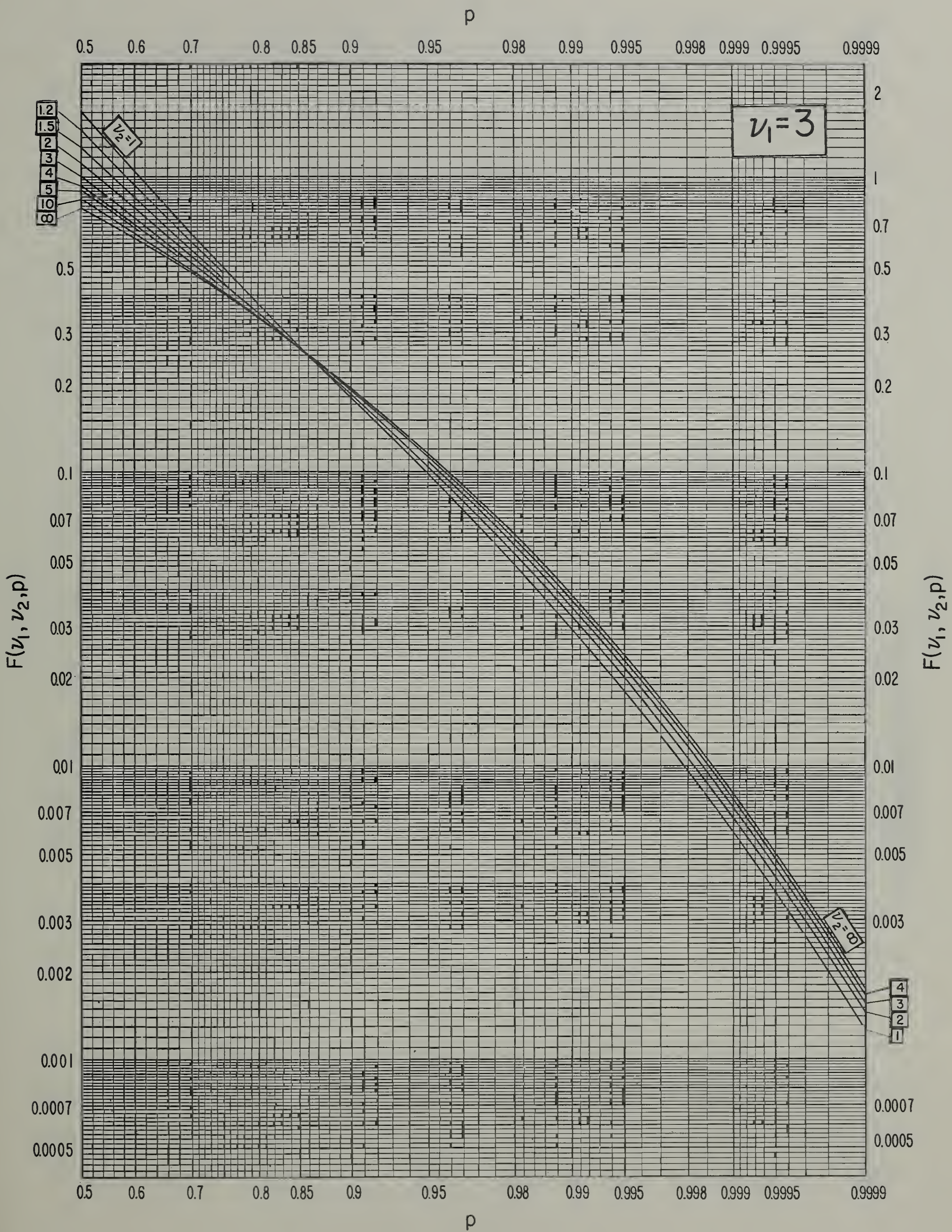
p
0.5
0.6
0.7
0.8
0.85
0.9
0.95
0.98
0.99
0.995
0.998
0.999
0.9995
0.9999
$\nu_1 = 1$
$\nu_2 = 1$
1.2
1.5
2
3
4
10
∞
$F(\nu_1, \nu_2, p)$
$F(\nu_1, \nu_2; p)$
2
0.3
0.2
0.1
0.07
0.05
0.03
0.02
0.01
0.007
0.005
0.003
0.002
0.001
0.0007
0.0005
10^{-3}
7
5
3
2
10^{-4}
10^{-5}
10^{-6}

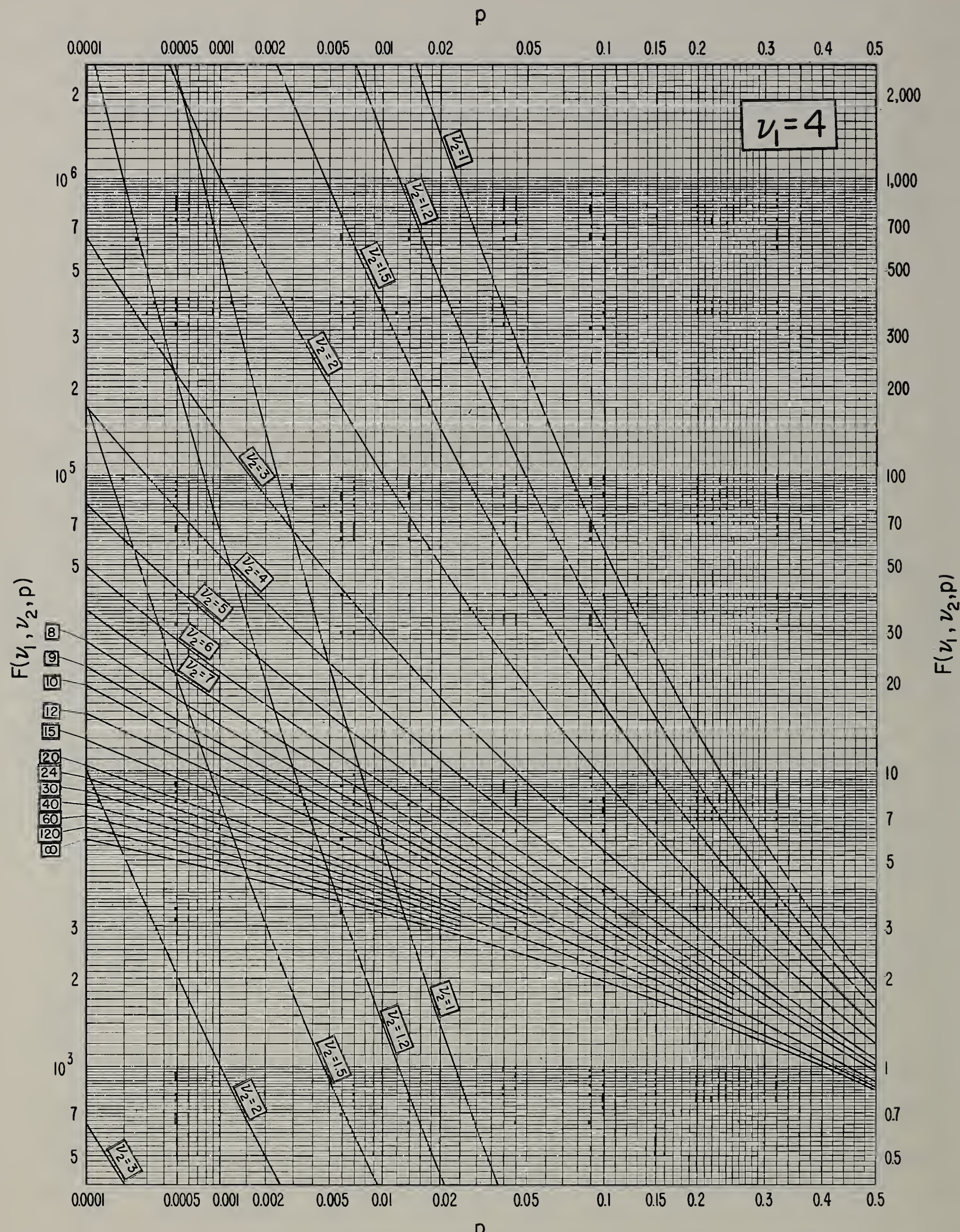


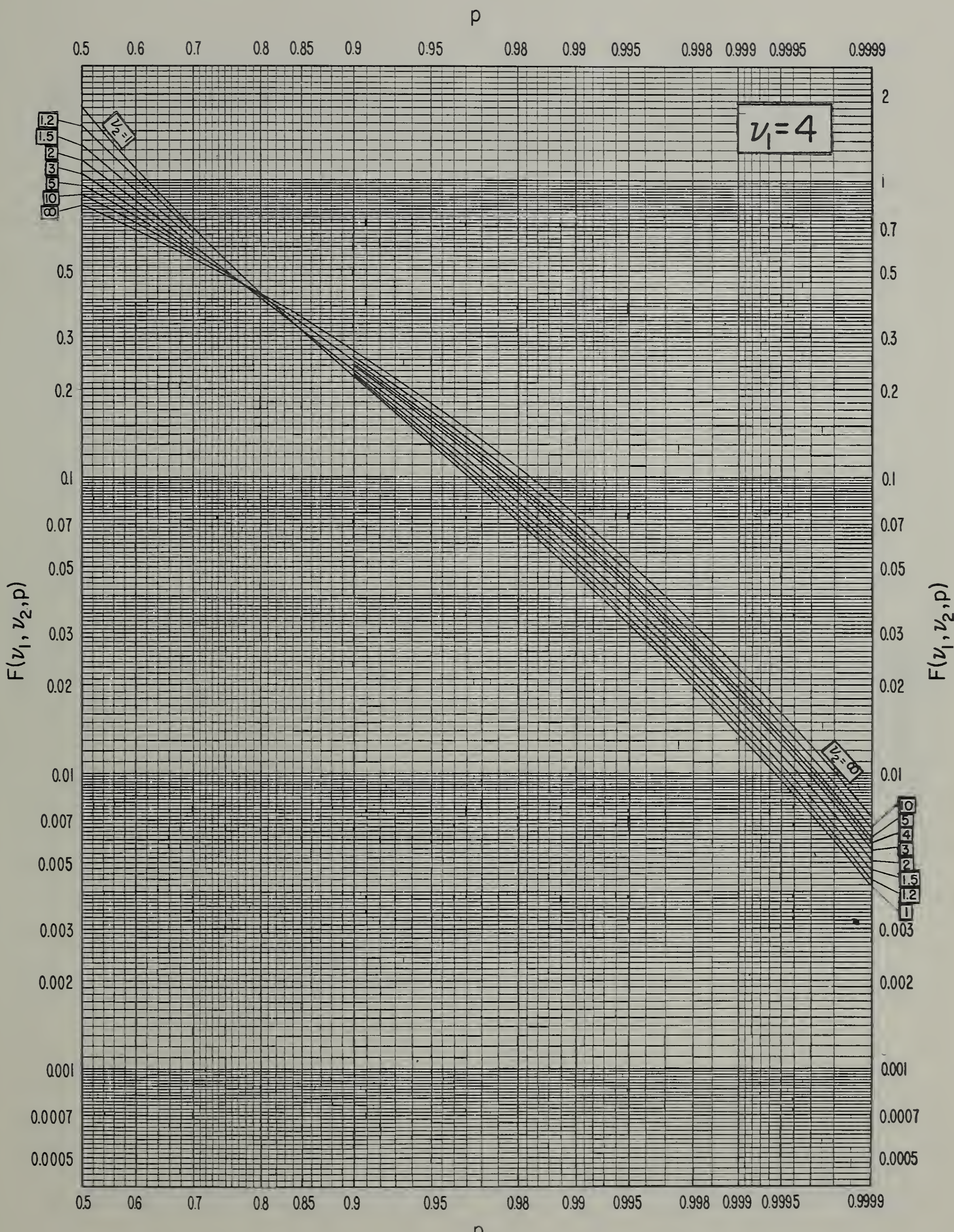
p
0.0001
0.0005
0.001
0.002
0.005
0.01
0.02
0.05
0.1
0.15
0.2
0.3
0.4
0.5
$\nu_1 = 2$
$F(\nu_1, \nu_2, p)$
2,000
1,000
700
500
300
200
100
70
50
30
20
10
7
5
3
2
1
0.7
0.5
2
10^6
7
5
3
2
10^5
7
10^3
7
5
$\nu_2 = 1$
$\nu_2 = 1.2$
$\nu_2 = 1.5$
$\nu_2 = 2$
$\nu_2 = 3$
$\nu_2 = 4$
$\nu_2 = 5$
$\nu_2 = 6$
$\nu_2 = 7$
8
9
10
12
15
20
24
30
40
60
120
∞

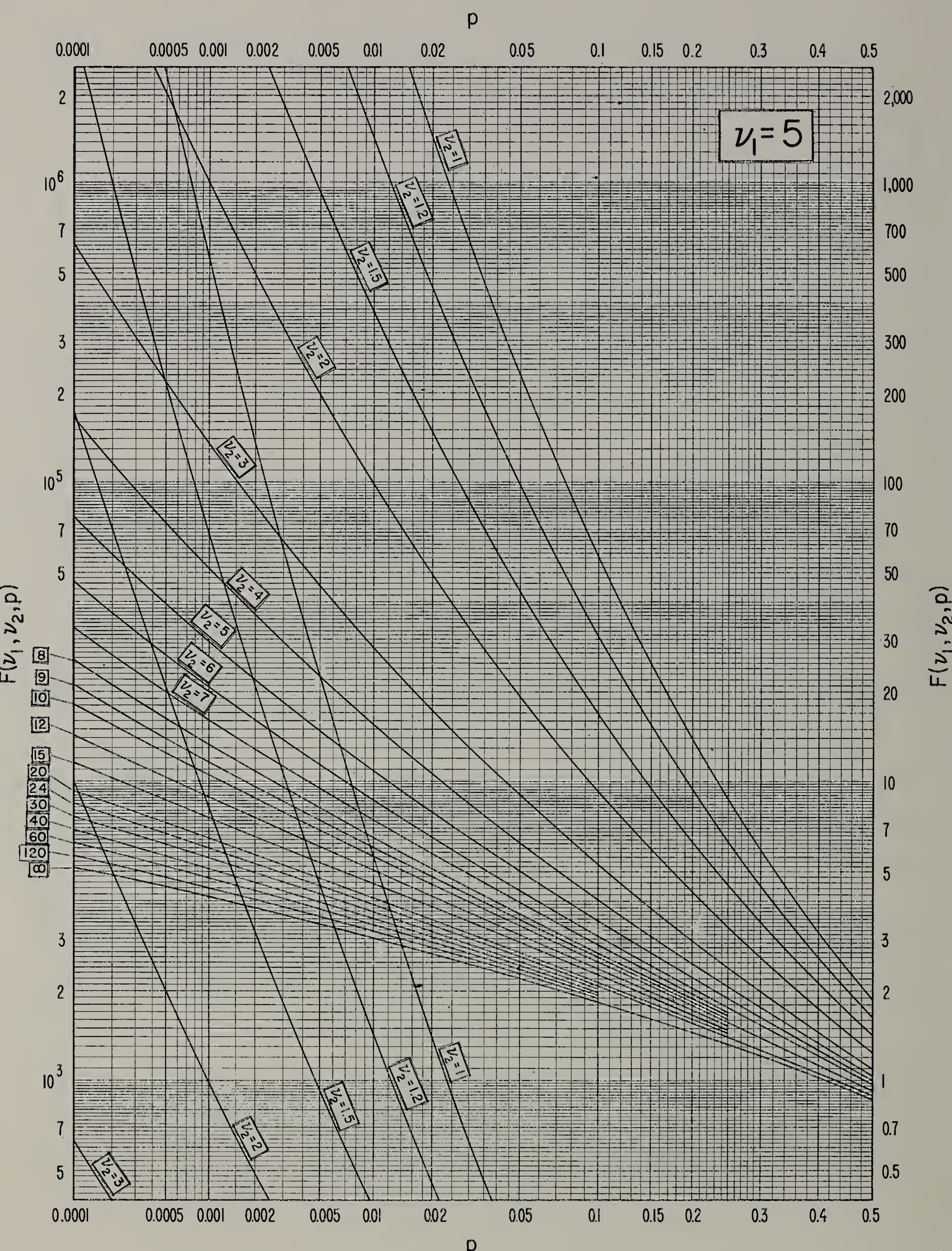

p
0.5
0.6
0.7
0.8
0.85
0.9
0.95
0.98
0.99
0.995
0.998
0.999
0.9995
0.9999
$\nu_1 = 2$
$\nu_2 = 1$
1.2
1.5
2
3
4
5
10
∞
$F(\nu_1, \nu_2, p)$
2
0.5
0.3
0.2
0.1
0.07
0.05
0.03
0.02
0.01
0.007
0.005
0.003
0.002
0.001
0.0007
0.0005
10^{-3}
7
5
3
2
10^{-4}
10^{-5}
10^{-6}


p
0.0001
0.0005
0.001
0.002
0.005
0.01
0.02
0.05
0.1
0.15
0.2
0.3
0.4
0.5
$\nu_1 = 3$
$F(\nu_1, \nu_2, p)$
10^6
10^5
10^3
2,000
1,000
700
500
300
200
100
70
50
30
20
10
7
5
3
2
1
0.7
0.5
$\nu_2 = 1$
$\nu_2 = 1.2$
$\nu_2 = 1.5$
$\nu_2 = 2$
$\nu_2 = 3$
$\nu_2 = 4$
$\nu_2 = 5$
$\nu_2 = 6$
$\nu_2 = 7$
8
9
10
12
15
20
24
30
40
60
120
∞

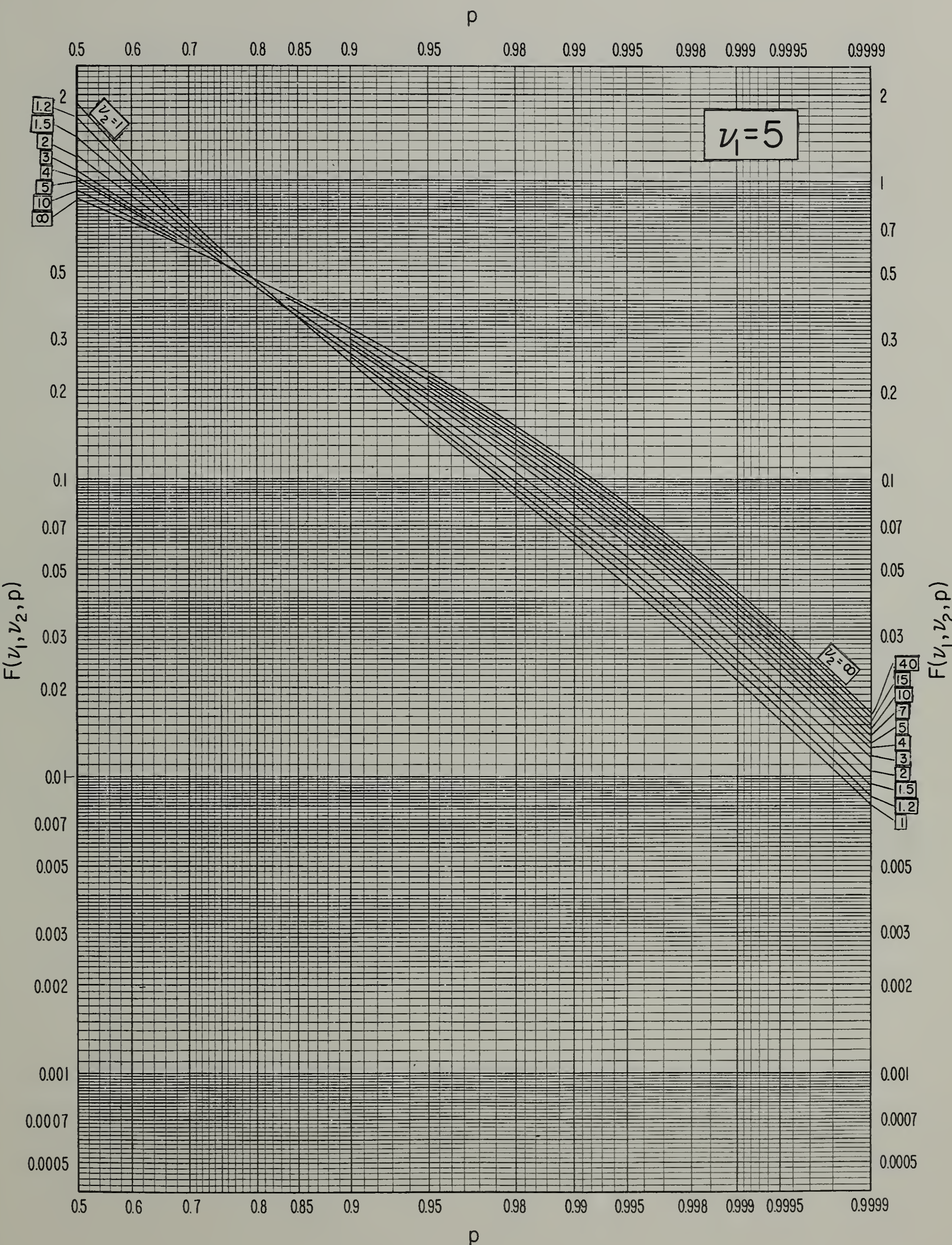

p
$\nu_1 = 3$
$\nu_2 = 1$
$\nu_2 = \infty$
1.2
1.5
2
3
4
5
10
∞
$F(\nu_1, \nu_2, p)$
0.5 0.6 0.7 0.8 0.85 0.9 0.95 0.98 0.99 0.995 0.998 0.999 0.9995 0.9999
2 1 0.7 0.5 0.3 0.2 0.1 0.07 0.05 0.03 0.02 0.01 0.007 0.005 0.003 0.002 0.001 0.0007 0.0005


p
0.0001 0.0005 0.001 0.002 0.005 0.01 0.02 0.05 0.1 0.15 0.2 0.3 0.4 0.5
$\nu_1 = 4$
10^6
10^5
10^3
2 7 5 3 2 7 5 3 2 7 5
2,000 1,000 700 500 300 200 100 70 50 30 20 10 7 5 3 2 1 0.7 0.5
$F(\nu_1, \nu_2, p)$
$\nu_2 = 1$
$\nu_2 = 1.2$
$\nu_2 = 1.5$
$\nu_2 = 2$
$\nu_2 = 3$
$\nu_2 = 4$
$\nu_2 = 5$
$\nu_2 = 6$
$\nu_2 = 7$
8 9 10 12 15 20 24 30 40 60 120 ∞

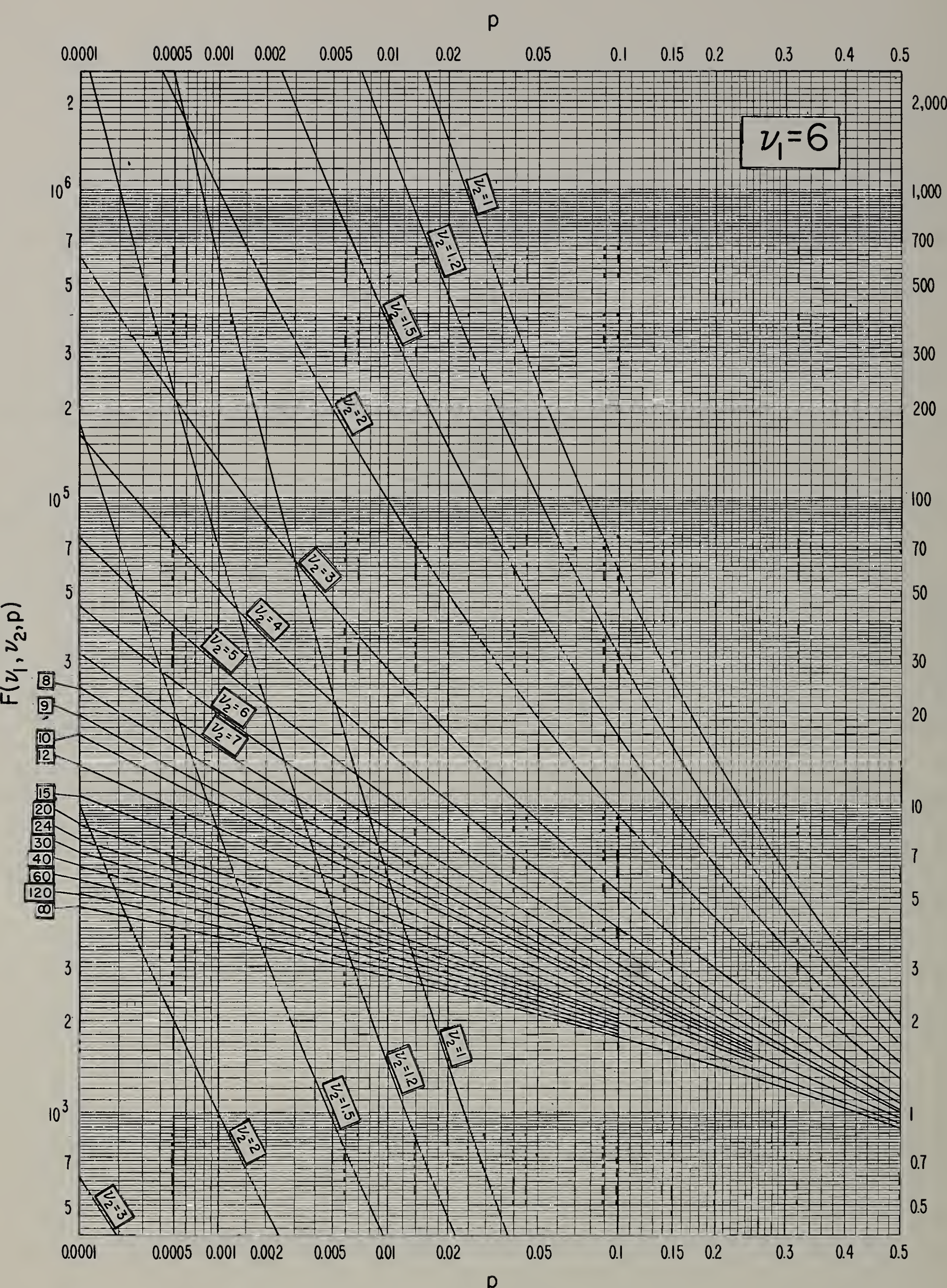

p
0.5
0.6
0.7
0.8
0.85
0.9
0.95
0.98
0.99
0.995
0.998
0.999
0.9995
0.9999
$\nu_1 = 4$
$\nu_2 = 1$
1.2
1.5
2
3
5
10
∞
$\nu_2 = \infty$
10
5
4
3
2
1.5
1.2
1
2
1
0.7
0.5
0.3
0.2
0.1
0.07
0.05
0.03
0.02
0.01
0.007
0.005
0.003
0.002
0.001
0.0007
0.0005
$F(\nu_1, \nu_2, p)$



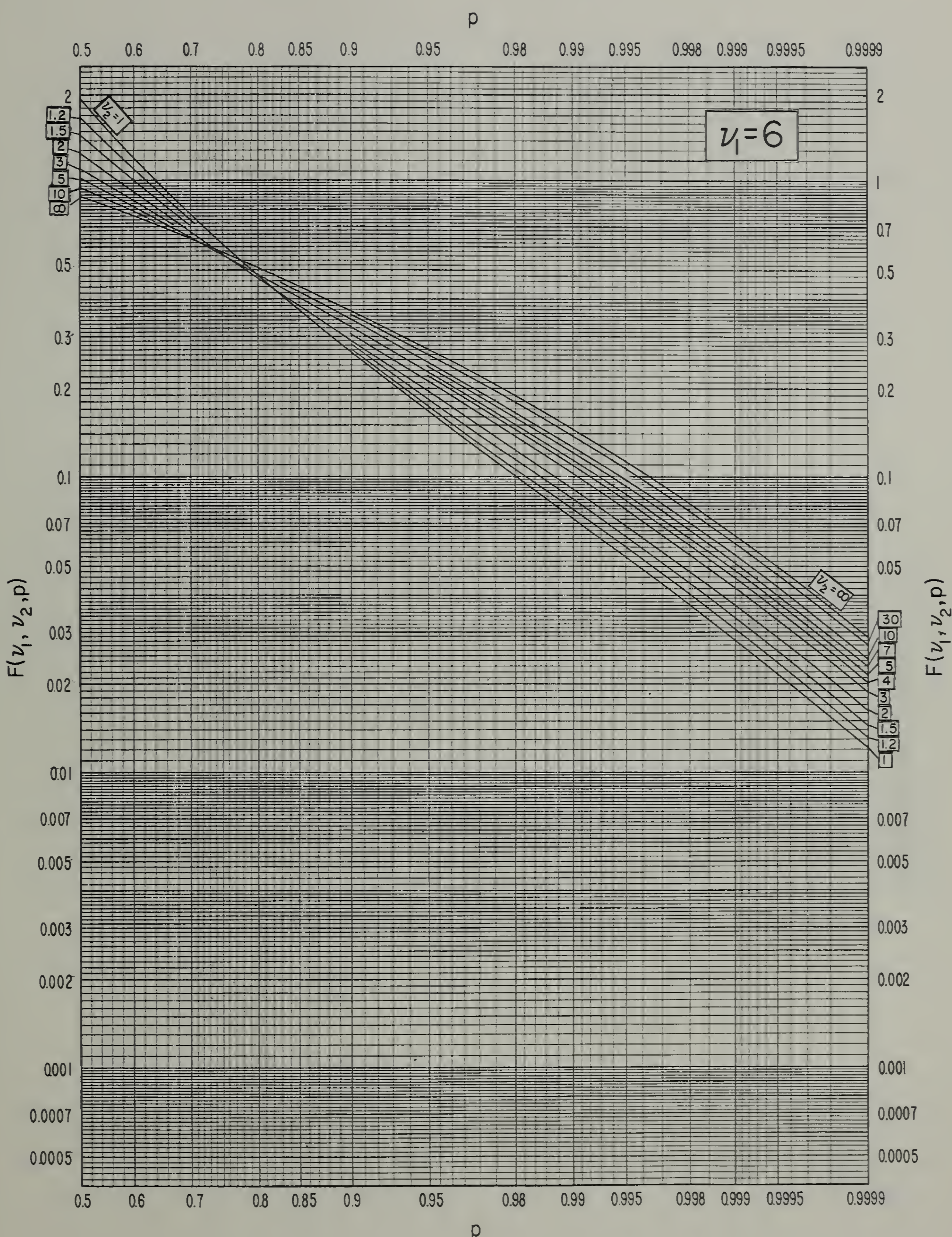
p
0.0001
0.0005
0.001
0.002
0.005
0.01
0.02
0.05
0.1
0.15
0.2
0.3
0.4
0.5
$\nu_1 = 5$
$F(\nu_1, \nu_2, p)$
10^6
10^5
10^3
2,000
1,000
700
500
300
200
100
70
50
30
20
10
7
5
3
2
1
0.7
0.5
$\nu_2 = 1$
$\nu_2 = 1.2$
$\nu_2 = 1.5$
$\nu_2 = 2$
$\nu_2 = 3$
$\nu_2 = 4$
$\nu_2 = 5$
$\nu_2 = 6$
$\nu_2 = 7$
8
9
10
12
15
20
24
30
40
60
120
∞



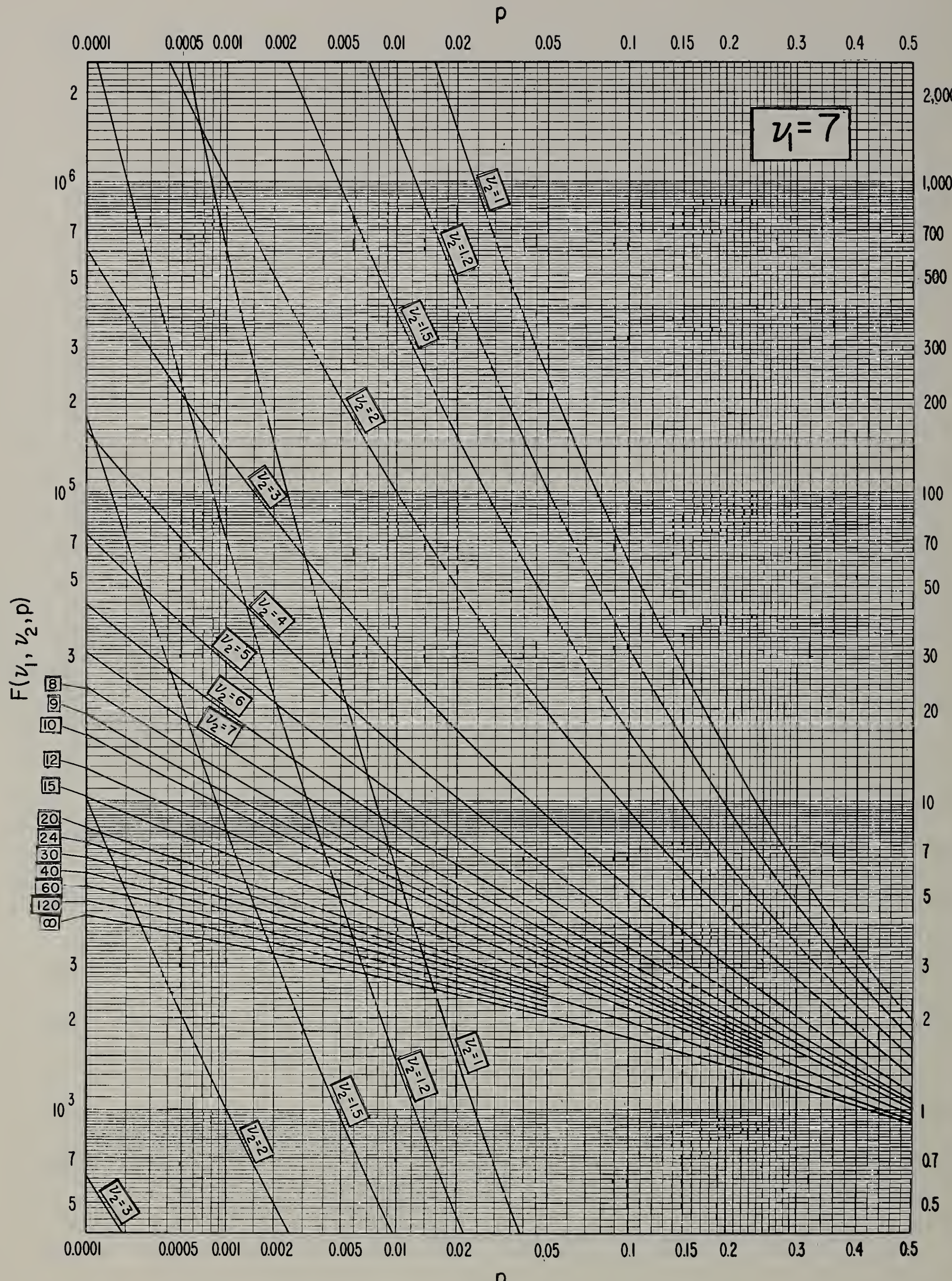
p
0.5
0.6
0.7
0.8
0.85
0.9
0.95
0.98
0.99
0.995
0.998
0.999
0.9995
0.9999
$\nu_1=5$
$\nu_2=1$
1.2
1.5
2
3
4
5
10
∞
$\nu_2=\infty$
40
15
10
7
5
4
3
2
1.5
1.2
1
2
1
0.7
0.5
0.3
0.2
0.1
0.07
0.05
0.03
0.02
0.01
0.007
0.005
0.003
0.002
0.001
0.0007
0.0005
$F(\nu_1, \nu_2, p)$

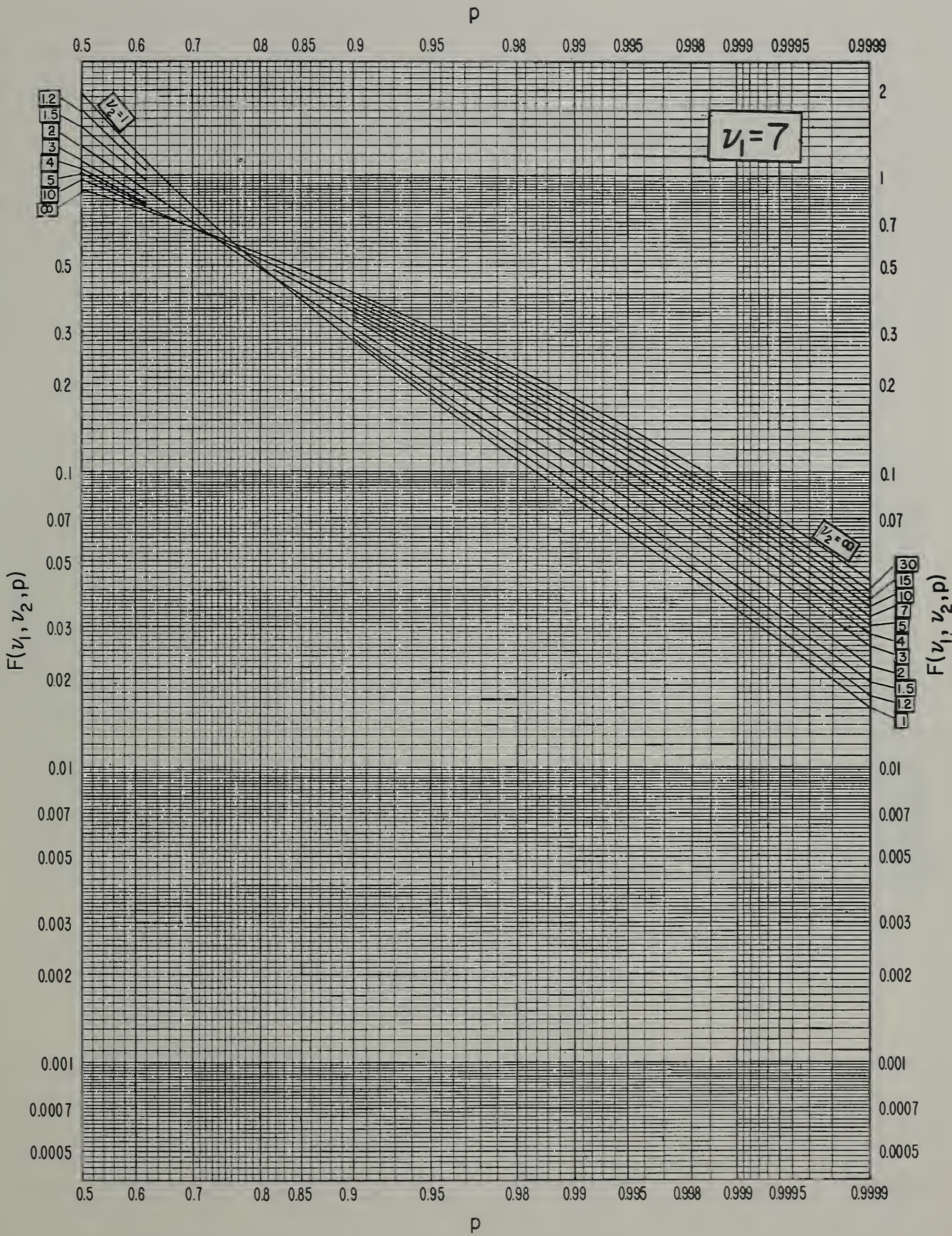


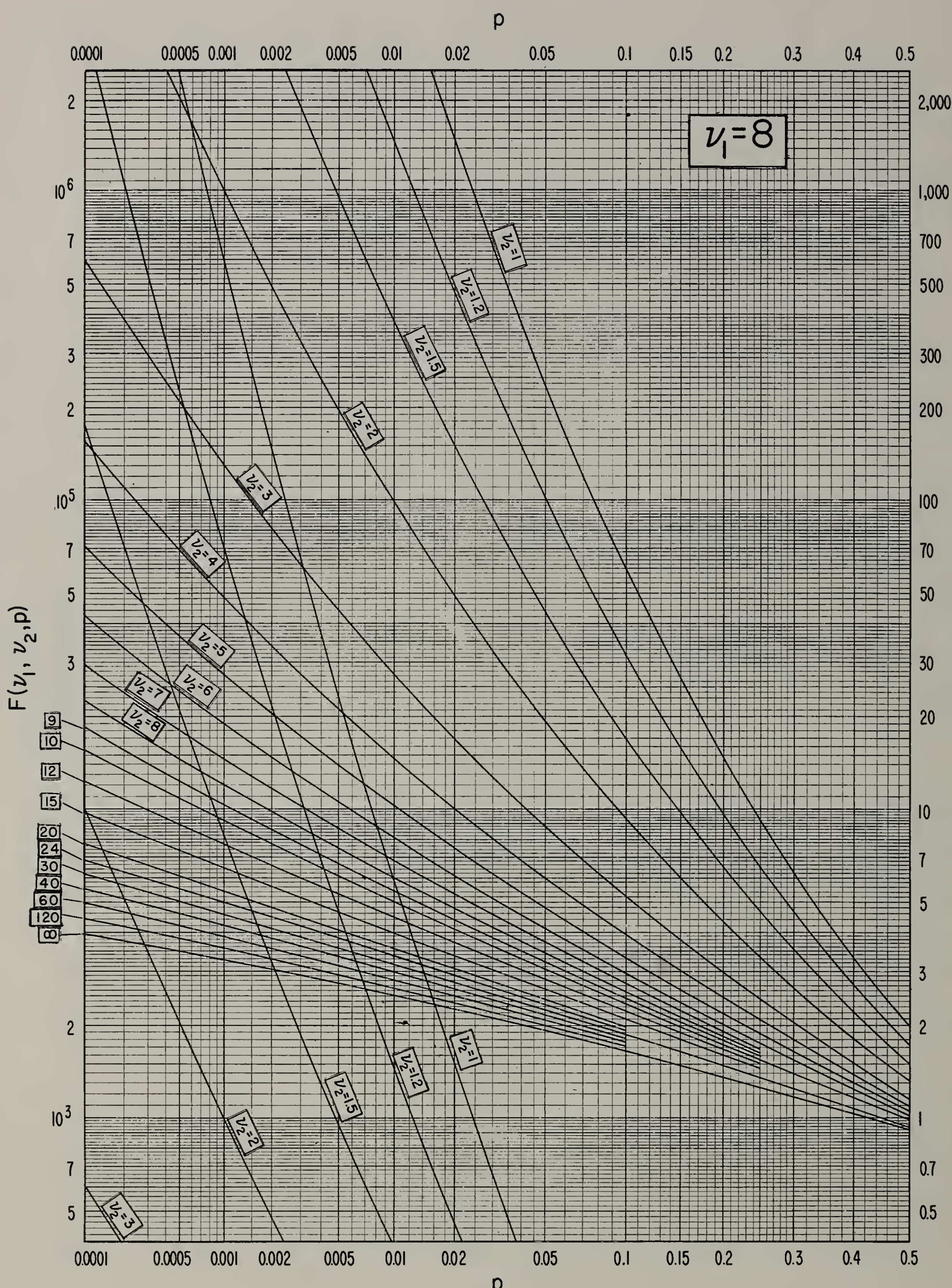
p
0.0001
0.0005
0.001
0.002
0.005
0.01
0.02
0.05
0.1
0.15
0.2
0.3
0.4
0.5
$\nu_1=6$
$F(\nu_1, \nu_2, p)$
$\nu_2=1$
$\nu_2=1.2$
$\nu_2=1.5$
$\nu_2=2$
$\nu_2=3$
$\nu_2=4$
$\nu_2=5$
$\nu_2=6$
$\nu_2=7$
8
9
10
12
15
20
24
30
40
60
120
∞
2
10^6
7
5
3
10^5
10^3
2,000
1,000
700
500
300
200
100
70
50
30
20
10
1
0.7
0.5



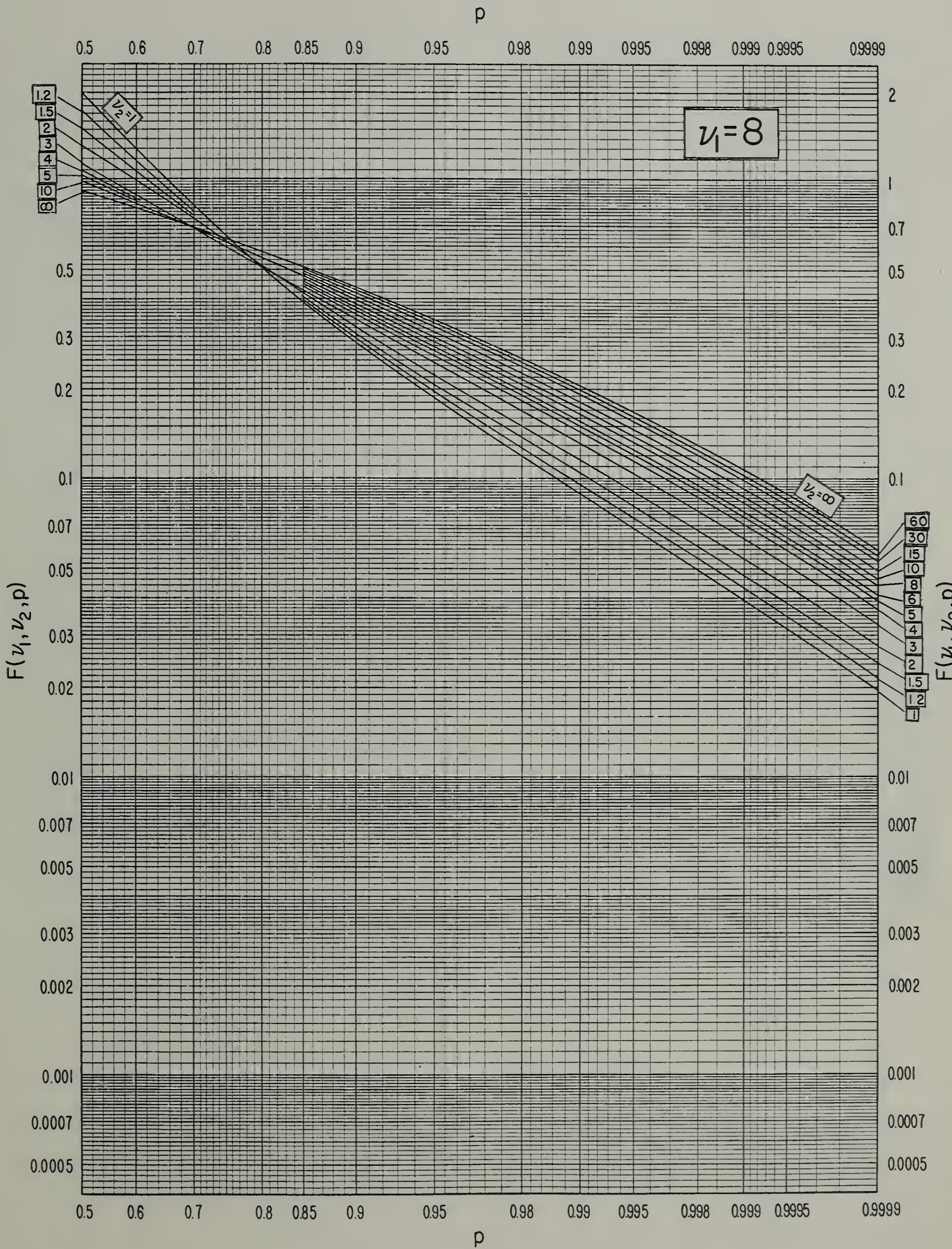
p
0.5 0.6 0.7 0.8 0.85 0.9 0.95 0.98 0.99 0.995 0.998 0.999 0.9995 0.9999
$\nu_1 = 6$
$\nu_2 = 1$
$\nu_2 = \infty$
1.2
1.5
2
3
5
10
∞
30
10
7
5
4
3
2
1.5
1.2
1
2 1 0.7 0.5 0.3 0.2 0.1 0.07 0.05 0.03 0.02 0.01 0.007 0.005 0.003 0.002 0.001 0.0007 0.0005
$F(\nu_1, \nu_2, p)$


p
0.0001 0.0005 0.001 0.002 0.005 0.01 0.02 0.05 0.1 0.15 0.2 0.3 0.4 0.5
$\nu_1 = 7$
$\nu_2=1$
$\nu_2=1.2$
$\nu_2=1.5$
$\nu_2=2$
$\nu_2=3$
$\nu_2=4$
$\nu_2=5$
$\nu_2=6$
$\nu_2=7$
8
9
10
12
15
20
24
30
40
60
120
∞
$F(\nu_1, \nu_2, p)$
2 10^6 7 5 3 2 10^5 7 5 3 2 10^3 7 5
2,000 1,000 700 500 300 200 100 70 50 30 20 10 7 5 3 2 1 0.7 0.5

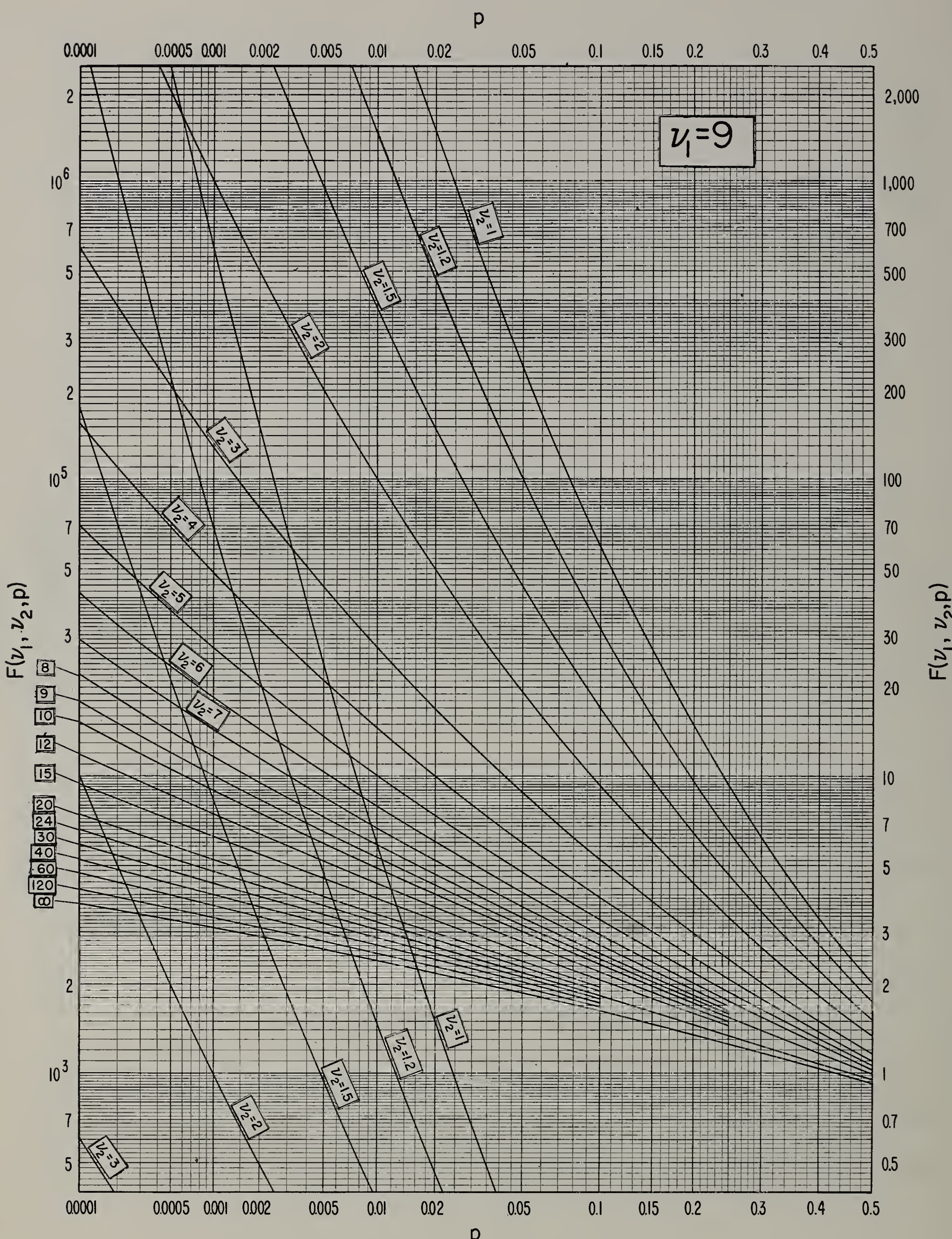

$\nu_1 = 7$
p
0.5
0.6
0.7
0.8
0.85
0.9
0.95
0.98
0.99
0.995
0.998
0.999
0.9995
0.9999
$F(\nu_1, \nu_2, p)$
2
1
0.7
0.5
0.3
0.2
0.1
0.07
0.05
0.03
0.02
0.01
0.007
0.005
0.003
0.002
0.001
0.0007
0.0005
$\nu_2 = 1$
1.2
1.5
2
3
4
5
10
∞
$\nu_2 = \infty$
30
15
10
7
5
4
3
2
1.5
1.2
1

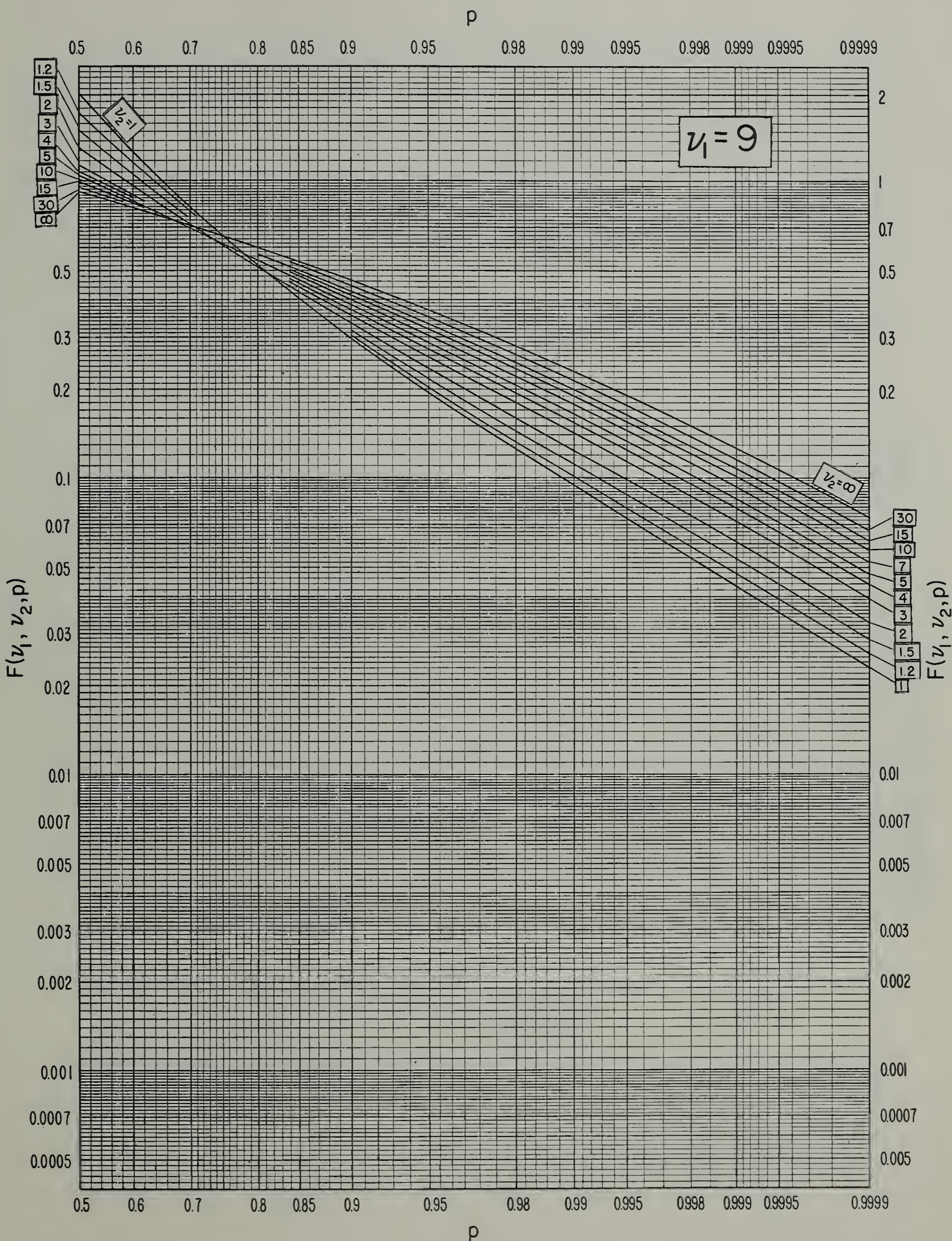


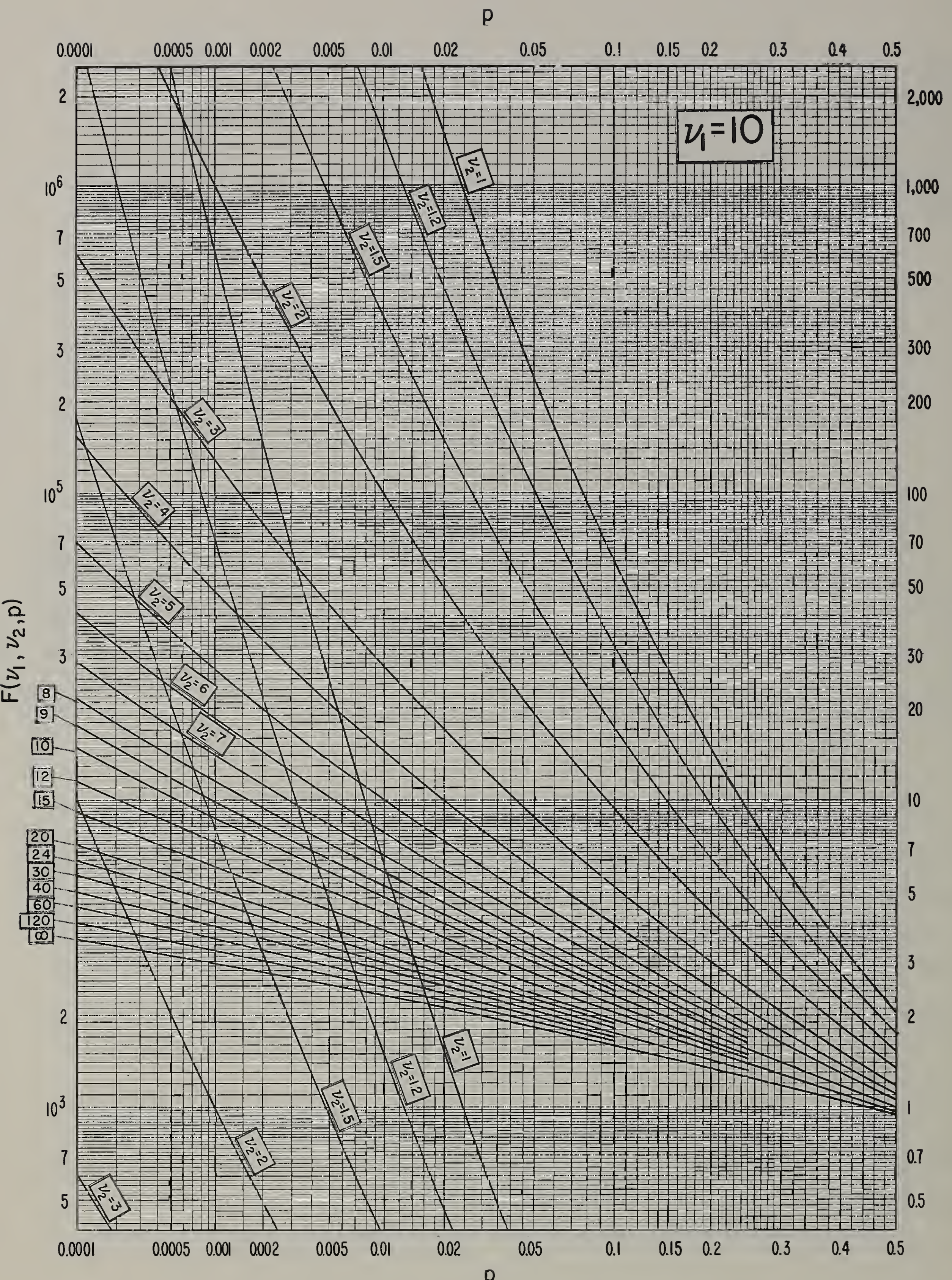
p
0.0001
0.0005
0.001
0.002
0.005
0.01
0.02
0.05
0.1
0.15
0.2
0.3
0.4
0.5
$\nu_1=8$
$F(\nu_1, \nu_2, p)$
10^6
10^5
10^3
2,000
1,000
700
500
300
200
100
70
50
30
20
10
7
5
3
2
1
0.7
0.5
$\nu_2=1$
$\nu_2=1.2$
$\nu_2=1.5$
$\nu_2=2$
$\nu_2=3$
$\nu_2=4$
$\nu_2=5$
$\nu_2=6$
$\nu_2=7$
$\nu_2=8$
9
10
12
15
20
24
30
40
60
120
∞



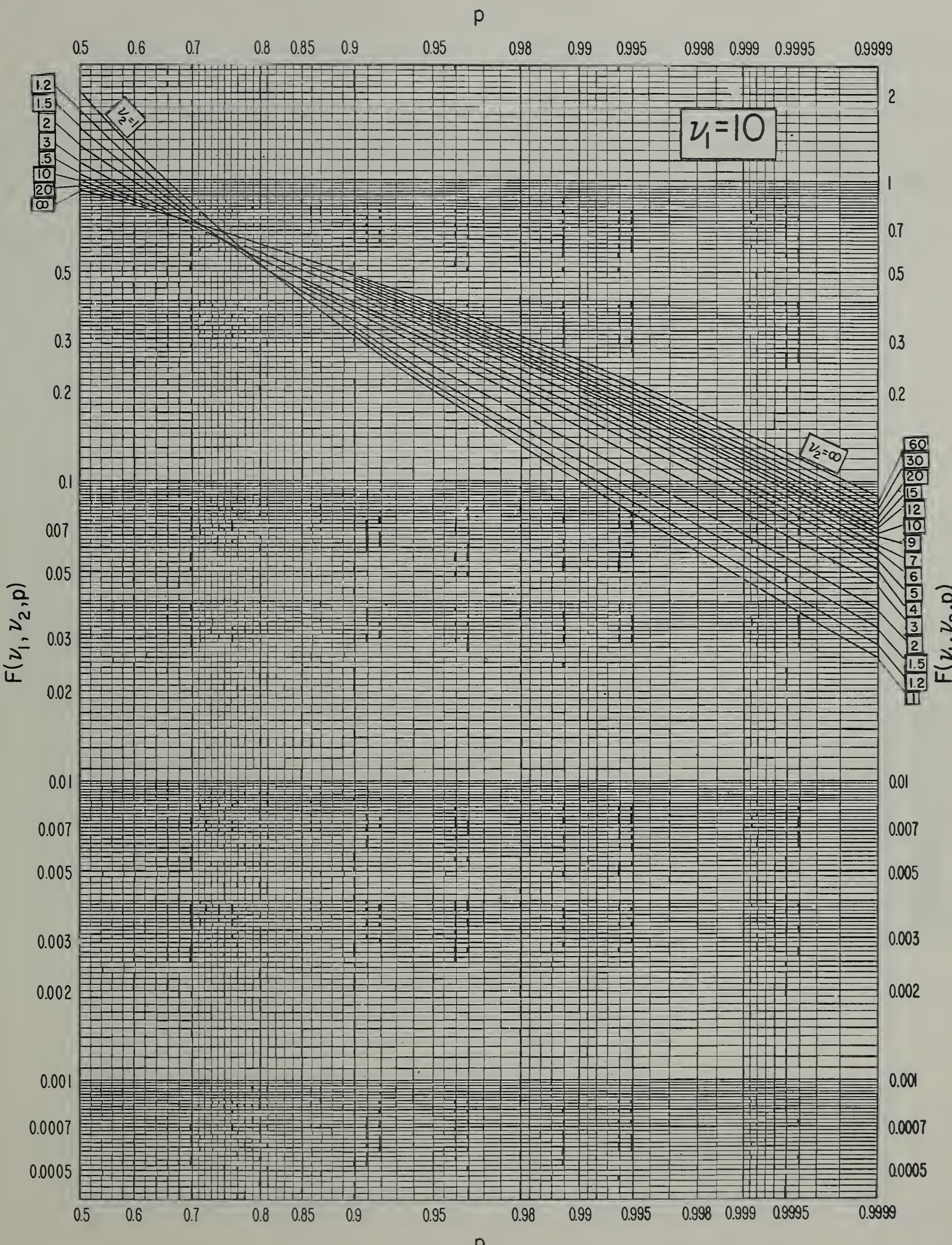
p
0.5 0.6 0.7 0.8 0.85 0.9 0.95 0.98 0.99 0.995 0.998 0.999 0.9995 0.9999
$\nu_1=8$
$\nu_2=1$
$\nu_2=\infty$
1.2
1.5
2
3
4
5
10
∞
60
30
15
10
8
6
5
4
3
2
1.5
1.2
1
$F(\nu_1,\nu_2,p)$
2 1 0.7 0.5 0.3 0.2 0.1 0.07 0.05 0.03 0.02 0.01 0.007 0.005 0.003 0.002 0.001 0.0007 0.0005

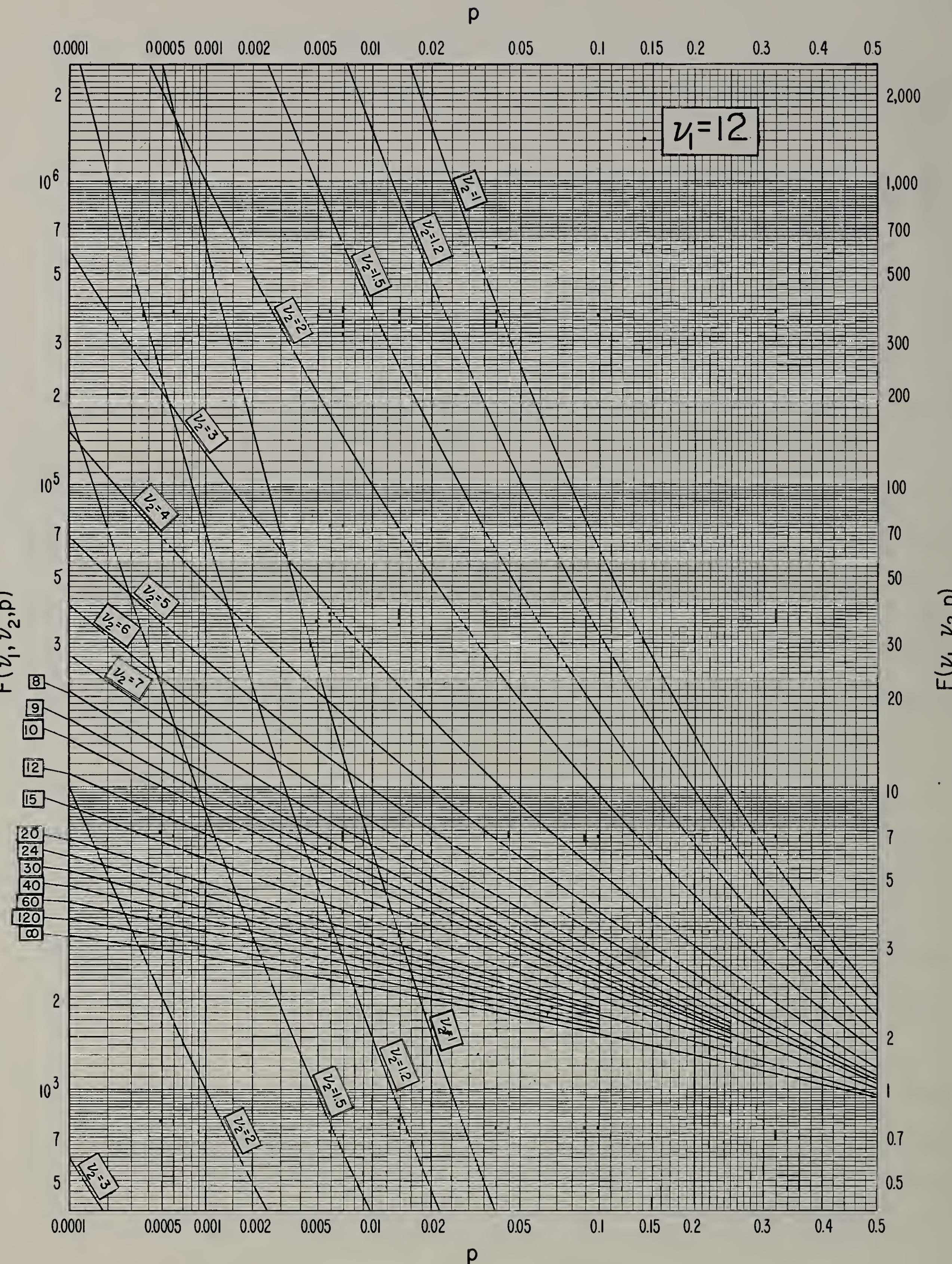

$\nu_1=9$
p
$F(\nu_1, \nu_2, p)$
$\nu_2=1$
$\nu_2=1.2$
$\nu_2=1.5$
$\nu_2=2$
$\nu_2=3$
$\nu_2=4$
$\nu_2=5$
$\nu_2=6$
$\nu_2=7$
8
9
10
12
15
20
24
30
40
60
120
∞
0.0001 0.0005 0.001 0.002 0.005 0.01 0.02 0.05 0.1 0.15 0.2 0.3 0.4 0.5
2,000 1,000 700 500 300 200 100 70 50 30 20 10 7 5 3 2 1 0.7 0.5
10^6 10^5 10^3

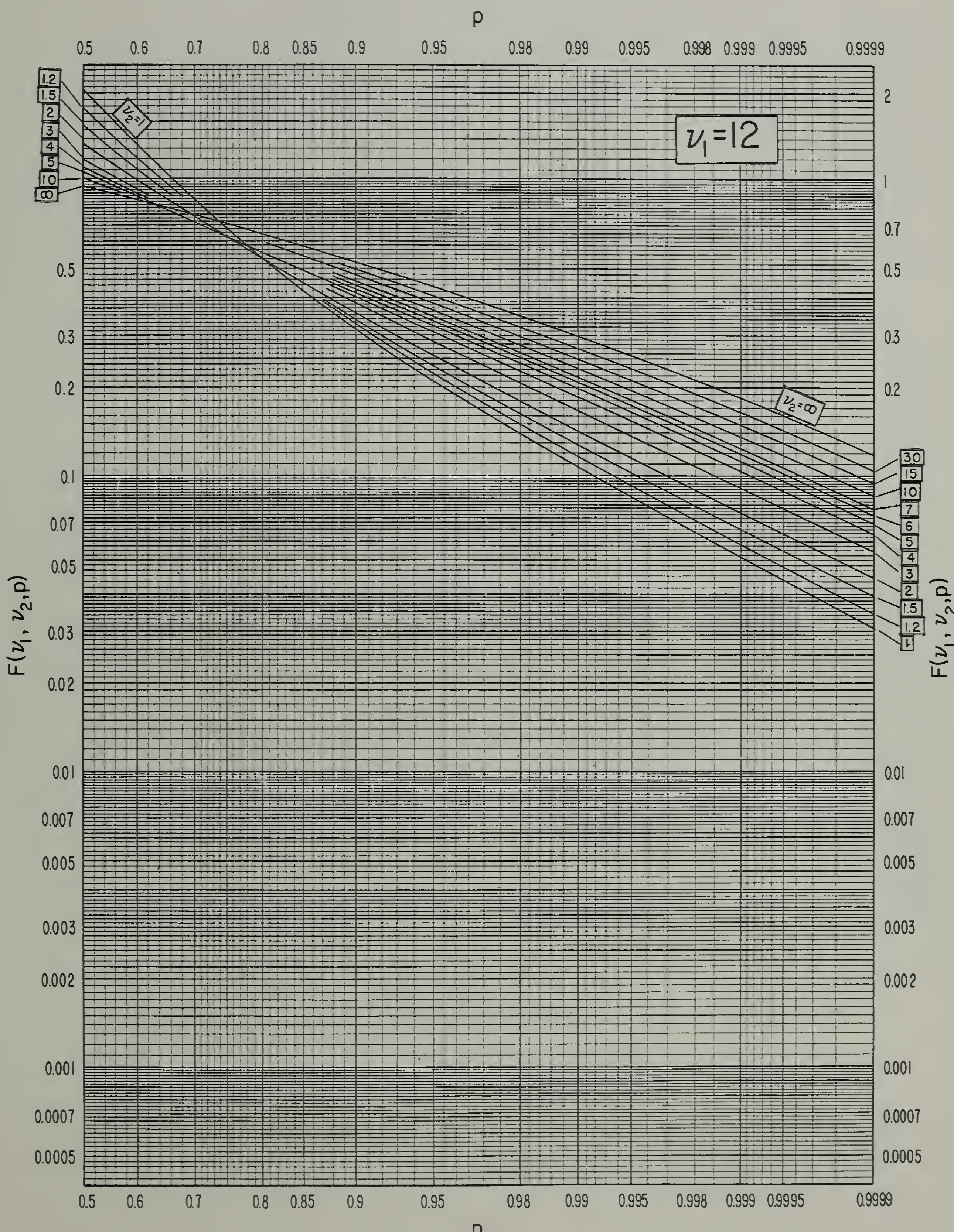

p
0.5 0.6 0.7 0.8 0.85 0.9 0.95 0.98 0.99 0.995 0.998 0.999 0.9995 0.9999
$\nu_1 = 9$
$\nu_2 = 1$
$\nu_2 = \infty$
1.2 1.5 2 3 4 5 10 15 30 ∞
30 15 10 7 5 4 3 2 1.5 1.2 1
$F(\nu_1, \nu_2, p)$
2 1 0.7 0.5 0.3 0.2 0.1 0.07 0.05 0.03 0.02 0.01 0.007 0.005 0.003 0.002 0.001 0.0007 0.0005
0.005

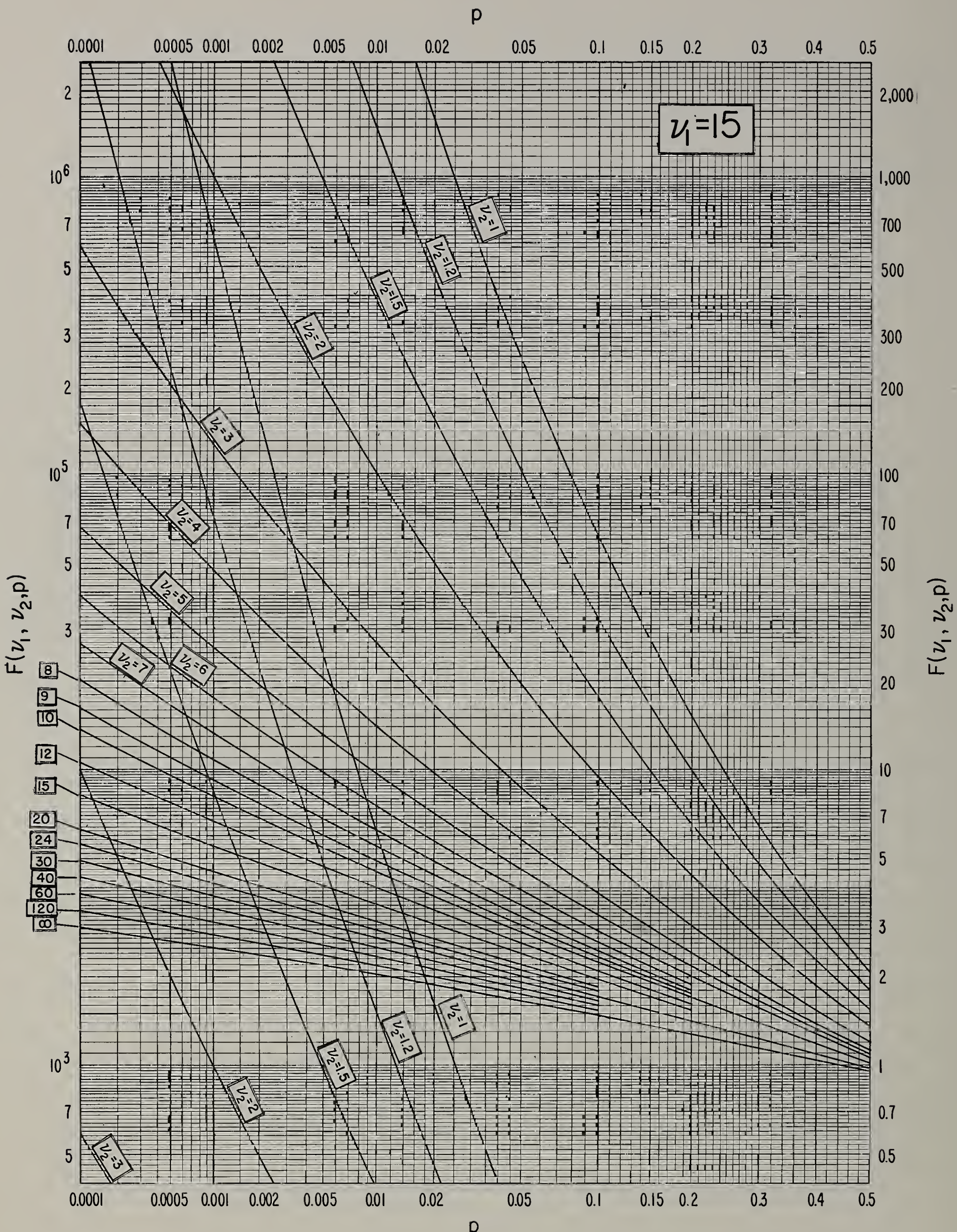


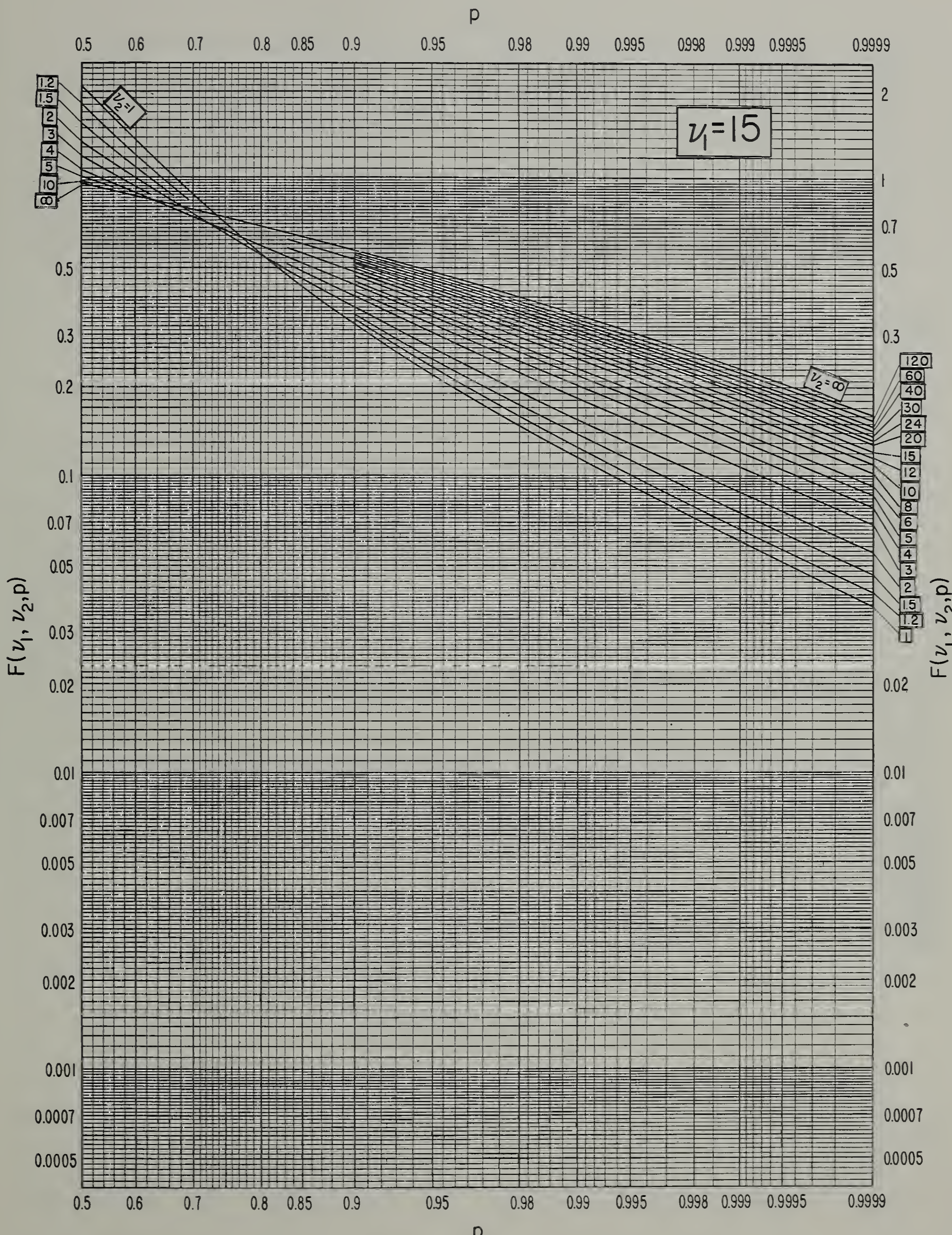
p
0.0001 0.0005 0.001 0.002 0.005 0.01 0.02 0.05 0.1 0.15 0.2 0.3 0.4 0.5
$\nu_1=10$
$\nu_2=1$
$\nu_2=1.2$
$\nu_2=1.5$
$\nu_2=2$
$\nu_2=3$
$\nu_2=4$
$\nu_2=5$
$\nu_2=6$
$\nu_2=7$
8
9
10
12
15
20
24
30
40
60
120
∞
$F(\nu_1, \nu_2, p)$
2,000 1,000 700 500 300 200 100 70 50 30 20 10 7 5 3 2 1 0.7 0.5
2 10^6 7 5 3 2 10^5 7 5 3 2 10^3 7 5


p
0.5 0.6 0.7 0.8 0.85 0.9 0.95 0.98 0.99 0.995 0.998 0.999 0.9995 0.9999
$\nu_1=10$
$\nu_2=1$
$\nu_2=\infty$
1.2 1.5 2 3 5 10 20 ∞
60 30 20 15 12 10 9 7 6 5 4 3 2 1.5 1.2 1
2 1 0.7 0.5 0.3 0.2 0.1 0.07 0.05 0.03 0.02 0.01 0.007 0.005 0.003 0.002 0.001 0.0007 0.0005
$F(\nu_1, \nu_2, p)$

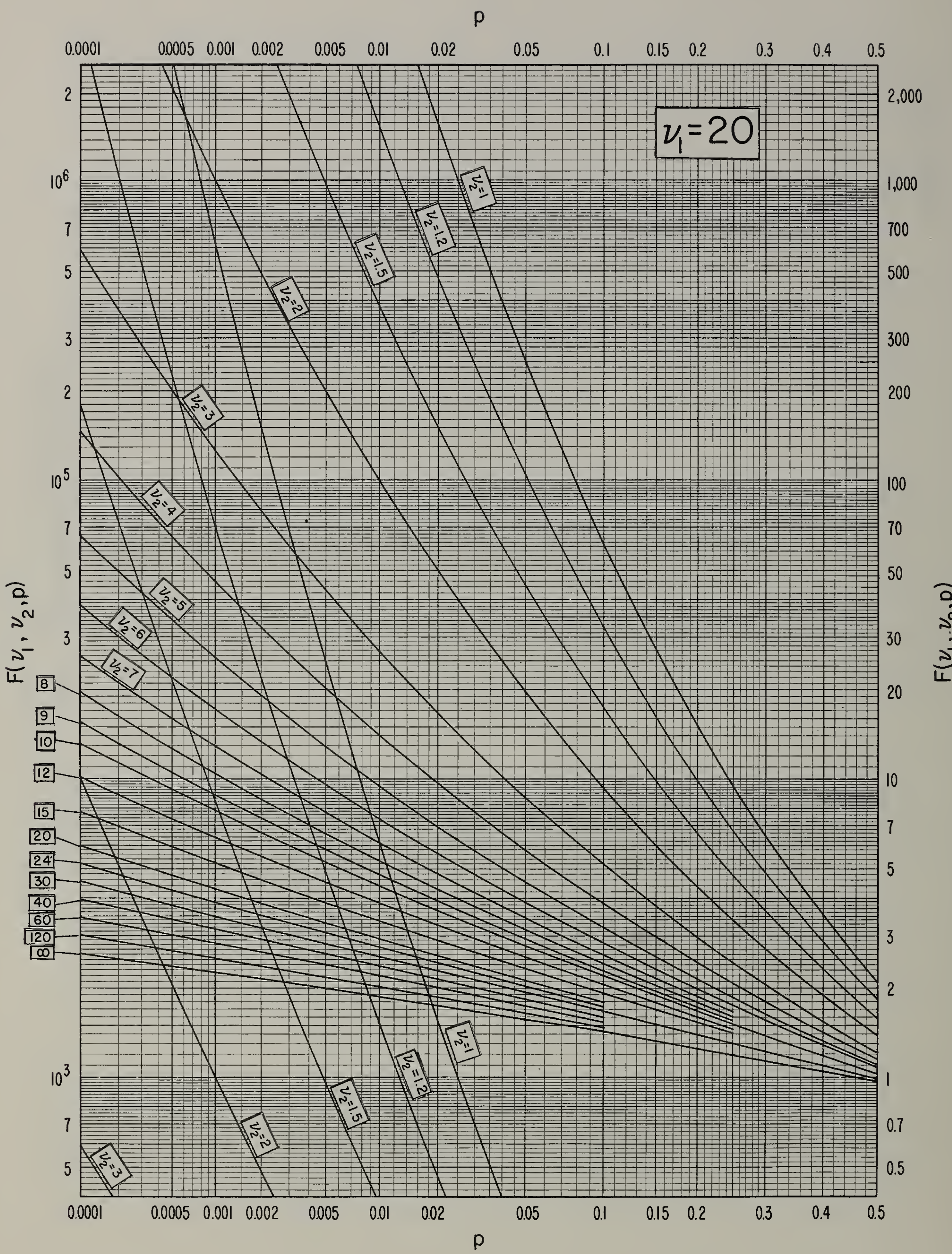

p
0.0001 0.0005 0.001 0.002 0.005 0.01 0.02 0.05 0.1 0.15 0.2 0.3 0.4 0.5
$\nu_1=12$
$F(\nu_1, \nu_2, p)$
$\nu_2=1$
$\nu_2=1.2$
$\nu_2=1.5$
$\nu_2=2$
$\nu_2=3$
$\nu_2=4$
$\nu_2=5$
$\nu_2=6$
$\nu_2=7$
8
9
10
12
15
20
24
30
40
60
120
∞
2
10^6
7
5
3
2
10^5
7
5
3
2
10^3
7
5
2,000
1,000
700
500
300
200
100
70
50
30
20
10
7
5
3
2
1
0.7
0.5


p
0.5 0.6 0.7 0.8 0.85 0.9 0.95 0.98 0.99 0.995 0.998 0.999 0.9995 0.9999
$\nu_1=12$
$\nu_2=1$
$\nu_2=\infty$
1.2
1.5
2
3
4
5
10
∞
30
15
10
7
6
5
4
3
2
1.5
1.2
1
2
1
0.7
0.5
0.3
0.2
0.1
0.07
0.05
0.03
0.02
0.01
0.007
0.005
0.003
0.002
0.001
0.0007
0.0005
$F(\nu_1, \nu_2, p)$

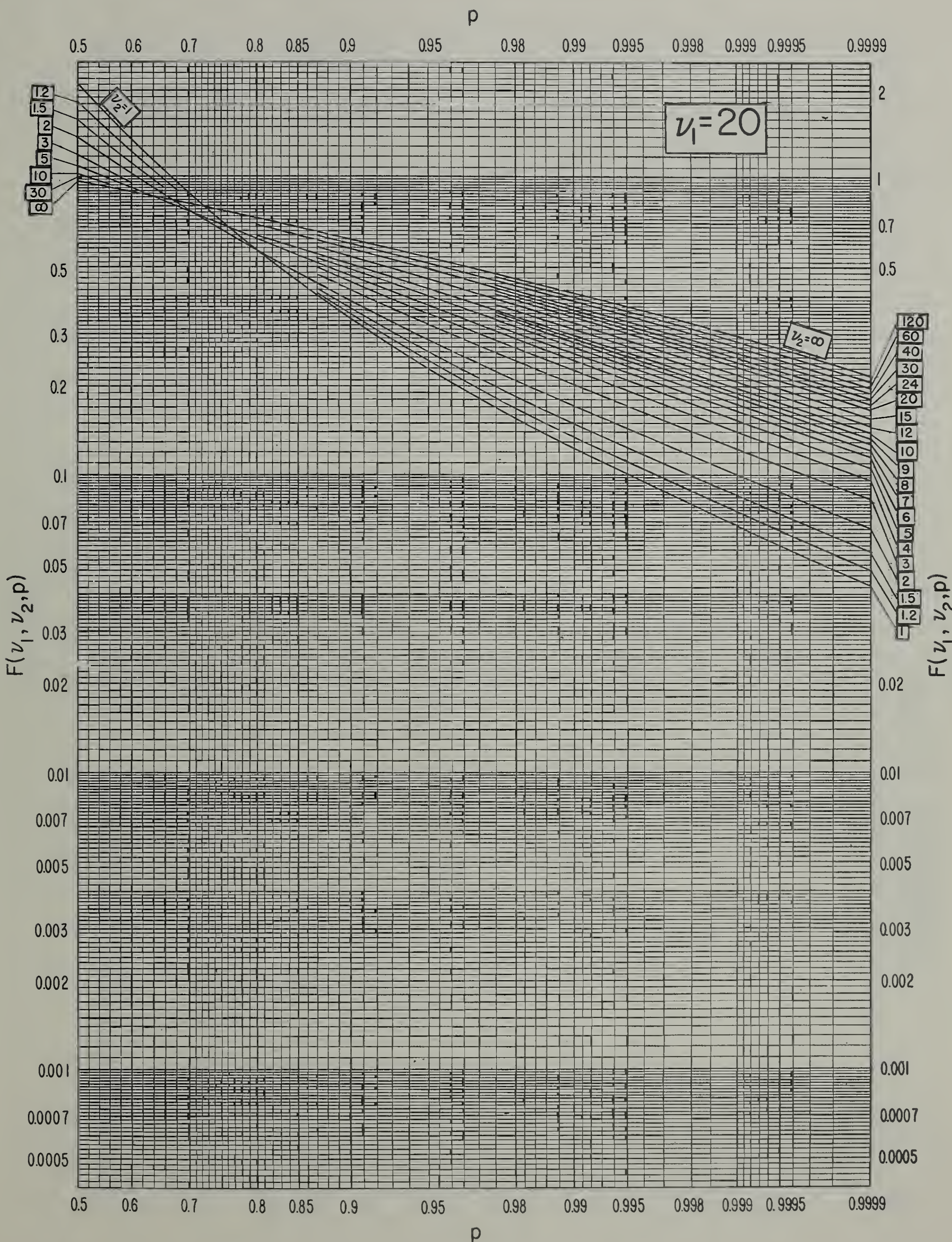

p
$\nu_1=15$
$\nu_2=1$
$\nu_2=1.2$
$\nu_2=1.5$
$\nu_2=2$
$\nu_2=3$
$\nu_2=4$
$\nu_2=5$
$\nu_2=6$
$\nu_2=7$
8
9
10
12
15
20
24
30
40
60
120
∞
$F(\nu_1, \nu_2, p)$
0.0001 0.0005 0.001 0.002 0.005 0.01 0.02 0.05 0.1 0.15 0.2 0.3 0.4 0.5
2 10^6 7 5 3 2 10^5 7 5 3 10^3 7 5
2,000 1,000 700 500 300 200 100 70 50 30 20 10 7 5 3 2 1 0.7 0.5



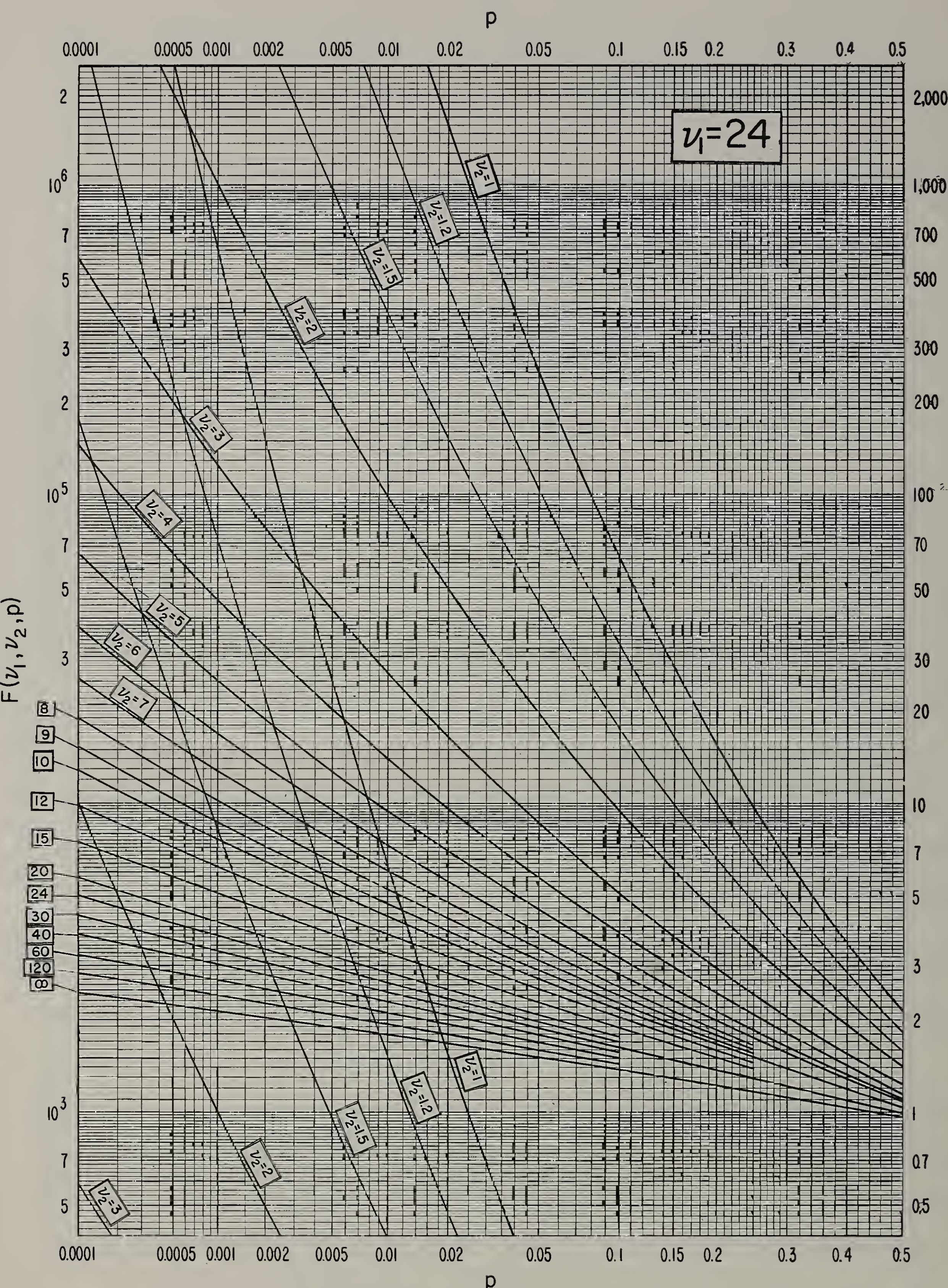
p
0.5
0.6
0.7
0.8
0.85
0.9
0.95
0.98
0.99
0.995
0.998
0.999
0.9995
0.9999
$\nu_1 = 15$
$\nu_2 = 1$
$\nu_2 = \infty$
1.2
1.5
2
3
4
5
10
∞
120
60
40
30
24
20
15
12
10
8
6
5
4
3
2
1.5
1.2
1
$F(\nu_1, \nu_2, p)$
2
1
0.7
0.5
0.3
0.2
0.1
0.07
0.05
0.03
0.02
0.01
0.007
0.005
0.003
0.002
0.001
0.0007
0.0005



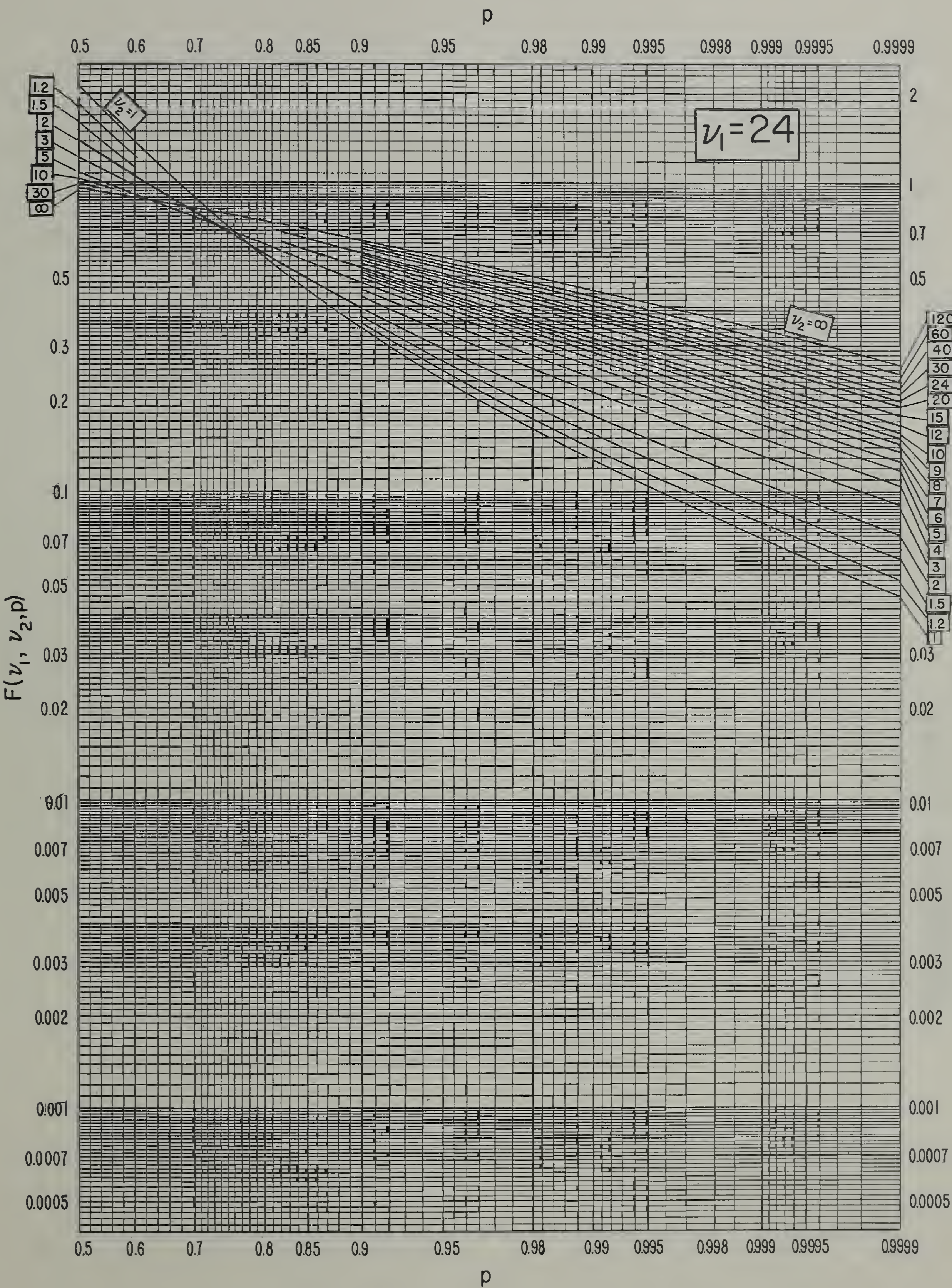
p
0.0001
0.0005
0.001
0.002
0.005
0.01
0.02
0.05
0.1
0.15
0.2
0.3
0.4
0.5
$\nu_1 = 20$
$F(\nu_1, \nu_2, p)$
$\nu_2=1$
$\nu_2=1.2$
$\nu_2=1.5$
$\nu_2=2$
$\nu_2=3$
$\nu_2=4$
$\nu_2=5$
$\nu_2=6$
$\nu_2=7$
8
9
10
12
15
20
24
30
40
60
120
∞
2
10^6
7
5
3
10^5
10^3
2,000
1,000
700
500
300
200
100
70
50
30
20
10
1
0.7
0.5

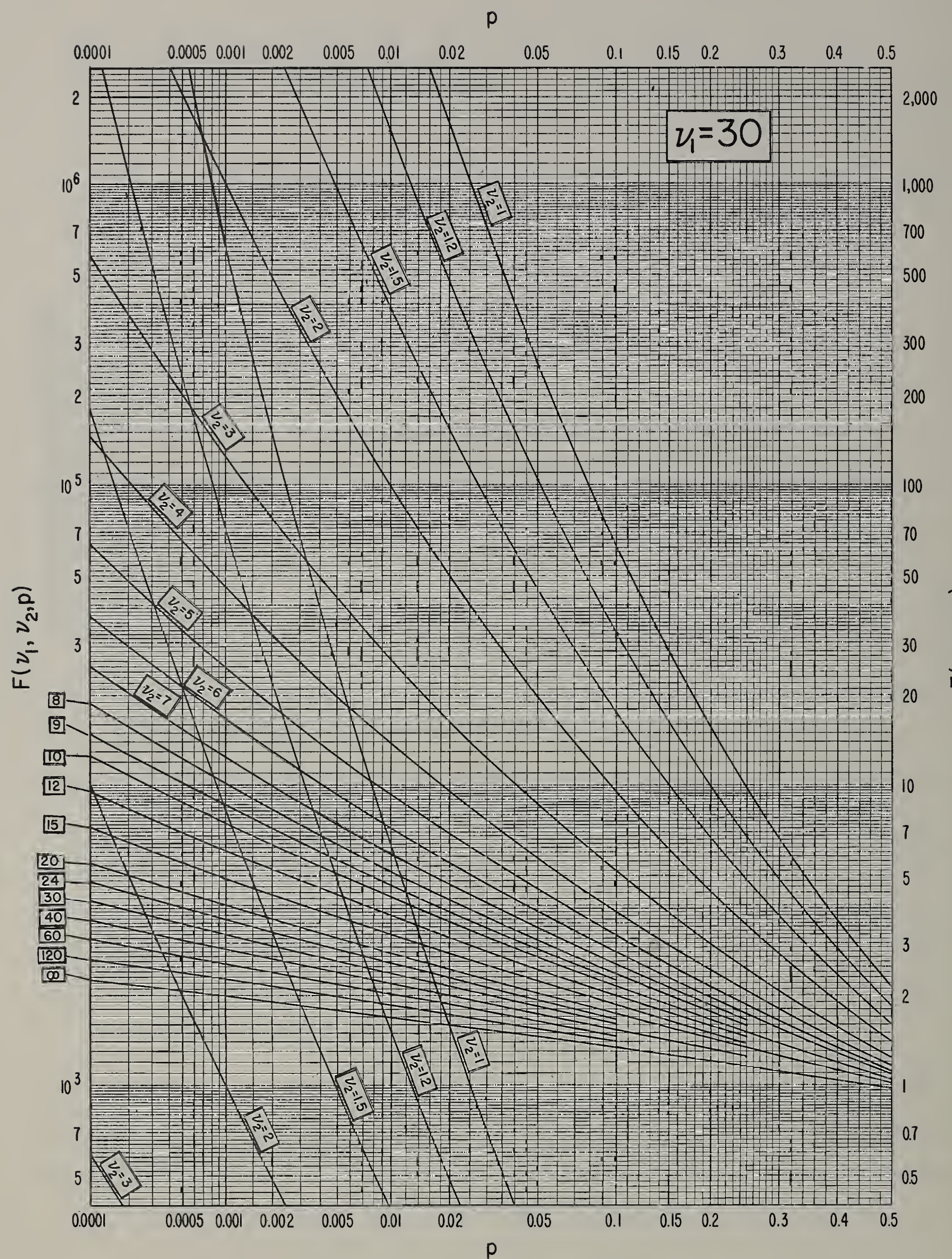


p
0.5
0.6
0.7
0.8
0.85
0.9
0.95
0.98
0.99
0.995
0.998
0.999
0.9995
0.9999
$\nu_1 = 20$
$\nu_2 = 1$
$\nu_2 = \infty$
$F(\nu_1, \nu_2, p)$

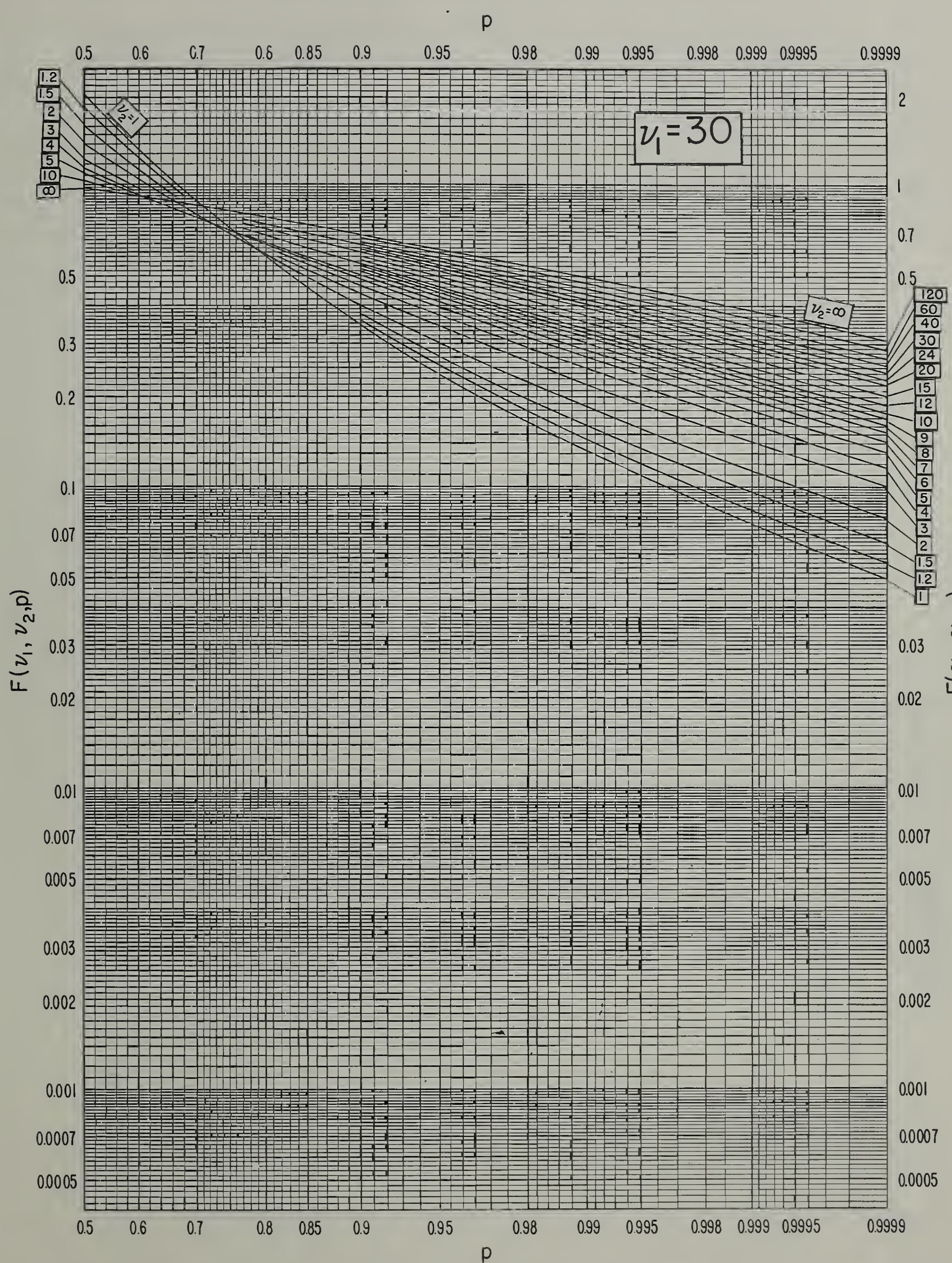


p
0.0001
0.0005
0.001
0.002
0.005
0.01
0.02
0.05
0.1
0.15
0.2
0.3
0.4
0.5
$\nu_1=24$
$F(\nu_1, \nu_2, p)$
$\nu_2=1$
$\nu_2=1.2$
$\nu_2=1.5$
$\nu_2=2$
$\nu_2=3$
$\nu_2=4$
$\nu_2=5$
$\nu_2=6$
$\nu_2=7$
8
9
10
12
15
20
24
30
40
60
120
∞
2
10^6
7
5
3
2
10^5
7
5
3
10^3
7
5
2,000
1,000
700
500
300
200
100
70
50
30
20
10
7
5
3
2
1
0.7
0.5

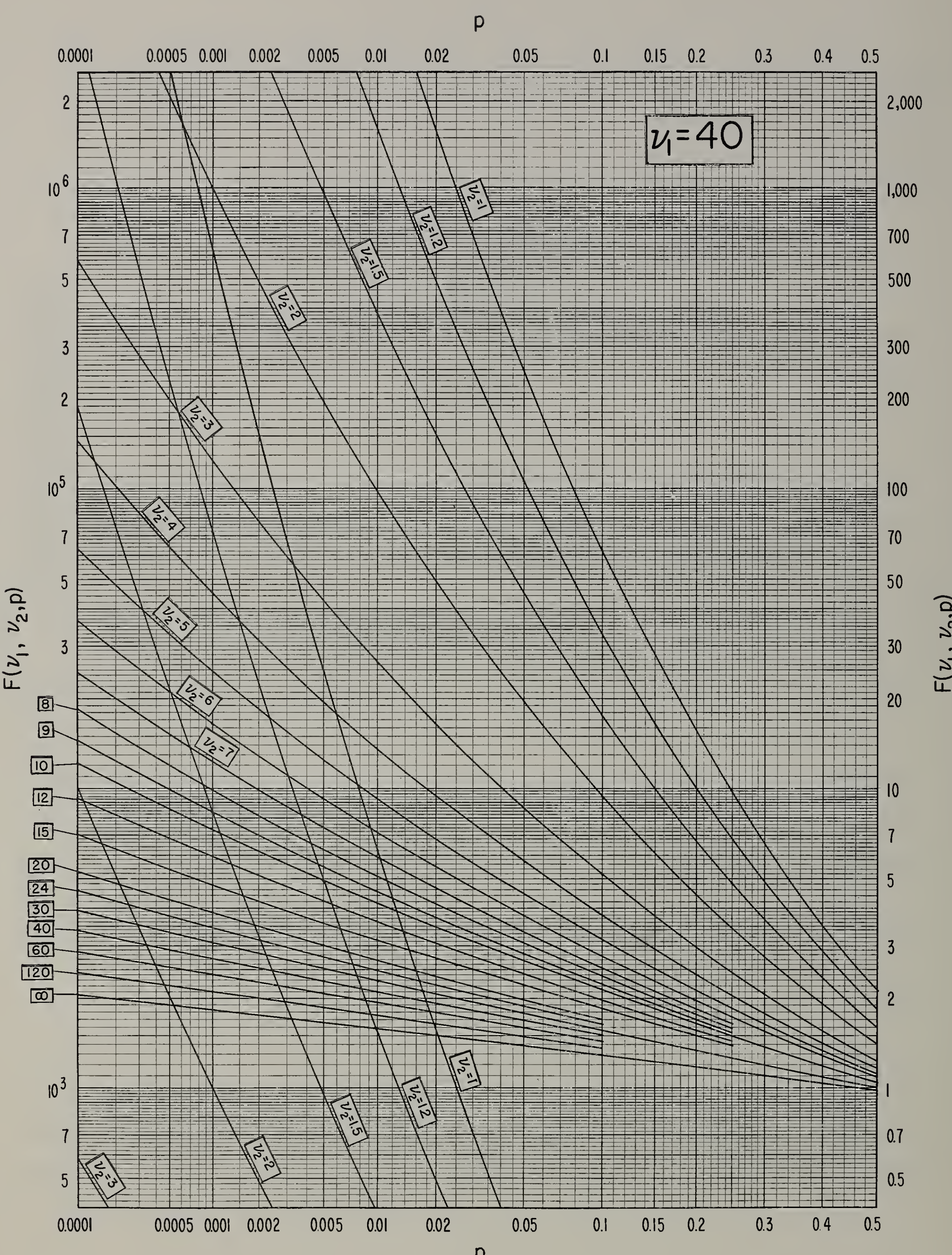

p
0.5 0.6 0.7 0.8 0.85 0.9 0.95 0.98 0.99 0.995 0.998 0.999 0.9995 0.9999
$\nu_1 = 24$
$\nu_2 = 1$
$\nu_2 = \infty$
1.2 1.5 2 3 5 10 30 ∞
120 60 40 30 24 20 15 12 10 9 8 7 6 5 4 3 2 1.5 1.2 1
$F(\nu_1, \nu_2, p)$
2 1 0.7 0.5 0.3 0.2 0.1 0.07 0.05 0.03 0.02 0.01 0.007 0.005 0.003 0.002 0.001 0.0007 0.0005



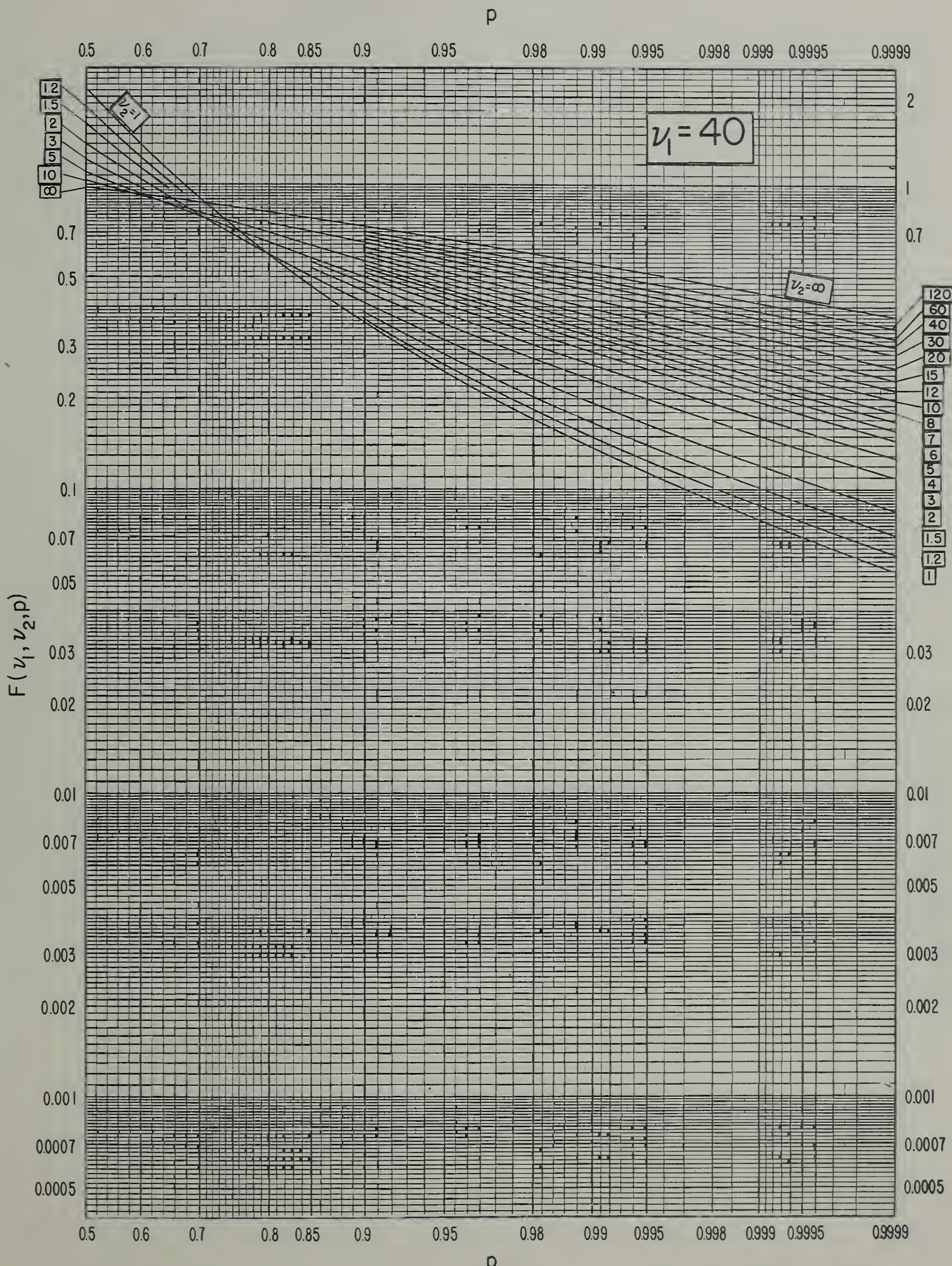
p
0.0001 0.0005 0.001 0.002 0.005 0.01 0.02 0.05 0.1 0.15 0.2 0.3 0.4 0.5
$\nu_1=30$
$F(\nu_1, \nu_2, p)$
$\nu_2=1$
$\nu_2=1.2$
$\nu_2=1.5$
$\nu_2=2$
$\nu_2=3$
$\nu_2=4$
$\nu_2=5$
$\nu_2=6$
$\nu_2=7$
8
9
10
12
15
20
24
30
40
60
120
∞
2,000
1,000
700
500
300
200
100
70
50
30
20
10
7
5
3
2
1
0.7
0.5

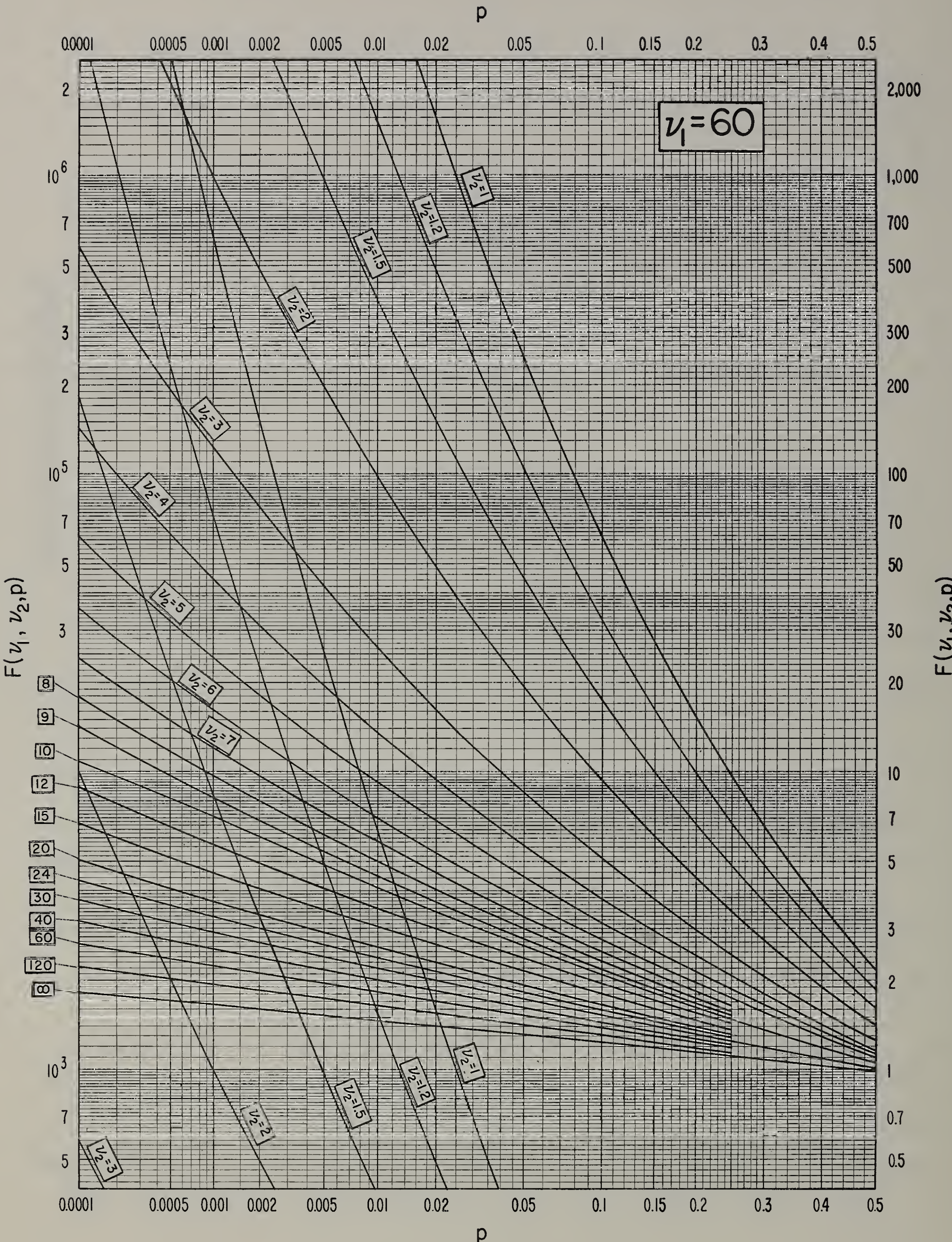


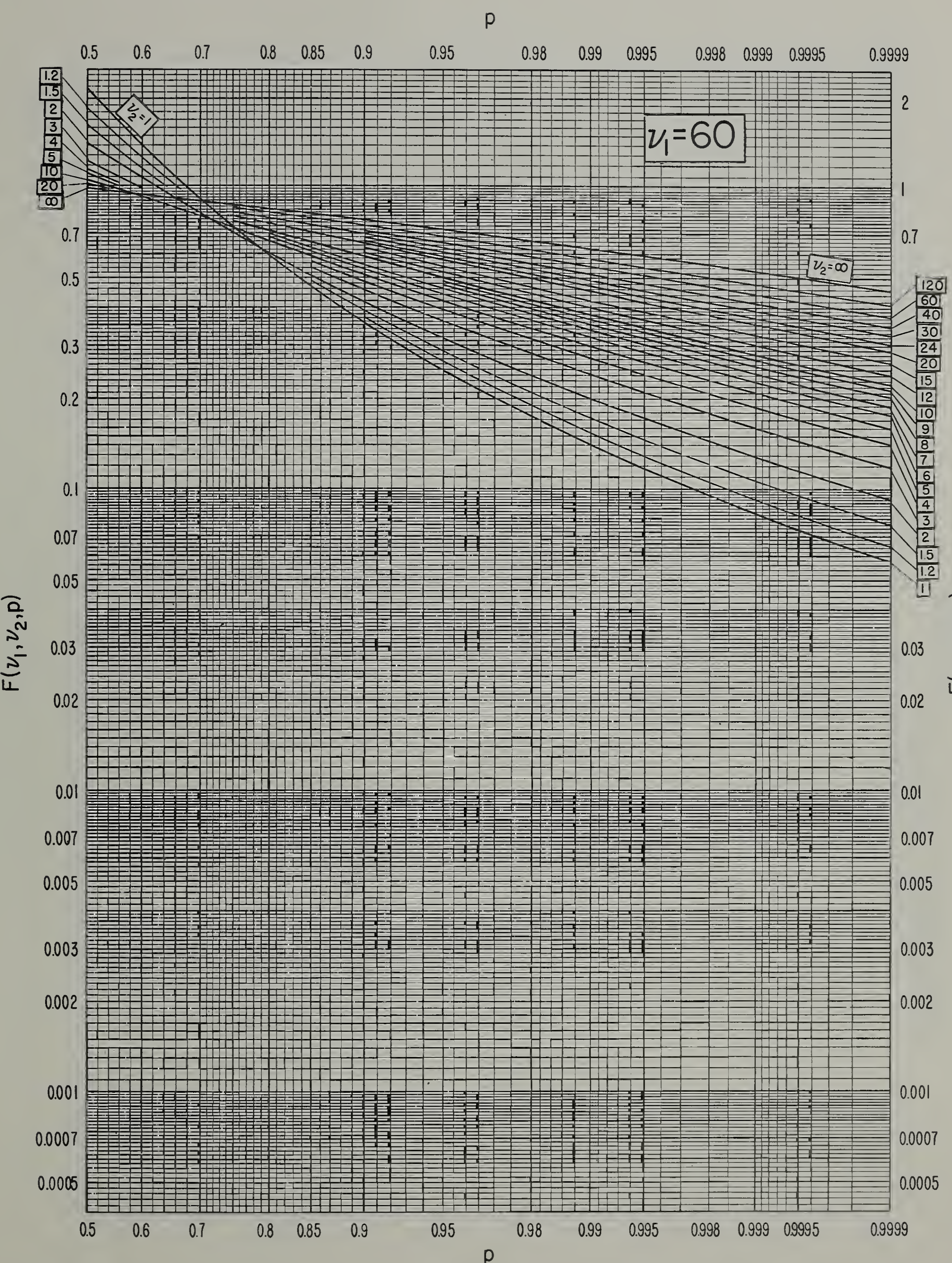
p
0.5
0.6
0.7
0.6
0.85
0.9
0.95
0.98
0.99
0.995
0.998
0.999
0.9995
0.9999
$\nu_1 = 30$
$\nu_2 = 1$
$\nu_2 = \infty$
$F(\nu_1, \nu_2, p)$
2
1
0.7
0.5
0.3
0.2
0.1
0.07
0.05
0.03
0.02
0.01
0.007
0.005
0.003
0.002
0.001
0.0007
0.0005
1.2
1.5
2
3
4
5
10
∞
120
60
40
30
24
20
15
12
10
9
8
7
6
5
4
3
2
1.5
1.2
1
0.8

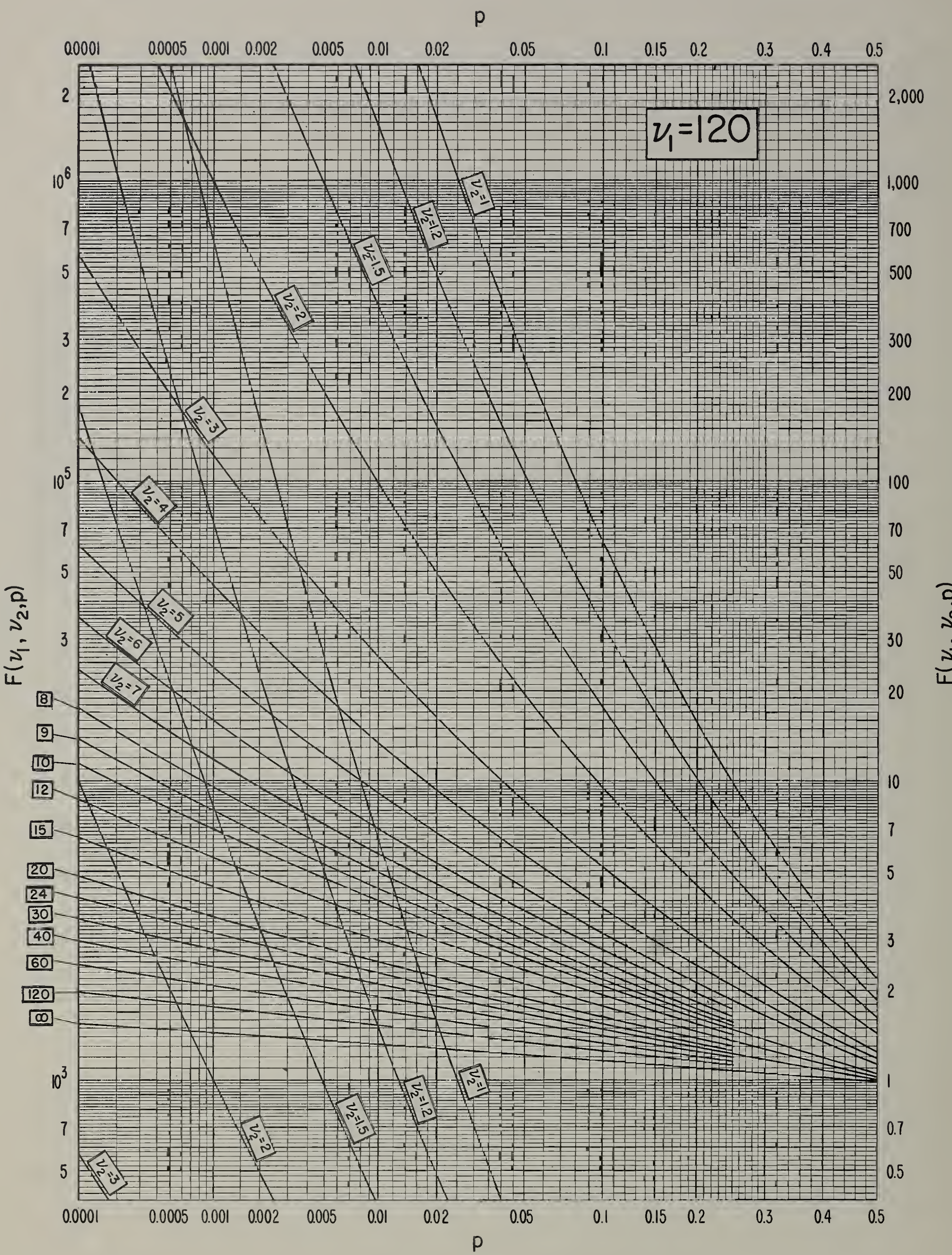


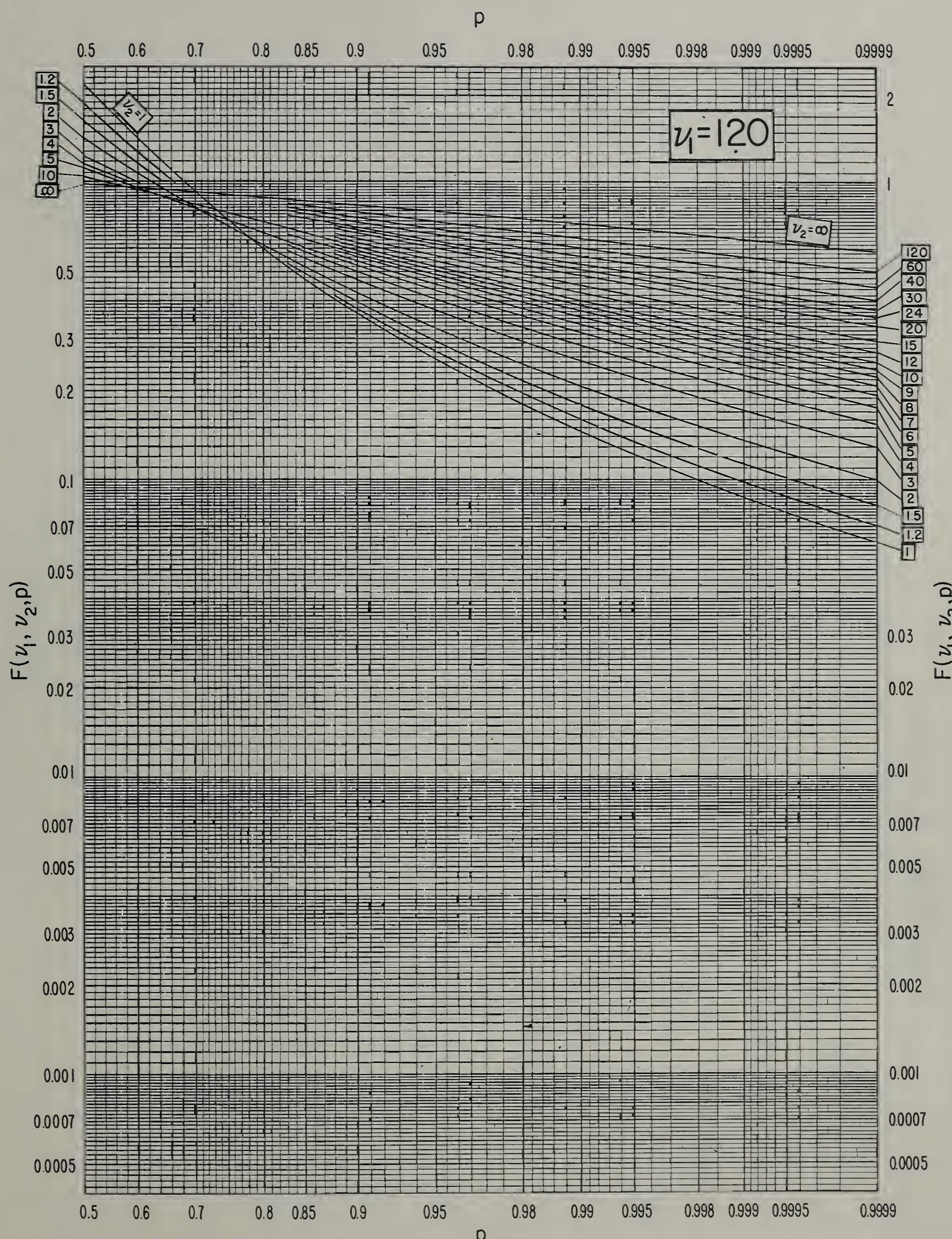
$\nu_1 = 40$
p
0.0001 0.0005 0.001 0.002 0.005 0.01 0.02 0.05 0.1 0.15 0.2 0.3 0.4 0.5
$F(\nu_1, \nu_2, p)$
2 10^6 7 5 3 2 10^5 7 5 3 10^3 7 5
2,000 1,000 700 500 300 200 100 70 50 30 20 10 7 5 3 2 1 0.7 0.5
$\nu_2=1$
$\nu_2=1.2$
$\nu_2=1.5$
$\nu_2=2$
$\nu_2=3$
$\nu_2=4$
$\nu_2=5$
$\nu_2=6$
$\nu_2=7$
8 9 10 12 15 20 24 30 40 60 120 ∞

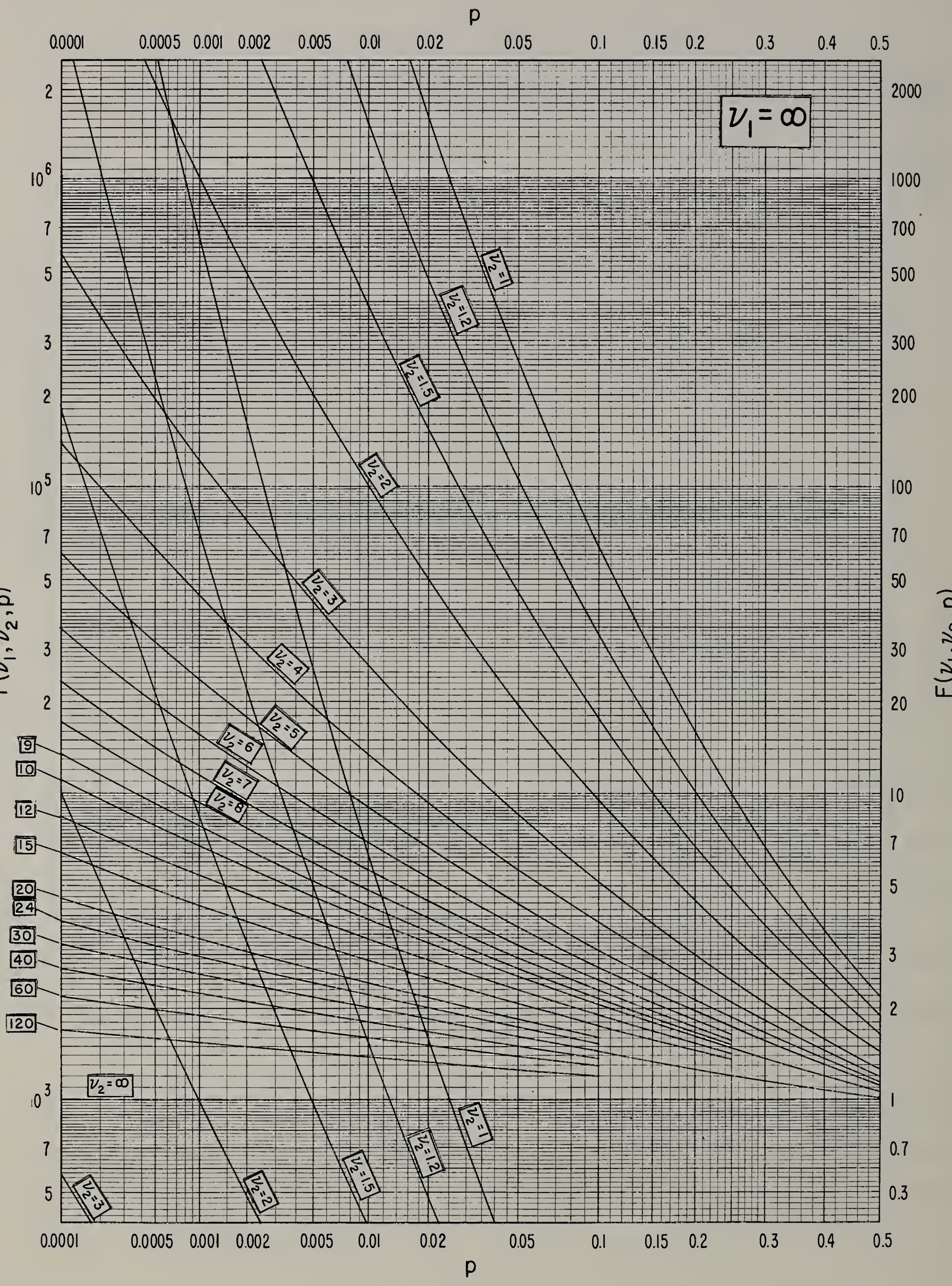

p
0.5
0.6
0.7
0.8
0.85
0.9
0.95
0.98
0.99
0.995
0.998
0.999
0.9995
0.9999
$\nu_1 = 40$
$\nu_2 = 1$
$\nu_2 = \infty$
1.2
1.5
2
3
5
10
∞
120
60
40
30
20
15
12
10
8
7
6
5
4
3
2
1.5
1.2
1
2
1
0.7
0.5
0.3
0.2
0.1
0.07
0.05
0.03
0.02
0.01
0.007
0.005
0.003
0.002
0.001
0.0007
0.0005
$F(\nu_1, \nu_2, p)$


p
0.0001 0.0005 0.001 0.002 0.005 0.01 0.02 0.05 0.1 0.15 0.2 0.3 0.4 0.5
$\nu_1 = 60$
$\nu_2 = 1$
$\nu_2 = 1.2$
$\nu_2 = 1.5$
$\nu_2 = 2$
$\nu_2 = 3$
$\nu_2 = 4$
$\nu_2 = 5$
$\nu_2 = 6$
$\nu_2 = 7$
8
9
10
12
15
20
24
30
40
60
120
∞
$F(\nu_1, \nu_2, p)$
2 10^6 7 5 3 2 10^5 7 5 3 10^3 7 5
2,000 1,000 700 500 300 200 100 70 50 30 20 10 7 5 3 2 1 0.7 0.5

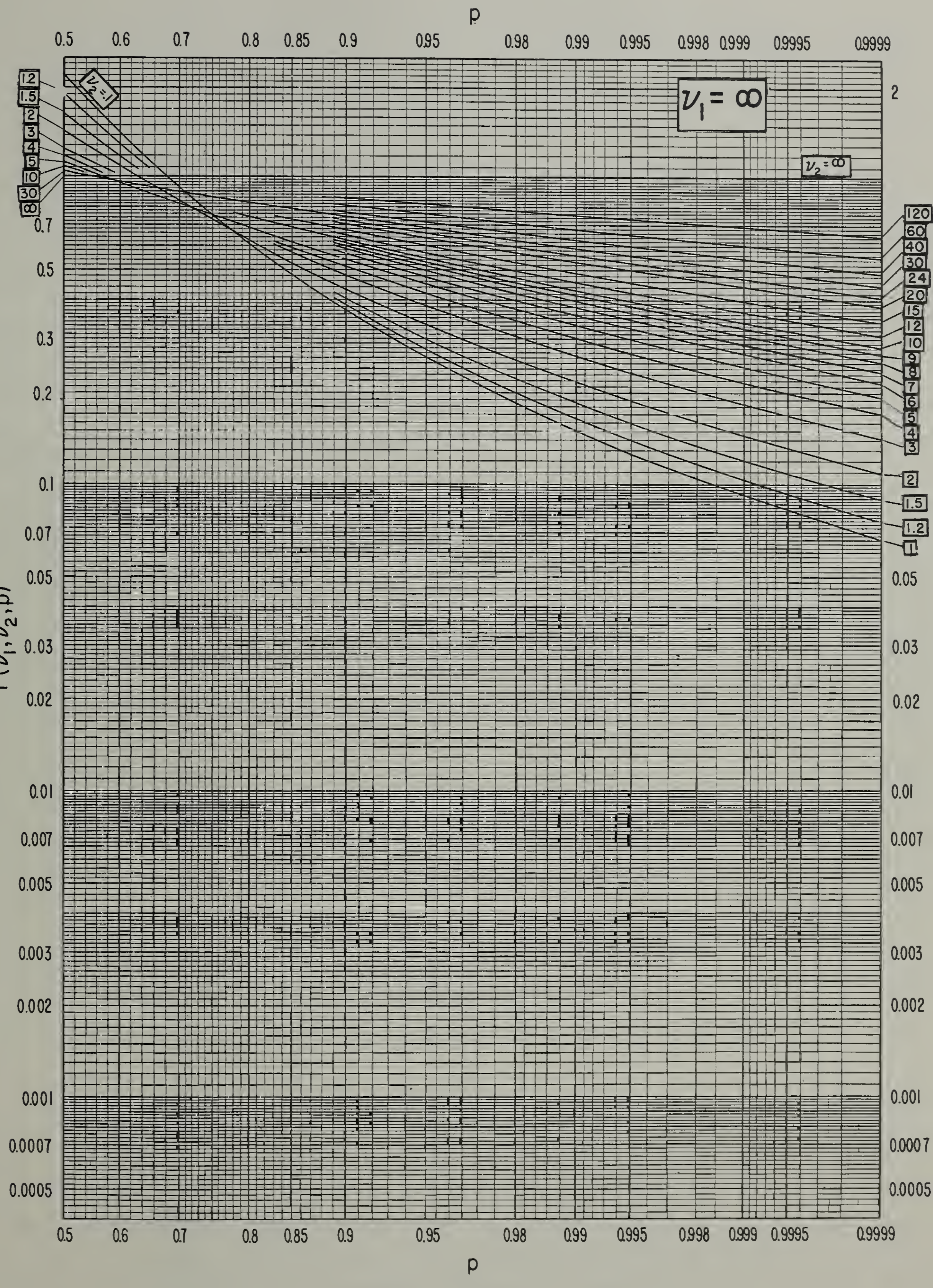

p
0.5 0.6 0.7 0.8 0.85 0.9 0.95 0.98 0.99 0.995 0.998 0.999 0.9995 0.9999
$\nu_1 = 60$
$\nu_2 = 1$
$\nu_2 = \infty$
1.2 1.5 2 3 4 5 10 20 ∞
120 60 40 30 24 20 15 12 10 9 8 7 6 5 4 3 2 1.5 1.2 1
$F(\nu_1, \nu_2, p)$
2 1 0.7 0.5 0.3 0.2 0.1 0.07 0.05 0.03 0.02 0.01 0.007 0.005 0.003 0.002 0.001 0.0007 0.0005


p
0.0001
0.0005
0.001
0.002
0.005
0.01
0.02
0.05
0.1
0.15
0.2
0.3
0.4
0.5
ν_1=120
ν_2=1
ν_2=1.2
ν_2=1.5
ν_2=2
ν_2=3
ν_2=4
ν_2=5
ν_2=6
ν_2=7
8
9
10
12
15
20
24
30
40
60
120
∞
$F(\nu_1, \nu_2, p)$
2,000
1,000
700
500
300
200
100
70
50
30
20
10
7
5
3
2
1
0.7
0.5


p
0.5 0.6 0.7 0.8 0.85 0.9 0.95 0.98 0.99 0.995 0.998 0.999 0.9995 0.9999
$\nu_1 = 120$
$\nu_2 = 1$
$\nu_2 = \infty$
1.2 1.5 2 3 4 5 10 ∞
120 60 40 30 24 20 15 12 10 9 8 7 6 5 4 3 2 1.5 1.2 1
2 1 0.5 0.3 0.2 0.1 0.07 0.05 0.03 0.02 0.01 0.007 0.005 0.003 0.002 0.001 0.0007 0.0005
$F(\nu_1, \nu_2, p)$



p
0.0001
0.0005
0.001
0.002
0.005
0.01
0.02
0.05
0.1
0.15
0.2
0.3
0.4
0.5
$\nu_1 = \infty$
$F(\nu_1, \nu_2, p)$
10^6
10^5
10^3
2000
1000
700
500
300
200
100
70
50
30
20
10
7
5
3
2
1
0.7
0.3
$\nu_2 = 1$
$\nu_2 = 1.2$
$\nu_2 = 1.5$
$\nu_2 = 2$
$\nu_2 = 3$
$\nu_2 = 4$
$\nu_2 = 5$
$\nu_2 = 6$
$\nu_2 = 7$
$\nu_2 = 8$
9
10
12
15
20
24
30
40
60
120
$\nu_2 = \infty$


p
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0.6
0.7
0.8
0.85
0.9
0.95
0.98
0.99
0.995
0.998
0.999
0.9995
0.9999
$\nu_1 = \infty$
$\nu_2 = 1$
$\nu_2 = \infty$
1.2
1.5
2
3
4
5
10
30
∞
120
60
40
30
24
20
15
12
10
9
8
7
6
5
4
3
2
1.5
1.2
1
2
0.7
0.5
0.3
0.2
0.1
0.07
0.05
0.03
0.02
0.01
0.007
0.005
0.003
0.002
0.001
0.0007
0.0005
$F(\nu_1, \nu_2, p)$

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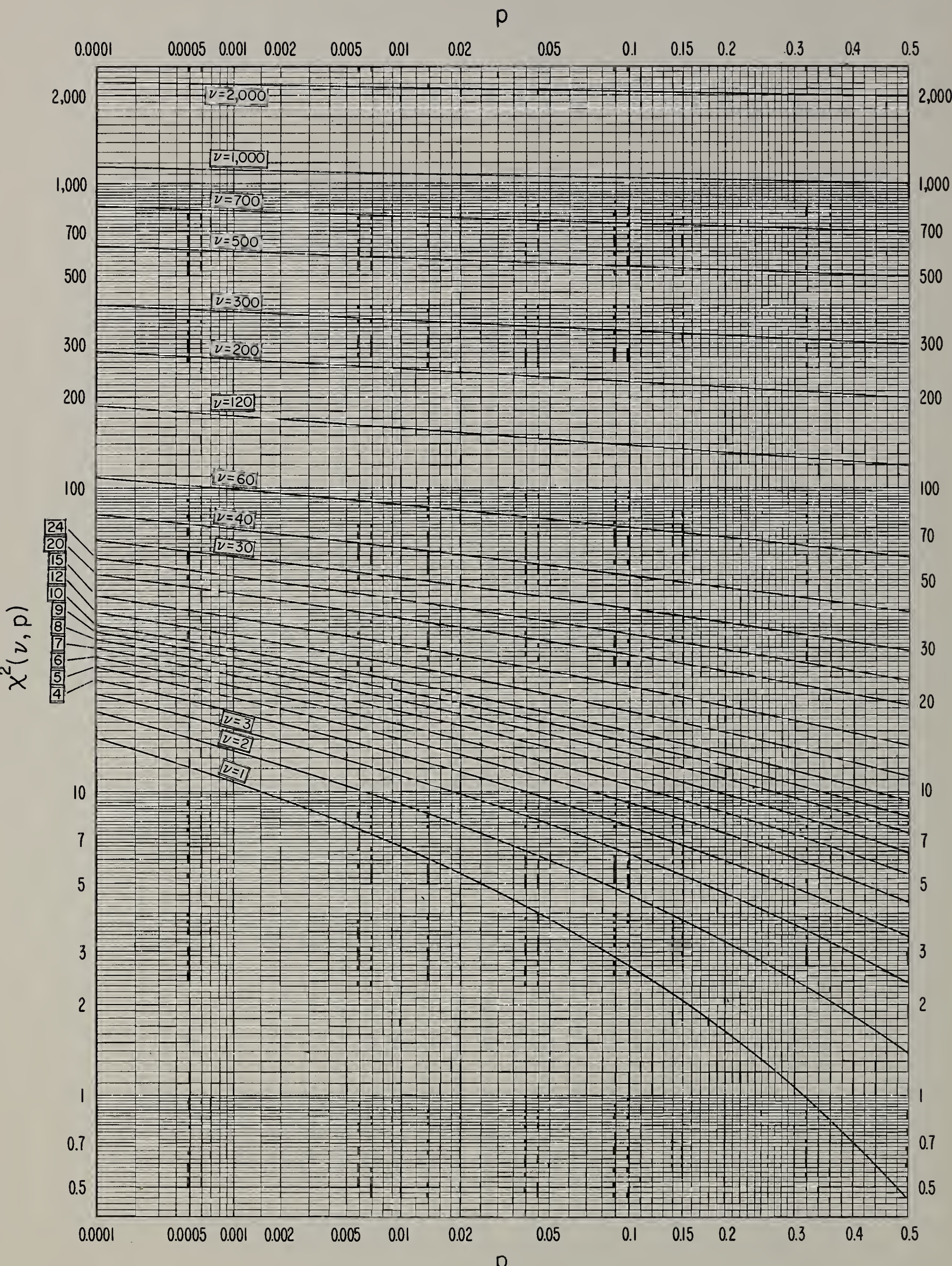
The Probability Distribution of χ^2

$\chi^2(\nu, p) = \nu F(\nu, \infty, p)$

ν	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5	p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν
1	(+1) 1.5137	(+1) 1.0828	7.8794	6.6349	5.0239	3.8415	2.7055	1.3233	(-1) 4.5494	(-1) 1.0153	(-2) 1.5791	(-3) 3.9321	(-4) 9.8207	(-4) 1.5709	(-5) 3.9270	(-6) 1.5708	(-8) 1.5708	1
2	(+1) 1.8421	(+1) 1.3816	(+1) 1.0597	9.2103	7.3778	5.9915	4.6052	2.7726	1.3863	(-1) 5.7536	(-1) 2.1072	(-1) 1.0259	(-2) 5.0636	(-2) 2.0101	(-2) 1.0025	(-3) 2.0010	(-4) 2.0002	2
3	(+1) 2.1108	(+1) 1.6266	(+1) 1.2838	(+1) 1.1345	9.3484	7.8147	6.2514	4.1084	2.3660	1.2125	(-1) 5.8438	(-1) 3.5185	(-1) 2.1580	(-1) 1.1483	(-2) 7.1721	(-2) 2.4298	(-3) 5.2149	3
4	(+1) 2.3513	(+1) 1.8467	(+1) 1.4860	(+1) 1.3277	(+1) 1.1143	9.4877	7.7794	5.3853	3.3567	1.9226	1.0636	(-1) 7.1072	(-1) 4.8442	(-1) 2.9711	(-1) 2.0699	(-2) 9.0804	(-2) 2.8418	4
5	(+1) 2.5745	(+1) 2.0515	(+1) 1.6750	(+1) 1.5086	(+1) 1.2832	(+1) 1.1070	9.2364	6.6257	4.3515	2.6746	1.6103	1.1455	(-1) 8.3121	(-1) 5.5430	(-1) 4.1174	(-1) 2.1022	(-2) 8.2175	5
6	(+1) 2.7856	(+1) 2.2458	(+1) 1.8548	(+1) 1.6812	(+1) 1.4449	(+1) 1.2592	(+1) 1.0645	7.8408	5.3481	3.4546	2.2041	1.6354	1.2373	(-1) 8.7208	(-1) 6.7573	(-1) 3.8107	(-1) 1.7235	6
7	(+1) 2.9877	(+1) 2.4322	(+1) 2.0278	(+1) 1.8475	(+1) 1.6013	(+1) 1.4067	(+1) 1.2017	9.0372	6.3458	4.2548	2.8331	2.1674	1.6899	1.2390	(-1) 9.8926	(-1) 5.9849	(-1) 2.9996	7
8	(+1) 3.1828	(+1) 2.6125	(+1) 2.1955	(+1) 2.0090	(+1) 1.7535	(+1) 1.5507	(+1) 1.3362	(+1) 1.0219	7.3441	5.0706	3.4895	2.7326	2.1797	1.6465	1.3444	(-1) 8.5712	(-1) 4.6359	8
9	(+1) 3.3720	(+1) 2.7877	(+1) 2.3589	(+1) 2.1666	(+1) 1.9023	(+1) 1.6919	(+1) 1.4684	(+1) 1.1389	8.3428	5.8988	4.1682	3.3251	2.7004	2.0879	1.7349	1.1519	(-1) 6.6081	9
10	(+1) 3.5564	(+1) 2.9588	(+1) 2.5188	(+1) 2.3209	(+1) 2.0483	(+1) 1.8307	(+1) 1.5987	(+1) 1.2549	9.3418	6.7372	4.8652	3.9403	3.2470	2.5582	2.1558	1.4787	(-1) 8.8890	10
12	(+1) 3.9134	(+1) 3.2909	(+1) 2.8300	(+1) 2.6217	(+1) 2.3337	(+1) 2.1026	(+1) 1.8549	(+1) 1.4845	(+1) 1.1340	8.4384	6.3038	5.2260	4.4038	3.5706	3.0738	2.2142	1.4275	12
15	(+1) 4.4264	(+1) 3.7697	(+1) 3.2801	(+1) 3.0578	(+1) 2.7488	(+1) 2.4996	(+1) 2.2307	(+1) 1.8245	(+1) 1.4339	(+1) 1.1036	8.5468	7.2609	6.2621	5.2294	4.6009	3.4827	2.4082	15
20	(+1) 5.2386	(+1) 4.5315	(+1) 3.9997	(+1) 3.7566	(+1) 3.4170	(+1) 3.1410	(+1) 2.8412	(+1) 2.3828	(+1) 1.9337	(+1) 1.5452	(+1) 1.2443	(+1) 1.0851	9.5908	8.2604	7.4339	5.9210	4.3952	20
24	(+1) 5.8613	(+1) 5.1179	(+1) 4.5558	(+1) 4.2980	(+1) 3.9364	(+1) 3.6415	(+1) 3.3196	(+1) 2.8241	(+1) 2.3337	(+1) 1.9037	(+1) 1.5659	(+1) 1.3848	(+1) 1.2401	(+1) 1.0856	9.8862	8.0849	6.2230	24
30	(+1) 6.7632	(+1) 5.9703	(+1) 5.3672	(+1) 5.0892	(+1) 4.6979	(+1) 4.3773	(+1) 4.0256	(+1) 3.4800	(+1) 2.9336	(+1) 2.4478	(+1) 2.0599	(+1) 1.8493	(+1) 1.6791	(+1) 1.4954	(+1) 1.3787	(+1) 1.1588	9.2580	30
40	(+1) 8.2064	(+1) 7.3402	(+1) 6.6766	(+1) 6.3691	(+1) 5.9342	(+1) 5.5758	(+1) 5.1805	(+1) 4.5616	(+1) 3.9335	(+1) 3.3660	(+1) 2.9050	(+1) 2.6509	(+1) 2.4433	(+1) 2.2164	(+1) 2.0706	(+1) 1.7916	(+1) 1.4883	40
60	(+2) 1.0950	(+1) 9.9607	(+1) 9.1952	(+1) 8.8379	(+1) 8.3298	(+1) 7.9082	(+1) 7.4397	(+1) 6.6981	(+1) 5.9335	(+1) 5.2294	(+1) 4.6459	(+1) 4.3188	(+1) 4.0482	(+1) 3.7485	(+1) 3.5535	(+1) 3.1738	(+1) 2.7497	60
120	(+2) 1.8633	(+2) 1.7362	(+2) 1.6364	(+2) 1.5895	(+2) 1.5221	(+2) 1.4657	(+2) 1.4023	(+2) 1.3006	(+2) 1.1933	(+2) 1.0922	(+2) 1.0062	(+1) 9.5701	(+1) 9.1576	(+1) 8.6926	(+1) 8.3851	(+1) 7.7750	(+1) 7.0728	120

$\chi^2(\nu, p) = \nu F(\nu, \infty, p)$

With ν degrees of freedom $\chi^2 > \chi^2(\nu, p)$ with probability p. χ^2 is the sum of the squares of ν independent values from a normal distribution with zero mean and unit standard deviation. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1) 1.2345 = 0.12345.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0.05
0.1
0.15
0.2
0.3
0.4
0.5
2,000
1,000
700
500
300
200
100
70
50
30
20
10
7
5
3
2
1
0.7
0.5
$\chi^2(\nu, p)$
ν=2,000
ν=1,000
ν=700
ν=500
ν=300
ν=200
ν=120
ν=60
ν=40
ν=30
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15
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8
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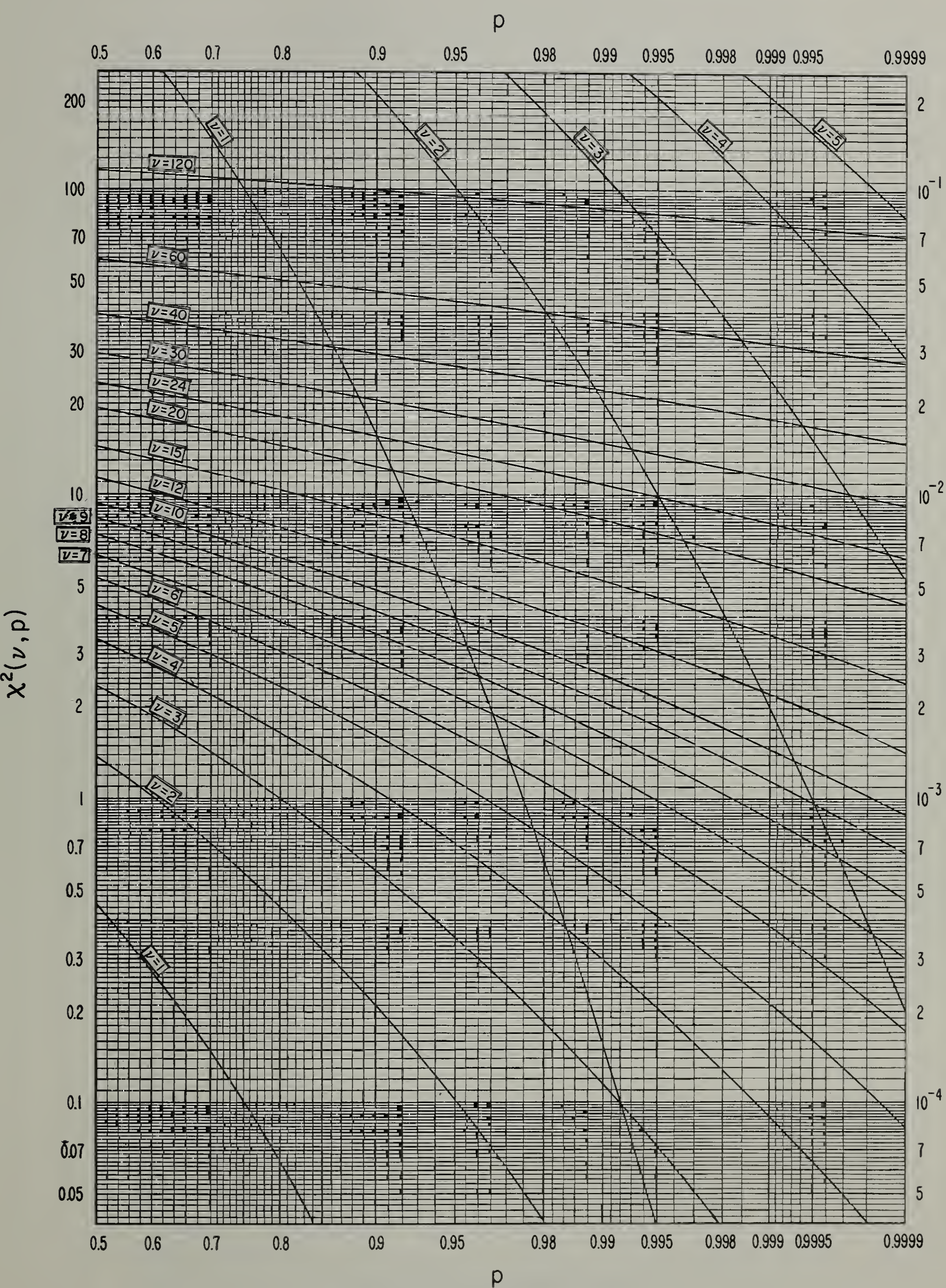
6
5
4
ν=3
ν=2
ν=1


p
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0.6
0.7
0.8
0.9
0.95
0.98
0.99
0.995
0.998
0.999
0.9995
0.9999
$\chi^2(\nu, p)$
200
100
70
50
30
20
10
5
3
2
1
0.7
0.5
0.3
0.2
0.1
0.07
0.05
2
10^{-1}
7
5
3
2
10^{-2}
7
5
3
2
10^{-3}
7
5
3
2
10^{-4}
7
5
ν=1
ν=2
ν=3
ν=4
ν=5
ν=6
ν=7
ν=8
ν=9
ν=10
ν=12
ν=15
ν=20
ν=24
ν=30
ν=40
ν=60
ν=120

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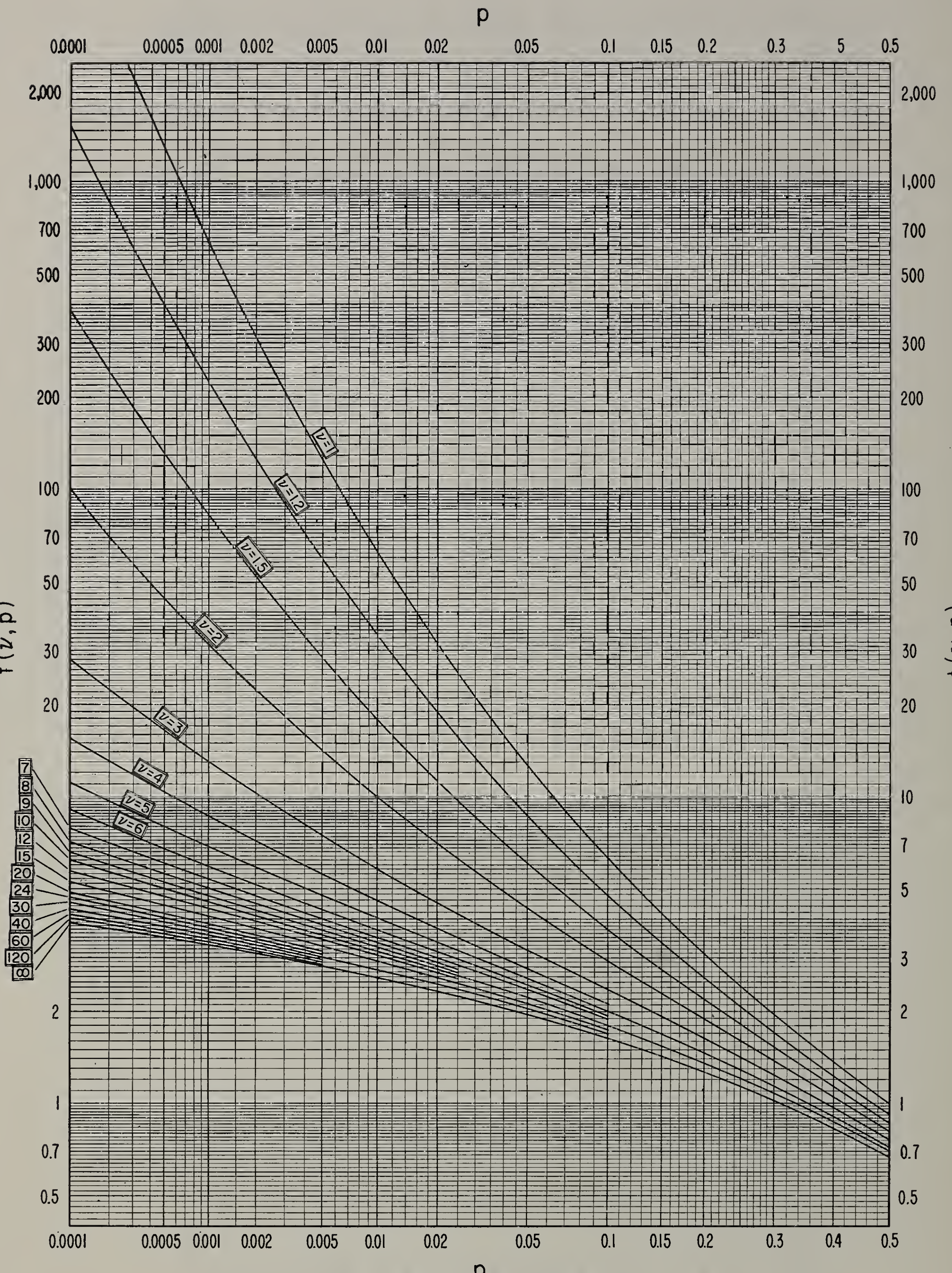
The Probability Distribution of Student's t

$t(\nu, p) = +\sqrt{F(1, \nu, p)}$

ν	p = 0.0001	p = 0.001	p = 0.005	p = 0.01	p = 0.025	p = 0.05	p = 0.1	p = 0.25	p = 0.5	p = 0.75	p = 0.9	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.999	p = 0.9999	ν
1	(+3) 6.3662	(+2) 6.3662	(+2) 1.2732	(+1) 6.3657	(+1) 2.5452	(+1) 1.2706	6.3138	2.4142	1.0000	(-1) 4.1421	(-1) 1.5838	(-2) 7.8701	(-2) 3.9290	(-2) 1.5709	(-3) 7.8541	(-3) 1.5708	(-4) 1.5708	1
1.2	(+3) 1.5434	(+2) 2.2654	(+1) 5.9240	(+1) 3.3239	(+1) 1.5468	8.6488	4.7958	2.0897	(-1) 9.3358	(-1) 3.9768	(-1) 1.5305	(-2) 7.6117	(-2) 3.8008	(-2) 1.5198	(-3) 7.5984	(-3) 1.5197	(-4) 1.5197	1.2
1.5	(+2) 3.8458	(+1) 8.2847	(+1) 2.8317	(+1) 1.7820	9.6353	6.0166	3.7051	1.8230	(-1) 8.7259	(-1) 3.8131	(-1) 1.4763	(-2) 7.3481	(-2) 3.6699	(-2) 1.4675	(-3) 7.3372	(-3) 1.4674	(-4) 1.4674	1.5
2	(+1) 9.9992	(+1) 3.1599	(+1) 1.4089	9.9249	6.2053	4.3027	2.9200	1.6036	(-1) 8.1650	(-1) 3.6514	(-1) 1.4213	(-2) 7.0799	(-2) 3.5367	(-2) 1.4143	(-3) 7.0711	(-3) 1.4142	(-4) 1.4142	2
3	(+1) 2.8000	(+1) 1.2924	7.4533	5.8409	4.1765	3.1825	2.3534	1.4226	(-1) 7.6489	(-1) 3.4921	(-1) 1.3660	(-2) 6.8087	(-2) 3.4018	(-2) 1.3604	(-3) 6.8018	(-3) 1.3604	(-4) 1.3603	3
4	(+1) 1.5544	8.6103	5.5976	4.6041	3.4954	2.7764	2.1319	1.3444	(-1) 7.4070	(-1) 3.4138	(-1) 1.3383	(-2) 6.6729	(-2) 3.3341	(-2) 1.3334	(-3) 6.6666	(-3) 1.3333	(-4) 1.3333	4
5	(+1) 1.1178	6.8688	4.7734	4.0321	3.1634	2.5706	2.0150	1.3010	(-1) 7.2668	(-1) 3.3672	(-1) 1.3217	(-2) 6.5915	(-2) 3.2936	(-2) 1.3172	(-3) 6.5858	(-3) 1.3172	(-4) 1.3172	5
6	9.0823	5.9588	4.3168	3.7074	2.9687	2.4469	1.9432	1.2733	(-1) 7.1756	(-1) 3.3365	(-1) 1.3108	(-2) 6.5374	(-2) 3.2666	(-2) 1.3064	(-3) 6.5321	(-3) 1.3064	(-4) 1.3064	6
7	7.8846	5.4079	4.0294	3.4994	2.8412	2.3646	1.8946	1.2543	(-1) 7.1114	(-1) 3.3145	(-1) 1.3029	(-2) 6.4988	(-2) 3.2475	(-2) 1.2988	(-3) 6.4936	(-3) 1.2987	(-4) 1.2987	7
8	7.1200	5.0413	3.8325	3.3554	2.7515	2.3060	1.8595	1.2403	(-1) 7.0639	(-1) 3.2983	(-1) 1.2971	(-2) 6.4701	(-2) 3.2331	(-2) 1.2930	(-3) 6.4651	(-3) 1.2930	(-4) 1.2930	8
9	6.5937	4.7809	3.6897	3.2498	2.6850	2.2622	1.8331	1.2297	(-1) 7.0272	(-1) 3.2856	(-1) 1.2925	(-2) 6.4477	(-2) 3.2220	(-2) 1.2886	(-3) 6.4427	(-3) 1.2885	(-4) 1.2885	9
10	6.2110	4.5869	3.5813	3.1692	2.6338	2.2281	1.8125	1.2213	(-1) 6.9981	(-1) 3.2755	(-1) 1.2889	(-2) 6.4299	(-2) 3.2131	(-2) 1.2850	(-3) 6.4250	(-3) 1.2850	(-4) 1.2850	10
12	5.6945	4.3178	3.4284	3.0545	2.5600	2.1788	1.7823	1.2088	(-1) 6.9548	(-1) 3.2605	(-1) 1.2835	(-2) 6.4030	(-2) 3.1997	(-2) 1.2797	(-3) 6.3984	(-3) 1.2797	(-4) 1.2797	12
15	5.2391	4.0727	3.2860	2.9467	2.4899	2.1315	1.7531	1.1967	(-1) 6.9119	(-1) 3.2456	(-1) 1.2781	(-2) 6.3764	(-2) 3.1865	(-2) 1.2744	(-3) 6.3719	(-3) 1.2744	(-4) 1.2744	15
20	4.8373	3.8495	3.1534	2.8453	2.4231	2.0860	1.7247	1.1848	(-1) 6.8696	(-1) 3.2306	(-1) 1.2727	(-2) 6.3499	(-2) 3.1732	(-2) 1.2691	(-3) 6.3454	(-3) 1.2691	(-4) 1.2691	20
24	4.6544	3.7454	3.0905	2.7969	2.3910	2.0639	1.7109	1.1789	(-1) 6.8485	(-1) 3.2232	(-1) 1.2700	(-2) 6.3366	(-2) 3.1667	(-2) 1.2665	(-3) 6.3321	(-3) 1.2665	(-4) 1.2665	24
30	4.4824	3.6460	3.0298	2.7500	2.3596	2.0423	1.6973	1.1731	(-1) 6.8276	(-1) 3.2157	(-1) 1.2673	(-2) 6.3234	(-2) 3.1601	(-2) 1.2638	(-3) 6.3190	(-3) 1.2638	(-4) 1.2638	30
40	4.3206	3.5509	2.9712	2.7045	2.3289	2.0211	1.6839	1.1673	(-1) 6.8066	(-1) 3.2084	(-1) 1.2646	(-2) 6.3102	(-2) 3.1535	(-2) 1.2612	(-3) 6.3059	(-3) 1.2612	(-4) 1.2612	40
60	4.1686	3.4602	2.9145	2.6603	2.2991	2.0003	1.6707	1.1616	(-1) 6.7862	(-1) 3.2011	(-1) 1.2619	(-2) 6.2969	(-2) 3.1469	(-2) 1.2586	(-3) 6.2928	(-3) 1.2585	(-4) 1.2585	60
120	4.0254	3.3734	2.8599	2.6174	2.2699	1.9799	1.6576	1.1559	(-1) 6.7656	(-1) 3.1937	(-1) 1.2593	(-2) 6.2839	(-2) 3.1404	(-2) 1.2559	(-3) 6.2796	(-3) 1.2559	(-4) 1.2559	120
∞	3.8906	3.2905	2.8070	2.5758	2.2414	1.9600	1.6449	1.1503	(-1) 6.7449	(-1) 3.1864	(-1) 1.2566	(-2) 6.2707	(-2) 3.1338	(-2) 1.2533	(-3) 6.2666	(-3) 1.2533	(-4) 1.2533	∞

$t(\nu, p) = +\sqrt{F(1, \nu, p)}$

With ν degrees of freedom, the absolute value $|t| > t(\nu, p)$ with probability p; Student's $t > t(\nu, p)$ with probability 0.5p; Student's $t > -t(\nu, p)$ with probability 1 - 0.5p; Student's $t < -t(\nu, p)$ with probability 0.5p. With $\nu = \infty$, t is normally distributed about zero with unit standard deviation. The numbers in parentheses indicate the power of ten by which the number following is to be multiplied, e.g., (-1)1.2345 = 0.12345.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0.15
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0.3
5
0.5
t (ν, p)
2,000
1,000
700
500
300
200
100
70
50
30
20
10
7
5
3
2
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0.7
0.5
ν=1
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ν=2
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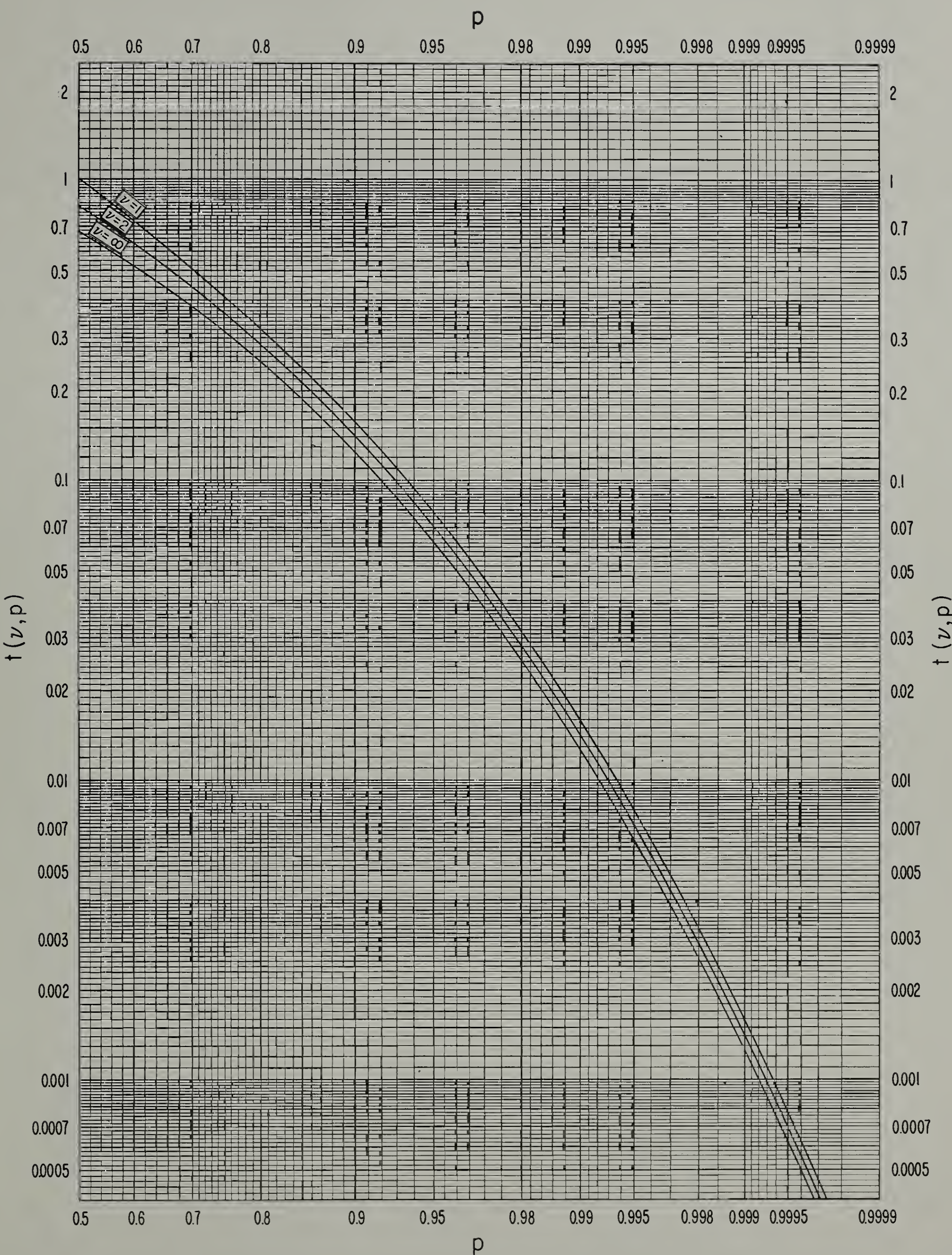
0.4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0.2
0.1
0.07
0.05
0.03
0.02
0.01
0.007
0.005
0.003
0.002
0.001
0.0007
0.0005
t (ν, p)
ν=1
ν=2
ν=∞

U. S. DEPARTMENT OF COMMERCE
Sinclair Weeks, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of the scientific program of the National Bureau of Standards at laboratory centers in Washington, D. C., and Boulder, Colorado, is given in the following outline:

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Electricity and Electronics. Resistance and Reactance. Electron Tubes. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat and Power. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology and Lubrication. Engine Fuels.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Nuclear Physics. Radioactivity. X-rays. Betatron. Nucleonic Instrumentation. Radiological Equipment. AEC Radiation Instruments.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Gas Chemistry. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Organic Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Concreting Materials. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Air Conditioning. Heating, and Refrigeration. Floor, Roof, and Wall Coverings. Codes and Specifications. Heat Transfer.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analogue Systems. Application Engineering.

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