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Progress Report for July-Sept. 1952

on

Applications of the Theory of Stochastic Processes to the Study of Trajectories

(NBS Project 1103-21-5119)

NBS

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

This is a special progress report to the U.S. Naval Ordnance Test Station which sponsors NBS Project 1103-21-5119, Application of the theory of stochastic processes to the study of trajectories.

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PROGRESS REPORT FOR JULY-SEPT. 1952

ON

APPLICATION OF THE THEORY OF STOCHASTIC PROCESSES TO THE STUDY OF TRAJECTORIES

(NBS Project 1103-21-5119)

I. Summary

This report contains a summary of the work done during the quarter. Results of this work are briefly stated, Technical reports written in connection with this project are mentioned but are transmitted separately.

II. Discussion of work done during the quarter.

The study of strongly continuous stochastic processes was continued during this quarter. A more intuitive formulation of the theorem proven in NBS report 1665 was found. Using a recent result [Sankhya 11, 1951] of Professor Mann it can be stated in the following manner: A stochastic process with independent increments which can be realized in the space of continuous functions has normally distributed increments. This reformulation has of course no theoretical significance, it should however appeal more to people interested in applications since it avoids the use of the formal definition of strong continuity. This formulation should also be useful in explaining why the assumption of normality of the increments can be made without hesitation on physical grounds if one deals with trajectory data. A characterization of the Wiener process was given by proving the following theorem:

A stochastic process $y(t)$ is a Wiener process if and only if the following three conditions are satisfied

(a) $y(t)$ is a strongly continuous process with independent increments and initial value $y(0) = 0$.

(b) The mean value function of the process is identically zero.

(c) The variance of the increments $y(t+\tau) - y(t)$ depends only on the length τ of the interval over which the increment is taken but is independent of the location of the interval on the time axis.

The question arose whether assumptions (c) is independent of assumptions (a) and (b). This was answered by constructing processes with factorable variance function. The following theorem was proved:

Let y(t) be a stochastic process, defined for t = 0 and denote by $f(t,\tau)$ the variance of the increment $[y(t+\tau) - y(t)]$. Assume that

(i) y(t) is a process with independent increments and is strongly continuous in every finite interval [a,b] where $0 \leqq a < b$,

(ii) the mean value function of y(t) is identically zero, i.e. Ey(t) = 0,

(iii) the initial value is zero, y(0) = 0,

(iv) $f(t,\tau) > 0$ for $t \geqq 0$ and $\tau > 0$

(v) $f(t,\tau) = h(t)\, g(\tau)$

Then y(t) is a Gaussian process with covariance function

$$(3) \quad \sigma_{y(t_1)y(t_2)} = \sigma^2_{y(t)} = c\,\frac{e^{Kt}-1}{e^K-1}$$

where $t = \min(t_1, t_2)$.

This theorem shows that there exist strongly continuous processes with independent increments which are different from the Wiener process. These processes could become useful because they offer an alternative to the Wiener process which is different from the well known Ornstein - Uhlenbeck process. However our study of the possible application of the theory of stochastic processes to trajectory data does not indicate at present any need for such an alternative.

It is intended to condense the results obtained in NBS reports 1665 and 1846 into a single paper on strongly continuous stochastic processes and to submit this paper to a technical journal for publication.

During the last visit of E. Lukacs and Professor H. B. Mann at Inyokern problems connected with the estimation of the mean value function of a Wiener process were discussed extensively. It was agreed that it would be desirable to demonstrate the procedure by giving a complete numerical example. Some preliminary work was done on constructing a suitable numerical example.