A WAVINESS TESTER

FOR

MICA

By

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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS
THE NATIONAL BUREAU OF STANDARDS

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- Office of Basic Instrumentation
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BOULDER, COLORADO


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by

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1. INTRODUCTION

In the spring of 1956, the National Bureau of Standards was presented with the problem of evaluating the waviness of mica. Previously several methods of evaluating and measuring mica waviness have been advocated. These methods, for the most part, have been mechanical with such criteria as peak surface rise in relation to gross surface plane, and mechanical deformation as a function of the applied force.

Careful evaluation of these methods shows that they are not actually measuring surface waviness. The method of using a gate under which the sample piece of mica is passed has a basic fault. The sample is rejected if it contacts the gate. The criterion used is the maximum surface height not surface waviness. The peak of the wave of a surface is not an accurate measure of surface waviness. A sample with one smooth continuous wave which has its peak at the center and slopes gradually to each edge would be rejected by a machine of the "gate" design because the center would contact the gate. While this type of surface is indeed curved, it is not as undesirably wavy as a piece with many waves, none of which rise to the peak of the rejected piece. The gate type of machine would pass the wavy piece and reject the bowed piece, while it should have done the reverse.

Another of the advocated mechanical methods of measuring waviness is based upon the deformation of a sample as a function of the force applied. This type of compression tester has two basic faults. First, it measures the compressibility of the sample piece which may or may not be a unique function of the waviness. Second, its measurements are made by destructive means, which should be avoided if possible. The compressibility of a piece of mica is a function of the amount and location of air stains, the number and arrangement of cleavage planes, and the inherent elasticity, Young's Modulus, of the individual sample. The proper evaluation of these diverse factors would require other tests which result in a complex test method.

The destructive method of testing, while sometimes the only possible method, should be avoided if practical alternative methods can be found. There is no basic advantage to testing the elasticity of a sample if the test may cause some degree of cracking, splitting, or permanent deformation, when waviness and not compressibility, is to be measured.
Another possible mechanical method could use a very delicate profile-measuring instrument with some form of stylus, except, that by its very nature, a stylus requires an activation pressure. Therefore, a stylus is inappropriate for measuring the surface of a material like mica, since it would scratch the mica.

Consideration of these facts makes apparent the basic disadvantages to the advocated mechanical methods of measuring waviness. There are, however, several basic advantages to the use of an optical method. A light beam cannot scratch or deform a surface, hence its use will not result in a destructive means of evaluation. The same sample may be evaluated continuously by an optical method without changing its properties. Optical measurements generally yield greater precision than do mechanical measurements. The waviness tester developed at NBS is an instrument which measures waviness by an optical curvature calculation.

2. CRITERIA OF WAVINESS

2.1 Definition

As a criterion of waviness we have adopted the mathematical sense of curvature as being synonymous with waviness.

If we look at two surfaces, one having many ripples and the other having just a few smooth undulations, we could easily decide that the first surface is more wavy than the second. We will develop a mathematical expression of waviness by means of which the geometric properties of the two surfaces may be compared.

As a means of comparison we have used the parabola. If we evaluate the curvature of a parabola at each point, we find that the apex is more sharply curved than the rest of the parabola and that the curvature decreases as we move away from the apex. If we now compare our two wavy surfaces with the parabola, and try to fit some part of the parabola to each unit of the two surfaces, we must use segments near the apex to match the tight ripples and segments further from the apex to match the gradual undulations. We may then conclude that the piece with the more wavy surface is more curved than the relatively smooth piece.
2.2 Curvature Formulation

If we now wish to derive a formula by means of which the curvature, or waviness, of a given surface may be numerically measured at any point, the methods of the calculus may be used.

Let \( F(x,y) = 0 \), be the given continuous curve, and \( P(x,y) \) be the point on the given curve at which the curvature is to be evaluated. See figure 1.

Choose a 2nd point, \( Q(x + \Delta x, y + \Delta y) \) on the curve and denote the arc length from \( P \) to \( Q \) by \( S \). Let \( \theta \) denote the inclination of tangent to the curve at \( P \). Then \( \theta + \Delta \theta \) will be the inclination of the tangent to the curve at \( Q \), where \( \Delta \theta \) is the angle through which the tangent line revolves when moving from \( P \) to \( Q \) along the arc \( S \). \( \Delta \theta / \Delta S \) is now the average change in direction of the curve per unit of arc between \( P \) and \( Q \).
As $\Delta x$ (and $\Delta y$) approach zero, $\Delta S$ and $\Delta \Theta$ approach zero and $\Delta \Theta / \Delta S$ approaches $\frac{d\Theta}{dS}$ which is defined as the curvature or waviness at point P, denoted by $K$, where

$$K = \frac{d\Theta}{dS}.$$ 

2.3 Sign Convention

In the mathematical sense, a curve that is concave upward is designated as positive and a curve that is concave downward as negative. The mathematician considers a curve always concave and denotes the direction for which it is concave by means of the sign. In a practical sense, surfaces are designated concave and convex. The convention that is followed throughout this report is: concave (hollowed) surfaces are negative and convex (bulged) surfaces are positive.

To recapitulate the preceding discussion, if two surface waves are equal in height but differ in length or are different in height and equal in length, for a given arc length they differ in waviness; while if two waves have equal lengths and heights for any given arc-length all along their surface then they have equal waviness.

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A rigorous solution for curvature, $K$, in rectilinar coordinates is:

$$K = \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}} \quad \text{or} \quad -\frac{\frac{d^2x}{dy^2}}{\left[1 + \left(\frac{dx}{dy}\right)^2\right]^{\frac{3}{2}}}.$$
3. MEASUREMENT OF WAVINESS BY USE OF THE CURVATURE CALCULATOR

3.1 General description

The Curvature Calculator, or Surface-Tangent Differentiator, measures the change in angle between the tangent and the surface for a given, surface arc-length. The curvature calculator uses a small beam of light to probe the surface of the test sample and measures the spread of the reflected beam. The beam of light is focused on the surface and is distorted upon reflection if the surface is curved. The reflected beam falls on a rotating mirror and passes through a slit onto a phototube. The greater the surface distortion the greater is the spread of the reflected beam. The measure of beam spread is then a correlate of the curvature or waviness.

3.2 Source

The source is a small "light-gun" bulb. Directly in front of the bulb envelope is a small rectangular aperture, approximately 1 x 4 mm, and a piece of heat absorbing glass. An achromatic lens is placed so that the aperture and image are equidistant from the lens. The lens is of sufficient size to include the entire cone of flux from the source through the aperture. The angle of convergence, \( \alpha \), of the beam, is (to a first order approximation) related to the diameter, \( d \), of the cone of flux at the lens divided by twice the focal length, \( F \), of the lens,

\[
\alpha = \frac{d}{2F}.
\]

The beam falls on the test surface at approximately 30° from the perpendicular and has a constant spot size, \( \Delta S \). See figure 2. If the change in tangent angle is \( \Delta \Theta \) from one side of the spot to the other, then the angle of the reflected beam, \( \beta \), will have a spread of \( \alpha + 2 \Delta \Theta \), thus:

\[
\beta = \alpha + 2 \Delta \Theta.
\]

As \( \alpha \) is constant, the change \( \Delta \beta \) is,

\[
\Delta \beta = 2 \Delta \Theta.
\]

Thus, by measuring \( \Delta \beta \), \( \Delta \Theta \) can be determined for a constant \( \Delta S \) (the 1 x 4 mm. spot) which gives us the average curvature \( K \).

\[
\bar{K} = \frac{\Delta \Theta}{\Delta S}.
\]

\(^2\) General Electric Mazda Light Gun Lamp No. 1489 6.5V-2.75 A
Figure 2

Change in beam spread as a function of a change in tangent angle, $\Delta \theta$, with constant spot spread, $\Delta s$. 

$\beta = \alpha + 0.2 \Delta \theta$

- Normal
- Tangent
- Incident beam
- Reflected beam
4. BEAM SPREAD VS. BEAM DISPLACEMENT

4.1 Calibration Surfaces

To illustrate more clearly the operating principle of the Curvature Calculator and to describe a calibration technique for the instrument, we have included several photographs. Each of these photographs shows two fluorescent-lamp tubes as seen reflected by curved and flat surfaces.

The flat surfaces provide a reference standard for intercomparing the curved surfaces. The first five photographs show the fluorescent tubes reflected from the front surfaces of five lenses in optical contact with a black glass background. Each picture was made in almost the same position – a position quite close to the fluorescent tubes to magnify the phenomena present.

The geometry when viewing fluorescent tubes reflected in curved surfaces is similar but not identical to the geometry of the Curvature Calculator. Referring to figure 3A, $\psi$ is the angle subtended at the eye by the curved surface at distance $D$. $\Delta \Omega$ is the change in tangent angle across the curved surface. $A$ is the original width of the fluorescent tubes. $E$ is their apparent width as seen reflected from the curved surface. The geometry permits us to set up the equalities:

\[
\frac{E}{E+C} = \frac{A}{A+B} \quad \frac{A}{2D} = \psi \\
E + C = A \\
\frac{B}{D} = 2 \Delta \Omega
\]

By solving these equations simultaneously we can show that the width of the image in a curved surface is to the width in a flat surface as the viewing angle $\psi$ is to itself plus the change in the tangent angle, $\Delta \Omega$, thus:

\[
\frac{E}{A} = \frac{\psi}{\psi + \Delta \Omega}
\]
Careful consideration of the geometry illustrated in figure 3A will show that $\psi$ and $\Delta \Omega$ may be considered in the same units. For a given distance, $D$, and a given width, $A$, the relative image width, $A/E$, is an inverse function of the lens power. Figure 3B is a graph of the relative image width as a function of lens power. $\Delta \Omega$ is empirically established as numerically equal to $4/3 \psi$ times the power in diopters of the curved surface. The four circled points on the graph are the measured relative image widths shown in figures 4, 5, 7, and 8. The points show good agreement with the empirically established diopter conversion factor, $4/3 \psi$. 
Relative Image Width = \frac{\psi}{\psi + \Delta \Omega}

Lens Power (Diopters)

Figure 3B
Figure 4 shows the reflection in a 0.5 diopter cylindrical, plano-convex lens, with the curvature from top to bottom. The image reflected by the lens is approximately 60% of the width of the images reflected by the flat black glass with each edge displaced toward the center (the lower edge being displaced more than the upper edge).
The second lens has a greater curvature, 1.0 diopter, which makes the apparent spread smaller. The effect of this lens on the image is shown in figure 5. The image formed by the lens is approximately 40% of the width of the image reflected by the flat black glass.
The third lens has the same curvature as the second but it is a spherical lens. This gives the sides a greater displacement than the center so that the top edge bows down and the bottom edge bows up. This effect of this lens is shown in figure 6.
The fourth lens is cylindrical and has a 2.0 diopter curvature. The fifth lens is cylindrical and has a 4.0 diopter curvature. The effects of these lens are shown in figures 7 and 8, respectively. The relative image widths are 30 and 20% respectively.
4.2 Test Surfaces.

The next five pictures show typical reflections from mica test-surfaces. (Background is black-glass.)

Figure No. 9

Figure 9 shows a slightly wavy piece of mica. If we compare the spread and displacement of the reflections in the lenses with the reflections in the mica samples, we can make a qualitative evaluation of the curvature of the mica.
Figure No. 10

Figure No. 10 shows a very thin piece of slightly wavy mica, illustrating that the reflection is not a function of the thickness.
Figure No. 11

Figure 11 is a photograph of a heavily waved piece of mica illustrated by large changes in the pattern of the reflection. Note the gross discontinuity and change in spread of the image of the lower tube.
Figures 12 and 13 are photographs of the same piece of medium-wavy mica. As with the other pieces of mica, the curvature at any place may be compared with the known curvature of the lenses by comparing the spread of the fluorescent tubes in each.

There is, however, a major point illustrated by these two pictures. In figure 13 the mica has been propped up by a square of cardboard faintly visible at the top of the mica. This cardboard is about the thickness of the book of mica. By being placed under one edge, the cardboard has tilted the gross plane of the mica. This, of course, does not change the curvature of the mica but it does change the position of the reflected image of the fluorescent tubes, since the displacement of the image is a function of the angle between the tangent to the surface and the reference plane. Thus it is quite apparent that a slight change in the gross plane will greatly change the reading of an instrument designed to read the curvature of a surface as a function of the displacement of the image. The arrows, however, illustrate that the spread of a spot at a given place on the mica is almost independent of the gross plane of the sample. In each picture it is apparent that the evaluation of the spread at a given place is an excellent measurement of the waviness, relatively independent of the orientation of the gross plane of the sample (providing, of course, that the beam stays within the optics of the instrument).
A change in displacement without change in curvature may also be illustrated with known curved surfaces. Figure 14 shows a cylindrical lens viewed from the side so that the surface is straight from top to bottom but is curved from right and left. Thus there is no curvature in the direction of the fluorescent tubes, yet it is easily seen that the right and left sides of the lens cause a noticeable displacement in portions of the image.

Taking these facts into consideration, we can see that spread, and not displacement, is the best indication of curvature.
5. RECOMMENDATIONS FOR FUTURE INSTRUMENTAL DEVELOPMENT OF THE CURVATURE CALCULATOR

5.1 General Description.

Figures 15 and 16 show the Curvature Calculator as it was operated in the laboratory. Figure 16 is a close-up of the working surface. The source housing is at the upper right and the mirror housing and phototube are at the left center of the photograph. The small white spot visible on the test surface is about four times the area of the actual light-probe and lies in about the same place. The internal construction is shown schematically in figure 17. Figure 18 is a schematic composite of the recommendation for future instrumental development.

5.2 Source.

The present source is a "light-gun" lamp that operates on 6.5V at 2.75A. With proper geometric arrangement, this source has sufficient intensity to trigger the Illuminant Duty-Cycle Meter (when using a 5819 photo-multiplier tube as the probe). There are, however, alternate light sources that could be used. The best practical source would be the brightest, smallest lamp obtainable. Since source stability is not of prime importance, we could consider using an arc for the illuminant.

There is commercially available a hundred watt "concentrated-arc" lamp which has a 1.5 mm source diameter. The dimensions and intensity of this source would provide wide flexibility. However, the power supply and ventilation requirements of the arc lamp make it so difficult to handle that its use may be limited to the laboratory.

3 The 5819 is a 10 stage, head on, photomultiplier, phototube made by Radio Corporation of America.

4 Sylvania, sealed, Zirconium, "Concentrated-Arc" lamp with the power supply manufactured by Burton Manufacturing Company, 11201 West Pico Blvd., Los Angeles 64, California.
FIGURE 17
THE CURVATURE CALCULATOR

[Diagram of a curvature calculator with various components labeled, including light source, heat absorbing glass, achromatic lens, stray light shield, test specimen, felt top, variable speed motor, circular rotating table, and a 5819 RCA shielded photo tube.]
FIGURE 18

RECOMMENDATIONS FOR DEVELOPMENT OF THE CURVATURE CALCULATOR
We have also tried to get more accurate readings by blocking off part of the source lens with particular geometric patterns. Preliminary experiment has revealed that this blocking may be a very effective means of magnifying and refining our readings.

5.3 Scan Table

Our primary experiments were made on a variable speed, rotating table. We found this arrangement quite convenient since we could vary table speed and scan circumference. On a later model we used an X,Y, scanning table which traveled 5 inches in 60 seconds. The major advantage of this type of table is its constant translation speed which allows readings to be independent of sample placement. As a further improvement, we would use a light-weight table traveling at about 1 inch per second. A table of this type could be easily constructed and would increase the speed of operation of the entire machine.

It is possible to incorporate automatically-variable table-travel controls and an automatic sorter, but with present considerations of cost and flexibility, these refinements seem economically unjustifiable. If, however, a high speed, automatic production model appears advantageous, automation could be provided.

5.4 Optical Evaluation of Beam Spread.

A. 90° Beam Rotation.

As the Curvature Calculator is currently designed and operated, the beam spread of the reflected spot is evaluated in only one direction. The sample is then rotated 90° to measure its waviness in the other direction. If rotation of the specimen becomes inconvenient, the effect of rotation could be accomplished optically. The beam could be separated into two components. One component could be rotated the required 90° and then the two components could be recombined. The use of this or other optical rotation methods would depend upon a need for speed and convenience.

B. Rotating Prism.

The first working model of the Curvature Calculator used a four-sided, rotating, front-surfaced mirror to pass the reflected beam across the slit. Since constructing the laboratory model, we have found that a transmitting, rotating, octangular prism will produce the required scanning more efficiently and with a great deal more flexibility. Any future model should include such a rotating prism.
C. Slit Widths

The size of the slit through which the beam spread is evaluated was experimentally established at about 0.5 mm. However, this slit width may not be ideal for all future instrumentation. The slit width chosen is dependent on phototube sensitivity and beam intensity, while the beam spread resolution is in turn a function of the slit width. An improved Curvature Calculator might require another size slit for optimum operation.

5.5 Illuminant Duty-Cycle Meter

A. General Description.

The Illuminant Duty-Cycle Meter\textsuperscript{5} (IDCM) evaluates the spread of the reflected beam, \( \beta \), (fig. 2) by measuring the fraction of the cycle for which light falls on the phototube. The upper portion of figure 19 shows the relative beam intensity-distribution as it moves across the phototube as a function of time. Since the source intensity is nearly constant, the product of intensity and time (area under the curve) is approximately constant. Thus, if the phototube's output is connected directly to a meter, a constant reading would be obtained. The lower portion of figure 19 shows the output of the IDCM for the spot intensity-distribution above. Note that the output is a square wave triggered by the spot, and that the duration is the same as that of the spot spread. By connecting this pulsing DC current to a meter, we obtain an average voltage reading proportional to the spot-spread traverse time, \( A \), divided by the cycle time, \( B \). Referring again to fig. 19.

\[
\text{Duty-cycle of spot 1} = \frac{A}{B}
\]

\[
\text{Duty-cycle of spot 2} = \frac{A'}{B'}
\]

\textsuperscript{5} Designed by Mr. Milton L. Kuder, N.B.S., Electronic Instrumentation Section, Electricity and Electronics Division. Circuit diagram Appendix 1.
We have found that the Illuminant Duty-Cycle Meter satisfies our needs quite adequately. The rest of the Curvature Calculator, in fact, does not make full use of the IDCM's speed as the meter is designed to respond to as many as 50,000 pulses each second (reaction time of 20μ-second). This reaction time is well beyond the operational speed of the other components. Further development of the Curvature Calculator, to utilize the potential operational-speed, will enable the machine to work faster, as well as more efficiently and accurately than an inspector.

B. Meter Output of the IDCM

There are several distinct ways to indicate the output of the IDCM. The output is a high frequency (spot scan frequency) square wave whose time duration is the beam spread as seen by the phototube. When this wave is averaged by a meter movement, we have the illuminant duty-cycle of the spot, which in turn, is a direct function of sample curvature as shown before. The most accurate and convenient method of reading the duty-cycle seems to be the use of a cathode-ray oscilloscope with a persistent screen. The initial cost of an oscilloscope is within reason, but it requires periodic maintenance.

A second method might use a meter movement with some form of reset type, peak-reading indicator. But as meters of this type usually require high wattage, making one of the proper range would be difficult if not impossible. Meter movements also have the basic disadvantage of long reaction time or large needle over-travel.

A third and possibly most rewarding approach would be to use an inexpensive, recording, electro-oscillograph which would have short reaction time and small over-travel. These and other methods should be evaluated to find the most practical one.

5.6 Standards.

All testing machines of this type should have permanent standards by which the machine may be periodically calibrated as well as spot-checked at any time. Sheets of graded mica might be used as standards, but they would be susceptible to curvature changes such as those caused by splitting, scratching, cracking, or warping. Mica standards would also vary from area to area and would be difficult to duplicate. We therefore recommend that the Curvature Calculator be supplied with standards made of a durable material, like black glass, that have been ground and polished to appropriate curvatures. Thus supplied with standards two machines operated independently could provide duplicable results.
THE NATIONAL BUREAU OF STANDARDS

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