NATIONAL BUREAU OF STANDARDS REPORT

4117

PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION

A Quarterly Report
January through March 1955

FOR OFFICIAL USE

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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- Office of Basic Instrumentation
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NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT NAML

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APPLIED MATHEMATICS DIVISION

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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

January 1 through March 31, 1955

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Status of Projects

March 31, 1955

I. NUMERICAL ANALYSIS SECTION

(Section 11.1)

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS

Task 1101-10-1104/55-55

Origin and Sponsor: NBS — Authorized 8/13/54
Managers: J. Todd, P. Davis — Revised 8/29/54
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. Further experiments were made by P. Davis to test the versatility of the orthonormalizing code developed by him and P. Rabinowitz.

The first 32 orthonormal polynomials for a square (in the sense of G. Szegö) have been obtained. Using the limit of the ratio k_{n+1}/k_n, the transfinite diameter of the square has probably been determined correctly to 4 decimal places. This value has hitherto been known only to about $1\frac{1}{2}$ decimal places. The same procedure was carried out for an arbitrarily drawn "bean shaped" domain. For this region the first 16 orthonormal polynomials were obtained and the transfinite diameter obtained to about 3 decimal places. A manuscript describing these matters is in preparation. Further plans for this code consist in its application to the Dirichlet problem by means of orthonormal harmonic functions and by the use of a method recently proposed by Z. Nehari which uses non-harmonic functions, but for which an estimate of error is available.

P. Davis tested various methods of generating random numbers for their suitability for the Monte Carlo evaluation of multiple integrals. A method based upon Fibonacci sequences was rejected as being too erratic. Successive multiplication by $5^{17}.2^{-42}$ yielded results which were (experimentally) $O(1/\sqrt{N})$ where N is the number of points sampled. Using multiples of certain irrational quantities yielded results which were $O(1/N)$. Tests were carried out up to dimension 12. A manuscript entitled "Some Monte Carlo experiments in computing multiple integrals" by P. Davis and P. Rabinowitz has been submitted to a technical journal and contains a complete description of these matters.

A code has been developed by P. Davis and P. Rabinowitz which yields the abscissas and weights of Gaussian quadrature formulas of high orders. Gaussian rules of the following orders were run off and are available for distribution; n = 16,20,24,32,40,48,64,80,96. The abscissas and weights have been computed to 21 decimal places. The code makes use of certain inequalities for the zeros of the Legendre polynomials in terms of the zeros of $J_0(x)$, which were established by G. Szegö in 1936. A manuscript describing the process and containing these values is in preparation.

P. Henrici is carrying out a program of automatizing certain formal manipulations with power series on SEAC. In order to avoid round-off

difficulties and to obtain exact results, the coefficients of the series are assumed to be rationals, and all arithmetic operations are performed in the field of rational numbers. The two following problems have been coded and tested:

(1) Raising a power series to an arbitrary power. Given the (rational) coefficients a_n of the power series

$$y = 1 + a_1x + a_2x^2 + \ldots$$

and an arbitrary rational (not necessarily positive) number α, this code computes the coefficients b_n of the power series

$$y^\alpha = 1 + b_1x + b_2x^2 + \ldots$$

$$= (1 + a_1x + a_2x^2 + \ldots)^\alpha$$

(2) Inverting a power series with arbitrary leading power. Given a series of the form

$$y = x^\alpha (1 + a_1x + a_2x^2 + \ldots),$$

where α and the a_n are again arbitrary rational numbers, this code computes the coefficients b_n in the expansion

$$x = y^{1/\alpha}(1 + b_1y^{1/\alpha} + b_2y^{2/\alpha} + \ldots) .$$

The code in problem (1) is used in (2) as a subroutine. It is based on an algorithm which has been suggested by J.C.P. Miller during a recent visit. Code (2) is based on the Lagrange-Bürmann formula. Both codes can handle at the present 16 coefficients. The codes can also apply to cases where the coefficients a_n and hence b_n are polynomials in a variable t, say, by computing the b_n for equidistant values of t and differencing.

Possible applications of the codes which are envisaged for the future include combinatorial analysis (code (1)) and computation of certain asymptotic expansions. As a first application the coefficients arising from the inversion of

$$y = x^3(1 + (3/5)x^2 + (3/7)x^4 + \ldots)$$

have been computed. They arise in the asymptotic expansion of $F(1-\beta;2;4\beta)$ for large values of β.

The basic operations for rationals, which had to be coded for the routines described, can also be used independently. At the moment their codes are limited to rationals with numerator and denominator not exceeding 2^{40}. Each result and each intermediate result is reduced to lowest terms by the Euclidean algorithm.

A study of the numerical evaluation of the integral

$$f(y) = \int_0^\infty (J_1(t/2))^2 J_2(ty)dt \qquad (0<y<2)$$

has been made by P. Henrici. It was found that this integral, whose direct numerical evaluation is difficult, can be expressed for all values of y in terms of certain hypergeometric series, which are readily computable.

To facilitate a new method of computing Coulomb wave functions of arbitrary order devised by M. Abramowitz and I. Stegun (see publication (13) below), P. Henrici has derived the expansion

$$\rho e^{i} = \frac{e^{2\pi\eta} - 1}{2\pi\eta} \sum_{L=0}^{\infty} (2L+1) e^{i\delta_L(\eta)} F_L(\eta,\rho)$$

where

$$\delta_0 = 0, \qquad \delta_L = \frac{\pi L}{2} + \sum_{k=1}^{L} + a_n^{-1} \frac{\eta}{k} .$$

P. Henrici has studied the asymptotic location of the zeros of the so-called Bessel polynomials

$$\theta_n(z) = \sum_{r=0}^{n} \frac{(n+r)!}{(n-r)!\,r!\,2^r} z^{n-r}$$

of large order n. Among other things, the following approximation was found for the (only) real zero z_n of the polynomials of odd order n:

$$z_n = - \frac{2n}{g + g^{-1}}$$

where

$$g = g_0\left(1 - \frac{.417}{n}\right),$$

and $g_0 = 3.31905$ is the (only) real zero of

$$g = \exp\left\{\frac{g^2 + 1}{g^2 - 1}\right\}.$$

Publications:

(1) Stable systems of differential equations with integrable forcing term. H. Antosiewicz. Submitted to a technical journal.
(2) Computation of vibration modes and frequencies on SEAC. W. Cahill and S. Levy (NBS 6.4). Accepted for publication in the Journal of the Institute of Aeronautical Sciences.
(3) On a problem in the theory of mechanical quadratures. P. Davis. To appear in the Pacific Journal of Mathematics.
(4) A multi-purpose orthonormalizing code and its uses. P. Davis and P. Rabinowitz. To appear in the Journal of the Association for Computing Machinery.
(5) Application of two methods of numerical analysis to the computation of the reflected radiation of a point source. P. Henrici.J.Washington Acad. Sciences 45, 38-45 (Feb. 1955).
(6) On the Lerch zeta function. F. Oberhettinger. To appear in the Pacific Journal of Mathematics.
(7) Generation and testing of random numbers on SEAC. O. Taussky, J. Todd, M. Newman, and J. Cameron. In manuscript.
(8) Motivation for working in numerical analysis. J. Todd. Communications on Pure and Applied Math. 8, 97-116 (1955).
(9) On the solution of the Lichtenstein-Gerschgorin integral equation in conformal mapping: II. Computational experiments. J. Todd and S. E. Warschawski (University of Minnesota). Appeared in Experiments in the Computation of Conformal Maps, NBS Applied Mathematics Series 42, pp. 31-44.
(10) On non-linear differential equations of the second order with integrable forcing term. H. A. Antosiewicz. To appear in the Journal of the London Mathematical Society.

(11) Evaluation of a definite integral. P. Henrici. In manuscript.
(12) Asymptotic location of the zero of the Bessel polynomials. P. Henrici. In manuscript.
(13) The generation of Coulomb wave functions by means of recurrence relations. I. Stegun and M. Abramowitz. Submitted to a technical journal.

TURBULENT ATMOSPHERIC CONTAMINATION
Task 1101-10-1104/55-59

Origin and Sponsor: Applied Physics Laboratory, Johns Hopkins University, and National Bureau of Standards — Authorized 7/22/54
Managers: E. Marden, G. Hawkins
Full task description: July-Sept 1954 issue, p. 3

Status: COMPLETED. Trial computations completed on SEAC demonstrate the feasibility of using digital computers for the speedy prediction of the transport of atmospheric contamination, if the distribution and intensity of sources are known and predicted meteorological conditions are given. The test computations deal with a single point source and "predict" air pollution resulting from such a source, hour by hour, up to 24 hours, for varying areas up to 30x30 miles, using prevailing rather than predicted weather conditions. Computing times range from one-half hour to several hours and allow the extrapolation that cases involving multiple point sources or area sources can be handled in similar reasonably short times by resorting to efficient arrangements of computations.

BASIC RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5116/50-2

Origin and Sponsor: Office of Scientific Research, ARDC, USAF, and Directorate of Management Analysis, USAF — Authorized 3/31/50
Managers: J. Todd, A. Hoffman
Full task description: Jan-Mar 1950 issue, p. 24

Status: CONTINUED. In the study of some simple scheduling problems, it has been observed that the principal item of importance is a partial ordering among a given set of jobs, i.e., $a \prec b$ if an individual can complete job a and begin job b. In this connection, A. J. Hoffman has found a simple necessary and sufficient condition for a given partially ordered set to be isomorphic to a set of intervals on the real line with their natural partial ordering. A further result on interval sets, relevant to scheduling problems, is a linear programming algorithm for finding the subcollection of a given collection of intervals (whose union covers (0,1)) that also covers (0,1) and minimizes the sum of weights assigned to the intervals. The algorithm depends on the lemma that the absolute value of the determinant of an incidence matrix in which the units of any column appear consecutively cannot exceed one. As a special case one can derive a linear programming proof via duality of the fact that the smallest cardinal number of a covering subcollection is the largest number of points in (0,1) no two of which are in the same subinterval. Similarly, for any collection of intervals, covering or not, the subcollection of largest cardinal number whose subintervals do not overlap is the smallest number of points containing a representative of each subinterval of the original

collection. These "duality" statements are, however, directly accessible.

Using a result of A. W. Tucker the following generalization of a result of Dantzig and Hirsch has been proved: If K is a non-empty intersection of half-spaces in Euclidean n-space, and if f is a concave function defined on K and bounded from below, then (i) the greatest lower bound f is attained, and (ii) if K has extreme points, then the greatest lower bound f is attained at an extreme point.

Publications:

(1) A theorem on alternatives for pairs of matrices. H. A. Antosiewicz. To appear in the Pacific Journal of Mathematics.
(2) Some metric inequalities in the space of matrices. Ky Fan and A. J. Hoffman. Proc. Am. Math. Soc. 6, 111-116 (1955).
(3) Discrete analogs of inequalities of Wirtinger. Ky Fan, O. Taussky, and J. Todd. To appear in Monatshefte für Mathematik.
(4) A determinantal inequality. Ky Fan and J. Todd. J. London Math. Soc. 30, 58-64 (Jan. 1955).
(5) On distinct systems of representatives. A. J. Hoffman and H. W. Kuhn (Bryn Mawr College). In manuscript.
(6) On the relevance of LeChatelier's principle to linear programming. A. J. Hoffman. In manuscript.
(7) An extremum property of sums of eigenvalues. H. Wielandt. Proc. Am. Math. Soc. 6, 106-110 (Feb. 1955).

SUPPORTING RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5116/54-9

Origin: Directorate of Management Analysis, USAF Authorized 9/25/53
Sponsor: Office of Scientific Research, Air Research and Development Command, USAF
Managers: J. Todd, A. J. Hoffman
Full task description: July-Sept 1953 issue, p. 35

Status: COMPLETED. A Linear Programming Symposium, sponsored by the Office of Scientific Research of the Air Research and Development Command, was presented by the National Bureau of Standards and the Directorate of Management Analysis, DCS/Comptroller, USAF, in Washington, D. C., on January 27,28,29. The sessions at the Pentagon and the National Bureau of Standards were attended by 364 registrants. The full program is listed under Lectures and Symposia, page 42, of this report.

Editorial work on the Proceedings of the symposium is now under way, and it is anticipated that the Proceedings will be published sometime this fall.

A final report on the work of this project is being prepared.

Publications:

(1) On "overshoot" in the "furthest hyperplane" method. R. Bryce. In manuscript.
(2) On the solution of the caterer problem. J. W. Gaddum, A. J. Hoffman, and D. Sokolowsky. Logistics Res. Quar. 1, 223-229 (1954).
(3) An algorithm for solving the transportation problem. A. Gleyzal. To appear in the Journal of Research of the NBS.
(4) On the optimal ordering of items for a two-stage process. A. J. Hoffman. In manuscript.
(5) On block relaxation. L. S. Joel. In manuscript.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-10-5116/55-56

Origin and Sponsor: Office of Naval Research — Authorized 8/13/54
Managers: O. Taussky-Todd, M. Newman — Revised 8/29/54
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. O. Taussky-Todd and M. Newman prepared a manuscript on their results concerning the uniqueness of the normal basis, and a generalization of the concept of normal basis, of algebraic number fields. They can now show that the normal basis in a cyclic field is not unique unless the degree is 2,3,4,6. Also Mrs. Todd prepared a manuscript concerning unimodular circulants. Mrs. Todd investigated some properties of group matrices, - in particular the cases where the group matrix is a normal matrix. She prepared a manuscript on these results.

M. Newman continued his investigations on the coefficients $P_r(n)$ defined by

$$\prod_{n=1} (1-x^n)^r = \sum_{n=0} P_r(n)x^n$$

Identities for these coefficients which are generalizations of previously proved identities have been discovered, and a general theorem concerning the existence of identities for all positive integral r has been proved. In connection with this work, a table of the coefficients $P_r(n)$ has been prepared on SEAC which gives the first 800 values of $P_r(n)$ for $1 \leq r \leq 13$, and correspondingly fewer values of $P_r(n)$ for larger values of r. It is believed that this table will be of fundamental importance to workers in fields dealing with the elliptic modular functions, and plans for its publication are under way.

J. Todd is making further experiments in connection with the discrepancies found in the Youden problem. In particular, a better approximation to the normal deviates is being used.

Mr. Todd is planning experiments on conformal mapping of regions defined numerically by an (r,θ) equation. As a control, the case of ellipses, which have already been examined using their equation in parametric form, will be handled first. (See J. Todd and S. E. Warschawski in NBS Applied Mathematics Series 42, p. 31.)

H. A. Antosiewicz is studying the various definitions of asymptotic stability for solutions of ordinary differential equations, viz., asymptotic stability in Lyapunov's sense, equiasymptotic stability, and uniform asymptotic stability, with a view toward unification of the great number of results presently available. For instance, asymptotic stability in the case of linear systems with constant coefficients implies uniform asymptotic stability; asymptotic stability in the case of linear systems with variable coefficients or autonomous systems implies equiasymptotic stability. In turn, these imply the existence of a so-called Lyapunov function and conversely the existence of a Lyapunov function for such systems implies these types of stability. A report (see publication (20) below is being prepared.

H. A. Antosiewicz and P. Rabinowitz are continuing their investigation of the differential equations of nerve fiber excitation. A new code, prepared by P. Rabinowitz, has been tested and is now being used in all computations on the original Hodgkin and Huxley system of first order equations as well as on the new system containing a second order equation for the membrane potential. While the study of the original system centered around the search for so-called threshold values of the input current, the investigations of the modified system concern the search

for parameter values which yield a desired behavior of the membrane potential. A comprehensive report (see publication (19) below on the results obtained to date in the investigation is being prepared jointly with K. S. Cole (Naval Medical Research Institute).

Publications:

(1) On a generalization of the normal basis in abelian algebraic number fields. M. Newman and O. Taussky-Todd. Accepted for publication in Communications on Pure and Applied Mathematics (New York University).
(2) A note on group matrices. O. Taussky-Todd. Submitted to a technical journal.
(3) The normalizer of certain modular subgroups. M. Newman. In manuscript.
(4) Generalizations of identities for the coefficients of certain modular forms. M. Newman. In manuscript.
(5) Compactification of completely regular spaces. Ky Fan and F. Wagner (University of Notre Dame). In manuscript.
(6) Some inequalities concerning positive-definite Hermitian matrices. Ky Fan. To appear in the Proceedings of the Cambridge Philosophical Society.
(7) A comparison theorem for eigenvalues of normal matrices. Ky Fan. Submitted to a technical journal.
(8) Power series for $\log(e^x \cdot e^y)$. K. Goldberg. Submitted to a technical journal.
(9) On certain series expansions involving Whittaker functions and Jacobi polynomials. P. Henrici. To appear in the Pacific Journal of Mathematics.
(10) On generating functions of the Jacobi polynomials. P. Henrici. To appear in the Pacific Journal of Mathematics.
(11) The number of absolute points of a correlation. A.Hoffman, M. Newman, E. Straus, and O. Taussky. Submitted to a technical journal.
(12) Structure theorems for modular subgroups. M. Newman. To appear in the Duke Journal of Mathematics, March 1955.
(13) The coefficients of certain modular forms. M. Newman. To appear in the Journal of the London Mathematical Society.
(14) Some computational problems in algebraic number theory. O. Taussky. To appear in the Proceedings of the American Mathematical Society. Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953.
(15) Error bounds for eigenvalues of symmetric integral equations. H. Wielandt. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953.
(16) On eigenvalues of sums of normal matrices. H. Wielandt. Submitted to a technical journal.
(17) Automatic computations of nerve fiber excitation. K. S. Cole (Naval Medical Research Institute), H. A. Antosiewicz, and P. Rabinowitz. In manuscript.
(18) On asymptotic stability. H. A. Antosiewicz. In manuscript.

ANALYTIC STUDY OF WAR GAMES
Task 1101-10-5116/55-83

Origin and Sponsor: Armament Branch, ARDC, USAF Authorized 12/29/54
Manager: H. A. Antosiewicz
Full task description: Oct-Dec 1954 issue, p. 7

Status: CONTINUED. H. A. Antosiewicz has prepared a report containing the results that have been obtained to date and has submitted it to the

sponsor. The majority of results concern the model as proposed under the assumption that defensive forces are absent. A necessary and sufficient condition has been proved for victory of one party over its opponent in which victory is described by the magnitudes of the ultimate values of the offensive forces and the rates of production. This condition, as might be expected, is satisfied, for example, when one party's initial offensive force and initial rate of production exceeds those of its opponent. Also, relations among the ultimate values of the strategic variables (including defensive forces) have been derived, which yield necessary conditions for victory in terms of the strategic parameters. Some of these relations can be used to obtain sufficient conditions so that one party's strategic variables do not ultimately exceed those of its opponent; that is, if these conditions are fulfilled for one party, its chances to defeat its opponent will be improved, but its victory is not guaranteed.

Publication:
(1) Analytic study of war games. H. A. Antosiewicz. In manuscript.

STUDIES IN THE THEORY OF ASYMPTOTIC EXPANSIONS
Task 1101-10-5116/55-116

Origin: NBS Authorized 3/31/55
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: P. Davis

Objective: To study the class of functions f that are regular in a fixed domain B, that possess finite norm

$$\iint_B f(z)\,^2 dA < \infty$$

and that, in addition, possess zero asymptotic expansions (in the sense of Poincaré) at fixed boundary points of B. Designate such a class by A. It is proposed to develop for A, representation theory, kernel function theory, and estimates of growth, and to study in this way the class of functions that possess preassigned asymptotic expansions at n fixed boundary points and which possess finite norm. The fundamental question to be answered in this direction is the following. Given a domain B and n fixed points $z_1,\ldots,z_n$ on its boundary. Given also n sequences of complex constants a_{jk} $(j=0,1,2,\ldots;\ k=1,2,\ldots,n)$. What are the necessary and sufficient conditions on a_{jk} in order that there exist a regular function of finite norm over B and which at the points z_k possesses the asymptotic expansion

$$f(z) = \sum_{j=0}^{\infty} a_{jk}(z-z_k)^j \qquad (k=1,2,\ldots,n).$$

These questions may be considered as a special case of a more general class of problems which can be outlined as follows. If, at a boundary point z* of a domain B, an analytical function f(z) possesses an asymptotic expansion,

$$f(z) = \sum_{k=0}^{\infty} a_k(z-z^*)^k,$$

each coefficient a_k may be considered as a certain linear functional L_k over the class of functions regular in D: $a_k = L_k(f)$. In view of the fact that the point z* lies on the boundary of D, L_k becomes an unbounded operator. The general problem therefore is one of the existence, the uniqueness, and the representation of solutions of the interpolation problem.

$$(1) \qquad L_k(f) = a_k \qquad (k = 0,1,2,\ldots)$$

for a preassigned sequence of unbounded linear functionals and for a preassigned sequence of constants a_k.

Background: The theory of asymptotic expansions (in the sense of Poincaré) of analytic functions lacks thus far both an interpolatory and an L^2 aspect. In reference [1] below, a method was developed which handled problems of type (1) where the linear functionals L_k were all bounded. This development was aided materially by the introduction of a certain set of orthogonal functions that possess, in addition, a property of biorthogonality with respect to L_k. These functions may be considered as a generalization of the set of doubly orthogonal functions introduced by S. Bergman (see, e.g., [4]). In reference [2] this method was further extended to the class of analytic functions in one (or in several) variables that satisfy a linear differential equation, and a satisfactory interpolatory theory was developed.

In passing from bounded to unbounded linear functionals, it may be anticipated that a number of questions of considerable difficulty will arise. It is proposed to study the problem (1), making use of the techniques of references [1], [2], [3], and appropriate modifications wherever possible, and to determine the extent to which a coherent theory can be developed.

As stressed above, such an approach to the subject of asymptotic series appears to be new, although in reference [5], we can already discern the beginnings of it. It would be of great value to have a purely function-theoretic approach to this subject where development has hitherto been attached principally to special function theory.

References: [1] J. L. Walsh and P. Davis, Interpolation and orthonormal systems, J. d'Analyse Mathematique (Jerusalem) 2, 1-28 (1952). [2] P. Davis and J. L. Walsh, Representations and extensions of bounded linear functionals defined on classes of analytic functions, Trans. Amer. Math. Soc. 76, 190-206 (1954). [3] P. Davis, Linear functional equations and interpolation series, Pacific Journal of Mathematics (to appear). [4] S. Bergman, The kernel function and conformal mapping (American Mathematical Society, 1950). [5] T. Carleman, Les fonctions quasi-analytiques (Paris, 1926).

Status: New.

II. COMPUTATION LABORATORY
(Section 11.2)

1. Mathematical Tables

1102-40-1110/43-3 TABLES OF $E_1(z)$, $(z=x+iy)$

Origin: Canadian National Research Council
Manager: I. Stegun
Full task description: Apr-June 1949 issue, p. 41
Status: Inactive. For status to date, see Oct-Dec 1953 issue, p. 28.

1102-40-1110/47-2 TABLES OF COULOMB WAVE FUNCTIONS
Origin: NBS
Manager: P. Rabinowitz
Full task description: Apr-June 1949 issue, p. 45
Status: Continued. Values of the regular and irregular functions and their derivatives were computed for $\rho=\eta$, L=0, $.5 \leq \rho \leq 14.5$.

1102-40-1110/50-7 WAVE FUNCTION FOR LITHIUM
Origin: NBS
Sponsor: Bureau of Ordnance, USN
Manager: W. F. Cahill
Full task description: Apr-June 1950 issue, p. 36
Status: Inactive. For status to date, see Oct-Dec 1954 issue, p. 9.

1102-40-1110/51-8 TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Origin: Section 11.3, NBS
Managers: A. J. Hoffman, S. Tsingou
Full task description: Apr-June 1951 issue, p. 49.
Status: Continued. Results for 19 values of (x,β) have been computed to date. Some test cases were run on SEAC for very large values of f_1 and f_2 in order to determine the feasibility of obtaining the results with the original code.

1102-40-1110/52-7 REVISION OF MATHEMATICAL TABLES
Origin: NBS
Managers: W. F. Cahill, I. Stegun
Full task description: July-Sept 1951 issue, p. 41.
Status: Continued. Following is the status of those mathematical tables the sales stock of which has been exhausted and for which reissue is planned:

"Tables of sines and cosines for radian arguments," NBS Applied Mathematics Series 43; available from U. S. Government Printing Office, Washington 25, D. C., $3.00. This table was formerly designated as Mathematical Tables MT4.

"Tables of natural logarithms," vol. IV, Mathematical Tables MT12 (1941); revision is in progress for reissue in the Applied Mathematics Series.

1102-40-1110/52-14 TABLE OF ARCSIN FOR COMPLEX ARGUMENTS
Origin: NBS
Manager: D. Liepman
Full task description: July-Sept 1951 issue, p. 41
Status: Continued. Checking of the tables is in process.

1102-40-1110/52-18 EXTENSION OF THE TABLE OF HYPERBOLIC SINES AND COSINES
Origin: NBS
Managers: W. F. Cahill, K. Nelson
Full task description: July-Sept 1951 issue, p. 41
Status: Continued. Checking of the manuscript of the tables has been completed, and the introduction and preface have been prepared.

1102-40-1110/52-23 TABLE OF THE MODIFIED AIRY INTEGRAL
Origin: NBS
Manager: P. Rabinowitz
Full task description: July-Sept 1951 issue, p. 42
Status: Continued. The preface has been written, and preparation of the manuscript for publication is in process.

1102-40-1110/52-25 TABLE OF ERROR FUNCTION FOR COMPLEX ARGUMENTS
Origin: NBS
Manager: W. Hall
Full task description: July-Sept 1951 issue, p. 42
Status: Inactive. For status to date, see Oct-Dec 1954 issue, p. 11.
See also the report under "Hypergeometric Functions," task 1102-40-1110/53-35, p. 12 of this issue.

1102-40-1110/52-31 EXTENSION OF TABLES OF THE EXPONENTIAL FUNCTION FOR NEGATIVE ARGUMENTS
Origin: NBS
Managers: E. Marden, S. Prusch
Full task description: July-Sept 1951 issue, p. 43
Status: Inactive. For status to date see Oct-Dec 1954 issue, p. 11.

1102-40-1110/52-37 SPHEROIDAL WAVE FUNCTIONS
Origin: NBS
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38
Status: Continued. Checking of computations has been resumed.

1102-40-1110/52-49 RADIAL MATHIEU FUNCTIONS
Origin: NBS
Manager: I. Rhodes
Full task description: Jan-Mar 1952 issue, p. 45
Status: Inactive. For status to date, see July-Sept 1954 issue, p. 10.

1102-40-1110/52-57 SIEVERT'S INTEGRAL
Origin: NBS
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46
Status: Inactive. For status to date, see Oct-Dec 1953 issue, p. 33.

1102-40-1110/53-35 HYPERGEOMETRIC FUNCTIONS
Origin: NBS
Managers: P. Rabinowitz, W. F. Cahill
Full task description: Jan-Mar 1953 issue, p. 36
Status: Completed. A code to compute the hypergeometric function for complex parameters and arguments has been written and checked out. The code computes this function by its power series expansion,

$$F(a,b;c;z) = 1+ \frac{ab}{c}\frac{z}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)}\frac{z^2}{2!} +\dots \ .$$

The code uses double precision operations and in general produces a result accurate to 10^{11-s} significant figures where F(a,b;c;z) and its largest term are both less than 10^s.

The code has been modified to compute the confluent hypergeometric function. Using this modified code ω=1.35218 220099 4 has been found as a zero of the function ${}_1F_1(\frac{1-\omega}{2};1;2\omega)$.

Tabulation of these functions is not contemplated at this time. The codes are kept on file for computation of values as needed.

A skeleton table of the complex error function (see publication 1 below)

$$\mathrm{Erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$

was completed for x,y=0(.2)1.2(.4)3.2, where z=x+iy, by use of the relation

$$\mathrm{Erf}(z) = \frac{2z}{\sqrt{\pi}}\ {}_1F_1(1/2;3/2;-z^2).$$

Publications: (1) A short table of the error function for complex arguments. W. F. Cahill. An NBS report. (2) Programs for computing the hypergeometric series. W. F. Cahill. MTAC VIII, 36-37 (Jan. 1954).

1102-40-1110/53-52 L-SHELL CONVERSION COEFFICIENTS
Origin: Oak Ridge National Laboratory
Managers: W. Hall, J. Wegstein
Full task description: Apr-June 1953 issue, p. 45
Status: Continued. Computations for z=75, z=65, and z=45 were completed in the case of the L_1 shell.

3711-60-0009/55-65 AUTOMATIC CODING
Origin: NBS
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11
Status: Continued. The routines for raising a number to an integral power and for computing sines or cosines of numbers in floating point representation have been written, code-checked, and incorporated into the Base 00 system. On an experimental basis the following subroutines and supervisory routines have also been included in the Base 00 system:

(1) a subroutine initiated by a single word that moves a block of information from one place in the memory to another.
(2) a supervisory routine for accelerating the convergence of certain iterative processes
(3) a supervisory routine for quadratic interpolation
(4) a service routine that enables the machine operator to print out or type in a series of decimal numbers by inserting a single word into the computer.

A technical memorandum describing the operation and use of Base 00 as a computing system is being written.

A service routine which will find all referands to any specific memory address has been written and is being code-checked. This routine uses only standard memory locations and thus may be used for code-checking.

A general purpose routine for generating iterative loops was developed, which appears to be useful in many codes. The n code words which are to be repeated with suitable modifications are preceded by a single word whose first three addresses are n, m, and p. This word indicates that the n succeeding words (instructions) are to be repeated either m times or until the number in cell p is less than the number in cell m. The addresses in each instruction being repeated can be modified each time the instruction is repeated, and the specifications for these modifications are carried by each instruction itself. As an example, a table of values $y_i=ax_i+z_i$, i=1(1)m, can be computed from tables of x_i and z_i using the iterative loop generator and only three computer words.

2. Mathematical Services

Note: The tasks under Mathematical Services are arranged serially according to the digits following the slant lines in the task number. The first two digits following the slant line designate the fiscal year in which the task was authorized.

1102-40-5126/50-13 RAY TRACING
Origin and Sponsor: NBS, Section 2.2
Manager: E. Marden
Full task description: Jan-Mar 1950 issue, p. 33
Status: Inactive.

1102-40-5126/51-37 MOLECULAR STRUCTURE, III
Origin: Naval Research Laboratory, USN
Manager: P. O'Hara
Full task description: July-Sept 1951 issue, p. 50
Status: Continued. Preliminary calculations for phase determination have been completed for the aureomycin crystal. Refinement procedures are being carried out to improve the coordinates of the 19 atoms of the spurite crystal.

1102-40-5126/52-20 SPHERICAL BLAST
Origin and Sponsor: Naval Ordnance Laboratory
Manager: D. Jirauch
Full task description: July-Sept 1951 issue, p. 56
Status: Continued. A preliminary report on the method used in coding the problem, together with a brief description of the problem, is being revised in accordance with comments from readers.

1102-40-5126/52-44 CALCULATIONS FOR d SPACINGS
Origin and Sponsor: NBS, Division 9
Manager: A. Futterman
Full task description: Oct-Dec 1951 issue, p. 47
Status: Continued. Calculations for d-spacings were performed for the following cases: (1) Tetragonal crystals $CaWO_4$, ZnF_2, BaO_2, SrO_2; (2) Hexagonal crystals Zn_2SiO_4, Be_2SiO_4, K_2PfF_6, Rb_2PfF_6, $(NH_4)_2PfF_6$.

1102-40-5126/53-27 COMPUTATION OF THERMODYNAMIC FUNCTIONS
Origin and Sponsor: NBS, Section 3.2
Manager: E. Marden
Full task description: Jan-Mar 1953 issue, p. 57
Status: Continued. Thermodynamic functions were calculated for a number of additional molecules. At present, two different codes are in use. One, corresponding to the original formulation of this task (see 11.2/33-49-5, Oct-Dec 1948 issue, p. 49) includes only the rigid rotator harmonic oscillator terms (cf. D. E. Mann, N. Acquista, and E.K. Plyler, J. Chem. Phys. 21, 1949 (1953)). The other includes first-order correction terms for an harmonicity rotation-vibration interaction, and centrifugal stretching, as well as certain second-order low-temperature correction terms (cf. A. S. Friedman and L. Haar, J. Chem. Phys. 22, 2051 (1954)). Comparisons were made in order to evaluate the importance of the added correction terms. To date such calculations have been carried out for over one hundred diatomic hydrides, denterides and tritides; the results are being prepared for issuance as a Bureau circular. More recently several polyatomic molecules have been treated, including H_2S, D_2S, T_2S, HDS, HTS, DTS, and their isotopes.

1102-40-5126/53-51 RADIATION DIFFUSION
Origin: NBS, Section 4.8
Sponsor: Armed Forces Special Weapons Project
Manager: A. Futterman
Full task description: Apr-June 1953 issue, p. 57 (Neutron Diffusion III)
Status: Continued. The boundary problem mentioned in the previous report is being pursued. The results of several Monte Carlo calculations (response functions for scintillation spectrometers, transmission and reflection of gamma radiation through finite barriers) have been analyzed.

3711-60-0009/54-11 COMPUTATION OF VIBRATION MODES AND FREQUENCIES
Origin: NBS, Section 6.4
Managers: I. Rhodes, W. F. Cahill
Full task description: Oct-Dec 1953 issue, p. 42
Status: Completed. The first five antisymmetric modes of an aircraft structure (using 26 mass points) were computed during this period. In all, the first five symmetric and antisymmetric modes of eight aircraft structures have been completed.
Publication: Computation of vibration modes and frequencies on SEAC. W. F. Cahill and S. Levy (NBS Section 6.4). To appear in the Journal of the Institute of Aeronautical Sciences.

1102-40-5126/54-13 AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: New York Quartermaster Procurement Agency
Manager: H. Bremer
Full task description: Oct-Dec 1953 issue, p. 43
Status: Continued. During the past quarter the Quartermaster Corps submitted three procurement problems. The analyses of these problems were done on SEAC, using the simplex method for solving the general transportation problem.

3711-60-0009/54-17 DEPOLYMERIZATION
Origin: NBS, Section 7.6
Manager: J. Bram
Full task description: Oct-Dec 1953 issue, p. 44
Status: Continued. Preparation of the code continued. It is being modified by an addition requested by the originator.

3711-60-0009/54-19 ENERGY LEVELS OF COMPLEX ATOMS
Origin: NBS, Section 4.1
Manager: W. Hall
Full task description: Jan-Mar 1954 issue, p. 41
Status: Inactive. For status to date, see July-Sept 1954 issue, page 14.

1102-40-5126/54-20 LOW TEMPERATURE PROPERTIES OF BORON COMPOUNDS
Origin and Sponsor: NBS, Section 3.2
Manager: J. Wegstein
Full task description: Jan-Mar 1954 issue, p. 41
Status: Terminated.

1102-40-5126/54-21 EXPERIMENTAL PROGRAM FOR MULTIPLE INPUT AND OUTPUT
Origin and Sponsor: NBS, Section 12.3
Manager: I. Rhodes
Full task description: Jan-Mar 1954 issue, p. 42
Status: Inactive.

3711-60-0009/54-22 ENERGY DISTRIBUTIONS ON OPTICAL IMAGE
Origin: NBS, Section 2.2
Manager: L. S. Joel
Full task description: Jan-Mar 1954 issue, p. 43
Status: Inactive.

1102-40-5126/54-25 DEFLECTED RADIATION FROM AN INFINITE LAMBERT PLANE
Origin and Sponsor: Armed Forces Special Weapons Group
Managers: P. Henrici, L. S. Joel
Full task description: Jan-Mar 1954 issue, p. 44
Status: Completed. The portion of the task dealing with reflected radiation from a horizontal surface has been completed. Computations for the vertical cases are being considered; if and when they are to be carried out they will appear as a new task.

A general subroutine for Gaussian quadrature, developed for this problem, is on file. The numerical analysis underlying the computation has been presented in the publication below.

Publication: Application of two methods of numerical analysis to the computation of the reflected radiation of a point source. P. Henrici. J. Wash. Acad. Sci. 45, 38-45 (Feb. 1955).

3711-60-0009/54-27 EMF OF CELLS AT HIGH TEMPERATURE
Origin: NBS, Section 1.8
Managers: E. Marden, G. Hawkins
Full task description: Jan-Mar 1954 issue, p. 45
Status: Completed. Computation of the electromotive forces for all iodides, chlorides, fluorides, bromides, and for several extraneous compounds has been completed.

3711-60-0009/54-28 LOVIBOND NETWORK FOR CIE SOURCE A
Origin: NBS, Section 2.1
Managers: E. Marden, G. Hawkins
Full task description: Jan-Mar 1954 issue, p. 46
Status: Completed. Computations were made on SEAC relating the Lovibond network to CIE sources A and C using the new data supplied from England, and the results for both sets of computations have been transmitted to the sponsor.

3711-60-0009/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Manager: S. Prusch
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. Preparation for publication of the table of arc and spark spectra for ruthenium on the card-controlled typewriter was completed.

1102-40-5126/54-33 AIR CONDITIONING IN UNDERGROUND STRUCTURES
Origin and Sponsor: NBS, Section 10.3
Manager: S. Tsingou
Full task description: Jan-Mar 1954 issue, p. 47
Status: Continued. The original code has been changed so that the following integral can be evaluated.

$$\frac{2}{\pi^3}\, ak^2 \int_0^{\infty} \frac{1 - e^{-\frac{\alpha t}{a^2} y^2}}{[Y_1(y)]^2 + [J_1(y)]^2} \frac{dy}{y^3}$$

for ak = 1, and $\alpha t/a^2$=.005(various)50.

1102-40-5126/54-34 EQUILIBRIUM CALCULATIONS FOR WATER
Origin and Sponsor: Naval Ordnance Laboratory
Manager: J. Wegstein
Full task description: Apr-June 1954 issue, p. 38
Status: Continued. During the period that SEAC was being moved, the tables of thermodynamic properties that were previously calculated with the SEAC were used to calculate entropy tables for water at high temperatures on an IBM 701 calculator. The results were transmitted to the sponsor.

1102-40-5126/54-35 THERMAL STRESSES IN STRUCTURES
Origin and Sponsor: NBS, Section 6.4
Manager: W. Hall
Full task description: Apr-June 1954 issue, p. 38
Status: Continued. Two problems were run using boundary conditions corresponding to those observed in tests conducted in the Engineering Mechanics Section. The observed conditions were approximated by a sequence of linear functions of time.

3711-60-0009/54-36 VELOCITY OF LIGHT
Origin: NBS, Section 11.3
Manager: P. Rabinowitz
Full task description: Jan-Mar 1954 issue, p. 47
Status: Continued. Several third degree polynomials were fitted to data submitted by the sponsor.

3711-60-0009/54-38 COMPRESSIBILITY FACTORS OF DRY AIR
Origin: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Continued. Because of the slowness of convergence of the iterative method of computation, the code is being revised to accelerate the iteration process.

1102-40-5126/55-39 MOLECULAR VIBRATIONS
Origin and Sponsor: NBS, Section 3.2
Manager: K. Goldberg
Full task description: July-Sept 1954 issue, p. 16
Status: Continued. A new code is being prepared to obtain the eigenvectors as well as the eigenvalues of a symmetric matrix up to order 40 by the Jacobi-Givens method.

1102-40-5126/54-43 CHARACTERISTICS OF CONDUCTING RESISTORS
Origin and Sponsor: NBS, Section 12.1
Manager: B. Gill
Full task description: Apr-June 1954 issue, p. 40
Status: Inactive.

1102-40-5126/54-46 STUDY IN OPTIMIZATION OF POWER OUTPUT
Origin: Raytheon Manufacturing Company
Sponsor: U. S. Corps of Engineers
Manager: L. S. Joel
Full task description: Apr-June 1954 issue, p. 41
Status: Completed. The problem described in the Apr-June 1954 issue, p. 41, have been solved.

1102-40-5126/55-58 ELECTROMAGNETIC RADIATION FROM LIGHTNING
Origin and Sponsor: NBS, Section 82.1
Manager: B. R. Gill
Objective: To perform Fourier analyses on time records of lightning flashes.
Background: The intense electrical discharge of a lightning flash produces a brief but intense electromagnetic radiation. It momentarily blots out radio reception in the near vicinity; the addition of many lightning flashes produces much of the steady noise at a distance that limits the receivable signal level. This task, however, is concerned with the vertical component of the electrical field of individual lightning flashes.

It is the purpose of these computations to obtain information about the appearance of the record of the field of lightning flashes, such as the distribution of the energy as a function of frequency at various distances and the propagation of waves at very low frequencies (1 to 50 kc). In particular, it is desired to study the attenuation and distortion of the frequency functions with respect to distance.
Status: New. Twenty-seven curves were scaled on SEAC for nine sets of data observed simultaneously at Fort Belvoir, Va., Boulder, Colo., and Stanford, Calif.; ten curves were scaled for five sets of data from Fort Belvoir and Stanford.

Status of Projects

1102-40-5126/55-61 **ELASTIC CROSS SECTION FOR NEUTRON SCATTERING**
Origin and Sponsor: Naval Research Laboratory
Managers: I. Stegun, R. Zucker
Full task description: Oct-Dec 1954 issue, p. 18
Status: Continued. The code for carrying out the computations has been written and checked. Theoretical results are being matched against experimental results for lead, copper, carbon, tin, and aluminum.

1102-40-5126/55-62 **INTEGRALS INVOLVED IN SUPERSONIC FLUTTER**
Origin and Sponsor: National Advisory Committee for Aeronautics
Manager: S. Tsingou
Full task description: July-Sept 1954 issue, p. 17
Status: Continued. The results are in the process of being checked.

1102-40-5126/55-64 **HIGH-TEMPERATURE THERMODYNAMIC TABLES (FMT)**
Origin and Sponsor: NBS, Section 3.2
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 18
Status: Inactive.

3711-60-0009/55-66 **RECONSTITUTION OF MONOCHROMATIC LIGHT INTENSITIES**
Origin: NBS, Division 30
Manager: H. Bremer
Full task description: July-Sept 1954 issue, p. 18
Status: Inactive. For status to date, see Oct-Dec 1954 issue, p. 19.

1102-40-5126/55-67 **DOSAGE INTEGRAL**
Origin and Sponsor: U. S. Weather Bureau
Manager: W. F. Cahill
Full task description: July-Sept 1954 issue, p. 19
Status: Completed. The original four-fold integral was reduced to

$$2\pi A\mu^{a-1}\int_{\mu}^{\infty}\int_{0}^{\infty}\frac{\operatorname{Exp}\left\{-\frac{(t-d)^2+r^2}{c^2t^b}-\gamma r\right\}\sinh\left\{\frac{2(d-t)}{c^2t^b}r\right\}}{t^{\beta}(d-t)r/c^2t^b}\,drdt$$

and was computed on SEAC for six sets of the parameters. A similar integral where the original integrand was multiplied by $\gamma[(d-ut-x)^2+y^2+z^2]$ was likewise reduced to a double integral and was computed on SEAC for one set of parameters. The results have been transmitted to the sponsor.

3711-60-0009/55-68 **CRYSTAL STRUCTURE CALCULATIONS**
Origin: NBS, Section 9
Manager: A. Futterman
Objective: To perform calculations necessary for the determination of crystal structures from X-ray diffraction patterns such as

(a) Fourier summations

$$\rho(x,y,z) = \frac{1}{V}\sum\sum\sum F(h,k,l)\exp[-2\pi i(hx+ky+lz)],$$

(b) structure factor calculations,

$$F(h,k,l) = \sum_{n=1}^{N} f_n \exp 2\pi i\,(hx_n+hy_n+lz_n),$$

(c) least squares refinement of approximate structure factors, and
(d) methods to obtain approximate structure factors.

Background: Let $\rho(x,y,z)$ be the electron density at the point (x,y,z) in the unit cell of a crystal. An X-ray diffraction pattern obtained from the crystal consists of a number of spots which can be characterized by means of the "indices" h,k,l. Let F(h,k,l) be the complex number denoting the amplitude and phase of the diffracted ray. Then the relation (a) above holds. Of the "structure factors" F, the amplitudes for a finite number of h,k,l may be obtained from measurements of the intensity of the diffracted rays, while the phases are unknown. If the N atoms in the crystal are idealized to points (x_n,y_n,z_n), the structure factors are approximated by (b) above, where the coefficient $f_n(h,k,l)$ is the "form factor" of the n-th atom. Form factors have been tabulated for most types of atoms. The locations (x_n,y_n,z_n) are sometimes approximately known.

The goal of X-ray diffraction analysis is to determine the electron density and structure factors completely. Each of these, if completely known, would determine the other by virtue of (a) or (b). The problem is to fit together the incomplete pieces of information on these two functions in such a way that (a) and (b) are satisfied.

Comment: This task is related to tasks 1102-40-5126/51-37, 1102-40-5126/52-44, and 1102-40-5126/55-87.

Status: New. A general code for the calculation of d-spacings for all crystals has been written and is in the process of being checked.

3711-60-0009/55-69 THEORY OF DIELECTRIC RELAXATION

Origin: NBS, Section 7.6

Manager: K. Goldberg

Full task description: July-Sept 1954 issue, p. 19

Status: Inactive. For status to date, see Oct-Dec 1954 issue, p. 19.

1102-40-5126/55-72 MARYLAND INTER-INDUSTRY STUDY

Origin and Sponsor: Office of Scientific Research, ARDC, USAF

Manager: L. S. Joel

Full task description: July-Sept 1954 issue, p. 20

Status: Continued. Problems relating to allocation of factors of production in an idealized economic model are being solved as linear programs by the simplex method.

1102-40-5126/55-73 HEAT CONVECTION

Origin: University of Minnesota

Sponsor: Atomic Energy Commission

Manager: G. Hawkins

Full task description: Oct-Dec 1954 issue, p. 20

Status: Inactive. For status to date, see Oct-Dec 1954 issue, p. 20.

1102-40-5126/55-74 LIQUID-VAPOR TRANSITION, II

Origin and Sponsor: Naval Medical Research Institute

Managers: R. Zucker, S. Prusch

Objective: To solve by numerical methods an integral equation arising in the statistical-mechanical theory of the transition zone between the liquid and vapor phases according to two approximate theories of liquid state.

Background: The problem was proposed by T. L. Hill of the Naval Medical Research Institute. It is a continuation of task 1102-40-1106/51-22, the principal difference being in the choice of the function Ø and the parameters.

Status: New. The integral equation was solved on punched card machines for one value of the parameter together with one given set of functional values. The solution for several similar equations arising in the same theory is in progress.

3711-60-0009/55-75 PARAMETER OF THE DISPERSION EQUATION FOR OPTICAL GLASS
Origin: NBS, Section 1.6
Manager: R. Zucker
Objective: To evaluate constants k_i of the equation

$$n^2-1 = \sum_{i=1}^{j} \frac{K_i \lambda^2}{\lambda^2 - \lambda_i^2}$$

by the method of least squares from given values of n, λ and λ_i; to compute n and the residuals for the specified values of n and λ: and to compute n by the formula for various values of the wavelength λ.
Background: The results will enable one to express more accurately the refractivities of glass for use in optical design throughout a longer interval of the spectrum than is now used in optical glass catalogues.
Status: New. The problem was coded for SEAC, and many cases for various values of the parameters were run.

1102-40-5126/55-78 NEUTRON TRANSPORT
Origin and Sponsor: Atomic Energy Commission, New York
Manager: S. Tsingou
Full task description: Oct-Dec 1954 issue, p. 21
Status: Inactive.

1102-40-5126/55-79 COMPLETE DEGRADATION IN THE NEUTRON
Origin and Sponsor: Atomic Energy Commission, New York
Manager: A. Futterman
Full task description: Oct-Dec 1954 issue, p. 21
Status: Continued. Several cases for $m<\frac{1}{2}$ were run. The case where $m=\frac{1}{2}$ has given erroneous answers, and the code is under study for possible revision.

1102-40-5126/55-80 ALPHA-ANALYSIS
Origin and Sponsor: Atomic Energy Commission, New York
Manager: W. G. Hall
Full task description: Oct-Dec 1954 issue, p. 22
Status: Inactive.

3711-60-5126/55-81 COMBINING TESTS FOR SIGNIFICANCE
Origin: NBS, Section 11.3
Manager: L. S. Joel
Full task description: Oct-Dec 1954, p. 23
Status: Continued. The code for computing the power points of combining the probabilities of two independent variance-ratio tests into a single test of significance has been written and is now being checked. Preliminary results have been obtained for the percentage points (under the null hypothesis) for the two probability points .05 and .01.

3711-60-0009/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: S. Prusch
Objective: To prepare tables of R_t/R_0 versus temperature for each thermometer tested. The temperature range is t = -183(10)630 degrees Centigrade.

The problem in calibrating a standard platinum resistance thermometer on the international scale is to determine the constants α, δ, β, and R_0 in the equation

$$t = \frac{1}{\alpha}\frac{R_t - R_0}{R_0} + \delta\left(\frac{t}{100} - 1\right)\left(\frac{t}{100}\right) + \beta\left(\frac{t}{100} - 1\right)\left(\frac{t}{100}\right)^3 .$$

The problem is attacked by determining the resistance R at four specified temperatures, namely, the normal boiling points of oxygen, water, sulfur, and the ice point.

Background: The results previously have been obtained by hand computing. A study is being made to see if the work can be systematized and results obtained by machine methods.

Status: New. Punched-card procedures for preparing calibration tables for various types of standard platinum resistance thermometers, excluding low temperature ones, have been set up.

3711-60-0009/55-86 **FLOW COEFFICIENTS FOR FLUIDS**

Origin: NBS, Section 6.7

Manager: P. J. Walsh

Objective: To compute the flow coefficient $K=K_0+b\lambda$ of an orifice from empirical formulae for K_0 and b such as

$$(1)\quad \begin{cases} K_0 = (.6014 - .01352D^{-1/4}) + (.376 - .07257D^{-1/4}) \\ \qquad\qquad \left[\dfrac{.00025}{D^2\beta^2 + .0025D} + \beta^4 + 1.5\beta^{16}\right], \\ b = (.0002 + \dfrac{.0011}{D}) + (.0038 + \dfrac{.0004}{D})\left[\beta^2 + (16.5 + 5D)\beta^{16}\right], \end{cases}$$

for a large number of values of the parameters D, β, and λ. D is the inside diameter of the pipe in which the orifice is installed concentrically.

$$\beta = \frac{d}{D} = \frac{\text{the diameter of the orifice}}{\text{the inside diameter of the pipe}} = \text{diameter ratio},$$

R_D = the Reynolds number referred to the diameter of the pipe, and $\lambda = \frac{10^3}{\sqrt{R_D}}$ is a convenient independent variable for use in coordinating observed values of K.

Background: These formulae were developed empirically from the original test data gathered by the ASME Research Committee on Fluid Meters and the A.G.A. Gas Measurement Committee at Ohio State University during 1931-1933. In addition to these two groups, the Committee on Flow Measurement of the International Standards Organization is now interested in the test data. The formulas are used for computing the flow coefficients of the orifices for various pipe diameters D when pressure taps are situated at assigned positions preceding and following the orifice plate.

Comment: This problem was proposed by H. S. Bean (Section 5.7).

Status: New. Using formula (1), the values K have been computed for various pipe diameters ($2.067 < D < 15.25$) with β ranging from .1 to .8 (41 values).

1102-40-5126/55-87 "ZERO" METHOD DETERMINATION OF CRYSTAL STRUCTURE
Origin and Sponsor: NBS, Section 9.7
Manager: J. Bram
Objective: To calculate the best (least squares) solution $x_1,y_1,z_1,\ldots,x_5,y_5,z_5$ of the equations

$$\sum_{j=1}^{5} f_j\, e^{2\pi i(h_i x_j + k_i y_j + l_i z_j)} = F(h_i,k_i,l_i), \quad i = 1,2,\ldots,85.$$

The f_j and $F(h_i,k_i,l_i)$ are known; all but three of the F's are taken to be zero.
Background: The data obtained by X-ray diffraction studies consist of intensity values proportional to

$$F(h,k,l) = \sum_j f_j\, e^{2\pi i(hx_j + ky_j + lz_j)}$$

where (x,y,z) are the coordinates of an individual atom. However, only |F| is known. The present problem arises in taking those nonspace group reflections which are so small that they can be assumed equal to zero; one must then solve the above system for the x_i,y_i,z_i. The actual equations used in this project are those that arise in the study of napthalene.

For other approaches to the problem of determining crystal structure from X-ray diffraction patterns, see tasks 1102-40-5126/51-37 and 1102-40-5126/52-44.
Status: New. A code for computation on SEAC has been prepared and checked.

1102-40-5126/55-88 STRESSES IN A WALL FOUNDATION
Origin and Sponsor: NBS, Section 10.1
Managers: E. Marden, G. Hawkins
Objective: To compute on SEAC stresses, σ_x, σ_y, σ_{xy}, in a wall foundation for given values of the length and height of the wall and the unit shortening due to shrinkage.
Background: If the interior of the wall is considered to be an elastic slab, the stress function $\emptyset$ satisfies the biharmonic equation

$$\frac{\partial^4 \emptyset}{\partial x^4} + 2\,\frac{\partial^4 \emptyset}{\partial x^2 \partial y^2} + \frac{\partial^4 \emptyset}{\partial y^4} = 0.$$

In a system of rectangular coordinates, x and y, in the plane of the wall,

$$\sigma_x = \partial^2\emptyset/\partial y^2 = \text{stress in x-direction}$$
$$\sigma_y = \partial^2\emptyset/\partial x^2 = \text{stress in y-direction}$$
$$\sigma_{xy} = -\partial^2\emptyset/\partial x\partial y = \text{sheer stress.}$$

The boundary conditions at the top of the wall, y=b, are

$$(\sigma_y)_{y=b} = 0; \quad (\tau_{xy})_{y=b} = 0$$

and along the bottom of the wall, y=b/2, the conditions are

$$(\epsilon_x)_{y=b/2} = S/E; \quad (v)_{y=b/2} = 0,$$

where S/E is the unit shortening due to shrinkage, E is Young's modulus, and v is the vertical displacement. Along the sides x=0 and x=a, the boundary conditions are

$$(\sigma_x)_{x=0,a} = 0; \quad (\tau_{xy})_{x=0,a} = 0.$$

It is assumed that the wall shrinks uniformly as it dries out, and that the footing restrains the shrinkage of the bottom of the wall. The solution is to be obtained in the form of series expansions.

Status: New. A general code has been written for the SEAC to obtain solutions for walls whose lengths and heights will be specified by the input data in each case. Code checking is now in progress.

3711-60-0009/55-89 TEMPERATURE DISTRIBUTION IN SOLID WINGS HEATED AERODYNAMICALLY

Origin: NBS, Section 6.4
Manager: P. Davis, W. F. Cahill
Full task description: Oct-Dec 1954 issue, p. 22
Status: Continued. A program to compute the roots B_n of the expression

$$\tan B_n = \frac{2\,AB_n}{A^2B_n^2-1}$$

and to evaluate the series

$$\frac{v}{T} = 1 - \sum_{n=1}^{\infty} e^{-B_n^2 t} R_n(AB_n \cos B_n\xi + \sin B_n\xi)$$

where

$$R_n = \frac{2(AB_n \sin B_n - \cos B_n + 1)}{B_n\left\{A^2B_n^2 + 2A + 1\right\}}$$

has been coded and is being checked out.

These computations will cover the simplified case of time independent boundary conditions. The more complicated case in which the boundary conditions are time varying is being studied to arrive at a computation scheme.

3711-60-0009/55-90 STUDY OF A DIFFERENCE EQUATION ARISING IN STATISTICAL MECHANICS

Origin: NBS, Section 3.2
Manager: M. Newman
Full task description: Oct-Dec 1954 issue, p. 23
Status: Inactive.

3711-60-0009/55-91 COMPUTATION OF VIRIAL COEFFICIENTS

Origin: NBS, Section 3.2
Manager: P. Davis
Full task description: Oct-Dec 1954 issue, p. 23
Status: Continued. Experimentation with the Monte Carlo method for

numerical integration continued. A discussion of the findings is included under task 1101-10-1104/55-55, p. 1 . A preliminary code has been written for evaluation, by the Monte Carlo method, of the nine-fold integral defined in the preceding report (Oct-Dec 1954 issue, p. 23).

3711-60-0009/55-92 CYLINDRICAL ELECTRON LENS CALCULATIONS
Origin: NBS, Section 4.5
Manager: W. G. Hall
Objective: To compute on SEAC the trajectories for an electron from the equations of motion

$$m\ddot{x} = -eE_x = e\frac{\partial \phi}{\partial x}$$
$$m\ddot{y} = -eE_y = e\frac{\partial \phi}{\partial y}$$

where ϕ is the electric potential distribution given by

$$\phi(x,y) = 1 - \frac{2A\cos x \cosh y}{\cos 2x + \cosh 2y} .$$

E_x = x-directed field,
E_y = y-directed field,
A = a parameter determined by the geometry of the lens.
The problem is to solve this system given the initial values $x_0, y_0, \dot{x}_0, \dot{y}_0$, and to compare trajectories having small changes in the initial conditions, such as when the initial velocity differs by one part in 10,000 or less, or when velocity direction differs by 3.10^{-5} radians. It is necessary to obtain the results to at least six significant figures.
Background: These calculations are being performed to investigate the possibilities of obtaining better resolution in an electron lens than is possible by the use of current procedures.
Comment: This computation was suggested by L. Marton (NBS Section 4.5) and relates to a problem of W. E. Waters (DOFL).
Status: New. A number of trajectories have been computed. A form of the Runge-Kutta method was used. To give an indication of the accuracy obtained, the difference in velocity as given by $v = (\dot{x}^2+\dot{y}^2)^{\frac{1}{2}}$ and $v\,[(e/m)\phi]^{\frac{1}{2}}$ was computed at each step and the maximum difference was printed along with the minimum v. In general, the maximum difference in velocity has been of the order of 10^{-10} and the minimum velocity has been of the order of 10^{-1}.
For the bulk of computation done so far, an interval $\Delta t = .16$ has been used. In the test case run using the intervals of .08, .04, and .02, results differed only in the eighth place from the largest interval to the smallest.

1102-40-5126/55-95 HEAT TRANSFER IN AN EXTENSIVE MEDIUM
Origin and Sponsor: NBS, Section 10.3
Manager: D. Liepman
Objective: To evaluate $e^n \operatorname{erf} \sqrt{n}$ for various values of n.
Background: This function is encountered in the theoretical analysis of the heat transfer in an extensive medium internally bounded by a hollow sphere. The surface conditions of the sphere are desired.
Status: Completed (New). The function has been evaluated and the results have been transmitted to the sponsor.

1102-40-5126/55-96 DISTRIBUTION OF LIGHT OUTPUT FROM NEUTRON SPECTROMETERS
Origin and Sponsor: NBS, Section 4.11
Manager: I. Rhodes
Objective: To calculate by random sampling, on SEAC, the slowing down of a fast neutron in a restricted volume of liquid scintillator. From this to calculate the light output for each incident neutron that remains in the volume. By following the paths of many neutrons the probability distribution of the light output for a given incident neutron may be obtained.
Background: It is the object of the calculation to design a spectrometer, based upon the principle of total absorption, in which the distribution of light output is as narrow as possible, at the same time maintaining a reasonably high detection efficiency. Hand Monte-Carlo calculations indicated that such a spectrometer was promising and would have quite high detection efficiency. All previous fast neutron detectors which have good energy resolution have efficiencies of about 10^{-4}. The proposed spectrometer may increase this efficiency by a factor of 100.
Comments: This problem was proposed by J. E. Leiss (NBS, Section 4.11).
Status: New. The problem has been coded for SEAC, and runs have been made for several different sizes of spectrometers at neutron energies of 7 mev and 14 mev.

1102-40-5126/55-98 THERMAL DIFFUSIVITY
Origin and Sponsor: NBS, Section 3.2
Manager: P. Davis
Objective: To obtain the temperature distribution in a cylinder and in an annular cylinder to which sinusoidal heating has been applied.
Background: This problem arises in the experimental determination of the thermal diffusivities of various specimens.
Status: New. The simplified case has been completely evaluated. Consideration of the complete case awaits further experimental work.

1102-40-5126/55-99 INTEGRAL OF A PRODUCT OF BESSEL FUNCTIONS
Origin and Sponsor: NBS, Section 6.4
Managers: P. Henrici, D. Jirauch
Objective: To evaluate the integral

$$f(k) = \int_0^{\infty} \left(J_i\left(\frac{x}{2}\right)\right)^2 J_2(kx)\, dx$$

for x = 0(.01)2.00.
Background: The evaluation of this integral is required for the solution of a problem in structural mechanics. Direct evaluation of the integral by numerical quadrature methods is difficult since for x large the integrand approaches zero only as $x^{-3/2}$. The integral can be transformed, however, by complex variable methods into a product of hypergeometric series, which can be composed rapidly.
Status: Completed (New). The numerical results obtained on SEAC, accurate to eight significant digits, have been transmitted to the sponsor. A report describing the method of computation is in preparation.

1102-40-5126/55-102 COST ACCOUNTING OPERATIONS FOR HIGH SPEED COMPUTERS
Origin and Sponsor: Bureau of Supplies and Accounts, Department of the Navy
Managers: G. Hawkins, E. Marden
Objective: To program certain accounting operations of the originating agency for performance on electronic computers; in particular, to distribute the monthly financial obligations for various U. S. Naval installations in this country and abroad according to different types

of supply suballotment functions carried on at these installations; to convert these data into cost per unit within each of the suballotment functions and combinations of functions; to compare identical functions between similar activities and to summarize obligations for all activities combined.

Background: The Bureau of Supplies and Accounts of the Navy Department in Washington receives monthly recapitulations of financial obligations for supply operations from more than 60 U. S. Naval supply depots, U. S. Naval supply centers, U. S. Naval shipyards, U. S. Naval stations, U.S. Naval submarine bases, U. S. Naval fuel depots, construction battalions, and other supply activities. At the same time there is reported the number of man hours employed in the activities connected with the Naval installations to account for the obligations to expend funds. These obligations fall into more than 30 different categories, depending upon the nature of the activities conducted at the Naval installation. These data are used for the administration of current funds and as a basis for projecting budget requests for future requirements.

Status: New. Coding the problem for UNIVAC is in progress.

1102-40-5126/55-103 MISSOURI RIVER PROBLEM

Origin and Sponsor: Missouri River Division, Corps of Engineers, U.S.Army

Manager: H. Bremer

Objective: To attempt to develop multiple regression equations to predict the amount of run-off in the Missouri River based upon such influencing factors as snow cover, snow-water content, precipitation, and possibly such other factors as spring temperature and precipitation from the previous years.

Background: The Army Corps of Engineers is interested in predicting the amount of water in the Missouri River flowing past a specific point for a fixed period of time (this is known as the run-off). Such data may aid in the determination of possible dam sites and sizes and in the prediction of spring floods. The method of multiple regression has been used commonly in forecasting seasonal run-off for other rivers. This approach has been shown to work well for data collected for the other river basins. In this particular problem, because of the insufficient length of reliable records, the method will have to include a considerable amount of preliminary independent decisions to be made on the basis of the physical characteristics of the basin. The regression analysis itself is to be performed on a high speed computer such as UNIVAC, which is most readily available to the Corps of Engineers.

Status: New. Two codes have been written for UNIVAC. One obtains the means, standard deviations, and simple cross correlations for any set of observations that may occur; the other does the major part of the computations involved in obtaining the multiple regression coefficients for 9 or less independent variables. The first code has been checked out and with it the following was obtained: (1) means, standard deviations and simple cross correlations for snow-water content for 37 observation stations, each with 20 observations for the 20 years over which records have been kept; (2) means, standard deviations, and simple cross correlations for precipitation taken at 33 stations each with eight different sets of observations extending also over 20 years.

1102-40-5126/55-105 AERONOMIC TIDAL WINDS OF THERMAL ORIGIN

Origin and Sponsor: NBS, Section 82.1

Managers: B. Gill, H. Howe

Objective: To find on SEAC the speed and direction of the horizontal tidal winds and the tidal pressure oscillations for several periods as a function of altitude above the earth's surface using (1) two atmospheric models (NACA and Rocket Panel), and (2) three different boundary

conditions at the top of the atmosphere.
Background: The earth's atmosphere exhibits tidal oscillations just as do the earth's oceans, with the difference that the atmospheric tides are strongly affected by solar radiation. These atmospheric tides manifest themselves as barometric variations, mainly of a semi-diurnal nature; the variations depend on the altitude.
Status: New. The code was written and checked out; trial values were computed.

1102-40-5126/55-106 REFLEX KLYSTRON
Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Department of Defense
Manager: J. Bram
Objective: To evaluate on SEAC the efficiency of a klystron

$$\eta = \frac{V_1}{2\pi V_0} \int_0^{2\pi} \cos\,(\theta + \omega T(\theta))\, d\theta$$

where $T(\theta) = \frac{3T_0}{2\sqrt{2}} \frac{2\sqrt{(V_1/V_0)\cos\theta}}{\sqrt{c}} \int_0^1 \frac{dx}{\sqrt{x^{\frac{1}{2}} + c}\sqrt{x+(V_1/V_0)\cos\theta}}$

if $0 \le \theta \le \frac{\pi}{2}$ or $\frac{3\pi}{2} \le \theta \le 2\pi$,

$$= \frac{3T_0}{2\sqrt{2}} \int_{(V_1/V_0)\cos\theta}^1 \frac{dx}{\sqrt{x^{\frac{1}{2}} + c}\sqrt{x+(V_1/V_0)\cos\theta}}$$

if $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$,

for various values of the parameters.
Background: For the given klystron, ωT_0 is the transit angle associated with the transit time T_0 that is required for an electron to leave and return to the gap; c is a parameter associated with the amount of space charge; V_1/V_0 is the ratio of peak RF voltage to the voltage of the d.c. power supply; η is the efficiency of the klystron.
Status: New. The program has been prepared and is ready to be code checked.

1102-40-5126/55-109 ELECTRON PULSE HEIGHT DISTRIBUTION
Origin and Sponsor: NBS, Section 4.11
Manager: R. Zucker
Objective: For given distributions $f(x_i)$ and $f(x_j)$, to find the sum $\sum_{x_i x_j} f(x_i)f(x_j)$ for assigned values of $x_i + x_j$. This represents a folding together of different mono-energetic electron pulse height distributions obtained from the NBS high energy X-ray spectrometer.
Background: This folding procedure is being performed to synthesize the response of the spectrometer to mono-energetic X-rays.
Comment: This problem was proposed by H. W. Koch (NBS Section 4.11).
Status: New. Calculations are performed as data are received from originator.

1102-40-5126/55-113 **REACTOR DESIGN**
Origin: Westinghouse Atomic Power Division
Sponsor: Atomic Energy Commission
Manager: U. Hochstrasser
Objective: To find the lowest eigenvalue λ and the corresponding solutions $\phi_j(x,y,z)$ [$j = 0,1,2,\ldots,n$] of the n+1 partial differential equations:

$$-\nabla\left\{h_o(x,y,z)\nabla\phi_0(x,y,z)\right\} + f_o(x,y,z)\,\phi_0(x,y,z) = \lambda\sum_{k=0}^{n} g_{ok}(x,y,z)\,\phi_k(x,y,z)$$

$$-\nabla\left\{h_j(x,y,z)\,\nabla\,\phi_j(x,y,z)\right\} + f_j(x,y,z)\,\phi_j(x,y,z) = \lambda\sum_{k=0}^{n} g_{jk}(x,y,z)\phi_k(x,y,z)+d_j(x,y,z)\phi_{j-1}(x,y,z),$$

$$j = 1,2,\ldots,n.$$

The x,y,z are the coordinates of any point lying in the interior R of a closed surface S_R, but not on given closed surfaces S_{ci}[$i=1,\ldots,m$], which are distributed in some way in R. $h_j(x,y,z)$, $f_j(x,y,z)$, $g_{jk}(x,y,z)$, $d_j(x,y,z)$ are given functions in R, which have discontinuities only on S_{ci}. The boundary conditions on S_R are given by

$$\phi_j(x,y,z)=0: \quad j=0,1,2,\ldots,n, \ (x,y,z) \in S_R.$$

In addition the "currents" normal to the surfaces S_{ci} have to be continuous when passing through S_{ci}:

$$\frac{\partial}{\partial n}[h_j(x,y,z)\,\phi_j(x,y,z)]\Big|_{x,y,z\to S_{ci+}} = \frac{\partial}{\partial n}[h_j(x,y,z)\,\phi_j(x,y,z)]\Big|_{x,y,z\to S_{ci-}}$$

where S_{ci+} means the outside of the closed surface S_{ci}, S_{ci-} the inside.
Background: The problem arises in reactor design. The function ϕ_j is the j-th energy level flux. The coefficients $h_j(x,y,z)$, $f_j(x,y,z)$, $g_{jk}(x,y,z)$, $d_j(x,y,z)$ are determined by measurements.
Status: New.

III. STATISTICAL ENGINEERING LABORATORY
(Section 11.3)

1. Fundamental Research in Mathematical Statistics

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-10-1107/49-1a

Origin: NBS Authorized 1/9/49
Manager: L. S. Deming
Full task description: Apr-June 1949 issue, p. 75

Status: CONTINUED. For a description of the continuing activity on this task, see the Jan-Mar 1954 issue, page 49.

MANUAL ON FITTING STRAIGHT LINES
Task 1103-10-1107/50-2

Origin: NBS Authorized 3/1/50
Manager: F. S. Acton
Full task description: Jan-Mar 1950 issue, p. 42

Status: CONTINUED. The manuscript is currently in the hands of technical advisors, for comment.

TABLES TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-10-1107/51-1

Origin: NBS Authorized 7/1/50
Managers: C. Eisenhart, L. S. Deming
Full task description: July-Sept 1950 issue, p. 57

Status: INACTIVE. For status to date see July-Sept 1952 issue, page 64.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-10-1107/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. For a description of the continuing activity on this task, see July-Sept 1954 issue, page 21.

Publications:
(1) Time-discrete stochastic processes in arbitrary sets, with applications to processes with absorbing regions and to the problem of loops in Markoff chains. D. van Dantzig. Accepted for publication (in French) in Annales de l'Institut Henri Poincaré (Paris).
(2) Inequalities for probabilities associated with the multivariate normal distribution. I. R. Savage. Submitted to a technical journal.
(3) On the variances and covariances of order statistics from the Weibull distribution. J. Lieblein. Accepted for publication in the Annals of Mathematical Statistics.

PROCEDURES OF NON-PARAMETRIC STATISTICS
Task 1103-10-1107/52-2

Origin: NBS Authorized 9/17/51
Manager: I. R. Savage
Full task description: July-Sept 1951 issue, p. 66

Status: INACTIVE. For the latest report on this task see July-Sept 1954 issue, page 22.

Publications:
(1) Easily used simultaneous confidence limits for a line. W. S. Connor. Submitted to a technical journal.
(2) Contributions to the theory of rank order statistics. I. R. Savage. Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-10-1107/53-1

Origin: NBS Authorized 10/15/52
Manager: W. H. Clatworthy
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. Work on the analysis of covariance applied to incomplete block designs is being continued. For details see Oct-Dec 1954 issue, page 25.

The material of publication (3) listed below is being reworked to include two-way elimination of heterogeneity.

Publications:
(1) Partially balanced incomplete block designs with two associate classes and two treatments per block. W. H. Clatworthy. Accepted for publication in the Journal of Research of the NBS.
(2) Partially replicated latin squares. W. J. Youden and J. S. Hunter (American Cyanamid Co.). Submitted to a technical journal.

(3) Some fractional factorial arrangements for factors at two levels. W. H. Clatworthy, W. S. Connor, and M. Zelen. In manuscript.

RESEARCH ON MATHEMATICAL ASPECTS OF ORDER STATISTICS METHODS
Task 1103-10-1107/55-110

Origin: NBS Authorized 3/3/55
Manager: J. Lieblein

Objective: To conduct mathematical research on problems involving the use of order statistics.

Background: Modern applications of statistical method in physical science experimentation make use of an increasing variety of "order statistics" procedures, i.e., those involving the size-order of observed quantities. In many cases this has made possible useful short cuts in the analysis of data compared with older methods which compute moments and other statistics of the observations without regard to order. Examples are the use of linear functions of order statistics to estimate the mean and standard deviation of a population, use of range in place of standard deviation in quality control, and use of criteria for rejection of outlying observations based on sample statistics such as $x_n-\bar{x}$, $x_n-\tilde{x}$, $(x_n-x_{n-i})/(x_n-x_1)$, where x_n, x_1, x_{n-i}, are respectively the largest, smallest, and (i+1)-st largest observations and $\bar{x}$ and $\tilde{x}$ are the sample mean and median. This project is intended to investigate mathematical problems that arise in the development and application of new methods concerned with the use of order statistics. It is also planned to provide an aid for the use of available methods by means of an up-to-date compilation of theoretical and tabular information on various types of order statistics.

Comments: Distributions of extremes such as the largest (smallest), second largest (second smallest), etc., sample values often approach limiting distributions when the sample size becomes indefinitely large. Such asymptotic distributions are generally referred to as "extreme value" distributions, and greatly facilitate the study of extreme values in large samples. Thus, as an outgrowth of the order statistics approach, there has arisen a wide area of statistical application concerned with extreme values. Illustrations are cases of breaking strength of materials and fatigue problems where a 'weakest-link' argument may be applied--that rupture is caused by failure of the weakest of the many hypothetical small 'elements' of which the material is considered composed,--and maximum wind velocities, used as a factor in building codes. Two recent publications of the Bureau, Applied Mathematics Series 22, "Probability Tables for the Analysis of Extreme-Value Data", and Applied Mathematics Series 33, "Statistical Theory of Extreme Values and Some Practical Applications", a series of lectures by E. J. Gumbel for which an additional printing became necessary, have stimulated wide-spread interest in the subject. The numerous inquiries on the part of users have indicated the desirability of further attention to various mathematical and statistical aspects, such as investigation of assumptions underlying the limiting distribution of extreme values.

Status: New. Correspondence has been exchanged with several workers in the field of extreme values concerning their application in fatigue problems and in breakdown voltage of capacitors.

Status of Projects

2. Applied Research in Mathematical Statistics

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. W. J. Youden has been consulting with J. B. Wachtman Section 9.1 (Porcelain and Pottery) on a study of the modulus of rupture of alumina briquettes as a function of temperature. An incomplete block design having a novel feature was employed. The material was available in blocks of sufficient size to furnish three specimens. A preliminary run was made at four different temperatures using 6 pairs of specimens, each pair from a different block. When the results from this incomplete block design were obtained, three additional test temperatures were chosen and the remaining test specimens so assigned that a second incomplete block design was obtained. The novel feature of the design was that all the results of the first design using blocks of size two were incorporated in the second design having blocks of size three. Thus at both stages of the investigation any differences between bricks were eliminated on the comparison of results at different temperatures.

Publication:
(1) Velocity of light from the molecular constants of carbon monoxide. E. K. Plyler, L. R. Blaine (4.2), and W. S. Connor. J. Opt. Soc. Amer. 45, 102-106 (Feb. 1955).

STATISTICAL ASPECTS OF NBS ADMINISTRATIVE OPERATIONS
Task 3737-60-0002/52-1

Origin: NBS Authorized 10/1/51
Manager: C. Eisenhart
Full task description: Oct-Dec 1951 issue, p. 56

Status: INACTIVE. For status to date see July-Sept 1952 issue, page 68.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: INACTIVE. For status to date see Oct-Dec 1954 issue, page 27.

RESEARCH IN APPLICATIONS OF MATHEMATICAL STATISTICS TO PROBLEMS OF THE CHEMICAL CORPS
Task 1103-40-5118/52-1

Origin and Sponsor: Biological Laboratories, Chemical Corps, Dept. of the Army — Authorized 10/1/51
Manager: C. Eisenhart
Full task description: Oct-Dec 1951 issue, p. 57

Status: CONTINUED. A report on a method of programming the analysis of variance for factorial experiments was submitted. Trial runs using the method have been made on SEAC and on punched card machines.

Extensive calculations have been made on SEAC of a function arising in the work of the Chemical Corps.

STATISTICAL ANALYSIS AND DESIGN OF EXPERIMENTS FOR THE U. S. GEOLOGICAL SURVEY
Task 1103-40-5140/54-1

Origin and Sponsor: U. S. Geological Survey, Department of Interior — Authorized 10/9/53
Managers: C. Eisenhart, W. J. Youden
Full task description: Oct-Dec 1953 issue, p. 50

Status: CONTINUED. W. C. Krumbein of Northwestern University was a Guest Worker in the Statistical Engineering Laboratory during the first several months of 1955. He was WAE with the Geochemistry and Petrology Branch of the U. S. Geological Survey as consultant with SEL staff members on geological problems requiring statistical advice or review. Among those availing themselves of consulting services during this quarter were W. C. Irving and A. Homka of the U.S.G.S. Division staff on publications, P. F. Narten and D. C. Alvord on biogeochemical exploration, R. Gulbrandson on phosphatic deposits, J. T. Hack on general geology of the Potomac Basin, Dorothy Carroll on Shenandoah Valley stream deposits, and W. H. Bradley on Maine tidal flat organisms.

STATISTICAL ANALYSIS OF BALL BEARING FATIGUE DATA
Task 1103-40-5145/54-1

Origin and Sponsor: American Standards Association — Authorized 1/13/54
Manager: J. Lieblein
Full task description: Jan-Mar 1954 issue, p. 54

Status: COMPLETED. The final report on this project, covering the methods of analysis and interpretation of the data, has been completed and transmitted to the sponsor.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-5146/55-93

Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: CONTINUED. Conferences were held with a number of interested persons on the form and content of the proposed manual. Active work on the manual is expected to get underway early in April, as soon as Paul N. Somerville, who is to be one of its authors, reports for duty in Washington.

IV. MATHEMATICAL PHYSICS SECTION

(Section 11.4)

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-10-1115/55-57

Origin and Sponsor: NBS — Authorized 9/1/54
Manager: E. W. Cannon
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. E. W. Cannon is continuing some basic investigations in the theory of high polymer structure. R. F. Dressler has resumed an investigation on the effect of local domain curvature on edge stresses in bent plates, for the second order approximation equations.

M. Abramowitz and W. Cahill have completed the preparation of a manuscript concerning the role of proper choice of difference expressions for boundary derivatives applied to the elastic clamped square plate problem.

Publications:

(1) Addition theorems for general Legendre and Gegenbauer functions. P. Henrici. Submitted to a technical journal.
(2) Kleine Bemerkung in asymptotischen Entwicklung des Fehlerintegrals. P. Henrici. Accepted for publication by Zeitschrift für angewandte Mathematik und Physik.
(3) On the vibration of a square clamped plate. M. Abramowitz and W. Cahill. Submitted to a technical journal.

RESEARCH IN MECHANICS OF CONTINUA
Task 1104-10-5160/54-23

Origin: NBS — Authorized 12/29/53
Sponsor: Office of Naval Research, USN
Manager: R. F. Dressler
Full task description: Oct-Dec 1953 issue, p. 27

Status: TERMINATED.

Publications:

(1) Comparison of theories and experiments for the hydraulic dam-break wave. R. F. Dressler. To appear in the Proceedings of the Tenth General Assembly of the International Union of Geodesy and Geophysics, held in Rome, Italy, September 1954.
(2) Entropy changes in the equations for rarefaction waves. R. F. Dressler. In manuscript.
(3) On helical springs of finite thickness. P. Henrici. Qu. App. Math. XIII, 106-110 (Apr. 1955).
(4) On rotational viscous flow through a tube. P. Henrici. ZAMP 5, 511-516 (1954).

(5) On the design of two-dimensional supersonic nozzles. B. Chaix (Federal Institute of Technology, Zurich, Switzerland) and P. Henrici. Submitted to a technical journal.
(6) The generation of Coulomb wave functions by means of recurrence relations. I. Stegun and M. Abramowitz. Submitted to a technical journal.
(7) Forced heat convection in laminar flow through a tube. M. Abramowitz. Submitted to a technical journal.

RESEARCH IN ELECTROMAGNETIC THEORY
Task 1104-10-5160/54-47

Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Department of the Army — Authorized 6/29/54, Revised 9/29/54
Manager: F. Oberhettinger
Full task description: July-Sept 1954 issue, p. 28

Status: CONTINUED. F. Oberhettinger is continuing an investigation on the defraction theory for the electromagnetic field generated by a dipole with arbitrary orientation.

RESEARCH IN MATHEMATICAL ELASTICITY
Task 1104-10-5160/55-85

Origin and Sponsor: Office of Scientific Research, ARDC, USAF — Authorized 12/27/54
Manager: R. F. Dressler
Full task description: Oct-Dec 1954 issue, p. 30

Status: CONTINUED. The first studies on vibrations of wings with free edges using finite difference methods were begun with a consideration of right triangular shapes (half delta wings) for various aspect ratios. One side of the wing is elastically clamped; the other side, the hypotenuse, and the included sharp corner are free. The problem has been transformed so that one fixed domain can be used for all wing shapes. This permits use of the same square mesh for all problems. The form for the special sharp corner boundary condition for corner angles not equal to 90° has been obtained for this fixed domain and utilized in the elimination of exterior mesh points near the corner.

R. Dressler, M. Abramowitz, and W. Cahill have completed a detailed study concerning the best methods for eliminating the exterior mesh points in terms of the free boundary conditions. These finite difference formulations present considerable practical difficulty for several reasons: (1) the differential operator is of high order (fourth), (2) the free edge conditions are of high order (second and third), (3) all possible third order cross derivatives occur in the shear conditions on the hypotenuse. About twelve difference patterns have been considered, for both square and oblique meshes, and with the differential equation expressed both on the boundary and off. Many of the schemes would lead to an excess of exterior points over available boundary conditions. A pattern has been chosen using slightly unsymmetric approximations for the boundary conditions and the differential equation on the hypotenuse. This has the great advantage of permitting elimination of exterior points locally without the use of the computing machine. At the free corner it

has been necessary to solve separately one ninth order algebraic system. This is now being done for special cases.

W. Cahill has completed the problem formulation for aspect ratio equal to 2 and $\sigma = .29$ for a mesh with ten points per side. This will serve as a preliminary test by solving its matrix of order 45 for the lowest eigenvalue and eigenvector using a code for SEAC which is already available. For the main results, however, U. Hochstrasser and I. Rhodes have begun preparation of a new machine code for larger, unsymmetric matrices and for higher eigenvalues and vectors.

For the linear deflections of corrugated diaphragms with rotational symmetry, R. F. Dressler has completed a review of existing theories. It is felt that a more rigorous formulation in terms of shell theory (combined bending and stretching) should prove more dependable than engineering-type approaches. The problem is being formulated for the case of constant normal loading, e.g., air pressure, for diaphragms of arbitrary meridial shape for (a) constant thickness and (b) with a thick stiffening disk of arbitrary radius at the center.

F. Oberhettinger is studying available analytic techniques for certain types of boundary value problems for the non-homogeneous biharmonic equation. Such problems arise in the stretching theory for the elastic edge layer. If practical, such methods will be applied to the solution of edge layer stresses for specific plate shapes where the interior solution is now known. Simultaneously, alternative approaches of numerical type, such as use of finite differences or particular orthonormal solutions, are being considered.

NATIONAL BUREAU OF STANDARDS AUTOMATIC COMPUTER (SEAC)

Reassembly of the SEAC computer and auxiliary units continued through the month of January in the new location. The first engineering test routines were run on the machine January 7, and scheduled computation was resumed January 22 for evening and early morning operation. Full three-shift computation was resumed February 21.

During the remainder of the quarter completely reliable operation was hampered by a number of intermittent troubles occurring in the central machine and in both the acoustic and electrostatic memories. This trouble was largely attributed to breakage of the solder connections during the move. There were also a number of cracked crystals in the cells of the acoustic memory.

During the portion of the quarter that SEAC was in operation, its overall efficiency was 73 per cent during scheduled operation compared with 81 per cent for the last quarter reported. Scheduled computation time was distributed as follows:

Total scheduled time available for computation . .	535 hours
Good operating time during scheduled time.	384 "
Idle-in-order time	7 "
Time lost during or following malfunction.	144 "

In addition, SEAC was utilized for 172 hours of nonscheduled computation,-- that is, good operation without a duty engineer in attendance for machine maintenance,-- and 434 hours for engineering and preventive maintenance.

The record of SEAC operations from January 22 through March 31 is as follows:

Task No.	Title	Hours Used: Code Checking	Productive Operation
NBS:			
1104/55-55	Research in Numerical Analysis	9	12
1110/47-2	Tables of Coulomb wave functions	5	8
1110/50-7	Wave function for lithium		1
1110/51-8	Table of power points of analysis-of-variance tests		1
1110/52-25	Table of error function for complex arguments	2	4
1110/53-52	L-shell conversion coefficients		35
5126/52-34	Molecular vibrations		1
5126/52-44	Calculations for d spacings		1
5126/53-15	Nerve fiber reaction	1	7
5126/53-27	Thermodynamic functions	2	
5126/53-29	Dynamic behavior of aircraft structure	2	10
5126/53-51	Radiation diffusion	4	25
5126/53-53	Heterogeneity model		45
0009/54-15	Matrix reduction	41	46
0009/54-17	Depolymerization	1	
0009/54-28	Lovibond network for CIE source A		3
5126/54-33	Air conditioning in underground structures		1
5126/54-35	Thermal stresses in structures		3
0009/54-38	Compressibility factors of dry air	5	
5126/54-45	Transient flow		2

Task No.	Title	Hours Used: Code Checking	Productive Operation
NBS:			
5126/54-36	Velocity of light		1
5126/55-39	Molecular vibrations		2
5126/55-63	Morse integrals		4
5126/55-65	Automatic coding	12	1
0009/55-68	Crystal structures	3	
5126/55-75	Dispersion equation for optical glass	4	1
5126/55-81	Combining tests for significance	1	
5160/55-85	Research in mathematical elasticity	1	5
0009/55-86	Flow coefficients for fluids	6	3
5126/55-87	"Zero" method determination of crystal structures	3	7
5126/55-88	Stresses in a wall foundation	1	
0009/55-89	Temperature distribution in solid wings	1	
0009/55-92	Cylindrical electron lens calculations	6	23
5126/55-96	Distribution of light output from neutron spectrometers	1	41
5126/55-97	High temperature properties of air	5	67
5126/55-99	Integral products of Bessel functions		1
5126/55-115	Adsorptions integrals	1	
0002	Statistical engineering	1	
Other:			
5126/51-3	Meteorology		6
5126/54-13	Award of procurement contracts by linear programming		2
5126/54-44	Flight performance		4
5126/55-61	Elastic neutron scattering	4	17
5126/55-62	Integrals involved in supersonic flutter	3	
5126/55-73	Heat convection	1	
5126/55-79	α-analysis		1
5126/55-101	Transcendental equations	1	
5126/55-104	Calculations of various fuzing systems	1	2
5126/55-105	Aeronomic tidal winds of thermal origin		1
5126/55-106	Reflex klystron	1	3
5126/55-107	Missile trajectory	2	
	Totals:	131	397

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Division Lecture

BRENNER, J. (State College of Washington and Ballistics Research Laboratory) Bounds for determinants and new methods for locating roots of matrices. March 28.

Seminar in Functional Analysis and Applications

ANTOSIEWICZ, H. (1) Linear spaces. February 16. (2) Normed linear spaces. February 23. (3) Banach-Steinhaus theorem and some of its applications. March 2.

HENRICI, P. (1) Iteration methods: Linear equations. March 9. (2) Iteration methods: Nonlinear equations. March 16.

DAVIS, P. (1) Approximate methods of analysis. March 23. (2) Infinite systems of linear equations: Integral equations. March 30.

Applied Statistics Seminar

TUKEY, J. W. (Princeton University and Bell Telephone Laboratories). Some basic problems in planning and interpreting experiments. January 20.

Algebra Seminar

(Offered by The American University in cooperation with the National Bureau of Standards)

JACKSON, I. (Bureau of Ships) A paper of Shoda (I). January 5.

MARSH, R. (Department of Defense) A paper of Shoda (II). January 12.

BRAM, Leila D. On a method of Todd and Coxeter for determining the subgroups of a given finite group. January 17.

GOLDBERG, K. On a paper of Etó. January 26.

TAUSSKY-TODD, O. Commutator sub-groups of p-groups. January 26.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

Papers presented at the meeting of the American Mathematical Society, Brooklyn, N. Y., April 15-16, 1955:

ABRAMOWITZ, M. Forced heat convection in laminar flow through a tube.

GOLDBERG, K. Bounds and asymptotic behavior of coefficients in the series $\log e^x e^y$.

NEWMAN, M., and O. TAUSSKY. Unimodular circulants of rational numbers.

STEGUN, I., and M. ABRAMOWITZ. The generation of Coulomb wave functions by means of recurrence relations. (Presented by title.)

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ALT, F. On the equation of burning. Presented at a meeting of the Institute of Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Md., January 14.

EISENHART, C. Review of some of the recent work of the Statistical Engineering Laboratory, NBS. Presented at a Biostatistics Seminar, Johns Hopkins University, Baltimore, Md., January 27.

TODD, J. Controlled experiments in computing and the coding program at the NBS. Presented to the AEC Computing Facility, New York University, N. Y., March 10.

YOUDEN, W. J. (1) The gun problem. Presented before the Chicago Section, American Society for Quality Control, Chicago, Illinois, January 18. (2) Design of experiments in the physical sciences. Presented before the Committee on Statistics, University of Chicago, Chicago, Illinois, January 18, and at the Eleventh Annual Quality Control Clinic, American Society for Quality Control, Rochester, New York, February 15. (3) Statistical techniques light the way. Presented before the Chicago Section, American Society for Quality Control, Chicago, Illinois, January 19. (4) Series of nine lectures on: Evaluation and control of experimental error; Principles of statistical design analysis of variance; and Factorial experiments and concept of interaction. Given as part of a short course in statistics at the Institute of Statistics, North Carolina State College, Raleigh, N.C., January 22-30. (5) Laboratory control and experimentation. Presented before the Eleventh Annual Quality Control Clinic, American Society

for Quality Control, Rochester, New York, February 15. (6) Design of experiments. Presented before the Esso Manufacturing Technical Committee, Buck Hill Falls, Pennsylvania, February 17; and before the Washington Section, American Society for Quality Control, Washington, D. C., February 23. (7) Statistical design in the development of test methods. Given at a meeting of Committee D-1, the American Society for Testing Materials, Roanoke, Virginia, February 22. (8) Interlaboratory evaluation of analytical methods. Presented at the Pittsburgh Conference on Analytical Chemistry and Applied Spectroscopy Exposition of Modern Laboratory Equipment, sponsored by the Analytical Chemistry Group, Pittsburgh Section, American Chemistry Society and the Spectroscopy Society of Pittsburgh, March 2. (9) Experimental design for problems in physics. Presented at a Nuclear Physics Seminar at the NBS, March 16. (10) Statistical treatment of data. Presented before the Akron Section, American Chemical Society, Akron, Ohio, March 17. (11) Opportunities of mathematics in science. Presented at Paul Junior High School, Washington, D. C., March 25.

Linear Programming Symposium
January 27, 28,29, 1955

The National Bureau of Standards and the Directorate of Management Analysis, DCS/Comptroller, USAF, presented a Linear Programming Symposium, which was sponsored by the Office of Scientific Research of the Air Research and Development Command, USAF. The program follows:

Thursday, January 27

Address of Welcome
Brigadier General Harold Q. Huglin, Directorate of Management Analysis, Headquarters, USAF

Session on Applications, I.
Chairman: Merle M. Andrew, Office of Scientific Research, Air Research and Development Command

Military applications of linear programming.
Walter W. Jacobs, Headquarters, USAF

Linear programming in bid evaluation
Leon Gainen, National Bureau of Standards

Linear programming under uncertainty: An example of a failure
John M. Danskin, Operations Evaluation Group

The assembly line balancing problem
Melvin Salveson, General Electric Company

A commercial use of linear programming
James L. Batchelor, Management Consultant

Session on Computation
Chairman: Max A. Woodbury, Logistics Research Project, George Washington University

How to solve a linear programming problem
Alan J. Hoffman, National Bureau of Standards

Cyclic projections
Charles B. Tompkins, University of California at Los Angeles

Reduction of systems of linear relations
Philip Wolfe, Princeton University

Optimizing a function of additively separated variables subject to simple restrictions
Andrew Vazsonyi, Ramo-Wooldridge Corporation

Some results in nonlinear programming
Robert M. Thrall, University of Michigan

The first feasible solution to the linear programming problem
Saul Gass, Headquarters, USAF

Concepts and computing procedures for certain x_{ij}
Harry Markowitz, RAND Corporation

Friday, January 28

Address of Welcome
Allen V. Astin, Director, National Bureau of Standards

Session on Applications, II
Chairman: Franz L. Alt, National Bureau of Standards

Application of linear programming to optical filter design
Alex Orden, Burroughs Corporation

Linear programming and structural design
John Foulkes, Brown University

A production smoothing problem
George B. Dantzig and Selmer Johnson, RAND Corporation

A model for optimizing production by reference to cost surrogates
Abraham Charnes and William Cooper, Carnegie Institute of Technology

Programming under conditions involving uncertainty
D. F. Votaw, Jr., Yale University

Stochastic linear programming with applications to agricultural economics
Gerhard Tintner, Iowa State College

Session on Economic Theory
Chairman: Oscar Morgenstern, Princeton University

Linear programming and economic theory
Paul Samuelson, Massachusetts Institute of Technology

On the Wald existence theorem for general economic equilibrium
Harold W. Kuhn, Bryn Mawr College and Princeton University

The linear team: An example of linear programming under uncertainty
Roy Radner, Cowles Commission

On competetive equilibrium with a finite linear technology and dependent consumer preferences
Lionel Mackenzie, Duke University

Limitationality and foreign trade
Nicholas Georgescu-Roegen, Vanderbilt University

An activity analysis approach to location
Martin Beckmann, Cowles Commission
Tom Marschak, RAND Corporation

Saturday, January 29

Session on Theory of Linear Inequalities
Chairman: Edward W. Cannon, National Bureau of Standards

Linear inequalities and convex polyhedral sets
Albert W. Tucker, Princeton University

Consistency conditions for finite and infinite systems of linear inequalities
Ky Fan, University of Notre Dame and American University

The probability for the existence of a solution of a system of linear inequalities
Theodore S. Motzkin, University of California at Los Angeles

Optimal rays for linear programs
Alan J. Goldman, Princeton University

The multi-dimensional transportation problem
Emil Schell, Ford Motor Company

The present status of the traveling salesman problem
Isidor Heller, Logistics Research Project, George Washington University

Linear programming activities in England
S. Vajda, Admiralty Research Laboratory, England

Linear programming's future
George B. Dantzig, RAND Corporation

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Tables of sines and cosines for radian arguments. NBS Applied Mathematics Series 43. (A reissue of NBS Mathematical Table 4.) Available from U. S. Government Printing Office, Washington 25, D. C., $3.00.

1.3 Technical Papers

(1) A determinantal inequality. K. Fan and J. Todd. J. Lond. Math. Soc., 30, 58-64 (Jan. 1955).

(2) Some metric inequalities in the space of matrices. K. Fan and A. J. Hoffman. Proc. Am. Math. Soc. 6, 111-116 (1955).

(3) On the solution of the caterer problem. J. W. Gaddum, A. J. Hoffman, and D. Sokolowsky. Log. Res. Quart. 1, 223-229 (1954).

(4) Application of two methods of numerical analysis to the computation of the reflected radiation of a point source. P. Henrici. J. Wash. Acad. Sci., 45, 38-45 (Feb. 1955).

(5) On helical springs of finite thickness. P. Henrici. Qu. App. Math. XIII, 106-110 (Apr. 1955).

(6) On rotational viscous flow through a tube. P. Henrici. ZAMP, 5, 511-516 (1954).

(7) Least p-th power polynomials on a real finite point set. T.S. Motzkin and J. L. Walsh. Trans. Am. Math. Soc. 78, 67-81 (Jan. 1955).

(8) On the linear iteration procedures for symmetric matrices, A. M. Ostrowski. Rend. Mat. App.{V}, VIII, 3-4 (1954).

(9) On the spectrum of a one parametric family of matrices. A. M. Ostrowski. J. Reine Angew. Math. 193, 143-160 (1954).

(10) Motivation for working in numerical analysis. J. Todd. Communications on Pure and Applied Math. VIII, 97-116 (1955).

(11) A method for the numerical integration of differential equations of second order without explicit first derivatives. R. de Vogelaere. J. Res. NBS 54, 119-125 (Mar. 1955), RP2572.

(12) An extremum property of sums of eigenvalues. H. Wielandt. Proc. Am. Math. Soc. 6, 106-110 (1955).

1.5 Miscellaneous Publications

(1) Experiments in the computation of conformal maps. NBS Applied Mathematics Series 42. Available from U. S. Government Printing Office, Washington 25, D. C., 40 cents.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION MARCH 31, 1955.

2.1 Mathematical Tables

(1) Table of characteristic values of Mathieu's equation for large values of the parameter. G. Blanch and I. Rhodes. Accepted for publication in the Journal of the Washington Academy of Sciences.

2.3 Technical Papers

(1) Forced heat convection in laminar flow through a tube. M. Abramowitz. Submitted to a technical journal.

(2) On the vibration of a square clamped plate. M. Abramowitz and W. F. Cahill. Submitted to a technical journal.

(3) Approximate method for rapid Loran computation. M. Abramowitz, D. H. Call, and J. C. Mathews. Submitted to a technical journal.

(4) A theorem on alternatives for pairs of matrices. H. Antosiewicz. Accepted for publication in the Pacific Journal of Mathematics.

(5) Asymptotic solution of linear differential equations with a parameter. H. Antosiewicz. Submitted to a technical journal.

(6) On a certain integral involving Bessel functions. H. Antosiewicz. Submitted to a technical journal.

(7) On nonlinear differential equations of the second order with integrable forcing term. H. Antosiewicz. Accepted for publication in the Journal of the London Mathematical Society.

(8) On the differential equation $\ddot{x} + k(f(x) + g(x)\dot{x})\dot{x} = ke(t)$. H. A. Antosiewicz. Submitted to a technical journal.

(9) Stable systems of differential equations with integrable forcing term. H. A. Antosiewicz. Submitted to a technical journal.

(10) Computation of vibration modes and frequencies on SEAC. W.F.Cahill and S. Levy (NBS 6.4). Accepted for publication in the Journal of the Institute of Aeronautical Sciences.

(11) On the design of two-dimensional nozzles by the method of characteristics. B. Chaix (Federal Institute of Technology, Zurich) and P. Henrici. Submitted to a technical journal.

(12) Partially balanced incomplete block designs with two associate classes and two treatments per block. W. H. Clatworthy. Accepted for publication in the Journal of Research of the NBS.

(13) Easily used simultaneous confidence limits for a line. W.S. Connor. Submitted to a technical journal.

(14) Time-discrete stochastic processes in arbitrary sets, with applications to processes with absorbing regions and to the problem of loops in Markoff chains. D. van Dantzig. Accepted for publication (in French) in Annales de l'Institut Henri Poincaré (Paris).

(15) On a problem in the theory of mechanical quadratures. P. Davis. Accepted for publication in the Pacific Journal of Mathematics.

(16) Some Monte Carlo experiments in computing multiple integrals. P. Davis and P. Rabinowitz. Submitted to a technical journal.

(17) A multi-purpose orthonormalizing code and its uses. P. Davis and P. Rabinowitz. Accepted for publication in the Journal of the Association for Computing Machinery.

(18) Some sampling results on the power of nonparametric tests against normal alternatives. W. J. Dixon (University of Oregon) and D. Teichroew. Submitted to a technical journal.

(19) Comparison of theories and experiments for the hydraulic dam-break wave. R. F. Dressler. To appear in the Proceedings of the Tenth General Assembly, of the International Union of Geodesy and Geophysics, held in Rome, Italy, September 1954.

(20) Heat flow in a fluid with eddying flow. W. H. Durfee (now with Operations Research Office, Johns Hopkins University). Submitted to a technical journal.

(21) On the optimal character of the (s,S) policy in inventory theory. A. Dvoretzky, J. Kiefer, and J. Wolfowitz. Submitted to a technical journal.

(22) A comparison theorem for eigenvalues of normal matrices. K. Fan. Submitted to a technical journal.

(23) On systems of linear inequalities. K. Fan. Submitted to a technical journal.

(24) Some inequalities concerning positive-definite Hermitian matrices. K. Fan. Accepted for publication in the Proceedings of the Cambridge Philosophical Society.

(25) Discrete analogs of inequalities of Wirtinger. K. Fan, O. Taussky, and J. Todd. To appear in Monatschefte für Mathematik.

(26) Asymptotic lower bounds for the fundamental frequency of convex membranes. G. E. Forsythe. Accepted for publication in the Pacific Journal of Mathematics.

(27) An algorithm for solving the transportation problem. A. Gleyzal. Accepted for publication in the Journal of Research of the NBS.

(28) The formal power series for $\log e^x e^y$. K. Goldberg. Submitted to a technical journal.

(29) Addition theorems for general Legendre and Gegenbauer functions. P. Henrici. Submitted to a technical journal.

(30) On certain series expansions involving Whittaker functions and Jacobi polynomials. P. Henrici. Accepted for publication in the Pacific Journal of Mathematics.

(31) On generating functions of the Jacobi polynomials. P. Henrici. Accepted for publication in the Pacific Journal of Mathematics.

(32) The number of absolute points of a correlation. A. J. Hoffman, M. Newman, E. G. Straus, and O. Taussky. Submitted to a technical journal.

Publication Activities

(33) Indentation pressure of a smooth punch. E. Levin. Accepted for publication in the Quarterly of Applied Mathematics.

(34) Acoustic radiation pressure on a circular disk. H. Levine. To appear in the Proceedings of the Fifth Symposium on Applied Mathematics of the American Mathematical Society, held in Pittsburgh, Pa., June 1952.

(35) On moments of order statistics from the Weibull distribution. J. Lieblein. Accepted for publication in the Annals of Mathematical Statistics.

(36) A proof of Hilbert's Nullstellensatz. T. S. Motzkin. Submitted to a technical journal.

(37) The assignment problem. M. Motzkin. To appear in the Proceedings of the American Mathematical Society's Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 1953.

(38) On Fejer sets in linear and spherical spaces. T. S. Motzkin and I. J. Schoenberg. Accepted for publication in Annals of Mathematics.

(39) Pairs of matrices with property L, II. T. S. Motzkin and O. Taussky. Submitted to a technical journal.

(40) An alternative proof of a theorem on unimodular groups. M. Newman. Submitted to a technical journal.

(41) The coefficients of certain modular forms. M. Newman. Submitted to a technical journal.

(42) Structure theorems for modular subgroups. M. Newman. Accepted for publication in the Duke Mathematical Journal.

(43) The diophantine equation $ax^m - by^n = c$. M. Newman. Submitted to a technical journal.

(44) On a generalization of the normal basis in abelian algebraic number fields. M. Newman and O. Taussky. To appear in Communications on Pure and Applied Mathematics of New York University.

(45) On asymptotic series for functions occurring in the theory of diffraction of waves by wedges. F. Oberhettinger. Submitted to a technical journal.

(46) On the Lerch zeta function. F. Oberhettinger. Accepted for publication in the Pacific Journal of Mathematics.

(47) Determinanten mit ueberwiegender Hauptdiagonale und die absolute Konvergenz von linearen Iterationsprozessen. A. M. Ostrowski. Accepted for publication in Comentarii Mathematici Helvetici.

(48) On Gauss' speeding up device in the theory of single step iteration. A. M. Ostrowski. Submitted to a technical journal.

(49) On the convergence of Gauss' alternating procedure in the method of the least squares, I. A. M. Ostrowski. Submitted to a technical journal.

(50) On spectra of second-order differential operators. D. Ray. Submitted to a technical journal.

(51) On the convergence of asymptotic solutions of linear differential equations. R. M. Redheffer (U.C.L.A.) and W. Wasow. Submitted to a technical journal.

(52) Osculatory interpolation in the complex plane. H. E. Salzer (Department of the Army). Accepted for publication in the Journal of Research of the NBS.

(53) Contributions to the theory of rank order statistics. I. R. Savage. Submitted to a technical journal.

(54) Inequalities for probabilities associated with the multivariate normal distribution. I. R. Savage. Submitted to a technical journal.

(55) The generation of Coulomb wave functions by means of recurrence relations. I. A. Stegun and M. Abramowitz. Submitted to a technical journal.

(56) A note on group matrices. O. Taussky. Submitted to a technical journal.

(57) Some computational problems in algebraic number theory. O. Taussky. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 1953.

(58) Generation and testing of pseudo-random numbers. O. Taussky and J. Todd. Submitted to a technical journal.

(59) An improved cathode ray tube storage system. R. Thorensen. To appear in the Proceedings of the Western Computer Conference of the AIEE-IRE-ACM held in Los Angeles, Calif., February 1953.

(60) Machine attacks on problems whose variables are permutations. C. B. Tompkins. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Numerical Analysis, held in Santa Monica, Calif., August 1953.

(61) An extension of a theorem of Dantzig's. C. Tompkins and I. Heller (George Washington University). Submitted to a technical journal.

(62) Best approximation polynomials of given degree. J. Walsh. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held in Santa Monica, Calif., August 1953.

(63) Discrete approximations to elliptic differential equations. W. Wasow. Accepted for publication in Zeitschrift für angewandte Mathematik und Physik.

(64) On the asymptotic transformation of certain distributions into the normal distribution. W. Wasow. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica, Calif., August 1953.

(65) Error bounds for eigenvalues of symmetric integral equations. H. Wielandt. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica, Calif., August 1953.

(66) On eigenvalues of sums of normal matrices. H. Wielandt. Submitted to a technical journal.

(67) Partially replicated latin squares. W. J. Youden and J. S. Hunter. Submitted to a technical journal.

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