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Progress Report for Jan. - March 1953

on

Applications of the Theory of Stochastic Processes to the Study of Trajectories

(NBS Project 1103-21-5119)

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APPLICATION OF THE THEORY OF STOCHASTIC PROCESSES TO THE STUDY OF TRAJECTORIES

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## I. Summary

This report contains a summary of the work done during the quarter. Results of this work are briefly stated. Technical reports written in connection with this project are mentioned but are transmitted separately.

## II. Discussion of work done during the quarter.

It was found desirable to construct an artificial trajectory which would permit to demonstrate how the mean value function of a fundamental random process could be estimated. A polynomial of degree four was selected to represent the "true" path and equidistant points on this curve were determined. Suitably chosen random numbers were attached as "observational errors" to the coordinates of this curve. In this manner "simulated observations" were obtained which will be used in studying the proposed estimation procedures.

A stochastic process y(t) is studied and it is assumed that

$$y(t) = x(t) + f(t) \text{ where}$$

$$f(t) = K_1\phi_1(t) + K_2\phi(t) + \ldots + K_s\phi(t)$$

and where x(t) is a fundamental random process (Wiener process). The first problem is the estimation of the parameters $K_1, K_2, \ldots, K_s$ on the basis of observations taken on y(t) over a time interval [0,T]. In the proposed example the "simulated observations" will represent the y(t) process.

It is known that estimates of the parameters $K_1 \ldots K_s$ can be obtained in the following manner

Let $||S^{jr}||$ be the inverse of the matrix $\left|\left|\frac{1}{j+r-1}\right|\right| \quad j,r=1,\ldots s$

and denote by $\phi^{jr} = \frac{S^{jr}}{jrT^{j+r-1}}$, the estimate $\hat{K}_j$ of $K_j$ is then found to be

$$\hat{K}_j = \sum_{r=1}^{s} \phi^{jr}\, r \int_0^T t^{r-1} dy(t) \qquad (j=1,2,\ldots s)$$

This method of estimating the mean value curve f(t) becomes quite convenient as soon as tables of the matrix $||S^{jr}||$ are available. It became therefore necessary to compute tables of this matrix. This was done by I. R. Savage and E. Lukacs in N.B.S. report No. 2279 entitled "Tables useful in estimating the mean value function of a fundamental random process". Since the tables and their method of computation are of some independent interest a second paper entitled "Tables of inverses of finite segments of the Hilbert Matrix" was prepared by the same authors. This is a condensed version of NBS report 2279 and it is intended to submit it to a technical journal.

It might be worthwhile to remark that it seems that the results obtained during this quarter will also be applicable to work under progress in the Bureau's Ordnance Development Division. A paper by Eugene Lukacs "On strongly continuous stochastic processes" was submitted to a technical journal. This paper is the result of the author's work performed in the previous quarter.

This is a special progress report to the U.S. Naval Ordnance Test Station which sponsors NBS Project 1103-21-5119, Application of the theory of stochastic processes to the study of trajectories.

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