NATIONAL BUREAU OF STANDARDS REPORT

3487

PROJECTS and PUBLICATIONS

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

A Quarterly Report

April through June 1954

FOR OFFICIAL USE

U. S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

U. S. DEPARTMENT OF COMMERCE
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The scope of activities of the National Bureau of Standards is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section is engaged in specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside of the back cover of this report.

Electricity. Resistance and Reactance Measurements. Electrical Instruments. Magnetic Measurements. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

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Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion.

Mineral Products. Porcelain and Pottery. Glass. Refractories. Enameled Metals. Concreting Materials. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Heating and Air Conditioning. Floor, Roof, and Wall Coverings. Codes and Specifications.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering.

Electronics. Engineering Electronics. Electron Tubes. Electronic Computers. Electronic Instrumentation. Process Technology.

Radio Propagation. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Frequency Utilization Research. Tropospheric Propagation Research. High Frequency Standards. Microwave Standards.

Office of Basic Instrumentation

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NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT

NAML

NBS REPORT

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PROJECTS and PUBLICATIONS

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NATIONAL APPLIED MATHEMATICS LABORATORIES

April through June 1954

NBS

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

NATIONAL APPLIED MATHEMATICS LABORATORIES

April 1 through June 30, 1954

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NATIONAL APPLIED MATHEMATICS LABORATORIES

The Institute for Numerical Analysis, in Los Angeles, California, ceased to exist at the end of the quarter, April-June 1954. However, many of its studies and projects will be continued under the sponsorship of the Office of Naval Research and of the Office of Ordnance Research under the administration of the Department of Mathematics of the University of California at Los Angeles. The physical facilities of the Institute for Numerical Analysis including SWAC and the library will be loaned by the National Bureau of Standards to the University of California. The status of the projects is reported here in the usual form with the understanding that tasks designated as in a "continued" status will be continued under the University administration. Future activity in these tasks will not be reported in future issue of these quarterly reports, but will be covered in communications of the University of California.

A new section, "Numerical Analysis," has been established in Washington as of July 1, 1954. At the same time, the Machine Development Laboratory has been replaced by a section, "Mathematical Physics".

The new organization of the National Applied Mathematics Laboratories therefore will be as follows:

1. Numerical Analysis. J. Todd, Chief.
2. Computation Laboratory. M. Abramowitz, Chief (Acting).
3. Statistical Engineering Laboratory. C. Eisenhart, Chief.
4. Mathematical Physics. E. W. Cannon, Chief.

The activities of this organization will continue to be reported in these quarterly reports.

Contents

Index of Active Research and Development Projects

Note: This index is not intended to cover the numerous special problem solutions, statistical analyses, and other ad hoc services to Government agencies, which form an important part of the work of the National Applied Mathematics Laboratories. These services are, however, fully represented in the body of the report.

A. Research: Pure Mathematics

B. Research: Numerical Analysis

C. Research: Applied Mathematics, Physics and Astronomy

D. Mathematical Statistics

E. Mathematical Tables

F. Manuals, Bibliographies, Indices, and Technical Information

G. Computing Machine Development

Status of Projects

June 30, 1954

I. INSTITUTE FOR NUMERICAL ANALYSIS

(Section 11.1)

1. Fundamental Research

SOLUTION OF SETS OF SIMULTANEOUS ALGEBRAIC EQUATIONS AND TECHNIQUES FOR THE INVERSION AND ITERATION OF MATRICES
Task 1101-10-5100/49-AE2

Origin: NBS — Authorized 2/15/49
Sponsor: Office of Naval Research, USN — Revised 9/15/49
Manager: G. E. Forsythe
Full task description: July-Sept 1949 issue, p. 2

Status: CONTINUED. Louise Straus' (binary) SWAC routine for improving the inverse of a matrix (see Jan-Mar 1954 issue) has been combined with an elimination routine (floating binary) to get the inverse.

Some preliminary steps have been taken by B. F. Handy, Jr., and G. E. Forsythe to assemble information on available SWAC matrix codes for the growing SWAC manual.

M. R. Hestenes, G. Lopez, and E. H. Modini have continued to experiment on SWAC with various modifications of the conjugate gradient method for solving linear systems. Running time is something like 3 or 4 minutes for 30 iterations with a matrix of order 30. This usually suffices to solve a linear system.

B. F. Handy has finished a preliminary code for solving n general simultaneous quadratic equations in connection with task 1101-40-5131/53-42 (see p. 12). The code employs a gradient method with the metric $\|x\| = \sum|x_i|$, and is designed for any $n \leqslant 16$. The first-power metric was chosen by G. E. Forsythe to avoid the scaling difficulties of squaring small numbers in fixed-point operation. More work is needed on the main program before the metric can be successfully used.

Publications: (1) "The conjugate gradient method for solving linear systems," by M. R. Hestenes; to appear in Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, in Santa Monica, 1953. (2) "Proof of an extension of a theorem of G. B. Dantzig's," by I. Heller and C. B. Tompkins; submitted to a technical journal.

CALCULATION OF EIGENVALUES, EIGENVECTORS, AND EIGENFUNCTIONS OF LINEAR OPERATORS
Task 1101-10-5100/50-3

Origin: NBS Authorized 7/1/49
Sponsor: Office of Naval Research, USN
Manager: G. E. Forsythe
Full task description: July-Sept 1949 issue, p. 13

Status: INACTIVE.

Publications: (1) "Asymptotic lower bounds for the frequencies of polygonal membranes," by G. E. Forsythe; to appear in the Pacific Journal of Mathematics. (2) "Asymptotic lower bounds for the fundamental frequency of convex membranes," by G. E. Forsythe; to appear in the Pacific Journal of Mathematics.

STUDIES IN PURE MATHEMATICS
Task 1101-10-5100/50-4

Origin: NBS Authorized 7/1/49
Sponsor: Office of Naval Research, USN
Manager: C. B. Tompkins
Full task description: July-Sept 1949 issue, p. 16

Status: CONTINUED. G. E. Forsythe used SWAC to generate a complete table of semigroups of order 4. A semigroup is a set of elements with a closed and associative multiplication defined over them. Two semigroups are here called distinct if they are neither isomorphic nor anti-isomorphic. Let f(n) be the number of distinct semigroups of order n. Prior tables had shown that f(1)=1, f(2)=4, f(3)=18, and had given 121 distinct semigroups of order 4. The present calculation, based on an exhaustive search for associative systems among all possible multiplication tables on four elements, established three new results: (1) The total number of semigroups with 4 given elements is 3492. Of these 126 are distinct, and 58 of the latter are commutative. (2) There is now a list on punched cards of the 3492 semigroups. (3) Publication (1) below contains a list of all 126 distinct semigroups in lexicographic order, each semigroup multiplication table being in its normal (that is, lexicographically first) form.

T. S. Motzkin and J. L. Selfridge have proved some theorems about the possible nature of the normal multiplication tables for semigroups of order n, with elements $0,1,\ldots,n-1$. They fall into $n+1$ classes $C_0,\ldots,C_n$. The first row of the table is $0\ldots0$ for C_0, starts with i zeros followed by an i for C_{n-i}, $i=2,\ldots,n-1$, starts with 011 for C_{n-1} and is $01\ldots n-1$ for C_n. The class C_n consists exactly of all groups. These results may make possible the listing of all semigroups of order 5. The preparation of a code for this listing has been started.

T. S. Motzkin developed asymptotic results for the number of semigroups of order n, and noted that the equality of the $(2n-2)!/(n-1)!n!$ meanings of $a_1a_2\ldots a_n$ in a general multiplication follows from but $\binom{n}{3}$ equalities.

T. S. Motzkin and J. L. Selfridge proved that the equation $a^{b^c}=b^{c^a}$ has no positive integral solutions when $a\neq b$. Many equations of a similar nature were studied. (See also Problem E1124, by L. J. Landen and J. L. Selfridge, Amer. Math. Mo. 61,423 (1954).) The general solution of a hyper-exponential diophantine equation can usually be obtained if

there are only two variables involved, or if the equation has no higher than second order exponentials and three variables. An exception to this seems to be the equation $a^{b^c}=c^a$. Many particular solutions have been found, including some in which a and c are not each powers of the same number, but the general solution seems rather elusive.

C. B. Tompkins, working with various colleagues, considered several problems involving discrete variables. This work was partially connected with project 1101-10-5150/53-1, p. 7 .

Publications: (1) "SWAC computes all 126 distinct semigroups of order 4," by G. E. Forsythe; IN MANUSCRIPT. (2) "Machine attacks on problems whose variables are permutations," by C. B. Tompkins; to appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, August 1953, Santa Monica, California.

COMPUTATION OF THE COMPLEX ZEROS OF THE RIEMANN ZETA FUNCTION
Task 1101-10-5100/50-13

Origin: NBS Authorized 6/1/50
Sponsor: Office of Naval Research, USN
Manager: R. Horgan
Full task description: Apr-June 1950 issue, p. 13

Status: INACTIVE. For status to date see July-Sept 1953 issue, p. 3.

STUDIES IN THE NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS
Task 1101-10-5100/51-1

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: W. Wasow
Full task description: July-Sept 1950 issue, p. 12

Status: CONTINUED. The solution of boundary value problems for differential equations of the form $\varepsilon y''=F_1(x,y,\varepsilon)y'+F_2(x,y,\varepsilon)$ for small positive ε has been studied by W. Wasow. If a solution u(x) of the "reduced" differential equation obtained for $\varepsilon=0$ is known which satisfies one of the two boundary conditions $y(a)=A$, $y(b)=B$, then, under certain additional assumptions the solution of the full equation satisfying both boundary conditions exists and can be represented by a convergent series of the form

$$y(x,\varepsilon) = u(x) + \int_{r=1}^{\infty} u_r(x,\varepsilon)\,\varepsilon^r$$

whose coefficients satisfy certain linear differential equations and can be approximately calculated by quadratures, if ε is small.

M. Muller's work on the numerical solution of Dirichlet's problem by means of random walks is described in task 1101-10-5100/51-2.

Publications: (1) "Discrete approximations to elliptic differential equations," by W. Wasow; to appear in Zeitschrift für angewandte Mathematik und Physik. (2) "Singular perturbations of boundary value problems for non-linear differential equations of the second order," by W. Wasow; IN MANUSCRIPT.

PROBABILITY METHODS AND SAMPLING TECHNIQUES
Task 1101-10-5100/51-2

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: D. Teichroew
Full task description: July-Sept 1950 issue, p. 13

Status: CONTINUED. A table of millile probability points of the t-distribution was computed and is available on punched cards.

A special distribution problem was studied. The results are given in publication (1) below.

M. Muller has prepared a manuscript which contains the results of his investigation of procedures for improving the efficiency of the Monte Carlo methods for solving boundary value problems. This research has involved two conceptually different approaches:

1. The underlying stochastic models utilize continuous parameter processes. This will enable direct statistical estimates of the unknown solution of the boundary value problem. This approach has been carried out in detail for the Dirichlet problem. A proof of the strong convergence of a very general class of processes has been given. In order that the procedures be feasible for high speed computing machines truncation schemes are considered. Attention is given to improving the efficiency of this approach by using statistical design. In essence one of the major advantages of this approach is that it is not necessary to approximate the given differential operators by difference operators. Further in all known methods of studying estimates of the unknown solution of the Dirichlet problem it has been necessary to introduce restrictions on the boundary and the function defined on the boundary which are restrictions in addition to those imposed by the Dirichlet problem; this is not necessary when using the approach introduced in this section.

2. This material follows the more classical approach of considering statistical estimates for the solution of the partial difference equations. Since almost all present numerical methods for solving partial difference equations use some sort of network of points a new approach is suggested to determine optimum operator approximations and admissible networks. These results go beyond Monte Carlo solutions and could be used for such techniques as relaxation and its many variations. This approach looks most promising for the Dirichlet problem in three or more dimensions. Some interesting conjectures are raised. Also, the popular belief that one always does better when the difference operator uses more neighboring points is shown not to be true. Results are given to show that increased efficiency is obtained by the use of statistical designs. This research has been carried out under the valuable guidance of George W. Brown of the University of California, Los Angeles. The contributions made by Robert Steinberg have also been most helpful.

Publications: (1) "The mixture of normal distributions with different variances," by D. Teichroew; IN MANUSCRIPT. (2) "Studies on the Monte Carlo method for solving boundary value problems," by M. Muller; IN MANUSCRIPT.

VARIATIONAL METHODS
Task 1101-10-5100/51-3

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: D. A. Pope
Full task description: July-Sept 1950 issue, p. 15

Status: CONTINUED. A SWAC program for computation of a minimizing curve to a one-dimensional calculus-of-variations problem has been completed and a test example computed. The problem of computing geodesics of a surface by direct methods on SWAC is also being investigated, and a program is being prepared to compute this for a torus. Theoretical investigations into the effectiveness of various methods for machine calculation are being continued.

STUDIES IN APPLIED MATHEMATICS
Task 1101-10-5100/51-4

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: T. H. Southard
Full task description: July-Sept 1950 issue, p. 16

Status: CONTINUED. The seminar on numerical analysis met twenty times during the quarter. A detailed list of sessions will be found under Lectures and Symposia in the back of this issue. Summaries of the sessions are being prepared under the editorship of C. B. Tompkins.

C. B. Tompkins and T. H. Southard have done some work on development of a routine for SWAC which will solve a two-person, zero-sum, rectangular game whose value is zero and whose typical matrix element is $A_{a\alpha} + VB_{a\alpha}$, where V is to be determined and $-1<B_{a\alpha}<0$. Such a game has applications in quantitative studies of warfare, and elsewhere. Some approximate results were obtained and improvement methods are being studied.

T. S. Motzkin and J. L. Walsh (Harvard University) continued their investigation of best approximating polynomials. This includes a special study of extremal distance polynomials (see publication 4 below). $P_n=P_n(z)=\prod_i \text{dist } zz_i$, $i=1,\ldots,n$, z and z_i points in d-space, $d\geq 3$, is a distance polynomial of degree n; on a symmetry hyperplane of the z_i it is a quasi distance polynomial Q_n. The expression $P^{-1}\sum (\partial P_n/\partial z^k)^2$, where z^k is a coordinate of z, can have more than n-1 real zeros and is thus not a P_{n-1} (nor a Q_{n-1}), in refutation of a conjecture of G. von Sz. Nagy. The number of real and complex "stats" (all $\partial P_n/\partial z^k=0$, no dist zz_i equal 0) for $n>d>1$ and general z_i is conjectured, and in several cases proved to be $2^n-2^{n-d+2}+(n-d+1)^2-d+2$. For the deviation $\sum \mu_j(P_n(\xi_j))^p$ on a given finite set $\xi_1,\ldots,$ results concerning stationary and minimizing polynomials generalize those obtained for $d\leq 2$. For continuous mass distributions μ, the behavior differs markedly from the case $d\leq 2$.

Publications: (1) "Numerical computation of low moments of order statistics from a normal population," by J. B. Rosser; submitted to a technical journal. (2) "Note on the Circle Theorem of Hydrodynamics," by E. Levin; accepted by Quarterly of Applied Mathematics. (3) "Indentation pressure of a smooth punch," by E. Levin; accepted by the Quarterly of Applied Mathematics. (4) "Extremal distance polynomials and the logarithmic potential in space," by T. S. Motzkin and J.L. Walsh; IN MANUSCRIPT.

MISCELLANEOUS STUDIES IN THEORETICAL PHYSICS
Task 1101-10-5100/51-5

Origin: Office of Naval Research, USN — Authorized 9/1/50
Sponsor: " "
Manager: R. D. Woods
Full task description: July-Sept 1950 issue, p. 19

Status: CONTINUED. W. Futterman is concluding theoretical and numerical work on calculations of the photo-disintegration of the deuteron. R. Woods is continuing computations of the elastic scattering of neutrons and protons by various elements, based on the optical model of the nucleus.

Publications: (1) "Application of variational methods to intermediate and high energy scattering," by E. Gerjuoy and D. S. Saxon; Physical Rev. 94, 478-491 (Apr. 15, 1954). (2) "Variational principles for the acoustic field," by E. Gerjuoy and D. S. Saxon; accepted by The Physical Review. (3) "On the diffuse surface optical model for nucleon-nuclei scattering," by R. Woods and D. S. Saxon; accepted as a Letter to the Editor of The Physical Review.

STUDY OF RUSSIAN MATHEMATICAL PROGRESS
Task 1101-10-5100/52-1

Origin: NBS — Authorized 3/15/52
Sponsor: Office of Naval Research, USN
Manager: G. E. Forsythe
Full task description: Jan-Mar 1952 issue, p. 11

Status: CONTINUED. Accumulation of bibliographical cards on Russian mathematical monographs continues (see Jan-Mar 1952 issue, p. 11, and Oct-Dec 1952 issue, p. 8). A "Bibliographical Survey of Russian Mathematical Monographs, 1930 to 1951," as well as a supplement, is available in limited quantities.

RESEARCH IN THE MATHEMATICAL THEORY OF PROGRAM PLANNING
Task 1101-10-5102/50-11

Origin: Office of Air Comptroller, USAF — Authorized 6/15/50
Sponsor: " "
Manager: T. S. Motzkin
Full task description: Apr-June 1950 issue, p. 12

Status: TERMINATED. An approximate solution of the assignment problem, by A. N. Gleyzal, was improved. Quadratic assignment problems and their applications were studied, in cooperation with J. C. Koopmans and M. Beckman (both Cowles Commission, Chicago). A special linear programming problem connected with scheduling was discussed with W. Karush, the shortest cycle problem with G. B. Dantzig (RAND Corporation) and I. Heller (George Washington University Logistics Project). At conferences in Washington with J. Todd and A. J. Hoffman (both NBS), plans for a tract or chapter on linear inequalities and (in conjunction with W. Jacobs, USAF) for a second symposium on linear inequalities and programming were developed. Other conferences connected with the termination of the project were held with J. W. Gaddum (Michigan State College), H. W. Kuhn (Bryn Mawr College), I. J. Schoenberg (University of Pennsylvania), R.M. Thrall

(University of Michigan), A. W. Tucker (Princeton University).

Publications: (1) "On the relaxation method for linear inequalities," by T. S. Motzkin and I. J. Schoenberg; accepted by the Canadian Journal of Mathematics. (2) "The assignment problem," by T. S. Motzkin; accepted as a Cowles Commission Discussion Paper and by the Proceedings of the Sixth Symposium for Applied Mathematics, held at Santa Monica, Calif., August 1953. (3) "The probability of solvability of linear inequalities," by T. S. Motzkin; IN MANUSCRIPT.

SCAMP
Task 1101-10-5150/53-1

Origin: Office of Naval Research, USN Authorized 6/10/53
Sponsor: " "
Manager: C. B. Tompkins
Full task description: Apr-June 1953 issue, p. 8

Status: CONTINUED. During the end of the quarter, when most of the participants had reached Los Angeles, the active summer phase of the program began. Much preliminary work for the session was carried out by M. Rees, E. H. Spanier, J. Swift, and C. B. Tompkins.

2. Development

NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER (SWAC)
Task 1101-20-5103/49-1

Origin: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF Authorized 11/1/48
Sponsor: " " "
Manager: H. D. Huskey
Full task description: Apr-June 1949 issue, p. 27

Status: CONTINUED. SWAC has continued to be used on a regularly scheduled two shift basis on both service and research problems. However, members of the University staff have occasionally operated the machine after the end of the regular shift at midnight and on weekends. During the quarter the machine was turned on for a total of 1189 hours, 925.2 of which were used for productive computing. This is about 78% of the total time. Approximately 38 hours was unscheduled maintenance time.

During the quarter the switch which controls hexidecimal or decimal output was transferred to the console. Part of the timing circuitry was simplified and the speed of the computer increased. Multiplication time was decreased from 384 μsec. to 368 μsec., and extract from 128 to 384 μsec. to 80 to 368 μsec.

STUDIES IN THE THEORY OF DIGITAL COMPUTING MACHINES
Task 1101-20-5103/53-1

Origin: NBS Authorized 9/30/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: H. D. Huskey
Full task description: July-Sept 1952 issue, p. 16

Status: CONTINUED. Studies on automatic coding using the interpretative floating subroutine system continued. A paper was presented on this at the Ann Arbor meeting of the Association for Computing Machinery. A more general interpretative subroutine which handles vector operations with single commands has been coded and checked.

3. Mathematical Services

COMPUTING SERVICES FOR RESEARCH STAFF OF THE INSTITUTE FOR NUMERICAL ANALYSIS
Task 1101-40-5130/54-1

Origin: NBS Authorized 9/2/48
Sponsor: Office of Naval Research, USN Revised 11/16/49
Manager: F. Hollander
Full task description: July-Sept 1949 issue, p. 25

Status: CONTINUED. Research staff problems involving the use of SWAC were as follows:

(1) For C. B. Tompkins: A search for two orthogonal latin squares of order 10 was continued, in connection with SCAMP, according to a code by F. Meek. No pairs were found.

(2) For W. Futterman: The approximate, closed form integrals occurring in the problem of the photo-electric disintegration of the deuteron are in the process of computation.

(3) For R. D. Woods: Computations of the scattering of neutrons and protons by various metals, using an optical model for the nucleus, were continued.

(4) For the Physical Chemistry Group of the Department of Chemistry, UCLA: Crystallographic calculations have been continued. Three-dimensional Fourier summations of electron density have been computed for four different crystalline substances; the amplitudes and phases ("structure factors") of the diffracted waves corresponding to various possible structures for each have also been calculated for comparison with experiment. A significant portion of the time has been devoted to refinement of atomic positions and of anisotropic atomic "temperature factors" by least squares and differential Fourier syntheses.

(5) For M. R. Hestenes and E. Mookini: Experiments were continued in the conjugate gradient method for solving systems of linear equations.

(6) For G. Forsythe: G. E. Forsythe used the SWAC to compute all semigroups of order 4 (see project 1101-10-5100/50-4, p. 2). He also did some research on machine methods of solving systems of linear and quadratic equations. Part of this was in connection with project 1101-40-5131/53-42, p. 12.

(7) For the Geophysics Department of UCLA: Calculations were continued and new codes were checked out in the problem of the high order tidal constituents for earth and ocean tides.

(8) For D. H. Lehmer and J. Selfridge: (a) The Ideal Waring Theorem code has been run up to exponent 100,000. Hence the Ideal Waring Theorem holds for all exponents less than 100,000. (b) The program for finding quadratic residues which are not squares was continued on a standby basis. (c) A recheck was made of some of the irregular prime runs between 2000 and 2550. One error was discovered. It was decided to recheck a few more values. (d) A code was checked out and run which computes the smallest prime $q=kp+1$ for a given prime p. (e) The Alway routine was revised to accommodate punched card input and output.

(9) For the Physics Department of UCLA: Completed calculation of q, the electric field gradient at the position of one of the nuclei of a Li_2 molecule using a proposed variational molecular wave function.

(10) For T. Southard: SWAC was used to obtain approximate solutions of some two-person, zero-sum, warlike games called HOTSPOT games (see 1101-10-5100/51-4, p. 5).

(11) For D. A. Pope: A code for a direct method of calculation of a one dimensional problem in the calculus of variations was checked out and a simple problem was calculated on SWAC.

PUNCHED CARD LIBRARY
Task 1101-40-5131/49-2

Origin: NBS Authorized 7/14/48
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: F. Hollander
Full task description: Apr-June 1949 issue, p. 26

Status: CONTINUED. Tables of random digits and random normal deviates were reproduced and shipped to the Naval Electronics Laboratory, San Diego, California. No new tables were added this quarter.

SIMPLIFIED ROLLING PULLOUT EQUATIONS
Task 1101-40-5131/51-34

Origin: Cornell Aeronautical Laboratory Authorized 6/22/51
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF
Manager: F. Hollander
Full task description: Apr-June 1951 issue, p. 26

Status: COMPLETED. All data received have been processed and results submitted to originator. "Report on the solution of the equations of flight during a rolling pull-out maneuver," by E. C. Yowell, will serve as a final report on this problem.

LOW MOMENTS OF ORDER STATISTICS
Task 1101-40-5131/51-36

Origin: University of Oregon — Authorized 6/22/51
Sponsor: Office of Naval Research, USN
Manager: H. D. Huskey
Full task description: Apr-June 1951 issue, p. 27

Status: INACTIVE. For status to date, see Jan-Mar 1954 issue, p.11.

SIERRA WAVE PROJECT
Task 1101-40-5131/52-36

Origin: Department of Meteorology, U.C.L.A. — Authorized 4/1/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: T. H. Southard
Full task description: Jan-Mar 1952 issue, p. 33

Status: COMPLETED. More theodolite data were reduced on SWAC. A routine for reduction of three-dimensional Raydist data was coded.

EIGENVALUES
Task 1101-40-5131/53-6

Origin: Consolidated Vultee Aircraft Corporation (Convair) — Authorized 9/26/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: F. Hollander
Full task description: July-Sept 1952 issue, p. 28

Status: TERMINATED.

METEOROLOGICAL MEANS
Task 1101-40-5131/53-10

Origin: Meteorology Department, UCLA — Authorized 9/30/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: F. H. Hollander
Full task description: July-Sept 1952 issue, p. 29 (See also project 11.1/31-50-17, Jan-Mar 1950 issue, p. 15.)

Status: CONTINUED. Work on the "Mountain Term" for the summer months of July-Aug 1949 was completed. Work is in progress on an evaluation of the non-geostropic flux of angular momentum for both the winter and summer months. This evaluation makes use of some previously obtained results, namely, the flux of total kinetic energy and the flux of the kinetic energy of zonal motion.

DISCRIMINANT FUNCTIONS
Task 1101-40-5131/53-18

Origin: School of Aviation Medicine, Randolph Field, USAF — Authorized 12/15/52
Manager: D. Teichroew
Full task description: Oct-Dec 1952 issue, p. 27

Status: COMPLETED. The SWAC code was extended to cover the range of p, the number of dimensions, from one to fifteen. Results have been submitted to the contractor.

BIO-ASSAY PROBLEM
Task 1101-40-5131/53-24

Origin: Stanford University — Authorized 3/31/53
Sponsor: Office of Naval Research, USN
Manager: D. Teichroew
Full task description: Jan-Mar 1953 issue, p. 22

Status: INACTIVE. Results are being examined by the contractor.

DISCRETE MINIMAL SPACES
Task 1101-40-5131/53-26

Origin: Gilfillan Brothers, Inc. — Authorized 3/31/53
Sponsor: Evans Signal Laboratories, Army Signal Corps
Manager: H. D. Huskey
Full task description: Jan-Mar 1953 issue, p. 23

Status: TERMINATED. For status to date see Apr-June 1953 issue, p. 23.

LINEAR EQUATIONS (CONVAIR)
Task 1101-40-5131/53-35

Origin: Consolidated Vultee Aircraft Corporation — Authorized 3/31/53
Sponsor: Bureau of Aeronautics, USN
Manager: F. Hollander
Full task description: Jan-Mar 1953 issue, p. 28

Status: TERMINATED. For status to date, see Oct-Dec 1953 issue, p. 16.

ACCEPTANCE TESTS
Task 1101-40-5131/53-41

Origin: Jet Propulsion Laboratory, California Institute of Technology — Authorized 6/29/53
Sponsor: Ordnance Corps, U. S. Army
Manager: H. D. Huskey
Full task description: Apr-June 1953 issue, p. 27

Status: TERMINATED. The computer will be delivered after the end of this quarter, and the test routines developed under this project will be used.

B.P.A. Studies
Task 1101-40-5131/53-42

Origin: Bonneville Power Administration — Authorized 6/29/53
Sponsor: " "
Manager: G. E. Forsythe
Full task description: Apr-June 1953 issue, p. 27

Status: CONTINUED. The Bonneville Power Administration asked for a gradient solution of three simultaneous quadratics in three complex variables--that is, six real quadratics. To solve this, a general quadratic code has been prepared, discussed under task 1101-10-5100/49-AE2, p. 1.

HELICOPTER STABILITY STUDIES
Task 1101-40-5131/53-44

Origin: J. B. Rea Company — Authorized 6/30/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: G. E. Forsythe
Full task description: July-Sept 1953 issue, p. 23

Status: CONTINUED. Almost all the real matrices of order 16 referred to in the Jan-Mar 1954 report have been successfully inverted. Another determinant of type det($\lambda F+G$) of order 13 is being evaluated with the Frame code. See task 1101-10-5100/50-3 in Jan-Mar 1954 issue, p. 2.

COMPUTATIONS FOR SCAMP
Task 1101-40-5131/54-1

Origin: Office of Naval Research, USN — Authorized 9/29/53
Sponsor: Office of Naval Research, USN
Manager: C. B. Tompkins
Full task description: July-Sept 1953 issue, p. 24

Status: CONTINUED. Computations continued on many discrete variable problems under the general supervision of J. Swift and C. B. Tompkins. F. B. Meek prepared a code for the reduction of a matrix of integers to a

triangular form (of Hermite), and reduced several small matrices to this form. This code was prepared primarily to facilitate some studies which are to be undertaken by G. Pall in connection with finite projective planes.

PARACHUTE DATA ANALYSIS
Task 1101-40-5131/54-7

Origin: U. S. Naval Auxiliary Air Station, El Centro, Calif. Authorized 2/25/54
Sponsor: U. S. Navy
Manager: D. Teichroew
Full task description: Jan-Mar 1954 issue, p. 16

Status: INACTIVE.

GALACTIC DISTRIBUTION STUDIES
Task 1101-40-5131/54-8

Origin: Statistical Laboratory, University of California, Berkeley Authorized 2/25/54
Manager: B. F. Handy, Jr.
Full task description: Jan-Mar 1954 issue, p. 16

Status: TERMINATED.

MATRIX INVERSIONS
Task 1101-40-5131/54-9

Origin: Northrop Aircraft, Inc. Authorized 2/25/54
Manager: F. H. Hollander
Full task description: Jan-Mar 1954 issue, p. 17

Status: TERMINATED.

SKEW RAYS
Task 1101-40-5131/54-10

Origin: Pacific Optical Company Authorized 2/25/54
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: F. H. Hollander
Full task description: Jan-Mar 1954 issue, p. 17

Status: CONTINUED. A small amount of code checking was done with data supplied by the originator, but no results were obtained.

II. COMPUTATION LABORATORY
(Section 11.2)

1. Research

RESEARCH IN CLASSICAL NUMERICAL ANALYSIS
Task 1102-10-1104/50-1

Origin: NBS Authorized 1/1/50
Managers: J. Todd, M. Abramowitz, H. A.Antosiewicz
Full task description: Jan-Mar 1950 issue, p. 21

Status: CONTINUED. J. Todd completed an expository paper (see publication (10) below), which was based on an invited address before the American Mathematical Society and Office of Ordnance Research Symposium on Applied Mathematics (Chicago, April 1954). In addition to classical numerical analysis, it contained material on modern and ultra-modern numerical analysis.

The seminar on Numerical Analysis, held in conjunction with American University, was continued. Participants during this quarter were W. F. Cahill, K. Goldberg, P. Henrici, M. Hyman (Westinghouse Corporation), L. S. Joel, N. Levine, and P. C. Rosenbloom (Minnesota University and Institute for Advanced Study).

Publications: (1) "Evaluation of $\int_0^\infty \exp(-u^2-(x/u))du$," by M. Abramowitz; J. Math. Phys. 32, 188-192 (July-Oct.1953). (2) "The practical evaluation of integrals," by M. Abramowitz; J. Soc. Indust. App. Math. 2, 20-35 (Mar.1954). (3) "Regular and irregular Coulomb wave functions expressed in terms of Bessel-Clifford functions," by M. Abramowitz; J. Math. Phys. 33, 111-116 (Apr. 1954). (4) "Coulomb wave functions along the transition line," by M. Abramowitz and P. Rabinowitz; to appear in The Physical Review. (5) "Some implications of Liapunov's conditions for stability," by H. A. Antosiewicz and P. Davis; accepted for publication in the Journal of Rational Mechanics and Analysis. (6) "Computation of vibration modes and frequencies on SEAC," by W. Cahill and S. Levy; IN MANUSCRIPT. (7) "On a problem in the theory of mechanical quadratures," by P. Davis; accepted by the Pacific Journal of Mathematics. (8) "On the estimation of quadrature errors for analytic functions," by P. Davis and P. Rabinowitz; submitted to a technical journal. (9) "The evaluation of the exponential integral for large complex arguments," J. Todd; J. Res. NBS 52, 313-318 (June 1954). (10) "Motivation for working in numerical analysis," by J. Todd; to appear in the Transactions of a Symposium in Applied Mathematics, held by the American Mathematical Society and the Office of Ordnance Research in Chicago, Ill., April 1954.

RESEARCH IN MODERN NUMERICAL ANALYSIS: INVESTIGATION OF BERGMAN'S METHOD FOR THE SOLUTION OF THE DIRICHLET PROBLEM FOR CERTAIN MULTIPLY CONNECTED DOMAINS
Task 1102-10-1104/50-2

Origin: NBS Authorized 3/1/50
Manager: P. Davis
Full task description: Jan-Mar 1950 issue, p. 22

Status: TERMINATED. The computations have been completed (see Oct-Dec 1951 issue, p. 26). There are no present plans for publication.

MISCELLANEOUS STUDIES IN PURE MATHEMATICS
Task 1102-10-1104/50-4

Origin: NBS Authorized 1/1/50
Managers: O. Taussky-Todd, J. Todd, M. Abramowitz, A. Hoffman
Full task description: Jan-Mar 1950 issue, p. 23

Status: CONTINUED. K. Fan has studied a class of Banach spaces closely related to uniformly convex spaces. He has proved for this class of spaces a generalization, not restricted to linear operators, of G. Birkhoff's mean ergodic theorem.

O. Taussky-Todd continued her investigation of normal matrices in certain problems of algebraic number theory. In this connection Mrs. Todd also studied the following question suggested to her by S. Gorn from Aberdeen Proving Ground: For what degrees is the normal basis for the integers of an algebraic number field unique, provided there exists one? She studied this question so far for cyclic fields of squarefree degrees and for n=4 only, and it was found that for n=2, 3,4,6 the normal basis is unique, but not for the other degrees. Also, for non-cyclic abelian fields of degree 4, the normal basis is unique.

K. Goldberg has revised his manuscript on the Baker-Hausdorff formula (previously reported under task 1102-10-1104/53-16) to exploit newly-discovered relations with Faber polynomials.

Publications: (1) "Linear functional equations and interpolation series," by P. Davis; to appear in the Pacific Journal of Mathematics. (2) "Some L^2 aspects of Faber polynomials," by P. Davis and H. Pollack; to appear in Duke Mathematical Journal. (3) "On representations and extensions of bounded linear functionals defined on classes of analytic functions," by P. Davis and J. L. Walsh; Trans. Amer. Math. Soc. 76, 190-206 (1954). (4) "Continuity in terms of connectedness," by K. Fan and R. A. Stuble; to appear in Proc. Nederl. Akad., Wetensch. Indagationes Math. (5) "Compactification of completely regular spaces," by K. Fan and F. Wagner; IN MANUSCRIPT. (6) "The representation of integers by binary quadratic rational forms," by K. Goldberg, M. Newman, E. Straus, J.D.Swift; Arch. Math. 5, 12-18 (1954). (7) "Ueber die Funktionen von Gegenbauer," by P. Henrici; Arch. Math. 5, 92-98 (1954). (8) "On certain series expansions involving Whittaker functions and Jacobi polynomials," by P. Henrici; accepted by the Pacific Journal of Mathematics. (9) "On generating functions of the Jacobi polynomials," by P. Henrici; accepted by the Pacific Journal of Mathematics. (10) "The number of absolute points of a correlation," by A. Hoffman, M. Newman, E. Straus, O.Taussky; submitted to a technical journal. (11) "A conjecture of Erdös" by M. Newman; submitted to a technical journal. (12) "Note on a certain determinant," by M. Newman; IN MANUSCRIPT. (13) "Structure theorems for modular subgroups," by M. Newman; to appear in the Duke Mathematical Journal. (14) "Some computational problems in algebraic number theory,"

by O. Taussky; submitted for inclusion in the Proceedings of the American Mathematical Society's Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953. (15) "Power series for $\log(e^x.e^y)$," by K. Goldberg; IN MANUSCRIPT.

NUMBER THEORETICAL TEST PROBLEMS FOR SEAC
Task 1102-10-1104/50-5a

Origin: NBS — Authorized 6/1/50
Managers: O. Taussky-Todd, K. Goldberg
Full task description: Apr-June 1950 issue, p. 31

Status: INACTIVE. For status to date, see Jan-Mar 1954 issue, p. 21.

SOLUTION OF LAPLACE EQUATION BY MONTE CARLO METHOD
Task 1102-10-1104/51-6

Origin: NBS — Authorized 9/28/50
Manager: M. Abramowitz
Full task description: July-Sept 1950 issue, p. 36

Status: CONTINUED. J. Todd and O. Taussky-Todd completed the manuscript of the paper on generation and testing of pseudo-random numbers. Further experiments and tests on the Fibonacci sequence have been carried out by J. M. Cameron and M. Newman.

I. Stegun continued the study of the location of the mean in samples from a normal population. J. Todd completed his paper on the solution of the Laplace equation by Monte Carlo methods.

Publications: (1) "Generation and testing of pseudo-random numbers," by O. Taussky and J. Todd; IN MANUSCRIPT. (2) "Experiments in the solution of differential equations by Monte Carlo methods," by J. Todd; IN MANUSCRIPT.

ANALYSIS OF GEOMAGNETIC FIELD
Task 1102-10-1104/52-8

Origin: NBS — Authorized 8/10/51
Manager: C. J. Swift
Full task description: July-Sept 1951 issue, p. 34

Status: TERMINATED. For status to date see Oct-Dec 1951 issue, p. 29.

SPECIAL PROBLEMS IN FINITE MATRIX THEORY
Task 1102-10-1104/52-34

Origin: NBS Authorized 11/6/51
Manager: O. Taussky-Todd
Full task description: Oct-Dec 1951 issue, p. 30

Status: CONTINUED. O. Taussky-Todd and T. S. Motzkin prepared a new and enlarged draft of their paper, "Classes of matrices with property L, II". Mrs. Todd studied a connection between sums of positive definite Hermitian matrices and sums of real (not necessarily symmetric) matrices with positive dominant main diagonal elements. It was known that, for sums of matrices of either type, $|A+B| \geq |A| + |B|$, and it is now shown that the inequality for matrices of the former type implies the inequality for matrices of the latter type.

Reference is also made here to the work by K. Fan, O. Taussky-Todd, and J. Todd in publication (5) of task 1102-10-5116/50-2, p. 18.

K. Fan has prepared manuscripts on Hermitian and normal matrices (see publications (3) and (4) below). It is shown in (4) that if the (r+1)th singular value of the difference of two normal matrices is not greater than ε, and if a circular disk contains at least p eigenvalues of one of the matrices, the concentric disk with radius increased by ε contains at least p-r eigenvalues of the second matrix. This generalizes earlier results of Ledermann and Wielandt. Results given in (3) for a positive definite Hermitian matrix H include: Minimum characterizations of det H/det C, where C is a principal submatrix of H; new inequalities for the eigenvalues of H in terms of its principal minors; and upper bounds for det H in terms of its principal minors.

A. J. Hoffman continued his study of minimax characterizations of functions of the eigenvalues of a positive definite Hermitian matrix, obtaining extensions of earlier results on sums and products.

Publications: (1) "Some remarks on commutators of matrices," by K. Fan, Arch. Math. 5, 102-107 (1954). (2) "Inequalities for eigenvalues of Hermitian matrices," by K. Fan; to appear in Contributions to the solutions of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39. (3) "Some inequalities concerning positive definite Hermitian matrices," by K. Fan; IN MANUSCRIPT. (4) "A comparison theorem for eigenvalues of normal operators," by K. Fan; IN MANUSCRIPT. (5) "Some metric inequalities in the space of matrices," by K. Fan and A. J. Hoffman; to appear in the Proceedings of the American Mathematical Society. (6) "Characteristic roots of quaternion matrices," by O. Taussky; Arch. Math. 5, 99-101 (1954). (7) "Generalized commutators of matrices and permutations of factors in a product of three matrices," by. O. Taussky; to appear in the von Mises Anniversary volume. (8) "The condition of the finite segments of the Hilbert matrix," by J. Todd; to be included in Contributions to the systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39. (9) "The conditions of certain matrices,II," by J. Todd; to appear in Archiv der Mathematik. (10) "Error bounds for eigenvalues of symmetric integral equations," by H. Wielandt; submitted for inclusion in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953. (11) "Einschliessung von Eigenwerten hermitescher Matrizen nach dem Abschnittsverfahren," by H. Wielandt; Arch. Math. 5, 108-114 (1954). (12) "On eigenvalues of sums of normal matrices," by H. Wielandt; submitted to a technical journal.

DIFFERENTIAL EQUATION FOR NERVE FIBER REACTION
Task 1102-10-1104/53-15

Origin: National Naval Medical Institute — Authorized 12/8/52
Sponsor: " "
Managers: H. A. Antosiewicz, P. Rabinowitz
Full task description: Oct-Dec 1952 issue, p. 34

Status: CONTINUED. A joint report with K. S. Cole of the Naval Medical Research Institute is being prepared.

BAKER-HAUSDORFF FORMULA
Task 1102-10-1104/53-16

Origin: NBS — Authorized 12/8/52
Managers: O. Taussky-Todd, K. Goldberg
Full task description: Oct-Dec 1952 issue, p. 35

Status: TERMINATED. See task 1102-10-1104/50-4, p. 15, for work performed during this quarter. Future work will be reported as part of that task.

BASIC RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5116/50-2

Origin: Office of Scientific Research, ARDC, USAF, and Office of the Air Comptroller, USAF — Authorized 3/31/50
Sponsor: Office of Scientific Research, ARDC, USAF
Managers: J. Todd, A. Hoffman
Full task description: Jan-Mar 1950 issue, p. 24

Status: CONTINUED. During this quarter, K. Fan continued his work on systems of linear inequalities, emphasizing infinite systems in a normed linear space. The results thus far developed concern existence theorems and characterizations of the solutions of minimum norm. He is also working on a monograph on the theory and application of convex sets.

K. Fan, O. Taussky-Todd and J. Todd studied the discrete analogs of a set of integral inequalities involving a function and its derivatives due to W. Wirtinger. New inequalities, involving a function and its second derivative, were established in the continuous and discrete cases. Discrete analogs were obtained for inequalities of Bellman and Northcott, and a complete solution was obtained for a geometrical extremal problem suggested by these results. The general tools used in deriving these results were extremal properties of eigenvalues of finite matrices. Best possible constants and characterizations of equality cases were also obtained. The investigation of additional problems to which the matrix techniques do not apply is proceeding.

A. J. Hoffman, in collabration with H. W. Kuhn (Bryn Mawr College), has applied the transportation problem of linear programming to the study of systems of distinct representatives. It is shown how old and new results on this combinational problem are special cases of the duality theorem applied to the transportation problem. New results include necessary and sufficient conditions that a system of distinct representatives for given subsets of a set intersect each summand of a partition of the set in a number of elements whose upper and lower bounds are prescribed.

Publications: (1) "A theorem on alternatives for pairs of matrices," by H. A. Antosiewicz; to appear in Pacific Journal of Mathematics. (2) "On a theorem of Ostrowski and Taussky," by R. Bellman and A. J. Hoffman; Arch. Math. 5, 123-127 (1954). (3) "Lower bounds for the rank and location of the eigenvalues of a matrix," by K. Fan and A. J. Hoffman; to appear in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39. (4) "Some metric inequalities in the space of matrices," by K. Fan and A. J. Hoffman; to appear in Proceedings of the American Mathematical Society. (5) "Discrete analogs of inequalities of Wirtinger," by K. Fan, O. Taussky, and J.Todd; IN MANUSCRIPT. (6) "A determinantal inequality," by K. Fan and J. Todd; to appear in the Journal of the London Mathematical Society. (7) "On the relevance of LeChatelier's principle to linear programming," by A. J. Hoffman; IN MANUSCRIPT. (8) "An extremum property of sums of eigenvalues," by H. Wielandt; to appear in the Proceedings of the American Mathematical Society.

COMPRESSIBLE FLOW -- METHOD OF ORTHOGONAL AND KERNEL FUNCTIONS
Task 1102-10-5116/52-16

Origin: Aeronautical Research Laboratory, Wright Air Development Center, USAF, and Harvard University — Authorized 9/29/51
Sponsor: Aeronautical Research Laboratory, WADC, USAF
Managers: P. Davis, F. L. Alt
Full task description: July-Sept 1951 issue, p. 36

Status: COMPLETED. Publication (1) has been transmitted to the sponsor.

Publications: (1) "Some SEAC computations of subsonic flow by Bergman's method of integral operators," by P. Davis and P. Rabinowitz; an NBS report. (2) "A multi-purpose orthonormalizing code and its uses," by P. Davis and P. Rabinowitz; submitted to a technical journal.

TABLES OF INTEGRALS INVOLVING THE HIGHER TRANSCENDENTAL FUNCTIONS
Task 1102-10-5116/52-33

Origin: NBS — Authorized 10/11/51
Manager: F. Oberhettinger
Full task description: July-Sept 1951 issue, p. 37

Status: CONTINUED. The article concerning Mathieu and Lamé wave functions for the Bateman project handbook is in preparation.

Publication: "On the Lerch zeta function," by F. Oberhettinger; submitted to a technical journal.

WATER WAVES
Task 1102-10-5116/53-54

Origin: NBS Authorized 6/1/53
Sponsor: Office of Naval Research, USN
Manager: P. Davis
Full task description: Apr-June 1953 issue, p. 38

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 25.

SUPPORTING RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5116/54-9

Origin: Office of the Air Comptroller, USAF Authorized 9/25/53
Sponsor: Office of Scientific Research, Air Research and Development Command, USAF
Managers: J. Todd, A. J. Hoffman
Full task description: July-Sept 1953 issue, p. 35

Status: CONTINUED. A seminar in linear programming was held fortnightly at American University and will be resumed in the fall. The speakers and topics are listed under "Lectures and Symposia", p. 49.

Thirteen linear programming problems were solved using the simplex method. It was found convenient to obtain the answers by doing the dual rather than the primal problem in each case. A feature of interest was the fact that 12 of these grouped naturally into three sets, in each of which the dual problem varied only in the objective function. For each set the four objective functions were carried and transformed, with the criteria for changing bases and arriving at a solution being applied to each objective function in turn. Less than half the number of iterations required to solve for the first objective function were needed in going from the first to second, and so forth.

A code for solving the personnel problem by the "Hungarian" method of H. W. Kuhn is being written.

Publications: (1) "On 'overshoot' on the furthest hyperplane method," by R. Bryce; IN MANUSCRIPT. (2) "Linear programming in bid evaluation," by L. Gainen, D. J. Honig, and E. D. Stanley; to appear in Logistics Research Quarterly. (3) "On the solution of the caterer problem," by J. W. Gaddum, A. J. Hoffman, D. Sokolowsky; submitted to a technical journal. (4) "An algorithm for solving the transportation problem," by A. Gleyzal; IN MANUSCRIPT. (5) "Smooth patterns of production," by A. J. Hoffman and W. Jacobs; submitted to a technical journal. (6) "On the optimal ordering of items for a two-stage process," by A. J. Hoffman; IN MANUSCRIPT. (7) "On block relaxation," by L. S. Joel; IN MANUSCRIPT. (8) "A remark on the smoothing problem," by H. A. Antosiewicz and A. J. Hoffman; submitted to a technical journal.

RESEARCH IN MECHANICS OF CONTINUA
Task 1102-10-5116/54-23

Origin: NBS Authorized 12/29/53
Sponsor: Office of Naval Research, USN
Manager: R. F. Dressler
Full task description: Oct-Dec 1953 issue, p. 27

Status: CONTINUED. The analysis concerning thermodynamical effects on dissipative flow in rarefaction waves has been completed by R. F. Dressler, and a manuscript has been prepared. Solution is possible, with the aid of the Riemann relations along the three Mach lines, for first-order effects on entropy, density, and velocity. Results are expressible as seventh degree polynomials plus second order pole terms. Evaluation indicates that only a small difference results for the back half of the wave compared with results from a simpler flow model in which heat generation accompanying frictional retardation is ignored.

R. F. Dressler is completing work on a manuscript comparing various theoretical and experimental results on the dam-break problem. For large t, this includes comparison of previous mathematical results due to Dressler and more recent formulas by G. B. Whitham with experiments done at NBS. For small t, an analysis by F. V. Pohle is considered, using Green's functions applied to Lagrangian coordinates to predict the sudden collapse of a free vertical wall of water. The necessary experimental work to check this theory has been completed this quarter using high-speed motion picture photography.

P. Henrici has completed an investigation on slightly rotational axisymmetrical flow of a viscous fluid through a cylindrical tube. This is an extension of the work of Collatz and Görtler, who obtained an eigenvalue problem for a function proportional to the angular velocity component. The present results produce explicit asymptotic expressions for the large eigenvalues and eigenfunctions, and derive the low ones by an alternative calculation. A manuscript describing this work has been completed.

A compilation of Green's functions for the heat equation, the wave equation, and the potential equation, for various regions is being continued by F. Oberhettinger.

Publications: (1) "Turbulent flow in shock tubes of varying cross-section," by R. F. Dressler; to appear in the Journal of Research of the NBS. (2) "Entropy changes in rarefaction waves," by R. F. Dressler; IN MANUSCRIPT. (3) "Comparison of theories and experiments for the hydraulic dam-break wave," by R. F. Dressler; IN MANUSCRIPT. (4) "On helical elastic springs of finite cross-section," by P. Henrici; to appear in the Quarterly of Applied Mathematics. (5) "On rotational viscous flow through a tube," by P. Henrici; IN MANUSCRIPT.

RESEARCH IN ELECTROMAGNETIC THEORY
Task 1102-10-5126/54-47

Origin: Diamond Ordnance Fuze Laboratory, Department of Defense Authorized 6/29/54
Sponsors: " "
Manager: F. Oberhettinger

Objective: (1) To investigate the asymptotic expansion of certain integrals. (2) To investigate the convergence of various Green's function representations for the electromagnetic field in homogeneous

and inhomogeneous enclosed media.

Background: (1) This research treats the problem of the diffraction of electromagnetic waves by a wedge at large distances from the diffracting edge of the wedge. It adapts the results of task 1102-40-5126/54-51 to the problems of the sponsor. (2) This research is directed toward media having tensor permeabilities, such as the ferrite rotator. It is hoped that results will facilitate the prediction of the amount of rotation, the propagation constants, and the energy distribution.

Status: NEW. The problems are under study.

DIFFRACTION OF WAVES
Task 1102-40-5126/54-51

Origin: NBS, Division 14 Authorized 6/29/54
Sponsor: " "
Manager: F. Oberhettinger

Objective: To investigate the diffraction of waves by a wedge.

Background: This problem considers the diffraction of acoustic or electromagnetic plane, cylindrical, or spherical waves due to an infinite wedge.

Status: NEW. The problem was reduced (see publication (1) below) to the determination of an infinite integral of the form

$$I(\delta,\alpha) = \frac{1}{2\alpha}\sin(\pi\frac{\delta}{\alpha})\int_0^\infty \frac{H(x)dx}{\cosh(\frac{\pi}{\alpha}x) - \cos(\frac{\pi}{\alpha}\delta)}$$

where

$$H(x) = \begin{cases} e^{-ik\rho\cosh x} \\ H_0^{(2)}[k(\rho^2 + \rho'^2 + 2\rho\rho'\cosh x)^{\frac{1}{2}}] \\ \dfrac{e^{-ik[\rho^2+\rho'^2+2\rho\rho'\cosh x + (z-z')^3]^{\frac{1}{2}}}}{[\rho^2+\rho'^2+2\rho\rho'\cosh x + (z-z')^2]^{\frac{1}{2}}} \end{cases}$$

according as the incident field is a plane, a cylindrical, or a spherical wave. An asymptotic expansion for $I(\delta,\alpha)$ for large values of $k\rho$ (large distances from the edge of the wedge) which is valid also for $\delta=0$ (which defines the boundaries of one of the geometric shadows) was derived (see publication (2) below). This expression consists of a sum of a Fresnel integral as leading term and an ordinary asymptotic series with elementary coefficients.

Publications: (1) "Diffraction of waves by a wedge," by F. Oberhettinger; to appear in Communications on Pure and Applied Mathematics. (2) "On asymptotic series for functions occurring in the theory of diffraction of waves by wedges," by F. Oberhettinger; IN MANUSCRIPT.

2. Mathematical Tables and Experimental Computations

TABLES OF $E_1(z)$, $(z = x + iy)$
Task 1102-10-1104/43-3

Origin: Canadian National Research Council — Authorized 7/1/47
Manager: I. A. Stegun
Full task description: Apr-June 1949 issue, p. 41

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 28.

TABLE OF THE GAMMA FUNCTIONS FOR COMPLEX ARGUMENTS
Task 1102-10-1104/46-1

Origin: NBS — Authorized 7/1/47
Manager: J. Todd
Full task description: Apr-June 1949 issue, p. 43

Status: CONTINUED. The volume is in press. The final proofs have been checked.

Publication: "Table of the gamma function for complex arguments," NBS Applied Mathematics Series 34; in press, U. S. Government Printing Office.

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-10-1104/47-2

Origin: NBS — Authorized 7/1/47
Managers: M. Abramowitz, P. Rabinowitz
Full task description: Apr-June 1949 issue, p. 45

Status: INACTIVE. For status to date, see Jan-Mar 1954 issue, p. 28.

BIBLIOGRAPHY OF CODING PROCEDURES, MATHEMATICAL TABLES AND NUMERICAL ANALYSIS
Task 1102-10-1104/50-5

Origin: NBS — Authorized 3/1/50
Managers: J. Todd, J. H. Wegstein, P. Rabinowitz — Revised 1/9/53
Full task description: Oct-Dec 1952 issue, p. 40

Status: CONTINUED. Preparation of the bibliographies continues. The following Technical Memoranda were issued:

47 Subroutine for conversion of decimal degrees to binary radians and vice versa.
50 Automonitor processing routine.
51 Matrix inversion routine (internal storage, maximum order 19): modified von Neumann-Goldstine method.
52 Subroutine for $\log_e N$; single precision, fixed binary point.
53 Subroutine for evaluation of Bessel functions of the first kind of integral order, fixed binary point.
54 Matrix multiplication routine (external storage, maximum order 104).
55 Subroutine for arcsin x and arccos x; single precision, fixed binary point.
59 Matrix inversion routine (external storage, maximum order 50): modified elimination method.
61 Base codes 1,2,3 (libraries of subroutines)
62 Binary to decimal conversion and transfer routine; single precision, fixed binary point.
63 Subroutine for Arc tan x; single precision, fixed binary point.
64 Routine for transforming quadratic forms.
65 Routine for accelerating convergence of sequences (Aitken's method).

WAVE FUNCTION FOR LITHIUM
Task 1102-10-1104/50-7

Origin: NBS — Authorized 6/1/50
Sponsor: Bureau of Ordnance, USN
Manager: W. F. Cahill
Full task description: Apr-June 1950 issue, p. 36

Status: CONTINUED. The iteration for the second approximation to the wave function has converged. A code to compute a new approximation to the eigenvalue is being checked out.

COLLECTED SHORT MATHEMATICAL TABLES OF THE COMPUTATION LABORATORY
Task 1102-10-1104/51-4

Origin: NBS — Authorized 9/28/50
Manager: J. Todd
Full task description: July-Sept 1950 issue, p. 43

Status: CONTINUED. The first volume is in press.

Publication: "Tables of functions and of zeros of functions," Volume I of Collected Short Tables of the Computation Laboratory, NBS Applied Mathematics Series 37; in press, U. S. Government Printing Office.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Task 1304-34-6351/51-8

Origin: Section 11.3, NBS — Authorized 3/26/51
Managers: A. Hoffman, S. Tsingou
Full task description: Apr-June 1951 issue, p. 49

Status: INACTIVE. For status to date, see Jan-Mar 1953 issue, p. 48.

REVISION OF MATHEMATICAL TABLES
Task 1102-10-1104/52-7

Origin: NBS Authorized 8/10/51
Managers: J. Todd, W. F. Cahill, I. Stegun
Full task description: July-Sept 1951 issue, p. 41

Status: CONTINUED. Following is the status of those mathematical tables the sales stock of which has been exhausted and for which reissue is planned:

"Table of sine and cosine integrals for arguments from 10 to 100," NBS Applied Mathematics Series 32; in press, U. S. Government Printing Office. This will be a reissue of the table formerly designated as Mathematical Table MT13 (1942).

"Tables of the error function and its derivative," NBS Applied Mathematics Series 41; in press, U. S. Government Printing Office. This was originally Mathematical Table MT8, "Tables of probability functions," vol. I (1941).

"Tables of sines and cosines for radian arguments," Applied Mathematics Series 43; in press, U. S. Government Printing Office. This table was formerly designated as Mathematical Table MT4.

"Tables of natural logarithms," vol. IV, Mathematical Table MT12 (1941); revision in progress for reissue in the Applied Mathematics Series.

TABLE OF ARCSIN FOR COMPLEX ARGUMENTS
Task 1102-10-1104/52-14

Origin: NBS Authorized 10/1/51
Manager: D. Liepman
Full task description: July-Sept 1951 issue, p. 41

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 31.

EXTENSION OF THE TABLE OF HYPERBOLIC SINES AND COSINES
Task 1102-10-1104/52-18

Origin: NBS Authorized 9/17/51
Manager: W. F. Cahill, K. Nelson
Full task description: July-Sept 1951 issue, p. 41

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 31.

TABLE OF THE MODIFIED AIRY INTEGRAL
Task 1102-10-1104/52-23

Origin: NBS Authorized 10/4/51
Manager: P. Rabinowitz
Full task description: July-Sept 1951 issue, p. 42

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 31.

TABLE OF ERROR FUNCTION FOR COMPLEX ARGUMENTS
Task 1102-10-1104/52-25

Origin: NBS Authorized 10/5/51
Manager: W. G. Hall
Full task description: July-Sept 1951 issue, p. 42

Status: INACTIVE. The 20-place table for x,y=0(.1)2 is available on punched cards.

EXTENSION OF TABLES OF THE EXPONENTIAL FUNCTION FOR NEGATIVE ARGUMENTS
Task 1102-10-1104/52-31

Origin: NBS Authorized 10/9/51
Manager: E. Marden, S. B. Prusch
Full task description: July-Sept 1951 issue, p. 43

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 32.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-10-1104/52-37

Origin: NBS Authorized 11/28/51
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE. For status to date, see Jan-Mar 1954 issue, p. 31.

VAN DER POL EQUATION
Task 1102-10-1104/52-43

Origin: NBS Authorized 11/28/51
Manager: W. F. Cahill
Full task description: Oct-Dec 1951 issue, p. 38

Status: TERMINATED. For status to date, see Apr-June 1952 issue, p. 49.

RADIAL MATHIEU FUNCTIONS
Task 1102-10-1104/52-49

Origin: NBS Authorized 2/1/52
Managers: J. Todd, I. Rhodes
Full task description: Jan-Mar 1952 issue, p. 45

Status: INACTIVE. For status to date, see Jan-Mar 1954 issue, p. 32.

SIEVERT'S INTEGRAL
Task 1102-10-1104/52-57

Origin: NBS Authorized 2/12/52
Managers: M. L. Paulsen, P. J. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 33.

PAINLEVÉ EQUATION
Task 1102-10-1104/53-3

Origin: NBS Authorized 8/11/52
Managers: J. Todd, H. A. Antosiewicz
Full task description: July-Sept 1952 issue, p. 49

Status: TERMINATED.

HYPERGEOMETRIC FUNCTIONS
Task 1102-10-1104/53-35

Origin: NBS Authorized 3/25/53
Managers: P. Rabinowitz, W. Cahill
Full task description: Jan-Mar 1953 issue, p. 36

Status: CONTINUED. The first three roots of the confluent hypergeometric function $F(1-B;2;4B)$ were computed for P. Henrici.

The hypergeometric function $F(26,26;53;\rho^2)$, $\rho^2=0.05,0.1(0.1)0.8$, 10S, was computed as exploratory work for a problem from the Air Force Cambridge Research Center.

L-SHELL CONVERSION COEFFICIENTS
Task 1102-10-5110/53-52

Origin: Oak Ridge National Laboratory Authorized 5/20/53
Manager: W. G. Hall, J. H. Wegstein
Full task description: Apr-June 1953 issue, p. 45

Status: CONTINUED. Machine codes for all required calculations

have been completed.

Present plans call for the calculation of conversion coefficients for the L_1,L_2 and L_3 shells for atomic numbers Z=15(10)95, for photon energies K=0.05(.05).2(.2)1.0(.5)2.0, for both electric and magnetic multipoles of multiplicities 2^x, x=1(1)5. In addition, a certain number of calculations for the K-shell, especially for low energies, are to be included. The computations proceed in three parts, of which Part I results in the screening function for the Thomas-Fermi-Dirac statistical model of the atom, and Part II in the eigenvalues for the ground state wave functions. Part III obtains the wave functions both for the ground state and for the disturbed state, based on the previously obtained eigenvalue, as well as the wave function of the photon, and evaluates the convolution integrals resulting in the conversion coefficients.

Of this program, the following portions have been completed to date:

Part I has been carried out for all required atomic numbers. The resulting screening functions are on file in manuscript form.

Part II was completed for the L_1 and L_2 shells for all desired Z, and for the K shell for all except Z=95. It was subsequently found possible to improve these values by a simple adaptation of the code for part III; this is currently being done simultaneously with the computations under Part III. For the L_3 shell, the method first employed in Part II failed; these eigenvalues will be obtained by means of the adapted code of Part III.

Part III has been completed for the L_1 shell for Z=15,25,55,75,85; and for the L_2 shell for Z=25,55,85. Values for the K shell have been completed for Z=25,35,55,75,85, for comparison with previously published results. Each set of coefficients requires almost three hours of automatic SEAC operation for its computation. During that time, 18 distinct codes, totalling 7,526 SEAC words, are read into the machine from wire cartridges.

3. Mathematical Services

Note: The tasks under Mathematical Services are arranged serially according to the digits following the slant lines in the task number. The first two digits following the slant line designate the fiscal year in which the task was authorized.

TABLES OF THERMODYNAMIC PROPERTIES OF GASES
Task 0302-40-2606/49-5

Origin: NBS, Section 3.2 Authorized 11/29/48
Sponsor: National Advisory Committee for Aeronautics
Manager: F. L. Alt
Full task description: Apr-June 1949 issue, p. 53

Status: TERMINATED. This task is superseded by task 1102-40-5126/53-27, p. 31.

RAY TRACING
Task 0202-10-2308/50-13

Origin: NBS, Section 2.2 Authorized 3/1/50
Sponsor: " "
Manager: R. K. Anderson
Full task description: Jan-Mar 1950 issue, p. 33

Status: CONTINUED. Completed ray tracing and spot diagram codes have been "debugged" and improvements have been incorporated. The Metrogon and Planigon aerial camera lenses, two triplet lenses, and a heliar lens have been analyzed.

MOLECULAR STRUCTURE CALCULATIONS, II
Task 1102-40-5126/50-16

Origin: Naval Research Laboratory, USN Authorized 3/31/50
Sponsor: " "
Manager: P. J. O'Hara
Full task description: Jan-Mar 1950 issue, p. 34

Status: TERMINATED. This task is superseded by task 1102-40-5126/51-37.

LIQUID-VAPOR TRANSITION
Task 1102-40-5126/51-22

Origin: Naval Medical Research Institute Authorized 2/1/51
Sponsor: " "
Manager: I. Stegun
Full task description: Jan-Mar 1951 issue, p. 44

Status: TERMINATED. The formulation of new work is being considered by the sponsor and will be reported under a new task when it is decided upon.

MOLECULAR STRUCTURE, III
Task 1102-40-5126/51-37

Origin: Naval Research Laboratory, USN Authorized 8/10/51
Managers: P. J. O'Hara, I. A. Stegun
Full task description: July-Sept 1951 issue, p. 50

Status: CONTINUED. Computations were performed as requested.

SHOCK WAVE PARAMETERS, II
Task 1102-40-5126/51-38

Origin: Bureau of Ordnance, USN — Authorized 6/28/51
Sponsor: " "
Manager: I. A. Stegun
Full task description: Apr-June 1951 issue, p. 46

Status: TERMINATED. Current work growing out of this task is being performed under task 1102-40-5126/54-34, p. 38.

PRECISE DETERMINATION OF THE PARAMETER OF DISPERSION EQUATION FOR SEVERAL TYPES OF OPTICAL GLASS
Task 1102-40-5126/52-17

Origin: NBS, Division 2 — Authorized 9/29/51
Sponsor: " "
Manager: I. A. Stegun
Full task description: July-Sept 1951 issue, p. 56

Status: TERMINATED. No further work is currently contemplated.

SPHERICAL BLAST
Task 1102-40-5126/52-20

Origin: Naval Ordnance Laboratory — Authorized 9/27/51
Sponsor: " "
Manager: D. H. Jirauch
Full task description: July-Sept 1951 issue, p. 56

Status: CONTINUED. A final report is in preparation.

CALCULATIONS FOR d SPACINGS
Task 1102-40-5126/52-44

Origin: NBS, Div. 9 — Authorized 12/7/51
Sponsor: " "
Managers: R. K. Anderson, A. M. Futterman
Full task description: Oct-Dec 1951 issue, p. 47

Status: CONTINUED. Programming for SEAC continued. Test cases are being checked in.

GAS ADSORPTION BY HIGH POLYMERS
Task 1102-40-5126/52-70

Origin: Naval Medical Center, Bethesda, Md. — Authorized 4/1/52
Sponsor: " "
Manager: I. Stegun
Full task description: Jan-Mar 1952 issue, p. 59

Status: TERMINATED. The formulation of future work is being considered by the sponsor and will be reported under a new task when it is decided upon.

NEUTRON DIFFUSION, II
Task 1102-40-5126/53-4

Origin: Atomic Energy Commission, New York Office, (NDA) Authorized 9/30/52
Sponsor: " " "
Manager: W. G. Hall
Full task description: July-Sept 1952 issue, p. 60

Status: CONTINUED. Computations are being performed on SEAC as requested, and the results are being sent to the sponsor as they are completed.

STRENGTH OF WING COMPONENTS
Task 1102-40-5126/53-11

Origin: National Advisory Committee for Aeronautics, Langley Field, Va. Authorized 12/8/52
Sponsor: " " "
Manager: E. Marden
Full task description: Oct-Dec 1952 issue, p. 54

Status: CONTINUED. Computations are being performed on SEAC as requested.

RADIANT HEATING OF SOLIDS
Task 1102-40-5126/53-20

Origin: NBS, Section 10.2 Authorized 12/15/52
Sponsor: " "
Manager: W. F. Cahill
Full task description: Oct-Dec 1952 issue, p. 55

Status: INACTIVE.

COMPUTATION OF THERMODYNAMIC FUNCTIONS
Task 1102-40-5126/53-27

Origin: NBS, Division 5 Authorized 3/18/53
Sponsor: " "
Manager: E. Marden
Full task description: Jan-Mar 1953 issue, p. 57

Status: CONTINUED. Calculations were performed with revised codes and the results were transmitted to the sponsor.

Status of Projects

STUDY OF TRICALCIUM ALUMINATE
Task 1102-40-5126/53-28

Origin: NBS, Division 9 — Authorized 3/30/53
Sponsor: " "
Manager: R. Anderson
Full task description: Jan-Mar 1953 issue, p. 58

Status: TERMINATED, pending suitable formulation by the problem originator.

DYNAMIC BEHAVIOR OF AIRCRAFT STRUCTURES
Task 1102-40-5126/53-29

Origin: NBS, Section 6.4 — Authorized 1/23/53
Sponsor: " "
Manager: I. Rhodes
Full task description: Jan-Mar 1953 issue, p. 58

Status: CONTINUED. Computations are being performed as requested, and results are being transmitted to the sponsor.

ACOUSTICAL IMPEDANCES
Task 1102-40-5126/53-39

Origin: NBS, Section 6.1 — Authorized 6/30/53
Sponsor: " "
Managers: S. Prusch, K. Nelson
Full task description: Apr-June 1953 issue, p. 55

Status: INACTIVE. For status to date, see Apr-June 1953 issue, p. 55.

LORAN UNIVAC CODE
Task 1102-40-5126/53-41

Origin: Hydrographic Office, U. S. Navy — Authorized 4/20/53
Sponsor: " "
Managers: I. Rhodes, D. H. Jirauch
Full task description: Apr-June 1953 issue, p. 56

Status: CONTINUED. The three routines are coded. The station pair code has been checked and at present is being run by the sponsor. The baseline code and the skywave correction and trainer code are awaiting time on the UNIVAC for checking.

RADIATION DIFFUSION
Task 1102-40-5126/53-51

Origin: NBS, Section 4.8 Authorized 6/9/53
Sponsor: Armed Forces Special Weapons Project
Manager: W. G. Hall
Full task description: Apr-June 1953 issue, p. 57 (Neutron Diffusion III)

Status: CONTINUED. The code for the first phase of this problem has been modified to incorporate a recent development. A code treating the spatial distribution of photons analytically is near completion.

HIGH TEMPERATURE PROPERTIES OF WATER
Task 1102-40-5126/54-4

Origin: NBS, Division 3.2 Authorized 12/7/53
Sponsor: Bureau of Ordnance, USN
Manager: J. H. Wegstein
Full task description: Oct-Dec 1953 issue, p. 41

Status: CONTINUED. H. W. Woolley has prepared a complete set of formulas for the computation of the thermal functions $Q,-(F^{O}-E^{O}_{O})/RT$, $(H^{O}-E^{O}_{O})/RT$, S^{O}/R, and C^{O}_{p}/R for one ionic species O_{VIII}. These equations, apart from changes in constants, can be applied to any ion which has only one bound electron. These rather elaborate formulas are being coded for evaluation with the SEAC using the interpretive floating point routines (Base 00).

Similar formulas are being prepared by Mr. Woolley for each of the other ionic species of oxygen.

DISTRIBUTION OF NORMAL MODES OF VIBRATION OF CUBIC LATTICES, II
Task 3711-60-0009/54-7

Origin: NBS, Division 30 Authorized 12/29/53
Sponsor: " "
Managers: W. F. Cahill
Full task description: Oct-Dec 1953 issue, p. 42

Status: CONTINUED. Two codes to compute the distribution of normal modes of vibration for body-centered and face-centered cubic lattices have been checked out. Each was run for 13,824 points (N=48). This involved generating 27,648 3x3 matrices, and finding their characteristic values.

NORMAL VIBRATIONS IN MOLECULES
Task 3711-60-0009/54-8

Origin: NBS, Section 3.2 Authorized 11/27/53
Sponsor: " "
Manager: M. Newman
Full task description: Oct-Dec 1953 issue, p. 42

Status: CONTINUED. During this quarter the eigenvalues of a symmetric matrix of order 14 were obtained for the originator.

COMPUTATION OF VIBRATION MODES AND FREQUENCIES
Task 3711-60-0009/54-11

Origin: NBS, Section 6.4 Authorized 11/27/53
Sponsor: " "
Managers: I. Rhodes, W. Cahill
Full task description: Oct-Dec 1953 issue, p. 42

Status: INACTIVE. For status to date, see Jan-Mar 1954 issue, p. 40.

Publication: "Computation of vibration modes and frequencies on SEAC," by W. F. Cahill and S. Levy (NBS Section 6.4); IN MANUSCRIPT.

SPACE AVERAGE OF TEMPERATURE DISTRIBUTION WITH A VOLUME DISTRIBUTION OF SOURCES
Task 3711-60-0009/54-12

Origin: NBS, Division 6 Authorized 12/11/53
Sponsor: " "
Manager: E. Marden
Full task description: Oct-Dec 1953 issue, p. 43

Status: COMPLETED. The results have been transmitted to the originator.

AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Task 1102-40-5126/54-13

Origin: New York Quartermaster Procurement Agency Authorized 11/27/53
Sponsor: " " "
Manager: H. Bremer
Full task description: Oct-Dec 1953 issue, p. 43

Status: CONTINUED. Ten bid evaluation problems were submitted by the NYQMPA during the quarter. The answers were obtained on SEAC and sent back usually within two days after receipt of the problem. One problem had to be solved using the general simplex method, all others used the special "transportation problem" routine. The code for UNIVAC is now being checked in cooperation with personnel at the Office of the Air Comptroller, USAF.

Status of Projects

FLOW IN SUPERSONIC NOZZLE
Task 1102-40-5126/54-14

Origin: National Advisory Committee for Aeronautics, Langley Field, Va. Authorized 12/16/53
Sponsor: " " "
Managers: P. Henrici, E. Marden, G. Hawkins
Full task description: Oct-Dec 1953 issue, p. 44

Status: CONTINUED. Computations for supersonic flow in nozzles for four additional Mach numbers have been completed and transmitted to the originator.

DEPOLYMERIZATION
Task 3711-60-0009/54-17

Origin: NBS, Section 7.6 Authorized 12/16/53
Sponsor: " "
Manager: R. Anderson
Full task description: Oct-Dec 1953 issue, p. 44

Status: INACTIVE. For Status to date, see Oct-Dec 1953 issue, p. 44.

ENERGY LEVELS OF COMPLEX ATOMS
Task 3711-60-0009/54-19

Origin: NBS, Section 4.1 Authorized 3/31/54
Sponsor: " "
Manager: W. G. Hall
Full task description: Jan-Mar 1954 issue, p. 41

Status: CONTINUED. Coding is now being performed by the sponsor.

LOW TEMPERATURE PROPERTIES OF BORON COMPOUNDS
Task 1102-40-5126/54-20

Origin: NBS, Section 3.2 Authorized 2/24/54
Sponsor: " "
Manager: J. H. Wegstein
Full task description: Jan-Mar 1954 issue, p. 41

Status: INACTIVE.

EXPERIMENTAL PROGRAM FOR MULTIPLE INPUT AND OUTPUT
Task 1102-40-5126/54-21

Origin: NBS, Section 12.3 Authorized 2/24/54
Sponsor: " "
Manager: I. Rhodes
Full task description: Jan-Mar 1954 issue, p. 42

Status: CONTINUED. The combined sorting-file merging program was adapted for use in an experimental interconnection of SEAC and DYSEAC and was demonstrated for the Bureau of Supplies and Accounts during the quarter.

ENERGY DISTRIBUTIONS ON OPTICAL IMAGE
Task 3711-60-0009/54-22

Origin: NBS, Section 2.2 Authorized 2/9/54
Sponsor: " " Revised 3/30/54
Manager: L. S. Joel
Full task description: Jan-Mar 1954 issue, p. 43

Status: CONTINUED. A code for the computation of $I(\xi,\eta)$ has been completed and checked for $(\xi,\eta)=(1,1),(1,6),(6,1),(6,6)$.

SOLUTION OF THE DIFFUSION EQUATION FOR ELECTRONS
Task 1102-40-5126/54-24

Origin: NBS, Division 4 Authorized 3/5/54
Sponsor: " "
Managers: M. Abramowitz, A. Futterman
Full task description: Jan-Mar 1954 issue, p. 43

Status: INACTIVE. This problem awaits further specifications from the originator. For status to date see Jan-Mar 1954 issue, p. 43.

DEFLECTED RADIATION FROM AN INFINITE LAMBERT PLANE
Task 1102-40-5126/54-25

Origin: Armed Forced Special Weapons Group Authorized 2/9/54
Sponsor: " " "
Managers: P. Henrici, L. S. Joel
Full task description: Jan-Mar 1954 issue, p. 44

Status: CONTINUED. Values of $\psi(u,v)$ are being computed for $u,v=.25(.25)16.0$.

EMF OF CELLS AT HIGH TEMPERATURE
Task 3711-60-0009/54-27

Origin: NBS, Section 1.8 Authorized 3/5/54
Sponsor: " "
Managers: E. Marden, G. Hawkins
Full task description: Jan-Mar 1954 issue, p. 45

Status: CONTINUED. The coding of this problem has been completed and the electromotive forces of all chlorides and bromides have been computed.

LOVIBOND NETWORK FOR CIE SOURCE A
Task 3711-60-0009/54-28

Origin: NBS, Section 2.1 Authorized 2/28/54
Sponsor: " "
Managers: E. Marden, G. Hawkins
Full task description: Jan-Mar 1954 issue, p. 46

Status: CONTINUED. The computation of the Lovibond networks for CIE sources A and C has been completed. In addition, slight modifications were made in the code to do special weekly calculations for the originator.

SPECTRUM ANALYSIS
Task 3711-60-0009/54-30

Origin: NBS, Division 4 Authorized 3/4/54
Sponsor: " "
Manager: S. Prusch
Full task description: Jan-Mar 1954 issue, p. 46

Status: CONTINUED. Differences between all known and predicted odd levels were each used to search for pairs of lines which confirm odd levels and predicted new even levels. This was done for Tc I. Data for over 6,000 wave lengths were punched for the element ruthenium.

AIR CONDITIONING IN UNDERGROUND STRUCTURES
Task 1102-40-5126/54-33

Origin: NBS, Section 10.3 Authorized 2/24/54
Sponsor: " "
Manager: S. Tsingou
Full task description: Jan-Mar 1954 issue, p. 47

Status: CONTINUED. The coding has been completed, and the computations are being performed in accordance with specifications of the problem originator.

EQUILIBRIUM CALCULATIONS FOR WATER
Task 1102-40-5126/54-34

Origin: Naval Ordnance Laboratory — Authorized 5/31/54
Sponsor: " "
Manager: J. Wegstein

Objective: To solve systems of simultaneous transcendental equations containing 40 or more unknowns describing the equilibrium among the species resulting from high-temperature decomposition of water.

Background: Thermodynamic properties of various gaseous mixtures are needed for temperatures up to 1,000,000°K and for gaseous densities of 0.5 to 2.0. For this range of temperatures and densities the familiar ideal-gas treatment is not applicable. It is necessary to account for the imperfect gas behavior and for the long distance forces between ionic particles and electronic excitation. In neither case a satisfactory theoretical approach is known, but various approximation procedures may be applied. A flexible procedure is necessary to account for the different approximation methods.

Status: NEW. The problem has been coded and a number of cases have been solved and submitted to the originator. Modifications of the original formulation are now being considered.

THERMAL STRESSES IN STRUCTURES
Task 1102-40-5126/54-35

Origin: NBS, Section 6.4 — Authorized 6/29/54
Sponsor: " "
Manager: W. Hall

Objective: To consider the effect of thermal transients on stress distribution in metal beams, especially beams of materials used in modern aircraft structures. A method for predicting temperature distribution due to conduction, radiation, and convection must first be found. Temperature dependent changes in elastic and thermal properties of the material and in the thermal resistance of joints are to be taken into account. The investigation is aimed at constructing mathematical models which will conform realistically to the physical phenomena.

Background: The large thermal stresses developed in high speed aircraft and missiles make it desirable to develop methods of thermal stress and temperature distribution analysis which will be useful as a basis for rational engineering design. It is believed that the use of numerical methods in conjunction with high-speed automatic computing machines may be successfully used in such analyses.

Status: NEW. Some results have been submitted to the sponsor.

VELOCITY OF LIGHT
Task 3711-60-0009/54-36

Origin: NBS, Section 11.3 Authorized 3/16/54
Sponsor: " "
Manager: P. Rabinowitz
Full task description: Jan-Mar 1954 issue, p. 47

Status: CONTINUED. Results are transmitted to the originator whenever required. A double precision floating matrix inversion routine, handling matrices of order up to 85, was coded for checking purposes.

CORRELATION OF IONOSPHERIC DATA
Task 3711-60-0009/54-37

Origin: NBS, Section 14.1 Authorized 3/12/54
Sponsor: " "
Managers: J. H. Wegstein, H. H. Howe
Full task description: Jan-Mar 1954 issue, p. 48

Status: COMPLETED. The computations were made successfully with the three-address SEAC. Besides developing several useful subroutines, H. H. Howe made several suggestions for improving the logic of the three-address system. The results of the computations were transmitted to the originator.

COMPRESSIBILITY FACTORS OF DRY AIR
Task 3711-60-0009/54-38

Origin: NBS, Section 3.2 Authorized 3/29/54
Sponsor: " "
Manager: M. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48

Status: INACTIVE. For status to date, see Jan-Mar 1954 issue, p. 48.

FREQUENCY CORRELATION
Task 1102-40-5126/54-41

Origin: NBS, Division 6 Authorized 6/29/51
Sponsor: " "
Manager: L. S. Joel

Objective: To evaluate

$$R = 0.200 + 0.800 \sum_{i=1}^{n} \sum_{j<i} \cos \frac{2\pi}{\lambda} [(k'-k)r_{ij}]$$

where k', k, r_{ij} are vectors:

$$k' = (\cos \theta', \sin \theta'), \quad k = (\cos \theta, \sin \theta)$$

$$r_{ij} = r_i - r_j, \quad r_i = (x_i, y_i);$$

input data: k or θ, k' or θ', λ, r_i.

Background: The problem is concerned with the evaluation of correlation coefficients for an array of microphones in a plane exposed to a sound wave.

Status: The first problem has been completed except for two values out of 1100. For the initial run R was evaluated for n=5, $\theta=-\pi/6$, $\theta'=0(\pi/50)2\pi$, for 11 values of λ.

RESOLUTION CORRECTION FOR SCINTILLATION SPECTROMETER
Task 1102-40-5126/54-42

Origin: NBS, Section 4.13 Authorized 6/4/54
Sponsor: " "
Manager: M. Newman

Objective: To invert a number of matrices of orders 9, 10 and 19, and to solve systems of equations having these matrices as coefficient matrices.

Background: The energy resolution of a scintillation spectrometer is strongly dependent on the quantum energy of the radiation. This fact causes a distortion of the continuous spectra measured by this instrument and necessitates a correction. This correction is made by solving systems of linear equations relating the true spectral distribution to the measured spectral distribution by means of a correction matrix embodying experimental resolution data.

Comment: This task was proposed by M. Ehrlich, NBS Section 4.13.

Status: NEW. Thirteen systems of equations of order 10, three of order 9, and two of order 19 have been solved. In addition, the coefficient matrices for these systems have been inverted.

CHARACTERISTICS OF CONDUCTING RESISTORS
Task 1102-40-5126/54-43

Origin: NBS, Section 12.1 Authorized 6/29/54
Sponsor: " "
Manager: R. K. Anderson

Objective: For each test resistor, experimental data gives values of I (direct current in milliamperes) and corresponding p (average of open circuit noise voltage squared per cycle per second). It is required to compute values of constants a and b in order to determine the best least-squares fit of

$$\log P = a + b \log I,$$

and to obtain further quantities needed for the analysis of variance.

Background: It is generally known that composition resistors, when conducting, generate a fluctuating potential which under normal conditions is greater than that due to thermal agitation. It is the purpose of this task to study commercial and experimental resistors and to develop standard test procedures and specifications for military use.

Comment: This task was proposed by G. T. Conrad (NBS Section 12.1).

Status: NEW. Programming is in progress.

COMPUTATION OF TRANSIENT HEAT FLOW PROBLEMS
Task 1102-40-5126/54-45

Origin: NBS, Section 10.2 Authorized 6/4/54
Sponsors: " "
Manager: W. F. Cahill

Objective: To obtain data on transient heat flow through various geometrical configurations which closely approximate structures tested by the NBS Fire Protection Section (10.2).

Background: For many years the Fire Protection Section has been testing the heat resistance of structures such as columns, floors, and panels. The necessity for theoretical studies of heat flow through these structures has been recognized by the staff of the section for a number of years, but many important calculations were beyond the realm of possibility until the invention of the high speed digital computer.

Status: NEW. A code, using finite differences, to compute two-dimensional heat flow has been composed and checked out.

STUDY IN OPTIMIZATION OF POWER OUTPUT
Task 1102-40-5126/54-46

Origin: Raytheon Manufacturing Company Authorized 5/18/54
Sponsor: U. S. Corps of Engineers
Manager: L. Gainen

Objective: To solve several linear programming problems by the simplex method.

Background: These problems are based on a linear approximation to the question of determining desirable levels of storage and rates of change of storage of water in a sequence of dams for the purpose of optimizing the output of electrical power.

Status: NEW. Two problems were submitted by the originator for solution, one involving the use of four dams and a second assuming only three existing dams. The first of these problems, solved on the SEAC by the simplex method as a set of 33 equations in 152 variables, is complete and the results have been transmitted to the originator. The second problem is being computed as a set of 22 equations in 114 variables. These problems are being solved in dualized form because of a limitation on the column size imposed by the SEAC coding, and because the dual allows use of a natural basis. Forty simplex iterations were required to compute the 33 by 152 set.

III. STATISTICAL ENGINEERING LABORATORY
(Section 11.3)

1. Fundamental Research in Mathematical Statistics

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-10-1107/49-1a

Origin: NBS — Authorized 1/9/49
Manager: L. S. Deming
Full task description: Apr-June 1949 issue, p. 75

Status: CONTINUED. For a description of the continuing activity on this task, see the Jan-Mar 1954 issue, p. 49.

MANUAL ON FITTING STRAIGHT LINES
Task 1103-10-1107/50-2

Origin: NBS — Authorized 3/1/50
Manager: F. S. Acton
Full task description: Jan-Mar 1950 issue, p. 42

Status: INACTIVE. For status to date see Oct-Dec 1952 issue, p. 58.

TABLES TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-10-1107/51-1

Origin: NBS — Authorized 7/1/50
Managers: C. Eisenhart and L. S. Deming
Full task description: July-Sept 1950 issue, p. 57

Status: INACTIVE. For status to date see July-Sept 1952 issue, p. 64.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-10-1107/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED.

Publications: (1) "On strongly continuous stochastic processes," by E. Lukacs; Sankhyā, 13, 219-228 (Mar. 1954). (2) "A property of the normal distribution," by E. Lukacs and E. P. King; Ann. Math. Stat. 25, 389-394 (June 1954). (3) "Nonnegative trigonometric polynomials and certain rational characteristic functions," by E. Lukacs and O. Szász; J. Res. NBS 52, 153-160 (Mar. 1954). (4) "Certain Fourier transforms of distribution, II," by E. Lukacs and O. Szász; Canadian J. Math. 6, 186-189 (Mar. 1954). (5) "On optimum grouping in one-criterion variance components analysis," by E. P. King; accepted for publication in the Journal of the American Statistical Association. (6) "Tables of inverses of finite segments of the Hilbert matrix," by I. R. Savage and E. Lukacs; to be included in NBS Applied Mathematics Series 39, "Contributions to the Solution of Systems of Linear Equations and the Determination of Eigenvalues." (7) "Bounds on a distribution function which are functions of moments to order four," by M. Zelen; accepted for publication in the Journal of Research of the NBS. (8) "Time-discrete stochastic processes in arbitrary sets, with applications to processes with absorbing regions and to the problem of loops in Markoff chains," by D. van Dantzig; accepted for publication (in French) in Annales de l'Institut Henri Poincaré (Paris). (9) "The use of extreme-value methods in engineering problems," by J. Lieblein; submitted to a technical journal. (10) "On the variances and covariances of order statistics from the Weibull distribution," by J. Lieblein; submitted to a technical journal. (11) "Two early papers on the relation between extreme-values and tensile strength," by J. Lieblein; submitted to a technical journal.

LAW OF PROPAGATION OF ERROR
Task 1103-10-1107/52-1

Origin: NBS Authorized 6/23/51
Managers: C. Eisenhart and I. R. Savage
Full task description: July-Sept 1951 issue, p. 65

Status: INACTIVE. For status to date see July-Sept 1951 issue, p. 65.

PROCEDURES OF NON-PARAMETRIC STATISTICS
Task 1103-10-1107/52-2

Origin: NBS Authorized 9/17/51
Manager: I. R. Savage
Full task description: July-Sept 1951 issue, p. 66

Status: CONTINUED. The work of I. R. Savage described in the preceding issue has been completed and has been prepared as an NBS report. After suitable revision it will be submitted to a technical journal.

Publications: (1) "Most probable rank orders," by I. R. Savage; withdrawn from publication. (2) "Easily used simultaneous confidence limits for a line," by W. S. Connor; submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-10-1107/53-1

Origin: NBS Authorized 10/15/52
Manager: W. S. Connor
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED.

Publications: (1) "New experimental designs for paired observations," by W. S. Connor and W. J. Youden; accepted for publication by Journal of Research of the NBS. (2) "Analysis for some partially balanced incomplete block designs having a missing block," by M. Zelen; accepted for publication in Biometrics. (3) "A note on partially balanced designs," by M. Zelen; accepted for publication in the Annals of Mathematical Statistics. (4) "Partially balanced incomplete block designs with two associate classes and two treatments per block," by W. H. Clatworthy; submitted to a technical journal. (5) "On the enumeration of partially balanced designs with two associate classes," by W. H. Clatworthy; submitted to a technical journal. (6) "Covariance analysis for incomplete block designs," by M. Zelen; IN MANUSCRIPT.

2. Applied Research in Mathematical Statistics

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden and J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. Activity under this project fell into two main categories:

A. Design of Experiments: In an experiment to determine the relative performance of 120 samples of cement on a gain in moisture test, only 15 samples could be accommodated in a run. The problem was to arrange samples to be assigned to each run so that the statistical analysis would largely remove the effects of different conditions from run-to-run from the average observed rates of gain for the samples. This was achieved by the use of a linked block design.

B. Development or Selection of the Appropriate Method for Analysis and Interpretation of Data: A method for providing mutually consistent conversion ratios for the activities of four national radium standards was devised. A further check upon the work was provided by comparing these adjusted ratios with those obtained from the weights of the samples to obtain the best estimates of a linear law relating activity to the mass of the sample in a situation where both measurements are

subject to errors.

Publications: (1) "Probability limits for the average chart when process standards are unspecified," by E. P. King; Industrial Quality Control X, 62-64 (May 1954). (2) "A comparison of four National Radium Standards, II," by W. J. Youden and W. S. Connor; IN MANUSCRIPT.

STATISTICAL ASPECTS OF NBS ADMINISTRATIVE OPERATIONS
Task 3737-60-0002/52-1

Origin: NBS Authorized 10/1/51
Manager: I. R. Savage
Full task description: Oct-Dec 1951 issue, p. 56

Status: INACTIVE. For status to date see July-Sept 1952 issue, p. 68.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin: Ship Structure Committee, NRC Authorized 12/1/51
Sponsor: " "
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: CONTINUED. The discriminant function for classifying steel plates as source, through, or end plates is being recomputed because of the redetermination of ferrite grain size by a conventional method of measurement.

The statistical analysis carried out on data obtained by thirteen cooperating laboratories in an interlaboratory study of notching and testing effects is being described for publication in a suitable technical journal.

RESEARCH IN APPLICATIONS OF MATHEMATICAL STATISTICS TO PROBLEMS OF THE CHEMICAL CORPS
Task 1103-40-5118/52-1

Origin: Biological Laboratories, Chemical Corps, Dept. of the Army Authorized 10/1/51
Sponsor: " " "
Manager: C. Eisenhart
Full task description: Oct-Dec 1951 issue, p. 57

Status: CONTINUED. A tabulation of fractional replications of the 2^n series of factorial designs arranged in incomplete blocks was nearing completion at the end of this quarter. A report containing papers presented at the Area Weapons Conference was prepared.

STATISTICAL ANALYSIS AND DESIGN OF EXPERIMENTS FOR THE U. S. GEOLOGICAL SURVEY
Task 1103-40-5140/54-1

Origin: U. S. Geological Survey, Department of Interior — Authorized 10/9/53
Sponsor: " "
Managers: C. Eisenhart and W. J. Youden
Full task description: Oct-Dec 1953 issue, p. 50

Status: CONTINUED. The study of a method of analysis of orientation data was continued. In addition correspondence was exchanged with individual members of the U. S. Geological Survey on such topics as design of experiments for determining precision and accuracy of certain chemical laboratory test methods and method of sampling for some lithology studies.

STATISTICAL ANALYSIS OF BALL BEARING FATIGUE DATA
Task 1103-40-5145/54-1

Origin: American Standards Association — Authorized 1/13/54
Sponsor: "
Manager: J. Lieblein
Full task description: Jan-Mar 1954 issue, p. 54

Status: CONTINUED. (1) At a meeting held May 28, 1954, in New York City between the Sub-Committee and Bureau representatives certain aspects of the nature of the ball bearing data at hand were clarified, which made it possible to identify statistically homogeneous subsets for study. (2) Research is being carried out on a method of deriving improved life estimates from the test groups of ball bearing data. The method is designed for use with "censored" data, in which only the lowest k out of n observations are available. For the case of ball bearing fatigue data this corresponds to a test group where the test is discontinued before all the bearings in the group have failed, so that the endurance lives of those remaining intact are necessarily unavailable. The method based on order statistics is derived from that developed in National Advisory Committee for Aeronautics Technical Note 3053, by Julius Lieblein, entitled "A new method of analyzing extreme-value data". After the necessary computations are completed the applicability of the method will be tested on the data at hand.

IV. MACHINE DEVELOPMENT LABORATORY

(Section 11.4)

in cooperation with

ELECTRONIC COMPUTER SECTION

(Section 12.3)

THE BUREAU OF THE CENSUS COMPUTING MACHINE
Task 1104-34-5107/47-1

Origin: The Bureau of the Census — Authorized 7/1/47
Sponsor: " "
Full task description: Apr-June 1949 issue, p. 58

Status: TERMINATED. Stockpiling of replacement parts for UNIVAC System No. 1, under Contract CST-10279, has been completed with the exception of a few minor items to be delivered within the next quarter. With the delivery of the latest versions of the maintenance and operating instruction books, the technical manual requirements of the contract have been met. It is expected that the contract will be terminated by September.

NATIONAL BUREAU OF STANDARDS EASTERN AUTOMATIC COMPUTER (SEAC)
Task 1104-34-5107/49-1

Origin: NBS — Authorized 12/15/48
Sponsor: Office of the Air Comptroller, USAF
Full task description: Apr-June 1949 issue, p. 59

Status: CONTINUED. During the past quarter, the SEAC installation has been operated as a general Bureau facility with an overall efficiency of 78% during scheduled operation. The computation time was distributed as follows:

Total time available for computation	1,239	hours
Good operating time.	1,005	"
Idle in order.	5	"
Down-time.	230	"

In addition the SEAC was utilized 360 hours for engineering and preventive maintenance. "Down-time" is defined as the time during which the computer was under repair or was rerunning a previously-run problem found to be in error as a result of machine malfunction. Preliminary planning and engineering layout have been under way in preparation for moving the SEAC installation to a permanent location, which is scheduled to occur in the fall.

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Numerical Analysis Colloquium
(Los Angeles, California)

MORREY, C. B. (University of California, Berkeley). On the derivation of the equations of hydrodynamics from statistical mechanics. July 12.

Informal Seminar on Problems Relating to Numerical Analysis
(Los Angeles, California)

ZIEGLER, J.R. (U.C.L.A.) Use of SWAC for item analysis of psychological instruments. April 1.

WOODS, R. SWAC computations of the scattering of protons, using the optical model of the nucleus. April 6.

FULLER, F.E. (Lockheed Aircraft Corporation). Some considerations of the flutter problem (with 10-minute movie of rocket flutter). April 8.

ORCHARD-HAYS, W. (RAND Corporation). Computational experience in solving linear programming problems. April 13.

PHILIPSON, L. Asymptotic character of the solutions of a class of ordinary linear differential equations depending on a parameter. April 15.

FORSYTHE, G.E. On direct methods of obtaining the characteristic values of a matrix. April 20.

GROSCH, H.R.J. (General Electric Company, Cincinnati). Description of General Electric computing activities. April 21.

SOKOLNIKOFF, I.S. (U.C.L.A.) Boundary value problems in elasticity, I. May 4.

POPE, D.A. On direct methods of solving problems in the calculus of variations. May 6.

HASSIG, H.J. (Lockheed Aircraft Corporation). Related digital and analog methods of flutter analysis. May 11.

SOKOLNIKOFF, I.S. (U.C.L.A.) Boundary value problems in elasticity, II. May 13.

TOMPKINS, C. B. Problems in numerical analysis related to the Morse theory in the calculus of variations. May 18.

BELLMAN, R. (RAND Corporation). Some problems in the theory of dynamic programming. May 20.

KLIEFORTH, H. (U.C.L.A.) SWAC computation of flight paths from photo-theodolite measurements. May 25.

LEVIN, E. Can a computing machine play chess? May 27.

SALZER, J.M. (Magnavox Company). Convergence and error studies by transform methods. June 1.

SPROWLS, R.C. (U.C.L.A.) Use of SWAC in the stochastic determination of cash balances. June 3.

FORSYTHE, G.E. SWAC computation of all semi-groups of order 4. June 22.

LOPEZ, G. Review of a new method by von Neumann for solving linear games. June 24.

THOMAS, T. Y. (Indiana University). A discussion of Luder's bands. June 29.

Special Lectures
(Los Angeles, California)

HALL, M., Jr. (Ohio State University) The Burnside problem. April 21.

RHODES, Ida (National Bureau of Standards, Washington). Recent developments in high speed computing. April 2.

HOLLANDER, F. H. Ten lectures on coding for the SWAC. May 1954.

SOUTHARD, T.H., F. H. HOLLANDER, and R. HORGAN. SWAC Coding Course, beginning June 21, 1954.

Linear Programming Seminar

HOFFMAN, A.J. An application of linear programming to a combinatorial problem. March 31.

JACOBS, W. (U. S. Air Force). The caterer problem. April 14.

FAN, K. Existence theorem for linear inequalities. April 28.

MOTZKIN, T. S. Linear inequalities in Statu Nascendi. May 5.

JOSEPH, J. (U. S. Air Force). Some military applications of linear programming. May 12.

GASS, S. (U. S. Air Force). Parametric minimization. May 12.

GAINEN, L., H. FISCHER (Captain, U. S. Air Force), and J. NATRELLA (U.S. Air Force). Symposium on computational experience, I. May 26.

KELLEY, J. (George Washington University). An extended transportation problem. June 9.

GAINEN, L., A. J. HOFFMAN, and L. S. JOEL. Symposium on computational experience, II. June 23.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

ANTOSIEWICZ, H. On nonlinear difference equations. Presented at the meeting of the American Mathematical Society, Chicago, Ill., April 21-May 1.

EISENHART, C. The Student-Fischer revolution in modern statistics. Presented at a Colloquium in Statistical Design, sponsored by the Naval Ordnance Laboratory in collaboration with the Office of Naval Research and the Bureau of Ordnance, held at the Naval Ordnance Laboratory, White Oak, Md., April 14.

FORSYTHE, G. E. SWAC computes all 126 distinct semigroups of order 4. Presented at a meeting of the American Mathematical Society, Portland, Oregon, June 19.

HUSKEY, H. D. (1) Undergraduate curriculum in machine computation. Presented at a Conference on Training Personnel for the Computing Machine Field, held at Wayne University, Detroit, Michigan, June 22-23. (2) Automatic coding, a progress report. Presented at the meeting of the Association for Computing Machinery, Ann Arbor, Mich. June 24, 1954.

TAUSSKY-TODD, O. Group theory and football forecasting. Presented at a meeting of Sigma Xi, Washington, D. C., May 24.

TODD, J. (1) Motivation for working in numerical analysis. Presented at a Symposium on Applied Mathematics held by the Office of Ordnance Research, held at Chicago, Ill., April 29. (2) Effect of high speed computers on the training of mathematicians. Presented at a Conference on Training Personnel for the Computing Machine Field, Wayne University, Detroit, Mich., June 22-23.

TOMPKINS, C. B. (1) Considerations involved in the formulation of a numerical attack on a game problem to be solved on a high speed automatic computer. Presented at a Mathematics Seminar, University of Rochester, Rochester, N. Y., April 27. (2) A theorem relating to the Kaczmarz Projection Scheme for solving linear equations and inequalities. Presented at a Logistics Research Project Seminar, Washington, D. C., April 29. (3) Some small matrix games of attrition and their numerical solution. Presented at the Princeton University Games Seminar, Princeton, N.J., May 7. (4) Computers in industrial research. Presented at the Fifth Annual Conference on Industrial Research under the auspices of Columbia University at Harriman, N. Y., June 11.

WASOW, W. R. Discrete approximations to elliptic differential equations. Presented at a meeting of the American Mathematical Society, Yosemite, California, May 1.

YOUDEN, W. J. Principles of experimental design. Presented at a meeting of the Milwaukee Chapter of the American Chemical Society, Milwaukee, Wis., April 23.

Papers presented at the meeting of the Mathematical Association of America, held at the University of Maryland, College Park, Md., May 1:

CAMERON, J., and M. NEWMAN. Pseudo random numbers for high speed computers.

HOFFMAN, J. Some practical and theoretical applications of the transportation problem.

Papers presented at the Annual Meeting of the Virginia Academy of Science, held at the University of Virginia, Charlottesville, Va., May 7:

CONNOR, W. S. On a problem in fitting a straight line.

LIEBLEIN, J. On a new method of analyzing extreme-value data.

Papers presented at the meeting of the Association for Computing Machinery, held at the University of Michigan, Ann Arbor, Michigan, June 22-25:

ABRAMOWITZ, M. Operation of a computation laboratory.

DAVIS, P., and P. RABINOWITZ. A multiple purpose orthonormalizing code and its use.

WEGSTEIN, J. H. A general purpose pseudo-code.

Papers presented at the meeting of the American Mathematical Society, New York, N. Y., April 23-24:

ABRAMOWITZ, M., and H. ANTOSIEWICZ. Coulomb wave functions in the transition region.

GOLDBERG, K. $\text{Log}(e^{xy})$ in a free associative ring.

Publication Activities

1. PUBLICATIONS WHICH APPEARED DURING THE QUARTER

1.3 Technical Papers

(1) Evaluation of the integral $\int_0^\infty e^{-u^2-(x/u)}du$. M. Abramowitz. J. Math. Phys. 32, 188-192 (July-Oct. 1953).

(2) Regular and irregular Coulomb wave functions expressed in terms of Bessel-Clifford functions. M. Abramowitz. J. Math. Phys. 33, 111-116 (Apr. 1954).

(3) On the practical evaluation of integrals. M. Abramowitz. J. Soc. Indust. App. Math. 2, 20-35 (Mar. 1954).

(4) On a theorem of Ostrowski and Taussky. R. Bellman (RAND Corporation) and A. J. Hoffman. Arch. Math. 5, 123-127 (1954).

(5) On representations and extensions of bounded linear functionals defined on classes of analytic functions. P. Davis and J. L. Walsh. Trans. Amer. Math. Soc. 76, 190-206 (1954).

(6) Some remarks on commutators of matrices. K. Fan. Arch. Math. 5, 102-107 (1954).

(7) A numerical solution of Schroedinger's equation in the continuum. W. Futterman, E. Osborne, and D. S. Saxon. J. Res. NBS 52, 259-264 (May 1954), RP2498.

(8) Application of variational methods to intermediate and high energy scattering. E. Gerjuoy and D. S. Saxon. Physical Rev. 94, 478-491 (Apr. 15, 1954).

(9) The representation of integers by binary quadratic rational forms. K. Goldberg, M. Newman, E. G. Straus, and J. D. Swift. Arch. Math. 5, 12-18 (1954).

(10) Ueber die Funktionen von Gegenbauer. P. Henrici. Arch. Math. 5, 92-98 (1954).

(11) Probability limits for the average chart when process standards are unspecified. E. P. King. Ind. Qual. Control 10, 62-64 (May 1954).

(12) On strongly continuous stochastic processes. E. Lukacs. Sankhyā 13, 219-228 (Mar. 1954).

(13) A property of the normal distribution. E. Lukacs and E. P. King. Ann. Math. Stat. 25, 389-394 (June 1954).

(14) Certain Fourier transforms of distributions (II). E. Lukacs and O. Szász. Canadian J. Math. 6, 186-189 (1954).

(15) Nonnegative trigonometric polynomials and certain rational characteristic functions. E. Lukacs and O. Szász. J. Res. NBS 52, 153-160 (Mar. 1954), RP2484.

(16) On nearly triangular matrices. A. Ostrowski. J. Res. NBS 52, 319-345 (June 1954), RP2509.

(17) Characteristic roots of quaternion matrices. O. Taussky. Arch. Math. 5, 99-101 (1954).

(18) Einschliessung von Eigenwerten hermitescher Matrizen nach dem Abschnittsverfahren. H. Wielandt. Arch. Math. 5, 108-114(1954).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION JUNE 30, 1954

2.1 Mathematical Tables

(1) Table of sine and cosine integrals for arguments from 10 to 100. NBS Applied Mathematics Series 32. (A reissue of NBS Mathematical Table MT13.) In press, Government Printing Office.

(2) Table of the Gamma function for complex arguments. NBS Applied Mathematics Series 34. In press, Government Printing Office.

(3) Tables of functions and of zeros of functions. Volume I of Collected short tables of the Computation Laboratory. NBS Applied Mathematics Series 37. In press, Government Printing Office.

(4) Tables of the error function and its derivative. NBS Applied Mathematics Series 41. (A reissue of NBS Mathematical Table 8, Tables of probability functions, vol. I.) In press, Government Printing Office.

(5) Tables of sines and cosines for radian arguments. NBS Applied Mathematics Series 43. (A reissue of NBS Mathematical Table 4.) In press, Government Printing Office.

(6) Tables of the inverses of finite segments of the Hilbert matrix. I. R. Savage and E. Lukacs. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(7) Table of salvo kill probabilities for square targets. NBS Applied Mathematics Series 43. In press, Government Printing Office.

2.3 Technical Papers

(1) Coulomb wave functions in the transition region. M. Abramowitz and H. Antosiewicz. Submitted to a technical journal.

(2) Approximate method for rapid Loran computation. M. Abramowitz, D. H. Call, and J. C. Mathews. Submitted to a technical journal.

(3) Coulomb wave functions along the transition line. M. Abramowitz and P. Rabinowitz. Accepted for publication in The Physical Review.

(4) The relaxation method for linear inequalities. S. Agmon. Accepted for publication in the Canadian Journal of Mathematics.

(5) A theorem on alternatives for pairs of matrices. H. Antosiewicz. Accepted for publication in the Pacific Journal of Mathematics.

Publication Activities

(6) Asymptotic solution of linear differential equations with a parameter. H. Antosiewicz. Submitted to a technical journal.

(7) On nonlinear differential equations of the second order with integrable forcing term. H. Antosiewicz. Submitted to a technical journal.

(8) On the differential equation $\ddot{x} + k(f(x) + g(x)\dot{x})\dot{x} = ke(t)$. H. A. Antosiewicz. Submitted to a technical journal.

(9) On a certain integral involving Bessel functions. H. Antosiewicz. Submitted to a technical journal.

(10) A representation for solutions of analytic systems of linear differential equations. H. A. Antosiewicz and M. Abramowitz. Accepted for publication in the Journal of the Washington Academy of Sciences.

(11) Some implications of Liapunov's conditions for stability. H. A. Antosiewicz and P. Davis. Accepted for publication in the Journal of Rational Mechanics and Analysis.

(12) On the enumeration of partially balanced designs with two associate classes. W. H. Clatworthy. Submitted to a technical journal.

(13) Partially balanced incomplete block designs with two associate classes and two treatments per block. W. H. Clatworthy. Accepted for publication in the Journal of Research of the NBS.

(14) Easily used simultaneous confidence limits for a line. W.S.Connor. Submitted to a technical journal.

(15) Experimental designs for paired observations. W. S. Connor and W. J. Youden. Accepted for publication in the Journal of Research of the NBS.

(16) Time-discrete stochastic processes in arbitrary sets, with applications to processes with absorbing regions and to the problem of loops in Markoff chains. D. van Dantzig. Accepted for publication (in French) in Annales de l'Institut Henri Poincaré (Paris).

(17) Linear functional equations and interpolation series. P. Davis. Accepted for publication in the Pacific Journal of Mathematics.

(18) On a problem in the theory of mechanical quadratures. P. Davis. Accepted for publication in the Pacific Journal of Mathematics.

(19) Some L^2 aspects of Faber polynomials. P. Davis and H. Pollack (Bell Telephone Laboratories). Accepted for publication in the Duke Mathematical Journal.

(20) A multiple purpose orthonormalizing code and its uses. P. Davis. and P. Rabinowitz. Submitted to a technical journal.

(21) On the estimation of quadrature errors for analytic functions. P. Davis and P. Rabinowitz. Submitted to a technical journal.

(22) Some SEAC computations of subsonic fluid flows by Bergman's method of integral operators. P. Davis and P. Rabinowitz. To be an appendix in the volume, "Bergman's Operator Method," by M. Z. v. Krzywoblocki (University of Illinois).

(23) Some sampling results on the power of nonparametric tests against normal alternatives. W. J. Dixon (University of Oregon) and D. Teichroew. Submitted to a technical journal.

(24) Turbulent flow in shock tubes of varying cross-section. R. F. Dressler. Accepted by the Journal of Research of the NBS.

(25) On the optimal character of the (s,S) policy in inventory theory. A. Dvoretzky, J. Kiefer, and J. Wolfowitz. Submitted to a technical journal.

(26) Inequalities for eigenvalues of Hermitian matrices. K. Fan. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(27) Lower bounds for the rank and location of the eigenvalues of a matrix. K. Fan and A. J. Hoffman. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(28) Some metric inequalities in the space of matrices. K. Fan and A. J. Hoffman. Accepted for publication in the Proceedings of the American Mathematical Society.

(29) A determinantal inequality. K. Fan and O. Taussky. Submitted to a technical journal.

(30) Punched-card experiments with accelerated gradient methods for linear equations. A. I. Forsythe and G. E. Forsythe. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(31) Asymptotic lower bounds for the frequencies of polygonal membranes. G. E. Forsythe. Accepted for publication in the Pacific Journal of Mathematics.

(32) Asymptotic lower bounds for the fundamental frequency of convex membranes. G. E. Forsythe. Accepted for publication in the Pacific Journal of Mathematics.

(33) Practical solution of linear equations and inversion of matrices. L. Fox. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues. NBS Applied Mathematics Series 39.

(34) On the solution of the caterer problem. J. W. Gaddum, A. J. Hoffman, and D. Sokolowsky. Submitted to a technical journal.

(35) Linear programming in bid evaluation. L. Gainen, D. J. Honig, and E. D. Stanley. Accepted for publication in Logistics Research Quarterly.

(36) Variational principles for the acoustic field. E. Gerjuoy and D. S. Saxon. Accepted for publication in The Physical Review.

(37) Equations of physics in general Newtonian space-time. A. N. Gleyzal. Submitted to a technical journal.

(38) The use of extreme-value methods in engineering problems. E. J. Gumbel and J. Lieblein. Submitted to a technical journal.

(39) Iterative methods of solving linear problems on Hilbert space. R. M. Hayes. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(40) On certain series expansions involving Whittaker functions and Jacobi polynomials. P. Henrici. Accepted for publication in the Pacific Journal of Mathematics.

(41) On generating functions of the Jacobi polynomials. P. Henrici. Accepted for publication in the Pacific Journal of Mathematics.

(42) On helical springs of finite thickness. P. Henrici. Accepted for publication in the Quarterly Journal of Applied Mathematics.

(43) On a combinatorial theorem. A. J. Hoffman. Submitted to a technical journal.

(44) Smooth patterns of production. A. J. Hoffman and W. Jacobs. Submitted to a technical journal.

(45) On the number of absolute points of a correlation. A. J. Hoffman, M. Newman, E. G. Straus, and O. Taussky. Submitted to a technical journal.

(46) On optimum grouping in one-criterion variance components analysis. E. P. King. Accepted for publication in the Journal of the American Statistical Association.

(47) Contractibility and convexity. H. W. Kuhn. Accepted for publication in the Proceedings of the American Mathematical Society.

(48) On certain character matrices. D. H. Lehmer. Submitted to a technical journal.

(49) Indentation pressure of a smooth punch. E. Levin. Accepted for publication in the Quarterly of Applied Mathematics.

(50) Acoustic radiation pressure on a circular disk. H. Levine. To appear in the Proceedings of the Fifth Symposium on Applied Mathematics of the American Mathematical Society, held in Pittsburgh, Pa., June 1952.

(51) On the variances and covariances of order statistics from the Weibull distribution. J. Lieblein. Submitted to a technical journal.

(52) A proof of Hilbert's Nullstellensatz. T. S. Motzkin. Submitted to a technical journal.

(53) The assignment problem. M. Motzkin. To appear in the Proceedings of the American Mathematical Society's Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 1953.

(54) On Fejér sets in linear and spherical spaces. T. S. Motzkin and I. J. Schoenberg. Accepted for publication in Annals of Mathematics.

(55) On the relaxation method for linear inequalities. T. S. Motzkin and I. J. Schoenberg. Accepted for publication in the Canadian Journal of Mathematics.

(56) Least p-th power polynomials on a real finite point set. T. S. Motzkin and J. L. Walsh. Submitted to a technical journal.

(57) A conjecture of Erdös. M. Newman. Submitted to a technical journal.

(58) Structure theorems for modular subgroups. M. Newman. Accepted for publication in the Duke Mathematical Journal.

(59) Diffraction of waves by a wedge. F. Oberhettinger. Submitted to a technical journal.

(60) On the Lerch zeta function. F. Oberhettinger. Submitted to a technical journal.

(61) On two problems in abstract algebra connected with Horner's rule. A. M. Ostrowski. To appear in the von Mises Anniversary Volume.

(62) Determinanten mit ueberwiegender Hauptdiagonale und die absolute Konvergenz von linearen Iterationsprozessen. A. M. Ostrowski. Submitted to a technical journal.

(63) On the convergence of Gauss' alternating procedure in the method of the least squares, I. A. M. Ostrowski. Submitted to a technical journal.

(64) On the linear iteration procedures for symmetric matrices. A. M. Ostrowski. Submitted to a technical journal.

(65) On the spectrum of a one parametric family of matrices. A. M. Ostrowski. Submitted to a technical journal.

(66) On Gauss' speeding up device in the theory of single step iteration. A. M. Ostrowski. Submitted to a technical journal.

(67) On absolute convergence of linear iteration processes. A. M. Ostrowski. Submitted to a technical journal.

(68) On spectra of second-order differential operators. D. Ray. Submitted to a technical journal.

(69) On the convergence of asymptotic solutions of linear differential equations. R. M. Redheffer (U.C.L.A.) and W. Wasow. Submitted to a technical journal.

(70) Numerical computation of low moments of order statistics from a normal population. J. B. Rosser. Submitted to a technical journal.

(71) Most probable rank orders. I. R. Savage. Submitted to a technical journal.

(72) An isoperimetric inequality for closed curves convex in even-dimensional Euclidean space. I. J. Schoenberg. Accepted for publication in Acta Mathematica.

(73) Generalized commutators of matrices and permutations of factors in a product of three matrices. O. Taussky. For publication in the von Mises Anniversary volume.

(74) Some computational problems in algebraic number theory. O. Taussky. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 26-28, 1953.

(75) An improved cathode ray tube storage system. R. Thorensen. To appear in the Proceedings of the Western Computer Conference of the AIEE-IRE-ACM held in Los Angeles, Calif., February 4,5,6,1953.

(76) Motivation for working in numerical analysis. J. Todd. To appear in the Transactions of a Symposium on Applied Mathematics sponsored by the Office of Ordnance Research and held in Chicago, Ill., April 29, 1954.

(77) The condition of certain matrices, II. J. Todd. To appear in Archiv der Mathematik.

(78) The condition of the finite segments of the Hilbert matrix. J. Todd. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(79) The evaluation of the exponential integral for large complex arguments. J. Todd. Accepted for publication in the Journal of Research of the NBS.

(80) A method for the numerical integration of differential equations of second order without explicit first derivatives. R. de Vogelaere. Accepted for the Journal of Research of the NBS.

(81) Discrete approximations to elliptic differential equations. W. Wasow. Accepted for publication in Zeitschrift für angewandte Mathematik und Physik.

(82) On the asymptotic transformation of certain distributions into the normal distribution. W. Wasow. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 26-28,1953.

(83) An extremum property of sums of eigenvalues. H. Wielandt. Accepted for publication in the Proceedings of the American Mathematical Society.

(84) Error bounds for the eigenvalues of symmetric integral equations. H. Wielandt. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 26-28, 1953.

(85) On eigenvalues of sums of normal matrices. H. Wielandt. Submitted to a technical journal.

(86) Statistics and planning tests at elevated temperatures. W. J. Youden. To appear in the Proceedings of the Society for Experimental Stress Analysis.

(87) A note on partially balanced designs. M. Zelen. Accepted for publication in Annals of Mathematical Statistics.

(88) Analysis for some incomplete block designs having a missing block. M. Zelen. Accepted for publication in Biometrics.

(89) Bounds on a distribution function which are functions of moments to order four. M. Zelen. Accepted by the Journal of Research of the NBS.

2.4 Reviews, Notes

(1) Note on the circle theorem of hydrodynamics. E. Levin. Accepted for publication by the Quarterly of Applied Mathematics.

(2) Two early papers on the relation between extreme values and tensile strength. (Formerly "A historical note on the application of the 'weakest link' idea to tensile strengths.") J. Lieblein. Submitted to a technical journal.

2.5 Miscellaneous Publications

(1) Contributions to the solution of systems of linear equations and the determination of eigenvalues. NBS Applied Mathematics Series 39. In press, Government Printing Office.

(2) Experiments in the computation of conformal maps. NBS Applied Mathematics Series 42. In press, Government Printing Office.

THE NATIONAL BUREAU OF STANDARDS

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The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

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Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($0.75), available from the Superintendent of Documents, Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.


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