NATIONAL BUREAU OF STANDARDS REPORT

4952

PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION

A Quarterly Report
July through September 1956

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS
THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major field laboratories in Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside back cover of this report.

WASHINGTON, D. C.


• Office of Basic Instrumentation
• Office of Weights and Measures

BOULDER, COLORADO


PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION

July through September 1956
APPLIED MATHEMATICS DIVISION

July 1 through September 30, 1956

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*Only unclassified projects are included in this report.
Status of Projects
September 30, 1956

1. NUMERICAL ANALYSIS

RESEARCH IN THEORIES OF DISCRETE STRUCTURES
Task 1100-11-5170/56-159

Origin: NBS
Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: O. Taussky-Todd
Full task description: July-Sept 1955 issue, p. 1

Status: CONTINUED. O. Taussky-Todd did further study on totally
positive matrices. She also investigated the feasibility of SEAC
computation of two numerical problems concerning class groups in quadratic
fields. Work on classes of matrices was continued,—in particular, the
discussion of the matrix class which corresponds to the product of the
ideal classes.

O. Taussky-Todd and J. Todd studied commutative unimodular
matrices in connection with a result of K. Goldberg. A new proof of
his result,—essentially that all abelian subgroups of the modular group
are cyclic,—was given, and a generalization to the case of 2x2 matrices
with elements in a complex quadratic field F, where the determinant is a
unit in F, was obtained.

A manuscript was prepared by E. Dade on "Abelian groups of
unimodular matrices." The paper offers a generalization of the theorems
of K. Goldberg, O. Taussky-Todd and J. Todd on the structure of abelian
subgroups of the group \( \Gamma_n(k) \) of nxn extended unimodular matrices over
an algebraic number field \( \mathbb{F}_k \). The problem is approached by considering,
not just the abelian subgroup, but the subalgebra of the full nxn matrix
algebra which it generates. The intersection \( \Gamma_n^h(k) \) with the subalgebra
is an abelian subgroup which is also the group of units for an order in
the algebra. The structure of the unit group of an order was investigated
and the results were applied to abelian subgroups.

O. Taussky-Todd continued earlier work concerning positive
definite quadratic forms and prepared a manuscript in collaboration with
G. Pall (Illinois Institute of Technology). The paper presents the exact
number of representations of a quadratic form with rational integral
coefficients as a sum of four squares of rational integral linear forms.
The treatment is by quaternions.

O. Taussky-Todd prepared a survey of computations on matrices with integral element for presentation to the Societa Italiana per il Progresso delle Scienze at Catania, Sicily.

E. Dade and K. Goldberg continued their investigation of incidence algebras. An application was found for symmetric block designs: A construction procedure was described for the parametric representation 

\[ r=p^2 \equiv 3 \pmod{4} \]

where \( p \) is a prime, \( k=(v-1)/2 \) and \( \lambda=(v-3)/4 \).

H. Cohn has computed units for "pure cubic" fields of type \( \sqrt[3]{N} \) by use of Dedekind's eta-function formula. The program is continuing in operation, and results to date include the discovery of two fields of class number 27, \( \sqrt[3]{217} \) and \( \sqrt[3]{342} \), and two of class number 21, \( \sqrt[3]{215} \) and \( \sqrt[3]{422} \). In each case a tabulation of residue and non-residue quadratic forms was obtained. A paper is in preparation.

Publications:


(2) Algebraic equations satisfied by roots of natural numbers. E. G. Straus (University of California at Los Angeles) and O. Taussky. Appeared: Pac. J. Math. 6, 97-98 (1956).


(5) Unimodular matrices of order two which commute. K. Goldberg. Submitted to a technical journal.

(6) Incidence algebras. E. C. Dade and K. Goldberg. In manuscript.


(9) The number of representations of a quadratic form as a sum of four squares. G. Pall (Illinois Institute of Technology) and O. Taussky. Submitted to a technical journal.

(10) Commuting bilinear transformations and matrices. O. Taussky and J. Todd. Submitted to a technical journal.

(11) Pairs of matrices of order two which generate free groups. K. Goldberg and M. Newman. Submitted to a technical journal.


(14) A composition of cyclic cubic units. H. Cohn and S. Gorn (Moore School of Engineering). In manuscript.
RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-10-1104/55-55

Origin: NBS
Managers: J. Todd, P. Davis
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. J. Todd prepared a survey of computational problems and experience connected with the Hilbert matrix. A paper on the subject was presented at the meeting of the Societa Italiana per il Progresso delle Scienze, Catania, Sicily, September 19.

P. Davis and I. Stegun have experimented with orthonormalizing codes to obtain uniform (Tschebyscheff) polynomial approximations to continuous functions. Good results were obtained. Within two or three passes on the orthonormalizing code, all the accuracy generally desired can be achieved. A description of the method is in preparation.

At the suggestion of M. Fekete, P. Davis computed on SEAC the transfinite diameter \( \hat{c}(E) \) of a set \( E \) consisting of two collinear line segments. The relationship

\[
\hat{c}(E) = \lim_{n \to \infty} \left( \frac{G_{n-1}}{G_n} \right)^{1/n}, \quad G_n = \int_E z^j z^k ds
\]

was employed, and the computation was carried out using a previously developed orthonormalizing code that can be instructed to print out the Gram determinant of a set of vectors. The computation was made in single-precision floating notation with \( n=10 \); two and a half significant figures were achieved. At this stage \( n=10 \), the effects of round-off were already significant. Further accuracy could probably not be achieved without going to higher values of \( n \) and double precision coding. The theoretical background for this problem is due to Fekete, Achiezer, and Fekete and Walsh.

P. Davis is preparing a bibliography in the field of domain polynomials.

R. Moore has prepared a routine which generates all permutations on 11 or fewer objects. This routine formed part of a larger problem whose purpose was to test the truth or falsity of a conjecture of G. A. Dirac and J. Todd concerning a problem of Sierpinski on equivalence of sets by finite decomposition. The conjecture was found to be false.

A paper on "Stability and Lyapunov's second method" was presented by H. A. Antosiewicz before the International Symposium on Algebraic Topology, Mexico City, August 15. During July and August Dr. Antosiewicz conducted a seminar on "Modern Topics in the Stability Theory of Differential Equations," before the staff of Instituto de Matematicas, University of Mexico, Mexico City.

H. Cohn has also studied the problem of stable configurations of electrons on a circle. This was solved completely in terms of stable and metastable states.

J. Gager has completed exploratory calculations of the Cauer
parameters. The final code, which incorporated several smaller routines, evaluated \( sn((m/n)K;k) \) for \( p=\sqrt{k}=.850(.001).999 \), where \( 1 \leq m < n \leq 12 \). The three smaller, preliminary routines can best be explained in the following manner. Code I computes \( k, k^2 \) and \( K \) for \( p=.850(.001).999 \) by using the arithmetic-geometric mean method. Code II computes \( q', k, q, \) and \( K \) for \( p=.850(.001).999; K \) was computed in two ways for comparison purposes. Code III computes \( \sin(2x+1)(m/n)(\pi/2), 0 \leq x \leq 30 \). The final code, Code IV, computes

\[
\frac{2\pi}{Kk} \sum_{n=0}^{n=30} \frac{q^{n+1/2}}{(1-q^{2n+1})} \sin(2x+1) \cdot \frac{m}{n} \cdot \frac{\pi}{2}
\]

and the Cauer parameters.

M. Marcus has studied the function

\[
\min_{\max} f[Ax_1,x_1], \ldots, (Ax_n,x_n)]
\]

\[
(x_i,x_j) = \delta_{ij}
\]

for \( f \) convex, where \( A \) is an \( nxn \) complex Hermitian matrix. He then obtained a generalization of Hadamard’s determinant theorem and the solution of a problem of Wielandt (Proc. Amer. Math. Soc. 6, 109(1955)). A paper has been submitted to a technical journal (see publication (7) below).

Dr. Marcus has generalized a theorem of v. Neumann and Fan on inequalities for singular values. He has also obtained the extreme values of \( \sum_{i=1}^{k} (Ax_i,x_j) \) where \( A \) is Hermitian indefinite and \( (x_i,x_j) = \delta_{ij} \).

This is equivalent to the solution of a problem on signed volumes of \( k \)-dimensional parallelepipeds. In addition, Dr. Marcus has obtained

\[
\min_{x_1, \ldots, x_k} \sum_{1 < r_1 < r_2 < k} \det(Ax_{r_1},x_{r_2})
\]

where \( (x_i,x_j) = \delta_{ij} \). This is then used to extend the Cauchy-Schwarz inequality to indefinite Hermitian transformations. Manuscripts on these studies are in preparation.

Publications:
(4) Note on bounds for certain determinants. E. V. Haynesworth. Submitted
to a technical journal.


(7) Convex functions of quadratic forms. M. Marcus. In manuscript.

(8) Some inequalities for quadratic forms and eigenvalues. M. Marcus. In manuscript.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-10-5116/55-56

Origin: NBS
Sponsor: Office of Naval Research
Managers: J. Todd, M. Newman

Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. M. Newman continued earlier work on classes of modular functions. He proved that appropriate linear combinations of these functions vanish and that they are finitely generated. A manuscript has been prepared. In connection with this work a routine to compute the coefficients of these functions on SEAC was prepared and about 1000 coefficients were determined.

K. Goldberg has completed a routine for computing the characteristic roots and vectors of symmetric matrices of order less than or equal to 23. The method for extracting the vectors is new and proved to be very fast on SEAC.

Mr. Goldberg has completed a file of floating subroutines for general use. The codes contain fewer orders than previously used without sacrificing time or accuracy.

Continuing recent work of H. Wielandt, K. Fan has studied normal matrices with preassigned eigenvalues. Necessary and sufficient conditions are obtained for (1) the existence of a normal matrix with all its eigenvalues and some columns preassigned, (2) the existence of two normal matrices M, N with preassigned eigenvalues and such that the rank of M-N is less than or equal to a preassigned number, and (3) the existence of two normal matrices M,N with preassigned eigenvalues and such that M+N (or MN) has a preassigned eigenvalue with preassigned multiplicity.

K. Fan has also been studying circular disks containing the eigenvalues of a matrix. He is making a critical survey of the literature on this topic. New results, generalizations of known results (of Brauer, Gershgorin, Taussky, Ostrowski, Kotelyanskii), have been obtained. It turns out that the topic is closely related to Perron-Frobenius' classical results on non-negative matrices.

Using topological method, K. Fan proved that if a real matrix $A=(a_{ij})$ has a dominant principal diagonal, then the elements in the
inverse matrix $A^{-1} = (b_{ij})$ satisfy the inequality

$$|b_{ij}| \leq \left( |a_{jj}| - \sum_{k \neq j} |a_{jk}| \right)^{-1}.$$  

A consequence of this relation is G.B. Price's inequality

$$|\det A| \geq s_1 s_2 \ldots s_n,$$

where

$$s_i = |a_{ii}| - \sum_{j > i} a_{ij}.$$

Publications:

(1) On dominant eigenvalues of positive matrices. T. Kato. In manuscript.


(7) The lowest frequency of a free square plate. H. Fujita (University of Tokyo), T. Kato, Y. Nakata (University of Tokyo), and M. Newman. In manuscript.


(11) Systems of inequalities involving convex functions. Ky Fan (University of Notre Dame), I. Glicksberg (RAND Corporation), and A. J. Hoffman. In manuscript.


Status of Projects

ANALYTIC STUDY OF WAR GAMES
Task 1101-10-5116/55-83

Origin and Sponsor: Armament Branch, ARDC, USAF
Manager: H. A. Antosiewicz
Full task description: Oct-Dec 1954 issue, p. 7

Status: TERMINATED.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-10-5116/56-148

Origin and Sponsor: National Institutes of Health, Authorized 9/30/55
Manager: H. A. Antosiewicz
Full task description: July-Sept 1955 issue, p. 7

Status: CONTINUED. At a particular temperature the search for solutions of the basic system of differential equations having a prescribed behavior has been completed. The length of the interval of admissible parameter has been determined to be less than $10^{-10}$. A number of cases for a lower temperature has been run.

E. Dade has completed and checked out two floating and two fixed-point routines for the evaluation of the function

$$\Lambda = \ln z_o \int_1^{z_o} y(z) \frac{dz}{z}$$

where

$$z^2y''(z) - iw(a+z)y(z) = iw,$$

$$y'(1) = y'(z_o) = 0,$$

$a, b, w, z_o$ being real constants. Although the general solution of this differential equation is known in terms of Bessel functions (of complex order and complex argument), the numerical integration is carried out by use of Taylor series expansions of $y(z)$ and the integral in $\Lambda$ evaluated by Simpson's rule. At present, tests are being made for various sets of the parameter values.
Status of Projects

NUMERICAL EXPERIMENTS ON POTENTIAL THEORY
USING THE NEHARI ESTIMATES
Task 1101-10-5116/56-189

Origin: NBS
Sponsor: Air Research and Development Command, USAF
Managers: P. Davis, U. Hochstrasser
Full task description: Apr–June 1956 issue, p. 6

Status: CONTINUED. Working with an irregular pentagon and n particular harmonic functions (usually Re and Im(z^k), k=0,1,...), the Dirichlet problem has been solved for the boundary data Re(cos z) and Re[1/(z+4)]. The cases n=3,4,...,9 have been covered, and comparisons have been made between the actual error and that given by Nehari's estimates. Computation of the harmonic measures of the sides of the pentagon is underway.
2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF \( E_1(z) \), \((z=x+iy)\)
Task 1102-40-1110/43-3

Manager: I. Stegun
Authorized 7/1/47
Full task description: Apr-June 1949 issue, p. 41

Status: CONTINUED. The final manuscript has been typed and checking is in progress.

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-40-1110/47-2

Origin: NBS
Manager: M. Abramowitz
Authorized 7/1/47
Full task description: Apr-June 1949 issue, p. 45

Status: CONTINUED. Work is continuing on the computation of \( F_0(\eta, \rho) \) and \( F'_0(\eta, \rho) \) for \( \eta = 0(.5)25, \rho = 0(.5)40 \). Checking has been started.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Task 1102-40-1110/51-8

Origin: Section 11.3, NBS
Manager: S. Peavy
Authorized 3/26/51
Full task description: Apr-June 1951 issue, p. 49

Status: INACTIVE. For status to date, see Oct-Dec 1955 issue, p. 8.
Status of Projects

REVISION OF MATHEMATICAL TABLES
Task 1102-40-1110/52-7

Origin: NBS Authorized 8/10/51
Managers: W. F. Cahill, I. Stegun
Full task description: July-Sept 1951 issue, p. 41

Status: INACTIVE. For status to date, see Jan-Mar 1956 issue, p. 6.

TABLE OF THE MODIFIED AIRY INTEGRAL
Task 1102-40-1110/52-23

Origin: NBS Authorized 10/4/51
Manager: I. Stegun
Full task description: July-Sept 1951 issue, p. 42

Status: CONTINUED. Checking of the completed manuscript is continuing.

TABLE OF ERROR FUNCTION FOR COMPLEX ARGUMENTS
Task 1102-40-1110/52-25

Origin: NBS Authorized 10/5/51
Manager: W. Hall
Full task description: July-Sept 1951 issue, p. 42

Status: INACTIVE. For status to date, see Oct-Dec 1954 issue, p. 11.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-40-1110/52-37

Origin: NBS Authorized 11/28/51
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE.
Status of Projects

SIEVERT'S INTEGRAL
Task 1102-40-1110/52-57

Origin: NBS
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46

Status: CONTINUED. Checking of the tabular values and preparation of the introductory material continued.

L-SHELL CONVERSION COEFFICIENTS
Task 1102-40-1110/53-52

Origin: Oak Ridge National Laboratory
Manager: W. Hall
Full task description: Apr-June 1953 issue, p. 45

Status: CONTINUED. The original project is completed except for K shell, Z=35, k=2.0. It is now believed desirable to consider the nucleus not as a point mass but as a sphere of finite radius for certain of the high atomic number cases for the K and L shells. These routines have not yet been written.

AUTOMATIC CODING
Task 3711-60-0009/55-65

Origin: NBS
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. A technical memo entitled "Integral Evaluation Code - Base 11" was written by John W. Cooper. This memo describes in detail the Base Code mentioned in the Jan-Mar 1956 issue, p. 8.

MATHEMATICAL SUBROUTINES
Task 3711-60-0009/56-160

Origin: NBS
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13
Status: CONTINUED. A subroutine has been prepared and checked for computing the serial correlation coefficients $r$ of order $k$:

$$r_k = \frac{\text{cov}(u_j, u_{j+k})}{(\text{var}^u_j \text{var}^b_{j+k})^{1/2}}$$

The code has been written in the normal mode with the series of values considered as integers times $2^{-42}$. 
3. PROBABILITY AND MATHEMATICAL STATISTICS

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-10-1107/49-1a

Origin: NBS
Manager: L. S. Deming
Full task description: Apr-June 1949 issue, p. 75

Status: CONTINUED. For a description of the continuing activity on this task, see the Jan-Mar 1954 issue, p. 49.

TABLES TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-10-1107/51-1

Origin: NBS
Managers: C. Eisenhart, L. S. Deming
Full task description: July-Sept 1950 issue, p. 57

Status: INACTIVE. For status to date, see July-Sept 1952 issue, p. 64.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-10-1107/51-2

Origin: NBS
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. For a description of the continuing activity on this task, see the Oct-Dec 1956 issue, page 12.

Publications:
(3) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. Joel (Computation Laboratory). Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-10-1107/53-1

Origin: NBS
Manager: W. S. Connor
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. R. C. Burton and W. S. Connor prepared a paper "On the identity relationship for fractional replicates of the $2^n$ series" (see publication (1) below). Regarding the collections of letters together with the identity element as the elements of an Abelian group, they gave necessary and sufficient conditions for the existence of an identity relationship which has specified numbers of letters in its generators and products. This work was reported in a contributed paper read at a meeting of the Institute of Mathematical Statistics in Seattle, Washington.

R. C. Bose developed some theory concerning the construction of optimum fractional designs of the $3^n$ series.

M. Zelen delivered an invited address on "The analysis of covariance for incomplete block designs" at the Detroit meeting of the American Statistical Association.

M. Zelen developed some theory concerning the use of incomplete block designs for factorial treatments.

Publications:
(2) Contributions on partially balanced incomplete block designs with two associate classes. W. H. Clatworthy. To appear as NBS Applied Mathematics Series 47.
(3) Fractional factorial experiment designs for factors at two levels. To appear as NBS Applied Mathematics Series 48.
Status of Projects

RESEARCH ON MATHEMATICAL ASPECTS OF ORDER STATISTICS METHODS
Task 1103-10-1107/55-110

Origin: NBS
Manager: C. Eisenhart
Full task description: Jan-Mar 1955 issue, p. 31

Status: CONTINUED. For a description of the continuing activity on this task, see the Apr-June 1956 issue, page 12.

Publication:
(1) Geological application of extreme-value theory to interpretation of cobbles and boulders in gravel deposits. W. C. Krumbein (Northwestern University) and J. Lieblein. Appeared: Trans. Geophysical Union 37, 313-319 (June 1956).

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-10-1107/56-170

Origin: NBS
Manager: Joan R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

Status: CONTINUED. The paper, "On the power of some rank order two-sample tests," by J. R Rosenblatt, is being prepared for submission to a technical journal. Work is underway on determining bounds for the variance of Kendall's τ.

Publications:
(1) Easily used simultaneous confidence limits for a line. W. S. Connor. Withdrawn from publication by the author.

MEASUREMENT OF RELIABILITY
Task 1103-10-1130/56-182

Origin: NBS
Manager: M. Zelen
Full task description: Jan-Mar 1956 issue, p. 13

Authorized 3/3/55
Authorized 12/15/55
Authorized 3/23/56
Status: CONTINUED. J. R. Rosenblatt has continued work on the properties of methods for obtaining confidence intervals for the product and related functions of binomial parameters.

M. Zelen is continuing work to determine how one can make effective use in practice of the relation between the distribution of failures and the conditional distribution of failures.


MANUAL ON EXPERIMENTAL STATISTICS
FOR ORDNANCE ENGINEERS
Task 1103-40-5146/55-93

Origin and Sponsor: Office of Ordnance Research
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: CONTINUED. With the completion of the section on "Sensitivity testing," Part II,--"Some statistical techniques for qualitative data,"-- has now been essentially completed. A section "Rejection of outliers" has been drafted, and in addition, work has been started on Part III, "Planning the collection and analysis of data."
4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS

Task 1104-10-1115/55-57

Origin: NBS
Manager: R. F. Dressler
Full task description: July-Sept 1954 issue, p. 27

Authorized 9/1/54

Status: CONTINUED. The study of the use of the Cartesian Diver balance as a weighing instrument is being continued by means of a computational program based on the theoretical analysis obtained by P. Chiarulli and F. Chilton, at the request of the Office of Basic Instrumentation (NBS). The program has been coded for SEAC and the computations are being run. Results will illustrate quantitatively the possible sensitivity of such a balance as a function of the various physical parameters and the effect of small variations in these parameters.

W. H. Pell has begun a study of the first and second boundary value problems of elasticity for certain bodies having axial symmetry. The principal tool being employed is the stress function approach of J. Boussinesq, which has been used extensively in recent times by E. Sternberg, R. A. Eubanks, and M. A. Sadowsky.

A. Ghaffari has begun a study concerning stability of solutions and existence of periodic solutions of second order non-linear ordinary differential equations of Cartwright-Littlewood type.

Publications:
(3) On the representation of a certain integral involving Bessel functions of hypergeometric series. P. Henrici. Submitted to a technical journal.
RESEARCH IN ELECTROMAGNETIC THEORY
Task 1104-10-5160/54-47

Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Department of the Army
Manager: F. Oberhettinger
Full task description: July-Sept 1954 issue, p. 28

Status: INACTIVE.

RESEARCH IN RADIATION THEORY
Task 1104-10-5160/56-175

Origin: NBS
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: Oct-Dec 1955 issue, p. 18

Status: CONTINUED. New additions to the existing collection of Mehler transforms have been made. At present the compilation embraces about 100 formulas.

The investigation concerning the propagation of electromagnetic and acoustic pulses is continuing. The first part of the results on diffraction of pulses by wedges has been submitted to a journal and accepted for publication. The diffracted field from cylindrical or plane pulses, with arbitrary pulse function, on wedges or edges has been obtained in a definite integral representation.

Publication:

RESEARCH IN MATHEMATICAL ELASTICITY
Task 1104-10-5160/55-85

Origin: NBS
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: R. F. Dressler
Full task description: Oct-Dec 1954 issue, p. 30

Status: CONTINUED. The lowest mode and frequency for $\sigma = .25$,
\[ T = \frac{1}{2} \] were computed for the vibrating triangular wing problem. There is no significant difference between this result and that for \( \sigma = .29, T = \frac{1}{2} \). The matrix for the same aspect ratio but with \( \sigma = .05 \) has been set up and computation for the lowest mode has been started.

The case for aspect ratio \( T = 1 \) has caused difficulty due to slowness of convergence, because it was necessary to scale the matrix four times smaller than the previous case. To overcome this trouble, U. Hochstrasser and I. Rhodes have modified the program. Previously, the code iterated on \( y(n) \) in \( y(n+1) = F(A) y(n) \), with normalization at every step, where \( F \) is the hypergeometric polynomial of degree 30, and the approximation to \( \lambda \), the lowest eigenvalue to \( A \), was obtained from \( \lambda_{n+1} y(n+1) = A y(n+1) \), where \( A \) is the matrix order 78. The new procedure approximates \( \mu \), the largest eigenvalue of \( F(A) \), from

\[ \mu \ y(n) = F(A) \ y(n), \]

and from \( \mu \), it then computes \( \lambda \).

A paper describing the method used and the results obtained thus far has been presented at the International Congress on Mechanics, held at Brussels in September.

On the corrugated diaphragm problems the solution for the thin diaphragm has been computed by using initial conditions computed from matching the three homogeneous and particular solutions at the end, rather than by forming the appropriate combination of these solutions. This has been done to avoid the error created from ill-conditioned matrices where the solution values are small quantities arrived at by linear combination of large quantities. Using this solution, the stresses have been computed and are being plotted. The results compared with solutions already obtained for the thick and medium diaphragms will reveal the relative importance of the interrelated stretching and bending effects in rotationally symmetric shells.

Publications:


Status of Projects

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-10-5160/56-154

Origin: NBS
Sponsor: Office of Naval Research
Managers: F. Oberhettinger, J. Lieblein
Full task description: July Sept 1955 issue, p. 20

Status: CONTINUED. F. Oberhettinger is continuing to expand the collection of Fourier transforms.

RESEARCH IN FLUID DYNAMICS OF TWO-PHASE FLOWS
Task 1104-10-5160/56-155

Origin: Office of Naval Research
Sponsor: "
Manager: R. F. Dressler
Full task description: July-Sept 1955 issue, p. 21

Status: CONTINUED. A paper describing a result of the first stage of the investigation dealing with interface conditions with provision for heat removal and cold liquid addition has been prepared and has been presented at the International Congress on Mechanics, Brussels.

A. Ghaffari and R. F. Dressler are undertaking a study of the thrust developed by condensible jet devices of the type previously considered. The one-dimensional flow conservation-law techniques developed for use with those models are applied to study the thrust under various conditions and, in addition, models are being considered which yield thrust augmentation as described by von Karman (see Reissner Anniversary Volume, pp. 461-468 (Edwards Brothers, Ann Arbor, Michigan, 1949)).

Publication:
RESEARCH IN MATHEMATICAL GEOPHYSICS
Task 1104-10-5160/56-156

Origin: The American University
Sponsor: Office of Naval Research
Manager: U. Hochstrasser
Full task description: July-Sept 1955 issue, p. 21

Status: TERMINATED. Dispersion curves for higher modes in the propagation of Rayleigh waves across an ocean floor have been computed by U. Hochstrasser on SEAC for the model mentioned in the April-June 1956 issue.

The code for solving the equations governing the deformation of the earth by gravitational forces has been finished and is being checked out. This code will permit a study of the deformation of the earth for a load with surface densities representable by solid spherical harmonics of any degree.

Subsequent research on this general topic will be reported in future issues under the heading, "Research in mathematical physics and related fields."

Publications:
(4) Lectures - Surface waves in an elastic media. R. Stoneley. A Monograph.
5. MATHEMATICAL AND COMPUTATIONAL SERVICES

1102-40-5126/51-37  MOLECULAR STRUCTURE, III
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. O'Hara
Full task description: July-Sept 1951 issue, p. 50
Status: Continued. A three-dimensional Fourier synthesis was computed for the Benzophenone crystals using about 300 reflections of known sign and intensity.

1102-40-5126/52-44  CALCULATIONS FOR d-SPACINGS
Origin and Sponsor: NBS, Division 9
Full task description: Oct-Dec 1951 issue, p. 47
Manager: R. Zucker
Status: Continued. About 20 calculations for d-spacings for tetragonal, hexagonal and orthorhombic crystals were performed. Also redetermination of unit cell constants by least squares fitting to measured d-spacings was carried out for about 31 crystals.
A general program that will calculate d-spacings for all crystallographic space groups, written by A. Futterman, was revised and completely code-checked. A general code was written to sort the d-spacings and their corresponding indices in descending order. Sorting can be done internally or by means of "block" sorting, using magnetic tapes for auxiliary storage. Test cases for all crystal groups were run and the codes were completely checked out.

1102-40-5126/53-27  COMPUTATION OF THERMODYNAMIC FUNCTIONS
Origin and Sponsor: NBS, Section 3.2
Manager: E. Marden
Full task description: Jan-Mar 1953 issue, p. 57
Status: Completed. Results have been transmitted to the sponsor.

1102-40-5126/53-51  RADIATION DIFFUSION, III
Origin: NBS, Section 4.8
Sponsor: Atomic Energy Commission
Manager: J. Doggett (4.8)
Full task description: Apr-June 1953 issue, p. 57 (Neutron Diffusion III)
Status: Continued. A sample set of photon "case histories" has been tabulated. The effect of singly scattered radiation for the two-medium problem is being measured.
1102-40-5126/54-13  AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: New York Quartermaster Procurement Agency
Manager: H. Bremer
Full task description: Oct-Dec 1953 issue, p. 43
Status: Continued. During the quarter solutions for 10 bid evaluation problems were carried out.

3711-60-0009/54-17  DEPOLYMERIZATION
Origin: NBS, Section 7.6
Manager: U. Hochstrasser
Full task description: Oct-Dec 1953 issue, p. 44
Status: Continued. Six additional cases have been run involving various changes of the parameters.

3711-60-0009/54-22  ENERGY DISTRIBUTIONS ON OPTICAL IMAGE
Origin: NBS, Section 2.2
Manager: L. S. Joel
Full task description: Jan-Mar 1954 issue, p. 43
Status: Inactive. For status to date, see Jan-Mar 1956 issue, p. 19

3711-60-0009/54-30  SPECTRUM ANALYSIS
Origin: NBS, Division 4
Manager: S. Prusch
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. The observed wavelengths of the spectrum of Thulium II were converted to vacuum wavenumbers on the SEAC. A code was prepared for SEAC to search for new energy levels. The code differed from that previously used in that there were no multiplets known in this spectrum, and it was necessary to form all possible sums between 3000 lines and 72 known terms. The code for the calculation has been checked, and approximately one-third of the data has been searched.

3711-60-0009/54-38  COMPRESSIBILITY FACTORS OF DRY AIR
Origin: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Inactive. For status to date, see Apr-June 1955 issue, p. 12

1102-40-5126/55-39  MOLECULAR VIBRATIONS
Origin and Sponsor: NBS, Section 3.2
Manager: K. Goldberg
Full task description: July-Sept 1954 issue, p. 16
Status: Completed.
1102-40-5126/54-43  CHARACTERISTICS OF CONDUCTING RESISTORS
Origin and Sponsor:  NBS, Section 1.6
Manager:  B. G. Urban
Full task description:  Apr-June 1954 issue, p. 40
Status:  Completed.

1102-40-5126/55-61  ELASTIC CROSS SECTION FOR NEUTRON SCATTERING
Origin and Sponsor:  Naval Research Laboratory
Manager:  I. Stegun
Full task description:  Oct-Dec 1954 issue, p. 18
Status:  Completed.

3711-60-0009/55-66  RECONSTITUTION OF MONOCHROMATIC LIGHT INTENSITIES
Origin:  NBS, Division 30
Manager:  H. Bremer
Full task description:  July-Sept 1954 issue, p. 18
Status:  Inactive.  For status to date, see Oct-Dec 1954 issue, p. 19

3711-60-0009/55-68  CRYSTAL STRUCTURE CALCULATIONS
Origin:  NBS, Division 9
Manager:  R. Prosen (Div. 9)
Full task description:  Jan-Mar 1955 issue, p. 18
Status:  Continued.  Structure factors were computed for the magnesium
borate crystal using three sets of atomic position parameters.  Codes
have been prepared to calculate structure factors for BAHPO$_4$ and
CA(OH)$_2$.

1102-40-5126/55-74  LIQUID-VAPOR TRANSITION, II
Origin and Sponsor:  Naval Medical Research Institute
Manager:  S. Prusch
Full task description:  Jan-Mar 1955 issue, p. 19
Status:  Completed.  Results have been transmitted to the sponsor.

3711-60-0009/55-75  PARAMETER OF THE DISPERSION EQUATION FOR OPTICAL GLASS
Origin:  NBS, Section 2.2
Manager:  R. Zucker
Full task description:  Jan-Mar 1955 issue, p. 20
Status:  Continued.  The constants $k_i$ were determined for AS$_2$S$_3$ with
$$\begin{align*}
\lambda_1^2 &= .0225 \\
\lambda_2^2 &= .0625 \\
\lambda_3^2 &= .1225 \\
\lambda_4^2 &= .2025 \\
\lambda_5^2 &= .3600
\end{align*}$$
1102-40-5126/55-81  COMBINING TESTS FOR SIGNIFICANCE
Origin:  NBS, Section 11.3
Manager:  L. S. Joel
Full task description:  Oct-Dec 1954 issue, p. 23
Status:  Inactive.

3711-60-0009/55-82  THERMOMETER CALibrATIONS
Origin:  NBS, Section 3.1
Manager:  S. Prusch
Full task description:  Jan-Mar 1955 issue, p. 20
Status:  Continued. Calibration tables were computed for two thermometers under test.

1102-40-5126/55-88  STRESSES IN A WALL FOUNDATION
Origin and Sponsor:  NBS, Section 10.1
Manager:  D. Jirauch
Full task description:  Jan-Mar 1955 issue, p. 22
Status:  Inactive.

1102-40-5126/55-103  MISSOURI RIVER PROBLEM
Origin and Sponsor:  Missouri River Division, Corps of Engineers, U.S.Amy
Manager:  H. Bremer
Full task description:  Jan-Mar 1955 issue, p. 26
Status:  Completed. All the codes have been written and checked for UNIVAC. Processing of the data is to be performed by the sponsor's staff.

1102-40-5126/55-113  REACTOR DESIGN
Origin:  Westinghouse Atomic Power Division
Sponsor:  Atomic Energy Commission
Manager:  U. Hochstrasser
Full task description:  Jan-Mar 1955 issue, p. 28
Status:  Continued. The code for the base reactor problem is being checked out on the SEAC. A report describing the problem and the experiments done so far is being written.

1102-40-5126/55-117  ATTENUATION OF PRESSURE PULSES OF FINITE AMPLITUDE
Origin:  NBS, Section 3.2
Sponsor:  Bureau of Aeronautics, U. S. Navy
Manager:  M. Paulsen
Full task description:  Apr-June 1955 issue, p. 18
Status:  Inactive.
Status of Projects

1102-40-5126/55-118  THERMOMETER CALIBRATION, II
Origin and Sponsor: NBS, Section 3.1
Manager: B. G. Urban
Full task description: Apr-June 1955 issue, p. 18
Status: Inactive. For status to date, see Jan-Mar 1956 issue, p. 22

1102-40-5126/55-121  ELECTRON PENETRATION
Origin: NBS, Section 4.8
Sponsor: Atomic Energy Commission
Manager: S. Peavy
Full task description: Apr-June 1955 issue, p. 19
Status: Continued. All the codes have been checked and some initial runs have been made.

1102-40-5126/55-122  SOLUTION OF NORMAL EQUATIONS
Origin: Advisory Committee on Weather Control
Sponsor: U. S. Weather Bureau
Manager: M. Newman
Full task description: Apr-June 1955 issue, p. 20
Status: Completed.

1102-40-5126/55-123  TEMPERATURE DISTRIBUTION
Origin: NBS, Section 6.4
Sponsor: NBS, Section 6.4
Manager: W. G. Hall
Full task description: Apr-June 1955 issue, p. 19
Status: Inactive. For status to date, see Oct-Dec 1955 issue, p. 24

1102-40-5126/55-126  AERODYNAMIC HEATING
Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Dept. of the Army
Managers: B. Walter, R. Capuano
Full task description: July-Sept 1955 issue, p. 31
Status: Continued. Work was continued on the heat transfer for a laminar boundary layer. New velocities ($V'$) and $V*$ were calculated and new $T_{AW}'$ and $T_{AW}*_{AW}$ were obtained (see Apr-June 1956 issue). Tables of $T_{W}'$ and $T_{W}*_{W}$ were calculated for $B = 7, 5, 4, 3, 2, 1, 7$, for the four trajectories. Intersections for the various values of $T_{W}'$, resp. $T_{W}*_{W}$ were computed for the various values of $B$.

1102-40-5126/55-127  VIBRATIONS OF CIRCULAR DISC
Origin and Sponsor: Evans Signal Laboratory
Manager: U. Hochstrasser
Full task description: July-Sept 1955 issue, p. 32
Status: Continued. Approximately 1600 additional values have been computed for a table of $\mathcal{E}$ vs. $d/h$, where $\mathcal{E}$ is the dimensionless resonant frequency, $d$ the diameter, and $h$ the thickness of the disc.

**3711-60-0009/56-131  **CALCULATIONS IN OPTICS  
**Origin:** NBS, Section 2.2  
**Manager:** D. P. Feder (2.2)  
**Full task description:** July-Sept 1955 issue, p. 33  
**Status:** Continued. The spot diagram code has been completely rewritten to take advantage of new coding techniques and the acquisition of a plotter and a high speed punch. This code has been operating successfully and is about four times as fast as the old code. Some success has been attained in using the steepest descent method for solving the set of non-linear equations arising in the design of optical systems. Numerical experimentation is being carried out using an accelerated optimum gradient method, using a method due to Crochett and Chernoff, and using a method of constraints.

**1102-40-5126/56-133  **COMPLEX EIGENVALUES  
**Origin:** Moore School of Engineering  
**Sponsor:** Office of Naval Research  
**Manager:** I. Stegun  
**Full task description:** July-Sept 1955 issue, p. 33  
**Status:** Completed. Results were transmitted to sponsor.

**1102-40-5126/56-136  **CALCULATION OF WAVE FUNCTIONS BY HARTREE METHOD  
**Origin:** Naval Research Laboratory  
**Sponsor:** "  
**Manager:** S. Peavy  
**Full task description:** July-Sept 1955 issue, p. 34  
**Status:** Continued. Runs have been made upon request of the sponsor.

**1102-40-5126/56-137  **STABILITY OF SUPPORTED PLATES  
**Origin and Sponsor:** National Advisory Committee on Aeronautics  
**Manager:** R. Zucker  
**Full task description:** July-Sept 1955 issue, p. 35  
**Status:** Completed. Results have been transmitted to sponsor.

**1102-40-5126/56-139  **STUDY OF INTERNUCLEAR POTENTIAL FOR $\text{H}_3$  
**Origin and Sponsor:** NBS, Section 3.2  
**Manager:** E. Haynsworth  
**Full task description:** July-Sept 1955 issue, p. 36  
**Status:** Continued. Wave functions and the total electronic energies for the ground state of the linear symmetric $\text{H}_3$ intermediate complex have
been calculated over a wide range of internuclear distances in the LCAO-MO approximation employing a simple variational treatment of (1) a single determinantal wave function, and (2) a complete configuration interaction (CI) wave function made up of a linear combination of determinants over all possible allowed combinations of MSO's made up of Slater functions. As a further refinement, the effective nuclear charges of the three centers a, b, c were varied \( z^a = z^b = z^c \) in both cases. The first calculation obtained the total energy in terms of the associated eigenvectors. The second computation involved the calculation of the eigenvalues and eigenvectors of 4x4 symmetric matrices.

1102-40-5126/56-140  MULTIPLE CORRELATION ROUTINES  
Origin and Sponsor:  Agricultural Economics Division, Department of Agriculture  
Managers:  H. Bremer, M. Paulsen  
Full task description:  Oct-Dec 1955 issue, p. 26  
Status:  Continued. By now the use of the codes involved in obtaining the multiple correlation has become routinized. A system is underway whereby the sponsor sets up the data to be analyzed and then prepares it himself in the proper input format for SEAC. At present the sponsor is using the input equipment for SEAC at the NBS Computation Laboratory. Future plans are to have similar equipment set up at his office to facilitate the handling.

1102-40-5126/56-143  CHARGED PARTICLE TRAJECTORIES IN THE MAGNETIC FIELD OF THE EARTH  
Origin and Sponsor:  Naval Research Laboratory  
Manager:  J. H. Wegstein  
Full task description:  Oct-Dec 1955 issue, p. 27  
Status:  Continued. The trajectories for 89 more sets of initial conditions were computed with the NORC and were sent to the sponsor.

1102-10-5126/56-151  MULTIPLE SCATTERING IN CROSS SECTION MEASUREMENTS  
Origin and Sponsor:  Naval Research Laboratory  
Manager:  I. Rhodes  
Full task description:  Oct-Dec 1955 issue, p. 30  
Status:  Completed. Results have been transmitted to the sponsor.

1102-40-5126/56-157  DAMAGE ASSESSMENT  
Origin and Sponsor:  Office of the Assistant Secretary of Defense (Supply and Logistics)  
Managers:  A. J. Hoffman, H. Bremer  
Full task description:  Oct-Dec 1955 issue, p. 31  
Status:  Completed. A program of a series of routines was set up which
takes the input of the nation's resources and correlates it with certain attack data, then prepares various outputs in the form of listings that summarize the extent of the damage to the nation as a consequence of such an attack. The routines have been written and code-checked, and they were used in two runs. Each of the runs involved over 200,000 items of information and required about 100 hours of UNIVAC time. Towards the end, because of lack of time on UNIVACS in the Washington area, the problem was finished at a UNIVAC installation in Boston.

1102-40-5126/56-162 STRESSES IN A WALL RESTING ON A FOOTING
Origin and Sponsor: NBS, Section 10.1
Manager: I. Stegun
Full task description: Jan-Mar 1956 issue, p. 26
Status: Continued. Data has been prepared for the solution of 100 equations in 100 unknowns. Use will be made of the matrix inversion codes carrying along a check vector.

1102-40-5126/56-163 ANGULAR DISTRIBUTIONS AND POLARIZATION EFFECTS IN NUCLEAR SCATTERING
Origin and Sponsor: Naval Research Laboratory
Manager: I. Stegun
Full task description: Oct-Dec 1955 issue, p. 32
Status: Continued. Changes are being made in the code to decrease time requirements.

1102-40-5126/56-165 INTEGRALS FOR SCATTERING FUNCTIONS
Origin and Sponsor: Naval Research Laboratory
Manager: R. Zucker
Full task description: Oct-Dec 1955 issue, p. 32
Status: Continued. At the request of the sponsor, the function \( \xi \cdot I(p, \xi) \) as \( \xi \to 0 \) was tabulated as a function of \( p \) for \( I \) (confluent) and \( I \) (Bessel) for purposes of making a graph. In the regions of maxima and minima and of discontinuities, additional values of the functions were calculated. (For \( I \) (confluent), see Oct-Dec 1955 issue, p. 32. For \( I \) (Bessel), see Apr-June 1956 issue, p. 27.)

1102-40-5126/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Manager: E. Haynsworth
Full task description: Jan-Mar 1956 issue, p. 27
Status: Inactive.
1102-40-5126/56-167 He$^3$ REACTIONS

**Origin and Sponsor:** Naval Research Laboratory

**Manager:** I. Stegun

**Objective:** To fit polynomial expansions to data by the method of least squares.

**Background:** It has recently been found that many deuteron induced nuclear reactions take place by a direct process rather than by the formation of a compound nucleus. It is of interest to determine whether He$^3$ induced nuclear reactions may also proceed by a direct interaction. A series of experiments have been performed at the Naval Research Laboratory in which the angular distributions of the reaction products have been studied from the bombardment of light nuclei with He$^3$ particles. From the theory of compound nucleus reactions these angular distributions should be capable of being expressed in terms of a rather simple expansion of Legendre polynomials. The theory can also give qualitative information about the coefficients in these expansions. In order to compare the theory with the experimental results, it is necessary to attempt to fit polynomial expansions to the data.

The problem was proposed by H. D. Holmgren, NRL.

**Status:** Completed (NEW). Results were transmitted to the sponsor.

1102-40-5126/56-169 CRYSTAL FIELD EFFECTS FOR ATOMS

**Origin and Sponsor:** NBS, Division 3

**Manager:** I. Stegun

**Full task description:** Jan-Mar 1956 issue, p. 28

**Status:** Inactive. For status to date, see Jan-Mar 1956 issue, p. 28

1102-40-5126/56-171 COLLISION INTEGRALS USED IN TRANSPORT THEORY

**Origin and Sponsor:** NBS, Section 3.2

**Manager:** J. Cooper

**Full task description:** Oct-Dec 1955 issue, p. 33

**Status:** Continued. 1. The code mentioned in the Apr-June issue, page 28, was used to obtain transport properties for a mixture of five gases at 20 different temperatures. 2. A new code was written to solve the linear homogeneous system of equations

$$\delta_{ij} - \frac{m_i c_{ij}(T)}{\sum_k m_k c_k(T)} = \sum_{k=1}^n c_{ik} m_{ik}(T) \left\{ \frac{a_{ij}}{m_i} - \frac{a_{kj}}{m_k} \right\}$$

$$\sum_{j=1}^n a_{ij} m_j = 0$$

where the $a_{ij}$'s are the unknowns and the $c_i$'s and $m_i$'s are the same
as in the code mentioned in (1) above.

\[ m_{ik} \text{ is found from the } \Omega_{1k}^{(1,1)} \text{ previously defined by the relation:} \]

\[ m_{ij}(T) = \frac{16}{3} n \cdot \frac{m_i m_j}{m_i + m_j} \Omega_{1j}^{(1,1)}(T) \]

Once the quantities \( a_{ij} \) are found, new results are found by the relation:

\[ C_{ij} = \frac{C_i}{m_i} n_o RT a_{ij} \]

The coding for this part of the problem was completed, using parts of the code mentioned in (1). The code was checked and run for the same input data as the first problem.

1102-40-5126/56-172 NUMERICAL EVALUATION OF SPECIAL INTEGRAL EXPRESSIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Department of the Army
Manager: L. Joel
Full task description: Jan-Mar 1956 issue, p. 29
Status: Inactive.

1102-40-5126/56-176 MODEL GOVERNMENT PAYROLL ON HIGH SPEED COMPUTERS
Origin and Sponsor: NBS, Section 12.5
Manager: G. H. Urban
Full task description: Jan-Mar 1956 issue, p. 30
Status: Inactive.

1102-40-5126/56-179 NORMAL PROPAGATION CONSTANT
Origin and Sponsor: NBS, Section 82.10
Manager: H. H. Howe (82.10)
Full task description: Apr-June 1956 issue, p. 30
Status: Completed. Two new attacks were used in the effort to find all of the modes. Solving equation (2) (Apr-June 1956 issue, page 31) for \( n \):

\[ n = 2Hc + \left( i \log \frac{R}{R_s} \right)/2\pi \]

For fixed values of the parameters \( H, K, G, \) and \( L \), we now look upon \( n \) as a complex function of the complex variable \( c \). \( n \) is multiple-valued, in that the \( n \) for fixed \( c \) may be augmented by any real integer. We need all values of \( c \) that give real integral \( n \). The integral \( n \) assigned to such a "mode" is arbitrary, but we make it definite by saying that
for fixed n, c should be a continuous function of L and G, and should approach \( c = n/2H \) as L and G approach zero.

The first new attack involved selecting values of H, K, G, L, and tracing in the c-plane the curve for which n is real; and spotting the points at which n is a real integer. n was assumed to vary continuously along the curve; this fixed n for all points of the curve once it is fixed for any point. For non-zero L and G, one integral n occurs for \( c = 0 \); not this one, but the next one is \( n = 0 \). This method worked well for very small L, smaller than are likely to occur in nature. As L gradually increases (e.g., for \( H = 4 \), \( G/H = 0.0001 \), \( K = 15 \), it occurs at \( L/H = 0.00277 \)), the curve of real n develops a bow near \( n = 0 \), which finally separates entirely from the main curve; hence, the method of numbering n along a continuous curve produces results that show a discontinuity when c for given n is listed as a function of L.

The next attack was to trace in the c-plane the curve for which n is a real integer, with variable L, by iterating equation (2). This worked easily and quickly for any n except \( n = 0 \). But for \( n = 0 \), it went slowly and eventually bogged down entirely. Then it was learned that the elusive mode 0 is, over much of the range of L, extremely close to the value of c for which \( R = 0 \), which is a singular point of n. I.e., over most of the range of L, there is an isolated "circle" of real n; at some point on the "circle" is the root \( n = 0 \); at the "center", the imaginary part of n is \(-\infty\). For much of the range of L, the "circle" is so small that the value of c at its center (which can be found by simple algebraic means) is to high accuracy also the value of c for \( n = 0 \). The imaginary part of c is much larger on this "circle" than for other integral n, with resulting large attenuation of the wave. For high values of L, the "circle" which includes \( n = 0 \) returns and rejoins the main curve; e.g., for \( H = 4 \), \( G/H = 0.0001 \), \( K = 15 \), \( L/H = 1 \), we must consider that integral values of n lie on the continuous curve of real n in the sequence \( 1, 2, 3, 4, 5, 0, 6, 7, 8 \).

For all except very small values of L, the dominant mode (the one having least attenuation) is found to be \( n = 1 \), rather than \( n = 0 \) as has commonly been supposed.

1102-40-5126/56-184  GAIN CALCULATIONS OF AN ITERATED TRANSISTOR AMPLIFIER
Origin and Sponsor:  Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: E. Haynsworth
Full task description: Apr-June 1956 issue, p. 32
Status:  Continued. The voltage gain calculations for a number of different transistors have been completed, and several variations in parameters have been introduced to give a more complete analysis for each transistor.
MECHANICAL MEASUREMENTS OF GAGE BLOCKS

Objective: To program test data obtained from mechanical measurements of gage blocks so that data reduction and formation of the final certification reports may be accomplished using automatic computing machines. In addition, the program is to be arranged to allow for periodic analyses of the test data to determine the condition of the master gage blocks.

Background: The reduction, analysis, and transfer of the test data to the certification report is now accomplished manually. Although the reduction of data is a simple arithmetic process, the quantity of data involved requires many man-hours of labor. In addition, much supervisory time is required to ascertain that the arithmetic processes are accomplished without error and that the final results are transferred to the certification report accurately. Processing of the data on automatic computing machines will be more efficient and more economical.

Status: NEW. Sample problems are being run to test the procedure.

CONTINUED FRACTIONS

Objective: To tabulate

\[
\frac{(\sqrt{2})^j}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} f(x) \, dx,
\]

where

\[ f(x) = [\sqrt{2} \cdot x]^j \{ \Phi(\sqrt{2} \cdot x+q) \}^n \]

and \[ \Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-z^2/2} \, dx \]

for \( q = 0(.25)4.5, \)
\( n = 2(1)50, \)
\( j = 0,1,2,3,4. \)

Background: This computation was undertaken at the request of R. C. Bose, who spent the summer of 1956 at the NBS Statistical Engineering Laboratory. The tables will be useful in connection with a statistical test concerning the choice of a population with the largest mean from among a set of normal populations having the same variance.

Status: NEW. The code has been written and checked out. The computations are under way.
1102-40-5126/57-206  EXPECTED VALUES OF ORDER STATISTICS

Origin and Sponsor: NBS, Section 11.3
Manager: R. Durrah
Objective: To compute

\[ \int_{-\infty}^{\infty} P_r(x) \exp \left\{ \frac{-rx^2}{2} \right\} \, dx, \quad r = 2, 3, 4, 5, \]

where

\[ P_r(x) = C_{n,k} \left[ \Phi(x) \right]^{k-r} [1 - \Phi(x)]^{n-k+1-r} [x_r(n,k)]; \]

and

\[ \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-x^2/2} \, dx \]

\[ x_r(n,k) = \sum_{i=0}^{r-1} (-1)^i \cdot \frac{(r-1)!}{(r-1-i)!i!} \cdot \frac{(k-1)!}{(k-r+i)!} \cdot \frac{(n-r+i)!}{(n-r)!} \cdot \Phi^i \]

Background: The computations were performed at the request of R. C. Bose, who spent the summer of 1956 at the NBS Statistical Engineering Laboratory. The tables will be used in connection with a statistical test concerning the choice of a population with the largest mean from among a set of normal populations having the same variance.

Status: NEW. The code has been written and checked out.

3711-60-0009/57-210  SOUND VELOCITY

Origin and Sponsor: NBS, Section 3.2
Manager: U. Hochstrasser
Objective: Tables of values for the pressure \( P \) and the density are given for fixed temperatures. It is required to fit these data by polynomials such that

\[ P = P(\rho) = a_0 + a_1 \rho + a_2 \rho^2 + \ldots, \]

and to evaluate

\[ a = \sqrt{101.325 \frac{dP}{d\rho}} \]

for a set of values of \( \rho \) by using these polynomials.

Background: This problem arises in the theory of sound. Here \( a \) is the velocity of sound.
Status: NEW. The code has been written.

1102-40-5126/57-215 COMPUTATION OF INTEGRALS INVOLVING BESSEL FUNCTIONS
Origin and Sponsor: NBS, Section 6.1
Manager: U. Hochstrasser
Objective: To compute tables of the functions,

\[ P(x, \theta) = \frac{2}{x} \int_{0}^{x} dv \int_{0}^{\infty} \cos(u \sin \theta) J_0(u)du, \]

\[ Q(x, \theta) = -\frac{2}{x} \int_{0}^{x} dv \int_{0}^{\infty} \cos(u \sin \theta) Y_0(u)du, \]

for \( x = 1(1)20, \theta = 15^\circ(15)90^\circ. \)

Background: These integrals appear when the diffraction of sound is considered at the edge of an absorbent material.

Status: NEW. The code has been written and checked out. The results on the work completed thus far have been transmitted to the sponsor.
6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Authorized 7/1/50

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Strength of glass: analysis of data, for M. J. Kerper, Refractories Section.
(3) Evaluation of waxes: analysis of data and methods for improving precision of results, for W. W. Walton, Surface Chemistry Section.
(5) Statistical analyses were made on SEAC for H. Allen, Radiometry Section; J. E. McKinney, Rubber Section; M. Burkhard, Sound Section; M. Greenspan, Sound Section; G. Conrad, Engineering Electronics Section; J. Mandel, Testing and Specifications; J. Hilsenrath, Thermodynamics Section; and for the Child Safety Project.

Publications:
Status of Projects

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC  Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: CONTINUED. An analysis of results from nine physical and chemical tests on ship steel plates from current production is being carried out. An incomplete block arrangement was used in the experiment, so balanced that variation of heats, ingot within a heat, and plates within an ingot could be evaluated.

STATISTICAL ANALYSIS AND DESIGN OF EXPERIMENTS
FOR THE U. S. GEOLOGICAL SURVEY
Task 1103-40-5140/54-1

Department of Interior
Managers: C. Eisenhart, W. J. Youden
Full task description: Oct-Dec 1953 issue, p. 50

Status: CONTINUED. Two manuscripts by Frank Flanagan of the Geochemistry and Petrology Branch, U. S. Geological Survey, were reviewed. One involved the mathematics of sample splitters, and the other rank correlation methods.

CATALOGUE OF FRACTIONAL REPLICATION DESIGNS
Task 1103-40-5147/57-213

Origin and Sponsor: Bureau of Ships  Authorized 8/30/56
Managers: W. S Connor, M. Zelen

Objective: To construct and prepare a catalogue of fractional replication designs for the 3rd factorial series. These designs are to be arranged so that heterogeneity between blocks can be eliminated.

Background: The use of factorial designs has now become widely accepted as an efficient way for carrying out experiments involving many different factors. However, one of the main difficulties with factorial designs is that the number of measurements required may be large and in some cases prohibitive. Another disadvantage is that in many experimental situations it is not practical to plan an entire
experimental program in advance, but to make a few smaller experiments which serve as a guide to further work. This latter condition is especially true when measurements are made singly or in small groups, such that the experimental results become known sequentially as they are taken. The work under this project extends to the $3^n$ series the work previously done on the $2^n$ series.

Status: NEW. The work of construction has been completed. This was done during the summer by R. C. Burton, H. M. Pettigrew, and F. L. Miller, Jr., with assistance from R. C. Bose, who undertook to develop the theory (see task 1103-10-1107/53-1, Studies in the mathematics of experiment design, p.14). The designs are now being typed and checked under the direction of Lola Deming. It is expected that the SEAC will be useful in checking.

The number of factors ranges from 4 to 10, and the fraction from $1/3$ to $1/243$. Alias relationships are presented for completely randomized designs, and block confounding relationships for the designs with blocks. The number of units per block ranges from 3 to 243, and the number of blocks from 3 to 27.
The record of SEAC operations for tasks of the Applied Mathematics Division for the period July 1 through September 30 is as follows:

<table>
<thead>
<tr>
<th>Task No.</th>
<th>Title</th>
<th>Code</th>
<th>Checking</th>
<th>Productive Operation</th>
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</thead>
<tbody>
<tr>
<td>1104/55-55</td>
<td>Research in numerical analysis</td>
<td>6</td>
<td>29</td>
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<tr>
<td>5116/55-56</td>
<td>Research in mathematical topics applicable to numerical analysis</td>
<td>3</td>
<td>40</td>
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<tr>
<td>5116/56-148</td>
<td>Study of differential equations for nerve fiber excitation</td>
<td>24</td>
<td>11</td>
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<tr>
<td>1110/47-2</td>
<td>Tables of Coulomb wave functions</td>
<td>6</td>
<td>115</td>
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<td>1110/53-51</td>
<td>Radiation diffusion</td>
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<td>70</td>
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<td>1110/53-52</td>
<td>L-Shell conversion coefficients</td>
<td>5</td>
<td>10</td>
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<tr>
<td>5126/52-44</td>
<td>Calculation for d-spacings</td>
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<td>10</td>
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<tr>
<td>5126/53-29</td>
<td>Dynamic behavior of aircraft structures</td>
<td>2</td>
<td>11</td>
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<td>5126/54-4</td>
<td>High temperature properties of water</td>
<td>5</td>
<td>11</td>
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<td>5126/54-30</td>
<td>Spectrometer analysis</td>
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<td>5126/55-68</td>
<td>Crystal structure calculations</td>
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<td>5126/55-97</td>
<td>High temperature properties of air</td>
<td>4</td>
<td>57</td>
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<td>5126/55-121</td>
<td>Electron penetration</td>
<td>9</td>
<td>8</td>
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<tr>
<td>5126/55-127</td>
<td>Vibration of a circular disc</td>
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<td>5126/56-128</td>
<td>Ground reflection coefficients</td>
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<tr>
<td>0009/56-131</td>
<td>Ray tracing, II</td>
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<tr>
<td>5126/56-139</td>
<td>Internuclear potential for H₃</td>
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<td>5126/56-144</td>
<td>Auto correlation</td>
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<tr>
<td>0009/56-160</td>
<td>Mathematical subroutines</td>
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<tr>
<td>5126/56-166</td>
<td>SCF-LCAO solution of some hydrides</td>
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<td>5126/56-167</td>
<td>Multiple regression</td>
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<td>5126/56-169</td>
<td>Crystal field effects for atoms</td>
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<tr>
<td>5126/56-171</td>
<td>Collision integrals used in transport theory</td>
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<td>Normal propagation constant</td>
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<td>5126/56-185</td>
<td>Matrix multiplication</td>
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<td>5126/56-192</td>
<td>Noise measurement</td>
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<td>5126/56-195</td>
<td>Continued fractions</td>
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<td>5126/57-198</td>
<td>Transient heat</td>
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<td>3</td>
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<td>5126/57-206</td>
<td>Value order statistics</td>
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<td>5126/57-214</td>
<td>Gears</td>
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<td>5160/55-85</td>
<td>Research in mathematical elasticity</td>
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<td>5160/56-156</td>
<td>Research in mathematical geophysics</td>
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<td>18</td>
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<tr>
<td>0002/52-1</td>
<td>Statistical engineering</td>
<td>14</td>
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Misc. Training
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<tr>
<th>Code</th>
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<th>Checking</th>
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<tr>
<td>5126/53-45</td>
<td>Application of game theory</td>
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<tr>
<td>5126/54-13</td>
<td>Award of procurement contracts for linear programming</td>
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<tr>
<td>5126/55-104</td>
<td>&quot;Fuze&quot; problem</td>
<td>1</td>
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<tr>
<td>5126/55-113</td>
<td>Reactor design</td>
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<tr>
<td>5126/56-130</td>
<td>Aircraft responses</td>
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<tr>
<td>5126/56-136</td>
<td>Calculation of wave functions</td>
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<tr>
<td>5126/56-140</td>
<td>Multiple correlation routines</td>
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<tr>
<td>5126/56-142</td>
<td>Matrix computation</td>
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<td>5126/56-151</td>
<td>Cross section measurements</td>
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<tr>
<td>5126/56-163</td>
<td>Nuclear scattering</td>
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<td>5126/56-165</td>
<td>Integral for scattering functions</td>
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<td>5126/56-184</td>
<td>Transistor amplifier</td>
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<td>5126/56-187</td>
<td>Moments</td>
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<td>5126/56-188</td>
<td>Gases</td>
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<td>5126/56-189</td>
<td>Numerical exponents</td>
<td>4</td>
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</table>
Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Division Seminar

BOSE, R. C. (Guest Worker from University of North Carolina). On a problem in abelian groups. August 8.


Numerical Analysis Seminar


Reliability Seminar


LIEBLEIN, J. How conditions of use affect the reliability and performance of electron tubes. July 5.


Statistical Theory Seminar

(Offered jointly by the National Institute of Health and the National Bureau of Standards)

Papers and Invited Talks
Presented by Members of the Staff
at Meetings of Outside Organizations

BOSE, R. C. (Guest Worker from University of North Carolina). On a
decision procedure for ranking means. Presented at a Seminar
at the Bureau of the Census, August 9.

BURTON, R. C. On the identity relationship for fractional replicates
of the $2^n$ series. Read by W. S. Connor at a meeting of the
Institute of Mathematical Statistics, Seattle, Washington,
August 20-27.

CAMERON, J. M. Use of SEAC for statistical calculation. Presented at
a meeting of the Statistical Summer Session, held jointly
with the American Statistical Association at Massachusetts
Institute of Technology, Cambridge, Massachusetts, August 17.

RHODES, I. Some sobering thoughts on handling data by means of electronic
computers. Presented at a meeting of the National Machine

SOMERVILLE, P. N. Selecting the "best" from k+1 populations. Presented
at the Gordon Conference on Statistics in Chemistry and
Chemical Engineering, held by the American Association for
the Advancement of Science, New Hampton, N. H., August 21.

YOUDE\N, W. J. Mathematics and the teaching of physics and chemistry.
Presented at a seminar of the Department of Chemistry, American
University, Washington, D. C., July 5.

ZELEN, M. Statistical concepts in life testing. Presented at a meeting
of the Professional Group in Component Parts, Institute
of Radio Engineers, Washington, D. C., September 12.

* * * * *

Papers presented at the meeting of the Association for Computing
Machinery, Los Angeles, California, August 27-29:

ABRAMOWITZ, M. Self-checking methods of integration.

ALT, F. Mathematical techniques in data processing problems.

Papers presented by P. Chiarulli at the Ninth International Congress on
Mechanics, Brussels, Belgium, September 5-13:
CHIARULLI, P., and R. F. DRESSLER. Condensation interfaces in two-phase flows.

DRESSLER, R. F. Vibrations of triangular wings.

* * * * * *

Papers presented at the meeting of the Societa Italiana per il Progresso delle Scienze, Catania, Sicily, September 19:

TAUSSKY-TODD, O. Computational problems for matrices of rational integers.

TODD, J. Computational problems concerned with the Hilbert matrix.

* * * * *

Papers presented at the Joint Annual Meeting of the American Statistical Association, the Institute of Mathematical Statistics, and the Biometrics Society, Detroit, Michigan, September 7-8:

ABRAMOWITZ, M., and J. M. CAMERON. Requirements of scientific computations arising in Government computing.

YOUDEN, W. J. Randomization and experimentation.

ZELEN, M. Analysis of covariance for incomplete block designs.
Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.1 Mathematical Tables


1.3 Technical Papers


(4) Geological application of extreme-value methods to interpretation of cobbles and boulders in gravel deposits. W. C. Krumbein (Northwestern University) and J. Lieblein. Trans. Amer. Geophysical Union 37, 313-319 (June 1956).


2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION SEPTEMBER 30, 1956

2.1 Mathematical Tables

(1) Tables of the bivariate normal distribution function and related functions. To appear as NBS Applied Mathematics Series 50.

2.2 Manuals, Bibliographies, and Indices


(2) Fractional factorial experimental designs for factors at two levels. To appear as NBS Applied Mathematics Series 48.

2.3 Technical Papers


(13) Unimodular matrices of order two which commute. K. Goldberg. Submitted to a technical journal.

(14) Pairs of matrices of order two which generate free groups. K. Goldberg and M. Newman. Submitted to a technical journal.

(15) Note on bounds for certain determinants. E. Haynsworth. Submitted to a technical journal.


(17) On the representation of a certain integral involving Bessel functions by hypergeometric series. P. Henrici. Submitted to a technical journal.


(25) Some theorems about \( P_r(n) \). M. Newman. Submitted to a technical journal.


(28) Application of quaternions to the representations of a binary quadratic form as a sum of four squares. G. Pall (Illinois Institute of Technology) and O. Taussky. Submitted to a technical journal.


(30) Pitfalls in computation. I. A. Stegun and M. Abramowitz. Submitted to a technical journal.


(37) Commutativity in finite matrices. O. Taussky. Submitted to a technical journal.

(38) Commuting bilinear transformations and matrices. O. Taussky and J. Todd. Submitted to a technical journal.


(44) Randomization and experimentation. W. J. Youden. Submitted to a technical journal.


(47) The analysis of covariance for incomplete block designs. M. Zelen. Submitted to a technical journal.

2.5 Miscellaneous items

(1) Contributions on partially balanced incomplete block designs with two associate classes. W. H. Clatworthy. To appear as NBS Applied Mathematics Series 47.

(2) Further contributions to the solution of simultaneous linear equations and the determination of eigenvalues. To appear as Applied Mathematics Series 49.
THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($0.75), available from the Superintendent of Documents, Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.