NATIONAL BUREAU OF STANDARDS REPORT

4795

SECOND EDITION AUGUST 8, 1972

SOME OBSERVATIONS ON THE USE OF UNDERGROUND RESERVOIRS AS HEAT SINKS

Report to

Office of the Chief of Engineers
Department of the Army
Washington, D. C. 20305

NBS

U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

NATIONAL BUREAU OF STANDARDS

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[1] Headquarters and Laboratories at Gaithersburg, Maryland, unless otherwise noted; mailing address Washington, D.C. 20234.
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Preface

This report is being reissued at this time because of the current national concern for energy conservation. The technical content is pertinent to those processes where available thermal energy must be stored for use at a later time. The best transfer analysis mathematics and techniques in this report were developed in regard to underground heat sinks but can be used for problems dealing with the storage of thermal energy.

SOME OBSERVATIONS ON THE USE OF UNDERGROUND RESERVOIRS AS HEAT SINKS

by

B. A. Peavy and R. S. Dill

Abstract

Disposal of waste heat from such equipment as Diesel engines and refrigerating or air conditioning machines is a practical necessity for the occupation of large underground installations and the operation of their facilities. Cooling water supply from outside may be interrupted during emergencies so the use of underground reservoirs for the disposal of waste heat is under consideration. Recirculation of the water and use of the reservoir as a heat sink have the advantages that the water is conserved for other purposes and that the total heat absorbing capacity is augmented by the capacity of the rock surrounding the reservoir to absorb heat. The objectives of the present work were (1) to find whether a reservoir provided for an existing underground installation is large enough to meet design criteria and (2) to determine for the general case the advantage, in terms of increased heat capacity, of recirculating the water and utilizing the heat absorbing capacity of the surrounding rock. An approximate analytical solution of the problem was sought and its precision was investigated by means of tests with the large reservoir and with a smaller reservoir provided for experimental purpose.

INTRODUCTION

This paper covers analytical and experimental work undertaken to establish a means of predicting, for engineering design purposes, the heat capacities of water-filled underground reservoirs. The capacity of the contained water is, of course, the product of its weight and temperature rise, but the total heat capacity of the reservoir is augmented by heat transferred to the surrounding rock and the rate of heat absorption by the surrounding rock is dependent upon the rate of temperature rise of the water in the reservoir.

It is probable that underground reservoirs will be useful as emergency heat sinks in connection with underground protective structures designed to resist attack in wartime, since heat from engines or from air conditioning equipment can be absorbed by such a reservoir if the outside water service is cut off due to enemy action or other causes.

An analytical method for predicting water temperature as a function of time and heat input rate was adapted from the literature, and experiments were conducted with an existing reservoir to corroborate the findings.

MATHEMATICAL ANALYSIS AND ASSUMPTIONS

To develop a theoretical approach to this problem, considerations are based on a hypothetical, infinitely long cylindrical cavity of radius, a, from which heat is transferred radially from its lateral surface to the surrounding material (rock) that is assumed to be boundless. The thermal properties of the surrounding material are thermal conductivity, k, and thermal diffusivity, α. The hollow cylinder contains a fluid (water) of specific heat, c', Btu per pound degree F and mass, M', pounds per foot of length of the cylinder. At any time, t, the temperature θ of the fluid is assumed to be the same as the temperature at the surface of the cylinder. A constant heat flux Q, Btu per hour per foot length of cylinder, is added to the fluid from an external source and the heat balance becomes:

$$M'c'\frac{d\theta}{dt} - k\frac{d\theta}{dr} = Q \qquad \text{at radius} = a$$

From Carslaw and Jaeger, "Conduction of Heat in Solids", p. 284, the temperature of the fluid at any time, t, is:

$$\theta = \frac{2Q}{\pi^3 k}\int_0^\infty \frac{1 - e^{-\frac{\alpha t}{a^2}y^2}}{\left[\frac{y}{G}Y_0(y) - Y_1(y)\right]^2 + \left[\frac{y}{G}J_0(y) - J_1(y)\right]^2}\,\frac{dy}{y^3}$$

The above equation, written in dimensionless groups is:

$$\frac{\theta k}{Q} = f\left(\frac{\alpha t}{a^2}, G\right) \qquad (1)$$

for which numerical solutions, evaluated by the National Bureau of Standards Computation Laboratory, appear in figures 1 and 1A. On these figures:

$$G = \frac{2\pi a^2 k}{M'c'\alpha}$$

Symbols appear at the end of this report.

Tunnels used for underground reservoirs have irregular or non-circular cross sections, and finite lengths. The equivalent radius of the tunnel is computed from the average perimeter of the actual irregular cross section. Except for cases in which the width to height ratio is large, this gives approximately the same heat flow path as the cylindrical case. For a reservoir or tunnel of finite length some heat is transferred into the rock at the closed end and into the rock surrounding the opening at the open end. This heat is not accounted for in the theoretical treatment and can often be neglected because (1) this heat provides a factor of safety in design computations, (2) this heat is negligible when the ratios of length to width or height are great, since in such a

case the surface area of the end is small in comparison to the lateral surface area, and (3) relative time for use of the reservoir is small.

EXPERIMENTAL UNDERGROUND RESERVOIR

The experimental underground reservoir is 66 feet long with an irregular cross section on the order of seven feet square. The equivalent cylindrical radius was computed to be 4.2 feet from averaging perimeter measurements taken every two feet along the length. The reservoir is surrounded by greenstone except for one end which was closed during tests by a three-foot concrete dam with a plywood diaphragm above it. The properties of the greenstone are: thermal conductivity, 1.45 Btu/hr-ft-(deg F); specific heat, 0.2 Btu/lb (deg F); and density, 166 lbs/ft^3. Water at the dam was pumped through a tank containing electric heaters and discharged back to the reservoir. Heat input to the water by electric heaters and pump was measured by a watthour meter. Temperature measurements were made at various positions in the rock, at the rock surface and in the water by means of thermocouples.

OBSERVATIONS WITH FOG-NOZZLE SPRAYING OF ROCK SURFACES ABOVE WATER LEVEL

For the duration of this test, approximately 11,390 gallons or 95,000 pounds of water was maintained in the reservoir. The water was discharged back to the reservoir through a two-inch header with fog-nozzles spaced every seven feet along the length of the reservoir. Nozzles were directed at the exposed rock surface above the water level.

Heat was added to the water at a rate of 55,000 Btu per hour for 256 hours. Figure 2 shows the time-temperature relationship for the water, rock surface and rock at depths of 2 and 5 feet. The values for the dimensionless groups of equation (1) are:

$$G = \frac{2\pi(4.2)^2 \times 1.45 \times 86}{95{,}000 \times 1.0 \times 0.0395} = 3.68$$

$$\frac{\theta k}{Q} = \frac{1.45 \times 86 \times \theta}{55{,}000} = 0.002270$$

$$\frac{\alpha t}{a^2} = \frac{0.0395t}{(4.2)^2} = 0.00224t$$

The result of substitution of experimental data in the above equations is shown on figure 3 along with the theoretical plot for G=3.5.

The experimental results agree closely with those yielded by equation (1) closely up to $\alpha t/a^2 = 0.2$. The discrepancy between the theoretical and experimental results for values of $\alpha t/a^2$ greater than 0.2, can be explained by the fact that the theoretical treatment does not take into account the heat transferred into the end of the reservoir. This can be demonstrated more clearly by comparing the heat balances as follows:

Heat Balance Based on Temperature Increases	At End of 258 Hours
heat in water	2.97×10^6 Btu
heat in rock cylinder	10.15×10^6 Btu
heat in the end	1.07×10^6 Btu
Total	14.20×10^6 Btu
Total heat added indicated by Watthour meters	14.27×10^6 Btu
Approx. Losses from System	70,000 Btu

Theoretical Heat Balance Based on Equation (1) At End of 258 Hours

heat in water	3.45×10^6 Btu
heat in rock cylinder	10.75×10^6 Btu
Total	14.2×10^6 Btu

The heat balances show that for the same amount of heat added to the system, the distribution assumed for the theoretical case was different from that which actually occurred during the experiment and that the water temperature rise was less for the experimental case. On the basis of the total amount of heat added, the theoretical heat absorbed by the water was approximately 3 percent greater than the experimental. An empirically determined relationship for values of $\alpha t/a^2$ greater than 0.2 can be written:

$$\left(\frac{\theta k}{Q}\right)_{actual} = \left(\frac{\theta k}{Q}\right)_{theoretical} - \frac{0.03kt}{M'} \qquad (2)$$

which closely follows the experimental plot beyond $\alpha t/a^2 = 0.2$.

Unaccountable heat losses from the system include water seepage into the reservoir from fissures in the rock and out the overflow pipe, and water vapor transfer and heat loss through openings above the dam. As a percentage of the total heat added these losses can be considered negligible.

OBSERVATIONS WITHOUT SPRAYING OF SURFACES ABOVE WATER LEVEL

For the duration of this test, approximately 15,200 gallons or 126,600 pounds of water was maintained in the reservoir. Circulating water was drawn from the reservoir near the bottom of the dam, passed through the heater and pump and returned to the reservoir through a long pipe which extended the length of the tunnel and delivered the water at the far end.

Heat was added to the circulating water at a rate of 64,500 Btu/hr for 258 hours and figure 4 shows the time-temperature relationship obtained for the water, rock surfaces and rock depths of 2 and 5 feet. The values for the dimensionless groups of equation (1) are:

$$G = \frac{2\pi(4.2)^2 \times 1.45 \times 86}{126,600 \times 1.0 \times 0.0395} = 2.76$$

$$\frac{\theta k}{Q} = \frac{1.45 \times 660}{64,500} = 0.00193\theta$$

$$\frac{\alpha t}{a^2} = \frac{0.0395t}{(4.2)^2} = 0.00224t$$

The result of substitution of experimental data in the above equations is shown on figure 5 along with the theoretical plots for G=2.5 and 3.0. The experimental results are slightly above those yielded by equation (1) for G=2.76 and for values of $\alpha t/a^2$ less than 0.4. For greater values of $\alpha t/a^2$ they fall below the results of equation (1). The discrepancy between the theoretical and experimental results when $\alpha t/a^2$ exceeds 0.4 can be reconciled as in the case of the spraying test by the following empirical relationship:

$$\left(\frac{\theta k}{Q}\right)_{actual} = \left(\frac{\theta k}{Q}\right)_{theoretical} - \frac{0.02kt}{M'} \qquad (3)$$

DISCUSSION AND CONCLUSIONS

Two assumptions are essential for this analytical study of the problem which cannot be met in a practical sense. They are as follows:

1. That heat is transferred radially from the lateral surface of the hollow cylinder and that the ends of the hollow cylinder are insulated from heat flow. This was not true during the experiments because there was heat flow into the ends and its effect gave lower water temperatures than predicted by theory.

2. That water is so well stirred that the temperature of the water is the same as the temperature of the surface of the reservoir. This implies that there is no thermal resistance between the water and the rock surface. In the two tests (figures 2 and 4) there was a temperature difference between the water and the surface. This effect was more evident when there was no spraying of rock surfaces above the water level.

The above two effects appear to counteract each other and the greatest effect after a period of time was shown to be heat stored in the end of the reservoir. This was borne out by lower water temperatures than predicted by the theoretical approach, as shown in the empirical equations (2) and (3).

COMPARISON OF RESULTS WITH AND WITHOUT SPRAYING OF ROCK SURFACES

Comparison between the two tests cannot be directly made because of the difference in heat input rates, but when equation (2) is subtracted from (3), the following relation results:

$$\theta_3 - \theta_2 = \frac{0.01Qt}{M'}$$

where θ_3 and θ_2 are the temperature rises for the non-spraying and spraying tests, respectively. By assuming a steady heat flux, Q, of 700 Btu/hr-ft, mass, M', of 1350 lbs/ft, and time, t, of 400 hours; the temperature difference between the two cases becomes 2F. This represents an increase of approximately 16 hours attributable to spraying water on rock surfaces above the water level.

DESIGN APPLICATIONS

The results of the experiments indicate that equation (1) and figure 1 can be used for estimating heat capacities of underground reservoirs with reasonable accuracy in most practical cases. For values of $\alpha t/a^2$ exceeding 0.2, the heat capacity of the rock surrounding the ends of a reservoir becomes appreciable and makes the total capacity greater than that predicted by equation (1) and figure 1. Correction factors are suggested in equations (2) and (3). However, underground reservoirs used as heat sinks are likely to be larger than the experimental reservoirs so that values of $\alpha t/a^2$ exceeding 0.2 are not likely in practice and the correction is expected to be unnecessary in most cases. For the sake of example, assume an equivalent radius of 10 feet and $\alpha = 0.04$ ft^2/hr, then the time necessary to reach a value of $\alpha t/a^2 = 0.2$ is 500 hours or 21 days. This is probably a considerably longer time than is anticipated for the use of a reservoir as an emergency heat sink.

For a sample problem, consider an underground reservoir with dimensions 230' long by 24' wide and 29' high holding 1.5 million gallons of water. The thermal properties of the rock are the same as that for the experiments and the initial water temperature is 50F. The value for the equivalent radius is:

$$a = \frac{2(34 + 29)}{2\pi} = 20 \text{ feet}$$

Assuming a maximum of 50F temperature rise of the water and a heat input rate to the water of 6×10^6 Btu/hr, the values for the dimensionless groups are:

$$G = \frac{2\pi (20)^2 \times 1.45 \times 230}{1,500,000 \times 0.33 \times 1.0 \times 0.0395} = 1.7$$

$$\frac{\alpha t}{a^2} = \frac{0.0395t}{(20)^2} = 9.88 \times 10^{-5}t$$

$$\frac{\theta k}{Q} = \frac{50 \times 1.45 \times 230}{6 \times 10^6} = 0.00278$$

Referring to figure 1A, $\alpha t/a^2 = 0.012$, and the time equals 121 hours or a savings of 16 hours due to heat transfer to the surrounding rock.

The above example is taken from an actual underground reservoir where results of a test on this reservoir closely substantiate the above figures.

The value of G with the reservoir completely filled is:

$$G = \frac{2\rho c}{\rho' c'}$$

with water in the reservoir:

$$G = \frac{\rho c}{31.2}$$

where ρ and c are the density and specific heat of the surrounding rock, respectively. For these experiments $\rho = 186$ lbs/ft^3 and c = 0.2 Btu/lb-°F, then G = 1.2. Therefore, optimum conditions are reached when the reservoir is completely filled with water.

SYMBOLS

Symbols and constants used for the underground reservoir are as follows:

a = radius of equivalent cylinder, feet
= 4.2 feet

c' = specific heat of fluid, Btu/lb - (deg F)
= 1.0 (water)

G = mathematical quantity, used in equation (1)

k = thermal conductivity of rock, Btu/hr - ft - (deg F)
= 1.45 (greenstone)

M' = weight of fluid per unit length of cylinder, lbs/ft.

Q = heat flux to fluid from external source, Btu/hr - ft

t = time, hrs

α = thermal diffusivity of rock, ft^2/hr
= 0.0395 for greenstone

ρc = volume specific heat of rock, Btu/ft^3 - (deg F)
= 36.7 (greenstone)

$\rho' c'$ = volume specific heat of fluid, Btu/ft^3 - (deg F)
= 62.4 (water)

θ = temperature rise of water above the initial temperature at time, t, deg F.

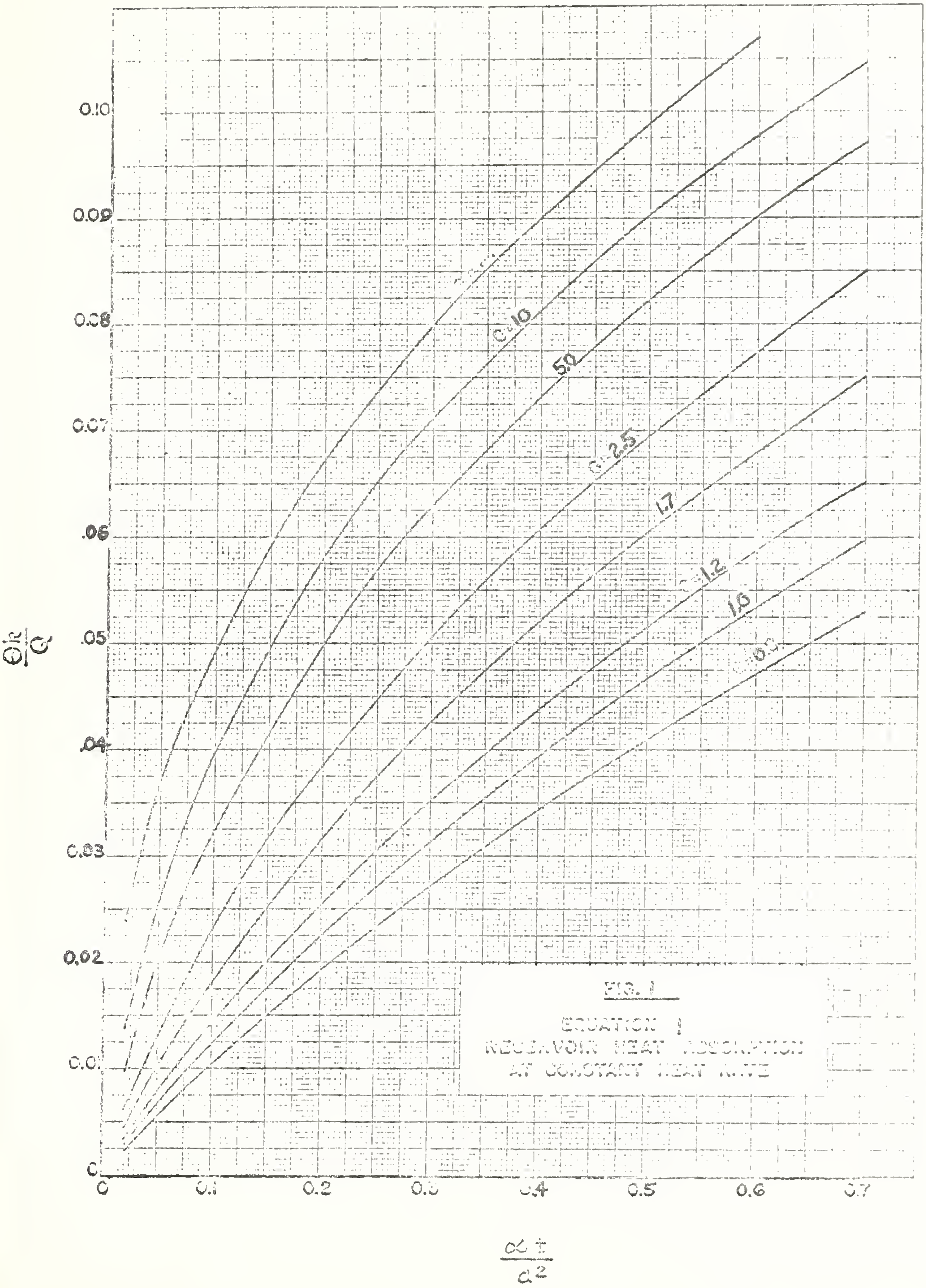

FIG. 1

EQUATION 1
RESERVOIR HEAT ABSORPTION
AT CONSTANT HEAT RATE

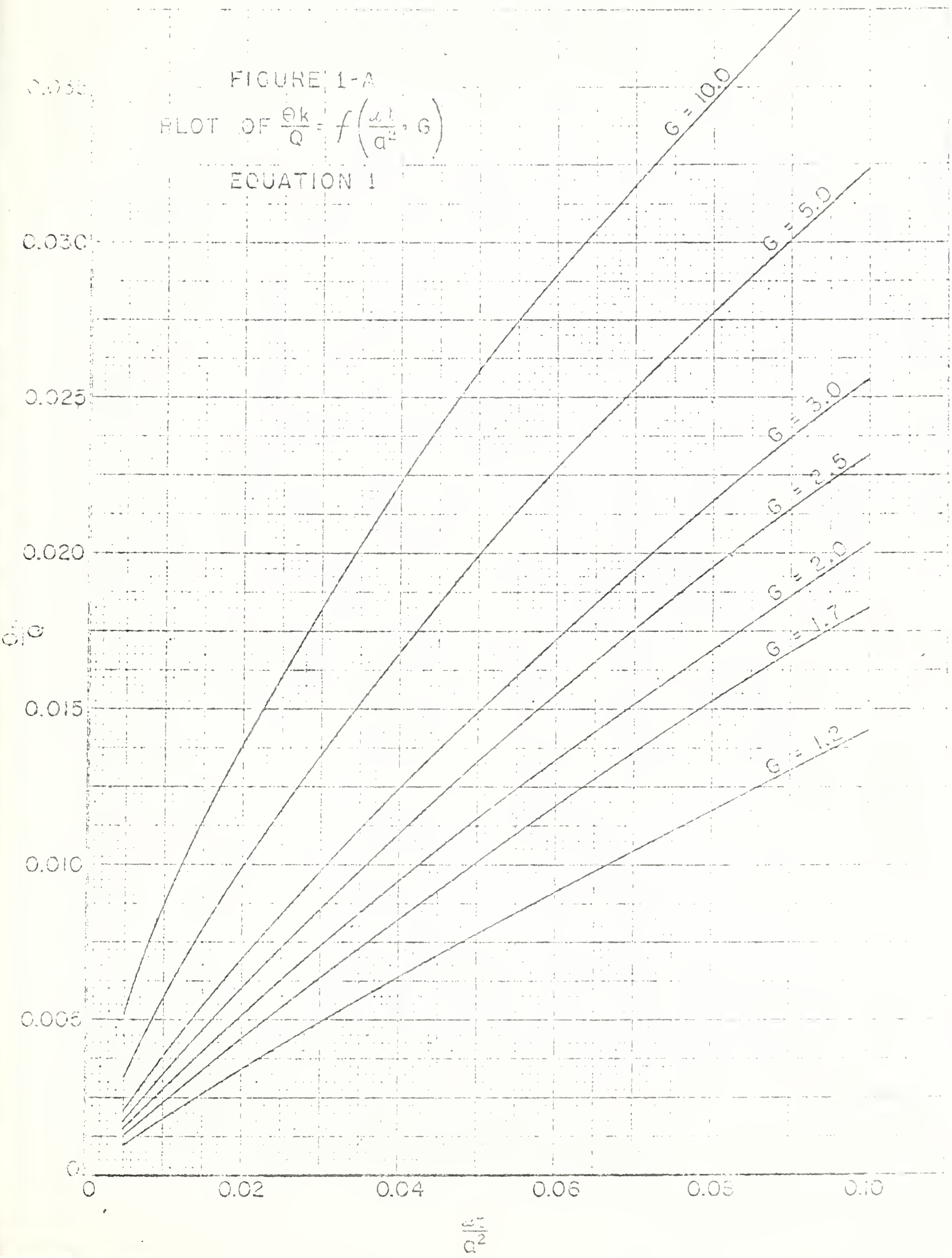
FIGURE 1-A
PLOT OF $\frac{\Theta k}{Q} = f\left(\frac{\alpha t}{a^2}, G\right)$
EQUATION 1
G = 10.0
G = 5.0
G = 3.0
G = 2.5
G = 2.0
G = 1.7
G = 1.2
0.030
0.025
0.020
0.015
0.010
0.005
0
0.02
0.04
0.06
0.08
0.10
$\frac{\alpha t}{a^2}$

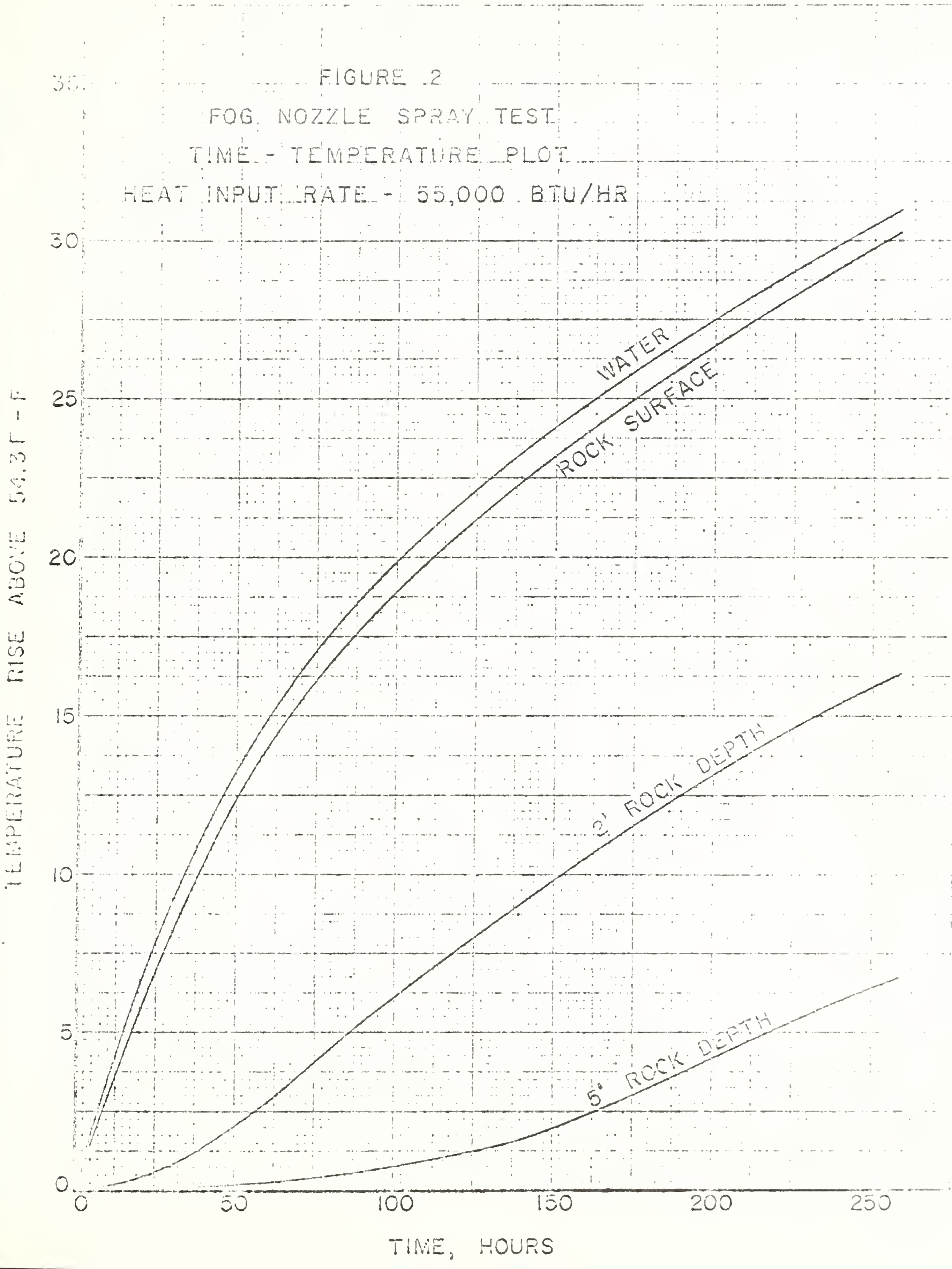

FIGURE 2
FOG NOZZLE SPRAY TEST
TIME - TEMPERATURE PLOT
HEAT INPUT RATE - 55,000 BTU/HR
WATER
ROCK SURFACE
2' ROCK DEPTH
5' ROCK DEPTH
TEMPERATURE RISE ABOVE 54.3F - F
TIME, HOURS
0
5
10
15
20
25
30
35
50
100
150
200
250

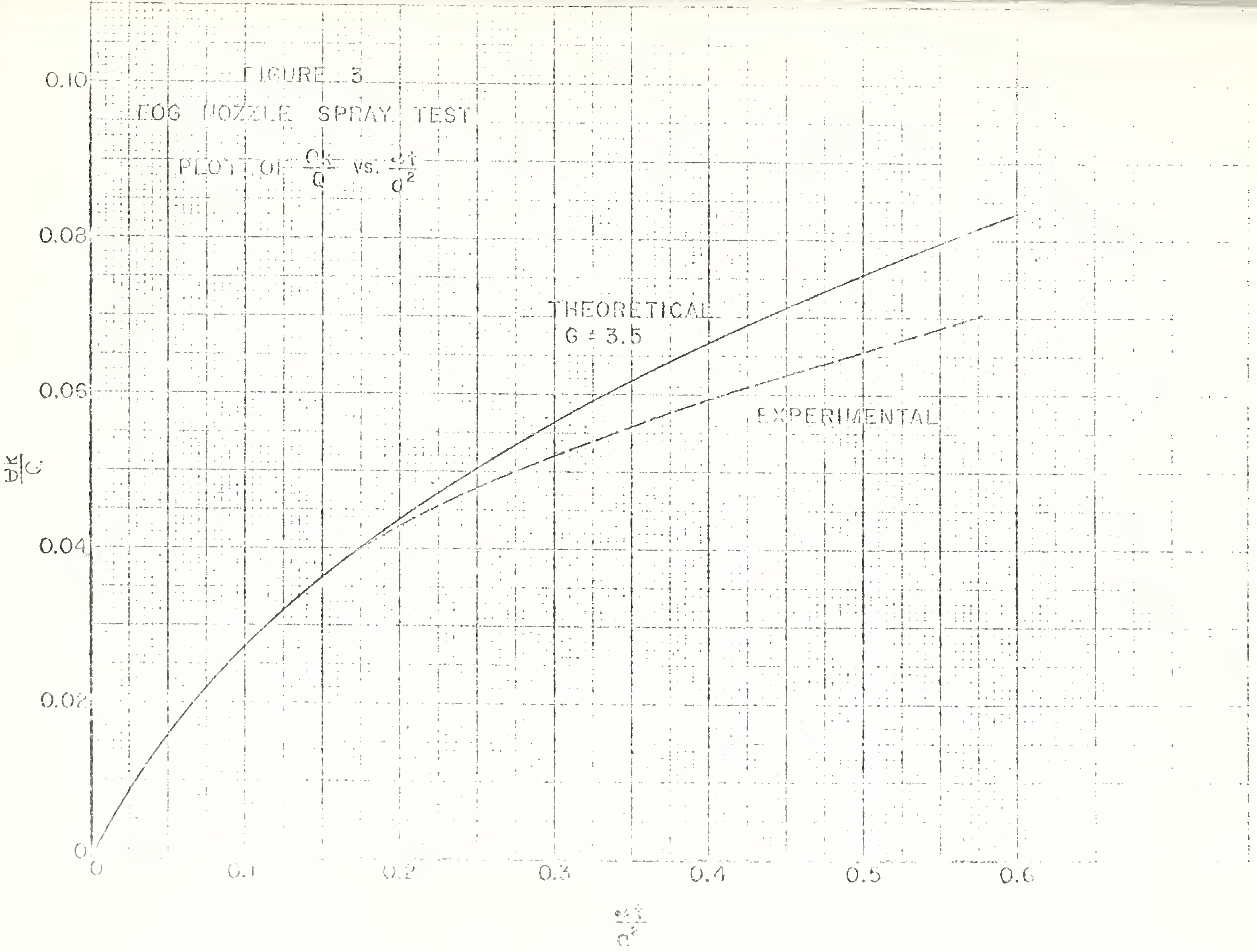
FIGURE 3
FOG NOZZLE SPRAY TEST
THEORETICAL
G = 3.5
EXPERIMENTAL
0.10
0.08
0.06
0.04
0.02
0
0
0.1
0.2
0.3
0.4
0.5
0.6

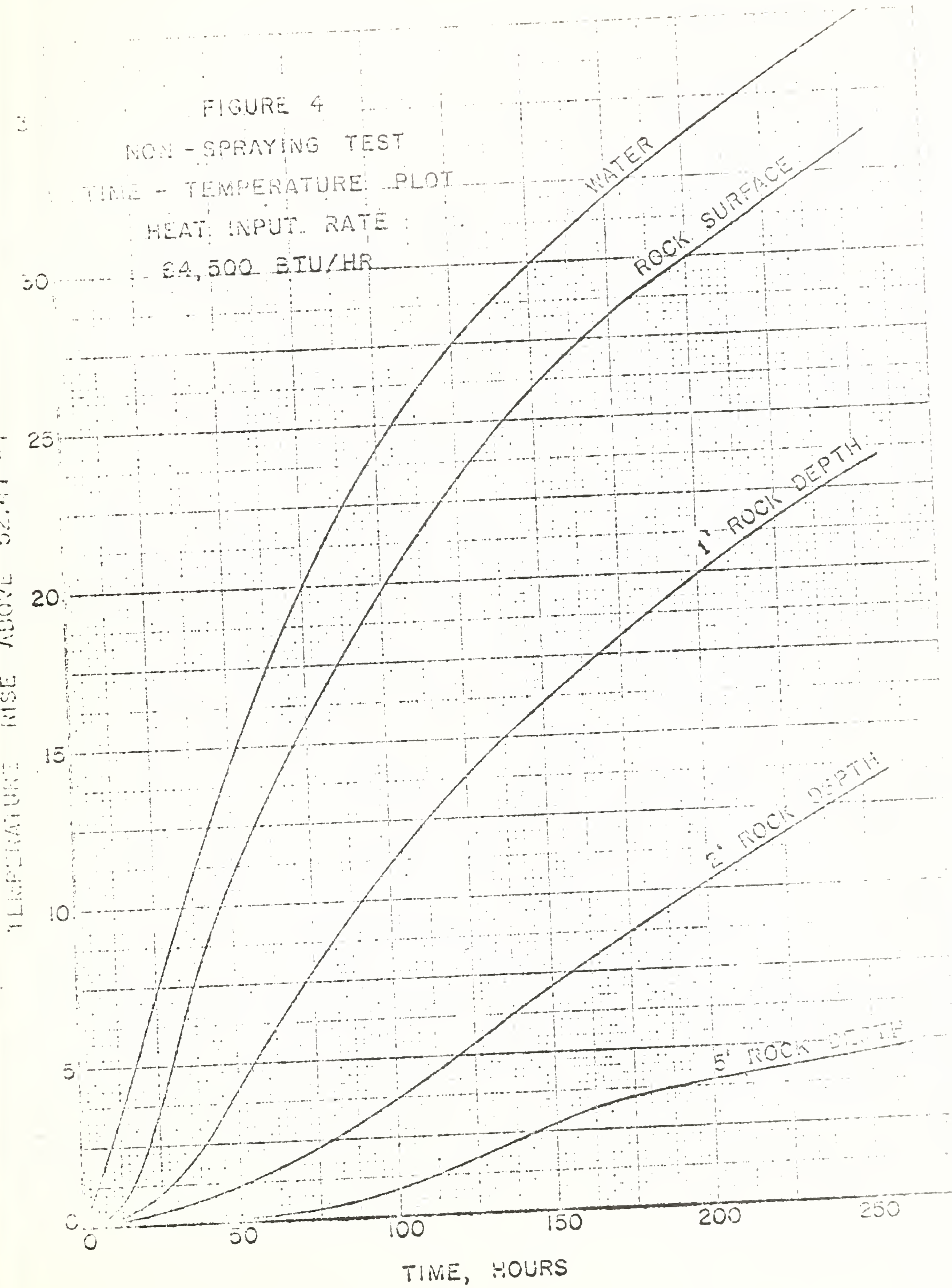
FIGURE 4
NON-SPRAYING TEST
TIME - TEMPERATURE PLOT
HEAT INPUT RATE
64,500 BTU/HR
WATER
ROCK SURFACE
1' ROCK DEPTH
2' ROCK DEPTH
5' ROCK DEPTH
TEMPERATURE RISE ABOVE 52.4T - F
TIME, HOURS
0
5
10
15
20
25
30
50
100
150
200
250

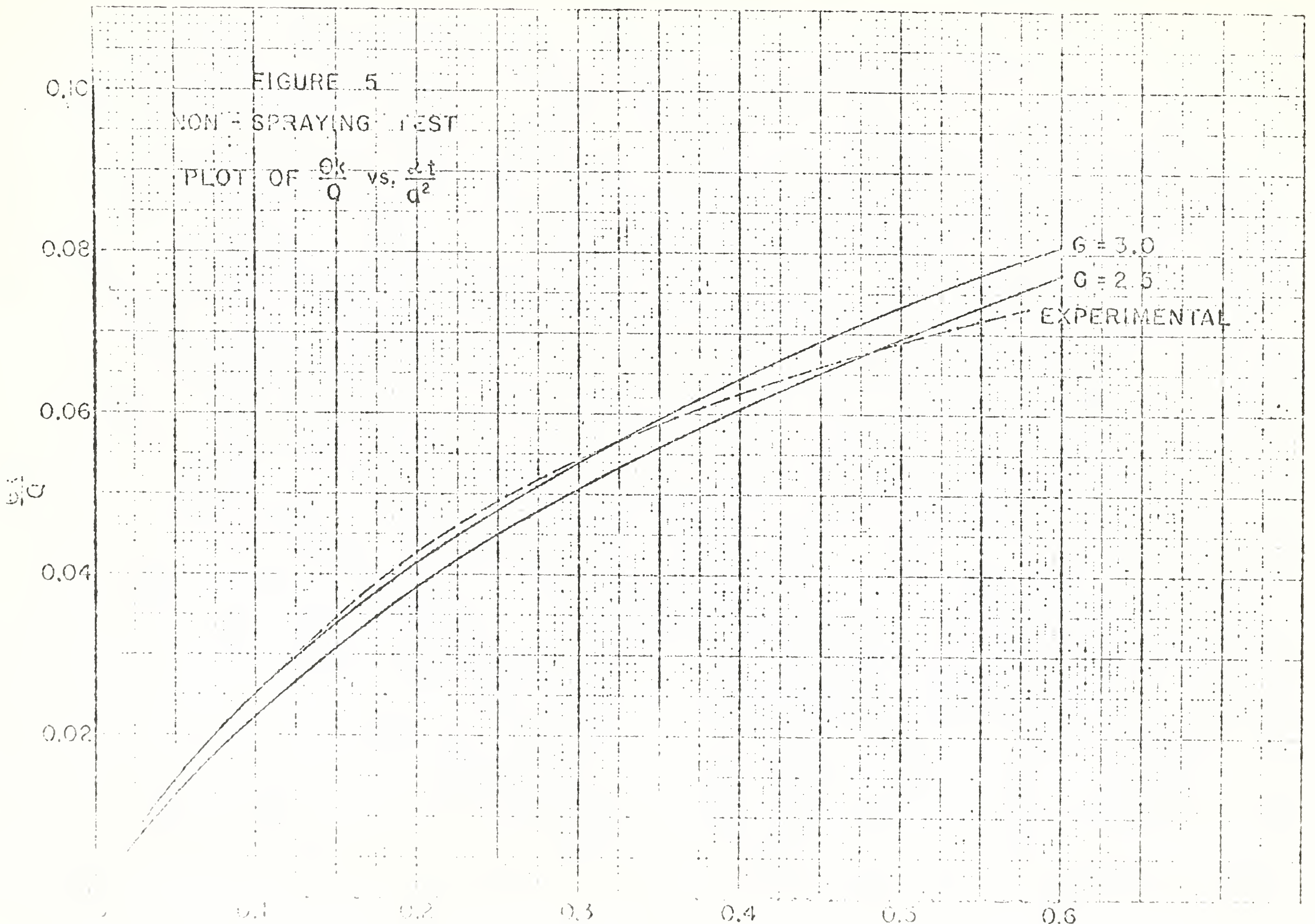
FIGURE 5
NON - SPRAYING TEST
PLOT OF Qk/Q vs. αt/a²
0.10
0.08
0.06
0.04
0.02
0.1
0.2
0.3
0.4
0.5
0.6
G = 3.0
G = 2.5
EXPERIMENTAL