NATIONAL BUREAU OF STANDARDS REPORT

3283

PROJECTS and PUBLICATIONS

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

A Quarterly Report

January through March 1954

FOR OFFICIAL USE

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

U. S. DEPARTMENT OF COMMERCE

Sinclair Weeks, *Secretary*

NATIONAL BUREAU OF STANDARDS

A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section is engaged in specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside of the back cover of this report.

Electricity. Resistance and Reactance Measurements. Electrical Instruments. Magnetic Measurements. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat and Power. Temperature Measurements. Thermodynamics. Cryogenic Physics. Engines and Lubrication. Engine Fuels. Cryogenic Engineering.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Neutron Measurements. Infrared Spectroscopy. Nuclear Physics. Radioactivity. X-Ray. Betatron. Nucleonic Instrumentation. Radiological Equipment. Atomic Energy Commission Radiation Instruments Branch.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Gas Chemistry. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Control.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Organic Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion.

Mineral Products. Porcelain and Pottery. Glass. Refractories. Enameled Metals. Concreting Materials. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Heating and Air Conditioning. Floor, Roof, and Wall Coverings. Codes and Specifications.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering.

Electronics. Engineering Electronics. Electron Tubes. Electronic Computers. Electronic Instrumentation. Process Technology.

Radio Propagation. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Frequency Utilization Research. Tropospheric Propagation Research. High Frequency Standards. Microwave Standards.

● Office of Basic Instrumentation

● Office of Weights and Measures.

NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT NAML

NBS REPORT 3283

PROJECTS and PUBLICATIONS

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

January through March 1954

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

NATIONAL APPLIED MATHEMATICS LABORATORIES

January 1 through March 31, 1954

ADMINISTRATIVE OFFICE

Franz L. Alt, Ph.D., Acting Chief
*Edward W. Cannon, Ph.D., Assistant Chief
Olga Taussky-Todd, Ph.D., Mathematics Consultant
W. J. Youden, Ph.D., Mathematics Consultant
Robert F. Dressler, Ph.D., Mathematics Consultant
*Eugene Lukacs, Ph.D., Mathematics Consultant
Ida Rhodes, M.A., Mathematician
Myrtle R. Kellington, M.A., Technical Aid
Luis O. Rodriguez, M.A., Chief Clerk
John B. Tallerico, B.C.S., Assistant Chief Clerk
*Billie R. Gill, M.S., Secretary
Louis J. Meyerle, Jr., Secretary
Esther L. Turner, Secretary

INSTITUTE FOR NUMERICAL ANALYSIS
(Los Angeles, California)

Charles B. Tompkins, Ph.D.....Acting Director of Research
Magnus R. Hestenes, Ph.D...Assistant Director and UCLA Liaison Officer
*Gertrude Blanch, Ph.D...Assistant to the Director (Numerical Analysis)

Research Staff

*Forman S. Acton, D.Sc.....Mathematician
George E. Forsythe, Ph.D.....Mathematician
Harry D. Huskey, Ph.D.....UCLA Research Associate (Mathematics)
Theodore S. Motzkin, Ph.D.....UCLA Research Associate (Mathematics)
Leopoldo Nachbin, Ph.D.....UCLA Research Associate (Mathematics)
Thomas H. Southard, Ph.D.....Mathematician
Daniel Teichroew, Ph.D.....Statistician
Wolfgang R. Wasow, Ph.D.....Mathematician

Graduate Fellows

Eugene Levin, M.A.
Genovevo C. Lopez, B.A. (UCLA)
Edwin Mookini, M.A. (UCLA)
Mervin Muller, M.A.
Lloyd Philipson, M.A.
David A. Pope, M.A. (UCLA)
Roger Woods, M.A. (UCLA)

Mathematical Services Unit

Frederick H. Hollander, M.A.....Mathematician
Robert R. Reynolds, M.S.....Mathematician

Computing Staff

Lillian Forthal, B.A.
Benjamin F. Handy, Jr., M.S.
Ruth B. Horgan, B.A.
Frank B. Meek, M.A.
Nan N. Reynolds, M.S.
Louise M. Straus, M.A.

Administrative Unit

Estelle H. Strauss.....Librarian Assistant
Mildred Vukmanic.....Administrative Assistant
Mildred B. Webb.....Administrative Clerk
Reve Vineyard.....Administrative Clerk
Leo Moskowitz.....Procurement Clerk
Vendla H. Gordanier.....Secretary
Dorothy M. Hibbard.....Secretary
Elsie L. Husman.....Secretary

COMPUTATION LABORATORY

John Todd, B.S.....Chief
Milton Abramowitz, Ph.D.....Assistant Chief
Ruth K. Anderson, M.A.....Mathematician
Henry A. Antosiewicz, Ph. D.....AU** Research Associate
Philip Davis, Ph.D.....Mathematician
Karl Goldberg, M.A.....Mathematician
Peter Henrici, Ph.D.....AU** Research Associate
Alan J. Hoffman, Ph.D.....Mathematician
Morris Newman, Ph. D.....Mathematician
Fritz Oberhettinger, Ph.D.....AU** Research Associate
Philip Rabinowitz, Ph.D.....Mathematician
Irene A. Stegun, M.A.....Mathematician
Charles J. Swift, Ph.D.....Physicist
Joseph H. Wegstein, M.S.....Physicist

Computing Staff

Hans O. Bremer, B.A.
William F. Cahill, M.S.
Ruth E. Capuano
Anne R. Cock, B.A.
Mary M. Dunlap, B.S.
Anne M. Futterman, B.A.
Leon Gainen, B.A.
Elizabeth F. Godefroy
Allen A. Goldstein, M.A.
Stanley D. Grant, Jr.
William G. Hall, B.S.
Genevie E. Hawkins, B.S.
John G. Hershberger, M.S.
Dorothea H. Jirauch, M.A.
Lambert S. Joel, B.A.
Terry W. Ledley, M.A.
Norman Levine, M.S.
David S. Liepman
Ethel C. Marden, B.A.
Kermit C. Nelson
Peter J. O'Hara, B.S.
Mary Orr
Maxine L. Paulsen, B.S.
B. Stanley Prusch
Otto T. Steiner, B.A.
Frank J. Stockmal, B.A.
Sally Tsingou, B.S.
Bertha H. Walter
Ruth Zucker, B.A.

Secretarial Staff

Gloria S. Ford, B.S.
Lillian Sloane
Marjorie E. Young

** American University

STATISTICAL ENGINEERING LABORATORY

Churchill Eisenhart, Ph.D.....Chief
Joseph M. Cameron, M.S.....Assistant Chief
Lola S. Deming, M.A.....Technical Aid
Willard H. Clatworthy, Ph.D.....Mathematical Statistician
William S. Connor, Jr., Ph.D.....Mathematician
Julius Lieblein, Ph.D.....Mathematician
Mary G. Natrella, B.A.....Mathematician
I. Richard Savage, M.S.....Mathematician
Marvin Zelen, M.A.....Mathematical Statistician
Marion T. Carson.....Computer
Yvette B. Cocozzella.....Secretary
Lela J. Hamilton.....Secretary

MACHINE DEVELOPMENT LABORATORY

*Edward W. Cannon, Ph.D.....Chief
Edith N. Reese, B.A.....Mathematician

*On leave of absence

Contents

Index of Active Research and Development Projects

Note: This index is not intended to cover the numerous special problem solutions, statistical analyses, and other ad hoc services to Government agencies, which form an important part of the work of the National Applied Mathematics Laboratories. These services are, however, fully represented in the body of the report.

A. Research: Pure Mathematics

B. Research: Numerical Analysis

C. Research: Applied Mathematics, Physics, and Astronomy

D. Mathematical Statistics

E. Mathematical Tables

F. Manuals, Bibliographies, Indices, and Technical Information

G. Computing Machine Development

Status of Projects

March 31, 1954

I. INSTITUTE FOR NUMERICAL ANALYSIS

(Section 11.1)

1. Fundamental Research

SOLUTION OF SETS OF SIMULTANEOUS ALGEBRAIC EQUATIONS AND TECHNIQUES FOR THE INVERSION AND ITERATION OF MATRICES

Task 1101-10-5100/49-AE2

Origin: NBS — Authorized 2/15/49
Sponsor: Office of Naval Research, USN — Revised 9/15/49
Manager: G. E. Forsythe
Full task description appears in July-Sept 1949 issue, p. 2.

Status: CONTINUED. Louise Straus has completed a convenient SWAC routine for improving the inverse of a matrix by the "Newton-Schulz" formula $X_{k+1}=(2I-X_kA)X_k$. The routine accepts a matrix A of orders up to 31 and an approximate inverse X_0, both expressed in floating binary notation. Using single-precision non-floating operation with triple-precision accumulation of inner products, the routine improves the inverse as far as possible and punches the answer.

In publication 2 below is a brief summary of the available SWAC codes for solving matrix problems, as of the beginning of the quarter. Since then one could add the above inverse-improvement code, and the Frame routine now reported under task 1101-10-5100/50-3, p.2.

M. R. Hestenes and E. H. Mookini have completed two SWAC codes for solving linear systems by two variants of the conjugate-gradient method, for orders up to 30. These employ M. R. Hestenes' "floating vectors," described under task 1101-10-5100/50-3. Publication 1 describes these and other variants of the conjugate-gradient method, with particular emphasis on non-hermitian matrices.

The projection method described in the July-Sept 1953 issue turns out to have been previously discovered by E. W. Purcell. See J. Math. Phys. vol. 32 (1953), pp. 180-183.

For work on the reduction of a matrix to Hermite normal form, see task 1101-10-5150/53-1, p. 7.

In publication 3, C. B. Tompkins and I. Heller (of George Washington University) extended a theorem of G. B. Dantzig. As a consequence,

they prove that solutions of some realistically defined transportation problems can be found in the ring of integers. Hence fractional ocean-going ships will not be demanded as solutionsto this abstraction of the problem of efficient shipping!

For work on linear programming related to this task, see the report of task 1101-10-5102/50-11, p.6.

The research of G. E. Forsythe and E. G. Straus (of U.C.L.A.) on best conditioned matrices has been submitted in an abstract to the International Congress of Mathematicians. The result reported in the Oct-Dec 1953 issue has been generalized to a fairly general class of preconditioning matrices.

Publications: (1) "The conjugate gradient method for solving linear systems," by M. R. Hestenes; in typescript. (2) "Seminar on numerical analysis, Summary of presentation of 7 January 1954," by G. E. Forsythe, in typescript. (3) "Proof of an extension of a theorem of G.B. Dantzig's," by I. Heller and C. B. Tompkins; in typescript.

CALCULATION OF EIGENVALUES, EIGENVECTORS, AND EIGENFUNCTIONS OF LINEAR OPERATORS
Task 1101-10-5100/50-3

Origin: NBS Authorized 7/1/49
Sponsor: Office of Naval Research, USN
Manager: G. E. Forsythe
Full task description appears in July-Sept 1949 issue, p. 13.

Status: CONTINUED. SWAC codes have been prepared for obtaining the characteristic polynomial of nonsymmetric real matrices A of order n by direct methods. Two methods were tried:

(i) Gaussian elimination with floating binary point to find the linear dependence between x, Ax, $A^2x,\ldots,A^nx$, using a code prepared by J. Pettit of the UCLA Geophysics Department.

(ii) An abridgment of the Leverrier method of calculating trace $(A^k)=\sum_{i=1}^{n} \lambda_i^k$ $(k=1,\ldots,n)$ and solving Newton's identities for the coefficients of the characteristic polynomial. The abridgment was devised independently by Frame, Souriau, and Sominskiĭ and Faddeev, and is described, for example, in Dwyer's "Linear Computations," p. 225.

Method (i) seemed to lead to inconsistent results when applied to A and A^T, and when x was varied. At best the elimination solution probably needed improvement by a separate routine; method (i) was abandoned in favor of method (ii).

Method (ii) was developed into a very convenient routine of limited applicability. The matrices were carried in the "floating row" form devised by Hestenes for the conjugate-gradient method (see task 1101-10-5100/49-AE2, p. 1). For example, each row of the matrices is written in n+1 cells, the 0th containing an integral exponent of 2 common to each of the n components 1,...,n. In this way one obtains both the speed of ordinary (non-floating) machine operation and the automatic scaling of floating operation. The routine deals with $n\leqslant 14$ (easily modifiable to $n\leqslant 30$), and for n=10 will type out one coefficient of the characteristic polynomial each 10 seconds.

Let the characteristic polynomial be written in the form $A^n+c_1A^{n-1}+c_2A^{n-2}- \ldots + c_nI$. Let the eigenvalues of the nonsingular matrix A be numbered so that $|\lambda_1|\leqslant|\lambda_2|\leqslant\ldots\leqslant|\lambda_n|$. Then $|c_n| = |\det(A)|=|\lambda_1\lambda_2\ldots\lambda_n|$. The round-off error in getting c_k with our routine has been found empiri-

cally to be approximately $2^{-40}|\lambda_n|^k$. (We use 36-bit word length throughout.) Thus the least accurate coefficient, c_n, is subject to an error e of about $2^{-40}|\lambda_n|^n$. Hence the relative error $|e/c_n|$ is approximately $2^{-40}\prod_{k=1}^{n}|\lambda_n/\lambda_k|$. Make the rough assumption that $|\lambda_k/\lambda_{k-1}|$ has a constant value r for k=2,3,...,n. Then $r^{n-1}=P=|\lambda_n/\lambda_1|$, the condition-number of A. Now, with this assumption, $|e/c_n| = 2^{-40}r^{n-1}r^{n-2}\ldots r^2 r = 2^{-40}r^{\frac{1}{2}n(n-1)} = 2^{-40}P^{\frac{1}{2}n}$. Suppose we want to achieve b binary places of accuracy in c_n. Then we need $|e/c_n| \leqslant 2^{-b}$, or $2^{-40}P^{\frac{1}{2}n} < 2^{-b}$, or

$$(*) \qquad P < 2^{(80-2b)/n} .$$

Perhaps b=10 is sufficient for ordinary engineering purposes. Thus we can state as a rough rule that our code will give the coefficients of the characteristic polynomial of A to 3-decimal accuracy for matrices with P condition-numbers not exceeding $2^{60/n}$.

For n=10, this limits P to be less than $2^6(=64)$. This is a fairly severe restriction on the matrices which can be handled by the method, and leaves one with the feeling that iterative methods are to be preferred for most matrices.

Participants in research on this task were G. E. Forsythe and L. Straus.

Publications: (1) "Asymptotic lower bounds for the frequencies of polygonal membranes," by G. E. Forsythe; to appear in the Pacific Journal of Mathematics. (2) "Asymptotic lower bounds for the fundamental frequency of convex membranes," by G. E. Forsythe; to appear in the Pacific Journal of Mathematics.

STUDIES IN PURE MATHEMATICS
Task 1101-10-5100/50-4

Origin: NBS Authorized 7/1/49
Sponsor: Office of Naval Research, USN
Manager: C. B. Tompkins
Full task description appears in July-Sept 1949 issue, p. 16.

Status: CONTINUED. Most work is reported under other task reports.

Dr. Leopoldo Nachbin of South America has been in residence at the INA during most of the period, and he has continued his researches on algebra. He has addressed seminars at the University of California at Los Angeles and the local peripatetic seminar in mathematics during the period.

COMPUTATION OF THE COMPLEX ZEROS OF THE RIEMANN ZETA FUNCTION
Task 1101-10-5100/50-13

Origin: NBS Authorized 6/1/50
Sponsor: Office of Naval Research, USN
Manager: R. Horgan
Full task description appears in Apr-June 1950 issue, p.13.

Status: INACTIVE. For status to date see July-Sept 1953 issue, p. 3.

STUDIES IN THE NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS
Task 1101-10-5100/51/1

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: W. Wasow
Full task description appears in July-Sept 1950 issue, p. 12.

Status: CONTINUED. Some minor work on the problem of solving differential equations that arise in the local embedding of surfaces was carried out by C. B. Tompkins. This work will continue.

The studies of L. Philipson on the asymptotic character of the solutions of

$$y^{(n)} + \sum_{r=1}^{n} a_r y^{(n-r)} + \sum_{s=0}^{m} (b_s x + c_s) y^{(m-s)} = 0$$

are reaching a conclusion. Fundamental systems of explicit solutions by Laplace contour integrals have been asymptotically evaluated for large λ. The exact sectors of validity of the various asymptotic expressions as well as the Stokes phenomena that describe their interrelations are manifest.

W. Wasow is working on a unified introduction to the asymptotic theory of non-linear analytic differential equations.

Dr. W. F. Brown and Prof. T. Y. Thomas, who are spending the winter at the Institute for Numerical Analysis as visitors, are contributing to the mathematical activities of the Institute.

Publications: (1) "On singular perturbation problems in the theory of non-linear vibrations," by W. Wasow; Actes du Colloque International des Vibrations Non Lineaires, Ile de Porquerolles 1951, pp. 207-222(1953). (2) "Singular perturbation methods for nonlinear oscillations," by W. Wasow; Proceedings of a Symposium on Nonlinear Circuit Analysis, Polytechnic Institute of Brooklyn, 1953, pp. 75-98. (3) "Discrete approximations to elliptic differential equations," by W. Wasow; IN MANUSCRIPT.

PROBABILITY METHODS AND SAMPLING TECHNIQUES
Task 1101-10-5100/51-2

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: D. Teichroew
Full task description appears in July-Sept 1950 issue, p. 13.

Status: CONTINUED. Two reports (1), (2) on probability points of the Incomplete Gamma distribution and one on the use of generalized normalization polynomials (3) have been completed.

M. Muller is continuing his investigation of procedures for improving the efficiency of the Monte Carlo method for solving boundary value problems by using predetermined networks or by using continuous stochastic processes.

T. Motzkin investigated neighbor sets in n dimensions (publication (4) below).

Participants in the research on this task were T. Motzkin, M. Muller, and D. Teichroew. Some of the coding was done by L. Forthal.

Publications: (1) "A table of Campbell's polynomials for the computation of probability points of the incomplete Gamma distribution,"

by D. Teichroew; IN MANUSCRIPT. (2) "A table of millile probability points of the incomplete Gamma distribution," by D. Teichroew; IN MANUSCRIPT. (3) "Generalized normalization polynomials for the t and incomplete Gamma distributions," by D. Teichroew; IN MANUSCRIPT. (4) "Neighbor sets for random walks and difference equations," by T. Motzkin; IN MANUSCRIPT.

VARIATIONAL METHODS
Task 1101-10-5100/51-3

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: M. R. Hestenes
Full task description appears in July-Sept 1950 issue, p. 15.

Status: CONTINUED. Various direct numerical methods in the calculus of variations are being studied and a SWAC procedure is being prepared for the numerical computation of a minimizing curve for one dimensional fixed end point problems, using a method of "steep descent" given by a generalized gradient. Theoretical investigations into the various methods are being undertaken with a view to comparing their effectiveness in multidimensional as well as one dimensional problems.

Participants in this task were D. A. Pope and C. B. Tompkins.

STUDIES IN APPLIED MATHEMATICS
Task 1101-10-5100/51-4

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: T. H. Southard
Full task description appears in July-Sept 1950 issue, p. 16.

Status: CONTINUED. The seminar on numerical analysis met 20 times during the quarter. A detailed list of sessions will be found under Lectures and Symposia in the back of this issue. Summaries of the sessions of January 7 and March 25 were prepared by G. E. Forsythe; other summaries are being prepared by C. B. Tompkins.

E. Levin did some work on punch indentation problems. An admissible velocity field is constructed for an arbitrary punch; hence, for any particular case, a limit design theorem of Drucker, Prager, and Greenberg may be used to compute an upper bound for the punch indentation pressure. Results of the present work are used to compute an upper bound for a punch with circular cross section.

C. B. Tompkins and T. H. Southard have done some work on development of a routine for SWAC which will solve a 2-person, zero-sum, rectangular game whose value is zero and whose typical matrix element is $A_{a\alpha} + VB_{a\alpha}$, where V is to be determined and $-1 \leqslant B_{a\alpha} < 0$. Such a game has applications in quantitative studies of warfare and elsewhere.

Publications: (1) "Numerical computation of low moments of order statistics from a normal population," by J. B. Rosser; submitted to a technical journal. (2) "Note on the circle theorem of hydrodynamics," by E. Levin; accepted by Quarterly of Applied Mathematics. (3) "Bounds for punch indentation problems," by E. Levin; being submitted to a technical journal. (4) "Summary of January 7 numerical analysis seminar,"

by G. E. Forsythe; multilithed typescript, 11 p. (5) "Summary of March 25 numerical analysis seminar," by G. E. Forsythe; multilithed typescript, 2 p.

MISCELLANEOUS STUDIES IN THEORETICAL PHYSICS
Task 1101-10-5100/51-5

Origin: Office of Naval Research, USN Authorized 9/1/50
Sponsor: " "
Manager: R. D. Woods
Full task description appears in July-Sept 1950 issue, p. 19.

Status: CONTINUED. W. Futterman is concluding theoretical and numerical work on calculations of the photo-disintegration of the deuteron. R. Woods is continuing computations of the elastic scattering of protons by various elements, based on the optical model of the nucleus.

Publications: (1) "Application of variational methods to intermediate and high energy scattering," by E. Gerjuoy and D. S. Saxon; accepted by Physical Review. (2) "Variational principles for the acoustic field," by E. Gerjuoy and D. S. Saxon; submitted to a technical journal.

STUDY OF RUSSIAN MATHEMATICAL PROGRESS
Task 1101-10-5100/52-1

Origin: NBS Authorized 3/15/52
Sponsor: Office of Naval Research, USN
Manager: G. E. Forsythe
Full task description appears in Jan-Mar 1952 issue, p. 11.

Status: CONTINUED. The task manager is continuing to accumulate bibliographical cards on Russian mathematical monographs (see Jan-Mar 1952 issue, p. 11, and Oct-Dec 1952 issue, p. 8). The following publication is available in limited quantities: "Bibliographical Survey of Russian Mathematical Monographs, 1930 to 1951," an NBS report. A supplement to the above is also available.

RESEARCH IN THE MATHEMATICAL THEORY OF PROGRAM PLANNING
Task 1101-10-5102/50-11

Origin: Office of Air Comptroller, USAF Authorized 6/15/50
Sponsor: " "
Manager: T. S. Motzkin
Full task description appears in Apr-June 1950 issue, p. 12.

Status: CONTINUED. In the assignment problem, various ways of preconditioning were examined. Under certain conditions on the elements, the expected value of a pseudo-maximum obtained by either of the two ordinarily used first guesses was evaluated. As a simplest example, in an n by n matrix whose elements are zero and one with independent equal probability, the expected value of a sum obtained by first choosing a greatest element, then in its minor a greatest element, etc., is $n-e_n$

with $e_n \uparrow .5785\ldots$. The question was studied when and how gradually more accurate and elaborate methods should be used.

In the more general equidistribution problem a new characterization of the vertices of the pertinent polyhedron was given, and adjacent vertices were determined in the case where all nonzeros in a vertex are equal. Polyhedra belonging to other generalizations of the assignment problem and to related problems of applicational interest were also studied.

A code of D. H. Lehmer for finding a pseudo-maximum for an n by n assignment problem with $n<16$ by examining random permutations was run on the SWAC.

Reference is made to a theorem proved by I. Heller and C. Tompkins and reported under task 1101-10-5100/49-AE2 (p. 1) which has immediate application to some transportation problems. It generalizes a theorem first used in this connection by G. B. Dantzig.

Publications: (See also July-Sept 1953 issue, p. 9-10.) "The assignment problem," by T. S. Motzkin (Cowles Commission Discussion Paper Mathematics No. 425); to appear in Proceedings of the American Mathematical Society Sixth Symposium for Applied Mathematics, held at Santa Monica City College, August 1953.

SCAMP
Task 1101-10-5150/53-1

Origin: Office of Naval Research, USN Authorized 6/10/53
Sponsor: " "
Manager: C. B. Tompkins
Full task description appears in Apr-June 1953 issue, p. 8.

Status: CONTINUED. The research concerning problems of discrete variables has continued. Studies of automatic methods for removing isomorphs have proceeded. An early paper by H. S. White, F. L. Cole, and L. D. Cummings (Memoirs of the National Academy of Sciences, 1919) has served as a guide in some of this work relating to Steiner triples.

A search for a pair of orthogonal Latin Squares of order 10 has been reduced to a code, and no orthogonal squares have been found.

A report involving a problem of exhaustive search for solutions of non-linear equations involving permutations has been submitted to the sponsor.

Plans are continuing for the active program during the summer of 1954.

F. B. Meek is coding a reduction of integral quadratic forms to the so-called normal forms of Hermite.

M. Newman completed the reduction to Hermite normal form of two matrices of orders 57 and 73, respectively. (cf. task 1101-10-5150/53-1).

Participants in this task included F. B. Meek, J. D. Swift, C. B. Tompkins.

2. Development

NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER (SWAC)
Task 1101-20-5103/49-1

Origin: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF — Authorized 11/1/48
Sponsor: " " " "
Manager: H. D. Huskey
Full task description appears in Apr-June 1949 issue, p. 27.

Status: CONTINUED. SWAC has continued to be used on a regularly scheduled two shift basis on both service and research problems. However, members of the University staff have occasionally operated the machine after the end of the regular shift at midnight and on weekends, so that since the end of January the SWAC has been used for an average of 111 hours per week. During the quarter the machine was turned on for a total of 1152 hours, 789 of which was productive computing. This is about 68% of the total time. Slightly over 31 hours was unscheduled maintenance.

During the quarter the Flexowriter input unit was replaced by a hand keyboard on the console. This simplified the input circuitry and removed from the computer the crystal input matrix, so that there should be a decrease in maintenance.

STUDIES IN THE THEORY OF DIGITAL COMPUTING MACHINES
Task 1101-20-5103/53-1

Origin: NBS — Authorized 9/30/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: H. D. Huskey
Full task description appears in July-Sept 1952 issue, p. 16.

Status: CONTINUED. During this quarter there has been an effort to consolidate information on all the various codes that are in existence for running SWAC. In this respect a new coding manual is in the process of preparation.

The floating point card control method of operating SWAC has been extended to give the possibility of using repetitive routines stored on the drum. Elementary automatic programming of formulas has been continued and the interpretative routines have been adapted for use with floating SWACPEC. This makes it possible to automatically program a simple problem and to immediately feed in the data and obtain results with no preliminary scaling and with a minimum of attention to the coding process. Test routines and input and output routines have been revised during the quarter to make the operation more efficient.

3. Mathematical Services

COMPUTING SERVICES FOR RESEARCH STAFF OF THE INSTITUTE FOR NUMERICAL ANALYSIS
Task 1101-40-5130/54-1

Origin: NBS — Authorized 9/2/48
Sponsor: Office of Naval Research, USN — Revised 11/16/49
Manager: F. Hollander
Full task description appears in July-Sept 1949 issue, p. 25.

Status: CONTINUED. Research staff problems involving the use of SWAC were as follows:

(1) For C. B. Tompkins: (a) Studies were continued concerning numerical embedding of surfaces. A further investigation was made of some inequalities generated in a study of organizations, and of some discrete problems. (b) A search for two orthogonal latin squares of order 10 was continued, in connection with SCAMP, according to a code by F. B. Meek. No pairs were found.

(2) For W. Futterman: (a) Matrix elements occurring in the problem of the photo-electric effect for the deuteron have been computed for a central Yukawa potential up to octupole interaction. This assumed ordinary as well as Serber force. (b) Dipole integrals for a tensor force ground state were also calculated.

(3) For R. D. Woods: Coding was checked and some exploratory computations were made of the scattering of protons by various metals, using an optical model for the nucleus.

(4) For the Physical Chemistry Group of the Department of Chemistry, UCLA: The physical chemistry group carried out a series of calculations on the SWAC applied to crystallographic structure research during the past quarter. A crystallographic interpretation routine, SWAXTL, was coded and first employed for structure factor calculations. These calculations predict the intensity of an X-ray beam reflected by a plane of atoms in a given crystal. This calculation has been applied to four unique crystal structures. A three-dimensional Fourier series calculation of the electron density for a given crystal has been coded. It is applicable to all crystallographic space groups and has been used in calculations for four different crystals, each with a different one of the 230 possible space groups.

(5) For M. R. Hestenes and E. Mookini: Experiments were continued in the conjugate gradient method for solving systems of linear equations. Several routines involving triple precision inner products were tested on 6x6 and 8x8 systems of linear equations. For well-conditioned matrices the results were very good. Experiments are now being carried out on ill-conditioned matrices.

(6) For G. Forsythe and L. Straus: (a) Coding was completed and experiments made on the problem of obtaining the characteristic polynomial of a real matrix up to order 14 by Frame's method. Details are reported under project 1101-10-5100/50-3, p. 2. (b) Coding was checked, and test cases run on the problem of improving an approximate inverse of a real matrix, up to order 32, by the Newton-Schulz iteration: $X_{n+1}=X_n(2I-AX_n)$. (See project 1101-10-5100/49-AE2 (p. 1) for further details.)

(7) For the Geophysics Department, UCLA: Calculations were continued and new codes checked out, in the problem of the high order tidal constituents for earth and ocean tides.

(8) For the School of Business Administration, UCLA: (a) The code was completed and checked, and some runs were made on SWAC on the problem

of the Gambling Game. The routine derives frequency distribution of the duration of play to the basic probability game, and moments to basic probability game. (b) A basic routine for the probability game in the problem of the Business Game has been written but not completely checked out on SWAC.

(9) For D. H. Lehmer and J. Selfridge: (a) The program to find penultimate remainders of the known Mersenne primes was completed. (b) The program for finding quadratic residues which are not squares was continued on a standby basis. This routine uses only the high speed memory and the output typewriter. (c) Testing primes for irregularity was completed to 2500. Kummer's criterion was checked for each irregular found and the result is that Fermat's last theorem has no counter-examples for exponents less than 2500. (d) The cyclotomic numbers of order 16 were computed for all primes less than 1000, and a general cyclotomic number routine using the drum was code checked. (e) A code was checked out to attack the assignment problem by a Monte Carlo method. (f) A code is checked out and ready to run which will settle the Ideal Waring Theorem for exponents up to 250,000. (g) The following is a result of calculations performed: No Fermat number whose character is unknown has a divisor less than 2^{36}.

(10) For the Physics Department, UCLA: Computation of a proposed wave function for the Li_2 molecule using three exponential variational parameters is proceeding. The coding for the binding energy has been completed using the floating SWACPEC coding scheme of M. Melkanoff. Preliminary runs have been made.

(11) For the Education Department, UCLA: Detailed "item analysis" of psychometric instruments, based on varying methods of identifying criterion groups, has been accomplished. Scoring keys resulting from these analyses have been developed, and differential methods of weighting to arrive at final scores are being studied.

PUNCHED CARD LIBRARY
Task 1101-40-5131/49-2

Origin: NBS Authorized 7/14/48
Sponsor: Aeronautical Research Laboratory, Wright Air Develpment Center, Air Research and Development Command, USAF
Manager: F. Hollander
Full task description appears in Apr-June 1949 issue, p. 26.

Comments: A catalog of tables on punched cards which are on file at the Institute may be obtained by addressing the Institute for Numerical Analysis, 405 Hilgard Avenue, Los Angeles 24, California. Within the limits of the program of the computation unit of the Institute, tables will be duplicated upon request, provided the requester furnishes the blank cards. Requests should be addressed directly to the Institute.

Status: CONTINUED. A request was received from the University of Utah for tables of sines and cosines (sin x, cos x, Δ, δ^2, x=0(.0001)1, 11D). Boeing Airplane Company requested tables of Bessel functions: $Y_0(x)$, $Y_1(x)$, $Y_0(ix)$, $Y_1(ix)$, x=0(.01)10, 10D; and $J_0(x)$, x real, x=0(.01)10, 10D. These were reproduced from our files and shipped.

A table of keypunched Bessel functions of fractional order was received from the U. S. Naval Air Missile Test Center, Point Mugu, California, and added to the punched card library. This table contains all J_ν functions listed in the NBS "Table of Bessel Functions of Fractional Order," vol. 1 (Columbia University Press, New York, 1948). An erratum

was noted in the table on page 139 for $\nu = -\frac{1}{4}$, x = 20.840: the correct δ^2 is 12305, not 12299 as listed. This table was checked for accuracy after keypunching by the staff at the Center.

SIMPLIFIED ROLLING PULLOUT EQUATIONS
Task 1101-40-5131/51-34

Origin: Cornell Aeronautical Laboratory Authorized 6/22/51
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF
Manager: F. Hollander
Full task description appears in Apr-June 1951 issue, p. 26.

Status: CONTINUED. The sets of data reported in the last issue have been used to compute the solutions of the remaining five conditions. This essentially completes the computation on this task. A final report remains to be written. A report describing the equations and the method of solution has been written by E. C. Yowell.

LOW MOMENTS OF ORDER STATISTICS
Task 1101-40-5131/51-36

Origin: University of Oregon Authorized 6/22/51
Sponsor: Office of Naval Research, USN
Manager: H. D. Huskey
Full task description appears in Apr-June 1951 issue, p. 27.

Status: CONTINUED. The coding of certain routines that can be used in this problem has been done in task 1101-40-5131/54-8, p. 16.

SIERRA WAVE PROJECT
Task 1101-40-5131/52-36

Origin: Department of Meteorology, U.C.L.A. Authorized 4/1/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: T. H. Southard
Full task description appears in Jan-Mar 1952 issue, p. 33.

Status: CONTINUED. Theodolite data for several more flights have been reduced on SWAC.

EIGENVALUES
Task 1101-40-5131/53-6

Origin: Consolidated Vultee Aircraft Corporation (Convair) Authorized 9/26/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: F. Hollander
Full task description appears in July-Sept 1952 issue, p. 28.

Status: INACTIVE.

METEOROLOGICAL MEANS
Task 1101-40-5131/53-10

Origin: Meteorology Department, UCLA Authorized 9/30/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: F. H. Hollander
Full task description appears in July-Sept 1952 issue, p. 29. (See also project 11.1/31-50-17, Jan-Mar 1950 issue, p. 15.)

Status: CONTINUED. Computations were suspended during January and February. Calculations were resumed during March on the "Mountain Term". No further work was accomplished on the 100 mb level for Jan-Feb 1949.

DISCRIMINANT FUNCTIONS
Task 1101-40-5131/53-18

Origin: School of Aviation Medicine, Randolph Field, USAF Authorized 12/15/52
Manager: D. Teichroew
Full task description appears in Oct-Dec 1952 issue, p. 27.

Status: CONTINUED. Preliminary results are being examined. As a reult of the preliminary computations the code is being modified to permit larger values of the parameters.

BIO-ASSAY PROBLEM
Task 1101-40-5131/53-24

Origin: Stanford University Authorized 3/31/53
Sponsor: Office of Naval Research, USN
Manager: D. Teichroew
Full task description appears in Jan-Mar 1953 issue, p. 22.

Status: INACTIVE. Results are being examined by the contractor.

DISCRETE MINIMAL SPACES
Task 1101-40-5131/53-26

Origin: Gilfillan Brothers, Inc. Authorized 3/31/53
Sponsor: Evans Signal Laboratories, Army Signal Corps
Manager: H. D. Huskey
Full task description appears in Jan-Mar 1953 issue, p. 23.

Status: INACTIVE. For status to date see Apr-June 1953 issue, p. 23.

LINEAR EQUATIONS (CONVAIR)
Task 1101-40-5131/53-35

Origin: Consolidated Vultee Aircraft Corporation Authorized 3/31/53
Sponsor: Bureau of Aeronautics, USN
Manager: F. Hollander
Full task description appears in Jan-Mar 1953 issue, p. 28.

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 16.

MONTE CARLO STUDIES
Task 1101-40-5131/53-39

Origin: RAND Corporation Authorized 3/31/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF Terminated 3/31/54
Manager: H. D. Huskey

Objective: To study the Ising problem and order-disorder phenomena using the Monte Carlo method.

Comments: This task originates with RAND's Air Force contract. RAND is coding the problem for SWAC, and the NBS will furnish SWAC time and such operator and consulting time as is required.

Status: TERMINATED.

RANKING PROBABILITIES
Task 1101-40-5131/53-40

Origin: Cornell University Authorized 3/31/53
Sponsor: Office of Naval Research Completed 3/31/54
Manager: D. Teichroew

Objective: To compute a table of the function

$$P(h|r,k) = r \int_{-\infty}^{\infty} [F(x + \sqrt{2}\, h)]^{k-r}[1 - F(x)]^{r-1} f(x)dx$$

where

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt \text{ and } f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

for $k=4(1)10$, $r=1(1)[\frac{k}{2}]$ where $[m]$ is the largest integer less than or equal to m, and $h= 0(.01)S_{r,k}$ where $S_{r,k}$ is the smallest two-decimal-place number for which $P(S_{r,k} \mid r,k) \geq .99$. Certain related tables will be obtained by inverse interpolation in the $P(h \mid r,k)$ table.

Background: The $P(h \mid r,k)$ integral arises in the study of power functions of tests based on order statistics in samples from a normal distribution. The present computations are designed as an aid in applying the theory developed by R. Bechhofer. "The probability of a correct ranking (Preliminary Report)," Annals of Mathematical Statistics, 139(Mar. 1952).

Status: COMPLETED. The final report below has been submitted for publication.

Publication: "A table giving a probability associated with order statistics in samples from two normal populations which have the same variance but different means," by D. Teichroew; IN MANUSCRIPT.

ACCEPTANCE TESTS
Task 1101-40-5131/53-41

Origin: Jet Propulsion Laboratory, California Institute of Technology — Authorized 6/29/53
Sponsor: Ordnance Corps, U. S. Army
Manager: H. D. Huskey
Full task description appears in Apr-June 1953 issue, p. 27.

Status: CONTINUED. Miss L. Forthal of the Institute and Mr. Carr of the Jet Propulsion Laboratory have completed the coding of the test procedure and are checking it. At the present time delivery of the computer to the JPL has been delayed and it is expected that the test routines will be run during the next quarter.

B.P.A. Studies
Task 1101-40-5131/53-42

Origin: Bonneville Power Administration — Authorized 6/29/53
Sponsor: " "
Manager: R. R. Reynolds
Full task description appears in Apr-June 1953 issue, p. 27.

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 17.

HELICOPTER STABILITY STUDIES
Task 1101-40-5131/53-44

Origin: J. B. Rea Company Authorized 6/30/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: G. E. Forsythe
Full task description appears in July-Sept 1953 issue, p. 23.

Status: CONTINUED. The main effort has been to obtain two polynomials in λ of type $\det(\lambda F+G)$, which govern the stability of a helicopter control system. The approach has been to calculate $A=G^{-1}F$ very carefully, and then to try to get the characteristic polynomial of A by one of the two methods currently reported under task 1101-10-5100/50-3, p. 2. The approach was successful for one of the two polynomials. The other proved too badly conditioned, however, and has not been evaluated.

A second series of problems, representing a different approach to a similar stability problem, requires the inversion of some forty-odd real matrices of order 16. The input cards are all prepared, and running on SWAC will commence soon.

COMPUTATIONS FOR SCAMP
Task 1101-40-5131/54-1

Origin: Office of Naval Research, USN Authorized 9/29/53
Sponsor: Office of Naval Research, USN
Manager: C. B. Tompkins
Full task description appears in July-Sept 1953 issue, p. 24.

Status: CONTINUED. The computations carried out during the period concerned a search for orthogonal Latin Squares of order 10 and continued computation of Steiner triples of order 15.

CRUISE CONTROLLER
Task 1101-40-5131/54-3

Origin: J. B. Rea Company Authorized 9/29/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF Terminated 3/31/54
Manager: F. H. Hollander

Objective: A cruise controller is an instrument which is designed to determine the appropriate throttle setting to obtain the maximum miles per pound. The associated SWAC problem is to determine the type of trottle input which results in the smallest hunting period.

Comments: The originator is analyzing this problem to determine its appropriate mathematical form.

Status: TERMINATED.

PARACHUTE DATA ANALYSIS
Task 1101-40-5131/54-7

Origin: U. S. Naval Auxiliary Air Station, El Centro, Calif. Authorized 2/25/54
Sponsor: U. S. Navy
Manager: D. Teichroew

Objective: To provide services to assist the Navy Parachute Unit by conducting a study designed to recommend statistical procedures and modern computing techniques applicable to the parachute test activity.

Status: NEW. A report containing a statistical analysis of several experiments has been submitted to the contractor.

GALACTIC DISTRIBUTION STUDIES
Task 1101-40-5131/54-8

Origin: Statistical Laboratory, University of California, Berkeley Authorized 2/25/54
Manager: B. F. Handy, Jr.

Objective: Given parameters: $r,s,t,\theta(a,b),\sigma,\beta$

$$A_{\sigma,\beta,\theta}(r,s,t) = \int_0^\infty \int_0^\infty \int_0^\infty x_1^r x_2^s x_3^t \theta(\sigma x_1)\theta(\sigma x_2)\theta(\sigma x_3) e^{-h} dx_1 dx_2 dx_3$$

where $h = \frac{1}{3}[x_1^2 + x_2^2 + x_3^2 - x_1x_2 - (x_1+x_2)\, x_3 \cos 2\beta\,]$

and $\theta(y) = \int_{-\infty}^{w} e^{-\frac{1}{2}u^2} du$

with $w=a-5\log_{10}y - 5.15\times10^{-9}y|b$.

Background: This computation originated as a result of studies being carried on at the Lick Observatory, University of California, Berkeley, in conjunction with the Statistical Laboratory, University of California, Berkeley, to find a mathematical model of the distribution of galaxies in a theoretically non-expanding universe. By comparing the computed results with observed phenomena it is hoped to be able to make certain statistical observations concerning the make-up of the universe. As a result of the present computations certain changes in the statistical model can be made and new computations performed.

Status: NEW. A code, written by S. Marks, for evaluating

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}\, dt$$

to 30 decimal places, for x= -12(.02)0 was checked out, and the table computed. Methods of interpolation in this table are being studied, and the code for computation of $\theta(y)$ is being written.

MATRIX INVERSIONS
Task 1101-40-5131/54-9

Origin: Northrop Aircraft, Inc. Authorized 2/25/54
Manager: F. H. Hollander

Objective: To invert matrices up to order 32, using an existing floating point routine.

Background: The originator occasionally will submit for solution sets of equations which arise in aircraft design. Results are to be transmitted as obtained.

Status: NEW. The inverse of a 15x15 matrix was calculated using a routine coded by J. Pettit of the UCLA Department of Geophysics.

SKEW RAYS
Task 1101-40-5131/54-10

Origin: Pacific Optical Company Authorized 2/25/54
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: F. H. Hollander

Objective: To trace bundles of skew rays through a centered lens system using the equations developed by D. Feder in J. Opt. Soc. Amer. 41, 630-635 (Sept. 1951). The code will trace a pattern of rays over half of the entrance pupil. Vignetting will be computed as part of the results.

Background: A centered lens system with 12 spherical air to glass surfaces was designed. Skew ray traces are needed to determine improvements to the system. The development of formulae suited to automatic computing systems makes the determination of skew rays a relatively simple problem.

Status: NEW. The coding has been completed for the tracing of skew rays through a system of six components, with 12 air-to-glass surfaces. Data are to be supplied as soon as possible, so that the computations can be started.

II. COMPUTATION LABORATORY

(Section 11.2)

1. Research

RESEARCH IN CLASSICAL NUMERICAL ANALYSIS
Task 1102-10-1104/50-1

Origin: NBS Authorized 1/1/50
Managers: J. Todd, M. Abramowitz, H. A. Antosiewicz
Full task description appears in Jan-Mar 1950 issue, p. 21.

Status: CONTINUED. M. Abramowitz and W. F. Cahill made a study of the application of the converging factor to the asymptotic expansion of the Coulomb wave functions.

H. A. Antosiewicz completed his investigation of the boundedness of every solution of equations of the type

$$\ddot{x} + p(x,\dot{x})\,\dot{x} + h(x) = e(t)$$

with integrable forcing term e(t). (See publication (1) below.)

M. Abramowitz and P. Rabinowitz have prepared a paper on the Coulomb wave function along the transition line. (See publication (2) below.)

In connection with the forthcoming table of $E_1(z)$ (task 1102-10-1104/43-3, p. 27), J. Todd prepared a paper on the use of the Laguerre approximate quadrature formula to evaluate $E_1(z)$ in the complex domain, either for large values of the arguments where the function is not tabulated, or within the table instead of using a two-dimensional interpolation. This method is more efficient than the use of the asymptotic series.

A seminar in Numerical Analysis held in conjunction with American University, is being conducted by J. Todd. The general topic is stability considerations in connection with the numerical solution of differential equations. Among those who have participated to date are M. Abramowitz, F. L. Alt, S. Gorn (Ballistics Research Laboratory), P. Rabinowitz, and J. Todd.

Publications: (1) "On nonlinear differential equations of the second order with integrable forcing term", by H. A. Antosiewicz; submitted to a technical journal. (2) "Coulomb wave functions along the transition line," by M. Abramowitz and P. Rabinowitz; IN MANUSCRIPT. (3) "Computation of vibration modes and frequencies on SEAC," by W. Cahill and S. Levy; IN MANUSCRIPT. (4) "On a problem in the theory of mechanical quadratures," by P. Davis; accepted by the Pacific Journal of Mathematics. (5) "On the estimation of quadrature errors for analytic functions," by P. Davis and P. Rabinowitz; submitted to a technical journal. (6) "The practical evaluation of integrals," by M. Abramowitz; submitted to a technical journal. (7) "The evaluation of the exponential integral for large complex arguments," J. Todd; to appear in NBS Journal of Research. (8) "Evaluation of the $\int_0^\infty \exp(-u^2-(x/u))du$," by M. Abramowitz; accepted

by the Journal of Mathematics and Physics. (9) "Regular and irregular Coulomb wave functions expressed in terms of Bessel-Clifford functions," by M. Abramowitz; accepted by the Journal of Mathematics and Physics. (10) "Asymptotic solution of linear differential equations with a parameter," by H. Antosiewicz; submitted to a technical journal. (11) "On the differential equation $\ddot{x} + k(f(x) + g(x)\dot{x})\dot{x} = ke(t)$," by H. Antosiewicz; submitted to a technical journal. (12) "On a certain integral involving Bessel functions," by H. Antosiewicz; submitted to a technical journal. (13) "Some implications of Liapunov's conditions for stability," by H. Antosiewicz and P. Davis; submitted to a technical journal.

RESEARCH IN MODERN NUMERICAL ANALYSIS: INVESTIGATION OF BERGMAN'S METHOD FOR THE SOLUTION OF THE DIRICHLET PROBLEM FOR CERTAIN MULTIPLY CONNECTED DOMAINS
Task 1102-10-1104/50-2

Origin: NBS Authorized 3/1/50
Manager: P. Davis
Full task description appears in Jan-Mar 1950 issue, p. 22.

Status: INACTIVE. For status to date, see Jan-Mar 1952 issue, p. 35.

MISCELLANEOUS STUDIES IN PURE MATHEMATICS
Task 1102-10-1104/50-4

Origin: NBS Authorized 1/1/50
Managers: O. Taussky-Todd, J. Todd, M. Abramowitz, A. Hoffman
Full task description appears in Jan-Mar 1950 issue, p. 23.

Status: CONTINUED. O. Taussky-Todd continued her study of the significance of normal matrices in certain aspects of algebraic number theory; in particular numerical examples in the case of cubic fields (normal or not normal) were investigated.

M. Newman completed two papers entitled "A conjecture of Erdös" and "Note on a certain determinant". In the first it is shown that for a large class of pairs m,n the diophantine equation $x^m - y^n = 1$ has only a finite number of rational integral solutions x,y. Later O. Taussky-Todd showed how to establish the result for all pairs m,n; further generalizations are under study by M. Newman. In the second paper a determinant related to the Vandermonde determinant is evaluated.

In this period a seminar in algebra, held in conjunction with the American University and directed by O. Taussky-Todd, was concluded. The general topic was computational problems in algebraic number theory and the speakers included N. C. Ankony (Johns Hopkins University), B. V. Dean (National Security Agency), K. Goldberg, A. J. Hoffman, M. Newman, J. Todd (National Bureau of Standards).

K. Fan continued work on topology. In particular he completed a paper with R. A. Stuble (Illinois Institute of Technology) on "Continuity in terms of connectedness" and is working on another with F. Wagner concerning "Compactification of completely regular spaces".

We report here, some of the minor consulting jobs carried out by P. Henrici for other workers in NBS. Similar activities have been carried out by other members of the Computation Laboratory staff. The purpose of this record is to indicate types of problems arising elsewhere in NBS that can be solved painlessly by CL staff members.

Status of Projects

1. Evaluation of

$$\sum_{m=1}^{k} \sum_{q=0}^{m} \frac{m!}{q!(m-q)!} p^q (1-p)^{m-q} (m + qr)(N-m-qr)$$

2. Estimation of error caused by truncating the range in the integral

$$\int_0^{\infty} \frac{1 - e^{-ny^2}}{[y\, Y_0(y) - nY_1(y)]^2 + [y\, J_0(y) - nJ_1(y)]^2} \frac{dy}{y}$$

3. Estimation of integrals of the form

$$F(p) = \int_0^{\infty} \sin px\, f(x)dx,$$

where $f(x) > 0$, $f(x)$ monotonically decreasing.

4. Evaluation of the integral

$$\int_a^{\infty} \int_{-\infty}^{\infty} \frac{e^{-\beta(x^2+y^2)}}{(x^2 + y^2)^2} dx\, dy$$

5. Asymptotic evaluations of the solution of the differential equation

$$\frac{dq}{dt} = q(ae^{-q} - 1) \qquad (a > 1)$$

$$q(0) = q_0 \qquad (0 < q_0 < \log a)$$

6. Determination of the complete solution of a system of four linear differential equations of the first order with constant coefficients.

7. Reduction of $\sum_{n=0}^{\infty} Z^{(2n+1)^2}$ to ϑ- functions.

Publications: (1) "Some computational problems in algebraic number theory," by O. Taussky; submitted for inclusion in the Proceedings of the American Mathematical Society's Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953. (2) "The number of absolute points of a correlation," by A. Hoffman, M. Newman, E. Straus, O. Taussky; IN MANUSCRIPT. (3) "The representation of integers by binary quadratic rational forms," by K. Goldberg, M. Newman, E. Straus, and D. Swift; accepted by Archiv der Mathematik. (4) "Ueber die Funktionen von Gegenbauer," by P. Henrici; accepted by Archiv der Mathematik. (5) "On certain series expansions involving Whittaker functions and Jacobi polynomials," by P. Henrici; accepted by Pacific Journal of Mathematics. (6) "Linear functional equations and interpolation series," by P. Davis; accepted for publication in the Pacific Journal of Mathematics. (7) "On generating functions of the Jacobi polynomials," by P. Henrici; IN MANUSCRIPT. (8) "A conjecture of Erdös" by M. Newman; submitted to a technical journal. (9) "Note on a certain determinant,"

by M. Newman; IN MANUSCRIPT. (10) "Continuity in terms of connectedness," by K. Fan and R. A. Stuble; IN MANUSCRIPT. (11) "Compactification of completely regular spaces," by K. Fan and F. Wagner; IN MANUSCRIPT. (12) "Some L^2 aspects of Faber polynomials," by P. Davis and H. Pollack; accepted by the Duke Mathematical Journal. (13) "On representations and extensions of bounded linear functionals defined on classes of analytic functions," by P. Davis and J. L. Walsh; accepted by the Transactions of the American Mathematical Society. (14) "Structure theorems for modular subgroups," by M. Newman; submitted to a technical journal.

NUMBER THEORETICAL TEST PROBLEMS FOR SEAC
Task 1102-10-1104/50-5a

Origin: NBS Authorized 6/1/50
Managers: O. Taussky-Todd, K. Goldberg
Full task description appears in Apr-June 1950 issue, p. 31.

Status: CONTINUED. The routine to determine primes $p \leqslant 100{,}000$ for which $2^{p-1}-1$ is divisible by p^2 is being run as time is available and is now extended to $p<83{,}400$ with no positive results other than the well-known cases $p=1093$ and $p=3511$.

The routine to compute $u \pmod p$, where $(t + u\sqrt{p})/2$ is the fundamental unit in the field $R(\sqrt{p})$, p a prime, has been run for all primes less than 18,000 and for all primes of the form $4n+1<100{,}000$. No cases of $u \equiv 0 \pmod p$ have been found.

The work previously reported on consecutive l-th power residues (for April-June 1950 and January-March 1951) has recently been successfully used by H. S. Vandiver. Further extension of this work is desirable.

SOLUTION OF LAPLACE EQUATION BY MONTE CARLO METHOD
Task 1102-10-1104/51-6

Origin: NBS Authorized 9/28/50
Manager: M. Abramowitz
Full task description appears in July-Sept 1950 issue, p. 36.

Status: CONTINUED. A study is in progress of new methods for generating pseudo-random numbers.

We report here a study begun by I. A. Stegun at the request of W. J. Youden. Let $x_1, x_2, \ldots, x_n$ be a sample of n observations from a normal population. Let $\bar{x}=(x_1+x_2+\ldots+x_n)/n$. Determine the probabilities p_i^n of the events $x_i \leqslant \bar{x} < x_{i+1}$. The following results have been obtained by taking 8000 observations in the case $n=4$:

$$p_1^4 = .2665, \quad p_2^4 = .4723, \quad p_2^4 = .2612.$$

Further work is in progress.

ANALYSIS OF GEOMAGNETIC FIELD
Task 1102-10-1104/52-8

Origin: NBS Authorized 8/10/51
Manager: C. J. Swift
Full task description appears in July-Sept 1951 issue, p. 34.

Status: INACTIVE. For status to date see Oct-Dec 1951 issue, p.29.

SPECIAL PROBLEMS IN FINITE MATRIX THEORY
Task 1102-10-1104/52-34

Origin: NBS Authorized 11/6/51
Manager: O. Taussky-Todd
Full task description appears in Oct-Dec 1951 issue, p. 30.

Status: CONTINUED. J. Todd studied the matrices that arise by the replacement of differential equations such as $u_{xx}=u_t$, $u_{xx}=u_{tt}$ by simple finite difference equations. The characteristic roots of these matrices can be found by methods similar to those used for equations of the form $u_{xx}+u_{yy}=ku$.

O. Taussky-Todd began a revision of her bibliography on bounds for characteristic roots of finite matrices.

K. Fan considered the following problem. For a positive-definite Hermitian matrix H of order n, it is well known that the inequality $\det H \leqslant (1,2,\ldots,p)(p+1,p+2,\ldots,n)$ holds, where $(1,2,\ldots,p)$ denotes the principal minor of H corresponding to the first p unit vectors $e_1,e_2,\ldots,e_p$. A more complete result than this inequality is obtained:

$$\frac{\det H}{(p+1,\ p+2,\ldots,n)}$$

is the minimum of p-rowed $\det((Hx_i,x_j))$, when the p vectors $x_1,x_2,\ldots,x_p$, $(1\leqslant i,\ j\leqslant p)$, vary under the biorthonormal condition $(x_i,e_j)=\delta_{ij}$ $(1\leqslant i,j\leqslant p)$. Other inequalities concerning positive definite Hermitian matrices are also obtained, among which we have the inequality

$$\frac{\det H}{(p+1,p+2,\ldots,n)} \leqslant \prod_{i=1}^{p} \frac{(i,\ p+1,p+2,\ldots,n)}{(p+1,p+2,\ldots,n)} .$$

Publications: (1) "Some remarks on commutators of matrices," by K. Fan; accepted by Archiv der Mathematik. (2) "Inequalities for eigenvalues of Hermitian matrices," by K. Fan; to appear in "Contributions to the solutions of systems of linear equations and the determination of eigenvalues," NBS Applied Mathematics Series 39. (3) "Some metric inequalities in the space of matrices," by K. Fan and A. Hoffman; submitted to a technical journal. (4) "The conditions of certain matrices, II," by J. Todd; accepted by Archiv der Mathematik. (5) "Error bounds for eigenvalues of symmetric integral equations," by H. Wielandt; submitted for inclusion in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953. (6) "Einschliessung von Eigenwerten hermitescher Matrizen nach dem Abschnittsverfahren," by H. Wielandt; accepted by Archiv der Mathematik. (7) "Characteristic roots of quaternion matrices," by O. Taussky; accepted by Archiv der Mathematik. (8) "On eigenvalues of sums of normal matrices," by H. Wielandt; submitted to a technical journal.

(9) "Generalized commutators of matrices and permutations of factors in a product of three matrices," by O. Taussky; to appear in the von Mises Anniversary volume. (10) "The condition of the finite segments of the Hilbert matrix," by J. Todd; to be included in Contributions to the systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

DIFFERENTIAL EQUATION FOR NERVE FIBER REACTION
Task 1102-10-1104/53-15

Origin: National Naval Medical Institute Authorized 12/8/52
Sponsor: " "
Managers: H. A. Antosiewicz, P. Rabinowitz
Full task description appears in Oct-Dec 1952 issue, p. 34.

Status: INACTIVE.

BAKER-HAUSDORFF FORMULA
Task 1102-10-1104/53-16

Origin: NBS Authorized 12/8/52
Managers: O. Taussky-Todd, K. Goldberg
Full task description appears in Oct-Dec 1952 issue, p. 35.

Status: CONTINUED. Work on the manuscript, "The equation $e^z=e^x.e^y$ in a free associative ring," by K. Goldberg, continued.

HYPERGEOMETRIC FUNCTIONS
Task 1102-10-1104/53-35

Origin: NBS Authorized 3/25/53
Managers: P. Rabinowitz, W. Cahill
Full task description appears in Jan-Mar 1953 issue, p. 36.

Status: INACTIVE.

Publication: "Programs for computing hypergeometric series," by W. F. Cahill; MTAC 8, 36-37 (Jan. 1954).

BASIC RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5116/50-2

Origin: Office of Scientific Research, ARDC, USAF Authorized 3/31/50
and Office of the Air Comptroller, USAF
Sponsor: Office of Scientific Research, ARDC, USAF
Managers: J. Todd, A. Hoffman
Full task description appears in Jan-Mar 1950 issue, p. 24.

Status: CONTINUED. Using theorems on convex sets, K. Fan has derived various classical results on Chebyshev approximation. The method

employed is a generalization of Rademacher and Schoenberg's use of Helly's theorem to attack Chebyshev's problem. He has also studied the intersection properties of a linear manifold of arbitrary dimension with the intersection of a finite number of closed half-spaces. From these properties, he derives existence theorems (some of which are known results of Tchernikow and Blumenthal) for systems of linear inequalities in a very simple way. Both the geometric part and the applications of this study are continuing.

K. Fan and J. Todd have generalized an inequality of J.B.Chassan, concerned with the estimation of the position of an object given by line-of-sight observations. The generalization (see publication (5) below) is based on geometric considerations, and the proof is algebraic, unlike Chassan's statistical proof.

H. Antosiewicz is studying methods for the systematic simplification and solution of systems of linear inequalities.

A. J. Hoffman has given simple proofs of some theorems of Frobenius and Ullman on properties of matrices with non-negative elements. The theorems of Frobenius are relevant to stability problems in various econometric studies, such as inter-industry analysis.

A seminar in linear programming and related problems was organized by A. J. Hoffman. The first meeting took place on March 31.

Publications: (1) "A theorem on alternatives for pairs of matrices," by H. A. Antosiewicz; submitted to a technical journal. (2) "On a theorem of Ostrowski and Taussky," by R. Bellman and A. J. Hoffman; accepted by Archiv der Mathematik. (3) "Lower bounds for rank and location of the eigenvalues of a matrix," by K. Fan and A. J. Hoffman; to appear in "Contributions to the solution of systems of linear equations and the determination of eigenvalues," NBS Applied Mathematics Series 39. (4) "Some metric inequalities in the space of matrices," by K. Fan and A. J. Hoffman; submitted to a technical journal. (5) "A determinantal inequality," by K. Fan and J. Todd; IN MANUSCRIPT. (6) "A characterization of normal matrices," by A. J. Hoffman and O. Taussky; J. Res. NBS 52, 17-19 (Jan. 1954). (7) "On the relevance of LeChatelier's principle to linear programming," by A. J. Hoffman; IN MANUSCRIPT. (8) "An extremum property of sums of eigenvalues," by H. Wielandt; submitted to a technical journal. (9) "On a combinatorial theorem," by A. J. Hoffman; submitted to a technical journal.

COMPRESSIBLE FLOW--METHOD OF ORTHOGONAL AND KERNEL FUNCTIONS

Task 1102-10-5116/52-16

Origin: Aeronautical Research Laboratory, Wright Air Development Center, USAF, and Harvard University — Authorized 9/29/51

Sponsor: Aeronautical Research Laboratory, WADC, USAF

Managers: P. Davis, F. L. Alt

Full task description appears in July-Sept 1951 issue, p. 36.

Status: CONTINUED. A report entitled "Some SEAC computations of subsonic fluid flows by Bergman's method of integral operators" is in manuscript. This report describes and discusses some computations that were made for a boundary value problem corresponding to a univalent flow of type S. These computations serve to indicate the degree of approximation which can be achieved with the first few terms of the various infinite processes occuring in the theory. The report contains a detailed description of the boundary value problem solved and extensive discussion of the numerical computation of the particular solutions and the singular solutions.

Publication: "A multiple-purpose ortho-normalizing code and its uses," by P. Davis and P. Rabinowitz; IN MANUSCRIPT.

TABLES OF INTEGRALS INVOLVING THE HIGHER TRANSCENDENTAL FUNCTIONS
Task 1102-10-5116/52-33

Origin: NBS Authorized 10/11/51
Manager: F. Oberhettinger
Full task description appears in July-Sept 1951 issue, p. 37.

Status: CONTINUED. Some additional material for the second volume of the tables of integrals, which is in the process of being varityped, has been furnished. Rechecking and correcting of a number of samples for this volume turned out to be necessary.

[We note that the first volume of this collection has now appeared: "Tables of Integral Transforms, Vol. I," edited by A. Erdelyi with W. Magnus, F. Oberhettinger, and F. G. Tricomi, has been published by McGraw-Hill.]

Publication: "On the Lerch zeta function," by F. Oberhettinger; submitted to a technical journal.

SUPPORTING RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5116/54-9

Origin: Office of the Air Comptroller, USAF Authorized 9/25/53
Sponsor: Office of Scientific Research, Air Research and Development Command, USAF
Managers: J. Todd, A. J. Hoffman
Full task description appears in July-Sept 1953 issue, p. 35.

Status: CONTINUED. A rapid method of solving the modification of the caterer problem described in the Oct-Dec 1953 issue, p. 26, second paragraph, has been found. The essential device consists of (i) a transformation reducing the number of variables, and (ii) a permutation of the variables so that the appearance of the matrix is sufficiently similar to that of the unmodified caterer problem for the techniques of publication (3) to apply. Although (ii) is not always possible for the most general problem of this type that could be proposed, it is possible for the specific case of interest to the sponsor. A noteworthy feature of the algorithmic solution is that there is no penalty for failure to estimate requirements in the distant future accurately, provided the correct requirements are known in time.

The construction of an optimal purchase pattern for an item for which there is limited storage space has been under study. The case in which requirements and prices are precisely known for the various time periods has been solved, and work is proceeding on situations in which less information is available.

The experiment to try to compute the expected value of the value of a game whose coefficients are only known probabilistically, but will be known exactly at the time the game is played, has thus far been unsuccessful. The tools used were various modifications of Brown's method, and machine trials of a small problem did not converge to the known answer.

Trials of the Brown-Koopmans method of solving a linear program revealed a difficulty when an iterate is on an intersection of two or more

bounding hyperplanes. A modification has been proved valid theoretically, and will be coded.

The code for performing the simplex method by systematic use of the inverse of the basic matrix has been completed and is undergoing tests.

Publications: (1) "A remark on the smoothing problem," by H. A. Antosiewicz and A. J. Hoffman; an NBS report. (2) "On 'overshoot' in the furthest hyperplane method," by R. Bryce; IN MANUSCRIPT. (3) "Linear programming in bid evaluation," by L. Gainen, D. J. Honig, and B. D. Stanley; to appear in Logistics Research Quarterly. (4) "A method of solving the transportation problem," by A. Gleyzal; submitted to a technical journal. (5) "Smooth patterns of production," by A. J. Hoffman and W. Jacobs; an NBS report. (6) "On the optimal ordering of items for a two-stage process," by A. J. Hoffman; IN MANUSCRIPT. (7) "On block relaxation," by L. S. Joel; IN MANUSCRIPT.

RESEARCH IN MECHANICS OF CONTINUA
Task 1102-10-5116/54-23

Origin: NBS Authorized 12/29/53
Sponsor: Office of Naval Research, USN
Manager: R. F. Dressler
Full task description appears in Oct-Dec 1953 issue, p. 27.

Status: CONTINUED. The study of resistive action and cross-section variations on shock tube flow for a purely mechanical model has been completed by R. F. Dressler, and a paper submitted for publication. Exact results for the wave front have been compared with first order effects for the full wave. These first order effects are obtained analytically by solving a mixed-type boundary value problem for a pair of linear partial differential equations with variable coefficients. Exponential ducts create an antisymmetrical effect on the sound speed; the resistance raises densities in the back wave and lowers densities in the forward portion.

An analysis involving the full thermodynamical effects on a dissipative aerodynamic flow has been initiated by R. F. Dressler. This study involves the energy equation with variable entropy as well as modified momentum and continuity equations. Using the Riemann relations along the three Mach line families, it has been found possible to solve explicitly for first order effects on entropy, velocity, density, and temperature for the back portion of a rarefaction wave. Research is almost completed on this problem and a manuscript will be prepared next quarter, "Entropy and temperature changes in rarefaction waves."

An improved method for obtaining higher approximations in the elastic problem of helical springs has been worked out by P. Henrici. A revised manuscript incorporating these new results is now being prepared. Results represent an improvement over the recent work of Freiberger on this problem.

A study has been started by M. Abramowitz and H.A. Antosiewicz concerning the behavior of solutions to a type of second order differential equation $y''+\phi(x)y=0$ connected with the mechanics of particles in a Coulomb field of force. The case of interest concerns the solutions in the neighborhood of a point x_0 where the sign of $\phi(x)$ changes. Some representations for these solutions have already been obtained in terms of Airy integrals.

Two problems in heat conduction proposed by P. Haas (14.8) have been solved by F. Oberhettinger. These, and similar requests, have suggested that a compilation of Green's function for the heat equation, the wave equation, and for the potential equation, for various regions be made. This has been started by F. Oberhettinger.

Publications: (1) "Turbulent flow in shock tubes of varying cross-section," by R. F. Dressler; to appear in the NBS Journal of Research. (2) "Helical elastic springs of finite cross-section," by P. Henrici; IN MANUSCRIPT.

WATER WAVES
Task 1102-10-5116/53-54

Origin: NBS Authorized 6/1/53
Sponsor: Office of Naval Research, USN
Manager: P. Davis
Full task description appears in Apr-June 1953 issue, p. 38.

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 25.

2. Mathematical Tables and Experimental Computations

TABLES OF $E_1(z)$, $(z = x + iy)$
Task 1102-10-1104/43-3

Origin: Canadian National Research Council Authorized 7/1/47
Manager: I. A. Stegun
Full task description appears in Apr-June 1949 issue, p. 41.

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 28.

TABLE OF THE GAMMA FUNCTIONS FOR COMPLEX ARGUMENTS
Task 1102-10-1104/46-1

Origin: NBS Authorized 7/1/47
Manager: J. Todd
Full task description appears in Apr-June 1949 issue, p. 43.

Status: CONTINUED. The volume is in press.

Publication: "Table of the gamma function for complex arguments," NBS Applied Mathematics Series 34; in press, U. S. Government Printing Office.

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-10-1104/47-2

Origin: NBS Authorized 7/1/47
Managers: M. Abramowitz, P. Rabinowitz
Full task description appears in Apr-June 1949 issue, p. 45.

Status: CONTINUED. Another short table has been issued as an NBS report. Computations were carried out for values at the turning point $\rho = 2\eta$.

TABLE OF LAGRANGIAN COEFFICIENTS FOR SEXAGESIMAL INTERPOLATION
Task 1102-10-1104/48-2

Origin: NBS Authorized 5/25/48
Manager: J. Todd Completed 3/31/54

Objective: Preparation of a volume of interpolation coefficients for the argument p at intervals of 1/3600 for 3-, 4-, 5-, 6-, 7-point interpolation and at intervals of 1/60 for 8-, 9-, 10-, 11-point interpolation.

Background: Functions tabulated with arguments in degrees occur in every branch of applied science. The present tables will simplify and reduce the labor of interpolation in such tables, whenever the functions are desired for minutes and seconds.

Status: COMPLETED.

Publication: "Tables of Lagrangian coefficients for sexagesimal interpolation," NBS Applied Mathematics Series 35 (Jan. 4, 1954). Available from U. S. Government Printing Office, Washington 25, D. C., $2.00.

BIBLIOGRAPHY OF CODING PROCEDURES, MATHEMATICAL TABLES AND NUMERICAL ANALYSIS
Task 1102-10-1104/50-5

Origin: NBS Authorized 3/1/50
Managers: J. Todd, J. H. Wegstein, P. Rabinowitz Revised 1/9/53
Full task description appears in the Oct-Dec 1952 issue, p. 40.

Status: CONTINUED. Preparation of the bibliographies continues. The following Technical Memoranda were issued:

44 Subroutine for binary to decimal conversion of a double precision number with fixed binary point.
45 Subroutine for binary to decimal conversion of a single precision number with floating binary point.
46 Subroutine for decimal to binary conversion of a single precision number with floating binary point.
48 Subroutine for the evaluation of polynomials, single precision, fixed binary point.
49 Automatic transfer routine.

56 Fast square root subroutine, single precision, fixed binary point.
57 Decimal to binary and binary to decimal conversion subroutines for integers and fractions.

WAVE FUNCTION FOR LITHIUM
Task 1102-10-1104/50-7

Origin: NBS Authorized 6/1/50
Sponsor: Bureau of Ordnance, USN
Manager: W. F. Cahill
Full task description appears in Apr-June 1950 issue, p. 36.

Status: CONTINUED. The iteration for the first approximation to the wave function is continued as machine time is available. A code to compute a new approximation to the eigenvalue is being written; this includes a double integration involving the first approximation.

COLLECTED SHORT MATHEMATICAL TABLES OF THE COMPUTATION LABORATORY
Task 1102-10-1104/51-4

Origin: NBS Authorized 9/28/50
Manager: J. Todd
Full task description appears in July-Sept 1950 issue, p. 43.

Status: CONTINUED. The first volume is in press.

Publication: "Tables of functions and of zeros of functions," Volume I of Collected Short Tables of the Computation Laboratory, NBS Applied Mathematics Series 37; in press, U. S. Government Printing Office.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Task 1304-34-6351/51-8

Origin: Section 11.3, NBS Authorized 3/26/51
Managers: A. Hoffman, S. Tsingou
Full task description appears in Apr-June 1951 issue, p. 49.

Status: INACTIVE. For status to date, see Jan-Mar 1953 issue, p. 45.

REVISION OF MATHEMATICAL TABLES
Task 1102-10-1104/52-7

Origin: NBS Authorized 8/10/51
Managers: J. Todd, W. F. Cahill, I. Stegun
Full task description appears in July-Sept 1951 issue, p. 41.

Status: CONTINUED. Following is the status of those mathematical tables the sales stock of which has been exhausted and for which reissue is planned:

"Table of sine and cosine integrals for arguments from 10 to 100," NBS Applied Mathematics Series 32; in press, U. S. Government Printing Office. This will be a reissue of the table formerly designated as Mathematical Table MT13 (1942).

"Tables of the error function and its derivative," NBS Applied Mathematics Series 41; in press, U. S. Government Printing Office. This was originally Mathematical Table MT8, "Tables of probability functions," vol. I (1941).

"Tables of sines and cosines for radian arguments," to be reissued in the Applied Mathematics Series; formerly designated as Mathematical Table MT4.

"Tables of natural logarithms," vol. IV, Mathematical Table MT12 (1941); revision in progress for reissue in the Applied Mathematics Series.

"Miscellaneous Physical Tables," Mathematical Table MT17. It has been decided not to reissue this. A revision and extension of the tables of Planck's radiation functions is under consideration by Tunis Wentink, Jr., General Electric Co. A revision of the second part, Tables of electronic functions, suggested by Dr. L. L. Marton, will be carried out on SEAC; publication of the tables will be handled by Division 4 of the NBS.

TABLE OF ARCSIN FOR COMPLEX ARGUMENTS
Task 1102-10-1104/52-14

Origin: NBS Authorized 10/1/51
Manager: A. A. Goldstein
Full task description appears in July-Sept 1951 issue, p. 41.

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 31.

EXTENSION OF THE TABLE OF HYPERBOLIC SINES AND COSINES
Task 1102-10-1104/52-18

Origin: NBS Authorized 9/17/51
Manager: W. F. Cahill
Full task description appears in July-Sept 1951 issue, p. 41.

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 31.

TABLE OF THE MODIFIED AIRY INTEGRAL
Task 1102-10-1104/52-23

Origin: NBS Authorized 10/4/51
Manager: P. Rabinowitz
Full task description appears in July-Sept 1951 issue, p. 42.

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 31.

TABLE OF ERROR FUNCTION FOR COMPLEX ARGUMENTS
Task 1102-10-1104/52-25

Origin: NBS Authorized 10/5/51
Manager: M. Abramowitz
Full task description appears in July-Sept 1951 issue, p. 42.

Status: INACTIVE. The 20-place table for x,y=0(.1)2 is available on punched cards.

EXTENSION OF TABLES OF THE EXPONENTIAL FUNCTION FOR NEGATIVE ARGUMENTS
Task 1102-10-1104/52-31

Origin: NBS Authorized 10/9/51
Manager: E. Marden
Full task description appears in July-Sept 1951 issue, p. 43.

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 32.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-10-1104/52-37

Origin: NBS Authorized 11/28/51
Manager: T. Ledley
Full task description appears in Oct-Dec 1951 issue, p. 38.

Status: CONTINUED. Computations of the characteristic values for the prolate and oblate spheroidal wave equations for $m=0(1)5$, $\ell=0(1)10$, $c^2=0(.01)1$, $\ell/c=0(.005)1$ have been completed except for a few values of ℓ/c for $m=5$, $\ell=9,10$. The values have been transferred to punched cards.

VAN DER POL EQUATION
Task 1102-10-1104/52-43
(formerly 1102-10-1110/52-43)

Origin: NBS Authorized 11/28/51
Manager: W. F. Cahill
Full task description appears in Oct-Dec 1951 issue, p. 38.

Status: INACTIVE. For status to date, see Apr-June 1952 issue, p. 49.

RADIAL MATHIEU FUNCTIONS
Task 1102-10-1104/52-49

Origin: NBS Authorized 2/1/52
Managers: J. Todd, I. Rhodes
Full task description appears in Jan-Mar 1952 issue, p. 45.

Status: CONTINUED. Checking of the values by E. Aparo and D. Dainelli at the Istituto Nazionale per le Applicazioni del Calcolo (Rome) is continuing.

SIEVERT'S INTEGRAL
Task 1102-10-1104/52-57

Origin: NBS Authorized 2/12/52
Managers: M. L. Paulsen, P. J. O'Hara
Full task description appears in Jan-Mar 1952 issue, p. 46.

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 33.

TABLE OF SECANTS AND COSECANTS
Task 1102-10-1104/52-81
(formerly 1102-10-1110/52-81)

Origin: NBS Authorized 7/1/52
Managers: K. C. Nelson, I. A. Stegun Completed 3/31/54

Objective: To prepare a table of secant x and cosecant x to nine significant figures for x = 0(.01°)90°.

Background: The tables have been in possession of the Computation Laboratory for several years and were calculated from the previously published tables of sines and cosines. The proposed volume is intended as a companion to the earlier volume. Comparison with Gifford's Table of Secants has disclosed a large number of errors in Gifford.

Status: COMPLETED.

Publication: "Table of secants and cosecants to nine significant figures at hundredths of a degree," NBS Applied Mathematics Series 40 (Mar. 26, 1954). Available from U. S. Government Printing Office, Washington 25, D. C., 35 cents.

PAINLEVÉ EQUATION
Task 1102-10-1104/53-3

Origin: NBS Authorized 8/11/52
Managers: J. Todd, H. A. Antosiewicz
Full task description appears in July-Sept 1952 issue, p. 49.

Status: INACTIVE. For status to date, see Jan-Mar 1953 issue, p. 48.

DYNAMIC BEHAVIOR OF AIRCRAFT STRUCTURES
Task 1102-40-5126/53-29

Origin: NBS, Section 6.4 Authorized 1/23/53
Sponsor: " "
Manager: I. Rhodes
Full task description appears in Jan-Mar 1953 issue, p. 58.

Status: CONTINUED. Computations are being performed as requested, and results are being transmitted to the sponsor.

L-SHELL CONVERSION COEFFICIENTS
Task 1102-10-5110/53-52

Origin: Oak Ridge National Laboratory Authorized 5/20/53
Manager: C. J. Swift
Full task description appears in Apr-June 1953 issue, p. 45.

Status: CONTINUED. Production is continued as SEAC time is available. The K-shell is about half done and the L_1-shell is about one-eighth done.

3. Mathematical Services

Note: The tasks under Mathematical Services are arranged serially according to the digits following the slant lines in the task numbers. The first two digits following the slant line designate the fiscal year in which the task was authorized.

TABLES OF THERMODYNAMIC PROPERTIES OF GASES
Task 0302-40-2606/49-5

Origin: NBS, Section 3.2 Authorized 11/29/48
Sponsor: National Advisory Committee for Aeronautics
Manager: F. L. Alt
Full task description appears in Apr-June 1949 issue, p. 53.

Status: INACTIVE.

RAY TRACING
Task 0202-10-2308/50-13

Origin: NBS, Section 2.2 Authorized 3/1/50
Sponsor: " "
Manager: R. K. Anderson
Full task description appears in Jan-Mar 1950 issue, p. 33.

Status: CONTINUED. SEAC codes for generating and tracing rays

through spheric and aspheric surfaces havebeen completed. Two optical systems have been analyzed with the new code.

MOLECULAR STRUCTURE CALCULATIONS, II
Task 1102-40-5126/50-16

Origin: Naval Research Laboratory, USN Authorized 3/31/50
Sponsor: " " " "
Manager: P. J. O'Hara
Full task description appears in Jan-Mar 1950 issue, p. 34.

Status: CONTINUED. Computations were performed as requested.

LIQUID-VAPOR TRANSITION
Task 1102-40-5126/51-22

Origin: Naval Medical Research Institute Authorized 2/1/51
Sponsor: " " " "
Manager: I. Stegun
Full task description appears in Jan-Mar 1951 issue, p. 44.

Status: INACTIVE. For status to date, see July-Sept 1952 issue, p. 52.

MOLECULAR STRUCTURE, III
Task 1102-40-5126/51-37

Origin: Naval Research Laboratory, USN Authorized 8/10/51
Managers: P. J. O'Hara, I. A. Stegun
Full task description appears in July-Sept 1951 issue, p. 50.

Status: CONTINUED. Computations were performed as requested.

SHOCK WAVE PARAMETERS, II
Task 1102-40-5126/51-38

Origin: Bureau of Ordnance, USN Authorized 6/28/51
Sponsor: " " "
Manager: I. A. Stegun
Full task description appears in Apr-June 1951 issue, p. 46.

Status: INACTIVE. For status to date, see Apr-June 1953 issue, p. 48.

POWDER DIFFRACTION
Task 1102-40-5126/52-6

Origin: NBS, Section 9.7 Authorized 9/17/51
Manager: E. Marden Terminated 3/31/54

Objective: To calculate interplanar spacings in crystals for postulated lattice parameters, in order to index x-ray powder diffraction patterns.

Background: The indexing of powder diffraction patterns is checked by comparison of observed and calculated values of $d_{hk\ell}$. The computation is done on SEAC, by means of a routine prepared by F. Ordway and E. C. Marden as a part of task 1102-21-1104/51-3 (see Jan-Mar 1951 issue, p. 37). Tapes containing lists of indices, suitable for requested use in this task, have been prepared by E. Tatge (section 9.7).

Status: TERMINATED, in favor of task 1102-40-5126/52-44 (see p. 35).

PRECISE DETERMINATION OF THE PARAMETER OF DISPERSION EQUATION FOR SEVERAL TYPES OF OPTICAL GLASS
Task 1102-40-5126/52-17

Origin: NBS, Division 2 Authorized 9/29/51
Sponsor: " "
Manager: I. A. Stegun
Full task description appears in July-Sept 1951 issue, p. 56.

Status: INACTIVE.

SPHERICAL BLAST
Task 1102-40-5126/52-20

Origin: Naval Ordnance Laboratory Authorized 9/27/51
Sponsor: " "
Manager: D. H. Jirauch
Full task description appears in July-Sept 1951 issue, p. 56.

Status: CONTINUED. Some work continued to reorganize the material prior to test runs on SEAC.

CALCULATIONS FOR d SPACINGS
Task 1102-40-5126/52-44

Origin: NBS, Div. 9 Authorized 12/7/51
Sponsor: " "
Managers: R. K. Anderson, A. M. Futterman
Full task description appears in Oct-Dec 1951 issue, p. 47.

Status: CONTINUED. Programming for SEAC is now in progress.

GAS ADSORPTION BY HIGH POLYMERS
Task 1102-40-5126/52-70

Origin: Naval Medical Center, Bethesda, Md. Authorized 4/1/52
Sponsor: " " " "
Manager: I. Stegun
Full task description appears in Jan-Mar 1952 issue, p. 59.

Status: INACTIVE. For status to date, see Apr-June 1952 issue, p. 59.

NEUTRON DIFFUSION, II
Task 1102-40-5126/53-4

Origin: Atomic Energy Commission, New York Office, (NDA) Authorized 9/30/52
Sponsor: " " "
Managers: O. Steiner, N. Levine
Full task description appears in July-Sept 1952 issue, p. 60.

Status: CONTINUED. Results are being sent to the sponsor as they are completed. New codes are being written and code-checked as required to cover various source geometries. Computations have been carried out for a number of materials for different sets of data.

STRENGTH OF WING COMPONENTS
Task 1102-40-5126/53-11

Origin: National Advisory Committee for Aeronautics, Langley Field, Va. Authorized 12/8/52
Sponsor: " "
Manager: E. Marden
Full task description appears in Oct-Dec 1952 issue, p. 54.

Status: CONTINUED. Computations are being performed on SEAC as requested.

RADIANT HEATING OF SOLIDS
Task 1102-40-5126/53-20

Origin: NBS, Section 10.2 Authorized 12/15/52
Sponsor: " "
Manager: W. F. Cahill
Full task description appears in Oct-Dec 1952 issue, p. 55.

Status: INACTIVE.

COMPUTATION OF THERMODYNAMIC FUNCTIONS
Task 1102-40-5126/53-27

Origin: NBS, Division 5 Authorized 3/18/53
Sponsor: " "
Manager: E. Marden
Full task description appears in Jan-Mar 1953 issue, p. 57.

Status: CONTINUED. Computations have continued on SEAC, and thermodynamic functions for several molecules have been calculated using the two previous codes. In addition, a new code has been written to compute polynomials in powers of 1/T (where T is temperature) for a range of temperatures from 50° to 5000°K.

STUDY OF TRICALCIUM ALUMINATE
Task 1102-40-5126/53-28

Origin: NBS, Division 9 Authorized 3/30/53
Sponsor: " "
Manager: R. Anderson
Full task description appears in Jan-Mar 1953 issue, p. 58.

Status: INACTIVE. For status to date, see Jan-Mar 1953 issue, p. 58.

REDUCTION OF ECLIPSE DATA
Task 1102-40-5126/53-34

Origin: Air Photographing and Charting Services, USAF Authorized 3/30/53
Sponsor: " " " " Completed 3/31/54
Manager: A. A. Goldstein

Objective: (1) To obtain all possible information from the photo-electric observation of the 1952 eclipse, particularly with reference to geodetic position. (2) From the results of the above to propose specific observational methods to determine geodetic position from the eclipse of 1954.

Background: Measurements of previous eclipses were directed toward obtaining the two times of inner contact of the sun and moon. This gave two equations of condition which would allow the correction of two parameters at each station. Photo-electric measurements provide a continuous curve of light intensity during the eclipse, with the consequent possibility of many equations of condition.

Status: COMPLETED. Measurements of light intensity taken during the 1952 total eclipse from three stations in Africa (Bangui, Khartoum, and Port Sudan) have been reduced and analyzed with the following results.

1. Information is available for certain corrections to the geodetic positions of the three stations.
2. A correction to the difference between the solar and lunar diameter has been found. The accuracy of the measurement as indicated by the probable error is roughly one part in a thousand. If the known solar diameter is taken as exact, we obtain for the geocentric lunar radius $S=(.272374\pm.000011)\pi$, where π is the lunar parallax. The values

in general use are $S=.27227\pi$ for eclipses, and
$S=.27250\pi$ for occultations.

3. The Solar Limb Darkening (for light of wave length 8000 angstroms) has been determined. The results obtained are over the last one percent of the solar edge. They are believed to be more consistent than previously published results.

4. The method developed and used here appears practical for large-scale applications and indicates the desirability of conducting light intensity measurements during the next total eclipse on a larger scale.

ACOUSTICAL IMPEDANCES
Task 1102-40-5126/53-39

Origin: NBS, Section 6.1 Authorized 6/30/53
Sponsor: " "
Managers: S. Prusch, K. Nelson
Full task description appears in Apr-June 1953 issue, p. 55.

Status: INACTIVE. For status to date, see Apr-June 1953 issue, p. 55.

LORAN UNIVAC CODE
Task 1102-40-5126/53-41

Origin: Hydrographic Office, U. S. Navy Authorized 4/20/53
Sponsor: " "
Managers: I. Rhodes, D. H. Jirauch
Full task description appears in Apr-June 1953 issue, p. 56.

Status: CONTINUED. The baseline code and the skywave correction and trainer code are ready for code checking.

RADIATION DIFFUSION
Task 1102-40-5126/53-51

Origin: NBS, Section 4.8 Authorized 6/9/53
Sponsor: Armed Forces Special Weapons Project
Managers: F. Stockmal, W. F. Cahill
Full task description appears in Apr-June 1953 issue, p. 57 (Neutron Diffusion III).

Status: CONTINUED. Computations on SEAC continue as time is available.

HIGH TEMPERATURE PROPERTIES OF WATER
Task 1102-40-5126/54-4

Origin: NBS, Division 3.2 Authorized 12/7/53
Sponsor: Bureau of Ordnance, USN
Manager: J. H. Wegstein
Full task description appears in Oct-Dec 1953 issue, p. 41.

Status: CONTINUED. Preliminary to finding formulas to represent the thermal functions of water at high temperatures such as specific heat and the Gibbs free energy, Mr. Woolley (Section 3.2) has written formulas for the partition function describing the high temperature properties of hydrogen. These formulas involve physical conditions and a parameter which affect the computed properties in an unknown manner. It is planned to evaluate these formulas with the SEAC in order to check them. If these formulas give a satisfactory description of hydrogen at high temperatures it should then be possible to extend the method to the complete system of fifteen or twenty molecules, atoms, and ions representing water at high temperatures.

The SEAC interpretive floating-point routines are being perfected in preparation for evaluating these formulas.

DISTRIBUTION OF NORMAL MODES OF VIBRATION OF CUBIC LATTICES, II
Task 3711-60-0009/54-7

Origin: NBS, Division 30 Authorized 12/29/53
Sponsor: " "
Managers: F. Stockmal, W. F. Cahill
Full task description appears in Oct-Dec 1953 issue, p. 42.

Status: CONTINUED. A new code to compute the distribution of normal modes of vibration for body centered cubic lattices has been composed and checked out.

NORMAL VIBRATIONS IN MOLECULES
Task 3711-60-0009/54-8

Origin: NBS, Section 3.2 Authorized 11/27/53
Sponsor: " "
Manager: M. Newman
Full task description appears in Oct-Dec 1953 issue, p. 42.

Status: CONTINUED. Seven eight by eight symmetric matrices were inverted. The programming for the computation of the eigenvalues and eigenvectors of a symmetric matrix by the method of Jacobi is underway.

COMPUTATION OF VIBRATION MODES AND FREQUENCIES
Task 3711-60-0009/54-11

Origin: NBS, Section 6.4 Authorized 11/27/53
Sponsor: " "
Managers: I. Rhodes, W. Cahill
Full task description appears in Oct-Dec 1953 issue, p. 42.

Status: CONTINUED. The first five antisymmetric and first four symmetric vibration modes and frequencies of an aircraft structure were computed using a 24x24 matrix of influence coefficients for the symmetric case and a 31x31 matrix of influence coefficients for the antisymmetric case.

SPACE AVERAGE OF TEMPERATURE DISTRIBUTION WITH A VOLUME DISTRIBUTION OF SOURCES
Task 3711-60-0009/54-12

Origin: NBS, Division 6 Authorized 12/11/53
Sponsor: " "
Manager: E. Marden
Full task description appears in Oct-Dec 1953 issue, p. 43.

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 43.

AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Task 1102-40-5126/54-13

Origin: New York Quartermaster Procurement Agency Authorized 11/27/53
Sponsor: " " " "
Manager: H. Bremer
Full task description appears in Oct-Dec 1953 issue, p. 43.

Status: CONTINUED. Approximately 17 contracts (for socks, raincoats, wiping cloth, soap, etc.) have been awarded in accordance with answers obtained on the SEAC. One of these problems required use of the general simplex method; all others were solved using the special version of the simplex method applicable to the so-called transportation problem. New codes to facilitate the computation of optimum shipping schedules have been completed and checked and will soon be put into operation. A code for UNIVAC is being prepared in collaboration with personnel of the Office of the Air Comptroller, USAF.

At present problems are arriving at the rate of about three every two weeks, and the sponsor receives the answers in one or two days.

FLOW IN SUPERSONIC NOZZLE
Task 1102-40-5126/54-14

Origin: National Advisory Committee for Aeronautics Authorized 12/16/53
Langley Field, Va.
Sponsor: " "
Managers: P. Henrici, E. Marden, G. Hawkins
Full task description appears in Oct-Dec 1953 issue, p. 44.

Status: CONTINUED. The SEAC code has been completed and computations for supersonic flow in nozzles for two different Mach numbers have been completed.

DEPOLYMERIZATION
Task 3711-60-0009/54-17

Origin: NBS, Section 7.6 Authorized 12/16/53
Sponsor: " "
Manager: R. Anderson
Full task description appears in Oct-Dec 1953 issue, p. 44.

Status: INACTIVE. For status to date, see Oct-Dec 1953 issue, p. 44.

ENERGY LEVELS OF COMPLEX ATOMS
Task 3711-60-0009/54-19

Origin: NBS, Section 4.1 Authorized 3/31/54
Sponsor: " "
Manager: W. G. Hall

Objective: To determine the characteristic values and characteristic vectors of matrices arising in the determination of energy levels of complex atoms. The orders of the matrices range from 5x5 to 30x30.

Background: Energy levels of complex atoms are obtainable as eigenvalues of a Schrödinger equation. By use of a perturbation procedure, these energies can be expressed as eigenvalues of matrices. The elements of these matrices are linear forms in several adjustable parameters. The eigenvalues are calculated using estimated values of the parameters, the estimate of parameters are improved by using a least squares calculation with observed values, and eigenvalues of the new matrix calculated.

Status: NEW. Some of the characteristic roots of several matrices have been computed using a code prepared by W. F. Cahill, which uses the power method.

LOW TEMPERATURE PROPERTIES OF BORON COMPOUNDS
Task 1102-40-5126/54-20

Origin: NBS, Section 3.2 Authorized 2/24/54
Sponsor: " "
Manager: J. H. Wegstein

Objective: (a) To obtain specific heat from primary electrical

data by utilizing electronic computers. (b) To correlate and compute thermodynamic properties (enthalpy, entropy, and free energy functions) from specific heat data using semi-empirical or theoretical methods such as the inversion of the partition function to obtain the frequency distribution of the solid from the measured specific heats. This also includes the smoothing of heat capacity data and integration of such to obtain the other thermodynamic properties.

Background: The present computation of thermodynamic properties by hand methods permits treatment of about five compounds a year. It is desired to analyze more compounds and to make additional computations on their thermodynamic properties.

Status: NEW.

EXPERIMENTAL PROGRAM FOR MULTIPLE INPUT AND OUTPUT
Task 1102-40-5126/54-21

Origin: NBS, Section 12.3 Authorized 2/24/54
Sponsor: " "
Manager: I. Rhodes

Objective: To provide expert programming assistance for experimental machine studies to evaluate new data-processing auxiliary storage equipment and to advise on desirable selection features for multiple input-output equipment which is under development, such as notched magnetic disks, wire storage arrays, and high-speed tape cartridges.

Background: The design of DYSEAC provides for input-output operations, which are concurrent with internal computations, to as many as 256 input-output units and for special buffering so that information transfers may be made automatically to a variety of input-output devices operating at varying speeds. These design features provide opportunities to evaluate new devices which have been proposed for use as permanent or auxiliary data-storage and to try out theoretical programs for sorting and searching operations which require many outputs or inputs operable in parallel. The experimental studies will include the programming of sample data-processing problems involving multiple input-output operations to simulate the use of new multiple input-output devices and to provide data for design decisions as to external selection equipment, desired switching speeds, and similar considerations, for improvements to SEAC and improved computer systems design.

Status: NEW. Several subroutines were written on a sorting-file merging problem using SEAC to simulate the use of 16 outputs for coarse sorting. A preliminary report of results has been issued, entitled "Preliminary report on a combined sorting file merging method for electronic data processing," by Ida Rhodes and M. Stevens.

ENERGY DISTRIBUTIONS ON OPTICAL IMAGE
Task 3711-60-0009/54-22

Origin: NBS, Section 2.2 Authorized 2/9/54
Sponsor: " " Revised 3/30/54
Manager: L. S. Joel

Objective: To compute

$$2\int_0^1 \left\{ \exp i\,(-10Z^2 + 200Z^4 - 200Z^6) \right\} Z\, J_0(\eta Z)\,dZ$$

for $\eta = 0(.5)9(1)40$.

Background: The light intensity of a point source formed by an optical system with aberration can be expressed as a double integral

$$I(\xi,\eta) = \iint \exp i \left\{ \xi x + \eta y + W(x,y) \right\} dx\, dy,$$

where $W(x,y)$ is the aberration expressed as an error of phase in the integral for $I(\xi,\eta)$ and ξ,η are coordinates in the image plane. Taking the x,y coordinates in the pupil, the integral is to be evaluated over the pupil. In the most general case $W(x,y)$ would be obtained by fitting a polynomial to ray tracing results. If we assume that $W(x,y)$ can be expressed as a polynomial of the sixth degree and that the pupil is circular the double integral reduces to a simple integral involving the Bessel function $J_0(z)$.

Comments: This problem was originated by Dr. Weinstein, guest worker in Section 2.2, from the Technical Optics Section, Imperial College of Science and Technology, London, England.

Status: NEW. Quadratures were carried out on SEAC, using a standard subroutine which divides the integration interval into k equal subintervals and applies 16-point Gaussian quadrature to each. Calculations were performed with k=6 and, for checking, k=7 for all η, and k=16 for $\eta = 0(1)9$. The standard subroutine for $J_n(X)$ was used for $\eta Z \leq 9$. This subroutine computes $J_n(X)$ by means of the series expansion. For values of $\eta Z < 9$ the asymptotic expansion was employed with three terms in the cosine factor and two in the sine factor. Initial examination of results indicates that for $\eta \sim 25$ it will be necessary to recompute using k=16, as the number of oscillations in the integrand is quite large.

SOLUTION OF THE DIFFUSION EQUATION FOR ELECTRONS
Task 1102-40-5126/54-24

Origin: NBS, Division 4 Authorized 3/5/54
Sponsor: " "
Managers: M. Abramowitz, A. Futterman

Objective: To calculate electron depth distributions by solving an integro-differential diffusion equation which describes fairly accurately the penetration of electrons.

Background: The diffusion equation governing electron penetration and diffusion in plane geometry contains three independent variables describing an electron's energy, location, and direction. This equation has

resisted solution largely because (a) the scattering cross section is strongly peaked in angle and (b) an asymptotic description of the spatial distribution has not been available. Integral methods have recently been developed and tested for calculating spatial moments of fairly high order. Partly with the help of these, the desired asymptotic description has been obtained. These integral methods, which reduce the calculation of spatial moments to a recursive system, are ideally suited to the capabilities of high speed computers. What is desired, then, is to calculate spatial moments up to the 30th or 40th order with the SEAC and to make a Legendre polynomial representation of the spatial distribution. This can be used to test the accuracy of simpler methods based on the knowledge of a spatial weight function. Following this, spatial moments or distributions for a number of interesting and practical problems may be obtained using the SEAC.

Status: NEW. The code is finished. The problem will be run on SEAC according to specifications given by the originator.

REFLECTED RADIATION FROM AN INFINITE LAMBERT PLANE
Task 1102-40-5126/54-25

Origin: Armed Forces Special Weapons Group Authorized 2/9/54
Sponsor: " "
Managers: P. Henrici, L. S. Joel

Objective: To evaluate the integral

$$\Phi(h,r,d) = \frac{hd^2}{\pi} \int_0^{\infty} \int_0^{2\pi} \frac{\rho\, d\rho\, d\phi}{(h^2+r^2+\rho^2-2r\rho\cos\phi)^{3/2}(\rho^2+d^2)^2}$$

for various values of the parameters.

Background: Let the plane $z=0$ in an (x,y,z)-space be a Lambert plane, and let a monochromatic source of light of intensity I be fixed at the point $(0,0,h)$. Let a small horizontal plane p be situated at (x,y,d). Then the illumination of p due to reflection at the Lambert plane is $\mu I \Phi(h,r,d)$, $r=\sqrt{x^2+y^2}$, where μ is a constant depending only on the physical properties of the Lambert plane.

Status: NEW. After some transformations, the double integral can be expressed as a finite simple integral, which depends only on the two variables $u=r/h$, $v=d/h$. Putting

$$\Phi(h,r,d) = \frac{1}{h^2}\,\Psi(u,v),$$

one finds

$$\Psi(u,v) = \frac{2v^2}{(1+u^2)^{3/2}} \int_0^1 \frac{\eta}{(1+\eta^2)^{3/2}} \left\{ \frac{1}{(q+\eta)^2} + \frac{\eta^3}{(1+q\eta)^2} \right\} F\left(\tfrac{3}{4},\tfrac{5}{4};1;\frac{4p\eta}{(1+\eta^2)^2}\right) d\eta.$$

where

$$p = \frac{u^2}{1+u^2}, \qquad q = \frac{v^2}{1+u^2},$$

and F denotes the hypergeometric function. This integral is being evaluated

by 16-point Gaussian quadratures. The hypergeometric function is computed from the power series, the convergence of which is sped up by Aitken's δ^2-method. The integration error has been estimated by a method due to P. Davis (J. Rational Mech. Anal. 2(1953), 303-313).

The code for the problem has been completed and $\Psi(u,v)$ has been computed for u=0(.05)1.60, v=.05(.05)1.60.

PHASE DETERMINATION FOR COLEMANITE BY PROBABILITY METHODS
Task 1102-40-5126/54-26

Origin: U. S. Geological Survey Authorized 3/4/54
Sponsor: " "
Manager: P. J. O'Hara

Objective: To determine the phases (or signs) of the structure factors of colemanite for later use in computing the electron density function.

Background: This project deals with the application of probability methods to the study of the structure factor equations for a centrosymmetric crystal. The magnitude of the crystal structure factor is computed using the observed X-ray intensities and known quantities involving the chemical composition and symmetry of the crystal. The phases (or signs) of the structure factors are regarded as unknown.

Status: COMPLETED (NEW). The results have been transmitted to the sponsor.

EMF OF CELLS AT HIGH TEMPERATURE
Task 3711-60-0009/54-27

Origin: NBS, Section 1.8 Authorized 3/5/54
Sponsor: "
Managers: E. Marden, G. Hawkins

Objective: To compute the electromotive force of galvanic cells of type M/MX/X, where M=metal, MX=compound, X=non-metal, for various temperatures, from the relationship $E_T = -\Delta F_T/n\mathcal{F}$, where E_T = emf at temperature T, ΔF_T = free energy of formation at temperature T, n = small integer, $\mathcal{F}$ = 23060.5.

Background: ΔF_T is computed from temperature by means of the following equation:

$$\Delta F_T = H_0 - \Delta aT \cdot \ln T - \frac{\Delta bT^2}{2} - \frac{\Delta c}{2T} + \mathcal{J}T.$$

The equations are computed over a given temperature range, which will vary with each problem. At the limits of the temperature ranges, $H_{T=\text{limit}}$ must be modified by a transition factor, and the temperatures of transition (allotropic change, fusion and vaporization) will vary from one problem to another. The computations are to be performed for all known fluorides, chlorides, bromides, iodides, oxides, sulfides, and possibly nitrides, phosphides, hydrides, silicides, and carbides.

Status: NEW. Coding of this problem for SEAC is in progress.

LOVIBOND NETWORK FOR CIE SOURCE A
Task 3711-60-0009/54-28

Origin: NBS, Section 2.1 Authorized 2/28/54
Sponsor: " "
Managers: E. Marden, G. Hawkins

Objective: To compute the Lovibond network for CIE source A, using the equations as noted below. (The Lovibond network for CIE sources B and C have been computed in England.)

Background: The numbers to be computed are tristimulus values X,Y, and Z; and chromaticity coordinates x', y', and z', defined as follows:

$$X = \sum T_R^r T_Y^y T_B^b (E\bar{x})\Delta\lambda \; ; \; Y = \sum T_R^r T_Y^y T_B^b (E\bar{y})\Delta\lambda \; ; \; Z = \sum T_R^r T_Y^y T_B^b (E\bar{z})\Delta\lambda \; ;$$

$$x' = \frac{X}{(X + Y + Z)} \; ; \quad y' = \frac{Y}{(X + Y + Z)} \; ; \quad z' = \frac{Z}{(X + Y + Z)} \; .$$

One of the exponents r,y,b is always zero, and the other two are integers as follows: r = 0(1)20(2)40(5)80(10)100,
y = 0(1)6,8,10,15,20,30,40,60,100,
b = 0(1)10(2)20,25,30,40,50 .

T_R, T_Y, and T_B are the spectral internal transmittances of the Lovibond unit red, yellow, and blue glasses, respectively, and the products $E\bar{x}$, $E\bar{y}$, and $E\bar{z}$ are functions of the wavelength.

Status: NEW. Coding of this problem for SEAC is in progress.

SPECTRUM ANALYSIS
Task 3711-60-0009/54-30

Origin: NBS, Division 4 Authorized 3/4/54
Sponsor: " "
Manager: S. Prusch

Objective: To analyse the atomic structure of Tc I and Tc II. Since the energy levels of atoms can be expressed as a function of the number of waves per cm(σ) of the light emitted by excited atoms, the wave numbers (σ) of over 4500 lines for experimentally measured values of wavelengths (λ) are to be computed to eight places using the formula of Edlen,

$$(n-1)\cdot 10^8 = 6432.8 + \frac{2949810}{146 - \sigma^2} + \frac{25540}{41 - \sigma^2} \; ,$$

where n is the index of refraction of air, and

$$\sigma_{vac} = \frac{1}{n\lambda_{air}} \; .$$

Many energy levels of an atom can be discovered due to the fact that transitions between levels give rise to groups of lines which have differences in common. Differences between all known and predicted even levels can be used to search for pairs of lines which would confirm the even levels and predict new odd levels. The differences between odd levels can

be used to search for pairs of lines to confirm the odd levels and predict new even levels.

Background: The calculations of wave numbers have previously been done from Kayser's Schwingungszahlen; however, a double interpolation has to be made, then a correction applied to bring the values up to date. The formula of Edlen gives the up to date values directly.

The searching for energy levels has formerly been done manually, requiring hundreds of hours combing a list of spectral lines for pairs of lines having a common difference.

Status: NEW. Differences between all known and predicted even levels were each used to search for pairs of lines which confirm even levels and predict new odd levels. This was done for Tc I.

AIR CONDITIONING IN UNDERGROUND STRUCTURES
Task 1102-40-5126/54-33

Origin: NBS, Section 10.3 Authorized 2/24/54
Sponsor: " "
Manager: S. Tsingou

Objective: To evaluate the integrals

$$\int_0^\infty \frac{1 - \exp(-\frac{\alpha t}{a^2} y^2)}{[yY_0(y)-ak\ Y_1(y)]^2 + [yJ_0(y)-ak\ J_1(y)]^2} \frac{dy}{y^3} .$$

and

$$\int_0^\infty \frac{1 - \exp(-\frac{\alpha t}{a^2} y^2)}{[yY_0(y)-ak\ Y_1(y)]^2 + [yJ_0(y)-ak\ J_1(y)]^2} \frac{dy}{y}$$

for various values of the parameters $\alpha t/a^2$ = .5,1,2,5,10,50, ak=.5,1,1.5, 2,3.

Background: The integrals in question arise in the problem of heat transfer from water to an underground reservoir.

Status: NEW. The code has been completed and checked. One case has been run, and others are to be run as requested.

VELOCITY OF LIGHT
Task 3711-60-0009/54-36

Origin: NBS, Section 11.3 Authorized 3/16/54
Sponsor: " "
Manager: P. Rabinowitz

Objective: To fit, in the sense of least squares, polynomials of the third and fourth degree to a set of data, to compute the experimental error, and to compute the variances and covariances of the coefficients

of the polynomials.

Background: These computations are required by the Radiometry Section (4.2) for reducing experimental observations in connection with the determination of the velocity of light.

Comments: This problem was proposed by E. K. Plyler (4.2) in cooperation with W. S. Connor (11.3).

Status: NEW. Least square polynomials of third and fourth degrees have been fitted to data submitted by the sponsor.

CORRELATION OF IONOSPHERIC DATA
Task 3711-60-0009/54-37

Origin: NBS, Section 14.1 Authorized 3/12/54
Sponsor: " "
Managers: J. H. Wegstein, H. H. Howe

Objective: To correlate certain ionospheric data (magnetic K-figures, maximum usable frequency, hours of fading) with sunspot number, both simultaneous and with time lag, in order to determine whether the other phenomena show any significant lag behind sunspots, and how much.

Background: This problem was tackled previously on the CPC from a different point of view.

Status: NEW. A code has been written and proved in on the three-address SEAC. About one hour of SEAC time is required to complete the calculations. This task has afforded an opportunity to test the performance of the three-address SEAC.

COMPRESSIBILITY FACTORS OF DRY AIR
Task 3711-60-0009/54-38

Origin: NBS, Section 3.2 Authorized 3/29/54
Sponsor: " "
Manager: M. Paulsen

Objective: To prepare SEAC routines for evaluating the dimensionless compressibility factor z and the density ρ from the relations

$$z = P/RT\rho$$
$$z = A + B\rho + C\rho^2 + D\rho^3 + \ldots$$

for prescribed values of the constants, P = pressure, T = temperature, and the virial coefficients A,B,C,D,... . When the programs have been checked, tables will be prepared as desired.

Background: The desired tables were computed heretofore either by hand or IBM equipment. It is hoped that the successful accomplishment of this task will make available a more comprehensive tabulation of those factors of importance in explaining the influence of imperfections on the thermodynamic properties of gases.

Status: NEW. A flow diagram has been prepared.

III. STATISTICAL ENGINEERING LABORATORY

(Section 11.3)

1. Fundamental Research in Mathematical Statistics

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-10-1107/49-1a

Origin: NBS Authorized 1/9/49
Manager: L. S. Deming
Full task description appears in Apr-June 1949 issue, p. 75.

Status: CONTINUED. Abstracts of statistical literature have been taken from the two main reviewing services of mathematical and statistical publications, namely, Zentralblatt für Mathematik (for the years 1931-1941) and Mathematical Reviews (beginning with 1939 and continuing with the current issues as they arrive). These abstracts are mounted individually on cards, are classified by subject, and are arranged in an alphabetical file by author. Because of the almost complete overlap of the two abstracting journals since 1941, only abstracts from Mathematical Reviews will be added to the file on a current basis.

MANUAL ON FITTING STRAIGHT LINES
Task 1103-10-1107/50-2

Origin: NBS Authorized 3/1/50
Manager: F. S. Acton
Full task description appears in Jan-Mar 1950 issue, p. 42.

Status: INACTIVE. For status to date see Oct-Dec 1952 issue, p. 58.

TABLE TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-10-1107/51-1

Origin: NBS Authorized 7/1/50
Managers: C. Eisenhart and L. S. Deming
Full task description appears in July-Sept 1950 issue, p. 57.

Status: INACTIVE. For status to date see July-Sept 1952 issue, p. 64.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-10-1107/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description appears in July-Sept 1950 issue, p. 58.

Status: CONTINUED.

Publications: (1) "Tables of expected values of 1/X for positive Bernoulli and Poisson variables," by E. Grab and I. R. Savage; J. Amer. Stat. Assoc., 49, 169-173 (Mar. 1954). (2) "A new method for analyzing extreme-value data," by J. Lieblein; NACA Technical Note 3053 (Jan.1954). (3) "Tables of the inverses of finite segments of the Hilbert matrix," by I. R. Savage and E. Lukacs; to be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39. (4) "A property of the normal distribution," by E. P. King and E. Lukacs; accepted for publication in the Annals of Mathematical Statistics. (5) "On strongly continuous stochastic processes," by E. Lukacs; accepted for publication in Sankhya. (6) "Certain Fourier transforms to distributions (II)," by E. Lukacs and O. Szász; accepted for publication in the Canadian Journal of Mathematics. (7) "Non-negative trigonometric polynomials and certain rational characteristic functions," by E. Lukacs and O. Szász; accepted for publication in the Journal of Research of the NBS. (8) "Time-discrete stochastic processes in arbitrary sets, with applications to processes with absorbing regions and to the problem of loops in Markoff chains," by D. van Dantzig; submitted to a technical journal. (9) "On the variances and covariances of order statistics from the Weibull distribution," by J. Lieblein; submitted to a technical journal. (10) "Two early papers on the relation between extreme values and tensile strength," by J. Lieblein; submitted to a technical journal. (11) "On optimum grouping in one-criterion variance components analysis," by E. P. King; accepted for publication in the Journal of the American Statistical Association. (12) "The use of extreme-value methods in engineering problems," by E. J. Gumbel and J. Lieblein; IN MANUSCRIPT.

LAW OF PROPAGATION OF ERROR
Task 1103-10-1107/52-1

Origin: NBS Authorized 6/23/51
Managers: C. Eisenhart and I. R. Savage
Full task description appears in July-Sept 1951 issue, p. 65.

Status: INACTIVE. For status to date see July-Sept 1951 issue, p. 65.

PROCEDURES OF NON-PARAMETRIC STATISTICS
Task 1103-10-1107/52-2

Origin: NBS Authorized 9/17/51
Manager: I. R. Savage
Full task description appears in July-Sept 1951 issue, p. 66.

Status: CONTINUED. The work of I. R. Savage on most probable rank orders and alternatives of the Lehmann type has been continued. In particular, the following testing situation has been studied intensively:

H_0 -- $X_1,\ldots,X_m,Y_1,\ldots,Y_n$ are mutually independent and identically random variables.

H_1 -- $X_1,\ldots,X_m,Y_1,\ldots,Y_n$ are mutually independent, and there exists a continuous cumulative distribution function $F(x)$ and positive constants Δ_1 and Δ_2 $(\Delta_2>\Delta_1)$ such that the X's are identically distributed with the c.d.f. F^{Δ_1} and the Y's are identically distributed with the c.d.f. F^{Δ_2}.

This situation has statistical interest since special cases involve the testing of the scale parameter for the exponential distribution and the location parameter for the extreme value distribution. For specific values of the parameters it is possible to find the exact probabilities for each rank order without the necessity of numerical integration. For all values of the parameters the construction of optimum test procedures for small samples is possible due to the fact that the probabilities of rank orders are partially ordered. Finally, the limiting test has been derived and it is interesting to note that this procedure gives results almost identical with those obtained by the best procedures for small samples.

W. S. Connor has studied the function $y=a+bx$ in the case when x is without error but y can be observed only through the random variable Y which has a continuous distribution function with a unique median y. When observations on y are taken at only two values of x, a simple nonparametric technique is available for determining simultaneous confidence intervals for all y's.

Publications: (1) "Bibliography of nonparametric statistics and related topics," by I. R. Savage; J. Amer. Stat. Assoc. 48, 844-906 (Dec. 1953). (2) "Most probable rank orders," by I. R. Savage; submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-10-1107/53-1

Origin: NBS Authorized 10/15/52
Manager: W. S. Connor
Full task description appears in the Oct-Dec 1952 issue, p. 60.

Status: CONTINUED. M. Zelen has continued the study of a new method for combining information from inter- and intra-block analyses for exact tests of significance. Preliminary results indicate that the new method is more powerful than any existing method.

M. Zelen (with John Mandel, Division 7, Section 5) has analyzed new designs for the two-way elimination of heterogeneity.

Publication: (1) "Some theorems for partially balanced designs," by W. S. Connor and W. H. Clatworthy; Ann. Math. Stat. 25, 100-112

(Mar. 1954). (2) "An embedding theorem for balanced incomplete block designs," by M. Hall, Jr. (Ohio State University) and W. S. Connor; Canadian J. Math. VI, No. 1, 35-41 (1954). (3) "Analysis for some incomplete block designs having a missing block," by M. Zelen; accepted for publication in Biometrics. (4) "A note on partially balanced designs," by M. Zelen; accepted for publication in the Annals of Mathematical Statistics. (5) "Partially balanced incomplete block designs with two associate classes and two treatments per block," by W. H. Clatworthy; submitted to a technical journal. (6) "On the enumeration of partially balanced designs with two associate classes," by W. H. Clatworthy; submitted to a technical journal. (7) "New experimental designs for paired observations," by W. S. Connor and W. J. Youden; IN MANUSCRIPT.

2. Applied Research in Mathematical Statistics

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden and J. Cameron
Full task description appears in July-Sept 1950 issue, p. 60.

Status: CONTINUED. Activity under this project fell into two main categories:

A. Design of Experiments: (a) A partially balanced design was constructed for an investigation of hygroscopic properties of cement in which 120 kinds of cement were to be tested in groups of 8. An experimental arrangement involving two replications was constructed from a balanced incomplete block design by means of its incidence matrix. (b) A 1/3 replicate of a 3x5x6 factorial design was developed for testing the effect of various factors on the warping of plastic laminates used in printed circuits.

B. Development or Selection of the Appropriate Method for Analysis and Interpretation of Data: (a) In corrosion studies of pitting of metal pipes the average maximum pit depth was shown by extreme-value methods to be a logarithmic function of the length of pipe. This result provides a theoretical treatment for a problem for which only empirical methods have previously been used. (b) An optimum assignment of a given fixed number of observations in order to minimize the variance of the intercept of a straight line was worked out for the case where two x values were used. This problem arose in connection with work on resistence thermometers.

Publications: (1) "Acceptance sampling of electroplated articles," by J. M. Cameron and F. Ogburn (5.6); Plating 41, 43-46 (Jan. 1954). (2) "Performance of inspectors and gasoline pumps," by W. J. Youden and M. Jensen (32.0); NBS Miscellaneous Publication 209, 82-88 (available from U. S. Government Printing Office, Washington 25, D. C., 40 cents). (3) "Probability limits for the average chart when process standards are unspecified," by E. P. King; submitted to a technical journal.

STATISTICAL ASPECTS OF NBS ADMINISTRATIVE OPERATIONS
Task 3737-60-0002/52-1

Origin: NBS Authorized 10/1/51
Manager: I. R. Savage
Full task description appears in Oct-Dec 1951 issue, p. 56.

Status: INACTIVE. For status to date see July-Sept 1952 issue, p. 68.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin: Ship Structure Committee, NRC Authorized 12/1/51
Sponsor: " "
Manager: W. J. Youden
Full task description appears in Oct-Dec 1951 issue, p. 58.

Status: CONTINUED. The method of estimating the transition temperature of a steel plate by linear interpolation between two test temperatures was evaluated by estimating the transition temperature from a second degree polynomial fitted to observations at several test temperatures. This was done for two each of "source", "through," and "end" plates.

RESEARCH IN APPLICATIONS OF MATHEMATICAL STATISTICS TO PROBLEMS OF THE CHEMICAL CORPS
Task 1103-40-5118/52-1

Origin: Biological Laboratories, Chemical Corps Authorized 10/1/51
Dept. of the Army
Sponsor: " " "
Manager: C. Eisenhart
Full task description appears in Oct-Dec 1951 issue, p. 57.

Status: CONTINUED. A study is in progress to construct and compile fractional replications of 2^n and 3^n factorial designs in incomplete blocks
A classified meeting was held at the Bureau on March 16 on the subject of Area Weapons. About 40 experts from various scientific fields and from various Government agencies and universities attended. The purposes of the meeting were to exchange current research information, to improve the exchange of information in the future, and to organize research in such a manner that the results will be of the greatest use.

STATISTICAL ANALYSIS AND DESIGN OF EXPERIMENTS FOR THE U. S. GEOLOGICAL SURVEY
Task 1103-40-5140/54-1

Origin: U. S. Geological Survey, Department of Interior — Authorized 10/9/53
Sponsor: " " "
Managers: C. Eisenhart and W. J. Youden
Full task description appears in Oct-Dec 1953 issue, p. 50.

Status: CONTINUED. Attention was devoted during this quarter principally to study and evaluation of methods advocated in earth-sciences literature for the analysis of orientation data, with a view to developing simplified procedures (a) for testing whether an observed distribution of points and a circumference departs sufficiently from a uniform distribution to warrant the conclusion that one or more "preferred directions" exist, (b) for estimating the azimuth of the "preferred direction" and determining the statistical uncertainty of the result, and (c) for testing whether two or more sets of orientation data may be regarded as random samples from distributions having one and the same preferred direction. A preliminary draft version of a note giving "A quick-and-easy test for the existence of preferred directions" was completed for limited circulation for comment as the quarter ended; another, giving a numerical evaluation of the bias of the so-called "empty-test" as commonly used, was essentially completed; and a third, giving "A quick-and-easy method for determining 'preferred direction'," was nearing completion in preliminary draft.

In addition, some correspondence was exchanged with certain individual members of the U. S. Geological Survey on such topics as random subsampling versus replicate subsampling, precision of chemical analyses, use of statistical methods of correlation and regression as tools in the study of geochemical associates, and the application of the log-normal distribution to problems of ore sampling.

STATISTICAL ANALYSIS OF BALL BEARING FATIGUE DATA
Task 1103-40-5145/54-1

Origin: American Standards Association — Authorized 1/13/54
Sponsor: " "
Manager: J. Lieblein

Objective: To study and evaluate available methods for analysis of data on fatigue-life of ball bearings; and to determine the load-life curve of ball bearings indicated by the test data obtained and made available by the cooperating ball bearing manufacturers.

Background: Sub-Committee 7 of the American Standards Association Committee B3 has assigned to it the task of analyzing a mass of test data compiled by the laboratories of prominent ball bearing manufacturers for the purpose of determining load-life relations. (The sub-committee has discussed various methods of analysis and there is some question as to the correct method. Variation in method would lead to radically different conclusions resulting in confusion and disagreement as to exactly what the data indicated.) It is the unanimous opinion of all concerned that the Statistical Engineering Laboratory of the National Bureau of Standards is best fitted to resolve the issue, in that it has the necessary competence and impartiality and is acceptable to all parties involved. The results are expected to be of widespread engineering importance and economic value

to ball bearing users representing almost every engineering field.

Status: NEW. A meeting between the Sub-Committee and Bureau representatives was held early in the quarter at which time various aspects of the project were discussed and available data were turned over to the Statistical Engineering Laboratory for study and analysis. A report of the meeting embodying background material for the project was drafted and submitted to the Sub-Committee for comments. A revised report embodying the suggestions received is in preparation. The endurance data for ball bearings that were received from the Sub-Committee were examined on an overall, exploratory basis as regards the suitability of various types of load-life relationships. The wide scatter in fatigue lives present required further study to be made of the best methods for treating the data.

IV. MACHINE DEVELOPMENT LABORATORY

(Section 11.4)

in cooperation with

ELECTRONIC COMPUTER SECTION

(Section 12.3)

THE BUREAU OF THE CENSUS COMPUTING MACHINE
Task 1104-34-5107/47-1

Origin: The Bureau of the Census — Authorized 7/1/47
Sponsor: " "
Full task description appears in Apr-June 1949 issue, p. 58.

Status: CONTINUED. Under contract between the NBS and the Eckert-Mauchly Division of Remington Rand Inc., stockpiling of replacement parts for UNIVAC System No. 1 has been continued.

NATIONAL BUREAU OF STANDARDS EASTERN AUTOMATIC COMPUTER (SEAC)
Task 1104-34-5107/49-1

Origin: NBS — Authorized 12/15/48
Sponsor: Office of the Air Comptroller, USAF
Full task description appears in Apr-June 1949 issue, p. 59.

Status: CONTINUED. Since January 1, 1954, the SEAC installation has been operated as an item of general Bureau equipment.

During this quarter SEAC was operated with an over-all efficiency of 85% during scheduled operation. The computation time was distributed as follows: Total time available -- 721 hours; Good operating time -- 612 hours; Down-time-- 109 hours. In addition, 186 hours were devoted to engineering and 137 hours to scheduled maintenance.

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Numerical Analysis Colloquium
(Los Angeles, California)

NEYMAN, J. (University of California, Berkeley). Studies of the spatial distribution of galaxies with particular reference to the hypothesis of the expanding universe. February 1.

LANCZOS, C. (North American Aviation Corporation). Spectroscopic eigenvalue analysis. March 1.

Numerical Analysis Seminar
(Los Angeles, California)

HOLMBOE, J. (University of California, Los Angeles). On the behavior of nonsteady two-dimensional rotational flow. January 5.

FORSYTHE, G.E. A survey of SWAC codes for solving matrix problems. January 7.

LANCZOS, C. (North American Aviation Corporation). On a quadrature method of solving eigenvalue problems for ordinary differential equations. January 12.

HUSKEY, H.D. Diagnosing SWAC failures. January 14.

WASOW, W.R. On improving approximate solutions of partial differential equations by h^2-extrapolation. January 19.

KAUS, P. (University of California, Los Angeles; graduate student). Machine solution of a non-linear eigenvalue problem of elementary particle theory. January 21.

MELKANOFF, M.A. (University of California, Los Angeles; graduate student). Machine computation of a proposed wave function for the Li_2 molecule. January 26.

MOTZKIN, T.S. Minimal number of multiplications for evaluating a polynomial. January 28.

HOLLANDER, F.H. Diagnosing SWAC failures, II. February 16.

TOMPKINS, C.B. SWAC results on isometric embedding of surfaces of revolution. February 18.

FUTTERMAN, W. A machine calculation of the photo-electric effect for the deuteron. February 23.

ROGERS, T.A.(University of California, Los Angeles). Electronic analogue computers. March 2.

LEBELL, D. (University of California, Los Angeles). Survey of UCLA analogue computers. March 4.

SELFRIDGE, J.L. (University of California, Los Angeles). Some applications of SWAC to number-theoretic computations. March 9.

MOOKINI, E. SWAC codes for solving systems of linear equations by the conjugate-gradient method. March 11.

MacNEAL, R. (California Institute of Technology). Solution of partial differential equations on C.I.T. analogue equipment. March 16.

KING, G.W. (International Telemeter Corporation). Three-dimensional random walks with excluded volume. March 18.

MULLER, M. Use of SWAC to study the behavior of such statistical theories as sequential analysis. March 23.

PETTIT, J.T. (University of California, Los Angeles). A matrix inversion code and its application to earth tides. March 25.

KRUSE, F.H. and SPARKS, R. (University of California, Los Angeles). Computing problems in chemical crystallography. March 30.

SPECIAL SEMINAR, sponsored jointly by the Department of Mathematics, University of California, Los Angeles, and the National Bureau of Standards. Los Angeles:

COURANT, R. (New York University). Problems of stability and soap film demonstrations. January 12.

Applied Mathematics Colloquim
(Washington)

BUSK, T.H. (University of Copenhagen). A class of polynomials. January 22.

Statistical Engineering Colloquia

ZELEN, M. Inter-block analysis for incomplete block designs. January 8.

CONNOR, W.S. Simultaneous confidence interval estimation. January 29.

Lectures and Symposia

Applied Statistics Colloquium

EISENHART, C. Statistical method in experimentation, II. January 15.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

ALT, F.L. A survey of computers and mathematical methods in management. Presented at the Computation Laboratory, Wayne University, Detroit, Mich., March 3.

ANTOSIEWICZ, H. Asymptotic solution of ordinary differential equations. Presented at the David Taylor Model Basin, Washington, D. C., January 15.

FORSYTHE, G.E. What are relaxation methods? Presented as part of the Lecture Series, "Mathematics for Modern Engineering," at the University of California: Los Angeles, February 1; Corona, February 2; Berkeley, February 4.

HENRICI, P. On certain equations related to the theory of generalized axially symmetric potentials. Presented at the Institute for Applied Mathematics and Fluid Dynamics, University of Maryland, College Park, Md., January 8.

HOFFMAN, A.J. The use of high-speed computers in awarding contracts. Presented to the Mobilization Designation Unit No. 1, Office of the Quartermaster General, Ft. Myer, Arlington, Va., March 15.

HUSKEY, H.D. A solution for automatic unit control. Presented at the IRE Computer Conference, Los Angeles, Calif., February 11.

SOUTHARD, T.H. Elements of nomography. Presented to the Pi Mu Epsilon Fraternity, University of California, Los Angeles, March 29.

TAUSSKY-TODD, O. Generation of pseudo-random numbers on high speed automatic digital computing machines. Presented at the Monte Carlo Symposium sponsored by the Wright Air Development Command, USAF, and held at Gainesville, Fla., March 15.

TODD, J. (1) What is numerical analysis? Presented at the Mathematics Colloquium, University of Maryland, College Park, Md., January 7. (2) Some tools of the numerical analyst. Presented at the NBS Scientific Staff meeting, Washington, March 12. (3) Tests of new systems of pseudo-random numbers. Presented at the Monte Carlo Symposium, sponsored by the Wright Air Development Command, USAF, and held at Gainesville, Fla., March 15.

TOMPKINS, C.B. (1) Some general objectives of a numerical analysis curriculum. Presented at the meeting of the Southern California Section of the Mathematical Association of America, held at the George Pepperdine College, Los Angeles, Calif., March 13. (2) Methods of steepest descent. Presented as part of the Lecture Series, "Mathematics for Modern Engineering," at the University of California: Los Angeles, March 29; Corona, March 30; Berkeley, April 1.

YOUDEN, W.J. (1) Control and measurement of experimental error. Presented to the local chapter of the American Statistical Association,

University of Illinois, Urbana, Ill., January 14. (2) Behavior of measurements. Special lecture presented at the University of Illinois Short Course on Quality Control, Urbana, Ill., January 14. (3) New experimental designs. Presented to the Central Illinois Section of the American Society for Quality Control, University of Illinois, Urbana, Ill., January 15. (4) Scheduling project work for greater returns. Presented to the Engineering Group, Bureau of Ships, Washington, D. C., February 10. (5-13) Interpretation of chemical data. Presented to the local sections of the American Chemical Society located at Kingsport, Knoxville, Nashville, Memphis, and Chattanooga, Tenn. and at Sheffield, Birmingham, and Auburn, Ala. and at Atlanta, Ga., February 16-26. (14) Design of experiments in operations research. Operations Research Office, The Johns Hopkins University, Md., March 9.

Publication Activities

1. PUBLICATIONS WHICH APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Tables of Lagrangian coefficients for sexagesimal interpolation. NBS Applied Mathematics Series 35. Available from U.S. Government Printing Office, $2.00.

(2) Table of secants and cosecants to nine significant figures at hundredths of a degree. NBS Applied Mathematics Series 40. Available from U. S. Government Printing Office, Washington 25, D.C., 35 cents.

(3) Tables of expected values of 1/X for positive Bernoulli and Poisson variables. E. Grab and I. R. Savage. J. Amer. Stat. Assoc. 49, 169-173 (Mar. 1954).

1.2 Manuals, Bibliographies, Indices

(1) The statistical theory of extreme values and some practical applications (A series of lectures by E. J. Gumbel). NBS Applied Mathematics Series 33. Available from U. S. Government Printing Office, Washington 25, D. C., 40 cents.

(2) Bibliography of nonparametric statistics and related topics. I. R. Savage. J. Amer. Stat. Assoc. 48, 844-906 (Dec. 1953).

1.3 Technical Papers

(1) On mildly nonlinear partial difference equations of elliptic type. L. Bers. J. Res. NBS 51, 229-236 (Nov. 1953).

(2) On modified divided differences. G. Blanch. MTAC 8, 1-11(Jan.1954).

(3) Programs for computing hypergeometric series. W. F. Cahill. MTAC 8, 36-37 (Jan. 1954).

(4) Acceptance sampling of electroplated articles. J. M. Cameron and F. Ogburn. Plating 41, 43-46 (Jan. 1954).

(5) Some theorems for partially balanced designs. W. S. Connor and W. H. Clatworthy. Ann. Math. Stat. 25, 100-112 (Mar. 1954).

(6) An embedding theorem for balanced incomplete block designs. M. Hall, Jr., and W. S. Connor. Canadian J. Math. 6, No. 1, 35-41 (1954).

(7) A characterization of normal matrices. A. J. Hoffman and O. Taussky. J. Res. NBS 52, 17-19 (Jan. 1954).

(8) A new method of analyzing extreme-value data. J. Lieblein. NACA Technical Note 3053 (Jan. 1954, Washington, D. C.). Available only from the NACA.

(9) On the accuracy of the numerical solution of the Dirichlet problem by finite differences. J.L. Walsh and D. Young. J. Res. NBS 51, 343-363 (Dec. 1953).

(10) Singular perturbation methods for nonlinear oscillations. W.Wasow. Proceedings of a Symposium on Nonlinear Circuit Analysis, held by the Polytechnic Institute of Brooklyn, N. Y., 1953, pp. 75-98.

(11) Performance of inspectors and gasoline pumps. W.J. Youden and M. W. Jensen. Report of the Thirty-Eighth National Conference on Weights and Measures 1953, National Bureau of Standards Miscellaneous Publication 209 (Dec. 21, 1953), p. 82-88. Available from U. S. Government Printing Office, 40 cents.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION MARCH 31, 1954

2.1 Mathematical Tables

(1) Table of sine and cosine integrals for arguments from 10 to 100. NBS Applied Mathematics Series 32. (A reissue of NBS Mathematical Table MT13.) In press, Government Printing Office.

(2) Table of the Gamma function for complex arguments. NBS Applied Mathematics Series 34. In press, Government Printing Office.

(3) Tables of functions and of zeros of functions. Volume I of Collected short tables of the Computation Laboratory. NBS Applied Mathematics Series 37. In press, Government Printing Office.

(4) Tables of the error function and its derivative. NBS Applied Mathematics Series 41. (A reissue of NBS Mathematical Table 8, Tables of probability functions, vol. I.) In press, Government Printing Office.

(5) Tables of sines and cosines for radian arguments. To appear in the NBS Applied Mathematics Series. (A reissue of NBS Mathematical Table 4.)

(6) Tables of the inverses of finite segments of the Hilbert matrix. I. R. Savage and E. Lukacs. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

2.3 Technical Papers

(1) Evaluation of the integral $\int_0^\infty e^{-u^2-(x/u)}du$. M. Abramowitz. Accepted for publication in the Journal of Mathematics and Physics.

(2) Regular and irregular Coulomb wave functions expressed in terms of Bessel-Clifford functions. M. Abramowitz. Accepted for publication in Journal of Mathematics and Physics.

(3) The practical evaluation of integrals. M. Abramowitz. Submitted to a technical journal.

(4) Approximate method for rapid Loran computation. M. Abramowitz, D.H. Call, and J. C. Mathews. Submitted to a technical journal.

(5) The relaxation method for linear inequalities. S. Agmon. Accepted for publication in the Canadian Journal of Mathematics.

(6) A theorem on alternatives for pairs of matrices. H. Antosiewicz. Submitted to a technical journal.

(7) Asymptotic solution of linear differential equations with a parameter. H. Antosiewicz. Submitted to a technical journal.

(8) On nonlinear differential equations of the second order with integrable forcing term. H. Antosiewicz. Submitted to a technical journal.

(9) On the differential equation $\ddot{x} + k(f(x) + g(x)\dot{x})\dot{x} = ke(t)$. H. A. Antosiewicz. Submitted to a technical journal.

(10) On a certain integral involving Bessel functions. H. Antosiewicz. Submitted to a technical journal.

(11) Some implications of Liapunov's conditions for stability. H. A. Antosiewicz and P. Davis. Submitted to a technical journal.

(12) On a theorem of Ostrowski and Taussky. R. Bellman and A. J. Hoffman. Accepted for publication in Archiv der Mathematik.

(13) On the enumeration of partially balanced designs with two associate classes. W. H. Clatworthy. Submitted to a technical journal.

(14) Partially balanced incomplete block designs with two associate classes and two treatments per block. W. H. Clatworthy. Submitted to a technical journal.

(15) Time-discrete stochastic processes in arbitrary sets, with applications to processes with absorbing regions and to the problem of loops in Markoff chains. D. van Dantzig. Submitted to a technical journal.

(16) Linear functional equations and interpolation series. P. Davis. Accepted for publication in the Pacific Journal of Mathematics.

(17) On a problem in the theory of mechanical quadratures. P. Davis. Accepted for publication in the Pacific Journal of Mathematics.

(18) Some L^2 aspects of Faber polynomials. P. Davis and H. Pollack. Accepted for publication in the Duke Mathematical Journal.

(19) On the estimation of quadrature errors for analytic functions. P. Davis and P. Rabinowitz. Submitted to a technical journal.

(20) On representations and extensions of bounded linear functionals defined on classes of analytic functions. P. Davis and J.L.Walsh. Accepted for publication in the Transactions of the American Mathematical Society.

(21) Some sampling results on the power of nonparametric tests against normal alternatives. W. J. Dixon and D. Teichroew. Submitted to a technical journal.

(22) Turbulent flow in shock tubes of varying cross-section. R. F. Dressler. Accepted by the Journal of Research of the NBS.

(23) On the optimal character of the (s,S) policy in inventory theory. A. Dvoretzky, J. Kiefer, and J. Wolfowitz. Submitted to a technical journal.

(24) Inequalities for eigenvalues of Hermitian matrices. K. Fan. To be included in Contributions to the solution of systems of linear

equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(25) Some remarks on commutators of matrices. K. Fan. Accepted for publication in Archiv der Mathematik.

(26) Lower bounds for the rank and location of the eigenvalues of a matrix. K. Fan and A. J. Hoffman. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(27) Some metric inequalities in the space of matrices. K. Fan and A. J. Hoffman. Submitted to a technical journal.

(28) Punched-card experiments with accelerated gradient methods for linear equations. A. I. and G. E. Forsythe. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(29) Asymptotic lower bounds for the frequencies of polynomial membranes. G. E. Forsythe. Accepted for publication in the Pacific Journal of Mathematics.

(30) Asymptotic lower bounds for the fundamental frequency of convex membranes. G. E. Forsythe. Accepted for publication in the Pacific Journal of Mathematics.

(31) SWAC. G. E. Forsythe. Submitted to a technical journal.

(32) Practical solution of linear equations and inversion of matrices. L. Fox. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues. NBS Applied Mathematics Series 39.

(33) A numerical solution of Schroedinger's equation in the continuum. W. Futterman, E. Osborne, and D. S. Saxon. Accepted for publication in the Journal of Research of the NBS.

(34) Linear programming in bid evaluation. L. Gainen, D. J. Honig, and E. D. Stanley. Accepted for publication in Logistics Research Quarterly.

(35) A nonlinear model for the composite pi-meson. S. G. Gasiorowicz. Submitted to a technical journal.

(36) Application of variational methods to intermediate and high energy scattering. E. Gerjuoy and D. S. Saxon. Accepted for publication in Physical Review.

(37) Variational principles for the acoustic field. E. Gerjuoy and D. S. Saxon. Submitted to a technical journal.

(38) A method of solving the transportation problem. A. N. Gleyzal. Submitted to a technical journal.

(39) Equations of physics in general Newtonian space-time. A.N.Gleyzal. Submitted to a technical journal.

(40) The representation of integers by binary quadratic rational forms. K. Goldberg, M. Newman, E. G. Straus, and J. D. Swift. Accepted for publication in Archiv der Mathematik.

(41) Iterative methods of solving linear problems on Hilbert space. R. M. Hayes. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(42) On certain series expansions involving Whittaker functions and Jacobi polynomials. P. Henrici. Accepted for publication in the Canadian Journal of Mathematics.

(43) Ueber die Funktionen von Gegenbauer. P. Henrici. To appear in Archiv der Mathematik.

(44) On a combinatorial theorem. A. J. Hoffman. Submitted to a technical journal.

(45) On optimum grouping in one-criterion variance components analysis. E. P. King. Accepted for publication in the Journal of the American Statistical Association.

(46) Probability limits for the average chart when process standards are unspecified. E. P. King. Submitted to a technical journal.

(47) A property of the normal distribution. E. P. King and E. Lukacs. Accepted for publication in the Annals of Mathematical Statistics.

(48) Contractibility and convexity. H. W. Kuhn. Accepted for publication in the Proceedings of the American Mathematical Society.

(49) On certain character matrices. D. H. Lehmer. Submitted to a technical journal.

(50) Bounds for punch indentation problems. E. Levin. Submitted to a technical journal.

(51) Acoustic radiation pressure on a circular disk. H. Levine. To appear in the Proceedings of the Fifth Symposium on Applied Mathematics of the American Mathematical Society.

(52) On the variances and covariances of order statistics from the Weibull distribution. J. Lieblein. Submitted to a technical journal.

(53) On strongly continuous stochastic processes. E. Lukas. Accepted for publication in Sankhya.

(54) Certain Fourier transforms of distributions (II). E. Lukacs and O. Szász. Accepted for publication in the Canadian Journal of Mathematics.

(55) Nonnegative trigonometric polynomials and certain rational characteristic functions. E. Lukacs and O. Szász. Accepted for publication in the Journal of Research of the NBS.

(56) A proof of Hilbert's Nullstellensatz. T. S. Motzkin. Submitted to a technical journal.

(57) On Fejér sets in linear and spherical spaces. T. S. Motzkin and I. J. Schoenberg. Accepted for publication in Annals of Mathematics.

(58) On the relaxation method for linear inequalities. T. S. Motzkin and I. J. Schoenberg. Submitted to a technical journal.

(59) Least p-th power polynomials on a real finite point set. T. S. Motzkin and J. L. Walsh. Submitted to a technical journal.

(60) A conjecture of Erdös. M. Newman. Submitted to a technical journal.

(61) Structure theorems for modular subgroups. M. Newman. Submitted to a technical journal.

(62) On the Lerch zeta function. F. Oberhettinger. Submitted to a technical journal.

(63) On two problems in abstract algebra connected with Horner's rule. A. M. Ostrowski. Submitted to a technical journal.

(64) On nearly triangular matrices. A. Ostrowski. Submitted to a technical journal.

(65) Determinanten mit ueberwiegender Hauptdiagonale und die absolute Konvergenz von linearen Iterationsprozessen. A. M. Ostrowski. Submitted to a technical journal.

(66) On the convergence of Gauss' alternating procedure in the method of the least squares, I. A. M. Ostrowski. Submitted to a technical journal.

(67) On the linear iteration procedures for symmetric matrices. A. M. Ostrowski. Submitted to a technical journal.

(68) On the spectrum of a one parametric family of matrices. A. M. Ostrowski. Submitted to a technical journal.

(69) On Gauss' speeding up device in the theory of single step iteration. A. M. Ostrowski. Submitted to a technical journal.

(70) On absolute convergence of linear iteration processes. A. M. Ostrowski. Submitted to a technical journal.

(71) On spectra of second-order differential operators. D. Ray. Submitted to a technical journal.

(72) On the convergence of asymptotic solutions of linear differential equations. R. M. Redheffer and W. Wasow. Submitted to a technical journal.

(73) Numerical computation of low moments of order statistics from a normal population. J. B. Rosser. Submitted to a technical journal.

(74) Most probable rank orders. I. R. Savage. Submitted to a technical journal.

(75) An isoperimetric inequality for closed curves convex in even-dimensional Euclidean space. I. J. Schoenberg. Submitted to a technical journal.

(76) Characteristic roots of quaternion matrices. O. Taussky. Accepted for publication in Archiv der Mathematik.

(77) Generalized commutators of matrices and permutations of factors in a product of three matrices. O. Taussky. For publication in the von Mises Anniversary volume.

(78) Some computational problems in algebraic number theory. O.Taussky. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 26-28, 1953.

(79) An improved cathode ray tube storage system. R. Thorensen. To appear in the Proceedings of the Western Computer Conference of the AIEE-IRE-ACM held in Los Angeles, Calif., February 4,5,6,1953.

(80) The condition of certain matrices, II. J. Todd. To appear in Archiv der Mathematik.

(81) The condition of the finite segments of the Hilbert matrix. J. Todd. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(82) The evaluation of the exponential integral for large complex arguments. J. Todd. Accepted for publication in the Journal of Research of the NBS.

(83) Discrete approximations to elliptic differential equations. W. Wasow. Submitted to a technical journal.

(84) On the asymptotic transformation of certain distributions into the normal distribution. W. Wasow. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 26-28,1953.

(85) An extremum property of sums of eigenvalues. H. Wielandt. Submitted to a technical journal.

(86) Einschliessung von Eigenwerten hermitescher Matrizen nach dem Abschnittsverfahren. H. Wielandt. To appear in Archiv der Mathematik.

(87) Error bounds for the eigenvalues of symmetric integral equations. H. Wielandt. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 26-28, 1953.

(88) On eigenvalues of sums of normal matrices. H. Wielandt. Submitted to a technical journal.

(89) A note on partially balanced designs. M. Zelen. Accepted for publication in Annals of Mathematical Statistics.

(90) Analysis for some incomplete block designs having a missing block. M. Zelen. Accepted for publication in Biometrics.

2.4 Reviews, Notes

(1) Note on the circle theorem of hydrodynamics. E. Levin. Accepted for publication by the Quarterly of Applied Mathematics.

(2) Two early papers on the relation between extreme values and tensile strength. (Formerly "A historical note on the application of the 'weakest link' idea to tensile strengths.") J. Lieblein. Submitted to a technical journal.

2.5 Miscellaneous Publications

(1) Contributions to the solution of systems of linear equations and the determination of eigenvalues. NBS Applied Mathematics Series 39. In press, Government Printing Office.

(2) Experiments in the computation of conformal maps. To appear in the NBS Applied Mathematics Series.

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($0.75), available from the Superintendent of Documents, Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.


NBS