

NATIONAL BUREAU OF STANDARDS REPORT

3481

SOME FRACTIONAL FACTORIAL ARRANGEMENTS
FOR FACTORS AT TWO LEVELS

by

W. H. Clatworthy
W. S. Connor
M. Zelen



U. S. DEPARTMENT OF COMMERCE
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SOME FRACTIONAL FACTORIAL ARRANGEMENTS FOR FACTORS AT TWO LEVELS

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Statistical Engineering Laboratory



**U. S. DEPARTMENT OF COMMERCE
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FOREWORD

This is a technical report on Research in Applications of Mathematical Statistics to Problems of the Chemical Corps, for the Biological Laboratories, Chemical Corps, U. S. Department of the Army carried out in the Statistical Engineering Laboratory (NBS Project Number 1103-40-5118/52-1).

F.L. Alt
Acting Chief,
National Applied Mathematics
Laboratories

A.V. Astin
Director,
National Bureau of Standards

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Some Fractional Factorial Arrangements for
Factors at Two Levels

by

W.H. Clatworthy, W.S. Connor, and M. Zelen

1. Introduction. This report contains 68 experimental arrangements or plans for fractional replication of factorial designs of the 2^n series, where n , the number of factors, ranges from 5 to 12 and each factor is at two levels. The plans include arrangements which are $1/2$, $1/4$, $1/8$, $1/16$, and $1/32$ of a full replication and which involve 8, 16, 32, 64, 128, and 256 measurements. With but two exceptions the experimental units have been assigned to blocks.

A bibliography which is not exhaustive is given at the end of this introduction. Several of the designs given in this report may be found in these references. However this report represents an independent effort.

A brief description of the plans follows. Each plan has a designation of the form

Plan $n.r.k$

which refers to the $1/r$ replication of the 2^n factorial design with $2^n/rk$ incomplete blocks of k experimental units each. For example, Plan 7.2.4 refers to the $1/2$ replicate of the 2^7 factorial design with 16 blocks of 4 experimental units each.

In all the designs the n factors, each at two levels, are designated by capital letters A, B, C, etc. Small letters refer to assignment of the factors to the experimental units, the presence of small letter x meaning that the high level of factor X is applied and the absence of small letter x meaning that the low level of factor X is applied. For example, in Plan 5.2.4 the five factors are A, B, C, D, and E. In block 1 of this design one of the experimental units receives the treatment combination de, i.e., low levels of factors A, B, and C and high levels of factors D and E are applied to the experimental unit. The symbol "(1)" is also found in block 1, indicating that low levels of all factors of the design are applied to one experimental unit of block 1.

A main effect or interaction is said to be estimable if it is confounded only with higher order interactions. In any plan in which a main effect is confounded with two-factor interactions, it is explicitly stated which two-factor interactions are involved.

For each design there is given a fundamental confounding relationship composed of groups of capital letters (representing main effects and interactions) connected with equal signs. The equal signs are read "is confounded with". The fundamental confounding relationship is useful in determining how the main effects and interactions are confounded with each other as a

result of the design being a fractional part of a complete replication. For example, in Plan 6.4.4 the fundamental confounding relationship is

$$I = ABCE = ABDF = CDEF .$$

To determine how the main effects and interactions are confounded by the $1/4$ replication of the full 2^6 factorial, each term of the fundamental confounding relationship is multiplied by the main effect or interaction in question, with the understanding that $X^2 = 1$ where $X = A, B, C, \text{ etc.}$ and $I(X...Z) = X \dots Z$. For example, the confounding of main effect A is given by

$$A = BCE = BDF = ACDEF .$$

Note that multiplication by BCE, BDF, and ACDEF give exactly the same result, except for order of the groups. The other confounding relationships are:

$$\begin{aligned} B &= ACE = ADF = BCDEF \\ C &= ABE = ABCDF = DEF \\ D &= ABCDE = ABF = CEF \\ E &= ABC = ABDEF = CDF \\ F &= ABCEF = ABD = CDE \\ AB &= CE = DF = ABCDEF \end{aligned}$$

AC = BE = BCDF = ADEF
AD = BCDE = BF = ACEF
AE = BC = BDEF = ACDF
AF = BCEF = BD = ACDE
CD = ABDE = ABCF = EF
CF = ABEF = ABCD = DE
ACD = BDE = BCF = AEF
ACF = BEF = BCD = ADE

The treatment combinations were assigned to the blocks so as to confound interactions AB, BC, and AC with the blocks. If b is the number of blocks, then certain $b-1$ of the confounding relationships designate which effects cannot be estimated because of block confounding. These are listed for each design as "Block confounding" wherein each group of letters designates a confounding relationship.

Examination of the totality of confounding relationships for Plan 6.4.4 reveals that the main effects A, B, C, D, E, and F are estimable, that the two-factor interactions which are not already confounded with blocks are confounded with other two-factor interactions and hence are not estimable, and that no three-factor interaction is estimable. Thus in this design it is impossible to estimate any interaction unless one is prepared to assume that some other interaction(s) of the same order is(are) negligible.

In some of the plans where the number of treatment combinations is large, the blocks are not written out in full. For such designs only the initial block is recorded in detail and one treatment combination is given for each of the remaining blocks. The other blocks are obtained by successively multiplying the treatment combinations in the initial block by each of the given treatment combinations with the proviso that $x^2 = 1$, ($x = a, b, c$, etc.). For example, Plan 10.4.8 is such a design. The initial block consists of the following eight treatment combinations

(1), abcd, efhj, abcdefhj, eghk, abcdeghk,
fgjk, abcdfgjk.

To obtain the treatment combinations for block 2, say, one multiplies these eight groups by ab (the treatment combination listed for block 2), obtaining

ab, cd, abefhj, cdefhj, abeghk, cdeghk,
abfgjk, cdfgjk.

Although the plans in this report have been carefully checked, there is always the possibility of errors. The authors would appreciate hearing from anyone discovering errors.

The authors wish to express their thanks to Mrs. Lola Deming for checking the plans and to Mrs. Yvette Cocozzella who did the typing.

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Plan 5.2.4. 5 factors, 1/2 replication, 4 blocks of 4 units each.

Factors: A,B,C,D,E

I = ABCDE

Block confounding: AB, AC, BC.

All main effects and all two-factor interactions except AB, AC, and BC are estimable.

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	ab	ac	ad
abcd	cd	bd	bc
de	abde	acde	ae
abce	ce	be	bcde

Plan 5.2.8. 5 factors, 1/2 replication, 2 blocks of 8 units each.

Factors: A,B,C,D,E

I = ABCDE

Block confounding: AB

All main effects and all two-factor interactions except AB are estimable.

<u>Blocks</u>	
<u>1</u>	<u>2</u>
(1)	ac
abcd	bd
de	acde
abce	be
ab	bc
cd	ad
abde	bcde
ce	ae

Plan 6.2.4. 6 factors, 1/2 replication, 8 blocks of 4 units each.

Factors: A,B,C,D,E,F

I = ABCDEF

Block confounding: ABF, ACF, BC, ABE, EF, BCEF, ACE.

All main effects, all two-factor interactions except AD, BC, and EF are estimable. Twelve of the three-factor interactions are confounded in pairs as follows:

ABC = DEF	ADE = BCF
ABD = CEF	ADF = BCE
ACD = BEF	AEF = BCD ,

while the remaining eight three-factor interactions are confounded with blocks.

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
(1)	ab	ac	bc	ae	be	ce	abce
abcd	cd	bd	ad	bcde	acde	abde	de
bcef	acef	abef	ef	abcf	cf	bf	af
adef	bdef	cdef	abodef	df	abdf	acdf	bcdf

Plan 6.2.8. 6 factors, 1/2 replication, 4 blocks of 8 units each.

Factors: A,B,C,D,E,F

I = ABCDEF

Block confounding: ABF, ACF, BC

All main effects and all two-factor interactions except BC are estimable. Sixteen of the twenty three-factor interactions are confounded in pairs as follows:

ABC = DEF	ACE = BDF
ABD = CEF	ADE = BCF
ABE = CDF	ADF = BCE
ACD = BEF	AEF = BCD ,

while the remaining four three-factor interactions are confounded with blocks.

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	ab	ac	bcd
abcd	cd	bd	ad
bcef	acef	abef	ef
adef	bdef	cdef	abcdef
abce	ce	be	ae
de	abde	acde	bcde
af	bf	cf	abcf
bcdf	acdf	abdf	df

Plan 6.2.16. 6 factors, 1/2 replication, 2 blocks of 16 units each.

Factors: A,B,C,D,E,F

I = ABCDEF

Block confounding: ABF

All main effects and all two-factor interactions are estimable. The three-factor interactions ABF and CDE are confounded with blocks while all others are confounded in pairs as follows:

ABC = DEF	ACD = BEF	ADE = BCF
ABD = CEF	ACE = BDF	ADF = BCE
ABE = CDF	ACF = BDE	AEF = BCD

Blocks

<u>1</u>		<u>2</u>	
(1)	ab	ac	bc
abcd	cd	bd	ad
bcef	acef	abef	ef
adef	bdef	cdef	abcdef
abce	ce	be	ae
de	abde	acde	bcde
af	bf	cf	abcf
bcdf	acdf	abdf	df

Plan 7.2.4. 7 factors, 1/2 replication, 16 blocks of 4 units each.

Factors: A,B,C,D,E,F,G

I = ABCDEFG

Block confounding: ABFG, ACFG, BC, ABEG, EF, BCEF, ACEG, ABEF, EG, BCEG, ACEF, FG, AB, AC, BCFG

All main effects, 15 of the 21 two-factor interactions, and 26 of the 35 three-factor interactions are estimable. The two-factor interactions

AB, AC, BC, EF, EG, FG

and the three-factor interactions

ADE, ADF, ADG, BDE, BDF, BDG, CDE, CDF, CDG

are confounded with blocks.

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
(1)	ab	ac	bc	ae	be
abcd	cd	bd	ad	bcde	acde
defg	abdefg	acdefg	bcdefg	adfg	bdfg
abcefg	cefg	befg	aefg	bcfg	acfg
<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>	
ce	abce	af	bf	cf	
abde	de	bcdf	acdf	abdf	
cdfg	abcdfg	adeg	bdeg	cdeg	
abfg	fg	bceg	aceg	abeg	
<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>	
abcf	ef	abef	acef	bcef	
df	abcdef	cdef	bdef	adef	
abcdeg	dg	abdg	acdg	bcdg	
eg	abcg	cg	bg	ag	

Plan 7.2.8. 7 factors, 1/2 replication, 8 blocks of 8 units each.

Factors: A,B,C,D,E,F,G

I = ABCDEFG

Block confounding: ABFG, ACF, BCG, BCEF, ACEG, ABE, EFG

All main effects, all two-factor interactions, and all three-factor interactions except

ABE, ACF, ADG, BCG, BDF, CDE, and EFG

are estimable.

<u>Blocks</u>							
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>7</u>	<u>6</u>	<u>8</u>
(1)	ab	ac	bc	ae	ce	be	abce
abcd	cd	bd	ad	bcde	abde	acde	de
bcef	acef	abef	ef	abcf	bf	cf	af
adef	bdef	cdef	abcdef	df	acdf	abdf	bcdf
cdfg	abcdfg	adfg	bdfg	acdefg	defg	bcdefg	abdefg
abfg	fg	bcfg	acfg	befg	abcefg	aefg	cefg
bdeg	adeg	abcdeg	cdeg	abdg	bcdg	dg	acdg
aceg	bceg	eg	abeg	cg	ag	abcg	bg

Plan 7.2.16. 7 factors, 1/2 replication, 4 blocks of 16 units each.

Factors: A,B,C,D,E,F,G

I = ABCDEFG

Block confounding: ABFG, ACF, BCG

All main effects, all two-factor interactions, and all three-factor interactions except ACF, BCG, and CDE are estimable.

<u>Blocks</u>			
<u>1</u>		<u>2</u>	
(1)	abce	ab	ce
abcd	de	cd	abde
bcef	af	acef	bf
adef	bcdf	bdef	acdf
cdfg	abdefg	abcdfg	defg
abfg	cefg	fg	abcefg
bdeg	acdg	adeg	bcdg
aceg	bg	bceg	ag
<u>3</u>		<u>4</u>	
ac	be	bc	ae
bd	acde	ad	bcde
abef	cf	ef	abcf
cdef	abdf	abcdef	df
adfg	bcdefg	bdfg	acdefg
bcfg	aefg	acfg	befg
abcdeg	dg	cdeg	abdg
eg	abcg	abeg	cg

Plan 7.2.32. 7 factors, 1/2 replication, 2 blocks of 32 units each.

Factors: A,B,C,D,E,F,G

I = ABCDEFG

Block confounding: ABFG

All main effects, all two-factor interactions and all three-factor interactions except CDE are estimable.

Blocks

1

(1)	abce	ab	ce
abcd	de	cd	abde
bcef	af	acef	bf
adef	bcdf	bdef	acdf
cdfg	abdefg	abcdfg	defg
abfg	cefg	fg	abcefg
bdeg	acdg	adeg	bcdg
aceg	bg	bceg	ag

2

ac	be	bc	ae
bd	acde	ad	bcde
abef	cf	ef	abcf
cdef	abdf	abcdef	df
adfg	bcdefg	bdfg	acdefg
bcfg	aefg	acfg	befg
abcdeg	dg	cdeg	abdg
eg	abcg	abeg	cg

Plan 8.2.8. 8 factors, 1/2 replication, 16 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCDEFGH

Block confounding: ABCD, ABCE, DE, ABDEG, CEG, CDG, ABG, BCH, ADH, AEH, BCDEH, ACDEGH, BEGH, BDGH, ACGH.

All main effects, all two-factor interactions except BF and DE, and all three-factor interactions except ABG, ADH, AEH, BCH, CDG, and CEG are estimable. None of the four-factor interactions is estimable.

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
(1)	ab	ac	bc	ad	bd
abfh	fh	bcfh	acfh	bdfh	adfh
bcfg	acfg	abfg	fg	abcdfg	cdfg
acgh	bcgh	gh	abgh	cdgh	abcdgh
cdeh	abcdeh	adeh	bdeh	aceh	bceh
abcdef	cdef	bdef	adef	bcef	acef
bdefgh	adefgh	abcdefgh	cdefgh	abefgh	efgh
adeg	bdeg	cdeg	abcdeg	eg	abeg
<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>	
cd	abcd	ae	be	ce	
abcdfh	cdfh	befh	aefh	abcefh	
bdfg	adfg	abcefg	cefg	befg	
adgh	bdgh	cegh	abcegh	aegh	
eh	abeh	acd	bcdh	dh	
abef	cf	bcd	acd	abdf	
bcefg	acefgh	abdfgh	dfgh	bcd	
aceg	bceg	dg	abd	cdg	

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Plan 8.2.8. (Continued).

Blocks

<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
abce	de	abde	acde	bcde
cefh	abdefh	defh	bcdefh	acdefh
aefg	bcdefg	acdefg	abdefg	defg
begh	acdegh	bcdegh	degh	abdegh
abdh	ch	abch	ah	bh
df	abcf	cf	bf	af
acdfgh	bfigh	afgh	abcfgh	cfg
bcdg	ag	bg	cg	abcg

Plan 8.2.16. 8 factors, 1/2 replication, 8 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCDEFGH

Block confounding: ABCD, ABEF, CDEF, BCEG, ADEG, ACFG, BDFG.

All main effects, all two-factor interactions, and all three-factor interactions are estimable. Fourteen of the 70 four-factor interactions are confounded with blocks while the remaining 56 are confounded in pairs.

<u>Blocks</u>			
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	ab	ac	bc
abcd	cd	bd	ad
abef	ef	bcef	acef
cdef	abcdef	adef	bdef
bceg	aceg	abeg	eg
adeg	bdeg	cdeg	abcdeg
acfg	bcfg	fg	abfg
bdfg	adfg	abcdfg	cdfg
efgh	abefgh	acefgh	bcefgh
abcdefgh	cdefgh	bdefgh	adefgh
abgh	gh	bcgh	acgh
cdgh	abcdgh	adgh	bdgh
bcfh	acfh	abfh	fh
adfh	bdfh	cdfh	abcdfh
aceh	bceh	eh	abeh
bdeh	adeh	abcdeh	cdeh

Plan 8.2.16. (Continued)

Blocks

<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
ae	be	ce	abce
bcde	acde	abde	de
bf	af	abcf	cf
acdf	bcdf	df	abdf
abcg	cg	bg	ag
dg	abdg	acdg	bcdg
cefg	abcefg	aefg	befg
abdefg	defg	bcdefg	acdefg
afgh	befg	cfgh	abcfgh
bcdfgh	acdfgh	abdfgh	dfgh
begh	aegh	abcegh	cegh
acdegh	bcdegh	degh	abdegh
abcefh	cefh	befh	aefh
defh	abdefh	acdefh	bcdefh
ch	abch	ah	bh
abdh	dh	bcdh	acdh

Plan 8.2.32. 8 factors, 1/2 replication, 4 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCDEFGH

Block confounding: ABCD, ABEF, CDEF

All main effects, all two-factor interactions, and all three-factor interactions are estimable. Six of the 70 four-factor interactions are confounded with blocks and the other 64 are confounded in pairs.

Blocks

Combine blocks in Plan 8.2.16 as follows:

1 and 2

3 and 4

5 and 6

7 and 8 .

Plan 8.2.64. 8 factors, 1/2 replication, 2 blocks of 64 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCDEFGH

Block confounding: ABCD

All main effects, all two-factor and all three-factor interactions are estimable. Two of the 70 four-factor interactions are confounded with blocks while the remaining 68 are confounded in pairs.

Blocks

Combine blocks in Plan 8.2.16 as follows:

1, 2, 3, and 4

5, 6, 7, and 8 .

Plan 9.2.8. 9 factors, 1/2 replication, 32 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCDEFGHJ

Block confounding: ABFGJ, ACFG, BCJ, ADG, BDFJ, CDF, ABCDGJ,
FGHJ, ABH, ACHJ, BCFGH, ADFHJ, BDGH, CDGHJ,
ABCDFH, ACE, BCEFGJ, EFG, ABEJ, CDEG, ABCDEFJ,
ADEF, BDEGJ, ACEFGHJ, BCEH, EHJ, ABEFGH,
CDEFHJ, ABCDEGH, ADEGHJ, BDEFH.

The main effects, 33 of the 36 two-factor interactions, 71 of the 84 three-factor interactions and 111 of the 126 four-factor interactions are estimable. The following interactions are confounded with blocks:

BD, FJ, GH, ABG, ABH, ACE, ADG, ADH, BCF, BCJ,
CDF, CDJ, EFG, EFH, EGJ, EHJ, ABEF, ABEJ, ACFG,
ACFH, ACGJ, ACHJ, ADEF, ADEJ, BCEG, BCEH, BDFJ,
BDGH, CDEG, CDEH, FGHJ.

Plan 9.2.8. (Continued).

Blocks

1.(Initial Block)

Block Multipliers

(1)	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>		
abcd	ab	ac	bc	ae	be	ce		
aegh								
bcdegh								
cefj	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	<u>13</u>		
abdefj	abce	af	bf	cf	abcf	ef		
acfghj								
bdfghj								
	<u>14</u>	<u>15</u>	<u>16</u>	<u>17</u>	<u>18</u>	<u>19</u>		
	abef	acef	bcef	ag	bg	cg		
	<u>20</u>	<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>	<u>25</u>		
	abcg	eg	abeg	aceg	bceg	fg		
	<u>26</u>	<u>27</u>	<u>28</u>	<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>	
	abfg	acfg	bcfg	aefg	befg	cefg	abcefg	

1910

1911

1912

1913

Year	Jan	Feb	Mar	Apr	May	June
1910	10	15	20	25	30	35
1911	12	18	22	28	32	38
1912	14	20	24	30	34	40
1913	16	22	26	32	36	42

1914

Plan 9.2.16. 9 factors, 1/2 replication, 16 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCDEFGHJ

Block confounding: ABCDE, ABCF, DEF, BDEFGH, ACFGH, ACDEGH, BGH, BCEH, ADH, AEFH, BCDFH, CDFG, ABEFG, ABDG, CEG.

All main effects, all two-factor interactions, all three-factor interactions except

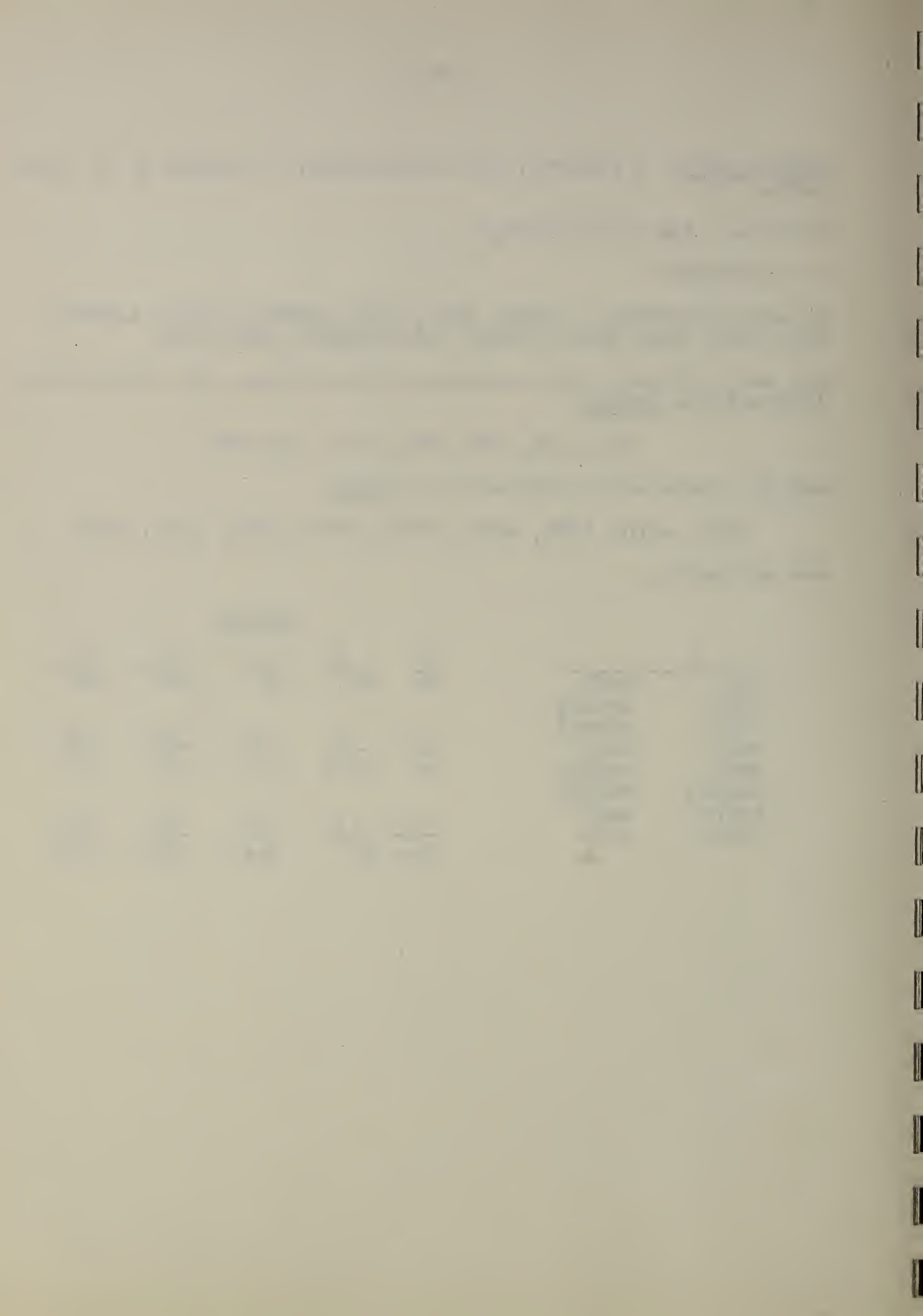
ACJ, ADH, BFJ, BGH, CEG, and DEF

and all four-factor interactions except

ABCF, ABDG, AEFH, AEGJ, BCEH, BDEJ, CDFG, CDHJ, FGHJ

are estimable.

		<u>Blocks</u>				
<u>1</u>		<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
(1)	befg	ab	ac	bc	ad	bd
acde	abcdfg					
adfj	abdegj					
cefj	bcgj	<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>
abhj	aefghj	cd	abcd	af	bf	cf
bcdehj	cdfghj					
bdfh	degh					
abcefh	acgh	<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
		abcf	df	abdf	acdf	bcdf



Plan 9.2.32. 9 factors, 1/2 replication, 8 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCDEFGHJ

Block confounding: ABCD, CDEF, ABEF, ACEG, BDEG, ADFG, BCFG

The main effects, all two-factor interactions, all three-factor interactions, and all four-factor interactions except those mentioned above are estimable.

Blocks

1. (Initial Block)

Block Multipliers

(1)	hj
abcd	abcdhj
abef	abefhj
cdef	cdefhj
bdeg	bdeghj
aceg	aceghj
adfg	adfghj
bcfg	bcfghj
bceh	bcej
adeh	adej
acfh	acfj
bdfh	bdfj
cdgh	cdgj
abgh	abgj
abcdefgh	abcdefghj
efgh	efgj

2.	ab
3.	ac
4.	bc
5.	ae
6.	be
7.	ce
8.	abce

THE UNIVERSITY OF CHICAGO

PHYSICS DEPARTMENT

PHYSICS 311

PROBLEM SET 1

1. A particle of mass m moves in a circular path of radius r with constant speed v . Find the magnitude of the centripetal force.

SOLUTION

The centripetal force is given by $F_c = \frac{mv^2}{r}$.

Plan 6.4.2. 6 factors, 1/4 replication, 8 blocks of 2 units each.

Factors: A,B,C,D,E,F

I = ABCE = ABDF = CDEF

Block confounding: AB, BC, AC, EF, ABEF, BCEF, ACEF

All main effects are estimable. All two-factor interactions have two- and four-factor interactions as aliases. Three-factor interactions are aliases of main effects or of other three-factor interactions:

ACD = BDE = BCF = AEF

ACF = BEF = BCD = ADE .

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1) abcdef	ab cdef	ce abdf	abce df
<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
acd bef	bcd aef	ade bcf	bde acf

Plan 6.4.4. 6 factors, 1/4 replication, 4 blocks of 4 units each.

Factors: A,B,C,D,E,F

I = ABCE = ABDF = CDEF

Block confounding: AB, BC, AC

The main effects are estimable, but all two factor-interactions have two- and four-factor aliases while all three-factor interactions are aliases of either main effects or three-factor interactions.

	<u>Blocks</u>			
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)		ab	acd	bcd
abcdef		cdef	bef	aef
abce		ce	bde	ade
df		abdf	acf	bcf

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The information provided in this document is for your information only. It is not intended to be used as a substitute for professional advice. Please consult your attorney for more information.

Item	Value	Category	Notes
1	100	Real Estate	Property in State A
2	200	Personal Assets	Automobile
3	300	Financial Assets	Bank Account
4	400	Real Estate	Property in State B
5	500	Personal Assets	Household Goods
6	600	Financial Assets	Investment Account
7	700	Real Estate	Property in State C
8	800	Personal Assets	Art Collection
9	900	Financial Assets	Retirement Account
10	1000	Real Estate	Property in State D

Plan 6.4.8. 6 factors, $1/4$ replication, 2 blocks of 8 units each.

Factors: A,B,C,D,E,F

I = ABCE = ABDF = CDEF

Block confounding: AB

The main effects are estimable. All two-factor interactions have two- and four-factor aliases while all three-factor interactions are aliases of either main effects or three-factor interactions.

	<u>Blocks</u>	
	<u>1</u>	<u>2</u>
	(1)	acd
	abcdef	bef
	abce	bde
	df	acf
	ab	bcd
	cdef	aef
	ce	ade
	abdf	bcf

1000

1000

1000

1000

Plan 7.4.4. 7 factors, 1/4 replication, 8 blocks of 4 units each.

Factors: A,B,C,D,E,F,G

I = ABCE = ABDFG = CDEFG

Block confounding: ACD, BEF, ABCDEF, BC, ABD, CEF, ADEF

The main effects, 12 of the 21 two-factor interactions, and 5 of the higher order interactions are estimable. The following two-factor interactions are estimable:

AD, AF, AG, BD, BF, BG, CD, CF, CG, DE, EF, EG.

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
(1)	ab	df	abdf	dg	abdg	fg	abfg
abce	ce	abcdef	cdef	abcdeg	cdeg	abcefg	cefg
adefg	bdefg	aeg	beg	aef	bef	ade	bde
bcdfg	acdfg	bcg	acg	bcf	acf	bcd	acd

THE UNIVERSITY OF CHICAGO

PHYSICS DEPARTMENT

PHYSICS 350

PROBLEM SET 1

Due: Monday, September 10, 2012

1. (10 points)

A particle of mass m moves in a potential $V(x) = \frac{1}{2}kx^2$.

(a)

Find the energy levels E_n and the corresponding wave functions $\psi_n(x)$.

(b) Calculate the expectation value of the position $\langle x \rangle$ for the ground state.

(c) Calculate the expectation value of the momentum $\langle p \rangle$ for the ground state.

(d) Calculate the expectation value of the energy $\langle E \rangle$ for the ground state.

Plan 7.4.8. 7 factors, 1/4 replication, 4 blocks of 8 units each.

Factors: A,B,C,D,E,F,G

I = ABCEG = ABDF = CDEFG

Block confounding: AB, AD, BD

The main effects and all of the two-factor interactions except (for those already confounded in pairs by the fundamental identity)

AB, AD, AF, BD, BF, DF

are estimable. Six of the three-factor interactions have as aliases six other three-factor interactions.

<u>Blocks</u>			
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	ab	bcd	acd
ce	abce	bde	ade
cg	abcg	bdg	adg
eg	abeg	bcdeg	acdeg
abdf	df	acf	bcf
abcdef	cdef	aef	bef
abcdfg	cdfg	afg	bfg
abdefg	defg	acefg	bcefg

1842

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Plan 7.4.16. 7 factors, 1/4 replication, 2 blocks of 16 units each.

Factors: A,B,C,D,E,F,G

I = ABCEG = ABDF = CDEFG

Block confounding: AB

All main effects and all two-factor interactions except

AB, AD, AF, BD, BF, DF

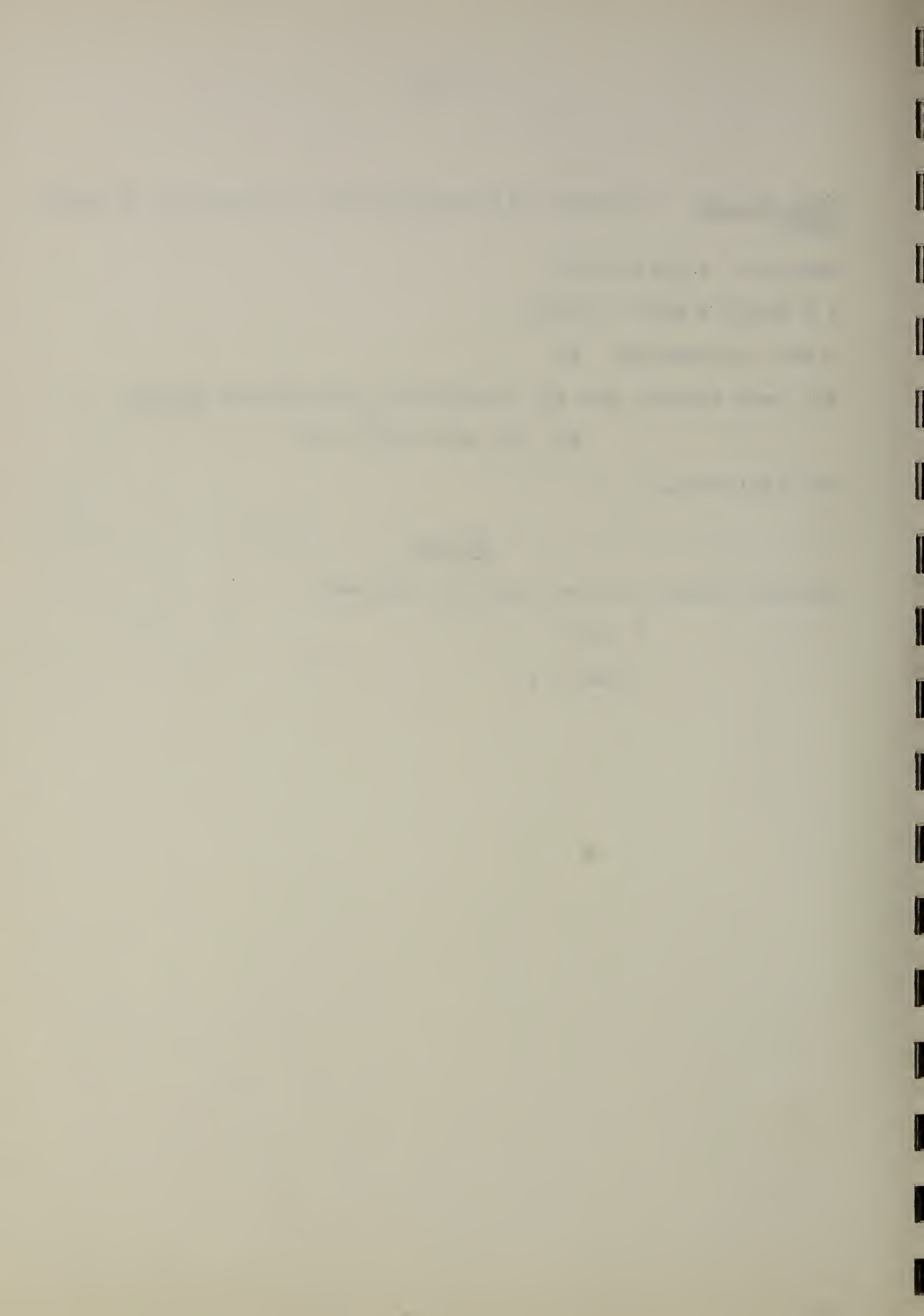
are estimable.

Blocks

Combine blocks in Plan 7.4.8 as follows:

1 and 2

3 and 4 .



Plan 8.4.4. 8 factors, 1/4 replication, 16 blocks of 4 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCEG = ABDFH = CDEFGH

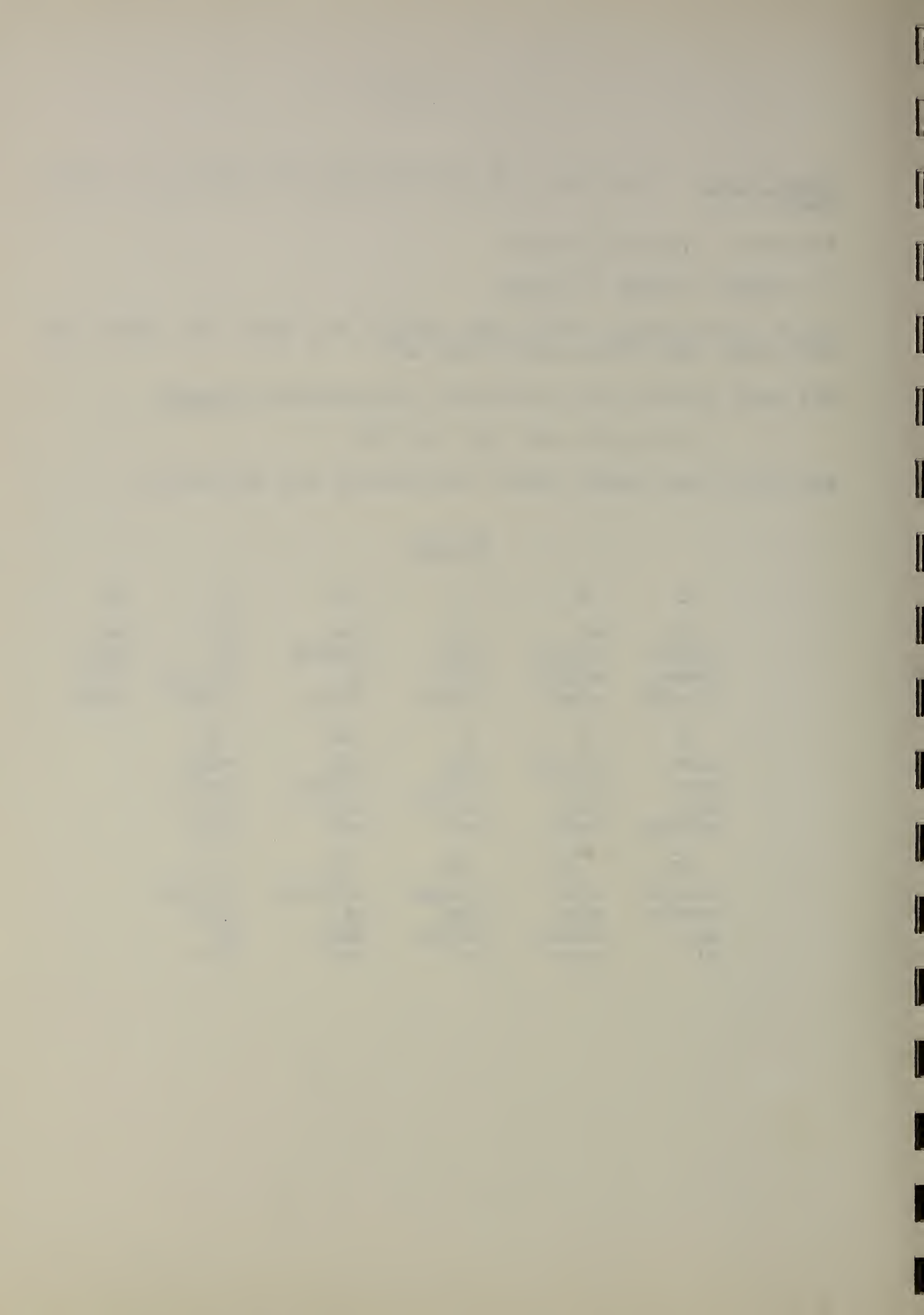
Block confounding: ACD, BEF, ABCDEF, BC, ABD, CEF, ADEF, DE, ACE, BDF, ABCF, BCDE, ABE, CDF, AF

The main effects, all two-factor interactions except

AF, AH, BC, BG, CG, DE, FH ,

and 19 of the higher order interactions are estimable.

<u>Blocks</u>					
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
(1)	ab	acd	bcd	aef	bef
adefh	bdefh	cefh	abcefh	dh	abdh
bcdeg	acdeg	abeg	eg	abcdfg	cdfg
abcfgh	cfgh	bdfgh	adfg	bcegh	acegh
<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>	
cdef	abcdef	agh	bgh	cdgh	
ach	bch	defg	abdefg	acefg	
bfg	afg	abcdeh	cdeh	beh	
abdegh	degh	bcf	acf	abdf	
<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>	
abcdgh	efgh	abefgh	acdefgh	bcdefgh	
bcefg	adg	bdg	cg	abcg	
aeh	bcdfh	ācdfh	abfh	fh	
df	abce	ce	bde	ade	



Plan 8.4.8. 8 factors, 1/4 replication, 8 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCEG = ABDFH = CDEFGH

Block confounding: ACD, BEF, ABCDEF, BC, ABD, CEF, ADEF

The main effects, all two-factor interactions except BC and FH, and 22 higher order interactions are estimable.

Blocks

Combine blocks in Plan 8.4.4 as follows:

1 and 13	5 and 9
2 and 14	6 and 10
3 and 15	7 and 11
4 and 16	8 and 12 .

The first part of the report deals with the general situation of the country, and the second part with the specific details of the project. The first part is divided into two sections: the first section deals with the general situation of the country, and the second section deals with the specific details of the project. The second part is divided into three sections: the first section deals with the specific details of the project, the second section deals with the specific details of the project, and the third section deals with the specific details of the project.

Table

Year	Value
1980	100
1981	100
1982	100
1983	100

Plan 8.4.16. 8 factors, $1/4$ replication, 4 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H

$I = ABCEG = ABDFH = CDEFGH$

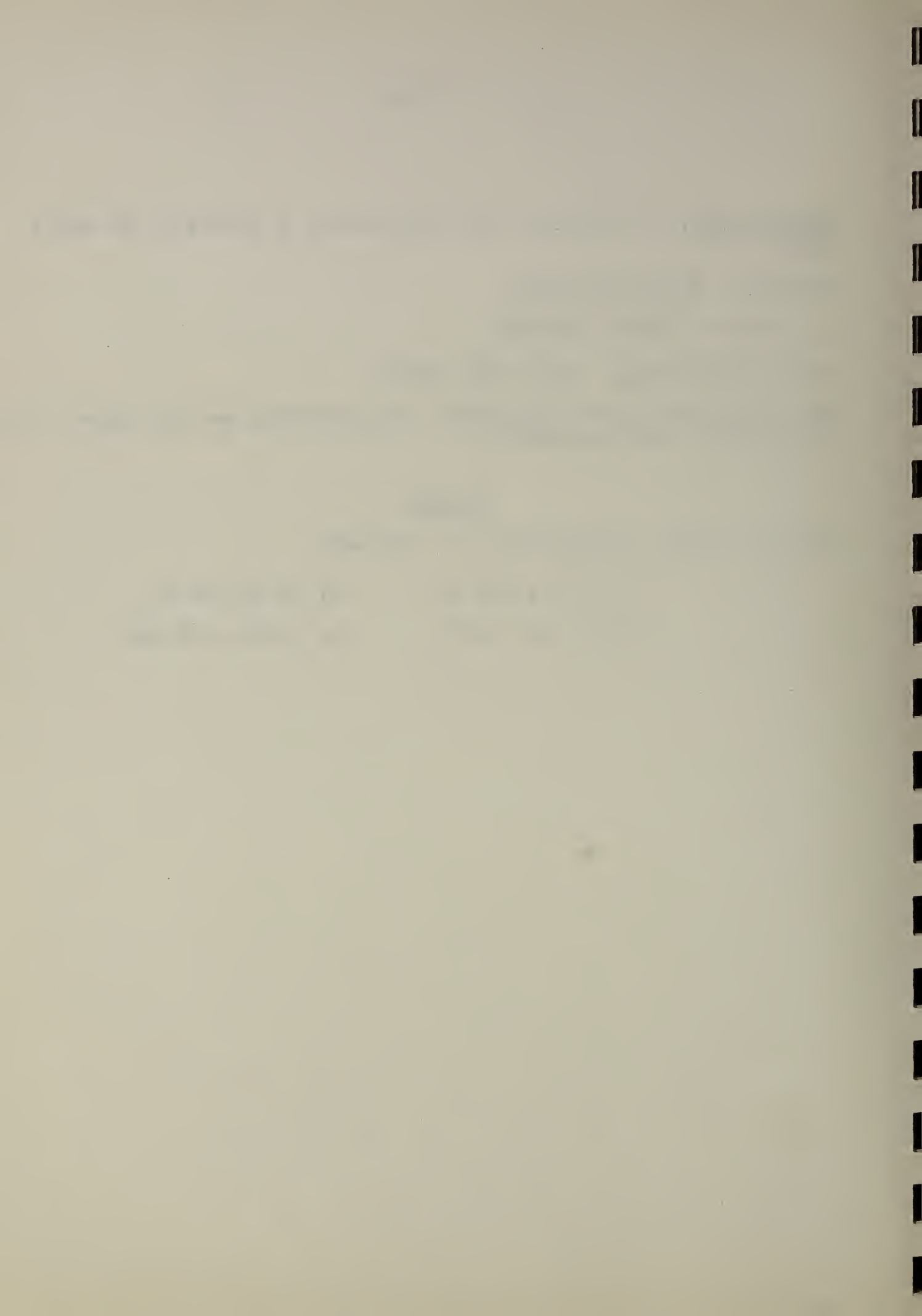
Block confounding: ACD, BEF, ABCDEF

The main effects, all two-factor interactions, and 2^4 higher order interactions are estimable.

Blocks

Combine blocks in Plan 8.4.4 as follows:

1, 7, 11, and 13	3, 5, 9, and 15
2, 8, 12, and 14	4, 6, 10, and 16.



Plan 8.4.32. 8 factors, 1/4 replication, 2 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCEG = ABDFH = CDEFGH

Block confounding: ACD

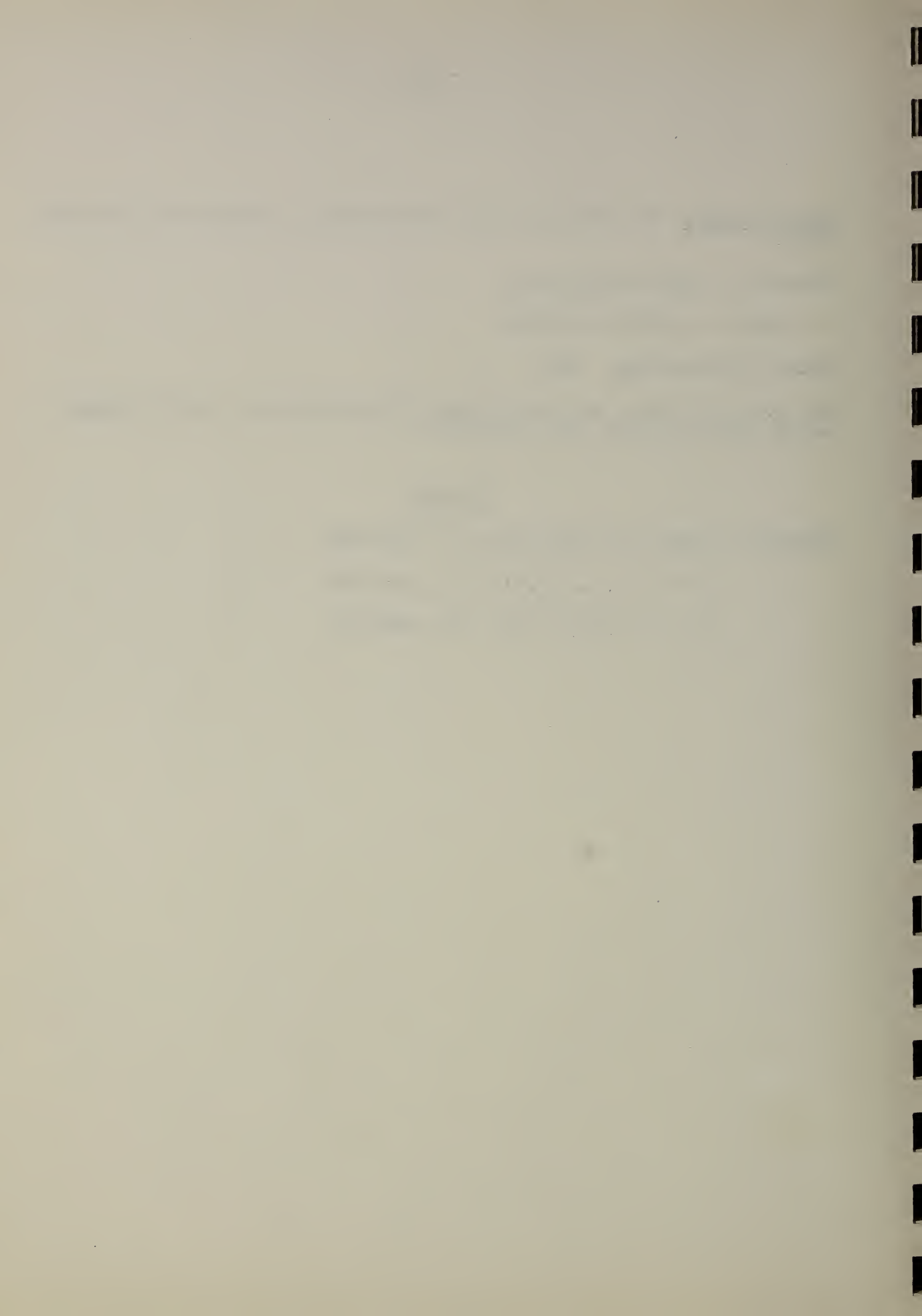
The main effects, all two-factor interactions, and 26 higher order interactions are estimable.

Blocks

Combine blocks in Plan 8.4.4 as follows:

1, 4, 6, 7, 10, 11, 13, and 16

2, 3, 5, 8, 9, 12, 14, and 15.



Plan 9.4.8. 9 factors, 1/4 replication, 16 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCEGJ = ABDFHJ = CDEFGH

Block confounding: ACDJ, BEFJ, ABCDEF, BCJ, ABD, CEF, ADEFJ, DEJ, ACE, BDF, ABCFJ, BCDE, ABEJ, CDFJ, AF.

The main effects, all two-factor interactions except AF and CG, and 69 of the higher order interactions are estimable.

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
(1)	ab	ce	abce	cg	abcg
bcdeg	acdeg	bdg	ad ^o	bde	ade
adefh	bdefh	acdfh	bcdfh	acdefgh	bcdefgh
abcfgh	cfgh	abefgh	efgh	abfh	fh
bdhj	adhj	bcdehj	acdehj	bcdghj	acdghj
ceghj	abceghj	ghj	abghj	ehj	abehj
abefj	efj	abcfj	cfj	abcefgj	cefgj
acdfgj	bcdfgj	adefgj	bdefgj	adfj	bdfj
<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>	
eg	abeg	df	abdf	cdef	
bcd	acd	bcefg	acefg	bfg	
adfgh	bdfgh	aeh	beh	ach	
abcefh	cefh	abcdgh	cdgh	abdegh	
bdeghj	adeghj	bfhj	afhj	bcefhj	
chj	abchj	cdefghj	abcdefghj	dfghj	
abfgj	fgj	abdej	dej	abcdj	
acdefj	bcdefj	acgj	bcgj	aegj	

1870

1871

1872

1873

1874

1875

Plan 9.4.8. (Continued)

Blocks

<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
abcdef	cdfg	abcdfg	defg	abdefg
afg	bef	aef	bcf	acf
bch	acegh	bcegh	agh	bgh
degh	bdh	dh	abcdeh	cdeh
acefhj	bcfghj	acfghj	befghj	aefghj
abdfghj	defhj	abdefhj	cdfhj	abcdfhj
cdj	abcdegj	cdegj	abdgj	dgj
begj	aj	bj	acej	bcej

100

100

100

100

100

100

100

Plan 9.4.16. 9 factors, 1/4 replication, 8 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCEGJ = ABDFHJ = CDEFGH

Block confounding: ACDJ, BEFJ, ABCDEF, BCJ, ABD, CEF, ADEFJ.

The main effects, all two-factor interactions, and 75 higher order interactions are estimable.

Blocks

Combine blocks in Plan 9.4.8 as follows:

1 and 4	9 and 12
2 and 3	10 and 11
5 and 8	13 and 16
6 and 7	14 and 15 .

1. The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that this is crucial for the company's financial health and for providing reliable information to stakeholders.

2. The second part of the document outlines the specific procedures for recording transactions. It details the steps from initial entry to final review, ensuring that all necessary information is captured and verified.

3. The third part of the document addresses the role of the accounting department in this process. It highlights the need for clear communication and collaboration between different departments to ensure the accuracy and completeness of the records.

Appendix A

Item	Description	Value
1	Office Supplies	\$150.00
2	Travel Expenses	\$250.00
3	Equipment Purchase	\$500.00
4	Professional Fees	\$300.00
5	Utilities	\$100.00

Plan 9.4.32. 9 factors, 1/4 replication, 4 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCEGJ = ABDFHJ = CDEFGH

Block confounding: ACDJ, BEFJ, ABCDEF

All main effects, all two-factor interactions, and 79 higher order interactions are estimable.

Blocks

Combine blocks in Plan 9.4.8 as follows:

1, 4, 10, and 11

2, 3, 9, and 12

5, 8, 14, and 15

6, 7, 13, and 16.

1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes the need for transparency and accountability in financial reporting.

2. The second part of the document outlines the various methods and techniques used to collect and analyze data. It includes a detailed description of the experimental procedures and the statistical tools employed.

3. The third part of the document presents the results of the study, showing the trends and patterns observed in the data. It includes several tables and graphs to illustrate the findings.

4. The fourth part of the document discusses the implications of the results and provides recommendations for future research. It also includes a conclusion summarizing the key points of the study.

Plan 10.4.8. 10 factors, 1/4 replication, 32 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K

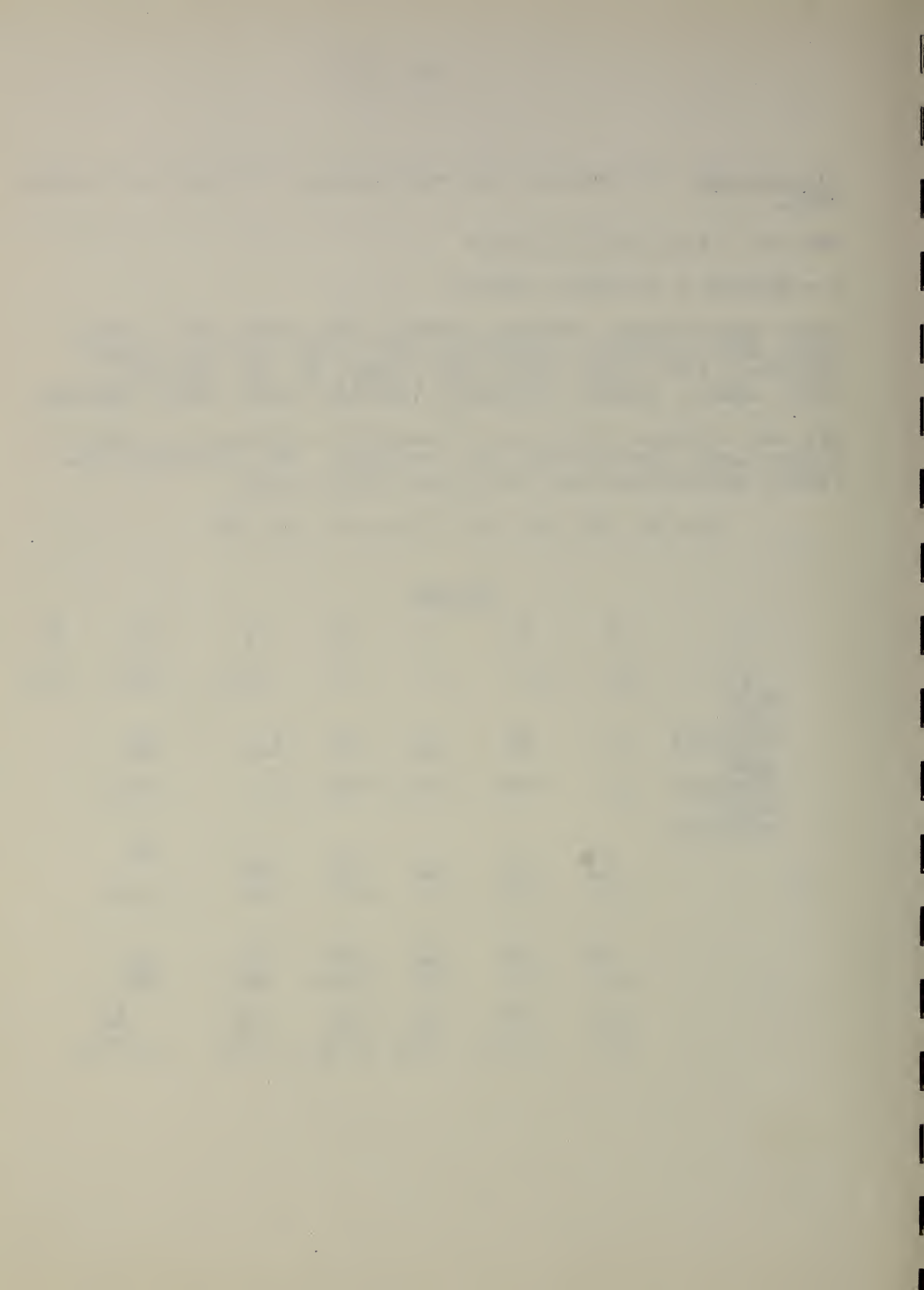
I = ABCDEFG = ABCDHJK = EFGHJK

Block confounding: ABEFHJ, CDEFHJ, ABCD, ACEH, BCFJ, ADFJ, BDEH, ACGK, BCEFGHJK, ADEFGHJK, BDGK, EGHK, ABFGJK, CDFGJK, ABCDEGHK, ACFJ, BCEH, ADEH, BDFJ, EFHJ, AB, CD, ABCDEFHJ, FGJK, ABEGHK, CDEGHK, ABCDFGJK, ACEFGHJK, BCGK, ADGK, BDEFGHJK

All main effects, 36 of the 45 two-factor interactions, and 178 higher order interactions are estimable. The following two-factor interactions are confounded with blocks:

AB, AC, AD, BC, BD, CD, EH, FJ, and GK.

	<u>Blocks</u>							
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
(1)		ab	ac	bc	ef	abef	acef	bcef
abcd								
efhj								
abcdefghj		<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>	
eghk								
abcdeghk		eg	abeg	aceg	bceg	fg	abfg	
fgjk								
abcdfgjk							<u>20</u>	
		<u>15</u>	<u>16</u>	<u>17</u>	<u>18</u>	<u>19</u>		
		acfg	bcfg	aeh	beh	ceh	abceh	
		<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>	<u>25</u>	<u>26</u>	
		afh	bfh	cfh	abcfh	agh	bgh	
		<u>27</u>	<u>28</u>	<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>	
		cgh	abcgh	aefgh	befgh	cefgh	abcefg	



Plan 10.4.16. 10 factors, 1/4 replication, 16 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K

I = ABCDEFG = ABCDHJK = EFGHJK

Block confounding: ABEFHJ, CDEFHJ, ABCD, ACEH, BCFJ, ADFJ, BDEH, ACGK, BCEFGHJK, ADEFGHJK, BDGK, EGHK, ABFGJK, CDFGJK, ABCDEGHK.

All main effects, 42 of the 45 two-factor interactions, and 188 higher order interactions are estimable. Interactions AD, BC, and FJ are confounded with blocks.

Blocks

Combine blocks in Plan 10.4.8 as follows:

1 and 12	17 and 28
2 and 11	18 and 27
3 and 10	19 and 26
4 and 9	20 and 25
5 and 16	21 and 32
6 and 15	22 and 31
7 and 14	23 and 30
8 and 13	24 and 29 .

1. Introduction
2. Methodology
3. Results
4. Discussion
5. Conclusion

6. Appendix
7. References
8. Acknowledgements
9. Author Biographies
10. Contact Information

Author	Year	Journal	Volume	Page
Smith, J.	2015	Journal of Business	42	123-145
Johnson, M.	2016	Journal of Marketing	38	234-256
Williams, K.	2017	Journal of Finance	50	345-367
Brown, L.	2018	Journal of Economics	45	456-478
Green, P.	2019	Journal of Law and Economics	30	567-589
White, R.	2020	Journal of Management	47	678-700
Black, S.	2021	Journal of Accounting	35	789-811
Grey, T.	2022	Journal of Information Systems	25	890-912
Blue, U.	2023	Journal of Operations Management	41	923-945
Gold, V.	2024	Journal of Human Resources	33	956-978

Plan 10.4.32. 10 factors, 1/4 replication, 8 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K

I = ABCDEFG = ABCDHJK = EFGHJK

Block confounding: ABEFHJ, CDEFHJ, ABCD, ACEH, BCFJ, ADFJ, BDEH.

All main effects, all two-factor interactions, and 193 higher order interactions are estimable.

Blocks

Combine blocks in Plan 10.4.8 as follows:

1, 6, 12, and 15

2, 5, 11, and 16

3, 8, 10, and 13

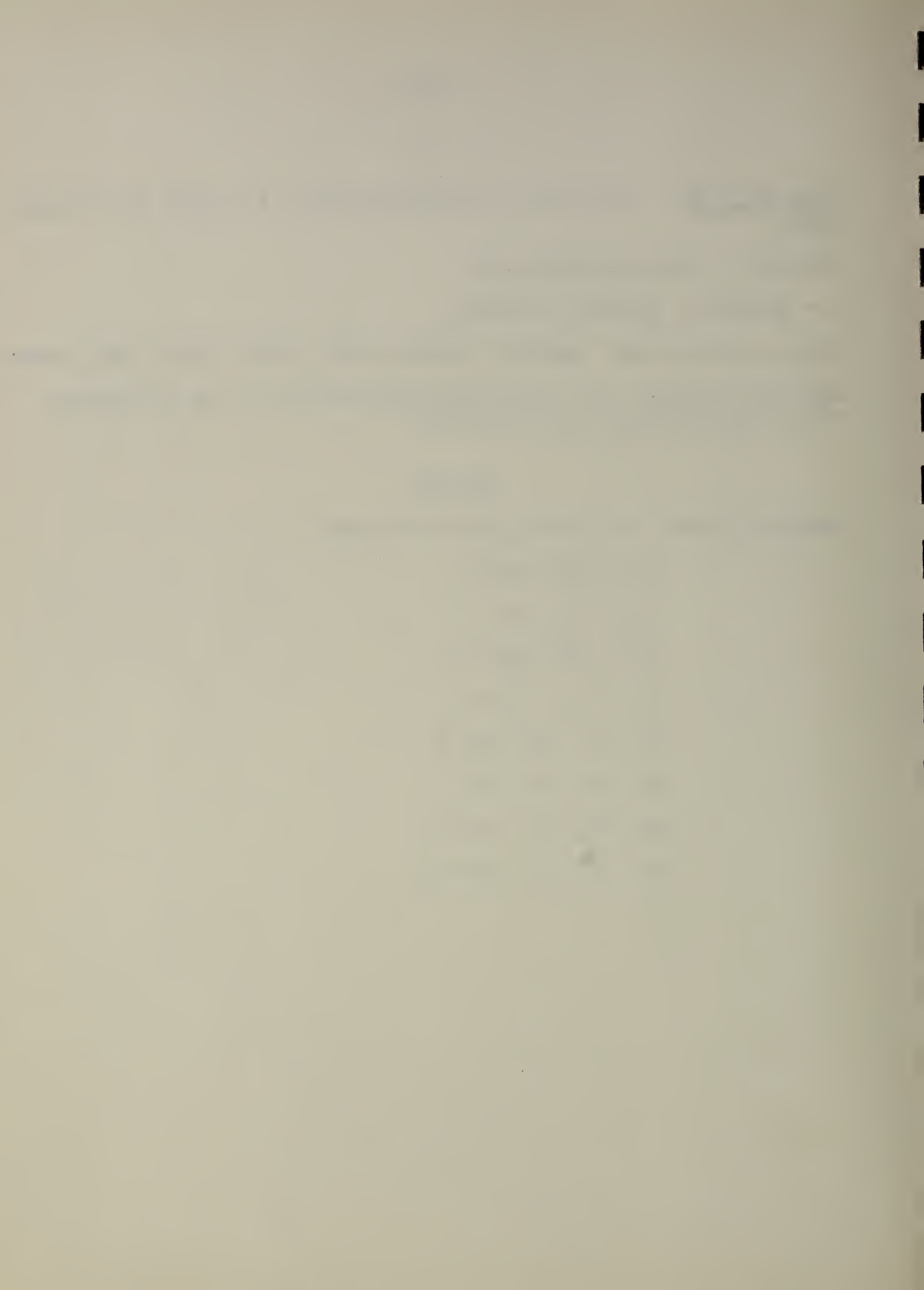
4, 7, 9, and 14

17, 22, 28, and 31

18, 21, 27, and 32

19, 24, 26, and 29

20, 23, 25, and 30.



Plan 8.8.4. 8 factors, 1/8 replication, 8 blocks of 4 units each.

Factors: A,B,C,D,E,F,G,H

I = ABEGH = ACFG = BCEFH = ABCD = CDEGH = BDFG = ADEFH

Block confounding: EGH, FG, EFH, BEH, BG, BEFGH, BF

All main effects and the following two-factor interactions are estimable:

AE, AH, BE, BH, CE, CH, DE, DH, EF,
EG, EH, FH, GH

<u>Blocks</u>							
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
(1)	abcd	acefgh	bdefgh	cdf	abf	adegh	bcegh
abcdefg	efg	bdh	ach	abeg	cdeg	bcfh	adfh
eh	abcdeh	acfg	ace	cdefh	abefh	adg	bcg
abcdfgh	fgh	bde	bdfg	abgh	cdgh	bcef	adef

Plan 8.8.8. 8 factors, 1/8 replication, 4 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H

I = ABEGH = ACFG = BCEFH = ABCD = CDEGH = BDFG = ADEFH

Block confounding: EGH, FG, EFH

All main effects and the following two-factor interactions are estimable:

AE, AH, BE, BH, CE, CH, DE, DH, EF, EG, EH, FH, GH .

The following two-factor interactions are confounded with one another:

AF = CG, AG = CF, BF = DG, BG = DF .

<u>Blocks</u>			
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	acefgh	cdf	adegh
abcdefg	bdh	abeg	bcfh
eh	acfg	cdefh	adg
abcdfgh	bde	abgh	bcef
abcd	bdefgh	abf	bcegh
efg	ach	cdeg	adfh
abcdeh	ace	abefh	bcg
fgh	bdfg	cdgh	adef

Plan 8.8.16. 8 factors, 1/8 replication, 2 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H

$I = ABEGH = ACFG = BCEFH = ABCD = CDEGH = BDFG = ADEFH$

Block confounding: EGH

All main effects are estimable and the following two-factor interactions are estimable:

AE, AH, BE, BH, CE, CH, DE, DH, EF, EG, EH, FH, GH .

The following two-factor interactions are confounded with one another:

AF=CG, AG=CF, BF=DG, BG=DF, AD=BC .

Blocks

<u>1</u>	<u>2</u>
(1)	cdf
abcdefg	abeg
eh	cdefh
abcdfgh	abgh
abcd	abf
efg	cdeg
abcdeh	abefh
fgh	cdgh
acefgh	adeqh
bdh	bcfh
acfg	adg
bde	bcef
bdefgh	bceqh
ach	adfh
ace	bcg
bdfg	adef

Plan 9.8.4. 9 factors, 1/8 replication, 16 blocks of 4 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABEGHJ = ACFGJ = BCEFH = ABCD = CDEGHJ = BDFGJ = ADEFH

Block confounding:

EGHJ, FGJ, EFH, AFHJ, AEFG, AGH, AEJ, ABEG,

ABHJ, ABFGH, ABEFJ, BEFGHJ, BF, BEH, BGJ

All main effects and all two-factor interactions are estimable with the exception of:

AB, AC, AD, AF, BC, BD, BF, CD, CF,

DF, EG, HJ .

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1) abcdefg eghj abcdfhj	fgh abcdeh efj abcdgj	abcdfgh eh abcdefj gj	abcd efg abcdeghj fhj
<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
bdefgh ach bdfj acegj	bde acfg bdghj acefhj	ace bdfg acghj bdefhj	bdegj acefgh bdh acfj

Plan 9.8.4. (Continued)

Blocks

<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>
abf	abgh	cdgh	cdf
cdeg	cdefh	abefh	abeg
abefghj	abej	cdej	cdefghj
cdhj	cdfgj	abfgj	abhj
<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
adegh	adef	bcef	bcegh
bcfh	bcg	adg	adfh
adj	adefghj	bcfghj	bcj
bcefgj	bcehj	adehj	adefgj

Plan 9.8.8. 9 factors 1/8 replication, 8 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABEGHJ = ACFGJ = BCEFH = ABCD = CDEGHJ = BDFGJ = ADEFH

Block confounding: EGHJ, FGJ, EFH, AFHJ, ACFG, AGH, AEJ

All main effects and all two-factor interactions are estimable with the exception of:

AB, AC, AD, BC, BD, CD

<u>Blocks</u>							
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
(1)	abcd	bdc	ace	abf	cdf	adef	bcef
abcdefg	efg	acfg	bdfg	cdeg	abeg	bcg	adg
fgh	abcdfgh	bdefgh	acefgh	abgh	cdgh	adegh	bcegh
abcdeh	eh	ach	bdh	cdefh	abefh	bcfh	adfh
efj	abcdefj	bdfj	acfj	abej	cdej	adj	bcj
abcdgj	gj	acegj	bdegj	cdfgj	abfgj	bcefgj	adefgj
eghj	abcdeghj	bdghj	acghj	abefghj	cdefghj	adefghj	bcfghj
abcdfhj	fhj	acefhj	bdefhj	cdhj	abhj	bcehj	adehj

Plan 9.8.16. 9 factors, 1/8 replication, 4 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABEGHJ = ACFGJ = BCEFH = ABCD = CDEGHJ = BDFGJ = ADEFH

Block confounding: EGHJ, FGJ, EFH

All main effects and all two-factor interactions are estimable with the exception of:

AB, AC, AD, BC, BD, CD .

Blocks

Combine blocks in Plan 9.8.8 as follows:

1 and 2

3 and 4

5 and 6

7 and 8 .

Plan 9.8.32. 9 factors, 1/8 replication, 2 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABEGHJ = ACFGJ = BCEFH = ABCD = CDEGHJ = BDFGJ = ADEFH

Block confounding: EGHJ

All main effects and all two-factor interactions are estimable with the exception of:

AB, AC, AD, BC, BD, CD .

Blocks

Combine blocks in Plan 9.8.8 as follows:

1, 2, 3, and 4

5, 6, 7, and 8 .

Plan 10.8.8. 10 factors, 1/8 replication, 16 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K

I = ABEGHJ = ACFGJK = BCEFHK = ABCDK = CDEGHJK = BDFGJ = ADEFH

Block confounding: GHJ, EHK, EGJK, FHJK, FGK, EFJ, EFGH, EG, EHJ, GHK, JK, EFGHJK, EFK, FGJ, FH

All main effects and all two-factor interactions are estimable with the exception of:

BD, EG, FH, JK

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	eghj	dfk	defghjk
abcd	abcdeghj	abcfk	abcefgghjk
aegjk	ahk	adefgj	adfh
bcdegjk	bcdhk	bcefgj	bcfh
acefgh	acfj	acdeghk	acdjk
bdefgh	bdfj	beghk	bjk
cfhjk	cefgk	cdhj	cdeg
abdfhjk	abdefgk	abhj	abeg
<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
dhjk	degk	fhj	efg
abchjk	abcegk	abcdfhj	abcdefg
adegh	adj	aefghk	afjk
bcegh	bcj	bcdefghk	bcdfjk
acdefgjk	acdfhk	acegj	ach
befgjk	bfhk	bdegj	bdh
cdf	cdefghj	ck	ceghjk
abf	abefghj	abdk	abdeghjk

Plan 10.8.8. (Continued).

Blocks

<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>
abej	abgh	abdefjk	abdfghk
cdej	cdgh	cefjk	cfgkh
bgk	behjk	bdfg	bdefhj
acdjk	acdehjk	acfg	acefhj
bcfghj	bcef	bcdghjk	bcdek
adefghj	adef	aghjk	aej
abcefhk	abcfgjk	abcdeh	abcdgjk
defhk	dfgjk	eh	gjk
<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
abdehk	abdgjk	abefh	abfgj
cehk	cgjk	cdefh	cdfgj
bdghj	bde	befghjk	befk
acghj	ace	acdfghjk	acdefk
bcdfgk	bcdefhjk	beg	bcehj
afgk	aefhjk	adg	adehj
abcdefj	abcdfgh	abcejk	abcghk
efj	fgh	dejk	dghk

Plan 10.8.16. 10 factors, 1/8 replication, 8 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K

I = ABEGHJ = ACFGJK = BCEFHK = ABCDK = CDEGHJK = BDFGJ = ADEFH

Block confounding: GHJ, EHK, EGJK, FHJK, FGK, EFJ, EFGH

All main effects and two-factor interactions are estimable.

Blocks

Combine blocks in Plan 10.8.8 as follows:

1 and 2	9 and 10
3 and 4	11 and 12
5 and 6	13 and 14
7 and 8	15 and 16

Plan 10.8.32. 10 factors, $1/8$ replication, 4 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K

I = ABEGHJ = ACFGJK = BCEFHK = ABCDK = CDEGHJK = BDFGJ = ADEFH

Block confounding: GHJ, EHK, EGJK

All main effects and two factor interactions are estimable.

Blocks

Combine blocks in Plan 10.8.8 as follows:

1, 2, 3, and 4

5, 6, 7, and 8

9, 10, 11, and 12

13, 14, 15, and 16 .

Plan 11.8.8. 11 factors, 1/8 replication, 32 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L

I = ABEGHJ = ACFGJK = BCEFHK = ABCDKL = CDEGHJKL = BDFGJL
= ADEFHL

Block confounding: GHJL, EHKL, EGJK, FHJK, FGKL, EFJL, EFGH,
EGL, EHJ, GHK, JKL, EFGHJKL, EFK, FGJ,
FHL, AEG, AEHJL, AGHKL, AJK, AEFGHJK,
AEFKL, AFGJL, AFH, AL, AGHJ, AEHK, AEGJKL,
AFHJKL, AFGK, AEFJ, AEFGL

All main effects and all two-factor interactions are estimable with the exception of:

AL, CJ, DG .

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
(1)	eghj	dfk	defghjk	dhjk	degk	fhj	efg
bcdegjk							
bdefgh							
cfhjk							
adfgkl							
abcefjl							
abehkl							
acdghjl							

Plan 11.8.8. (Continued).

Blocks

<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>
abej	abgh	abdefjk	abdfghk
<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
abdehk	abdgjk	abefh	abfgj
<u>17</u>	<u>18</u>	<u>19</u>	<u>20</u>
abdfi	abdefghjl	abkl	abeghijkl
<u>21</u>	<u>22</u>	<u>23</u>	<u>24</u>
abfhijkl	abefgkl	abdhjl	abdegl
<u>25</u>	<u>26</u>	<u>27</u>	<u>28</u>
defjl	dfghl	ejkl	ghkl
<u>29</u>	<u>30</u>	<u>31</u>	<u>32</u>
efhkl	fgjkl	dehl	dgjl

Plan 11.8.16. 11 factors, 1/8 replication, 16 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L

I = ABEGHJ = ACFGJK = BCEFHK = ABCDKL = CDEGHJKL = BDFGJL
 = ADEFHL

Block confounding: GHJL, EHKL, EGJK, FHJK, FGKL, EFJL, EFGH,
 EGL, EHJ, GHK, JKL, EFGHJKL, EFK, FGJ, FHL

All main effects and two-factor interactions are estimable.

	<u>Blocks</u>			
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)		eghj	dfk	defghjk
bcdegjk				
bdefgh	<u>5</u>		<u>6</u>	<u>7</u>
cfhjk		dhjk	degk	fhj
adfgkl				
abcdfjl	<u>8</u>		<u>9</u>	<u>10</u>
abehkl		efg	abej	abgh
acdghjl				
defjl	<u>11</u>		<u>12</u>	<u>13</u>
bcfgkl		abdefjk	abdfghk	abdehk
bghjl				
cdehkl				
aegjk	<u>14</u>		<u>15</u>	16
abcd		abdgjk	abefh	abfgj
abdfhjk				
acefgh				

Plan 11.8.32. 11 factors, 1/8 replication, 8 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L

I = ABFGHJ = ACFGJK = BCEFHK = ABCDIL = CDEGHJKL
 = BDFGJL = ADEFHL

Block confounding: GHJL, EHKL, EGJK, FHJK, FGKL, EFJL, EFGH

All main effects and two-factor interactions are estimable.

Blocks

1			
(1)			abfgj
bcdegjk			acdefk
bdefgh			adehj
cfhjk			abceghk
adfgkl			bdjkl
abcefjl			cegl
abehkl			efghjkl
acdghjl			bcdfnl
defjl			abdegl
bcfgkl			acjkl
bghjl			afhl
cdehkl			abcdefghjkl
aegjk			befk
abcd			cdfgj
abdfhjk			dghk
acefgh			bcehj
<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
eghj	dfk	defghjk	dhjk
6	<u>7</u>	<u>8</u>	
degk	fhj	efg	

The first part of the paper discusses the general theory of the subject. It is shown that the theory is based on the principle of least action. The action is defined as the integral of the Lagrangian over time. The Lagrangian is a function of the coordinates and velocities. The equations of motion are derived from the principle of least action.



Plan 7.16.4. 7 factors, 1/16 replication, 2 blocks of 4 units each.

Factors: A,B,C,D,E,F,G

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFG
= EFG = CDG = ABG = ADF = BCF = BDE = ACE

Block confounding: A

All main effects except A are estimable. They are identified with two-factor interactions as follows:

B = AG = CF = DE	E = FG = BD = AC
C = DG = BF = AE	F = EG = AD = BC
D = CG = AF = BE	G = EF = CD = AB

Blocks

<u>1</u>	<u>2</u>
(1)	acfg
cdef	adeg
bdfg	abcd
bceg	abef

Plan 7.16.8. 7 factors, 1/16 replication, 1 block of 8 units each.

Factors: A,B,C,D,E,F,G

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFG
= EFG = CDG = ABG = ADF = BCF = BDE = ACE

All main effects are estimable. They are identified with two-factor interactions as follows:

A = BG = DF = CE	E = FG = BD = AC
B = AG = CF = DE	F = EG = AD = BC
C = DG = BF = AE	G = EF = CD = AB
D = CG = AF = BE	

Block

(1)	abcd
acfg	bdfg
adeg	bceg
cdef	abef

Plan 8.16.4. 8 factors, 1/16 replication, 4 blocks of 4 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFGH
= EFGH = CDGH = ABGH = ADFH = BCFH = BDEH = ACEH

Block confounding: AB, AC, BC

All main effects are estimable.

	<u>Blocks</u>			
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)		acfg	cdef	adeg
abcd		bdfg	abef	bceg
abcdefgh		bdeh	abgh	bcfh
efgh		aceh	cdgh	adfh

Plan 8.16.8. 8 factors, 1/16 replication, 2 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFGH
= EFGH = CDGH = ABGH = ADFH = BCFH = BDEH = ACEH

Block confounding: AB

All main effects are estimable.

<u>Blocks</u>	
<u>1</u>	<u>2</u>
(1)	acfg
cdef	adeg
abcd	bdfg
abef	bceg
abcdefgh	bdeh
abgh	bcfh
efgh	aceh
cdgh	adfh

Plan 8.16.16. 8 factors 1/16 replication, 1 block of 16 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFGH
= EFGH = CDGH = ABGH = ADFH = BCFH = BDEH = ACEH

All main effects are estimable.

Block

(1)	abcdefgh
acfg	bdeh
adeg	bcfh
cdef	abgh
abcd	efgh
bdfg	aceh
bceg	adfh
abef	cdgh

Plan 9.16.4. 9 factors, 1/16 replication, 8 blocks of 4 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFGHJ
= EFGHJ = CDGHJ = ABGHJ = ADFHJ = BCFHJ = BDEHJ = ACEHJ

Block confounding: AB, AC, BC, AE, BE, CE, ABCE

All main effects and the following two-factor interactions are estimable: AH, AJ, BH, BJ, CH, CJ, DH, DJ, EH, EJ, FH, FJ, GH, GJ, HJ.

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	aceh	cdgh	adeg
abcdefgh	bdfg	abef	bcfh
abcdefgj	bdfghj	abefhj	bcfj
hj	acej	cdgj	adeghj
<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
abcd	bdeh	abgh	bceg
efgh	acfg	cdef	adfh
efgj	acfghj	cdefhj	adfj
abcdhj	bdej	abgj	bceghj

Plan 9.16.8. 9 factors, 1/16 replication, 4 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFGHJ
= EFGHJ = CDGHJ = ABGHJ = ADFHJ = BCFHJ = BDEHJ = ACEHJ

Block confounding: AB, AC, BC

All main effects and the following two-factor interactions are estimable: AH, AJ, BH, BJ, CH, CJ, DH, DJ, EH, EJ, FH, FJ, GH, GJ, HJ.

	<u>Blocks</u>			
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)		aceh	cdgh	adeg
abcd		bdeh	abgh	bceg
abcdefgh		bdfg	abef	bcfh
efgh		acfg	cdef	adfh
efgj		acfghj	cdefhj	adfj
abcdefghj		bdfghj	abefhj	bcfj
abcdhj		bdej	abgj	bceghj
hj		acej	cdgj	adeghj

Plan 9.16.16. 9 factors, 1/16 replication, 2 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFGHJ
= EFGHJ = CDGHJ = ABGHJ = ADFHJ = BCFHJ = BDEHJ = ACEHJ

Block confounding: AB

All main effects and the following two-factor interactions are estimable: AH, AJ, BH, BJ, CH, CJ, DH, DJ, EH, EJ, FH, FJ, GH, GJ, HJ.

Blocks

	<u>1</u>		<u>2</u>
(1)	cdgh	aceh	adeg
abcd	abgh	bdeh	bceg
abcdefgh	abef	bdfg	bcfh
efgh	cdef	acfg	adh
efgj	cdefhj	acfghj	adfj
abcdefgj	abefhj	bdfghj	bcfj
abcdhj	abgj	bdej	bceghj
hj	cdgj	acej	adeghj

Plan 10.16.8. 10 factors, 1/16 replication, 8 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K

I = ABCDJK = ABEFJ = CDEFK = BCEGJK = ADEG = ACFGK = BDFGJ
 = ABCDEFGH = EFGHJK = CDGHJ = ABGHK = ADFHJK = BCFH
 = BDEHK = ACEHJ

Block confounding: AD, AE, DE, BC, ABCD, ABCE, BCDE

All main effects are estimable. All two-factor interactions are estimable, except AD=EG, AE=DG, AG=DE, BC=EH, BF=CH, BH=CF, and JK.

<u>Blocks</u>			
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	aek	abcd	bhj
adeg	dgk	bceg	abdeghj
bcfh	abcefhk	adfh	cfj
abcdefgh	bcdfghk	efgh	acdefgj
bcjk	abcej	adjk	chk
abcdegjk	bdgj	egjk	acdeghk
fhjk	aefhj	abcdfhjk	bfk
adefghjk	dfghj	bcefhjk	abdefgk
<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
dej	abehjk	acdhj	bdeh
agj	bdghjk	ceghj	abgh
bcdefhj	acefjk	abdfj	cdef
abcfghj	cdfgjk	befgj	acfg
bcdek	aceh	abdhk	cdehjk
abcgk	cdgh	beghk	acghjk
defhk	abef	acdfk	bdefjk
afghk	bdfg	cefgk	abfgjk

Plan 10.16.16. 10 factors, 1/16 replication, 4 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K

I = ABCDJK = ABEFJ = CDEFK = BCEGJK = ADEG = ACFGK = BDFGJ

= ABCDEFGH = EFGHJK = CDGHJ = ABGHK = ADFHJK = BCFH = BDEHK = ACEHJ

Block confounding: AD, AE, DE

All main effects are estimable. All two-factor interactions are estimable, except AD=EG, AE=DG, AG=DE, BC=EH, BF=CH, and BH=CF.

	<u>Blocks</u>			
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)		aek	acdhj	dej
adeg		dgk	ceghj	agj
bcfh		abcefhk	abdfj	bcdefhj
abcdefgh		bcdfghk	befgj	abcfghj
bcjk		abcej	abdhk	bcdek
abcdegjk		bcdgj	beghk	abcgk
fhjk		aefhj	acdfk	defhk
adefghjk		dfghj	cefgk	afghk
bhj		abehjk	abcd	bdeh
abdeghj		bdghjk	bceg	abgh
cfj		acefjk	adfh	cdef
acdefgj		cdfgjk	efgh	acfg
chk		aceh	adjk	cdehjk
acdeghk		cdgh	egjk	acghjk
bfk		abef	abcdfhjk	bdefjk
abdefgk		bdfg	bcefghjk	abfgjk

Plan 11.16.8. 11 factors, 1/16 replication, 16 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L

I = ABCDJK = ABEFJL = CDEFKL = BCEGJKL = ADEGL = ACFGK = BDFGJ
= ABCDEFGH = EFGHJK = CDGHJL = ABGHKL = ADFHJKL = BCFHL = BDEHK
= ACEHJ

Block confounding: DEFG, BCFG, BCDE, ACEF, ACDG, ABEG, ABDF, ABCDEFGHJKL, ABCHJKL, ADEHJKL, AFGHJKL, BDGHJKL, BEFHJKL, CDFHJKL, CEGHJKL.

All main effects are estimable. All two-factor interactions except the following are estimable:

EF, DG, AC, BH .

<u>Blocks</u>			
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	abdfj	abcd	cfj
abcdefgh	ceghj	efgh	abdeghj
defgjl	abegl	abcefgjl	cdegl
abchjl	cdfhl	dhjl	abfhl
acefjk	bcdek	bdefjk	aek
bdghjk	afghk	acghjk	bcdfghk
acdgkl	bcfgjkl	bgkl	adfgjkl
befhkl	adehjkl	acdefhkl	bcehjkl

Plan 11.16.8. (Continued).

Blocks

<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
bceg	bhj	abef	dej
adfh	acdefgj	cdgh	abcfghj
bcd fjl	bdefghl	abdgjl	fgl
aeghjl	acl	cefhjl	abcdehl
abfgjk	abcefhk	bcjk	acdfk
cdehjk	dgk	adefghjk	beghk
abdekl	abcdghjkl	bcdefgkl	acegjkl
cfghkl	efjkl	ahkl	bdfhjkl
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>
adeg	acdhj	cdef	abcej
bcfh	befgj	abgh	dfghj
afjl	acefghl	cgjl	abcdfgl
bcdeghjl	bdl	abdefhjl	ehl
cd f g jk	defhk	adjk	bfk
abehjk	abcgk	bce f g h jk	acdeghk
cekl	ghjkl	aefgkl	bdegjkl
abdfghkl	abcdefjkl	bcdhkl	acfhjkl
<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
acfg	bcdgj	bdfg	agj
bdeh	aefhj	aceh	bcdefhj
acdejl	bcefl	bejl	adefl
bfghjl	adghl	acdfghjl	bcghl
egjk	abdefgk	abcdegjk	cefgk
abcdfhjk	chk	fhjk	abdhk
dfkl	abjkl	abcfkl	cdjkl
abceghkl	cdefghjkl	deg hkl	abefghjkl

Plan 11.16.16. 11 factors, 1/16 replication, 8 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L

I = ABCDJK = ABEFJL = CDEFKL = BCEGJKL = ADEGL = ACFGK = BDFGJ
= ABCDEFGH = EFGHJK = CDGHJL = ABGHKL = ADFHJKL = BCFHL
= BDEHK = ACEHJ

Block confounding: DEFG, BCFG, BCDE, ACEF, ACDG, ABEG, ABDF

All main effects and two-factor interactions are estimable.

Blocks

Combine blocks in Plan 11.16.8 as follows:

1 and 2	9 and 10
3 and 4	11 and 12
5 and 6	13 and 14
7 and 8	15 and 16 .

Plan 12.16.8. 12 factors, 1/16 replication, 32 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M

I = ABCEJK = ABEFJL = CDEFKL = BCEGJKLM = ADEGLM = ACFGKM
= BDFGJM = ABCDEFGH = EFGHJK = CDGHJL = ABGHKL = ADFHJKLM
= BCFHLM = BDEHKM = ACEHJM

Block confounding: DEFG, BCFG, BCDE, ACEF, ACDG, ABEG, ABDF, ABCDEFGHJKL, ABCHJKL, ADEHJKL, AFGHJKL, BDGHJKL, BEFHJKL, CDFHJKL, CEGHJKL, LM, DEFGLM, BCFGLM, BCDELM, ACEFLM, ACDGLM, ABEGLM, ABDFLM, ABCDEFGHJKM, ABCHJKM, ADEHJKM, AFGHJKM, BDGHJKM, BEFHJKM, CDFHJKM, CEGHJKM

All main effects are estimable. All two factor interactions except the following are estimable:

AF, GH, CE, BD .

<u>Blocks</u>			
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1) abcdefg acefjk bdghjk bcdejlm afghjlm abdfklm ceghklm	defgjl	abcd	bgkl
<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
bceg	bcdfjl	abef	ahkl
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>
abdfj	abegl	adeg	afjl

Plan 12.16.8. (Continued).

Blocks

<u>13</u> cdef	<u>14</u> cgjl	<u>15</u> cfj	<u>16</u> cdeg1
<u>17</u> acfg	<u>18</u> dfkl	<u>19</u> bhj	<u>20</u> acl
<u>21</u> dej	<u>22</u> fgl	<u>23</u> bdfg	<u>24</u> bejl
<u>25</u> acdhj	<u>26</u> bd1	<u>27</u> bfk	<u>28</u> ehl
<u>29</u> chk	<u>30</u> bcefl	<u>31</u> agj	<u>32</u> adefl

Plan 12.16.16. 12 factors, 1/16 replication, 16 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M

I = ABCDJK = ABEFJL = CDEFKL = BCEGJKLM = ADEGLM = ACFGKM
= BDFGJM = ABCDEFGH = EFGHJK = CDGHJL = ABGHKL = ADFHJKLM
= BCFHLM = BDEHKM = ACEHJM

Block confounding: DEFG, BCFG, BCDE, ACEF, ACDG, ABEG, ABDF, ABCDEFGHJKL, ABCHJKL, ADEHJKL, AFGHJKL, BDGHJKL, BEFHJKL, CDFHJKL, CEGHJKL

All main effects and two-factor interactions are estimable.

Blocks

Combine blocks in Plan 12.16.8 as follows:

1 and 2	17 and 18
3 and 4	19 and 20
5 and 6	21 and 22
7 and 8	23 and 24
9 and 10	25 and 26
11 and 12	27 and 28
13 and 14	29 and 30
15 and 16	31 and 32 .

Plan 10.32.4. 10 factors, 1/32 replication, 8 blocks of 4 units each.

Factors: A,B,C,D,E,F,G,H,J,K

I = ABCD = ABEF = CDEF = ABGH = CDGH = EFGH = ABCDEFGH = ABJK
 = CDJK = EFJK = ABCDEFJK = GHJK = ABCDGHJK = ABefGHJK = CDEFghJK
 = ACEGJ = BDEGJ = BCFGJ = ADFGJ = BCEHJ = ADEHJ = ACFHJ = BDFHJ
 = BCEGK = ADEGK = ACFGK = BDFGK = ACEHK = BDEHK = BCFHK = ADFHK

Block confounding: CD, CE, DE, ACEF, ADEF, AF, ACDF

All main effects are estimable, but no two-factor interactions are estimable.

<u>Blocks</u>			
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	cdef	abcd	abef
abcdefgh	abgh	efgh	cdgh
ghjk	cdefghjk	abcdghjk	abefghjk
abcdefjk	abjk	efjk	cdjk
<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
bcegj	bdfgj	adegj	acfgj
adfhj	acehj	bcfhj	bdehj
bcehk	bdfhk	adehk	acfhk
adfgk	acegk	bcfgk	bdegk

Plan 10.32.8. 10 factors, $1/32$ replication, 4 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K

I: Same as Plan 10.32.4

Block confounding: CD, CE, DE

All main effects are estimable, but no two-factor interactions are estimable.

Blocks

Combine blocks in Plan 10.32.4 as follows:

1 and 2

3 and 4

5 and 6

7 and 8 .

Plan 10.32.16. 10 factors, 1/32 replication, 2 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K

I: Same as Plan 10.32.4

Block confounding: CD

All main effects are estimable, but no two-factor interactions are estimable.

Blocks

Combine blocks in Plan 10.32.4 as follows:

1, 2, 3, and 4

5, 6, 7, and 8 .

Plan 11.32.8. 11 factors, 1/32 replication, 8 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L

I = ABCDL = ABEFL = CDEF = ABGH = CDGHL = EFGHL = ABCDEFGH
 = ABJK = CDJKL = EFJKL = ABCDEFJK = GHJK = ABCDGHJKL
 = ABEFGHJKL = CDEFGHJK = ACEGJL = BDEGJ = BCFGJ = ADFGJL
 = BCEHJL = ADEHJ = ACFHJ = BDFHJL = BCEGKL = ADEGK = ACFGK
 = BDFGKL = ACEHKL = BDEHKL = BCFHKL = ADFHKL

Block confounding: CD, CE, DE, ACEF, ADEF, AF, ACDF

All main effects are estimable; the following two-factor interactions are estimable:

AL, BC, BD, BE, BF, BL, CG, CH, CJ, CK, CL,
 DG, DH, DJ, DK, DL, EG, EH, EJ, EK, EL, FG,
 FH, FJ, FK, FL, GL, HL, JL, KL

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	cdef	abcd	abef
abcdefgh	abgh	efgh	cdgh
ghjk	cdefghjk	abcdghjk	abefghjk
abcdefjk	abjk	efjk	cdjk
bhjl	bcdefhjl	acdhjl	aefhjl
acdefgjl	agjl	befgjl	bcdgjl
bgkl	bcdefgkl	acdgkl	aefgkl
acdefhkl	ahkl	befhkl	bcdhkl

Plan 11.32.8. (Continued).

<u>Blocks</u>			
<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
bcegj	bdfgj	adegj	acfgj
adfhj	acehj	bcfhj	bdehj
bcehk	bdfhk	adehk	acfhk
adfgk	acegk	bcfgk	bdegk
ceghl	dfghl	abdeghl	abcfghl
abdf1	abcel	cfl	del
cejkl	dfjkl	abdejkl	abcfjkl
abdfghjkl	abceghjkl	cfghjkl	deghjkl

Plan 11.32.16. 11 factors, 1/32 replication, 4 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L

I: Same as Plan 11.32.8

Block confounding: CD, CE, DE

All main effects and the following two-factor interactions are estimable:

AC, AD, AE, AF, AL, BC, BD, BE, BF, BL, CG,
CH, CJ, CK, CL, DG, DH, DJ, DK, DL, EG, EH,
EJ, EK, EL, FG, FH, FJ, FK, FL, GL, HL, JL,
KL

Blocks

Combine blocks in Plan 11.32.8 as follows:

- 1 and 2
- 3 and 4
- 5 and 6
- 7 and 8 .

Plan 11.32.32. 11 factors, 1/32 replication, 2 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L

I: Same as Plan 11.32.8

Block confounding: CD

All main effects and the following two-factor interactions are estimable:

AC, AD, AE, AF, AL, BC, BD, BE, BF, BL, CG,
CH, CJ, CK, CL, DG, DH, DJ, DK, DL, EG, EH,
EJ, EK, EL, FG, FH, FJ, FK, FL, GL, HL, JL,
KL

Blocks

Combine blocks in Plan 11.32.8 as follows:

1, 2, 3, and 4

5, 6, 7, and 8 .

Plan 12.32.8. 12 factors, 1/32 replication, 16 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M

I = ABCDLM = ABEFL = CDEFM = ABGHM = CDGHL = EFGHLM = ABCDEFGH
 = ABJKM = CDJKL = EFJKLM = ABCDEFJK = GHJK = ABCDGHJKLM
 = ABEFGHJKL = CDEFGHJKM = ACEGJL = BDEGJM = BCFGJ = ADFGJLM
 = BCEHJLM = ADEHJ = ACFHJM = BDFHJL = BCEGKLM = ADEGK = ACFGKM
 = BDFGKL = ACEHKL = BDEHKM = BCFHK = ADFHKLM

Block confounding: CD, CE, DE, ACEF, ADEF, AF, ACDF, KLM,
 CDKLM, CEKLM, DEKLM, ACEFKLM, ADEFKLM,
 AFKLM, ACDFKLM

All main effects and two-factor interactions are estimable except for the following:

AF, BG, CD, CE, CH, DE, DH, EH, GH, GJ, HJ,
 JK, JM .

<u>1</u>	<u>Blocks</u> <u>2</u>	<u>3</u>	<u>4</u>
(1)	bcdem	abcd	aem
abcdefgh			
afjkm			
bcddeghjkm			
bgkl			
acdefhkl			
abfgjlm			
cdehjlm			

Plan 12.32.8. (Continued).

Blocks

<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
cdef	bfm	abef	acdfm
<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>
del	bclm	abcel	adlm
<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
cfl	bdeflm	abdf1	aceflm

Plan 12.32.16. 12 factors, 1/32 replication, 8 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M

I: Same as Plan 12.32.8

Block confounding: CD, CE, DE, ACEF, ADEF, AF, ACDF

All main effects and two-factor interactions are estimable except for the following:

AF, CD, CE, DE, GH, GJ, HJ, JK

Blocks

Combine blocks in Plan 12.32.8 as follows:

1 and 2	9 and 10
3 and 4	11 and 12
5 and 6	13 and 14
7 and 8	15 and 16 .

Plan 12.32.32. 12 factors, 1/32 replication, 4 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M

I: Same as Plan 12.32.8

Block confounding: CD, ACEF, ADEF

All main effects and two factor interactions are estimable except for CD.

Blocks

Combine blocks in Plan 12.32.8 as follows:

1, 2, 3, and 4

5, 6, 7, and 8

9, 10, 11, and 12

13, 14, 15, and 16 .

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