NATIONAL BUREAU OF STANDARDS REPORT

3948

PROJECTS and PUBLICATIONS

of the

APPLIED MATHEMATICS DIVISION

A Quarterly Report

October through December 1954

◇ NBS ◇

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section is engaged in specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside of the back cover of this report.

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●Office of Basic Instrumentation   ●Office of Weights and Measures.

PROJECTS and PUBLICATIONS

of the

APPLIED MATHEMATICS DIVISION

October through December 1954

<NBS>

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

October 1 through December 31, 1954

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CONTENTS

Status of Projects

December 31, 1954

I. NUMERICAL ANALYSIS SECTION
(Section 11.1)

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-10-1104/55-55

Origin and Sponsor: NBS   Authorized 8/13/54
Managers: J. Todd, P. Davis   Revised   8/29/54
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. In planning future activity within the project in connection with "controlled" experiments in computation, a survey of earlier work of this type on SEAC has been made. Among the non-trivial computational experiments carried out were the following:

    (1) Monte Carlo
        *Generation and testing of pseudo-random numbers
        *Inversion of a matrix
        *Solution of Laplace equation in various dimensions
        Evaluation of multiple integrals
    (2)*Inversion of matrices
    (3)*Conformal mapping
        *Mapping of ellipses on circle using Lichtenstein-Gerschgorin
          integral equation
        Computation of exterior mapping function and transfinite
          diameter using orthogonal polynomials
    (4)*Game theory and linear programming
    (5) Determination of characteristic roots of matrices

Reports of papers have been published on the starred items; reports on other topics are in preparation. Further experiments are planned as soon as SEAC becomes available again.

      J. Todd has been preparing an extended version of publication (15) below, which will appear (in German) in Jahresbericht der Deutscher Mathematiker Verein.

      P. Davis and P. Rabinowitz have computed the first 12 orthonormal polynomials for a certain simply connected non-convex region, in order to study the convergence of the method of the kernel function for obtaining the interior and exterior mapping functions of the region and its electrostatic capacity (transfinite diameter). The computation used a general orthonormalizing code for complex vectors written by P. Rabinowitz. The boundary was specified at 43 points and the total computation time, including the tape movement, was half an hour. Eleven orthonormal polynomials have apparently yielded the electrostatic capacity to three figures. A report presenting these results is in preparation.

      The manuscript (12) by J. Cameron and others, is being revised; further experiments will be undertaken whenever SEAC becomes available.

1

P. Davis has undertaken experimental computations on the evaluation of multiple integrals by sampling. For further details see task 3711-60-0009/55-91, page 23.

Further studies have been made by I. Stegun with pseudo-random numbers of the multiplicative type. If we let $x_1, x_2, \ldots, x_n$ be a sample of n observations from a normal population and $\bar{x} = (x_1 + x_2 + \ldots + x_n)/n$, the probabilities p_i^n of the events $x_i \leq \bar{x} < x_{i+1}$ have been obtained for the cases when n equal 5 through 8. The results obtained in the case n = 4 were reported erroneously (in the Jan-Mar 1954 issue); the corrected values are as follows.

Number of observations	n	p_1^n	p_2^n	p_3^n	p_4^n	p_5^n	p_6^n	p_7^n
7000	4	.17	.65	.17				
7000	5	.048	.448	.457	.048			
7000	6	.012	.220	.535	.223	.011		
5000	7	.0028	.087	.408	.406	.092	.0036	
5000	8	.0014	.027	.238	.468	.234	.031	.0004

M. G. Kendall (Biometrika 41, 560-564, 1944) has discussed part of this problem theoretically. His results give the following values of p_1^n:

n=4, .174;   n=5, .078;   n=6, .040;   n=7, .024;   n=8, .015.

Several possible reasons for the discrepancy between the observed results and the theoretical ones are being investigated. In particular, the experiments will be repeated with the pseudo-random numbers of the Fibonacci type and with different methods of obtaining normal deviates from them.

Extensive work by H. Antosiewicz on the analysis of non-linear differential equations simulating a war game is described in task 1102-10-5116/55-83, page 7.

H. Antosiewicz and P. Rabinowitz are continuing their work on the differential equations of nerve fiber reactions. In the original Hodgkin-Huxley system the first order equation for the membrane potential has now been replaced by a second order equation which, it is hoped, will give rise to threshold phenomena that will approximate more closely the observed nerve fiber behavior. P. Rabinwoitz has completed a new and more flexible code for this system which will be tested as soon as SEAC is available again.

M. Abramowitz and I. Stegun are investigating the numerical stability of a method for generating the regular and irregular Coulomb wave functions of arbitrary order from their recurrence relations.

Publications:
(1) Coulomb wave functions along the transition line. M. Abramowitz and P. Rabinowitz. Phys. Rev. 96, 77-79 (Oct. 1, 1954).
(2) Coulomb wave functions in the transition region. M. Abramowitz and H. Antosiewicz. Phys. Rev. 96, 75-77 (Oct. 1, 1954).
(3) A representation for solutions of analytic systems of differential equations. H. Antosiewicz and M. Abramowitz. J. Washington Acad. Sci. 44, 382-384 (Dec. 1954).
(4) Automatic computation of nerve excitation. K. S. Cole (Naval Medical Research Institute), H. Antosiewicz, and P. Rabinowitz. In manuscript.
(5) Stable systems of differential equations with integrable forcing term. H. Antosiewicz. Submitted to a technical journal.
(6) Computation of vibration modes and frequencies on SEAC. W. Cahill and S. Levy (NBS 6.4). Submitted to a technical journal.

(7) On a problem in the theory of mechanical quadratures. P. Davis. To appear in the Pacific Journal of Mathematics.

(8) On the estimations of quadrature errors for analytic functions. P. Davis and P. Rabinowitz. MTAC $\underline{8}$, 193-203 (Oct. 1954).

(9) A multi-purpose orthonormalizing code and its uses. P. Davis and P. Rabinowitz. To appear in the Journal of the Association for Computing Machinery.

(10) Application of two methods of numerical analysis to the computation of the reflected radiation of a point source. P. Henrici. To appear in the Journal of the Washington Academy of Sciences.

(11) On the Lerch zeta function. F. Oberhettinger. To appear in the Pacific Journal of Mathematics.

(12) Generation and testing of random numbers on SEAC. O. Taussky, J. Todd, M. Newman, and J. Cameron. In manuscript.

(13) Motivation for working in numerical analysis. J. Todd. To appear in the Transactions of a Symposium on Applied Mathematics, held by the American Mathematical Society and the Office of Ordnance Research in Chicago, Ill., April 29, 1954.

(14) Obituary notice on L. F. Richardson. J. Todd. MTAC $\underline{8}$, 242-245 (Oct. 1954).

(15) Experiments in the solution of differential equations by Monte Carlo methods. J. Todd. J. Washington Acad. Sci. $\underline{44}$, 377-381 (Dec. 1954).

(16) On the solution of the Lichtenstein-Gerschgorin integral equation in conformal mapping: II. Computational experiments. J. Todd and S. E. Warschawski (University of Minnesota). To be included in Experiments in the Computation of Conformal Maps, NBS Applied Mathematics Series 42.

(17) The condition of matrices, II. J. Todd. Arch. Math. $\underline{5}$, 249-257 (1954).

(18) On non-linear differential equations of the second order with integrable forcing term. H. A. Antosiewicz. To appear in the Journal of the London Mathematical Society.

TURBULENT ATMOSPHERIC CONTAMINATION
Task 1101-10-1104/55-59

Origin and Sponsor: Applied Physics Laboratory,   Authorized 7/22/54
                      Johns Hopkins University, and
                      National Bureau of Standards
Managers: E. Marden, G. Hawkins
Full task description: July-Sept 1954 issue, p. 3

    Status: CONTINUED. Computations were completed for the pilot problem, as described in the July-Sept 1954 issue, page 3.

BASIC RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5116/50-2

Origin and Sponsor: Office of Scientific Research,   Authorized 3/31/50
                      ARDC, USAF, and Directorate of
                      Management Analysis, USAF
Managers: J. Todd, A. Hoffman
Full task description: Jan-Mar 1950 issue, p. 24

    Status: CONTINUED. Further applications of linear programming (more specifically, the transportation problem) to combinatorial problems

in the spirit of the general theorem on distinct representatives of subsets
by Hoffman and Kuhn (see Apr-June 1954 issue, p. 18) have been the subject
of investigation this quarter.

In collaboration with G. Dantzig (RAND Corporation), A. Hoffman
has shown how the smallest number C of disjunct chains covering a finite
partially ordered set P can be computed (and the chains found) as a
special case of the transportation problem. Consideration of the dual
yields the result that C is also the largest number of mutually indifferent
elements of P, a result originally due to R. P. Dilworth.

A variation of this device can be used to compute and locate
the largest number p of mutually disjunct paths joining vertices of two
disjunct sets of vertices in a finite graph. The dual yields Menger's
theorem that p is also the smallest number of vertices that intersect
each such path. The corresponding situation for arcs rather than vertices
also is amenable to the same treatment. The analogue of Menger's theorem
in this case had recently been established by Fulkerson and Ford (RAND
Corporation). These theorems hold whether the graph is oriented or not.

Finally, a new method of finding the distance between two disjunct
sets of vertices in a graph has been developed. The dual of this problem
does not seem to have a natural formulation.

Publications:
(1) A theorem on alternatives for pairs of matrices. H. A. Antosiewicz.
    To appear in the Pacific Journal of Mathematics.
(2) Some metric inequalities in the space of matrices. Ky Fan and
    A. J. Hoffman. To appear in the Proceedings of the American Mathematical
    Society.
(3) Discrete analogs of inequalities of Wirtinger. Ky Fan, O. Taussky,
    and J. Todd. To appear in Monatshefte für Mathematik.
(4) A determinantal inequality. Ky Fan and J. Todd. To appear in the
    Journal of the London Mathematical Society.
(5) On distinct systems of representatives. A. J. Hoffman and H. W. Kuhn
    (Bryn Mawr College). In manuscript.
(6) On the relevance of LeChatelier's principle to linear programming.
    A. J. Hoffman. In manuscript.
(7) An extremum property of sums of eigenvalues. H. Wielandt. To appear
    in the Proceedings of the American Mathematical Society.

SUPPORTING RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5116/54-9

Origin: Directorate of Management Analysis, USAF   Authorized 9/25/53
Sponsor: Office of Scientific Research, Air Research
          and Development Command, USAF
Managers: J. Todd, A. J. Hoffman
Full task description: July-Sept 1953 issue, p. 35

Status: CONTINUED. The principal activity of the quarter has been
organization of the Linear Programming Symposium to be held in Washington
on January 27, 28, 29, 1955. The formal program consists   of five one-
hour addresses and 26 shorter talks.

In preparation for his address on computation procedures
A. Hoffman has considered the relative merits of the inverse of the basis
and the "product" form of the inverse for performing simplex computations.
The study, while not yet complete, seems to favor the product form.

During this quarter the fortnightly Linear Programming Seminar
conducted by interested personnel in the Washington area was reactivated.
The first lecture, given on December 15, was delivered by A. Hoffman on

"The solution of the generalized caterer problem."

        Publications:
(1) On "overshoot" in the "furthest hyperplane" method. R. Bryce. In
    manuscript.
(2) On the solution of the caterer problem. J. W. Gaddum, A. J. Hoffman,
    and D. Sokolowsky. To appear in the Logistics Research Quarterly.
(3) An algorithm for solving the transportation problem. A. Gleyzal.
    To appear in the Journal of Research of the NBS.
(4) Smooth patterns of production. A. J. Hoffman and W. Jacobs.
    Management Science 1, 86-91 (1954).
(5) On the optimal ordering of items for a two-stage process. A. J. Hoffman.
    In manuscript.
(6) On block relaxation. L. S. Joel. In manuscript.
(7) A remark on the smoothing problem. H. A. Antosiewicz and A.J. Hoffman.
    Management Science 1, 92-95 (1954).

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO
NUMERICAL ANALYSIS
Task 1101-10-5116/55-56

Origin and Sponsor: Office of Naval Research   Authorized 8/13/54
Managers: O. Taussky-Todd, M. Newman   Revised   8/29/54
Full task description: July-Sept 1954 issue, p. 5

        Status: CONTINUED. O. Taussky-Todd, in collaboration with
M. Newman, continued an investigation of the uniqueness and generalization
of the normal basis of algebraic number fields. In particular, it was
shown that if an n x n unimodular circulant ($\underline{n}$ a power of a prime) is of
the form AA', then AA'=BB', where B is an integral circulant. This had
previously been shown for $n \leq 6$. Some known results on circulants were
extended to general group matrices.
        M. Newman has derived a simple proof of his result that, if
H is a subgroup of the modular group G which contains the subgroup $G_0(n)$
(the totality of elements $\begin{pmatrix} a & b \\ nc & d \end{pmatrix}$), then $H=G_0(d)$, d/n. He has also shown
that the normalizer of $G_0(n)$ is $G_0(n/\delta)$, where $\delta/24$ and is precisely
determined by the highest powers of 2 and 3 dividing n. Further work by
Dr. Newman, in support of an investigation in statistical mechanics,
is reported under task 3711-60-0009/55-90 (see p.23).
        E. C. Dade completed a routine to compute the coefficients of
the polynomials

$$\prod_{r=1}^{n} (1 - x^r) = \sum_{j=0}^{\frac{n^2+n}{2}} A_j^n x^j$$

by means of the recursion formula

$$A_s^n = A_s^{n-1} - A_{s-n}^{n-1} .$$

All the coefficients were calculated for n=1(1)8, and the least
and greatest coefficients together with their locations were calculated
for the first 100 values of n. In addition the central terms were
computed and some interesting empirical results obtained, some of which

were later verified by Mr. Dade by means of an integral formula for A_s^n which he found.

A. Hoffman has applied linear programming to combinatorial problems in partially ordered sets and graphs. For further details, see task 1102-10-5116/50-2, page 3.

K. Goldberg has continued his investigations of the coefficients in the formal power series expansion of log $(e^x \cdot e^y)$. New results include bounds for the coefficients and asymptotic information.

J. Bram has derived an alternative proof of the bounded case of the well-known theorem of Fuglede that an operator commuting with a normal operator A on a Hilbert space commutes with A*. The new proof is based on his previous result that, in the closed subspace S generated by the images of a fixed vector f under polynomials in A and A*, there is a vector g such that S is the closure of the images of g under polynomials in A above.

P. Davis and H. Pollak (Bell Telephone Laboratories) are preparing a manuscript "On the analytic continuation of mapping functions". The object of the investigation is to relate the continuation properties of a region B with an analytic boundary to the geometry of the region as expressed through the sequence of moments $\int_B \int x^m y^n dx dy$.

Two seminars were conducted. One was in Algebra, held in conjunction with American University, under the direction of M. Newman and O. Taussky-Todd; for details see page 35. The other was concerned with miscellaneous topics in mathematics of current interest to the staff of the division; a list of topics appears on page 34.

A book on combinatorial analysis is in preparation by M. Newman. The book is the outgrowth of a series of lectures in this subject that were given by him at American University in the spring of 1953. The object of the book is to present the various methods which are of use in combinatorial analysis, together with important examples to illustrate these methods. The planned table of contents is as follows:

Chapter 1. Binomial coefficients, permutations, combinations
Chapter 2. Methods of enumeration by recurrence formulas
Chapter 3. Theorems in logic and number theory
            The Sylvester sieve formula
            The Möbius function
            Ramsey's theorem
            Symbolic methods of Kaplansky
Chapter 4. Partitions
Chapter 5. Selected Topics
            Systems of distinct representatives
            Finite projective geometrics
            Incidence matrices
            Hadamard matrices
            Difference sets
            Latin squares
            Triple systems.

Publications:
(1) Linear functional equations and interpolation series. P. Davis. Pac. J. Math. 4, 503-532 (1954).
(2) Complex biorthogonality for certain sets of polynomials. P. Davis and H. Pollak (Bell Telephone Laboratories). Duke Math. J. 21, 653-668 (Dec. 1954).
(3) Continuity in terms of connectedness. Ky Fan and R. A. Stuble (University of Notre Dame). Proc. Nederl. Akad., Wetensch. A57, 161-164.
(4) Compactification of completely regular spaces. Ky Fan and F. Wagner (University of Notre Dame). In manuscript.
(5) Some inequalities concerning positive-definite Hermitian matrices. Ky Fan. To appear in the Proceedings of the Cambridge Philosophical Society.

(6) A comparison theorem for eigenvalues of normal operators. Ky Fan. Submitted to a technical journal.
(7) Power series for log $(e^x \cdot e^y)$. K. Goldberg. In manuscript.
(8) On certain series expansions involving Whittaker functions and Jacobi polynomials. P. Henrici. To appear in the Pacific Journal of Mathematics.
(9) On generating functions of the Jacobi polynomials. P. Henrici. To appear in the Pacific Journal of Mathematics.
(10) The number of absolute points of a correlation. A. Hoffman, M. Newman, E. Straus, and O. Taussky. Submitted to a technical journal.
(11) The diophantine equation $ax^m - by^n = c$. M. Newman. Submitted to a technical journal.
(12) Note on a certain determinant. M. Newman. In manuscript.
(13) Structure theorems for modular subgroups. M. Newman. To appear in the Duke Journal of Mathematics.
(14) The coefficients of certain modular forms. M. Newman. Submitted to a technical journal.
(15) Some computational problems in algebraic number theory. O. Taussky. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953.
(16) Generalized commutators of matrices and permutations of factors in a product of three matrices. O. Taussky. Studies in Mathematics and Mechanics Presented to R. von Mises, pp. 67-68 (Academic Press, Inc., New York, 1954).
(17) Error bounds for eigenvalues of symmetric integral equations. H. Wielandt. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953.
(18) On eigenvalues of sums of normal matrices. H. Wielandt. Submitted to a technical journal.

ANALYTIC STUDY OF WAR GAMES
Task 1101-10-5116/55-83

Origin and Sponsor: Armament Branch, ARDC, USAF   Authorized 12/29/54
Manager: H. A. Antosiewicz

Objective: To study a mathematical model of a war game between two opposing parties involving a system of six ordinary non-linear differential equations in which the dependent variables are the strengths of the opposing forces and the rates of each party's production. The choice of coefficients in this system of equations together with the initial values of the dependent variables determines the dynamic evolution of the war model. The central problem is the determination of coefficients in the system of differential equations in such a way as to guarantee victory of one party over its opponent. This amounts essentially to a decision of how best to divide one party's attacking effort between the opponent's actual force-in-being and his productive capacity. A victory is to be characterized by analytic conditions upon the coefficients of the system. Although the concept of victory is well understood intuitively, it is necessary to formulate it here precisely in analytic terms.

To simplify matters, once a choice of coefficients has been made, the same coefficients are used throughout for a particular war game. Thus, the final outcome of the game is completely determined by the coefficients of the system and the initial values of the dependent variables. Numerical evidence obtained from previous investigations on SEAC has pointed up the rather insignificant role played by the defensive forces

in this model. Therefore, the present study is initially based upon the assumptions that defensive forces are absent. The system is thereby reduced to a system of four non-linear differential equations containing six essential parameters. In the future it is planned to extend this investigation to the full system of six equations which is governed by ten parameters.

Background: The model under investigation was proposed by General L. I. Davis, USAF. Earlier numerical investigations were carried out on REAC and on SEAC; also, this model of a war game was used for demonstration purposes on DYSEAC, at its dedication. Results obtained in this way were somewhat inconclusive as regards the optimum choice of coefficients to guarantee victory for one party and pointed up the desirability of further analytic investigation.

Status: NEW. Numerical evidence has pointed up the very insignificant role played by the defensive forces in this model. Therefore the analytic investigation is presently based upon the assumption that defensive forces are absent. The system is thereby reduced to four non-linear differential equations containing six essential parameters.

The following results have been obtained to date. If the strength of A's offensive forces tends to a non-zero finite limit as time increases indefinitely so does the strength of B's offensive forces, and the rates of production of A and B approach finite limits whose ratio is equal to the ratio of the constants determining the fraction of one party's offensive forces sent against his opponents offensive forces. If these constants are equal, victory is determined solely by the initial values of the strengths of offensive forces and the initial rates of production. In particular, if A and B attack only one another's offensive forces, their production rates both approach the same (finite) limit, and the strengths of the offensive forces approach (finite) limits, which are given explicitly in terms of the initial values.

(Section 11.2)

1. Mathematical Tables

1102-40-1110/43-3   TABLES OF $E_1(z)$, $(z=x+iy)$

Origin: Canadian National Research Council
Manager: I. Stegun
Full task description: Apr-June 1949 issue, p. 41
Status: Inactive. For status to date, see Oct-Dec 1953 issue, page 28.

1102-40-1110/47-2   TABLES OF COULOMB WAVE FUNCTIONS
Origin: NBS
Manager: P. Rabinowitz
Full task description: Apr-June 1949 issue, p. 45
Status: Inactive. For status to date, see July-Sept 1954 issue, page 8.

1102-40-1110/50-7   WAVE FUNCTION FOR LITHIUM
Origin: NBS
Sponsor: Bureau of Ordnance, USN
Manager: W. F. Cahill
Full task description: Apr-June 1950 issue, p. 36
Status: Continued. The computation of the atomic wave function of the
  Li^+ 1s2s state has been completed, and the results are being checked.

1102-40-1110/51-4   COLLECTED SHORT MATHEMATICAL TABLES OF THE
                     COMPUTATION LABORATORY
Origin: NBS
Full task description: July-Sept 1950 issue, p. 43
Status: Completed. This volume brings together 18 short tables of functions
  and of zeros of functions. Seven of these tables were published
  originally in the Journal of Mathematics and Physics; others appeared
  in the Bulletin of the American Mathematical Society, the Quarterly
  of Applied Mathematics, the Philosophical Magazine, and Mathematical
  Tables and Other Aids to Computation; and three (marked by asterisks)
  are published here for the first time. The tables included are as follows:
  On the function $H(m,a,x)=exp(-ix)F(m+1-ia,2m+2;2ix)$. A. N. Lowan and
    W. Horenstein.
  Table of the integrals $\int_0^x J_0(t)dt$ and $\int_0^x Y_0(t)dt$. A. N. Lowan and
    M. Abramowitz.
  Table of $Ji_0(x) = \int_x^\infty [J_0(t)/t]dt$ and related functions. A. N. Lowan,
    G. Blanch, and M. Abramowitz.
  Table of $f_n(x)=(x/2)^{-n}n!J_n(x)$.
  The functions $E_n(x)=\int_1^\infty e^{-xu}u^{-n}du$. G. Placek.

Table of the Struve functions $L_\nu(x)$ and $H_\nu(x)$.
Table of Fourier coefficients. A. N. Lowan and J. Laderman.
*Table of sines and cosines for radian arguments between 100 and 1,000.
*Radix table for finding logarithms to 25 decimal places. H. E. Salzer.
*Table of $x^n/n!$.
Table of the zeros of the Legendre polynomials of order 1–16 and the
   weight coefficients for Gauss' mechanical quadrature formula.
   A. N. Lowan, N. Davids, and A. Levinson.
Table of the zeros and weight factors of the first fifteen Laguerre
   polynomials. H. E. Salzer and R. Zucker.
A short table of the first five zeros of the transcendental equation
   $J_0(x)Y_0(kx)-J_0(kx)Y_0(x)=0$. A. N. Lowan and A. Hillman.

Zeros of certain Bessel functions of fractional order. M. Abramowitz.
More zeros of certain Bessel functions of fractional order.
Zeros of the derivative of Bessel functions of fractional order.
   H. E. Salzer.
Complex zeros of $Y_0(z)$, $Y_1(z)$, and $Y_1'(z)$. A. Hillman and I. Sherman.

Roots of sin $z = z$. A. P. Hillman and H. E. Salzer.
Publication: "Tables of functions and of zeros of functions: Collected
   short tables of the Computation Laboratory," NBS Applied Mathematics
   Series 37; available from U. S. Government Printing Office, Washington
   25, D. C., $2.25.

1102–40–1110/51–8   TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Origin: Section 11.3, NBS
Managers: A. J. Hoffman, S. Tsingou
Full task description: Apr–June 1951 issue, p. 49
Status: Inactive. For status to date, see July–Sept 1954 issue, page 9.

1102–40–1110/52–7   REVISION OF MATHEMATICAL TABLES
Origin: NBS
Managers: W. F. Cahill, I. Stegun
Full task description: July–Sept 1951 issue, p. 41
Status: Continued. Following is the status of those mathematical tables
   the sales stock of which has been exhausted and for which reissue is
   planned:
      "Tables of the error function and its derivative," NBS Applied
   Mathematics Series 41; available from U. S. Government Printing Office,
   Washington 25, D. C., $3.25. This was originally Mathematical Table MT8,
   "Tables of probability functions," vol. I (1941).
      "Tables of sines and cosines for radian arguments," NBS Applied
   Mathematics Series 43; in press, U. S. Government Printing Office. This
   table was formerly designated as Mathematical Table MT4.
      "Tables of natural logarithms," vol. IV, Mathematical Table MT12
   (1941); revision in progress for reissue in the Applied Mathematics Series.

1102–40–1110/52–14   TABLE OF ARCSIN FOR COMPLEX ARGUMENTS
Origin: NBS
Manager: D. Liepman
Full task description: July–Sept 1951 issue, p. 41
Status: Inactive. For status to date, see Oct–Dec 1953 issue, p. 31.

1102–40–1110/52–18   EXTENSION OF THE TABLE OF HYPERBOLIC SINES AND COSINES
Origin: NBS
Managers: W. F. Cahill, K. Nelson
Full task description: July–Sept 1951 issue, p. 41

Status: Continued. The manuscript of the table has been typed and checked. An introduction has been written, and the tables are being prepared for publication.

1102-40-1110/52-23   TABLE OF THE MODIFIED AIRY INTEGRAL
Origin: NBS
Manager: P. Rabinowitz
Full task description: July-Sept 1951 issue, p. 42
Status: Inactive. For status to date, see Oct-Dec 1953 issue, page 31.

1102-40-1110/52-25   TABLE OF ERROR FUNCTION FOR COMPLEX ARGUMENTS
Origin: NBS
Manager: W. Hall
Full task description: July-Sept 1951 issue, p. 42
Status: Inactive. The 20-place table for x,y=0(.1)2 is available on punched cards. In view of the publication of the Russian Academy of Science tables of this function, the program of this task is being reconsidered.

1102-40-1110/52-31   EXTENSION OF TABLES OF THE EXPONENTIAL FUNCTION FOR NEGATIVE ARGUMENTS
Origin: NBS
Managers: E. Marden, S. Prusch
Full task description: July-Sept 1951 issue, p. 43
Status: Continued. The manuscript has been typed and is in the process of being checked.

1102-40-1110/52-37   SPHEROIDAL WAVE FUNCTIONS
Origin: NBS
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38
Status: Inactive. For status to date, see Jan-Mar 1954 issue, page 31.

1102-40-1110/52-49   RADIAL MATHIEU FUNCTIONS
Origin: NBS
Manager: I. Rhodes
Full task description: Jan-Mar 1952 issue, p. 45
Status: Inactive. For status to date, see July-Sept 1954 issue, page 10.

1102-40-1110/52-57   SIEVERT'S INTEGRAL
Origin: NBS
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46
Status: Inactive. For status to date, see Oct-Dec 1953 issue, page 33.

1102-40-1110/53-35   HYPERGEOMETRIC FUNCTIONS
Origin: NBS
Managers: P. Rabinowitz, W. F. Cahill
Full task description: Jan-Mar 1953 issue, p. 36
Status: Inactive. For status to date, see Apr-June 1954 issue, page 27.

1102-40-1110/53-52   L-SHELL CONVERSION COEFFICIENTS
Origin: Oak Ridge National Laboratory
Managers: W. Hall, J. Wegstein
Full task description: Apr-June 1953 issue, p. 45
Status: Continued. Further exploratory computations (see July-Sept 1954 issue, p. 10) were made to overcome singularities at r=0, and a combination of series expansions and stepwise integrations, which appears satisfactory, was arrived at. Computations for the L_1 shell of z=45 and z=65 were completed for several energy levels.

3711-60-0009/55-65   AUTOMATIC CODING
Origin: NBS
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11
Status:Continued. The general purpose interpretive floating decimal point routine (Base 00) now in use requires only one instruction word to execute each of the following on floating decimal point numbers: addition, subtraction, multiplication, division, logarithm, exponential, square root, binary to decimal conversion, decimal to binary conversion, and comparison of absolute values. The interpretive routine also permits treating numbers in their usual SEAC form as well as automonitoring. This routine is being modified to include the following operations: sine, cosine, raising numbers to integral powers. A set of instructions for its use is also being prepared.

2. Mathematical Services

Note: The tasks under Mathematical Services are arranged serially according to the digits following the slant lines in the task number. The first two digits following the slant line designate the fiscal year in which the task was authorized.

1102-40-5126/50-13   RAY TRACING
Origin and Sponsor: NBS, Section 2.2
Manager: E. Marden
Full task description: Jan-Mar 1950 issue, p. 33
Status: Continued. Seven additional aerial camera lens systems were analyzed, and spot diagrams were computed for them. Several hundred rays were traced through each such lens system.

1102-40-5126/51-37   MOLECULAR STRUCTURE, III
Origin: Naval Research Laboratory, USN
Manager: P. O'Hara
Full task description: July-Sept 1951 issue, p. 50
Status: Continued. Computations to aid in the determination of the molecular structure of the spurite crystal were performed, as requested.

1102-40-5126/52-20   SPHERICAL BLAST
Origin and Sponsor:Naval Ordnance Laboratory
Manager: D. Jirauch
Full task description: July-Sept 1951 issue, p. 56
Status: Continued. A preliminary report on the method used in coding the problem, together with a brief description of the problem, was drafted.

1102-40-5126/52-44   CALCULATIONS FOR d SPACINGS
Origin and Sponsor: NBS, Division 9
Manager: A. Futterman
Full task description: Oct-Dec 1951 issue, p. 47
Status: Continued. Computations for several different crystals were
   performed in accordance with sets of parameters furnished by the sponsor.

1102-40-5126/53-11   STRENGTH OF WING COMPONENTS
Origin and Sponsor: National Advisory Committee for Aeronautics,
                    Langley Field, Va.
Manager: G. Hawkins
Full task description: Oct-Dec 1952 issue, p. 54
Status: Completed. The computations were completed according to the code
   set up by the originator and were transmitted to that agency.

1102-40-5126/53-27   COMPUTATION OF THERMODYNAMIC FUNCTIONS
Origin and Sponsor: NBS, Section 3.2
Manager: E. Marden
Full task description: Jan-Mar 1953 issue, p. 57
Status: Continued. Computations were performed for several additional
   molecules. In addition, thermodynamic functions were re-calculated for
   for several molecules for which improved physical constants were obtained.

1102-40-5126/53-29   DYNAMIC BEHAVIOR OF AIRCRAFT STRUCTURES
Origin and Sponsor: NBS, Section 6.4
Manager: I. Rhodes
Full task description: Jan-Mar 1953 issue, p. 58
Status: Completed. The results have been transmitted to the sponsor.

1102-40-5126/53-39   ACOUSTICAL IMPEDANCES
Origin and Sponsor: NBS, Section 6.1
Manager: S. Prusch
Full task description: Apr-June 1953 issue, p. 55
Status: Completed. The computations were performed as requested by the
   originator.

1102-40-5126/53-41   LORAN UNIVAC CODE
Origin and Sponsor: Hydrographic Office, U. S. Navy
Managers: I. Rhodes, D. Jirauch
Full task description: Apr-June 1953 issue, p. 56
Status: Completed. The Loran UNIVAC codes have been turned over to the
   sponsor.

1102-40-5126/53-51   RADIATION DIFFUSION
Origin: NBS, Section 4.8
Sponsor: Armed Forces Special Weapons Project
Manager: A. Futterman
Full task description: Apr-June 1953 issue, p. 57 (Neutron Diffusion III)
Status: Continued. The response functions for several sodium iodide
   crystals ($\frac{1}{2}$"x$\frac{1}{2}$" to 5"x9") have been obtained over an energy range .279
   to 4.45 Mev. by means of random sampling techniques. Approximately
   50,000 photon histories have been generated to date.
      Monte Carlo methods are being applied to investigate photon scattering
   in non-homogeneous media. Present interest is being fixed on photon
   crossing the plane interface of two media with different densities.

1102-40-5126/54-4   HIGH TEMPERATURE PROPERTIES OF WATER
Origin: NBS, Division 3.2
Sponsor: Bureau of Ordnance, USN
Managers: J. Wegstein, J. Cooper
Full task description: Oct-Dec 1953 issue, p. 41
Status: Completed. The calculations of thermal functions for singly
   ionized O_{VIII} and H were completed for 50 points in the temperature
   range 10^3 to 10^8 degrees Kelvin.
      The code was modified to compute the same functions in the same range
   for all of the other polyelectric ions of oxygen except O^+ using a
   modified formulation developed by H. Wooley. The computation for these
   thermal functions was completed. In addition all of the polyelectric
   ionic species for another element were completed using the same procedure.

3711-60-0009/54-7   DISTRIBUTION OF NORMAL MODES OF VIBRATION OF CUBIC
                         LATTICES, II
Origin: NBS, Division 30
Manager: W. F. Cahill
Full task description: Oct-Dec 1953 issue, p. 42
Status: Completed. The distribution of normal modes of vibration in
   cubic lattices for various crystal structures were computed using a
   48x48x48 grid.

3711-60-0009/54-8   NORMAL VIBRATIONS IN MOLECULES
Origin: NBS, Section 3.2
Manager: M. Newman
Full task description: Oct-Dec 1953 issue, p. 42
Status: Completed. A symmetric matrix was calculated, and its eigenvalues
   and eigenvectors were computed. The results have been transmitted to
   the sponsor.

3711-60-0009/54-11   COMPUTATION OF VIBRATION MODES AND FREQUENCIES
Origin: NBS, Section 6.4
Managers: I. Rhodes, W. F. Cahill
Full task description: Oct-Dec 1953 issue, p. 42
Status: Continued. The first five symmetric modes for a fourth aircraft
   structure (using 31 mass points) were computed. Also, the first five
   antisymmetric modes for three aircraft structures (using 26 mass points)
   were computed.
Publication: "Computation of vibration modes and frequencies on SEAC,"
   by W. F. Cahill and S. Levy (NBS Section 6.4); to appear in the Journal
   of the Institute of Aeronautical Sciences.

1102-40-5126/54-13   AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: New York Quartermaster Procurement Agency
Manager: H. Bremer
Full task description: Oct-Dec 1953 issue, p. 43
Status: Continued. At the present time contract awards are being
   evaluated at the rate of about two per week using UNIVAC routines. The
   UNIVAC codes that are used for this work were written by M. DeCarlo-Cottone
   in the Office of the Directorate of Management Analysis, U.S.A.F.
   Computations on SEAC will be resumed as soon as the machine is available
   (see p. 32).

1102-40-5126/54-14   FLOW IN SUPERSONIC NOZZLE
<u>Origin and Sponsor</u>: National Advisory Committee for Aeronautics,
                        Langley Field, Va.
<u>Managers</u>: E. Marden, G. Hawkins
<u>Full task description</u>: Oct-Dec 1953 issue, p. 44
<u>Status</u>: Completed. The boundary contours of supersonic nozzles have
  been calculated for given Mach numbers. The increment used in the
  angle of expansion between successive Mach lines was .25 degree.

3711-60-0009/54-17   DEPOLYMERIZATION
<u>Origin</u>: NBS, Section 7.6
<u>Manager</u>: J. Bram
<u>Full task description</u>: Oct-Dec 1953 issue, p. 44
<u>Status</u>: Continued. The coding for SEAC has been completed and is ready
  to be code-checked.

3711-60-0009/54-19   ENERGY LEVELS OF COMPLEX ATOMS
<u>Origin</u>: NBS, Section 4.1
<u>Manager</u>: W. Hall
<u>Full task description</u>: Jan-Mar 1954 issue, p. 41
<u>Status</u>: Inactive. For status to date, see July-Sept 1954 issue, page 14.

1102-40-5126/54-20   LOW TEMPERATURE PROPERTIES OF BORON COMPOUNDS
<u>Origin and Sponsor</u>: NBS, Section 3.2
<u>Manager</u>: J. Wegstein
<u>Full task description</u>: Jan-Mar 1954 issue, p. 41
<u>Status</u>: Inactive. For status to date, see Jan-Mar 1954 issue, page 41.

1102-40-5126/54-21   EXPERIMENTAL PROGRAM FOR MULTIPLE INPUT AND OUTPUT
<u>Origin and Sponsor</u>: NBS, Section 12.3
<u>Manager</u>: I. Rhodes
<u>Full task description</u>: Jan-Mar 1954 issue, p. 42
<u>Status</u>: Continued. Further studies are being made in the application
  to chemical warfare.

3711-60-0009/54-22   ENERGY DISTRIBUTIONS ON OPTICAL IMAGE
<u>Origin</u>: NBS, Section 2.2
<u>Manager</u>: L. S. Joel
<u>Full task description</u>: Jan-Mar 1954 issue, p. 43
<u>Status</u>: Continued. Studies are being undertaken to improve the speed
  and precision of the integrations.

1102-40-5126/54-24   SOLUTION OF THE DIFFUSION EQUATION FOR ELECTRONS
<u>Origin and Sponsor</u>: NBS, Division 4
<u>Manager</u>: A. Futterman
<u>Full task description</u>: Jan-Mar 1954 issue, p. 43
<u>Status</u>: Completed. The results have been sent to the sponsor.

1102-40-5126/54-25   DEFLECTED RADIATION FROM AN INFINITE LAMBERT PLANE
<u>Origin and Sponsor</u>: Armed Forces Special Weapons Group
<u>Managers</u>: P. Henrici, L. S. Joel
<u>Full task description</u>: Jan-Mar 1954 issue, p. 44
<u>Status</u>: Inactive. For status to date, see July-Sept 1954 issue, page 14.

3711-60-0009/54-27   EMF OF CELLS AT HIGH TEMPERATURE
Origin: NBS, Section 1.8
Managers: E. Marden, G. Hawkins
Full task description: Jan-Mar 1954 issue, p. 45
Status: Continued. Electromotive forces for several additional compounds
  were computed.

3711-60-0009/54-28   LOVIBOND NETWORK FOR CIE SOURCE A
Origin: NBS, Section 2.1
Managers: E. Marden, G. Hawkins
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. Changes were made in the SEAC code to recompute the
  network for CIE sources A and C using new data supplied from England.

3711-60-0009/54-30   SPECTRUM ANALYSIS
Origin: NBS, Division 4
Manager: S. Prusch
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. Approximately 15 differences for Tc I were used.
  A table of arc and spark spectra of ruthenium is being prepared for
  publication. Differences between all known and predicted levels were
  used to search for pairs of lines which confirm known levels and predict
  new levels.
    A rectangular array of known levels and corresponding pairs of lines
  was prepared for Ru I.

1102-40-5126/54-33   AIR CONDITIONING IN UNDERGROUND STRUCTURES
Origin and Sponsor: NBS, Section 10.3
Manager: S. Tsingou
Full task description: Jan-Mar 1954 issue, p. 47
Status: Continued. Results for aG=2,3,4,5, and t/a^2=.05,.1,.2,.3,.4,.5,
  1,2,5,10,30,40 have been computed with the supplementary code.

1102-40-5126/54-34   EQUILIBRIUM CALCULATIONS FOR WATER
Origin and Sponsor: Naval Ordnance Laboratory
Manager: J. Wegstein
Full task description: Apr-June 1954 issue, p. 38
Status: Continued. Convergence was obtained in the iterative solution
  of 88 sets of 46 simultaneous transcendental equations. The results
  were transmitted to the sponsor.

1102-40-5126/54-35   THERMAL STRESSES IN STRUCTURES
Origin and Sponsor: NBS, Section 6.4
Manager: W. Hall
Full task description: Apr-June 1954 issue, p. 38
Status: Continued. The code was modified to change boundary conditions,
  and it will be run as soon as machine time becomes available.

3711-60-0009/54-36   VELOCITY OF LIGHT
Origin: NBS, Section 11.3
Manager: P. Rabinowitz
Full task description: Jan-Mar 1954 issue, p. 47
Status: Continued. Six polynomials were fitted to data submitted by
  sponsor. At the same time the experimental errors were computed.

3711-60-0009/54-38   COMPRESSIBILITY FACTORS OF DRY AIR
Origin: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Continued. Several test cases have been run in checking the code.

1102-40-5126/55-39   MOLECULAR VIBRATIONS
Origin and Sponsor: NBS, Section 3.2
Manager: K. Goldberg
Full task description: July-Sept 1954 issue, p. 16
Status: Continued. The routine for obtaining the eigenvalues of a
    symmetric matrix up to order 30, internally and floating, using the
    "Givens Variation" on the Jacobi method, was put into production in
    the month before SEAC was moved. It takes five minutes to find all
    the eigenvalues of a matrix of order 12.

1102-40-5126/54-41   FREQUENCY CORRELATION
Origin and Sponsor: NBS, Division 6
Manager: L.S. Joel
Full task description: Apr-June 1954 issue, p. 39
Status: Completed. The results have been sent to the sponsor.

1102-40-5126/54-42   RESOLUTION CORRECTION FOR SCINTILLATION SPECTROMETER
Origin and Sponsor: NBS, Section 4.13
Manager: M. Newman
Full task description: Apr-June 1954 issue, p. 40
Status: Completed. Matrices of orders 9, 10, and 19 were inverted, and
    systems of equations having these matrices as coefficients were solved.
    The results have been transmitted to the sponsor.

1102-40-5126/54-43   CHARACTERISTICS OF CONDUCTING RESISTORS
Origin and Sponsor: NBS, Section 12.1
Manager: B. Gill
Full task description: Apr-June 1954 issue, p. 40
Status: Continued. Sixty cases were run with revisions of former data.

1102-40-5126/54-45   COMPUTATION OF TRANSIENT HEAT FLOW PROBLEMS
Origin and Sponsor: NBS, Section 10.2
Manager: W. F. Cahill
Full task description: Apr-June 1954 issue, p. 41
Status: Completed. The problem has been run for all the heat diffusivities
    and heat transfer coefficients requested, and the results have been
    transmitted to the sponsor.

1102-40-5126/54-46   STUDY IN OPTIMIZATION OF POWER OUTPUT
Origin: Raytheon Manufacturing Company
Sponsor: U. S. Corps of Engineers
Manager: L. S. Joel
Full task description: Apr-June 1954 issue, p. 41
Status: Inactive. For status to date, see Apr-June 1954 issue, page 41.

1102-40-5126/55-49   WIECHERT DISTRIBUTION FUNCTION
Origin and Sponsor: NBS, Section 7.1
Manager: I. A. Stegun
Full task description: July-Sept 1954 issue, p. 16
Status: Completed. The integrals have been evaluated, and the results
    have been transmitted to the sponsor.

1102-40-5126/55-50   AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: Navy Purchasing Office
Manager: H. Bremer
Full task description: July-Sept 1954 issue, p. 17
Status: Terminated.

1102-40-5126/55-61   ELASTIC CROSS SECTION FOR NEUTRON SCATTERING
Origin and Sponsor: Naval Research Laboratory
Managers: I. A. Stegun, R. Zucker
Objective: To determine values of kR, v_0/E and ξ such that the function
$|f(\theta)|^2$ best approximates given curves.

$$f(\theta) = \frac{1}{2ik} \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\theta)[\eta_\ell - 1]$$

$$-\eta_\ell = \frac{\tan \delta_\ell(kr)[\tan \alpha_\ell(kR)-Z_\ell(pR)]-i[Z_\ell(pR)-\tan \beta_\ell(kR)]}{\tan \delta_\ell(kR)[\tan \alpha_\ell(kR)-Z_\ell(pR)]+i[Z_\ell(pR)-\tan \beta_\ell(kR)]}$$

$$Z_\ell(pR)=1-pR\,\sigma_\ell(pR); \quad pR=kR[1+\frac{v_0}{E}(1+i\xi)]^{\frac{1}{2}}=a+ib$$

where for $\ell=0$, $\sigma_0(a+ib) = \dfrac{\sin 2a - i\,\sinh 2b}{\cosh 2b - \cos 2a}$

$\ell \geq 1$, $x\,\sigma_\ell(x) = \dfrac{x^2 + \ell[x\sigma_{\ell-1}(x)] - \ell^2}{\ell - [x\sigma_{\ell-1}(x)]}$.

P_ℓ is the Legendre function and α_ℓ, β_ℓ and δ_ℓ are defined in terms of
spherical Bessel functions. (Scattering and Radiation from Circular
Cylinder and Spheres - U. S. Navy Dept. O.R.I. July 1946.)
Background: The function arises in the study of the scattering of neutrons.
  The functions to be calculated represent the differential elastic cross
  section for neutron scattering; an experimental determination of this
  function, to be matched against the theoretical results obtained here,
  is under way at the Naval Research Laboratory.
Status: New.

1102-40-5126/55-62   INTEGRALS INVOLVED IN SUPERSONIC FLUTTER
Origin and Sponsor: National Advisory Committee for Aeronautics
Manager: S. Tsingou
Full task description: July-Sept 1954 issue, p. 17
Status: Continued. All the results necessary to complete the tables
  have been computed and printed. A code to difference the results has
  been completed and will be checked as soon as SEAC time becomes
  available.

1102-40-5126/55-63   VIRIAL INTEGRAL INVOLVING A MORSE POTENTIAL FUNCTION
Origin and Sponsor: NBS, Section 3.2
Manager: J. Cooper
Full task description: July-Sept 1954 issue, p. 17
Status: Completed. The integrals listed have been computed for values
  of the parameters corresponding to the diatomic species OH, H_2, O_2,
  O_2^{++}, OH^+, OH^-, H_2^+, O_2^+ for temperatures from 10^5 to 10^7 degrees
  Kelvin. The results have been transmitted to the sponsor.

1102-40-5126/55-64   HIGH-TEMPERATURE THERMODYNAMIC TABLES (FMT)
Origin and Sponsor: NBS, Section 3.2
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 18
Status: Inactive.

3711-60-0009/55-66   RECONSTITUTION OF MONOCHROMATIC LIGHT INTENSITIES
Origin: NBS, Division 30
Manager: H. Bremer
Full task description: July-Sept 1954 issue, p. 18
Status: Continued. Several trials have been run on the SEAC, as explained
  in the previous issue, and the results are being studied.

1102-40-5126/55-67   DOSAGE INTEGRAL
Origin and Sponsor: U. S. Weather Bureau
Manager: W. F. Cahill
Full task description: July-Sept 1954 issue, p. 19
Status: Continued. The dosage integral was evaluated for three sets of
  parameters. A new integral, in which the original integrand was
  multiplied by $[(d-ut-x)^2+y^2+x^2]$, was evaluated for one set of parameters.

3711-60-0009/55-69   THEORY OF DIELECTRIC RELAXATION
Origin: NBS, Section 7.6
Manager: K. Goldberg
Full task description: July-Sept 1954 issue, p. 19
Status: Continued. No new results on the distinctness of the roots have
  been obtained, but work towards this end continues.

1102-40-5126/55-70   BALL BEARING FATIGUE DATA
Origin: NBS, Section 11.3
Sponsor: American Standards Association
Manager: A. Futterman
Objective: To compute

$$(1) \quad T_j^{(k)} = \sum_{i=1}^{nj} w_{ij}^{(k)} \ln x_{ij}, \quad k=1,2,3,4; \quad j=1(1)260,$$

and to find e^{T_1}, $1/T_2$, e^{T_3}, e^{T_4}. Here $\{x_{ij}\}$, i=1,2,...,n, represent
260 sets of data submitted by the sponsor under task 1103-40-5145/54-1
(see p.28) and then randomized by SEAC. The $w_{ij}^{(k)}$ are given weights.
   After the results (1) are obtained, the variances $(T_j^{(k)})$ are to be
found by the usual formula for the variance of a sum of independent
variables.
Background: This task furnishes one phase of the computations necessary
  in connection with task 1103-40-5145/54-1 (see Jan-Mar 1954 issue, p.54).
Status: Completed (New). The code was prepared and checked, and the values
  $T_j^{(k)}$ were obtained for the 260 sets of data.

1102-40-5126/55-71   LORAN BASE LINE COMPUTATIONS
Origin and Sponsor: Hydrographic Office, U. S. N.
Manager: D. Jirauch
Objective: To complete baseline extension and skywave correction tables
  for three rates.

Background: Baseline data and skywave correction tables are necessary
 for use with the standard Loran navigation tables to enable navigators
 to determine their positions with the aid of special electronic equipment.
Status: Completed (New). The results have been sent to the sponsor.

1102-40-5126/55-72   MARYLAND INTER-INDUSTRY STUDY
Origin and Sponsor: Office of Scientific Research, ARDC, USAF
Manager: L. S. Joel
Full task description: July-Sept 1954 issue, p. 20
Status: Continued. The matrix (I-AK) where K is scalar was inverted,
 and (I-AK)x=y was solved for five vectors y.

1102-40-5126/55-73   HEAT CONVECTION
Origin: University of Minnesota
Sponsor: Atomic Energy Commission
Manager: G. Hawkins
Objective: To consider the unsteady state performance of a cooling system
 having the form of a simple natural circulation loop, where boiling
 occurs in the hot portion of the loop. The goal of the investigation
 is the development of a mathematical model which conforms to transient
 one-phase or two-phase flow and which uses partial differential equations
 that can be solved numerically.
Background: Natural circulation loops are taking an added importance as
 the use of atomic reactors for power generation increases. The analysis
 of transient flows occurring in a simple natural circulation loop where
 no boiling is permitted to occur has been successfully treated, using
 a system of difference equations which were developed from the equation
 of motion, the continuity equation, and the energy equation. The
 analysis is now being expanded to include the case in which some boiling
 occurs in the hot portion of the loop.
Status: New.

1102-40-5126/55-77   LONGITUDINAL WAVES IN A STRING
Origin and Sponsor: NBS, Section 7.2
Manager: J. Bram
Objective: To evaluate the expression

$$P(\xi,\tau) = e^{-\xi/2}(\tau-\xi) + \frac{i\xi}{2} \int_{\xi}^{\tau} (\tau-t)\, e^{-t/2} \cdot \frac{J_1(\frac{i}{2}\sqrt{t^2-\xi^2})}{\sqrt{t^2-\xi^2}}\, dt$$

for various values of the arguments ξ,τ with $\xi<\tau$.
Background: The above expression is the solution of the polygraph line
 equation

$$P_{\xi\xi} - P_{\tau\tau} - P_{\tau} = 0$$

under particular boundary and initial conditions. This equation
describes the longitudinal waves that are set up in a semi-infinite
string, made of an elastic, viscous material, when one end begins
moving at a uniform velocity from an initial position of rest.
Status: Completed (New). The expression $P(\xi,\tau)$ was evaluated on SEAC
 for the various values of the arguments requested, and the results
 were transmitted to the sponsor.

1102-40-5126/55-78   NEUTRON TRANSPORT
Origin and Sponsor: Atomic Energy Commission, New York
Manager: S. Tsingou
Objective: To solve the integral equations

$$(1) \quad F_\ell(u)D_{n\ell}(u) = \int_u^M D_{n\ell}(u') \, P_\ell \left\{ e^{\frac{1}{2}(u-u')} \right\} du' + \alpha G(u)\delta_{n0}\,\delta_{\ell 0}$$

$$+ \frac{1}{2\ell+1} \frac{\alpha_0}{\mathcal{E}_h(u)} \left\{ (\ell+1) \, D_{n-1,\ell+1}(u) + D_{n-1,\ell-1}(u) \right\},$$

where $G(u)$ and $F_\ell(u)$ are given,

$$(2) \quad F_\ell(u) \, D_{n\ell}(u) = \int_u^M D_{n\ell}(u') \, P_\ell \left\{ e^{\frac{1}{2}(u-u')} \right\} du' - B_\ell(u)D_{n\ell}(u+h)$$

$$+ \alpha G(u) + \frac{1}{(2\ell+1)} \frac{\alpha_0}{\mathcal{E}_h(u)} \left\{ (\ell+1)D_{n-1,\ell+1}(u) + \ell D_{n-1,\ell-1}(u) \right\}$$

$B_\ell(u)$ and $F_\ell(u)$ are given.
Background: These equations arise through transformation of the neutron
  transport equation by the moments method. Scattering from all elements
  other than hydrogen is calculated by assuming in (1) a zero order and
  in (2) a first order expansion of the associated scattering integrals
  in powers of the reciprocal of the atomic mass. For further details
  see, "A solution of the neutron transport equation," by J. Certaine,
  NYO 3081 (Nuclear Development Associates 15C-43).
Status: New. Computations were performed for approximately 100 combinations
  of values of the parameters and were transmitted to the sponsor.

1102-40-5126/55-79   COMPLETE DEGRADATION IN THE NEUTRON
Origin and Sponsor: Atomic Energy Commission, New York
Manager: A. Futterman
Objective: To compute

$$\psi_{pk} = \sum_{j=0}^p P_{pj}^k \, G_{jk}$$

for $0 \leqslant p < P$, $0 \leqslant k \leqslant N+1 = [\frac{1}{h} \ln(\frac{1+m}{1-m})^2]+1$, where the quantities
P_{pj} and G_{jk} are obtained recursively from relations

$$P_{p+1,j+1}^k = (2p+3) \left(\frac{P_{p-1,j+1}^k}{2p-1} + \frac{A_k \, P_{pj}^k}{j+3} \right)$$

$$P_{p0}^k = \frac{2p+1}{p} \left(\mu_k \, P_{p-1,0}^k - \frac{p-1}{2p-3} \, P_{p-2,0}^k \right)$$

$$G_{jk} = \sum_{n=0}^{\infty} d_{j,j+s} \, h^s, \quad \begin{array}{l} s = 2n \text{ for } j \text{ even} \\ s = 2n+1 \text{ for } j \text{ odd.} \end{array}$$

and $d_{00}=1$, $d_{0n}= \frac{d_{0,n-1}}{n+2}$, $d_{jn} = \frac{j+2}{(n+1)(n+2)} \sum_{\lambda=0}^{n-j} \frac{n+1-\lambda}{\lambda!} d_{j-1,n-1-\lambda}$

$$\text{for } j > 0.$$

The quantities A_k and μ_k are functions of the parameters h and m.

The calculations are to be done for assigned values of these parameters.

Background: The matrix $\psi_0 = (\psi_{pk})$ computed above is the initial stage for a new treatment of the neutron transport equation. The elements ψ_{pk} have the property that if $f(u')$ is a polygonal function with values f_k at $u'_k = u + kh$, and $u' = u + 2\ln(1+m) - \ln(1 + 2m\mu + m^2)$, then

$$\frac{2p+1}{2} \int_{-1}^{1} f(u') P_p(\mu) d\mu = \sum_{k=0}^{N+1} f_k e^{-kh} \psi_{pk}.$$

Status: New. The code was revised and tested, and several runs made. Continued runs will be made as soon as SEAC is available.

1102-40-5126/55-80   ALPHA-ANALYSIS
Origin and Sponsor: Atomic Energy Commission, New York
Manager: W. G. Hall
Objective: To compute the function

$$F(r,u;\alpha) = e^{-2r} \sum_{n=0}^{N} D_{n0}(u) \left(\frac{\alpha}{\alpha_0}\right)^{n+1} Q_n^N (\alpha r)$$

where α_0 is a previously determined constant, Q_n^N is a polynomial of degree N, and the coefficients D_{n0} have been calculated in the neutron transport problem (task 1102-40-5126/55-78). Calculations are to be done for assigned values of the parameters α, r and u.

Background: It is desired to compute a function with quasi-exponential behavior whose first N+1 even moments are given by $D_{n0}(u)$. Specifically, this procedure is used to reconstruct the point isotropic neutron flux density from the previously computed moments D_{n0}.

Status: New. A number of cases were run, and the results were sent to the sponsor.

3711-60-0009/55-81   COMBINING TESTS FOR SIGNIFICANCE
Origin: NBS, Section 11.3
Manager: L. S. Joel
Objective: To compute tables of the power function for combining two independent tests of significance when the underlying distribution of both tests is a variance ratio distribution.

Background: In the analysis of incomplete block designs it has recently been found that it is possible to obtain two independent tests of significance for testing the null hypothesis that all treatment effects are the same. Since the power of the individual tests are different, both tests of significance are combined into a single test by weighting each test in proportion to its power.

Status: New. The code is being prepared. Test runs will be made as soon as SEAC is available.

3711-60-0009/55-89   TEMPERATURE DISTRIBUTION IN SOLID WINGS HEATED
                      AERODYNAMICALLY
Origin: NBS, Section 6.4
Manager: P. Davis, W. F. Cahill
Objective: To determine the temperature distributions in solid wings heated aerodynamically for a range of possible heating environments assuming heat flow in thickness directions only.

Background: The study will show the extent to which it is necessary to
   know the flight conditions in order to predict the operating range of
   a solid wing.
Status: New.

3711-60-0009/55-90   STUDY OF A DIFFERENCE EQUATION ARISING IN STATISTICAL
                                MECHANICS
Origin: NBS, Section 3.2
Manager: M. Newman
Objective: To study the solutions of the difference equation

$$(1) \quad g_{n+2} + a g_{n+1} + b g_n + c g_{n-1} + d g_{n-2} = e \lambda^n g_n, \quad n \geq 3, \quad |\lambda| < 1,$$

   which arises in the study of counting problems of the "Ising" type.
   The problem differs from other problems of this type in that the quantities
   g_1, g_2, g_3, g_4 are not arbitrary but depend on the solution. Thus a
   circular type of reasoning is involved.
Background: The problem is of application in the theory of ferromagnetism,
   order-disorder problems, statistical mechanics, etc.
Status: New. The solution of the difference equation has been obtained
   as a certain rational function of several hyper-elliptic functions.
   Further study is under way.

3711-60-0009/55-91   COMPUTATION OF VIRIAL COEFFICIENTS
Origin: NBS, Section 3.2
Manager: P. Davis
Objective: To calculate

$$\beta_3 = \int f_{43} \, f_{32} \, f_{21} \, f_{41} \, (\tfrac{1}{2} + f_{31} + \tfrac{1}{6} f_{31} f_{42}) \, d\vec{r}_2 \, d\vec{r}_3 \, d\vec{r}_4,$$

   where

$$f_{ij} = \exp\left[-\beta V(|\vec{r}_i - \vec{r}_j|)\right] - 1$$

   and

$$V(r) = 4 (r^{-12} - r^{-6}), \quad \vec{r}_1 \equiv 0.$$

   Since r_i are vectors in three-dimensional space, β_3 is a nine-fold
   integral. It is intended to evaluate it by means of the Monte Carlo
   method.
Background: This is the fourth virial coefficient of a Lennard-Jones gas.
Status: New. The fourth virial coefficient of a Lennard-Jones gas
   involves a 9-fold integral and the possibility of evaluating it by
   Monte Carlo methods is being studied. Preliminary tests to gain
   experience with the method were begun on the computation of the volume
   of an n-dimensional sphere. Values have been obtained for n=2,3,4,5,6,9
   and indicate the need for more tests.

III. STATISTICAL ENGINEERING LABORATORY
(Section 11.3)

1. Fundamental Research in Mathematical Statistics

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-10-1107/49-1a

Origin: NBS   Authorized 1/9/49
Manager: L. S. Deming
Full task description: Apr-June 1949 issue, p. 75

      Status: CONTINUED. For a description of the continuing activity
on this task, see the Jan-Mar 1954 issue, page 49.

MANUAL ON FITTING STRAIGHT LINES
Task 1103-10-1107/50-2

Origin: NBS   Authorized 3/1/50
Manager: F. S. Acton
Full task description: Jan-Mar 1950 issue, p. 42

      Status: CONTINUED. The manuscript is currently in the hands of
technical advisors, for comment.

TABLES TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-10-1107/51-1

Origin: NBS   Authorized 7/1/50
Managers: C. Eisenhart, L. S. Deming
Full task description: July-Sept 1950 issue, p. 57

      Status: INACTIVE. For status to date see July-Sept 1952 issue,
page 64.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-10-1107/51-2

Origin: NBS   Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. For a description of the continuing activity
on this task, see July-Sept 1954 issue, page 21.

Publications:
(1) Some applications of extreme-value theory. E. J. Gumbel and
    J. Lieblein. Am. Stat. $\underline{8}$, 14-17 (Dec. 1954).
(2) Two early papers on the relation between extreme values and tensile
    strength. J. Lieblein. Biometrika $\underline{41}$, Pts. 3 and 4, 559-560
    (Dec. 1954).
(3) Bounds on a distribution function that are functions of moments to
    order four. M. Zelen. J. Res. NBS $\underline{53}$, 377-381 (Dec. 1954).
(4) Time-discrete stochastic processes in arbitrary sets, with applications
    to processes with absorbing regions and to the problem of loops in
    Markoff chains. D. van Dantzig. Accepted for publication (in French)
    in Annales de l'Institut Henri Poincaré (Paris).
(5) Inequalities for probabilities associated with the multivariate normal
    distribution. I. R. Savage. Submitted to a technical journal.
(6) On the variances and covariances of order statistics from the Weibull
    distribution. J. Lieblein. Submitted to a technical journal.

PROCEDURES OF NON-PARAMETRIC STATISTICS
Task 1103-10-1107/52-2

Origin: NBS   Authorized 9/17/51
Manager: I. R. Savage
Full task description: July-Sept 1951 issue, p. 66

Status: INACTIVE. For the latest report on this task see
July-Sept 1954 issue, page 22.

Publications:
(1) Easily used simultaneous confidence limits for a line. W. S. Connor.
    Submitted to a technical journal.
(2) Contributions to the theory of rank order statistics. I. R. Savage.
    Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-10-1107/53-1

Origin: NBS   Authorized 10/15/52
Manager: W. S. Connor
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. Work is continuing on the analysis of
covariance applied to incomplete block designs. The general inter-block
analysis has been derived and seems to be quite useful as it gives
additional estimates not only for the treatments but for the unknown
regression coefficients as well. Another aspect of the covariance

investigation is development of the analysis for observations having the underlying mathematical model

$$y_{ij} = \mu + t_i + b_j + \sum_{j=1}^{b} \beta_j x_{ij} + \epsilon_{ij} \quad (i=1,2,\ldots,v; \quad j=1,2,\ldots,b)$$

where μ, t_i, b_j and β_j are fixed constants and ϵ_{ij} are independent identically distributed normal variates with $E(\epsilon_{ij})=0$, $\mathrm{var}(\epsilon_{ij})= \sigma^2$. This model differs from the usual covariance model in that it allows for the existence of different regression coefficients for each block.

Publications:
(1) New experimental designs for paired observations. W. J. Youden and W. S. Connor. J. Res. NBS 53, 191-196 (Sept. 1954).
(2) Partially balanced incomplete block designs with two associate classes and two treatments per block. W. H. Clatworthy. Accepted for publication in the Journal of Research of the NBS.
(3) On the enumeration of partially balanced designs with two associate classes. W. H. Clatworthy. Submitted to a technical journal.
(4) Partially replicated Latin squares. W. J. Youden and J. S. Hunter; submitted to a technical journal.
(5) Some fractional factorial arrangements for factors at two levels. W. H. Clatworthy, W. S. Connor, and M. Zelen. In manuscript.

2. Applied Research in Mathematical Statistics

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS   Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. W. H. Clatworthy and W. J. Youden are collaborating with D. C. Schubert (Section 4.5, Electron Physics) on the statistical aspects of his study of magnetron space charge density. Repeat determinations of the several quantities of interest display variability and consequently require statistical analysis and interpretation. The statistical parts of the magnetron space charge experiment involved (1) an investigation of drift in the experimental equipment, and (2) the determination of the least squares equation of the electrostatic field within the magnetron by use of orthogonal polynomials.
In the investigation of drift in the experimental apparatus two approaches were used. In the first approach statistical techniques were applied to test whether different "runs" were significantly different. In the second approach the data were recorded according to a predetermined schedule that permits estimation of the trend, provided it exists, and adjustment for it before proceeding with the curve fitting.

Publications:
(1) Acceptance sampling of electroplated articles. J. M. Cameron and F. Ogburn (NBS Section 5.6). Proc. Am. Electroplaters Soc. 41, 19-22 (1954); also appeared in Plating 41, 43-46 (Jan. 1954).
(2) Instrumental drift. W. J. Youden. Science 120, 627-631 (Oct. 22, 1954).

STATISTICAL ASPECTS OF NBS ADMINISTRATIVE OPERATIONS
Task 3737-60-0002/52-1

Origin: NBS   Authorized 10/1/51
Manager: C. Eisenhart
Full task description: Oct-Dec 1951 issue, p. 56

     Status: INACTIVE. For status to date see July-Sept 1952 issue,
page 68.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

     Status: CONTINUED. A program of sampling of production of ship
steel to give information on sources of variability in manufacturing
was devised. Methods of selection of specimens for v-notch tests are
being investigated.

RESEARCH IN APPLICATIONS OF MATHEMATICAL STATISTICS TO PROBLEMS OF THE CHEMICAL CORPS
Task 1103-40-5118/52-1

Origin and Sponsor: Biological Laboratories,   Authorized 10/1/51
                    Chemical Corps, Dept. of the Army
Manager: C. Eisenhart
Full task description: Oct-Dec 1951 issue, p. 57

     Status: CONTINUED. Work was continued on methods of programming
analysis of variance computations for large scale computers.

STATISTICAL ANALYSIS AND DESIGN OF EXPERIMENTS FOR THE U. S. GEOLOGICAL SURVEY
Task 1103-40-5140/54-1

Origin and Sponsor: U. S. Geological Survey,   Authorized 10/9/53
                     Department of Interior
Managers: C. Eisenhart, W. J. Youden
Full task description: Oct-Dec 1953 issue, p. 50

     Status: CONTINUED. Studies are continuing on data on the variation
in lithology of collections of pebbles taken at different sites, and
the variations of samples taken at the same site. Work was done for
L. R. Stieff and T. W. Stern (of the Geochemistry and Petrology Branch,
U.S.G.S. Naval Gun Factory Laboratory, Washington, D. C.) on statistical
aspects of a problem on radioactive emission. Conferences were held with
F. Flanigan (of the Geochemistry and Petrology Branch, U.S.G.S. Naval
Gun Factory Laboratory, Washington, D. C.) on evaluation of precision
of three types of sample dividers, and with O. Raup (of the U.S.G.S.
Geologic Studies Group, Grand Junction, Colorado) on his double-sampling
technique for achieving preassigned precision of results in orientation
studies.

STATISTICAL ANALYSIS OF BALL BEARING FATIGUE DATA
Task 1103-40-5145/54-1

Origin and Sponsor: American Standards Association   Authorized 1/13/54
Manager: J. Lieblein
Full task description: Jan-Mar 1954 issue, p. 54

    Status: CONTINUED. Improved estimates for the L_{10} and L_{50} life values were calculated on the SEAC for all the sets of data. These new estimates served as a basis for a series of analyses designed to obtain values for the unknown parameters f_c, a_1, a_2, a_3 in the life equation

$$L = \left[\frac{f_c (i \cos \alpha)^{a_1} \, Z^{a_2} \, D^{a_3}}{p} \right]^p .$$

Most of the results from these analyses have been obtained and it is expected that all results will be known shortly.

MANUAL ON EXPERIMENTAL STATISTICS
FOR ORDNANCE ENGINEERS
Task 1103-40-5146/55-93

Origin and Sponsor: Office of Ordnance Research   Authorized 12/29/54
Manager: C. Eisenhart

    Objective: To prepare a section on Experimental Statistics for the Ordnance Engineers Handbook.

    Background: The Office of Ordnance Research is preparing an Ordnance Engineering Handbook. An important section of this handbook will be the section on Experimental Statistics in which it is proposed to include descriptions, and criteria for the selection of statistical techniques useful in the programming and analysis of data from experiments.
    The contents and level at which the manual should be written should be based on a survey of the current practices and needs of Ordnance engineers and should have realistic examples from the field. No similar manual is in existence and the available handbooks on statistics do not have criteria to assist the non-statistican in the selection of the appropriate methods and on this account their usefulness is seriously limited. In addition nearly all presentations of statistical methods in the literature are directed towards the needs of the agricultural and biological sciences and there is a real need for translation of the concepts and methods into the language of the physical sciences.
    The Statistical Engineering Laboratory was requested to prepare this section on Experimental Statistics because there is no comparable statistical group working in the field of application of statistics in the physical sciences.

    Status: NEW.

IV. MATHEMATICAL PHYSICS SECTION

(Section 11.4)

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-10-1115/55-57

Origin and Sponsor: NBS   Authorized 9/1/54
Manager: E. W. Cannon
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. E. W. Cannon continued exploratory investigations in the structure of high polymers.

P. Henrici completed a paper on the generalization of the classical addition theorems for Legendre functions and Gegenbauer polynomials. A study of the application of the results to new potential problems involving tores and spheres is under way.

P. Henrici also wrote a note on the remainder term in the expression of the error integral for complex values of the argument. It was shown that the remainder can be made smaller in magnitude if the last considered term of the asymptotic formula is multiplied by a suitable complex constant, which is given explicitly.

Publications:
(1) Addition theorems for general Legendre and Gegenbauer functions. P. Henrici. Submitted to a technical journal.
(2) Kleine Bemerkung in asymptotischen Entivichlung des Fehlerintegrals. P. Henrici. In manuscript.

RESEARCH IN MECHANICS OF CONTINUA
Task 1104-10-5160/54-23

Origin: NBS   Authorized 12/29/53
Sponsor: Office of Naval Research, USN
Manager: R. F. Dressler
Full task description: Oct-Dec 1953 issue, p. 27

Status: CONTINUED. R. F. Dressler has begun the study of the behavior in rarefied regions of the local sound speed of a polytropic gas in unsteady flow, subject to frictional dissipation and entropy increase. Methods used in a previous study are not applicable here to domains neighboring a vacuum, since energy and momentum equations are singular on such wave fronts. It is hoped that a modified type of boundary layer treatment will reveal the implications of the dynamical model used for these low density regions.

The manuscript on "Entropy changes in rarefaction waves" by R. F. Dressler has been rewritten.

M. Abramowitz has completed a study of heat convection in laminar flow. The usual Graetz assumption is not used, i.e. assuming heat conduction in the axial direction to be negligible. Results are obtained

for small and large ranges of a significant parameter.
        M. Abramowitz and I. Stegun have completed a study concerning
some solutions of the Coulomb wave equation, and a manuscript has been
prepared.

        Publications:
(1) Comparison of theories and experiments for the hydraulic dam-break
    wave. R. F. Dressler. To appear in the Proceedings of the Tenth
    General Assembly of the International Union of Geodesy and Geophysics,
    held in Rome, Italy, September 1954.
(2) Turbulent flow in shock tubes of varying cross section. R. F. Dressler.
    J. Res. NBS 53, 253-260 (Oct. 1954), RP2541.
(3) Entropy changes in the equations for rarefaction waves. R. F. Dressler.
    In manuscript.
(4) On helical springs of finite thickness. P. Henrici. To appear in
    the Quarterly of Applied Mathematics.
(5) On rotational viscous flow through a tube. P. Henrici. To appear in
    Zeitschrift für angewandte Mathematik und Physik.
(6) On the design of two-dimensional supersonic nozzles. B. Chaix(Federal
    Institute of Technology, Zurich, Switzerland) and P. Henrici. In
    manuscript.
(7) Generation of solutions of the Coulomb wave equation. M. Abramowitz
    and I. Stegun. In manuscript.
(8) Forced heat convection in laminar flow through a tube. M. Abramowitz.
    In manuscript.

RESEARCH IN ELECTROMAGNETIC THEORY
Task 1104-10-5160/54-47

Origin and Sponsor: Diamond Ordnance Fuze Laboratory,   Authorized 6/29/54
                      Department of the Army   Revised   9/29/54
Manager: F. Oberhettinger
Full task description: July-Sept 1954 issue, p. 28

        Status: CONTINUED. F. Oberhettinger continued work on the problem
of diffraction of a wave field generated by a dipole with arbitrary
orientation on an ideally conducting half-plane.

RESEARCH IN MATHEMATICAL ELASTICITY
Task 1104-10-5160/55-85

Origin and Sponsor: Office of Scientific Research;   Authorized 12/27/54
                    ARDC, USAF
Manager: R. F. Dressler

        Objective: To carry on research in the fields of elastic equilibrium
and vibrations, including investigations of a basic research nature and
solution of problems of a more specific character. Analytical approaches are
envisaged for the basic problems, chosen from among such topics as edge-
layers in generalized plane stress and shell theory, and nonlinear
effects in plate bending; whereas the other problems will require convenient
mathematical formulation for use in high-speed computational procedures.
In the latter category it is hoped to obtain specific numerical results
for problems such as vibrations of delta wings, edge layer stress distributions
in bent plates of the more common shapes, and design data for corrugated
diaphragms in aeronautical instruments. Topics mentioned represent a list

already approved by the sponsor, and related or alternative topics may be investigated as subsequent progress dictates.

Background: Earlier studies concerning the role of boundary condition approximations on convergence rapidity using finite differences for plate vibrations are described in M. Abramowitz and W. Cahill, "On the vibration of a square clamped plate" (1953, an NBS Report). This will be amplified and applied to delta wings, and compared with results to be obtained by variational techniques. Edge layers stress computations will be based upon a method described in K. O. Friedrichs and R. F. Dressler, "A boundary layer theory for elastic bending of plates" (Proceedings Eighth International Congress on Mechanics, Istanbul, 1952), which leads to biharmonic and related equations in semi-infinite strips. Plane stress edge-layer effect represents the complementary problems to bent plates, already investigated by the stretching technique. Suitable extensions may also be possible for shell theory and nonlinear plate bending.

Status: NEW.

NATIONAL BUREAU OF STANDARDS EASTERN AUTOMATIC COMPUTER (SEAC)

SEAC continued in productive operation for the first half of the second quarter of the fiscal year 1955 while the new site for the machine was still in preparation. From mid-November through the end of the quarter the entire installation was moved to its new location. By December 23 all of the signal leads had been connected and the signals were being checked. Debugging of the computer proper and the acoustic and Williams memories was well under way and was progressing satisfactorily as of the end of the quarter.*

During the approximately seven weeks that SEAC was in operation its overall efficiency was 81% during scheduled operation, which compared quite favorably with 89% for the first quarter of fiscal year 1955 and was considerably better than the 75% average of the first four years of operation. Scheduled computation time was distributed as follows:

```
Total scheduled time available for computation . . 545 hours
Good operating time. . . . . . . . . . . . . . . . 440   "
Idle-in-order time . . . . . . . . . . . . . . . .   4   "
Time lost during or following malfunction. . . . . 101   "
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In addition, SEAC was utilized for 348 hours of nonscheduled computation (nonscheduled computation being defined as operation without a duty engineer in attendance for machine maintenance) and 182 hours for engineering and preventive maintenance.

The record of SEAC operations from October 1 through November 20 is as follows:

		Hours Used:	
Task No.	Title	Code Checking	Productive Operation
NBS:			
1104/55-55	Research in numerical analysis	11	3
5116/54-9	Supporting research in linear programming	3	
1110/47-2	Tables of Coulomb wave functions		3
1110/50-7	Wave function for lithium		27
1110/52-37	Spheroidal wave functions	2	1
1110/53-52	Internal conversion coefficients	3	51
1110/55-65	Automatic coding	2	
2308/50-13	Ray tracing	3	1
0009/52-44	Calculations for d spacing		1
5126/53-27	Thermodynamic functions		1
5126/53-29	Dynamic behavior of aircraft structures		28
5126/53-51	Radiation diffusion	8	91
5126/54-4	High temperature properties of water	7	105
0009/54-8	Normal vibrations in molecules	3	5
0009/54-15	Matrix reduction	3	6
0009/54-17	Depolymerization	1	1
0009/54-19	Energy levels of complex atoms	2	1
0009/54-22	Energy distribution on optical image		11
5126/54-33	Air conditioning in underground structures		1
0009/54-36	Velocity of light		1
5126/54-43	Characteristics of conducting resistors		7
5126/54-49	Wiechert distributing function	1	27
5125/55-39	Molecular vibrations	3	8

Task No.	Title	Hours Used: Code Checking	Productive Operation
NBS:			
5125/55-63	Morse integrals		1
5126/55-64	High-temperature thermodynamic tables	2	
0009/55-66	Reconstitution of light intensities	2	4
5126/55-70	Ball bearing fatigue data	8	21
5126/55-77	Evaluation of integrals	7	
0009/55-81	Combining tests for significance	1	
	Totals:	69	418
Other:			
1104/55-59	Turbulent atmospheric contamination		5
5126/53-14	Neutron diffusion	3	18
5126/53-45	Theory of games	2	11
5126/54-13	Award of procurement contracts by linear programming	1	45
5126/54-25	Reflected radiation from an infinite Lambert plane	1	3
5126/54-34	Equilibrium calculations for water	8	60
5126/54-40	Flight simulator	1	14
5126/54-44	Flight performance	1	22
5126/55-52	Atmospheric waves	2	9
5126/55-60	Detonator voltage and current oscillograms		3
5126/55-61	Elastic neutron scatterings	1	5
5126/55-62	Integrals involved in supersonic flutter	6	53
5126/55-67	Dosage integrals	1	7
5126/55-72	Maryland inter-industry study		1
5126/55-73	Heat Convection	16	3
5126/55-79	α-Analysis - NDA	.	5
	Totals:	43	264

*Operation of SEAC was resumed on a part-time basis on January 22, 1955.

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Division Lectures

WILKINSON, J. (National Physical Laboratory, Teddington, England)
(1) Linear algebra on the pilot A.C.E. October 19.
(2) Latent roots of matrices. October 26.

MORSE, P. M. (Massachusetts Institute of Technology) Computing needs in operations research. December 3.

Mathematics Seminar

NEWMAN, M. The coefficients of certain modular forms. October 25.

ANTOSIEWICZ, H. Stability of differential equations. November 1.

HENRICI, P. The quotient difference algorithm of H. Rutishauser, and a remark on Aitken's δ^2-method. November 8.

RABINOWITZ, P. Paracompact spaces. November 22.

DAVIS, P. How shall we evaluate multiple integrals? December 6.

HOFFMAN, A. Incidence matrices, linear programming, and combinatorial problems. December 13.

ABRAMOWITZ, M. Mathieu functions. December 20.

Statistical Engineering Seminars

BERGER, M.J. (Nuclear Physics Section, NBS) A study of Gamma ray diffusion by random sampling. November 24.

LUKACS, E. (Office of Naval Research) On the characterization of
populations by properties of suitable statistics. December 16.

Algebra Seminar

(Offered by The American University in cooperation with the National
Bureau of Standards

NEWMAN, M. (1) Commutator subgroups. October 13.
(2) Modular groups. October 27, 29.
(3) Automorphisms of the modular group. November 17, 24.

MARSH, R. (Department of Defense) p-groups. November 3.

GOLDBERG, K. p-adic numbers. November 10.

JOEL, L. Commutators of matrices. December 1.

JACKSON, I. (Bureau of Ships) Commutator subgroups of infinite permutation
matrices. December 8, 15.

Papers and Invited Talks
Presented by Members of the Staff
at Meetings of Outside Organizations

ALT, F. L. Scientific computation on high speed machines. Presented at
a Seminar on Automatic Computation at the Battelle Memorial
Institute, Columbus, Ohio, November 17.

EISENHART, C. (1-3) Statistical engineering at the National Bureau of
Standards. Presented to the Graduate Seminar Group,
Department of Mathematics, Princeton University, October 19;
also at a Seminar at Stanford University, November 3; and at
a Seminar at the University of Chicago, November 7. (4-5)
Scales, scale readings, and information. Presented before
the Central New Jersey Chapter, of the American Statistical
Association, Princeton, N. J., October 19; and at a Seminar
at the Magnolia Petroleum Company, Dallas, Texas, October 29.

FAN, K., O. TAUSSKY, and J. TODD. Discrete analogs of inequalities of
Wirtinger. Presented at the annual meeting of the American
Mathematical Society, Pittsburgh, Pennsylvania, December 27-28.

LIEBLEIN, J. Estimation of 'censored' samples of extreme data. Presented
at the Third Berkeley Symposium on Mathematical Statistics
and Probability, Institute of Mathematical Statistics, Berkeley,
California, December 29.

TAUSSKY, O. Estimation of characteristic values of finite matrices.
Presented at a Physics Seminar at the University College,
London, October 21.

TODD, J.   (1) The theory and practice of the solution of large systems
           of linear equations. Presented at a Physics Seminar at the
           University College, London, October 21. (2) The applied
           mathematics program of the National Bureau of Standards. Presented
           at the Admiral Research Laboratory, Teddington, England, November 1.

YOUDEN, W. J.   (1,2) Principles of experimental design. Presented before
           the Madison Section of the American Chemical Society, Madison,
           Wisconsin, October 4; and at the U. S. Forest Products Laboratory,
           Madison, Wisconsin, October 5. (3-11) Statistical design.
           A series of nine lectures presented before the New York Section,
           American Chemical Society, New York, N. Y., October 6, 13, 20,
           27, November 3, 10, 17, 23, and December 8. (12) Statistical
           methods in the design of experiments and interpretation of data.
           Presented before a meeting of the Association of Research
           Directors, Kearney, N. J., October 21. (13) Analysis with
           experimental data. Presented before the National Printing Ink
           Research Institute, Bethlehem, Pa., October 22. (14) Getting
           the most out of your experiments. Presented before the Sixteenth
           Annual Forum, Packaging Institute, New York, N. Y., October 27.
           (15) Sophisticated chemistry. Presented before the Student
           Chemical Society, Lehigh University, Bethlehem, Pa., December 9.
           (16) The gun problem. Presented at the Princeton Conference,
           American Society for Quality Control, Princeton, N. J., December
           10. (17) Opportunities of mathematics in science. Presented
           before the Science Club, Woodrow Wilson High School, Washington,
           D. C., December 14.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Tables of functions and of zeros of functions. Collected short tables of the Computation Laboratory. NBS Applied Mathematics Series 37. Available from U. S. Government Printing Office, Washington 25, D. C., $2.25.

(2) Tables of the error function and its derivative. NBS Applied Mathematics Series 41. (A reissue of NBS Mathematical Table 8, Tables of probability functions, vol. I.) Available from U. S. Government Printing Office, Washington 25, D. C., $3.25.

(3) Table of salvo kill probabilities for square targets. NBS Applied Mathematics Series 43. Available from U. S. Government Printing Office, Washington 25, D. C., 30 cents.

1.3 Technical Papers

(1) Coulomb wave functions in the transition region. M. Abramowitz and H. Antosiewicz. Phys. Rev. 96, 75-77 (Oct. 1, 1954).

(2) Coulomb wave functions along the transition line. M. Abramowitz and P. Rabinowitz. Phys. Rev. 96, 77-79 (Oct. 1, 1954).

(3) A representation for solutions of analytic systems of linear differential equations. H. A. Antosiewicz and M. Abramowitz. J. Wash. Acad. Sci. 44, 382-384 (Dec. 1954).

(4) A remark on the smoothing problem. H. Antosiewicz and A. J. Hoffman. Management Science 1, 92-95 (Oct. 1954).

(5) Acceptance sampling of electroplated articles. J. M. Cameron and F. Ogburn (NBS Section 5.6). Proc. Am. Electroplaters Soc. 41, 19-22 (1954). Also appeared in Plating 41, 43-46 (Jan. 1954).

(6) A comparison of four national radium standards. Part II: Statistical procedures and survey. W. S. Connor and W. J. Youden. J. Res. NBS 53, 273-275 (Nov. 1954).

(7) New experimental designs for paired observations. W. S. Connor and W. J. Youden. J. Res. NBS 53, 191-196 (Sept. 1954).

(8) Linear functional equations and interpolation series. P. Davis. Pac. J. Math. 4, 503-532 (1954).

(9) Complex biorthogonality for certain sets of polynomials. P. Davis and H. Pollak (Bell Telephone Laboratories). Duke Math. J. 21, 653-668 (Dec. 1954).

(10) On the estimation of quadrature errors for analytic functions. P. Davis and P. Rabinowitz. MTAC 48, 193-203 (Oct. 1954).

(11) Turbulent flow in shock tubes of varying cross-section. R.F.Dressler. J. Res. NBS 53, 253-260 (Oct. 1954), RP2541.

(12) Some applications of extreme-value theory. E. J. Gumbel and J. Lieblein. Amer. Stat. 8, 14-17 (Dec. 1954).

(13) Smooth patterns of production. A. J. Hoffman and W. Jacobs. Management Science 1, 86-91 (1954).

(14) Contractibility and convexity. H. W. Kuhn (Bryn Mawr College). Proc. Am. Math. Soc. 5, 777-779 (Oct. 1954).

(15) On two problems in abstract algebra connected with Horner's rule. A. M. Ostrowski. Studies in Mathematics and Mechanics Presented to Richard von Mises, pp. 40-48 (Academic Press Inc., New York, 1954).

(16) An isoperimetric inequality for closed curves convex in even-dimensional Euclidean space. I. J. Schoenberg. Acta Math. 91, 143-164 (1954).

(17) Generalized commutators of matrices and permutations of factors in a product of three matrices. O. Taussky. Studies in Mathematics and Mechanics presented to Richard von Mises, pp. 67-68 (Academic Press Inc., New York, 1954).

(18) Experiments in the solution of differential equations by Monte Carlo methods. J. Todd. J. Wash. Acad. Sci. 44, 377-381 (Dec. 1954).

(19) The condition of certain matrices, II. J. Todd. Arch. Math. V, 249-257 (1954).

(20) Instrumental drift. W. J. Youden. Science 120, 627-631 (Oct. 22, 1954).

(21) Bounds on a distribution function which are functions of moments to order four. M. Zelen. J. Res. NBS 53, 377-381 (Dec. 1954).

1.4 Reviews, Notes

(1) Note on the circle theorem of hydrodynamics. E. Levin. Qu. App. Math. XII, 315-316 (Oct. 1954).

(2) Two early papers on the relation between extreme values and tensile strength. J. Lieblein. Biometrika 41, 559-560 (Dec. 1954).

(3) Obituary notice on L. F. Richardson. J. Todd. MTAC 8, 242-245 (1954).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION DECEMBER 31, 1954

2.1 Mathematical Tables

(1) Tables of sines and cosines for radian arguments. NBS Applied Mathematics Series 43. (A reissue of NBS Mathematical Table 4.) In press, Government Printing Office.

2.3 Technical Papers

(1) Approximate method for rapid Loran computation. M. Abramowitz, D. H. Call, and J. C. Mathews. Submitted to a technical journal.

(2) A theorem on alternatives for pairs of matrices. H. Antosiewicz.
Accepted for publication in the Pacific Journal of Mathematics.

(3) Asymptotic solution of linear differential equations with a
parameter. H. Antosiewicz. Submitted to a technical journal.

(4) On a certain integral involving Bessel functions. H. Antosiewicz.
Submitted to a technical journal.

(5) On nonlinear differential equations of the second order with
integrable forcing term. H. Antosiewicz. Accepted for publication
in the Journal of the London Mathematical Society.

(6) On the differential equation $\ddot{x} + k(f(x) + g(x)\dot{x})\dot{x} = ke(t)$.
H. A. Antosiewicz. Submitted to a technical journal.

(7) Stable systems of differential equations with integrable forcing
term. H. Antosiewicz. Submitted to a technical journal.

(8) Computation of vibration modes and frequencies on SEAC. W.F.Cahill
and S. Levy (NBS 6.4). Accepted for publication in the Journal
of the Institute of Aeronautical Sciences.

(9) On the enumeration of partially balanced designs with two associate
classes. W. H. Clatworthy. Submitted to a technical journal.

(10) Partially balanced incomplete block designs with two associate
classes and two treatments per block. W. H. Clatworthy. Accepted
for publication in the Journal of Research of the NBS.

(11) Easily used simultaneous confidence limits for a line. W.S.Connor.
Submitted to a technical journal.

(12) Time-discrete stochastic processes in arbitrary sets, with
applications to processes with absorbing regions and to the
problem of loops in Markoff chains. D. van Dantzig. Accepted
for publication (in French) in Annales de l'Institut Henri Poincaré
(Paris).

(13) On a problem in the theory of mechanical quadratures. P. Davis.
Accepted for publication in the Pacific Journal of Mathematics.

(14) A multi-purpose orthonormalizing code and its uses. P. Davis and
P. Rabinowitz. Accepted for publication in the Journal of the
Association for Computing Machinery.

(15) Some sampling results on the power of nonparametric tests against
normal alternatives. W. J. Dixon (University of Oregon) and
D. Teichroew. Submitted to a technical journal.

(16) Comparison of theories and experiments for the hydraulic dam-break
wave. R. F. Dressler. To appear in the Proceedings of the Tenth
General Assembly, of the International Union of Geodesy and
Geophysics, held in Rome, Italy, September 1954.

(17) Heat flow in a fluid with eddying flow. W. H. Durfee (now with
Operations Research Office, Johns Hopkins University). Submitted
to a technical journal.

(18) On the optimal character of the (s,S) policy in inventory theory.
A. Dvoretzky, J. Kiefer, and J. Wolfowitz. Submitted to a
technical journal.

(19) A comparison theorem for eigenvalues of normal matrices. K. Fan. Submitted to a technical journal.

(20) On systems of linear inequalities. K. Fan. Submitted to a technical journal.

(21) Some inequalities concerning positive-definite Hermitian matrices. K. Fan. Accepted for publication in the Proceedings of the Cambridge Philosophical Society.

(22) Some metric inequalities in the space of matrices. K. Fan and A. J. Hoffman. Accepted for publication in the Proceedings of the American Mathematical Society.

(23) Discrete analogs of inequalities of Wirtinger. K. Fan, O. Taussky, and J. Todd. To appear in Monatschefte für Mathematik.

(24) A determinantal inequality. K. Fan and J. Todd. Accepted for publication in the Journal of the London Mathematical Society.

(25) Asymptotic lower bounds for the fundamental frequency of convex membranes. G. E. Forsythe. Accepted for publication in the Pacific Journal of Mathematics.

(26) On the solution of the caterer problem. J. W. Gaddum, A. J. Hoffman, and D. Sokolowsky. To appear in the Logistics Research Quarterly.

(27) An algorithm for solving the transportation problem. A. Gleyzal. Accepted for publication in the Journal of Research of the NBS.

(28) Addition theorems for general Legendre and Gegenbauer functions. P. Henrici. Submitted to a technical journal.

(29) Application of two methods of numerical analysis to the computation of the reflected radiation of a point source. P. Henrici. Accepted for publication in the Journal of the Washington Academy of Sciences.

(30) On certain series expansions involving Whittaker functions and Jacobi polynomials. P. Henrici. Accepted for publication in the Pacific Journal of Mathematics.

(31) On generating functions of the Jacobi polynomials. P. Henrici. Accepted for publication in the Pacific Journal of Mathematics.

(32) On helical springs of finite thickness. P. Henrici. Accepted for publication in the Quarterly of Applied Mathematics.

(33) On rotational viscous flow through a tube. P. Henrici. Accepted for publication in ZAMP.

(34) The number of absolute points of a correlation. A. J. Hoffman, M. Newman, E. G. Straus, and O. Taussky. Submitted to a technical journal.

(35) Indentation pressure of a smooth punch. E. Levin. Accepted for publication in the Quarterly of Applied Mathematics.

(36) Acoustic radiation pressure on a circular disk. H. Levine. To appear in the Proceedings of the Fifth Symposium on Applied Mathematics of the American Mathematical Society, held in Pittsburgh, Pa., June 1952.

(37) On the variances and covariances of order statistics from the Weibull distribution. J. Lieblein. Submitted to a technical journal.

(38) A proof of Hilbert's Nullstellensatz. T. S. Motzkin. Submitted to a technical journal.

(39) The assignment problem. M. Motzkin. To appear in the Proceedings of the American Mathematical Society's Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 1953.

(40) On Fejér sets in linear and spherical spaces. T. S. Motzkin and I. J. Schoenberg. Accepted for publication in Annals of Mathematics.

(41) Pairs of matrices with property L, II. T. S. Motzkin and O. Taussky. Submitted to a technical journal.

(42) Least p-th power polynomials on a real finite point set. T. S. Motzkin and J. L. Walsh. To appear in the Transactions of the American Mathematical Society.

(43) The coefficients of certain modular forms. M. Newman. Submitted to a technical journal.

(44) The diophantine equation $ax^m - by^n = c$. M. Newman. Submitted to a technical journal.

(45) Structure theorems for modular subgroups. M. Newman. Accepted for publication in the Duke Mathematical Journal.

(46) On asymptotic series for functions occurring in the theory of diffraction of waves by a wedge. F. Oberhettinger. Submitted to a technical journal.

(47) On the Lerch zeta function. F. Oberhettinger. Accepted for publication in the Pacific Journal of Mathematics.

(48) Determinanten mit ueberwiegender Hauptdiagonale und die absolute Konvergenz von linearen Iterationsprozessen. A. M. Ostrowski. Submitted to a technical journal.

(49) On the convergence of Gauss' alternating procedure in the method of the least squares, I. A. M. Ostrowski. Submitted to a technical journal.

(50) On the linear iteration procedures for symmetric matrices. A. M. Ostrowski. Accepted for publication in Rendiconti di Matematico, Rome.

(51) On the spectrum of a one parametric family of matrices. A. M. Ostrowski. Accepted for publication in Journal für die reine und angewandte Mathematik.

(52) On Gauss' speeding up device in the theory of single step iteration. A. M. Ostrowski. Submitted to a technical journal.

(53) On absolute convergence of linear iteration processes. A.M.Ostrowski. Submitted to a technical journal.

(54) On spectre of second-order differential operators. D. Ray. Submitted to a technical journal.

(55) On the convergence of asymptotic solutions of linear differential equations. R. M. Redheffer (U.C.L.A.) and W. Wasow. Submitted to a technical journal.

(56) Osculatory interpolation in the complex plane. H. E. Salzer (Department of the Army). Accepted for publication in the Journal of Research of the NBS.

(57) Contributions to the theory of rank order statistics. I. R. Savage. Submitted to a technical journal.

(58) Inequalities for probabilities associated with the multivariate normal distribution. I. R. Savage. Submitted to a technical journal.

(59) Some computational problems in algebraic number theory. O. Taussky. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 1953.

(60) Generation and testing of pseudo-random numbers. O. Taussky and J. Todd. Submitted to a technical journal.

(61) An improved cathode ray tube storage system. R. Thorensen. To appear in the Proceedings of the Western Computer Conference of the AIEE-IRE-ACM held in Los Angeles, Calif., February 4,5,6, 1953.

(62) Motivation for working in numerical analysis. J. Todd. To appear in the Transactions of a Symposium on Applied Mathematics sponsored by the Office of Ordnance Research and held in Chicago, Ill., April 29, 1954.

(63) An extension of a theorem of Dantzig's. C. Tompkins and I. Heller. (George Washington University). Submitted to a technical journal.

(64) A method for the numerical integration of differential equations of second order without explicit first derivatives. R. de Vogelaere. Accepted for the Journal of Research of the NBS.

(65) Discrete approximations to elliptic differential equations. W. Wasow. Accepted for publication in Zeitschrift für angewandte Mathematik und Physik.

(66) On the asymptotic transformation of certain distributions into the normal distribution. W. Wasow. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 26-28, 1953.

(67) An extremum property of sums of eigenvalues. H. Wielandt. Accepted for publication in the Proceedings of the American Mathematical Society.

(68) Error bounds for eigenvalues of symmetric integral equations. H. Wielandt. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 1953.

(69) On eigenvalues of sums of normal matrices. H. Wielandt. Submitted to a technical journal.

(70) Partially replicated latin squares. W. J. Youden and J. S. Hunter. Submitted to a technical journal.

2.5 Miscellaneous Publications

(1) Experiments in the computation of conformal maps. NBS Applied Mathematics Series 42. In press, Government Printing Office.

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.00). Information on calibration services and fees can be found in NBS Circular 483, Testing by the National Bureau of Standards (25 cents). Both are available from the Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.

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