NATIONAL BUREAU OF STANDARDS REPORT

3773

DETERMINATION OF COLOR OF MAXIMUM CONTRAST

By

Deane B. Judd

To

Aerial Reconnaissance Laboratory
Wright Air Development Center
Wright-Patterson Air Force Base
Department of the Air Force

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Photometry and Colorimetry Section
Optics and Metrology Division

To

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PREFACE

This is one of a series of reports prepared under National Bureau of Standards Project No. 0201-20-2325, "Color Reconnaissance Studies". This project is financed by the Aerial Reconnaissance Laboratory. Wright Air Development Center, Wright-Patterson Air Force Base, Ohio (Air Force Contract No. AF 33(616) 52-21); and it is coordinated with Air Force Contract No. AF 33(616)-262 (Suppl. Agreement S3(54-583)) under Dr. Hugh T. O'Neill, O'Neill Associates, Annapolis, Maryland, who supplies all requests for studies, and all specimens and materials for spectrophotometric and colorimetric analyses, except those directly requested or supplied by the Optics Division and others at the Wright Air Development Center.

The present report resulted from one of Dr. O'Neill's work requests, which was assigned an Oral Work Request Serial Number; WADC-24/53.

H. J. Keegan, Project Leader

DETERMINATION OF COLOR

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I. REQUEST

1. A formula for calculating for any given color whose chromaticity coordinates are known, the chromaticity coordinates of that color (producible from photographic film such as Kodachrome of Ansco color) which will give the greatest contrast to the eye (including non-spectral colors).

2. Are two such colors calculated according to this formula properly or conventionally called "Complementary colors".?

3. A formula for calculating how much greater is the contrast between the darker complementary color and black than with its lighter complementary color. Also the name for the darker complementary color and white.

II. INTRODUCTION

The basic information required for problems 1 and 3 is a method of predicting reliably from the fundamental specifications of the two colors what the magnitude of the color contrast or difference, visually appraised, will be. There have been a number of such formulas derived and verified within about a factor of 2 for various specific observing conditions; see formulas 28, 29, 30, 31, 32, 33, 34, and 35 given by Judd (1952) 1 *

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* Figures in brackets indicate the index reference at the end of this report.

Formulas 28 and 29 apply only to color differences involving a change of Munsell hue by 10 steps or less. For problem 1, involving the largest possible color difference between some one color and some member of a

specified color gamut, these formulas are not applicable, but may be replaced for specimens of large angular subtense by the Godlove (1951) [2] formula:

$$\Delta E = \left[2C_1C_2(1 - \cos 3.6° \ \Delta H) + (\Delta C)^2 + (4\Delta V)^2\right]^{1/2} \quad (1)$$

where ΔE is the predicted magnitude of the color difference, visually appraised between two colors specified by Munsell renotations, $H_1\ V_1/C_1$ and $H_2\ V_2/C_2$, $\Delta H = H_1 - H_2$, $\Delta V = V_1 - V_2$, $\Delta C = C_1 - C_2$. The unit for ΔE predicted in this way has the size of one Munsell chroma step or one fourth of a value step. To obtain the prediction in NBS units multiply by 5.0.

The other formulas cited (30 to 35) are applicable to all color differences of whatever size between samples of large angular extent. If the two colors are specified by tristimulus values, X, Y, Z, in the 1931 CIE system, formula 34 is recommended (Adams, 1942 [3] ; Nickerson and Stultz, 1944 [4]).

$$\Delta E = 50\left\{(0.23\ \Delta V_y)^2 + \left[\Delta(V_x - V_y)\right]^2 + \left[0.4\ \Delta(V_z - V_y)\right]^2\right\}^{1/2} \quad (2)$$

where V_x, V_y, V_z are functions, respectively, of X, Y, Z given by Nickerson (1950) and Judd (1952) [5] . The predictions are given in NBS units (Judd, 1939) [6].

If one of the two colors whose contrast must be evaluated subtends an angle of the order of 0.1°, blue-yellow differences evaluated by $0.4\ \Delta\ (V_z - V_y)$ are found to be of negligible importance. This phenomenon is known as small-field tritanopia (Judd, 1949 [7]; Wright, 1949 [8] ; Middleton, 1949 [9]). For this angular condition, the constant, 0.4, should be changed to zero. If, furthermore, the angular extent of one of the colors is of the order of 0.01°, all chromatic differences come to be of negligible importance, and the formula reduces simply to a factor multiplied by ΔV_y. All of these phenomena can be represented by a generalized formula:

$$\Delta E = 50\left\{(k_1\ \Delta V_y)^2 + \left[k_r\ \Delta(V_x - V_y)\right]^2 + \left[k_b\ \Delta(V_z - V_y)\right]^2\right\}^{1/2} \quad (3)$$

where k_1, k_r and k_b are functions of the angular size of the smaller of the two colors. These functions are not precisely known but have values approximating those given in Table 1.

Table 1, Approximate values of the lightness constant, k_l, the red-green constant, k_r, and the blue-yellow constant, k_b, in the generalized Adams chromatic-value formula for size of color difference, visually appreciated.

Size of field, degrees	Lightness constant k_l	Red-green constant k_r	Blue-yellow constant k_b
10	0.23	1.0	0.4
1	.20	.9	.3
0.1	.12	.5	.0
0.01	.02	.0	.0

Problem 1

To solve the problem requires a method of trial and error by means of the applicable formula (1, 2, or 3).

One method would involve computation of the color difference between the given color and a succession of colors chosen from the extremes of the color gamut defined by the three primary dyes of the system being studied (Kodachrome, Ansco color, or whatever special system may be devised). By making perhaps 10 such calculations a color from the ganut not contrasting from the given color by significantly less than the maximum might be found. Similarly, by a succession of applications of this method, the special system yielding a maximum out of a group of special systems might be found.

Another method depends on the property, held by formulas 1, 2, and 3 in common, that a space array of colors is possible such that the distance between the locations of any two colors is proportional to the color difference, E, between them computed by formula. By this method a space diagram of the limiting colors of the gamut being studied would be made, that is, the shell enclosing the colors in this space would be defined by a sufficient number of points. Then, the point within the shell corresponding to the given color would be located in this space diagram, and trial radii extending from this point until they cut the shell would be drawn. The longest such radius would identify the desired color of maximum contrast.

IV. Problem 2

It may now be noted that if the shell had a perfectly spherical shape with the black-white axis passing through the center, the color contrasting maximally from a given non-neutral color would necessarily be an exact complementary; that is, would give by additive mixture in suitable proportions the chromaticity of the source. If, indeed, the shape of the shell is any figure of revolution about the black-white axis of the color solid generated by the applicable formula, this same conclusion would hold, except that more than one complementary color might be found to have maximal contrast. However, there is no three-dye color gamut having such a shell, and it seems, in fact, impossible to imagine how to devise a three-dye system whose gamut would have a shell of such shape. The answer to problem 2 is, therefore, that the color found to be maximally in contrast with any given color will only by chance be an exact complementary of that color. Many such pairs will be found approximately complementary, and some may be found that differ from the complementary by as much as one-half of the hue circuit; that is, are of the same rather than opposite hues. An example of this extreme deviation from complementarism is a reddish gray. The primary red of the three-dye system will usually contrast more strongly with the reddish gray than any other member of the color gamut, and therefore by more than any complementary of the reddish gray.

V. Problem 3

This problem is the easiest of the three. It may be solved by direct use of the applicable formula (1, 2, or 3).

VI. Reference.

[1] D. B. Judd, Color in Business, Science, and Industry, pp. 256 - 265, New York, John Wiley & Sons, 1952

[2] I. H. Godlove, Improved color-difference formula with application to the perceptibility of fadings, J. Optical Soc. Am. **41**, 760; 1951

[3] E. Q. Adams, X-Z planes in the 1931 ICI system of colorimetry, J. Optical Soc. Am. **32**, 168; 1942

[4] D. Nickerson and K. F. Stultz, Color Tolerance Specification, J. Optical Soc. Am. 34, 550; 1944.

[5] D. Nickerson, Tables for use in computing small color differences, Am. Dyestuff Reptr., 39, 541 (Aug. 21, 1950). See also, D. B. Judd, Reference 1 (above), p. 268, and Appendix Tables A, B, and C.

[6] D. B. Judd, Specification of uniform color tolerances for textiles, Textile Research, 9 253, 292; 1939. See also, D. B. Judd, Specification of color tolerances at the National Bureau of Standards, Am. J. Psychol., 52, 418; 1939.

[7] D. B. Judd, Response functions for types of vision according to the Müller theory, J. Research NBS, 42, 1; 1949, RP1946.

[8] W. D. Wright, Les courbes de mélange des couleurs enregistrées sur la fovéa sous de très petits champs visuels, Rev. optique, 28, 174; 1949.

[9] W. E. K. Middleton and M. C. Holmes, The apparent colors of surfaces of small subtense - A preliminary report, J. Optical Soc. Am., 39, 582; 1949.

THE NATIONAL BUREAU OF STANDARDS

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The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

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Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.00). Information on calibration services and fees can be found in NBS Circular 483, Testing by the National Bureau of Standards (25 cents). Both are available from the Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.

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