

# NATIONAL BUREAU OF STANDARDS REPORT

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Generation and Testing of Pseudo-random Numbers

by

Olga Taussky and John Todd



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# Generation and Testing of Pseudo-random Numbers

Olga Taussky and John Todd

National Bureau of Standards

## 1. INTRODUCTION

We shall confine our attention to generation and testing of sequences of pseudo-random numbers by arithmetical\* processes on automatic high speed digital computers. We shall also confine our attention mainly to a uniform distribution\*\* of random numbers not random digits\*\*\* The approximation of normal deviates and other random variates by polynomials in uniform variates has been discussed in detail by Teichroew [13]; for other methods e.g., acceptance or rejection methods, see von Neumann [6, 36-38] and Votaw and Rafferty [16].

We also confine our attention to the results of testing, not to the design of tests. Apart from the "quality" of the number generated we are mainly concerned with the speed of production.

\*There seems to be no published information about the testing of <sup>numbers</sup> ~~generated by~~ physical processes incorporated in automatic high speed computers such as the Ferranti or ERA machines.

\*\*For practical purposes we have found it satisfactory to approximate a normal deviate by the addition of some 8 or 12 uniform deviates; for a report on experiments concerning this, see Cameron and Newman [3]. See also Juncosa [17].

\*\*\* The following caution is necessary. It might be supposed that the digits in particular fixed positions of pseudo-random numbers would be satisfactory pseudo-random digits. Tests carried out by Juncosa [17] show that this is not the case; sequences which were very good pseudo-random numbers, gave rise to sequences of pseudo-random digits which could at best be classed as fair.

The written words about this topic seem to begin with Lehmer's paper [7] presented at Harvard 14 September 1949, shortly followed by Mauchly's paper [8] presented to the American Statistical Association on 29 December 1949). A fairly comprehensive account was included in a thesis of Teichroew [13].

Lehmer's definition of a pseudo-random sequence is worth repeating: it is "a vague notion embodying the idea of a sequence in which each term is unpredictable to the uninitiated and whose digits pass a certain number of tests, traditional with statisticians and depending somewhat on the uses to which the sequence is to be put."

## 2. MID-SQUARE METHODS

Lehmer mentioned the so-called mid-square method used on the ENIAC and due to von Neumann and Metropolis [see N. Metropolis (10)]. This can be described as follows, in a special case. Take a 4 digit number  $x_0$ , e.g.  $x_0 = 2061$ . Square it to obtain  $04247721$ . Define  $x_1 = 2477$ , the middle four digits of  $x_0^2$ . Next  $x_1^2 = 06135529$  and  $x_2 = 1355$ . Similarly  $x_3 = 8360$ ,  $x_4 = 8896$ , etc.

The detailed steps necessary to obtain  $x_1$ , for instance, on SEAC, the National Bureau of Standards Eastern Automatic Computer are as follows: Take the low product of  $x_0$  by itself to obtain 7721; then take the high product of this by 0100 to obtain 0077. Take the high product of  $x_0$  by itself to obtain 0424; then take the low product of this by 0100 to obtain 2400. Add 0077 and 2400 to obtain

$x_1 = 2477$ . This process can be shortened and speeded up in the case of machines which have e.g., shift-orders.

For the results of some tests on numbers generated this way see Mauchly [8] and Votaw and Rafferty [16]. For tests carried out by punched card equipment see Hammer [6, p. 33], and Forsythe [6, pp. 34-35]. Satisfactory results have been obtained by these methods in certain cases.

### 3. CONGRUENTIAL METHODS - MULTIPLICATIVE

These also are first mentioned by Lehmer. He used the relation

$$x_{n+1} = k x_n \pmod{M}$$

with  $k = 23$ ,  $M = 10^8 + 1$  for ENIAC. This sequence produces 8-decimal digit numbers with period 5882352. The choice of 23 is best possible for this modulus in so far as that no larger multiplier produces a longer period and no smaller multiplier produces a period more than half as long.

Tests on 5000 numbers generated this way were carried out, using punched card equipment by L. E. Cunningham, and they were found satisfactory.

In using ENIAC it was possible to sample these pseudo-random numbers at random; this additional precautionary measure is not convenient on other machines.



In 1950, when the National Bureau of Standards Eastern Automatic Computer came into operation it was decided to carry out a series of experiments in the Monte Carlo method: solution of partial differential equations [15], inverting of matrices [14], etc. For these experiments we have used random numbers generated as follows:

$$(1) \quad x_0 = 1, x_{n+1} = \rho x_n \pmod{2^{42}}$$

where  $\rho$  is any odd power of 5. In practice  $\rho = 5^{17}$ , (the largest power of 5 acceptable by the machine) and  $x_0$  could be any integer satisfying  $x_0 = 1 \pmod{5}$ . This sequence has period  $2^{40} \sim 10^{12}$ . It is generated by a single order: low multiplication.

We shall now illustrate the behavior of a similar sequence in a simple case. We take the residues mod  $2^6$  of powers of 5; these have period 16. They are, in decimal notation,

1, 5, 25, 61, 49, 53, 9, 45, 33, 37, 57, 29, 17, 21, 41, 13, 1,...

or in binary

000001, 000101, 011001, 111101, 110001, 110101, 001001, 101101,  
100001, 100101, 111001, 011101, 010001, 010101, 101001, 001101....

We note that the period of the digits in a particular position in numbers increases as we move to the left: the last binary digit is always 1, the next is always zero, the next is alternatively 0 and 1, the next has period 4, and so on. In the case of the numbers (1) it is only the digit in the 42nd place which has the full period  $2^{40}$ . This phenomenon was noted by R. Kersh and J. B. Rosser.



In practice this behavior is not troublesome, for we usually only require random numbers with a few binary digits. For instance, in the case of random walks on a plane lattice [15] we have to decide in which of the intervals

$$[0, \frac{1}{4}), [\frac{1}{4}, \frac{1}{2}), [\frac{1}{2}, \frac{3}{4}), [\frac{3}{4}, 1],$$

the random number lies, and we therefore only use the first two binary digits.

A set of  $16,384$  of these numbers (not individual digits) were subjected to a series of tests [1], [2]. The results obtained indicate that this sequence is satisfactory; they are described in § 4 below.

Later, similar processes were used for other machines. Teichroew used on SWAC the numbers produced by

$$x_0 = 1, x_{n+1} = \rho x_n \pmod{2^{36}}, \rho = 5^{13}$$

Again which have period  $2^{34} \div 2 \times 10^{10}$ , the sequence

$$x_0 = 1, x_{n+1} = \rho x_n \pmod{10^{10}}$$

has period

$$2^7 \cdot 5^8 = 5 \times 10^7$$

for  $\rho = 7$ . This sequence is suitable for use in the OARAC.

The sequence

$$(3.2) \quad x_0 \text{ any odd number, } x_{n+1} = 5^{13} x_n \pmod{2^{39}}$$

has been used on  $\overline{\text{ORDAC}}$  [17]; the sequence

$$x_0 \text{ any odd number, } x_{n+1} = 5^{17} x_n \pmod{2^{43}}$$

has been used on EDVAC [17].

The sequence

$$x_0 = 1, x_{n+1} = 7^{4k+1} x_n \pmod{10^{11}}$$

is suitable for UNIVAC. The period for this is

$$5 \times 10^8.$$

These have been tested by J. Moshman [9]. Moshman examined (the first six digits of) 10,000 numbers as<sup>a</sup>/whole, and in groups of 2,000. Reasonable results were obtained, apart from the sixth digit, which appeared "too random". An account of processes for obtaining pseudo-random numbers on ENIAC, for which the convenient moduli are  $10^5$  and  $10^{10}$ , are given by Juncosa [17].

A systematic account of the periods of sequences obtained by congruence methods has been given by Duparc, Lekkerkerker and Peremans (1953). We shall not attempt to summarize this paper.

#### 4. RESULTS OF TESTS

A series of tests on the numbers generated on SEAC by the relation (3.1) were carried out in collaboration with J. M. Cameron and B. F. Handy, Jr., [1]. A summary of the results obtained is given below: all these results are satisfactory.

# A. Frequency Test

- a) of all 16,384 numbers (32 intervals). Test of goodness of fit to rectangular distribution:

$$\chi^2 = 22.18, \text{d.f.} = 31, \text{Pr}\{\chi^2 > 22.18\} = .9;$$

- b) of 128 sets of 128 numbers (32 intervals). Test of goodness of fit of 128 values of  $\chi^2$  to the  $\chi^2$  distribution:

$$\chi^2 = 8.07, \text{d.f.} = 7, \text{Pr}\{\chi^2 > 8.07\} = .3.$$

# B. Conformance of distribution of certain statistics to expectation

Statistic	Observed average*	Expected average	Observed variance*	Expected variance
$\sum x_i / 128$	.49829	.50000	.000 6792	.000 6510
$\sum x_i^2 / 128$	.33163	.33333	.000 7344	.000 6944
$\sum (x_i - x_{i+1})^2 / 128$	.16508	.16536	.000 3272	.000 3869

\*Based on 128 values.

Distribution of mean ( $\sum x_i / 128$ ) should be approximately normal. Goodness-of-fit test of means to normal distribution gave:

$$\chi^2 = 5.87, \text{d.f.} = 9, \text{Pr}\{\chi^2 > 5.87\} = .75.$$

# C. Runs up and down

Length of run	1	2	3	4	$\geq 5$	Any length
Observed average* number of runs	425.3	187.9	55.4	10.4	2.7	681.8
Expected average number of runs	426.8	187.5	53.9	11.7	2.4	682.3
Observed variance of number of runs	400.9	97.3	24.1	15.2	2.1	181.7
Expected variance of number of runs	433.3	115.2	42.8	10.9	2.4	181.0

\*Based on 16 sets of 1024.

# D. Runs above and below the mean

Length of run	1	2	3	4	5	6	7	8	9	10	Any length
Observed average* number of runs	251.2	122.4	65.5	32.7	17.7	8.4	3.9	1.9	1.1	.62	505.5
Expected average number of runs	256	128	64	32	16	8	4	2	1	.1	513

\*Based on 16 sets of 1024.



A further series of tests were carried out on numbers generated on ORDVAC by the relation (3.2) by M. L. Juncosa [17]. Among these was a test for serial correlation with lag 3. All results were satisfactory.

## 5. CONGRUENTIAL METHODS - ADDITIVE

The only practical reason to search further for processes to generate random numbers is to gain speed. The obvious suggestion is to try using addition instead of multiplication. This has been discussed by Duparc, Lekkerkerker and Peremans [4]. For instance, consider the (reduced) Fibonacci sequence

$$F_0 = 0, F_1 = 1, F_{n+2} = F_{n+1} + F_n \pmod{M} \quad n = 0, 1, \dots$$

If we take  $M = 2^{44}$ , as is appropriate for SEAC, we find that the Fibonacci sequence has period

$$3 \times 2^{43} \sim 2.5 \times 10^{13}$$

The speed of generation, and the period of these numbers seem satisfactory. However, the numbers are obviously not independent.

The reduction mod  $2^{44}$  is accomplished merely by disregarding overflow in the addition

$$F_{n+2} = F_{n+1} + F_n ;$$

SEAC operates with numbers of  $44$  binary digits. To illustrate the behavior of this type of sequence consider the simple case  $M = 2^3$ : the resulting sequence has period 12 and is obtained by repetition of

$$0, 1, 1, 2, 3, 5, 0, 5, 5, 2, 7, 1.$$



The following heuristic arguments indicate that the sequence may, nevertheless, give satisfactory pseudo-random numbers. It is well known that if  $f_0 = 0$ ,  $f_1 = 1$ ; then

$$f_n = (\lambda^n - \mu^n) / \sqrt{5}$$

where

$$\lambda = \frac{1}{2} (\sqrt{5} + 1), \mu = \frac{1}{2} (-\sqrt{5} + 1)$$

Now, clearly,

$$F_n \equiv f_n \pmod{2^{44}}$$

but, since  $\mu < 1$ , we have

$$F_n \doteq (\lambda^n / \sqrt{5}) \pmod{2^{44}}$$

and we are again dealing with residues of powers.

We therefore began an investigation of this system. Some of the results obtained to date are reported in detail in the next section: here we summarize our results. The sequence  $\{F_n\}$  gave satisfactory results as far as the frequency and moment test were concerned; however, the results for runs were unsatisfactory, there being a preponderance of runs of length 2. This suggested that instead we use the sequence  $\{F_{2n}\}$  of alternate members of the sequence  $\{F_n\}$ . The results of the frequency, moments and run test appear satisfactory, but not as good as the power residues.

## 6. RESULTS OF SOME TESTS ON THE REDUCED FIBONACCI SEQUENCE

At present we have not completed a comprehensive series of tests. We report some of the results obtained; a full report will appear in [2].

### A. Frequency test

The distribution of three sets of 16384 numbers  $\{F_{2n}\}$  in 32 intervals were the following:

516,462,525,507,506,516,512,517,487,488,466,506,512,482,538,  
539,558,487,523,503,519,524,512,519,509,522,551,522,501,508,  
524,523.

513,499,508,497,507,563,525,511,534,487,500,542,497,506,545,  
491,521,505,503,515,480,487,491,510,501,548,525,540,522,494,  
480,537.

561,533,488,520,514,551,493,504,492,509,513,482,549,516,546,  
531,503,522,536,511,540,483,530,473,504,515,504,479,469,526,  
503,484.

The corresponding values of  $\chi^2$  are

$$26.88, 27.16, 35.17, \text{ d. f. } = 31.$$

### B. Moments

Here are ten sets of values of the moments\* of 128 numbers of the sequence  $F_n$ .

.5073	.5169	.5518	.5104	.4819	.5051	.4912	.4562	.5028	.4907	$\Sigma x_i / 128$
.3414	.3474	.3896	.3409	.3087	.3356	.3265	.2840	.3321	.3316	$\Sigma x_i^2 / 128$
.2594	.2620	.3032	.2549	.2222	.2492	.2439	.2013	.2469	.2533	$\Sigma x_i^3 / 128$
.2103	.2100	.2486	.2028	.1712	.1968	.1937	.1532	.1964	.2062	$\Sigma x_i^4 / 128$
.1922	.1495	.1581	.1596	.1559	.1705	.1656	.1512	.1421	.1920	$\Sigma (x_i - x_{i+1}) / 128$
.1664	.1813	.1776	.1538	.1332	.1687	.1813	.1469	.1733	.1608	$\Sigma (x_i - x_{i+2}) / 128$
.1641	.1762	.1940	.1674	.1676	.1698	.1763	.1721	.1601	.1850	$\Sigma (x_i - x_{i+3}) / 128$
.1752	.1465	.2191	.1574	.1518	.1417	.1636	.1628	.1737	.1696	$\Sigma (x_i - x_{i+4}) / 128$

\* The theoretical expected values are : .5000, .3333, .2500, .2000, .1654, .1641, .1628, .1615.

### C. RUNS UP AND DOWN

In the table below the first three columns record the numbers of runs up and down in a series of three sets of  $1024$  numbers of the sequence  $\{F_n\}$ ; the next three columns give similar results for the sequence  $\{F_{2n}\}$ ; the last column gives the theoretical expected results.

results.		$F_n$			$F_{2n}$			
UP	1	87	83	80	206	211	204	213.4
	2	112	118	112	77	96	107	93.8
	3	35	36	30	44	24	23	27.0
	4	11	9	17	8	5	3	5.9
	5	5	6	5	0	2	0	
	6	1	0	3	0	0	0	1.2
	7	1	1	0	0	0	0	
	8	1	1	1	0	0	0	
DOWN	1	79	84	76	213	199	200	213.4
	2	116	119	119	89	101	99	93.8
	3	40	32	29	24	31	31	27.0
	4	12	10	17	9	6	7	5.9
	5	6	6	7	0	0	1	
	6	1	2	1	0	0	0	1.2
	7	0	1	0	0	0	0	
	8	0	1	0	0	0	0	

### D. RUNS ABOVE and BELOW THE MEANS

In the table below we record the number of runs above and below the mean in a series of six sets of  $1024$  numbers of the sequence  $\{F_{2n}\}$ ; the last column gives the theoretical expected results.

ABOVE	1	134	140	133	125	128	132	128
	2	52	48	54	64	52	44	64
	3	45	32	45	38	30	48	32
	4	16	28	16	27	35	23	16
	5	10	10	11	9	10	11	8
	6	0	3	4	3	2	2	4
	7	0	0	1	1	0	0	2
	8	1	0	0	0	0	0	2



BELOW	1	119	141	154	150	137	134	128
	2	55	44	36	53	47	56	64
	3	49	45	42	38	39	44	32
	4	22	14	19	18	26	16	16
	5	10	13	10	6	7	9	8
	6	1	2	3	1	2	2	4
	7	1	2	0	0	0	0	2
	8	0	0	0	0	0	0	2

The numbers of the observed averages, in a series of 16 sets of  $102^4$  numbers from  $\{F_{2n}\}$ , with runs above and below the mean of the same length added, were

Length:	1	2	3	4	5	6	7	8
Observed count:	261.9	103.4	82	43.1	18.9	4.6	.9	.1
Theoretical count:	256	128	64	32	16	8	4	4

The observed averages, in a series of 16 sets of  $102^4$  numbers from  $\{F_{2n}\}$ , with the runs up and down of the same length added together were:

Length:	1	2	3	4	5
Observed count:	425.6	187.1	58.9	10.6	1.32
Theoretical count:	426.8	187.5	53.9	11.7	2.4

## 7. MISCELLANEOUS METHODS

7.1 Forsythe [5] discusses a scheme suggested by Rosser for the generation of random digits. We describe a simple example. Take four "random" numbers with say 3,4,5,7 binary digits and repeat these numbers, as indicated below:

```

1 0 1 1 0 1 1 0 1 1 0 1 1 0 1 1 0 ...
1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 ...
0 1 1 0 0 0 1 1 0 0 0 1 1 0 0 0 1 ...
1 0 1 1 0 1 1 0 1 1 0 1 1 0 1 1 0 ...

```



The first line contains repetitions of the number 101, the second repetitions of 1001, and so on. Consider the sequence obtained by adding (modulo 2) successive columns of this array. This is

1 1 1 1 1 0 1 0 1 0 0 0 0 0 0 1 0 1 0 1 ...

This sequence has period not greater than  $3 \times 4 \times 5 \times 7 = 420$ . Forsythe examined on SWAC, the National Bureau of Standards Western Automatic Computer, a series of 1217370 digits obtained in this way from four random numbers with 31, 33, 34 and 35 binary digits. Among the tests which he applied was the following: let  $s_j$  be the sum of 100 consecutive digits, then he examined twelve groups each of 1000 sums  $s_j$ . Of these eleven were in reasonable fit with the theoretical binomial distribution; the twelfth was a bad fit.

7.2 The sequence of digits in certain algebraic and transcendental numbers have been tested. For a summary and references see Teichroew [13]. Some pass and some do not pass the standard test e.g.,  $e$  is apparently bad. Apart from the difficulty in generating these, this seems sufficient reason to discard this method.

We note here that Richtmyer [11] has used algebraic numbers in connection with a quasi-Monte Carlo problem on SEAC. Roughly speaking, an integral is evaluated by "systematic" sampling at points depending on certain quadratic surds; satisfactory deterministic error bounds can be obtained from the theory of algebraic numbers.

7.3 In certain recent investigations e.g., in connection with the assignment problem, the generating of random permutations has

become of interest. It is possible to use any of the pseudo-random sequence described above to generate pseudo-random permutations.\* More direct constructions have been suggested by T. S. Motzkin and D. H. Lehmer.

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\*Cf. f.n.<sup>xxx</sup> on p. 1.

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Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards (\$1.00). Information on calibration services and fees can be found in NBS Circular 483, Testing by the National Bureau of Standards (25 cents). Both are available from the Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.

