NATIONAL BUREAU OF STANDARDS REPORT

4546

PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION

A Quarterly Report

October through December 1955

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section is engaged in specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside of the back cover of this report.

Electricity and Electronics. Resistance and Reactance. Electron Tubes. Electrical Instruments. Magnetic Measurements. Process Technology. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

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Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services.

Radio Propagation Engineering. Frequency Utilization Research. Tropospheric Propagation Research.

Radio Standards. High Frequency Standards. Microwave Standards.

● Office of Basic Instrumentation   ● Office of Weights and Measures

NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT

11.0

NBS REPORT

4546

PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION

October through December 1955

<NBS>

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION
October 1 through December 31, 1955

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CONTENTS

*Only unclassified projects are included in this report.

Status of Projects

December 31, 1955

1. NUMERICAL ANALYSIS

RESEARCH IN THEORIES OF DISCRETE STRUCTURES
Task 1100-10-5116/56-159

Origin: NBS
Sponsor: Office of Naval Research
Manager: O. Taussky-Todd
Full task description: July-Sept 1955 issue, p. 1

Authorized 9/30/55

Status: CONTINUED. In connection with the work of this task,
K. Goldberg and M. Newman have studied certain subgroups of the modular
group G and have determined sufficient conditions for two elements of G
to generate a free subgroup of G. A manuscript on the subject has been
prepared (see item (14) below). This work was started through a particular
problem raised by J. L. Brenner (State College of Washington), from which
O. Taussky suggested a generalization.
  The two-dimensional analog of the Wirtinger inequality was
further studied by Ky Fan, O. Taussky, and J. Todd. It was noted earlier
that in the symmetrical case (n^2 variables) equality occurs for vectors
that are linear combinations of a certain set of four n^2 vectors. In
the unsymmetrical case of mn variables, however, there is equality only
for vectors that are linear combinations of two mn-vectors.
  T. Kato and O. Taussky have given several proofs of the theorem
that a bounded operator A in Hilbert space is normal if and only if A*A-AA*
commutes with A. For finite matrices two proofs are given without the
use of characteristic roots. The theorem is established in the case when
the elements of the matrix are elements of an abstract field provided it
is formally real or that the relation $\sum a_i \bar{a}_i = 0$ implies $a_i = 0$ when $\bar{}$ is

an involution in the field. This enables the theorem to be used for
matrices whose elements are polynomials in several indeterminates, e.g.,
group matrices. Generalizations of the theorem were also found. The first
vanishing commutator of the form (A, (A,...(A,(A,A*))...)) has been examined
for some special types of finite matrices. A paper on commutators of A and
A* will appear (see (13)).
  T. Kato has proved that the (infinite) Hilbert matrix (a_{ik}),
(i,k=1,2,...), with $a_{ik} = (i+k)^{-1}$ has a positive eigenvector belonging

to the eigenvalue π, thus answering in the affirmative a question raised
by O. Taussky (Research Problem 12, Bull. Amer. Math. Soc. 60, 290 (1954)).
The methods of proof were suggested by an examination of the actual numerical
values of the eigenvectors of the finite segments of the matrix. Some of
the material examined was published by R. A. Fairthorne and J.C.P. Miller
(MTAC 3, 399-400 (1948-9)) and also by O. Taussky (Quart. J. Math., Oxford
Ser. 20, 80-83 (1949)); relevant computations had been carried out previously
at New York University Institute of Mathematical Sciences on the UNIVAC,
and special computations were carried out by K. Goldberg on SEAC. A
manuscript has been prepared (12).

Publications:

(1) On a generalization of the normal basis in abelian algebraic number fields. M. Newman and O. Taussky. To appear in Communications on Pure and Applied Mathematics (New York University).

(2) A note on group matrices. O. Taussky. To appear in the Proceedings of the American Mathematical Society.

(3) The formal power series for log ($e^x \cdot e^y$). K. Goldberg. To appear in the Duke Mathematical Journal.

(4) The number of absolute points of a correlation. A. Hoffman, M. Newman, E. Straus, and O. Taussky. To appear in the Pacific Journal of Mathematics.

(5) Some computational problems in algebraic number theory. O. Taussky. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953.

(6) Bounds and asymptotic behavior of the coefficients in the series $\log(e^x e^y)$. K. Goldberg. In manuscript.

(7) Unimodular integral circulants. O. Taussky. Appeared in Schur Memorial Volume, Math. Zeitsch. <u>63</u>, 286-289 (1955).

(8) Algebraic equations satisfied by roots of natural numbers. E. G. Straus (University of California at Los Angeles) and O. Taussky. To appear in Pacific Journal of Mathematics.

(9) Pairs of matrices with property L, II. T. S. Motzkin and O. Taussky. To appear in Transactions of the American Mathematical Society.

(10) Generation and testing of pseudo-random numbers. O. Taussky and J. Todd. Submitted to a technical journal.

(11) An algebraic proof of the isoperimetric property of polygons. Ky Fan. O. Taussky and J. Todd. J. Washington Acad. Sci. <u>45</u>, 339-342 (1955).

(12) On the Hilbert matrix. T. Kato. In manuscript.

(13) Commutators of A and A*. T. Kato and O. Taussky. To appear in the Journal of the Washington Academy of Sciences.

(14) Some free subgroups of the modular group. K. Goldberg and M. Newman. In manuscript.

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-10-1104/55-55

Origin: NBS   Authorized 8/13/54
Managers: J. Todd, P. Davis   Revised   8/29/54
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. H. A. Antosiewicz has continued his investigation of Lyapunov's second method. The concept of quasi-asymptotic stability, which means the asymptotic stability of a solution that is not (simply) stable relative to neighboring solutions, is shown to play a role similar to the quasi-uniform asymptotic stability introduced by J. Massera. It is distinct from the concept of asymptotic stability in Lyapunov's sense but implies it for certain classes of differential equations. A number of converses to the classical Lyapunov theorems are proved. (See publication (14) below.)

P. Davis and K. Fan have continued their study of certain inequalities in normed linear spaces and their application to problems of interpolation and approximation.

Completeness of order p has been introduced and studied. A sequence $\{f_n\}$ is called complete of order p, $p \geqslant 1$, if $\sum_{n=1}^{\infty} |\emptyset(f_n)|^p < \infty$ implies $\emptyset = 0$. Making use of the space $X \oplus \ell^p$, that is, the direct sum of X with the sequential Banach space ℓ^p, it has been found possible to derive necessary and sufficient conditions in order that $\{f_n\}$ be complete of order p. Such conditions again appear as statements on the possibility of approximation with

coefficients restricted in size. New theorems of Paley-Wiener type have also been obtained.

P. Davis has been conducting numerical experiments on SEAC for the determination of the least p-th power approximation ($p \geqslant 2$). The method is an iterative one that makes use of an orthonormalizing code ($p = 2$), already developed.

The test program for pseudo random numbers prepared by T. Kato is being elaborated and exploited by J. M. Cameron.

J. Todd has been preparing a survey of various topics belonging to the so-called constructive theory of functions that are of interest or of actual or potential use in numerical analysis. Among these are the theorems of Weierstrass, Bernstein, the Markoffs, Chebyshev, Zolotareff.

K. Goldberg has continued his study of generating functions and their induced convolutions. Specific attention has been given to the generating function $\sum_{s=1}^{\infty} f(sx)$ and the generalized convolution $\sum f_1(x_1)...f_n(x_n)$ summed for all positive integers x_i such that $(x_1,...,x_n) = x$ for a fixed n. Several identities have been found when the functions considered are multiplicative. The formula $f(m)f(n) = f(mn/(m,n))f((m,n))$, $f(1) = 1$, which is an equivalent definition for multiplicity, is basic to these considerations.

Publications:
(1) Stable systems of differential equations with integrable perturbation term. H. Antosiewicz. To appear in the Journal of the London Mathematical Society.
(2) Computation of vibration modes and frequencies on SEAC. W. Cahill and S. Levy (NBS 6.4). J. Aeronautical Sci. 22, 837-843 (1955).
(3) On a problem in the theory of mechanical quadratures. P. Davis. Pac. J. Math. 5, Suppl. 1, 669-674 (1955).
(4) On the Lerch zeta function. F. Oberhettinger. To appear in the Pacific Journal of Mathematics.
(5) A subroutine for computations with rational numbers. P. Henrici. To appear in the Journal of the Association for Computing Machinery.
(6) A direct approach to the problem of stability in the numerical solution of partial differential equations. J. Todd. In manuscript.
(7) Some Monte Carlo experiments for computing multiple integrals. P. Davis and P. Rabinowitz. To appear in Mathematical Tables and Other Aids to Computation.
(8) Abscissas and weights for Gaussian quadratures of high order. P. Davis and P. Rabinowitz. To appear in the Journal of Research (NBS).
(9) Numerical experiments in potential theory using orthonormal functions. P. Davis and P. Rabinowitz. To appear in the Journal of the Washington Academy of Sciences.
(10) Inequalities and approximation in function spaces. P. Davis and K. Fan. In manuscript.
(11) A matrix with real characteristic roots. K. Goldberg. To appear in Journal of Research (NBS).
(12) Note on bounds for determinants. E. V. Haynsworth. In manuscript.
(13) On asymptotic stability. H. A. Antosiewicz. In manuscript.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO
NUMERICAL ANALYSIS
Task 1101-10-5116/55-56

Origin: NBS   Authorized 8/13/54
Sponsor: Office of Naval Research   Revised   8/29/54
Managers: J. Todd, M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. A numerical computation for the determination
of the lowest frequency ν of a free square plate has been carried out on SEAC
by T. Kato in collaboration with M. Newman. The computation is based on a
formula giving upper and lower bounds of eigenvalues of positive definite
operators (see T. Kato, Math. Ann. $\underline{126}$, 253-262 (1953)). It makes use of
many of the basic codes for matrix operations that were prepared by M. Newman,
some of which were described in a previous report and some below, and a
pair of trial functions introduced by Y. Nakata and H. Fujita were used
containing 6 and 8 adjustable parameters. The result, in a suitable unit,
is $3.5257 < \nu < 3.5292$. This is a rigorous estimate.

Some inequalities have been deduced by T. Kato for estimating the
dominant eigenvalue of a positive matrix. It is believed that these improve
earlier results of various authors.

The following new programs have been prepared by M. Newman:

MI2. External floating point matrix inversion and solution of equations
routine. Computes $A^{-1}B$, where A,B are given matrices of dimensions
not exceeding 384. Solving time for two sets of 40 equations in 40
unknowns is approximately one hour. Inversion of a 100x100 matrix
would require approximately 20 hours. A large part of the time is tape
running time. Data is in standard form.

MO3. General purpose output routine for all external matrix codes in this
collection.

MM4. External floating point multiplication routine. Computes the product
AB, where A,B are given matrices (not necessarily square) of dimensions
not exceeding 256.

MA2. External floating point addition routine. Computes $\alpha A + \beta B$, where
α, β are scalars and A,B are given matrices (not necessarily square)
of dimensions not exceeding 256.

A. J. Hoffman and J. B. Kruskal (Princeton University) have continued
their study of conditions under which a rectangular incidence matrix has
the unimodular property (every square submatrix is singular or unimodular).
Let G be a directed graph; a path in G is an ordered sequence of nodes
$v_1, \ldots, v_r$ such that the arc $v_i v_{i+1}$ is positive; a loop in the associated
undirected graph $|G|$ is said to be alternating if its arcs are traversed
alternately in positive and negative directions. Let A be the incidence
matrix of nodes vs. all paths of G. Then A has the unimodular property
if and only if all loops in $|G|$ are alternating. This result is applied to
the study of "cross sections" of families of subsets. A family of subsets
of a given set has the "inclusive property" if, whenever two of the sets have
a non-empty intersection, one is a subset of the other. Let R be a finite
set, $S = \{S_1, \ldots, S_m\}$, $T = \{T_1, \ldots, T_n\}$ be two families of subsets each with
the inclusive property. Let $0 \leqslant a_i \leqslant b_i$ $(i = 1, \ldots, m)$, $0 \leqslant c_j \leqslant d_j$
$(j = 1, \ldots, n)$ be given integers. Necessary and sufficient conditions are
given for the existence of a cross section $C \subseteq R$ such that $a_i \leqslant \overline{C \cap S_i} \leqslant b_i$
$(i = 1, \ldots, m)$ and $c_j \leqslant \overline{C \cap T_j} \leqslant d_j$ $(j = 1, \ldots, n)$. ($\overline{K}$ denotes the cardinality
of the set K.) The proof depends on the foregoing result on incidence
matrices. A special case of the cross section theorem proves the following:

Let a set R of nk elements be partitioned in two ways into $\{S_1, \ldots, S_k\}$, $\{T_1, \ldots, T_k\}$, $\bar{S}_i = \bar{T}_i = n$ ($i = 1, \ldots, k$). Then for any m with $0 \leqslant m \leqslant n$, there exists a set $C \subseteq R$ such that $\bar{C} = mk$, $\overline{C \cap S_i} = m = \overline{C \cap T_i}$ ($i = 1, \ldots, k$).

M. Newman and J. Todd have considered a problem raised by C. Störmer about the solutions in integers of certain equations of the form $m\ p^{-1}(x) + n\ p^{-1}(y) = k\omega$ where p is the Weierstrassian elliptic function, with certain invariants. Some new solutions were obtained, and effective methods for computation of all solutions are being investigated.

In connection with some work undertaken previously, a certain class of elliptic modular functions is being studied by M. Newman. The object of the study is to determine a polynomial basis for the entire modular functions on $\Gamma_0(p)$.

Professor Stiefel (Zurich) has revised his manuscript, "Kernel Polynomials in Linear Algebra", in preparation for publication.

Publications:
(1) The normalizer of certain modular subgroups. M. Newman. To appear in the Canadian Mathematical Journal.
(2) Generalizations of identities for the coefficients of certain modular forms. M. Newman. To appear in Journal of the London Mathematical Society.
(3) Compactification of completely regular spaces. Ky Fan and F. Wagner (University of Notre Dame). In manuscript.
(4) On dominant eigenvalues of positive matrices. T. Kato. In manuscript.
(5) A comparison theorem for eigenvalues of normal matrices. Ky Fan. To appear in the Pacific Journal of Mathematics.
(6) On certain series expansions involving Whittaker functions and Jacobi polynomials. P. Henrici. Pac. J. Math. 5, Suppl. 1, 725-744 (1955).
(7) On generating functions of the Jacobi polynomials. P. Henrici. To appear in the Pacific Journal of Mathematics.
(8) An identity for the coefficients of certain modular forms. M. Newman. J. London Math. Soc. 30, 488-493 (1955).
(9) Error bounds for eigenvalues of symmetric integral equations. H. Wielandt. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953.
(10) Fully convex normed linear spaces. Ky Fan and I. Glicksberg (RAND Corporation). Proc. Nat. Acad. Sci. 41, 947-953 (1955).
(11) On eigenvalues of sums of normal matrices. H. Wielandt. Pac. J. Math. 5, 633-638 (1955).
(12) A theorem on alternatives for pairs of matrices. H. A. Antosiewicz. Pac. J. Math. 5, 641-642 (1955).
(13) On systems of distinct representatives. A. J. Hoffman and H. W. Kuhn (Bryn Mawr College). To appear in Annals of Mathematics Study 38.
(14) Systems of linear inequalities. Ky Fan. To appear in the Annals of Mathematics Study 38.
(15) Systems of distinct representatives and linear programming. A. J. Hoffman and H. W. Kuhn (Bryn Mawr College). To appear in the American Mathematical Monthly.
(16) On a theorem of Dilworth. G. B. Dantzig (RAND Corporation) and A. J. Hoffman. To appear in Annals of Mathematics Study 38.
(17) On "overshoot" in the "furthest hyperplane" method. R. Bryce. In manuscript.
(18) Notes on some scheduling programs. A. J. Hoffman. In manuscript.
(19) On block relaxation. L. S. Joel. In manuscript.
(20) On the minimization of concave and convex functionals. G. B. Dantzig (RAND Corporation), A. J. Hoffman, W. Hirsch (New York University). In manuscript.
(21) The lowest frequency of a free square plate. H. Fujita (University of Tokyo), T. Kato, Y. Nakata (University of Tokyo) and M. Newman. In manuscript.

(22) An inclusion theorem for modular subgroups. M. Newman. Submitted
     to a technical journal.
(23) A table of the coefficients of the powers of $\eta(\tau)$. M. Newman.
     To appear in the Proceedings of the Royal Netherlands Academy of Science.
(24) On the existence of identities for the coefficients of certain modular
     forms. M. Newman. To appear in Journal of the London Mathematical
     Society.
(25) An alternative proof of a theorem on unimodular groups. M. Newman.
     To appear in the Proceedings of the American Mathematical Society.

ANALYTIC STUDY OF WAR GAMES
Task 1101-10-5116/55-83

Origin and Sponsor: Armament Branch, ARDC, USAF   Authorized 12/29/54
Manager: H. A. Antosiewicz
Full task description: Oct-Dec 1954 issue, p. 7

     Status: CONTINUED. Dr. Antosiewicz attended the Michigan Conference
on War Game Simulation for Senior Air Force Officer Training, held at
Ann Arbor, November 11, 12, and participated in a panel discussion on
mathematical models for strategic war games.

     Publication:
(1) Analytic study of war games. H. A. Antosiewicz. To appear in the
     Naval Research Logistics Quarterly.

STUDIES IN THE THEORY OF ASYMPTOTIC EXPANSIONS
Task 1101-10-5116/55-116

Origin: NBS   Authorized 3/31/55
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: P. Davis
Full task description: Jan-Mar 1955 issue, p. 8

     Status: COMPLETED. A report has been submitted to the sponsor.

     Publication:
(1) Uniqueness theory for asymptotic expansion in arbitrary domains.
     P. Davis. In manuscript.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-10-5116/56-148

Origin and Sponsor: National Institutes of Health,   Authorized 9/30/55
                     Bethesda, Md.
Manager: H. A. Antosiewicz
Full task description: July-Sept 1955 issue, p. 7

     Status: CONTINUED. In searching for values of the parameter K for
which solutions of the basic system of differential equations have a
prescribed behavior, 21 cases have been run so far. The results indicate
a great sensitivity of the system to small changes in the parameter K
as well as in the integration steps used.

Publication:
(1) Automatic computation of nerve excitation. K. S. Cole (Naval Medical
    Research Institute), H. A. Antosiewicz and P. Rabinowitz. J. Soc. Ind.
    Appl. Math. 3, 153-172 (1955).

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF $E_1(z)$, $(z=x+iy)$
Task 1102-40-1110/43-3

Manager: I. Stegun   Authorized 7/1/47
Full task description: Apr-June 1949 issue, p. 41

     Status: CONTINUED. Preparation of an introduction to accompany publication of the completed tables continued.

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-40-1110/47-2

Origin: NBS   Authorized 7/1/47
Manager: M. Abramowitz
Full task description: Apr-June 1949 issue, p. 45

     Status: CONTINUED. A code has been prepared for generating the functions F_L, F_L', G_L, G_L' for given values of ρ and η and varying L. In the range $.5 \leqslant \eta \leqslant 2$ five-figure accuracy is obtained and for $\eta < .5$ and $\eta > 2$ seven-figure accuracy is possible.
     A table of the functions G_0 and G_0' was prepared and transferred to punched cards. The values are given for $\rho = 0(.5)40$, $\eta = 1(.5)19.5$ and $\rho = 0(.5)50$, $\eta = 20(.5)25$.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Task 1102-40-1110/51-8

Origin: Section 11.3, NBS   Authorized 3/26/51
Manager: S. Peavy
Full task description: Apr-June 1951 issue, p. 49

     Status: CONTINUED. Computations for the main body of the table have been completed on SEAC. An investigation is under way to determine the most desirable method for calculating the remainder of the table.

REVISION OF MATHEMATICAL TABLES
Task 1102-40-1110/52-7

Origin: NBS   Authorized 8/10/51
Managers: W. F. Cahill, I. Stegun
Full task description: July-Sept 1951 issue, p. 41

     Status: INACTIVE.

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TABLE OF ARCSIN FOR COMPLEX ARGUMENTS
Task 1102-40-1110/52-14

Origin: NBS   Authorized 10/1/51
Manager: B. Urban
Full task description: July-Sept 1951 issue, p. 41

   Status: INACTIVE. For status to date, see Jan-Mar 1955 issue, p. 11.

EXTENSION OF THE TABLE OF HYPERBOLIC SINES AND COSINES
Task 1102-40-1110/52-18

Origin: NBS   Authorized 9/17/51
Manager: K. Nelson
Full task description: July-Sept 1951 issue, p. 41

   Status: COMPLETED.

   Publication: Table of hyperbolic sines and cosines (x = 2(.001)10, 9S).
Applied Mathematics Series 45. Available from U. S. Government Printing
Office, Washington 25, D. C., 55 cents.

TABLE OF THE MODIFIED AIRY INTEGRAL
Task 1102-40-1110/52-23

Origin: NBS   Authorized 10/4/51
Manager: I. Stegun
Full task description: July-Sept 1951 issue, p. 42

   Status: CONTINUED. Preparation of the manuscript for publication
continued.

TABLE OF ERROR FUNCTION FOR COMPLEX ARGUMENTS
Task 1102-40-1110/52-25

Origin: NBS   Authorized 10/5/51
Manager: W. Hall
Full task description: July-Sept 1951 issue, p. 42

   Status: INACTIVE. For status to date, see Oct-Dec 1954 issue, p. 11.

EXTENSION OF TABLES OF THE EXPONENTIAL FUNCTION FOR
NEGATIVE ARGUMENTS
Task 1102-40-1110/52-31

Origin: NBS   Authorized 10/9/51
Managers: E. Marden, S. Prusch
Full task description: July-Sept 1951 issue, p. 43

   Status: COMPLETED.

   Publication: Table of the descending exponential (x=2.5 to x=10, 20D).
Applied Mathematics Series 46. Available from U. S. Government Printing
Office, Washington 25, D. C., 50 cents.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-40-1110/52-37

Origin: NBS   Authorized 11/28/51
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

    Status: INACTIVE.

SIEVERT'S INTEGRAL
Task 1102-40-1110/52-57

Origin: NBS   Authorized 2/12/52
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46

    Status: CONTINUED. The tabular values have been typed on the card-
controlled typewriter and checking is in progress. Preparation of the
introductory material continued.

L-SHELL CONVERSION COEFFICIENTS
Task 1102-40-1110/53-52

Origin: Oak Ridge National Laboratory   Authorized 5/20/53
Manager: W. Hall
Full task description: Apr-June 1953 issue, p. 45

    Status: CONTINUED. Computations for L_{III} shell, $z = 35$, were completed.
For $z = 45$ and $z = 95$, eigenvalues were determined and computations were
made for some k values. Computations for K shell, $z = 65$, were completed.

AUTOMATIC CODING
Task 3711-60-0009/55-65

Origin: NBS   Authorized 9/29/54
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

    Status: CONTINUED. A Base 01 code was placed in service containing the
following service routines and subroutines (in addition to those in the
Base 00 code): Cube Root, Equation Solver, Move Routine, Input-Output Service
Routines, Iterator. The "base code" system is facilitated by a four-address
computer and a fast input unit (magnetic wire on the SEAC). The Base 00 and
Base 01 systems are used with the four-address SEAC entirely, in lieu of an
automatic compiler system.
    H. H. Howe (82.10) has completed the preparation of a "Manual for
SEAC coding in the three-address system" (in manuscript form).
    The following Technical Memoranda by J. H. Wegstein are completed:
        (1) Input-Output service routines for Base 00 codes
        (2) Move subroutine
        (3) Experimental iteration routine

MATHEMATICAL SUBROUTINES
Task 3711-60-0009/56-160

Origin: NBS   Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

Status: CONTINUED. A Base 00 floating routine for evaluating the
modified Bessel function of the second kind $K_n(x)$ was written, which
employs the integral representation $K_n(x) = \int_0^\infty e^{-x \cosh t} \cosh nt \, dt$.

The integral representation may be used for all values of the variable x.
The routine employs the trapezoidal rule, and the integration interval
is automatically chosen to give results correct to within one or two units
in the eighth significant figure.

3. PROBABILITY AND MATHEMATICAL STATISTICS

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-10-1107/49-1a

Origin: NBS   Authorized 1/9/49
Manager: L. S. Deming
Full task description: Apr-June 1949 issue, p. 75

    Status: CONTINUED. For a description of the continuing activity on this task, see the Jan-Mar 1954 issue, p. 49.

MANUAL ON FITTING STRAIGHT LINES
Task 1103-10-1107/50-2

Origin: NBS   Authorized 3/1/50
Manager: F. S. Acton
Full task description: Jan-Mar 1950 issue, p. 42

    Status: CONTINUED. The manuscript is currently with technical advisers for comment.

TABLES TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-10-1107/51-1

Origin: NBS   Authorized 7/1/50
Managers: C. Eisenhart, L. S. Deming
Full task description: July-Sept 1950 issue, p. 57

    Status: INACTIVE. For status to date, see July-Sept 1952 issue, p. 64.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-10-1107/51-2

Origin: NBS   Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

    Status: CONTINUED. A first draft of a paper on "The probability of non-failure of several components" by Joan R. Rosenblatt was prepared.

    Publications:
(1) Time-discrete stochastic processes in arbitrary sets, with applications to processes with absorbing regions and to the problem of loops in Markoff chains. D. van Dantzig. Accepted for publication (in French) in Annales de l'Institut Henri Poincaré (Paris).

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(2) A comparison between the mean successive difference and the root
    mean square successive difference. S. Geisser. Submitted to a
    technical journal.
(3) A note on the normal distribution. S. Geisser. To appear in the
    Annals of Mathematical Statistics.
(4) Inequalities for probabilities associated with the multivariate
    normal distribution. I. R. Savage. Submitted to a technical journal.

PROCEDURES OF NON-PARAMETRIC STATISTICS
Task 1103-10-1107/52-2

Origin: NBS   Authorized 9/17/51
Manager: I. R. Savage
Full task description: July-Sept 1951 issue, p. 66

    Status: TERMINATED. For latest report on this task see July-Sept
1954 issue, p. 22. Further work under this task will be reported under
task 1103-10-1107/56-170 (see p.14).

    Publications:
(1) Easily used simultaneous confidence limits for a line. W. S. Connor.
    Submitted to a technical journal.
(2) Contributions to the theory of rank order statistics. I. R. Savage.
    Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-10-1107/53-1

Origin: NBS   Authorized 10/15/52
Manager: W. H. Clatworthy
Full task description: Oct-Dec 1952 issue, p. 60

    Status: CONTINUED. Readying of the work on partially balanced
incomplete block (PBIB) designs for publication continued. The publication
will include (1) W. H. Clatworthy's enumeration and construction of a
large number of PBIB designs, which considerably extends the list given
in Bose, Clatworthy, and Shrikhande, (2) the duals of a number of these
designs that were worked out by C. Burton and Dr. Clatworthy, and (3)
proofs by Mr. Burton of some theorems regarding the existence of such duals.

    Publications:
(1) Partially replicated Latin squares. W. J. Youden and J. S. Hunter
    (American Cyanamid Co.). To appear in Biometrics.
(2) On partially balanced incomplete block designs with two associate
    classes. W. H. Clatworthy. In manuscript.
(3) Exact tests of significance for combining inter- and intra-block
    information. M. Zelen. Submitted to a technical journal.
(4) On the analysis of covariance. M. Zelen. Submitted to a technical
    journal.

RESEARCH ON MATHEMATICAL ASPECTS OF ORDER
STATISTICS METHODS
Task 1103-10-1107/55-110

Origin: NBS   Authorized 3/3/55
Manager: J. Lieblein
Full task description: Jan-Mar 1955 issue, p. 31

     Status: CONTINUED. Attention was given to several types of problems
in the asymptotic theory of extreme values that had arisen in practical
applications. One problem involved the relationship of alternative
limiting processes to the selection of an asymptotic type for fitting
a given set of extreme data. A second problem was to investigate the
behavior of an extreme event as a function of duration, for example,
the relationship between the peak five-minute wind intensity and the
considerably higher peak one-minute intensity. The results of this work
were included in notes for a talk on developments in the application of
extreme value theory presented before the Annual Meeting of the Institute
of Mathematical Statistics in New York City on December 27.

     Publication:
(1) Application of extreme-value theory to cobbles and boulders in gravel
     deposits. W. C. Krumbein (Northwestern University) and J. Lieblein.
     Submitted to a technical journal.

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-10-1107/56-170

Origin: NBS   Authorized 12/15/55
Manager: Joan R. Rosenblatt

     Objective: To conduct research on the properties and interpretation
of so-called non-parametric statistical methods. This includes, in
particular, "distribution-free" tests and the analysis of rankings.

     Background: There is great need for better understanding of
techniques of statistical analysis which may be used when only very general
assumptions can be made about the underlying stochastic process which
generates data. Criteria are needed for choosing among various distribution-
free methods which have been proposed for similar experimental situations.
Many experimental situations yield data in the form of paired comparisons
or rankings; there is great need for work on models which will lead to the
efficient design and analysis of such experiments.

     Comment: This task supersedes task 1103-10-1107/52-2 (Procedures of
non-parametric statistics), terminated this quarter, which had a somewhat
different objective.

     Status: NEW. An investigation is being made, in collaboration with
G. L. Burrows of the Department of Agriculture, of the distribution properties
of certain statistics which measure agreement among judges, when m judges
have assigned rankings to n objects.
     A paper on the asymptotic efficiency of certain types of non-
parametric tests is nearly completed.

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-10-1115/55-57

Origin: NBS   Authorized 9/1/54
Manager: R. F. Dressler
Full task description: July-Sept 1954 issue, p. 27

   Status: CONTINUED. At the invitation of the Research Committee on
Pressure Elements of the American Society of Mechanical Engineers,
R. F. Dressler prepared a discussion consisting of an analysis and evaluation
of the Haringx linear and non-linear theories of corrugated elastic diaphragms.
   P. Henrici solved a problem concerning generalized axially
symmetric potentials. A. Weinstein and others have shown that every solution
u(x,y) of the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{2\nu}{y}\frac{\partial u}{\partial x} = 0 \qquad (\nu \neq -1, -2, -3, \ldots),$$

which is regular in a domain containing a segment of the axis y = 0,
is uniquely determined by its values on y = 0. The present study answers
the question to what extent the domain of regularity of the function
g(z)=u(z,0) (considered as a function of complex z) determines the domain
of regularity of u(x,y). This generalized earlier results of A. Erdélyi
and G. Szégo.
   P. Henrici also continued his investigation of the Quotient-
Difference algorithm of H. Rutishauser. Basically a method for determining
the poles of a meromorphis function from the coefficients of one of its
Taylor expansions, the algorithm can be applied to problems such as the
computation of the eigenvalues and eigenvectors of a matrix, factorization
of polynomials, interpolation by sums of exponentials, and others. A
comprehensive report on this subject has now been prepared.
   A new addition theorem has been derived by P. Henrici for the
product of two Jacobi polynomials, and a useful series expansion has been
derived for the integral

$$f_n(x,y,z) = \text{Im} \int_0^\infty e^{-z\sqrt{t^2-y^2}}\, J_n(xt)\, \frac{t\, dt}{\sqrt{t^2-y^2}},$$

which for n = 0 is related to Sommerfeld's integral.
   P. Chiarulli has initiated a study on axially symmetric propagation
of strong shocks in gases. A similarity analysis has been started to explore
the feasibility of considering, as an approximation to the actual shock, a
shock of ellipsoidal shape with the center of the ellipsoid in motion.

   Publications:
(1) A discussion of the Haringx diaphragm theories. R. F. Dressler. To be
    published in the Transactions of the American Society of Mechanical
    Engineers.
(2) Addition theorems for general Legendre and Gegenbauer functions.
    P. Henrici. J. Rational Mech. Anal. 4, 983-1018 (1955).

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(3) On the representation of a certain integral involving Bessel functions
    of hypergeometric series. P. Henrici. Submitted to a technical journal.
(4) Review by P. Henrici of "Kernel functions and elliptic differential
    equations in mathematical physics," by S. Bergman and M. Schiffer.
    Bull. Amer. Math. Soc. <u>61</u>, 596-600 (1955).
(5) Automatic computations with power series. P. Henrici. To appear in
    the Journal of the Association for Computing Machinery.
(6) On the domain of regularity of generalized axially symmetric potentials.
    P. Henrici. Submitted to a technical journal.
(7) The quotient-difference algorithm. P. Henrici. In manuscript.

RESEARCH IN ELECTROMAGNETIC THEORY
Task 1104-10-5160/54-47

Origin and Sponsor: Diamond Ordnance Fuze Laboratory,   Authorized 6/29/54
                    Department of the Army   Revised   9/29/54
Manager: F. Oberhettinger
Full task description: July-Sept 1954 issue, p. 28

        Status: INACTIVE.

RESEARCH IN MATHEMATICAL ELASTICITY
Task 1104-10-5160/55-85

Origin: NBS   Authorized 12/27/54
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: R. F. Dressler
Full task description: Oct-Dec 1954 issue, p. 30

        Status: CONTINUED. The most difficult phase of this project has
been successfully completed, namely, the derivation of a practical and
accurate analysis and accompanying computational procedure for determining
all stresses under combined stretching and bending, and all displace-
ments, for any corrugated pressure elements of arbitrary profile and
thickness, under any type of lateral loading and boundary conditions. Full
results for the clamped edge and semi-free edge problems together with a
full description of the method have been presented in a paper, "Properties
of corrugated diaphragms," by R. F. Dressler, at the American Society of
Mechanical Engineers National Meeting, Chicago, Ill., November 13-18.
(See publication (1) below.) The second phase of the project is now in
progress,-- to solve a sequence of problems for a diaphragm of fixed shape
for different values of the thickness parameter, in order to settle the
important question concerning the relative importance of stretching versus
bending effects. Two thicknesses corresponding to h (= .007 in.) and 3h
have now been computed, and cases h/3 and h/9 remain to be done. A third
phase of the project will then be an analysis of the effect on stress
concentrations by addition of a central stiffening disk, frequently used
in instrumentation applications. All of these results should follow easily
from the basic procedure now available.
        On the vibrating triangular wing problem a method for determining
the lowest mode and frequency has been successfully completed. Several cases
have now been computed for different values of Poisson's ratio, which for this
application to plates with free edges remains an essential parameter since
it appears explicitly in the boundary conditions. It has been shown that a
matrix of order 45 is sufficiently large for obtaining the lowest mode.

For the second mode the method has not yet been successful. Here our use
of the hypergeometric polynomial matrix function to spread the eigenvalues
for faster convergence produces values which converge rapidly, but not to
the correct result. Likewise the results for the bi-orthogonal left- and right-
hand eigenvectors are not now correct. Intensive checking of the code
has therefore been in progress by I. Rhodes and U. Hochstrasser by studying
small matrices having known spectra.

   An investigation has been started by R. F. Dressler on the theory
of the edge effect in generalized plane stress, utilizing some of the
stretching methods for the boundary layer domain already successfully
applied to the bending problem for plates. The basic three-dimensional
equations have been set up in the interior space variables and in the
transformed variables for the edge domain half-space. One object is to
obtain the correct simplified differential equations and associated boundary
conditions for describing stresses near the edge in thin and thick plates.

  Publication:
(1) Properties of corrugated diaphragms. R. F. Dressler. To appear in
  the Transactions of the American Society of Mechanical Engineers (now
  available as ASME Preprint No. 55-A-181).

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-10-5160/56-154

Origin: NBS   Authorized 9/30/55
Sponsor: Office of Naval Research
Managers: F. Oberhettinger, J. Lieblein
Full task description: July-Sept 1955 issue, p. 20

   Status: CONTINUED. Slightly more than 600 Fourier transforms for
distribution functions have been compiled so far. The compilation includes
some previously underived results that have been generated to increase its
scope. The results are now arranged in a systematic card index, which is
immediately applicable for publication in book form.

RESEARCH IN FLUID DYNAMICS OF TWO-PHASE FLOWS
Task 1104-10-5160/56-155

Origin: Office of Naval Research   Authorized 9/30/55
Sponsor: "   "
Manager: R. F. Dressler
Full task description: July-Sept 1955 issue, p. 21

   Status: CONTINUED. Appropriate discontinuity relations have been
derived that are valid for the states on either side of an idealized plane
interface with a condensible gas on one side and condensed liquid on the other.
The theory has been set up for a general case where discontinuities are
permitted in all thermodynamic and kinetic quantities. Some typical calculations
for the specific application to steam and water have been carried out. An
investigation is now being made to find typical initial conditions which would
lead to such a condensation interface, both with and without heat conduction
through the duct walls. The question of the extent of the analogy between
this phenomena and flame-fronts is also under investigation.

   A study is also in progress on the quasi-one-dimensional steady flow
of a submerged steam jet. The model is as follows: Steam flows from an

infinite reservoir of steam at given state into another water reservoir,
thus forming a steam pocket surrounded by water. All the steam is assumed
to condense on a vertical end wall of water with given pressure and
temperature. The side walls of the jet therefore consist of a non-condensing
water interface. The equations of motion and energy balance including
friction at the side walls, heat-conduction through the side walls, and
heat conduction in the steam have been studied. The boundary conditions
at the end wall will depend upon the outcome of the discontinuity study
outlined above. This particular model has been chosen to study the flow
which occurs with various combinations of the initial parameters, and to
ascertain whether such a flow is possible in a continuous manner, or
whether it would be necessary to include shock or condensation phenomena
within the steam pocket itself.

RESEARCH IN MATHEMATICAL GEOPHYSICS
Task 1104-10-5160/56-156

Origin: The American University   Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: R. Stoneley
Full task description: July-Sept 1955 issue, p. 21

        Status: CONTINUED. A problem on the variation of amplitude of
Rayleigh waves with depth in an elastic medium with a double surface layer
has been formulated by R. Stoneley and programmed for SEAC by U. Hochstrasser.
This is now awaiting computation.
        The mathematical part of an investigation of the propagation of
Rayleigh waves in an elastic medium with a double surface layer covered
with uniform thickness of water has been completed by Dr. Stoneley and
has been written up for publication. The equation giving the wave velocity
as a function of wave length appears as an 11-row determinant, which is
now being programmed for numerical solution on the SEAC.
        An investigation has begun of the hydrodynamical problem
concerning the turbidity current initiated by a submarine earthquake.
        Dr. Stoneley is continuing the preparation of a monograph on
elastic surface waves.

        Publications:
(1) The transmission of Rayleigh waves across an ocean floor with two-
    surface layers. R. Stoneley. To appear in the Bulletin of the
    Seismological Society of America.

RESEARCH IN RADIATION THEORY
Task 1104-10-5160/56-175

Origin: NBS   Authorized 12/28/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger

        Objective: (a) To study the rigorous solutions to certain important
problems in electromagnetics, such as diffraction from a linear antenna in
front of a semi-infinite reflecting screen or a semi-infinite conducting
wedge having arbitrary angle α. It is aimed to obtain valid representations
for rigorous solutions to such problems to serve as a standard for evaluating
the usual approximate results, which neglect the full exact boundary conditions
in the formulation. (b) To attempt to obtain an improved representation for

the Green's function G(P) associated with a rectangular wave guide to
overcome the disadvantage of the present double series expansion, which
is not convergent when the radiation point source Q is in the same cross
sectional plane as P.   (c) To determine modes in ferrite elements.

    Background: (a) The excitation problem for an infinite wedge of
arbitrary angle has appeared in F. Oberhettinger's paper, "Diffraction of
waves by a wedge," (Comm. Pure Appl. Math. $\underline{7}$, (1955)), but the semi-infinite
case appears to present more difficulty and should be studied in general.
Only the case $\alpha = \pi$ has so far been treated (see A. E. Heins, Carnegie
Institute of Technology Tech. Report No. 17 (1955)).   Results concerning
the edge effects for reflecting screens would be useful.   (b) To determine
the interior and exterior field of a slotted wave guide antenna, a
representation of Green's function is needed that would hold for all relative
positions of the points P and Q.   (c) The ferrites, because of their high
resistivities, exhibit new and useful magnetic properties at frequencies
well above those at which the usual magnetic alloys are applicable.   A
basic problem is that of a cylindrical ferrite rod in free space.

    Status: NEW.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

1102-40-5126/50-13   RAY TRACING
Origin and Sponsor: NBS, Section 2.2
Manager: E. Marden
Full task description: Jan-Mar 1950 issue, p. 33
Status: Inactive

1102-40-5126/51-37   MOLECULAR STRUCTURE, III
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. O'Hara
Full task description: July-Sept 1951 issue, p. 50
Status: Continued. Signs of the normalized structure factor   E   were
  determined for 300 reflections of the dimethoxybenzophenone crystal.
  A three-dimensional Fourier synthesis was computed for this crystal
  using E as amplitude.

1102-40-5126/52-44   CALCULATIONS FOR d SPACINGS
Origin and Sponsor: NBS, Division 9
Manager: R. Zucker
Full task description: Oct-Dec 1951 issue, p. 47
Status: Continued. Calculations for d-spacings were performed continuously
  for various tetragonal, hexagonal and orthorhombic crystals; about 20
  were carried out during the past quarter. Also redetermination of
  unit cell constants by least squares fitting to measured d-spacings was
  performed for about 20 crystals.

1102-40-5126/53-27   COMPUTATION OF THERMODYNAMIC FUNCTIONS
Origin and Sponsor: NBS, Section 3.2
Manager: E. Marden
Full task description: Jan-Mar 1953 issue, p. 57
Status: Continued. Using different physical constants, thermodynamic
  functions were re-computed for several molecules.

1102-40-5126/53-51   RADIATION DIFFUSION
Origin: NBS, Section 4.8
Sponsor: Atomic Energy Commission
Manager: J. Doggett (4.8)
Full task description: Apr-June 1953 issue, p. 57 (Neutron Diffusion III)
Status: Continued. The Mott formula for the elastic scattering of electrons
  and positrons by nuclei has been evaluated for Al, Cu, Sn, Pb and U at
  various energies in the range $.05 \leq E \leq 10$ Mev.

1102-40-5126/54-13   AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: New York Quartermaster Procurement Agency
Manager: H. Bremer
Full task description: Oct-Dec 1953 issue, p. 43
Status: Continued. Analysis of contract award problems are processed on SEAC
  as they are submitted by the sponsor.

3711-60-0009/54-17   **DEPOLYMERIZATION**
<u>Origin</u>: NBS, Section 7.6
<u>Manager</u>: J. Bram
<u>Full task description</u>: Oct-Dec 1953 issue, p. 44
<u>Status</u>: Continued. Several cases were run, and the results transmitted
   to the originator.

3711-60-0009/54-22   **ENERGY DISTRIBUTIONS ON OPTICAL IMAGE**
<u>Origin</u>: NBS, Section 2.2
<u>Manager</u>: L. S. Joel
<u>Full task description</u>: Jan-Mar 1954 issue, p. 43
<u>Status</u>: Continued. The new code, which uses Filons method for quadrature
   of an oscillating function, was completed and is being checked.

3711-60-0009/54-30   **SPECTRUM ANALYSIS**
<u>Origin</u>: NBS, Division 4
<u>Manager</u>: S. Prusch
<u>Full task description</u>: Jan-Mar 1954 issue, p. 46
<u>Status</u>: Continued. SEAC codes have been written and checked out for
   computing wave numbers from given wavelengths and for searching for
   pairs of lines having a common difference. A code is being prepared to
   search for differences between all known and predicted levels for pairs
   of lines which confirm known levels and predict new levels. Searching
   for pairs of lines for approximately 76 common differences was carried
   out for ruthenium I.

3711-60-0009/54-38   **COMPRESSIBILITY FACTORS OF DRY AIR**
<u>Origin</u>: NBS, Section 3.2
<u>Manager</u>: M. Paulsen
<u>Full task description</u>: Jan-Mar 1954 issue, p. 48
<u>Status</u>: Inactive. For status to date, see Apr-June 1955 issue, p. 12.

1102-40-5126/55-39   **MOLECULAR VIBRATIONS**
<u>Origin and Sponsor</u>: NBS, Section 3.2
<u>Manager</u>: K. Goldberg
<u>Full task description</u>: July-Sept 1954 issue, p. 16
<u>Status</u>: Inactive.

1102-40-5126/54-43   **CHARACTERISTICS OF CONDUCTING RESISTORS**
<u>Origin and Sponsor</u>: NBS, Section 1.6
<u>Manager</u>: B. G. Urban
<u>Full task description</u>: Apr-June 1954 issue, p. 40
<u>Status</u>: Inactive. For status to date, see Apr-June 1955 issue, p. 12.

3711-60-0009/55-53   **ELECTRONIC FUNCTIONS**
<u>Origin</u>: NBS, Section 4.5
<u>Manager</u>: W. G. Hall
<u>Full task description</u>: Apr-June 1955 issue, p. 12
<u>Status</u>: Continued. Editing of the card deck into format suitable for
   printing was completed. An introduction to the tables, by L. L. Marton,
   C. Marton, and W. G. Hall, has been prepared.

1102-40-5126/55-58   ELECTROMAGNETIC RADIATION FROM LIGHTNING
Origin and Sponsor: NBS, Section 82.1
Manager: B. G. Urban
Full task description: Jan-Mar 1955 issue, p. 17
Status: Inactive. For status to date, see Jan-Mar 1955 issue, p. 17.

1102-40-5126/55-61   ELASTIC CROSS SECTION FOR NEUTRON SCATTERING
Origin and Sponsor: Naval Research Laboratory
Manager: I. Stegun
Full task description: Oct-Dec 1954 issue, p. 18
Status: Inactive. For status to date, see Apr-June 1955 issue, p. 13.

3711-60-0009/55-66   RECONSTITUTION OF MONOCHROMATIC LIGHT INTENSITIES
Origin: NBS, Division 30
Manager: H. Bremer
Full task description: July-Sept 1954 issue, p. 18
Status: Inactive. For status to date, see Oct-Dec 1954 issue, p. 19.

3711-60-0009/55-68   CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Division 9
Manager: I. Stegun
Full task description: Jan-Mar 1955 issue, p. 18
Status: Continued. A general code has been written by D. R. Prosen that
   will calculate two- and three-dimensional Fourier and Patterson functions.
   This routine can be used for all crystallographic space groups and
   has the further advantage that it can calculate selectively certain
   Fourier or Harker sections.

1102-40-5126/55-74   LIQUID-VAPOR TRANSITION, II
Origin and Sponsor: Naval Medical Research Institute
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 19
Status: Continued. Solution of the integral equation on punched card
   machines continued for one more value of the parameter and one given set
   of functional values.

3711-60-0009/55-75   PARAMETER OF THE DISPERSION EQUATION FOR OPTICAL GLASS
Origin: NBS, Section 2.2
Manager: R. Zucker
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. Further calculations for the refractive index of
   KRS-5 were carried out for n at 19°C, 25°C, and 31°C.

1102-40-5126/55-81   COMBINING TESTS FOR SIGNIFICANCE
Origin: NBS, Section 11.3
Manager: L. S. Joel
Full task description: Oct-Dec 1954 issue, p. 23
Status: Continued. The code is complete, and results are being transmitted
   to the originator as they are computed (about 5000 values to date).

3711-60-0009/55-82   THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. Calibrations were completed for constants α, $\circ$ and β for
   for approximately 25 thermometers under test.

1102-40-5126/55-87   "ZERO" METHOD DETERMINATION OF CRYSTAL STRUCTURE
Origin and Sponsor: NBS, Section 9.7
Manager: J. Bram
Full task description: Jan-Mar 1955 issue, p. 22
Status: Terminated. Trial runs had indicated the inadequacy of the
  present method of computation.

1102-40-5126/55-88   STRESSES IN A WALL FOUNDATION
Origin and Sponsor: NBS, Section 10.1
Managers: E. Marden, G. Hawkins
Full task description: Jan-Mar 1955 issue, p. 22
Status: Continued. Using new coefficients, stresses for another wall
  foundation were computed, and results were transmitted to the sponsor.

1102-40-5126/55-103   MISSOURI RIVER PROBLEM
Origin and Sponsor: Missouri River Division, Corps of Engineers, U. S. Army
Manager: H. Bremer
Full task description: Jan-Mar 1955 issue, p. 26
Status: Continued. Several cases have been run on UNIVAC, and the routines
  have been rewritten to accommodate larger amounts of data.

1102-40-5126/55-105   AERONOMIC TIDAL WINDS OF THERMAL ORIGIN
Origin and Sponsor: NBS, Section 82.1
Managers: B. G. Urban, H. H. Howe (82.1)
Full task description: Jan-Mar 1955 issue, p. 26
Status: Inactive. For status to date, see Jan-Mar 1955 issue, p. 26 .

1102-40-5126/55-113   REACTOR DESIGN
Origin: Westinghouse Atomic Power Division
Sponsor: Atomic Energy Commission
Manager: U. Hochstrasser
Full task description: Jan-Mar 1955 issue, p. 28
Status: Continued. Checking of the codes mentioned in the previous issue
  has been completed. Several runs have been made with various over-
  relaxation factors in order to determine the influence of this factor on
  the convergence of the approximations to the solution of the system of
  difference equations. Further runs are planned for investigating the
  effect of scanning in different ways through the points in the grid.

1102-40-5126/55-117   ATTENUATION OF PRESSURE PULSES OF FINITE AMPLITUDE
Origin: NBS, Section 3.2
Sponsor: Bureau of Aeronautics, U. S. Navy
Manager: M. Paulsen
Full task description: Apr-June 1955 issue, p. 18
Status: Inactive. For status to date, see July-Sept 1955 issue, p. 30.

1102-40-5126/55-118   THERMOMETER CALIBRATION, II
Origin and Sponsor: NBS, Section 3.1
Manager: B. G. Urban
Full task description: Apr-June 1955 issue, p. 18
Status: Continued. The code is being expanded to compute certain constants
  and differences formerly computed by hand; then it will be revised to
  produce calibration tables at one-fifth the former interval.

1102-40-5126/55-121   ELECTRON PENETRATION
Origin: NBS, Section 4.8
Sponsor: Atomic Energy Commission
Manager: S. Peavy
Full task description: Apr-June 1955 issue, p. 19
Status: Continued. A new code is in the process of being written for a
    simpler method devised by L. V. Spencer (4.8).

1102-40-5126/55-122   SOLUTION OF NORMAL EQUATIONS
Origin: Advisory Committee on Weather Control
Sponsor: U. S. Weather Bureau
Manager: M. Newman
Full task description: Apr-June 1955 issue, p. 20
Status: Inactive.

1102-40-5126/55-123   TEMPERATURE DISTRIBUTION
Origin: NBS, Section 6.4
Sponsor: NBS, Section 6.4
Manager: W. G. Hall
Full task description: Apr-June 1955 issue, p. 19
Status: Continued. The code was used on a test problem especially devised
    to try out all parts of the code. The results were as expected.

1102-40-5126/55-126   AERODYNAMIC HEATING
Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Dept. of the Army
Managers: B. Walter, R. Capuano
Full task description: July-Sept 1955 issue, p. 31
Status: Continued. The aerodynamic heating T_W has been calculated for
    four missile trajectories, with each trajectory calculated for four
    different values of missile shell heat capacity (1/B),

$$\Delta T_W = \frac{\overline{T_{AW}} - T_W}{\dfrac{1}{\overline{Bh_e}\Delta t} + \dfrac{1}{2}}$$

Then T_W was calculated for the above with the following modifications:
(1) $\overline{Bh_e}\Delta t$ increased by $+ 1.6°/°$, (2) $\overline{T_{AW}}$ increased by $+ 8°F$, and $\overline{Bh_e}\Delta t$
increased by $.4°/°$. In each case the modifications were originated at
various positions of the missile trajectory.

1102-40-5126/55-127   VIBRATIONS OF CIRCULAR DISC
Origin and Sponsor: Evans Signal Laboratory
Manager: U. Hochstrasser
Full task description: July-Sept 1955 issue, p. 32
Status: Continued. The solutions for the preliminary problem mentioned
    in the July-September 1955 issue were computed on SEAC and transmitted
    to the sponsor. Codes for solving the main equations have been written
    and checked out.

3711-60-0009/56-131   RAY TRACING
<u>Origin</u>: NBS, Section 2.2
<u>Manager</u>: E. Marden
<u>Full task description</u>: July-Sept 1955 issue, p. 33
<u>Status</u>: Inactive. For status to date, see July-Sept 1955 issue, p. 33.

1102-40-5126/56-133   COMPLEX EIGENVALUES
<u>Origin</u>: Moore School of Engineering
<u>Sponsor</u>: Office of Naval Research
<u>Manager</u>: I. Stegun
<u>Full task description</u>: July-Sept 1955 issue, p. 33
<u>Status:</u> Continued. The largest eigenvalues have been obtained for seven
  matrices of order 10 and one matrix of order 6.

1102-40-5126/56-134   TRANSMISSION-DELAY TIMES
<u>Origin</u>: NBS, Section 82.1
<u>Sponsor</u>: NBS, Section 82.1
<u>Managers</u>: B. G. Urban, H. H. Howe (82.1)
<u>Full task description</u>: July-Sept 1955 issue, p. 34
<u>Status</u>: Inactive.

1102-40-5126/56-136   CALCULATION OF WAVE FUNCTIONS BY HARTREE METHOD
<u>Origin</u>: Naval Research Laboratory
<u>Sponsor</u>:   "   "   "
<u>Manager</u>: S. Peavy
<u>Full task description</u>: July-Sept 1955 issue, p. 34
<u>Status</u>: Continued. After the code was checked out, several runs were made.
  However, since this code used considerable time on SEAC, new approximations
  for V(r) were derived by the sponsor and a new code is to be written.
        The new equations for V(r) are:

For $r < \frac{1}{\mu}$,

$$V(r) = 3I\left[-\int_0^{\frac{1}{\mu}-r} |R(z)|^2\, dz + \frac{1}{4r}\int_{\frac{1}{\mu}-r}^{\frac{1}{\mu}+r} |R(z)|^2\left[z+\frac{1}{z}\left(r^2-\frac{1}{\mu^2}\right)-2r\right]dz\right];$$

for $r \geq \frac{1}{\mu}$,

$$V(r) = \frac{3I}{4r}\int_{r-\frac{1}{\mu}}^{\frac{1}{\mu}+r} |R(z)|^2\left[z + \frac{1}{z}\left(r^2 - \frac{1}{\mu^2}\right) - 2r\right]dz.$$

1102-40-5126/56-137   STABILITY OF SUPPORTED PLATES
<u>Origin and Sponsor</u>: National Advisory Committee on Aeronautics
<u>Manager</u>: R. Zucker
<u>Full task description</u>: July-Sept 1955 issue, p. 35
<u>Status:</u> Continued. Computations for r=3 and r=2 were completed for various
  combinations of k_c, k_s, β.· As answers are obtained, they are submitted to
  the sponsor and graphed. From time to time additional sets of parameters
  are calculated around maxima and minima.

1102-40-5126/56-138   CRYSTAL COUNTER EFFICIENCY
Origin and Sponsor: Naval Research Laboratory
Manager: L. Joel
Full task description: July-Sept 1955 issue, p. 36
Status: Continued (Reactivated). The sponsor requested data for two
  additional distances: 0 and 3 cm. These were computed.

1102-40-5126/56-139   STUDY OF INTERNUCLEAR POTENTIAL FOR H_3
Origin and Sponsor: NBS, Section 3.2
Manager: E. Haynsworth
Full task description: July-Sept 1955 issue, p. 36
Status: Continued. The main code was checked out and several problems
  were run. An additional code was written to compute the total electronic
  energy of the system, which is a function of the vector coefficients
  c_{ij} obtained by the previous calculation. That is,

$$E = T_r \left[(\rho^{1\alpha} + \rho^{1\beta} + \rho^{2\alpha}) H \right.$$

$$\left. + \sum_{p,r,q,t=1}^{3} J_{prqt} \left[\rho_{rt}^{2\alpha} (\rho_{pq}^{1\alpha} + \rho_{pq}^{1\beta}) + \rho_{pq}^{1\alpha} \rho_{rt}^{1\beta} - \rho_{pr}^{1\alpha} \rho_{qt}^{1\beta} \right] \right].$$

The 3x3 matrices $\rho = (\rho_{ij})$ are found as products C'C of the vectors
$C_1^\alpha, C_1^\beta, C_2^\alpha$, and the J_{prqt} are given two-electron integrals over atomic
orbitals.

1102-40-5126/56-140   MULTIPLE CORRELATION ROUTINES
Origin and Sponsor: Agricultural Economics Division, Department of
                          Agriculture
Managers: H. Bremer, M. Paulsen
Objective: To analyze on SEAC economic data on natural commodities.
  This involves the preparation of routines to obtain the normal equations,
  the regression coefficients and their standard errors, the multiple
  correlation coefficients, the standard error of estimate and the constant
  term for sets of commodity data, and the development of a program toward
  fitting systems of equations relating to the data by the limited information
  approach.
Background: The Agricultural Marketing Service of the Agricultural
  Economics Division undertakes a program to determine the extent to which
  economic factors influence the utilization and prices of natural commodities
  in the nation, such as cotton, corn, wheat, hogs, beef cattle, and others.
  The analysis for each commodity will be used to predict estimates of
  future utilization and price, or for analyzing the effects of alternative
  Government policies with respect to these commodities.
Status: New. The codes were written, and checking is in progress.

1102-40-5126/56-141   X-RAY SPECTRA CALIBRATION
Origin and Sponsor: NBS, Section 4.11
Manager: I. A. Stegun
Objective: To obtain energy and angular distributions for the X-rays
  produced in thin targets by high energy electrons. The electron energies
  are 2.5, 5, and 10 Mev, and the target materials are beryllium, aluminum,
  and gold. A theoretical expression for this distribution, given by
  Gluckstern and Hull (Physical Review 90, 1030 (1953)), is to be used.

Background: The X-radiation from all high-energy electron accelerators
has a continuous distribution in energy. Every experiment performed
with such a machine depends upon the shape of this distribution.
Although these experiments have been in progress for the last fifteen
years, no rigorous interpretation of them has been possible because
the shape was not known accurately. Recently techniques have been
developed at NBS with which to measure high-energy X-ray energies and
a well-controlled experiment has been performed to measure the shape
and to compare with theory. The experiment was performed in considerable
detail at a variety of angles and energies. Therefore, in order to make
a complete comparison with theory, the theoretical expression must be
evaluated at the same energies and angles.

        The problem was requested by H. W. Koch of the NBS Betatron
Section (4.1).

Status: Completed (New). The results were transmitted to the sponsor.

1102-40-5126/56-143   CHARGED PARTICLE TRAJECTORIES IN THE MAGNETIC FIELD
                            OF THE EARTH

Origin and Sponsor: Naval Research Laboratory
Manager: J. H. Wegstein
Objective: To integrate numerically the differential equations which
determine the trajectories of a charged particle in the magnetic field
of the earth.

        The particles are primarily protons and the field of the earth
is represented as that of a magnetic dipole. Using Störmer units in a
cartesian coordinate system, the equations of motion for the particles are

$$\ddot{x} = -(x^2+y^2+z^2)^{-5/2}[(2z^2-x^2-y^2)\ \dot{y} - 3yz\dot{z}]$$

$$\ddot{y} = -(x^2+y^2+z^2)^{-5/2}[\ (x^2+y^2-2z^2)\ \dot{x} + 3xz\dot{z}]$$

$$\ddot{z} = 3z[h(x^2+y^2+z^2)^{-5/2}+(x^2+y^2)(x^2+y^2+z^2)^{-4}]$$

where $h = (x\dot{y}-y\dot{x}) - (x^2+y^2)(x^2+y^2+z^2)^{-3/2}$.

        These equations are to be numerically integrated from given
initial conditions, using the Runge-Kutta method for simultaneous
differential equations. The accuracy of the calculation is to be
checked periodically by observing the values of $\dot{x}^2+\dot{y}^2+\dot{z}^2$ which should
equal 1. The integration interval is determined automatically from the
curvature C obtainable by the formula $C^2 = \ddot{x}^2+\ddot{y}^2+\ddot{z}^2$.

Background: Observations on the time and space variation of the earth's
magnetic field suggest the presence of large electrical currents in the
upper atmosphere. The existence of these currents is also indicated
by recent experiments of W. H. Bennett on the Störmertron, a device which
may be described as an analogue computer for the determination of
trajectories in a dipole field. In the present calculations those
trajectories will be investigated which appear favorable for the production
of large currents. The Störmertron results and the earlier calculations
of Störmer will be used as guides in the choice of initial conditions for
these trajectories.

        This problem is being programmed for solution with the NORC.

Status: New. The code has been written and checked out on the NORC. The
trajectories for 111 cases have been computed, and the results were sent
to the sponsor.

1102-40-5126/56-145   GAMMA-RAY SPECTROSCOPY
Origin and Sponsor: NBS, Section 4.1
Manager: M. Newman
Objective: To compute the inverse of a 56x56 matrix arising in gamma-ray
   spectroscopy, and to find the solutions of several systems of equations
   in 56 unknowns with this matrix as ground matrix.
Background: The problem arises in connection with scintillation spectrometers.
   A recent experiment performed in the Betatron Laboratory resulted in a
   large number of pulse height distributions obtained with the scintillation
   spectrometer. The conversion of these distributions into Bremstrahlung
   spectra that can be compared with theory requires the use of the inverse
   of a 56x56 matrix. The computation was requested by M. Starfeld (4.1).
Status: Completed (New). The inverse of the 56x56 matrix and the solutions
   of the corresponding equations (about 15) were computed. The results
   were transmitted to the sponsor.

1102-40-5126/56-146   SPORADIC-E THEORY BY TURBULENCE
Origin and Sponsor: NBS, Section 82.1
Managers: B. G. Urban, H. H. Howe, (82.1)
Objective: To compute tables showing the transmission, t, of a parabolic
   E-region of thickness 2Y, subject to uniform turbulence, as a function
   of three parameters ρ, k, and P. Given f_c = critical frequency of

   E-region, and f = frequency of wave under study, then $\rho = f/f_c$, which is

   greater than 1. Given ℓ = linear dimension of the turbulence (definable
   only on a statistical basis) and λ_c = wavelength in free space of a

   wave of the critical frequency, then $k = 4\pi \ell/\lambda_c$, and k is of the order

   of unity. The mean ion-density N varies as $1-z^2$ between z=-1 and z=1,
   and is zero elsewhere; z is the altitude, suitably modified to make it
   dimensionless. Turbulence causes the ion-density to change from N to
   $N+\Delta N$; ΔN varies with position and time, but we here assume that the
   mean square of $\Delta N/N$ is constant, of the order of .0001. P is defined
   as $(\Delta N/N)^2 \cdot (Y/2\ell)$, and is of the order of .01. The transmission t of
   the layer is computed by the following formulas in reverse order:

$$t = e^{-\tau}, \quad \tau = P \int_{-1}^{1} \left(\frac{1-z^2}{\rho^2 - (1-z^2)}\right)^2 F \, dz,$$

$$F = \frac{(\beta^2 + 2)^2}{2(\beta^2 + 1)} - \frac{\beta^2 + 2}{\beta^2} \log_e (\beta^2 + 1), \quad \beta^2 = k^2(\rho^2 - 1 + z^2).$$

   The transmission is tabulated both as t and in decibels, where the
   latter is computed as 10 Mτ. The integration is performed by the nine-
   point Gaussian formula.
Background: Sometimes the E-region of the ionosphere reflects radio waves
   of a frequency much higher than normal. This is called Sporadic E and
   implies a much greater ion-density than normal. The theory that is here
   being studied assumes that sporadic E occurs because of irregularities
   in the ion-density caused by turbulence. This investigation covers only
   the first of a number of models: It is assumed that the general level
   of ion-density is a parabolic function of height, and that the turbulence
   is uniform throughout the region (i.e., percent fluctuation from general
   level is the same throughout the region, and linear dimensions of fluctuations
   are the same throughout). At this time, only the amount of radiation
   that passes through the E-region to the F-region is computed.
Status: Completed (New). The required tables were computed and transmitted
   to the originator.

1102-40-5126/56-147   GROUND-WAVE ATTENUATION
<u>Origin and Sponsor</u>: NBS, Section 82.1
<u>Managers</u>: B.G.Urban, H. H. Howe (82.1)
<u>Objective</u>: To compute the intensity τ of a radio ground-wave as a function
of four parameters D, f, σ, and k. Here D is the distance in km; f
is the frequency in kc/sec; σ is the conductivity of the ground (assumed
the same throughout the path) in mhos/meter; k is the ratio by which
the radius of the earth is increased to allow for atmospheric refraction.
The result is a complex number and is to be given in both polar and
rectangular forms, $T=P+iQ=Re^{i\theta}$.
      The formulas involve double summation and involve certain
auxiliary parameters; certain peculiarities of form arise from the fact
that they are only approximations to other more involved formulas.
The formulas are:

$$K = 830.45 \sqrt{\sigma}/\sqrt{f} \sqrt[3]{f}, \quad \chi = .0008022 \; D \; f^{1/3}/k^{2/3},$$

$$\delta = (K/k^{1/3})e^{i(135°)}, \quad T = \sqrt{2\pi i \chi} \sum_{s=0}^{\infty} \frac{e^{i\tau_s \chi}}{2\tau_s - (1/\delta)^2} ,$$

$$\pi_s = \sum_{j=0}^{\infty} A_{s,j} \quad \text{or} \quad \sum_{j=0}^{\bullet} B_{s,j}$$

where the A-series is used for large δ and the B-series for small δ.
The terms of the series are found by iteration formulas:
 If $j > 1$,

$$A_{s,j} = (-\frac{1}{2A_{s,o}}) [\sum_{n=1}^{j-1} A_{s,n} A_{s,j-n} - \frac{1-2}{j} (\frac{1}{\delta^2}) A_{s,j-2}];$$

$$B_{s,j} = (\frac{j-2}{j})(\delta^2) [\sum_{n=0}^{j-2} B_{s,n} B_{s,j-n-2}]$$

For j=1,

$$A_{s,1} = (-\frac{1}{2A_{s,o}}) (\frac{1}{\delta}), \quad B_{s,1} = -\delta .$$

For j = 0, the formula if s is large is:

$$A_{s,o} = \tfrac{1}{2} [3\pi(s + \tfrac{1}{4})]^{2/3} [e^{i\pi/3}],$$

and the formula for $B_{s,o}$ is identical, except that the 1/4 is replaced
by 3/4. But for small s, a more accurate value is to be obtained from
the following table:

s:	0	1	2	3	4	5
$A_{s,o}$:	0.808	2.577	3.824	4.892	5.851	6.737
$B_{s,o}$:	1.856	3.245	4.382	5.386	6.305	7.161

where all values are multiplied by $e^{i\pi/3}$.

The dividing point between the A and B series, at which neither
one converges, occurs where $\left| \delta^2 \, \tau_s \right| = \frac{1}{2}$. When this equation is nearly
satisfied convergence is so slow that it is impracticable, since all
values of A or B must be retained in the memory until they become
negligibly small. Hence, for small D, where many values of s must be
used, there is quite a range of values of δ (and hence of f) for which
the computations cannot be carried out by these formulas. It is probable
that they could be computed by analytic continuation.

Background: In connection with studies of low-frequency radio propagation,
   information is needed relating the field strength broadcast by a source
   to that of the ground wave reaching a distant point. The tables we
   prepared in this task express the theoretical relation between them for
   various values of four parameters.

Status: New. A code was prepared to a suitably scaled parameter D, and
   computations were made for 12 values of f and for three values each of
   D and δ.

1102-40-5126/56-149   LOGICAL MATRIX MULTIPLICATION
Origin and Sponsor: Signal Corps Engineering Laboratories
Manager: D. Jirauch
Objective: To multiply logically according to certain rules the rows of an
   incidence matrix of order 170. The results desired are (a) a certain
   symmetric incidence matrix, and (b) certain n-tuples of numbers, where n
   should be as large as 16, if possible.
Background: The problem arises in the assignment of broadcast frequencies
   to transmitters in a given area so that interference is at a minimum.
   The elements (0 or 1) of the symmetric incidence matrix indicate the
   presence or absence of interference between transmitters operating on
   two frequencies; the desired results are sets of 16 frequencies no two
   of which interfere with each other.
Status: Completed (New). The results were transmitted to the sponsor.

1102-10-5126/56-151   MULTIPLE SCATTERING IN CROSS SECTION MEASUREMENTS
Origin and Sponsor: Naval Research Laboratory
Manager: I. Rhodes
Objective: To compute corrections for multiple scattering of neutrons in
   thick target measurements by Monte Carlo techniques.
Background: In cross section measurements with thick targets, allowance
   must be made for the possibility that the incident particle will scatter
   more than once before emerging from the target. These multiple scattering
   events may make a major contribution to the detected flux. Their magnitude
   may be determined by an iterative Monte Carlo computation, using the
   single scattering results as the starting point in the iteration. The
   program has been developed for the geometry of a ring target, and for
   specific application to measurements on the angular distribution of
   elastically scattered neutrons in the medium energy region. The problem
   was proposed by R. Jastrow and H. Wagman of the Naval Research Laboratory.
Status: New.

1102-40-5126/56-152   COST ACCOUNTING OPERATIONS FOR HIGH SPEED COMPUTERS, II
Origin and Sponsor: Bureau of Supplies and Accounts, Department of the Navy
Manager: G. Hawkins
Full task description: July-Sept 1955 issue, p. 38
Status: Continued. A code to distribute expenditures and obligations was
   written, and code-checking of this on UNIVAC was started. A code to
   handle special shipyard installations was also written. The code for
   presenting and editing the output is almost completed.

1102-40-5126/56-153   VIBRATIONAL EIGENVALUES OF HYDROGEN, DEUTERIUM AND
                          TRITIUM COMPOUNDS

Origin and Sponsor: NBS, Section 3.2
Manager: E. Haynsworth
Objective: To determine the eigenvalues of the non-symmetric product, FG,
   of positive definite, symmetric, nxn matrices, F and G, which arise in
   the study of molecular spectra.
Background: The eigenvalues obtained from this routine are related simply
   to the normal vibrations of the atoms in the molecule. The group of
   molecules for which the frequencies are presently being calculated are
   ammonia, phosphine, arsine, and stibine and all of the possible isotopes
   formed by substitutions of hydrogen, deuterium and tritium. This constitutes
   a group of 40 molecules now under consideration. The secular equations
   are sixth degree corresponding to the six vibrational degrees of freedom
   which exist for these molecules. The normal frequencies are unique in
   that any vibration of a system can be expressed as a superposition of
   normal frequencies. The ultimate goal is to use these normal frequencies
   to compute thermodynamic functions, namely, specific heats, free energy,
   enthalpy, and entropy, for all 40 molecules. The plan is to continue
   this type of calculation for molecules associated with 18th degree secular
   equations and perhaps with equations of even   higher degree.
Status: Completed (New). Some of the matrices had enough elements alike to
   satisfy the conditions of a theorem by E. Haynsworth (not yet published)
   by use of which the matrices could be reduced by inspection to matrices
   of smaller order which had the same roots. In this particular problem
   n=6, and these matrices were reduced to three 2x2 matrices. So the
   problem of finding the roots was trivial.
        For the non-reducible matrices, a general routine was set up to
   factor the matrix F into the product C'C, where C is a real upper
   triangular matrix, and C' is its transpose. Then the symmetric matrix
   CGC' was formed, and its eigenvalues found, as CGC' had the same roots
   as FG.
        The completed computations were transmitted to the sponsor.

1102-40-5126/56-157   DAMAGE ASSESSMENT
Origin and Sponsor: Office of the Assistant Secretary of Defense (Supply
                       and Logistics)
Managers: A. J. Hoffman, H. Bremer
Objective: To develop programs for obtaining, by means of high-speed
   computers, estimates of the damage to the country from enemy air attack.
Background: For purposes of analysis, it is vital that the Department of
   Defense be able to deduce, from a knowledge of the weapon characteristics
   and ground zeros of an enemy air attack, estimates of the damage inflicted
   on the population and productive facilities of the country. Previous
   work on this problem has produced a set of UNIVAC codes, and it is the
   object of the present task to reformulate the procedures in order to
   obtain programs for UNIVAC that will (1) permit a more realistic assessment
   of the damage and (2) produce output in a more usable form, yet (3) keep
   the computing time within reasonable bounds.
Status: New. Some conferences on weapon effects have been held, and
   preliminary flow charting for part of the program has been started.

1102-40-5126/56-158   FEDERAL RESERVE DATA
Origin and Sponsor: Federal Reserve Board
Manager: E. Marden
Objective: To program and code for UNIVAC a series of tabulations based on
   data collected from the 12 Federal Reserve districts. The tabulations
   will furnish such information as numbers of loans, amounts outstanding,
   etc., and will make use of cross classifications according to such

characteristics of the loan data as business of borrower, assets of
borrower, maturity dates, geographical location of borrower, and others.
Background: The processing of data for this survey, which occurs from time
to time at the Federal Reserve Board, is being carried out on a digital
computing machine for the first time. This tabulation will be used to
assess the application of digital computing machines to the Board's work.
Status: New.

1102-40-5126/56-163   ANGULAR DISTRIBUTIONS AND POLARIZATION EFFECTS IN
                       NUCLEAR SCATTERING
Origin and Sponsor: Naval Research Laboratory
Manager: I. Stegun
Objective: To calculate differential cross sections and polarization effects
in the nuclear scattering of neutrons and protons by numerical integration
of the wave equation and exact computation of asymptotic phase shifts.
Background: Experiments have been performed on the angular distribution
of neutrons and protons elastically scattered by nuclei at energies up
to 300 million volts. Polarization effects in double scattering have
also been measured, primarily at high energies. The theoretical
interpretation of these measurements provides information regarding
several aspects of nuclear structure, among these the mean radius and
density profile of the nucleus, the absorption coefficient and refractive
index of nucleons in nuclear matter, and the strength of the spin-orbit
coupling.
        In the present calculations the nucleus is represented by a
complex potential well of arbitrary radial form, with the coulomb field
and a Thomas spin-orbit term included. The phase shifts which determine
the cross section are computed exactly by numerical integration of the
wave equation in this potential. The calculations will include a survey
of cross sections for neutrons and protons over the full range of energies
and atomic weights.
        Calculations have been carried out at energies of 150 and 300
million volts, in which the same nuclear model is assumed, but phase
shifts are computed in WKB approximation. Below 100 million volts the
WKB approximation is inadequate and the present analysis based on exact
computation of the phase shifts is required. Exact calculations will also
be carried out at the higher energies, however, for comparison with the
WKB results.
        The problem was proposed by Dr. Robert Jastrow of the Naval Research
Laboratory.
Status: New. The code was written and is being checked for a sample problem
describing the scattering of protons by aluminum at 300 Mev.

3711-60-0009/56-164   TRANSIENT HEAT FLOW PROBLEM, II
Origin: NBS, Section 10.2
Manager: W. F. Cahill
Objective: To obtain data on transient heat flow through various geometrical
configurations which closely approximate structures tested by the NBS
Fire Protection Section (10.2).
Background: This task is a continuation of task 1102-40-5126/54-45 for
additional values of the parameters.
Status: Completed (New). The cases for the new parameters were computed and
were sent to the originator.

1102-40-5126/56-165   INTEGRALS FOR SCATTERING FUNCTIONS
Origin and Sponsor: Naval Research Laboratory
Manager: R. Zucker
Objective: To evaluate 11 mathematical expressions involving integrals that
are functions of six parameters. The integrands involve the function

$$I = \cfrac{1}{\sum\limits_{n=0}^{\infty} (n+1) \sin^2 (\delta_n - \delta_{n+1})} = f(\xi, p),$$

where

$$\tan \delta_n = \frac{-J_n(\xi^{\frac{1}{2}})}{Y_n(\xi^{\frac{1}{2}})} \cdot \frac{\tan \zeta_n - \xi^{\frac{1}{2}} \dfrac{J_n'(\xi^{\frac{1}{2}})}{J_n(\xi^{\frac{1}{2}})}}{\tan \zeta_n - \xi^{\frac{1}{2}} \dfrac{Y_n'(\xi^{\frac{1}{2}})}{Y_n(\xi^{\frac{1}{2}})}}$$

$$\tan \zeta_n = \frac{1}{2}\frac{p}{(p-\xi)^{\frac{1}{2}}} - (p-\xi)^{\frac{1}{2}} + \left(n+1 - \frac{p}{2(p-\xi)^{\frac{1}{2}}}\right)$$

$$\cdot \frac{F\left(n+2 - \dfrac{p}{2(p-\xi)^{\frac{1}{2}}} \mid 2n+2 \mid 2(p-\xi)^{\frac{1}{2}}\right)}{F\left(n+1 - \dfrac{p}{2(p-)^{\frac{1}{2}}} \mid 2n+2 \mid 2(p-\xi)^{\frac{1}{2}}\right)}$$

J_n is a spherical Bessel function and Y_n is a spherical Neumann function.

Background: In the measuring of the transport properties of semiconductors, it has become necessary to evaluate the influence of the scattering of free charge carriers by ionized impurities. The existing theoretical treatments amount to calculations in Born approximations and are thus restricted to the condition that the wavelength of the charge carrier be much smaller than the size of the scattering center. This excludes from the allowed range most of the concentrations of impurities and regions of temperature where ionized impurity scattering is dominant. In order to get the correct picture over the range realizable by experiment, it is necessary to obtain the above exact calculations.

Status: New. The code was written, and checking was started.

1102-40-5126/56-171   COLLISION INTEGRALS USED IN TRANSPORT THEORY

Origin and Sponsor: NBS, Section 3.2

Manager: J. Cooper

Objective: To compute a table of transport properties as a function of temperature. This involves three steps:
(a) To calculate the angle of deflection of a particle scattered by a central force,

$$\chi(g,b) = \pi - 2b \int_{r_m}^{\infty} \frac{dr/r^2}{\sqrt{1-\left(\dfrac{b}{r}\right)^2 - \dfrac{\emptyset(r)}{g^2}}}$$

where χ is the angular deviation measured from the initial line of approach of the particle,
    g is the relative velocity of the particle and the force center,
    s is the impact parameter or distance of the line of approach from a line drawn parallel to it through the force center,
    r_m is the distance of closest approach,
    $\emptyset(r)$ is the potential of the force center.

This involves solution by an iterative process of the equation

$$\sqrt{1 - (\tfrac{b}{r})^2 - \frac{\phi(r)}{g^2}} = 0$$

to find the largest root and then integration to find the value of χ as a function of g and b for various potential functions.
(b) To evaluate the collision integrals for parameters l and s (integers < 5), and T (ranging from .1 to 1),

$$\Omega(l,s,T) = \frac{4}{(s+1)_0^1 \; T^{s+2}(1 + \tfrac{1}{2} \cdot \frac{1+(-1)l}{1+l})} \quad \cdot$$

(1)

$$\cdot \int_0^\infty \int_0^\infty \exp - \frac{q^2}{T} [1 - \cos^l \chi(g,b)] g^{2s+3} \; b \; db \; dg$$

(c) To combine the collision integrals for each value of T to compute the transport properties.
<u>Background</u>: The problem is communicated by Dr. Melville S. Green, NBS Thermodynamics Section.
<u>Status</u>: New.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS
RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS   Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

    Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

    (1) Fitting of polynomials to data on atomic spectra was carried out with the aid of SEAC, for E. K. Plyler and H. C. Allen, Jr., of Section 4.2.

    (2) Least squares determination of the virial coefficients of air was done on SEAC, for A. Friedman of Section 3.2.

    (3) Experimental data were analyzed for evaluating the modulus of rupture of 0^0 sapphire rods over a range of temperatures, for J.B. Wachtman of Section 9.1.

    (4) The experimental results of full-scale field trials, which made use of a 1/4 fractional replication experimental design, were analyzed for P. Haas of Section 33.1, DOFL, with the aid of SEAC.

    (5) An investigation of the precision of measuring the sound absorption coefficient was carried out for R. Waterhouse of Section 6.1.

    (6) Two exploratory experiments were planned and analyzed for evaluating how well judges can discriminate distinctness of image and brightness characteristics for standard wood samples, for I. Nimeroff of Section 2.1.

    (7) Some exploratory work on the distribution of properties of fuels which are mixtures of two other fuels was done for R. S. Jessup of Section 3.2.

    Publications:
(1) National physical standards and design of experiments. W. J. Youden. To appear in Revue de L'Institut International de Statistique (The Hague).
(2) More scientists. W. J. Youden. Submitted to a technical journal.
(3) A simplified calibration of platinum versus platinum-rhodium thermocouples. B. E. Squires (3.1) and M. Zelen. In manuscript.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC   Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

    Status: CONTINUED. A multiple regression study of the effect of chemical composition on ship steel is nearly completed.

RESEARCH IN APPLICATIONS OF MATHEMATICAL STATISTICS TO
PROBLEMS OF THE CHEMICAL CORPS
Task 1103-40-5118/52-1

Origin and Sponsor: Biological Laboratories,   Authorized 10/1/51
               Chemical Corps, Dept. of the Army
Manager: C. Eisenhart
Full task description: Oct-Dec 1951 issue, p. 57

      Status: CONTINUED. A report giving fractional replicated designs of the 2^n series for n = 13, 14, 15, 16 factors was submitted.
      The SEAC code for evaluating the power function of combining two independent tests of significance has been written and code checked. The computation of the power points is now underway.

STATISTICAL ANALYSIS AND DESIGN OF EXPERIMENTS
FOR THE U. S. GEOLOGICAL SURVEY
Task 1103-40-5140/54-1

Origin and Sponsor: U. S. Geological Survey,   Authorized 10/9/53
               Department of Interior
Managers: C. Eisenhart, W. J. Youden
Full task description: Oct-Dec 1953 issue, p. 50

      Status: CONTINUED. Some additional work was done on the use of extreme value techniques in the sampling and analysis of data on pebble deposits. Correspondence was carried on with Helen Cannon of Colorado Plateau District on the problem of the association between the incidence of certain plants and uranium deposits.

MANUAL ON EXPERIMENTAL STATISTICS
FOR ORDNANCE ENGINEERS
Task 1103-40-5146/55-93

Origin and Sponsor: Office of Ordnance Research   Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

      Status: CONTINUED. Further progress has been made in the writing of the manual. An important section, "Statistical Tests Concerning Averages," was written and revised, and copies were forwarded to the Office of Ordnance Research. Other sections are in various stages of development.

APPLICATION

of

NATIONAL BUREAU OF STANDARDS AUTOMATIC COMPUTER (SEAC)

The record of SEAC operations for tasks of the Applied Mathematics Division for the period October 1 through December 31 is as follows:

Task No. NBS:	Title	Hours Used: Code Checking	Hours Used: Productive Operation
1104/55-55	Research in numerical analysis	22	1
5116/55-56	Research in mathematical topics applicable to numerical analysis		1
5116/56-148	Nerve fiber reaction	4	13
1110/47-2	Tables of Coulomb wave functions	30	16
1110/52-31	Exponential function for negative argument		1
1110/52-57	Sieverts integral	1	
1110/53-51	Radiation diffusion III	13	56
1110/53-52	L-Shell conversion coefficients	1	62
1110/55-65	Automatic coding	7	10
5126/51-8	Tables of power points of analysis of variance tests		9
5126/52-44	Calculations for d-spacings	1	8
5126/53-27	Thermodynamics functions		1
5126/53-29	Dynamic behavior of aircraft structure	2	12
0009/54-17	Depolymerization	2	22
0009/54-19	Energy levels of complex atoms		2
5126/54-30	Spectrometer analysis	8	17
0009/55-53	Electronic functions		2
0009/55-75	Parameter of the dispersion equation for optical glass		1
5126/55-81	Combining tests for significance	3	25
5126/55-88	Stresses in a wall foundation	4	7
5126/55-96	Monte Carlo		7
5126/55-97	High temperature properties of air	7	103
5126/55-105	Aeronomic tidal winds of thermal origin	3	1
5126/55-115	Adsorption integrals		1
5126/55-118	Thermometer calibrations	1	
5126/56-121	Electron penetration	3	
5126/55-123	Temperature distribution	2	
5126/56-129	Processing of Public Housing data		1
0009/56-131	Ray tracing	18	22
5126/56-134	Transmission delay times		1
5126/56-139	Study of internuclear potential for H_3	6	1
5126/56-141	Spectrometer calibration	3	14
5126/56-145	Gamma-ray spectroscopy	1	21
5126/56-146	Sporadic E-Theory by turbulence	8	2
5126/56-147	Ground-wave attenuation	12	33
5126/56-150	Mathematical expression	2	3
5126/56-153	Vibrational eigenvalues	6	4
5126/56-160	Matrix factoring	9	2
0009/56-164	Transient heat flow problem		13

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Task No.	Title	Hours Used: Code Checking	Hours Used: Productive Operation
NBS:			
5126/56-167	Multiple regression		1
Miscellaneous	Training	3	
0002/52-1	Statistical aspects of NBS administrative operations	6	47
5160/55-85	Research in mathematical elasticity	1	24
Other:			
5126/53-45	Application of game theory	2	
5126/54-13	Award of procurement contracts for linear programming	2	15
5126/55-104	"Fuse" problem	7	11
5126/55-113	Reactor design	24	4
5126/55-119	Field rocket problem	13	5
5126/55-122	Solution of normal equation		2
5126/55-127	Vibrations of a circular disc		3
5126/56-130	Aircraft responses	2	11
5126/56-133	Complex eigenvalues	1	11
5126/56-136	Calculation of wave functions	3	33
5126/56-137	Stability of supported plates	3	34
5126/56-138	Crystal counter efficiency		9
5126/56-140	Multiple correlation routines	10	
5126/56-151	Multiple scattering in cross section measurements	1	19
5126/56-163	Angular distributions and polarization effects in nuclear scattering		3
Miscellaneous		1	23
	Totals:	258	740

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Seminar: Special Topics in Numerical Analysis

(Offered by the Department of Mathematics of The American University in collaboration with the Graduate School of the National Bureau of Standards)

KATO, T. (University of Tokyo) (1) Perturbation theory of eigenvalues: 1. Operators in Banach space. October 17. 2. Finite Matrices. October 24. (2) On helium wave functions. October 31. (3) On eigenvalues of free plates. November 7.

TODD, J. Approximation theory: 1. The theorems of Weierstrass, Bernstein and Chebyshev. November 15. 2. Theorems of Bernstein, Voronowskaja, Chebyshev and Markoff. December 6.

DAVIS, P. Extremal problems and criteria for closure: Three lectures, November 22, December 13, December 20.

Papers and Invited Talks
Presented by Members of the Staff
at Meetings of Outside Organizations

Papers presented at the Office of Ordnance Research Conference on the Design and Experiments in Army Research, Development, and Testing held at the National Bureau of Standards, Washington, D. C., October 19-21.

CAMERON, J. M. Some examples of the use of high speed computers in statistics.

EISENHART, C. The principle of randomization in the design of experiments.

YOUDEN, W. J. Design of experiments in industrial research and development.

* * * * *

Papers presented at the Joint Meeting of the American Statistical Association and the Institute of Mathematical Statistics, New York, N.Y., December 27-30:

BURTON, R. C. Some combinatorial relationships arising in the dualization of incomplete block designs.

LIEBLEIN, J. Recent developments in the application of extreme value theory.

MILLER, F. L., Jr. The construction of fractional factorial designs for the 2^n series.

NATRELLA, M. G. Unpublished tables prepared in the Statistical Engineering Laboratory, National Bureau of Standards.

PETTIGREW, H. The use of the 2^n fractional factorial designs for factors
        at 4 levels.

ROSENBLATT, J. R. On the efficiency of certain classes of tests based on
        U-statistics.

YOUDEN, W. J. (1) Special training arranged by Local Sections of the
        American Chemical Society. (2) Who makes designs?

ZELEN, M. On the analysis of incomplete block designs.

                              *   *   *   *   *

ALT, F. L. Development of automatic computers. Presented at the Institute
        on Electronics in Management held by the Department of Government
        and Public Administration of The American University, Washington,
        D. C., November 14.

ANTOSIEWICZ, H.A. Stability theory of nonlinear differential equations.
        Presented at the Department of Mathematics, Princeton University,
        N. J., December 9.

DAVIS, P. Orthonormalizing codes and their uses. Presented at the Moore
        School of Engineering, University of Pennsylvania, Philadelphia,
        Pa., November 10.

EISENHART, C. Random notes. Presented before the Washington Chapter,
        American Statistical Society, Washington, D. C., November 28.

HOFFMAN, A. J. Mathematical approach to production scheduling. Presented
        before the American Institute of Industrial Engineers, Washington,
        D. C., October 11.

KATO, T. (University of Tokyo) and O. TAUSSKY-TODD. Normal matrices.
        Presented by title at the meeting of the American Mathematical
        Society, Houston, Texas, December 30.

NEWMAN, M. The coefficients of modular form. Presented at the meeting of
        the American Mathematical Society, University of Maryland, College
        Park, Md., October 29.

TODD, J. The problem of stability in the numerical solution of ordinary
        and partial differential equations. Presented at a Seminar of
        the Moore School of Engineering, University of Pennsylvania,
        Philadelphia, Pa., December 14.

YOUDEN, W. J. Making good measurements. (1) Presented before the RESA
        Society, Camp Detrick, Frederick, Maryland, November 17, and (2)
        before the St. Louis Section of the American Chemical Society, St.
        Louis, Missouri, December 5.

        Programming measurements in experimentation. Presented before
        the Washington Chapter, Institute of Radio Engineers, Washington,
        D. C., November 21.

        Marbles, measurements, and mathematics. Presented at Northwestern
        High School, Hyattsville, Maryland, December 14.

ZELEN, M. Exact tests of significance for the recovery of inter-block
        information. Presented at the Department of Agriculture, Washington,
        D. C., October 14.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Table of hyperbolic sines and cosines, x=2 to x=10, 9S. Applied Mathematics Series 45. Available from U. S. Government Printing Office, Washington, D. C., 55 cents.

(2) Table of the descending exponential, x=2.5 to x=10. Applied Mathematics Series 46. Available from U. S. Government Printing Office, Washington, D. C., 50 cents.

1.3 Technical Papers

(1) A theorem on alternatives for pairs of matrices. H. Antosiewicz. Pac. J. Math. $\underline{5}$, Suppl. 1, 641-642 (1955).

(2) Computation of vibration modes and frequencies on SEAC. W. F. Cahill and S. Levy (NBS 6.4). J. Aeronautical Sciences $\underline{22}$, 837-843 (Dec. 1955).

(3) Automatic computation of nerve excitation. K. Cole (National Institutes of Health), H. Antosiewicz, and P. Rabinowitz. J. Soc. Ind. Appl. Math. $\underline{3}$, 153-172 (1955).

(4) On a problem in the theory of mechanical quadratures. P. Davis. Pac. J. Math. $\underline{5}$, Suppl. 1, 669-674 (1955).

(5) Fully convex normed linear spaces. K. Fan and I. Glicksburg (RAND Corporation). Proc. Nat. Acad. Sci. $\underline{41}$, 947-953 (1955).

(6) An algebraic proof of the isoperimetric inequality for polygons. K. Fan, O. Taussky and J. Todd. Washington Acad. Sci. $\underline{45}$, 334-342 (1955).

(7) Addition theorems for general Legendre and Gegenbauer functions. P. Henrici. J. Rational Mech. Anal. $\underline{4}$, 983-1018 (1955).

(8) On certain series expansions involving Whittaker functions and Jacobi polynomials. P. Henrici. Pac. J. Math. $\underline{5}$, Suppl. 1, 725-744 (1955).

(9) An identity for the coefficients of certain modular forms. M. Newman. J. London Math. Soc. $\underline{30}$, 488-493 (1955).

(10) Unimodular circulants. O. Taussky. Math. Zeitschr. $\underline{63}$, 286-289 (1955).

(11) Computation of atomic levels: the spectrum of singly ionized tantulum. R. E. Trees (NBS Section 4.1), W. F. Cahill, and P. Rabinowitz. J. Res. NBS $\underline{55}$, 335-341 (Dec. 1955), RP2639.

(12) On eigenvalues of sums of normal matrices. H. Wielandt. Pac. J. Math. $\underline{5}$, 633-638 (1955).

1.4 Notes

    (1) Kleine Bemerkung zur asymptotischen Entwicklung des Fehlerintegrals.
        P. Henrici. Zeitschr. angew. Math. Phys. $\underline{6}$, 145-146 (1955).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION DECEMBER 31, 1955

2.1 Mathematical Tables

    (1) A table of the coefficients of the powers $\eta(\tau)$. M. Newman.
        To appear in the Proceedings of the Royal Netherlands Academy
        of Sciences.

2.3 Technical Papers

    (1) Analytic study of war games. H. Antosiewicz. To appear in the
        Naval Research Logistics Quarterly.

    (2) On the differential equation $\ddot{x} + k(f(x) + g(x)\dot{x})\dot{x} = ke(t)$.
        H. A. Antosiewicz. Submitted to a technical journal.

    (3) Stable systems of differential equations with integrable forcing
        term. H. A. Antosiewicz. To appear in the Journal of the London
        Mathematical Society.

    (4) Easily used simultaneous confidence limits for a line. W. S. Connor.
        Submitted to a technical journal.

    (5) Time-discrete stochastic processes in arbitrary sets, with applications
        to processes with absorbing regions and to the problem of loops in
        Markoff chains. D. van Dantzig. To appear (in French) in Annales
        de l'Institut Henri Poincaré (Paris).

    (6) On the theorem of Dilworth. G. B. Dantzig (RAND) and A. J. Hoffman.
        To appear in "Papers on linear inequalities and related systems,"
        Annals of Mathematics Study 38.

    (7) Uniqueness theory for asymptotic expansions in general regions.
        P. Davis. Submitted to a technical journal.

    (8) Abscissas and weights for Gaussian quadratures of high order.
        P. Davis and P. Rabinowitz. To appear in the Journal of Research
        (NBS).

    (9) Numerical experiments in potential theory using orthonormal functions.
        P. Davis and P. Rabinowitz. To appear in the Journal of the Washington
        Academy of Sciences.

   (10) Some Monte Carlo experiments in computing multiple integrals.
        P. Davis and P. Rabinowitz. To appear in Mathematical Tables and
        Other Aids to Computation.

   (11) Comparison of theories and experiments for the hydraulic dam-break
        wave. R. F. Dressler. To appear in the Proceedings of the Tenth
        General Assembly of the International Union of Geodesy and Geophysics,
        held in Rome, Italy, September 1954.

   (12) Properties of currugated diaphragms. R. F. Dressler. To appear in
        the Transactions of the American Society of Mechanical Engineers.

(13) Heat flow in a fluid with eddying flow. W. H. Durfee (now at
     Mt. Holyoke College). Submitted to a technical journal.

(14) A comparison theorem for eigenvalues of normal matrices. K. Fan.
     To appear in Pacific Journal of Mathematics.

(15) On systems of linear inequalities. K. Fan. To appear in "Papers
     on linear inequalities and related systems," Annals of Mathematics
     Study 38, ed. by H. W. Kuhn and A. W. Tucker.

(16) Systems of inequalities involving convex functions. Ky Fan.
     Submitted to a technical journal.

(17) A comparison between the mean successive difference and the root
     mean square successive difference. S. Geisser. Submitted to a
     technical journal.

(18) A note on the normal distribution. S. Geisser. To appear in Annals
     of Mathematical Statistics.

(19) A matrix with real characteristic roots. K. Goldberg. To appear
     in the Journal of Research (NBS).

(20) The formal power series for $\log e^x e^y$. K. Goldberg. To appear in
     the Duke Journal of Mathematics.

(21) A subroutine for computations with rational numbers. P. Henrici.
     To appear in the Journal of the Association for Computing Machinery.

(22) Automatic computations with power series. P. Henrici. To appear
     in the Journal of the Association for Computing Machinery.

(23) On generating functions of the Jacobi polynomials. P. Henrici.
     To appear in Pacific Journal of Mathematics.

(24) On the domain of regularity of generalized axially symmetric
     potentials. P. Henrici. Submitted to a technical journal.

(25) On the representation of a certain integral involving Bessel
     functions by hypergeometric series. Submitted to a technical
     journal.

(26) On systems of distinct representatives. A. J. Hoffman and H. W. Kuhn
     (Bryn Mawr College). To appear in "Papers on linear inequalities
     and related systems," Annals of Mathematics Study 38.

(27) Systems of distinct representatives and linear programming.
     A. J. Hoffman and H. W. Kuhn (Bryn Mawr College). To appear in
     the American Mathematical Monthly.

(28) The number of absolute points of a correlation. A. J. Hoffman,
     M. Newman, E. G. Straus, O. Taussky. To appear in the Pacific
     Journal of Mathematics.

(29) Commutators of A and A*. T. Kato and O. Taussky. To appear in the
     Journal of the Washington Academy of Sciences.

(30) Applications of extreme value theory to cobbles and boulders in
     gravel pits. W. C. Krumbein (Northwestern University) and
     J. Lieblein. Submitted to a technical journal.

(31) Solvability and consistency for linear equations and inequalities.
H. W. Kuhn (Bryn Mawr College). To appear in the American Mathematical
Monthly.

(32) Pairs of matrices with property L, II. T. S. Motzkin and O. Taussky.
To appear in the Transactions of the American Mathematical Society.

(33) An alternative proof of a theorem on unimodular groups. M. Newman.
To appear in the Proceedings of the American Mathematical Society.

(34) An inclusion theorem for modular groups. M. Newman. Submitted to
a technical journal.

(35) Generalizations of identities for the coefficients of certain
modular forms. M. Newman. To appear in the Journal of the London
Mathematical Society.

(36) On the existence of identities for the coefficients of certain
modular forms. M. Newman. Submitted to a technical journal.

(37) The diophantine equation $ax^m - by^n = c$. M. Newman. Submitted to a
technical journal.

(38) The normalizer of certain modular subgroups. M. Newman. To
appear in Canadian Journal of Mathematics.

(39) On a generalization of the normal basis in abelian algebraic number
fields. M. Newman and O. Taussky. To appear in Communications on
Pure and Applied Mathematics (New York University).

(40) On asymptotic series for functions occurring in the theory of
diffraction of waves by wedges. F. Oberhettinger. To appear in
the Journal of Mathematics and Physics.

(41) On the Lerch zeta function. F. Oberhettinger. To appear in the
Pacific Journal of Mathematics.

(42) Determinanten mit ueberwiegender Hauptdiagonale und die absolute
Konvergenz von linearen Iterationsprozessen. A. M. Ostrowski.
To appear in Comentarii Mathematici Helvetici.

(43) On Gauss' speeding up device in the theory of single step iteration.
A. M. Ostrowski. Submitted to a technical journal.

(44) On the convergence of Gauss' alternating procedure in the method of
the least squares, I. A. M. Ostrowski. Submitted to a technical
journal.

(45) Contributions for inverse osculatory interpolation. H. E. Salzer.
(Department of the Army). To appear in the Journal of Research of the
NBS.

(46) Contributions to the theory of rank order statistics. I. R. Savage.
Submitted to a technical journal.

(47) Inequalities for probabilities associated with the multivariate
normal distribution. I. R. Savage. Submitted to a technical
journal.

(48) Simultaneous tests of randomness and other hypotheses. I. R. Savage.
Submitted to a technical journal.

(49) Algebraic equations satisfied by roots of natural numbers.
E. G. Straus (University of California at Los Angeles) and O. Taussky.
To appear in the Pacific Journal of Mathematics.

(50) A note on group matrices. O. Taussky. To appear in the Proceedings
of the American Mathematical Society.

(51) Some computational problems in algebraic number theory. O. Taussky.
To appear in the Proceedings of the American Mathematical Society
Sixth Symposium on Applied Mathematics, held at Santa Monica City
College, August 1953.

(52) Generation and testing of pseudo-random numbers. O. Taussky and
J. Todd. Submitted to a technical journal.

(53) Machine attacks on problems whose variables are permutations.
C. B. Tompkins. To appear in the Proceedings of the American
Mathematical Society Sixth Symposium on Numerical Analysis, held
in Santa Monica, Calif., August 1953.

(54) An extension of a theorem of Dantzig's. C. Tompkins and I. Heller
(George Washington University). To appear in "Papers on linear
inequalities and related systems," Annals of Mathematics Study 38.

(55) Best approximation polynomials of given degree. J. Walsh. To
appear in the Proceedings of the American Mathematical Society
Sixth Symposium on Applied Mathematics, held in Santa Monica, Calif.,
August 1953.

(56) On the asymptotic transformation of certain distributions into
the normal distribution. W. Wasow. To appear in the Proceedings
of the American Mathematical Society Sixth Symposium Applied
Mathematics, held at Santa Monica, Calif., August 1953.

(57) Error bounds for eigenvalues of symmetric integral equations.
H. Wielandt. To appear in the Proceedings of the American Mathematical
Society Sixth Symposium on Applied Mathematics, held at Santa Monica,
Calif., August 1953.

(58) National physical standards and design of experiment. W. J. Youden.
To appear in Revue de L'Institut International de Statistique
(The Hague).

(59) Partially replicated latin squares. W. J. Youden and J. S. Hunter
(American Cyanimid Co.). To appear in Biometrics.

(60) Exact tests of significance for combining intra- and inter-block
information. M. Zelen. Submitted to a technical journal.

(61) On the analysis of covariance. M. Zelen. Submitted to a technical
journal.

2.5 Miscellaneous

(1) More scientists. W. J. Youden. Submitted to a technical journal.

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($0.75), available from the Superintendent of Documents, Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.