NATIONAL BUREAU OF STANDARDS REPORT

3119

PROJECTS and PUBLICATIONS

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

A Quarterly Report

October through December 1953

FOR OFFICIAL USE

U. S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

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●Office of Basic Instrumentation

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NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT
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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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Index of Active Research and Development Projects

Note: This index is not intended to cover the numerous special problem solutions, statistical analyses, and other ad hoc services to Government agencies, which form an important part of the work of the National Applied Mathematics Laboratories. These services are, however, fully represented in the body of the report.

A. Research: Pure Mathematics

B. Research: Numerical Analysis

C. Research: Applied Mathematics, Physics, and Astronomy

*New task

D. Mathematical Statistics

E. Mathematical Tables

F. Manuals, Bibliographies, Indices, and Technical Information

G. Computing Machine Development

Status of Projects

December 31, 1953

I. INSTITUTE FOR NUMERICAL ANALYSIS

(Section 11.1)

1. Fundamental Research

SOLUTION OF SETS OF SIMULTANEOUS ALGEBRAIC EQUATIONS AND TECHNIQUES FOR THE INVERSION AND ITERATION OF MATRICES
Task 1101-10-5100/49-AE2
(formerly 11.1/1-49-AE2)

Origin: NBS — Authorized 2/15/49
Sponsor: Office of Naval Research, USN — Revised 9/15/49
Manager: G. E. Forsythe
Full task description appears in July-Sept 1949 issue.

Status: CONTINUED. As part of a service problem (task 1101-40-5131/53-6, q.v., p.14) it was necessary to get the inverse of a certain 43×43 minor A_1 of a 45×45 matrix A. The matrix A was symmetric, and all its eigenvalues and eigenvectors had been previously obtained for another purpose. A general technique was devised with which A_1^{-1} was obtained from the known information by matrix multiplications alone. A preliminary draft of a note explaining the technique has been prepared.

A by-product of the above problem was experience with a standard iteration for improving the inverse of a matrix. Let X_k be an approximation to A^{-1}. The iteration proceeds from the formula

$$X_{k+1} = (2I - X_k A)X_k .$$

Each element of $2I-X_kA$, whose accuracy is crucial to the method, is obtained by a triple-precision accumulation of single-precision products. The element is then shifted and converted to a single-precision number in the range (0,2). The multiplication $(2I-X_kA)X_k$ is performed with similar care. Without such care it was found that round-off errors made successive X_k worse and worse!

For any symmetric positive definite matrix A with eigenvalues

$0 < \lambda_1 \leqslant \ldots \leqslant \lambda_n$, let P(A) denote the condition number λ_n / λ_1 of A. Let D be any nonsingular diagonal matrix. A common practical method of scaling A before computing A^{-1} consists in forming DAD. The question arises: how should D be chosen so that P(DAD) is minimized? Stimulated by a conjecture of D. M. Young, the question has been studied at the Institute for Numerical Analysis and by E. G. Straus of U.C.L.A. Matrices A are said to be "best conditioned" when $P(A) \leqslant P(DAD)$ for all D. Best conditioned matrices have now been characterized in terms of a certain geometrical property of the invariant sets belonging to λ_1 and λ_n. If λ_n and λ_1 are simple eigenvalues, the characterization is that the corresponding invariant lines be reflections of each other in the plane in which certain coordinates vanish. This result proves Young's conjecture that matrices with his Property A are best conditioned when their diagonal elements are all equal.

A new code for using the conjugate gradient method on SWAC is now being written. The major subroutines, including a triple precision inner product, have already been written and their testing has just begun. Several variations of the conjugate gradient method will be tried with more use than before of the error function $E(x) = |x-h|^2$.

Participants in the research on this task were G. E. Forsythe, M. R. Hestenes, F. H. Hollander, E. H. Mookini, T. S. Motzkin, Louise Straus, and C. B. Tompkins.

Publications: No new ones.

CALCULATION OF EIGENVALUES, EIGENVECTORS, AND EIGENFUNCTIONS OF LINEAR OPERATORS

Task 1101-10-5100/50-3
(formerly 11.1/1-50-3)

Origin: NBS
Authorized 7/1/49
Sponsor: Office of Naval Research, USN
Manager: M. R. Hestenes
Full task description appears in July-Sept 1949 issue.

Status: CONTINUED. A manuscript has been completed by G. E. Forsythe entitled "Asymptotic lower bounds for the fundamental frequency of convex membranes." This manuscript has been presented for publication.

A report has been prepared by E. E. Osborne entitled "Solution of the matrix equation $(M - \Omega D)X = 0$." This report describes a computational scheme for finding the eigenvalues of this system. It includes a complete code for carrying out the necessary computations. The matrices computed are 6x6 matrices with complex coefficients.

Participants in the research on this task were G. E. Forsythe, M. R. Hestenes, and Louise Straus.

Publications: (1) "Completely continuous normal operators with property L," by I. Kaplansky; Pac. J. Math. 3, 721-724 (Dec. 1953).
(2) "Asymptotic lower bounds for the frequencies of polygonal membranes," by G. E. Forsythe; submitted to a technical journal.

STUDIES IN PURE MATHEMATICS
Task 1101-10-5100/50-4
(formerly 1101-11-5101/50-4)

Origin: NBS Authorized 7/1/49
Sponsor: Office of Naval Research, USN
Manager: C. B. Tompkins
Full task description appears in July-Sept 1949 issue.

Status: CONTINUED. T. S. Motzkin studied incompatible algebraic equations and gave a new proof of Hilbert's Nullstellensatz. He furnished bounds on the degrees and on the number of polynomials needed (also if the coefficients are polynomials in additional indeterminates) and developed another incompatibility criterion. Exact bounds for the degree in a particular case and for the number of polynomials are given. The "Helly dimension" of the set of all hypersurfaces of degree d in n-space is shown to be less than or equal to $\binom{n+d}{n}-1$, with equality for an infinite coordinate field.

Related work is reported under various other tasks. Problems in differential geometry and in the calculus of variations gave rise to some problems involving differential equations reported under task 1101-10-5100/51-1, (p. 3). Discrete variable problems are reported under 1101-10-5100/53-1, (p. 7), 1101-40-5131/54-1 (p.17), and 1101-40-5130/54-1 (p.10).

Publications: (1) "A proof of Hilbert's Nullstellensatz," by T. S. Motzkin; submitted to a technical journal. (2) "An isoperimetric inequality for closed curves convex in even-dimensional Euclidean space," by I. J. Schoenberg; submitted to a technical journal.

COMPUTATION OF THE COMPLEX ZEROS OF THE RIEMANN-ZETA FUNCTION
Task 1101-10-5100/50-13
(formerly 1101-11-5101/50-13)

Origin: NBS Authorized 6/1/50
Sponsor: Office of Naval Research, USN
Manager: R. Horgan
Full task description appears in Apr-Jun 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1953 issue.

STUDIES IN THE NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS
Task 1101-10-5100/51-1
(formerly 1101-11-5100/51-1)

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: W. Wasow
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. The problem of embedding an intrinsically defined surface isometrically into three-dimensional space leads to an initial value problem for a non-linear system of three partial differential equations. C. B. Tompkins has proposed a finite difference method for the step-by-step solution of this problem. A preliminary study by G. Blanch of the truncation and round-off errors inherent in this method in the case

of the sphere indicates that the procedure is practicable. It is planned to prepare a code and to perform exploratory calculations on the SWAC.

Several techniques for the numerical solution of Plateau's problem on the SWAC are under study. The emphasis is on the parametric case.

For use in the setting-up of difference equations or equivalent random walks, finite neighbor sets with certain properties were investigated by T. S. Motzkin (see publication 5 below). The main properties considered are (1) every point P of a given n-dimensional lattice has s neighbors Q, s fixed; (2) Q-P is a fundamental vector of the lattice; (3) a finite number of steps leads from P to every lattice point; (4) the notion of neighbor is invariant under an n-dimensional discrete translation group. Examples are given to show that (1)-(4) can be fulfilled for any $s \geqslant 3$ and $s \leqslant 2n$. Also, a set of neighbors fulfilling (1), (2), (4) with $s=n+1$ is described for which the neighbors of P form a regular simplex with center P.

The asymptotic properties for large λ of differential equations of the form

$$y^{(n)} + \sum_{r=1}^{n} a_r y^{(n-r)} + \sum_{s=0}^{m} (b_s x + c_s) y^{(m-s)} = 0$$

(a_r, b_s, c_s constants) is being studied by L. Philipson. Particular emphasis is given to the Stokes phenomena in the complex x-plane.

The methods of Gershgorin and Collatz for the appraisal of the truncation error in the solution of Dirichlet's problem by finite differences has been generalized by W. Wasow so as to apply to general types of nets and to a general class of interpolation schemes, and also to the "discretization error" connected with solutions of Dirichlet's problem by random walks that are not restricted to the points of a net. With the help of the results obtained the extrapolation technique of J. F. Richardson for the improvement of approximations obtained by finite difference methods can be analyzed mathematically, justifying it in certain applications, and showing in others that it is not reliable.

Participants in the research on this project were G. Blanch, T. S. Motzkin, L. Philipson, C. B. Tompkins, W. Wasow.

Publications: (1) "Asymptotic solution of the differential equation of hydrodynamic stability in a domain containing a transition point," by W. Wasow; Ann. Math. 58, 222-252(Sept. 1953). (2) "An expansion method for parabolic partial differential equations," by J. W. Green; J. Res. NBS 51, 127-132(Sept. 1953). (3) "On small disturbances of plane Couette flow," by W. Wasow; J. Res. NBS 51, 195-202(Oct. 1953). (4) "On the convergence of asymptotic solutions of linear differential equations," by R. M. Redheffer and W. Wasow; submitted to a technical journal. (5) "Neighbor sets for random walks and difference equations," by T. S. Motzkin; IN MANUSCRIPT.

PROBABILITY METHODS AND SAMPLING TECHNIQUES
Task 1101-10-5100/51-2
(formerly 1101-11-5100/51-2)

Origin: NBS — Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: D. Teichroew
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. The asymptotic expansion for expressing t- and chi-square variates in terms of normal deviates with the same probability value has been generalized by the use of an auxiliary parameter. The

generalized expansion permits approximation with greater accuracy than was possible before. A code has been prepared to compute the necessary coefficients.

The SWAC has been used to compute two tables of the Incomplete Gamma Function

$$p = \frac{1}{\Gamma(a)} \int_0^{x_p} e^{-t}\, t^{a-1}\, dt .$$

The first table gives p to 10 decimals for a=1(1)20 and the second gives x_p for p=.001(.001).999 for selected values of a between 5 and 100.

Initial runs on the SWAC have been made for determining the Power Function and the Expected Sample Size for Sequential Tests of Linear Hypotheses. The Efficiency of this procedure is being compared to that of the classical fixed sample size approach for the case of Analysis of Variance.

M. Muller is investigating procedures for improving the efficiency of the Monte Carlo method for solving boundary value problems.

A SWAC code was prepared by G. Forsythe for the solution of a certain Dirichlet problem in a square by random walks.

Two new procedures for generating random numbers were outlined by T. S. Motzkin.

Participants in this task research were G. Forsythe, T. Motzkin, M. Muller, D. Teichroew, and W. Wasow. Some of the coding was done by L. Forthal.

Publications: "On the asymptotic transformation of certain distributions into the normal distribution," by W. Wasow; to appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 26-28, 1953.

VARIATIONAL METHODS
Task 1101-10-5100/51-3

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: M. R. Hestenes
Full task description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue.

STUDIES IN APPLIED MATHEMATICS
Task 1101-10-5100/51-4

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: T. H. Southard
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. The seminar on numerical analysis met 22 times during the quarter. A detailed list of sessions will be found under Lectures and Symposia in the back of this issue. The third summary of presentations, covering the period from June 15 to August 13, was prepared by T. S. Motzkin.

Status of Projects

As a preliminary to coding of numerically or analytically given functions, T. S. Motzkin investigated the efficiency of operational sequences, measured by the number k of parameters of the family obtained. It was shown that for a sequence of n two-entry operations O_i of any kind other than addition and an unlimited number of additions, $k \leqslant 2n+1$. If all O_i are divisions, the algorithm of recurrent fractions gives, even for one variable, $k=2n+1$; the same is true for partial fractions, with a limitation on the resulting family of rational functions by the reality of the roots of the denominator. If all O_i are multiplications, the bound for one variable is $k=2n$, an improvement on the Newton-Horner synthetic division procedure. If a number is "forgotten" when used, the bound for m multiplications and n divisions is $2(m+n)-[(m-1)/(n+1)]$, where [ ] denotes the largest integer function. The determination, by Tchebyshev methods, of the best coefficients to be used for these approximating rational functions was also studied.

E. Levin did some work on the Circle Theorem of Hydrodynamics. One of the hypotheses of the Circle Theorem was that there be no rigid boundaries in the plane. This restriction may be somewhat relaxed and one can obtain a necessary and sufficient condition for admissible rigid boundaries.

Publications: (1) "Numerical computation of low moments of order statistics from a normal population," by J. B. Rosser; submitted to a technical journal. (2) "Changes of sign of sums of random variables," by P. Erdös and G. Hunt; Pac. J. Math. 3, 673-687 (Dec. 1953). (3) "A numerical analyst's fifteen-foot shelf," by G. E. Forsythe; MTAC VII, 221-228 (Oct. 1953). (4) "Seminar on numerical analysis--summary of presentations between April 20 and May 13, 1953," by C. B. Tompkins; multilithed typescript, 10 p. (5) "Seminar on numerical analysis--summary of presentations between May 18 and June 8, 1953," by G. E. Forsythe; multilithed typescript, 38 p. (6) "Third Summary--Seminar on Numerical Analysis, June 15-August 13, 1953," by T. S. Motzkin; multilithed typescript, 38 p. (7) "Note on the circle theorem of hydrodynamics," by E. Levin; submitted to a technical journal.

MISCELLANEOUS STUDIES IN THEORETICAL PHYSICS
Task 1101-10-5100/51-5

Origin: Office of Naval Research, USN — Authorized 9/1/50
Sponsor: " "
Manager: R. D. Woods
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. W. Futterman is concluding theoretical and numerical work on calculations of the photo-disintegration of the deuteron. R. Woods is continuing programming and preliminary computations of the elastic scattering of protons by various elements, based on the optical model of the nucleus.

Publications: (1) "Modes of vibrations of a suspended chain," by D. S. Saxon and A. S. Cahn; Qu. J. Mech. App. Math. (Oxford) VI, Pt. 3, 273-285(1953). (2) "Application of variational methods to intermediate and high energy scattering," by E. Gerjuoy and D. S. Saxon; submitted to a technical journal. (3) "Variational principles for the acoustic field," by E. Gerjuoy and D. S. Saxon; submitted to a technical journal.

STUDY OF RUSSIAN MATHEMATICAL PROGRESS
Task 1101-10-5100/52-1

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: G. E. Forsythe
Authorized 3/15/52
Full task description appears in Jan-Mar 1952 issue.

Status: CONTINUED. The task manager is continuing to accumulate bibliographical cards on Russian mathematical monographs (see Jan-Mar 1952 issue, p. 11, and Oct-Dec 1952 issue, p. 8). The translations formerly reported under this task are now reported in connection with the pertinent tasks of the Institute for Numerical Analysis.

The following report has been re-run and is available in limited quantities: "Bibliographical survey of Russian mathematical monographs, 1930 to 1951." A supplement to the above is also available.

GENERALIZED RAYLEIGH-RITZ METHOD FOR EIGENVALUES OF A CLAMPED PLATE
Task 1101-10-5100/53-1

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: C. B. Tompkins
Authorized 12/29/52
Terminated 12/31/53

Objective: To test the method developed by N. Aronszajn, by use of the theory of Hilbert space, for eigenvalue problems. Specifically, let there be given a square plate, clamped at the edges, and defined by $|x| \leqslant 1$; $|y| \leqslant 1$. Let u be a function satisfying

(1) $$\Delta^2 u = \mu u; \quad u = \frac{\partial u}{\partial n} = 0, \quad \text{on the boundary.}$$

The objective is to find approximations to the eigenvalues μ for which (1) has nontrivial solutions. In the above Δ is the Laplacian operator so that

$$\Delta^2 u = \frac{\partial^4 u}{\partial x^4} + \frac{\partial^4 u}{\partial y^4} .$$

Background: Upper bounds for the eigenvalues μ can be obtained by the Rayleigh-Ritz method, which involves the solution of a system of algebraic equations. The disadvantage of the method is that at any stage upper bounds are obtained only for the first n eigenvalues, where n is the order of the determinant associated with the approximation. In the present study, an attempt will be made to apply Aronszajn's generalized Rayleigh-Ritz method for differential problems. The advantage of the method is that at every stage upper bounds can be obtained for all eigenvalues. Moreover, it is hoped that at the n-th stage, the approximation obtained from the generalized method is better than the corresponding approximation obtainable from the n-th stage of the ordinary Rayleigh-Ritz method. However, the generalized method involves a computing process which is considerably more involved than the ordinary method, and one of the prime objects of the study will be to test whether programming of the process for high-speed equipment is feasible. The method involves using as auxiliary functions the known solutions for the clamped circular plate (for the circle inscribed in the square).

Status: TERMINATED.

RESEARCH IN THE MATHEMATICAL THEORY OF PROGRAM PLANNING
Task 1101-10-5102/50-11
(formerly 11.1/1-50-11)

Origin: Office of Air Comptroller, USAF Authorized 6/15/50
Sponsor: " "
Manager: T. S. Motzkin
Full task description appears in Apr-June 1950 issue.

Status: CONTINUED. The fundamental problem of the existence of a solution of a problem of linear programming or linear inequalities was investigated from a probabilistic point of view. Concerning a system of n linear inequalities in m unknowns, it is shown, under rather general suppositions concerning the probability distribution of the coefficients, that the probability p for compatibility is

$$\left(\binom{n+1}{m} + \binom{n+1}{m-2} + \binom{n+1}{m-4} + \ldots\right) \Big/ 2^n.$$

It follows readily that $p=1/2$ as $n=2m+1$.

The analogy was studied between the theory of linear inequalities (in particular, theorems on implication, incompatibility, duality, and Helly dimension) and those of linear diaphantine equations and of algebraic equations (see task 1101-10-5100/50-4, p. 3).

Continued attention was given to the assignment problem (see July-Sept 1953 issue, p. 7-8) and its ramifications.

Progress was made in the write-up of the Bibliography on Linear Inequalities and Programming, and of the chapter on linear inequalities for the Handbook of Numerical Analysis.

Reference is made to related work of R. Horgan and C. B. Tompkins reported under task 1101-40-5130/54-1, p. 10.

Publications: Same as given in July-Sept 1953 issue, p. 9-10.

SCAMP
Task 1101-10-5150/53-1

Origin: Office of Naval Research, USN Authorized 6/10/53
Sponsor: " "
Manager: C. B. Tompkins
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. Research concerning problems with discrete variables continued. Attention was still directed toward problems in which variables are permutations. M. Hall, Jr., and J. D. Swift made suggestions which have led to a variation in the code which will probably permit determination of all Steiner triples of order 15 without inacceptable repetition of isomorphs. A study of results of a problem of exhaustive search for solutions of non-linear equations involving permutations has been made and a report will be submitted to the sponsor. F. B. Meek, J. D. Swift, and C. B. Tompkins have initiated a systematic search for a pair of orthogonal Latin Squares of order 10. Computations for SCAMP are reported as task 1101-40-5131/54-1, p. 17.

Prospective participants in the more active phase of the program during the summer of 1954 have received letters requesting them to establish clearance and to take care of other formalities involved in their participation.

M. Newman has completed and checked a SEAC code for the com-

putation of the Hermite normal form of a matrix with rational integral elements, of order not exceeding 200. Several small matrices have been run, and the calculation of the Hermite normal form of a 91x91 incidence matrix for a non-Desarguian finite plane projective geometry of order 9 is in progress.

2. Development

NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER (SWAC)
Task 1101-20-5103/49-1
(formerly 1101-34-5103/49-1)

Origin: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF — Authorized 11/1/48
Sponsor: " " "
Full task description appears in Apr-June 1949 issue.

Status: CONTINUED. The SWAC has been used on a two-shift basis on both service and research problems. In recent weeks approximately 55 hours out of an available 60 have been used for productive computing. Approximately three hours per day are used for preventative maintenance.

There have been no engineers on the staff at the Institute since November 1; all maintenance is being done by contract with Magnavox Company.

LOGICAL NOTATION AND BLOCK DIAGRAM SYMBOLISM FOR A.D.C.M.
Task 1101-20-5103/49-2
(formerly 11.1/22-49-2)

Origin: NBS — Authorized 2/15/49
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF — Terminated 12/31/53
Manager: H. D. Huskey

Objective: To establish a suitable and consistent terminology, notation, and symbolism for the automatic computing machine (SWAC) being constructed under task 1101-20-5103/49-1 (formerly 11.1/22-49-1), with a view to developing thereby an acceptance proposal for a standard language for describing logical aspects of a.d.c.m. in general.

Background: Because of the diversity of groups working on computing machines a number of different sets of conventions regarding terminology and symbolism are currently in use. Criticism has developed among the mathematicians of the Institute for Numerical Analysis regarding the consistency, compactness, and suitability of some of the systems of notation and symbolism now in use. It has seemed to be definitely worthwhile from the viewpoint of the research program of the Institute to develop a notation and symbolism for the Institute machine which will be acceptable to the research staff, and it is hoped that if this task is accomplished

thoughtfully a standard of wide applicability may result.

Status: TERMINATED.

STUDIES IN THE THEORY OF DIGITAL COMPUTING MACHINES
Task 1101-20-5103/53-1

Origin: NBS Authorized 9/30/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: H. D. Huskey
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. New systems of coding are being studied. A floating point card controlled method of using the SWAC was perfected by Melkanoff of UCLA. Elementary automatic programming of formulae was started and more elaborate interpretive routines are being developed by Huskey.

A simple foundation of automatic coding, outlined by T. S.Motzkin, involves the standardized use of closing brackets as evaluation command.

Techniques for expediting code checking are being studied by several staff members.

3. Mathematical Services

COMPUTING SERVICES FOR RESEARCH STAFF OF THE INSTITUTE FOR NUMERICAL ANALYSIS
Task 1101-40-5130/54-1
(formerly 1101-40-1111/49-1a)

Origin: NBS Authorized 9/2/48
Sponsor: Office of Naval Research, USN Revised 11/16/49
Manager: F. Hollander
Full task description appears in July-Sept 1949 issue (see task 11.1/32-49-1).

Status: CONTINUED. Research staff problems involving the use of SWAC were:

(1) For D. H. Lehmer and J. Selfridge: (a) Runs were made on several Mersenne primes of large size to obtain the penultimate remainders of the numbers. The routine was a modification of the one originally coded to determine primality of numbers of form $2^{24}-1$ by a theorem of Lucas. (b) A difference set of order 127 was checked. (c) All solutions were found for: $a^{16}-b^{16}\equiv 1 \pmod p$, $a^{24}-b^{24}\equiv 1 \pmod p$ where $p=nk+1$, n is the exponent, and k is an arbitrary integer equal to 1,2,..., for all primes less than 1000. (d) Certain prime numbers were tested by Alway's method. (e) A backlog problem has been set up which consists of tests of irregular primes in the investigation of Fermat's last theorem.

(2) For R. Woods: Code checking and calculations were continued on an optical model for calculating the scattering of protons by heavy nuclei.

(3) For the Geophysics Department of U.C.L.A.: Data processing was begun on the problem of the high order tidal constituents for earth and ocean tides.

(4) For the Physics Department of U.C.L.A.: (a) Computation has been started on the solution of the molecular wave function for the Li_2 molecule. (b) Computations have been done to obtain the eigenvalues of a system of non-linear differential equations in a problem of nuclear scattering. A method involving a "corrector-predictor" was used. (See MTAC VI, 253-254, Oct. 1952.) Coding was done by P. Kaus and B. F. Handy.

(5) For the Education Department of U.C.L.A.: Computations were continued on item analysis of certain psychological instruments.

(6) For the Business Administration Department of U.C.L.A.: Determination of working capital needs by stochastic methods, on various simplified models of business organization.

(7) For the Chemistry Department of U.C.L.A.: Calculations in the problem of crystal structure, involving a three-dimensional Fourier analysis, and calculations on Fourier transforms.

(8) For D. Teichroew: Tables of the incomplete Γ-function were calculated.

(9) For T. Thomas: Several algebraic and exponential equations were solved.

(10) For C. B. Tompkins: (a) Two large sets of linear inequalities derived from a linear programming problem have been attacked by cyclic projections with acceleration. A stable cycle indicating no possible solution was found for one and a cycle probably stable within roundoff error was found for the other. These sets each involved 128 inequalities in 80 unknowns, and the solution was coded for SWAC by R. B. Horgan. (b) Solution of games has been attempted in which the payoff matrix has a parameter which must be set to make the value of the game zero. It is hoped to solve games up to ten by ten in size derived from studies of attrition processes. The method is the method of fictitious play due to Brown and von Neumann accelerated by a scheme of cyclic under-relaxed projections.

(11) For W. Futterman: Code checking was begun on a routine to calculate the matrix elements occurring in the photoelectric disintegration of the deuteron.

PUNCHED CARD LIBRARY
Task 1101-40-5131/49-2
(formerly 1101-53-1101/49-2)

Origin: NBS Authorized 7/14/43
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: F. Hollander
Full task description appears in Apr-June 1949 issue.

Comments: A catalog of tables on punched cards which are on file at the Institute may be obtained by addressing the Institute for Numerical Analysis, 405 Hilgard Avenue, Los Angeles 24, California. Within the limits of the program of the computation unit of the Institute, tables will be duplicated upon request, provided the requester furnishes the blank cards. Requests should be addressed directly to the Institute.

Status: CONTINUED. Requests were received for tables of Bessel functions $Y_0(x)$, $Y_1(x)$ and $K_0(x)$, $K_1(x)$, δ^2, from Stanford University; and

for $E_2(x) = \int_1^{\infty} (e^{-xu}/u^2)du$ from Iowa State. These were reproduced from our files and shipped.

The table of the volume under the bivariate normal surface and auxiliary tables relating to this function were added to the library. The functions tabulated are:

(1) $$L(h,k,r) = \frac{1}{2\pi\sqrt{1-r^2}} \int_h^{\infty} \int_k^{\infty} \exp\left\{-\frac{1}{2(1-r^2)}(x^2+y^2-2rxy)\right\} dxdy$$

for h=0(.1)4.0, k=0(.1)h, and r=0(.05).95(.01)1.00; values are given to at least 6D, for positive and negative values of r.

(2) $$V(h,k) = \frac{1}{2\pi} \int_0^h dx \int_0^{\frac{k}{h}x} e^{-\frac{1}{2}(x^2+y^2)} dy,$$

and

(3) V(k,h), where $k/h = \lambda$, for h=0(.01)4.0, λ=0.1(.1)1.0.

STATISTICAL SMOOTHING
Task 1101-40-5131/51-19
(formerly 1101-53-1101/51-19)

Origin: Stanford Research Institute, Stanford University — Authorized 1/15/51
Sponsor: Office of Research Operations, U.S.Army — Completed 12/31/53
Manager: N. Reynolds

Objective: To perform the statistical smoothing of empirical two-dimensional data.

Status: COMPLETED. Three cases were completed this quarter.

TABLES OF THE BIVARIATE NORMAL DISTRIBUTION FUNCTION
Task 1101-40-5131/51-32
(formerly 1101-53-1101/51-32)

Origin: Division 13, NBS — Authorized 5/31/51
Sponsor: Office of Chief of Ordnance, U.S. Army — Completed 12/31/53
Manager: C. B. Tompkins

Objective: To compile a table of the bivariate normal distribution function, defined by

$$L(h,k,r) = \frac{1}{2\pi\sqrt{1-r^2}} \int_h^{\infty} \int_k^{\infty} \exp\left[\frac{-(x^2+y^2-2rxy)}{2(1-r^2)}\right] dx\, dy.$$

Background: The function L(h,k,r) has been tabulated and published in Karl Pearson's Tables for Statisticians and Biometricians, part II,(1931) p. 78-137, Biometric Laboratory, University College, London, for the following range of parameters: $\pm$r=0(.05)1; h,k=0(.1)2.6. This table has

been extended by Evelyn Fix to cover a range of h and k up to 4, for the same range of r. This extension is unpublished, but available in typewritten form at the University of California Statistical Laboratory, Berkeley. The major portion of the compilation will comprise these two basic tables. They will be unified and perhaps augmented by suitable auxiliary tables to increase their usefulness. Also the values will be keypunched, so as to be useful in conjunction with IBM computations.

Status: COMPLETED. The tables have been completed and some distribution in preliminary draft and by means of punched cards has been completed. Dr. Gertrude Blanch was the chief participant in the accomplishment of this task.

SIMPLIFIED ROLLING PULLOUT EQUATIONS
Task 1101-40-5131/51-34
(formerly 1101-53-1101/51-34)

Origin: Cornell Aeronautical Laboratory Authorized 6/22/51
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF
Manager: F. Hollander
Full task description appears in Apr-June 1951 issue.

Status: CONTINUED. Six sets of observed data were received, and modifications of the code are being made to integrate the equations using this information. Some computation on the first set of data has been done.

LOW MOMENTS OF ORDER STATISTICS
Task 1101-40-5131/51-36
(formerly 1101-53-1101/51-36)

Origin: University of Oregon Authorized 6/22/51
Sponsor: Office of Naval Research, USN
Manager: H. D. Huskey
Full task description appears in Apr-June 1951 issue.

Status: CONTINUED. The routine for the integral $(1/\sqrt{2\pi})\int_{-\infty}^{x} e^{-t^2/2}dt$ [x = -12.00(.02)12.00] has been coded by S. Marks and the code has been checked out. Conversion routines have been coded by B. F. Handy, Jr.

SIERRA WAVE PROJECT
Task 1101-40-5131/52-36
(formerly 1101-53-1101/52-36)

Origin: Department of Meteorology, U.C.L.A. Authorized 4/1/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: T. H. Southard
Full task description appears in Jan-Mar 1952 issue.

Status: CONTINUED. A code is being developed for the reduction on SWAC of theodolite data for several more flights.

EIGENVALUES
Task 1101-40-5131/53-6

Origin: Consolidated Vultee Aircraft Corporation (Convair) Authorized 9/26/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: F. Hollander
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. The 45x45 matrix, whose eigenvalues and eigenvectors were determined previously was inverted after deletion of two rows and columns to remove two singularities. An approximation to the inverse, with an accuracy of four decimals, was forwarded to the proposer of the problem.

METEOROLOGICAL MEANS
Task 1101-40-5131/53-10

Origin: Meteorology Department, UCLA Authorized 9/30/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: F. H. Hollander
Full task description appears in July-Sept 1952 issue. (See also project 11.1/31-50-17, Jan-Mar 1950 issue, p. 15.)

Status: CONTINUED. All computations for the 850 mb level have been completed. Coding for the "Mountain Term" is finished and checked, and some computing has been done. ("Mountain Term" represents the equations used to compute the pressure differences across mountain ranges, where these intersect a given plane of elevation above sea level, at a given latitude.)

Coding for and computation of the time averages of the "Mountain Term" remain to be done. Vertical integrations of these time averages, as well as of the pressure differences for each day and latitude, are also not yet computed. The 100 mb level for Jan-Feb 1949 is still to be done.

Computation on this problem has been stopped until further funds are available.

DISCRIMINANT FUNCTIONS
Task 1101-40-5131/53-13

Origin: School of Aviation Medicine, Randolph Field, USAF Authorized 12/15/52
Sponsor: "
Manager: D. Teichroew
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED. Preliminary results are being examined.

BIO-ASSAY PROBLEM
Task 1101-40-5131/53-24

Origin: Stanford University — Authorized 3/31/53
Sponsor: Office of Naval Research, USN
Manager: D. Teichroew
Full task description appears in Jan-Mar 1953 issue.

Status: INACTIVE. Results are being examined by the contractor.

SINGLE SHOT PROBABILITIES
Task 1101-40-5131/53-25

Origin: Naval Air Missile Test Center, Point Mugu — Authorized 3/31/53
Sponsor: Bureau of Aeronautics, USN — Terminated 12/31/53
Manager: D. Teichroew

Objective: To investigate the distribution of random variables in populations in which parameters themselves have a probability distribution and in particular to determine the effect of such distributions on single shot hit probabilities.

Background: In testing a hypothesis by means of a sample of individuals one usually assumes that the individuals are a random sample from a polulation with fixed values for its parameters. Sometimes it is more realistic to assume that the values of the parameters may be different for the different individuals, in particular, that in the whole population the parameter values have a certain given distribution. For example, it has been found that accident frequencies sometimes resemble a negative binomial distribution more closely than they do a Poisson distribution; the negative binomial distribution would be expected if one assumed that the number of accidents happening to one person is a Poisson variate and that over a number of individuals the mean has a Gamma distribution.

Status: TERMINATED.

DISCRETE MINIMAL SPACES
Task 1101-40-5131/53-26

Origin: Gilfillan Brothers, Inc. — Authorized 3/31/53
Sponsor: Evans Signal Laboratories, Army Signal Corps
Manager: H. D. Huskey
Full task description appears in Jan-Mar 1953 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue.

ROOTS OF DETERMINANTS
Task 1101-40-5131/53-32

Origin: North American Aviation Company — Authorized 3/31/53
Sponsor: Atomic Energy Commission — Completed 12/31/53
Manager: H. D. Huskey

Objective: To determine the value of a variable such that a determinant whose elements are functions of this variable should vanish.

Background: This problem arises in connection with the work of the AEC group at North American Aviation Company. Several sets of these determinants are to be submitted for solution.

Status: COMPLETED.

LINEAR EQUATIONS (CONVAIR)
Task 1101-40-5131/53-35

Origin: Consolidated Vultee Aircraft Corporation — Authorized 3/31/53
Sponsor: Bureau of Aeronautics, USN
Manager: F. Hollander
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. The double precision code was completed but has not been checked out. The system of equations was withdrawn by the originator of the problem. A new set of equations will be submitted for solution.

MONTE CARLO STUDIES
Task 1101-40-5131/53-39

Origin: RAND Corporation — Authorized 3/31/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: H. D. Huskey
Full task description appears in Jan-Mar 1953 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue.

RANKING PROBABILITIES
Task 1101-40-5131/53-40

Origin: Cornell University — Authorized 3/31/53
Sponsor: Office of Naval Research, USN
Manager: D. Teichroew
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. A final report is being prepared.

ACCEPTANCE TESTS
Task 1101-40-5131/53-41

Origin: Jet Propulsion Laboratory, California Institute of Technology — Authorized 6/29/53
Sponsor: Ordnance Corps, U. S. Army
Manager: H. D. Huskey
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. E. C. Yowell prepared a report on acceptance testing entitled "Recommendations for an Acceptance Test for a Consolidated Engineering Corporation Model 30-201 Computer." L. Forthal of INA and M. Carr of JPL have begun coding the procedure.

B. P. A. Studies
Task 1101-40-5131/53-42

Origin: Bonneville Power Administration — Authorized 6/29/53
Sponsor: " "
Manager: R. R. Reynolds
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. Reports 1, 3, and 5, by R. J. Cypser, P. R. Johannessen, and J. J. Carey of the Economy Loading Research Group, Massachusetts Institute of Technology, were reviewed. A report was submitted to the sponsor.

HELICOPTER STABILITY STUDIES
Task 1101-40-5131/53-44

Origin: J. B. Rea Company — Authorized 6/30/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: G. E. Forsythe
Full task description appears in July-Sept 1953 issue.

Status: CONTINUED. The first phase of the problem is to compute two polynomials in λ of type det $(\lambda F+G)$. Here F, G are of orders 9 or 10, real, non-symmetric. Since F is singular, we are going to compute $\det(G^{-1}F+\lambda^{-1}I)$ as a polynomial in λ^{-1}. Let $A=G^{-1}F$. It is hoped to find the characteristic polynomial of A by elimination from a sequence of vectors $x, Ax, A^2x,\ldots,A^nx$, where n is the order of A.

Participants in the solution of this problem are G. E. Forsythe, F. H. Hollander, Louise Straus.

COMPUTATIONS FOR SCAMP
Task 1101-40-5131/54-1

Origin: Office of Naval Research, USN — Authorized 9/29/53
Sponsor: Office of Naval Research, USN
Manager: C. B. Tompkins
Full task description appears in July-Sept 1953 issue.

Status: CONTINUED. E. Lehmer and M. Hall, Jr., studied the use of multipliers in seeking difference sets. E. Lehmer coded the problems and ran several cases of interest; no new difference sets were found, but the possibilities were exhausted in some cases where there was doubt as to whether a set exists. L. E. Rickard computed some probability tables upon which rough estimates of the extent of some problems involving search through all permutations can be based. The code of L. E. Rickard for the computation of Steiner triples was modified to decrease the number of isomorphic outputs, and computation of Steiner triples of order 15 was undertaken by R. B. Horgan and J. D. Swift. A problem involving exhaustive search through permutations was solved by C. B. Tompkins with special attention being paid to experience concerning the amount of data needed and the size of batches of permutations which could be rejected from competition on the basis of one piece of information. B. F. Handy, Jr., coded a continuous variable approach to a problem closely related to the one computed by C. B. Tompkins, but experience with computing showed that time requirements were excessive and a new code is being prepared.

CRUISE CONTROLLER
Task 1101-40-5131/54-3

Origin: J. B. Rea Company — Authorized 9/29/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: F. Hollander
Full task description appears in July-Sept 1953 issue.

Status: INACTIVE. The originator is continuing analysis of the problem.

RAM ROCKET TRAJECTORIES - PARAMETER STUDY
Task 1101-40-5131/54-4

Origin: Marquardt Aircraft Company — Authorized 9/29/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: F. H. Hollander

Objective: To determine engine characteristics for optimum performance over a wide range of trajectory variables. The analysis and coding is to be performed by the originator. INA will supply SWAC time and coding consultation as is necessary.

Status: COMPLETED. All work was performed by Marquardt Aircraft Company employees, except for necessary coding assistance and operating assistance on SWAC.

LINEAR EQUATIONS (BERKELEY)
Task 1101-40-5131/54-6

Origin: Statistical Laboratory, University of California, Berkeley — Authorized 12/29/53
Manager: H. D. Huskey

Objective: To utilize existing codes to solve sets of linear equations.

Background: Sixteen sets of 10 linear equations in 10 unknowns, and 16 sets of 11 linear equations in 11 unknowns, were submitted for solution on the SWAC.

Status: COMPLETED (NEW). The equations were solved using a routine coded by J. Pettit of the Geophysics Department of U.C.L.A. This code operates in floating binary notation, and utilizes a method of direct elimination of the unknowns. Computing time was approximately three hours.

II. COMPUTATION LABORATORY

(Section 11.2)

1. Research

RESEARCH IN CLASSICAL NUMERICAL ANALYSIS
Task 1102-10-1104/50-1
(formerly 11.2/11-50-1)

Origin: NBS — Authorized 1/1/50
Managers: J. Todd, M. Abramowitz, and H. A. Antosiewicz
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. H. Antosiewicz investigated the boundedness of every solution of equations of the type $\ddot{x}+\emptyset(x,\dot{x})\dot{x}+h(x)=e(t)$ whenever the forcing term e(t) is integrable over the entire positive half line.

M. Abramowitz and W. F. Cahill are continuing their work on the vibration of plates. A preliminary report on the use of difference equations is being prepared.

M. Abramowitz prepared a paper on the practical evaluation of integrals.

In continuation of earlier work of Gohner (Ing. Arch. 1, 619 (1930)) and Freiberger (Australian J. Sc. Res. {A} 2, 354 (1949)), P. Henrici investigated the stress distribution in a helical spring of finite circular cross-section. It is shown that Freiberger's solution of the problem can be developed in terms of powers of the spring index. The first three terms of this power series reproduces Gohner's earlier approximation.

Publications: (1) "On a certain integral involving Bessel functions" by H. A. Antosiewicz; submitted to a technical journal. (2) "Forced heat convection in laminar flow through a tube" by M. Abramowitz; IN MANUSCRIPT. (3) "On helical springs of finite thickness" by P. Henrici; IN MANUSCRIPT. (4) "Computation of vibration modes and frequencies on SEAC" by W. Cahill and S. Levy; IN MANUSCRIPT. (5) "On a problem in the theory of mechanical quadratures" by P. Davis; accepted by the Pacific Journal of Mathematics. (6) "On the estimation of quadrature errors for analytic functions" by P. Davis and P. Rabinowitz; submitted to a technical journal. (7) "The practical evaluation of integrals" by M. Abramowitz; IN MANUSCRIPT.

RESEARCH IN MODERN NUMERICAL ANALYSIS: INVESTIGATION OF BERGMAN'S METHOD FOR THE SOLUTION OF THE DIRICHLET PROBLEM FOR CERTAIN MULTIPLY CONNECTED DOMAINS
Task 1102-10-1104/50-2
(formerly 11.2/11-50-2)

Origin: NBS — Authorized 3/1/50
Manager: P. Davis
Full task description appears in Jan-Mar 1950 issue.

Status: INACTIVE. For status to date see Jan-Mar 1952 issue, p. 35.

MISCELLANEOUS STUDIES IN PURE MATHEMATICS
Task 1102-10-1104/50-4
(formerly 11.2/11-50-4)

Origin: NBS Authorized 1/1/50
Managers: O. Taussky-Todd, J. Todd, M. Abramowitz, and A. Hoffman
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. O. Taussky-Todd prepared a written report of her talk at the Santa Monica Symposium 1953 (see item 1 under publications below).

The authors of publication (2) have extended their work to include a comparison of the number of absolute points of a correlation ρ with the number of absolute points of ρ^k, where k is prime to the order of ρ.

O. Taussky-Todd is studying the significance of normal matrices in certain aspects of algebraic number theory where only symmetric matrices had been considered previously.

Publications: (1) "Some computational problems in algebraic number theory" by O. Taussky-Todd; submitted for inclusion in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953. (2) "The number of absolute points of a correlation" by A. Hoffman, M. Newman, E. Straus, O. Taussky; IN MANUSCRIPT. (3) "The representation of integers by binary quadratic rational forms" by K. Goldberg, M. Newman, E. Straus, and D. Swift; accepted by Archiv der Mathematik. (4) "Ueber die Funktionen von Gegenbauer" by P. Henrici; accepted by Archiv der Mathematik. (5) "On certain series expansions involving Whittaker functions and Jacobi polynomials" by P. Henrici; submitted to a technical journal. (6) "Linear functional equations and interpolation series" by P. Davis; submitted to a technical journal.

NUMBER THEORETICAL TEST PROBLEMS FOR SEAC
Task 1102-10-1104/50-5a
(formerly 11.2/11-50-5)

Origin: NBS Authorized 6/1/50
Managers: O. Taussky-Todd and K. Goldberg
Full task description appears in Apr-Jun 1950 issue.

Status: CONTINUED. The routine to determine the primes $p \leq 100{,}000$ for which $2^{p-1}-1$ is divisible by p^2 is being run as time is available and is now extended to $p<75{,}000$ with no positive results other than the well-known cases $p=1093$ and $p=3511$.

The routine to determine certain congruence properties of the fundamental unit $t+u\sqrt{p}$ in the field $R(\sqrt{p})$ where p is a prime has been used to verify the conjecture that $u \not\equiv 0 \pmod{p}$ for all primes of the form $4n+1$ less than 100,000. (See N. C. Ankeny, E. Artin, S. Chowla, "The class number of real quadratic fields," Ann. Math. 59, 480, 1952.)

The table of least positive primitive roots is being extended as time is available.

SOLUTION OF LAPLACE EQUATION BY MONTE CARLO METHOD
Task 1102-10-1104/51-6

Origin: NBS — Authorized 9/28/50
Manager: M. Abramowitz
Full task description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see Apr-June 1952 issue, p.34.

THREE-BODY PROBLEM
Task 1102-21-1104/52-4

Origin: NBS — Authorized 10/5/51
Manager: A. Goldstein — Terminated 12/31/53

Objective: To develop and test a machine method for calculations of particular orbits in the restricted planar three-body problem.

Background: The "restricted" three-body problem is that of the motion of an infinitesimally small body in the gravitational field of two bodies of finite size. It has long been known that the problem cannot be solved in closed form; therefore, numerical integration of the differential equations of motion is appropriate. Among the many applications of the problem is that to a missile moving under the influence of earth and moon; particular orbits of interest are, for instance, (1) orbits starting at the earth and ending at the moon, or (2) stable orbits remaining between earth and moon. In this project some orbits of type (1) will be tabulated.

Status: TERMINATED.

ANALYSIS OF GEOMAGNETIC FIELD
Task 1102-10-1104/52-8

Origin: NBS — Authorized 8/10/51
Manager: C. J. Swift
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see Oct-Dec 1951 issue, p. 29.

SPECIAL PROBLEMS IN FINITE MATRIX THEORY
Task 1102-10-1104/52-34

Origin: NBS — Authorized 11/6/51
Manager: O. Taussky-Todd
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. H. Wielandt prepared two reports on his work concerning (a) the location of the eigenvalues of A+B, where A and B are normal matrices with known spectra; and (b) a maximum-minimum characteristic of sums of eigenvalues leading to linear inequalities for the eigenvalues of three hermitian matrices A,B,C satisfying A+B=C.

A. Hoffman has shown that one may adapt the methods of Wielandt to derive minimax characterizations of the product of singular values of a matrix or the product of eigenvalues of a hermitian positive semi-definite matrix. One such characterization yields inequalities relating the singular values of matrices A,B,C where C=AB.

Publications: (1) "Some remarks on commutators of matrices," by K. Fan; accepted by Archiv der Mathematik. (2) "Inequalities for eigenvalues of Hermitian matrices," by K. Fan; to be included in "Contributions to the solutions of systems of linear equations and the determination of eigenvalues," NBS Applied Mathematics Series 39. (3) "Some metric inequalities in the space of matrices," by K. Fan and A. Hoffman; submitted to a technical journal." (4) "The condition of certain matrices, II" by J. Todd; accepted by Archiv der Mathematik. (5) "Error bounds for eigenvalues of symmetric integral equations," by H. Wielandt; submitted for inclusion in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953. (6) "Einschliessung von Eigenwerten hermitescher Matrizen nach dem Abschnittsverfahren," by H. Wielandt; accepted by Archiv der Mathematik. (7) "Characteristic roots of quaternion matrices," by O. Taussky; accepted by Archiv der Mathematik. (8) "On eigenvalues of sums of normal matrices," by H. Wielandt; submitted to a technical journal.

DIFFERENTIAL EQUATION FOR NERVE FIBER REACTION
Task 1102-10-1104/53-15

Origin: National Naval Medical Institute Authorized 12/8/52
Sponsor: " "
Managers: H. A. Antosiewicz and P. Rabinowitz
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED. Computations are performed as requested by the sponsor.

BAKER-HAUSDORFF FORMULA
Task 1102-10-1104/53-16

Origin: NBS Authorized 12/8/52
Manager: O. Taussky-Todd and K. Goldberg
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED. Work on the publication mentioned below continued.

Publication: "The equation $e^{z}=e^{x}\cdot e^{y}$ in a free associative ring" by K. Goldberg; IN MANUSCRIPT.

Status of Projects

HYPERGEOMETRIC FUNCTIONS
Task 1102-10-1104/53-35

Origin: NBS Authorized 3/25/53
Managers: P. Rabinowitz, W. Cahill
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED.

Publication: "Programs for computing hypergeometric series" by W. F. Cahill; to appear in Mathematical Tables and Other Aids to Computation.

BASIC RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5116/50-2

Origin: Office of Scientific Research, ARDC, USAF and Office of the Air Comptroller, USAF Authorized 3/31/50
Sponsor: Office of Scientific Research, ARDC, USAF
Managers: J. Todd and A. Hoffman
Full task description appears in Jan-Mar 1950 issue, see 11.2/12-50-1.

Status: CONTINUED. A new proof of equivalence of two theorems of the alternative for pairs of matrices has been obtained (see publication item 1 below). The proof is purely algebraic, making no use of the hyperplane separation principle. It is based, instead, on the relation $A^{++} = A$, where A is a submanifold of a linear space.

The relevance of the second part of the Le Chatelier Principle to problems in linear programming is considered in publication item 7 below. Of the four reasonable interpretations in the linear programming context of this part of the principle, three are shown to be false, and a fourth is true, but essentially trivial. These results clarify a heuristic discussion given elsewhere.

The method used in proving an extension of an inequality of Hardy, Littlewood and Pólya (see publication 7 below) has led to the following problem. Let $r_1, r_2, \ldots, r_n$ be given constants. Determine the vertices of the set of all vectors $x=(x_1,\ldots,x_n)$ satisfying $x_i \leqslant x_{i+1}$, $\sum_{i=1}^{k} x_i \geqslant \sum_{i=1}^{k} r_i$, $k=1,\ldots,n$. A geometric characterization has been obtained.

Publications: (1) "A theorem on alternatives for pairs of matrices," by H. A. Antosiewicz; IN MANUSCRIPT. (2) "On a theorem of Ostrowski and Taussky" by R. Bellman and A. J. Hoffman; accepted by Archiv der Mathematik. (3) "Lower bounds for the rank and location of the eigenvalues of a matrix," by K. Fan and A. J. Hoffman; to be included in "Contributions to the solution of systems of linear equations and the determination of eigenvalues," NBS Applied Mathematics Series 39. (4) "Some metric inequalities in the space of matrices," by K. Fan and A. J. Hoffman; submitted to a technical journal. (5) "A characterization of normal matrices," by A. J. Hoffman and O. Taussky; to appear in the Journal of Research of the NBS. (6) "On an inequality of Hardy, Littlewood and Pólya," by A. J. Hoffman; an NBS report. (7) "On the relevance of Le Chatelier's principle to linear programming," by A. J. Hoffman; IN MANUSCRIPT. (8) "An extremum property of eigenvalues," by H. Wielandt; submitted to a technical journal.

COMPRESSIBLE FLOW - METHOD OF ORTHOGONAL AND KERNEL FUNCTIONS
Task 1102-10-5116/52-16

Origin: Aeronautical Research Laboratory, Wright Air Development Center, USAF, and Harvard University — Authorized 9/29/51
Sponsor: Aeronautical Research Laboratory, WADC, USAF
Managers: P. Davis and F. L. Alt
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Particular solutions ψ^* of the equation $\Delta\psi^*+4F(\lambda)\psi^*=0$ have now been computed in the range $-.25(.01)(-.15)$ for λ and $0(.01).20$ for θ. Computation of the singular solution $\psi^{(L,2)}(\lambda,\theta;\lambda_0,\theta_0)$ over the above range is nearly completed. Coding for the solution of the boundary value problem in the pseudo-logarithmic plane using least square procedures with particular solutions is completed and this boundary value problem is being run.

A report, discussing the computations carried out, is being prepared.

TABLES OF INTEGRALS INVOLVING THE HIGHER TRANSCENDENTAL FUNCTIONS
Task 1102-10-5116/52-33
(formerly 1102-21-5117/52-33)

Origin: NBS — Authorized 10/11/51
Manager: F. Oberhettinger
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. F. Oberhettinger has prepared a chapter on Mathieu functions for inclusion in a future volume of "Higher Transcendental Functions" of the Bateman Manuscript Project.

WATER WAVES
Task 1102-10-5116/53-54

Origin: NBS, Division 6 — Authorized 6/1/53
Sponsor: Office of Naval Research, USN
Manager: P. Davis
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. A joint paper by P. Davis and R. F. Dressler entitled "Numerical integration of a non-linear equation of motion of a fluid bounded by a free surface," has been prepared in draft form.

SUPPORTING RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5116/54-9

Origin: Office of the Air Comptroller, USAF — Authorized 9/25/53
Sponsor: Office of Scientific Research, Air Research and Development Command, USAF
Managers: J. Todd and A. J. Hoffman
Full task description appears in July-Sept 1953 issue.

Status: CONTINUED. The problem of contract awards, described in the July-Sept 1953 issue, was further studied for the purpose of solving the difficulties arising from the necessity of distributing the sizes and monthly schedules of the various bidders to meet the depot requirements. It was decided to follow a procedure which (1) ignored the difficulty for the purpose of deciding the total each bidder was to receive, then (2) distributed these amounts proportionately and optimally for each size-month combination. Other alternatives considered involved an inordinate amount of computing for what appeared to be small savings to the Government. Experiments so far have disclosed that this method will result in some net financial gain to the Government, possibly small, apart from the collateral advantages of speed and confidence. The work prior to the latest experiments is described in publication (4).

Further work on the caterer problem--principally a recasting of the demonstrations, which shed new light not only on this problem but on the properties of solutions of the general multiparameter linear programming problem--was incorporated in publication (3). Work has now begun on a modification of the caterer problem, involving a caterer with several establishments but only one laundry.

Publication (6) presents what is apparently the only known example to illustrate the disturbing phenomenon of "cycling" in a degenerate simplex problem. It is shown that cycling may arise in any one of these circumstances: (a) the minimum has been attained, (b) there is no minimum, (c) there is a minimum not yet attained. In all three cases, the algorithm cycles, and thus thwarts discovery of the facts.

Some moderately general circumstances under which a dynamic linear program breaks up into smaller problems have been shown to be illustrated by the smoothing problem. These are reported in publication (7), along with a formula for the solution in the special case that the monthly requirements increase. Publication (1) derives a formula in the case that the requirements increase up to a given month, then decrease.

A perspicuous proof of the result of S. Johnson on the optimal ordering of items for a two-machine production process is presented in publication (8). The result is shown to be a consequence of the fact that the time it takes to process a set of items is not less than the time it takes to process a subset.

A code has been written for the purpose of studying linear programs when some of the coefficients--either of the inequalities or the objective function--are only known probabilistically, a situation arising in most military applications. Several aspects are to be considered. For example, one may wish to know the distribution of the optimal value of the objective function if, at the time of actual application, the coefficients <u>are</u> known exactly. Or they may never have been known exactly, and one may still wish to discover the vector which will optimize the expected value of the objective. The code provides for various modifications of Brown's method, which seems to be a suitable tool.

Publications: (1) "A remark on the smoothing problem," by H. A. Antosiewicz and A. J. Hoffman; IN MANUSCRIPT. (2) "On 'overshoot' in the furthest hyperplane method," by R. Bryce; IN MANUSCRIPT. (3) "On the solution of the caterer problem," by J. W. Gaddum, A. J. Hoffman and D. Sokolowsky; an NBS report. (4) "Linear programming in bid evaluations," by L. Gainen, D. J. Honig and E. D. Stanley; to appear in Logistics Research Quarterly. (5) "A method of solving the transportation problem," by A. Gleyzal; submitted to a technical journal. (6) "Cycling in the simplex method," by A. J. Hoffman; IN MANUSCRIPT. (7) "Smooth patterns of production," by A. J. Hoffman and W. Jacobs; IN MANUSCRIPT. (8) "On the optimal ordering of items for a two-stage process," by A. J. Hoffman; IN MANUSCRIPT. (9) "On block relaxation," by L. S. Joel; IN MANUSCRIPT. (10) "The double description method on the SEAC," by S. Pollack; an NBS report.

RESEARCH IN MECHANICS OF CONTINUA
Task 1102-10-5116/54-23

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: R. F. Dressler

Authorized 12/29/53

Objective: To investigate a class of problems arising from the systems of partial differential equations describing the Newtonian mechanics of continuous media. The main emphasis will be upon the fields of linear and non-linear elasticity, aerodynamics, hydrodynamics, and theoretical hydraulics. Where possible, it is desired to obtain results first in closed analytic form rather than by numerical integration of specific cases. Activities will embrace numerical analysis, however, when results contain expressions difficult to evaluate or include types of problems with novel features in computational technique, such as free-boundary problems in hydrodynamics. In the initial phases of this program, topics will be selected from among those listed below; subsequent activity may branch out, depending upon implications of early results.

Elasticity: The elastic boundary layer ("edge effect") in generalized plane stress; edge effect for non-linear boundary of plates; vibrations of delta wings by combined Rayleigh-Ritz-Weinstein methods; best computational techniques for $\nabla^4\phi=f(x,y)$ in semi-infinite strip domains, connected with problems of edge stress in plates; stress distributions in helical springs of finite cross-section.

Aerodynamics: Rarefaction- and compression-wave propagation subject to dissipation and cross-section variations, from standpoint of Mach line theory.

Hydrodynamics and Hydraulics: Stability criteria for interface behavior of two liquids in density-current flow; behavior of resistance controlled wave tips by developing better fitting procedures and by a study of the asymptotic tip equation $\phi_x^3+\lambda\phi_t^2=0$; the use of the asymptotic fitting procedure in other classes of wave problems; development of feasible methods for numerical computation of free-boundary water wave problems arising from the exact non-linear theory.

Background: Elasticity: The boundary layer effect for bent plates has been handled by Friedrichs and Dressler; the problem now proposed is the supplementary one in the linear theory. An attempt should also be made to extend this technique to the vonKarman non-linear theory for bent plates.

Although the Rayleigh-Ritz-Weinstein methods have been broadly generalized in theory, the application to vibration modes has been accomplished thus far only in a few isolated instances; shapes for fixed and fixed-free edges, and to ascertain the feasibility of employing high-speed computers to carry it out.

Although the helical spring problem is a classical one, it is believed that the present results of Gohner and Freiberger can be extended and improved.

Aerodynamics: It is highly desirable to know quantitatively how the Mach number and pressure are actually affected by frictional dissipation, and how they might be controlled by varying cross-section. This problem applies not only to shock tubes and wind tunnels, but to various types of pneumatic control systems and air brakes; experimental investigations on such controls are now active in the Thermodynamics Section of NBS.

Hydrodynamics: The problem of attaching a wave tip to a centered simple wave has been handled by Whitham and Dressler; in a manner which is satisfactory except in the transition region. A better fitting procedure is needed for this problem and the method should be applied to other types of waves featuring in practical engineering applications.

The interfacial stability problem is one of significance in rivers and oceans, where such underwater wave movement may exist between different thermal strata or between layers of different salinity.

Status: NEW.

Status of Projects

2. Mathematical Tables and Experimental Computations

TABLES OF $E_1(z)$, $(z = x + iy)$

Task 1102-10-1104/43-3
(formerly 1102-10-1110/43-3)

Origin: Canadian National Research Council — Authorized 7/1/47
Manager: I. A. Stegun
Full task description appears in Apr-June 1949 issue, see task 11.2/2-43-3.

Status: CONTINUED. The final manuscript has been completed and the introduction is in process of revision.

TABLE OF THE GAMMA FUNCTIONS FOR COMPLEX ARGUMENTS

Task 1102-10-1104/46-1
(formerly 1102-10-1110/46-1)

Origin: NBS — Authorized 7/1/47
Manager: J. Todd
Full task description appears in Apr-June 1949 issue, see task 11.2/2-46-1.

Status: CONTINUED. The page proofs have been received for this volume.

Publication: "Table of the gamma function for complex arguments," NBS Applied Mathematics Series 34; in press, U. S. Government Printing Office.

TABLES OF COULOMB WAVE FUNCTIONS

Task 1102-10-1104/47-2
(formerly 1102-10-1110/47-2)

Origin: NBS — Authorized 7/1/47
Managers: M. Abramowitz and P. Rabinowitz
Full task description appears in Apr-June 1949 issue, see task 11.2/2-47-2.

Status: CONTINUED. A short table of the regular and irregular solutions for L=0,5,10 and ρ and η in the range from 0 to 10 has been computed. An introduction to these tables is being prepared.

TABLE OF ANTILOGARITHMS

Task 1102-10-1104/47-3
(formerly 1102-10-1110/47-3)

Origin: NBS — Authorized 7/1/47
Manager: J. Todd — Completed 12/31/53

Objective: To prepare a table of 10^x to ten decimal places for $x = 0(.00001)1$.

Background: The function 10^x is of basic importance. The best existing table is that of J. Dodson, "Antilogarithmic Canon," London (1742), which is out of print. The proposed publication, which was suggested by H. E. Salzer of the Computation Laboratory, will be an improvement over Dodson's table from the standpoint of accuracy and format.

Status: COMPLETED.

Publication: "Tables of 10^x," NBS Applied Mathematics Series 27 (U. S. Government Printing Office, Washington 25, D. C., $3.50).

TABLE OF LAGRANGIAN COEFFICIENTS FOR SEXAGESIMAL INTERPOLATION
Task 1102-10-1104/48-2
(formerly 1102-10-1110/48-2)

Origin: NBS Authorized 5/25/48
Manager: J. Todd
Full task description appears in Apr-June 1949 issue, see task 11.2/2-48-2.

Status: CONTINUED. Checking of the page proof for this volume continued.

Publication: "Tables of Lagrangian coefficients for sexagesimal interpolation," NBS Applied Mathematics Series 35; in press, U.S. Government Printing Office.

BIBLIOGRAPHY OF CODING PROCEDURES, MATHEMATICAL TABLES AND NUMERICAL ANALYSIS
Task 1102-10-1104/50-5
(formerly 1102-10-1110/50-5)

Origin: NBS Authorized 3/1/50
Managers: J. Todd, J. H. Wegstein, and P. Rabinowitz Revised 1/9/53
Full task description appears in the Oct-Dec 1952 issue.

Status: CONTINUED. Preparation of the bibliographies continues.

WAVE FUNCTION FOR LITHIUM
Task 1102-10-1104/50-7
(formerly 1102-10-1110/50-7)

Origin: NBS Authorized 6/1/50
Sponsor: Bureau of Ordnance, USN
Manager: J. H. Wegstein
Full task description appears in Apr-June 1950 issue.

Status: CONTINUED. Computation on SEAC continues as time is available.

COLLECTED SHORT MATHEMATICAL TABLES OF THE COMPUTATION LABORATORY
Task 1102-10-1104/51-4
(formerly 1102-10-1110/51-4)

Origin: NBS Authorized 9/28/50
Manager: J. Todd
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. The first volume of such tables is in press.

Publication: "Tables of functions and of zeros of functions," Volume I of Collected Short Tables of the Computation Laboratory, NBS Applied Mathematics Series 37; in press, U. S. Government Printing Office.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Task 1304-34-6351/51-8

Origin: Section 11.3, NBS Authorized 3/26/51
Managers: A. Hoffman and L. Joel
Full task description appears in Apr-June 1951 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue, p. 45.

REVISION OF MATHEMATICAL TABLES
Task 1102-10-1104/52-7
(formerly 1102-10-1110/52-7)

Origin: NBS Authorized 8/10/51
Managers: J. Todd, W. F. Cahill, and I. Stegun
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Following is the status of those mathematical tables the sales stock of which has been exhausted and for which reissue is planned:

"Table of natural logarithms for arguments between zero and five to sixteen decimal places," NBS Applied Mathematics Series 31 (1953); available from U. S. Government Printing Office, Washington 25, D. C., $3.25. This is a reissue of NBS Mathematical Table MT10, "Table of natural logarithms," vol. III (1941).

"Table of sine and cosine integrals for arguments from 10 to 100," NBS Applied Mathematics Series 32; in press, U. S. Government Printing Office. This will be a reissue of the table formerly designated as Mathematical Table MT13 (1942).

"Tables of circular and hyperbolic sines and cosines for radian arguments," NBS Applied Mathematics Series 36; available from U. S. Government Printing Office, Washington 25, D. C., $3.00. This is a reissue of the tables formerly designated as Mathematical Table MT3 (1939, 2d ed. 1949).

"Tables of the error function and its derivative," NBS Applied Mathematics Series 41; in press, U. S. Government Printing Office. This was originally Mathe-

matical Table MT8, "Tables of probability functions," vol. I (1941).

"Tables of sines and cosines for radian arguments," to be reissued in the Applied Mathematics Series; formerly designated as Mathematical Table MT4.

"Table of natural logarithms," vol. IV, Mathematical Table MT12 (1941); revision in progress for reissue in the Applied Mathematics Series.

"Tables of sine, cosine, and exponential integrals," vol. I, Mathematical Table MT5 (1940); out of print, not to be reissued.

TABLE OF ARCSIN FOR COMPLEX ARGUMENTS
Task 1102-10-1104/52-14
(formerly 1102-10-1110/52-14)

Origin: NBS Authorized 10/1/51
Manager: A. A. Goldstein
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The information has been put on punched cards for checking and editing.

EXTENSION OF THE TABLE OF HYPERBOLIC SINES AND COSINES
Task 1102-10-1104/52-18
(formerly 1102-10-1110/52-18)

Origin: NBS Authorized 9/17/51
Manager: W. F. Cahill
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Processing of the punched cards for typing continued.

TABLE OF THE MODIFIED AIRY INTEGRAL
Task 1102-10-1104/52-23
(formerly 1102-10-1110/52-23)

Origin: NBS Authorized 10/4/51
Manager: P. Rabinowitz
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The manuscript has been typed and is in process of being checked. An introduction to the volume has been prepared.

TABLE OF ERROR FUNCTION FOR COMPLEX ARGUMENTS
Task 1102-10-1104/52-25
(formerly 1102-10-1110/52-25)

Origin: NBS Authorized 10/5/51
Managers: M. Abramowitz and F. J. Stockmal
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The table for x,y=0(.1)2 has been computed and is available on punch cards. In addition a short table of the error function for complex arguments for x,y=0(.2)2(.4)3.2 to 9 significant figures is in preparation as an NBS report.

EXTENSION OF TABLES OF THE EXPONENTIAL FUNCTION FOR NEGATIVE ARGUMENTS
Task 1102-10-1104/52-31
(formerly 1102-10-1110/52-31)

Origin: NBS Authorized 10/9/51
Manager: E. Marden
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Functions have been differenced and punched cards prepared for typing of the final manuscript.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-10-1104/52-37
(formerly 1102-10-1110/52-37)

Origin: NBS Authorized 11/28/51
Manager: T. Ledley
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. The machine computation is practically completed for m=0(1)5, ℓ=0(1)10. A complete description is being written up.

VAN DER POL EQUATION
Task 1102-10-1104/52-43
(formerly 1102-10-1110/52-43)

Origin: NBS Authorized 11/28/51
Manager: W. F. Cahill
Full task description appears in Oct-Dec 1951 issue.

Status: INACTIVE. For status to date see Apr-June 1952 issue, p. 49.

RADIAL MATHIEU FUNCTIONS
Task 1102-10-1104/52-49
(formerly 1102-10-1110/52-49)

Origin: NBS Authorized 2/1/52
Managers: J. Todd and I. Rhodes
Full task description appears in Jan-Mar 1952 issue.

Status: CONTINUED. All tabulations have been computed and are being checked at the Istituto Nazionale per le Applicazioni del Calcolo (Rome) by E. Aparo and D. Dainelli, who assisted in the analysis and coding-programming during their stay at the Computation Laboratory as guest workers.

SIEVERT'S INTEGRAL
Task 1102-10-1104/52-57
(formerly 1102-10-1110/52-57)

Origin: NBS Authorized 2/12/52
Managers: M.L. Paulsen and P. J. O'Hara
Full task description appears in Jan-Mar 1952 issue.

Status: CONTINUED. Computations have been completed and differencing of the results, which have been transferred to punch cards, is nearly completed.

SCATTERING FUNCTIONS
Task 1102-10-1104/52-63
(formerly 1102-10-1110/52-63)

Origin: NBS Authorized 3/10/52
Manager: A. Goldstein Terminated 12/31/53

Objective: To develop practical numerical and analytical methods for determining the scattering of plane waves by obstacles (or force centers in quantum mechanical cases). Mathematically this requires the solution of the wave equation

$$\nabla^2 x + k^2(r)x = 0$$

with proper behavior at large distances from the scatterer. The possibility of using high-speed digital computers for this problem will be investigated.

Background: There is considerable technological interest in the scattering of electromagnetic waves by large molecules, fogs, dust particles, etc. The interpretation of experimental results in the scattering of high energy particles by nuclei is one of the central problems in nuclear physics.

Status: TERMINATED, in favor of tasks of higher priority.

TABLE OF SECANTS AND COSECANTS
Task 1102-10-1104/52-81
(formerly 1102-10-1110/52-81)

Origin: NBS Authorized 7/1/52
Managers: K. C. Nelson and I. A. Stegun
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. In press.

Publication: "Table of secants and cosecants to nine significant figures at hundredths of a degree," NBS Applied Mathematics Series 40; in press, U. S. Government Printing Office.

PAINLEVÉ EQUATION
Task 1102-10-1104/53-3
(formerly 1102-10-1110/53-3)

Origin: NBS Authorized 8/11/52
Managers: J. Todd and H. A. Antosiewicz
Full task description appears in July-Sept 1952 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue, p. 48.

L-SHELL CONVERSION COEFFICIENTS
Task 1102-10-5110/53-52
(formerly 1102-10-1110/53-52)

Origin: Oak Ridge National Laboratory Authorized 5/20/53
Manager: C. J. Swift
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. Some answers have been obtained and the code is in production.

3. Mathematical Services

Note: The tasks under Mathematical Services are arranged serially according to the digits following the slant lines in the task numbers. The first two digits following the slant line designate the fiscal year in which the task was authorized.

TABLES OF THERMODYNAMIC PROPERTIES OF GASES
Task 0302-40-2606/49-5
(formerly 11.2/33-49-5)

Origin: NBS, Section 3.2 Authorized 11/29/48
Sponsor: National Advisory Committee for Aeronautics
Manager: F. L. Alt
Full task description appears in Apr-June 1949 issue.

Status: INACTIVE.

RAY TRACING
Task 0202-10-2308/50-13
(formerly 11.2/33-50-13)

Origin: NBS, Section 2.2 Authorized 3/1/50
Sponsor: "
Manager: R. K. Anderson
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Computations are being performed as requested.

MOLECULAR STRUCTURE CALCULATIONS, II
Task 1102-40-5126/50-16
(formerly 1102-53-1106/50-16)

Origin: Naval Research Laboratory, USN Authorized 3/31/50
Sponsor: " "
Manager: P. J. O'Hara
Full task description appears in Jan-Mar 1950 issue, see task 11.2/33-50-16.

Status: CONTINUED. Computations were performed as requested.

LIQUID-VAPOR TRANSITION
Task 1102-40-5126/51-22
(formerly 1102-53-1106/51-22)

Origin: Naval Medical Research Institute Authorized 2/1/51
Sponsor: " "
Manager: I. Stegun
Full task description appears in Jan-Mar 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue, p. 52.

MOLECULAR STRUCTURE, III
Task 1102-40-5126/51-37
(formerly 1102-53-1106/51-37)

Origin: Naval Research Laboratory, USN Authorized 8/10/51
Managers: P. J. O'Hara and I. A. Stegun
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Computations were performed as requested.

SHOCK WAVE PARAMETERS, II
Task 1102-40-5126/51-38
(formerly 1102-53-1106/51-38)

Origin: Bureau of Ordnance, USN Authorized 6/28/51
Sponsor: " "
Manager: I. A. Stegun
Full task description appears in Apr-June 1951 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue, p.48.

POWDER DIFFRACTION
Task 1102-40-5126/52-6
(formerly 1102-53-1106/52-6)

Origin: NBS, Section 9.7 Authorized 9/17/51
Manager: E. Marden
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1951 issue, p. 53.

PRECISE DETERMINATION OF THE PARAMETER OF DISPERSION EQUATION FOR SEVERAL TYPES OF OPTICAL GLASS
Task 1102-40-5126/52-17
(formerly 1102-53-1106/52-17)

Origin: NBS, Division 2 Authorized 9/29/51
Sponsor: "
Manager: I. A. Stegun
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE.

SPHERICAL BLAST
Task 1102-40-5126/52-20
(formerly 1102-53-1106/52-20)

Origin: Naval Ordnance Laboratory Authorized 9/27/51
Sponsor: " "
Manager: D. H. Jirauch
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Some work was done to reorganize the material prior to test runs on SEAC.

CALCULATIONS FOR d SPACINGS
Task 1102-40-5126/52-44
(formerly 1102-53-1106/52-44)

Origin: NBS, Div. 9 Authorized 12/7/51
Sponsor: "
Manager: I. Stegun
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. During this quarter d-spacing calculations were computed for various compounds and least squares calculations were made.

GAS ADSORPTION BY HIGH POLYMERS
Task 1102-40-5126/52-70
(formerly 1102-53-1106/52-70)

Origin: Naval Medical Center, Bethesda, Md. Authorized 4/1/52
Sponsor: " "
Manager: I. Stegun
Full task description appears in Jan-Mar 1952 issue.

Status: INACTIVE. For status to date see Apr-June 1952 issue, p. 59.

CHEMICAL TRANSITION PROBABILITIES
Task 1102-40-5126/52-82
(formerly 1102-53-1106/52-82)

Origin: NBS Section 3.2 and Hydrocarbon Research Corp. Authorized 6/1/52
Completed 12/31/53
Sponsor: Office of Naval Research, USN
Manager: A. A. Goldstein

Objective: To compute quantum-mechanical transition probabilities related to the chemical reactions: $H_2 + O \rightarrow OH + H$, $O_2 + H \rightarrow OH + O$. The detailed computations involve the evaluation of integrals, frequently referred to as matrix elements, that appear in the time-dependent theory of quantum mechanics which has been applied to chemical kinetics. (See for example. S. Golden and A. M. Peiser, J. Chem. Phys. 17, 630 (1949)). In addition, the computations involve averaging the squared magnitudes of the matrix elements over distributions of the parameters upon which the former depend. The distributions are related to the probability of finding the initial reactant molecules in specified energy and momentum states.

Background: The results of these computations are needed to provide a theoretical description of certain chemical reactions which are presumed to be important hydrogen-oxygen flames. Of special interest is the possibility that the distribution of the OH radical may be determined in its various rotational and vibrational energy states. The results of these computations will enable the correlation to be made of the experimental results which may be obtained from spectroscopic studies of these flames. This task was proposed and suggested by Drs. Peiser and Golden, Hydrocarbon Research Corporation, and by H. P. Broida, NBS 3.2.

Status: COMPLETED.

NEUTRON DIFFUSION, II
Task 1102-40-5126/53-4

Origin: Atomic Energy Commission, New York Office, (NDA) Authorized 9/30/52
Sponsor: " " "
Managers: O. Steiner and N. Levine
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. Results are being sent to the sponsor as they are completed. New codes are being written to cover various source geometries, including pulse sources, rectangular sources, fission sources, and conical sources. Computations have been carried out for a number of materials including hydrogen, lithium, oxygen, water.

STRENGTH OF WING COMPONENTS
Task 1102-40-5126/53-11

Origin: National Advisory Committee for Aeronautics, Langley Field, Va. Authorized 12/8/52
Sponsor: " "
Manager: E. Marden
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED.

RADIANT HEATING OF SOLIDS
Task 1102-40-5126/53-20

Origin: NBS, Section 10.2 Authorized 12/15/52
Sponsor: "
Manager: W. F. Cahill
Full task description appears in Oct-Dec 1952 issue.

Status: INACTIVE.

COMPUTATION OF THERMODYNAMIC FUNCTIONS
Task 1102-40-5126/53-27

Origin: NBS, Division 5 Authorized 3/18/53
Sponsor: "
Manager: E. Marden
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. The previous code was improved to obtain more significant figures in the final results, and thermodynamic functions (heat capacity, enthalpy, free energy, and entropy) and their differences were computed for several additional molecules.

STUDY OF TRICALCIUM ALUMINATE
Task 1102-40-5126/53-28

Origin: NBS, Division 9 Authorized 3/30/53
Sponsor: "
Manager: R. Anderson
Full task description appears in Jan-Mar 1953 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue, p. 58.

DYNAMIC BEHAVIOR OF AIRCRAFT STRUCTURES
Task 1102-40-5126/53-29

Origin: NBS, Section 6.4 Authorized 1/23/53
Sponsor: "
Manager: I. Rhodes
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. Computations are being performed as requested, and results are being transmitted to the sponsor.

REDUCTION OF ECLIPSE DATA
Task 1102-40-5126/53-34

Origin: Air Photographing and Charting Services, USAF Authorized 3/30/53
Sponsor: " "
Manager: A. Goldstein
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. The final report is being prepared.

SPECTRAL ANALYSIS OF STATIONARY TIME SERIES
Task 1102-40-5126/53-37

Origin: Statistical Research Center, University of Chicago Authorized 4/20/53
Completed 12/31/53
Sponsor: Office of Naval Research, USN
Manager: I. Stegun

Objective: To compute the serial products

$$c_\nu = \sum_{n=1}^{N-|\nu|} x_n x_{n+\nu} \quad \text{and} \quad \gamma_s = \sum_{n=1}^{N-|s|} x_n^2 x_{n+|s|}^2$$

which are used in estimating the spectral distribution function and its kurtosis, respectively.

Background: The present statistical theory of analysis of stationary time series has assumed complete knowledge of the covariance sequence or, equivalently, of the spectrum of the process. It is, therefore, important to be able to estimate one of these. Knowledge of the spectrum seems to yield greater immediate insight into the structure of the process.

Comments: This problem was proposed by M. Rosenblatt and U.Grenander, SRC, University of Chicago.

Status: COMPLETED.

Publication: "Tables of autoregressive series" by Computation Laboratory; an NBS report.

ACOUSTICAL IMPEDANCES
Task 1102-40-5126/53-39

Origin: NBS, Section 6.1 Authorized 6/30/53
Sponsor: "
Managers: S. Prusch and K. Nelson
Full task description appears in Apr-June 1953 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue, p. 55.

LORAN UNIVAC CODE
Task 1102-40-5126/53-41

Origin: Hydrographic Office, U. S. Navy Authorized 4/20/53
Sponsor: " "
Managers: I. Rhodes and D. H. Jirauch
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. The computation of Loran tables has been coded and successfully proved in on the UNIVAC. The Skywave correction and trainer code was coded and is awaiting code checking. The baseline routine is in process of being coded for UNIVAC.

OPTIMUM SECTIONS FOR DELTA WING
Task 1102-40-5126/53-49

Origin: National Advisory Committee for Aeronautics, Langley Field, Va. Authorized 4/29/53
Completed 12/31/53
Sponsor: " "
Manager: O. Steiner

Objective: To perform computations to determine optimum sections for delta wings at supersonic speeds.

Status: COMPLETED. Results were transmitted to the sponsor.

RADIATION DIFFUSION
Task 1102-40-5126/53-51

Origin: NBS, Section 4.8 Authorized 6/9/53
Sponsor: Armed Forces Special Weapons Project
Manager: F. Stockmal
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. The code for part 1 (Photon random walk in an infinite Compton scattering medium) has been completely checked and used to generate 1000 random walks.

These random walks have been used for the calculation of 3 problems in gamma-ray diffusion: (1) the reflection and transmission of radiation by finite plane parallel barriers; (2) the albedo of semi-infinite media; (3) the penetration of radiation in an infinite medium.

TRANSPORTATION PROBLEM II
Task 1102-40-5126/53-55

Origin: Logistics Research Project, George Washington University — Authorized 6/9/53, Terminated 12/31/53
Sponsor: Office of Naval Research
Managers: A. J. Hoffman and L. Gainen

Objective: To solve systems of equations arising in the problem of allocating shipments from bidders to depots minimizing the total cost of the operation. To investigate problems arising from conditions imposed by bidders by time phasing of requirements, etc., and to formulate these conditions for computation.

Background: The technique of G. B. Dantzig in applying the simplex method in finding solutions of the transportation problem of Hitchcock-Koopmans is used in solving standard allocation problems.

Status: TERMINATED. This work is now being carried on as part of task 1102-40-5126/54-13 (see p.43).

STANDARD LORAN TABLES - Rates 1L4, 1L5
Task 1102-40-5126/54-2

Origin: Hydrographic Office, U. S. Navy — Authorized 9/16/53
Sponsor: " " — Completed 12/31/53
Manager: D. H. Jirauch

Objective: To prepare tables giving coordinates of hyperbolic lines of position.

Background: Standard Loran navigation tables are necessary for preparation of charts used by navigators in determining their positions with the aid of certain electronic equipment.

Status: COMPLETED.

HIGH TEMPERATURE PROPERTIES OF WATER
Task 1102-40-5126/54-4

Origin: NBS, Division 3.2 — Authorized 12/7/53
Sponsor: Bureau of Ordnance, USN
Manager: J. H. Wegstein

Objective: To derive equations describing the high temperature thermodynamic properties of water and to solve these equations numerically over a very large temperature range and over the maximum pressure range that can be treated theoretically.

Background: The properties of water at high temperature are of interest to the Naval Ordnance Laboratory. Such properties are not now known in the range under consideration.

Status: NEW.

DISTRIBUTION OF NORMAL MODES OF VIBRATION OF CUBIC LATTICES, II
Task 3711-60-0009/54-7

Origin: NBS, Division 30 Authorized 12/29/53
Sponsor: "
Manager: F. Stockmal

Objective: To compute the distribution of normal modes of vibration in cubic lattices, on which depends the vibrational contribution to the thermodynamic properties of polyatomic molecules and crystals.

Background: This task is a continuation of task number 1102-21-1104/52-62, a complete description of which appeared in the Jan-Mar 1952 issue. The code for the earlier problem had been run on the SEAC, and distribution densities obtained for cubic lattices of various sizes.

Status: NEW. Evaluation was made of earlier programs in order to modify the procedures for suitability in the newly formulated problem.

NORMAL VIBRATIONS IN MOLECULES
Task 3711-60-0009/54-8

Origin: NBS, Section 3.2 Authorized 11/27/53
Sponsor: "
Manager: M. Newman

Objective: Given the nxn+3 matrix S and the nxn symmetric matrix F, to calculate the symmetric matrix A = S'FS, and then to compute the eigenvalues and eigenvectors of A.

Background: This problem arises in molecular spectroscopy. The object of the problem is the calculation of the best force constants between the atoms of a molecule. With these constants the normal frequencies of vibrations are calculated.

Status: NEW. Fifty-eight 6x6 matrices and four 14x14 matrices M were computed from M=S'FS, where S is an nx(n+3) matrix, F an nxn symmetric matrix. The eigenvalues and eigenvectors of these matrices M were then computed. In addition three 6x6 matrices and one 7x7 matrix were computed, and then inverted.

COMPUTATION OF VIBRATION MODES AND FREQUENCIES
Task 3711-60-0009/54-11

Origin: NBS, Section 6.4 Authorized 11/27/53
Sponsor: NBS, Section 6.4
Managers: I. Rhodes and W. Cahill

Objective: To compute the free vibration modes and frequencies of aircraft structures. The codes are to be general in nature and apply without modification to any aircraft structure whose influence coefficients, mass distribution, and geometry are specified.

Background: A knowledge of the vibration modes and frequencies of aircraft is of primary importance in determining their flutter speed and in making dynamic analyses of the response of the structure to landing impact and other shock loads.

Status: NEW.

SPACE AVERAGE OF TEMPERATURE DISTRIBUTION WITH A VOLUME DISTRIBUTION OF SOURCES
Task 3711-60-0009/54-12

Origin: NBS, Division 6 Authorized 12/11/53
Sponsor: "
Manager: E. Marden

Objective: To evaluate

$$A = \frac{32}{\pi^2}\beta^2 \sum_{n}^{\infty} \sum_{\nu}^{\infty} \frac{1}{x_n^2(2\nu+1)^2\{[(x_n/a)^2 + ((2\nu+1)\frac{\pi}{\ell})^2]^2 + \beta^2\}}$$

$$B = \frac{32}{\pi^2}\beta \sum_{n}^{\infty} \sum_{\nu}^{\infty} \frac{(x_n/a)^2 + [(2\nu+1)\frac{\pi}{\ell}]^2}{x_n^2(2\nu+1)^2\{[(x_n/a)^2 + ((2\nu+1)\frac{\pi}{\ell})^2]^2 + \beta^2\}}$$

$$\rho = (A^2+B^2)^{\frac{1}{2}}$$

$$\tan\theta = \frac{B}{A}$$

where $x_n, n=1,2,\ldots$ runs over the zeros of J_0 (the Bessel functions of order 0), $\nu = 0,1,\ldots$, and β assumes values from .20 to 4500. Several values are taken for the parameters a and ℓ.

Background: This problem arises in measuring the pressure response of condenser microphones.

Status: NEW. Computations were carried out to evaluate the performance of several condenser microphones from data furnished defining their shapes. A SEAC code is available for running of additional cases.

AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Task 1102-40-5126/54-13

Origin: New York Quartermaster Procurement Agency Authorized 11/27/53
Sponsor: " " "
Manager: H. Bremer

Objective: To solve the problems, arising in the sponsor's work, of awarding purchase contracts at minimum cost to the Government.

Background: This task is a generalization of the investigations that were undertaken in task 1102-40-5126/53-55.

Status: NEW. The work of task 1102-40-5126/53-55 (see p.41), now terminated, is being applied to actual problems arising in the sponsor's operations. The results of three computations have been transmitted to the sponsor for use in awarding contracts. A new code is being written to expedite the machine handling of the problem of phasing the successful contractor's shipments to the various depots.

FLOW IN SUPERSONIC NOZZLE
Task 1102-40-5126/54-14

Origin: National Advisory Committee for Aeronautics, Langley Field, Va. Authorized 12/16/53
Sponsor: " " "
Managers: E. Marden, G. Hawkins

Objective: To calculate the boundary contour of a supersonic nozzle for a given Mach number.

Background: It is assumed that the sonic surface is a plane normal to the axis of the nozzle. The Mach net is then computed by the Prandtl-Busemann method (see L. Prandtl and A. Busemann: Stodala-Festschrift, Zurich, 1929, p. 499), starting from the first Mach line and from the given convex part of the boundary. In order to compute the slopes of the two Mach lines at each point of the net it is necessary to solve for ε the following equation:

$$\theta = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\varepsilon\right) - \tan^{-1}\varepsilon$$

where θ is given and $\gamma=1.4$ is the adiabatic constant. This could be done by making a table of the functions and using inverse interpolation, but for SEAC computation it is simpler to solve the equation at each step by Newton's method using the solution of the previous point as a first approximation. The net is then built up progressively by computing the intersection of the left-running and right-running Mach lines.

After the boundary has reached its maximum slope it must be modified in such a way that uniform flow is obtained in the downstream region of the nozzle.

This problem has been solved for several nozzles using the Bell relay computer at Langley Field. However, an increment in the angle of expansion between two subsequent Mach lines of one degree was used there. It is proposed in the SEAC calculation to use increments of twenty-five hundredths of a degree.

Status: NEW. Programming and coding of this problem for SEAC have been completed and checking in of the code has begun.

DEPOLYMERIZATION
Task 3711-60-0009/54-17

Origin: NBS, Section 7.6 Authorized 12/16/53
Sponsor: "
Manager: R. Anderson

Objective: To obtain numerical solutions for the system of differential equations

$$\frac{dQ_{N-1}}{dT} = -(1+\sigma)(N-4)Q_{N-1}$$

$$\frac{dQ_n}{dT} = -(1+\sigma)(n-3)Q_n + 2\left(\frac{K_0}{k_1} - \frac{1}{4}\sigma\right)Q_{n+2} + 2\sum_{i=1}^{\frac{N-n-3}{2}}\frac{K_i}{k_1}Q_{n+2i+2},\quad 4\leq n\leq N-3,$$

with boundary conditions at T=0, $\begin{cases} Q_{N-1} = 1 \\ Q_N = 0, \end{cases}$

where K_i is defined in terms of the parameters σ and ε by the equation

$$\frac{K_i}{k_1} = 2(1+\tfrac{1}{2}\sigma)\left\{1-\tfrac{1}{4}[1+3(1-\varepsilon)](1+\varepsilon)^i\right\} + \sigma .$$

Background: This system arises in the theory of chain reactions which describes the depolymerization of long polymer molecules as developed by R. Simha, L. A. Wall, and P. J. Blatz; J. Polymer Science, 1950. (N-1) represents the initial number of units in a chain, Q_n the fraction of molecules containing n units, and T is a dimensionless variable related to time. $1/\varepsilon$ is the average kinetic chain length of reaction and σ the ratio between the probabilities of transfer and initiation of a free radical chain.

Status: NEW. Analysis of the problem for computation on SEAC has been started.

III. STATISTICAL ENGINEERING LABORATORY

(Section 11.3)

1. Fundamental Research in Mathematical Statistics

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE

Task 1103-10-1107/49-1a
(formerly 11.3/2-49-1)

Origin: NBS — Authorized 1/9/49
Manager: L. S. Deming
Full task description appears in Jan-Mar 1949 issue.

Status: CONTINUED. The file of abstracts of statistical literature taken from Mathematical Reviews is on a current basis. The preparation of abstracts from Zentralblatt für Mathematik is continuing. These Zentralblatt abstracts start at 1940 and work back through the years; 1933 is the earliest volume at this date from which material for this file has been drawn.

MANUAL ON FITTING STRAIGHT LINES

Task 1103-10-1107/50-2
(formerly 11.3/2-50-2)

Origin: NBS — Authorized 3/1/50
Manager: F. S. Acton
Full task description appears in Jan-Mar 1950 issue.

Status: INACTIVE. For status to date see Oct-Dec 1952 issue, p.58.

TABLE TO FACILITATE DRAWING RANDOM SAMPLES

Task 1103-10-1107/51-1

Origin: NBS — Authorized 7/1/50
Managers: C. Eisenhart and L. S. Deming
Full task description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue, p.64.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-10-1107/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. J. Lieblein prepared a working paper dealing with a probability problem arising in connection with counter data.

Publications: (1) "A property of the normal distribution," by E. P. King and E. Lukacs; submitted to a technical journal. (2) "A new method of analyzing extreme-value data," by J. Lieblein; accepted for publication as a NACA Technical Note. (3) "On strongly continuous stochastic processes," by E. Lukacs; accepted for publication in Sankhya. (4) "Certain Fourier transforms of distributions (II)," by E. Lukacs and O. Szász; accepted for publication in the Canadian Journal of Mathematics. (5) "Nonnegative trigonometric polynomials and certain rational characteristic functions," by E. Lukacs and O. Szász; accepted for publication in the Journal of Research of the NBS. (6) "Two early papers on the relation between extreme values and tensile strength" (formerly "A historical note on the application of the 'weakest-link' idea to tensile strengths") by J. Lieblein; submitted to a technical journal. (7) "Tables of expected values of 1/X for positive Bernoulli and Poisson variables," by E. Grab and I. R. Savage; accepted for publication in the Journal of the American Statistical Association. (8) "Tables of the inverses of finite segments of the Hilbert matrix," by I. R. Savage and E. Lukacs; to be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39. (9) "On optimum grouping in one-criterion variance components analysis," by E. P. King; submitted to a technical journal. (10) "On the variances and covariances of order statistics from the Weibull distribution," by J. Lieblein; submitted to a technical journal. (11) "Time-discrete stochastic processes in arbitrary sets, with applications to processes with absorbing regions and to the problem of loops in Markoff chains," by D. van Dantzig; submitted to a technical journal. (12) "Bounds on a distribution function which are functions of moments to order four," by M. Zelen; submitted to a technical journal.

LAW OF PROPAGATION OF ERROR
Task 1103-10-1107/52-1

Origin: NBS Authorized 6/23/51
Managers: C. Eisenhart and I. R. Savage
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1951 issue, p.65.

PROCEDURES OF NON-PARAMETRIC STATISTICS
Task 1103-10-1107/52-2

Origin: NBS Authorized 9/17/51
Manager: I. R. Savage
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. An investigation has been started to find the most probable rank orders in various sampling situations arising in

nonparametric problems. These results are of use in the construction of optimum nonparametric test procedures. Preliminary work suggests that it will be possible to construct uniformly most powerful rank order tests for a nonparametric situation discussed recently by Lehman.

Publications: (1) "Bibliography of nonparametric statistics and related topics," by I. R. Savage; accepted for publication by the Journal of the American Statistical Association. (2) "Most probable rank orders," by I. R. Savage; IN MANUSCRIPT.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-10-1107/53-1

Origin: NBS Authorized 10/15/52
Manager: W. S. Connor
Full task description appears in the Oct-Dec 1952 issue.

Status: CONTINUED. (1) W. H. Clatworthy prepared a working paper which presents some new partially balanced designs with two associate classes. (2) M. Zelen has obtained preliminary results on exact tests of significance for combining information from both inter and intra block analysis of incomplete block designs. The power functions of different critical regions are under investigation. (3) W. S.Connor has begun a study of the combinational properties of triangular designs.

Publications: (1) "Partially balanced incomplete block designs with two associate classes and two treatments per block," by W. H. Clatworthy; submitted to a technical journal. (2) "Some theorems for partially balanced designs," by W. S.Connor and W. H. Clatworthy; accepted for publication in Annals of Mathematical Statistics. (3) "An embedding theorem for balanced incomplete block designs," by M. Hall, Jr. (Ohio State University) and W. S. Connor; accepted for publication in the Canadian Journal of Mathematics. (4) "A note on partially balanced designs," by M. Zelen; accepted for publication in the Annals of Mathematical Statistics. (5) "Analysis for some incomplete block designs having a missing block," by M. Zelen; accepted for publication in Biometrics. (6) "Designs for paired observations," by W. S. Connor and W. J. Youden; IN MANUSCRIPT. (7) "On the enumeration of partially balanced designs with two associate classes," by W. H. Clatworthy; IN MANUSCRIPT.

2. Applied Research in Mathematical Statistics

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1
(formerly 3011-60-0002/51-1)

Origin: NBS Authorized 7/1/50
Managers:W. J. Youden and J. Cameron
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. Activity under this task fell into two main

categories:

A. Design of Experiments: A special partially balanced design was constructed for use in calibrating thermocouple wires. Because of the construction of the apparatus, only differences between wires could be measured. This necessitated a modification of the usual incomplete block design to account for the fact that within a block only a subset of the usual treatment comparisons could be measured at any one time.

B. Development or Selection of the Appropriate Method for Analysis and Interpretation of Data: A sampling plan for insuring that inversion thermometers meet a desired tolerance specification (i.e., in repeat determinations, a given proportion must repeat within .01° C of the resulting mean) was constructed. This plan is implemented by using the range of these repeat determinations.

Publications: (1) "Rejection of outlying observations," by F.Proschan; Am. J. Phys. 21, 520-525 (Oct. 1953). (2) "The principles of experimental design," by W. J. Youden; appeared as a chapter in Research Operations in Industry, p. 317-329(Kings Crown Press, Columbia University, New York, 1953). (3) "Making one measurement do the work of two," by W. J. Youden and W. S. Connor; Chem. Eng. Prog. 49, 549-552 (Oct. 1953). (4) "Acceptance sampling of electroplated articles," by J. M. Cameron and F. Ogburn; accepted for publication in Plating. (5) "Probability limits for the average chart when process standards are unspecified," by E. P. King; submitted to a technical journal. (6) "Performance of inspectors and gasoline pumps," by W. J. Youden and M. W. Jensen; submitted to a technical journal.

STATISTICAL ASPECTS OF NBS ADMINISTRATIVE OPERATIONS
Task 3737-60-0002/52-1
(formerly 3011-60-0002/52-1)

Origin: NBS Authorized 10/1/51
Manager: I. R. Savage
Full task description appears in Oct-Dec 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue, p.68.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin: Ship Structure Committee, NRC Authorized 12/1/51
Sponsor: " "
Manager: W. J. Youden
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. Work in progress is directed towards improving the discriminant function for predicting plate performance, and establishing confidence limits associated with coefficients in the formula relating transition temperature and plate composition.

RESEARCH IN APPLICATIONS OF MATHEMATICAL STATISTICS TO PROBLEMS OF THE CHEMICAL CORPS
Task 1103-40-5118/52-1

Origin: Biological Laboratories, Chemical Corps, Dept. of the Army — Authorized 10/1/51
Sponsor: " " "
Manager: C. Eisenhart
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. Plans for special computations were worked out.

STATISTICAL ANALYSIS AND DESIGN OF EXPERIMENTS FOR THE U. S. GEOLOGICAL SURVEY
Task 1103-40-5140/54-1

Origin: U. S. Geological Survey, Department of Interior — Authorized 10/9/53
Sponsor: " " "
Managers: C. Eisenhart and W. J. Youden

Objective: The survey of problems in field geology in AEC-sponsored projects at the U. S. Geological Survey offices in Grand Junction, Colorado; the application of statistical methods to field sampling and field observations in geology; and the application of statistical methods to laboratory work in connection with field geology.

Background: Problems encountered in large and complex projects of the U. S. Geological Survey necessitate the application of statistical techniques to ensure that the observations are taken properly and interpreted correctly for significant conclusions to be drawn.

Status: NEW. C. Eisenhart spent a day at the Trace Elements Laboratory of the U. S. Geological Survey at Denver, Colorado, and W. J. Youden and C. Eisenhart spent four days with personnel of the Grand Junction District, U. S. Geological Survey, making a survey of the statistical problems in AEC-sponsored Geological studies of the Colorado Plateau.

After their return to Washington, conferences were held with those Geological Survey personnel in Washington concerned with the geologic studies of the Colorado Plateau, in which the trip to Colorado was reviewed and further activities outlined.

As a member of the Panel on Earth Sciences of the Committee on Statistics in the Physical Sciences of the American Statistics Association, Dr. Eisenhart participated as a discusser at a Symposium on Statistics in Geology held in conjunction with the Annual Meeting of the American Statistical Association, Washington, D. C., December 29, 1953; and then met informally to discuss the application of statistics in geology with a number of geologists assembled in Washington for this Symposium.

IV. MACHINE DEVELOPMENT LABORATORY

(Section 11.4)

in cooperation with

ELECTRONIC COMPUTER SECTION

(Section 12.3)

THE BUREAU OF THE CENSUS COMPUTING MACHINE

Task 1104-34-5107/47-1
(formerly 11.4/21-47-1)

Origin: The Bureau of the Census — Authorized 7/1/47
Sponsor: " "
Full task description appears in Apr-June 1949 issue.

Status: CONTINUED. Under contract between the NBS and the Eckert-Mauchly Division of Remington Rand Inc., stockpiling of replacement parts for UNIVAC System No. 1 has been continued. Additional auxiliaries, particularly high-speed printing equipment, have been also investigated and evaluated at the request of the Bureau of the Census.

THE AIR COMPTROLLER'S COMPUTING MACHINE

Task 1104-34-5107/47-3
(formerly 11.4/24-47-3)

Origin: Office of the Air Comptroller, USAF — Authorized 7/1/47
Sponsor: " " — Completed 12/31/53

Objective: To develop specifications for, and to construct an automatic-sequenced electronic digital computing machine suitable for use by the U. S. Air Force in program planning and control.

Background: The Air Comptroller's Office requires a high-speed and flexible computing machine to calculate detailed programs consistent with general policy decisions, and to facilitate rapid recomputation of programs to meet budgetary and other limitations. The problems involved are of wide applicability, and a part of the present project consists in formulating them mathematically. It is expected that the primary computation problem to be solved by the machine will consist of finding rapidly the solutions of large systems of simultaneous equations containing up to 1000 unknowns. The computer must be able to store and classify large quantities of data, and to refer rapidly for needed items to huge tables of organization, equipment, supply and other similar data. These tables will contain millions of items. It is required, in addition, that the printing devices associated with the computer will be capable of extremely high-speed printing of the complete details of the Air Force's programs that have been computed.

Status: COMPLETED. With the continued successful operation of the machine by the Air Comptroller's Office, this Bureau's responsibility is considered to have been discharged.

NATIONAL BUREAU OF STANDARDS EASTERN AUTOMATIC COMPUTER (SEAC)
Task 1104-34-5107/49-1
(formerly 11.4/24-49-1)

Origin: NBS Authorized 12/15/48
Sponsor: Office of the Air Comptroller, USAF
Full task description appears in Apr-June 1949 issue.

Status: CONTINUED. During the past quarter, SEAC was operated with an over-all efficiency of 74.5% during scheduled computation, as compared with 81% for the previous quarter. With the completion of debugging of the electrostatic memory, regular computation using 1,024 words of storage was begun. It is believed that the requirement to return the full memory, 512 words of acoustic and 512 words of electrostatic, to operation following a failure has been at least partially responsible for the drop in operating efficiency from the earlier level of around 80%.

The policy has been adopted that beginning January 1, 1954, the SEAC be utilized principally as an item of general Bureau equipment rather than as a service facility for other Federal agencies. Maintenance costs will be distributed to all technical projects at the Bureau, which benefit from the constant availability of such equipment.

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Numerical Analysis Colloquium
(Los Angeles, California)

FEKETE, M. (Hebrew University). Asymptotic behavior of certain polynomials of least deviation on point sets of given transfinite diameter. October 20.

Numerical Analysis Seminar
(Los Angeles, California)

TOMPKINS, C. B. Numerical local embedding of a surface with riemannian metric into euclidean 3-space. I. Geometrical introduction and classical formulas. September 29.

BLANCH, G. Numerical local embedding of a surface with riemannian metric into euclidean 3-space. II. Some analysis of the partial differential equations involved. October 1.

SWIFT, J. D. Steiner triples and their calculation on SWAC. October 6.

TEICHROEW, D. The generation of random variables on SWAC. October 8.

CODDINGTON, E. A. (University of California, Los Angeles). On perturbations of linear systems possessing periodic solutions. October 13.

RUTHRAUFF, R. (Douglas Aircraft Company). I. Symbolic coding for an IBM 701 computer. II. Experience with the conjugate-gradient and elimination methods for linear systems. October 15.

REYNOLDS, R. R. Review of Caratheodory's 1937 paper on sweeping-out process for Dirichlet's problem. October 20.

WASOW, W. R. On the truncation error for Dirichlet's problem, I. October 22. II. November 19.

HUSKEY, H. D. New topics in coding for SWAC. October 27.

HESTENES, M. R. (University of California, Los Angeles). A variational approach to linear partial differential equations of elliptic type, I. October 29. II. November 3. III. November 5.

SOUTHARD, T. H. On the military evaluation function for the Hotspot game (a war game of attrition). November 10.

TOMPKINS, C. B. A computer looks at Plateau's problem. I. November 12. II. November 17.

REDHEFFER, R. M. (University of California, Los Angeles). Operators and initial-value problems. November 24.

MOTZKIN, T. S. (University of California, Los Angeles). Preliminaries to the over-all coding of functions. December 1.

TEICHROEW, D. Use of "asymptotic" normalization polynomials in the computation of probability functions and probability points. December 3.

BUSK, T. (University of Copenhagen). Aspects of a generalized difference calculus. December 8.

MILES, J. W. (University of California, Los Angeles). Structural fatigue under random loading. December 10.

KAHN, L. B. (Naval Supply Center, Oakland, California). Forecasting activities and managing inventories at the U. S. Naval Supply Center, Oakland. December 15.

LEVIN, E. Review of Douglas' 1927 paper on numerical solution of Plateau's problem. December 17.

Statistical Engineering Colloquia

EISENHART, C. Some notes on the evolution of the statistical method as a research tool. October 16.

ZELEN, M. Methods of measuring useful life of equipment under operational conditions. October 30.

CONNOR, W. S. On the experimental attainment of optimum conditions, a discussion of the methods of G.E.P.Box of the Imperial Chemical Industries, Manchester, England. December 11.

SAVAGE, I. R. Most probable rank orders. December 18.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

DAVIS, P. Program in the theory of mechanical quadrature. Presented at the Mathematics Colloquium, University of Maryland, College Park, Md., December 18.

EISENHART, C. (1) Statistical method in research and development. A series of three lectures presented as part of "Mathematics for Modern Engineering" at the University of California, October 26-29. (2) A new look at the probable error--it never was what it used to be. Presented at a meeting of the Mathematics Division, Naval Ordnance Test Station, China Lake, California, October 28; at a meeting of Sigma Xi, Oregon State College, Corvallis, Oregon, November 3; and at a Mathematics Seminar, University of Washington,

Seattle, Washington, November 4. (3) The role of statistical method in scientific research. Presented at a Mathematics Seminar, University of Oregon, Eugene, Oregon, November 3. (4) Statistical method for the chemical, physical, and engineering sciences. Presented at the Engineering Institute, University of Wisconsin, Madison, Wisconsin, November 16. (5) Precision and accuracy. Presented to the Basic Science Division Study Group of the American Institute of Electrical Engineers, New York, N. Y., November 18. (6) Statistical method in experimentation. Presented at a Scientific Staff Meeting at the NBS, Washington, D. C., December 18.

FEKETE, M. The radii of plane domains and their applications. Presented by The American University in cooperation with the National Bureau of Standards at the Bureau, Washington, D. C., December 11.

FORSYTHE, G. E. (1) Best conditioned matrices. Presented at a meeting of the Mathematics Department, University of California, Los Angeles, November 25. (2) How should we solve a system of linear equations? Presented to the Institute for Certificated Personnel of the Los Angeles City Schools, Los Angeles City College, December 4.

SAVAGE, I. R. Statistics without assumptions. Presented to the Columbian Society, George Washington University, Washington, D. C., December 7.

TODD, J. Comparative theory of numerical solutions of differential equations. Presented at the Applied Physics Laboratory Colloquium, Johns Hopkins University, Silver Spring, Md., October 30.

TOMPKINS, C. B. The mathematician in Government establishments. Presented at the meeting of the American Mathematical Society cosponsored by the National Research Council, New York, N.Y., October 23.

YOUDEN, W. J. (1) Experimental design. Presented at a staff meeting of the Logistics Research Project, George Washington University, Washington, D. C., October 1; at a Symposium of the D-13 Committee of the American Society for Testing Materials, New York, N. Y., October 15; to the Study Group of the Basic Science Division of the American Institute of Electrical Engineers, New York, N. Y., November 25; and at a meeting of the local chapter of the American Chemical Society, York, Pa., December 3. (2) Behavior of measurements. Presented at a meeting of the Washington Area Section of the American Society for Stress Analysis, Washington, D. C., October 26; and at a Seminar at the Naval Ordnance Laboratory, White Oak, Silver Spring, Md., December 10. (3) Statistics and experimental work. Presented at the Squibb Institute for Medical Research, New Brunswick, N. J., November 4.

Papers presented at the meeting of the American Mathematical Society, Baltimore,Md., December 28-31:

HOFFMAN, A.J. (1) On the singular values of the product of two operators. (2) On an inequality of Hardy, Littlewood, and Pólya. Both presented by title.

HOFFMAN, A. J., M. NEWMAN, E. G. STRAUS, O. TAUSSKY. The number of absolute points of a correlation.

Papers presented at the joint annual meetings of the American Statistical Association, the Biometric Society, and the Institute of Mathematical Statistics, Washington, D. C., December 27-30:

CONNOR, W. S. New experimental designs for paired observations.

Dixon, W. J., and D. TEICHROEW. Some sampling results on the power of non-parametric tests against normal alternatives.

LIEBLEIN, J. A historical note on the relation between extreme values and tensile strength. Presented by title.

SAVAGE, I. R. Optimum nonparametric tests for small samples.

TEICHROEW, D. A method for generating random variates on electronic computers.

Publication Activities

1. PUBLICATIONS WHICH APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Tables of 10^x. NBS Applied Mathematics Series 27. Available from U. S. Government Printing Office, Washington 25, D. C., $3.50.

(2) Table of natural logarithms for arguments between zero and five to sixteen decimal places. NBS Applied Mathematics Series 31. Available from U. S. Government Printing Office, Washington 25, D. C., $3.25. (This is a reissue of NBS Mathematical Table MT10, Table of natural logarithms, vol. III (1941)).

(3) Tables of circular and hyperbolic sines and cosines for radian arguments. NBS Applied Mathematics Series 36. Available from U.S. Government Printing Office, Washington 25, D. C., $3.00. (This is a reissue of NBS Mathematical Table MT3 (1939, 2d ed. 1949)).

1.3 Technical Papers

(1) On the solution of the differential equation occurring in the problem of heat convection in laminar flow through a tube. M. Abramowitz. J. Math. Phys. XXXII, 184-187(July-Oct.1953).

(2) The convergence of numerical iteration. H. A. Antosiewicz and J. M. Hammersley. Am. Math. Mo. LX, 604-607(Nov. 1953).

(3) Changes of sign of sums of random variables. P. Erdös and G. A. Hunt. Pac. J. Math. 3, 673-687(Dec. 1953).

(4) A numerical analyst's fifteen-foot shelf. G. E. Forsythe. M.T.A.C. VII, 221-228(Oct. 1953).

(5) An expansion method for parabolic partial differential equations. J. W. Green. J. Res. NBS 51, 127-132(Sept. 1953).

(6) The SWAC--design features and operating experience. H. Huskey, R. Thorensen, B. F. Ambrosio, and E. C. Yowell. Proc. IRE 41, 1294-1299(Oct. 1953).

(7) Completely continuous normal operators with property L. I. Kaplansky. Pac. J. Math. 3, 721-724(Dec. 1953).

(8) Pairs of matrices with property L, II (summary). T. S. Motzkin and O. Taussky. Proc. Nat. Acad. Sci. 39, 961-963(Sept. 1953).

(9) Rejection of outlying observations. F. Proschan. Am. J. Phys. 21, 520-525(Oct. 1953).

(10) Modes of vibration of a suspended chain. D. S. Saxon and A. S. Cahn. Qu. J. Mech. App. Math. VI, Pt. 3, 273-285(1953).

(11) The magnetic field of cylindrical coils. C. Snow. NBS Applied Mathematics Series 38. Available from U. S. Government Printing Office, Washington 25, D. C., 25 cents.

(12) Asymptotic solution of the differential equation of hydrodynamic stability in a domain containing a transition point. W. Wasow. Ann. Math. 58, 222-252(Sept. 1953).

(13) On small disturbances of plane Couette flow. W. Wasow. J. Res. NBS 51, 195-202(Oct. 1953).

(14) The principles of experimental design. W. J. Youden. Appeared as a chapter in "Research Operations in Industry," (Papers delivered at the Third Annual Conference on Industrial Research, June 1953), pp. 317-329(Kings Crown Press, Columbia University, New York, N. Y., 1953).

(15) Making one measurement do the work of two. W. J. Youden and W. S. Connor. Chem. Eng. Prog. 49, 549-552(Oct. 1953).

1.4 Notes

(1) On a computation of the capacity of a cube. P. Davis and W. F. Cahill. M.T.A.C. VII, 272-273(Oct. 1953).

1.5 Miscellaneous Publications

(1) Mathematical services useful in industry. A. S. Cahn, Jr. Proceedings of Symposium on Industrial Applications of Automatic Computing Equipment held by the Midwest Research Institute, Kansas City, Mo., January 8, 1953; pp. 19-27.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION DECEMBER 31, 1953

2.1 Mathematical Tables

(1) Table of sine and cosine integrals for arguments from 10 to 100. NBS Applied Mathematics Series 32. (A reissue of NBS Mathematical Table MT13.) In press, Government Printing Office.

(2) Table of the Gamma function for complex arguments. NBS Applied Mathematics Series 34. In press, Government Printing Office.

(3) Tables of Lagrangian coefficients for sexagesimal interpolation. NBS Applied Mathematics Series 35. In press, Government Printing Office.

(4) Tables of functions and of zeros of functions. Volume I of Collected short tables of the Computation Laboratory. NBS Applied Mathematics Series 37. In press, Government Printing Office.

(5) Table of secants and cosecants to nine significant figures at hundredths of a degree. NBS Applied Mathematics Series 40. In press, Government Printing Office.

(6) Tables of the error function and its derivative. NBS Applied Mathematics Series 41. (A reissue of NBS Mathematical Table 8, Tables of probability functions, vol. I.) In press, Government Printing Office.

(7) Tables of expected values of 1/X for positive Bernoulli and Poisson variables. E. Grab and I. R. Savage. Accepted for publication in the Journal of the American Statistical Association.

(8) Tables of the inverses of finite segments of the Hilbert matrix. I. R. Savage and E. Lukacs. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

2.2 Manuals, Bibliographies, Indices

(1) The statistical theory of extreme values and some practical applications (A series of lectures by E. J. Gumbel). NBS Applied Mathematics Series 33; in press, Government Printing Office.

(2) Bibliography of nonparametric statistics and related topics. I. R. Savage. Accepted for publication in the Journal of the American Statistical Association.

2.3 Technical Papers

(1) Evaluation of the integral $\int_0^\infty e^{-u^2-(x/u)}du$. M. Abramowitz. Accepted for publication in the Journal of Mathematics and Physics.

(2) Regular and irregular Coulomb wave functions expressed in terms of Bessel-Clifford functions. M. Abramowitz. Accepted for publication in Journal of Mathematics and Physics.

(3) Approximate method for rapid Loran computation. M. Abramowitz, D. H. Call, and J. C. Mathews. Submitted to a technical journal.

(4) The relaxation method for linear inequalities. S. Agmon. Accepted for publication in the Canadian Journal of Mathematics.

(5) On the differential equation $\ddot{x} + k(f(x) + g(x)\dot{x})\dot{x} = ke(t)$. H. A. Antosiewicz. Submitted to a technical journal.

(6) On a certain integral involving Bessel functions. H. Antosiewicz. Submitted to a technical journal.

(7) Some implications of Liapunov's conditions for stability. H. A. Antosiewicz and P. Davis. Submitted to a technical journal.

(8) On a theorem of Ostrowski and Taussky. R. Bellman and A. J. Hoffman. Accepted for publication in Archiv der Mathematik.

(9) On mildly nonlinear partial difference equations of elliptic type. L. Bers. Accepted for publication in the Journal of Research of the NBS.

(10) On modified divided differences. G. Blanch. Accepted for publication in Mathematical Tables and Other Aids to Computation.

(11) A general-purpose control panel for a model II CPC. P. B. Bremer, D. Teichroew, and E. C. Yowell. To be published in the IBM Newsletter.

(12) Programs for computing hypergeometric series. W. F. Cahill. Accepted for publication in Mathematical Tables and Other Aids to Computation.

(13) Acceptance sampling of electroplated articles. J. M. Cameron and F. Ogburn. Accepted for publication in Plating.

(14) Partially balanced incomplete block designs with two associate classes and two treatments per block. W. H. Clatworthy. Submitted to a technical journal.

(15) Some theorems for partially balanced designs. W. S. Connor and W. H. Clatworthy. Accepted for publication in the Annals of Mathematical Statistics.

(16) Time-discrete stochastic processes in arbitrary sets, with applications to processes with absorbing regions and to the problem of loops in Markoff chains. D. van Dantzig. Submitted to a technical journal.

(17) Linear functional equations and interpolation series. P. Davis. Submitted to a technical journal.

(18) On a problem in the theory of mechanical quadratures. P. Davis. Accepted for publication in the Pacific Journal of Mathematics.

(19) Some L^2 aspects of Faber polynomials. P. Davis and H. Pollack. Submitted to a technical journal.

(20) On representations and extensions of bounded linear functionals defined on classes of analytic functions. P. Davis and J. L. Walsh. Submitted to a technical journal.

(21) On the optimal character of the (s,S) policy in inventory theory. A. Dvoretzky, J. Kiefer, and J. Wolfowitz. Submitted to a technical journal.

(22) A test for statistical control applicable to a short series of observations. C. Eisenhart and E. P. King. Submitted to a technical journal.

(23) Inequalities for eigenvalues of Hermitian matrices. K. Fan. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(24) Some remarks on commutators of matrices. K. Fan. Accepted for publication in Archiv der Mathematik.

(25) Lower bounds for the rank and location of the eigenvalues of a matrix. K. Fan and A. J. Hoffman. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(26) Some metric inequalities in the space of matrices. K. Fan and A. J. Hoffman. Submitted to a technical journal.

(27) Punched-card experiments with accelerated gradient methods for linear equations. A. I. and G. E. Forsythe. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(28) Asymptotic lower bounds for the frequencies of polynomial membranes. G. E. Forsythe. Submitted to a technical journal.

(29) SWAC. G. E. Forsythe. Submitted to a technical journal.

(30) Practical solution of linear equations and inversion of matrices. L. Fox. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues. NBS Applied Mathematics Series 39.

(31) A numerical solution of Schroedinger's equation in the continuum. W. Futterman, E. Osborne, and D. S. Saxon. Accepted for publication in the Journal of Research of the NBS.

(32) Linear programming in bid evaluation. L.Gainen, D. J. Honig, and E. D. Stanley. Accepted for publication in Logistics Research Quarterly.

(33) A nonlinear model for the composite pi-meson. S. G. Gasiorowicz. Submitted to a technical journal.

(34) Application of variational methods to intermediate and high energy scattering. E. Gerjuoy and D. S. Saxon. Submitted to a technical journal.

(35) Variational principles for the acoustic field. E. Gerjuoy and D. S. Saxon. Submitted to a technical journal.

(36)A method of solving the transportation problem. A. N. Gleyzal. Submitted to a technical journal.

(37) Equations of physics in general Newtonian space-time. A. N. Gleyzal. Submitted to a technical journal.

(38) The representation of integers by binary quadratic rational forms. K. Goldberg, M. Newman, E. G. Straus, and J. D. Swift. Accepted for publication in Archiv der Mathematik.

(39) An embedding theorem for balanced incomplete block designs. M. Hall, Jr., and W. S. Connor. Accepted for publication in the Canadian Journal of Mathematics.

(40) Iterative methods of solving linear problems on Hilbert space. R. M. Hayes. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(41) On certain series expansions involving Whittaker functions and Jacobi polynomials. P. Henrici. Submitted to a technical journal.

(42) Ueber die Funktionen von Gegenbauer. P. Henrici. To appear in Archiv der Mathematik.

(43) On a combinatorial theorem. A. J. Hoffman. Submitted to a technical journal.

(44) A characterization of normal matrices. A. J. Hoffman and O. Taussky. Accepted for publication in the Journal of Research of the NBS.

(45) On optimum grouping in one-criterion variance components analysis. E. P. King. Submitted to a technical journal.

(46) Probability limits for the average chart when process standards are unspecified. E. P. King. Submitted to a technical journal.

(47) A property of the normal distribution. E. P. King and E. Lukacs. Submitted to a technical journal.

(48) On certain character matrices. D. H. Lehmer. Submitted to a technical journal.

(49) Acoustic radiation pressure on a circular disk. H. Levine. To appear in the Proceedings of the Fifth Symposium on Applied Mathematics of the American Mathematical Society.

(50) On the variances and covariances of order statistics from the Weibull distribution. J. Lieblein. Submitted to a technical journal.

(51) On strongly continuous stochastic processes. E. Lukacs. Accepted for publication in Sankhya.

(52) Certain Fourier transforms of distributions (II). E. Lukacs and O. Szász. Accepted for publication in the Canadian Journal of Mathematics.

(53) Nonnegative trigonometric polynomials and certain rational characteristic functions. E. Lukacs and O. Szász. Accepted for phblication in the Journal of Research of the NBS.

(54) A proof of Hilbert's Nullstellensatz. T. S. Motzkin. Submitted to a technical journal.

(55) On Fejér sets in linear and spherical spaces. T. S. Motzkin and I. J. Schoenberg. Accepted for publication in Annals of Mathematics.

(56) On the relaxation method for linear inequalities. T. S. Motzkin and I. J. Schoenberg. Submitted to a technical journal.

(57) Least p-th power polynomials on a real finite point set. T. S. Motzkin and J. L. Walsh. Submitted to a technical journal.

(58) Structure theorems and identities for modular subgroups. M. Newman. Submitted to a technical journal.

(59) The arithmetic structure of certain modular subgroups. M. Newman. Submitted to a technical journal.

(60) On the Lerch zeta function. F. Oberhettinger. Submitted to a technical journal.

(61) On two problems in abstract algebra connected with Horner's rule. A. M. Ostrowski. Submitted to a technical journal.

(62) On nearly triangular matrices. A. Ostrowski. Submitted to a technical journal.

(63) Determinanten mit ueberwiegender Hauptdiagonale und die absolute Konvergenz von linearen Iterationsprozessen. A. M. Ostrowski. Submitted to a technical journal.

(64) On the convergence of Gauss' alternating procedure in the method of the least squares, I. A. M. Ostrowski. Submitted to a technical journal.

(65) On the linear iteration procedures for symmetric matrices. A.M. Ostrowski. Submitted to a technical journal.

(66) On the spectrum of a one parametric family of matrices. A. M. Ostrowski. Submitted to a technical journal.

(67) On Gauss' speeding up device in the theory of single step iteration. A. M. Ostrowski. Submitted to a technical journal.

(68) On absolute convergence of linear iteration processes. A. M. Ostrowski. Submitted to a technical journal.

(69) On spectra of second-order differential operators. D. Ray. Submitted to a technical journal.

(70) On the convergence of asymptotic solutions of linear differential equations. R. M. Redheffer and W. Wasow. Submitted to a technical journal.

(71) Numerical computation of low moments of order statistics from a normal population. J. B. Rosser. Submitted to a technical journal.

(72) An isoperimetric inequality for closed curves convex in even-dimensional Euclidean space. I. J. Schoenberg. Submitted to a technical journal.

(73) Characteristic roots of quaternion matrices. O. Taussky. Accepted for publication in Archiv der Mathematik.

(74) Generalized commutators of matrices and permutations of factors in a product of three matrices. O. Taussky. For publication in the von Mises Anniversary volume.

(75) Some computational problems in algebraic number theory. O.Taussky. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 26-28, 1953.

(76) An improved cathode ray tube storage system. R. Thorensen. To appear in the Proceedings of the Western Computer Conference of the AIEE-IRE-ACM held in Los Angeles, Calif., February 4,5,6,1953.

(77) The condition of certain matrices, II. J. Todd. To appear in Archiv der Mathematik.

(78) The condition of the finite segments of the Hilbert matrix. J. Todd. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39.

(79) On the accuracy of the numerical solution of the Dirichlet problem by finite differences. J. L. Walsh and D. Young. Accepted for publication in the Journal of Research of the NBS.

(80) On the asymptotic transformation of certain distributions into the normal distribution. W. Wasow. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 26-28, 1953.

(81) Singular perturbation methods for nonlinear oscillations. W. Wasow. To appear in the Proceedings of a Symposium on Nonlinear Circuit Analysis, held by the Polytechnic Institute of Brooklyn, N. Y.

(82) An extremum property of sums of eigenvalues. H. Wielandt. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 26-28, 1953.

(83) Einschliessung von Eigenwerten hermitescher Matrizen nach dem Abschnittsverfahren. H. Wielandt. To appear in Archiv der Mathematik.

(84) Error bounds for the eigenvalues of symmetric integral equations. H. Wielandt. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 26-28, 1953.

(85) On eigenvalues of sums of normal matrices. H. Wielandt. Submitted to a technical journal.

(86) On the eigenvalues of A+B and AB. H. Wielandt. Submitted to a technical journal.

(87) Performance of inspectors and gasoline pumps. W. J. Youden and M. W. Jensen. Submitted to a technical journal.

(88) A note on partially balanced designs. M. Zelen. Accepted for publication in Annals of Mathematical Statistics.

(89) Analysis for some incomplete block designs having a missing block. M. Zelen. Accepted for publication in Biometrics.

(90) Bounds on a distribution function, which are functions of moments to order four. M. Zelen. Submitted to a technical journal.

2.4 Reviews, Notes

(1) Note on the circle theorem of hydrodynamics. E. Levin. Submitted to a technical journal.

(2) Two early papers on the relation between extreme values and tensile strength. (Formerly "A historical note on the application of the 'weakest link' idea to tensile strengths.") J. Lieblein. Submitted to a technical journal.

2.5 Miscellaneous Publications

(1) Contributions to the solution of systems of linear equations and the determination of eigenvalues. NBS Applied Mathematics Series 39. In press, Government Printing Office.

(2) Experiments in the computation of conformal maps. To appear in the NBS Applied Mathematics Series.

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($0.75), available from the Superintendent of Documents, Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.


NBS