NATIONAL BUREAU OF STANDARDS REPORT

4274

PROJECTS and PUBLICATIONS of the APPLIED MATHEMATICS DIVISION

A Quarterly Report

April through June 1955

FOR OFFICIAL DISTRIBUTION

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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- Office of Basic Instrumentation
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NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT	NBS REPORT
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PROJECTS and PUBLICATIONS of the APPLIED MATHEMATICS DIVISION

April through June 1955

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

April 1 through June 30, 1955

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*Only unclassified projects are included in this report.

Status of Projects

June 30, 1955

I. NUMERICAL ANALYSIS SECTION

(Section 11.1)

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-10-1104/55-55

Origin: NBS — Authorized 8/13/54
Managers: J. Todd, P. Davis — Revised 8/29/54
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. A library of matrix codes has been completed by M. Newman. These are general purpose matrix routines which perform any of the arithmetical operations AB, $A \pm B$, A^{-1} in any desired order. All the routines are for the internal memory of SEAC, the data being taken as single precision floated decimal numbers. The floating subroutines for these codes were written by K. Goldberg.

The routine for A^{-1}, for example, produces the inverse of a 28 by 28 matrix in approximately 15 minutes. In addition, there are available routines for the solution of equations, A'A, A'BA, and other routines of a special nature.

The routines all have the characteristic that the data input and data output are in the same standard form (either decimal or binary) so that the output of one routine may be used as the input of another. It is planned to add to this library other basic matrix codes, such as the finding of characteristic values and characteristic vectors.

P. Henrici continued his program of automatizing certain complicated arithmetic operations, in particular, computations with power series with rational coefficients, on SEAC. Two reports have been prepared which describe the results obtained and discuss future applications.

P. Henrici derived a new proof of the asymptotic expansion of the function

$$G(x) = \int_0^{\infty} e^{-\frac{u^3}{3}} (1 - e^{-xu}) \frac{du}{u} .$$

(to be included in the introduction of the Tables of integrals related to the Airy functions, to be submitted for publication in the NBS Applied Mathematics Series.)

J. Todd completed a paper giving a direct approach to the problem of the numerical stability of discrete schemes for the solution of partial differential equations, by various schemes, implicit and explicit, for such equations as $u_{xx}=u_t$, $u_{xxxx}+u_{tt}=0$, $u_{xx}+u_{yy}=u_t$, $u_{xx}=u_{tt}$. The main tools used are (1) the facts that the characteristic roots of a polynomial P in

a matrix C are the same polynomials $P(\lambda)$ of the characteristic roots λ of C and (2) certain facts about partitioned matrices.

Publications:

(1) Stable systems of differential equations with integrable perturbation term. H. Antosiewicz. To appear in the Journal of the London Mathematical Society.
(2) Computation of vibration modes and frequencies on SEAC. W. Cahill and S. Levy (NBS 6.4). Accepted, Journal of the Institute of Aeronautical Sciences.
(3) On a problem in the theory of mechanical quadratures. P. Davis. To appear in the Pacific Journal of Mathematics.
(4) A multi-purpose orthonormalizing code and its uses. P. Davis and P. Rabinowitz. J. Assoc. Comp. Mach. 1, 183-191 (1954).
(5) On the Lerch zeta function. F. Oberhettinger. To appear in the Pacific Journal of Mathematics.
(6) Generation and testing of random numbers on SEAC. O. Taussky, J. Todd, M. Newman, and J. Cameron. In manuscript.
(7) On nonlinear differential equations of the second order with integrable forcing term. H. A. Antosiewicz. J. London Math. Soc. 30, 64-67 (1955).
(8) Evaluation of a definite integral. P. Henrici. In manuscript.
(9) Asymptotic location of the zero of the Bessel polynomials. P. Henrici. In manuscript.
(10) The generation of Coulomb wave functions by means of recurrence relations. I. Stegun and M. Abramowitz. Phys. Rev. 98, 1851-1852 (June 15, 1955).
(11) A subroutine for computations with rational numbers. P. Henrici. In manuscript.
(12) Automatic computations with power series. P. Henrici. In manuscript.
(13) A direct approach to the problem of stability in the numerical solution of partial differential equations. J. Todd. In manuscript.
(14) Some Monte Carlo experiments for computing multiple integrals. P. Davis and P. Rabinowitz. Submitted to a technical journal.
(15) Abscissas and weights for Gaussian quadratures of high order. P. Davis and P. Rabinowitz. In manuscript.
(16) Numerical experiments in potential theory using orthonormal functions. P. Davis and P. Rabinowitz. In manuscript.
(17) Begründung für die Beschäftigung mit numerischer analysis. J. Todd. To appear in the Jahresbericht d. D.M.V.

BASIC RESEARCH IN LINEAR PROGRAMMING
Task 1101-10-5116/50-2

Origin: NBS — Authorized 3/31/50
Sponsor: Office of Scientific Research, ARDC, USAF, and Directorate of Management Analysis, USAF
Managers: J. Todd, A. Hoffman
Full task description: Jan-Mar 1950 issue, p. 24

Status: COMPLETED. H. Antosiewicz completed the editing of the Proceedings of the Linear Programming Symposium held in January. The exact date of publication is not yet known.

A. J. Hoffman continued the study of combinatorial applications of linear programming. An incidence matrix A with m rows and n columns is said to have the "unimodular property" if every square submatric of A has a determinant with absolute value 0 or 1. Extending an earlier result, he has shown that A has the unimodular property if

$$a_{ij_1}=a_{ij_2}=1 \longrightarrow \text{for all } k>j, \text{ either } a_{kj_1}=1 \longrightarrow a_{kj_2}=1 \text{ or } a_{kj_2}=1 \longrightarrow a_{kj_1}=1.$$

This lemma can be applied to the study of partially ordered sets in which $x \geq z$, $y \geq z$ implies x and y are comparable. Further this lemma seems to be the underlying reason for the integrality of solutions to linear programming problems (in which special properties of the "right-hand side" are not employed) for all known cases not covered by the lemma (used in the transportation problem and its generalizations) that the incidence matrix for points and arcs of a linear graph with no odd cycles has the unimodular property.

Additional work in the field of linear programming will be reported under task 1101-10-5116/55-56, Research in Mathematical Topics Applicable to Numerical Analysis.

Publications:

(1) A theorem on alternatives for pairs of matrices. H. A. Antosiewicz. To appear in the Pacific Journal of Mathematics.
(2) Discrete analogs of inequalities of Wirtinger. Ky Fan, O. Taussky, and J. Todd. Monatsh. Math. 59, 73-90 (1955).
(3) On distinct systems of representatives. A. J. Hoffman and H. W. Kuhn. To appear in the Annals of Mathematical Study 38.
(4) On "overshoot" in the "furthest hyperplane" method. R. Bryce. In manuscript.
(5) An algorithm for solving the transportation problem. A. Gleyzal. J. Res. NBS 54, 213-216 (1955).
(6) Notes on some scheduling programs. A. J. Hoffman. In manuscript.
(7) On block relaxation. L. S. Joel. In manuscript.
(8) Systems of linear inequalities. Ky Fan. To appear in an Annals of Mathematics Study.
(9) Systems of distinct representatives and linear programming. A.J. Hoffman. To appear in Annals of Mathematics Study 38.
(10) On a theorem of Dilworth. A. J. Hoffman and G. B. Dantzig (RAND Corporation). To appear in Annals of Mathematics Study 38.
(11) On the minimization of concave and convex functionals. G. B. Dantzig, A. J. Hoffman, W. Hirsch. In manuscript.
(12) How to solve a linear programming problem. A. J. Hoffman. To appear in the Proceedings of a Symposium on Linear Programming held in Washington, D. C., January 1955.
(13) Linear programming in bid evaluation. L. Gainen. To appear in the Proceedings of a Symposium on Linear Programming held in Washington, D. C., in January 1955.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-10-5116/55-56

Origin: NBS
Sponsor: Office of Naval Research
Managers: O. Taussky-Todd, M. Newman
Full task description: July-Sept 1954 issue, p. 5

Authorized 8/13/54
Revised 8/29/54

Status: CONTINUED. M. Newman completed a paper entitled "An inclusion theorem for modular subgroups," which generalizes certain results previously obtained. Let G be the multiplicative group of matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where a,b,c,d are rational integers and ad-bc=1. Let G(m,n) be the subgroup of G characterized by $b \equiv 0 \pmod{m}$ and $c \equiv 0 \pmod{n}$. Then if (m,n)=1 and H is a subgroup of G containing G(m,n), $H=G(m_1,n_1)$ where $m_1 \mid m$ and $n_1 \mid n$. If $(m,n) > 1$, the theorem is false and explicit counterexamples are given. A further generalization is given to the case where the underlying ring is any ring of algebraic integers.

Status of Projects

Matrices with a property which is shared by each of the principal minors were studied by K. Goldberg. If there is a transformation P of such a matrix A such that each principal minor of A is transformed into the corresponding principal minor of A^P and A and A^P have equal determinants, then A and A^P have equal roots. This procedure was used in a case where the transformation matrix A^P was symmetric so that the roots of A were shown to be real.

A problem in graph theory was studied and partially solved by K. Goldberg together with E. Dade. The problem is to find the number of ways that a certain set of graphs can be combined in order to give graphs with a prescribed number of polygons.

A study of defining combinatorial operators to simplify the procedures used in the method of generating functions is in progress. These operators have been used to generalize and give a more direct proof of previous work done in finding the coefficients of products of formal power series in non-commutative variables.

Olga T. Todd, jointly with E. G. Straus of the University of California at Los Angeles, studied algebraic equations satisfied by roots of natural numbers. A manuscript was prepared. The paper generalizes some of the work of paper (11) below.

Dr. Todd studied the connection between certain theorems which hold both for symmetric and for non-negative non-symmetric matrices.

Working jointly with I. L. Glicksberg of the RAND Corporation, Ky Fan continued the study of fully k-convex normed linear spaces.

Publications:

(1) On a generalization of the normal basis in abelian algebraic number fields. M. Newman and O. Taussky-Todd. Accepted, Communications on Pure and Applied Mathematics (New York University).

(2) A note on group matrices. O. Taussky-Todd. To appear in the Proceedings of the American Mathematical Society.

(3) The normalizer of certain modular subgroups. M. Newman. To appear in the Canadian Mathematical Journal.

(4) Generalizations of identities for the coefficients of certain modular forms. M. Newman. To appear in Journal of the London Mathematical Society.

(5) Compactification of completely regular spaces. Ky Fan and F. Wagner (University of Notre Dame). In manuscript.

(6) Some inequalities concerning positive-definite Hermitian matrices. Ky Fan. To appear in the Proceedings of the Cambridge Philosophical Society.

(7) A comparison theorem for eigenvalues of normal matrices. Ky Fan. To appear in the Pacific Journal of Mathematics.

(8) Power series for $\log(e^x.e^y)$. K. Goldberg. To appear in the Duke Journal of Mathematics.

(9) On certain series expansions involving Whittaker functions and Jacobi polynomials. P. Henrici. To appear in the Pacific Journal of Mathematics.

(10) On generating functions of the Jacobi polynomials. P. Henrici. To appear in the Pacific Journal of Mathematics.

(11) The number of absolute points of a correlation. A. Hoffman, M. Newman, E. Straus, and O. Taussky-Todd. To appear in the Pacific Journal of Mathematics.

(12) Structure theorems for modular subgroups. M. Newman. Duke Math. J. 22, 25-32 (1955).

(13) The coefficients of certain modular forms. M. Newman. To appear in the Journal of the London Mathematical Society.

(14) Some computational problems in algebraic number theory. O. Taussky. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953.

(15) Error bounds for eigenvalues of symmetric integral equations. H. Wielandt. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953.

(16) On eigenvalues of sums of normal matrices. H. Wielandt. Submitted to a technical journal.
(17) Automatic computations of nerve fiber excitation. K. S. Cole (Naval Medical Research Institute), H. A. Antosiewicz, and P. Rabinowitz. In manuscript.
(18) On asymptotic stability. H. A. Antosiewicz. In manuscript.
(19) An inclusion theorem for modular subgroups. M. Newman. In manuscript.
(20) Unimodular circulants. O. Taussky-Todd. Accepted for the Schur Memorial Volume, Mathematische Zeitschrift.
(21) Algebraic equations satisfied by roots of natural numbers. E. G. Straus (U.C.L.A.) and O. Taussky. In manuscript.

ANALYTIC STUDY OF WAR GAMES
Task 1101-10-5116/55-83

Origin and Sponsor: Armament Branch, ARDC, USAF Authorized 12/29/54
Manager: H. A. Antosiewicz
Full task description: Oct-Dec 1954 issue, p. 7

Status: CONTINUED. H. A. Antosiewicz continued his investigation of the war games model. As originally proposed, this model included as one of the strategic variables the rate of production of each side in such a manner that, in a game of attack of offensive forces upon offensive forces only, the rates of production of the two opposing sides would tend ultimately to the same goal. This was felt to be unrealistic and unnecessarily restrictive. To achieve more realism, H. A. Antosiewicz proposed a modification of the original model which, for the same situation, permits two different limiting values for the rates of production. At the same time it is possible to include what might be called overshoot of rate of production; that is, the rates of production are allowed to increase above their preassigned goals before ultimately approaching these (from above). This representation is based upon the reasoning that either side, in case of an attack upon its offensive forces, would attempt to increase its rate of production to a set goal as rapidly as possible and that, in doing so, the rate of production, when first reaching its goal, would tend to exceed it for some time thereafter. A report on this modification has been prepared and submitted to the sponsor.

Publication:
(1) Analytic study of war games. H. A. Antosiewicz. Submitted to a technical journal.

STUDIES IN THE THEORY OF ASYMPTOTIC EXPANSIONS
Task 1101-10-5116/55-116

Origin: NBS Authorized 3/31/55
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: P. Davis
Full task description: Jan-Mar 1955 issue, p. 8

Status: CONTINUED. Let $z=1$ lie on the boundary of a simply connected region D. For $f(z)$ regular in D, write

$$f(z) \sim \sum_{n=0}^{\infty} a_n (z-1)^n$$

to indicate that the right-hand series is the asymptotic expansion (in the sense of Poincaré) for $f(z)$ at $z=1$. Write,

$$R_n(z) = f(z)-a_0-a_1(z-1)-\ldots-a_{n-1}(z-1)^{n-1}, \quad f_n(z)=(z-1)^{-n}R_n(z).$$

For a given sequence of positive quantities $\{m_n\}$ we consider the class $A(m_n)$ of functions analytic in D for which

$$\|f_n\|^2 \leq M\, k^n\, m_n^2 \qquad (n=0,1,\ldots).$$

Here $\|\ \|$ designates some conveniently chosen norm. The norm

$$\|f\|^2 = \int_c |f(z)|^2\, ds$$

has been studied in detail. Carleman's idea of introducing a minimum problem has been combined with the theory of conformal kernel functions to arrive at a necessary and sufficient condition on $\{m_n\}$ in order that $A(m_n)$ be a uniqueness class for asymptotic expansions.

II. COMPUTATION LABORATORY

(Section 11.2)

1. Mathematical Tables

1102-40-1110/43-3 TABLES OF $E_1(z)$, $(z=x+iy)$

Manager: I. Stegun
Full task description: Apr-June 1949 issue, p. 41
Status: Continued. The manuscript of the tables is completed. Preparation of the introduction for publication continued.

1102-40-1110/47-2 TABLES OF COULOMB WAVE FUNCTIONS
Origin: NBS
Manager: P. Rabinowitz
Full task description: Apr-June 1949 issue, p. 45
Status: Continued. Computations of F_0, F_0', G_0, G_0' were made for $\rho=4\eta$, $\rho=3\eta$, $\rho=\eta$, $\rho=\eta/2$, with $0.5<\rho<50$. These tables will provide initial values to obtain results for the values of η and ρ by integration of the differential equation. Values for $L>0$ can be generated from the recurrence relations for the Coulomb wave functions, cf., I.A.Stegun and M.Abramowitz, "The generation of Coulomb wave functions by means of recurrence relations," The Physical Review, June 15, 1955.

1102-40-1110/50-7 WAVE FUNCTION FOR LITHIUM
Origin: NBS
Sponsor: Bureau of Ordnance, USN
Manager: W. F. Cahill
Full task description: Apr-June 1950 issue, p. 36
Status: Completed. The objective of this task was the calculation of the wave function for ionized lithium in the $1s2s\ ^3S$ state, and of the corresponding eigenvalue. The function $\phi=r_1 r_2 \psi$, where ψ is the wave function desired and r_1 and r_2 are the respective distances of the two electrons from the nucleus, was expressed as a series:

$$(1) \qquad \phi(r_1,r_2,\theta)=\sum_{\ell=0} \sqrt{2\ell+1}\ \Phi_\ell P_\ell(\cos\theta)$$

where P_ℓ are Legendre functions and θ is the angle subtended at the nucleus by the two electrons. The coefficients Φ_ℓ are functions of r_1 and r_2 satisfying a set of coupled partial differential equations which are obtained by substituting from (1) into the Schroedinger equation for the problem. The fact that Φ_0 dominates $\Phi_{\ell\neq 0}$ was made use of in the computation. The functions $\Phi_{\ell=0}$ as well as a small correction to Φ_0 were computed from difference equations obtained from the differential equations, using an iteration process. The functions Φ_1 and Φ_2 were computed on the SEAC and used in computing corrections to the eigenvalue.

The results are substantially in agreement with computations carried out by other methods at Yale University, and they demonstrate the suitability of the method used here. The SEAC codes developed for this problem are usable for similar computations for other atoms.

1102-40-1110/51-8 **TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS**
Origin: Section 11.3, NBS
Managers: A. J. Hoffman, S. Tsingou
Full task description: Apr-June 1951 issue, p. 49
Status: Inactive. For status to date, see Jan-Mar 1955 issue, p. 10.

1102-40-1110/52-7 **REVISION OF MATHEMATICAL TABLES**
Origin: NBS
Managers: W. F. Cahill, I. Stegun
Full task description: July-Sept 1951 issue, p. 41
Status: Continued. Revision of "Tables of natural logarithms," vol. IV, Mathematical Tables MT12 (1941), for reissue in the Applied Mathematics Series continued.

1102-40-1110/52-14 **TABLE OF ARCSIN FOR COMPLEX ARGUMENTS**
Origin: NBS
Manager: B. Gill
Full task description: July-Sept 1951 issue, p. 41
Status: Inactive. For status to date, see Jan-Mar 1955 issue, p. 11.

1102-40-1110/52-18 **EXTENSION OF THE TABLE OF HYPERBOLIC SINES AND COSINES**
Origin: NBS
Manager: K. Nelson
Full task description: July-Sept 1951 issue, p. 41
Status: Continued. The manuscript has been transmitted to the Government Printing Office for publication.
Publication: "Table of hyperbolic sines and cosines," (x=2 to x=10, 9S), Applied Mathematics Series 45; in press.

1102-40-1110/52-23 **TABLE OF THE MODIFIED AIRY INTEGRAL**
Origin: NBS
Manager: P. Rabinowitz
Full task description: July-Sept 1951 issue, p. 42
Status: Continued. The tables and the introduction have been completed and are being prepared for publication.

1102-40-1110/52-25 **TABLE OF ERROR FUNCTION FOR COMPLEX ARGUMENTS**
Origin: NBS
Manager: W. Hall
Full task description: July-Sept 1951 issue, p. 42
Status: Inactive. For status to date, see Oct-Dec 1954 issue, p. 11.

1102-40-1110/52-31 **EXTENSION OF TABLES OF THE EXPONENTIAL FUNCTION FOR NEGATIVE ARGUMENTS**
Origin: NBS
Managers: E. Marden, S. Prusch
Full task description: July-Sept 1951 issue, p. 43
Status: Continued. The table is in press at the U. S. Government Printing Office.
Publication: "Table of the descending exponential," (x=2.5 to x=10, 20D), Applied Mathematics Series 46; in press.

1102-40-1110/52-37 **SPHEROIDAL WAVE FUNCTIONS**
Origin: NBS
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38
Status: Inactive.

1102-40-1110/52-49 **RADIAL MATHIEU FUNCTIONS**
Origin: NBS
Manager: I. Rhodes
Full task description: Jan-Mar 1952 issue, p. 45
Status: Inactive. For status to date, see July-Sept 1954 issue, p. 10.

1102-40-1110/52-57 SIEVERT'S INTEGRAL
Origin: NBS
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46
Status: Continued. Preparation of the manuscript in a form suitable for publication has been started on the IBM card-controlled typewriter.

1102-40-1110/53-52 L-SHELL CONVERSION COEFFICIENTS
Origin: Oak Ridge National Laboratory
Managers: W. Hall
Full task description: Apr-June 1953 issue, p. 45
Status: Continued. Computations for the L_I shell have been completed. Computations for the L_{II} shell were completed except for z=15. The following eigenvalues were run for the L_{III} shell: z=65, z=75.

3711-60-0009/55-65 AUTOMATIC CODING
Origin: NBS
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11
Status: Continued. J. Cooper investigated a method for modifying the order for conversion from binary to decimal notation. The aim of the modification is to use the order for editing output data automatically.

A routine which will automatically "scan" any elementary function over a given range of a single variable was written by J. Cooper and is being checked. This routine is intended primarily to serve as a tool in planning integration procedures for integrals involving complicated elementary functions.

Experiments are being made by J. Wegstein with a routine for automatically determining the minimum point of a function $S(x_1,x_2,...,x_n)$ where $S \geq 0$. Such a routine would be useful, among other things, for fitting curves to arbitrarily chosen functions and for solving sets of simultaneous (not necessarily linear) equations.

A compiler routine has been written by J. Bram. It takes a program written in a pseudo code and prepares a standard three-address SEAC code. The pseudo code consists of symbols which correspond almost exactly with mathematical and flow diagram symbolism. In particular, any mathematical expression suitably furnished with parentheses to avoid ambiguity and consisting only of the four rational arithmetical operations and the elementary functions can be written by a symbol-for-symbol transliteration of the mathematical symbols into the pseudo-code symbols. The final compiled code is a fixed-point three-address code.

A technical memorandum on "The Base 00 computational system" was completed during this quarter.

2. Mathematical Services

Note: The tasks under Mathematical Services are arranged serially according to the digits following the slant lines in the task number. The first two digits following the slant line designate the fiscal year in which the task was authorized.

1102-40-5126/50-13 RAY TRACING
Origin and Sponsor: NBS, Section 2.2
Manager: E. Marden
Full task description: Jan-Mar 1950 issue, p. 33
Status: Inactive.

1102-40-5126/51-37 MOLECULAR STRUCTURE, III
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. O'Hara
Full task description: July-Sept 1951 issue, p. 50
Status: Continued. Refinement procedures were continued for improving the structure of the spurite crystal.

1102-40-5126/52-20 SPHERICAL BLAST
Origin and Sponsor: Naval Ordnance Laboratory
Manager: D. Jirauch
Full task description: July-Sept 1951 issue, p. 56
Status: Completed. A report entitled "Dynamic model of a spherically symmetric explosion" is being prepared. The report is concerned with the numerical solution of the partial differential equation of hyperbolic type describing gas flow behind a shock wave. It describes a new method of programming the numerical integration of such equations for automatic computors. The method is adapted to cases in which memory space in the machine is a limiting factor; it is applicable to all partial differential equations for which stepwise integration is appropriate.

1102-40-5126/52-44 CALCULATIONS FOR d SPACINGS
Origin and Sponsor: NBS, Division 9
Manager: A. Futterman
Full task description: Oct-Dec 1951 issue, p. 47
Status: Continued. Calculations for d-spacings were performed for the following cases: (1) orthorhombic crystals: Cs_2CrO_4, $NgSiO_3$, NH_4ClO_4; (2) hexagonal crystals: $(NH_4)GeF_6$, K_2GeF_6, Ne_2PtF_6, $NaFeO_2$; (3) tetragonal crystal $CaWO_4$. Redetermination of unit cell constants by least squares fitting to measured d-spacings was carried out for the following crystals: (1) orthorhombic: Cs_2CrO_4, NH_4ClO_4; (2) hexagonal: AsI_3, $(NH_4)GeF_6$, K_2GeF_6; (3) tetragonal: $CaWO_4$.

1102-40-5126/53-27 COMPUTATION OF THERMODYNAMIC FUNCTIONS
Origin and Sponsor: NBS, Section 3.2
Manager: E. Marden
Full task description: Jan-Mar 1953 issue, p. 57
Status: Continued. The SEAC code for computing thermodynamic functions to include anharmonicity, rotation-vibration coupling, centrifugal stretching, and non-classical rotational terms has been modified to accommodate molecules with a greater number of frequencies. Calculations of thermodynamic functions from 50°K to 15,000°K were performed for the following molecules: HCN, DCN, TCN, and CF_2Cl_2.

In addition, calculations were made for (C_2Cl_4) and (C_2Br_4) using the original code for this problem.

1102-40-5126/53-51 RADIATION DIFFUSION
Origin: NBS, Section 4.8
Sponsor: Atomic Energy Commission
Manager: A. Futterman
Full task description: Apr-June 1953 issue, p. 57 (Neutron Diffusion III)
Status: Continued. At present attention is fixed on a study of the diffusion of radiation from a source in the plane interface between two different homogeneous media. A calculation of elastic scattering cross sections for electrons is being carried out. A code for the first phase of this problem, viz., the evaluation of the gamma function for complex arguments, has been completed.

1102-40-5126/54-13 AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: New York Quartermaster Procurement Agency
Manager: H. Bremer
Full task description: Oct-Dec 1953 issue, p. 43
Status: Continued. Analyses of contract award problems were carried out on SEAC at the rate of about one per week. The contracts covered items such as overcoats, trousers, shirts, pillow cases, tents, and insignia.

3711-60-0009/54-17 DEPOLYMERIZATION
Origin: NBS, Section 7.6
Manager: J. Bram
Full task description: Oct-Dec 1953 issue, p. 44
Status: Inactive. For status to date, see Jan-Mar 1955 issue, p. 15

3711-60-0009/54-19 ENERGY LEVELS OF COMPLEX ATOMS
Origin: NBS, Section 4.1
Manager: W. Hall
Full task description: Jan-Mar 1954 issue, p. 41
Status: Continued. A code to form matrices of electrostatic and magnetic interactions for four electron spectra with d- and s-electrons ($d^4+d^3s+d^2s^2$), given 20 parameters which are the Slater integrals of electrostatic interaction and the integrals of magnetic interaction, has been checked out. This code has been used to generate matrices for the spectrum of Tantalum II and Ruthenium I.

1102-40-5126/54-21 EXPERIMENTAL PROGRAM FOR MULTIPLE INPUT AND OUTPUT
Origin: NBS, Section 12.3
Sponsor: Bureau of Supplies and Accounts, Department of the Navy
Manager: I. Rhodes
Full task description: Jan-Mar 1954 issue, p. 42
Status: Terminated. No further studies of a general nature are contemplated at this time. Specific applications growing out of this task are being reported under other task headings, especially 1102-40-5126/55-102, p. 16.

3711-60-0009/54-22 ENERGY DISTRIBUTIONS ON OPTICAL IMAGE
Origin: NBS, Section 2.2
Manager: L. S. Joel
Full task description: Jan-Mar 1954 issue, p. 43
Status: Inactive.

3711-60-0009/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Manager: S. Prusch
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. A SEAC code was prepared for computing wave numbers (σ) from wave lengths (λ). Also a code is being prepared for the searching of pairs of lines.

1102-40-5126/54-33 AIR CONDITIONING IN UNDERGROUND STRUCTURES
Origin and Sponsor: NBS, Section 10.3
Manager: S. Tsingou
Full task description: Jan-Mar 1954 issue, p. 47
Status: Completed. The desired integral (cf. Jan-Mar 1955 issue, p. 16) has been evaluated for about 20 values of the parameter $\alpha t/a^2$.

1102-40-5126/54-34 EQUILIBRIUM CALCULATIONS FOR WATER
Origin and Sponsor: Naval Ordnance Laboratory
Manager: J. Wegstein
Full task description: Apr-June 1954 issue, p. 38
Status: Inactive.

1102-40-5126/54-35 THERMAL STRESSES IN STRUCTURES
Origin and Sponsor: NBS, Section 6.4
Manager: W. Hall
Full task description: Apr-June 1954 issue, p. 38
Status: Inactive. For status to date, see Jan-Mar 1955 issue, p. 16.

3711-60-0009/54-36 VELOCITY OF LIGHT
Origin: NBS, Section 11.3
Manager: P. Rabinowitz, J. M. Cameron
Full task description: Jan-Mar 1954 issue, p. 47
Status: Continued. Additional third degree polynomials were fitted to data submitted by the sponsor to determine rotational constants of HCN and DCN. The method of least squares was used.

3711-60-0009/54-38 COMPRESSIBILITY FACTORS OF DRY AIR
Origin: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Continued. The code was completed and a test run was made. This code, using floating-point routines, computes a table of compressibility factors z and densities ρ where $z=A+B\rho+C\rho^2+D\rho^3+\ldots$ and $\rho = P/RzT$. For a given table the temperature T remains constant and the pressure P varies.

The virial coefficients A,B,C,D,... for a given temperature are furnished by the sponsor. The test runs carried out thus far have been concerned with a third degree polynomial with A=1. The present code can handle polynomials up to the eleventh degree with A arbitrary.

1102-40-5126/55-39 MOLECULAR VIBRATIONS
Origin and Sponsor: NBS, Section 3.2
Manager: K. Goldberg
Full task description: July-Sept 1954 issue, p. 16
Status: Continued. Nineteen symmetric matrices of the form A'BA were computed, given the rectangular matrix A and the symmetric matrix B. The eigenvalues and eigenvectors of these matrices were then calculated.

1102-40-5126/54-43 CHARACTERISTICS OF CONDUCTING RESISTORS
Origin and Sponsor: NBS, Section 12.1
Manager: B. Gill
Full task description: Apr-June 1954 issue, p. 40
Status: Continued. The code was adjusted to process smaller blocks of data; 20 sets were run and the results were transmitted to the sponsor.

3711-60-0009/55-53 ELECTRONIC FUNCTIONS
Origin: NBS, Section 4.5
Manager: W. G. Hall
Objective: To prepare eight tables of the quantities

$$V$$

$$V^* = V + \frac{e}{2m_0c^2} V^2$$

$$H\rho = \sqrt{\frac{2m_0}{e}}\sqrt{V^*}$$

$$\lambda = \frac{h}{\sqrt{2m_0 e}} \frac{1}{\sqrt{V^*}}$$

$$p^* = \frac{e}{m_0 c} H\rho = \beta W^*$$

$$E^* = \sqrt{p^{*2}+1}-1 = \frac{e}{m_0c^2} V = W^*-1$$

$$W^* = E^*+1$$

$$\beta = \frac{p^*}{w^*} = \sqrt{1 - \left(\frac{m_0c^3}{eV\cdot 10^8+m_0c^3}\right)^2}$$

where each of the eight variables V, V*, $H\rho$, λ, p*, E*, W*, β is taken in turn as independent variable. Then to merge the tables of these functions into one table arranged according to increasing values of V. The functions are to be tabulated so that each of the variables takes on the values $ax10^P$, for a=1(.1)3(.2)5(.5)9.5 for integral values of P ranging from 1 to as high as 19 for some of the variables.

Background: These tables of electronic functions will be a revision of those included in NBS Mathematical Table MT17, "Miscellaneous Physical Tables," (1941; now out of print). The present computations are being carried out in accordance with theoretical investigations of L. Marton of the NBS Electron Physics Section (4.5).

Status: New. Coding of the problem was started.

1102-40-5126/55-58 ELECTROMAGNETIC RADIATION FROM LIGHTNING

Origin and Sponsor: NBS, Section 82.1

Manager: B. Gill

Full task description: Jan-Mar 1955 issue, p. 17

Status: Inactive. For status to date, see Jan-Mar 1955 issue, p. 17.

1102-40-5126/55-61 ELASTIC CROSS SECTION FOR NEUTRON SCATTERING

Origin and Sponsor: Naval Research Laboratory

Manager: I. Stegun

Full task description: Oct-Dec 1954 issue, p. 18

Status: Continued. Computations were carried out on SEAC for 335 sets of parameters. Comparison of theoretical results with experimental results for lead, copper, carbon, tin, and aluminum continued.

1102-40-5126/55-62 INTEGRALS INVOLVED IN SUPERSONIC FLUTTER

Origin and Sponsor: National Advisory Committee for Aeronautics

Manager: S. Tsingou

Full task description: July-Sept 1954 issue, p. 17

Status: Completed. A code to check the tables was completed, and the tables were checked out. A manuscript was prepared and results were sent to the sponsor.

1102-40-5126/55-64 HIGH-TEMPERATURE THERMODYNAMIC TABLES (FMT)

Origin and Sponsor: NBS, Section 3.2

Manager: J. Wegstein

Full task description: July-Sept 1954 issue, p. 18

Status: Inactive.

3711-60-0009/55-66 RECONSTITUTION OF MONOCHROMATIC LIGHT INTENSITIES

Origin: NBS, Division 30

Manager: H. Bremer

Full task description: July-Sept 1954 issue, p. 18

Status: Inactive. For status to date, see Oct-Dec 1954 issue, p. 19.

3711-60-0009/55-68 CRYSTAL STRUCTURE CALCULATIONS

Origin: NBS, Division 9

Manager: A. Futterman

Full task description: Jan-Mar 1955 issue, p. 18

Status: Continued. Checking of the code for the calculations of d-spacings continued. This code is related to the one used in task 1102-40-5126/52-44 (p. 10) but is more general in that it includes the monoclinic and triclinic systems to which the older code does not apply.

3711-60-0009/55-69 **THEORY OF DIELECTRIC RELAXATION**
Origin: NBS, Section 7.6
Manager: K. Goldberg
Full task description: July-Sept 1954 issue, p. 19
Status: Continued. A paper explaining the method used to prove the reality of the roots of certain matrices and applying this method to the matrices arising in the theory of dielectric relaxation is in preparation.

1102-40-5126/55-72 **MARYLAND INTER-INDUSTRY STUDY**
Origin and Sponsor: Directorate of Intelligence, USAF*
Manager: L. S. Joel
Full task description: July-Sept 1954 issue, p. 20
Status: Completed. All desired computations, consisting of the inversion of certain Leontieff matrices, were carried out and the results were transmitted to the originator.

1102-40-5126/55-73 **HEAT CONVECTION**
Origin: University of Minnesota
Sponsor: Atomic Energy Commission
Manager: G. Hawkins
Full task description: Oct-Dec 1954 issue, p. 20
Status: Completed. Several programs were written and solutions were obtained corresponding to minor variants in the mathematical formulation of the problem. Results have been communicated to the University of Minnesota where a decision on publication will be made.

1102-40-5126/55-74 **LIQUID-VAPOR TRANSITION, II**
Origin and Sponsor: Naval Medical Research Institute
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 19
Status: Continued. The integral equation was solved on punched-card machines for two values of the parameter and two given sets of functional values.

3711-60-0009/55-75 **PARAMETER OF THE DISPERSION EQUATION FOR OPTICAL GLASS**
Origin: NBS, Section 1.6
Manager: R. Zucker
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. The constants K_i of the equation

$$n^2-1 = \sum_{i=1}^{5} \frac{K_i \lambda^2}{\lambda^2-\lambda_i^2}$$

were evaluated by the method of least squares for given values of n, λ and λ_i. Then a table of n was calculated for the given values of λ and λ_i and the obtained values of K_i. A table of the absolute differences of the observed and calculated values of n was also obtained. Finally, a table of n was submitted to the sponsor for λ=0(.02)1(.04)2(.2)3(.5)10(.2)50.

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*The origin and sponsor of this task was incorrectly reported in previous issues.

1102-40-5126/55-78 NEUTRON TRANSPORT
Origin and Sponsor: Atomic Energy Commission, New York
Manager: S. Tsingou
Full task description: Oct-Dec 1954 issue, p. 21
Status: Completed. No SEAC computations beyond those already reported appear to be required at this time.

1102-40-5126/55-79 COMPLETE DEGRADATION IN THE NEUTRON
Origin and Sponsor: Atomic Energy Commission, New York
Manager: A. Futterman
Full task description: Oct-Dec 1954 issue, p. 21
Status: Completed. No further computations are required at this time.

1102-40-5126/55-80 ALPHA-ANALYSIS
Origin and Sponsor: Atomic Energy Commission, New York
Manager: W. G. Hall
Full task description: Oct-Dec 1954 issue, p. 22
Status: Completed. No further computations are required at this time.

3711-60-5126/55-81 COMBINING TESTS FOR SIGNIFICANCE
Origin: NBS, Section 11.3
Manager: L. S. Joel
Full task description: Oct-Dec 1954, p. 23
Status: Continued. Checking of the code continued.

3711-60-0009/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Inactive. For status to date, see Jan-Mar 1955 issue, p. 21.

3711-60-0009/55-86 FLOW COEFFICIENTS FOR FLUIDS
Origin: NBS, Section 6.7
Manager: P. J. Walsh
Full task description: Jan-Mar 1955 issue, p. 21
Status: Continued. Using empirical formulae similar to (1) in the previous report (Jan-Mar 1955 issue, p. 21), a code was prepared to evaluate flow coefficients for 10 values of D ranging between 2.067 and 15.25 and for approximately 40 values of β where $.1<\beta<.82$. The code is now being checked.

1102-40-5126/55-87 "ZERO" METHOD DETERMINATION OF CRYSTAL STRUCTURE
Origin and Sponsor: NBS, Section 9.7
Manager: J. Bram
Full task description: Jan-Mar 1955 issue, p. 22
Status: Inactive. For status to date, see Jan-Mar 1955 issue, p. 22.

1102-40-5126/55-88 STRESSES IN A WALL FOUNDATION
Origin and Sponsor: NBS, Section 10.1
Managers: E. Marden, G. Hawkins
Full task description: Jan-Mar 1955 issue, p. 22
Status: Continued. Stresses in a wall foundation were computed for a given set of data corresponding to a single wall.

3711-60-0009/55-89 TEMPERATURE DISTRIBUTION IN SOLID WINGS HEATED AERODYNAMICALLY
Origin: NBS, Section 6.4
Manager: P. Davis, W. F. Cahill
Full task description: Oct-Dec 1954 issue, p. 22
Status: Inactive. For status to date, see Jan-Mar 1955 issue, p. 23.

3711-60-0009/55-90 STUDY OF A DIFFERENCE EQUATION ARISING IN STATISTICAL MECHANICS
Origin: NBS, Section 3.2
Manager: M. Newman
Full task description: Oct-Dec 1954 issue, p. 23
Status: Terminated. It appears to be unnecessary at this time to pursue the study of this problem beyond the solution previously reported (cf. Oct-Dec 1954 issue, p. 23).

3711-60-0009/55-91 COMPUTATION OF VIRIAL COEFFICIENTS
Origin: NBS, Section 3.2
Manager: P. Davis
Full task description: Oct-Dec 1954 issue, p. 23
Status: Terminated.

3711-60-0009/55-92 CYLINDRICAL ELECTRON LENS CALCULATIONS
Origin: NBS, Section 4.5
Manager: W. G. Hall
Full task description: Jan-Mar 1955 issue, p. 24
Status: Continued. Approximately 200 trajectories were computed. Two values of A were used corresponding to two systems. Except for prints the same routine was used for all cases. Only x_0 and the direction and magnitude of the initial velocity were changed.

1102-40-5126/55-96 DISTRIBUTION OF LIGHT OUTPUT FROM NEUTRON SPECTROMETERS
Origin and Sponsor: NBS, Section 4.11
Manager: I. Rhodes
Full task description: Jan-Mar 1955 issue, p. 25
Status: Completed. The results were transmitted to the sponsor. They show that it is feasible to construct a spectrometer 100 times as powerful as those previously available.

1102-40-5126/55-98 THERMAL DIFFUSIVITY
Origin and Sponsor: NBS, Section 3.2
Manager: P. Davis
Full task description: Jan-Mar 1955 issue, p. 25
Status: Terminated. Present needs have been taken care of with standard solutions.

1102-40-5126/55-102 COST ACCOUNTING OPERATIONS FOR HIGH SPEED COMPUTERS
Origin and Sponsor: Bureau of Supplies and Accounts, Dept. of the Navy
Managers: G. Hawkins, E. Marden
Full task description: Jan-Mar 1955 issue, p. 25
Status: Continued. Three main routines were written and checked on UNIVAC. In addition, several input data handling routines and an edit routine were written and checked.

1102-40-5126/55-103 MISSOURI RIVER PROBLEM
Origin and Sponsor: Missouri River Division, Corps of Engineers, U.S.Army
Manager: H. Bremer
Full task description: Jan-Mar 1955 issue, p. 26
Status: Continued. Codes have been written and checked to obtain simple and multiple regression coefficients. These codes have been written for UNIVAC.

1102-40-5126/55-105 AERONOMIC TIDAL WINDS OF THERMAL ORIGIN
Origin and Sponsor: NBS, Section 82.1
Managers: B. Gill, H. Howe
Full task description: Jan-Mar 1955 issue, p. 26
Status: Inactive. For status to date, see Jan-Mar 1955 issue, p. 26.

1102-40-5126/55-106 REFLEX KLYSTRON
Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Department of the Army
Manager J. Bram
Full task description: Jan-Mar 1955 issue, p. 27
Status: Completed. The desired integral n, representing the efficiency of a reflex klystron, was evaluated for about 300 combinations of the parameters c, ωT_0 and V_1/V_0 (for definitions, see previous report, p. 27). Specifically, for each of the values c =100, 10,1,.5,.2,.1, values of ωT_0 were chosen so that the expression $\theta_0=3\sqrt{2}\ \omega T_0(\sqrt{c+1}-\sqrt{c})$ has one of the values $2\pi(n+3/4)$, n=3,5,7,9,11; for each of these combinations of c and ωT_0, n was computed for about ten values of V_1/V_0 lying between 0 and $(V_1/V_0)^*$, the latter being approximately the value at which n=0. (For fixed c and ωT_0, n increases monotonically from 0 at $V_1/V_0=0$ to a maximum, then decreases and crosses the V_1/V_0 axis.)

1102-40-5126/55-109 ELECTRON PULSE HEIGHT DISTRIBUTION
Origin and Sponsor: NBS, Section 4.11
Manager: R. Zucker
Full task description: Jan-Mar 1955 issue, p. 27
Status: Completed. The results were transmitted to the sponsor.

1102-40-5126/55-113 REACTOR DESIGN
Origin: Westinghouse Atomic Power Division
Sponsor: Atomic Energy Commission
Manager: U. Hochstrasser
Full task description: Jan-Mar 1955 issue, p. 28
Status: Continued. Methods for the numerical solution of this problem have been investigated. A simple test problem has been set up and a set of difference equations approximating the system of differential equations has been derived. A code is being prepared for finding the solution of this set of 600 difference equations using D. Young's overrelaxation method. This test problem will be used to investigate certain variations of the overrelaxation method and to work out procedures and codes for problems more general in the geometry of the reactor and in the type of system of differential equations.

1102-40-5126/55-115 ADSORPTION INTEGRALS
Origin and Sponsor: NBS, Section 3.1
Manager: J. Cooper
Objective: To compute the integrals

$$T(\nu) = \int_{\nu-\Delta}^{\nu+\Delta} (\Delta\nu,x)\ \exp\left(-\frac{s\alpha/\pi}{x^2+\alpha^2}\right)dx$$

for a given range of values of ν and for various values of S_2, α, and $\Delta\nu$. For the initial phase of the problem

$$\sigma(\Delta\nu,x) = \frac{\sin^2 y}{y^2} \text{ where } y = \frac{(\nu-x)\pi}{\Delta\nu}\ .$$

Background: The function T represents the adsorption of radiant energy as it passes through a slit of width $\Delta\nu$ whose geometry is specified by the $\sigma(\Delta\nu,x)$ function. The results are to be compared with experimentally observed values to judge the accuracy of such computations.
Comment: The problem was requested specifically by A. Bass of the NBS Temperature Measurements Section (3.1).
Status: New. A code to compute the integrals was written using 16-point Gaussian quadrature. Computations were carried out for values of $\Delta\nu$=0.04, 0.2, 0.4, 0.8, 4.0, with $s/\pi\alpha$ =0.36 and α=.4. The integrals were plotted for ν=0(.04)2.0.

1102-40-5126/55-117 **ATTENUATION OF PRESSURE PULSES OF FINITE AMPLITUDE**
Origin: NBS, Section 3.2
Sponsor: Bureau of Aeronautics, U. S. Navy
Manager: M. Paulsen
Objective: To investigate the attenuation of finite-amplitude pressure pulses in a simple pneumatic system with an enlarged section in the transmission line. This involves solutions to the second-order partial differential equation of the hyperbolic type,

$$(a^2-u^2)\frac{\partial^2\phi}{\partial x^2} - 2u\cdot\frac{\partial^2\phi}{\partial x\partial z} - \frac{\partial^2\phi}{\partial z^2} = 0$$

with $u = \frac{\partial\phi}{\partial x}$, by the method of characteristics for this type of equation.
Background: In many pneumatic systems, attenuators are often inserted in the system in order to reduce the transmission of pressure pulses from the actuator upstream to the pressure regulator. The present investigation studies the characteristics of such an attenuator as affected by the size and geometry of the attenuator. To simplify the computation, the flow is assumed to be one-dimensional and isentropic.
Comment: This problem was requested by D. H. Tsai of the Pneumatics Laboratory (NBS, Section 3.2).
Status: New. Routines are in the process of preparation for computing the particle velocity μ and sound velocity a at the boundaries between various sections of the attenuator in a simulated pneumatic system. A study is being made which will select the appropriate scheme for computing these quantities at the boundaries of the various sections of the attenuator.

1102-40-5126/55-118 **THERMOMETER CALIBRATION**
Origin and Sponsor: NBS, Section 3.1
Manager: B. Gill
Objective: To fit polynomials to temperature-resistance measurements for platinum-resistance thermometers and to use the polynomials obtained to compute calibration tables.
Background: In 1939 E. W. Hogue and F. Brickwedde, of the NBS Temperature Measurements Section (3.1), established a temperature scale in the region from 10 to 90°K. This scale has been maintained at the NBS by means of a series of capsule-type platinum resistance thermometers. The scale is generally accepted as the unofficial standard in this country, and it has been distributed throughout this country by calibrating similar platinum thermometers against our standards. This calibration involves comparison of resistances at 15 different temperatures. Previously the data from these comparisons have been reduced to a smooth table of resistance vs. temperature entirely by hand, involving about three weeks of computation for a group of three thermometers.
Comment: This problem was requested by J. L. Riddle (NBS Section 3.1).
Status: New. The code for computing the table has been prepared. Experimentation to determine a minimum-degree polynomial of satisfactory fit is in progress.

1102-40-5126/55-120 **SOLUTIONS TO MATRICES**
Origin and Sponsor: NBS, Section 5.9
Manager: K. Goldberg
Objective: To find the characteristic roots of several small ($n \leq 13$) symmetric matrices.
Background: The problem originates in the calculation of the electronic energy levels of the methane molecule.
Comment: The problem was requested by M. B. Wallenstein of the NBS Thermochemistry Section (5.9).
Status: Completed (New). The results were transmitted to the sponsor.

1102-40-5126/55-121 ELECTRON PENETRATION
Origin: NBS, Section 4.8
Sponsor: Atomic Energy Commission
Managers: P. Walsh, A. Futterman
Objective: To calculate a number of electron depth distributions by solving an integro-differential diffusion equation which describes fairly accurately the penetration of electrons.
Background: The diffusion equation governing electron penetration and diffusion in plane geometry contains three independent variables describing an electron's energy, location, and direction. Integral methods have recently been developed and tested for calculating spatial moments of fairly high order of the electron flux. These integral methods, which reduce the calculation of spatial moments to a recursive system, are ideally suited to the capabilities of high speed computers. Spatial moments up to the 9th, 10th, or 12th order will be calculated with the SEAC. A functional representation of the desired distribution which agrees with these spatial moments will then be constructed. The latter requires a numerical solution of a set of simultaneous equations, which can also be easily accomplished with the SEAC. The whole procedure is to be applied to enough situations, i.e., scattering media, source geometries, and source energies, so that interpolation will be feasible to obtain almost any situation of interest to experimentalists up to electron source energies of 10 Mev or so. Perhaps as many as 150 individual problems will be solved in the course of this program, i.e., involving 8 to 10 source energies, 6 to 8 scattering materials, and 2 to 3 geometries. Publication of standard tables of the results is planned.
Comment: This task is an extension of task 1102-40-5126/54-24. It is requested by L. V. Spencer of the Nuclear Physics Section (4.8).
Status: New. The problem was analyzed and coded. The code is now being checked on SEAC.

1102-40-5126/55-122 SOLUTION OF NORMAL EQUATIONS
Origin: Advisory Committee on Weather Control
Sponsor: U. S. Weather Bureau
Manager: M. Newman
Objective: To compute $(S'S - v_1 v_2)^{-1}$, where S, v_1, v_2 are rectangular matrices, for several sets of data.
Background: The problem arises in weather prediction. The data given represents annual precipitation averages throughout various parts of Canada. The results are used to determine regression coefficients which are employed in predicting the weather for those areas.
Comment: The problem was specifically requested by H.C.S.Thom, Chief Climatologist of the Advisory Committee.
Status: New. Eight runs consisting of four 40x6 and four 30x6 S-matrices were completed, and the results were transmitted to the sponsor.

1102-40-5126/55-123 TEMPERATURE DISTRIBUTION
Origin: NBS, Section 6.4
Sponsor: NBS, Section 6.4
Manager: W. G. Hall
Objective: To compute non-steady temperature distributions for an I-beam, taking into consideration heat convection and radiation. The structure of any cross section is divided into 32 or fewer rectangular elements, and the temperature of each element is given by the finite-difference analog of the heat transfer equation.
The transfer of heat through any side is expressed by one of the following relations

$$(1) \qquad q = \left[C+D\left(\frac{t_m+t_n}{2}\right)\right] \frac{A_{mn}}{l_{mn}} (t_m-t_n)$$

$$(2) \qquad q = \left[(G+Ht_n)A_n'(t_n^4-t_a^4)F_n\right]$$

$$(3) \qquad q = \sigma A_n' (G+Ht_n) \sum_{m=1}^{32} F_{mn} (t_m^4-t_n^4)$$

where t_m is the temperature in the m-th element adjoining the n-th element with temperature t_n. The remaining parameters are constants depending on the geometry of the structure and the material.

Background: This task is a generalization of a problem that was done earlier on SEAC; see task 1102-40-5126/54-3. The problem was requested by S. Goodman of the NBS Engineering Mechanics Section (6.4).

Status: New. Coding of the problem was started.

1102-40-5126/55-124 EVALUATION OF INTEGRALS RELATED TO GAS FLOW

Origin: NBS, Section 3.2

Sponsor: Bureau of Aeronautics, U.S. Navy

Manager: J. Cooper

Objective: To evaluate the integrals

$$R_1 = \int_\alpha^\beta \frac{dr}{(1+ar)^{6/7}\sqrt{1-r^2}}$$

and

$$R_2 = \int_\alpha^\beta \frac{dr}{(1+ar)^{6/7}\sqrt{r-r^2}}$$

for various values of β and r. $\alpha = 0.528$, and $a>0$, $\alpha<\beta\leq 1$.

Background: These integrals represent the time taken to fill a reservoir of gas from a container at a higher pressure. The variable of integration r represents the ratio of pressure in the two vessels, and integration is performed from a lower limit where these equations apply to an upper limit which is one or less. An upper limit of unity represents a complete filling of the container, whereas a limit lower than one represents only a partial filling of the container.

The two integrals R_1 and R_2 are needed to represent both turbulent and non-turbulent flow between the two containers.

The factor a in the equations depends on the relative sizes of the reservoirs and on the deviation of the flow from the adiabatic case due to heat losses.

Status: New. The code has been written and checked using 16-point Gaussian quadrature. Values of R_1 and R_2 were computed for a=0.491 and β ranging between 0.528 and approximately 0.900 in steps of .005, and from 0.900 to 1 in steps of .002. Additional values at the same range of β were computed for a=.1(.05)1.3 for R_1 only.

1102-40-5126/55-129 PROCESSING OF PUBLIC HOUSING DATA

Origin: NBS, Section 12.1

Sponsor: U. S. Public Housing Authority

Manager: I. Rhodes

Objective: To attempt to program the tabulation of certain statistical data from reports on families in low rent housing, with special attention to the handling of errors due to inconsistencies in the information reported.

Background: A report on the reexamination of families in low rent housing must be drawn up regularly at the U. S. Public Housing Authority. This

is a compilation of data from detailed individual family reports, consisting of answers to approximately 35 questions for each person in the family and a comment column. Heretofore, mechanization of the compilation of the data has been considered to be an impossible task because of errors and inconsistencies in the data reported and because of the non-numerical comments that contain pertinent statistical information.

Status: New. A few sample reports involving almost all possible types of errors were drawn up for study. All types were listed and each type was assigned a number. A code for tabulating inconsistent data was worked out whereby the machine would be instructed to detect inconsistencies and to indicate each by its number. At present, the code has been written for the SEAC, but codes for other available machines will be considered. A demonstration was held for the benefit of the staff of the Public Housing Authority and other interested parties.

1102-40-5126/55-128 **GROUND REFLECTION COEFFICIENTS**

Origin and Sponsor: NBS, Section 82.1

Managers: B. Gill, H. H. Howe (82.1)

Objective: To prepare tables showing the sum of the intensities of the directly incoming wave and its reflection for each of six components of electromagnetic field, for various values of dielectric constant, angle of incidence, and ratio of conductivity to frequency.

Background: At very low frequencies such as predominate in Sferics, the earth is usually a good reflector. It is impossible to measure directly the intensity of an incoming radio wave because a receiver receives both the directly incoming wave and the wave reflected from the ground. Under reasonable simplifying assumptions the formula for the ground reflection coefficient, which is the ratio of the intensities of the direct and reflected waves, is known: it is given as a complex number, representing both attentuation and change of phase. The coefficient depends upon (1) angle of incidence, (2) dielectric constant of ground, (3) conductivity of ground, and (4) frequency of wave. Items (3) and (4) enter the formula for the coefficient only by their ratio, so that only three essential parameters are involved. The formula is rather intricate, nevertheless; moreover, there are two reflection coefficients, according to the plane of polarization of the incoming wave. The tables, giving the sum of the intensities of the direct and reflected waves will be useful in the interpretation and analysis of observations in Sferics.

Status: New. The code was written and is now being checked.

III. STATISTICAL ENGINEERING LABORATORY

(Section 11.3)

1. Fundamental Research in Mathematical Statistics

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-10-1107/49-1a

Origin: NBS — Authorized 1/9/49
Manager: L. S. Deming
Full task description: Apr-June 1949 issue, p. 75

Status: CONTINUED. For a description of the continuing activity on this task, see the Jan-Mar 1954 issue, p. 49.

MANUAL ON FITTING STRAIGHT LINES
Task 1103-10-1107/50-2

Origin: NBS — Authorized 3/1/50
Manager: F. S. Acton
Full task description: Jan-Mar 1950 issue, p. 42

Status: CONTINUED. The manuscript is currently in the hands of technical advisors, for comment.

TABLES TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-10-1107/51-1

Origin: NBS — Authorized 7/1/50
Managers: C. Eisenhart, L. S. Deming
Full task description: July-Sept 1950 issue, p. 57

Status: INACTIVE. For status to date see July-Sept 1952 issue, p. 64.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-10-1107/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. Dr. Geisser prepared a manuscript giving necessary and sufficient conditions for the independence of the mean and mean square succession difference (see (3) below).

Publications:
(1) On moments of order statistics from the Weibull distribution. J. Lieblein. Ann. Math. Stat. 26, 330-333 (June 1955).
(2) Time-discrete stochastic processes in arbitrary sets, with applications to processes with absorbing regions and to the problem of loops in Markoff chains. D. van Dantzig. Accepted for publication (in French) in Annales de l'Institut Henri Poincaré (Paris).
(3) A note on the normal distribution. S. Geisser. Submitted to a technical journal.
(4) Inequalities for probabilities associated with the multivariate normal distribution. I. R. Savage. Submitted to a technical journal.

PROCEDURES OF NON-PARAMETRIC STATISTICS
Task 1103-10-1107/52-2

Origin: NBS Authorized 9/17/51
Manager: I. R. Savage
Full task description: July-Sept 1951 issue, p. 66

Status: INACTIVE. For the latest report on this task see July-Sept 1954 issue, p. 22.

Publications:
(1) Easily used simultaneous confidence limits for a line. W. S. Connor. Submitted to a technical journal.
(2) Contributions to the theory of rank order statistics. I. R. Savage. Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-10-1107/53-1

Origin: NBS Authorized 10/15/52
Manager: W. H. Clatworthy
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. M. Zelen's work on the analysis of covariance as applied to incomplete block designs is now in manuscript (see (4) below).

Mr. Zelen has recently been able to show that when one replaces the treatments in a partially balanced design (two associate classes) with a balanced incomplete block design, the resulting design is a partially balanced design with not more than five associate classes.

The reworking of publication (3) listed below to include two-way elimination of heterogeneity is in progress. The revised manuscript is intended for publication in the Applied Mathematics Series.

Publications:
(1) Partially balanced incomplete block designs with two associate classes and two treatments per block. W. H. Clatworthy. J. Res. NBS 54, 177-190 (Apr. 1955).
(2) Partially replicated latin squares. W. J. Youden and J. S. Hunter (American Cyanmid Co.) Accepted for publication in Biometrics.

(3) Some fractional factorial arrangements for factors at two levels. W. H. Clatworthy, W. S. Connor, and M. Zelen. In manuscript.
(4) On the analysis of covariance. M. Zelen. In manuscript.

RESEARCH ON MATHEMATICAL ASPECTS OF ORDER STATISTICS METHODS
Task 1103-10-1107/55-110

Origin: NBS Authorized 3/3/55
Manager: J. Lieblein
Full task description: Jan-Mar 1955 issue, p. 31

Status: CONTINUED. A talk was presented on the theory and application of extreme values before the U. S. Department of Agriculture Departmental Committee on statistical design and analysis. A talk on the subject was also presented at a Building Technology staff meeting (NBS), and discussions were carried on with members of the Division regarding the application of extreme-value methods to wind velocity data in connection with building codes. In connection with these discussions, methods were worked out for appropriate use of return period information in conjunction with probability data. Advice was furnished in connection with a request from the David Taylor Model Basin concerning the most suitable method of studying data on extreme stress variations observed on a vessel during its winter voyages.

2. Applied Research in Mathematical Statistics

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Fitting of polynomials to data on atomic spectra for E. K. Plyler and H. C. Allen, Jr., of section 4.2.

(2) Analysis of data on a test of fire extinguishers for A. F. Robertson and H. Shoub of 10.2.

(3) Presentation of material to personnel of Division 10 on the applicability and uses of extreme value methods.

(4) Analysis of data to determine effects of size and shape on noise of composition type resistors for G. T. Conrad, Jr., of section 1.6.

(5) Planning and analysis of experiment to determine the radiant energy absorption by water vapor on the output of total radiation pyrometers for S. T. Lonberger, 3.1.

(6) Analysis of data to determine precision of thermocouple calibrations for B. E. Squires of section 3.1.

(7) Planning of fractional factorial experiment to determine effects of various factors on fatigue characteristics of steel for H.E. Frankel of 8.3.

(8) An analysis of data from an interlaboratory study of the effect of cathode melts on transconductance of vacuum tubes for C.P. Marsden, of 1.2.

STATISTICAL ASPECTS OF NBS ADMINISTRATIVE OPERATIONS
Task 3737-60-0002/52-1

Origin: NBS Authorized 10/1/5
Manager: C. Eisenhart
Full task description: Oct-Dec 1951 issue, p. 56

Status: TERMINATED. Activities under this task will be reported together with the activities of the major task under this project, 3737-60-0002/51-1

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: INACTIVE. For status to date see Oct-Dec 1954 issue, p. 27.

RESEARCH IN APPLICATIONS OF MATHEMATICAL STATISTICS TO PROBLEMS OF THE CHEMICAL CORPS
Task 1103-40-5118/52-1

Origin and Sponsor: Biological Laboratories, Chemical Corps, Dept. of the Army Authorized 10/1/51
Manager: C. Eisenhart
Full task description: Oct-Dec 1951 issue, p. 57

Status: CONTINUED. Computations are proceeding to find the operating characteristics of a test involving the combination of two tests of significance in the analysis of variance. Work on fractional factorial designs with two way elimination of error continues.

STATISTICAL ANALYSIS AND DESIGN OF EXPERIMENTS FOR THE U. S. GEOLOGICAL SURVEY
Task 1103-40-5140/54-1

Origin and Sponsor: U. S. Geological Survey, Department of Interior Authorized 10/9/53
Managers: C. Eisenhart, W. J. Youden
Full task description: Oct-Dec 1953 issue, p. 50

Status: CONTINUED. (1) A statistically based sampling plan was worked out for the sampling of fine-grained deposits in streams for use in a study of mineral content for Dr. Dorothy Carroll of the Sedimentary Petrology Project of the Geochemistry and Petrology Branch.

(2) An investigation was made of the applicability of statistical methods in establishing relationships between location, gradient, and other factors with size distribution of pebbles in studies in soil hydrology and erosion for J. T. Hack of the General Geology Branch.

(3) A mixed model analysis of variance was worked out for the analysis of data from a study of the effect of solid state phenomena on the fluorescence of uranium flux. This study is being conducted by the Geochemistry and Petrology Branch in Denver.

(4) A sampling plan based on the theory of extreme values was worked out in collaboration with W. C. Krumbein for use in an exploratory study of particle sizes in sedimentary deposits, such as certain gravel pits in Maryland. The data were analyzed with considerable success by means of extreme value methods and a joint paper on the outcome of the study was prepared for publication by J. Lieblein and Dr. Krumbein as noted below.

Publication:

(1) Application of extreme value theory to cobbles and boulders in gravel deposits. W. C. Krumbein (Northwestern University) and J. Lieblein. In manuscript.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-5146/55-93

Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: Paul N. Somerville, who arrived April 1, has begun active work on the manual. Conferences were held at several Ordnance establishments on various aspects of the manual.

IV. MATHEMATICAL PHYSICS SECTION

(Section 11.4)

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-10-1115/55-57

Origin: NBS — Authorized 9/1/54
Manager: E. W. Cannon
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. P. Henrici revised and enlarged a manuscript on Legendre and Gegenbauer functions. He has also completed a report on the evaluation of the integral

$$f(y) = \int_0^\infty (J_1(t/2))^2 J_2(ty)\, dt,$$

connected with problems in structural analysis. A network analysis has also been completed, which demonstrates the averaging property for electrical controls connected with NBS combustion experiments.

R. F. Dressler is continuing an investigation on local curvature effects on edge stresses in bent plates.

Publications:

(1) Addition theorems for Legendre and Gegenbauer functions. P. Henrici. Accepted for publication in Journal of Rational Mechanics and Analysis.
(2) On the vibration of a square clamped plate. M. Abramowitz and W. Cahill. Accepted for publication in Journal of the Association for Computing Machinery.
(3) Kleine Bemerkung in asymptotischen Entwicklung des Fehlerintegrals. P. Henrici. Accepted for publication in ZAMP.
(4) Evaluation of a definite integral. P. Henrici. An NBS report (April 1954).

RESEARCH IN ELECTROMAGNETIC THEORY
Task 1104-10-5160/54-47

Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Department of the Army — Authorized 6/29/54, Revised 9/29/54
Manager: F. Oberhettinger
Full task description: July-Sept 1954 issue, p. 28

Status: CONTINUED. F. Oberhettinger continued a study of problems for electromagnetic fields. Expressions for the two Green's functions of the equation $\Delta u + k^2 u = 0$, different from the representations by modes, have been obtained for an infinitely long cylinder with the rectangular cross section. This study is connected with the radiation problem for a slotted wave guide. A report on this task is in preparation.

RESEARCH IN MATHEMATICAL ELASTICITY
Task 1104-10-5160/55-85

Origin: NBS Authorized 12/27/54
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: R. F. Dressler
Full task description: Oct-Dec 1954 issue, p. 30

Status: CONTINUED. R. F. Dressler and W. F. Cahill have formulated an unsymmetric matrix of order 45 to obtain the lowest eigenvalue and eigenvector of a triangular vibrating plate with aspect ratio 2 and Poisson's ratio σ equal to .29. The complete calculation was carried out on SEAC. The resulting frequency obtained is 19 °/o higher than the corresponding experimental result reported in Gustafson, Stokey, and Zorowski (J. Aer. Sci., May 1953). This is to be expected since imperfect clamping and aerodynamic loading both have the effect of lowering an experimental result from the mathematical value.

R. F. Dressler and W. F. Cahill have also computed the lowest frequency and mode for this shape using an order 28 matrix, to determine the dependence on mesh size. The order 28 yields a frequency only 7°/o higher than the order 45 result, and the corresponding vectors differ by a negligible amount. This result is encouraging since the order 28 formulation has a mesh so sparse that only one interior point can retain its differential stencil unmodified by the effect of the boundary conditions. It is expected that the principal results will be based upon matrices of order 78. For this size a new code had to be developed. This has now been completed by I. Rhodes, and pilot models to test the code are being run. This code incorporates a technique with a hypergeometric polynomial of degree 19 to separate eigenvalues for faster convergence and uses a power method for unsymmetric real matrices to obtain higher modes. Experience with the previous computations showed that convergence was so slow, due to the proximity of adjacent eigenvalues, that some accelerating technique was necessary.

For the corrugated diaphragm problem, R. F. Dressler modified linear elastic shell equations with rotational symmetry for arbitrary meridial shape to include lateral loading (air pressure). In the first form, the two unknowns are meridial rotation and a modified shear force, leading to a non-homogeneous system of two ordinary differential equations of total order four, requiring additional integrations to obtain displacements. Secondly, equations have been derived using displacements and shear as unknowns, giving three equations of total order six. For both formulations, meridial arc length is taken as independent variable. The first set is not directly applicable for regions where the shell is perpendicular to its axis, and it possesses no advantage for electronic computation in requiring two successive integration steps. Therefore the second formulation has been chosen as the basis for analyses. These shell equations reduce to plate bending and plane stress equations with circular symmetry over the central flat portion of the diaphragm. The known solutions for these have been used to define initial values at the first corrugation for the computer calculations.

The Runge-Kutta method has been coded for this sixth order system, and a separate code has also been completed for calculating the many variable coefficients appearing in the equations. The first diaphragm shape now being solved is the one used by Grover and Bell (Soc. Exp. Stress Anal. 5, 2, 1948), in which the meridian consists of arcs of circles and straight line segments. Three independent homogeneous solutions and one non-homogeneous solution are needed to satisfy any boundary value problem at the outer edge. W. Cahill has now obtained the first homogeneous solution, which is simultaneously serving as a code-check. Independently of the machine, A. Cock is checking the code by appropriate hand calculations. Approximate machine time for each solution is 45 minutes, using 150 integration points over the diaphragm radius.

The shell equation coefficients contain first and second derivatives of the meridial curvature. At each of the eleven transition points between circular and straight sections, where curvature is discontinuous, a suitable smoothing function has been calculated and inserted in order to take account of the effect on the solutions of these delta functions and their derivatives.

APPLICATION

of

NATIONAL BUREAU OF STANDARDS AUTOMATIC COMPUTER (SEAC)

The record of SEAC operations for unclassified tasks of the Applied Mathematics Division for the period April 1 through June 30 is as follows:

Task No.	Title	Hours Used: Code Checking	Productive Operations
NBS:			
1104/55-55	Numerical analysis	5	17
1110/47-2	Tables of Coulomb wave functions	1	6
1110/50-7	Wave function for lithium		1
1110/53-52	L-shell conversion coefficients		90
5126/52-34	Molecular vibrations		42
5126/52-44	Calculations for d spacings	1	4
5126/53-15	Nerve fiber reaction	1	3
5126/53-27	Thermodynamics functions	1	3
5126/53-29	Dynamic behavior of aircraft structure		5
5126/53-51	Radiation diffusion	7	52
5126/53-53	Heterogeneity model		40
0009/54-15	Matrix reduction	5	1
0009/54-19	Energy levels of complex atoms		5
5126/54-33	Air conditioning in underground structures		2
0009/54-38	Compressibility factors of dry air	4	
5126/54-43	Characteristics of conducting resistors		1
0009/55-39	Molecular vibrations		51
0009/55-53	Electronic functions		5
0009/55-65	Automatic coding	18	11
5126/55-75	Dispersion equation for optical glass	1	2
5126/55-81	Combining tests for significance	3	3
5126/55-82	Thermometer calibrations	3	1
5126/55-86	Flow coefficients for fluids	3	
5126/55-87	"Zero" method determination of crystal structures	1	2
5126/55-88	Stresses in a wall foundation	2	9
0009/55-92	Cylindrical electron lens calculations		16
5126/55-96	Distribution of light output from neutron spectrometers	1	25
5126/55-97	High temperature properties of air	5	29
5126/55-115	Adsorption integrals	4	2
5126/55-117	Attenuation of pressure pulses of finite amplitude	2	
5126/55-118	Thermometer calibrations		7
5126/55-121	Electron penetration	2	
5126/55-124	Evaluation of integrals related to gas flow	2	3
0002/51-1	Collaboration on statistical aspects of NBS research and testing	3	32
5160/55-85	Research in mathematical elasticity	5	61
Misc.	Roots of polynomials	24	2
Misc.	Training, subroutines, etc.	22	3

Task No.	Title	Hours Used: Code Checking	Productive Operation
Other:			
5126/51-3	Meteorology	1	7
5126/54-13	Award of procurement contracts by linear programming	1	16
5126/54-44	Flight performance	1	17
5126/55-61	Elastic neutron scattering	1	12
5126/55-62	Integrals involved in supersonic flutter	2	6
5126/55-72	Maryland inter-industry study		8
5126/55-73	Heat convection	11	4
5126/55-101	Transcendental equations	2	12
5126/55-104	Calculations of various fuzing systems	1	158
5126/55-105	Aeronomic tidal winds of thermal origin	2	3
5126/55-106	Reflex klystron	2	26
5126/55-107	Missile trajectory	10	4
	Totals:	160	809

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Numerical Analysis Lectures

(Offered by the American University in cooperation with the National Bureau of Standards)

STIEFEL, E. (Federal Institute of Technology, Zurich). (1) Strategy in solving linear equations. Methods using orthogonal polynomials, conjugate gradients. April 4. (2) Application of the Fredholm integral equation of conformal mapping. April 11. (3) Determination of eigenvalues by vector iteration. Method of Lanczos. April 18. (4) Determination of eigenvalues: Quotient-difference algorithm. April 25.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

ABRAMOWITZ, M. (1) Computations on mathematical tables, and (2) Pitfalls in computation. Presented at a Physics Seminar at the Massachusetts Institute of Technology, Cambridge, Mass., June 23 and June 24.

ALT, F. L. (1) Problems exceeding the capacity of present day computers. Presented at a Mathematics Seminar at the Massachusetts Institute of Technology, Cambridge, Mass., May 24. (2) Computing machines: Organization, functions, and components. A series of six lectures presented at Wayne University, Detroit, Mich., June 6-11.

EISENHART, C. Some antecedents of modern experiment design. Presented at a joint meeting of the Statistics Section of the Virginia Academy of Science and the Virginia Chapter of the American Statistical Association, Harrisonburg, Va., May 13.

HOFFMAN, A. J. Computing linear programs. Presented before the Mathematics Division of the American Society for Engineering Education, at Pennsylvania State College, June 20-24.

LIEBLEIN, J. (1) Theory and application of extreme value methods. Presented at the U. S. Department of Agriculture, Departmental Committee on Statistical Design and Analysis, Washington, D. C., May 4. (2) Some topics of interest in the application of extreme values. Presented at the staff meeting of the Building Technology Division, NBS, May 31.

TAUSSKY-TODD, O. (1) Some practical bounds for eigenvalues of finite matrices. Presented at the AEC Computing Facility, Institute of Mathematical Sciences, New York University, New York, N. Y., April 21. (2) Matrix methods in algebraic number theory. Presented at the Conference on Number Theory at California Institute of Technology, Pasadena, California, June 22-24. (3) Integral group matrices. Presented before the Numerical Analysis Research Group, University of California at Los Angeles, California, June 27.

TODD, J. (1) The practical significance of stability in the solution of partial differential equations. Presented at the Berkeley Conference on Partial Differential Equations, University of California, Berkeley, Calif., June 20-July 1. (2) Controlled experiments in high-speed computations. Presented to the California Chapter, Society of Industrial and Applied Mathematics, San Francisco, Calif., June 23.

YOUDEN, W. J. (1) Statistics for skeptics. Presented before the Cleveland Section, American Society for Quality Control, Cleveland, O., April 1. (2) Interlaboratory evaluation of analytical methods. Presented as part of the Camp Detrick Course in Statistics, Camp Detrick, Md., April 7. Also presented at the Naval Research Laboratory, Washington, D. C., April 12. (3) The gun problem. Presented at the Marshall Laboratory, E. I. DuPont de Nemours and Co., Philadelphia, Pennsylvania, April 15.

(4) Dice, data, and deductions. Presented at a joint meeting of the Virginia and Hampton Roads Sections, American Chemical Society, William and Mary College, Williamsburg, Virginia, April 23. (5) Statistical applications in the chemical industries. Presented before a joint meeting of the Buffalo Section, American Society for Quality Control and Buffalo Chapter of the American Statistical Association, Buffalo, N. Y., April 25. (6) Problems arising in engineering investigations. Delivered at a symposium on Statistical Engineering, Chemical Corps Engineering Agency, Edgewood, Md., April 28.

(7) Ten talks given before local sections of the American Chemical Society on: Principles of experimental design; Interpretation of chemical data; and Statistical aspects of analytical determinations; presented in Kansas City, Mo.; Manhattan, Kan.; Wichita, Kan.; Ponca City, Okla.; Stillwater, Okla.; Tulsa, Okla.; Bartlesville, Okla.; Pittsburg, Kan.; Columbia, Mo.; and Macomb, Ill.; May 2-13.

ZELEN, M. Some new techniques in the design of experiments. Presented before the George Washington University Logistics Group, George Washington University, Washington, D. C., April 28.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Table of characteristic values of Mathieu's equation for large values of the parameter. G. Blanch and I. Rhodes. J. Wash. Acad. Sci. 45, 166-196 (June 1955).

1.3 Technical Papers

(1) On nonlinear differential equations of the second order with integrable perturbation term. H. Antosiewicz. J. London Math. Soc. 30, 65-67 (1955).

(2) Partially balanced incomplete block designs with two associate classes and two treatments per block. W. H. Clatworthy. J. Res. NBS 54, 177-190 (Apr. 1955).

(3) A multi-purpose orthonormalizing code and its uses. P. Davis and P. Rabinowitz. J. Assoc. Comp. Mach. 1, 183-191 (Oct. 1954).

(4) Discrete analogs of inequalities of Wirtinger. K. Fan, O. Taussky, and J. Todd. Monatsh. Math. 59, 73-90 (1955).

(5) An algorithm for solving the transportation problem. A. Gleyzal. J. Res. NBS 54, 213-216 (Apr. 1955), RP2583.

(6) Indentation pressure of a smooth punch. E. Levin. Qu. App. Math. 13, 134-137 (July 1955).

(7) On moments of order statistics from the Weibull distribution. J. Lieblein. Ann. Math. Stat. 26, 330-333 (June 1955).

(8) Structure theorems for modular subgroups. M. Newman. Duke Math. J. 22, 25-32 (1955).

(9) Osculatory interpolation in the complex plane. H. E. Salzer (now Department of the Army). J. Res. NBS 54, 263-266 (May 1955), RP2587.

(10) The generation of Coulomb wave functions by means of recurrence relations. I. A. Stegun and M. Abramowitz. Phys. Rev. 98, 1851-1852 (June 15, 1955).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION JUNE 30, 1955

2.1 Mathematical Tables

(1) Table of hyperbolic sines and cosines, x=2 to x=10, 9S. Applied Mathematics Series 45. In press, U. S. Government Printing Office.

(2) Table of the descending exponential, x=2.5 to x=10. Applied Mathematics Series 46. In press, U. S. Government Printing Office.

2.3 Technical Papers

(1) Forced heat convection in laminar flow through a tube. M. Abramowitz. Submitted to a technical journal.

(2) On the vibration of a square clamped plate. M. Abramowitz and W. F. Cahill. Accepted, Journal of the Association for Computing Machinery.

(3) Approximate method for rapid Loran computation. M. Abramowitz, D. H. Call, and J. C. Mathews. Submitted to a technical journal.

(4) A theorem on alternatives for pairs of matrices. H. Antosiewicz. Accepted, Pacific Journal of Mathematics.

(5) Analytic study of war games. H. Antosiewicz. Submitted to a technical journal.

(6) Asymptotic solution of linear differential equations with a parameter. H. Antosiewicz. Submitted to a technical journal.

(7) On a certain integral involving Bessel functions. H. Antosiewicz. Submitted to a technical journal.

(8) On the differential equation $\ddot{x} + k(f(x) + g(x)\dot{x})\dot{x} = ke(t)$. H. A. Antosiewicz. Submitted to a technical journal.

(9) Stable systems of differential equations with integrable forcing term. H. A. Antosiewicz. Accepted, Journal of the London Mathematical Society.

(10) Computation of vibration modes and frequencies on SEAC. W. F. Cahill and S. Levy (NBS 6.4). Accepted; Journal of the Institute of Aeronautical Sciences.

(11) On the design of two-dimensional nozzles by the method of characteristics. B. Chaix (Federal Institute of Technology, Zurich) and P. Henrici. Submitted to a technical journal.

(12) Easily used simultaneous confidence limits for a line. W. S. Connor. Submitted to a technical journal.

(13) Time-discrete stochastic processes in arbitrary sets, with applications to processes with absorbing regions and to the problem of loops in Markoff chains. D. van Dantzig. Accepted for publication (in French) in Annales de l'Institut Henri Poincaré (Paris).

(14) On the theorem of Dilworth. G. B. Dantzig (RAND) and A. J. Hoffman. To appear in Annals of Mathematics Study 38.

(15) On a problem in the theory of mechanical quadratures. P. Davis. Accepted, Pacific Journal of Mathematics.

(16) Some Monte Carlo experiments in computing multiple integrals. P. Davis and P. Rabinowitz. Submitted to a technical journal.

(17) Some sampling results on the power of nonparametric tests against normal alternatives. W. J. Dixon (University of Oregon) and D. Teichroew. Submitted to a technical journal.

(18) Comparison of theories and experiments for the hydraulic dam-break wave. R. F. Dressler. To appear in the Proceedings of the Tenth

General Assembly, of the International Union of Geodesy and Geophysics, held in Rome, Italy, September 1954.

(19) Heat flow in a fluid with eddying flow. W. H. Durfee (now with Operations Research Office, Johns Hopkins University). Submitted to a technical journal.

(20) A comparison theorem for eigenvalues of normal matrices. K. Fan. Accepted, Pacific Journal of Mathematics.

(21) On systems of linear inequalities. K. Fan. Submitted to a technical journal.

(22) Some inequalities concerning positive-definite Hermitian matrices. K. Fan. Accepted, Proceedings of the Cambridge Philosophical Society.

(23) Systems of inequalities involving convex functions. Ky Fan. Submitted to a technical journal.

(24) Asymptotic lower bounds for the fundamental frequency of convex membranes. G. E. Forsythe. Accepted, Pacific Journal of Mathematics.

(25) A note on the normal distribution. S. Geisser. Submitted to a technical journal.

(26) Power series for log $e^x e^y$. K. Goldberg. Accepted, Duke Journal of Mathematics.

(27) Addition theorems for general Legendre and Gegenbauer functions. P. Henrici. Accepted, Journal of Rational Mechanics and Analysis.

(28) On certain series expansions involving Whittaker functions and Jacobi polynomials. P. Henrici. Accepted, Pacific Journal of Mathematics.

(29) On generating functions of the Jacobi polynomials. P. Henrici. Accepted, Pacific Journal of Mathematics.

(30) On systems of distinct representatives. A. J. Hoffman and H. W. Kuhn (Bryn Mawr College). To appear in Annals of Mathematics Study 38.

(31) Systems of distinct representatives and linear programming. A. J. Hoffman and H. W. Kuhn (Bryn Mawr College). To appear in the American Mathematical Monthly.

(32) Acoustic radiation pressure on a circular disk. H. Levine. To appear in the Proceedings of the Fifth Symposium on Applied Mathematics of the American Mathematical Society, held in Pittsburgh, Pa., June 1952.

(33) A proof of Hilbert's Nullstellensatz. T. S. Motzkin. Submitted to a technical journal.

(34) The assignment problem. M. Motzkin. To appear in the Proceedings of the American Mathematical Society's Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 1953.

(35) On Fejer sets in linear and spherical spaces. T. S. Motzkin and I. J. Schoenberg. Accepted, Annals of Mathematics.

(36) Pairs of matrices with property L, II. T. S. Motzkin and O. Taussky. Accepted, Transactions of the American Mathematical Society.

(37) An alternative proof of a theorem on unimodular groups. M. Newman. Submitted to a technical journal.

(38) An identity for the coefficients of certain modular forms. M. Newman. Accepted, Journal of the London Mathematical Society.

(39) Generalizations of identities for the coefficients of certain modular forms. M. Newman. Accepted, Journal of the London Mathematical Society.

(40) The diophantine equation $ax^m-by^n=c$. M. Newman. Submitted to a technical journal.

(41) The normalizer of certain modular subgroups. M. Newman. Accepted, Canadian Journal of Mathematics.

(42) On a generalization of the normal basis in abelian algebraic number fields. M. Newman and O. Taussky. To appear in Communications on Pure and Applied Mathematics of New York University.

(43) On asymptotic series for functions occurring in the theory of diffraction of waves by wedges. F. Oberhettinger. Submitted to a technical journal.

(44) On the Lerch zeta function. F. Oberhettinger. Accepted, Pacific Journal of Mathematics.

(45) Determinanten mit ueberwiegender Hauptdiagonale und die absolute Konvergenz von linearen Iterationsprozessen. A. M. Ostrowski. Accepted, Comentarii Mathematici Helvetici.

(46) On Gauss' speeding up device in the theory of single step iteration. A. M. Ostrowski. Submitted to a technical journal.

(47) On the convergence of Gauss' alternating procedure in the method of the least squares , I. A. M. Ostrowski. Submitted to a technical journal.

(48) On spectra of second-order differential operators. D. Ray. Submitted to a technical journal.

(49) On the convergence of asymptotic solutions of linear differential equations. R. M. Redheffer (U.C.L.A.) and W. Wasow. Submitted to a technical journal.

(50) Contributions to the theory of rank order statistics. I. R. Savage. Submitted to a technical journal.

(51) Inequalities for probabilities associated with the multivariate normal distribution. I. R. Savage. Submitted to a technical journal.

(52) A note on group matrices. O. Taussky. Accepted, Proceedings of the American Mathematical Society.

(53) Unimodular circulants. O. Taussky. To appear in the I. Schur Memorial Volume to be published by Mathematische Zeitschrift.

Publication Activities

(54) Some computational problems in algebraic number theory. O. Taussky. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 1953.

(55) Generation and testing of pseudo-random numbers. O. Taussky and J. Todd. Submitted to a technical journal.

(56) An improved cathode ray tube storage system. R. Thorensen. To appear in the Proceedings of the Western Computer Conference of the AIEE-IRE-ACM held in Los Angeles, Calif., February 1953.

(57) Machine attacks on problems whose variables are permutations. C. B. Tompkins. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Numerical Analysis, held in Santa Monica, Calif., August 1953.

(58) An extension of a theorem of Dantzig's. C. Tompkins and I. Heller (George Washington University). Submitted to a technical journal.

(59) Best approximation polynomials of given degree. J. Walsh. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held in Santa Monica, Calif., August 1953.

(60) Discrete approximations to elliptic differential equations. W. Wasow. Accepted, Zeitschrift für angewandte Mathematik und Physik.

(61) On the asymptotic transformation of certain distributions into the normal distribution. W. Wasow. To appear in the Proceedings of the American Mathematical Society Sixth Symposium Applied Mathematics, held at Santa Monica, Calif., August 1953.

(62) Error bounds for eigenvalues of symmetric integral equations. H. Wielandt. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica, Calif., August 1953.

(63) On eigenvalues of sums of normal matrices. H. Wielandt. Submitted to a technical journal.

(64) National physical standards and design of experiment. W. J. Youden. To appear in Revue de L'Institut International de Statistique (The Hague).

(65) Partially replicated latin squares. W. J. Youden and J. S. Hunter (American Cyanmid Co.) Accepted, Biometrics.

(66) The number of absolute points of a correlation. A. J. Hoffman, M. Newman, E. G. Straus, and O. Taussky. Accepted, Pacific Journal of Mathematics.

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THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

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Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($0.75), available from the Superintendent of Documents, Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.


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