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TO A SHORT SERIES OF OBSERVATIONS

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Churchill Eisenhart and Edgar P. King

U. S. DEPARTMENT OF COMMERCE
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Introduction

In what follows we shall call attention to a useful test for statistical control based on the ratio $\left(r = \frac{R_1}{R_2}\right)$ of two ranges. This test has the attractive features that it is simple and it is applicable to a small amount of data.

Procedure

Given a sequence of 2n measurements $x_1, x_2, \ldots, x_{2n}$ taken in time-order (space-order, etc.) by a particular measurement process, form the sums and differences of successive, non-overlapping pairs from this sequence--obtaining the two columns:

sum	difference
$s_1 = x_1 + x_2$	$d_1 = x_1 - x_2$
$s_2 = x_3 + x_4$	$d_2 = x_3 - x_4$
. . . . . .	. . . .
. . . . . .	. . . .
$s_n = x_{2n-1} + x_{2n}$	$d_n = x_{2n-1} - x_{2n}$

Now compute the range of the sums (R_1), the range of the differences (R_2), and form their ratio $r = R_1/R_2$. If the computed value of r is less than the critical value given in Table I (attached) corresponding to n, accept the hypothesis that the measurement process is in a state of statistical control; otherwise, reject this hypothesis.

If $x_1, x_2, \ldots, x_{2n}$ is a sequence of independent measurements on a process in statistical control, then the sums should exhibt the same variability as the differences. We would hence except the ratio "r" to be "near" unity. On the other hand, if some disturbance is present that tends to make x's that are close together in time (or in space, etc.) agree more

closely than x's that are more widely separated in time (or space, etc.), then there will be a corresponding tendency for "unusually large" values of r to result.

Examples

1. Suppose we apply this test to the following sequence taken from a table of random numbers.

   $x_1 = 19$, $x_2 = 39$, $x_3 = 64$, $x_4 = 92$, $x_5 = 3$, $x_6 = 61$, $x_7 = 14$

   $x_8 = 75$, $x_9 = 90$, $x_{10} = 55$

   From this sequence we compute the following sums and differences:

sum	difference
58	-20
156	-28
64	-58
89	-61
145	35

   We then obtain $R_1 = 156-58 = 98$ and $R_2 = 35 - (-61) = 96$, and hence

   $$r = \frac{98}{96} = 1.02$$

   The critical value for r at the 5 percent level corresponding to n=5 is found in Table I to be 2.5. Since 1.02 is less than 2.5 we conclude that the sequence exhibts statistical control.

2. The following sequence was obtained from a table of random normal numbers after "building in" a linear trend.

   $x_1 = .46$, $x_2 = 1.06$, $x_3 = 3.49$, $x_4 = 4.02$
   $x_5 = 5.39$, $x_6 = 5.14$, $x_7 = 3.47$, $x_8 = 6.65$,
   $x_9 = 7.53$, $x_{10} = 8.44$

Applying the same test in this case yields:

sums	differences
1.52	- .60
7.51	- .53
10.53	.25
10.12	-3.18
15.97	- .91

$$R_1 = 15.97 - 1.52 = 14.45$$

$$R_2 = .25 - (-3.18) = 3.43$$

$$r = \frac{14.45}{3.43} = 4.21$$

Since 4.21 exceeds the critical value 2.5, we correctly conclude that a disturbance of some sort is present, and the figures themselves suggest an upward trend.

Concluding Remarks

A few remarks about the above procedure should be made.

1) The critical values in Table I assume normality of the underlying distribution.

2) Systematic errors that tend to make the x's that are near together in the sequence more dissimilar than x's that are more widely separated will ordinarily make "r" unusually small. This situation was neglected in the above discussion because it seldom occurs.

3) The "r" test uses an even number of measurements. With an odd number you should, of course, neglect one.