# NATIONAL BUREAU OF STANDARDS REPORT

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STATISTICAL INVESTIGATION OF FATIGUE LIFE OF BALL BEARINGS

by

Julius Lieblein and Marvin Zelen

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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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# NATIONAL BUREAU OF STANDARDS REPORT

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## STATISTICAL INVESTIGATION OF FATIGUE LIFE OF BALL BEARINGS

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Julius Lieblein and Marvin Zelen

Statistical Engineering Laboratory

Final report to the
American Standards Association
Committee B3, Subcommittee 7

NBS

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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# P R E F A C E

This report describes a statistical analysis of ball bearing fatigue-life data conducted at the National Bureau of Standards. The work was carried out under the sponsorship of American Standards Association Committee B-3 (Ball and Roller Bearings), Subcommittee 7 (Load Ratings).

Towards the close of 1953, this group requested the National Bureau of Standards to investigate the relationship between fatigue life and load implied by the ball bearing test data compiled by the respective manufacturers of such bearings over many years. The principal aim of the study was to be the determination of the slope of this relationship when the variables are expressed in certain appropriate logarithmic units. A project with this aim was initiated at the Bureau early in 1954, with responsbility for its execution assigned to the Bureau's Statistical Engineering Laboratory.

This report, representing the outcome of the investigations, completes the work under the above project. It describes the results of the undertaking, and in a series of appendices provides a technical explanation of the statistical analyses used.

Churchill Eisenhart
Chief, Statistical Engineering
Laboratory

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# STATISTICAL INVESTIGATION OF FATIGUE LIFE OF BALL BEARINGS

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National Bureau of Standards

## SUMMARY AND PRINCIPAL CONCLUSIONS

The primary aim of this study was to determine a best value for the parameter p in the life formula

$$L = \left[ \frac{f_c \, Z^{a_1} \, D_a^{a_2} \, (i \cos\alpha)^{a_3}}{P} \right]^p ,$$

where L denotes the number of revolutions (millions) that a specified percentage of ball bearings will fail to survive due to fatigue. (If the percentage is 10 per cent, then L is denoted by $L_{10}$ and is called the rating life; if the percentage is 50 per cent, then $L_{50}$ is used and is termed the median life.) The values for the parameters $f_c$, $a_1$, $a_2$, and $a_3$ are unknown and have to be estimated from the data; P represents the constant bearing load and the quantities Z, $D_a$, i, $\alpha$, denote the number of balls, ball diameter, number of rows, and contact angle, respectively.

The statistical analysis based on all deep-groove ball bearing data from SKF, New Departure, and Fafnir* yielded the final values for p shown in Table I, together with an indication of their uncertainties (i.e., 95 per cent confidence limits).

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* The data furnished by the Marlin-Rockwell Corporation were too few to be included in the analysis.

Table I. Final over-all values of p for deep-groove bearings.

| | $L_{10}$ | $L_{50}$ |
|---|---|---|
| p | $2.87 \pm 0.35$* | $2.80 \pm 0.31$ |

These values are in conformity with the value p = 3 when the uncertainty intervals are taken into account.

The data showed no evidence that any of the three companies have different values for *p*. Separate analyses for each company resulted in the values for *p* shown in Table II.

Table II. Individual estimates of p for deep-groove bearings by company.

| Company | Number of test groups | $L_{10}$ | $L_{50}$ |
|---|---|---|---|
| SKF | 50 | $3.00 \pm 0.64$* | $3.05 \pm 0.60$ |
| N. Dep. | 148 | $2.75 \pm 0.48$ | $2.62 \pm 0.40$ |
| Fafnir | 12 | $3.12 \pm 0.88$ | $2.88 \pm 1.02$ |

An analysis for self-aligning ball bearings from SKF only (40 test groups) gave the following values of *p*:

| | $L_{10}$ | $L_{50}$ |
|---|---|---|
| SKF, self-aligning only: | $p = 1.77 \pm 0.46$* | $1.93 \pm 0.44$ |

* The associated uncertainties are 95 per cent confidence limits.

Details concerning the other parameters in the life formula, as well as a description of the methods of analysis and their underlying assumptions are given in the body and appendices of this report.

## I. INTRODUCTION AND STATEMENT OF THE PROBLEM

The experience of ball bearing manufacturers over many years has led to the use of an equation of the form (reference [4]*, page 15, equation (53)--this reference will hereafter be cited as A/P 1947)

$$L = (C/P)^p \tag{1}$$

relating fatigue life L to load P when other factors are kept constant. In the above equation C is termed the "basic (dynamic) capacity", and is defined (cf. A/P 1947, p. 48) as the constant bearing load (in pounds) which 90 per cent of a group of similar bearings can endure for one million revolutions under the given running conditions.

The quantity C in equation (1) depends upon the characteristics of the bearing type, as indicated in A/P 1947, page 32, equation (120). When the expression cited is substituted in equation (1), the fatigue-life formula for ball bearings takes the form

$$L = \left[\frac{f_c\, Z^{a_1}\, D_a^{a_2}\, (i \cos \alpha)^{a_3}}{P}\right]^p \tag{2}$$

The symbols are defined as follows:

$Z$ = number of balls

$D_a$ = ball diameter in inches

$i$ = number of rows

$\alpha$ = contact angle

$P$ = bearing load in pounds

*Numbers in brackets denote references listed at the end of main text.

L = number of million revolutions that a specified percentage of bearings will fail to survive on account of fatigue causes. If the percentage is 10, then $L = L_{10}$, the rating life; if the percentage is 50, then $L = L_{50}$, the median life.

$p$, $a_1$, $a_2$, $a_3$, $f_c$ are taken as unknown parameters whose values have to be estimated from the data.

The main investigation carried out by the Statistical Engineering Laboratory, National Bureau of Standards, involved the determination of the unknown exponent $p$ from data on past ball bearing endurance tests. Earlier studies resulted in two values for $p$: the value $p = 3$, given in A/P 1947, p. 19, is commonly used by major ball bearing manufactures with the exception of New Departure which uses the value $p = 4$. The primary aim of this study was to determine which value, if either, is appropriate for use in the life formula.

The data available for analysis consisted of summaries of ball bearing endurance tests made in past years by SKF, New Departure, Fafnir, and the Marlin-Rockwell Corporation.* These constituted the basic data given to the Statistical Engineering Laboratory by the American Standards Association. Each endurance test consisted of a number of bearings of the same type (the number varying from test to test) which were tested simultaneously under the same load and running conditions. The worksheets summarizing the tests gave the number of millions of revolutions reached by each bearing in the test group before fatigue failure. Information was also given for those tests terminated

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*Data from the Marlin-Rockwell Corporation (hereafter referred to as M.R.C.) were too few to be included in the analysis.

before all bearings in the test group failed. In addition to the test results, the worksheets included information on the characteristics of the bearing type (e.g. values for Z, $D_a$, i, $\alpha$) and load P, as well as other items of descriptive and identifying information. A specimen worksheet is reproduced in Appendix A. Almost all of the data were for deep-groove bearings except for 40 test groups of self-aligning bearings supplied by SKF.

All necessary quantities for evaluating the unknown parameters in the life equation (2) were given directly on the worksheets except the fatigue life L.* This quantity can be estimated from the observed fatigue lives of individual bearings within a test group. As already noted, two concepts of "fatigue life" are used for L, namely, the <u>rating life</u> $L_{10}$, and the <u>median life</u> $L_{50}$. Separate analyses have been carried out with regard to each throughout.

Appendix A summarizes the data taken from the original worksheets that were used in the statistical analysis. Also given are the computed values for $L_{10}$, $L_{50}$, and the "Weibull slope" <u>e</u> (which relates to the dispersion of fatigue lives). The methods for obtaining these quantities from the bearing data are given in detail in Appendix B.

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* Estimates of $L_{10}$ and $L_{50}$ had been entered on the worksheets for many of the tests. However these were not regarded as part of the data submitted for analysis.

## II. ASSUMPTIONS

All conclusions reached in this report, and all statistical analyses employed, are based upon the following principal assumptions:

(a) The life formula (2) is the proper functional form for describing fatigue life in ball bearings.

(b) Differences in the measured life of bearings classed as identical, tested at the same load, reflect only the inherent variability of fatigue life, and are free from systematic errors which may arise from different test conditions, materials, manufacturing methods, etc.

(c) All the bearings in a test group can be regarded as a random sample from a homogeneous population of ball bearings.

(d) The probability distribution (in millions) of revolutions to fatigue failure is of the same form for each test group, though its parameters may differ from group to group.

(e) This fatigue-life distribution is of the type known as the "Weibull distribution".

The purely statistical assumptions, (c)-(e), served as the basis for the determination of $L_{10}$, $L_{50}$, and $\underline{e}$ for each test group. Assumption (e), however, is not involved in the methods used to evaluate the parameters in the life formula (2) from given values of $L_{10}$ or $L_{50}$. A different assumed form for the distribution of fatigue life might give somewhat different values

for $L_{10}$, and possibly different values for $L_{50}$, but the same methods could then be used to evaluate the unknown parameters in the life formula (2).

Other assumptions of a more technical nature were necessary in the course of the analyses. These are discussed in Appendices B and C.

As in all cases where inferences are made from given data, the conclusions reached here pertain only to the population from which the given data can be regarded as constituting a random sample.

## III. OUTLINE OF STATISTICAL ANALYSES

The purpose of this section is to indicate briefly, in general terms, the philosophy and nature of the methods used in the statistical analyses. These methods are discussed in Appendices B and C in considerably fuller detail, froma more technical viewpoint.

The statistical analyses were divided into two phases. The first phase considered the problem of finding estimates of $L_{10}$, $L_{50}$, and the Weibull slope $e$ from the given test data; the second phase used these estimates of $L_{10}$ and $L_{50}$ to evaluate the unknown values of the parameters in the life formula.

### 1. The estimation of $L_{10}$ and $L_{50}$.

The quantity L depends upon the existence of an underlying probability distribution of bearing lives. Selection of a distribution or population is equivalent to specifying the probability that a bearing selected at random from such a population will survive any given number of revolutions, L, or, conversely, that if $c$ is a specified probability, then L is the life period

that will be survived with this probability, e.g.

$$\text{Probability}\ \{\text{Life} \geq L\} = c = \begin{cases} .90, & \text{for } L = L_{10} \\ .50, & \text{for } L = L_{50} \end{cases} .$$

Accordingly, any L such as $L_{10}$ or $L_{50}$ must be obtained by estimating a characteristic of the assumed distribution, here taken as the "Weibull". Thus, this problem is one of statistical estimation.

One method of estimation makes use of an ingenious graphical device by means of which a true (theoretical) Weibull distribution plots as a straight line, and treats the problem as one of straight-line fitting by conventional least squares procedures. However, the procedure usually followed does not take into full account the number of bearings that remain intact when tests are incomplete, nor of the interdependence of successive points. Because of these and other limitations it was decided to use an alternative approach in the estimation of $L_{10}$ and $L_{50}$ for each test group. (See Appendix B.)

Instead, as part of the research carried out in connection with this investigation, a method was developed that takes into account explicitly the number of bearings remaining intact at the termination of a test, and also possesses several other advantages. This method makes use of certain specially-determined linear functions of the observed failure times (in logarithms), $x_i$, arranged in order of size. These functions have the general form

(3) $$T = \sum_{j=1}^{k} c_j x_j \quad .$$

The fact that failure time for the last bearing that fails is, say, the kth in an ordered sequence of n potentially available failure times is equivalent to making use of the fact that (n-k)

bearings remain intact. Since the method makes intimate use of the ordered arrangement of the data, it is termed an "order statistics" method.

The coefficients $c_j$ in (3) allow great flexibility. They have been determined in such a manner that the method will have certain desirable objective characteristics, e.g. freedom from systematic error and a minimum standard error.

In developing the method, it was found possible to make use of certain results obtained in previous research at the National Bureau of Standards in the field of the statistical theory of extreme values (Lieblein [3]).

Calculations based on such previous results were carried out to the extent to which available funds permitted. This necessitated certain modifications involving the use of sub-groups, explained in Appendix B, which, while not seriously curtailing the effectiveness of the order statistics method, prevented the attainment of the maximum benefits of which the method is capable.

## 2. Evaluation of the parameters in the life formula.

Once the estimates for $L_{10}$ and $L_{50}$ are obtained, it is possible to evaluate the exponent p in the life formula. However, in order to make the most efficient use of the given data, it is necessary to estimate the other parameters $f_c$, $a_1$, $a_2$, and $a_3$ as well*.

The methods for estimating the values of $L_{10}$ and $L_{50}$ for each test group actually yield results for $\ln L_{10}$ and $\ln L_{50}$.

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* For deep-groove bearings, $i = 1$, $\alpha = 0^\circ$; thus $i \cos \alpha = 1$ throughout and $a_3$ cannot be estimated.

Thus, taking logarithms* of the life equation (2) results in

$$\begin{aligned} \ln L &= (p \ln f_c) + (pa_1) \ln Z + (pa_2) \ln D_a \\ &\quad + (pa_3) \ln (i \cos \alpha) - p \ln P. \end{aligned} \tag{4}$$

Taking account of the fact that $i \cos \alpha = 1$ for deep-groove bearings (with which the study is chiefly concerned), equation (4) can be written more simply as

$$y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 \tag{5}$$

where

$$\left\{ \begin{array}{l} y = \ln L \qquad \text{(for either } L_{10} \text{ or } L_{50}\text{)} \\ b_0 = p \ln f_c, \quad b_1 = pa_1, \; b_2 = pa_2, \; b_3 = -p \\ x_1 = \ln Z, \; x_2 = \ln D_a, \; x_3 = \ln P \end{array} \right. \tag{6}$$

The quantities $x_1$, $x_2$, and $x_3$ depend on the characteristics of the bearing type and test conditions, and can be regarded as known exactly. On the other hand, the variable y which depends on the outcome of the bearing tests, is subject to considerable dispersion. Thus, estimates can be found for the parameters $b_0$, $b_1$, $b_2$, $b_3$, using standard least squares methods based on minimizing the sums of squared deviations in the y-direction. These methods are discussed in detail in Appendix C.

After the parameters $b_0$, $b_1$, $b_2$, and $b_3$ are estimated, values for $a_0$, $a_1$, $a_2$, and p, can be found from the relations

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* Natural logarithms to the base $\epsilon$ are used throughout as they arise directly in the evaluation of $L_{10}$ and $L_{50}$ (cf. Appendix B). The relation between logarithms to the base 10 and natural logarithms is $\log_{10} a = 2.302581 \ln a$.

$$a_0 = \ln f_c = -\frac{b_0}{b_3} \qquad a_1 = -\frac{b_1}{b_3}$$

$$a_2 = -\frac{b_2}{b_3} \qquad p = -b_3 \quad .$$

It is clear that the values for $a_0$, $a_1$, and $a_2$ depend on the value of $p$.

The estimates for $p$ and the $a$'s are subject to some uncertainties since they are based on test results which themselves are subject to considerable variability. Hence with every value of $p$ and of the $a$'s calculated from the life data, there is given also an interval of uncertainty to indicate its precision.

The intervals of uncertainty given with each parameter are essentially "95 per cent confidence limits". (Detailed explanation of confidence limits can be found in many elementary statistical texts, e.g. Dixon and Massey [1], Hoel [2], Mood [3], etc.) Briefly, confidence intervals describe the compatibility of the observations with an unknown parameter estimated from them; 95 per cent confidence limits are limits such that on the average, in repeated applications of the same procedure, 95 per cent of intervals so calculated will contain the unknown true value of the parameter. The confidence limits associated with $p$ are symmetric. However, the confidence limits associated with the $a$'s are asymmetric because of the dependence of the $a$'s on $p$.

A large interval of uncertainty associated with an estimate indicates an estimate having poor precision; a small interval of uncertainty is evidence of high precision. These intervals of uncertainty not only reflect the inherent variability of the test data, but are also affected by

(a) how well the life equation (2) is the proper functional form for bearing life;

(b) the suitability of the data (including the number of test groups) for estimating the parameters in the life formula.

Further technical details concerning the evaluation of the parameters in the life formula are given in Appendix C.

## IV. ANALYSES FOR DEEP-GROOVE BEARINGS

### 1. Value of p.

The overall values of p with respect to $L_{10}$ and $L_{50}$ for all deep-groove data from SKF, New Departure, and Fafnir have already been presented in the SUMMARY AND PRINCIPAL CONCLUSIONS (Table I) and need not be repeated here. The separate values for each of the three companies were also given in that section (Table II). The intervals of uncertainty specified by "$\pm$" quantities in those tables refer to intervals within which, with reasonable assurance, the true value of the parameter is located. (Probability that interval includes the true value = .95.) The fact that all of the intervals of uncertainty exhibit considerable overlap shows that the data are consistent with the supposition that all three companies have a common value of p for deep-groove bearings. The fact that all of the intervals include 3 indicates that all of the estimates of p are consistent with the practice of taking p = 3. Moreover the value of p for $L_{10}$ was not significantly different from that for $L_{50}$.

The values given for p are based on analyses of all deep-groove ball bearing data, irrespective of bearing type. As such, the parameter estimates represent "omnibus" values. In order to investigate the dependence of the exponent p on bear-

ing type, the data from New Departure, which was made up of three bearing types (Extra light, Light, and Medium), were analyzed separately. The results for the exponent p are shown in Table III. These results are all compatible with the value of $p = 3$.

Table III. Value of p by bearing type for New Departure deep-groove bearings.

| Type | Series | Number of test groups | Value of p | |
|---|---|---|---|---|
| | | | $L_{10}$ | $L_{50}$ |
| Extra light | 3L00 | 37 | $3.36 \pm 0.68$ | $3.23 \pm 0.47$ |
| Light | 3200 | 94 | $2.65 \pm 0.91$ | $2.13 \pm 0.79$ |
| Medium | 3300 | 17 | $1.89 \pm 1.28$ | $2.82 \pm 1.10$ |
| Total | | 148 | | |

2. Evaluation of the parameters $f_c$, $a_1$, $a_2$.

The computations that give estimates for the exponent p, also yield estimates for the quantities $\ln f_c$, $a_1$, and $a_2$. From the relations (6) it is clear that the values for these parameters depend on the value for p. Thus associated with every value of p there will be corresponding values for $\ln f_c$, $a_1$, and $a_2$. Tables IVa and IVb summarize these parameter estimates associated with the final values of p. The estimates for $a_0 = \ln f_c$, rather than $f_c$, are given here, because this is the parameter that arises naturally in the life formula.

Table IVa. Final values of $a_0$, $a_1$, $a_2$ for $\underline{L_{10}}$

| Company | p | $a_0$ | Interval of Uncertainty | $a_1$ | Interval of Uncertainty | $a_2$ | Interval of Uncertainty |
|---|---|---|---|---|---|---|---|
| SKF | 2.87 | 9.02 | (7.31, 10.79) | 0.380 | (-0.454, 1.201) | 1.72 | (1.51, 1.92) |
| N. Dep. | 2.87 | 8.55 | (7.98, 9.14) | 0.670 | ( 0.418, 0.920) | 1.81 | (1.70, 1.92) |
| Fafnir | 2.87 | 9.56 | (6.85, 12.42) | -0.174 | (-1.750, 1.352) | 1.37 | (0.09, 2.67) |

Table IVb. Final values of $a_0$, $a_1$, $a_2$ for $\underline{L_{50}}$

| Company | p | $a_0$ | Interval of Uncertainty | $a_3$ | Interval of Uncertainty | $a_2$ | Interval of Uncertainty |
|---|---|---|---|---|---|---|---|
| SKF | 2.80 | 10.36 | (8.81, 11.98) | 0.015 | (-0.741, 0.751) | 1.69 | (1.50, 1.88) |
| N. Dep. | 2.80 | 9.05 | (8.54, 9.60) | 0.695 | ( 0.470, 0.920) | 1.91 | (1.81, 2.01) |
| Fafnir | 2.80 | 9.05 | (6.61, 11.58) | 0.475 | (-0.921, 1.847) | 1.76 | (0.60, 2.93) |

The analyses conducted separately for each company resulted in other values than those in the previous paragraph for $a_0$, $a_1$, $a_2$. These results are summarized in Tables Va and Vb. They show excellent agreement with the results in Tables IVa and IVb, even though the values for p are somewhat different.

Simiarly, the values for $a_0$, $a_1$, and $a_2$, arising from separate analyses made on the three types of bearings from New Departure, resulted in still other estimates for these parameters. Tables VIa and VIb summarize these estimates. These estimates are less precise than the corresponding "omnibus" values given for New Departure in Tables Va and Vb. This is a consequence of the fact that within a bearing type, the quantities Z and $D_a$ hardly vary at all. This condition makes the data unsuitable for estimating the associated unknown parameters, $a_0$, $a_1$, and $a_2$.

## 3. Redetermination of the estimates for $f_c$

The uncertainty intervals associated with estimates for the parameter $a_0 = \ln f_c$ are quite large. This is primarily because the uncertainty associated with the estimate of $a_0$, also depends on how well the other parameters, $a_1$, $a_2$, and p are estimated. Another way to evaluate $a_0$, which may result in smaller intervals of uncertainty, is to assume *a priori* values for $a_1$, $a_2$, and p, and then determine the estimate for $a_0$. This procedure was followed using the values for the parameters given in A/P 1947, namely

$$a_1 = 2/3, \quad a_2 = 1.8, \quad p = 3 \quad .$$

Table Va. Values of $a_0$, $a_1$, $a_2$ for $\underline{L_{10}}$ based on independent analyses for each company

| Company | p | $a_0$ | Interval of Uncertainty | $a_1$ | Interval of Uncertainty | $a_2$ | Interval of Uncertainty |
|---|---|---|---|---|---|---|---|
| SKF | 3.00 | 8.97 | (7.18, 10.90) | 0.390 | (-0.507, 1.249) | 1.73 | (1.50, 1.94) |
| N. Dep. | 2.75 | 8.59 | (7.99, 9.24) | 0.666 | ( 0.398, 0.928) | 1.80 | (1.67, 1.92) |
| Fafnir | 3.12 | 9.21 | (7.29, 11.84) | -0.041 | (-1.326, 0.992) | 1.36 | (0.49, 2.30) |

Table Vb. Values of $a_0$, $a_1$, $a_2$ for $\underline{L_{50}}$ based on independent analyses for each company

| Company | p | $a_0$ | Interval of Uncertainty | $a_1$ | Interval of Uncertainty | $a_2$ | Interval of Uncertainty |
|---|---|---|---|---|---|---|---|
| SKF | 3.05 | 10.13 | (8.48, 12.00) | 0.072 | (-0.768, 0.855) | 1.71 | (1.50, 1.91) |
| N. Dep. | 2.62 | 9.15 | (8.61, 9.76) | 0.690 | ( 0.456, 0.922) | 1.90 | (1.79, 2.00) |
| Fafnir | 2.88 | 8.93 | (6.58, 12.39) | 0.510 | (-1.055, 1.810) | 1.75 | (0.66, 3.05) |

Table VIa. Values of $a_0$, $a_1$, $a_2$ for $\underline{L_{10}}$ by bearing type (N. Dep. only).

| Type | $a_0$ | Interval of Uncertainty | $a_1$ | Interval of Uncertainty | $a_2$ | Interval of Uncertainty |
|---|---|---|---|---|---|---|
| Extra light | 7.25 | ( 5.21, 9.39) | 1.07 | ( 0.35, 1.76) | 1.68 | (1.27, 2.08) |
| Light | 7.34 | ( 5.54, 9.33) | 1.21 | ( 0.43, 2.00) | 1.69 | (1.38, 1.93) |
| Medium | 2.50 | (-7.95, 16.04) | 3.70 | (-2.25, 9.06) | 1.27 | (0.30, 1.65) |

Table VIb. Values of $a_0$, $a_1$, $a_2$ for $\underline{L_{50}}$ by bearing type (N. Dep. only).

| Type | $a_0$ | Interval of Uncertainty | $a_1$ | Interval of Uncertainty | $a_2$ | Interval of Uncertainty |
|---|---|---|---|---|---|---|
| Extra light | 7.39 | ( 5.92, 8.92) | 1.23 | ( 0.73, 1.73) | 1.79 | (1.50, 2.08) |
| Light | 9.00 | ( 7.10, 11.67) | 0.87 | (-0.05, 1.68) | 1.77 | (1.46, 2.03) |
| Medium | 1.03 | (-4.34, 6.53) | 4.50 | ( 1.97, 7.19) | 1.48 | (1.21, 1.70) |

However if, on such a calculation, the values assumed for the parameters $a_1$, $a_2$, and p are not compatible with the given data, then values of $a_0$ (or $f_c$) so calculated will not be correct determinations for these data. Accordingly, an analysis was made to determine whether the A/P 1947 parameter values were compatible with the given data.

This analysis showed that the A/P 1947 parameter values are compatible with the data, with respect to all individual companies for rating life ($L_{10}$), but NOT for median life ($L_{50}$). (SKF was the only company for which the A/P 1947 parameter values are suitable for median life.) A further analysis, by bearing type for New Departure, showed that the above parameter values are not suitable for the rating life ($L_{10}$) with respect to Medium type bearings; with respect to median life ($L_{50}$) both the Extra light and Medium type bearings are not consistent with the A/P 1947 parameter values.

In the light of this last analysis, redetermined values of $a_0$, taking $a_1 = 2/3$, $a_2 = 1.8$, and $p = 3$, are only strictly valid with respect to the SKF, New Departure (Extra light and Light), and Fafnir Companies for rating life ($L_{10}$). These values are summarized in Table VII. (For convenience these new estimates are given for $f_c = \ln^{-1} a_0 = \exp a_0$.)

Table VII. Values for $f_c$ assuming $a_1=2/3$, $a_2=1.8$, $p=3.0$ for $L_{10}$ (deep-groove).

| Company | Number of test groups | $f_c$ | Interval of Uncertainty |
|---|---|---|---|
| SKF | 50 | 4538 | (4273, 4817) |
| N. Dep. (overall value) | 148 | 4925 | (4750, 5105) |
| Extra light | 37 | 4709 | (4403, 5034) |
| Light | 94 | 5033 | (4885, 5187) |
| Medium* | 17 | ---- | --------- |
| Fafnir | 12 | 3294 | (3029, 3583) |
| M.R.C. | 3 | 4639 | (3478, 6187) |

* Assumed values of parameters $a_1$, $a_2$, and p not compatible with test results for bearings of this series.

4. Analysis for self-aligning bearings.

An analysis, similar to that made on deep-groove bearings, was also made for the 40 test groups of self-aligning bearings from SKF. For this analysis it was also necessary to estimate the parameter $a_3$ in the life equation (2). Thus equation (5) derived from equation (4) now takes the form

$$y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4$$

where the quantities $y$; $x_1$, $x_2$, $x_3$; and $b_0$, $b_1$, $b_2$, $b_3$ have the same meaning as for the deep-groove bearings, and

$$b_4 = pa_3, \quad x_4 = i \cos \alpha .$$

Corresponding to the respective values for the exponent p given in the SUMMARY AND PRINCIPAL CONCLUSIONS, there will also be values for $a_0$, $a_1$, $a_2$, $a_3$. These are summarized in Table VIII. The intervals of uncertainty associated with these estimates are so large that no significance should be attached to these results. They are included in this report, only because the necessary calculations were made.

Table VIII. Values for $a_0$, $a_1$, $a_3$ for self-aligning bearings (SKF only)

| | p | $a_0$ | Interval of Uncertainty | $a_1$ | Interval of Uncertainty | $a_2$ | Interval of Uncertainty | $a_3$ | Interval of Uncertainty |
|---|---|---|---|---|---|---|---|---|---|
| $L_{10}$ | 1.77 | 21.95 | (-2.51,45.61) | 2.60 | (-0.88,5.81) | 0.691 | (0.097,1.247) | -28.74 | (-71.47,16.40) |
| $L_{50}$ | 1.93 | 22.35 | ( 1.54,42.63) | 1.92 | (-1.09,4.68) | 0.728 | (0.226,1.206) | -25.62 | (-62.16,13.04) |

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