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EFFECTIVE INTENSITY

OF

FLASHING AIRCRAFT LIGHTS

By

Theodore H. Projector

NBS

U. S. DEPARTMENT OF COMMERCE

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THE EFFECTIVE INTENSITY OF FLASHING AIRCRAFT LIGHTS

ABSTRACT

Since the classical work of Blondel and Rey in 1911 established the relation between the duration of a flash and its effective intensity, a considerable amount of research has been done on various aspects of the problem. This body of research is summarized and critically evaluated in this report. It is shown that the Blondel-Rey equation is still the best expression of the relation, and covers flash durations down to the microsecond range. The integral form of the equation, proposed by Blondel and Rey in 1911, appears valid for flashes of any wave shape and a method for handling this form, recently proposed by C. A. Douglas, is recommended for use in computing the effective intensity of all flashing lights.

1. INTRODUCTION

Flashing lights are in widespread use on aircraft because of their distinctiveness and their superiority to steady-burning lights in attracting attention. Navigation lights have been flashed in various configurations for many years, and the newer rotating beacons ("anti-collision" lights) are high-intensity flashing lights. In addition, flashing lights have been used as warning lights and for other purposes. Many novel lighting schemes for aircraft lighting are now in the experimental stage, and involve the use of flashing lights with characteristics, particularly in respect to duration of flash, differing markedly from previously used lights. The complete evaluation of most flashing light systems requires an analysis of many complex factors, many of them of a subtle nature. However, the fundamental element of all such systems, the apparent or "effective" intensity of flashing lights, can be dealt with in a relatively straightforward manner, and has been the subject of a considerable amount of research since 1911, when the classical work of Blondel and Rey[1] was published. It is the purpose of this paper to review existing knowledge in the field of the effective intensity of flashing lights and to make recommendations regarding methods of computation and specification.

Many variables may affect the apparent intensity of a flashing light, including the following:

1) The color of the light source.

2) The size of the light source.

3) The intensity-time distribution of the light pulse.

4) The illuminance at the observer's eye.

5) The adaptation state of the observer.

6) The portion of the observer's retina -- fovea or periphery -- used in observation. Foveal or central vision is mediated exclusively by cone receptors while both cones and rods mediate peripheral vision, with the density of cones decreasing as the distance from the fovea increases.

7) The luminance of the areas surrounding the light source.

Most of the research work which has been done on effective intensity has been under the following conditions: point sources, white light, threshold illuminance, abrupt flash or intensity-time distributions square in shape, dark adaptation, foveal vision, and dark surrounds. These experimental conditions will be assumed in this report except where otherwise specified.

2. CLASSICAL WORK OF BLONDEL AND REY

The photometry of steady burning lights at threshold had been studied intensively prior to 1911, and there had been some investigation of the intensity of flashing lights, but it was not until 1911, when Blondel and Rey[1] reported their classical research, that the problem first received systematic treatment. To obtain flashing lights of controlled duration, Blondel and Rey used a sector disc whose rotational speed could be adjusted to give flash durations from about 0.001 to 3 seconds. The intensity of the light source was varied while the observers recorded whether or not they could see the light. In order to express the results of their experiments, Blondel and Rey developed the concept of the equivalent steady intensity or effective intensity of the flashing light. The effective intensity is defined as the intensity of a steady-burning light which will be seen at threshold under the same conditions as the flashing light under study.

Fig. 1 gives the results of Blondel and Rey's work. The ordinates are the ratio, $Et/3E_3$, of the energy in the test flash to the energy in a 3-second flash wherein E is the instantaneous illuminance at the eye during the test flash of duration t, and E_3 is the illuminance at the eye during the 3-second flash. A 3-second flash was chosen as representing the minimum duration at which a flashing light would be equal

in effectiveness to a steady light providing an illuminance at the eye equal to the instantaneous illuminance of the flashing light. The abscissae are the duration of the flashes. Blondel and Rey found that a straight line could be fitted very well to the experimental points plotted on the graph, and the equation of the line could be expressed in the form:

$$\frac{E_o}{E} = \frac{t}{.21 + t} \tag{1}$$

where E_o is the threshold illuminance for a steady source and E is the illuminance during the time t of a flashing light at threshold. If it is kept in mind that the equation applies only to constant viewing conditions (threshold) then intensity may be used interchangeably with illuminance:

$$\frac{E_o}{E} = \frac{I_o}{I} = \frac{t}{.21 + t} \tag{2}$$

and one may refer to the effective intensity as a property of the flashing light.

3. ANALYSIS OF THE BLONDEL-REY EQUATION

The Blondel-Rey equation is fundamental. Although a considerable amount of work has been done since 1911, the equation still expresses as well as any other the relation between effective intensity and the parameters of a flash. In its general form:

$$\frac{I_e}{I} = \frac{t}{a + t} \tag{3}$$

where I_e is the effective intensity and I is the instantaneous intensity during the flash, it has been found to fit data taken above threshold as well as at threshold, with the value of "a" determined by the magnitude of the illuminance at the observer. It is important therefore to go into some detail about the equation in order to obtain a full understanding of it.

Let us consider three basic methods of flashing a light and then consider what the implications are for these three methods. One method still used in certain kinds of signaling lights is to put a shutter in front of a steady burning source and flash the light by opening and closing the shutter. A second method

is to use an electrical switch to turn a lamp on and off, as is done for example on aircraft position lights. A condenser discharge lamp is of this type in that a discrete quantity of electrical energy is discharged into the lamp, causing it to flash briefly and then go out. The third method of flashing a light is to rotate or oscillate a steady burning projector. In this case, as the light beam sweeps past the observer's eye, he gets the effect of a flashing light.

In Fig. 2, curves expressing the Blondel-Rey equation for the case of shutter flashing are given. A steady burning light of intensity, I, is periodically occulted to give a flashing light of effective intensity I_e. Effective intensity never exceeds the steady intensity in this kind of flashing. The curves first rise steeply, then level off, eventually reaching the value of steady intensity. Strictly speaking, the curves would not reach the steady intensity level until the flash duration is infintely long although for practical purposes they would be there much before that.

Plotted on this graph are curves for some other values of a. As will be shown later these values of a represent viewing conditions above threshold. In other words as the intensity of the light increases the relative effectiveness of its flash increases also. The value of a goes down as illuminance level goes up so that well above threshold a flashing light is much more effective relatively than it is at threshold. This is a very significant point and some apparently anomalous field observations may be explained by this factor. If an observer is standing close to a light and evaluates its effective intensity by comparing it with another light which is also quite bright, he is not making a valid comparison in terms of what would be seen if they were both difficult to see. If one of the lights has a much shorter flash duration than the other one, it will be benefited by being viewed close up where it becomes very intense. This benefit will be lost to a great extent when the lights are viewed close to threshold.

Fig. 3 is similar to Fig. 2 except that the beginning of the curves has been expanded to show the short duration flash characteristics in more detail. It may be seen in this figure that at or near threshold the curves are nearly linear, i.e., the effective intensity is approximately proportional to the duration of the flash. As the level of illuminance increases above threshold the portion of the curves which is approximately linear decreases.

Fig. 4 shows curves of the Bondel-Rey equation in a different way. This time instead of a light which is flashed by a shutter, the light is flashed in such a way that each flash has equal energy. For example, if the duration of the flash is cut in half, the intensity required to maintain equal energy in the flash is doubled. By rewriting the equation in the form shown in the figure the numerator becomes luminous energy and is a constant for any curve since we are postulating that each flash has equal energy. These curves show that the energy in a flash is more efficiently utilized as the time of the flash is reduced. The increase in efficiency with reduction of time of flash is considerably greater for the above-threshold condition than for the threshold condition. These curves further clarify the apparently anomalous observations on flashing lights of very different flash durations. If two lights of equal energy, one very short in duration, the other long, appear nearly equal in intensity at threshold viewing conditions, the short duration flashing light will appear much more intense when the lights are viewed well above threshold.

The curves of Fig. 4 are plotted on a linear time scale. A more informative set of curves is obtained when the time scale is logarithmic, as in Fig. 5, in which it may be seen that curves level off at short durations and the intensity-time relation is reciprocal. For a given viewing condition, if the time of flash is reduced to the point where the curve levels off, any further reduction in time produces no change in effective intensity as long as the energy per flash is constant. As one views a flashing light at higher and higher illuminance levels, the knee of the curve moves to shorter and shorter times, but in every case leveling occurs, beyond which the relation is reciprocal. In the reciprocal region, the instantaneous intensity by itself is not a measure of effectiveness; the effectiveness is determined only by the energy in the flash. Thus, as with condenser-discharge lamps, enormous instantaneous candlepowers may be produced for extremely short times, but the effectiveness of such flashes is related only to the energy contained in the flash, which may be very modest.

Figs. 2 and 3 are curves of the Blondel-Rey relation plotted for flashes of constant instantaneous intensity, while Figs. 4 and 5 are curves for flashes of equal energy. Another instructive way of presenting the same relationship is shown in Fig. 6, where the energy in the flash required to give constant effective intensity is plotted against time. Each curve therefore expresses the amount of energy at different flash durations required to produce a given visual effect. It is again shown that more energy is required at long flash durations to produce a given effect than at short durations. As before it is evident that a point is reached

in reducing flash time at which further reduction produces no gain in efficiency. The intersections of the dashed lines in the figure illustrate another factor which has been intensively studied - the "critical duration" or the time at which the relationship changes from one of reciprocity, $It = K$, to one in which $I = k$. At threshold the critical duration is about 0.1 second. A considerable body of subsequent research has confirmed the Blondel-Rey relationship in this form.

In Fig. 6, unlike the earlier figures, the curves for lower values of a lie below the curve for a = 0.2 (threshold). It will be helpful in understanding the relationship among these curves to consider an experimental procedure whereby they might be obtained. With the observers stations at some arbitrary distance from the light source, the intensity of the flash at very long duration is adjusted until the flashes appear to be at threshold. The curve for a =0.2 is then obtained by reducing the flash duration by set amounts and determining the relative energy required to maintain the light at threshold. When that is done the light is readjusted to the intensity and duration prevailing at the beginning of the experiment and the observers are stationed at a point closer to the light by an amount sufficient to raise the illuminance to a level above threshold corresponding to a = 0.05. A curve for a = 0.05 is then obtained as before (except that a comparison technique, as explained later, is used instead of threshold observations) and this procedure is repeated for any additional values of a desired. As noted previously, relatively smaller energies are required above threshold than at threshold with short duration flashes.

2. SUBSEQUENT RESEARCH

Shortly after the publication of their classic paper, Blondel and Rey published a second paper[2] in which they took up the question of flashes having time distributions other than square. On purely intuitive grounds they proposed the following modification of their original equation for effective intensity:

$$I_e = \frac{\int_{t_1}^{t_2} I\,dt}{.21 + (t_2 - t_1)} \qquad (4)$$

In this equation the numerator of the right side represents the light energy contained in the flash between the time

limits of integration, t_1 and t_2. For the case where the flash is abrupt (square in shape) this reduces to the original equation:

$$I_e = \frac{I\,(t_2 - t_1)}{.21 + (t_2 - t_1)} \tag{5}$$

The difficulty with the integral equation proposed by Blondel and Rey is that there is an ambiguity about the choice of the time limits, t_1 and t_2. Blondel and Rey recognized this difficulty and could offer no rigorous solution for it. They did however suggest that t_1 and t_2 should be chosen as the times when I was equal to the threshold value of I for steady illumination. As it turned out much later, this was an excellent choice for the case they were considering, e.g., where I_e is the effective candle-power for threshold viewing.

In 1916, Blondel and Rey[3] reported field tests on actual rotating beacons. The effective intensity threshold for abrupt flashes, and for conventional beacons with time distributions that rose and fell gradually, were determined. In the latter category they tested beacons with very short flash durations and with long durations. Within their experimental error, which was fairly large, they confirmed their earlier work, including the proposed integral form of the equation for non-abrupt flashes. Unfortunately, their experimental error was too large to resolve the ambiguity about the limits of integration.

In 1918, Reeves[4] did some work on the effective intensity of flashing lights using a spot of 5 degrees subtense. He used times from 2 milliseconds to 4 seconds and he generally confirmed the Blondel-Rey formula with one exception. At very fast flashing times he found a failure of reciprocity analogous to that found in photography. At the extremes of exposure, either extremely short or extremely long, it takes a little more exposure to get the same density on the film. Reeves found an analogous visual situation at very short flash times. Similarly, in 1920, Piéron[5] did some work with flashing lights using foveal, point sources. His duration range was from 2/3 millisecond to over 3 seconds, at threshold, and he also found that the curve fitted the Blondel-Rey relation except at the short duration end, where he found a failure of reciprocity. Both Reeves' and Piéron's work has been criticized in regard to their findings of a failure of reciprocity for short flash durations. Both may have been biased by an acquaintance with photographic reciprocity failure. In neither work was the magnitude of the reciprocity failure large nor did it extend over an appreciable number of observation points. Their work was done at a time when there was no adequate experimental equipment for verifying the actual course of exposure. It is quite

probable that the actual exposure was different and more than likely less than they supposed it to be at short durations. Both were undoubtedly trying to get as short exposure times as they could with their apparatus and may not have accounted fully for the finite time it took for their flashes to come to full intensity and to decay to zero intensity. At longer durations this error was negligible but may have been appreciable at very short durations. In view of all this and the fact that subsequent careful research extending to even shorter flash durations showed no measurable failure of reciprocity, it seems likely that Reeves' and Piéron's apparent failure of reciprocity at short flash durations is attributable either to experimental error or inadequate equipment. It should be emphasized here that such failures of reciprocity, if they are genuine, imply that extremely short flash durations are less efficient than longer but still moderately short durations in producing effective intensity. This would be contrary to the claim sometimes made for extremely fast-flashing condenser discharge lamps that they are <u>more</u> efficient than flashes of longer duration.

Piéron[6] also studied the effective intensity of flashing lights for rod and for cone vision. To do this, he set his experiment up so that observations were made with peripheral vision 20 degrees from the fovea. He then interposed a deep red or a deep blue filter between the flashing lights and the observers. In accordance with the Purkinje effect, only the rods are sensitive to the deep blue light and the cones to the deep red light. The rod sensitivity was greater than that of the cones, but both receptors operated essentially in accordance with the Blondel-Rey law, except of course for the apparent failure of reciprocity at short flash durations as noted above.

In 1931, Langmuir and Westendorp[8] reported the results of an encyclopedic investigation into various aspects of light signaling. Some of their work pertained to the effective intensity of flashing lights and generally confirmed the Blondel-Rey law.

In 1933, Toulmin-Smith and Green[9] took up the question of what happens above threshold. All the previous work had been at threshold and almost all of it had confirmed exactly Blondel-Rey's value of about 0.2 for a. One reason for their interest in working above threshold was that they and others had felt that in practical navigation the "useful" threshold was higher than laboratory measured thresholds or thresholds measured in the field under ideal conditions. They had postulated a value of 0.425 mile-candle as the "useful" threshold. They therefore were interested in how raising the threshold might affect the Blondel-Rey formula or, in general, the effective intensity of flashing lights.

In making threshold measurements an observer is asked to look at a light and report whether he can see it or not. This is a simple idea but difficult to do. Above threshold the observational criteria become more subtle and the observations even more difficult to make.

Above threshold, the essential idea developed by Blondel and Rey of defining the effective intensity of a flashing light as the intensity of an equivalent steady light is retained. The experimental procedure requires the observer to report not merely when he can or cannot see the flashing light but rather when a flashing light and a steady light, burning side by side, appear equal in intensity. Toulmin-Smith and Green, using this technique, investigated illuminance levels from 0.2 mile-candle (approximately threshold) to 4 mile-candles. Fig. 7 shows the data obtained at 0.5 mile-candle, and indicates the variability of the observations. The flash durations covered the range from 0.05 to 0.5 seconds. Fig. 8 shows the family of curves obtained at five illuminance levels from 0.2 to 4 mile-candles. Inasmuch as the shortest duration used in the experiments was 0.05 seconds, well above the region of reciprocity, the curves should not be considered as well-determined in that region.

Toulmin-Smith and Green attempted to fit an equation to their data, as shown in Fig. 9. In the figure, the dashed curve represents their test results for an illuminance of 0.425 mile-candle, (the "useful" threshold), interpolated from their actual test data. The Blondel-Rey equation

$$\frac{I_o}{I} = \frac{t}{.21 + t} \tag{6}$$

is represented by the lower broken curve. The upper curve represents the equation,

$$\frac{I_o}{I} = 1.4 \frac{t}{.2 + t} \tag{7}$$

proposed earlier by Van Vloten[7]. Toulmin-Smith and Green found an excellent fit with the equation,

$$\frac{I_o}{I} = 1.1 \frac{t}{.15 + t} \tag{8}$$

as represented by the solid curve. In additon, two sets of points are plotted on the graph, representing the equation,

$$\frac{I_o}{I} = \frac{t}{a + t} \qquad (9)$$

with values for a of 0.11 and 0.12. This last equation is the Blondel-Rey equation, except for the change in the value of the constant a. While Toulmin-Smith and Green's equation fits their curve better, it is doubtful that the somewhat poorer fit of the Blondel-Rey form is significantly different, in view of the inherent variability of the test data.

Hampton,[10] in 1934, objected to the Toulmin-Smith and Green equation because of the coefficient 1.1, asserting that an adequate fit to all of their data could be obtained by an equation in the Blondel-Rey form with the constant a treated as a function of the illuminance level. On this assumption, he plotted Toulmin-Smith and Green's data as shown in Fig. 10, and from this obtained the equation,

$$\frac{I_o}{I} = \frac{t}{\left(\frac{.0255}{E}\right)^{.81} + t} \qquad (10)$$

where E, the illuminance at the observer, (designated conspicuity by Hampton), is given in mile-candles. In this equation, a has the following values at the illuminance levels used in the original experiments.

Illuminance, Mile-Candles	a
.2	.19
.5	.09
1.0	.05
2.0	.03
4.0	.017

Hampton then replotted the original curves, using his equation, and obtained the curves of Fig. 11, not significantly different from those of Toulmin-Smith and Green.

In Fig. 12, Hampton compared his equation (Equation 3 in his paper), Toulmin-Smith and Green's formula (attributed to Green), and the experimental data for 0.425 mile-candle. None of the differences is significant.

In 1937, Stiles, Bennett, and Green[11] reviewed all the previous work on the effective intensity of flashing lights and concluded that the Blondel-Rey equation was fundamentally sound. They considered the apparent reciprocity failure discovered by Reeves and Piéron, but felt that the work on which it was based was of doubtful validity. They noted the integral form of the Blondel-Rey equation that had been proposed in 1911, and discussed the difficulty about the ambiguous time limits, emphasizing that although the formula appeared logically sound, there had been little or no experimental work to confirm it.

Neeland, Laufer, and Schaub,[12] in 1938, reported the results of measurement of the effective intensities of rotating beacons with square wave characteristics and with curved characteristics. The results of their tests with abrupt flashes confirmed the Blondel-Rey law and very roughly confirmed Toulmin-Smith and Green's data. However, for flashes with curved time distributions, their results were quite different. They found values of a which not only were higher than those found by Toulmin-Smith and Green, but increased with illuminance level rather than decreased. Furthermore their values of a were in some cases appreciably higher than 0.21, the threshold value found by Blondel and Rey and widely confirmed by subsequent research. Neeland et al, in computing I_e used the abrupt flash form of the Blondel-Rey equation rather than the integral form. They used the peak intensity as the value of I in the equation, and the time interval between the points on the distribution where the intensity had fallen to 10% of the peak intensity as the value of t. In so doing they were attributing considerably more energy to the flash than was actually there. Their paper did not give sufficiently detailed information to permit an exact recomputation by the integral form of the equation. However by postulating a candlepower-time distribution that is reasonable for the searchlight data they gave and using this distribution in the Blondel-Rey integral equation, results much more nearly in line with those of Toulmin-Smith and Green are obtained. It is felt therefore that the latter work was not seriously challenged.

In 1940, Schuil[13] reported an investigation into the effect on effective intensity of the repetition rate of flashes. He used two flash durations, 1/10 and 1/40 second, and varied the repetition rate from a rate high enough to merge one flash into the next so that the light was in effect steady burning, down to one per second, at which flash rate there appeared to be no significant effect of one flash on the apparent intensity of the next. Fig. 13 gives Schuil's results. At frequencies high enough so that the light appeared steady, Talbot's law applied; the apparent intensity was equal to the average intensity computed from a time distribution. At frequencies low enough to permit

the flashes to be seen as distinctly separate, the Blondel-Rey law held, and it appeared that there was no interaction between adjacent flashes. At intermediate frequencies, when the lights appeared as flickering lights the effective intensities were intermediate between those computed by Talbot's law and those computed by the Blondel-Rey law. The results reported were based on tests at an illuminance level of 0.5 mile-candle, about the "useful" threshold. Schuil noted in his report that he had obtained essentially similar results at 2 mile-candles and that he had not confirmed the value of a at the higher illuminance obtained by Toulmin-Smith and Green. He said however that since that was not the subject of the paper he would not go into it and unfortunately gave no details.

In 1949, Baumgardt[14] reported the results of an investigation of effective intensity extending flash durations to as short as 4 microseconds. Reciprocity down to about 1/3 millisecond had been well established prior to 1949. Baumgardt covered the range from 4 microseconds to 1/10 millisecond. The results of his experiments with four subjects are shown in Fig. 14. The viewing conditions were 12 degrees peripheral with a spot size of about 1 degree. It may readily be seen in the figure that, within experimental error, the energy, it, required for threshold excitation is constant over the range, and thus that the Blondel-Rey law is confirmed down to the microsecond region.

Baumgardt's work thus confirms the applicability of the Blondel-Rey law to virtually every type of flashing light now being used in aircraft lighting or being considered for it, including condenser-discharge lamps. Such lamps, depending on their firing circuits and energies, have time durations either within the range studied by Baumgardt or slightly longer. No research work has shown that extremely short flashes are more efficient than longer ones, except in accordance with the Blondel-Rey law. Some work has suggested that, if anything, there is a loss of efficiency, but this work is in considerable doubt, and the best information confirms the reciprocity relation predicted by the Blondel-Rey law throughout the range of flash durations now being realized in flashing aircraft lights. In the reciprocity range, as noted before, only the energy contained in the flash, it, is the measure of the effectiveness of the flash. Condenser discharge lamps reach extremely high peak intensities, but since these high intensities are maintained for correspondingly brief intervals, the energies, and consequently, the effective intensities, are moderate.

As a matter of fact, condenser-discharge lamps are not particularly efficient producers of light energy. Typical lamps, not counting their auxiliary equipment, produce about 30-40 lumen-seconds per watt-second of electrical energy input. This may be compared with efficiencies of about 60-70 lumens per watt for fluorescent lamps and about 15-20 for incandescent lamps. The efficiency of the auxiliary equipment used to operate condenser-discharge lamps is of the order of 50% or less, and thus the overall efficiency is no better than that of incandescent lamps. In addition to this, the auxiliary equipment is generally heavy and bulky so that it is clear that condenser-discharge lamps should be used on aircraft only when their unique property of extremely short duration flashing is an imperative requirement.

In 1951, Long[15] reported an investigation of the relation between the wave form of the flash and the effective intensity. He used seven wave forms ranging from a square wave to a triangular wave. Fig. 15 shows four of the seven wave forms. The other three were intermediate in shape. The wave forms were so designed as to contain the same light energy per flash. The viewing condition was 15 degrees peripheral. Long did not use the integral form of the Blondel-Rey equation or concern himself with the choice of the limits of integration. He computed the total energy under the curve for each wave shape. Long felt that he was well within the range of reciprocity since his flash durations were quite short, and therefore concerned himself only with the relation between the total energy in the flash and the effective intensity. His results are shown in Fig. 16. (Long identified the different wave shapes by the time it took for the intensity to rise to a maximum.) It will be noted in the figure that the constancy of the energy required for threshold excitation with the different wave shapes was rather remarkable, in view of the experimental errors usually found in work of this kind. As will be shown later, there is some question as to whether, in applying the Blondel-Rey integral equation, one should include all the energy in the flash. In the case of square waves or steep-sided trapezoids (see Fig. 15) the uncertainty is negligible. However, for curves such as the triangular wave of Fig. 15, the uncertainty may be significant. To check this, I have computed the effective intensity of the square wave and the triangular wave by the method to be described later, using the Blondel-Rey integral equation, and found that the effective intensity of the triangular wave is about 10% less than that of the square wave. This amounts to about .05 log unit of energy in Fig. 16. It is evident therefore that this does not significantly affect Long's finding that reciprocity

is valid for any of the wave shapes in his experiment. Long also investigated the relation between effective intensity and flash duration over the range from about 0.02 to 0.25 seconds with the results shown in Fig. 17. Again his results seem remarkably good. They conform closely to the Blondel-rey law with perhaps rather too sharp a break between the right side of the curve where I is constant, and the left side where It is constant.

The major research work in the field of measurement of the intensity of flashing lights has been described. A great deal more has been done, and it all leads to the same conclusion, that the essential validity of the relation discovered by Blondel and Rey in 1911 is still unchallenged. Even research work in problems where the effective intensity plays only a contributory role has yielded results in strict consonance with the Blondel-Rey law. For example, studies of the ability to locate random lights in space, or the ability to discriminate the direction of a moving spot, etc., have all produced results in the form of the Blondel-Rey law. One may be cited as an illustration. Fig. 18 shows the results reported by Brown[16] in 1955, of an experiment to determine the direction sensing ability of four subjects. The similarity between this data and the curves of Figs. 6 and 17 is evident.

5. THE INTEGRAL FORM OF THE BLONDEL-REY EQUATION

As noted before, the integral form of the Blondel-Rey equation, as shown in Fig. 19, is logically attractive, but involves an uncertainty, not resolved since the equation was first proposed in 1911, about the choice of time limits. Douglas[17] recently proposed a method whereby the equation may be used in a manner which eliminates this uncertainty. Douglas' method is elegant and straightforward and permits the unambiguous computation of effective intensity even with lights whose time distributions are complex in form. As with the equation itself, Douglas' method of computation is not validated by any experimental data, but does yield results which are unambiguous and in accord with experience within experimental error. Douglas proposed that the time limits, t_1 and t_2, be so chosen as to yield the maximum value of I_e. In his report describing the application of his computational method to distributions of various shapes, Douglas gives details as to the procedure which are an aid in minimizing the work necessary to obtain the result and in handling complex distributions. It appears that the general use of the method is highly desirable, especially in writing specifications, where its lack of ambiguity is of considerable value.

Douglas shows that the maximum value of I_e is obtained when the time limits are chosen as those times when the instantaneous intensity is equal to the effective intensity, and this turns out to be a considerable convenience in reducing the number of computations required to determine I_e. An interesting corollary of this is that, for a flashing light at threshold, the time limits are those when the instantaneous intensity is the threshold intensity of a steady light. These are exactly the limits proposed intuitively by Blondel and Rey in 1911!

In computing I_e by Douglas' method it is not necessary to ascertain the correct limits, t_1 and t_2, with great precision. It may be seen in Fig. 19, where I_e is the effective intensity, that moving up the curve for example to I' with corresponding limits t_1' and t_2' involves a reduction in the magnitude of the numerator of the equation since less area is included under the curve. But $t_2'-t_1'$ is less than t_2-t_1 so that the denominator is also reduced in magnitude and the net change in the right side of the equation is not very great. A similar compensation occurs if the time limits t_1'' and t_2'' are used.

Some numerical examples may be instructive. In Fig. 20 are plotted two representative intensity-time distributions, one for a flashing light with a duration of about 1/4 second, the other for a light with a duration of about 1/20 second. The values of I_e computed for five sets of time intervals are indicated on each curve at the level of the time limits. The middle value of each group of five is the maximum I_e computed by Douglas' method, and it will be noted that this value is equal to the instantaneous value of candlepower at the corresponding time limits. Typically, the maximum I_e occurs lower down on the curve for a short duration flash than a long one, and the computed values of I_e for different time limits tend to differ less from the maximum for the short duration flashes. As previously noted the change of I_e for different time limits on either curve is not very great, so that high precision in determining the time limits is not necessary.

To compute I_e by Douglas' method, the first step is to estimate I_e. The limits of integration, t_1 and t_2, are then the times when the instantaneous intensity is equal to the estimated I_e. Let us suppose that the given distribution is the narrower one in Fig. 20, and that the initial guess for I_e was 1.5 kilocandles, resulting in a computed value for I_e

of 5.4 kilocandles. The computed value, 5.4, characteristically lies between the estimated value, 1.5, and the maximum value, 5.6, but much closer to the latter. Thus the next guess is to be slightly higher than 5.4, and it is evident that the maximum will be reached in a very few successive approximations. When the initial guess corresponds to a level for I_e higher than the maximum, then the computed value again falls slightly below the maximum and the second guess is again made by estimating I_e slightly higher than the computed value of the first guess. With a little experience in such computations, it should seldom be necessary to make more than two or three guesses in order to obtain I_e with the necessary precision.

If computations for I_e are to be made for the sole purpose of determining whether a light unit meets the requirements of a specification, then only a single computation need be made. The limits of integration are chosen to correspond with the points on the distribution at which the instantaneous intensity is equal to the specified value of effective intensity. The computed I_e will always exceed the specification value if the actual maximum I_e is in excess of the specification. Likewise, if the maximum I_e is less than the specification minimum, then the computed I_e will always be less than the specified minimum. This procedure is convenient for determining compliance with a specification, but it should be noted that it results in computed values of I_e which are less than the maximum I_e, except when the maximum I_e is equal to the specified minimum when obviously they are equal.

The above sample computations were based on the curves of Fig. 20, with a = 0.2 in the Blondel-Rey equation, and therefore imply threshold viewing conditions. The effect of changing the viewing conditions to levels above threshold and correspondingly lower values of a is of interest. The following table gives the effective intensity in kilocandles for the two distributions of Fig. 20, for values of a equal to 0.2 (threshold), 0.1 and 0.05:

	I_e for a = 0.2	0.1	0.05
Narrow beam	5.6	9.7	16.3
Wide beam	7.5	10.3	12.9

It may also be of interest to consider the effect on I_e of changing the duration of the flash without otherwise changing

the distribution. Suppose for example that the lights of Fig. 20 are rotating beacons and it is desired to compute I_e if the speed of rotation is doubled. In this case, the ordinate scale is unchanged - the peak intensity and the shapes of the curves are unchanged - but the time scale is multiplied by 1/2. The following table gives I_e in kilocandles for this condition:

	I_e for a = 0.2	0.1	0.05
Narrow beam	3.2	5.7	9.7
Wide beam	4.9	7.5	10.3

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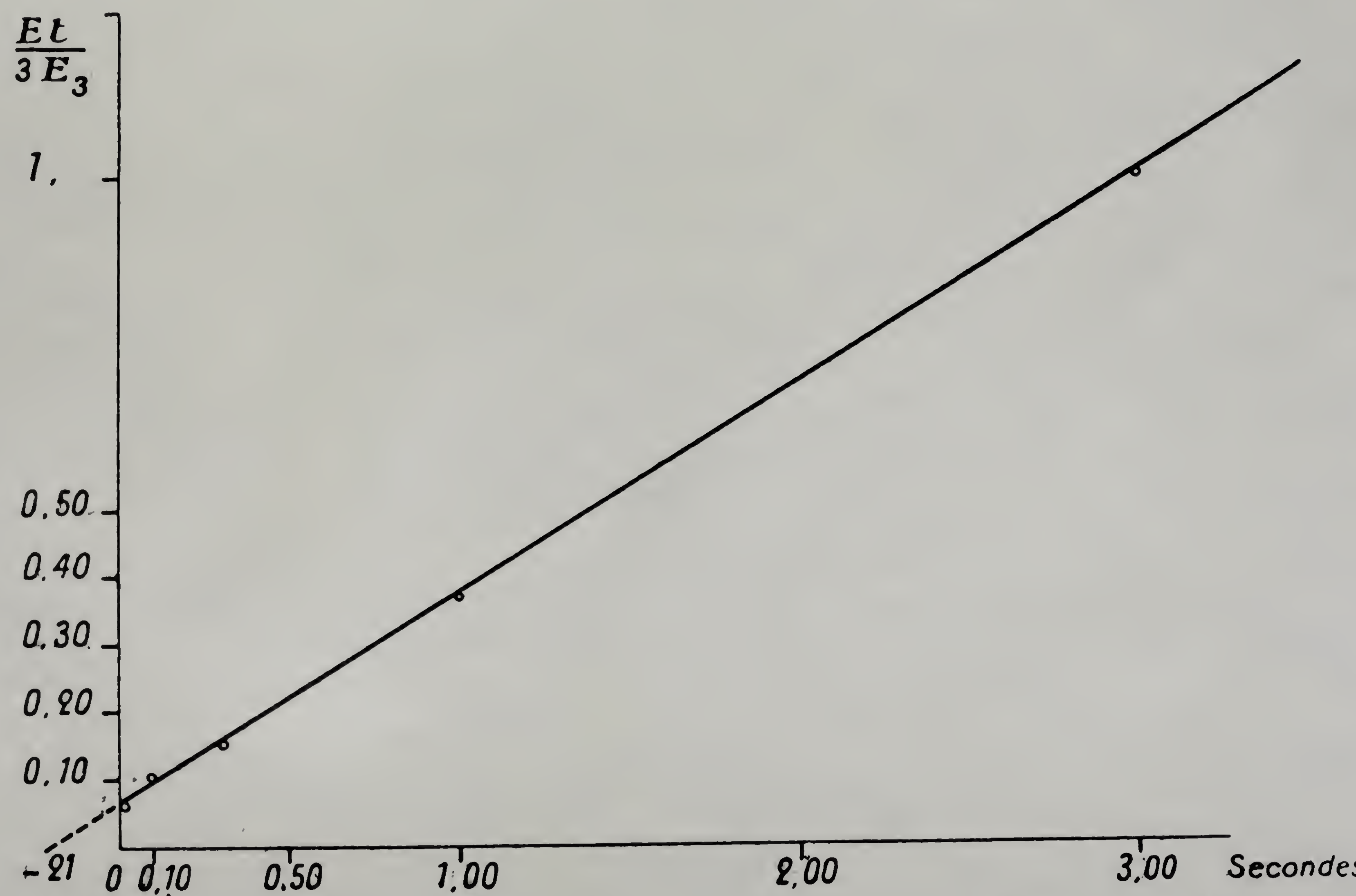
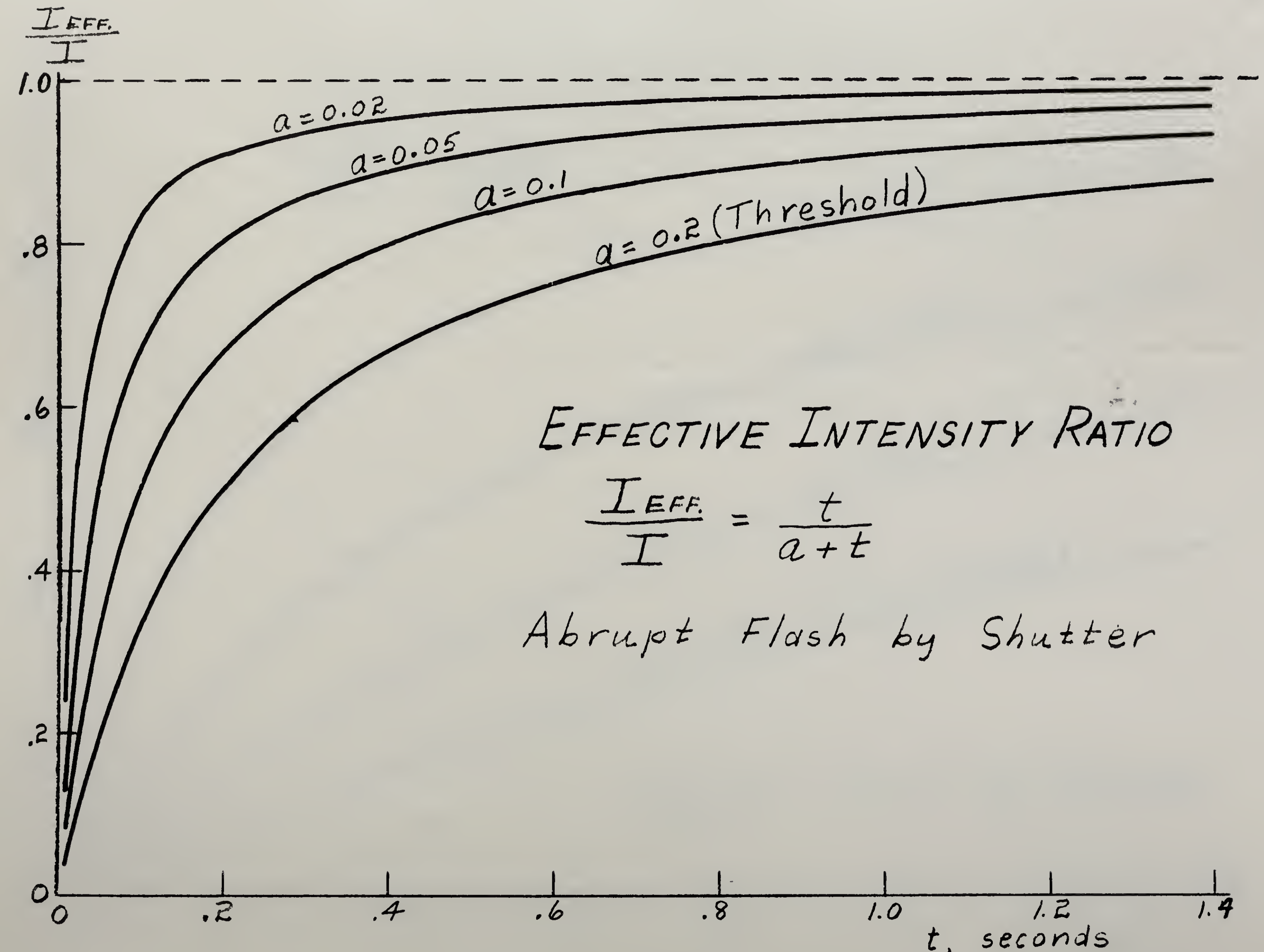
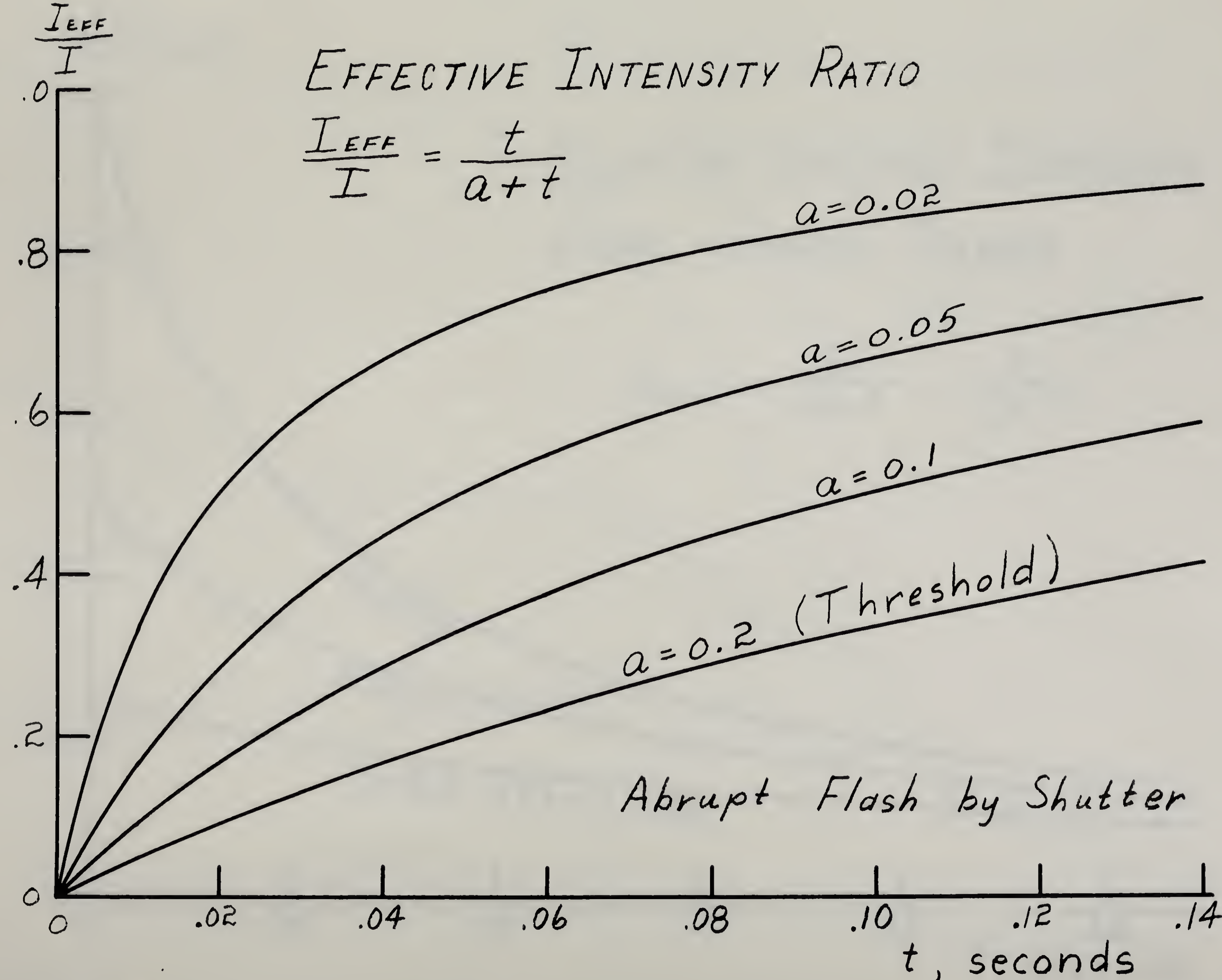


Fig. 1 — Loi de variation du produit Et en fonction du temps d'après les résultats des expériences.



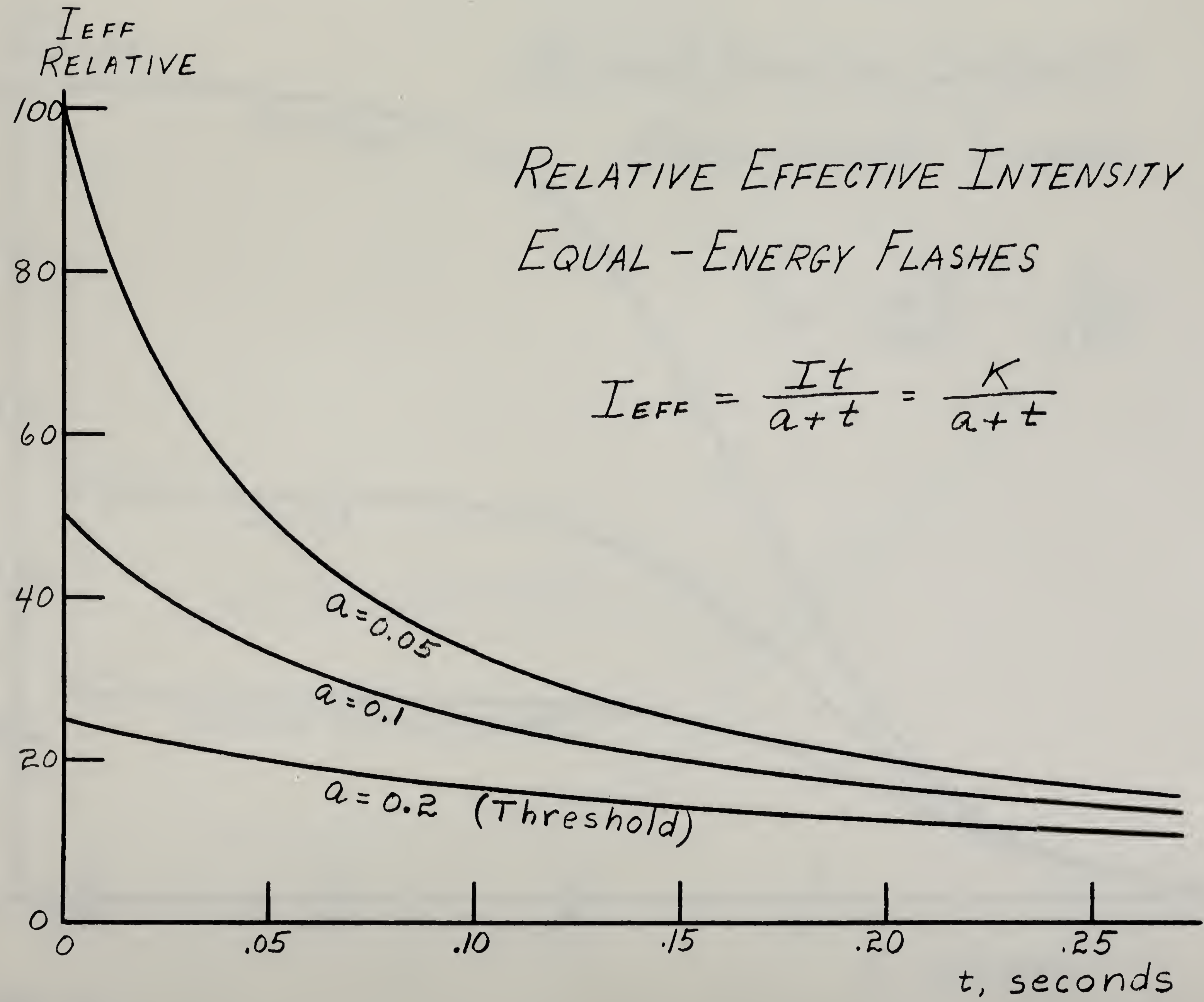
I EFF. / I
1.0
.8
.6
.4
.2
0
a = 0.02
a = 0.05
a = 0.1
a = 0.2 (Threshold)
EFFECTIVE INTENSITY RATIO
I EFF. / I = t / (a + t)
Abrupt Flash by Shutter
0
.2
.4
.6
.8
1.0
1.2
1.4
t, seconds

Fig. 2



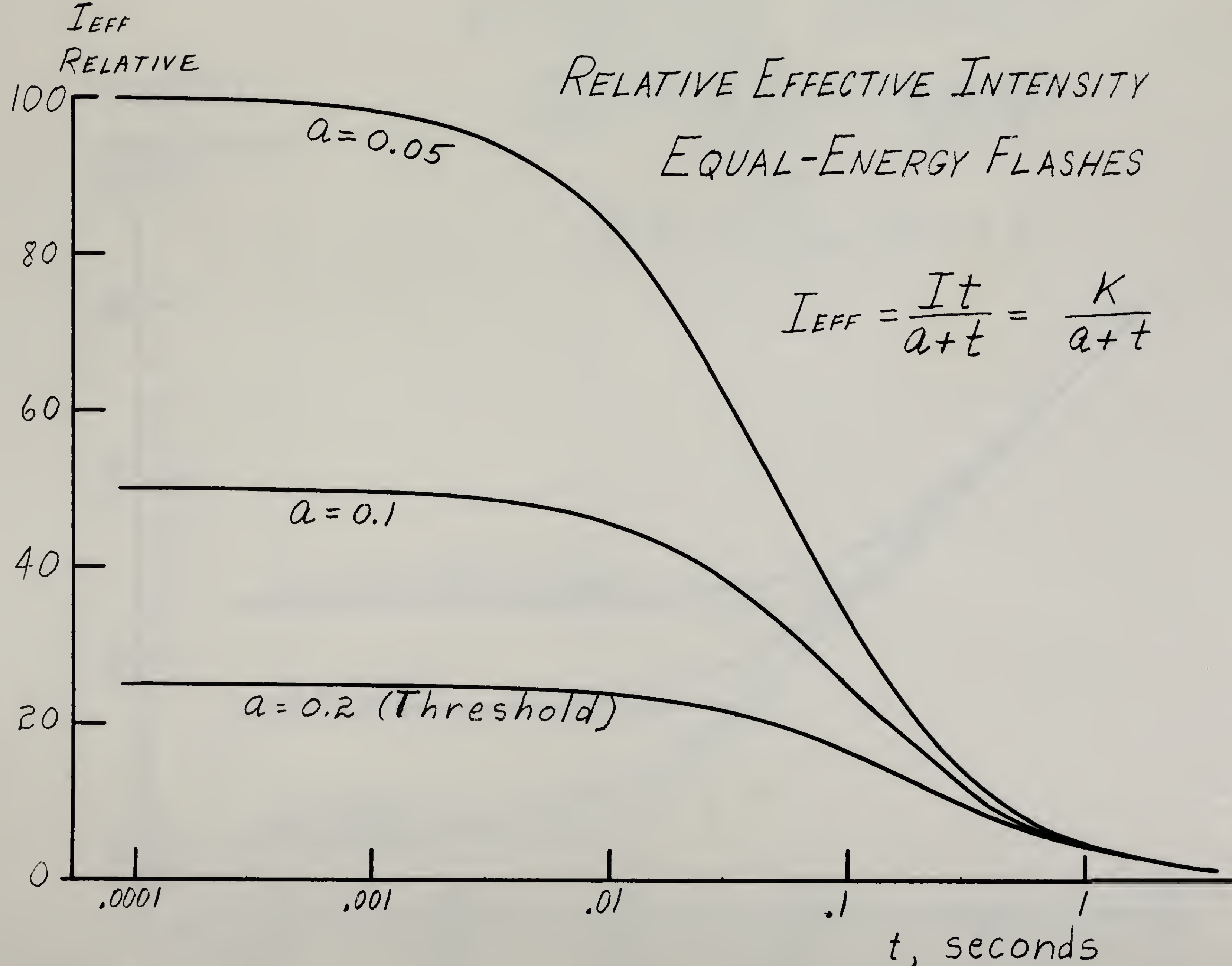
EFFECTIVE INTENSITY RATIO
$\frac{I_{EFF}}{I} = \frac{t}{a+t}$
$\frac{I_{EFF}}{I}$
.0
.8
.6
.4
.2
0
a=0.02
a=0.05
a=0.1
a=0.2 (Threshold)
Abrupt Flash by Shutter
0
.02
.04
.06
.08
.10
.12
.14
t, seconds

Fig. 3



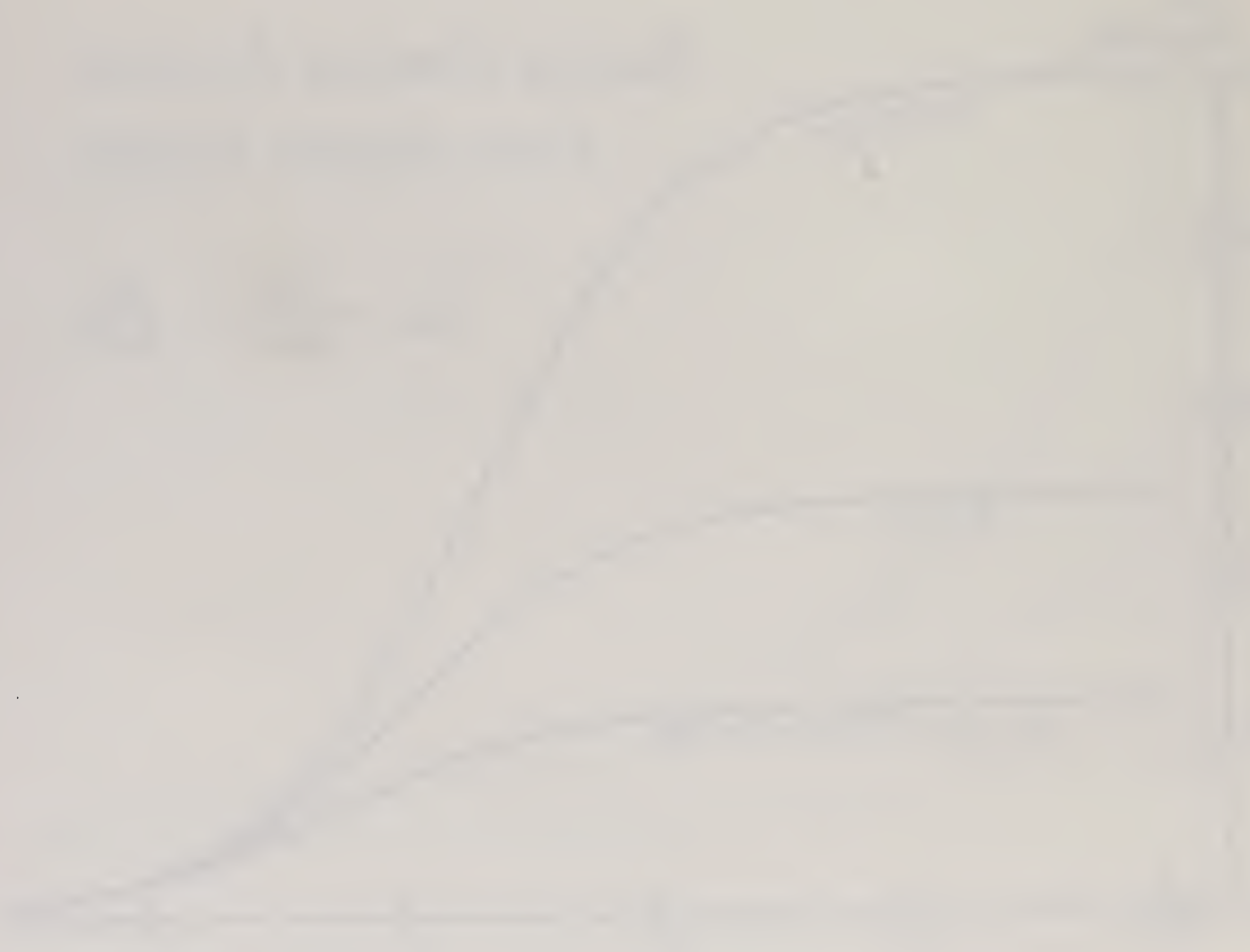
I_{EFF} RELATIVE
100
80
60
40
20
0
RELATIVE EFFECTIVE INTENSITY
EQUAL - ENERGY FLASHES
$I_{EFF} = \frac{It}{a+t} = \frac{K}{a+t}$
a = 0.05
a = 0.1
a = 0.2 (Threshold)
0
.05
.10
.15
.20
.25
t, seconds

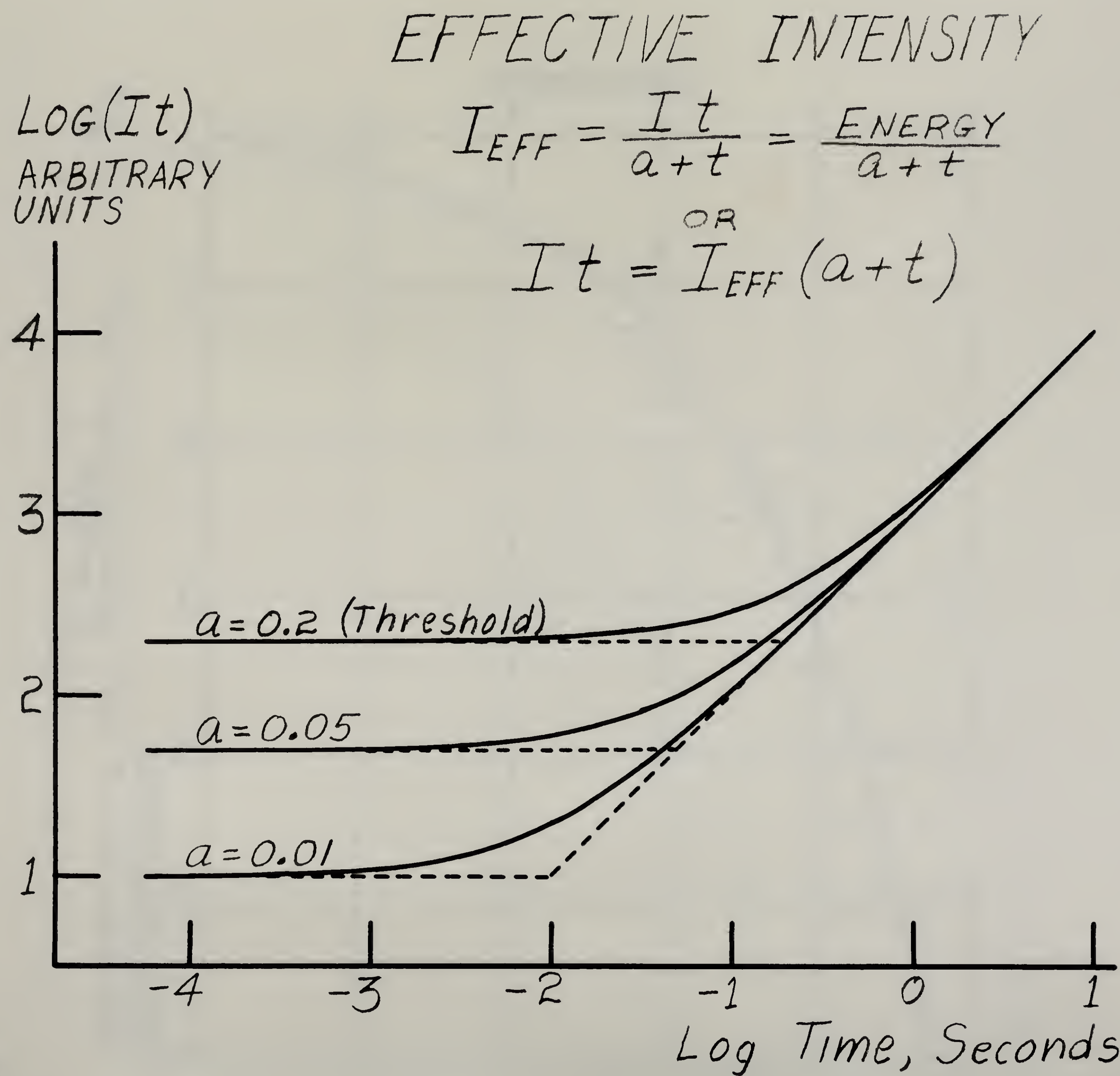
Fig. 4



RELATIVE EFFECTIVE INTENSITY
EQUAL-ENERGY FLASHES
$I_{EFF} = \frac{It}{a+t} = \frac{K}{a+t}$
I_{EFF} RELATIVE
100
80
60
40
20
0
a = 0.05
a = 0.1
a = 0.2 (Threshold)
.0001
.001
.01
.1
1
t, seconds

Fig. 5





EFFECTIVE INTENSITY
$I_{EFF} = \frac{It}{a+t} = \frac{\text{ENERGY}}{a+t}$
OR
$It = I_{EFF}(a+t)$
LOG(It) ARBITRARY UNITS
4
3
2
1
a=0.2 (Threshold)
a=0.05
a=0.01
-4
-3
-2
-1
0
1
Log Time, Seconds

Fig. 6

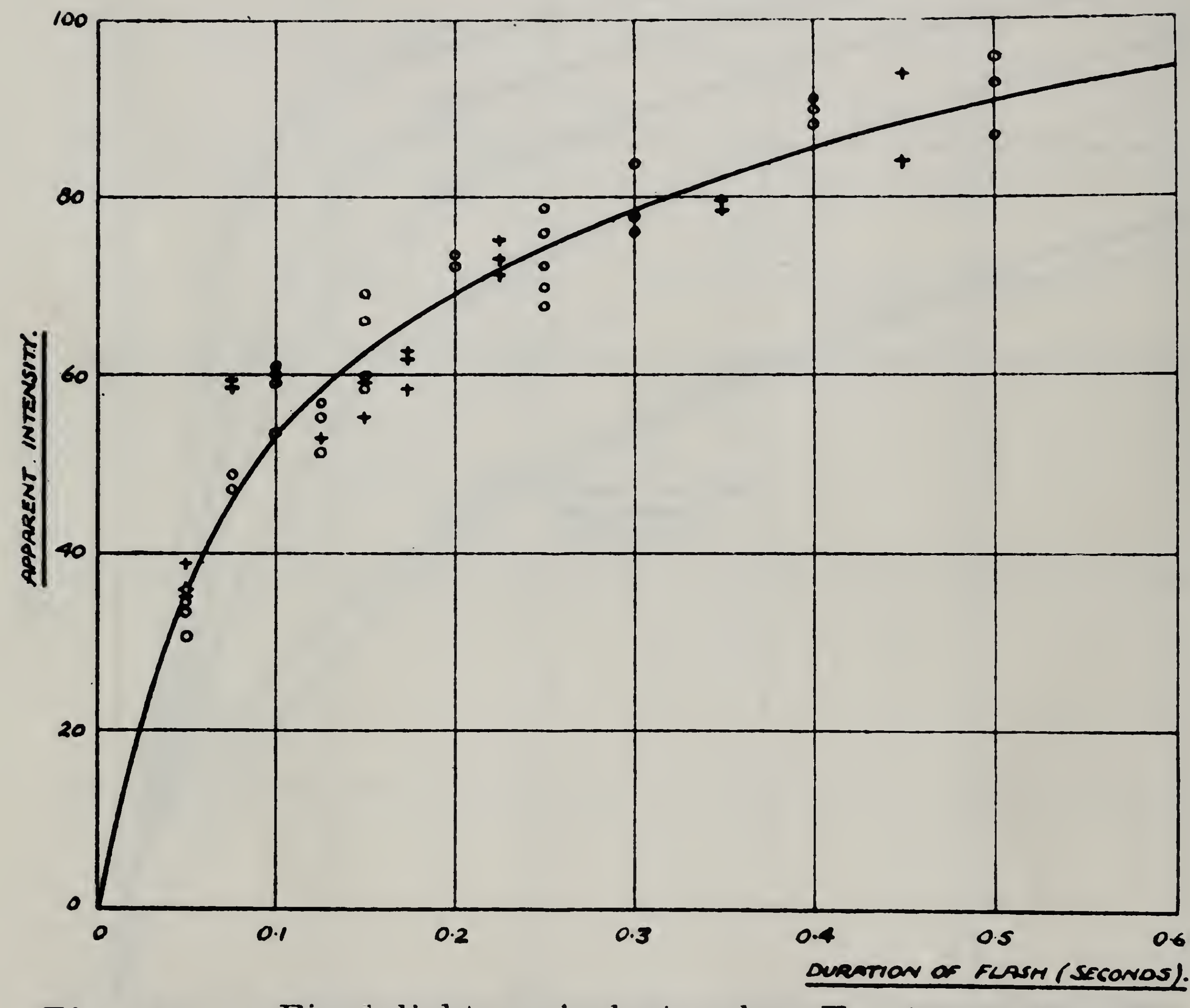


Fig. 7 —Fixed light equivalents when $E_c = 0.5$ candle-mile

o Observer B.

\+ ,, C.

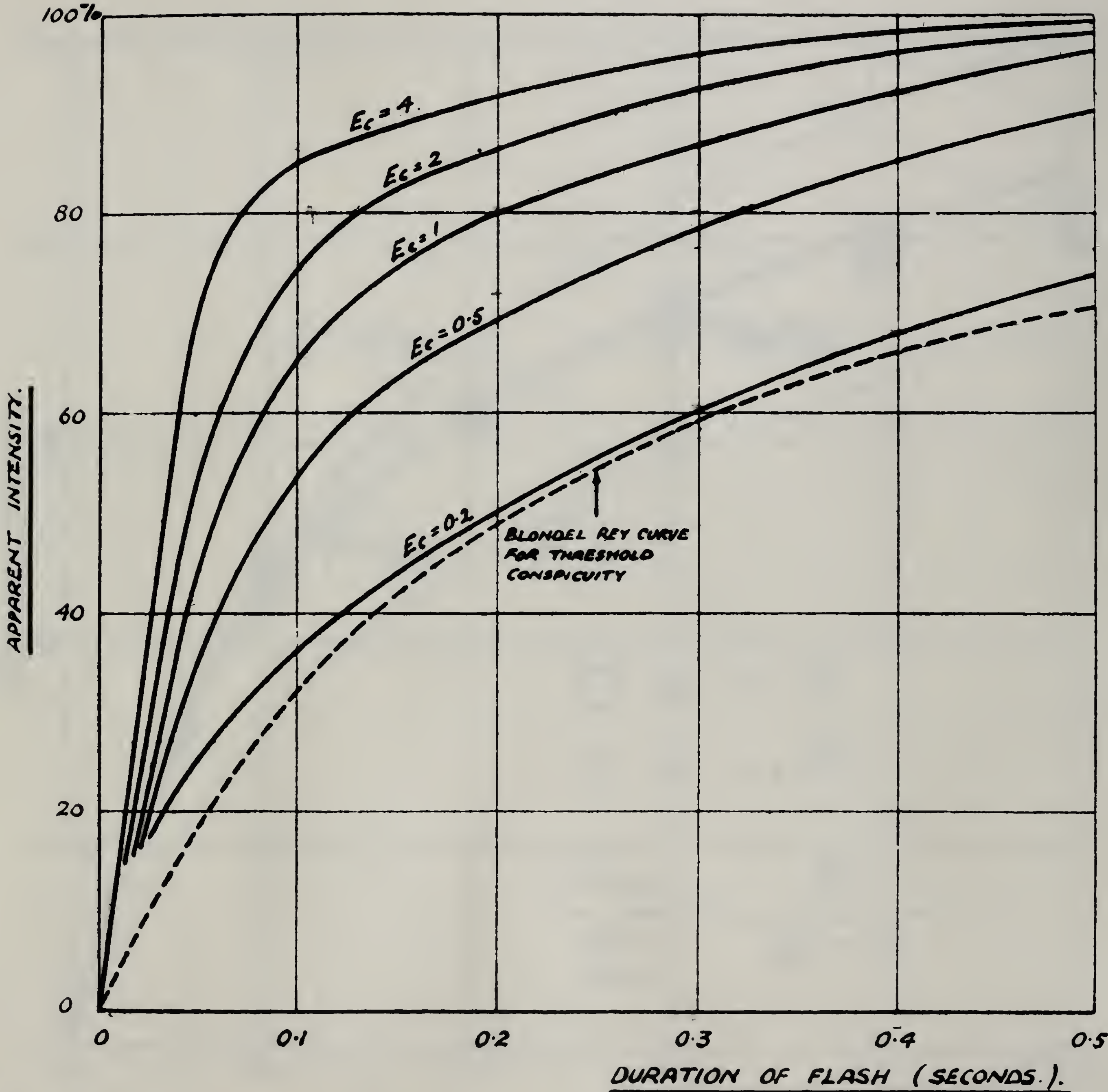
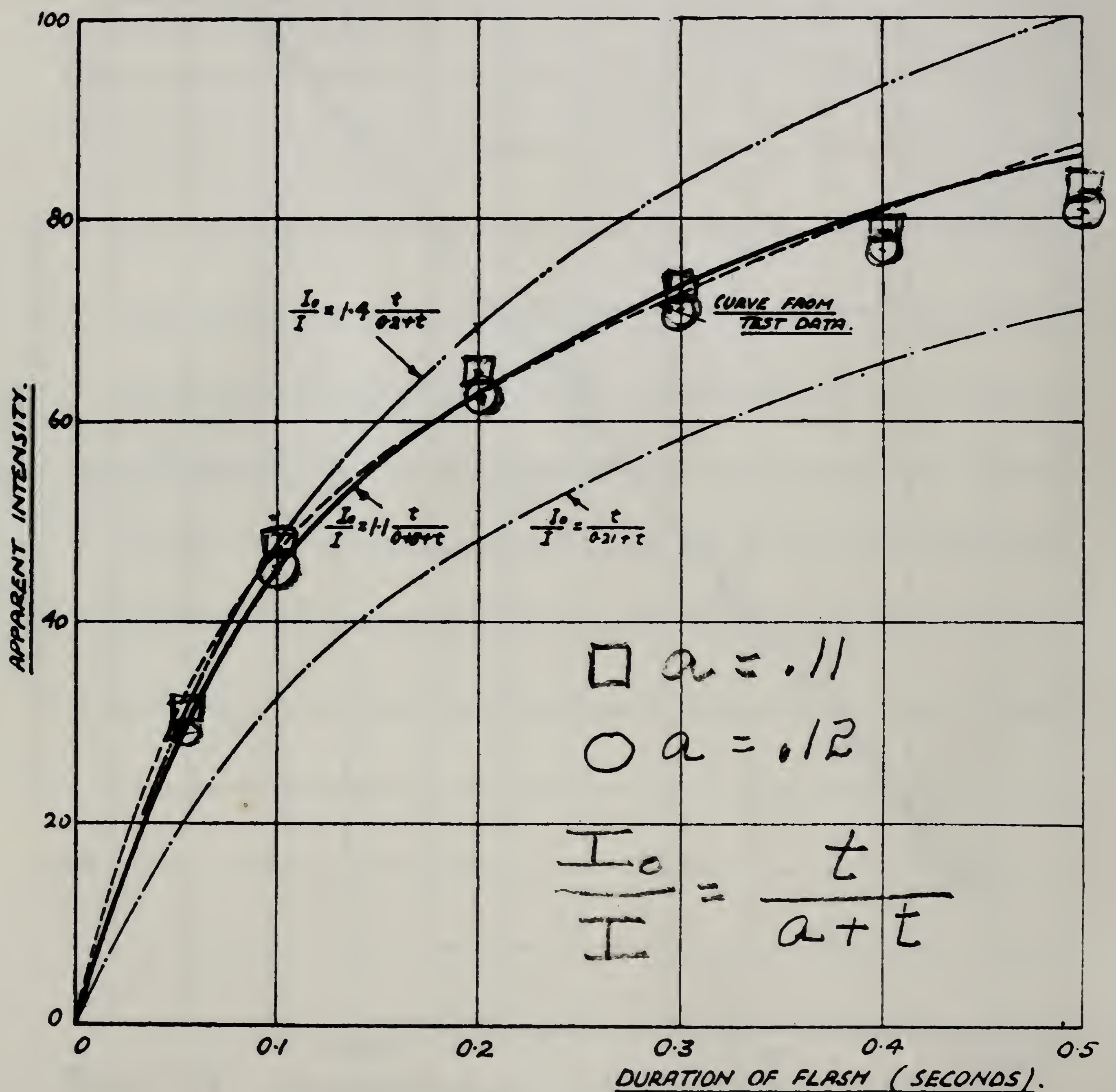


Fig. 8 —Fixed light equivalents at different conspicuity levels.



100
80
60
40
20
0
0
0·1
0·2
0·3
0·4
0·5
APPARENT INTENSITY.
DURATION OF FLASH (SECONDS).
$\frac{I_0}{I} = 1{\cdot}4\frac{t}{0{\cdot}2+t}$
CURVE FROM TEST DATA.
$\frac{I_0}{I} = 1{\cdot}1\frac{t}{0{\cdot}18+t}$
$\frac{I_0}{I} = \frac{t}{0{\cdot}21+t}$
□ a = .11
○ a = .12
$\frac{I_0}{I} = \frac{t}{a+t}$

Fig. 9

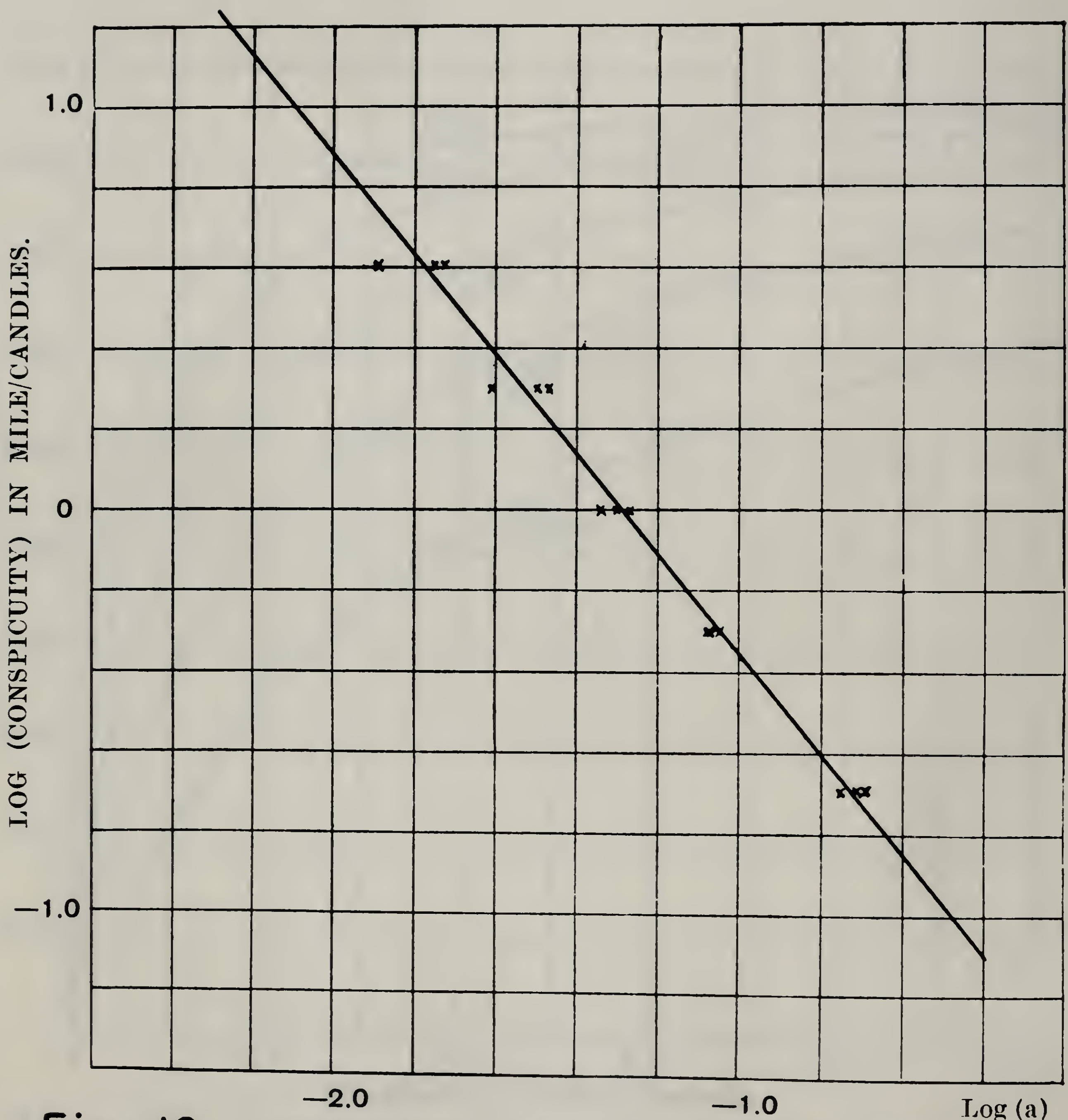


Fig. 10 –Relation between the value of (a) and Conspicuity.

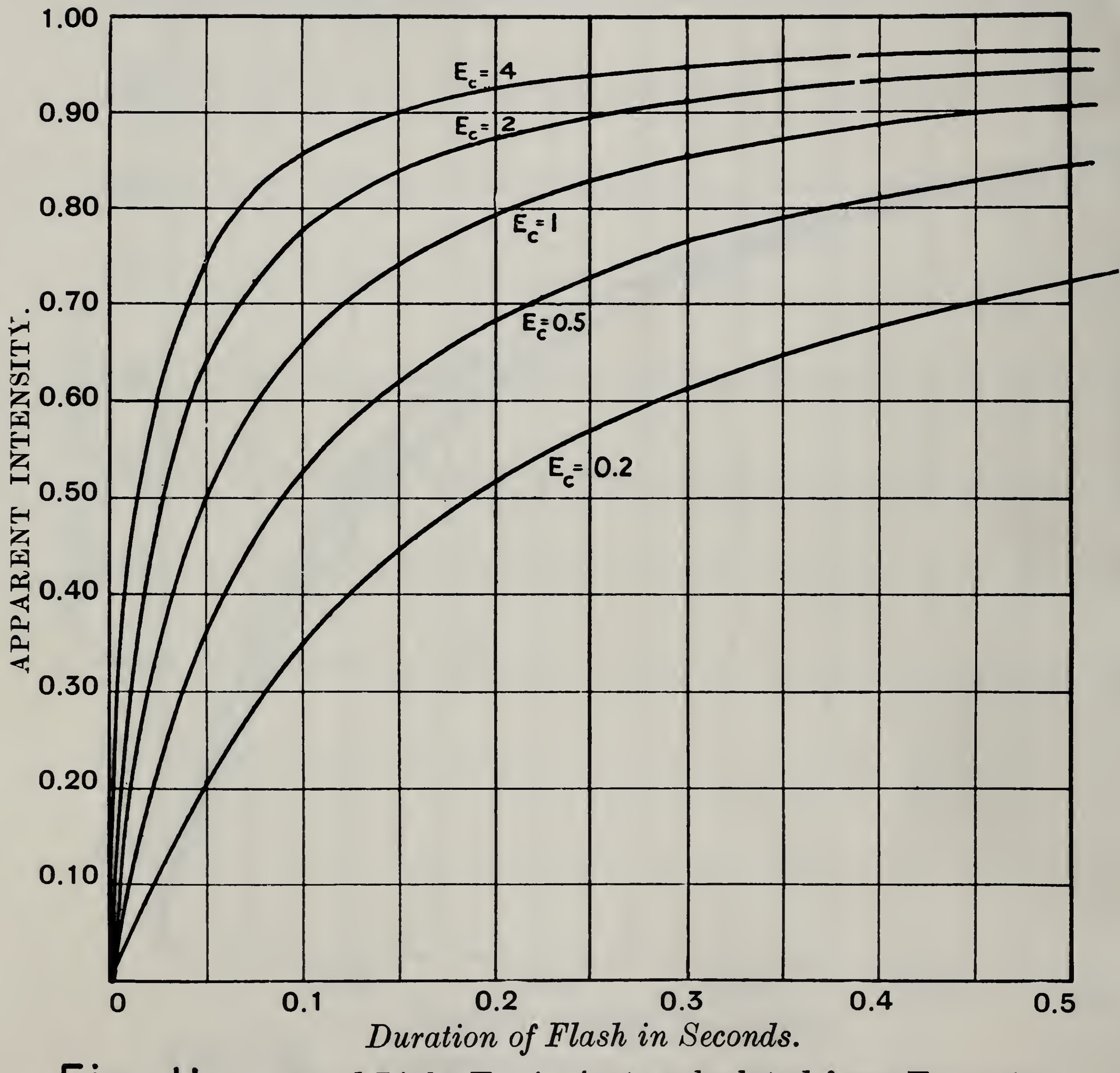


Fig. 11 —Fixed Light Equivalents calculated from Equ^{n} (3)

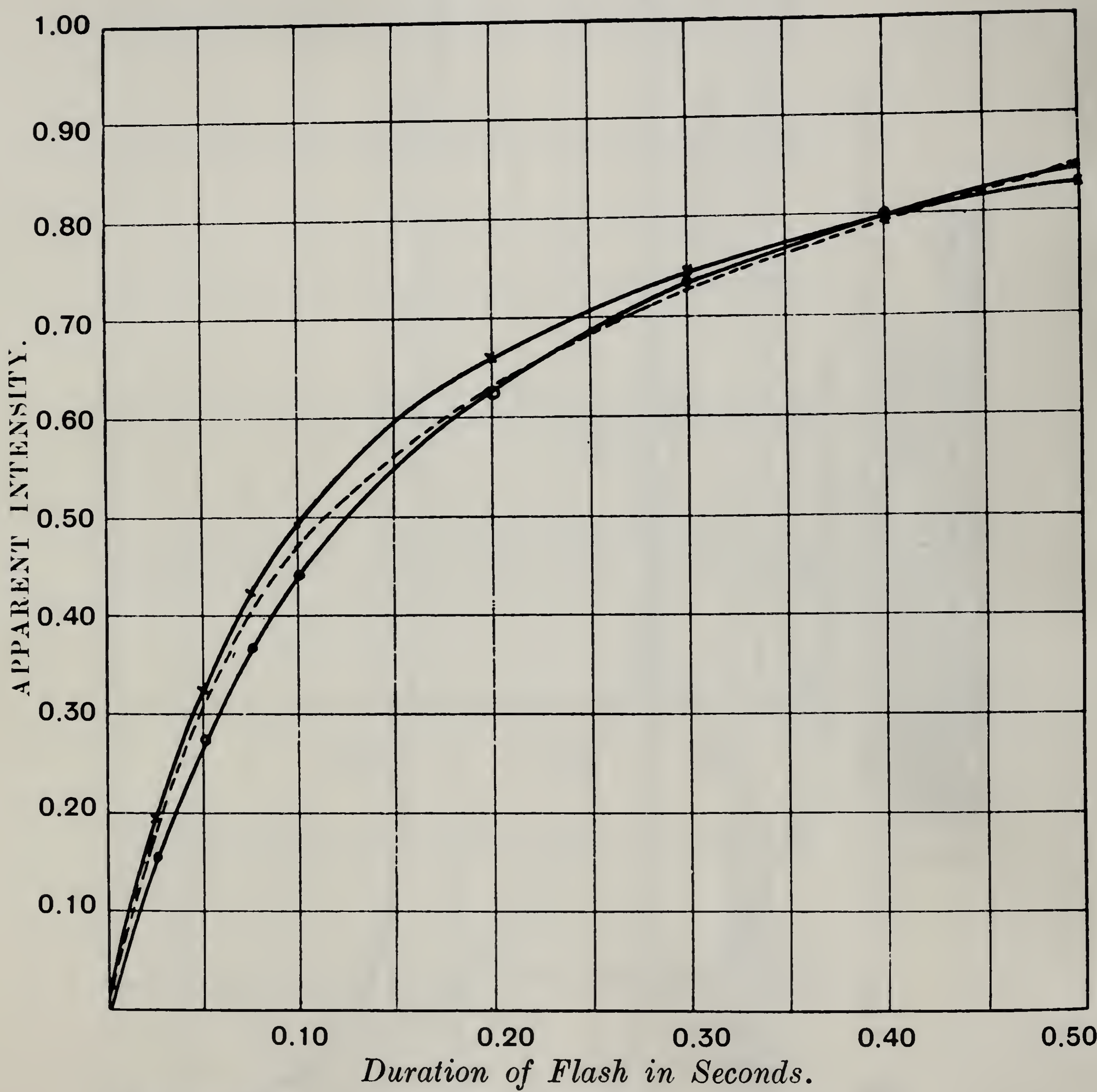


Fig. 12 –Green's Formula (1)......O ——— O

Eq^{n} (3) .... × ——— ×

Experimental .... – – – –

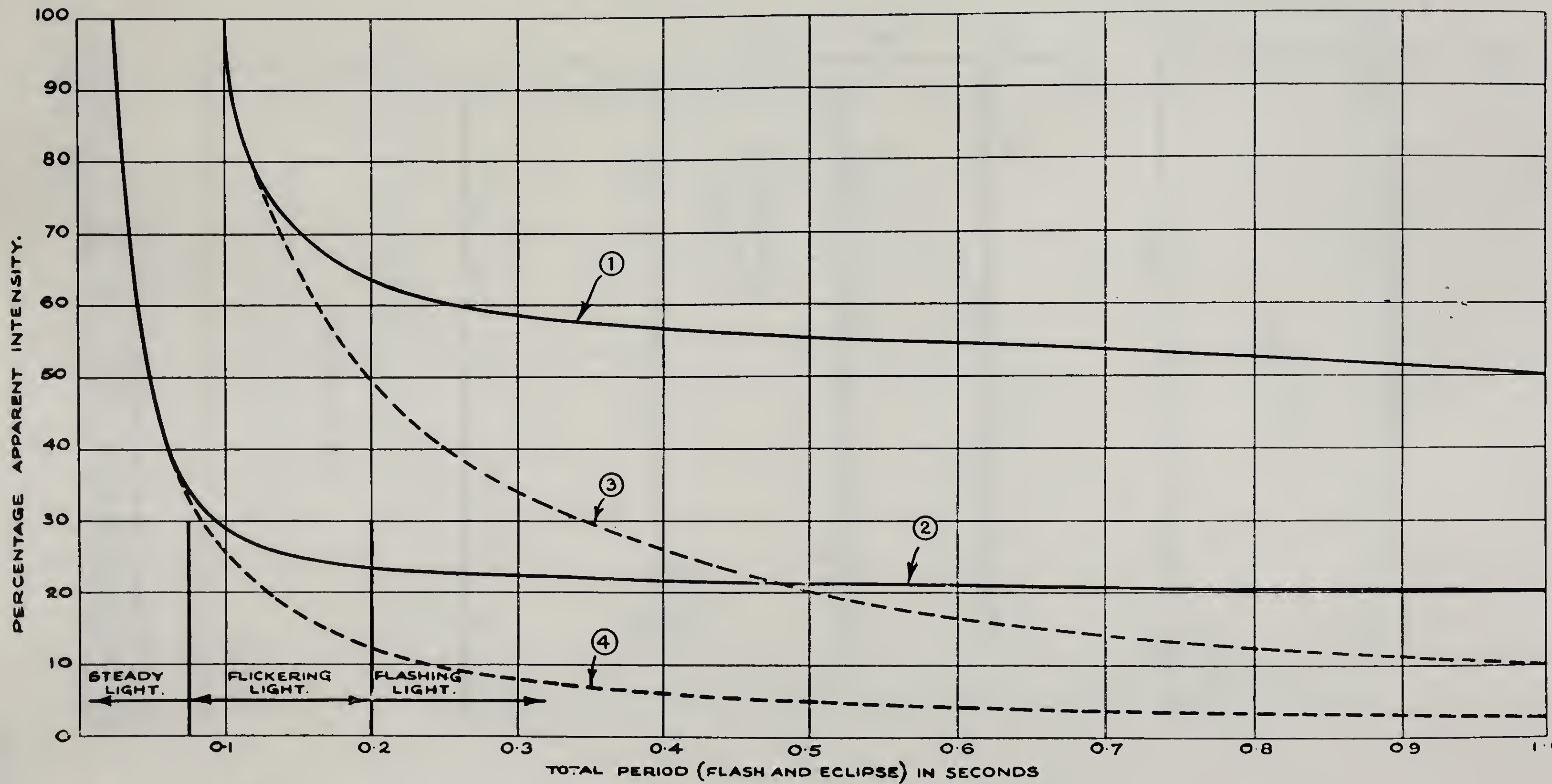
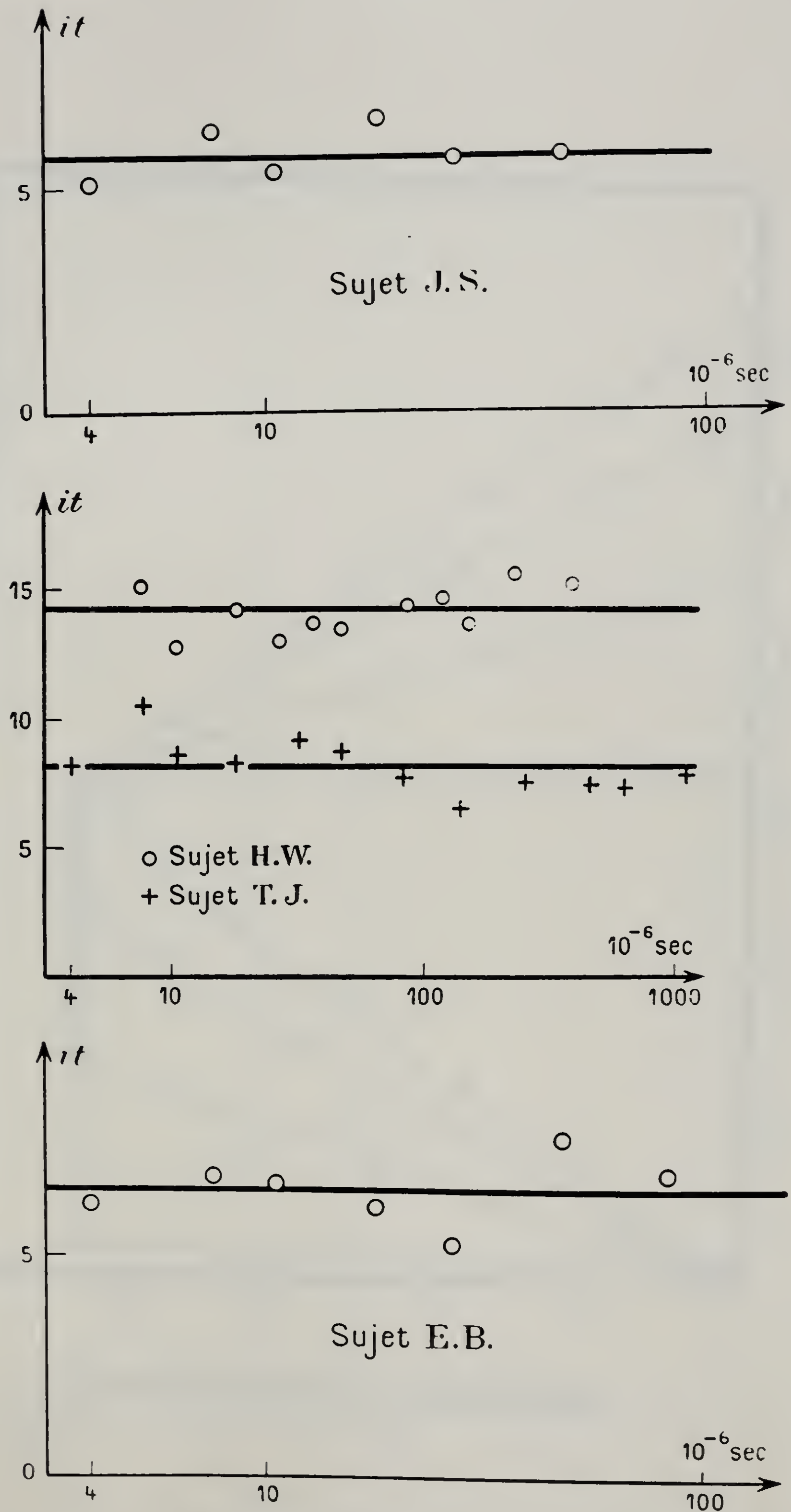


Fig. 13 Effect of varying the eclipse time of a flashing light keeping the flash duration constant.

1. Experimental curve for flash of 1/10th second.
2. Experimental curve for flash of 1/40th second.
3. Curve calculated from Talbot's Law (1/10th second flash).
4. Curve calculated from Talbot's Law (1/40th second flash).



— La quantité liminaire pour l'excitation des bâtonnets à 12° de la fovéa, en dépendance avec la durée de stimulation, entre 4.10^{-6} et $1,1.10^{-3}$ sec. Angle visuel : 1°2′.

Fig. 14

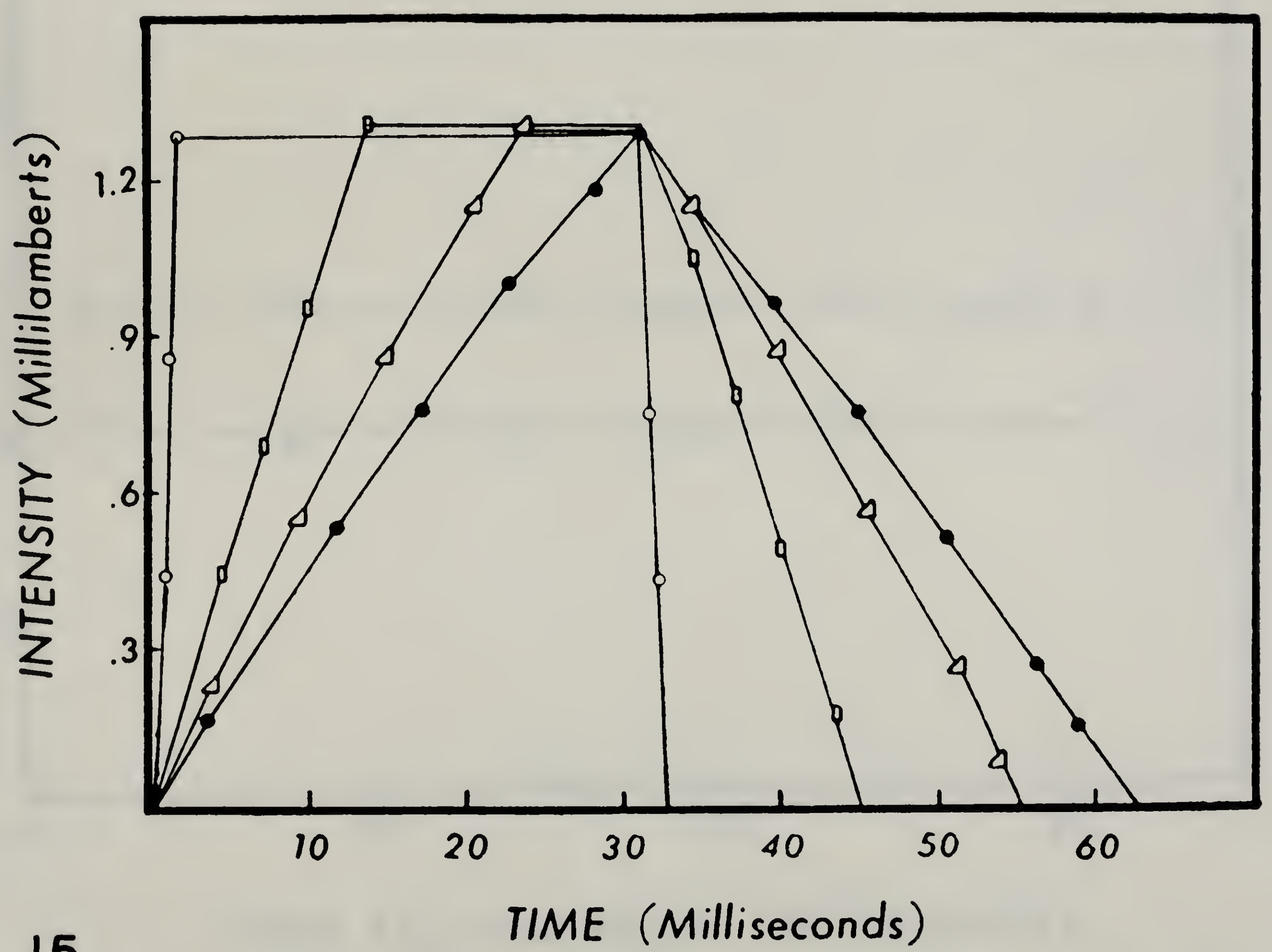


Fig. 15

Curves showing wave form of flash. Intensity is plotted against time for four of the stimulus flashes used in Experiment II.

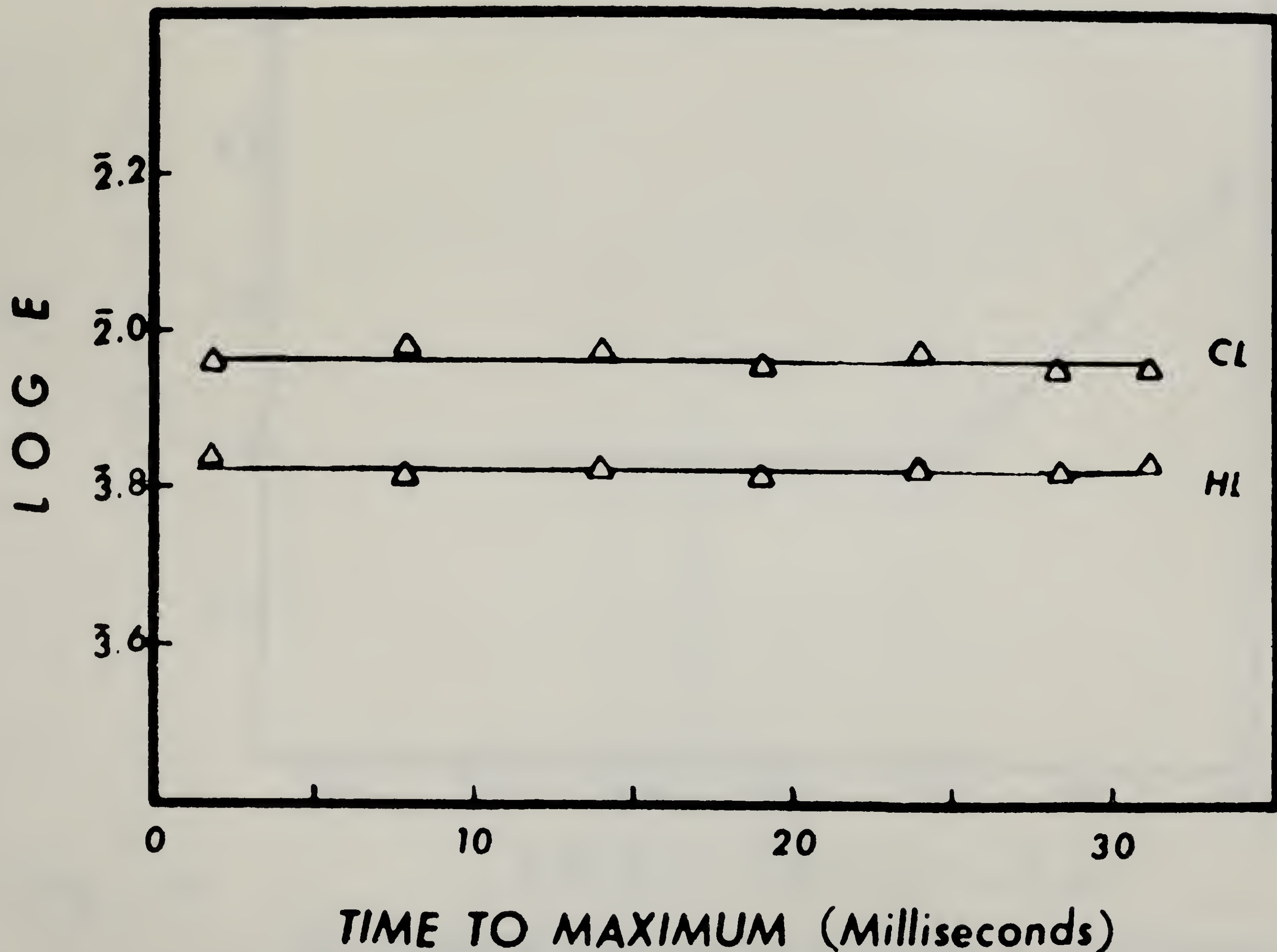
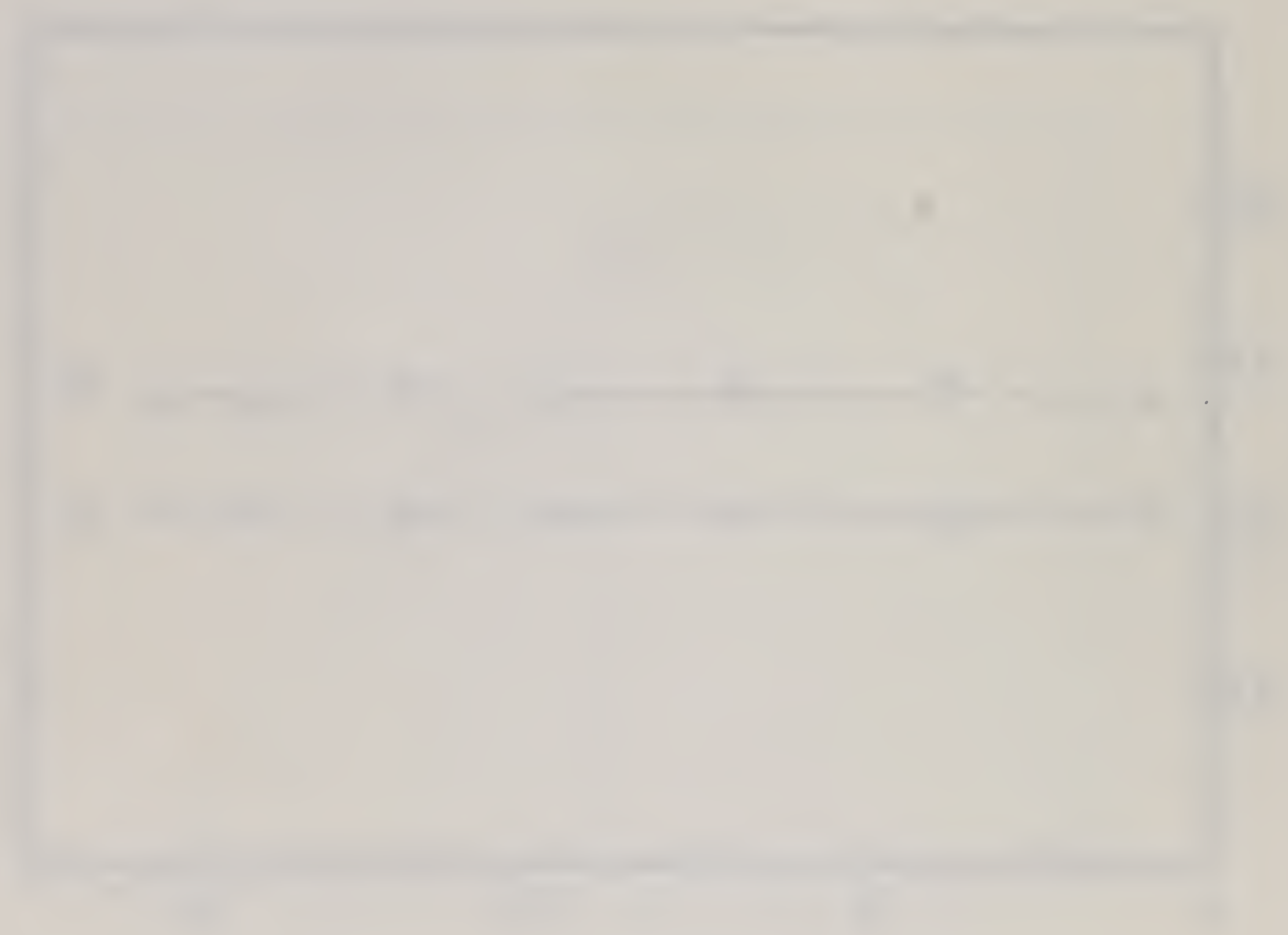


Fig. 16

Curves obtained in Experiment II showing the logarithm of the energy, E, required for the stimulus to reach maximum intensity



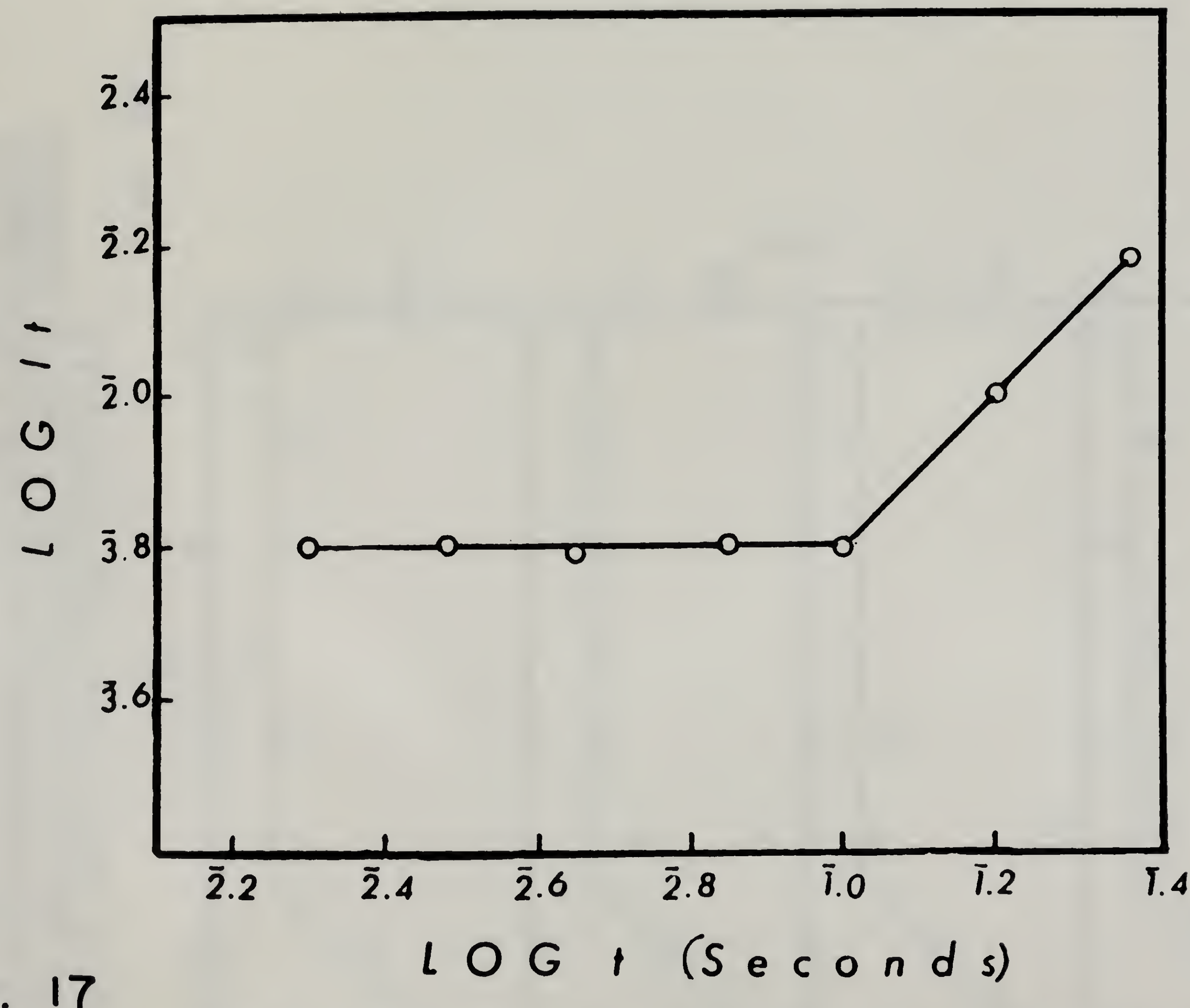


Fig. 17

The intensity-time curve obtained in Experiment I showing the logarithm of the product of intensity and duration, *It*, required for threshold as a function of the logarithm of the duration, *t*, of the stimulus.

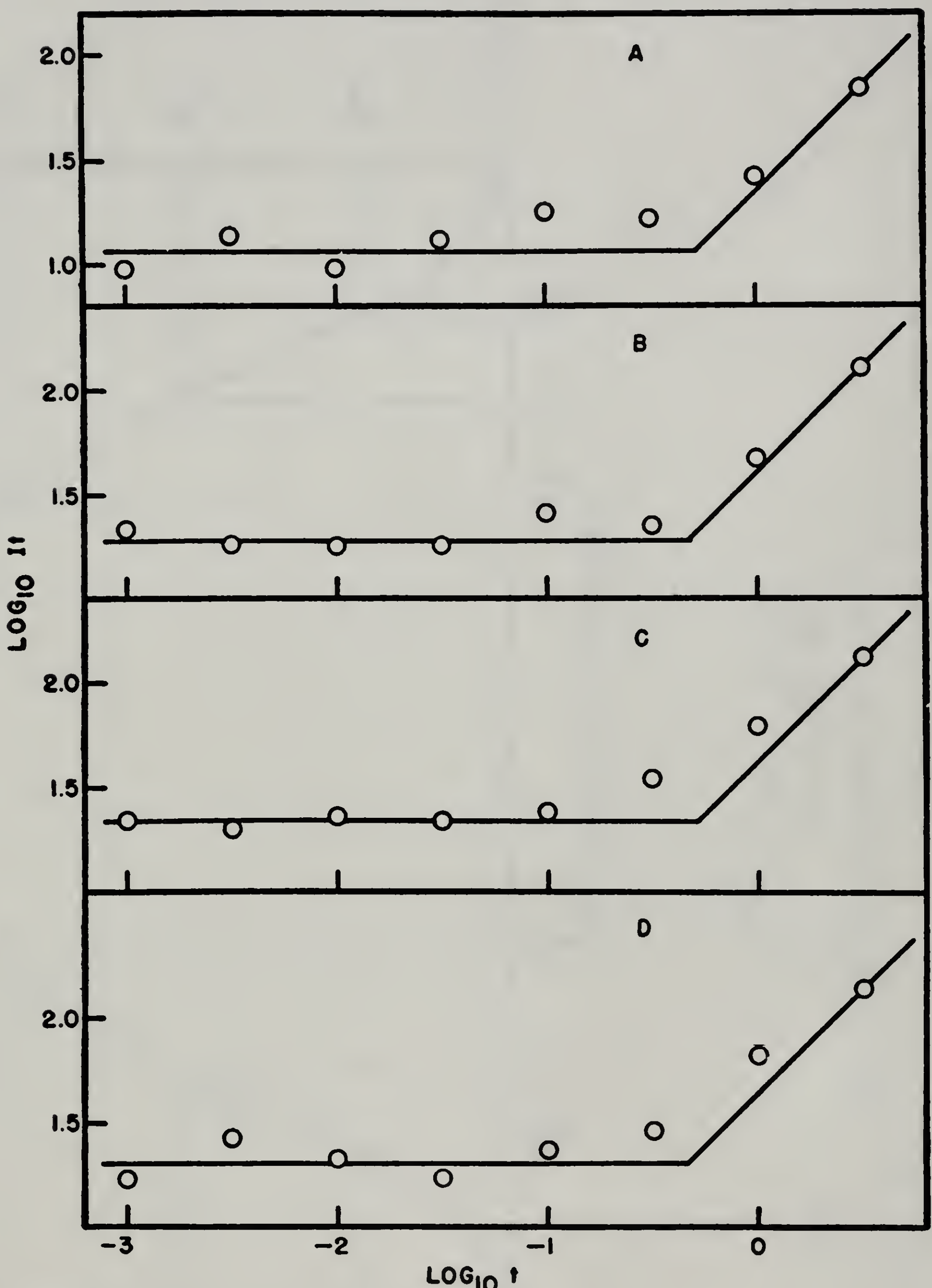
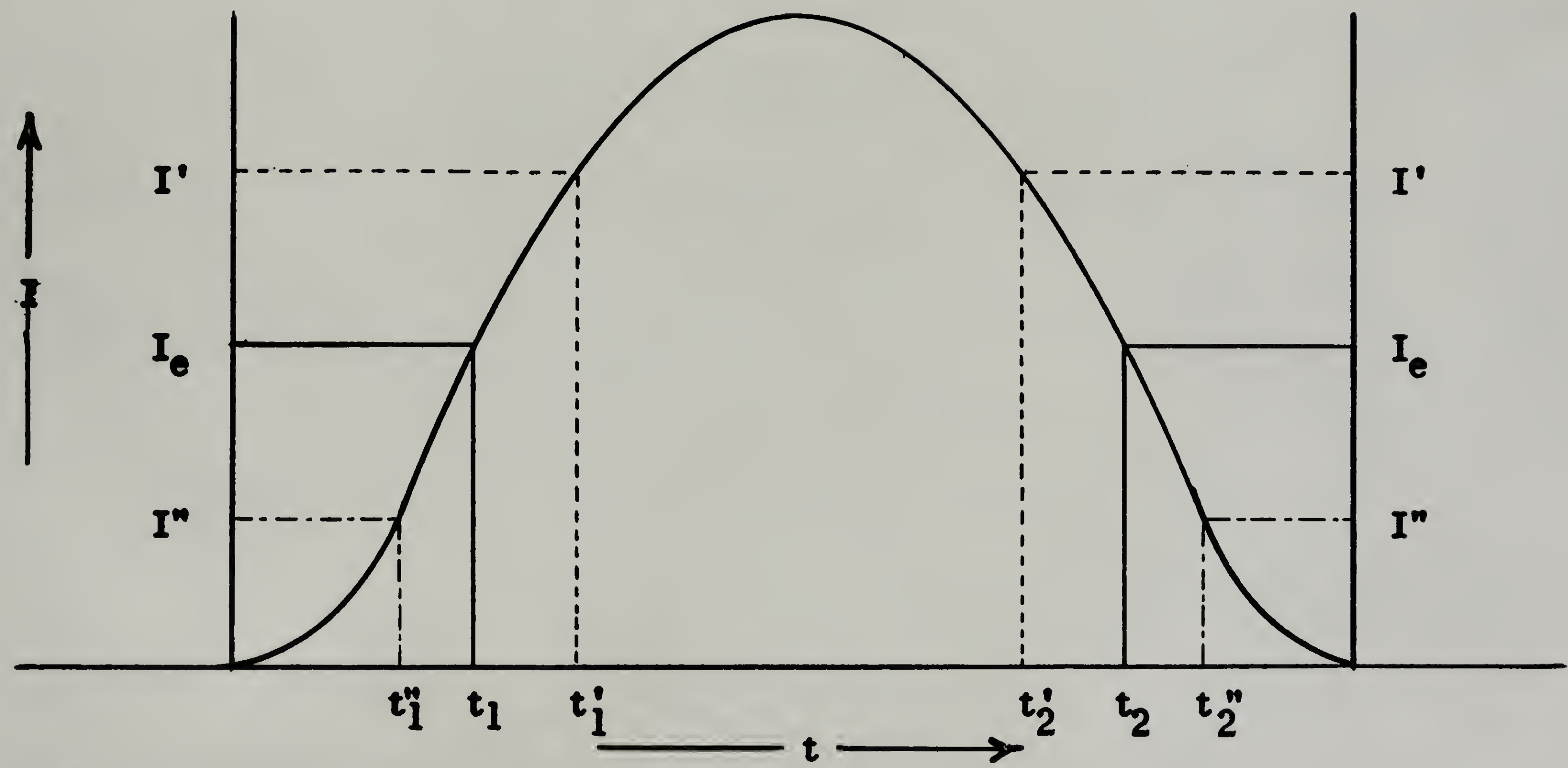


Fig. 18

The intensity-time relation based on the threshold luminance required to discriminate the direction of a velocity in a middle range of speeds for each stimulus duration. The logarithm of the product of intensity and duration (It) is plotted as a function of the logarithm of the exposure time (t). Each point is the mean of 16 measurements. The curves represent the anticipated theoretical relationships with the horizontal line $It=C$ and the inclined line with unit slope $I=K$.



$$I_e = \frac{\int_{t_1}^{t_2} I\,dt}{a + (t_2 - t_1)}$$

Fig. 19