ACTIVITIES IN APPLIED MATHEMATICS

1946-1947

NATIONAL APPLIED MATHEMATICS LABORATORIES
OF THE
NATIONAL BUREAU OF STANDARDS

PREFACE

This is a summary of the mathematical activities carried on during the fiscal year 1947 in a unit of the Office of the Director of the National Bureau of Standards. This unit of the Office of the Director was known within the Bureau by the code designation DA-2, and included as a component the well-known Mathematical Tables Project in New York City. The entire unit has since been incorporated into a new Division of the Bureau known as the National Applied Mathematics Laboratories.

The projects described in this report are representative of the present work of the National Applied Mathematics Laboratories. It is the mission of the Laboratories to perform research and provide services in various quantitative branches of mathematics, placing special emphasis on the development and exploitation of high-speed numerical analysis and modern statistical methodology. The Laboratories maintain an expert computing service of large capacity, and provide consulting services in classical applied mathematics and in mathematical statistics. These services are available primarily to other federal agencies, but under certain circumstances it is possible to perform work for industrial laboratories and universities.

Inquiries concerning the availability of the services of the National Applied Mathematics Laboratories, or concerning the details of any of the projects described in this report, should be addressed to the National Applied Mathematics Laboratories, 415 South Building, National Bureau of Standards, Washington 25, D.C.

J. H. Curtiss, Chief
National Applied Mathematics
Laboratories

Approved:

E. U. Condon, Director
National Bureau of Standards
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U. S. DEPARTMENT OF COMMERCE
W. Averell Harriman, Secretary

NATIONAL BUREAU OF STANDARDS
E. U. Condon, Director

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CONTENTS

1. ADMINISTRATIVE AND PLANNING ACTIVITIES

The Office of the Assistant to the Director, DA-2, consisted during the fiscal year 1947 of the organization set forth in Table I. It will be noted from the table that certain units were activated during the fiscal year. The growth in the Mathematical Tables Project was the result of taking over the New York Project of the Navy Department Hydrographic Office, which throughout the war had been supervised by personnel of the Mathematical Tables Project on loan.

Table I. Organization of DA-2

Unit	Number of Persons	
	1 July 1946	30 June 1947
1. Administrative (Washington)	3	5
2. Mathematical Tables Project (150 Nassau St., NYC)	48	60
3. Statistical Engineering Group (Washington)	0	6
4. Machine Development Group (Washington)	0	4

The administrative head of these mathematical activities was Dr. J. H. Curtiss, Assistant to the Director. The Director of the Mathematical Tables Project was Dr. Arnold N. Lowan, and his two

chief assistants were Dr. Gertrude Blanch and Mr. Milton Abramowitz. The chief of the statistical group was Dr. Churchill Eisenhart, who reported for duty on 1 October 1946. The head of the Machine Development Group was Dr. E. W. Cannon, who reported for duty on 1 July 1946. His chief assistant was Mr. Albert Cahn, who joined the Bureau staff on 13 January 1947.

The efforts of the administrative unit were focused during the fiscal year on the task of making plans for a new division of the Bureau, which was to contain the above organization, and which was to be so designed that it could serve as a central applied mathematics laboratory for the entire Federal scientific program.

The idea of a central computation laboratory with an auxiliary mission of developing high speed computing machines had arisen in the Planning Division of the Office of Research and Invention (now the Office of Naval Research) of the Navy Department in 1945. In the early spring of 1946, the Chief of Naval Research had enlisted the aid of the Bureau of Standards in studying the problem of establishing such a facility.

The study soon revealed that something more fundamental than merely a large computation laboratory was needed, and that the development of high-speed tools for computation carried with it an obligation to perform research in the theory and disciplines needed for the most effective use of these tools. The investigation was brought to a conclusion in the fiscal year 1947 with the publication of a document entitled "The National Applied Mathematics Laboratories - A Prospectus". The functions of the Laboratories, as set forth in the Prospectus, would be as follows:

(a) to conduct basic research in various fields of mathematics heavily drawn upon by the physical and engineering sciences, placing special emphasis on the development and exploitation of high-speed numerical analysis in these fields;

(b) to provide a consulting service in special problems in such fields of mathematics;

(c) to develop and construct tools (such as tables and automatic high-speed computing machines) for work in such fields of mathematics;

(d) to conduct theoretical and in-service training in such fields of mathematics, and in particular, in the disciplines needed for the effective use of automatic high-speed computing machines;

(e) to provide an extensive general-purpose computing service specializing in digital methods of calculation;

(f) to prepare reports, monographs, manuals, indices, and other types of publications, setting forth the results of the work of the Laboratories and providing expository treatments of technical material lying within the scope of the Laboratories.

As a corollary to these activities, the Laboratories would provide facilities for competent scholars to investigate and develop in their own special fields of research the mathematical techniques studied by the Laboratories. The major items of equipment would be three automatic electronic digital computing machines. One of these would be the machine being constructed by the Bureau for the Office of Naval Research (see Section 3 of this Report).

Table 2. Organization of the Proposed National Applied Mathematics Laboratories

Section	Man years	
	Fiscal 1948	Fiscal 1949 and thereafter
0. Administration (Washington, D.C.)	6.00	6.00
1. Institute for Numerical Analysis (California)	25.00*	28.50*
2. Computation Laboratory (New York City and Washington, D.C.)	61.00	47.00
3. Statistical Engineering Laboratory (Washington, D.C.)	6.32	8.32
4. Machine Development Laboratory (Mathematics Group) (Washington, D.C.)	4.00	4.00
Totals	102.32	93.82

*It is expected that there will be some part-time employees, so the number of persons on duty in Section I will be somewhat in excess of these figures.

An outline of the organizational plan proposed in the "Prospectus" is given in Table 2. A part of the plan, not shown in the table, consisted in forming an "Applied Mathematics Executive Council" composed of representatives of interested Government agencies and private organizations. The members were to be appointed by the Director of the Bureau. The functions of the Council would be to give advice and guidance at the policy level

and to approve the budget.

It was decided after a prolonged investigation of sites for Section 1 to place this unit on the campus of the University of California at Los Angeles. The controlling factors were a favorable offer from UCLA for permanent quarters, and the interest of the Army Air Forces in having a computing center available in Southern California.

The "Prospectus" was approved by the appropriate authorities in the Commerce Department and Navy Department. Representatives of a number of other agencies and private institutions also indicated general concurrence. The "National Applied Mathematics Laboratories" were accordingly established along the lines of the "Prospectus" as Division 11 of the National Bureau of Standards on 1 July 1947.

Other important activities in the field of mathematics not mentioned specifically in the later sections of this report were the following:

The Establishing of the Applied Mathematics Colloquium Series

This is a series of lectures on applied mathematics intended primarily for Bureau personnel, and given by outstanding applied mathematicians connected with other institutions. The fiscal year 1947 was devoted to applied mathematical statistics. The lectures and attendance were as follows:

Lecture	Attendance
"The Combination of Data from Tests Conducted at Different Laboratories" by Prof. P.C. Mahalanobis, F.R.S.	100
"Statistical Quality Control" by Prof. Holbrook Working	65

Lecture (continued)	Attendance
"Some Problems in the Analysis of Sets of Consecutive Measurements" by Dr. Merrill M. Flood	104
"Random Processes" by Prof. Harald Cramer	97
"A Statistical Technique for Analytical Data" by Dr. William J. Youden	75

The Initiation of The Applied Mathematics Series

This new series of publications of the Bureau is intended to serve as a vehicle for manuals, expository treatises, and tables produced by the National Applied Mathematics Laboratories. The first number of the series, which should appear in the fall of 1947, is a mathematical table particularly useful in nuclear physics in the construction of certain types of chain-reacting piles. The title is

"Tables of the Bessel Functions $Y_0(x), Y_1(x), K_0(x), K_1(x), 0 \leq x \leq 1$"

Courses given in the Graduate School of the National Bureau of Standards

"Elements of Statistical Inference" by Churchill Eisenhart; two terms; attendance: 74 students, in 1st term; 59 students in 2nd term.

"Correlation and Regression" by Dr. J.H. Curtiss; one term; attendance: 18 students.

"Vector Analysis" by Dr. E.W. Cannon; one term; attendance: 28 students.

"Modern Operational Methods in Engineering" by Dr. E.W. Cannon; one term; attendance: 8 students.

"The Differential Equations of Mathematical Physics" by Dr. E.W. Cannon; two terms; attendance: 8 students, first term; 9 students, 2nd term.

In addition, a seminar on numerical analysis and related

aspects of function theory was conducted at the Mathematical Tables Project.

2. RESEARCH AND DEVELOPMENT PROJECTS COMPLETED DURING THE FISCAL YEAR 1947

(1) Tables of Spherical Scattering Functions for Complex Arguments, for Naval Research Laboratory.

(2) Computations related to the hydraulic analogy of shock wave intersections, for the Bureau of Ordnance, Navy Department.

(3) Estimation of parameters of distribution of maximum wind velocities at various stations, for the Weather Bureau.

(4) Various sets of Loran Tables for the Hydrographic Office, Navy Department.

(5) Table of Spherical Bessel Functions, Vol. II.

(6) Tables of the function

$$E_n(x) = \int_1^\infty (e^{-xu}/u^n)\,du$$

(7) Tables of circumferences and areas of circles to six decimal places, for the Bureau of Ordnance, Navy Department.

(8) Tables of the altitude and azimuth for selected groups of six stars, for the Hydrographic Office, Navy Department.

(9) Checking of tables in forthcoming OSRD book entitled "Selected Techniques of Statistical Analysis".

(10) Tables of

$$E_1(z) = \int_z^\infty (e^{-u}/u)\,du$$

(11) Solutions of the equation of the "Human Centrifuge"

$$\dot{w}^2 + w^4 = (w_o + \alpha t)^4$$

for various values of w_o and α. (The project included research

leading to the reduction of the problem to the solution of

$$\dot{w}^2 + w^4 = t^4$$

for which an asymptotic expansion was obtained.) For Special Devices Center, Office of Naval Research, Navy Department.

(12) Tables of the confluent hypergeometric function. (The Project included research leading to new approximations to the function.) For the Statistical Research Group, Columbia University (OSRD contractors).

(13) Tables of the Bessel Functions $J_\nu(x)$ for fractional values of ν.

(14) Tables of operating characteristics of certain double sampling plans, for the War Department.

(15) Tables of conversion angles for converting a rhumb line course into a great circle course, for the Hydrographic Office of the Navy Department

(16) Table of intensity functions for complex indices of refraction.

(17) Formulas for the percentage points of the distribution of the arithmetic mean in random samples from certain symmetrical universes.

(18) "Acceptance Sampling by Variables, with Special Reference to the Case in which Quality is Measured by Average or Dispersion" by J. H. Curtiss. To be published in the Journal of Research and in a Supplement to the Journal of the American Statistical Association.

(19) "The Assumptions Underlying the Analysis of Variance" by Churchill Eisenhart. Published in Biometrics, March 1947.

3. RESEARCH AND DEVELOPMENT PROJECTS UNDER WAY AT THE END OF THE FISCAL YEAR 1947

A. Projects on Which Substantial Progress Was Made During the Fiscal Year.

(1) *Mathieu Functions I.*

An 8-place table of the first fifteen odd and sixteen even characteristic values b of the Mathieu differential equation

$$\ddot{y} + (b - s\cos^2 t)\, y = 0$$

for s ranging from 0 to 100 at various intervals, was published for limited circulation in AMP report 165.1R in September, 1945. The manuscript of the Fourier coefficients for periodic solutions of period π and 2π has been completed. This manuscript will be combined in one volume with the table of characteristic values, and certain joining factors. An introduction to this volume was in process of preparation at the end of the fiscal year 1947.

(2) *Mathieu Functions II*

The periodic solutions

$$Se_r(s,t) = \sum_{n=0}^{\infty} De_{2n+p} \cos(2n+p)\,t$$

$$So_r(s,t) = \sum_{n=1}^{\infty} Do_{2n-p} \sin(2n-p)\,t$$

corresponding to the characteristic values under (1) are being tabulated for p = 0 or 1; r = 0(1)15; t = 0°(1°)90°, to six decimal places over the same range of the parameter s as in the tables under (1). This work is being carried out on IBM machines and was approximately 35% completed at the end of the fiscal year.

(3) *Tables of Bessel Functions $I_\nu(x)$; $\pm\nu$ = 1/3, 2/3, 1/4, 3/4.*

This is intended as a companion volume to the one under

COMPLETED PROJECTS (13), containing values of $J_\nu(x)$, and will be of similar scope. Differencing of the tables of $I_\nu(x)$ was 25 per cent completed at the end of the fiscal year.

(4) *Tables of Intensity Functions*

Computation for certain given complex indices of refraction have been completed. A final manuscript of the results was being prepared at the end of the fiscal year. A detailed description of the material for both real and complex indices of refraction with the definition of the tabulated functions is contained in the July, 1946 issue of *Mathematical Tables and Other Aids to Computation.*

(5) *Tables of* $E_1(z)$, $(z = x + iy)$

$$E_1(z) = \int_z^\infty (e^{-u}/u)\,du$$

Table I. $E_1(z) + \log_e z$, $x = [0(.01)1]$; $y = [0(.02)1;$ 6D]

Table II. $E_1(z)$, $x = [0(.02)4]$; $y = [0(.02)3(.05)10;$ 6D]

Table III. $e^x E_1(z)$, $x = [0(.02)4]$; $y = [0(.02)3(.05)10;$ 6D]

Table I is completed. Tables II and III were about 75 per cent completed at the end of the fiscal year.

(6) *Table of Jacobi Elliptic Functions*

Work on tables of $sn(u,k)$, $cn(u,k)$, $dn(u,k)$ for $k^2 = 0(.01)1$,

$$u = pK,\ p = 0(.01)1,\ K = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1 - K^2 \sin^2\varphi}}$$ was

about 50 percent completed at the end of the fiscal year.

(7) *Tables of Bessel Functions* $Y_0(z)$ *and* $Y_1(z)$.

The tables give the real and imaginary parts of the functions $Y_0(z)$ and $Y_1(z)$ for $z = \rho e^{i\varphi}$, $\rho = [0(.01)10]$; $\varphi = [0^\circ(5^\circ)90^\circ]$.

Punch cards for all entries have been prepared. Differencing of the table was being carried out on the tabulator during June, 1947.

(8) *Table of Gamma Functions for Complex Arguments*

Computations of $\log\Gamma(z)$, $z = x + iy$ for $x = [9(.1)10]$; $y = [0(.1)10]$ were about 35 per cent completed at the end of the fiscal year. They will serve as pivotal values to obtain $\log\Gamma(z)$ for $x = [0(.1)10]$; $y = [0(.1)10]$.

(9) *Tables of Characteristic Values for Spheroidal Wave Functions*

Systematic work was in progress on the determination of the characteristic values of the wave equation for the case of the prolate spheroid during the latter part of the fiscal year. The differential equation under consideration is:

$$(1-x^2)\ w'' - 2(m+1)\ xw' + (b - c^2x^2)\ w = 0$$

In June, 1947, work was proceeding on the characteristic values for $m = 1$, for orders $l = 0,1,2,3$. For $m = 1$, $l = 0$, the characteristic values have been computed up to $c^2 = 600$ at various intervals in c^2. For $l = 1,2,3$, the values have been computed up to $c^2 = 60$. Preparatory work had been carried out on the characteristic values for $m = 0$.

(10) *Tables of the Coulomb Wave Functions*

Systematic work is in progress on the tabulation of the regular solution $F_L(\eta,\rho)$ of the differential equation

$$\frac{d^2y}{d\rho^2} + \left(1 - \frac{2\eta}{\rho} - \frac{L(L+1)}{\rho^2}\right) y = 0$$

which arises in the quantum mechanical treatment of the problem of two particles moving in a Coulomb field of force. Calculations are now being carried out for the regular solution

$$F_L = C_L\rho^{L+1}\varphi_L,\quad F_L' = C_L\rho^L\varphi_L^* \text{ for } L = 0,\ \eta = 0(1)10$$

$\rho = 0(.1)10$. For the definition of φ_L^* and φ_L see the article "Coulomb Wave Functions in Repulsive Fields"- Physical Review, vol. 49, Jan. 1936, by Yost, Wheeler and Breit. Tables of the normalizing factor

$$C_o = \exp(\pi\eta/2)\,|\Gamma(1 + i\eta)| \quad \text{and} \quad \sigma_o = \arg\Gamma(1 + i\eta)$$

have been prepared. The values were computed up to the point where the expressions

$$\sqrt{2\pi\eta}\,\exp(-3\pi\eta/2) \quad \text{and} \quad (\pi/4) + \eta(\log\eta - 1) - 1/(12\eta)$$

yield approximations to C_o and σ_o, respectively, accurate to 8 significant figures. Values of $\rho\varphi_o$ and φ_o^* for $\eta = 0(.1)10$, $\rho = 0(.05)1.5$ to 7 significant figures were being computed in June, 1947. Values of $\rho\varphi_o$, φ_o^* $f = F_o'/F_o$ for $\eta = 3(1)10$, $\rho = 0(.1)10$ have been computed. Work was started for the same range of ρ and η on the tabulation of

$$\frac{df}{d\eta}, \qquad \frac{1}{2!}\frac{d^2f}{d\eta^2}, \qquad \frac{1}{3!}\frac{d^3f}{d\eta^3}, \qquad \frac{1}{4!}\frac{d^4f}{d\eta^4}.$$

These derivatives will be used for interpolation in $f(\eta,\rho)$ as a function of η.

(11) *Table of Antilogarithms*

A table of 10^x to 10 places of decimals for x from 0 to 1 at intervals of 10^{-5} was being prepared from a photographed copy of J. Dodson's *Antilogarithmic Canon,* London 1742. The values from 0 to .9 have been transferred to punch cards and differenced.

(12) *Table of Sines and Cosines to Hundredths of a Degree*

Tables of Sines and Cosines to 15 decimal places with second central differences. The proofreading and differencing of final manuscript was started in June, 1947.

(13) *Tables of the "τ" Integral*

The integral defined by

$$\tau = \frac{\mu}{x^2} \int_0^{x\sqrt{\frac{2}{\mu}}} \frac{w\,dw}{1 + w^2 \left[\cos \mu w - \frac{w}{2} \sin \mu w\right]^2}$$

was being computed for various values of the parameters x and μ. The case μ = .1 was being carried out by numerical integration. The task was laborious due to the large number of sharp peaks in the integrand, For μ = .001 about 80% of the work was completed by the end of the fiscal year by a process of obtaining analytic expansions for the integrand in the neighborhood of the peaks. The remainder of the integral will be evaluated by numerical integration.

(14) *The Arithmetic Mean and Median of a Set of N Measurements as Estimators of Parameters of Location, with Particular Reference to the Problem of Anomalous Observations.*

A statistical research project being performed by the Statistical Engineering Laboratory. This project is aimed at the studying of the sampling distributions of these statistical estimators, through the evaluation of the percentiles of their distributions, which represents a departure from the more usual method of studying such variables, by attempting to determine their moments. At the end of the fiscal year, the work was 50 per cent completed. Estimated time to complete the project 1 year.

(15) *The 2 x 2 Table, from the Viewpoint of Design of Experiments, Estimation, and Tests of Significance.*

A paper by Dr. Churchill Eisenhart on the tests-of-significance aspects of contingency tables, to be presented at the September 1947 meeting of the Institute of Mathematical Statistics. The design-of-experiments and estimation aspects were being covered by collaboration outside of the Bureau.

(16) *Revision of Statistical Phases of Federal Specification ZZ-R-601 ("Rubber Goods: General Specifications")*

This project was undertaken in cooperation with the organic and Fibrous Materials Division of the Bureau. Progress during fiscal year 1947 consisted of the following: A list of definitions of sampling and inspection terms, and three sets of sampling inspection plans for different consumer requirements, were prepared for inclusion in the proposed revised specification. A procedure was developed which, while not leading to outright rejection of apparently anomalous observations, greatly reduces their influence on the final results upon which decisions are based. The statistical parameters of the proposed procedure were determined from test data. Future work involves extensive editorial work on the specification and preparation of instructions pertaining to the selection and use of the various sets of sampling inspection plans. The project was about 80 per cent completed at the end of the fiscal year, and it is estimated to take about 3 months to complete it.

(17) *Sampling Plan for Testing Clinical Thermometers*

Sampling procedures currently used by Thermometry Section of the Bureau for testing Veterans Administration clinical thermometers consist essentially of 100 per cent inspection. However, the specification referenced in the current Veterans Administration contract calls for the termination of inspection and rejection of a delivery when the fraction defective among the items tested exceeds a specified limit. The first task performed under this project was to determine the operating characteristic curve of the inspection procedure of the referenced specification. Then a sequential inspection procedure permitting only a rejection of a delivery prior to complete inspection and having approximately the same operating characteristic curve was developed. In addition a sequential inspection procedure permitting acceptance or

rejection prior to complete inspection and having the same operating characteristic curve was also developed. Special studies of the reproducibility of the tests were undertaken. It is estimated that the work was about 50 per cent completed at the end of the fiscal year, and that time needed for completion will be about six months.

(18) *The Theory and Practice of Hitting a Straight Line*

This is a manual on certain statistical methods. Work on the project was initiated by Dr. J. H. Curtiss during the fiscal year in connection with the course in Correlation and Regression which he was giving in the NBS Graduate School. A mimeographed draft of the Introduction and first chapter, dealing with the case in which one variable is fixed without error, was prepared and distributed to students in the course.

(19) *Automatic Digital Computing Machine for the Bureau of the Census.*

This project has the objective of developing and constructing a large-scale, automatically-sequenced digital computing machine suitable for the preparation of Census reports and the performance of calculations arising in connection with Census work. In accordance with an agreement worked out in consultation with the Bureau of the Census and with the Office of Naval Research, during the fiscal year 1947, the project was limited to the preparation of design specifications for this machine, together with the performance of such component research as may be necessary to insure sufficiency of these design specifications. On October 24, 1946, a contract was entered into with the Electronic Control Company, Dr. J. W. Mauchly, Jr., and Mr. J. Presper Eckert, Jr., Proprietors, for the performance of this work. The Electronic Control Company was furnished by the Bureau with detailed performance requirements for the machine, and began work immediately upon recruitment of

personnel, procurement of equipment, and applied research on materials. Although the contract was scheduled to run only six months, it was deemed in the interest of the Bureau to extend the contract so that further research on components could be performed by the contractors. At the end of the fiscal year, work on the contract was about 90 per cent completed and the preparation of a detailed report was in progress.

The most important items of component research performed by the Electronic Control Company during the course of this contract were as follows:

(a) Studies of the optimum frequency of operation of mercury delay tanks.

(b) Tests on magnetic tape materials and reading and recording heads.

(c) Development of a servo device for intermittent tape feed.

(d) Studies of wireless typewriters to determine the maximum speed of reliable operation.

(e) Development of a deflection type electrostatic memory tube.

In connection with this project, a study of the programming of Census problems on proposed computing machines was inaugurated at the Bureau. Mrs. Ida Rhodes of the Mathematical Tables Project was transferred temporarily from New York City and was placed in charge of this project. The work was concerned largely with the study of the techniques of sorting and collating on a large scale. In connection with this activity a weekly meeting was held at the Bureau on programming, attended by representatives of the Bureau of the Census, the Office of Naval Research, the Social Security Administration, and members of the Bureau staff. In addition to the Census type of problem, basic specific mathematical

processes were investigated, including the solution of various types of differential equations.

It is estimated that the entire Census machine project was about 50 per cent complete at the end of the fiscal year, and that it will take about 16 months to complete the project.

(20) *Automatic Digital Computing Machine for the Office of Naval Research.*

This project has the objective of developing and constructing a large-scale automatically sequenced digital computing machine suitable for general mathematical calculations. As in the case of the Census machine, it was agreed that the fiscal year 1947 should be devoted to the design phase of the project. A letter containing performance specifications for the machine and the form of contract desired by the National Bureau of Standards was sent to all firms known to be competent in the field of electronics and to have personnel known to be qualified to design electronic digital computing machines. The letter solicited an expression of interest in contracting with the National Bureau of Standards for the design of the Navy's electronic digital machine. Either because of the contract form or the nature of the work, most of the manufacturers declined to participate in the program and the field soon narrowed down to the Raytheon Manufacturing Company of Waltham, Massachusetts, and the Hughes Aircraft Company of Culver City, California.

An investigation by Bureau personnel resulted in the conclusion that the Raytheon Manufacturing Company was better qualified to perform the work, and accordingly a contract effective 27 January 1947 was entered into with the Raytheon Company for design specifications together with the necessary component experimentation. Within the Raytheon Company the contract was under the supervision of Dr. N. E. Edlefsen, with Dr. C. V. L. Smith having responsibility for the mathematical phases of the work. Dr. J. Barnes of Tufts College

and Dr. W. R. Fuller of MIT were working as consultants on the contract.

The contractor immediately began work along two general lines: (1) Mathematical study of requirements that computing machines must fulfill and (2) Component research. In (1) the emphasis was upon (a) the relative frequency of the basic arithmetical operations (b) the number of references to and depositions in the memory (c) the number of transfers, both within the internal memory and between external and internal memory, (d) the frequency of reading in data and of reading out final results, (e) the need for using scale factors. In component research, the company performed advanced development work on electrostatic tubes and on mercury delay registers.

At the end of the fiscal year, work on the Raytheon design contract was about 80 per cent completed. It was expected that the design specifications will be available for study by the Bureau electronic engineers and mathematicians in the early part of August. These design specifications will be built about the mercury delay line as an internal memory device. However, some auxiliary work of this contractor in the deflection type electrostatic tube memory device was so successful that an extension of the contract to develop this device was being considered at the end of the fiscal year.

At the end of the fiscal year the work on the entire Navy machine project was about 40 per cent completed. Estimated time to complete the project, about 16 months.

B. New Projects Which Were Still in the Planning Stage at the End of the Fiscal Year

(1) *Tables of Value in the Development and Application of Mathematical Statistics*

Tables to facilitate the construction of random samples. Detailed specifications were drawn up by Messrs. Abramowitz,

Eisenhart, Laderman and Lowan of the Bureau in cooperation with Professor J. W. Tukey of Princeton University at a conference held in New York City. The tables so formulated are not tables of random numbers but are special arrangements of probability deviates or percentage points of the populations concerned.

(2) *Tables for the Occasional Computer*

The size, scope, and objectives of such a set of tables were discussed in general terms in the conference mentioned in (1). No final decisions were reached. The general idea of a handbook for computers was considered at various other times during the year. Dr. Wigner of the Oak Ridge Laboratories of the Atomic Energy Commission submitted, at the invitation of Dr. Lowan, a detailed outline for such a volume. Members of the staff of the Mathematical Tables Project have drawn up an alternative proposal somewhat different in scope from Dr. Wigner's outline. The general idea is to produce an enlarged and revised version of the well-known Jahnke-Emde handbook.

(3) *Table of Surfaces of Constant Pressure*

To find surfaces of constant barometric pressure of the form

$$\varphi(x,y) = \Sigma a_{ij}\, x^i y^j \quad 0 \leq i + j \leq 3$$

by principle of least squares to fit observations of pressure, altitude, and wind conditions, taken at various weather stations. Surfaces are to be found such that the following expressions are minimized:

$$1)\quad 10\Sigma(\varphi - \varphi_s)^2 + \Sigma\left(u\frac{\partial\varphi}{\partial x} + \frac{v\partial\varphi}{\partial y}\right)^2$$

$$2)\quad \frac{3}{10}\,\Sigma(\varphi - \varphi_s)^2 + \Sigma\left(v - \frac{1}{\lambda}\frac{\partial\varphi}{\partial x}\right)^2 + \Sigma\left(u + \frac{1}{\lambda}\frac{\partial\varphi}{\partial y}\right)^2$$

In the above φ_s, u, v, and λ are observed values and the summations extend over given stations.

(4) *Standard Sampling-Inspection Procedures*

At the invitation of the War Department, Bureau personnel participated in a two-day conference on the proposed standard sampling-acceptance inspection procedure developed by a Task Group of the War Department Inspection Advisory Council. The preparation of standard sampling-inspection procedures and tables for Federal Specifications was discussed subsequently by the Director of the National Bureau of Standards, Dr. Mina Rees, Head, Mathematics Branch, Office of Naval Research, and Professor W. Allen Wallis, University of Chicago, formerly Director, Statistical Research Group, Columbia University. It was decided to proceed with the preparation of such tables at the Bureau, and steps were taken to recruit additional personnel in connection with the program.

(5) *Computing Machines for the Army Air Forces*

During the last quarter of the fiscal year, the Bureau made commitments to extend the scope of the work on the Navy and Census machines to include such copies of these machines as may be required by the Army Air Forces. Progress during the fiscal year consisted largely of conferences with Air Forces personnel regarding the feasibility of such a program. Arrangements were made for the transfer of funds to carry on the work.

4. CONSULTATION AND ADVISORY SERVICES

(A) Statistical Engineering

Dr. Churchill Eisenhart and his staff spent about 85% of their time during the past year in statistical advisory work. In addition, Dr. J. H. Curtiss gave some time to this type of work. The clients of this service were for the most part personnel of the National Bureau of Standards. The more important jobs

were as follows:

(1) *Preparation and Analysis of Industrial Sampling Plans*

The usual acceptance sampling plans for hardwood plywood were analyzed, and the results were placed in an appendix to a Commercial Standard for this material. The operating characteristic of a proposed sampling plan for microscopic analysis of laundry starch, to be incorporated in a Federal Specification, was determined. A new method of sampling and testing tuning forks for the Veterans Administration was devised. Sampling plans for various types of thermometers were set up. The sampling plan in a proposed Commercial Standard for perforated clay pipe was analyzed and an appendix to the Standard prepared. Operating characteristics of sampling plans in a proposed Army Specification for the VT fuze were worked out.

Note: Two major projects in this category which began as consulting jobs are Nos. (16) and (17) in Section 3, part A, of this Report.

(2) *Analysis of Experimental and Test Data*

(i) Studies of the Validity of Stated Inferences

A study was made of the accuracy and precision of the standard method of determining the wool content of blankets, for reference in a pending FTC lawsuit. Advice was given on a number of occasions to various scientists in the Central Radio Propagation Laboratory relative to their applications of statistical methods. About fifteen (15) research papers submitted to the Research and Testing Editorial Committee of the Bureau were reviewed from a statistical viewpoint, and minor corrections or revisions were suggested to the authors. An extended study was made of a paper

on the effect of magnification on the precision of telescope pointing; it was determined that a significant serial correlation ignored by the author existed.

(ii) Analyses of raw data

True ranges of preignition ratings of different types of automatic spark plugs were estimated from sample ranges, and an appropriate graphical representation was devised. A regression analysis was performed on experimental data obtained in studies of the physical properties of leather. A regression analysis of data from 863 samples of manila rope was performed, using the computing facilities of the Mathematical Tables Project, and a paper was written around the results in collaboration with the person who obtained the raw data.

(3) Advice on probability theory

Assistance was given to various personnel of the Bureau from time to time on minor points in the theory of errors and tests of significance. In response to a request from the Naval Research Laboratory, a problem involving the distribution of a quotient of Poisson variables was solved.

(B) Computing Machinery

Dr. E. W. Cannon and his assistant, Mr. Albert S. Cahn, spent about 75% of their time during the year in advising representatives of other Government agencies concerning the applicability of automatic digital computing machinery to their activities. The work on the Census machine was carried on in continuous consultation with that Bureau. Many consultations were also held with the Office of Naval Research in connection with the machine under construction for the Navy.

Dr. Eisenhart participated as a formal discussant in the Symposium on Statistical Methods in Experimental and Industrial Chemistry, held in conjunction with the April 1947 meeting of the American Chemical Society, and again as a discussion leader in the May and June meetings of the Washington Quality Control Society.

Presented by title at meetings of the American Mathematical Society:

"The Approximation of Numbers as Sums of Reciprocals", by H. E. Salzer

"Table of Coefficients for Interpolating in Functions of Two Variables", by H. E. Salzer

"Alternative Formulas for Direct Interpolation of a Complex Function Tabulated Along Equidistant Circular Arcs", by H. E. Salzer

"An Alternative Definition of Reciprocal Differences", by H. E. Salzer

"Tables for Facilitating the Use of Chebyshev's Quadrature Formula", by H. E. Salzer