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NATIONAL BUREAU OF STANDARDS REPORT

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FRACTIONAL FACTORIAL DESIGNS FOR THE $1/2^S$ x 2^n SERIES
FOR $n = 12(1)16$ AND $s = 6,7,8.$

by

R. C. Burton,
F. L. Miller, Jr.,
and H. M. Pettigrew

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U. S. DEPARTMENT OF COMMERCE
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FRACTIONAL FACTORIAL DESIGNS FOR THE $1/2^S$ x 2^n SERIES
FOR   n = 12(1)16   AND   s = 6,7,8.

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FOREWARD

This report brings together work on the construction of two-level fractional factorial designs begun during the summer of 1955, and is an extension of the work of Clatworthy, Connor, and Zelen [1] . The experimental designs catalogued here may be useful in reducing the amount of experimentation for those experimental situations where the joint effects of many factors are to be evaluated.

This work is part of a continuing program of research on mathematical statistics and its applications carried out at the Statistical Engineering Laboratory, National Bureau of Standards for the Chemical Corps, U. S. Department of the Army (NBS Project Number 11030-40-5118/52-1).

C. Eisenhart
Chief, Statistical Engineering Laboratory

E. W. Cannon
Chief, Applied Mathematics Division
National Bureau of Standards

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Fractional Factorial Designs for the

$\frac{1}{2^s}$ x 2^n Series for n = 12(1)16 and s = 6, 7, 8

by

R.C. Burton, F.L. Miller, and H.M. Pettigrew

Introduction. This report is an extension of the catalogue of two-level fractional factorial replicate designs compiled by W.H. Clatworthy, W.S. Connor, and M. Zelen [1]. The designs are of the 2^n series, where n, the number of factors, ranges from 12 to 16 and each factor is at two levels. This report includes designs which are 1/64, 1/128, and 1/256 of a full replication.

Each design has a designation r. n. k., where r is the replication, n the number of factors, and k the number of experimental units per block. For example, plan 64.14.8 refers to a 1/64 replication of 14 factors in (32) blocks of 8 units each.

A main effect or interaction is said to be measurable if it is confounded only with higher-order interactions. In all designs, the main effects of each of the factors are confounded with interactions involving three or more factors. The information as to which two-factor interactions are not measurable is given for each design. Those two-factor interactions lost in choosing the fraction of treatment combinations are indicated under the heading "Without blocks." Additional two-factor interactions lost as a result of blocking are given under the heading "With blocks."

In all designs, capital letters are used to refer to factors, their main effects, and interactions. The lower case letters are used to denote the various treatment combinations and indicate the levels of each factor applied; the

absence of a letter indicates application of the lower level of the given factor, and the presence of a letter indicates the application of the upper level of that factor. For example, in a design of five factors A, B, C, D, E; the treatment combination acd denotes the higher level of A, C and D, and the lower level of B and E. The symbol "(1)" denotes the treatment combination consisting of the low levels of all the factors. These are the usual conventions, cf. Cochran and Cox [2], Davies [3], and Kempthorne [4] .

With each design is given the fundamental identity (denoted by I) used to choose a subset of the set of all possible treatment combinations. Also given are the block confounding relationships used to subdivide this subset of treatment combinations into experimental blocks. The equal signs are read, "is confounded with." Additional details concerning the construction and statistical analysis of the designs may be found in the references cited above.

In each design, all the treatment combinations of the first block are given explicitly. One treatment combination for every other block is given. This enables the remaining treatment combinations for any other block to be found by multiplying the first treatment of the block by all the treatment combinations in the first block. Multiplication, which is commutative, is defined such that $a \cdot b = ab$, $a \cdot a = 1$, and $1 \cdot a = a$. For example, the treatment combinations in block 2 of plan 64.14.8 are found by multiplying the corresponding treatments in block 1 by abfhkl:

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abfhkl	aghjmo
abcdfmn	acdgjklno
bceghln	cefhjkmno
bdegkm	defjlo

These designs represent a considerable effort on the part of the authors to retain as many measurable two-factor interactions as possible while at the same time allowing main effects to be only confounded with three-factor or higher order interactions. The designs have been carefully checked but there is always a possibility of error. We would appreciate hearing from anyone who improves the designs or finds errors in them.

The authors wish to express their thanks to Mr. Marvin Zelen who supervised this project, and to Mrs. Mary E. McKinley and Miss Caroline Yick for their painstaking efforts in typing the manuscript for reproduction.

Conversion of Two-level Designs to Four Levels.

It is possible to adapt the two-level designs in this catalogue to experiments in which the number of levels of the factors is a power of two, or where there is a mixture of levels each a power of two, such as experiments having some factors at two levels and some at four. To illustrate the conversion procedure, a 1/64 replication of 14 factors at two levels will be converted to a completely randomized four-level design

involving seven factors. The fundamental identity in plan 64.14.8 (and its
resulting treatment combinations) will be used without regard to the
grouping of treatments into blocks.

Let the seven new factors be denoted by α, β, γ, δ, Θ, ϕ and $\varkappa$, and
the four levels of each new factor be called the 0th (lowest) level, the
1st level, the 2nd level, and the 3rd (highest) level. To show the conven-
tions used in writing the four-level treatment combinations, the following
table has been made, (with the factors listed down the side and the levels
across the top):

Levels			
0th	1st	2nd	3rd
α $\quad \alpha^0 = (1)$	$\alpha' = \alpha$	α^2	α^3
β $\quad \beta^0 = (1)$	$\beta' = \beta$	β^2	β^3
γ $\quad \gamma^0 = (1)$	$\gamma' = \gamma$	γ^2	γ^3

(left side label: Factors)

The application of the 0th level of a factor x is denoted by x^0 or (1).
The application of the 1st level of x is denoted by x^1, or simply x,
and the applications of the 2nd and 3rd levels of x are denoted by
x^2 and x^3, respectively.

The treatment combination (1) represents that treatment where all
seven factors are at their lowest, or 0th, level. For all other treatment
combinations, the absence of a factor from the written treatment combination
indicates that that factor is present in its lowest level, and the (1) is

not written in the treatment. As an example, the treatment combination $\alpha^2 \gamma^3 \delta \phi$ implies $\alpha^2 \beta^0 \gamma^3 \delta^1 \theta^0 \phi^1 \psi^0$, and represents the combination of the 0th levels of β , θ , and ψ , the 1st levels of δ and ϕ , the 2nd level of α , and the 3rd level of γ . (It should be noted here that in the two-level designs capital letters were used to refer to the factors and their main effects and interactions, and lower case letters to treatment combinations, but in the four-level designs a given letter can refer to any of these).

Now if the fourteen factors of the two-level design are thought of as "pseudofactors" of the proposed four-level plan, a correspondence can be set up by grouping the pseudofactors in pairs, one pair with each four-level factor. One way to group them would be $[A, B: \alpha]$, (read "A and B with α"), $[C, D: \beta]$, $[E, F: \gamma]$, etc. As will be seen later, however, this is not the best grouping in this case, and instead the following groupings have been used:

A,J: α E,L: θ
B,C: β K,N: ϕ
D,G: γ F,M: ψ
H,O: δ

Now a correspondence must be set up between the four-level treatment combinations and the treatment combinations of the pseudofactors. This is done as follows. The application of the 0th level of factor α may be thought of as the application of the lower levels of both of the pseudofactors A and J, the application of the 1st level of α as the application of the upper level of pseudofactor A, the 2nd level of α as the upper level

of J, and the 3rd and highest level as the upper levels of both A and J.
In other words, α^0 or (1) in the four-level design will correspond to $\underline{a^0 j^0}$
or (1) in the two-level design, designating the lower level of both pseudo-
factors A and J, α will correspond to $\underline{a}$, α^2 to $\underline{j}$, and α^3 to $\underline{aj}$. This is
done in a similar manner for the other factors, as has been done in the
following table:

Four-level Treatments in Terms of Pseudofactors

		Levels			
		0th	1st	2nd	3rd
Factors	α	(1)	$\alpha = a$	$\alpha^2 = j$	$\alpha^3 = aj$
	β	(1)	$\beta = b$	$\beta^2 = c$	$\beta^3 = bc$
	γ	(1)	$\gamma = d$	$\gamma^2 = g$	$\gamma^3 = dg$
	δ	(1)	$\delta = h$	$\delta^2 = o$	$\delta^3 = ho$
	θ	(1)	$\theta = e$	$\theta^2 = l$	$\theta^3 = el$
	ϕ	(1)	$\phi = k$	$\phi^2 = n$	$\phi^3 = kn$
	ψ	(1)	$\psi = f$	$\psi^2 = m$	$\psi^3 = fm$

Using this table we can "express" any four-level treatment combination in
pseudofactors having two levels, and vice-versa. For example (1), represent-
ing the lower level of all fourteen pseudofactors, corresponds to the (1)
representing the 0th level of the seven four-level factors; $\alpha \gamma^3 \delta^2 \psi$,
representing the 0th level of factors β, θ, and ϕ, the 1st level of factors
α and ψ , the 2nd level of δ and the 3rd level of γ , corresponds to
adgof, or adfgo. The two-level treatment combination abdefghl corresponds
to $\alpha \beta \gamma^3 \delta \theta^3 \psi$. This last translation can be done in either of two ways:
either by taking each two-level letter in order and writing the correspond-
ing four-level letter, $\alpha \beta \gamma \theta \psi \gamma^2 \delta \theta^2$, and multiplying algebraically

giving $\alpha\beta\gamma^3\delta\,\theta^3\,\psi$, or by noting the pairs that occur in the two-level treatment, in this case dg and el , and immediately writing the highest levels of the corresponding four-level factors, $\gamma^3\theta^3$.

Thus there is a one to one correspondence between the two-level and the four-level treatment combinations. Therefore the 256 two-level treatment combinations, when "translated" into four-level combinations, will give the proper subset for the 1/64 replicate of a 4^7 factorial design.

One may either obtain all 256 treatments in terms of pseudofactors and then translate them one at a time into four-level terms, or translate only the treatments of the initial block and the block multipliers. If the latter method is used, multiplication by the block multipliers must follow these rules:

$$x^0 \times x = x$$
$$x^0 \times x^2 = x^2$$
$$x^0 \times x^3 = x^3$$
$$x \times x^2 = x^3$$
$$x \times x^3 = x^2$$
$$x^2 \times x^3 = x$$
$$x \times x = x^2 \times x^2 = x^3 \times x^3 = x^0 = (1)$$

Given below are the eight treatments in the initial block obtained by translating the treatments, (of the two-level design), and the second group of eight treatments found by multiplying by abfhkl $= \alpha\beta\delta\,\theta^2\phi\,\psi$:

(1)	$\alpha\beta\delta\,\theta^2\phi\,\psi$
$\beta^2\gamma\delta\,\theta^2\phi^3\,\psi^2$	$\alpha\beta^3\gamma\,\phi^2\,\psi^3$
$\alpha\beta^2\gamma^2\theta\,\phi^3\,\psi$	$\beta^3\gamma^2\delta\,\theta^3\phi^2$
$\alpha\gamma^3\delta\,\theta^3\,\psi^3$	$\beta\gamma^3\theta\,\phi\,\psi^2$
$\alpha^2\beta\gamma^2\delta^2\theta^2\phi\,\psi^3$	$\alpha^3\gamma^2\delta^3\,\psi^2$
$\alpha^2\beta^3\gamma^3\delta^3\,\phi^2\psi$	$\alpha^3\beta^2\gamma^3\delta^2\theta^2\phi^3$
$\alpha^3\beta^3\delta^2\theta^3\phi^2\,\psi^2$	$\alpha^2\beta^2\delta^3\theta\,\phi^3\,\psi^3$
$\alpha^3\beta\gamma\delta^3\theta\,\phi$	$\alpha^2\gamma\delta^2\theta^3\,\psi$

The block groupings here are used only as a convenient means of generating all 256 treatment combinations. The treatments are of course not grouped into blocks in this completely randomized design.

In the four-level designs, as in the two-level designs, the fundamental identity is used to determine what information is lost through confounding. In the two-level designs each main effect and each two-factor interaction, having only one degree of freedom, was either completely measurable or completely lost. In the four-level design, each of the seven main effects has three degrees of freedom and each of the twenty-one two-factor inter-actions has nine degrees of freedom. The three degrees of freedom for main effects can be broken up into three individual degrees of freedom, given by the "main effects" of the two pseudofactors and their "interaction", e.g. for α by A, J, and AJ. Similarly the nine degrees of freedom for each two-factor interaction can be broken up into nine individual degrees of freedom, given by multiplying the individual degrees of freedom of the main effects of the two factors, e.g., for $\alpha\beta$ by (A,J,AJ)x(B,C,BC) or AB, AC, ABC, BJ, CJ, BCJ, ABJ, ACJ, and ABCJ.

Each individual degree of freedom may or may not be measurable, so it is possible to have partial information on a main effect or two-factor interaction. To determine which are measurable, the aliases of each are found from the fundamental identity. For example, by checking AJ through the first few terms of the identity:

$$AJ = BCDJO = BEFJLNO = ACDEFJLN = \ldots$$

it is seen that it is confounded with BCDJO (from $\alpha\beta\gamma\delta$), BEFJLNO (from $\alpha\beta\delta\theta\phi\psi$) and ACDEFJLN (from $\alpha\beta\gamma\theta\phi\psi$). If AJ is checked through each remaining term of the identity, it will be found that in all cases its aliases are composed of three-factor or higher inter-actions. Therefore AJ is termed measurable. Similarly, the other twenty individual degrees of freedom for main effects are measurable. However, when the degrees of freedom for the two-factor interactions are checked, it is found that some are confounded with each other and hence are not measurable. The following table shows what information is measurable, and how lost information is confounded:

<u>Summary of Confounding in 1/64 x 4^7</u>

Main effects	measurable degrees of freedom	Lost d.f.	confounded with:	from 2-factor interaction:
α	A, J, AJ	none		
β	B, C, BC	"		
γ	D, G, DG	"		
δ	H, O, HO	"		
θ	E, L, EL	"		
ϕ	K, N, KN	"		
ψ	F, M, FM	"		
2-Factor inter-actions				
$\alpha\beta$	AB, CJ, ACJ, ABCJ	AC ABC BJ BCJ ABJ	EFM DO DEGL EH FHM, KL	$\theta\psi$ $\gamma\delta$ $\gamma\theta$ $\delta\theta$ $\delta\psi, \theta\phi$

2-Factor interactions	measurable degrees of freedom	Lost d.f.	confounded with:	from 2-factor interaction:
αγ	AG, DJ, AGJ, ADGJ	AD ADG GJ DGJ ADJ	BCO EK HKO BEL EHO	βδ θφ δφ βθ δθ
αδ	AH, AHO, JO, AJO	AO HJ HJO AHJ AHJO	BCD BCE GK BFM DE	βγ βθ γφ βψ γθ
αθ	AL, AEL, JL, AEJL	AE EJ EJL AEJ AJL	CFM, DGK BCH BDG DHO BK	βψ, γφ βδ βγ γδ βφ
αφ	AN, AKN, JN, JKN, AJN, AJKN	AK JK AJK	DEG GHO BL	γθ γδ βθ
αψ	AF,AM,FJ,JM,FJM AFJ,AJM	AFM AFJM	CE BH	βθ βδ
βγ	BD,BG,CD,CG,BCG, BCDG	BDG BCD CDG	EJL AO HL, FKM	αθ αδ δθ, φψ
βδ	BO, BHO, CO, CHO	BH CH BCH BCO BCHO	AFJM DGL EJ AD FKN	αψ γθ αθ αγ φψ
βθ	BE, CEL, BCL, BCEL	EL BEL CE CL BCE	AJK DGJ AFM DGH HJ	αφ αγ αψ γδ αδ

2-Factor inter-actions	measurable degrees of freedom	Lost d.f.	confounded with:	from 2-factor interaction:
$\beta\phi$	BN, BKN, CN, CKN, BCK, BCN	BK CK BCKN	AJL DFGM FHO	$\alpha\theta$ $\gamma\psi$ $\delta\psi$
$\beta\psi$	BF, BM, CF, CM, BCM, BCFM	BFM CFM BCF	AHJ DGK, AE HKNO	$\alpha\delta$ $\gamma\phi, \alpha\theta$ $\delta\phi$
$\gamma\delta$	DH, GH, GO, DGO, DGHO	DO DHO GHO DGH	ABC AEJ JK CL	$\alpha\beta$ $\alpha\theta$ $\alpha\phi$ $\beta\theta$
$\gamma\theta$	DL, DEL, EG, GL	DE EGL DEG DGL DEGL	AHJO KMN AK CH BJ	$\alpha\delta$ $\phi\psi$ $\alpha\phi$ $\beta\delta$ $\alpha\beta$
$\gamma\phi$	DK, DN, DKN, GN, DGN, DGKN	GK GKN DGK	HJO ELM AE, CFM	$\alpha\delta$ $\theta\psi$ $\alpha\theta, \beta\psi$
$\gamma\psi$	DF, DM, DFM, FG, FGM, DFG, DGM	GM DFGM	EKLN CK	$\theta\phi$ $\beta\phi$
$\delta\theta$	EHL, EO, LO, ELO, HLO, EHLO	EH HL EHO	BCJ CDG, FKM ADJ	$\alpha\beta$ $\beta\gamma, \phi\psi$ $\alpha\gamma$
$\delta\phi$	HN, HKN, KO, NO, KNO, HNO	HK HKO HKNO	FLM GJ BCF	$\theta\psi$ $\alpha\gamma$ $\beta\psi$
$\delta\psi$	FH, HM, FO, MO, FMO, HMO, FHMO	FHM FHO	KL, ABJ BCKN	$\theta\phi, \alpha\beta$ $\beta\phi$

2-Factor inter-actions	measurable degrees of freedom	Lost d.f.	confounded with:	from 2-factor interaction:
$\theta\phi$	EN, EKN, LN, KLN, EKL, ELN	EK KL EKLN	ADG ABJ, FHM GM	$\alpha\gamma$ $\alpha\beta$, $\delta\psi$ $\gamma\psi$
$\theta\psi$	EF, EM, FL, LM, EFL, EFLM	EFM FLM ELM	AC HK GKN	$\alpha\beta$ $\delta\phi$ $\gamma\phi$
$\phi\psi$	FK, KM, FN, MN, FMN, FKMN	FKM FKN KMN	HL, CDG BCHO EGL	$\delta\theta$, $\beta\gamma$ $\beta\delta$ $\gamma\theta$

In general it is possible to adapt any two-level design of the form 2^{2m} into a four-level design of form 4^m. The two-level factors should be paired off so as to furnish, if possible, all information on main effects, and then to provide as much information as possible on two-factor interactions, as determined by the fundamental identity. (To illustrate the importance of proper choice, consider the term AB. If we had grouped our letters $[AB:\alpha]$, $[CD:\beta]$, etc., AB would have been part of the main effect of factor α, and the first term of the identity shows AB to be confounded with CDO, part of a two-factor interaction. Thus at least one degree of freedom for a main effect would not have been measurable.) Finally after pairing the two-level factors, the treatment combinations of the two-level design are translated into terms of the four-level factors.

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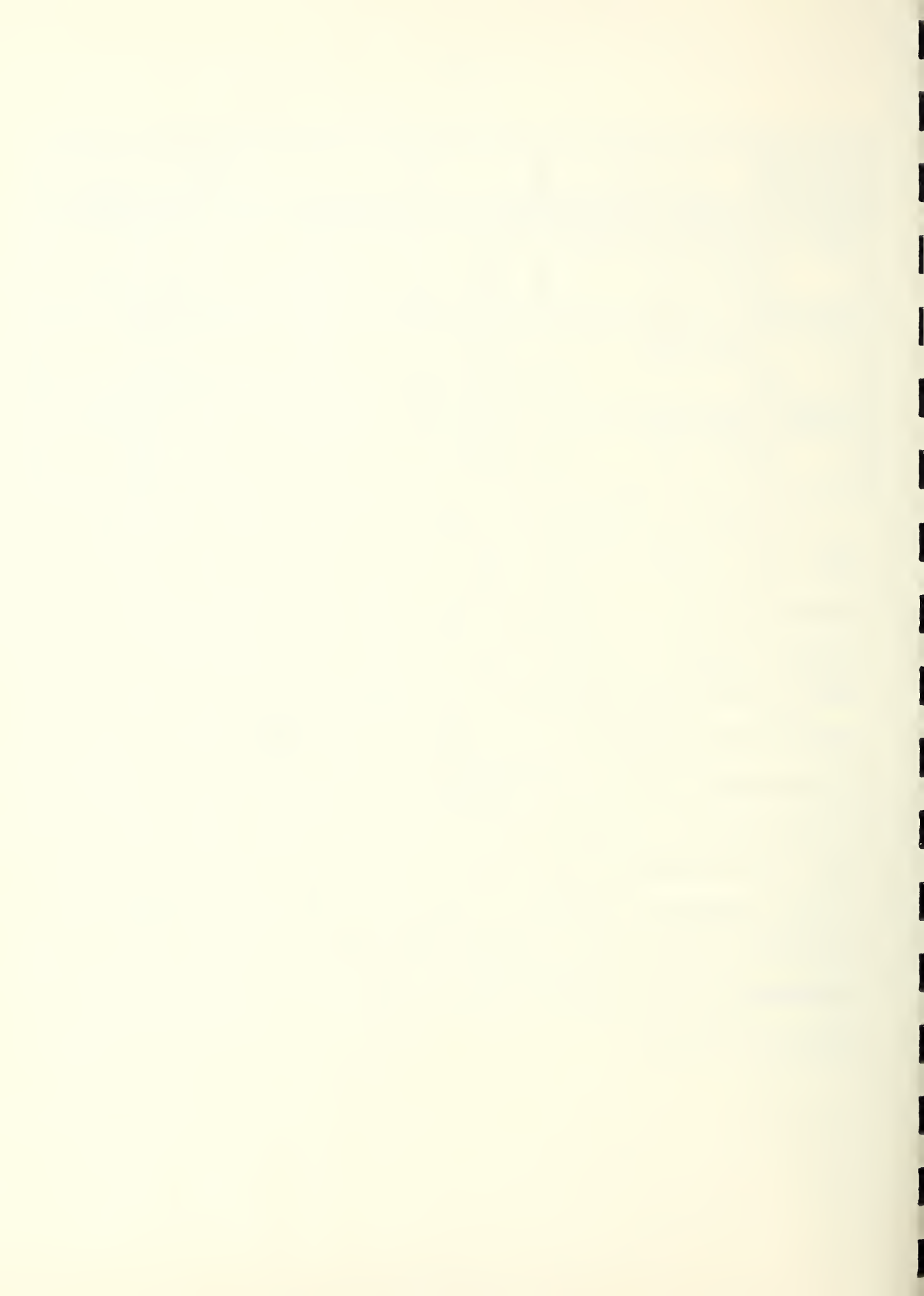
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Plan 64.12.8. 1/64 replication of 12 factors in 8 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M.

I = ABCD = ABEFL = CDEFL = ABGHL = CDGHL = EFGH = ABCDEFGH = ABJKL = CDJKL

  = EFJK = ABCDEFJK = GHJK = ABCDGHJK = ABEFGHJKL = CDEFGHJKL = ACEGJL

  = BDEGJL = BCFGJ = ADFGJ = BCEHJ = ADEHJ = ACFHJL = BDFHJL = BCEGK

  = ADEGK = ACFGKL = BDFGKL = ACEHKL = BDEHKL = BCFHK = ADFHK = ADLM = BCLM

  = BDEFM = ACEFM = BDGHM = ACGHM = ADEFGHLM = BCEFGHLM = BDJKM = ACJKM

  = ADEFJKLM = BCEFJKLM = ADGHJKLM = BCGHJKLM = BDEFGHJKM = ACEFGHJKM

  = CDEGJM = ABEGJM = ABCDFGJLM = FGJLM = ABCDEHJLM = EHJLM = CDFHJM

  = ABFHJM = ABCDEGKLM = EGKLM = CDFGKM = ABFGKM = CDEHKM = ABEHKM

  = ABCDFHKLM = FHKLM.
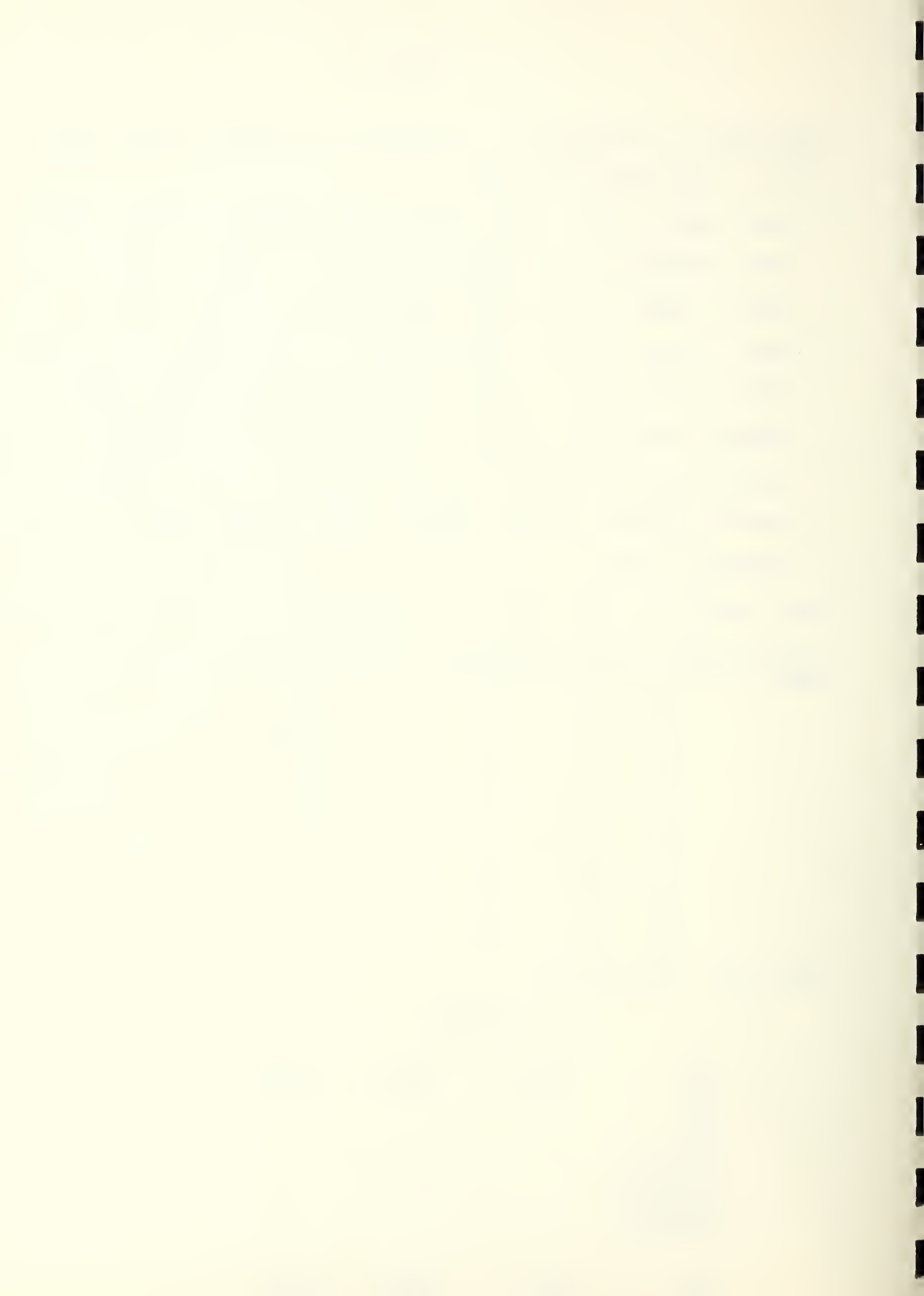
Block confounding: AB, AC, BC, CL, ABCL, AL, BL.

Without blocks: All main effects, and all two-factor interactions
except the following are measurable:

AB,	AC,	AD,	AL,	AM,
BC,	BD,	BL,	BM,	CD,
CL,	CM,	DL,	DM,	EF,
EG,	EH,	EJ,	EK,	FG,
FH,	FJ,	FK,	GH,	GJ,
GK,	HJ,	HK,	JK,	LM.

With blocks: Same as above.

Blocks:

1 (I)	2 abcd	3 cdfhkl	4 abfhkl
efgh			
ghjk			
efjk			
abcdehklm			
abcdfgklm			
abcdegjlm			
abcdfhjlm			

5 acghl	6 bdghl	7 adfgk	8 bcfgk

Plan 64.12.16. 1/64 replication of 12 factors in 4 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M.

I: Same as plan 64.12.8.

Block confounding: AB, AC, BC.

Without blocks: All main effects, and all two-factor interactions except the following are measurable:

AB,AC,AD,AL,AM,BC,BD,BL,

BM,CD,CL,CM,DL,DM,EF,EG,

EH,EJ,EK,FG,FH,FJ,FK,

GH,GJ,GK,HJ,HK,JK,LM.

With blocks: Same as above.

Blocks:

1 (I)	2 cdfhkl
efgh	
ghjk	3 acghl
efjk	
abcdehklm	
abcdfgklm	4 adfgk
abcdegjlm	
abcdfhjlm	
abcd	
abcdefgh	
abcdghjk	
abcdefjk	
ehklm	
fgklm	
egjlm	
fhjlm	

<u>Plan 64.12.32.</u> 1/64 replication of 12 factors in 2 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M.

I: Same as plan 64.12.8.

Block confounding: AB.

Without blocks: All main effects, and all two-factor interactions <u>except</u>
the following are measurable:

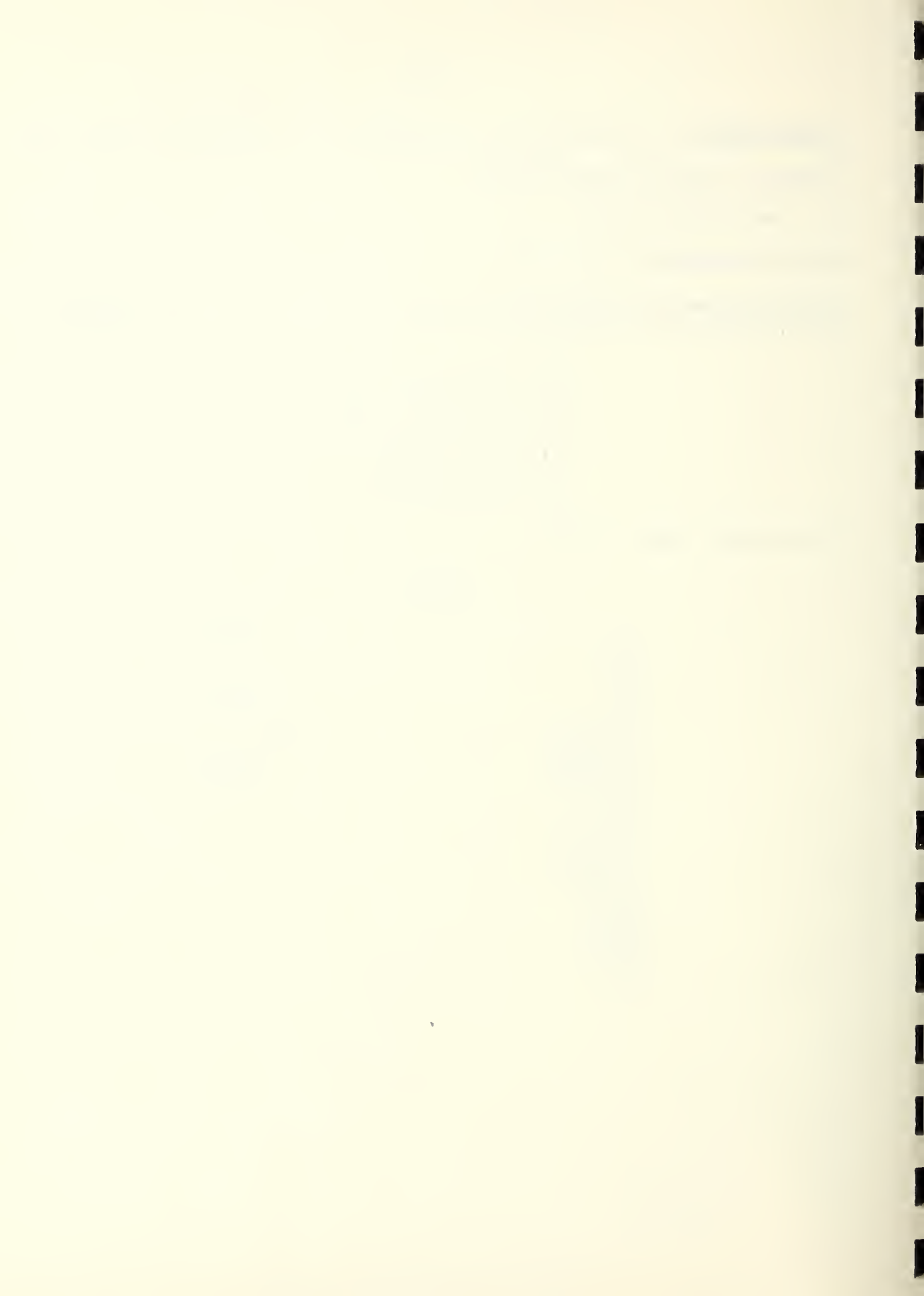$$AB,AC,AD,AL,AM,BC,BD,BL,$$

$$BM,CD,CL,CM,DL,DM,EF,EG,$$

$$EH,EJ,EK,FG,FH,FJ,FK,$$

$$GH,GJ,GK,HJ,HK,JK,LM.$$

With blocks: Same as above.

<div align="center">

<u>Blocks</u>

</div>

<u>1</u> (Ī)	<u>2</u> acghl
efgh	
ghjk	
efjk	
abcdehklm	
abcdfgklm	
abcdegjlm	
abcdfhjlm	
abcd	
abcdefgh	
abcdghjk	
abcdefjk	
ehklm	
fgklm	
egjlm	
fhjlm	
cdfhkl	
cdegkl	
cdfgjl	
cdehjl	
abefm	
abghm	
abefghjkm	
abjkm	
abfhkl	
abegkl	
abfgjl	
abehjl	
cdefm	
cdghm	
cdefghjkm	
cdjkm	
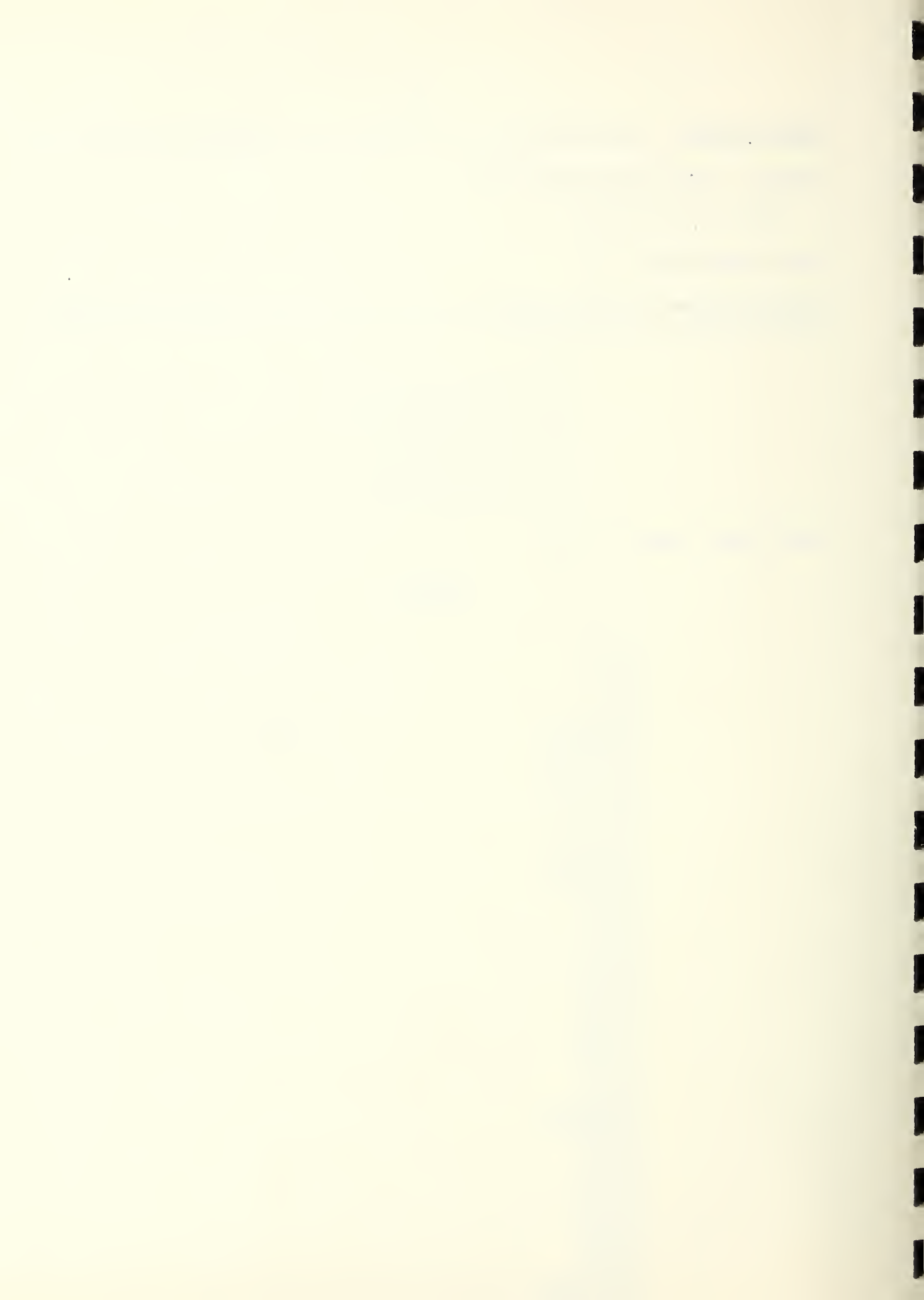
Plan 64.13.8. 1/64 replication of 13 factors in 16 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N.

I = ABCD = ABEFLN = CDEFLN = ABGHL = CDGHL = EFGHN = ABCDEFGHN = ABJKL

   = CDJKL = EFJKN = ABCDEFJKN = GHJK = ABCDGHJK = ABEFGHJKLN = CDEFGHJKLN

   = ACEGJL = BDEGJL = BCFGJN = ADFGJN = BCEHJ = ADEHJ = ACFHJLN = BDFHJLN

   = BCEGK = ADEGK = ACFGKLN = BDFGKLN = ACEHKL = BDEHKL = BCFHKN = ADFHKN

   = ADLMN = BCLMN = BDEFM = ACEFM = BDGHMN = ACGHMN = ADEFGHLM = BCEFGHLM

   = BDJKMN = ACJKMN = ADEFJKLM = BCEFJKLM = ADGHJKLMN = BCGHJKLMN = BDEFGHJKM

   = ACEFGHJKM = CDEGJMN = ABEGJMN = ABCDFGJLM = FGJLM = ABCDEHJLMN = EHJLMN

   = CDFHJM = ABFHJM = ABCDEGKLMN = EGKLMN = CDFGKM = ABFGKM = CDEHKMN

   = ABEHKMN = ABCDFHKLM = FHKLM.

Block confounding: AB,AC,BC,ABCFN,CFN,BFN,AFN,ABCDEFGJKLMN,CDEFGJKLMN,

BDEFGJKLMN,ADEFGJKLMN,DEGJKLM,ABDEGJKLM,ACDEGJKLM,BCDEGJKLM.

Without blocks: All main effects, and all two-factor interactions <u>except</u>
the following are measurable:
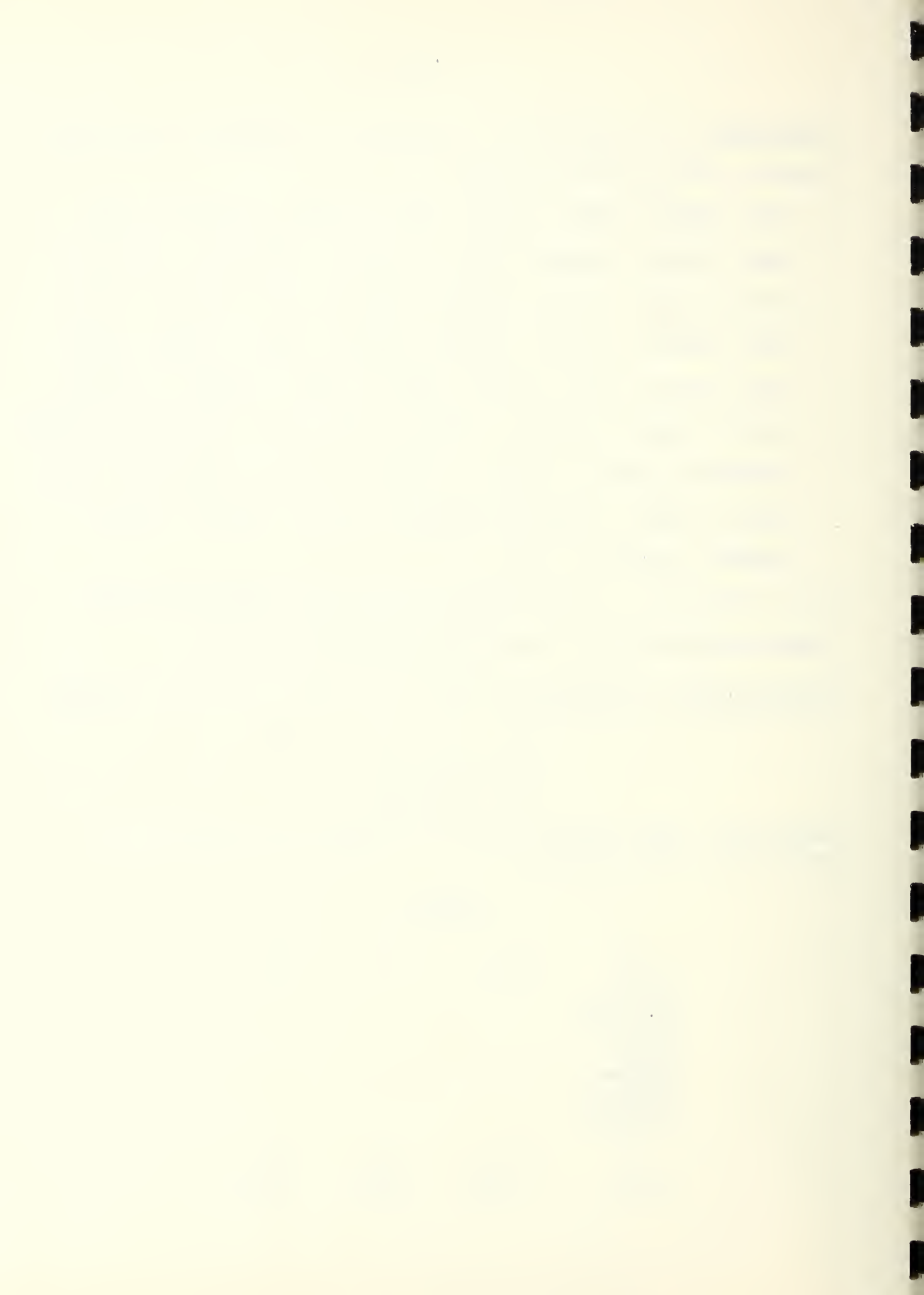
<div align="center">

AB,AC,AD,BC,BD,CD,

GH,GJ,GK,HJ,HK,JK.

</div>

With blocks: Same as above, except the two-factor interactions FJ,GN,
and HM also are <u>not</u> measurable.

<div align="center">

<u>Blocks:</u>

</div>

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	ghjk	efjk	efgh
ehklm			
abcdfhjlm			
abcdefjk			
efgjln			
fghjkmn			
abcdeghmn			
abcdgkln			

<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
cdfgjl	cdfhkl	cdegkl	cdehjl

Blocks (Continued):

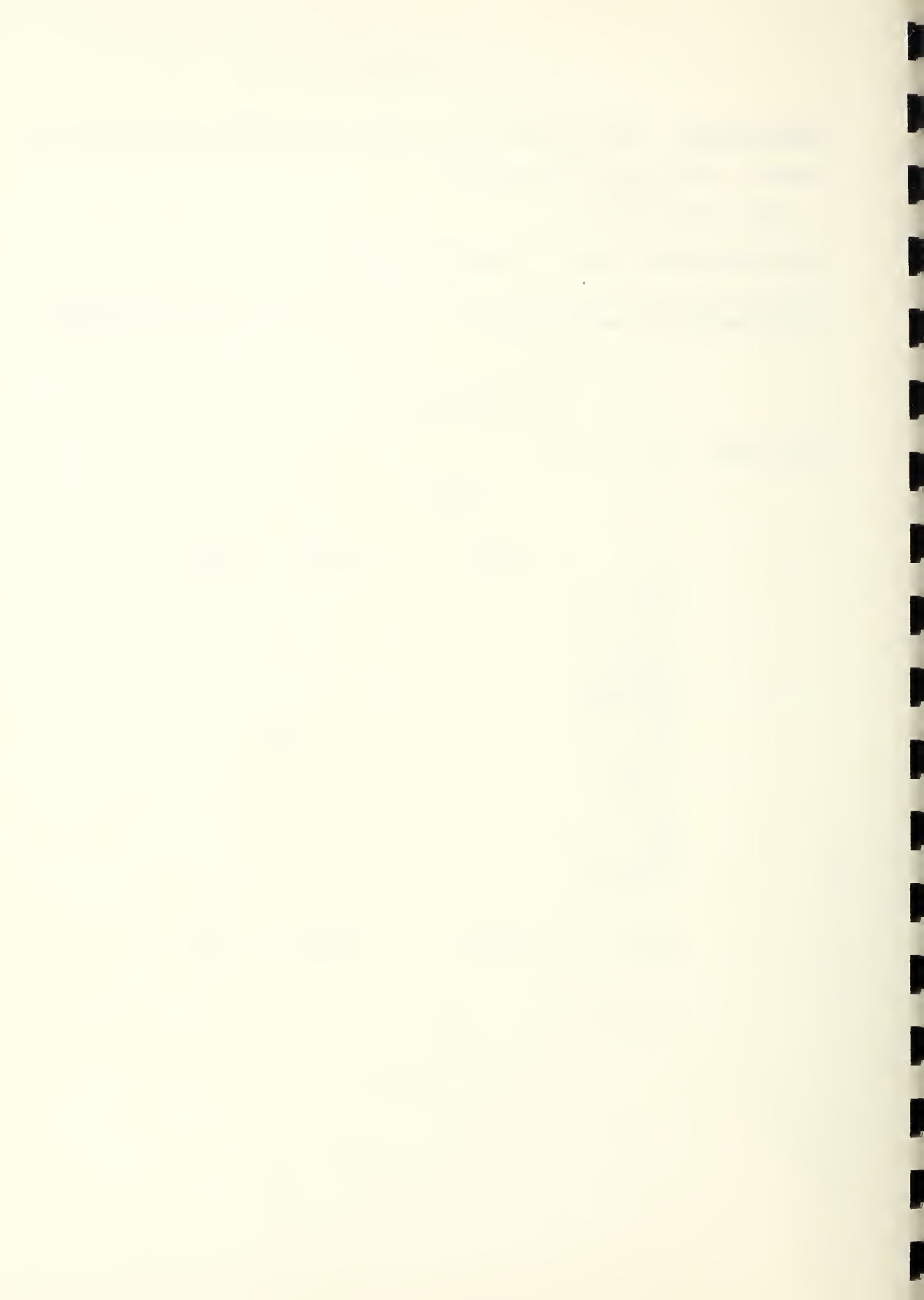| 9	10	11	12
acghl	acjkl	acefghjkl	acefl

13	14	15	16
adfhj	adfgk	adehk	adegj

Plan 64.13.32. 1/64 replication of 13 factors in 4 blocks at 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N.

I: Same as plan 64.13.8.

Block confounding: AB,AC,BC.

Without blocks: All main effects, and all two-factor interactions except the following are measurable:

AB,AC,AD,BC,BD,CD,

GH,GJ,GK,HJ,HK,JK.

With blocks: Same as above.

Blocks:

1		2
(1)	efjk	cdfgjl
ehklm	fhjlm	
abcdfhjlm	abcdehklm	
abcdefjk	abcd	
efgjln	gkln	
fghjkmn	eghmn	
abcdeghmn	abcdfghjkmn	
abcdgkln	abcdefgjln	
ghjk	efgh	
egjlm	fgklm	
abcdfgklm	abcdegjlm	
abcdefgh	abcdghjk	
efhkln	hjln	
fmn	ejkmn	
abcdejkmn	abcdfmn	
abcdhjln	abcdefhkln	

3	4
acghl	adfhj

Plan 64.13.64. 1/64 replication of 13 factors in 2 blocks of 64 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N.

I: Same as plan 64.13.8.

Block Confounding: AB.

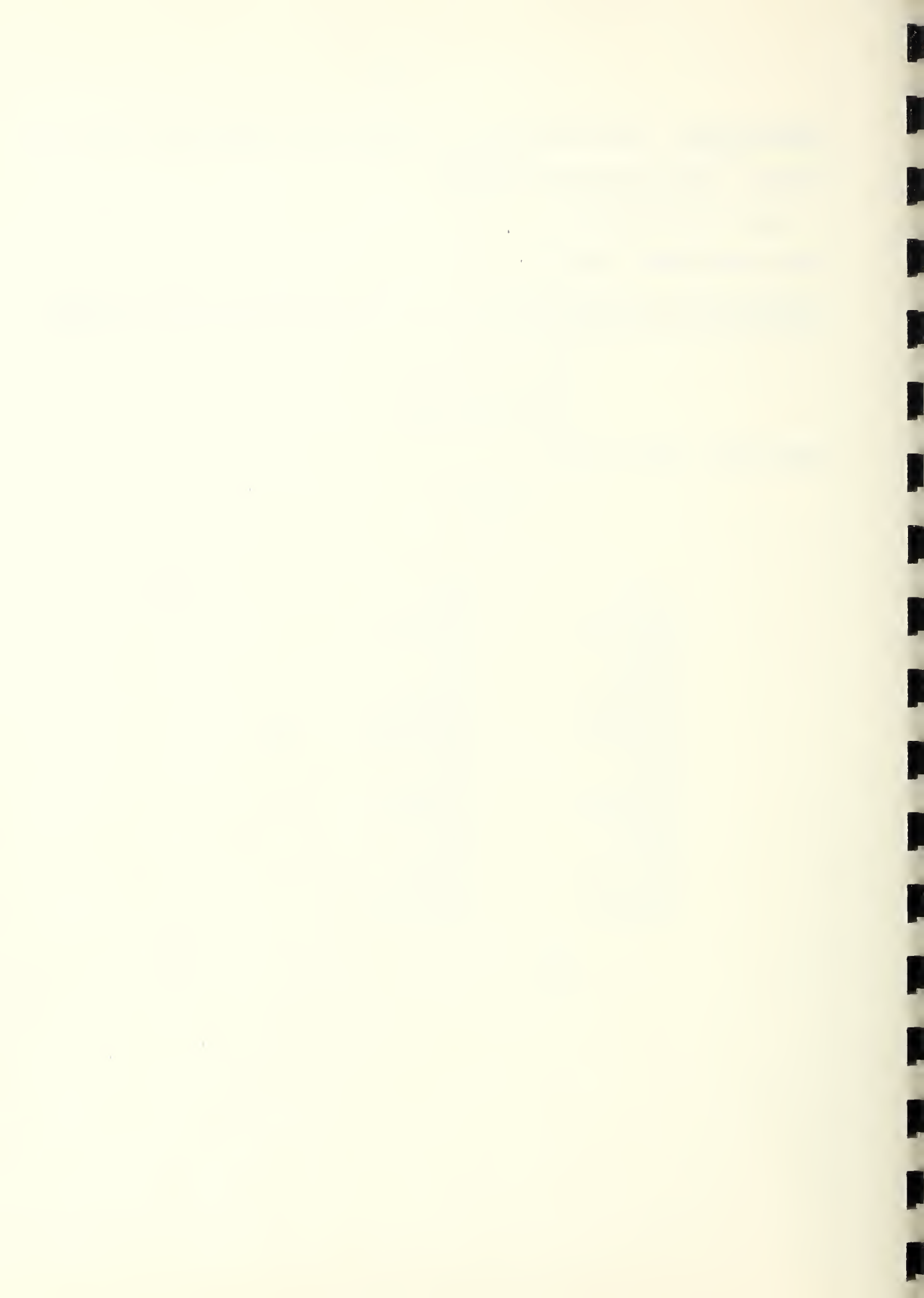Without blocks: All main effects, and all two-factor interactions except the following are measurable:

AB,AC,AD,BC,BD,CD,

GH,GJ,GK,HJ,HK,JK.

With blocks: Same as above.

Blocks:

1

(1)	efjk̄	cdfgjl	cdegkl
ehklm	fhjlm	cdefghjkm	cdghm
abcdfhjlm	abcdehklm	abghm	abefghjkm
abcdefjk	abcd	abegkl	abfgjl
efgjln	gkln	cden	cdfjkn
fghjkmn	eghmn	cdhklmn	cdefhjlmn
abcdeghmn	abcdfghjkmn	abefhjlmn	abhklmn
abcdgkln	abcdefgjln	abfjkn	aben
ghjk	efgh	cdfhkl	cdehjl
egjlm	fgklm	cdefm	cdjkm
abcdfgklm	abcdegjlm	abjkm	abefm
abcdefgh	abcdghjk	abehjl	abfhkl
efhkln	hjln	cdeghjkn	cdfghn
fmn	ejkmn	cdgjlmn	cdefgklmn
abcdejkmn	abcdfmn	abefgklmn	abgjlmn
abcdhjln	abcdefhkln	abfghn	abeghjkn

2
acghl
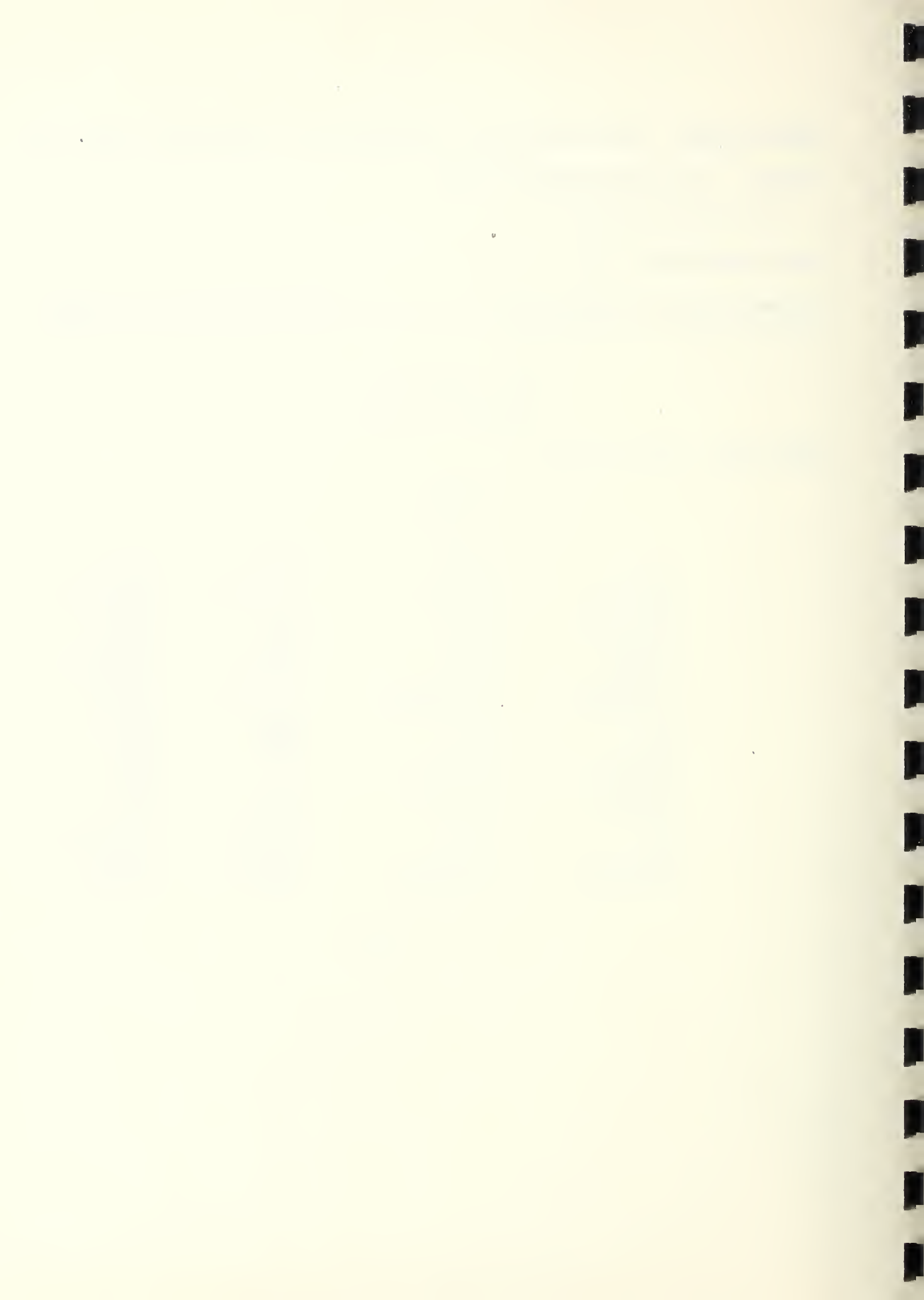
Plan 64.14.8. 1/64 replication of 14 factors in 32 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O.

I = ABCDO = ABEFLNO = CDEFLN = ABGHLO = CDGHL = EFGHN = ABCDEFGHNO = ABJKL

  = CDJKLO = EFJKNO = ABCDEFJKN = GHJKO = ABCDGHJK = ABEFGHJKLN = CDEFGHJKLNO

  = ACEGJLO = BDEGJL = BCFGJN = ADFGJNO = BCEHJ = ADEHJO = ACFHJLNO = BDFHJLN

  = BCEGKO = ADEGK = ACFGKLN = BDFGKLNO = ACEHKL = BDEHKLO = BCFHKNO = ADFHKN

  = ADLMN = BCLMNO = BDEFMO = ACEFM = BDGHMNO = ACGHMN = ADEFGHLM = BCEFGHLMO

  = BDJKMN = ACJKMNO = ADEFJKLMO = BCEFJKLM = ADGHJKLMNO = BCGHJKLMN = BDEFGHJKM

  = ACEFGHJKMO = CDEGJMNO = ABEGJMN = ABCDFGJLM = FGJLMO = ABCDEHJLMN = EHJLMNO

  = CDFHJMO = ABFHJM = ABCDEGKLMNO = EGKLMN = CDFGKM = ABFGKMO = CDEHKMN

  = ABEHKMNO = ABCDFHKLMO = FHKLM.

Block confounding: ABEO, BCK, ACEKO, ABCM, CEMO, AKM, BEKMO, ACLO, BCEL,

ABKLO, EKL, BLMO, AELM, CKLMO, ABCEKLM, AE, BO, ABCEK, CKO, BCEM, ACMO, EKM,

ABKMO, CELO, ABCL, BEKLO, AKL, ABELMO, LM, ACEKLMO, BCKLM.

Without blocks: All main effects and all two-factor interactions are measurable.

With blocks: Same as above, except that the following two-factor inter-
actions   are not measurable: AE, BJ, BO, CN, DH, FG, LM, JO.

<u>Blocks</u>

1 (I)	2 abfhkl	3 cdjkm	4 abcdfhjlm
cdhklmn			
acefgkn			
adefghlm			
bfgjklmo			
bcdfghjno			
abcejlmno			
abdehjko			

5 fmn	6 abhklmn	7 cdfgkn	8 abcdhjln

9 ghjk	10 abfgjl	11 cdghm	12 abcdfgklm

13 fghjkmn	14 abgjlmn	15 cdfghn	16 abcdgkln

Blocks (Continued)

<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>
abcd$\overline{\text{e}}$ghmn	cde$\overline{\text{fg}}$klmn	abeg$\overline{\text{h}}$jkn	efg$\overline{\text{j}}$ln
<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>
abc$\overline{\text{de}}$fgh	cdeg$\overline{\text{k}}$l	abe$\overline{\text{fg}}$hjkm	eg$\overline{\text{j}}$lm
<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>
ab$\overline{\text{cd}}$ejkmn	cde$\overline{\text{f}}$hjlmn	a$\overline{\text{be}}$n	ef$\overline{\text{hk}}$ln
<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>
abc$\overline{\text{de}}$fjk	cd$\overline{\text{eh}}$jl	ab$\overline{\text{e}}$fm	e$\overline{\text{hk}}$lm
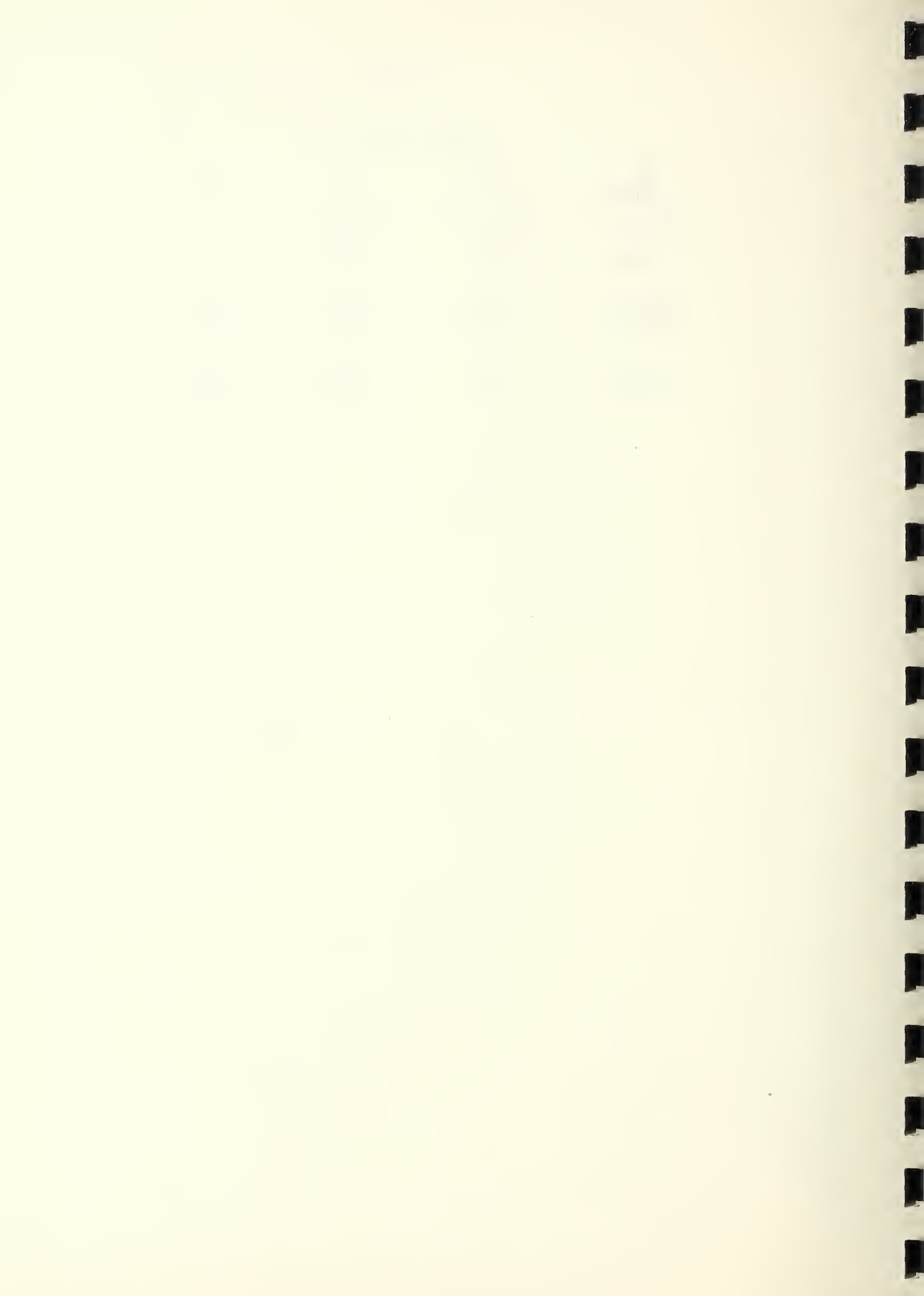
Plan 64.14.16. 1/64 replication of 14 factors in 16 blocks of 16 units each.
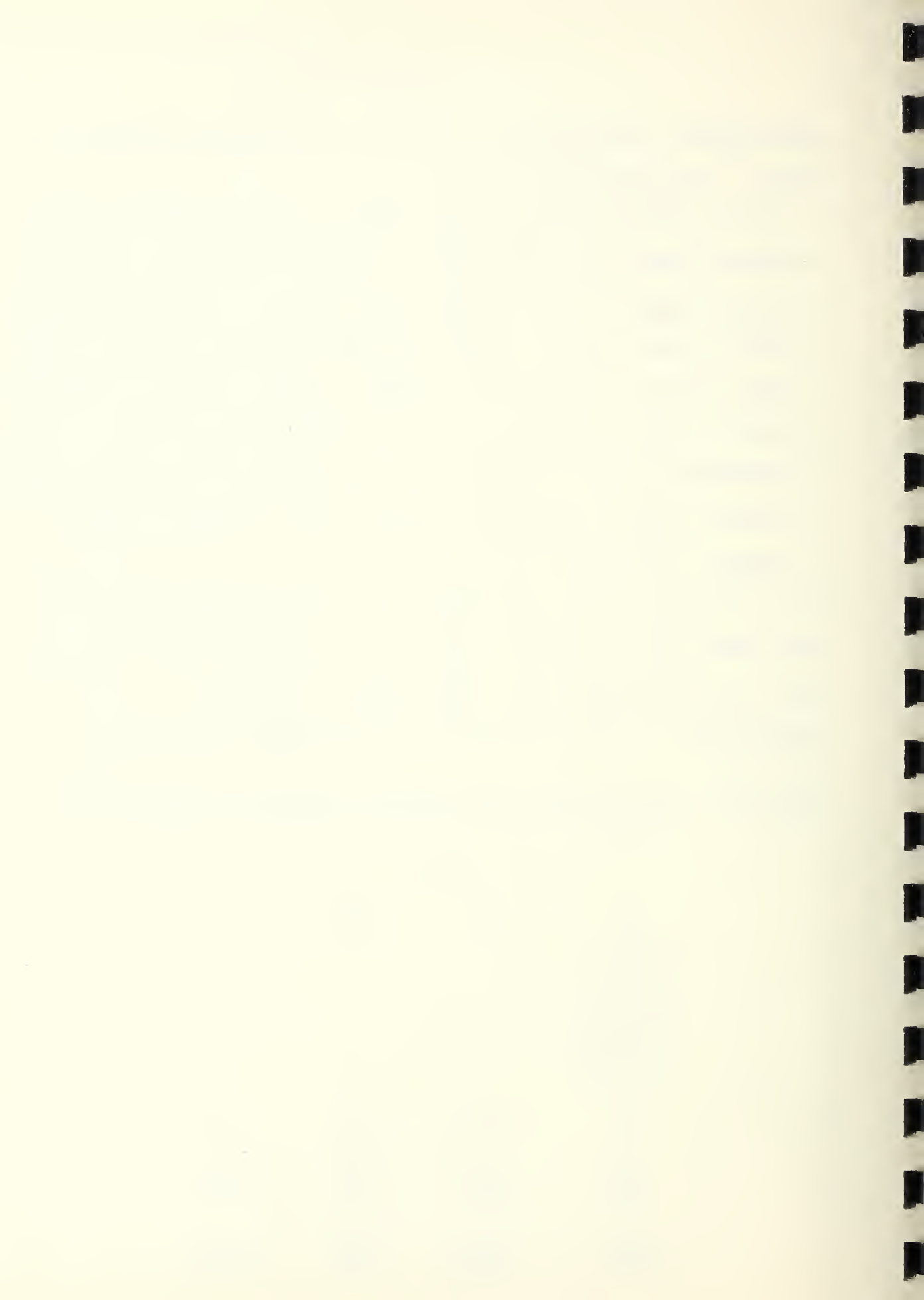
Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O.

I: Same as plan 64.14.8.

Block confounding: ABEO, BCK, ACEKO, ABCM, CEMO, AKM, BEKMO, ACLO, BCEL,

ABKLO, EKL, BLMO, AELM, CKLMO, ABCEKLM.

Without blocks: All main effects and all two-factor interactions are measurable.

With blocks: Same as above, except the following two-factor interactions
are not measurable:

CN, JO.

Blocks:

1		2
(1)	abfhkl	cd$\overline{jk}$m
cdhklmn	abcdfmn	
acefgkn	bceghln	
adefghlm	bdegkm	
bfgjklmo	aghjmo	
bcdfghjno	acdgjklno	
abcejlmno	cefhjkmno	
abdehjko	defjlo	

3	4	5	6
f$\overline{mn}$	cdf$\overline{jk}$n	g$\overline{h}$jk	cd$\overline{g}$hm

7	8	9	10
fgh$\overline{jk}$mn	cdf$\overline{gh}$n	abcde$\overline{gh}$mn	abeg$\overline{hj}$kn

11	12	13	14
abc$\overline{def}$gh	abe$\overline{fg}$hjkm	abcde$\overline{jk}$mn	ab$\overline{e}$n

15	16
abc$\overline{def}$jk	ab$\overline{ef}$m
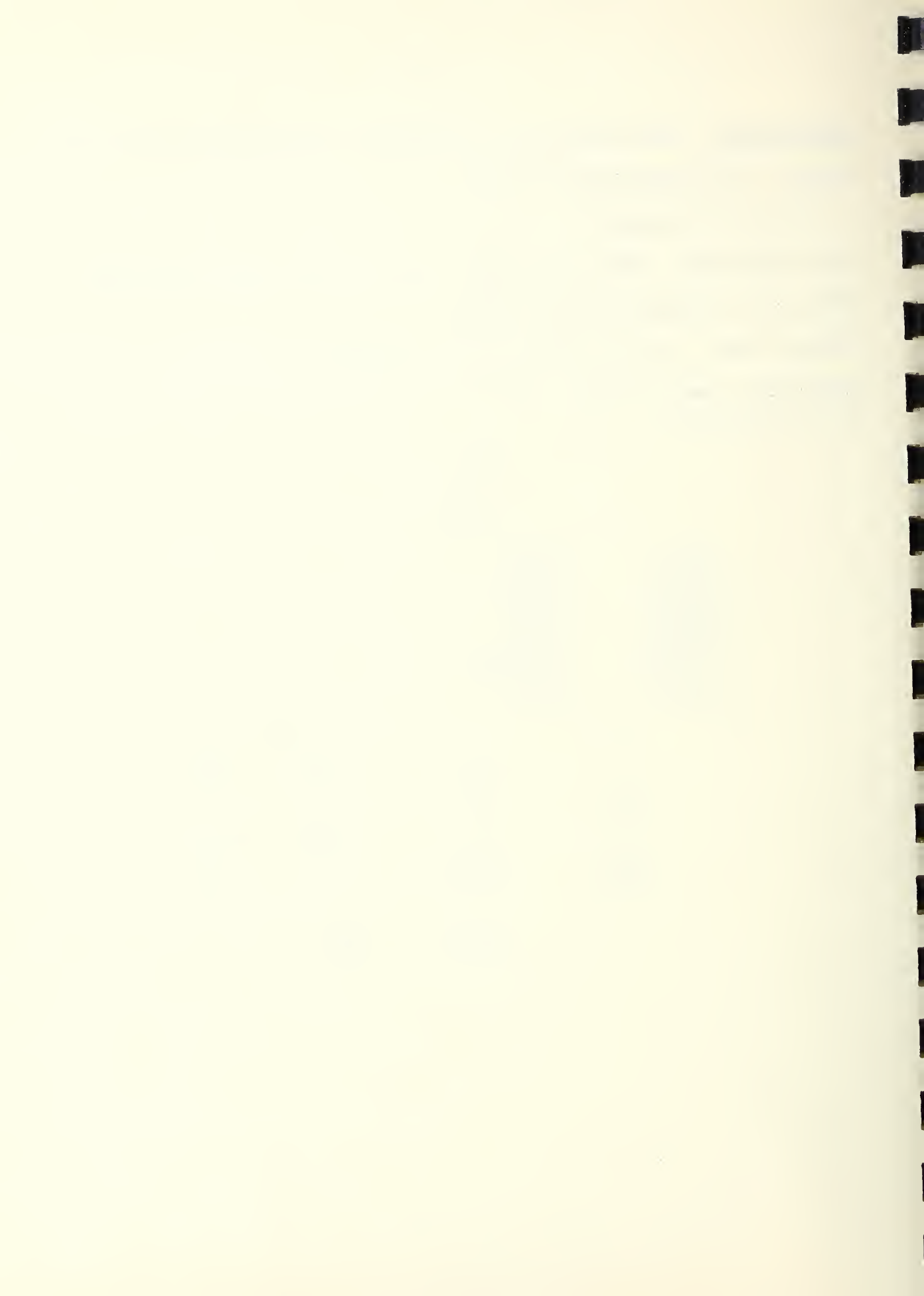
Plan 64.14.32. 1/64 replication of 14 factors in 8 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O.

I: Same as plan 64.14.8.

Block confounding: ABEO,BCK,ACEKO,ABCM,CEMO,AKM,BEKMO.

Without blocks: All main effects and two-factor interactions are measurable.

With blocks: Same as above.

Blocks:

1		2
(1)	cdjkm	f̄mn
cdhklmn	hjln	
acefgkn	adefgjmn	3
adefghlm	acefghjkl	ghj̄k
bfgjklmo	bcdfglo	
bcdfghjno	bfghkmno	4
abcejlmno	abdeklno	fghj̄kn
abdehjko	abcehmo	
abfhkl	abcdfhjlm	5
abcdfmn	abfjkn	abcdeghmn
bceghln	bdeghjklmn	
bdegkm	bcegj	6
aghjmo	acdghko	abcdefgh
acdgjklno	aglmno	
cefhjkmno	defhno	7
defjlo	cefklmo	abcdejkmn
		8
		abcdefjk

<u>Plan 64.14.64</u>. 1/64 replication of 14 factors in 4 blocks of 64 units each.
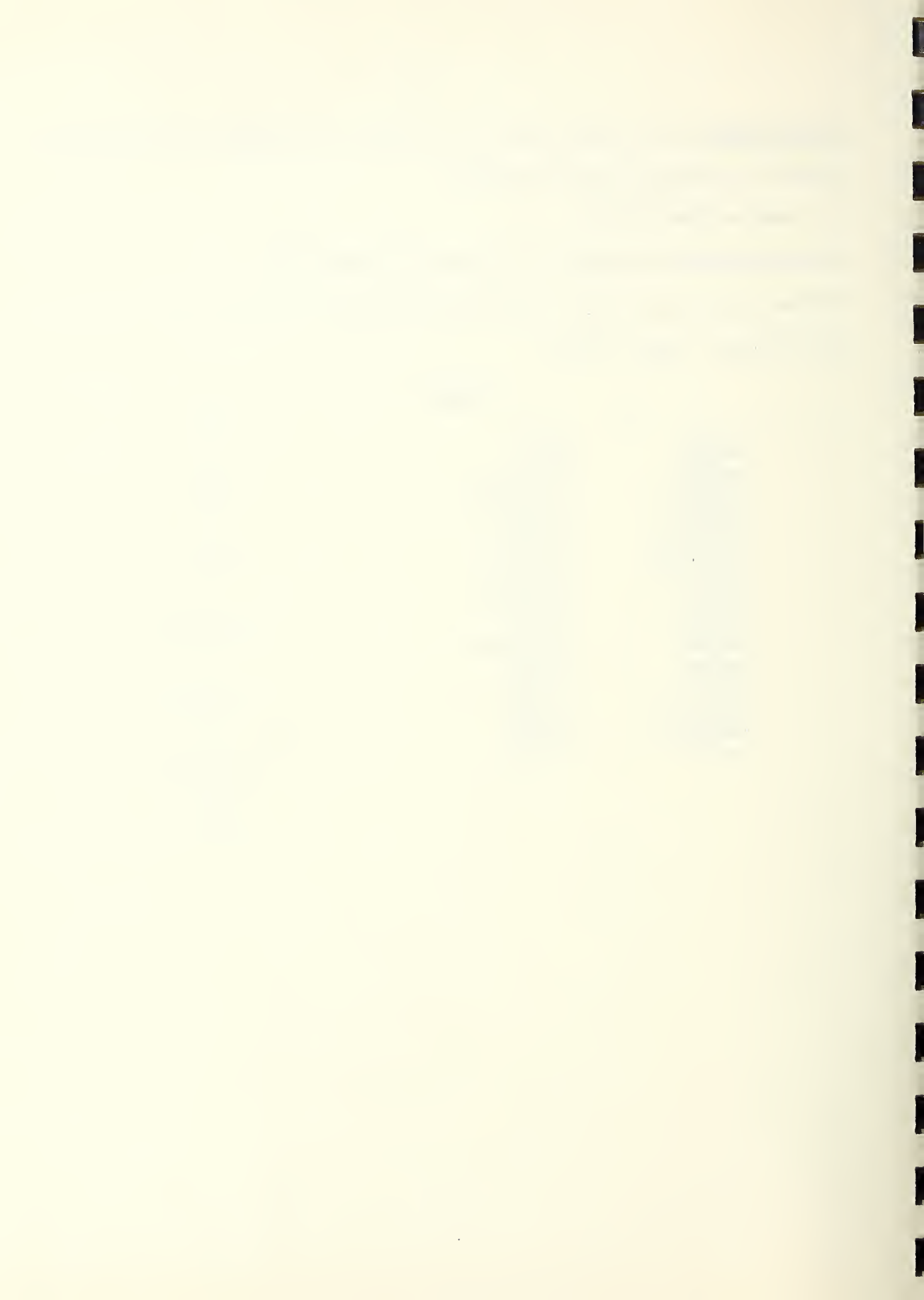
Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O.

I: Same as plan 64.14.8.

Block confounding: ABEO,BCK,ACEKO.

Without blocks: All main effects and all two-factor interactions are measurable.

With blocks: Same as above.

<div align="center">

Blocks

</div>

<u>1</u>		<u>2</u>
(1)	acegkm	ghjk
cdhklmn	adeghln	
acefgkn	bgjklno	<u>3</u>
adefghlm	bcdghjmo	abcdeghmn
bfgjklmo	abcefjlo	
bcdfghjno	abdefhjkmno	<u>4</u>
abcejlmno	abhklmn	abcdejkmn
abdehjko	abcd	
abfhkl	bcefghlm	
abcdfmn	bdefgkn	
bceghln	afghjno	
bdegkm	acdfgjklmo	
aghjmo	cehjko	
acdgjklno	dejlmno	
cefhjkmno	cdfjkn	
defjlo	fhjlm	
cdjkm	adegj	
hjln	aceghjklmn	
adefgjmn	bcdglmno	
acefghjkl	bghko	
bcdfglo	abdefklmo	
bfghkmno	abcefhno	
abdeklno	abcdhjln	
abcehmo	abjkm	
abcdfhjlm	bdefghjkl	
abfjkn	bcefgjmn	
bdeghjklmn	acdfghkmno	
bcegj	afglo	
acdghko	dehmo	
aglmno	ceklno	
defhno		
cefklmo		
fmn		
cdfhkl		

Plan 64.14.128. 1/64 replication of 14 factors in two blocks of 128 units each.
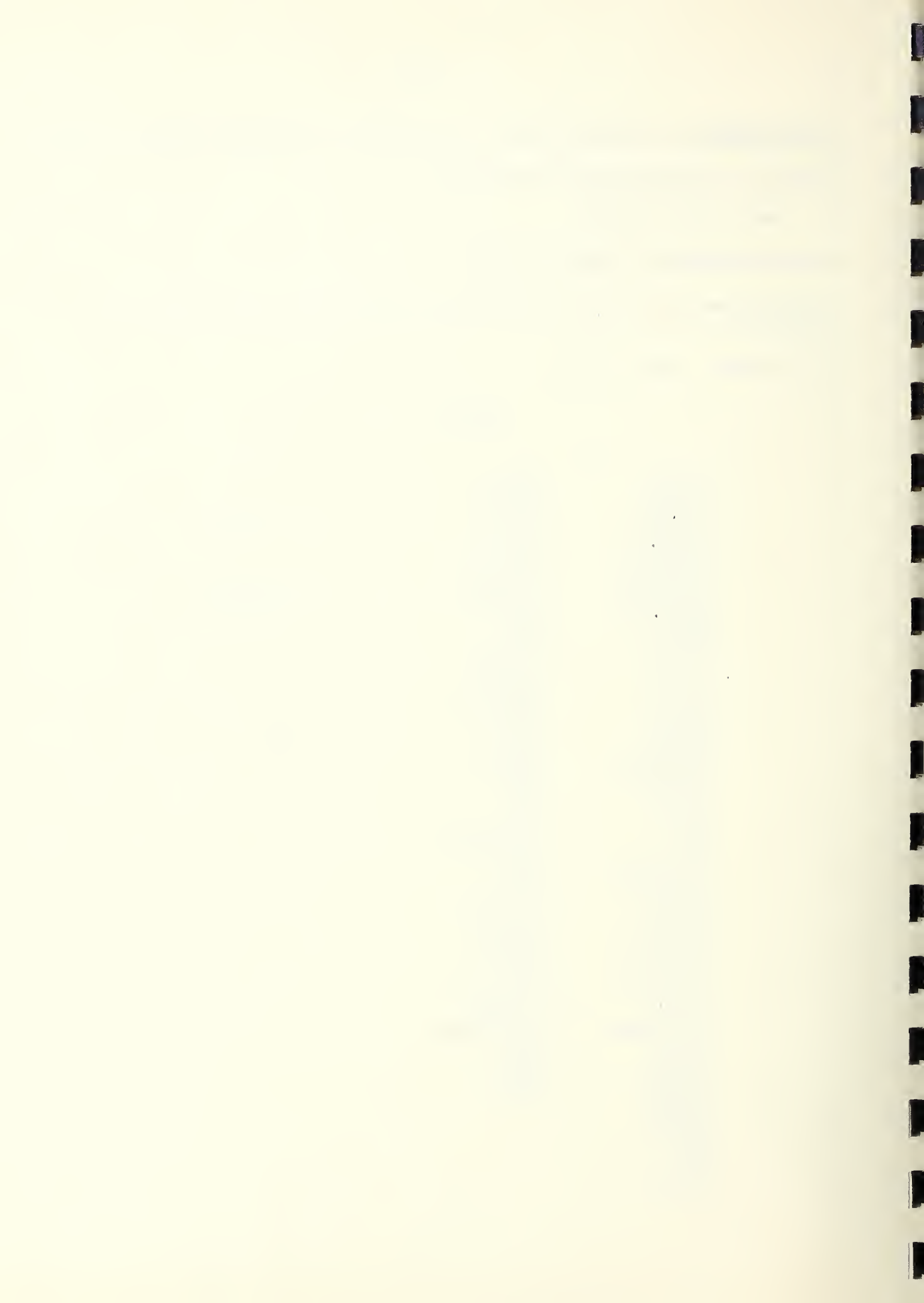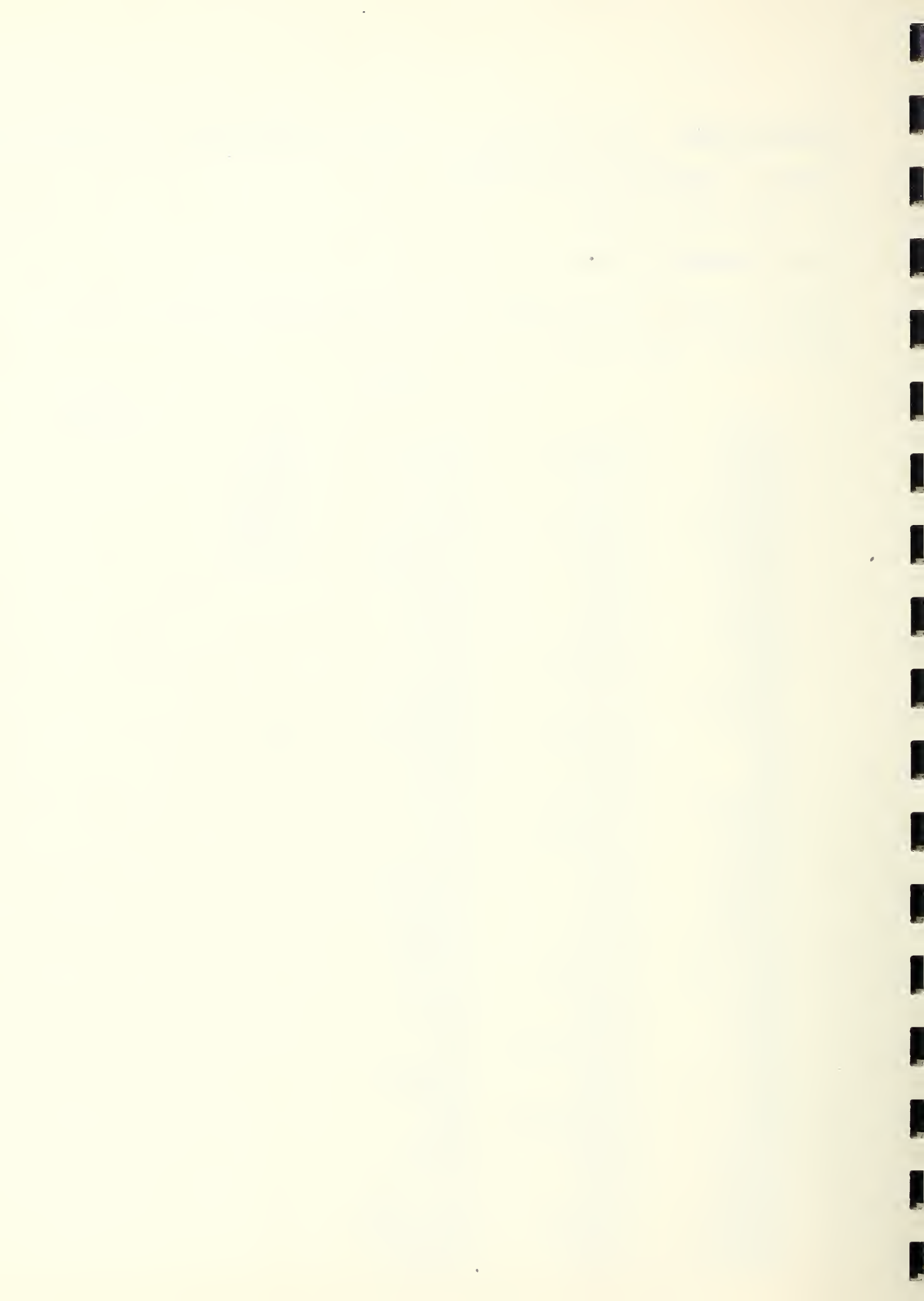
Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O.

I: Same as plan 64.14.8.

Block confounding: ABEO.

Without blocks: All main effects and two-factor interactions are measurable.

With blocks: Same as above.

Blocks

	1		2
(1)			abcdeghmn
cdhklmn	abcd	gkln	abghm
acefgkn	bcefghlm	adefhkmn	bdefl
adefghlm	bdefgkn	acefl	bcehkmn
bfgjklmo	afghjno	bcdfhjklo	acdfjmno
bcdfghjno	acdfgjklmo	bfjmno	afhjklo
abcejlmno	cehjko	abdeghjlno	degjkmo
abdehjko	dejlmno	abcegjkmo	ceghjlno
abfhkl	cdfjkn	abcdfgklm	
abcdfmn	fhjlm	abfghn	
bceghln	adegj	bdelmn	
bdegkm	aceghjklmn	bcehk	
aghjmo	bcdglmno	acdjo	
acdgjklno	bghko	ahjklmno	
cefhjkmno	abdefklmo	defgjkno	
defjlo	abcefhno	cefghjlmo	
cdjkm	abcdhjln	fghjkmn	
hjln	abjkm	cdfgjl	
adefgjmn	bdefghjkl	acehjm	
acefghjkl	bcefgjmn	adejkln	
bcdfglo	acdfghkmmo	bhlno	
bfghkmno	afglo	bcdkmo	
abdeklno	dehmo	abcefghklo	
abcehmo	ceklno	abdefgmno	
abcdfhjlm	ghjk	abgjlmn	
abfjkn	cdgjlmn	abcdghjk	
bdeghjklmn	acefhjn	bcefjklm	
bcegj	adefjklm	bdefhjn	
acdghko	bfhlmo	afkno	
aglmmo	bcdfkno	acdfhlmo	
defhno	abceghklmno	cego	
cefklmo	abdego	deghklmno	
fmn	abfgjl	cdfghn	
cdfhkl	abcdfghjkmn	fgklm	
acegkm	bcejkln	adehk	
adeghln	bdehjm	acelmn	
bgjklno	akmo	bcdhjklmno	
bcdghjmo	acdhlno	bjo	
abcefjlo	cefgmno	abdefghjlmo	
abdefhjkmno	defghklo	abcefgjkno	
abhklmn	cdghm	abcdgkln	

Plan 128.12.8. 1/128 replication of 12 factors in 4 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M.

I = ABEG = ACEF = BCFG = DEFG = ABDF = ACDG = BCDE = ADHK = BDEGHK = CDEFHK

  = ABCDFGHK = AEFGHK = BFHK = CGHK = ABCEHK = BCHJ = ACEGHJ = ABEFHJ = FGHJ

  = BCDEFGHJ = ACDFHJ = ABDGHJ = DEHJ = ABCDJK = CDEGJK = BDEFJK = ADFGJK

  = ABCEFGJK = CFJK = BGJK = AEJK = ABKL = EGKL = BCEFKL = ACFGKL = ABDEFGKL

  = DFKL = BCDGKL = ACDEKL = BDHL = ADEGHL = ABCDEFHL = CDFGHL = BEFGHL

  = AFHL = ABCGHL = CEHL = ACHJKL = BCEGHJKL =EFHJKL = ABFGHJKL = ACDEFGHJKL

  = BCDFHJKL = DGHJKL = ABDEHJKL = CDJL = ABCDEGJL = ADEFJL = BDFGJL = CEFGJL

  = ABCFJL = AGJL = BEJL = CDGHJM = ABCDEHJM = ADEFGHJM = BDFHJM = CEFHJM

  = ABCFGHJM = AHJM = BEGHJM = ACGJKM = BCEJKM = EFGJKM = ABFJKM = ACDEFJKM

  = BCDFGJKM = DJKM = ABDEGJKM = BDGM = ADEM = ABCDEFGM = CDFM = BEFM = AFGM

  = ABCM = CEGM = ABGHKM = EHKM = BCEFGHKM = ACFHKM = ABDEFHKM = DFGHKM

  = BCDHKM = ACDEGHKM = ABCDGHJKLM = CDEHJKLM = BDEFGHJKLM = ADFHJKLM

  = ABCEFHJKLM = CFGHJKLM = BHJKLM = AEGHJKLM =.BCGJLM = ACEJLM = ABEFGJLM

  = FJLM = BCDEFJLM = ACDFGJLM = ABDJLM = DEGJLM = ADGKLM = BDEKLM = CDEFGKLM

  = ABCDFKLM = AEFKLM = BFGKLM = CKLM = ABCEGKLM = GHLM = ABEHLM = ACEFGHLM = BCFHLM

  = DEFHLM = ABDFGHLM = ACDHLM = BCDEGHLM.

Block confounding: DG, DH, GH.

Without blocks: All main effects, but no two-factor interactions, are measurable.

With blocks: Same as above.

<u>Blocks</u>

1 (I)	2 hjkl	3 cegjm	4 ceghklm
abcklm			
bdghlm			
acdghk			
acefjk			
befjlm			
abcdefghjklm			
defghj			

Plan 128.12.16. 1/128 replication of 12 factors in 2 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M.

I: Same as plan 128.12.8.
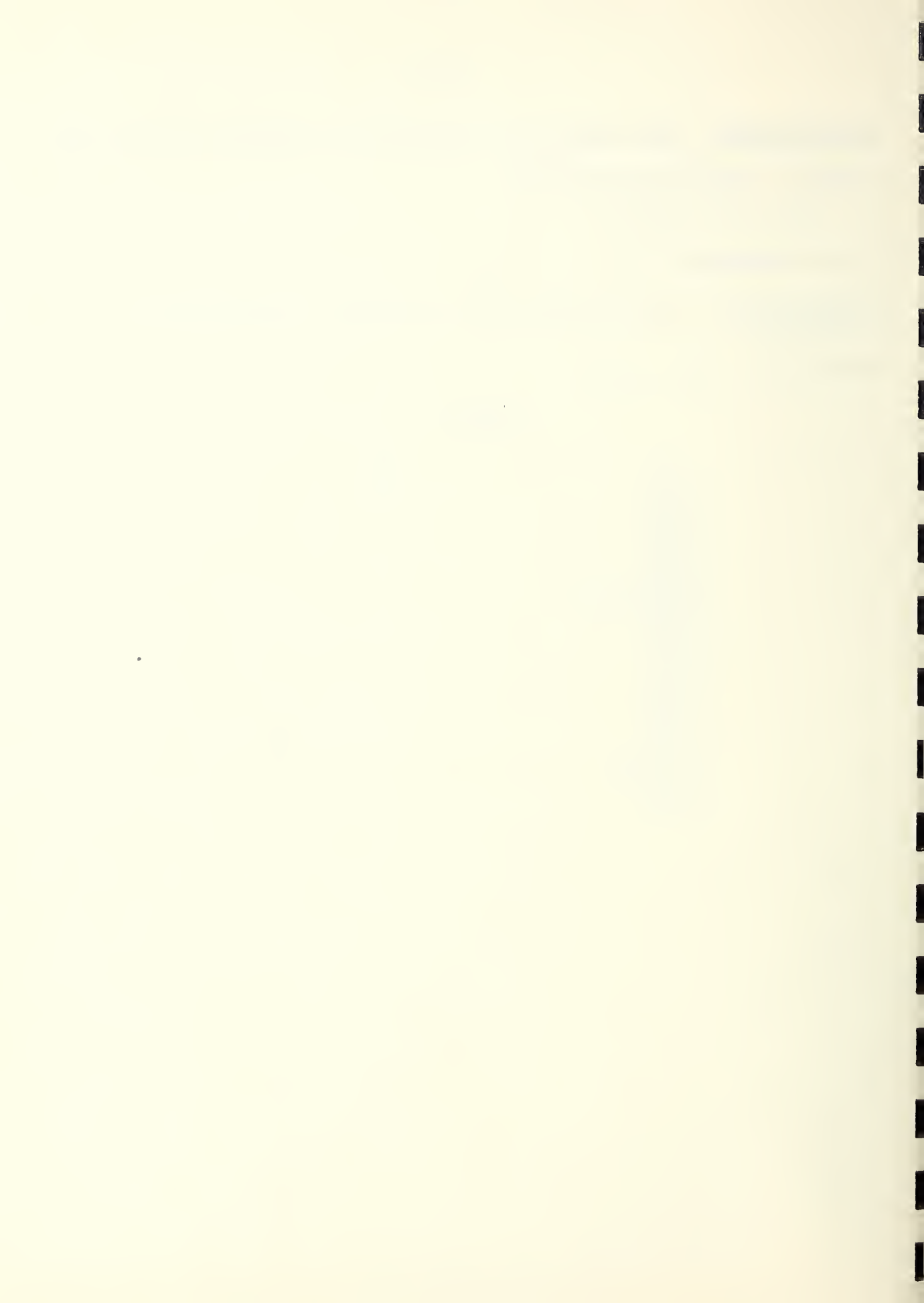
Block confounding: DG.

Without blocks: All main effects, but no two-factor interactions, are
measurables.

With blocks: Same as above.

<u>Blocks</u>

1 (I)	2
abcklm	cegjm
bdghlm	
acdghk	
acefjk	
befjlm	
abcdefghjklm	
defghj	
hjkl	
abchjm	
bdgjkm	
acdgjl	
acefhl	
befhkm	
abcdefgm	
defgkl	

Plan 128.13.8. 1/128 replication of 13 factors in 8 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N.

I = ABCD = ABEFL = CDEFL = ABGHL = CDGHL = EFGH = ABCDEFGH = ABJKL
  = CDJKL = EFJK = ABCDEFJK = GHJK = ABCDGHJK = ABEFGHJKL = CDEFGHJKL
  = ACEGJL = BDEGJL = BCFGJ = ADFGJ = BCEHJ = ADEHJ = ACFHJL = BDFHJL
  = BCEGK = ADEGK = ACFGKL = BDFGKL = ACEHKL = BDEHKL = BCFHK = ADFHK
  = ADLM = BCLM = BDEFM = ACEFM = BDGHM = ACGHM = ADEFGHLM = BCEFGHLM
  = BDJKM = ACJKM = ADEFJKLM = BCEFJKLM = ADGHJKLM = BCGHJKLM = BDEFGHJKM
  = ACEFGHJKM = CDEGJM = ABEGJM = ABCDFGJLM = FGJLM = ABCDEHJLM = EHJLM
  = CDFHJM = ABFHJM = ABCDEGKLM = EGKLM = CDFGKM = ABFGKM = CDEHKM
  = ABEHKM = ABCDFHKLM = FHKLM = EFGHJKLMN = ABCDEFGHJKLMN = ABGHJKMN
  = CDGHJKMN = ABEFJKMN = CDEFJKMN = JKLMN = ABCDJKLMN = ABEFGHMN
  = CDEFGHMN = GHLMN = ABCDGHLMN = EFLMN = ABCDEFLMN = ABMN = CDMN
  = ACFHKMN = BDFHKMN = BCEHKLMN = ADEHKLMN = BCFGKLMN = ADFGKLMN = ACEGKMN
  = BDEGKMN = BCFHJLMN = ADFHJLMN = ACEHJMN = BDEHJMN = ACFGJMN = BDFGJMN
  = BCEGJLMN = ADEGJLMN = ADEFGHJKN = BCEFGHJKN = BDGHJKLN = ACGHJKLN
  = BDEFJKLN = ACEFJKLN = ADJKN = BCJKN = BDEFGHLN = ACEFGHLN = ADGHN
  = BCGHN = ADEFN = BCEFN = BDLN = ACLN = CDFHKLN = ABFHKLN = ABCDEHKN
  = EHKN = ABCDFGKN = FGKN = CDEGKLN = ABEGKLN = ABCDFHJN = FHJN = CDEHJLN
  = ABEHJLN = CDFGJLN = ABFGJLN = ABCDEGJN = EGJN.

Block confounding: AB, AC, BC, CL, ABCL, AL, BL.

Without blocks: All main effects and the following two-factor interactions are measurable.

> AE, AF, AG, AH, AJ, AK, BE, BF, BG,
> BH, BJ, BK, CE, CF, CG, CH, CJ, CK,
> DE, DF, DG, DH, DJ, DK, EL, EM, FL,
> FM, GL, GM, HL, HM, JL, JM, KL, KM.

With blocks: Same as above.

Blocks:

1	2	3	4
(1)	abcd	cdfhkl	abfhkl
efgh			
ghjk			
efjk			
abcdehklmn			
abcdfgklmn			
abcdegjlmn			
abcdfhjlmn			

5	6	7	8
bdegkm	acegkm	bcefghlm	adefghlm

Plan 128.13.16. 1/128 replication of 13 factors in 4 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N.

I: Same as plan 128.13.8.

Block confounding: AB, AC, BC.

Without blocks: All main effects and the following two-factor interactions are measurable.

<div align="center">

AE, AF, AG, AH, AJ, AK, BE, BF, BG,
BH, BJ, BK, CE, CF, CG, CH, CJ, CK,
DE, DF, DG, DH, DJ, DK, EL, EM, FL,
FM, GL, GM, HL, HM, JL, JM, KL, KM.

</div>

With blocks: Same as above.

<div align="center">Blocks:</div>

1	2	3	4
(1)	cdfhkl	bdegkm	bcefghlm
efgh			
ghjk			
efjk			
abcdehklmn			
abcdfgklmn			
abcdegjlmn			
abcdfhjlmn			
abcd			
abcdefgh			
abcdghjk			
abcdefjk			
ehklmn			
fgklmn			
egjlmn			
fhjlmn			

Plan 128.13.32. 1/128 replication of 13 factors in 2 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N.

I: Same as plan 128.13.8.
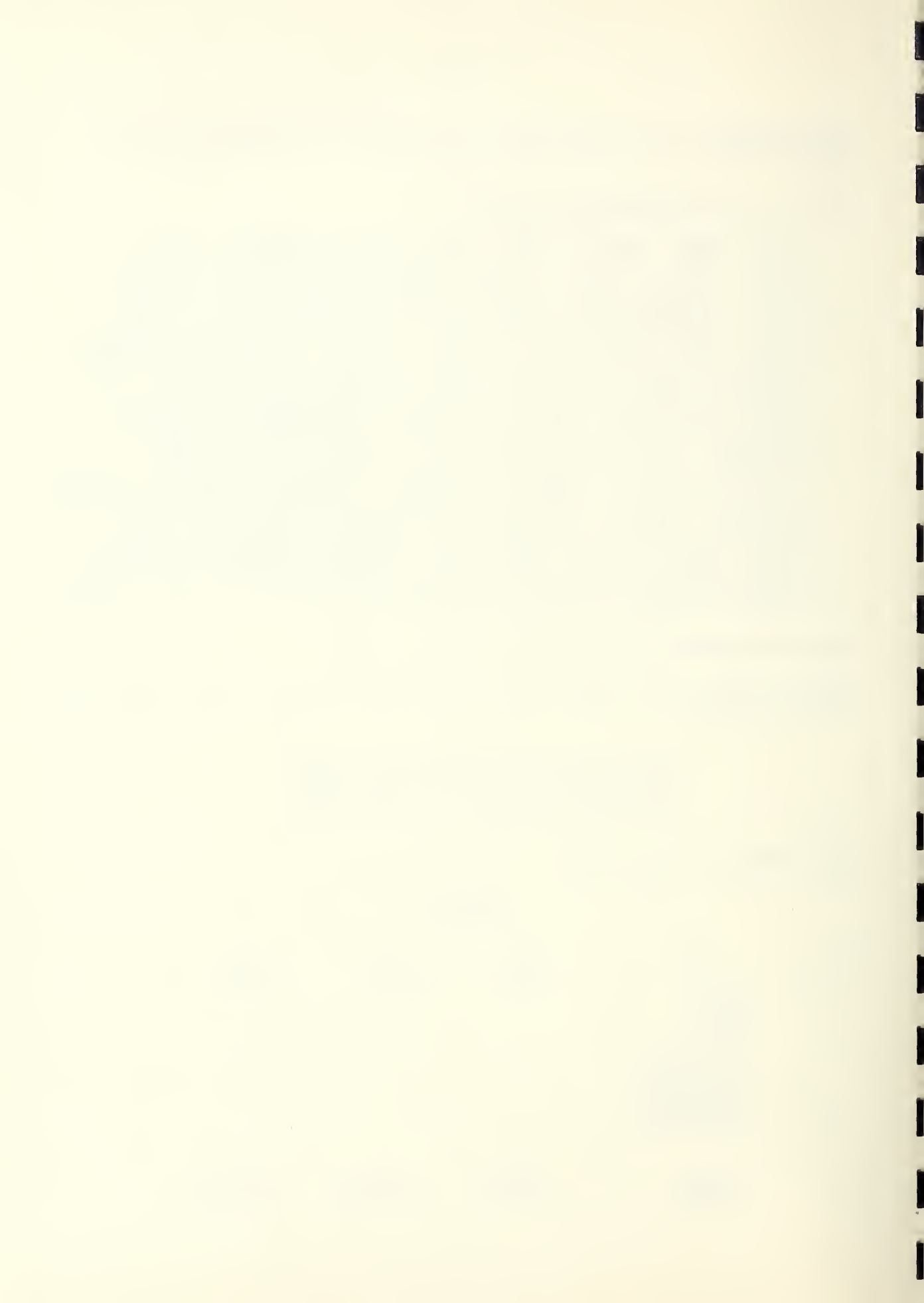
Block confounding: AB.

Without blocks: All main effects and the following two-factor interactions are measurable.

<div align="center">

AE, AF, AG, AH, AJ, AK, BE, BF, BG,
BH, BJ, BK, CE, CF, CG, CH, CJ, CK,
DE, DF, DG, DH, DJ, DK, EL, EM, FL,
FM, GL, GM, HL, HM, JL, JM, KL, KM.

</div>

With blocks: Same as above.

<div align="center">

Blocks:

</div>

1		2
(1)		bdegkm
efgh	cdefmn	
ghjk	cdghmn	
efjk	cdefghjkmn	
abcdehklmn	cdjkmn	
abcdfgklmn		
abcdegjlmn		
abcdfhjlmn		
abcd		
abcdefgh		
abcdghjk		
abcdefjk		
ehklmn		
fgklmn		
egjlmn		
fhjlmn		
cdfhkl		
cdegkl		
cdfgjl		
cdehjl		
abefmn		
abghmn		
abefghjkmn		
abjkmn		
abfhkl		
abegkl		
abfgjl		
abehjl		
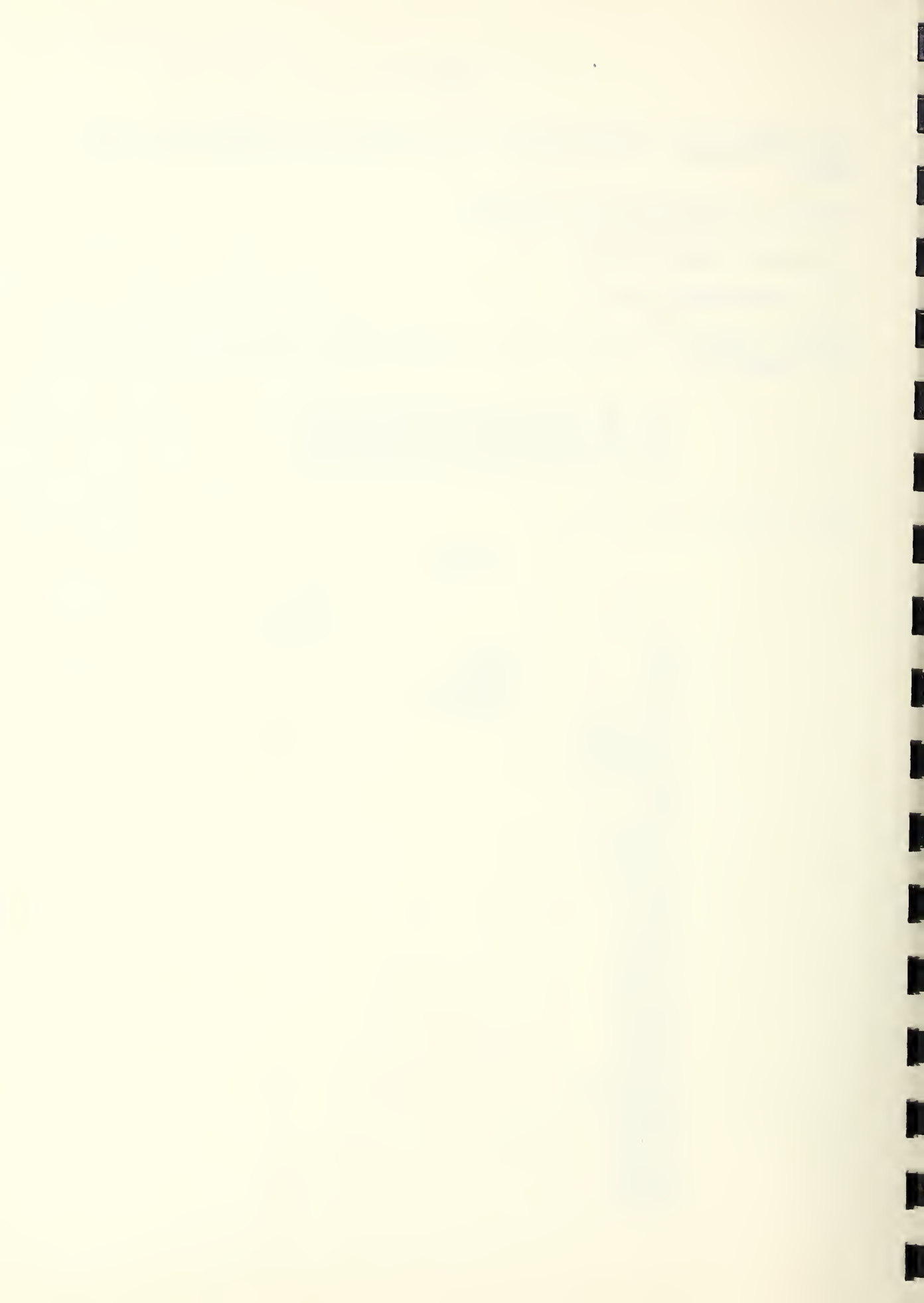
Plan 128.14.8. 1/128 replication of 14 factors in 16 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O.

I = ABCD = ABEFL = CDEFl = ABGHL = CDGHL = EFGH = ABCDEFGH = ABJKL = CDJKL

= EFJK = ABCDEFJK = GHJK = ABCDGHJK = ABEFGHJKL = CDEFGHJKL = ACEGJL

= BDEGJL = BCFGJ = ADFGJ = BCEHJ = ADEHJ = ACFHJL = BDFHJL = BCEGK = ADEGK

= ACFGKL = BDFGKL = ACEHKL = BDEHKL = BCFHK = ADFHK = ADLM = BCLM = BDEFM

= ACEFM = BDGHM = ACGHM = ADEFGHLM = BCEFGHLM = BDJKM = ACJKM = ADEFJKLM

= BCEFJKLM = ADGHJKLM = BCGHJKLM = BDEFGHJKM = ACEFGHJKM = CDEGJM = ABEGJM

= ABCDFGJLM = FGJLM = ABCDEHJLM = EHJLM = CDFHJM = ABFHJM = ABCDEGKLM

= EGKLM = CDFGKM = ABFGKM = CDEHKM = ABEHKM = ABCDFHKLM = FHKLM = EFGHJKLMNO

= ABCDEFGHJKLMNO = ABGHJKMNO = CDGHJKMNO = ABEFJKMNO = CDEFJKMNO = JKLMNO

= ABCDJKLMNO = ABEFGHMNO = CDEFGHMNO = GHLMNO = ABCDGHLMNO = EFLMNO

= ABCDEFLMNO = ABMNO = CDMNO = ACFHKMNO = BDFHKMNO = BCEHKLMNO = ADEHKLMNO

= BCFGKLMNO = ADFGKLMNO = ACEGKMNO = BDEGKMNO = BCFHJLMNO = ADFHJLMNO

= ACEHJMNO = BDEHJMNO = ACFGJMNO = BDFGJMNO = BCEGJLMNO = ADEGJLMNO

= ADEFGHJKNO = BCEFGHJKNO = BDGHJKLNO = ACGHJKLNO = BDEFJKLNO = ACEFJKLNO

= ADJKNO = BCJKNO = BDEFGHLNO = ACEFGHLNO = ADGHNO = BCGHNO = ADEFNO

= BCEFNO = BDLNO = ACLNO = CDFHKLNO = ABFHKLNO = ABCDEHKNO = EHKNO

= ABCDFGKNO = FGKNO = CDEGKLNO = ABEGKLNO = ABCDFHJNO = FHJNO = CDEHJLNO

= ABEHJLNO = CDFGJLNO = ABFGJLNO = ABCDEGJNO = EGJNO.

Block confounding: AB,AC,BC,CL,ABCL,AL,BL,GO,ABGO,ACGO,BCGO,CGLO,ABCGLO, AGLO,BGLO.

Without blocks: All main effects, and all two-factor interactions except the following are measurable:

<div style="text-align:center">

AB,AC,AD,AL,AM,BC,

BD,BL,BM,CD,CL,CM,

DL,DM,EF,EG,EH,EJ,

EK,FG,FH,FJ,FK,GH,

GJ,GK,HJ,HK,JK,LM.

</div>
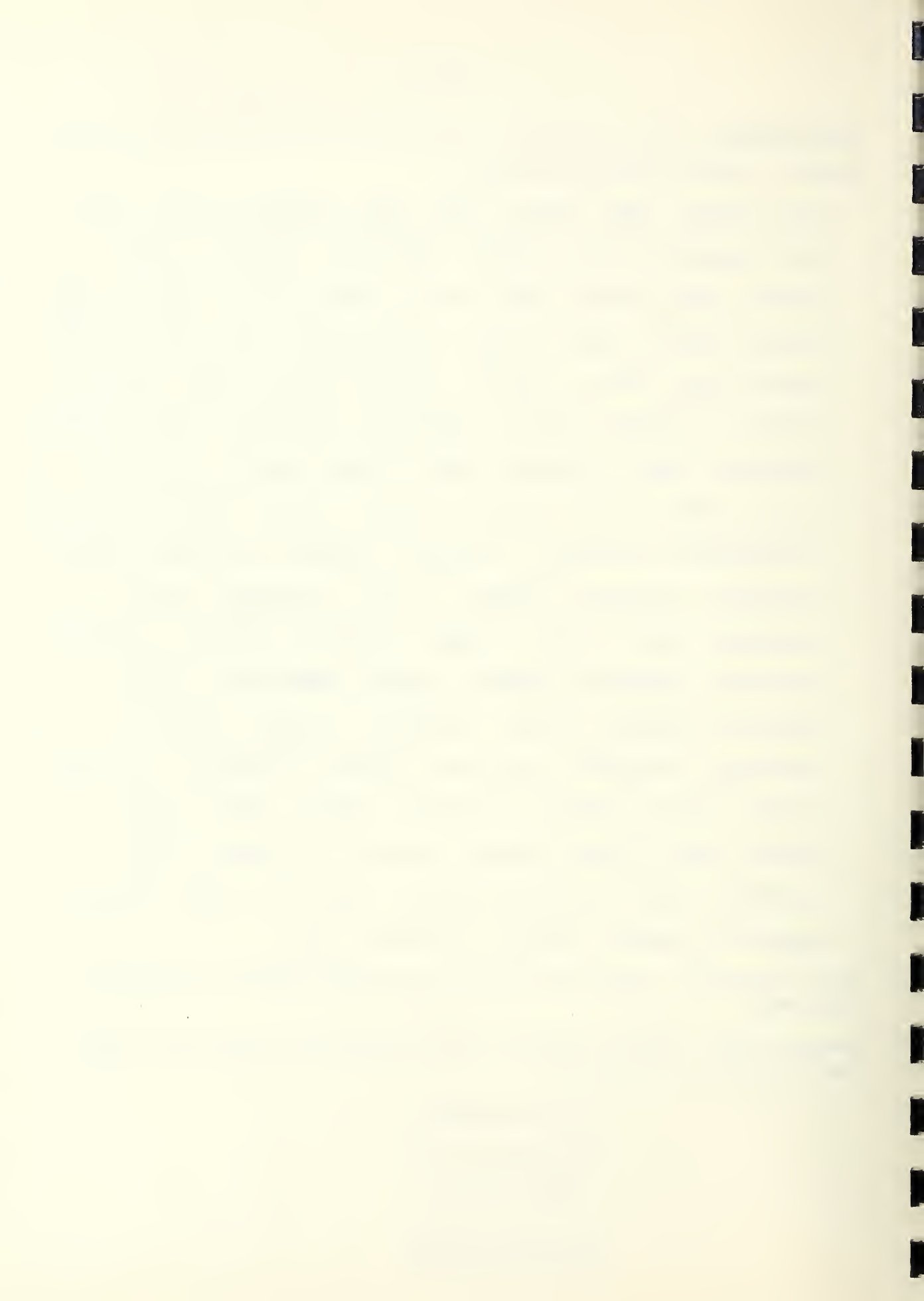
With blocks: Same as above, except that the two-factor interactions GO
and HN also are not measurable:

<div align="center">

Blocks:

$\underline{1}$ $\underline{2}$ $\underline{3}$ $\underline{4}$

</div>

<div align="center">

$\underline{1}$ $\underline{2}$ $\underline{3}$ $\underline{4}$
(I)   no   abcd   abcdno

</div>

efjk
abcdehklmn
abcdfhjlmn
efghno
ghjkno
abcdfgklmo
abcdegjlmo

<div align="center">

$\underline{5}$ $\underline{6}$ $\underline{7}$ $\underline{8}$
cdfhkl   cdfhklno   abfhkl   abfhklno

$\underline{9}$ $\underline{10}$ $\underline{11}$ $\underline{12}$
bdegkmno   bdegkm   acegkmno   acegkm

$\underline{13}$ $\underline{14}$ $\underline{15}$ $\underline{16}$
bcefghlmno   bcefghlm   adefghlmno   adefghlm

</div>

Plan 128.14.16. 1/128 replication of 14 factors in 8 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O.

I: Same as plan 128.14.8.

Block confounding: AB,AC,BC,CL,ABCL,AL,BL.

Without blocks: All main effects, and all two factor interactions <u>except</u> the following are measurable:

<div align="center">

AB,AC,AD,AL,AM,BC,

BD,BL,BM,CD,CL,CM,

DL,DM,EF,EG,EH,EJ,

EK,FG,FH,FJ,FK,GH,

GJ,GK,HJ,HK,JK,LM.

</div>

With blocks: Same as above.

<div align="center">

<u>Blocks:</u>

</div>

<u>1</u> (Ī)	<u>2</u> abcd	<u>3</u> cdfh̄kl	<u>4</u> abfh̄kl
efjk			
abcdehklmn			
abcdfhjlmn			
efghno			
ghjkno			
abcdfgklmo			
abcdegjlmo			
no			
efjkno			
abcdehklmo			
abcdfhjlmo			
efgh			
ghjk			
abcdfgklmn			
abcdegjlmn			

<u>5</u> bdegkmno	<u>6</u> acegkmno	<u>7</u> bcefgh̄lmno	<u>8</u> adefgh̄lmno

Plan 128.14.32. 1/128 replication of 14 factors in 4 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O.

I: Same as plan 128.14.8.

Block confounding: AB,AC,BC.

Without blocks: All main effects, and all two-factor interactions except the following are measurable:

$$AB,AC,AD,AL,AM,BC,$$

$$BD,BL,BM,CD,CL,CM,$$

$$DL,DM,EF,EG,EH,EJ,$$

$$EK,FG,FH,FJ,FK,GH,$$

$$GJ,GK,HJ,HK,JK,LM.$$

With blocks: Same as above.

Blocks:

1		2
(1)	abcd	cdfhkl
efjk	abcdefjk	
abcdehklmn	ehklmn	3
abcdfhjlmn	fhjlmn	bdegkmno
efghno	abcdefghno	
ghjkno	abcdghjkno	4
abcdfgklmo	fgklmo	bcefghlmno
abcdegjlmo	egjlmo	
no	abcdno	
efjkno	abcdefjkno	
abcdehklmo	ehklmo	
abcdfhjlmo	fhjlmo	
efgh	abcdefgh	
ghjk	abcdghjk	
abcdfgklmn	fgklmn	
abcdegjlmn	egjlmn	

Plan 128.14.64. 1/128 replication of 14 factors in 2 blocks of 64 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O.

I: Same as plan 128.14.8.

Block confounding: AB.

Without blocks: All main effects, and all two-factor interactions <u>except</u> the following are measurable:

<div align="center">

AB,AC,AD,AL,AM,BC,

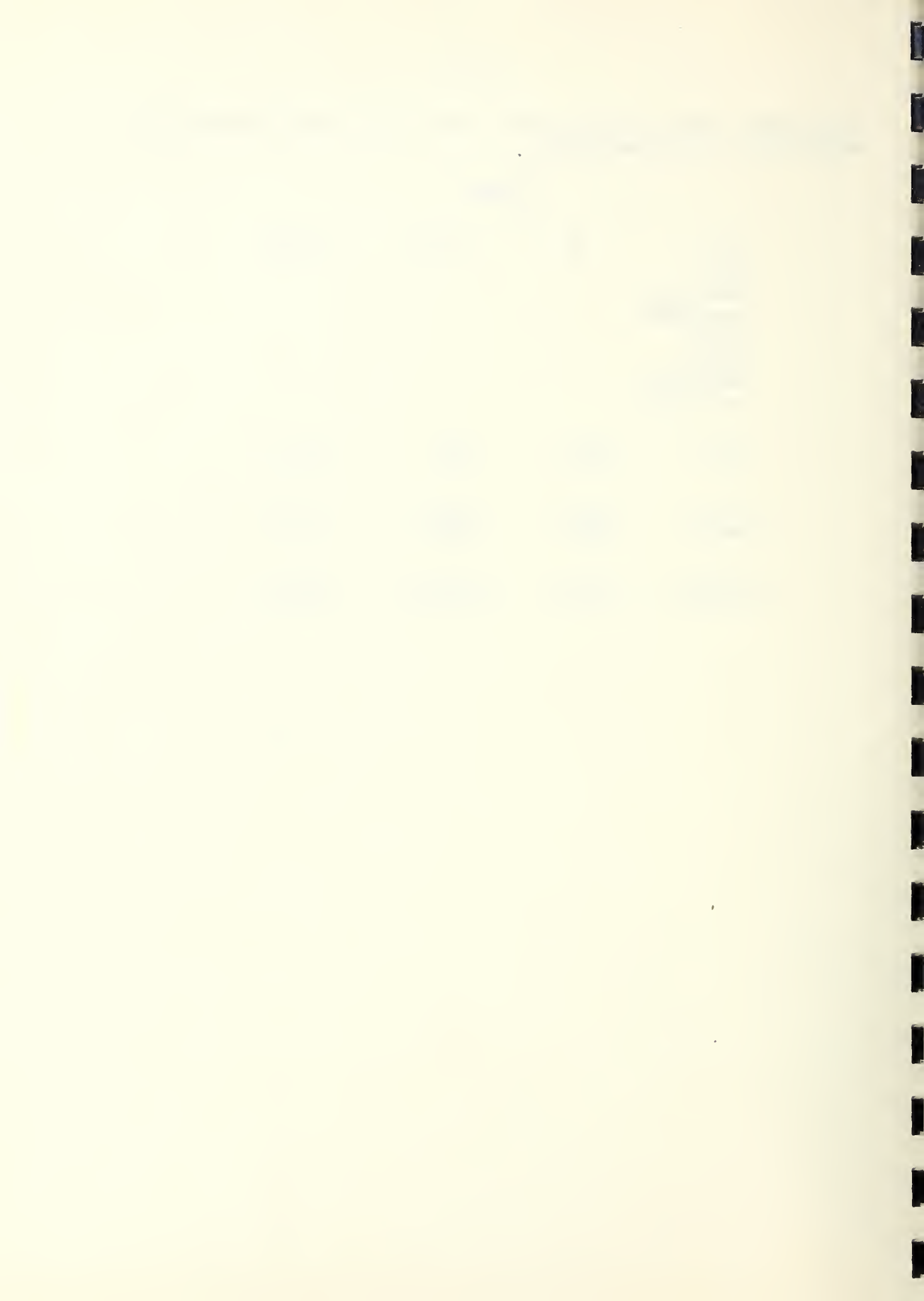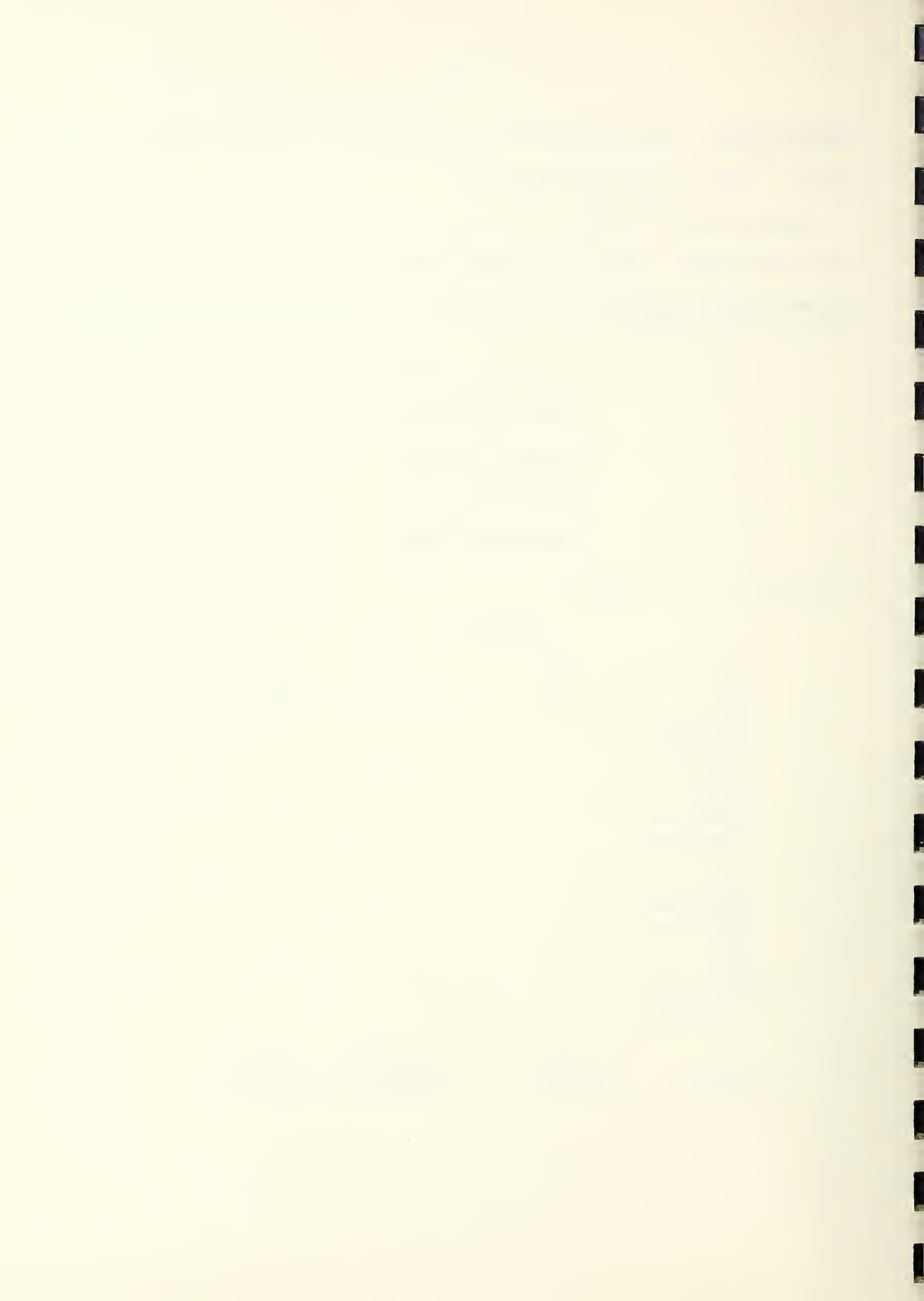BD,BL,BM,CD,CL,CM,

DL,DM,EF,EG,EH,EJ,

EK,FG,FH,FJ,FK,GH,

GJ,GK,HJ,HK,JK,LM.

</div>

With blocks: Same as above.

<div align="center">

Blocks:

1

</div>

(1)	abcd	cdfhkl	abfhkl
efjk	abcdefjk	cdehjl	abehjl
abcdehklmn	ehklmn	abefmn	cdefmn
abcdfhjlmn	fhjlmn	abjkmn	cdjkmn
efghno	abcdefghno	cdegklno	abegklno
ghjkno	abcdghjkno	cdfgjlno	abfgjlno
abcdfgklmo	fgklmo	abghmo	cdghmo
abcdegjlmo	egjlmo	abefghjkmo	cdefghjkmo
no	abcdno	cdfhklno	abfhklno
efjkno	abcdefjkno	cdehjlno	abehjlno
abcdehklmo	ehklmo	abefmo	cdefmo
abcdfhjlmo	fhjlmo	abjkmo	cdjkmo
efgh	abcdefgh	cdegkl	abegkl
ghjk	abcdghjk	cdfgjl	abfgjl
abcdfgklmn	fgklmn	abghmn	cdghmn
abcdegjlmn	egjlmn	abefghjkmn	cdefghjkmn

<div align="center">

2

bdegkmno

</div>

<u>Plan 128.15.8.</u> 1/128 replication of 15 factors in 32 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O,P.

I = ABEGN = ACEFNP = BCFGP = DEFGO = ABDFNO = ACDGNOP = BCDEOP = ADHKO

 = BDEGHKNO = CDEFHKNOP = ABCDFGHKOP = AEFGHK = BFHKN = CGHKNP = ABCEHKP

 = BCHJNOP = ACEGHJOP = ABEFHJO = FGHJNO = BCDEFGHJNP = ACDFHJP = ABDGHJ

 = DEHJN = ABCDJKNP = CDEGJKP = BDEFJK = ADFGJKN = ABCEFGJKNOP = CFJKOP

 = BGJKO = AEJKNO = ABKLOP = EGKLNOP = BCEFKLNO = ACFGKLO = ABDEFGKLP

 = DFKLNP = BCDGKLN = ACDEKL = BDHLP = ADEGHLNP = ABCDEFHLN = CDFGHL

 = BEFGHLOP = AFHLNOP = ABCGHLNO = CEHLO = ACHJKLN = BCEGHJKL = EFHJKLP

 = ABFGHJKLNP = ACDEFGHJKLNO = BCDFHJKLO = DGHJKLOP = ABDEHJKLNOP

 = CDJLNO = ABCDEGJLO = ADEFJLOP = BDFGJLNOP = CEFGJLN = ABCFJL = AGJLP

 = BEJLNP = CDGHJMO = ABCDEHJMNO = ADEFGHJMNOP = BDFHJMOP = CEFHJM

 = ABCFGHJMN = AHJMNP = BEGHJMP = ACGJKM = BCEJKMN = EFGJKMNP = ABFJKMP

 = ACDEFJKMO = BCDFGJKMNO = DJKMNOP = ABDEGJKMOP = BDGMNP = ADEMP

 = ABCDEFGM = CDFMN = BEFMNOP = AFGMOP = ABCMO = CEGMNO = ABGHKMNOP

 = EHKMOP = BCEFGHKMO = ACFHKMNO = ABDEFHKMNP = DFGHKMP = BCDHKM = ACDEGHKMN

 = ABCDGHJKLMP = CDEHJKLMNP = BDEFGHJKLMN = ADFHJKLM = ABCEFHJKLMOP

 = CFGHJKLMNOP = BHJKLMNO = AEGHJKLMO = BCGJLMOP = ACEJLMNOP = ABEFGJLMNO

 = FJLMO = BCDEFJLMP = ACDFGJLMNP = ABDJLMN = DEGJLM = ADGKLMNO = BDEKLMO

 = CDEFGKLMOP = ABCDFKLMNOP = AEFKLMN = BFGKLM = CKLMP = ABCEGKLMNP

 = GHLMN = ABEHLM = ACEFGHLMP = BCFHLMNP = DEFHLMNO = ABDFGHLMO = ACDHLMOP

 = BCDEGHLMNOP.

Block confounding: ABD,ACF,BCDF,ABCE,CDE,BEF,ADEF,FJ,ABDFJ,ACJ,BCDJ,

ABCEFJ,CDEFJ,BEJ,ADEJ,EH,ABDEH,ACEFH,BCDEFH,ABCH,CDH,BFH,ADFH,EFHJ,

ABDEFHJ,ACEHJ,BCDEHJ,ABCFHJ,CDFHJ,BHJ,ADHJ.

Without blocks: All main effects and all two-factor interactions are
measurable.

With Blocks: Same as above, except the following two-factor interactions are <u>not</u> measurable.

AP,BG,CM,DO,EH,EL,FJ,HL,KN.

<u>Blocks:</u>

1 (I)	2 acdghk	3 acdgjl	4 hjkl
cefhjklmn adefhjlop acdkmnop bcdfgjmo bdeghklno abceghlmp abfgjknp			

5 abcklm	6 bdghlm	7 bdgjkm	8 abchjm

9 ceghklm	10 adelm	11 adehjkm	12 cegjm

13 abegh	14 bcdek	15 bcdehjl	16 abegjkl

17 dgknp	18 achnp	19 acjklnp	20 dghjlnp

21 abcdglmnp	22 bhklmnp	23 bjmnp	24 abcdghjkmnp

25 cdehlmnp	26 aegklmnp	27 aeghjmnp	28 cdejkmnp

29 abdehknp	30 bcegnp	31 bceghjklnp	32 abdejlnp

Plan 128.15.16. 1/128 replication of 15 factors in 16 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O,P.

I: Same as plan 128.15.8.

Block confounding: ABD, ACF, BCDF, ABCE, CDE, BEF, ADEF, FJ, ABDFJ, ACJ, BCDJ, ABCEFJ, CDEFJ, BEJ, ADEJ.

Without blocks: All main effects and all two-factor interactions are measurable.

With blocks: Same as above, except the two-factor interactions EL and FJ are not measurable.

<u>Blocks:</u>

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	acdgjl	abcklm	bdgjkm
cefhjklmn			
adefhjlop	<u>5</u>	<u>6</u>	<u>7</u>
acdkmnop	ceghklm	adehjkm	abegh
bcdfgjmo			
bdeghklno	<u>8</u>	<u>9</u>	<u>10</u>
abceghlmp	bcdehjl	dgknp	acjklnp
abfgjknp			
acdghk	<u>11</u>	<u>12</u>	<u>13</u>
adefgjlmn	abcdglmnp	bjmnp	cdehlmnp
cefgjklop			
ghmnop	<u>14</u>	<u>15</u>	<u>16</u>
abfhjkmo	aeghjmnp	abdehknp	bceghjklnp
abcelno			
bdeklmp			
bcdfhjnp			

Plan 128.15.32. 1/128 replication of 15 factors in 8 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O,P.

I: Same as plan 128.15.8.

Block confounding: ABD, ACF, BCDF, ABCE, CDE, BEF, ADEF.

Without blocks: All main effects and all two-factor interactions are measurable.

With blocks: Same as above.

<div align="center">

Blocks:

</div>

1	2	3	4
(1)	abcklm	ceghklm	abegh
cefhjklmn			
adefhjlop	5	6	7
acdkmnop	dgknp	abcdglmnp	cdehlmnp
bcdfgjmo			
bdeghklno		8	
abceghlmp		abdehknp	
abfgjknp			
acdghk			
adefgjlmn			
cefgjklop			
ghmnop			
abfhjkmo			
abcelno			
bdeklmp			
bcdfhjnp			
acdgjl			
adefghkmn			
cefghop			
gjklmnop			
abflmo			
abcehjkno			
bdehjmp			
bcdfklnp			
hjkl			
cefmn			
adefkop			
acdhjlmnop			
bcdfghklmo			
bdegjno			
abcegjkmp			
abfghlnp			

Plan 128.15.64. 1/128 replication of 15 factors in 4 blocks of 64 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O,P.

I: Same as plan 128.15.8.

Block confounding: ABD, ACF, BCDF.

Without blocks: All main effects and all two-factor interactions are measurable.

With blocks: Same as above.

Blocks:

1

(1)	acdgjl	abcklm	bdgjkm
cefhjklmn	adefghkmn	abefhjn	bcdefghln
adefhjlop	cefghop	bcdefhjkmop	abefghklmop
acdkmnop	gjklmnop	bdlnop	abcgjnop
bcdfgjmo	abflmo	adfgjklo	cfko
bdeghklno	abcehjkno	acdeghmno	ehjlmno
abceghlmp	bdehjmp	eghkp	acdehjklp
abfgjknp	bcdfklnp	cfgjlmnp	adfmnp
acdghk	hjkl	bdghlm	abchjm
adefgjlmn	cefmn	bcdefgjkn	abefkln
cefgjklop	adefkop	abefgjmop	bcdeflmop
ghmnop	acdhjlmnop	abcghklnop	bdhjknop
abfhjkmo	bcdfghklmo	cfhjlo	adfgho
abcelno	bdegjno	ekmno	acdegjklmno
bdeklmp	abcegjkmp	acdep	egjlp
bcdfhjnp	abfghlnp	adfhjklmnp	cfghkmnp

2

ceghklm

3

dgknp

4

cdehlmnp

Plan 128.15.128. 1/128 replication of 15 factors in 2 blocks of 128 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O,P.

I: Same as plan 128.15.8.

Block confounding: ABD.

Without blocks: All main effects and all two-factor interactions are measurable.

With blocks: Same as above.

Blocks:

1

(1)	acdgjl	abcklm	bdgjkm
cefhjklmn	adefghkmn	abefhjn	bcdefghln
adefhjlop	cefghop	bcdefhjkmop	abefghklmop
acdkmnop	gjklmnop	bdlnop	abcgjnop
bcdfgjmo	abflmo	adfgjklo	cfko
bdeghklno	abcehjkno	acdeghmno	ehjlmno
abceghlmp	bdehjmp	eghkp	acdehjklp
abfgjknp	bcdfklnp	cfgjlmnp	adfmnp
acdghk	hjkl	bdghlm	abchjm
adefgjlmn	cefmn	bcdefgjkn	abefkln
cefgjklop	adefkop	abefgjmop	bcdeflmop
ghmnop	acdhjlmnop	abcghklnop	bdhjknop
abfhjkmo	bcdfghklmo	cfhjlo	adfgho
abcelno	bdegjno	ekmno	acdegjklmno
bdeklmp	abcegjkmp	acdep	egjlp
bcdfhjnp	abfghlnp	adfhjklmnp	cfghkmnp
ceghklm	adehjkm	abegh	bcdehjl
fgjn	acdfln	abcfgjklmn	bdfkmn
acdfgjkmop	fklmop	bdfgjlop	abcfop
adeghlnop	cehjnop	bcdeghkmnop	abehjklmnop
bdefhjklo	abcefghko	acdefhjmo	efghlmo
bcdmno	abgjlmno	adklno	cgjkno
abkp	bcdgjklp	clmp	adgjmp
abcefhjlmnp	bdefghmnp	efhjknp	acdefghklnp
adelm	cegjm	bcdek	abegjkl
acdfhjkn	fghkln	bdfhjlmn	abcfghmn
fhjmop	acdfghlmop	abcfhjklop	bdfghkop
ceklnop	adegjknop	abemnop	bcdegjlmnop
abcefgjlo	bdefo	efgjkmo	acdefklmo
abghkmno	bcdhjklmno	cghlno	adhjno
bcdghp	abhjlp	adghklmp	chjkmp
bdefgjklmnp	abcefkmnp	acdefgjnp	eflnp

<u>Plan 256.13.8.</u> 1/256 replication of 13 factors in 4 blocks of 8 units
each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N.

I = ABCEGHJKMN = ADFGJN = BCDEFHKM = AEFKLN = BCFGHJLM = DEGJKL
  = ABCDHLMN = ACFHJKMN = BEFG = CDGHKM = ABDEJN = CEHJLM = ABGKLN
  = ACDEFGHLMN = BDFJKL = ABCDFGHKMN = DEFJ = BCHJKM = AEGN = BCDEGHLM
  = ADJKLN = ABCEFHJLMN = FGKL = BDGJ = ACDEHKMN = ABFN = CEFGHJKM
  = ABDEFGJKLN = CDFHLM = BEKL = ACGHJLMN = ADEGHJLN = BCDKLM = EFHL
  = ABCFGJKLMN = DFGHJK = ABCDEFMN = AHKN = BCEGJM = CDEFGKLM = ABDFHJLN
  = ACEJKLMN = BGHL = ACDGMN = BDEHJK = CFJM = ABEFGHKN = BCEFJKLM
  = AFGHLN = ABCDEGKLMN = DHJL = ABCJMN = EGHK = BCDFGM = ADEFHJKN
  = ABEHLN = CGJKLM = BDEFGHJL = ACDFKLMN = BFHK = ACEFGJMN = ABDGHJKN
  = CDEM = AEFGHJKLMN = BCFL = DEHKLM = ABCDGJLN = GHJM = ABCEKN = ADFHMN
  = BCDEFGJK = CEGL = ABHJKLMN = ACDEFJLN = BDFGHKLM = ACFGKN = BEFHJM
  = CDJK = ABDEGHMN = BCDEJL = ADGHKLMN = ABCEFGLN = FHJKLM = ABCDFJKN
  = DEFGHM = BCGK = AEHJMN = ABDEFHKLMN = CDFGJL = BEGHJKLM = ACLN = BDHM
  = ACDEGJKN = ABFGHJMN = CEFK = DFKM = ABCDEFGHJN = AGJKMN = BCEH = ADELMN
  = BCDGHJKL = EFGJLM = ABCFHKLN = ACDHJN = BDEGKM = CFGH = ABEFJKMN
  = CDEFHJKL = ABDFGLMN = ACEGHKLN = BJLM = ABCGHN = EJKM = BCDFHJ
  = ADEFGKMN = BCEFGHKL = AFJLMN = ABCDEHJKLN = DGLM = BFGJKM = ACEFHN
  = ABDKMN = CDEGHJ = ABEGJLMN = CHKL = BDEFLM = ACDFGHJKLN = AFHJ
  = BCEFGKMN = DGHN = ABCDEJKM = EHJKLN = ABCGLM = ADEFGHKL = BCDFJLMN
  = CKMN = ABEGHJ = ACDFGJKM = BDEFHN = ACEFLM = BFGHJKLN = CDEGJLMN
  = ABDHKL = BCDGJKMN = ADEH = ABCFKM = EFGHJN = ABCDEFGJLM = DFHKLN
  = BCELMN = AGHJKL = ABDFGH = CDEFJKMN = BHJN = ACEGKM = BDEGHKLN = ACDJLM
  = ABEFHJKL = CFGLMN = DEFGLN = ABCDFHJKLM = AEJL = BCGHKLMN = ADGK
  = BCDEHJMN = FJKN = ABCEFGHM = ACDEGHJKLM = BDLN = CEFHKLMN = ABFGJL
  = CDFGHJMN = ABDEFK = ACHM = BEGJKN = ABCEHKLM = GJLN = BCDEFGHJKLMN
  = ADFL = BCFHMN = AEFGJK = ABCDGHJM = DEKN = BEFJLN = ACFGHKLM = ABDEGL
  = CDHJKLMN = ABJK = CEGHMN = BDFGKN = ACDEFHJM = EGKLMN = ABCHJL
  = ADEFJKLM = BCDFGHLN = AFGM = BCEFHJKN = DJMN = ABCDEGHK = ACEFGHJL
  = BFKLMN = CDEHLN = ABDGJKLM = CGHJKN = ABEM = ACDFHK = BDEFGJMN
  = ABCDEFHL = DFGJKLMN = BCEGHJLN = AKLM = BCDHKN = ADEGJM = ABCFGHJK
  = EFMN = BDEJKLMN = ACDGHL = ABEFGKLM = CFHJLN = ABDFJM = CDEFGHKN
  = BGMN = ACEHJK = ADHJKM = BCDEGN = FGHKMN = ABCEFJ = DEFHJLMN = ABCDFGKL
  = AEGHLM = BCJKLN = CDFN = ABDEFGHJKM = ACGJ = BEHKMN = ACDEKL = BDGHJLMN
  = CEFGJKLN = ABFHLM = BCFGJN = AEFHKM = ABCD = DEGHJKMN = ABCEGJKL
  = HLMN = BCDEFKLN = ADFGHJLM = ABGHKM = CEJN = BDFHJKMN = ACDEFG
  = BEFGHLMN = ACFJKL = ABDEHJLM = CDGKLN.

Block confounding: FG, GH, FH.

Without blocks: All main effects, but no two-factor interactions, are
measurable.

With blocks: Same as above.

Plan 256.13.8. (Continued).

<div align="center">

Blocks:

</div>

1	2	3	4
(1)	fgjkln	eghjkm	efhlmn
abcdefghjklmn			
adjmn			
bcefghkl			
cdfghn			
abejklm			
acfghjm			
bdekln			

<u>Plan 256.13.16.</u> 1/256 replication of 13 factors in 2 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N.

I: Same as plan 256.13.8.

Block confounding: FG.

Without blocks: All main effects, but no two-factor interactions, are measurable.

With blocks: Same as above.

<center>

Blocks:

</center>

1	2
(1)	eghjkm
abcdefghjklmn	
adjmn	
bcefghkl	
cdfghn	
abejklm	
acfghjm	
bdekln	
fgjkln	
abcdehm	
adfgklm	
bcehjn	
cdhjkl	
abefgmn	
achklmn	
bdefgj	

Plan 256.14.8. 1/256 replication of 14 factors in 8 blocks of 8 units
each.

Factors: A, B, C, D, E, F, G, H, J, K, L, M, N, O.

I = ABCEGHJKMNO = ADFGJNO = BCDEFHKM = AEFKLNO = BCFGHJLM = DEGJKL
  = ABCDHLMNO = ACFHJKMNO = BEFG = CDGHKM = ABDEJNO = CEHJLM = ABGKLNO
  = ACDEFGHLMNO = BDFJKL = ABCDFGHKMNO = DEFJ = BCHJKM = AEGNO = BCDEGHLM
  = ADJKLNO = ABCEFHJLMNO = FGKL = BDGJ = ACDEHKMNO = ABFNO = CEFGHJKM
  = ABDEFGJKLNO = CDFHLM = BEKL = ACGHJLMNO = ADEGHJLNO = BCDKLM = EFHL
  = ABCFGJKLMNO = DFGHJK = ABCDEFMNO = AHKNO = BCEGJM = CDEFGKLM = ABDFHJLNO
  = ACEJKLMNO = BGHL = ACDGMNO = BDEHJK = CFJM = ABEFGHKNO = BCEFJKLM
  = AFGHLNO = ABCDEGKLMNO = DHJL = ABCJMNO = EGHK = BCDFGM = ADEFHJKNO
  = ABEHLNO = CGJKLM = BDEFGHJL = ACDFKLMNO = BFHK = ACEFGJMNO = ABDGHJKNO
  = CDEM = AEFGHJKLMNO = BCFL = DEHKLM = ABCDGJLNO = GHJM = ABCEKNO
  = ADFHMNO = BCDEFGJK = CEGL = ABHJKLMNO = ACDEFJLNO = BDFGHKLM = ACFGKNO
  = BEFHJM = CDJK = ABDEGHMNO = BCDEJL = ADGHKLMNO = ABCEFGLNO = FHJKLM
  = ABCDFJKNO = DEFGHM = BCGK = AEHJMNO = ABDEFHKLMNO = CDFGJL = BEGHJKLM
  = ACLNO = BDHM = ACDEGJKNO = ABFGHJMNO = CEFK = DFKM = ABCDEFGHJNO
  = AGJKMNO = BCEH = ADELMNO = BCDGHJKL = EFGJLM = ABCFHKLNO = ACDHJNO
  = BDEGKM = CFGH = ABEFJKMNO = CDEFHJKL = ABDFGLMNO = ACEGHKLNO = BJLM
  = ABCGHNO = EJKM = BCDFHJ = ADEFGKMNO = BCEFGHKL = AFJLMNO = ABCDEHJKLNO
  = DGLM = BFGJKM = ACEFHNO = ABDKMNO = CDEGHJ = ABEGJLMNO = CHKL = BDEFLM
  = ACDFGHJKLNO = AFHJ = BCEFGKMNO = DGHNO = ABCDEJKM = EHJKLNO = ABCGLM
  = ADEFGHKL = BCDFJLMNO = CKMNO = ABEGHJ = ACDFGJKM = BDEFHNO = ACEFLM
  = BFGHJKLNO = CDEGJLMNO = ABDHKL = BCDGJKMNO = ADEH = ABCFKM = EFGHJNO
  = ABCDEFGJLM = DFHKLNO = BCELMNO = AGHJKL = ABDFGH = CDEFJKMNO = BHJNO
  = ACEGKM = BDEGHKLNO = ACDJLM = ABEFHJKL = CFGLMNO = DEFGLNO = ABCDFHJKLM
  = AEJL = BCGHKLMNO = ADGK = BCDEHJMNO = FJKNO = ABCEFGHM = ACDEGHJKLM
  = BDLNO = CEFHKLMNO = ABFGJL = CDFGHJMNO = ABDEFK = ACHM = BEGJKNO
  = ABCEHKLM = GJLNO = BCDEFGHJKLMNO = ADFL = BCFHMNO = AEFGJK = ABCDGHJM
  = DEKNO = BEFJLNO = ACFGHKLM = ABDEGL = CDHJKLMNO = ABJK = CEGHMNO
  = BDFGKNO = ACDEFHJM = EGKLMNO = ABCHJL = ADEFJKLM = BCDFGHLNO = AFGM
  = BCEFHJKNO = DJMNO = ABCDEGHK = ACEFGHJL = BFKLMNO = CDEHLNO = ABDGJKLM
  = CGHJKNO = ABEM = ACDFHK = BDEFGJMNO = ABCDEFHL = DFGJKLMNO = BCEGHJLNO
  = AKLM = BCDHKNO = ADEGJM = ABCFGHJK = EFMNO = BDEJKLMNO = ACDGHL
  = ABEFGKLM = CFHJLNO = ABDFJM = CDEFGHKNO = BGMNO = ACEHJK = ADHJKM
  = BCDEGNO = FGHKMNO = ABCEFJ = DEFHJLMNO = ABCDFGKL = AEGHLM = BCJKLNO
  = CDFNO = ABDEFGHJKM = ACGJ = BEHKMNO = ACDEKL = BDGHJLMNO = CEFGJKLNO
  = ABFHLM = BCFGJNO = AEFHKM = ABCD = DEGHJKMNO = ABCEGJKL = HLMNO
  = BCDEFKLNO = ADFGHJLM = ABGHKM = CEJNO = BDFHJKMNO = ACDEFG = BEFGHLMNO
  = ACFJKL = ABDEHJLM = CDGKLNO.

Block confounding: FG, GH, FH, DJ, DFGJ, DGHJ, DFHJ.

Without blocks: All main effects and the following two-factor interactions
are measurable:

AN, AO, BN, BO, CN,
CO, DN, DO, EN, EO,
FN, FO, GN, GO, HN,
HO, JN, JO, KN, KO,
LN, LO, MN, MO, NO.

With blocks: Same as above.

<p style="text-align:center">Blocks:</p>

1	2	3	4
(1)	cdfghn	cdhjkl	fgjkln
abcdefghjklmn			
adjmn			
bcefghkl			
no			
abcdefghjklmo			
adjmo			
bcefghklno			

5	6	7	8
cdefjkmn	eghjkm	efhlmn	cdeglm

Plan 256.14.16. 1/256 replication of 14 factors in 4 blocks of 16 units each.

Factors: A, B, C, D, E, F, G, H, J, K, L, M, N, O.

I : Same as plan 256.14.8.

Block confounding: FG, GH, FH.

Without blocks: All main effects and the following two-factor interactions are measurable:

$$AN, \ AO, \ BN, \ BO, \ CN,$$
$$CO, \ DN, \ DO, \ EN, \ EO,$$
$$FN, \ FO, \ GN, \ GO, \ HN,$$
$$HO, \ JN, \ JO, \ KN, \ KO,$$
$$LN, \ LO, \ MN, \ MO, \ NO.$$

With blocks: Same as above.

Blocks:

1	2	3	4
(1)	cdhjkl	cdefjkmn	efhlmn
abcdefghjklmn			
adjmn			
bcefghkl			
no			
abcdefghjklmo			
adjmo			
bcefghklno			
cdfghn			
abejklm			
acfghjm			
bdekln			
cdfgho			
abejklmno			
acfghjmno			
bdeklo			

Plan 256.14.32. 1/256 replication of 14 factors in 2 blocks of 32 units each.

Factors: A, B, C, D, E, F, G, H, J, K, L, M, N, O.

I : Same as plan 256.14.8.

Block confounding: FG.

Without blocks: All main effects and the following two-factor interactions are measurable:

$$
\begin{array}{lllll}
AN, & AO, & BN, & BO, & CN, \\
CO, & DN, & DO, & EN, & EO, \\
FN, & FO, & GN, & GO, & HN, \\
HO, & JN, & JO, & KN, & KO, \\
LN, & LO, & MN, & MO, & NO.
\end{array}
$$

With blocks: Same as above.

<div align="center">

Blocks:

</div>

1		2
(1)		cdefjkmn
abcdefghjklmn	bcehjn	
adjmn	fgjklo	
bcefghkl	abcdehmno	
no	adfgklmno	
abcdefghjklmo	bcehjo	
adjmo		
bcefghklno		
cdfghn		
abejklm		
acfghjm		
bdekln		
cdfgho		
abejklmno		
acfghjmno		
bdeklo		
cdhjkl		
abefgmn		
achklmn		
bdefgj		
cdhjklno		
abefgmo		
achklmo		
bdefgjno		
fgjkln		
abcdehm		
adfgklm		

Plan 256.15.8. 1/256 replication of 15 factors in 16 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O,P.

I = ABCEGHJKMNO = ADFGJNP = BCDEFHKMOP = AEFKLNP = BCFGHJLMOP = DEGJKL

 = ABCDHLMNO = ACFHJKMNO = BEFG = CDGHKMOP = ABDEJNP = CEHJLMOP

 = ABGKLNP = ACDEFGHLMNO = BDFJKL = ABCDFGHKMNP = DEFJOP = BCHJKM = AEGNO

 = BCDEGHLM = ADJKLNO = ABCEFHJLMNP = FGKLOP = BDGJOP = ACDEHKMNP = ABFNO

 = CEFGHJKM = ABDEFGJKLNO = CDFHLM = BEKLOP = ACGHJLMNP = ADEGHJLNOP

 = BCDKLMP = EFHLO = ABCFGJKLMN = DFGHJKO = ABCDEFMN = AHKNOP = BCEGJMP

 = CDEFGKLMP = ABDFHJLNOP = ACEJKLMN = BGHLO = ACDGMN = BDEHJKO = CFJMP

 = ABEFGHKNOP = BCEFJKLMO = AFGHLN = ABCDEGKLMNOP = DHJLP = ABCJMNOP

 = EGHKP = BCDFGMO = ADEFHJKN = ABEHLN = CGJKLMO = BDEFGHJLP = ACDFKLMNOP

 = BFHKP = ACEFGJMNOP = ABDGHJKN = CDEMO = AEFGHJKLMNP = BCFLOP = DEHKLM

 = ABCDGJLNO = GHJM = ABCEKNO = ADFHMNP = BCDEFGJKOP = CEGLOP = ABHJKLMNP

 = ACDEFJLNO = BDFGHKLM = ACFGKNO = BEFHJM = CDJKOP = ABDEGHMNP = BCDEJL

 = ADGHKLMNO = ABCEFGLNP = FHJKLMOP = ABCDFJKNP = DEFGHMOP = BCGK

 = AEHJMNO = ABDEFHKLMNO = CDFGJL = BEGHJKLMOP = ACLNP = BDHMOP = ACDEGJKNP

 = ABFGHJMNO = CEFK = DFKMO = ABCDEFGHJN = AGJKMNOP = BCEHP = ADELMNOP

 = BCDGHJKLP = EFGJLMO = ABCFHKLN = ACDHJN = BDEGKMO = CFGHP = ABEFJKMNOP

 = CDEFHJKLP = ABDFGLMNOP = ACEGHKLN = BJLMO = ABCGHNOP = EJKMP = BCDFHJO

 = ADEFGKMN = BCEFGHKLO = AFJLMN = ABCDEHJKLNOP = DGLMP = BFGJKMP

 = ACEFHNOP = ABDKMN = CDEGHJO = ABEGJLMN = CHKLO = BDEFLMP = ACDFGHJKLNOP

 = AFHJOP = BCEFGKMNP = DGHNO = ABCDEJKM = EHJKLNO = ABCGLM = ADEFGHKLOP

 = BCDFJLMNP = CKMNP = ABEGHJOP = ACDFGJKM = BDEFHNO = ACEFLM = BFGHJKLNO

 = CDEGJLMNP = ABDHKLOP = BCDGJKMNO = ADEH = ABCFKMOP = EFGHJNP

 = ABCDEFGJLMOP = DFHKLNP =BCELMNO = AGHJKL = ABDFGH = CDEFJKMNO = BHJNP

 = ACEGKMOP = BDEGHKLNP = ACDJLMOP = ABEFHJKL = CFGLMNO = DEFGLN

I (Continued) = ABCDFHJKLMO = AEJLP = BCGHKLMNOP = ADGKP = BCDEHJMNOP = FJKN

= ABCEFGHMO = ACDEGHJKLMO = BDLN = CEFHKLMNOP = ABFGJLP = CDFGHJMNOP

= ABDEFKP = ACHMO = BEGJKN = ABCEHKLMP = GJLNOP = BCDEFGHJKLMN = ADFLO

= BCFHMN = AEFGJKO = ABCDGHJMP = DEKNOP = BEFJLNOP = ACFGHKLMP = ABDEGLO

= CDHJKLMN = ABJKO = CEGHMN = BDFGKNOP = ACDEFHJMP = EGKLMNO = ABCHJL

= ADEFJKLMOP = BCDFGHLNP = AFGMOP = BCEFHJKNP = DJMNO = ABCDEGHK

= ACEFGHJL = BFKLMNO = CDEHLNP = ABDGJKLMOP = CGHJKNP = ABEMOP = ACDFHK

= BDEFGJMNO = ABCDEFHLOP = DFGJKLMNP = BCEGHJLNO = AKLM = BCDHKNO

= ADEGJM = ABCFGHJKOP = EFMNP = BDEJKLMNP = ACDGHLOP = ABEFGKLM

= CFHJLNO = ABDFJM = CDEFGHKNO = BGMNP = ACEHJKOP = ADHJKMP = BCDEGNOP

= FGHKMN = ABCEFJO = DEFHJLMN = ABCDFGKLO = AEGHLMP = BCJKLNOP = CDFNOP

= ABDEFGHJKMP = ACGJO = BEHKMN = ACDEKLO = BDGHJLMN = CEFGJKLNOP

= ABFHLMP = BCFGJN = AEFHKMO = ABCDP = DEGHJKMNOP = ABCEGJKLP = HLMNOP

= BCDEFKLN = ADFGHJLMO = ABGHKMO = CEJN = BDFHJKMNOP = ACDEFGP

= BEFGHLMNOP = ACFJKLP = ABDEHJLMO = CDGKLN.

Block confounding: AK,EK,AE,ACEK,CE,AC,CK,BC,ABCK,BCEK,ABCE,ABEK,BE,AB,BK.

Without blocks: All main effects and all two-factor interactions except the following are measurable:

AD,AE,AH,AK,AL,AM,

BC,BD,BE,BF,BG,BK,

BL,BN,CE,CF,CG,CJ,

CK,CN,DE,DH,DL,DN,

EF,EG,EH,EJ,EK,EN,

FG,FJ,FK,FN,GH,GJ,

GK,GM,HJ,HM,JK,JM,

JN,KL,KM,KN,LM,LN.

With blocks: Same as above, except that the following two-factor interactions also are not measurable:

AB,AC,AF,AG.

Blocks:

1
(I)
abcdefghjklmn
dhlmp
abcefgjknp
abcdefghko
jlmno
abcefgklmop
dhjnop

2
bdgho

3
bcfjlm

4
cdfghjlmo

5
bcehjn

6
cdegjno

7
efhlmn

8
bdefglmno

9
abcdfln

10
acfghlno

11
adjmn

12
abghjmno

13
adefhjl

14
abefgjlo

15
abcdehm

16
acegmo

Plan 256.15.16. 1/256 replication of 15 factors in 8 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O,P.

I: Same as plan 256.15.8.

Block confounding: AK,EK,AE,ACEK,CE,AC,CK.

Without blocks: All main effects, and all two-factor interactions except the following are measurable:

AD,AE,AH,AK,AL,AM,

BC,BD,BE,BF,BG,BK,

BL,BN,CE,CF,CG,CJ,

CK,CN,DE,DH,DL,DN,

EF,EG,EH,EJ,EK,EN,

FG,FJ,FK,FN,GH,GJ,

GK,GM,HJ,HM,JK,JM,

JN,KL,KM,KN,LM,LN.

With blocks: Same as above except that the following two-factor interactions also are not measurable:

AC,AF.

Blocks:

1 (I̅)	2 bcfj̲lm	3 bcehj̲n	4 efh̲l̲mn
abcdefghjklmn			
dhl̄mp			
abcefgjknp			
abcdefghko			
jlmno			
abcefgklmop			
dhjnop			
bdgho			
acefjklmno			
bglmop			
acdefhjknop			
acefk			
bdghjlmn			
acdefhklmp			
bgjnp			

Blocks (Continued):

5	6	7	8
abcd̲fln	adj̲mn	adef̲hjl	abcd̲ehm

Plan 256.15.32. 1/256 replication of 15 factors in 4 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O,P.

I: Same as plan 256.15.8.

Block confounding: AK,EK,AE.

Without blocks: All main effects, and all two-factor interactions except the following are measurable:

<div align="center">

AD,AE,AH,AK,AL,AM,

BC,BD,BE,BF,BG,BK,

BL,BN,CE,CF,CG,CJ,

CK,CN,DE,DH,DL,DN,

EF,EG,EH,EJ,EK,EN,

FG,FJ,FK,FN,GH,GJ,

GK,GM,HJ,HM,JK,JM,

JN,KL,KM,KN,LM,LN.

</div>

With blocks: Same as above.

<div align="center">

Blocks:

</div>

1 (I)	2 befgjln	2 bcehjn	3 abcdfln	4 adefhjl
abcdefghjklmn	adeghkn			
dhlmp	bcdfhjp			
abcefgjknp	aegklmnp			
abcdefghko	adeghjklmo			
jlmno	bcfno			
abcefgklmop	aegjkop			
dhjnop	bcdfhlmnop			
bdgho	cdfghjlmo			
acefjklmno	abekno			
bglmop	cfgjop			
acdefhjknop	abefhklmnop			
acefk	abejklm			
bdghjlmn	cdfghn			
acdefhklmp	abdghjlp			
bgjnp	cfgjlmnp			

Plan 256.15.64. 1/256 replication of 15 factors in 2 blocks of 64 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O,P.

I: Same as plan 256.15.8.

Block confounding: AK.

Without blocks: All main effects, and all two-factor interactions except the following are measurable:

AD,AE,AH,AK,AL,AM,

BC,BD,BE,BF,BG,BK,

BL,BN,CE,CF,CG,CJ,

CK,CN,DE,DH,DL,DN,

EF,EG,EH,EJ,EK,EN,

FG,FJ,FK,FN,GH,GJ,

GK,GM,HJ,HM,JK,JM,

JN,KL,KM,KN,LM,LN.

With blocks: Same as above.

Blocks:

1

(1)	bcfjlm	bcehjn	efhlmn
abcdefghjklmn	adeghkn	adfgklm	abcdgjk
dhlmp	bcdfhjp	bcdejlmp	defnp
abcefgjknp	aegklmnp	afghkp	abcghjklmp
abcdefghko	adeghjklmo	adfgjkno	abcdgklmno
jlmno	bcfno	bcehlmo	efhjo
abcefgklmop	aegjkop	afghjklmnop	abcghknop
dhjnop	bcdfhlmnop	bcdeop	defjlmop
bdgho	cdfghjlmo	cdegjno	bdefglmno
acefjklmno	abekno	abfhklmo	achjko
bglmop	cfgjop	ceghjlmnop	befghnop
acdefhjknop	abdehklmnop	abdfkop	acdjklmop
acefk	abejklm	abfhjkn	achklmn
bdghjlmn	cdfghn	cdeglm	befgj
acdefhklmp	abdehjkp	abdfjklmnp	acdknp
bgjnp	cfglmnp	ceghp	befghjlmp

2

abcdfln

<u>Plan 256.16.8.</u> 1/256 replication of 16 factors in 32 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O,P,S.

I = ABCEGHJKMNOS = ADFGJNP = BCDEFHKMOPS = AEFKLNPS = BCFGHJLMOP = DEGJKLS

  = ABCDHLMNO = ACFHJKMNO = BEFGS = CDGHKMOP = ABDEJNPS = CEHJLMOPS = ABGKLNP

  = ACDEFGHLMNOS = BDFJKL = ABCDFGHKMNP = DEFJOPS = BCHJKM = AEGNOS = BCDEGHLMS

  = ADJKLNO = ABCEFHJLMNPS = FGKLOP = BDGJOP = ACDEHKMNPS = ABFNO = CEFGHJKMS

  ⊛ ABDEFGJKLNOS = CDFHLM = BEKLOPS = ACGHJLMNP = ADEGHJLNOP = BCDKLMPS = EFHLO

  = ABCFGJKLMNS = DFGHJKOS = ABCDEFMN = AHKNOPS = BCEGJMP = CDEFGKLMP

  = ABDFHJLNOPS = ACEJKLMN = BGHLOS = ACDGMNS = BDEHJKO = CFJMPS = ABEFGHKNOP

  = BCEFJKLMO = AFGHLNS = ABCDEGKLMNOP = DHJLPS = ABCJMNOPS = EGHKP

  = BCDFGMOS = ADEFHJKN = ABEHLN = CGJKLMOS = BDEFGHJLP = ACDFKLMNOPS

  = BFHKPS = ACEFGJMNOP = ABDGHJKNS = CDEMO = AEFGHJKLMNP = BCFLOPS = DEHKLM

  = ABCDGJLNOS = GHJMS = ABCEKNO = ADFHMNPS = BCDEFGJKOP = CEGLOP

  = ABHJKLMNPS = ACDEFJLNO = BDFGHKLMS = ACFGKNOS = BEFHJM = CDJKOPS

  = ABDEGHMNP = BCDEJL = ADGHKLMNOS = ABCEFGLNP = FHJKLMOPS = ABCDFJKNPS

  = DEFGHMOP = BCGKS = AEHJMNO = ABDEFHKLMNO = CDFGJLS = BEGHJKLMOP = ACLNPS

  = BDHMOPS = ACDEGJKNP = ABFGHJMNOS = CEFK = DFKMO = ABCDEFGHJNS = AGJKMNOP

  = BCEHPS = ADELMNOPS = BCDGHJKLP = EFGJLMOS = ABCFHKLN = ACDHJN = BDEGKMOS

  = CFGHP = ABEFJKMNOPS = CDEFHJKLPS = ABDFGLMNOP = ACEGHKLNS = BJLMO

  = ABCGHNOP = EJKMPS = BCDFHJO = ADEFGKMNS = BCEFGHKLOS = AFJLMN

  = ABCDEHJKLNOPS = DGLMP = BFGJKMP = ACEFHNOPS = ABDKMN = CDEGHJOS

  = ABEGJLMNS = CHKLO = BDEFLMPS = ACDFGHJKLNOP = AFHJOP = BCEFGKMNPS = DGHNO

  = ABCDEJKMS = EHJKLNOS = ABCGLM = ADEFGHKLOPS = BCDFJLMNP = CKMNP = ABEGHJOPS

  = ACDFGJKM = BDEFHNOS = ACEFLMS = BFGHJKLNO = CDEGJLMNPS = ABDHKLOP

  = BCDGJKMNO = ADEHS = ABCFKMOP = EFGHJNPS = ABCDEFGJLMOPS = DFHKLNP

  ⊛ BCELMNOS = AGHJKL = ABDFGH = CDEFJKMNOS = BHJNP = ACEGKMOPS = BDEGHKLNPS

  = ACDJLMOP = ABEFHJKLS = CFGLMNO = DEFGLN = ABCDFHJKLMOS = AEJLP

I (Continued): BCGHKLMNOPS = ADGKPS = BCDEHJMNOP = FJKNS = ABCEFGHMO

= ACDEGHJKLMO = BDLNS = CEFHKLMNOP = ABFGJLPS = CDFGHJMNOPS = ABDEFKP

= ACHMOS = BEGJKN = ABCEHKLMP = GJLNOPS = BCDEFGHJKLMN = ADFLOS = BCFHMNS

= AEFGJKO = ABCDGHJMPS = DEKNOP = BEFJLNOP = ACFGHKLMPS = ABDEGLO

= CDHJKLMNS = ABJKOS = CEGHMN = BDFGKNOPS = ACDEFHJMP = EGKLMNO = ABCHJLS

= ADEFJKLMOP = BCDFGHLNPS = AFGMOPS = BCEFHJKNP = DJMNOS = ABCDEGHK

= ACEFGHJL = BFKLMNOS = CDEHLNP = ABDGJKLMOPS = CGHJKNPS = ABEMOP = ACDFHKS

= BDEFGJMNO = ABCDEFHLOP = DFGJKLMNPS = BCEGHJLNO = AKLMS = BCDHKNOS

= ADEGJM = ABCFGHJKOPS = EFMNP = BDEJKLMNP = ACDGHLOPS = ABEFGKLM = CFHJLNOS

= ABDFJMS = CDEFGHKNO = BGMNPS = ACEHJKOP = ADHJKMP = BCDEGNOPS = FGHKMN

= ABCEFJOS = DEFHJLMNS = ABCDFGKLO = AEGHLMPS = BCJKLNOP = CDFNOP

= ABDEFGHJKMPS = ACGJO = BEHKMNS = ACDEKLOS = BDGHJLMN = CEFGJKLNOPS

= ABFHLMP = BCFGJN = AEFHKMOS = ABCDP = DEGHJKMNOPS = ABCEGJKLPS = HLMNOP

= BCDEFKLNS = ADFGHJLMO = ABGHKMO = CEJNS = BDFHJKMNOP = ACDEFGPS

= BEFGHLMNOPS = ACFJKLP = ABDEHJLMOS = CDGKLN.

Block confounding: CE,CF,EF,DFG,CDEFG,CDG,DEG,OS,CEOS,CFOS,EFOS,DFGOS,

CDEFGOS,CDGOS,DEGOS,LM,CELM,CFLM,EFLM,DFGLM,CDEFGLM,CDGLM,DEGLM,LMOS,CELMOS,

CFLMOS,EFLMOS,DFGLMOS,CDEFGLMOS,CDGLMOS,DEGLMOS.

Without blocks: All main effects, and all two-factor interactions <u>except</u>
the following are measurable:

CE,CF,CK,EF,EK,FK.

With blocks: Same as above, except that the following two-factor interactions
also are not measurable:

BN,CP,DH,EP,FP,GJ,KP,LM,OS.

Blocks:

1 (1) bdghjlmn cefgjklmops bcdefhknops acdefhklmp abcefgjknp adghjos ablmnos	2 bcefghkl	3 jlmno	4 bcefghjkmno
5 abcdefghjklmn	6 adjmn	7 abcdefghko	8 adlo
9 cdeglm	10 bdfhkm	11 cdegjno	12 bdfhjklno
13 abfhjkn	14 acegjln	15 abfhklmo	16 acegmo
17 cfglmnp	18 behkmnp	19 cfgjop	20 behjklop
21 abdehjkp	22 acdfgjlp	23 abdehklmnop	24 acdfgmnop
25 defnp	26 bcdghklnp	27 defjlmop	28 bcdghjkmop
29 abcghjklmp	30 aefjmp	31 abcghknop	32 aeflnop

Plan 256.16.16. 1/256 replication of 16 factors in 16 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O,P,S.

I: Same as plan 256.16.8.

Block confounding: CE,CF,EF,DFG,CDEFG, CDG,DEG,OS,CEOS,CFOS,EFOS,DFGOS,
CDEFGOS,CDGOS,DEGOS.

Without blocks: All main effects, and all two-factor interactions except the
following are measurable:

$$CE,CF,CK,EF,EK,FK.$$

With blocks: Same as above, except the two-factor interaction OS also is
not measurable.

<div align="center">

Blocks:

1
($\overline{1}$)
bdghjlmn
cefgjklmops
bcdefhknops
acdefhklmp
abcefgjknp
adghjos
ablmnos
bcefghkl
cdefjkmn
bhjmops
dglnops
abdgmp
ahjlnp
abcdefjklos
acefghkmnos

</div>

2 jlmno	3 abcdefghjklmn	4 abcdefghko
5 cdeglm	6 cdegjno	7 abfhjkn
8 abfhklmo	9 cfglmmp	10 cfgjop
11 abdehjkp	12 abdehklmnop	13 defnp
14 defjlmop	15 abcghjklmp	16 abcghknop

Plan 256.16.32. 1/256 replication of 16 factors in 8 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O,P,S.

I: Same as plan 256.16.8.

Block confounding: CE,CF,EF,DFG, CDEFG,CDG,DEG.

Without blocks: All main effects, and all two-factor interactions except

the following are measurable:

CE,CF,CK,EF,EK,FK.

With blocks: Same as above.

Blocks:

1

(1)	jlmno
bdghjlmn	bdgho
cefgjklmops	cefgknps
bcdefhknops	bcdefhjklmps
acdefhklmp	acdefhjknop
abcefgjknp	abcefgklmop
adghjos	adghlmns
ablmnos	abjs
bcefghkl	bcefghjkmno
cdefjkmn	cdefklo
bhjmops	bhlnps
dglnops	dgjmps
abdgmp	abdgjlnop
ahjlnp	ahmop
abcdefjklos	abcdefkmns
acefghkmnos	acefghjkls

2	3	4
abcdefghjklmn	cdeglm	abfhjkn

5	6	7	8
cfglmnp	abdehjkp	defnp	abcghjklmp

Plan 256.16.64. 1/256 replication of 16 factors in 4 blocks of 64 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O,P,S.

I: Same as plan 256.16.8.

Block confounding: CE, CF, EF.

Without blocks: All main effects, and all two-factor interactions <u>except</u> the following are measurable:

$$CE, CF, CK, EF, EK, FK.$$

With blocks: Same as above.

<u>Blocks</u>:

<u>1</u>

(1)	jlmno	abcdefghjklmn	abcdefghko
bdghjlmn	bdgho	acefk	acefjklmno
cefgjklmops	cefgkmps	abdhnops	abdhjlmps
bcdefhknops	bcdefhjklmps	agjlmops	agnps
acdefhklmp	acdefhjknop	bgjnp	bglmop
abcefgjknp	abcefgklmop	dhlmp	dhjnop
adghjos	adghlmns	bcefklmnos	bcefjks
ablmnos	abjs	cdefghjkos	cdefghklmns
bcefghkl	bcefghjkmno	adjmn	adlo
cdefjkmn	cdefklo	abghl	abghjmno
bhjmops	bhlnps	acdefgklnops	acdefgjkmps
dglnops	dgjmps	abcefhjkmops	abcefhklnps
abdgmp	abdgjlnop	cefhjklnp	cefhkmop
ahjlnp	ahmop	bcdefgkmp	bcdefgjklnop
abcdefjklos	abcdefkmns	ghmnos	ghjls
acefghkmnos	acefghjkls	bdjlos	bdmns

<u>2</u>
cdeglm

<u>3</u>
cfglmnp

<u>4</u>
defnp

Plan 256.16.128. 1/256 replication of 16 factors in 2 blocks of 128 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M,N,O,P,S.

I: Same as plan 256.16.8.

Block confounding: CE.

Without blocks: All main effects, and all two-factor interactions except the
following are measurable:

$$CE,CF,CK,EF,EK,FK.$$

With blocks: Same as above.

<div align="center">Blocks:</div>

<div align="center">1</div>

(1)	abcdefghjklmn	cdeglm	abfhjkn
bdghjlmn	acefk	bcehjn	adfgklm
cefgjklmops	abdhnops	dfjkops	abceghlmnops
bcdefhknops	agjlmops	bfghklmnops	acdejops
acdefhklmp	bgjnp	afghkp	bcdejlmnp
abcefgjknp	dhlmp	abdfjklmnp	ceghp
adghjos	bcefklmnos	acehjlmos	bdfgknos
ablmnos	cdefghjkos	abcdegnos	fhjklmos
bcefghkl	adjmn	bdfhkm	acegjln
cdefjkmn	abghl	fgjkln	abcdehm
bhjmops	acdefgklnops	bcdeghjlops	afkmnops
dglnops	abcefhjkmops	cemnops	abdfghjklops
abdgmp	cefhjklnp	abcelp	dfghjkmnp
ahjlnp	bcdefgkmp	acdeghjmnp	bfklp
abcdefjklos	ghmnos	abfgjkmos	cdehlnos
acefghkmnos	bdjlos	adfhklnos	bcegjmos
jlmno	abcdefghko	cdegjno	abfhklmo
bdgho	acefjklmno	bcehlmo	adfgjkno
cefgknps	abdhjlmps	dfklmmps	abceghjps
bcdefhjklmps	agnps	bfghjkps	acdelmnps
acdefhjknop	bglmop	afghjklmnop	bcdeop
abcefgklmop	dhjnop	abdfkop	ceghjlmnop
adghlmns	bcefjks	acehns	bdfgjklms
abjs	cdefghklmns	abcdegjlms	fhkns
bcefghjkmno	adlo	bdfhjklno	acegmo
cdefklo	abghjmno	fgkmo	abcdehjlno
bhlnps	acdefgjkmps	bcdeghmnps	afjklps
dgjmps	abcefhklnps	cejlps	abdfghkmnps
abdgjlnop	cefhkmop	abcejmnop	dfghklop
ahmop	bcdefgjklnop	acdeghlop	bfjkmnop
abcdefkmns	ghjls	abfgklns	cdehjms
acefghjkls	bdmns	adfhjkms	bceglns

<div align="center">2</div>
<div align="center">cfglmnp</div>

THE NATIONAL BUREAU OF STANDARDS

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