NATIONAL BUREAU OF STANDARDS REPORT

3388

SHRINKAGE STRESSES IN MASONRY WALLS

PROGRESS REPORT NO. 1

by

T. W. Reichard, S. Levy and D. Watstein

Report to
Office of the Chief of Engineers
Department of the Army
and
Bureau of Yards and Docks
Department of the Navy

NBS

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● Office of Basic Instrumentation

● Office of Weights and Measures.

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Abstract

A method has been developed for obtaining a solution for the stresses and deformations in a masonry wall undergoing drying shrinkage while its lower edge is restrained from shrinking by a rigid foundation. The theoretical solution obtained for a wall whose length is twice its height is being verified by tests of rubber and plaster of Paris models.

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1. INTRODUCTION

During the past year, the National Bureau of Standards has been engaged in a study of stresses in masonry walls which are restrained from shortening at the foundation level while undergoing drying shrinkage or thermal contraction. The theoretical analysis in this study is being carried out by Mr. S. Levy and his associates in the Engineering Mechanics Section, while a concurrent experimental study is being made by T. W. Reichard and D. Watstein in the Structural Engineering Section. Both the theoretical and experimental studies

have been confined thus far to a wall whose height is half its length.

2. THEORETICAL ANALYSIS

A method has been developed for obtaining a solution for the stresses and deformations in a wall whose lower edge is restrained from shrinking. The method is general in form and can be applied to walls having any prescribed ratio of length to height. The solution is based on a stress function φ given below:

$$\varphi = \sum_{m=1,3\ldots} \frac{\pi^2 b}{16ma} k_m \left\{ \frac{\sinh \frac{m\pi y}{a}}{\cosh^2 \frac{m\pi b}{2a}} + \frac{2y}{b} \frac{\sinh \frac{m\pi b}{2a}\left(1-\frac{2y}{b}\right)}{\cosh \frac{m\pi b}{2a}} \right\} \sin \frac{m\pi x}{a}$$

$(0<x<a)$

$\left(\frac{b}{2}<y<b\right)$

$$+ \sum_{n=1,2,3\ldots} \frac{\pi^2 a}{16nb} t_n \left\{ \frac{\sinh \frac{n\pi x}{b}}{\cosh^2 \frac{n\pi a}{2b}} + \frac{2x}{a} \frac{\sinh \frac{n\pi a}{2b}\left(1-\frac{2x}{a}\right)}{\cosh \frac{n\pi a}{2b}} \right\} \sin \frac{n\pi y}{b}$$

$$+ \sum_{m=1,3\ldots} \frac{\pi^2}{16m^2} p_m \left\{ -\frac{\cosh \frac{m\pi b}{a}\left(\frac{y}{b}-\frac{1}{2}\right)}{\cosh^2 \frac{m\pi b}{2a}} + \left(2\frac{y}{b}-1\right) \frac{\cosh \frac{m\pi b}{a}\left(1-\frac{y}{b}\right)}{\cosh \frac{m\pi b}{2a}} + \frac{2a}{m\pi b} \frac{\sinh \frac{m\pi b}{a}\left(1-\frac{y}{b}\right)}{\cosh \frac{m\pi b}{2a}} \right\} \sin \frac{m\pi x}{a}$$

$$+ \sum_{m=1,3\ldots} \frac{\pi^2 b}{16ma} \ell_m \left\{ -\frac{\sinh \frac{m\pi y}{a}}{\sinh^2 \frac{m\pi b}{2a}} + \frac{2y}{b} \frac{\cosh \frac{m\pi b}{2a}\left(1-\frac{2y}{b}\right)}{\sinh \frac{m\pi b}{2a}} \right\} \sin \frac{m\pi x}{a}$$

$$+ \sum_{m=1,3\ldots} \frac{\pi^2 b}{16ma} s_m \left\{ \frac{\left(\frac{y}{b}-\frac{1}{2}\right) \sinh \frac{m\pi b}{a}\left(\frac{3}{2}-\frac{y}{b}\right) - \left(\frac{3}{2}-\frac{y}{b}\right) \sinh \frac{m\pi b}{a}\left(\frac{y}{b}-\frac{1}{2}\right)}{\sinh^2 \frac{m\pi b}{2a}} \right\} \sin \frac{m\pi x}{a}$$

The normal stresses on any element of the wall, σ_x and σ_y, and the shear stress τ_{xy} , are obtained from the stress function as follows:

$$\sigma_x = \frac{\partial^2 \varphi}{\partial y^2}$$

$$\sigma_y = \frac{\partial^2 \varphi}{\partial x^2}$$

and

$$\tau_{xy} = -\frac{\partial^2 \varphi}{\partial x \partial y}$$

The stress function has five infinite sequences of coefficients. The values of the terms in three of these sequences (i.e., k, t and l) are determined from the boundary condition that the shearing stress on the three free edges is zero. The boundary condition that the stress normal to these edges be zero is automatically satisfied. The fourth and fifth sequences of coefficients (i.e., p and s) are determined from the boundary condition that the lower edge is constrained by the foundation to have no deformation.

Two solutions have been obtained for a wall whose height is half its length. In one solution nine terms were used to fit the boundary conditions and in a second 15 terms were used together with estimates for five additional terms. The convergence of the series with increasing number of terms can be estimated from the plot in figures 1 and 2. On the basis of these results it is estimated that a solution using 40 terms should give good accuracy. Such a solution for a wall with a height equal to half its length is now being obtained.

3. DESCRIPTION OF THE MODELS

In order to verify the theoretical values of stresses and displacements in a wall undergoing drying shrinkage, several models were constructed and tested. These small scale models were made of either rubber or plaster of Paris.

The rubber models illustrated in figures 3 and 4 consisted essentially of a center strip representing the foundation to which were attached two wall panels. The foundation strip was 2- by 4-in. in cross section and 22-in. long. The wall panels were 9-in. high, 18-in. long and 1-in. thick. The three components of the model were molded separately and were cemented together with Pliobond cement. The Pliobond cement diluted to contain about 15 percent solids was applied to the surfaces in two coats. The first coat was allowed to dry for an hour and the second coat was dried for a minimum of 24 hours. The three units were then lightly clamped together and the whole model was placed in a 175° F oven for 12 hours. This method of cementing the rubber components together was quite satisfactory even though the tearing action at the ends of the wall panels during the test was quite severe.

Steel loading plates 2- by 4-in.in plan and 1/4-in. thick were bonded to the ends of the rubber foundation strip to permit application of tensile loads during the test. The load was applied by means of 3/8-in. diameter pull rods threaded

into the loading plates. The pull rods were turned down to a diameter of 1/8 in. at two sections in order to minimize the effect of any eccentricity in application of loads.

The state of stress which results in a wall undergoing shrinkage was simulated in the rubber model by applying a tensile load to the ends of the foundation strip. The displacements of the boundaries and interior points were observed by comparing the grids of white lines on the strained and unstrained wall panels as shown in figures 3 and 4. The white grid lines on the rubber were produced during the molding process by titanium oxide powder placed in grooves 0.002-in. deep milled in the steel plates of the mold.

The rubber used in the foundation strip consisted of 67 percent natural rubber and 33 percent NBS furnace black. The rubber in the wall units had the same proportions of natural rubber to five thermal black. The rubber in the wall had a modulus of elasticity of about 650 psi and the foundation strip had a slightly higher modulus. Poisson's ratio for both types of rubber was 0.5. The plaster models were cast as single units in which the center foundation strip was 3- by 4 1/4-in. in cross section and 56-in. long. The wall sections attached to the center strip along the 4 1/4-in. faces were 16-in. high, 32-in. long and 1 1/4-in. thick. The neat plaster of Paris used in the model was proportioned in the ratio of 2.5 parts of plaster to 1 part

of water, by weight. These proportions were chosen to give a good casting mix. The compressive strength of the plaster at the age of six weeks, when the model was tested, was 3400 psi as determined by 2- by 4-in. cylinders. The faces of the wall sections were cast against 1/4 in. glass plates in order to provide a smooth plane surface for attachment of bonded wire strain gages.

The magnitude and orientation of principal stresses were determined at 28 points with 3-element rosettes which can be seen in figure 5. The rosettes which clustered in the vicinity of each lower corner of the wall, were placed symmetrically with respect to the transverse centerline and the longitudinal midplane of the wall. Thus the strains at each of the seven points near the lower corner are given by the average of four 3-element rosettes. The elements of the rosettes were spaced 45° apart, with the middle element placed parallel to the lower edge of the wall.

The average modulus of elasticity of the plaster of Paris was 2,060,000 psi, the modulus of rigidity was 811,000 psi and the Poisson's ratio was 0.27. It is recalled in this connection that the value of Poisson's ratio assumed in the derivation of the theoretical solution was 0.3.

During the test of the plaster model, a compressive load of 20,000 lb was applied to the ends of the foundation strip and the readings of the strain gages obtained at this load were assumed to correspond to a state of zero stress. The load was then lowered to 500 lb and the resulting increment of strain indicated by the gages mounted at the midpoint of the foundation was taken to represent in magnitude the "shrinkage" of the wall section. The increments of strain observed in the wall section could thus be considered as being strains caused by stresses set up in a wall attached to a rigid foundation while undergoing shrinkage of specified magnitude. The value of "shrinkage" observed at the midpoint of foundation under conditions specified above was about 350×10^{-6} in./in.

COMPARISON OF COMPUTED VALUES OF SHEAR STRESS AT FREE EDGES OF WALL WITH ASSUMED BOUNDARY STRESSES ($\tau_{xy} = 0$)

LEGEND

⊙ 9 terms used in solution

X 15 " " " "

ε_s = shrinkage

E = modulus of elasticity of masonry

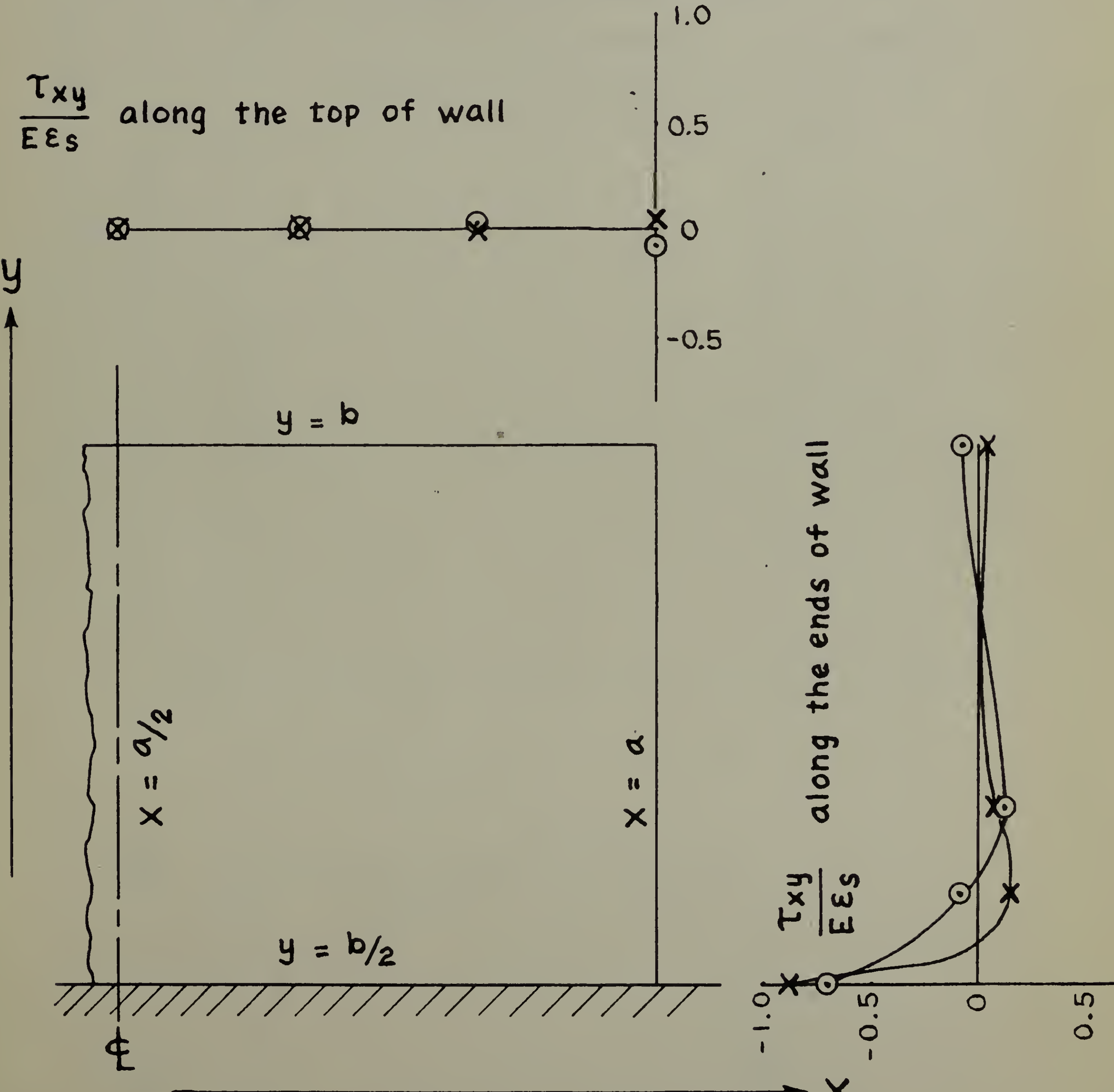

Figure 1.

COMPARISON OF COMPUTED AND ASSUMED STRESSES AND DISPLACEMENTS ALONG LOWER EDGE OF WALL

(TRUE VALUES: $\frac{\varepsilon_x}{\varepsilon_s} = 1$ and $\frac{V/a}{E} = 0$)

LEGEND

⊙ 9 terms in solution
X 15 " " "
ε_x = strain along lower edge of wall
ε_s = shrinkage
V = vertical displacement of foundation
E = modulus of elasticity of masonry

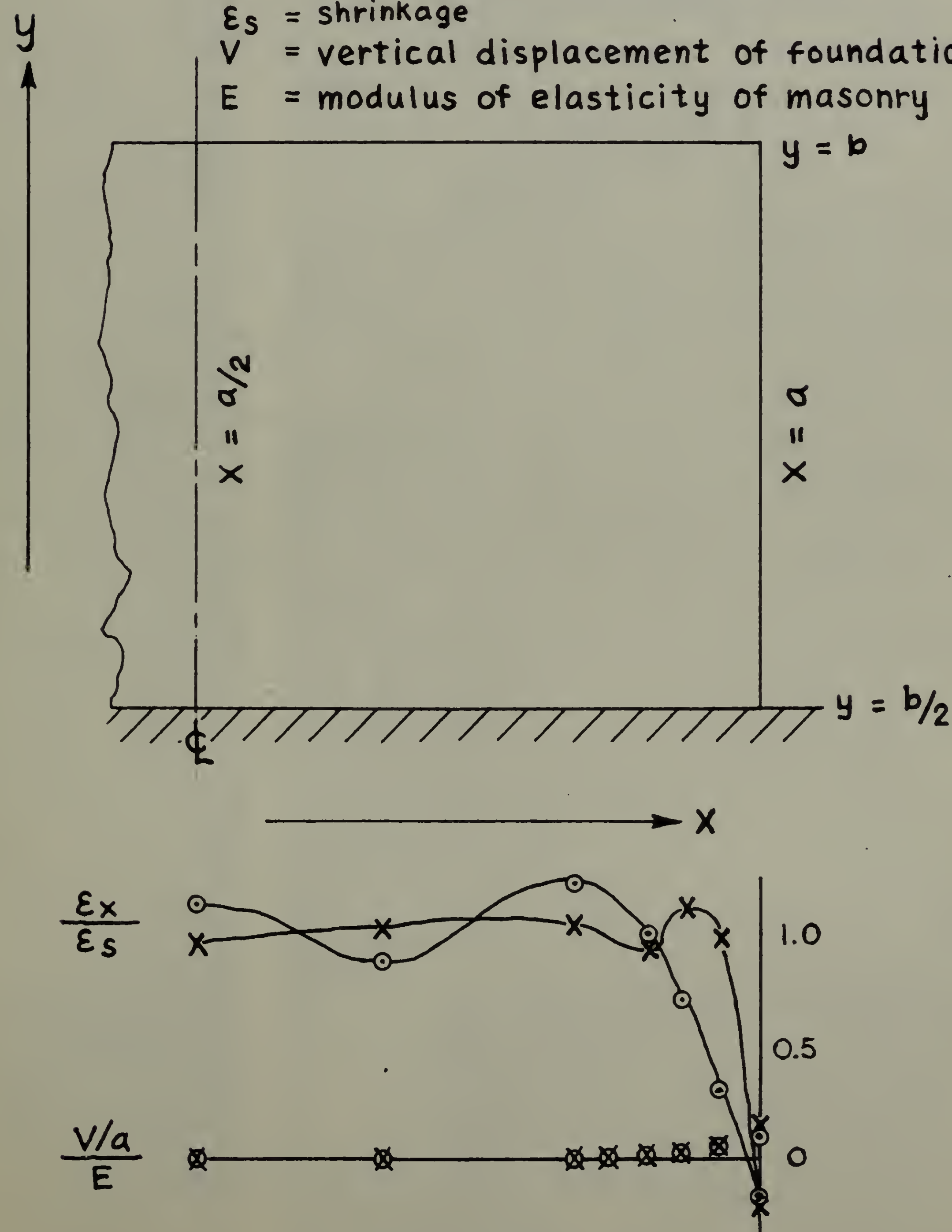

Figure 2.

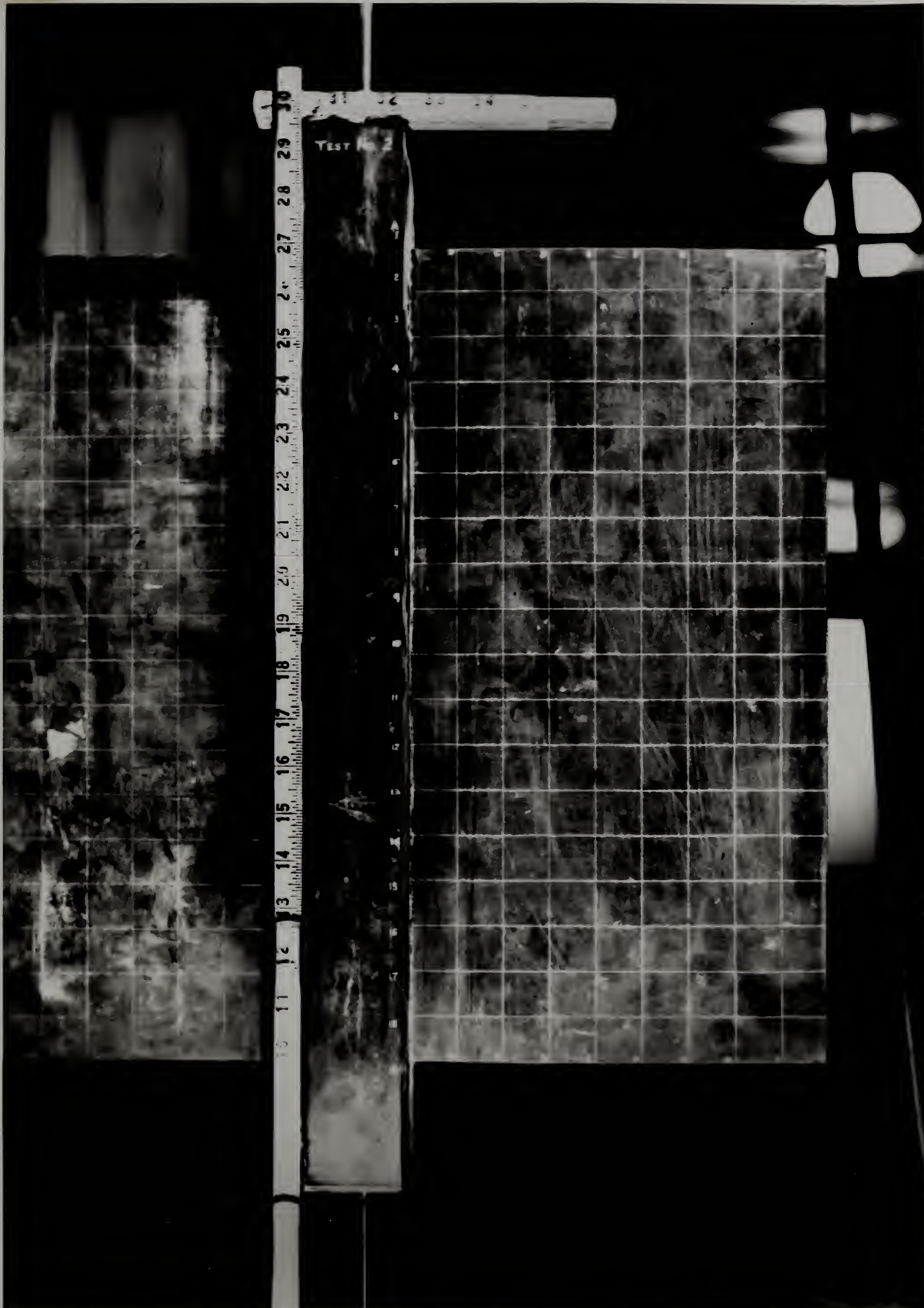

FIGURE 3. UNSTRAINED RUBBER MODEL

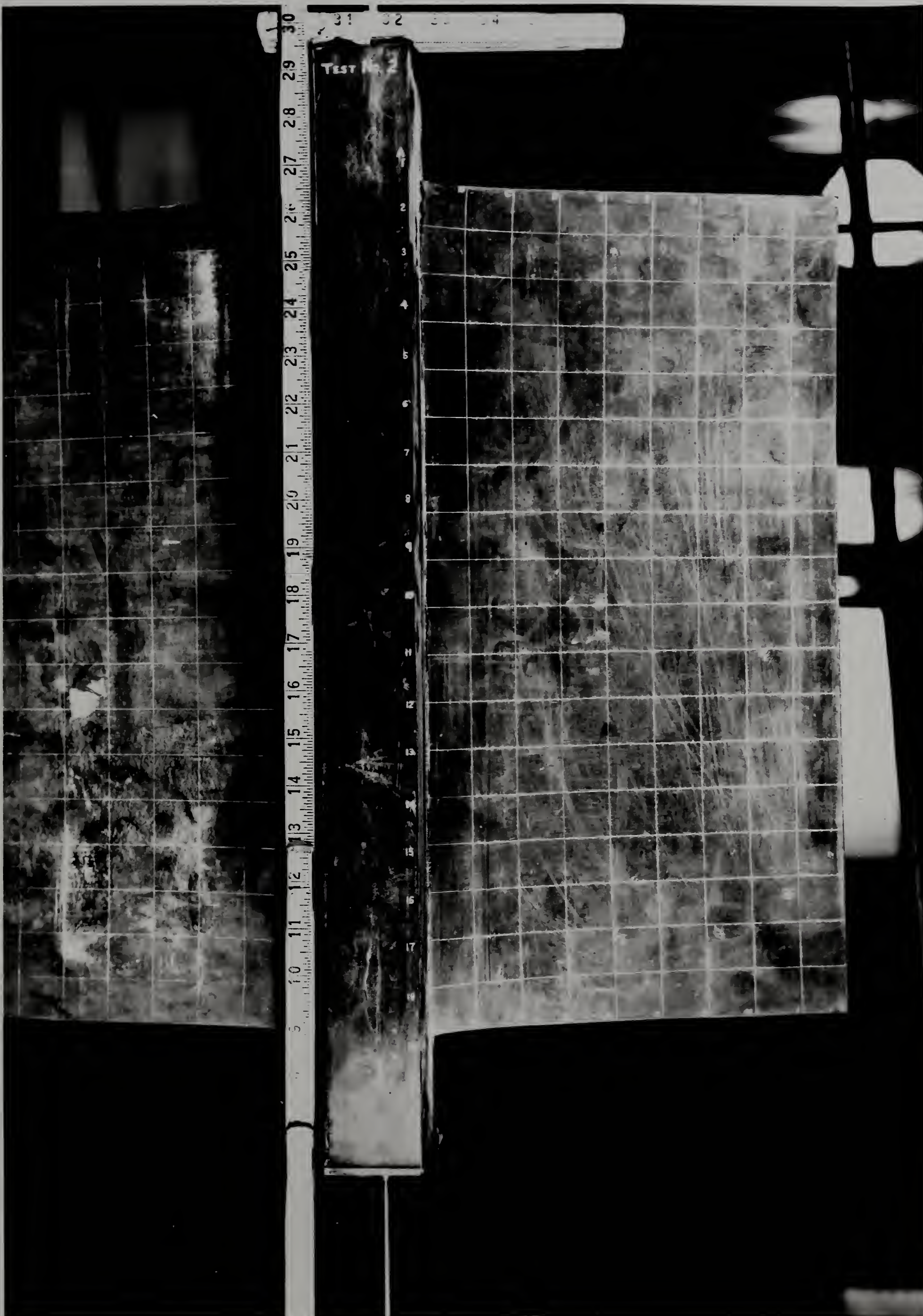

FIGURE 4. STRAINED RUBBER MODEL

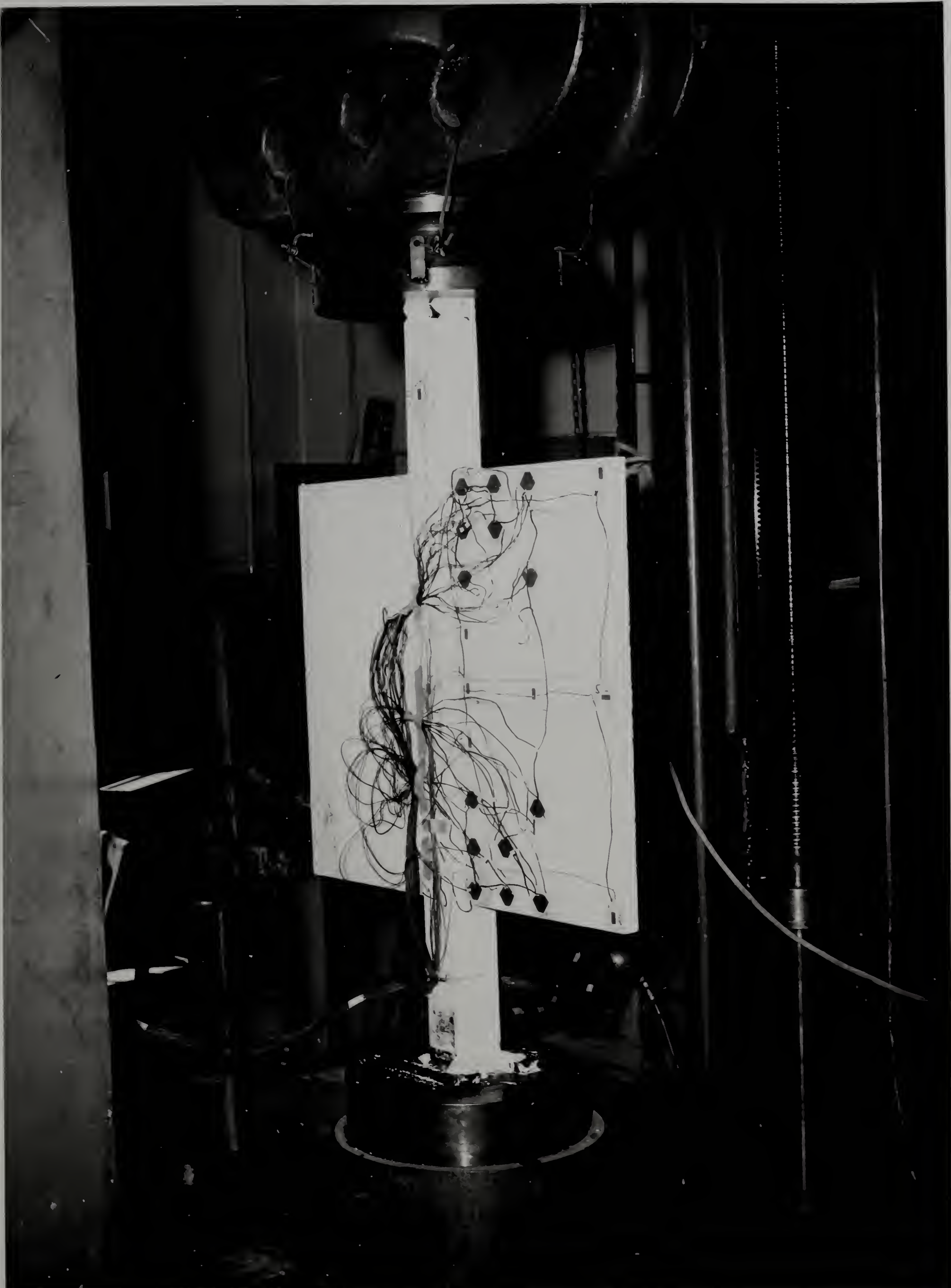

FIGURE 5. PLASTER MODEL UNDER TEST

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.00). Information on calibration services and fees can be found in NBS Circular 483, Testing by the National Bureau of Standards (25 cents). Both are available from the Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.

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