NATIONAL BUREAU OF STANDARDS REPORT

4952

PROJECTS and PUBLICATIONS of the APPLIED MATHEMATICS DIVISION

A Quarterly Report

July through September 1956

U. S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

U. S. DEPARTMENT OF COMMERCE
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THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major field laboratories in Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside back cover of this report.

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Electricity and Electronics. Resistance and Reactance. Electron Tubes. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat and Power. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology and Lubrication. Engine Fuels.

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Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analogue Systems. Application Engineering.

- Office of Basic Instrumentation
- Office of Weights and Measures

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Systems. Navigation Systems. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Radio Systems Application Engineering.

Radio Standards. Radio Frequencies. Microwave Frequencies. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Calibration Center. Microwave Physics. Microwave Circuit Standards.

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NBS PROJECT	NBS REPORT
11.0	4952

PROJECTS and PUBLICATIONS of the APPLIED MATHEMATICS DIVISION

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U. S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

July 1 through September 30, 1956

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CONTENTS

*Only unclassified projects are included in this report.

Status of Projects

September 30, 1956

1. NUMERICAL ANALYSIS

RESEARCH IN THEORIES OF DISCRETE STRUCTURES
Task 1100-11-5170/56-159

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: O. Taussky-Todd
Full task description: July-Sept 1955 issue, p. 1

Status: CONTINUED. O. Taussky-Todd did further study on totally positive matrices. She also investigated the feasibility of SEAC computation of two numerical problems concerning class groups in quadratic fields. Work on classes of matrices was continued,--in particular, the discussion of the matrix class which corresponds to the product of the ideal classes.

O. Taussky-Todd and J. Todd studied commutative unimodular matrices in connection with a result of K. Goldberg. A new proof of his result,--essentially that all abelian subgroups of the modular group are cyclic,--was given, and a generalization to the case of 2x2 matrices with elements in a complex quadratic field F, where the determinant is a unit in F, was obtained.

A manuscript was prepared by E. Dade on "Abelian groups of unimodular matrices." The paper offers a generalization of the theorems of K. Goldberg, O. Taussky-Todd and J. Todd on the structure of abelian subgroups of the group $\Gamma_n(k)$ of nxn extended unimodular matrices over an algebraic number field k. The problem is approached by considering, not just the abelian subgroup, but the subalgebra of the full nxn matrix algebra which it generates. The intersection $\Gamma_n(k)$ with the subalgebra is an abelian subgroup which is also the group of units for an order in the algebra. The structure of the unit group of an order was investigated and the results were applied to abelian subgroups.

O. Taussky-Todd continued earlier work concerning positive definite quadratic forms and prepared a manuscript in collaboration with G. Pall (Illinois Institute of Technology). The paper presents the exact number of representations of a quadratic form with rational integral coefficients as a sum of four squares of rational integral linear forms.

The treatment is by quaternions.

O. Taussky-Todd prepared a survey of computations on matrices with integral element for presentation to the Societa Italiana per il Progresso delle Scienze at Catania, Sicily.

E. Dade and K. Goldberg continued their investigation of incidence algebras. An application was found for symmetric block designs: A construction procedure was described for the parametric representation $r=p^n \equiv 3 \pmod 4$ where p is a prime, $k=(v-1)/2$ and $\lambda=(v-3)/4$.

H. Cohn has computed units for "pure cubic" fields of type $\sqrt[3]{N}$ by use of Dedekind's eta-function formula. The program is continuing in operation, and results to date include the discovery of two fields of class number 27, ($\sqrt[3]{217}$ and $\sqrt[3]{342}$), and two of class number 21, ($\sqrt[3]{215}$ and $\sqrt[3]{422}$). In each case a tabulation of residue and non-residue quadratic forms was obtained. A paper is in preparation.

Publications:

(1) The number of absolute points of a correlation. A. Hoffman, M. Newman, E. Straus (University of California at Los Angeles), O. Taussky. Appeared: Pac. J. Math. 6, 83-96 (1956).

(2) Algebraic equations satisfied by roots of natural numbers. E. G. Straus (University of California at Los Angeles) and O. Taussky. Appeared: Pac. J. Math. 6, 97-98 (1956).

(3) On the Hilbert matrix. T. Kato. To appear in the Proceedings of the American Mathematical Society.

(4) Classes of positive definite unimodular circulants. M. Newman and O. Taussky. To appear in the Canadian Journal of Mathematics.

(5) Unimodular matrices of order two which commute. K. Goldberg. Submitted to a technical journal.

(6) Incidence algebras. E. C. Dade and K. Goldberg. In manuscript.

(7) Commutativity in finite matrices. O. Taussky. To appear in American Mathematical Monthly.

(8) Classes of matrices. O. Taussky. To appear in the Illinois Journal of Mathematics.

(9) The number of representations of a quadratic form as a sum of four squares. G. Pall (Illinois Institute of Technology) and O. Taussky. Submitted to a technical journal.

(10) Commuting bilinear transformations and matrices. O. Taussky and J. Todd. Submitted to a technical journal.

(11) Pairs of matrices of order two which generate free groups. K. Goldberg and M. Newman. Submitted to a technical journal.

(12) A method for computing eigenvectors. K. Goldberg. In manuscript.

(13) Some algebraic number theory estimates based on the Dedekind eta-function. H. Cohn. Submitted to a technical journal.

(14) A composition of cyclic cubic units. H. Cohn and S. Gorn (Moore School of Engineering). In manuscript.

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-10-1104/55-55

Origin: NBS — Authorized 8/13/54
Managers: J. Todd, P. Davis — Revised 8/29/54
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. J. Todd prepared a survey of computational problems and experience connected with the Hilbert matrix. A paper on the subject was presented at the meeting of the Societa Italiana per il Progresso delle Scienze, Catania, Sicily, September 19.

P. Davis and I. Stegun have experimented with orthonormalizing codes to obtain uniform (Tschebyscheff) polynomial approximations to continuous functions. Good results were obtained. Within two or three passes on the orthonormalizing code, all the accuracy generally desired can be achieved. A description of the method is in preparation.

At the suggestion of M. Fekete, P. Davis computed on SEAC the transfinite diameter $\hat{\tau}(E)$ of a set E consisting of two collinear line segments. The relationship

$$\hat{\tau}(E)= \lim_{n \to \infty} (G_{n-1}/G_n)^{1/n}, \quad G_n = \left| \int\int_E z^i \bar{z}^k ds \right|_{j,k=1}^{n}$$

was employed, and the computation was carried out using a previously developed orthonormalizing code that can be instructed to print out the Gram determinant of a set of vectors. The computation was made in single-precision floating notation with n=10; two and a half significant figures were achieved. At this stage (n=10), the effects of round-off were already significant. Further accuracy could probably not be achieved without going to higher values of n and double precision coding. The theoretical background for this problem is due to Fekete, Achiezer, and Fekete and Walsh.

P. Davis is preparing a bibliography in the field of domain polynomials.

R. Moore has prepared a routine which generates all permutations on 11 or fewer objects. This routine formed part of a larger problem whose purpose was to test the truth or falsity of a conjecture of G. A. Dirac and J. Todd concerning a problem of Sierpinski on equivalence of sets by finite decomposition. The conjecture was found to be false.

A paper on "Stability and Lyapunov's second method" was presented by H. A. Antosiewicz before the International Symposium on Algebraic Topology, Mexico City, August 15. During July and August Dr. Antosiewicz conducted a seminar on "Modern Topics in the Stability Theory of Differential Equations," before the staff of Instituto de Matematicas, University of Mexico, Mexico City.

H. Cohn has also studied the problem of stable configurations of electrons on a circle. This was solved completely in terms of stable and metastable states.

J. Gager has completed exploratory calculations of the Cauer

parameters. The final code, which incorporated several smaller routines, evaluated sn((m/n)K;k) for $p=\sqrt{k}= .850(.001).999$, where $1 \leqslant m < n < 12$. The three smaller, preliminary routines can best be explained in the following manner. Code I computes k, k^2 and K for p=.850(.001).999 by using the arithmetic-geometric mean method. Code II computes q', k, q, and K for p=.850(.001).999; K was computed in two ways for comparison purposes. Code III computes $\mathrm{Sin}(2x+1)\cdot(m/n)\cdot(\pi/2)$, $0 \leqslant x \leqslant 30$. The final code, Code IV, computes

$$\mathrm{sn}\left(\frac{m}{n}\cdot K;k\right) = \frac{2\pi}{Kk} \sum_{n=0}^{n=30} \frac{q^{n+1/2}}{1-q^{2n+1}} \mathrm{Sin}(2x+1)\cdot \frac{m}{n} \cdot \frac{\pi}{2}$$

and the Cauer parameters.

M. Marcus has studied the function

$$\min_{\max (x_i,x_j)= \delta_{ij}} f[(Ax_1,x_1),\ldots,(Ax_n,x_n)]$$

for f convex, where A is an nxn complex Hermitian matrix. He then obtained a generalization of Hadamard's determinant theorem and the solution of a problem of Wielandt (Proc. Amer. Math. Soc. 6, 109(1955)). A paper has been submitted to a technical journal (see publication (7) below).

Dr. Marcus has generalized a theorem of v. Neumann and Fan on inequalities for singular values. He has also obtained the extreme values of $\prod_{j=1}^{k} (Ax_i,x_j)$ where A is Hermitian indefinite and $(x_i,x_j)= \delta_{ij}$. This is equivalent to the solution of a problem on signed volumes of k-dimensional parallelopipeds. In addition, Dr. Marcus has obtained

$$\min_{x_1,\ldots,x_k} \sum_{1<r_1<r_2<k} \det(Ax_{r_1},x_{r_2})$$

where $(x_i,x_j) = \delta_{ij}$. This is then used to extend the Cauchy-Schwarz inequality to indefinite Hermitian transformations. Manuscripts on these studies are in preparation.

Publications:

(1) On the Lerch zeta function. F. Oberhettinger. Pac. J. Res. 6, 117-120 (1956).
(2) A direct approach to the problem of stability in the numerical solution of partial differential equations. J. Todd. To appear in the Proceedings of a Symposium on Partial Differential Equations, held at Berkeley, California,1955.
(3) Complete sequences and approximations in normed linear spaces. P. Davis and K. Fan. Submitted to a technical journal.
(4) Note on bounds for certain determinants. E. V. Haynesworth. Submitted

to a technical journal.
(5) A survey of Lyapunov's second method. H. A. Antosiewicz. To appear in Annals of Mathematics Studies.
(6) Obituary notice on A. M. Turing. O. Taussky and J. Todd. Appeared: Mathematical Tables and Other Aids to Computation 10, 180-181(1956).
(7) Convex functions of quadratic forms. M. Marcus. In manuscript.
(8) Some inequalities for quadratic forms and eigenvalues. M. Marcus. In manuscript.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-10-5116/55-56

Origin: NBS — Authorized 8/13/54
Sponsor: Office of Naval Research — Revised 8/29/54
Managers: J. Todd, M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. M. Newman continued earlier work on classes of modular functions. He proved that appropriate linear combinations of these functions vanish and that they are finitely generated. A manuscript has been prepared. In connection with this work a routine to compute the coefficients of these functions on SEAC was prepared and about 1000 coefficients were determined.

K. Goldberg has completed a routine for computing the characteristic roots and vectors of symmetric matrices of order less than or equal to 23. The method for extracting the vectors is new and proved to be very fast on SEAC.

Mr. Goldberg has completed a file of floating subroutines for general use. The codes contain fewer orders than previously used without sacrificing time or accuracy.

Continuing recent work of H. Wielandt, K. Fan has studied normal matrices with preassigned eigenvalues. Necessary and sufficient conditions are obtained for (1) the existence of a normal matrix with all its eigenvalues and some columns preassigned, (2) the existence of two normal matrices M, N with preassigned eigenvalues and such that the rank of M-N is less than or equal to a preassigned number, and (3) the existence of two normal matrices M,N with preassigned eigenvalues and such that M+N (or MN) has a preassigned eigenvalue with preassigned multiplicity.

K. Fan has also been studying circular disks containing the eigenvalues of a matrix. He is making a critical survey of the literature on this topic. New results, generalizations of known results (of Brauer, Gershgorin, Taussky, Ostrowski, Kotelyanskii), have been obtained. It turns out that the topic is closely related to Perron-Frobenius' classical results on non-negative matrices.

Using topological method, K. Fan proved that if a real matrix $A=(a_{ij})$ has a dominant principal diagonal, then the elements in the

inverse matrix $A^{-1}=(b_{ij})$ satisfy the inequality

$$|b_{ij}| \leqslant (|a_{jj}| - \sum_{k\neq j} |a_{jk}|)^{-1}.$$

A consequence of this relation is G.B. Price's inequality

$$|\det A| \geqslant s_1 s_2 \ldots s_n,$$

where

$$s_i = |a_{ii}| - \sum_{j>i} a_{ij}.$$

Publications:

(1) On dominant eigenvalues of positive matrices. T. Kato. In manuscript.
(2) On systems of distinct representatives. A. J. Hoffman and H.W. Kuhn (Bryn Mawr College). To appear in Annals of Mathematics Study 38.
(3) Systems of linear inequalities. Ky Fan. To appear in Annals of Mathematics Study 38.
(4) Systems of distinct representatives and linear programming. A. J. Hoffman and H. W. Kuhn (Bryn Mawr College). To appear in the American Mathematical Monthly.
(5) Dilworth's theorem on partially ordered sets. G. B. Dantzig (RAND Corporation) and A. J. Hoffman. To appear in Annals of Mathematics Study 38.
(6) On the minimization of concave and convex functionals. G. B. Dantzig (RAND Corporation), A. J. Hoffman, W. Hirsch (New York University). In manuscript.
(7) The lowest frequency of a free square plate. H. Fujita (University of Tokyo), T. Kato, Y. Nakata (University of Tokyo), and M. Newman. In manuscript.
(8) An inclusion theorem for modular subgroups. M. Newman. Submitted to a technical journal.
(9) A table of the coefficients of the powers of $\eta(\tau)$. M. Newman. Koninkl. Nederl. Akademie van Wetenschappen (Amsterdam), Proceedings {A}, 59, No. 2, and Indag. Math. 18, 204-216(1956).
(10) Integral boundary points of convex polyhedra. A. J. Hoffman and J. B. Kruskal (Princeton University). To appear in the Annals of Mathematics Study 38.
(11) Systems of inequalities involving convex functions. Ky Fan (University of Notre Dame), I. Glicksberg (RAND Corporation), and A. J. Hoffman. In manuscript.
(12) Generalization of a theorem of Konig. A. J. Hoffman. J. Wash. Acad. Sci. 46, 211-212 (July 1956).
(13) Some theorems on $P_r(n)$. M. Newman. Submitted to a technical journal.
(14) On the existence of identities for the coefficients of certain modular forms. M. Newman. J. London Math. Soc. 31, 350-359 (1956).

ANALYTIC STUDY OF WAR GAMES
Task 1101-10-5116/55-83

Origin and Sponsor: Armament Branch, ARDC, USAF Authorized 12/29/54
Manager: H. A. Antosiewicz
Full task description: Oct-Dec 1954 issue, p. 7

Status: TERMINATED.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-10-5116/56-148

Origin and Sponsor: National Institutes of Health, Bethesda, Md. Authorized 9/30/55
Manager: H. A. Antosiewicz
Full task description: July-Sept 1955 issue, p. 7

Status: CONTINUED. At a particular temperature the search for solutions of the basic system of differential equations having a prescribed behavior has been completed. The length of the interval of admissible parameter has been determined to be less than 10^{-10}. A number of cases for a lower temperature has been run.

E. Dade has completed and checked out two floating and two fixed-point routines for the evaluation of the function

$$\Lambda = \ln z_o \Big/ \int_1^{z_o} y(z)\,\frac{dz}{z}$$

where

$$z^2 y''(z) - iw(a+z)\,y(z) = iwb,$$

$$y'(1) = y'(z_o) = 0,$$

a,b,w,z_o being real constants. Although the general solution of this differential equation is known in terms of Bessel functions (of complex order and complex argument), the numerical integration is carried out by use of Taylor series expansions of $y(z)$ and the integral in Λ evaluated by Simpson's rule. At present, tests are being made for various sets of the parameter values.

NUMERICAL EXPERIMENTS ON POTENTIAL THEORY USING THE NEHARI ESTIMATES
Task 1101-10-5116/56-189

Origin: NBS Authorized 7/10/56
Sponsor: Air Research and Development Command, USAF
Managers: P. Davis, U. Hochstrasser
Full task description: Apr-June 1956 issue, p. 6

Status: CONTINUED. Working with an irregular pentagon and n particular harmonic functions (usually Re and Im(z^k), k=0,1,...), the Dirichlet problem has been solved for the boundary data Re(cos z) and Re[1/(z+4)]. The cases n=3,4,...,9 have been covered, and comparisons have been made between the actual error and that given by Nehari's estimates. Computation of the harmonic measures of the sides of the pentagon is underway.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF $E_1(z)$, $(z=x+iy)$
Task 1102-40-1110/43-3

Manager: I. Stegun — Authorized 7/1/47
Full task description: Apr-June 1949 issue, p. 41

Status: CONTINUED. The final manuscript has been typed and checking is in progress.

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-40-1110/47-2

Origin: NBS — Authorized 7/1/47
Manager: M. Abramowitz
Full task description: Apr-June 1949 issue, p. 45

Status: CONTINUED. Work is continuing on the computation of $F_o(\eta,\rho)$ and $F_o'(\eta,\rho)$ for $\eta = 0(.5)25$, $\rho = 0(.5)40$. Checking has been started.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Task 1102-40-1110/51-8

Origin: Section 11.3, NBS — Authorized 3/26/51
Manager: S. Peavy
Full task description: Apr-June 1951 issue, p. 49

Status: INACTIVE. For status to date, see Oct-Dec 1955 issue, p. 8.

REVISION OF MATHEMATICAL TABLES
Task 1102-40-1110/52-7

Origin: NBS Authorized 8/10/51
Managers: W. F. Cahill, I. Stegun
Full task description: July-Sept 1951 issue, p. 41

Status: INACTIVE. For status to date, see Jan-Mar 1956 issue, p. 6.

TABLE OF THE MODIFIED AIRY INTEGRAL
Task 1102-40-1110/52-23

Origin: NBS Authorized 10/4/51
Manager: I. Stegun
Full task description: July-Sept 1951 issue, p. 42

Status: CONTINUED. Checking of the completed manuscript is continuing.

TABLE OF ERROR FUNCTION FOR COMPLEX ARGUMENTS
Task 1102-40-1110/52-25

Origin: NBS Authorized 10/5/51
Manager: W. Hall
Full task description: July-Sept 1951 issue, p. 42

Status: INACTIVE. For status to date, see Oct-Dec 1954 issue, p. 11.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-40-1110/52-37

Origin: NBS Authorized 11/28/51
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE.

SIEVERT'S INTEGRAL
Task 1102-40-1110/52-57

Origin: NBS Authorized 2/12/52
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46

Status: CONTINUED. Checking of the tabular values and preparation of the introductory material continued.

L-SHELL CONVERSION COEFFICIENTS
Task 1102-40-1110/53-52

Origin: Oak Ridge National Laboratory Authorized 5/20/53
Manager: W. Hall
Full task description: Apr-June 1953 issue, p. 45

Status: CONTINUED. The original project is completed except for K shell, Z=35, k=2.0. It is now believed desirable to consider the nucleus not as a point mass but as a sphere of finite radius for certain of the high atomic number cases for the K and L_I shells. These routines have not yet been written.

AUTOMATIC CODING
Task 3711-60-0009/55-65

Origin: NBS Authorized 9/29/54
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. A technical memo entitled "Integral Evaluation Code - Base 11" was written by John W. Cooper. This memo describes in detail the Base Code mentioned in the Jan-Mar 1956 issue, p. 8.

MATHEMATICAL SUBROUTINES
Task 3711-60-0009/56-160

Origin: NBS Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

Status: CONTINUED. A subroutine has been prepared and checked for computing the serial correlation coefficients r of order k:

$$r_k = \frac{\mathrm{cov}(u_j, u_{j+k})}{(\mathrm{var}_j^u \ \mathrm{var}_{j+k}^b)^{1/2}}$$

The code has been written in the normal mode with the series of values considered as integers times 2^{-42}.

3. PROBABILITY AND MATHEMATICAL STATISTICS

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-10-1107/49-1a

Origin: NBS Authorized 1/9/49
Manager: L. S. Deming
Full task description: Apr-June 1949 issue, p. 75

Status: CONTINUED. For a description of the continuing activity on this task, see the Jan-Mar 1954 issue, p. 49.

TABLES TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-10-1107/51-1

Origin: NBS Authorized 7/1/50
Managers: C. Eisenhart, L. S. Deming
Full task description: July-Sept 1950 issue, p. 57

Status: INACTIVE. For status to date, see July-Sept 1952 issue, p. 64.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-10-1107/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. For a description of the continuing activity on this task, see the Oct-Dec 1956 issue, page 12.

Publications:
(1) A note on the normal distribution. S. Geisser. Appeared: Ann. Math. Stat. 27, 858-859 (Sept. 1956).
(2) Statistical investigation on the fatigue life of deep groove ball bearings. J. Lieblein and M. Zelen. To appear in the Journal of Research, NBS.

(3) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. Joel (Computation Laboratory). Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-10-1107/53-1

Origin: NBS Authorized 10/15/52
Manager: W. S. Connor
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. R. C. Burton and W. S. Connor prepared a paper "On the identity relationship for fractional replicates of the 2^n series" (see publication (1) below). Regarding the collections of letters together with the identity element as the elements of an Abelian group, they gave necessary and sufficient conditions for the existence of an identity relationship which has specified numbers of letters in its generators and products. This work was reported in a contributed paper read at a meeting of the Institute of Mathematical Statistics in Seattle, Washington.

R. C. Bose developed some theory concerning the construction of optimum fractional designs of the 3^n series.

M. Zelen delivered an invited address on "The analysis of covariance for incomplete block designs" at the Detroit meeting of the American Statistical Association.

M. Zelen developed some theory concerning the use of incomplete block designs for factorial treatments.

Publications:

(1) On the identity relationship for fractional replicates of the 2^n series. R. C. Burton and W. S. Connor. Submitted to a technical journal.
(2) Contributions on partially balanced incomplete block designs with two associate classes. W. H. Clatworthy. To appear as NBS Applied Mathematics Series 47.
(3) Fractional factorial experiment designs for factors at two levels. To appear as NBS Applied Mathematics Series 48.
(4) Design of experiments in research and development. W. J. Youden. To appear in Proceedings of the 1955 Conference on the Design of Experiments in Army Research, Development, and Testing, Washington, D. C., October 1955.
(5) The analysis of covariance for incomplete block designs. M. Zelen. To appear in Biometrics.
(6) Exact tests of significance for combining inter- and intra-block information. M. Zelen. Submitted to a technical journal.

RESEARCH ON MATHEMATICAL ASPECTS OF ORDER STATISTICS METHODS
Task 1103-10-1107/55-110

Origin: NBS Authorized 3/3/55
Manager: C. Eisenhart
Full task description: Jan-Mar 1955 issue, p. 31

Status: CONTINUED. For a description of the continuing activity on this task, see the Apr-June 1956 issue, page 12.

Publication:
(1) Geological application of extreme-value theory to interpretation of cobbles and boulders in gravel deposits. W. C. Krumbein (Northwestern University) and J. Lieblein. Appeared: Trans. Geophysical Union 37, 313-319 (June 1956).

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-10-1107/56-170

Origin: NBS Authorized 12/15/55
Manager: Joan R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

Status: CONTINUED. The paper, "On the power of some rank order two-sample tests," by J. R Rosenblatt, is being prepared for submission to a technical journal. Work is underway on determining bounds for the variance of Kendall's τ.

Publications:
(1) Easily used simultaneous confidence limits for a line. W. S. Connor. Withdrawn from publication by the author.
(2) Contributions to the theory of rank order statistics: Two sample case. I. R. Savage. Appeared: Annals of Mathematical Statistics 27, 590-615 (Sept. 1956).
(3) On the independence of tests of randomness and other hypotheses. I. R. Savage. To appear in the Journal of the American Statistical Association.

MEASUREMENT OF RELIABILITY
Task 1103-10-1130/56-182

Origin: NBS Authorized 3/23/56
Manager: M. Zelen
Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. J. R. Rosenblatt has continued work on the properties of methods for obtaining confidence intervals for the product and related functions of binomial parameters.

M. Zelen is continuing work to determine how one can make effective use in practice of the relation between the distribution of failures and the conditional distribution of failures.

Three seminars were held on the "Reliability of Complex Equipment": (1) The planning and interpretation of life tests. M. Zelen. July 2. (2) How conditions of use affect the reliability and performance of electron tubes. J. Lieblein. July 5. (3) Predicting performance of complex equipment. J. R. Rosenblatt. July 6.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-5146/55-93

Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: CONTINUED. With the completion of the section on "Sensitivity testing," Part II,--"Some statistical techniques for qualitative data,"-- has now been essentially completed. A section "Rejection of outliers" has been drafted, and in addition, work has been started on Part III, "Planning the collection and analysis of data."

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-10-1115/55-57

Origin: NBS Authorized 9/1/54
Manager: R. F. Dressler
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. The study of the use of the Cartesian Diver balance as a weighing instrument is being continued by means of a computational program based on the theoretical analysis obtained by P. Chiarulli and F. Chilton, at the request of the Office of Basic Instrumentation (NBS). The program has been coded for SEAC and the computations are being run. Results will illustrate quantitatively the possible sensitivity of such a balance as a function of the various physical parameters and the effect of small variations in these parameters.

W. H. Pell has begun a study of the first and second boundary value problems of elasticity for certain bodies having axial symmetry. The principal tool being employed is the stress function approach of J. Boussinesq, which has been used extensively in recent times by E. Sternberg, R. A. Eubanks, and M. A. Sadowsky.

A. Ghaffari has begun a study concerning stability of solutions and existence of periodic solutions of second order non-linear ordinary differential equations of Cartwright-Littlewood type.

Publications:

(1) Entropy changes in rarefaction waves. R. F. Dressler. To appear in the Journal of Research, NBS.
(2) A discussion of the paper, "Design of corrugated diaphragms," by J. A. Haringx. R. F. Dressler. To appear in the Transactions of the American Society of Mechanical Engineers.
(3) On the representation of a certain integral involving Bessel functions of hypergeometric series. P. Henrici. Submitted to a technical journal.
(4) On the domain of regularity of generalized axially symmetric potentials. P. Henrici. To appear in the Proceedings of the American Mathematical Society.
(5) The quotient-difference algorithm. P. Henrici. To appear in NBS Applied Mathematics Series 49, "Further contributions to the solution of simultaneous linear equations and the determination of eigenvalues."

RESEARCH IN ELECTROMAGNETIC THEORY
Task 1104-10-5160/54-47

Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Department of the Army — Authorized 6/29/54, Revised 9/29/54
Manager: F. Oberhettinger
Full task description: July-Sept 1954 issue, p. 28

Status: INACTIVE.

RESEARCH IN RADIATION THEORY
Task 1104-10-5160/56-175

Origin: NBS — Authorized 12/28/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: Oct-Dec 1955 issue, p. 18

Status: CONTINUED. New additions to the existing collection of Mehler transforms have been made. At present the compilation embraces about 100 formulas.

The investigation concerning the propagation of electromagnetic and acoustic pulses is continuing. The first part of the results on diffraction of pulses by wedges has been submitted to a journal and accepted for publication. The diffracted field from cylindrical or plane pulses, with arbitrary pulse function, on wedges or edges has been obtained in a definite integral representation.

Publication:

(1) On the propagation of elextromagnetic and acoustic pulses. Part I: The diffraction of pulses by wedges. F. Oberhettinger. To appear in Zeitschrift für Physik.

RESEARCH IN MATHEMATICAL ELASTICITY
Task 1104-10-5160/55-85

Origin: NBS — Authorized 12/27/54
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: R. F Dressler
Full task description: Oct-Dec 1954 issue, p. 30

Status: CONTINUED. The lowest mode and frequency for $\sigma = .25$,

$T = \frac{1}{2}$ were computed for the vibrating triangular wing problem. There is no significant difference between this result and that for $\sigma = .29$, $T = \frac{1}{2}$. The matrix for the same aspect ratio but with $\sigma = .05$ has been set up and computation for the lowest mode has been started.

The case for aspect ratio $T = 1$ has caused difficulty due to slowness of convergence, because it was necessary to scale the matrix four times smaller than the previous case. To overcome this trouble, U. Hochstrasser and I. Rhodes have modified the program. Previously, the code iterated on $y^{(n)}$ in $y^{(n+1)} = F(A)\ y^{(n)}$, with normalization at every step, where F is the hypergeometric polynomial of degree 30, and the approximation to λ, the lowest eigenvalue to A, was obtained from $\lambda_{n+1}\ y^{(n+1)} = A\ y^{(n+1)}$, where A is the matrix order 78. The new procedure approximates μ, the largest eigenvalue of F(A), from

$$\mu y^{(n)} = F(A)\ y^{(n)},$$

and from μ, it then computes λ.

A paper describing the method used and the results obtained thus far has been presented at the International Congress on Mechanics, held at Brussels in September.

On the corrugated diaphragm problems the solution for the thin diaphragm has been computed by using initial conditions computed from matching the three homogeneous and particular solutions at the end, rather than by forming the appropriate combination of these solutions. This has been done to avoid the error created from ill-conditioned matrices where the solution values are small quantities arrived at by linear combination of large quantities. Using this solution, the stresses have been computed and are being plotted. The results compared with solutions already obtained for the thick and medium diaphragms will reveal the relative importance of the interrelated stretching and bending effects in rotationally symmetric shells.

Publications:

(1) Investigations of the properties of corrugated diaphragms. W. A. Wildhack. (NBS OBI), R. F. Dressler, and E. C. Lloyd (NBS Mechanics Division). To appear in the Transactions of the American Society of Mechanical Engineers. (Now available as ASME Preprint No. 55-A-181).
(2) Stationary principles for forced vibrations in elasticity and electromagnetism. J. L. Synge. To appear in the Proceedings of the Eighth Symposium in Applied Mathematics of the American Mathematical Society, held at Chicago, Ill., April 1956.
(3) Elastic waves in anisotropic media. J. L. Synge. Submitted to a technical journal.
(4) The torsion of a hollow square. J. L. Synge and W. F. Cahill. Submitted to a technical journal.
(5) The vibration of triangular wings. R. F. Dressler. In manuscript. Abstract to appear in the Proceedings of the Ninth International Congress on Mechanics, held at Brussels, September 1956.

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-10-5160/56-154

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Managers: F. Oberhettinger, J. Lieblein
Full task description: July Sept 1955 issue, p. 20

Status: CONTINUED. F. Oberhettinger is continuing to expand the collection of Fourier transforms.

RESEARCH IN FLUID DYNAMICS OF TWO-PHASE FLOWS
Task 1104-10-5160/56-155

Origin: Office of Naval Research Authorized 9/30/55
Sponsor: " "
Manager: R. F. Dressler
Full task description: July-Sept 1955 issue, p. 21

Status: CONTINUED. A paper describing a result of the first stage of the investigation dealing with interface conditions with provision for heat removal and cold liquid addition has been prepared and has been presented at the International Congress on Mechanics, Brussels.

A. Ghaffari and R. F. Dressler are undertaking a study of the thrust developed by condensible jet devices of the type previously considered. The one-dimensional flow conservation-law techniques developed for use with those models are applied to study the thrust under various conditions and, in addition, models are being considered which yield thrust augmentation as described by von Karman (see Reissner Anniversary Volume, pp. 461-468 (Edwards Brothers, Ann Arbor, Michigan, 1949)).

Publication:

(1) Condensation interfaces in two-phase flows. P. Chiarulli and R. F. Dressler. In manuscript. Abstract to appear in the Proceedings of the Ninth International Congress on Mechanics, Brussels, Belgium, September 1956.

RESEARCH IN MATHEMATICAL GEOPHYSICS
Task 1104-10-5160/56-156

Origin: The American University Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: U. Hochstrasser
Full task description: July-Sept 1955 issue, p. 21

Status: TERMINATED. Dispersion curves for higher modes in the propagation of Rayleigh waves across an ocean floor have been computed by U. Hochstrasser on SEAC for the model mentioned in the April-June 1956 issue.

The code for solving the equations governing the deformation of the earth by gravitational forces has been finished and is being checked out. This code will permit a study of the deformation of the earth for a load with surface densities representable by solid spherical harmonics of any degree.

Subsequent research on this general topic will be reported in future issues under the heading, "Research in mathematical physics and related fields."

Publications:

(1) The transmission of Rayleigh waves across an ocean floor with two surface layers. Part I. R. Stoneley. To appear in the Bulletin of the Seismological Society of America.
(2) On the propagation of turbidity currents. R. Stoneley. To appear in the Vening Meinesz Jubilee Volume.
(3) The attenuation of Rayleigh waves with depth in a medium with two surface layers. R. Stoneley and U. Hochstrasser. To appear in the Geophysical Supplement, Monthly Notices, Royal Astronomical Society.
(4) Lectures - Surface waves in an elastic media. R. Stoneley. A Monograph.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

1102-40-5126/51-37 MOLECULAR STRUCTURE, III
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. O'Hara
Full task description: July-Sept 1951 issue, p. 50
Status: Continued. A three-dimensional Fourier synthesis was computed for the Benzophenone crystals using about 300 reflections of known sign and intensity.

1102-40-5126/52-44 CALCULATIONS FOR d-SPACINGS
Origin and Sponsor: NBS, Division 9
Full task description: Oct-Dec 1951 issue, p. 47
Manager: R. Zucker
Status: Continued. About 20 calculations for d-spacings for tetragonal, hexagonal and orthorhombic crystals were performed. Also redetermination of unit cell constants by least squares fitting to measured d-spacings was carried out for about 31 crystals.

A general program that will calculate d-spacings for all crystallographic space groups, written by A. Futterman, was revised and completely code-checked. A general code was written to sort the d-spacings and their corresponding indices in descending order. Sorting can be done internally or by means of "block" sorting, using magnetic tapes for auxiliary storage. Test cases for all crystal groups were run and the codes were completely checked out.

1102-40-5126/53-27 COMPUTATION OF THERMODYNAMIC FUNCTIONS
Origin and Sponsor: NBS, Section 3.2
Manager: E. Marden
Full task description: Jan-Mar 1953 issue, p. 57
Status: Completed. Results have been transmitted to the sponsor.

1102-40-5126/53-51 RADIATION DIFFUSION, III
Origin: NBS, Section 4.8
Sponsor: Atomic Energy Commission
Manager: J. Doggett (4.8)
Full task description: Apr-June 1953 issue, p. 57 (Neutron Diffusion III)
Status: Continued. A sample set of photon "case histories" has been tabulated. The effect of singly scattered radiation for the two-medium problem is being measured.

1102-40-5126/54-13 AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: New York Quartermaster Procurement Agency
Manager: H. Bremer
Full task description: Oct-Dec 1953 issue, p. 43
Status: Continued. During the quarter solutions for 10 bid evaluation problems were carried out.

3711-60-0009/54-17 DEPOLYMERIZATION
Origin: NBS, Section 7.6
Manager: U. Hochstrasser
Full task description: Oct-Dec 1953 issue, p. 44
Status: Continued. Six additional cases have been run involving various changes of the parameters.

3711-60-0009/54-22 ENERGY DISTRIBUTIONS ON OPTICAL IMAGE
Origin: NBS, Section 2.2
Manager: L. S. Joël
Full task description: Jan-Mar 1954 issue, p. 43
Status: Inactive. For status to date, see Jan-Mar 1956 issue, p. 19

3711-60-0009/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Manager: S. Prusch
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. The observed wavelengths of the spectrum of Thulium II were converted to vacuum wavenumbers on the SEAC. A code was prepared for SEAC to search for new energy levels. The code differed from that previously used in that there were no multiplets known in this spectrum, and it was necessary to form all possible sums between 3000 lines and 72 known terms. The code for the calculation has been checked, and approximately one-third of the data has been searched.

3711-60-0009/54-38 COMPRESSIBILITY FACTORS OF DRY AIR
Origin: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Inactive. For status to date, see Apr-June 1955 issue, p. 12

1102-40-5126/55-39 MOLECULAR VIBRATIONS
Origin and Sponsor: NBS, Section 3.2
Manager: K. Goldberg
Full task description: July-Sept 1954 issue, p. 16
Status: Completed.

1102-40-5126/54-43 CHARACTERISTICS OF CONDUCTING RESISTORS
Origin and Sponsor: NBS, Section 1.6
Manager: B. G. Urban
Full task description: Apr-June 1954 issue, p. 40
Status: Completed.

1102-40-5126/55-61 ELASTIC CROSS SECTION FOR NEUTRON SCATTERING
Origin and Sponsor: Naval Research Laboratory
Manager: I. Stegun
Full task description: Oct-Dec 1954 issue, p. 18
Status: Completed.

3711-60-0009/55-66 RECONSTITUTION OF MONOCHROMATIC LIGHT INTENSITIES
Origin: NBS, Division 30
Manager: H. Bremer
Full task description: July-Sept 1954 issue, p. 18
Status: Inactive. For status to date, see Oct-Dec 1954 issue, p. 19

3711-60-0009/55-68 CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Division 9
Manager: R. Prosen (Div. 9)
Full task description: Jan-Mar 1955 issue, p. 18
Status: Continued. Structure factors were computed for the magnesium borate crystal using three sets of atomic position parameters. Codes have been prepared to calculate structure factors for $BAHPO_4$ and $CA(OH)_2$.

1102-40-5126/55-74 LIQUID-VAPOR TRANSITION, II
Origin and Sponsor: Naval Medical Research Institute
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 19
Status: Completed. Results have been transmitted to the sponsor.

3711-60-0009/55-75 PARAMETER OF THE DISPERSION EQUATION FOR OPTICAL GLASS
Origin: NBS, Section 2.2
Manager: R. Zucker
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. The constants k_i were determined for AS_2S_3 with

$$\lambda_1^2 = .0225 \qquad \lambda_4^2 = .2025$$

$$\lambda_2^2 = .0625 \qquad \lambda_5^2 = .3600$$

$$\lambda_3^2 = .1225$$

1102-40-5126/55-81 COMBINING TESTS FOR SIGNIFICANCE
Origin: NBS, Section 11.3
Manager: L. S. Joel
Full task description: Oct-Dec 1954 issue, p. 23
Status: Inactive.

3711-60-0009/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. Calibration tables were computed for two thermometers under test.

1102-40-5126/55-88 STRESSES IN A WALL FOUNDATION
Origin and Sponsor: NBS, Section 10.1
Manager: D. Jirauch
Full task description: Jan-Mar 1955 issue, p. 22
Status: Inactive.

1102-40-5126/55-103 MISSOURI RIVER PROBLEM
Origin and Sponsor: Missouri River Division, Corps of Engineers, U.S.Army
Manager: H. Bremer
Full task description: Jan-Mar 1955 issue, p. 26
Status: Completed. All the codes have been written and checked for UNIVAC.Processing of the data is to be performed by the sponsor's staff.

1102-40-5126/55-113 REACTOR DESIGN
Origin: Westinghouse Atomic Power Division
Sponsor: Atomic Energy Commission
Manager: U. Hochstrasser
Full task description: Jan-Mar 1955 issue, p. 28
Status: Continued. The code for the base reactor problem is being checked out on the SEAC. A report describing the problem and the experiments done so far is being written.

1102-40-5126/55-117 ATTENUATION OF PRESSURE PULSES OF FINITE AMPLITUDE
Origin: NBS, Section 3.2
Sponsor: Bureau of Aeronautics, U. S. Navy
Manager: M. Paulsen
Full task description: Apr-June 1955 issue, p. 18
Status: Inactive.

1102-40-5126/55-118 THERMOMETER CALIBRATION, II
Origin and Sponsor: NBS, Section 3.1
Manager: B. G. Urban
Full task description: Apr-June 1955 issue, p. 18
Status: Inactive. For status to date, see Jan-Mar 1956 issue, p. 22

1102-40-5126/55-121 ELECTRON PENETRATION
Origin: NBS, Section 4.8
Sponsor: Atomic Energy Commission
Manager: S. Peavy
Full task description: Apr-June 1955 issue, p. 19
Status: Continued. All the codes have been checked and some initial runs have been made.

1102-40-5126/55-122 SOLUTION OF NORMAL EQUATIONS
Origin: Advisory Committee on Weather Control
Sponsor: U. S. Weather Bureau
Manager: M. Newman
Full task description: Apr-June 1955 issue, p. 20
Status: Completed.

1102-40-5126/55-123 TEMPERATURE DISTRIBUTION
Origin: NBS, Section 6.4
Sponsor: NBS, Section 6.4
Manager: W. G. Hall
Full task description: Apr-June 1955 issue, p. 19
Status: Inactive. For status to date, see Oct-Dec 1955 issue, p. 24

1102-40-5126/55-126 AERODYNAMIC HEATING
Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Dept. of the Army
Managers: B. Walter, R. Capuano
Full task description: July-Sept 1955 issue, p. 31
Status: Continued. Work was continued on the heat transfer for a laminar boundary layer. New velocities (V') and V* were calculated and new $T'_{AW'}$ and T_{AW*} were obtained (see Apr-June 1956 issue). Tables of $T_{W'}$ and T_{W*} were calculated for B = 7,5,4,3,2,1.7, for the four trajectories. Intersections for the various values of $T_{W'}$ resp. T_{W*} were computed for the various values of B.

1102-40-5126/55-127 VIBRATIONS OF CIRCULAR DISC
Origin and Sponsor: Evans Signal Laboratory
Manager: U. Hochstrasser
Full task description: July-Sept 1955 issue, p. 32

Status: Continued. Approximately 1600 additional values have been computed for a table of ϵ vs. d/h, where ϵ is the dimensionless resonant frequency, d the diameter, and h the thickness of the disc.

3711-60-0009/56-131 CALCULATIONS IN OPTICS
Origin: NBS, Section 2.2
Manager: D. P. Feder (2.2)
Full task description: July-Sept 1955 issue, p. 33
Status: Continued. The spot diagram code has been completely rewritten to take advantage of new coding techniques and the acquisition of a plotter and a high speed punch. This code has been operating successfully and is about four times as fast as the old code.

Some success has been attained in using the steepest descent method for solving the set of non-linear equations arising in the design of optical systems. Numerical experimentation is being carried out using an accelerated optimum gradient method, using a method due to Crochett and Chernoff, and using a method of constraints.

1102-40-5126/56-133 COMPLEX EIGENVALUES
Origin: Moore School of Engineering
Sponsor: Office of Naval Research
Manager: I. Stegun
Full task description: July-Sept 1955 issue, p. 33
Status: Completed. Results were transmitted to sponsor.

1102-40-5126/56-136 CALCULATION OF WAVE FUNCTIONS BY HARTREE METHOD
Origin: Naval Research Laboratory
Sponsor: " " "
Manager: S. Peavy
Full task description: July-Sept 1955 issue, p. 34
Status: Continued. Runs have been made upon request of the sponsor.

1102-40-5126/56-137 STABILITY OF SUPPORTED PLATES
Origin and Sponsor: National Advisory Committee on Aeronautics
Manager: R. Zucker
Full task description: July-Sept 1955 issue, p. 35
Status: Completed. Results have been transmitted to sponsor.

1102-40-5126/56-139 STUDY OF INTERNUCLEAR POTENTIAL FOR H_3
Origin and Sponsor: NBS, Section 3.2
Manager: E. Haynsworth
Full task description: July-Sept 1955 issue, p. 36
Status: Continued. Wave functions and the total electronic energies for the ground state of the linear symmetric H_3 intermediate complex have

been calculated over a wide range of internuclear distances in the LCAO-MO approximation employing a simple variational treatment of (1) a single determinantal wave function, and (2) a complete configuration interaction (CI) wave function made up of a linear combination of determinants over all possible allowed combinations of MSO's made up of Slater functions. As a further refinement, the effective nuclear charges of the three centers a, b, c were varied ($z_a = z_b = z_c$) in both cases. The first calculation obtained the total energy in terms of the associated eigenvectors. The second computation involved the calculation of the eigenvalues and eigenvectors of 4x4 symmetric matrices.

1102-40-5126/56-140 MULTIPLE CORRELATION ROUTINES
Origin and Sponsor: Agricultural Economics Division, Department of Agriculture
Managers: H. Bremer, M. Paulsen
Full task description: Oct-Dec 1955 issue, p. 26
Status: Continued. By now the use of the codes involved in obtaining the multiple correlation has become routinized. A system is underway whereby the sponsor sets up the data to be analyzed and then prepares it himself in the proper input format for SEAC. At present the sponsor is using the input equipment for SEAC at the NBS Computation Laboratory. Future plans are to have similar equipment set up at his office to facilitate the handling.

1102-40-5126/56-143 CHARGED PARTICLE TRAJECTORIES IN THE MAGNETIC FIELD OF THE EARTH
Origin and Sponsor: Naval Research Laboratory
Manager: J. H. Wegstein
Full task description: Oct-Dec 1955 issue, p. 27
Status: Continued. The trajectories for 89 more sets of initial conditions were computed with the NORC and were sent to the sponsor.

1102-10-5126/56-151 MULTIPLE SCATTERING IN CROSS SECTION MEASUREMENTS
Origin and Sponsor: Naval Research Laboratory
Manager: I. Rhodes
Full task description: Oct-Dec 1955 issue, p. 30
Status: Completed. Results have been transmitted to the sponsor.

1102-40-5126/56-157 DAMAGE ASSESSMENT
Origin and Sponsor: Office of the Assistant Secretary of Defense (Supply and Logistics)
Managers: A. J. Hoffman, H. Bremer
Full task description: Oct-Dec 1955 issue, p. 31
Status: Completed. A program of a series of routines was set up which

takes the input of the nation's resources and correlates it with certain attack data, then prepares various outputs in the form of listings that summarize the extent of the damage to the nation as a consequence of such an attack. The routines have been written and code-checked, and they were used in two runs. Each of the runs involved over 200,000 items of information and required about 100 hours of UNIVAC time. Towards the end, because of lack of time on UNIVACS in the Washington area, the problem was finished at a UNIVAC installation in Boston.

1102-40-5126/56-162 STRESSES IN A WALL RESTING ON A FOOTING
Origin and Sponsor: NBS, Section 10.1
Manager: I. Stegun
Full task description: Jan-Mar 1956 issue, p. 26
Status: Continued. Data has been prepared for the solution of 100 equations in 100 unknowns. Use will be made of the matrix inversion codes carrying along a check vector.

1102-40-5126/56-163 ANGULAR DISTRIBUTIONS AND POLARIZATION EFFECTS IN NUCLEAR SCATTERING
Origin and Sponsor: Naval Research Laboratory
Manager: I. Stegun
Full task description: Oct-Dec 1955 issue, p. 32
Status: Continued. Changes are being made in the code to decrease time requirements.

1102-40-5126/56-165 INTEGRALS FOR SCATTERING FUNCTIONS
Origin and Sponsor: Naval Research Laboratory
Manager: R. Zucker
Full task description: Oct-Dec 1955 issue, p. 32
Status: Continued. At the request of the sponsor, the function $\xi \cdot I(p,\xi)$ as $\xi \rightarrow 0$ was tabulated as a function of p for I (confluent) and I (Bessel) for purposes of making a graph. In the regions of maxima and minima and of discontinuities, additional values of the functions were calculated. (For I (confluent), see Oct-Dec 1955 issue, p. 32. For I (Bessel), see Apr-June 1956 issue, p. 27.)

1102-40-5126/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Manager: E. Haynsworth
Full task description: Jan-Mar 1956 issue, p. 27
Status: Inactive.

1102-40-5126/56-167 He^3 REACTIONS
Origin and Sponsor: Naval Research Laboratory
Manager: I. Stegun
Objective: To fit polynomial expansions to data by the method of least squares.
Background: It has recently been found that many deuteron induced nuclear reactions take place by a direct process rather than by the formation of a compound nucleus. It is of interest to determine whether He^3 induced nuclear reactions may also proceed by a direct interaction. A series of experiments have been performed at the Naval Research Laboratory in which the angular distributions of the reaction products have been studied from the bombardment of light nuclei with He^3 particles. From the theory of compound nucleus reactions these angular distributions should be capable of being expressed in terms of a rather simple expansion of Legendre polynomials. The theory can also give qualitative information about the coefficients in these expansions. In order to compare the theory with the experimental results, it is necessary to attempt to fit polynomial expansions to the data.

The problem was proposed by H. D. Holmgren, NRL.
Status: Completed (NEW). Results were transmitted to the sponsor.

1102-40-5126/56-169 CRYSTAL FIELD EFFECTS FOR ATOMS
Origin and Sponsor: NBS, Division 3
Manager: I. Stegun
Full task description: Jan-Mar 1956 issue, p. 28
Status: Inactive. For status to date, see Jan-Mar 1956 issue, p. 28

1102-40-5126/56-171 COLLISION INTEGRALS USED IN TRANSPORT THEORY
Origin and Sponsor: NBS, Section 3.2
Manager: J. Cooper
Full task description: Oct-Dec 1955 issue, p. 33
Status: Continued. 1. The code mentioned in the Apr-June issue, page 28, was used to obtain transport properties for a mixture of five gases at 20 different temperatures. 2. A new code was written to solve the linear homogeneous system of equations

$$\delta_{ij} - \frac{m_i C_j(T)}{\sum_k m_k C_k(T)} = \sum_{k=1}^{n} C_k m_{ik}(T) \left\{ \frac{a_{ij}}{m_i} - \frac{a_{kj}}{m_k} \right\}$$

$$\sum_{j=1}^{n} a_{ij} m_j = 0$$

where the a_{ij}'s are the unknowns and the c_i's and m_i's are the same

as in the code mentioned in (1) above.

m_{ik} is found from the $\Omega_{ik}^{(1,1)}$ previously defined by the relation:

$$m_{ij}(T) = \frac{16}{3} n_o \cdot \frac{m_i m_j}{m_i + m_j} \Omega_{i,j}^{(1,1)}(T)$$

Once the quantities a_{ij} are found, new results are found by the relation:

$$C_{ij} = \frac{C_i}{m_i} n_o RT a_{ij}$$

The coding for this part of the problem was completed, using parts of the code mentioned in (1). The code was checked and run for the same input data as the first problem.

1102-40-5126/56-172 NUMERICAL EVALUATION OF SPECIAL INTEGRAL EXPRESSIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Department of the Army
Manager: L. Joel
Full task description: Jan-Mar 1956 issue, p. 29
Status: Inactive.

1102-40-5126/56-176 MODEL GOVERNMENT PAYROLL ON HIGH SPEED COMPUTERS
Origin and Sponsor: NBS, Section 12.5
Manager: G. H. Urban
Full task description: Jan-Mar 1956 issue, p. 30
Status: Inactive.

1102-40-5126/56-179 NORMAL PROPAGATION CONSTANT
Origin and Sponsor: NBS, Section 82.10
Manager: H. H. Howe (82.10)
Full task description: Apr-June 1956 issue, p. 30
Status: Completed. Two new attacks were used in the effort to find all of the modes. Solving equation (2) (Apr-June 1956 issue, page 31) for n:

$$n = 2Hc + (i \operatorname{Log} R_s R_g)/2\pi$$

For fixed values of the parameters H,K,G, and L, we now look upon n as a complex function of the complex variable c. n is multiple-valued, in that the n for fixed c may be augmented by any real integer. We need all values of c that give real integral n. The integral n assigned to such a "mode" is arbitrary, but we make it definite by saying that

for fixed n, c should be a continuous function of L and G, and should approach c = n/2H as L and G approach zero.

The first new attack involved selecting values of H, K, G, L, and tracing in the c-plane the curve for which n is real; and spotting the points at which n is a real integer. n was assumed to vary continuously along the curve; this fixed n for all points of the curve once it is fixed for any point. For non-zero L and G, one integral n occurs for c = 0; not this one, but the next one is n = 0. This method worked well for very small L, smaller than are likely to occur in nature. As L gradually increases (e.g., for H = 4, G/H = .0001, K = 15, it occurs at L/H = .00277), the curve of real n develops a bow near n = 0, which finally separates entirely from the main curve; hence, the method of numbering n along a continuous curve produces results that show a discontinuity when c for given n is listed as a function of L.

The next attack was to trace in the c-plane the curve for which n is a real integer, with variable L, by iterating equation (2). This worked easily and quickly for any n except n = 0. But for n = 0, it went slowly and eventually bogged down entirely. Then it was learned that the elusive mode 0 is, over much of the range of L, extremely close to the value of c for which $R_s = 0$, which is a singular point of n. I.e., over most of the range of L, there is an isolated "circle" of real n; at some point on the "circle" is the root n = 0; at the "center", the imaginary part of n is $-\infty$. For much of the range of L, the "circle" is so small that the value of c at its center (which can be found by simple algebraic means) is to high accuracy also the value of c for n = 0. The imaginary part of c is much larger on this "circle" than for other integral n, with resulting large attenuation of the wave. For high values of L, the "circle" which includes n = 0 returns and rejoins the main curve; e.g., for H = 4, G/H = .0001, K = 15, L/H = 1, we must consider that integral values of n lie on the continuous curve of real n in the sequence 1,2,3,4,5,0,6,7,8.

For all except very small values of L, the dominant mode (the one having least attenuation) is found to be n = 1, rather than n = 0 as has commonly been supposed.

1102-40-5126/56-184 GAIN CALCULATIONS OF AN ITERATED TRANSISTOR AMPLIFIER
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: E. Haynsworth
Full task description: Apr-June 1956 issue, p. 32
Status: Continued. The voltage gain calculations for a number of different transistors have been completed, and several variations in parameters have been introduced to give a more complete analysis for each transistor.

1102-40-5126/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: S. Prusch
Objective: To program test data obtained from mechanical measurements of gage blocks so that data reduction and formation of the final certification reports may be accomplished using automatic computing machines. In addition, the program is to be arranged to allow for periodic analyses of the test data to determine the condition of the master gage blocks.
Background: The reduction, analysis, and transfer of the test data to the certification report is now accomplished manually. Although the reduction of data is a simple arithmetic process, the quantity of data involved requires many man-hours of labor. In addition, much supervisory time is required to ascertain that the arithmetic processes are accomplished without error and that the final results are transferred to the certification report accurately. Processing of the data on automatic computing machines will be more efficient and more economical.
Status: NEW. Sample problems are being run to test the procedure.

1102-40-5126/57-195 CONTINUED FRACTIONS
Origin and Sponsor: NBS, Section 11.3
Manager: R. Durrah
Objective: To tabulate

$$\frac{(\sqrt{2})^j}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} f(x)dx,$$

where

$$f(x) = [\sqrt{2}\cdot x]^j \left\{ \Phi (\sqrt{2}\cdot x+q)\right\}^n$$

and $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-z^2/2} dx$

for q = 0(.25)4.5,
n = 2(1)50,
j = 0,1,2,3,4.

Background: This computation was undertaken at the request of R. C. Bose, who spent the summer of 1956 at the NBS Statistical Engineering Laboratory. The tables will be useful in connection with a statistical test concerning the choice of a population with the largest mean from among a set of normal populations having the same variance.
Status: NEW. The code has been written and checked out. The computations are under way.

1102-40-5126/57-206 EXPECTED VALUES OF ORDER STATISTICS
Origin and Sponsor: NBS, Section 11.3
Manager: R. Durrah
Objective: To compute

$$\int_{-\infty}^{\infty} P_r(x) \exp\left\{\frac{-rx^2}{2}\right\} dx, \quad r = 2,3,4,5,$$

where

$$P_r(x) = C_{n,k} [\Phi(x)]^{k-r} [1-\Phi(x)]^{n-k+1-r} [x_r(n,k)];$$

$$\begin{cases} n = 1(1)20, \\ k = n/2(n)n, \end{cases}$$

and

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-x^2/2} dx$$

$$x_r(n,k) = \sum_{i=0}^{r-1} (-1)^i \cdot \frac{(r-1)!}{(r-1-i)!(i)!} \cdot \frac{(k-1)!}{(k-r+i)!} \cdot \frac{(n-r+i)!}{(n-r)!} \cdot \Phi^i$$

Background: The computations were performed at the request of R. C. Bose, who spent the summer of 1956 at the NBS Statistical Engineering Laboratory. The tables will be used in connection with a statistical test concerning the choice of a population with the largest mean from among a set of normal populations having the same variance.
Status: NEW. The code has been written and checked out.

3711-60-0009/57-210 SOUND VELOCITY
Origin and Sponsor: NBS, Section 3.2
Manager: U. Hochstrasser
Objective: Tables of values for the pressure P and the density are given for fixed temperatures. It is required to fit these data by polynomials such that

$$P = P(\rho) = a_o + a_1 \rho + a_2 \rho^2 + \ldots,$$

and to evaluate

$$a = \sqrt{101.325 \frac{dP}{d\rho}}$$

for a set of values of ρ by using these polynomials.
Background: This problem arises in the theory of sound. Here a is the velocity of sound.

Status: NEW. The code has been written.

1102-40-5126/57-215 COMPUTATION OF INTEGRALS INVOLVING BESSEL FUNCTIONS
Origin and Sponsor: NBS, Section 6.1
Manager: U. Hochstrasser
Objective: To compute tables of the functions,

$$P(x,\theta) = \frac{2}{x} \int_0^x dv \int_v^\infty \cos (u \sin \theta)\, J_0(u)du,$$

$$Q(x,\theta) = -\frac{2}{x} \int_0^x dv \int_v^\infty \cos (u \sin \theta)\, Y_0(u)du,$$

for $x = 1(1)20$, $\theta = 15°(15)90°$.
Background: These integrals appear when the diffraction of sound is considered at the edge of an absorbent material.
Status: NEW. The code has been written and checked out. The results on the work completed thus far have been transmitted to the sponsor.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Strength of glass: analysis of data, for M. J. Kerper, Refractories Section.

(2) Heat of combustion of fuels: method of production of heat of combustion from other properties, for G. T. Armstrong and R. S. Jessup, Organic Chemistry Section.

(3) Evaluation of waxes: analysis of data and methods for improving precision of results, for W. W. Walton, Surface Chemistry Section.

(4) Water turbulence: statistical aspects of experimental arrangement, for G. C. Sherlin, Fluid Mechanics Section.

(5) Statistical analyses were made on SEAC for H. Allen, Radiometry Section; J. E. McKinney, Rubber Section; M. Burkhard, Sound Section; M. Greenspan, Sound Section; G. Conrad, Engineering Electronics Section; J. Mandel, Testing and Specifications; J. Hilsenrath, Thermodynamics Section; and for the Child Safety Project.

Publications:

(1) National physical standards and design of experiments. W. J. Youden. To appear in Revue de L'Institut International de Statistique (The Hague).
(2) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.
(3) Statistics: Engineering viewpoint. W. J. Youden. To appear in the Journal of Engineering Education.
(4) More scientists. W. J. Youden. Submitted to a technical journal.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: CONTINUED. An analysis of results from nine physical and chemical tests on ship steel plates from current production is being carried out. An incomplete block arrangement was used in the experiment, so balanced that variation of heats, ingot within a heat, and plates within an ingot could be evaluated.

STATISTICAL ANALYSIS AND DESIGN OF EXPERIMENTS FOR THE U. S. GEOLOGICAL SURVEY
Task 1103-40-5140/54-1

Origin and Sponsor: U. S. Geological Survey, Department of Interior Authorized 10/9/53
Managers: C. Eisenhart, W. J. Youden
Full task description: Oct-Dec 1953 issue, p. 50

Status: CONTINUED. Two manuscripts by Frank Flanagan of the Geochemistry and Petrology Branch, U. S. Geological Survey, were reviewed. One involved the mathematics of sample splitters, and the other rank correlation methods.

CATALOGUE OF FRACTIONAL REPLICATION DESIGNS
Task 1103-40-5147/57-213

Origin and Sponsor: Bureau of Ships Authorized 8/30/56
Managers: W. S Connor, M. Zelen

Objective: To construct and prepare a catalogue of fractional replication designs for the 3^n factorial series. These designs are to be arranged so that heterogeneity between blocks can be eliminated.

Background: The use of factorial designs has now become widely accepted as an efficient way for carrying out experiments involving many different factors. However, one of the main difficulties with factorial designs is that the number of measurements required may be large and in some cases prohibitive. Another disadvantage is that in many experimental situations it is not practical to plan an entire

experimental program in advance, but to make a few smaller experiments which serve as a guide to further work. This latter condition is especially true when measurements are made singly or in small groups, such that the experimental results become known sequentially as they are taken. The work under this project extends to the 3^n series the work previously done on the 2^n series.

Status: NEW. The work of construction has been completed. This was done during the summer by R. C. Burton, H. M. Pettigrew, and F. L. Miller, Jr., with assistance from R. C. Bose, who undertook to develop the theory (see task 1103-10-1107/53-1, Studies in the mathematics of experiment design, p.14). The designs are now being typed and checked under the direction of Lola Deming. It is expected that the SEAC will be useful in checking.

The number of factors ranges from 4 to 10, and the fraction from 1/3 to 1/243. Alias relationships are presented for completely randomized designs, and block confounding relationships for the designs with blocks. The number of units per block ranges from 3 to 243, and the number of blocks from 3 to 27.

APPLICATION
OF
NATIONAL BUREAU OF STANDARDS AUTOMATIC COMPUTER (SEAC)

The record of SEAC operations for tasks of the Applied Mathematics Division for the period July 1 through September 30 is as follows:

Task No. NBS:	Title	Code Checking	Productive Operation
1104/55-55	Research in numerical analysis	6	29
5116/55-56	Research in mathematical topics applicable to numerical analysis	3	40
5116/56-148	Study of differential equations for nerve fiber excitation	24	11
1110/47-2	Tables of Coulomb wave functions		14
1110/53-51	Radiation diffusion	4	70
1110/53-52	L-Shell conversion coefficients		115
5126/52-44	Calculation for d-spacings	6	10
5126/53-29	Dynamic behavior of aircraft structures	2	
5126/54-4	High temperature properties of water		2
5126/54-30	Spectrometer analysis	5	11
5126/55-68	Crystal structure calculations	4	11
5126/55-97	High temperature properties of air	4	57
5126/55-121	Electron penetration	9	8
5126/55-127	Vibration of a circular disc	7	61
5126/56-128	Ground reflection coefficients	1	6
0009/56-131	Ray tracing, II		25
5126/56-139	Internuclear potential for H_3	1	11
5126/56-144	Auto correlation		15
0009/56-160	Mathematical subroutines	6	
5126/56-166	SCF-LCAO solution of some hydrides		17
5126/56-167	Multiple regression		3
5126/56-169	Crystal field effects for atoms	1	5
5126/56-171	Collision integrals used in transport theory	15	25
5126/56-179	Normal propagation constant	14	42
5126/56-185	Matrix multiplication		4
5126/56-192	Noise measurement	3	1
5126/56-195	Continued fractions	8	7
5126/57-198	Transient heat	1	3
5126/57-206	Value order statistics	2	
5126/57-214	Gears	2	
5160/55-85	Research in mathematical elasticity		90
5160/56-156	Research in mathematical geophysics	12	7
0002/52-1	Statistical engineering	3	18
Misc.	Training	14	6

	Title	Code Checking	Productive Operation
OTHER:			
5126/53-45	Application of game theory		3
5126/54-13	Award of procurement contracts for linear programming		35
5126/55-104	"Fuze" problem		184
5126/55-113	Reactor design	1	5
5126/56-130	Aircraft responses	1	5
5126/56-136	Calculation of wave functions		5
5126/56-140	Multiple correlation routines		4
5126/56-142	Matrix computation		4
5126/56-151	Cross section measurements		73
5126/56-163	Nuclear scattering	6	3
5126/56-165	Integral for scattering functions	3	10
5126/56-184	Transistor amplifier	12	16
5126/56-187	Moments	1	3
5126/56-188	Gases	5	1
5126/56-189	Numerical exponents	4	2

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Division Seminar

BOSE, R. C. (Guest Worker from University of North Carolina). On a problem in abelian groups. August 8.

COHN, H. (Guest Worker from Washington University). Equilibrium configurations of charged particles on a sphere. August 15.

Numerical Analysis Seminar

MARCUS, M. (NBS-NRC Fellowship). Functions of quadratic forms. September 28.

Reliability Seminar

ZELEN, M. The planning and interpretation of life tests. July 2.

LIEBLEIN, J. How conditions of use affect the reliability and performance of electron tubes. July 5.

ROSENBLATT, J. R. Predicting performance of complex equipment. July 6.

Statistical Theory Seminar

(Offered jointly by the National Institute of Health and the National Bureau of Standards)

BOSE, R. C. A problem in ranking concerning multiple comparisons. August 3.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

BOSE, R. C. (Guest Worker from University of North Carolina). On a decision procedure for ranking means. Presented at a Seminar at the Bureau of the Census, August 9.

BURTON, R. C. On the identity relationship for fractional replicates of the 2^n series. Read by W. S. Connor at a meeting of the Institute of Mathematical Statistics, Seattle, Washington, August 20-27.

CAMERON, J. M. Use of SEAC for statistical calculation. Presented at a meeting of the Statistical Summer Session, held jointly with the American Statistical Association at Massachusetts Institute of Technology, Cambridge, Massachusetts, August 17.

RHODES, I. Some sobering thoughts on handling data by means of electronic computers. Presented at a meeting of the National Machine Accountants Association, Washington, D. C., September 19.

SOMERVILLE, P. N. Selecting the "best" from k+1 populations. Presented at the Gordon Conference on Statistics in Chemistry and Chemical Engineering, held by the American Association for the Advancement of Science, New Hampton, N. H., August 21.

YOUDEN, W. J. Mathematics and the teaching of physics and chemistry. Presented at a seminar of the Department of Chemistry, American University, Washington, D. C., July 5.

ZELEN, M. Statistical concepts in life testing. Presented at a meeting of the Professional Group in Component Parts, Institute of Radio Engineers, Washington, D. C , September 12.

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Papers presented at the meeting of the Association for Computing Machinery, Los Angeles, California, August 27-29:

ABRAMOWITZ, M. Self-checking methods of integration.

ALT, F. Mathematical techniques in data processing problems.

Papers presented by P. Chiarulli at the Ninth International Congress on Mechanics, Brussels, Belgium, September 5-13:

CHIARULLI, P., and R. F. DRESSLER. Condensation interfaces in two-phase flows.

DRESSLER, R. F. Vibrations of triangular wings.

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Papers presented at the meeting of the Societa Italiana per il Progresso delle Scienze, Catania, Sicily, September 19:

TAUSSKY-TODD, O. Computational problems for matrices of rational integers.

TODD, J. Computational problems concerned with the Hilbert matrix.

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Papers presented at the Joint Annual Meeting of the American Statistical Association, the Institute of Mathematical Statistics, and the Biometrics Society, Detroit, Michigan, September 7-8:

ABRAMOWITZ, M., and J. M. CAMERON. Requirements of scientific computations arising in Government computing.

YOUDEN, W. J. Randomization and experimentation.

ZELEN, M. Analysis of covariance for incomplete block designs.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) A table of the coefficients of the powers $\eta(\tau)$. M. Newman. Koninkl. Nederl. Akademie van Wetenschappen (Amsterdam), Proceedings {A} , 59, No. 2, and Indag. Math. 18, 204-216 (1956).

1.3 Technical Papers

(1) A note on the normal distribution. S. Geisser. Ann.Math. Stat. 27, 858-859 (Sept. 1956).

(2) Generalization of a theorem of Konig. A. J. Hoffman. J. Wash. Acad. Sci. 46, 211-212 (July 1956).

(3) The number of absolute points of a correlation. A. J. Hoffman, M. Newman, E. G. Straus, O. Taussky. Pac. J. Math. 6, 83-96, (1956).

(4) Geological application of extreme-value methods to interpretation of cobbles and boulders in gravel deposits. W. C. Krumbein (Northwestern University) and J. Lieblein. Trans. Amer. Geophysical Union 37, 313-319 (June 1956).

(5) On the existence of identities for the coefficients of certain modular forms. M. Newman. J. London Math. Soc. 31, 350-359 (1956).

(6) On the Lerch zeta function. F. Oberhettinger. Pac. J. Res. 6, No. 1, 117-120 (1956).

(7) Contributions to the theory of rank order statistics: Two sample case. I. R. Savage. Ann. Math. Stat. 27, 590-615 (Sept. 1956).

(8) Algebraic equations satisfied by roots of natural numbers. E. G. Straus (University of California at Los Angeles) and O. Taussky. Pac. J. Math. 6, 97-98 (1956).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION SEPTEMBER 30, 1956

2.1 Mathematical Tables

(1) Tables of the bivariate normal distribution function and related functions. To appear as NBS Applied Mathematics Series 50.

2.2 Manuals, Bibliographies, and Indices

(1) Analyzing straight line data. F. S. Acton. To appear as a book in the Applied Statistics Series of the Wiley Publications in Statistics.

(2) Fractional factorial experimental designs for factors at two levels. To appear as NBS Applied Mathematics Series 48.

2.3 Technical Papers

(1) A survey of Lyapunov's second method. H. Antosiewicz. To appear in Annals of Mathematics Studies.

(2) On the identity relationship for fractional replicates of the 2^n series. R. C. Burton and W. S. Connor. Submitted to a technical journal.

(3) Some examples of the use of high speed computers in statistics. J. Cameron. To appear in the Proceedings of the 1955 Conference on Design, Development, and Testing, Washington, D. C., October 1955.

(4) Condensation interfaces in two-phase flows. P. Chiarulli and R. F. Dressler. Submitted to a technical journal.

(5) Some algebraic number theory estimates based on the Dedekind eta-function. H. Cohn. Submitted to a technical journal.

(6) Dilworth's theorem on partially ordered sets. G. B. Dantzig (RAND) and A. J. Hoffman. To appear in "Papers on linear inequalities and related systems," Annals of Mathematics Study 38.

(7) Uniqueness theory for asymptotic expansions in general regions. P. Davis. To appear in the Pacific Journal of Mathematics.

(8) Complete sequences and approximations in normed linear spaces. P. Davis and K. Fan. Submitted to a technical journal.

(9) A discussion of "Design of corrugated diaphragms" by J. A. Haringx. R. F. Dressler. To appear in the Transactions of the American Society of Mechanical Engineers.

(10) Entropy changes in rarefaction waves. R. F. Dressler. To appear in the Journal of Research, NBS.

(11) On systems of linear inequalities. K. Fan. To appear in "Papers on linear inequalities and related systems," Annals of Mathematics Study 38, ed. by H. W. Kuhn and A. W. Tucker.

(12) Systems of inequalities involving convex functions. Ky Fan. I. Glicksburg (University of Notre Dame), and A. J. Hoffman. To appear in the Proceedings of the American Mathematical Society.

(13) Unimodular matrices of order two which commute. K. Goldberg. Submitted to a technical journal.

(14) Pairs of matrices of order two which generate free groups. K. Goldberg and M. Newman. Submitted to a technical journal.

(15) Note on bounds for certain determinants. E. Haynsworth. Submitted to a technical journal.

(16) On the domain of regularity of generalized axially symmetric potentials. P. Henrici. To appear in the Proceedings of the American Mathematical Society.

(17) On the representation of a certain integral involving Bessel functions by hypergeometric series. P. Henrici. Submitted to a technical journal.

(18) Integral boundary points of convex polyhedra. A. J. Hoffman and J. B. Kruskal (Princeton University). To appear in Annals of Mathematics Study 38, "Papers on linear inequalities and related systems."

(19) On systems of distinct representatives. A. J. Hoffman and H. W. Kuhn (Bryn Mawr College). To appear in Annals of Mathematics Study 38, "Papers on linear inequalities and related systems."

(20) Systems of distinct representatives and linear programming. A. J. Hoffman and H. W. Kuhn (Bryn Mawr College). To appear in the American Mathematical Monthly.

(21) On the Hilbert matrix. T. Kato. To appear in the Proceedings of the American Mathematical Society.

(22) Solvability and consistency for linear equations and inequalities. H. W. Kuhn (Bryn Mawr College). To appear in the American Mathematical Monthly.

(23) Statistical investigation of the fatigue life of deep groove ball bearings. J. Lieblein and M. Zelen. To appear in the Journal of Research, NBS.

(24) An inclusion theorem for modular groups. M. Newman. Submitted to a technical journal.

(25) Some theorems about $P_r(n)$. M. Newman. Submitted to a technical journal.

(26) Classes of positive definite unimodular circulants. M. Newman and O. Taussky. To appear in the Canadian Journal of Mathematics.

(27) On the propagation of electromagnetic and acoustic pulses. Part I: Diffraction of pulses by wedges. F. Oberhettinger. Submitted to a technical journal.

(28) Application of quaternions to the representations of a binary quadratic form as a sum of four squares. G. Pall (Illinois Institute of Technology) and O. Taussky. Submitted to a technical journal.

(29) On the independence of tests of randomness and other hypotheses. I. R. Savage. To appear in the Journal of the American Statistical Association.

(30) Pitfalls in computation. I. A. Stegun and M. Abramowitz. Submitted to a technical journal.

(31) The transmission of Rayleigh waves across an ocean floor with two surface layers. R. Stoneley. To appear in the Bulletin of the Seismological Society of America.

(32) The attenutation of Rayleigh waves with depth in a medium with two surface layers. R. Stoneley and U. Hochstrasser. To appear in the Geophysical Supplement, Monthly Notices, Royal Astronomical Society.

(33) Elastic waves in anistropic media. J. L. Synge. Submitted to a technical journal.

(34) Stationary principles for forced vibrations in elasticity and electromagnetism. J. L. Synge. To appear in the Proceedings of the Eighth Symposium in Applied Mathematics held by the American Mathematical Society, Chicago, Ill., April 1956.

(35) The torsion of a hollow square. J. L. Synge and W. F. Cahill. Submitted to a technical journal.

(36) Classes of matrices. O. Taussky. To appear in the Illinois Journal of Mathematics.

(37) Commutativity in finite matrices. O. Taussky. Submitted to a technical journal.

(38) Commuting bilinear transformations and matrices. O. Taussky and J. Todd. Submitted to a technical journal.

(39) A direct approach to the problem of stability in the numerical solution of partial differential equations. J. Todd. To appear in the Proceedings of a Symposium on Partial Differential Equations held at Berkeley, California, 1955.

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(41) Investigations of the properties of corrugated diaphragms. W. A. Wildhack (NBS Office of Basic Instrumentation), R. F. Dressler, and E. C. Lloyd (NBS Mechanics Division). To appear in the Transactions of the American Society of Mechanical Engineers.

(42) Design of experiments in research and development. W J. Youden. To appear in the Proceedings of the Office of Ordnance Research Conference on Design and Experiments in Army Research, Development, and Testing, Washington, D. C., October 1955.

(43) National physical standards and design of experiment. W. J. Youden. To appear in Revue de L'Institut International de Statistique (The Hague).

(44) Randomization and experimentation. W. J. Youden. Submitted to a technical journal.

(45) Statistics--Engineering viewpoint. W. J. Youden. Submitted to a technical journal.

(46) Exact tests of significance for combining intra- and inter-block information. M. Zelen. Submitted to a technical journal.

(47) The analysis of covariance for incomplete block designs. M. Zelen. Submitted to a technical journal.

(48) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. S. Joel. Submitted to a technical journal.

2.5 Miscellaneous items

(1) Contributions on partially balanced incomplete block designs with two associate classes. W. H. Clatworthy. To appear as NBS Applied Mathematics Series 47.

(2) Further contributions to the solution of simultaneous linear equations and the determination of eigenvalues. To appear as Applied Mathematics Series 49.

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