NATIONAL BUREAU OF STANDARDS REPORT

1843

EFFECT OF TOTAL SAMPLE SIZE ON LEVEL OF SIGNIFICANCE AT THE START OF A CONTROL CHART

by

F. Proschan and I. R. Savage

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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NATIONAL BUREAU OF STANDARDS
A. V. Astin[redacted] *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section is engaged in specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside of the back cover of this report.

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F. Proschan and I. R. Savage
Statistical Engineering Laboratory
Applied Mathematics Division

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FOREWORD

This report was prepared as part of a continuing program of research on mathematical statistics and its applications carried out at the National Bureau of Standards under the general supervision of Dr. Churchill Eisenhart, Chief of the Statistical Engineering Laboratory. The Statistical Engineering Laboratory is Section 11.3 of the National Applied Mathematics Laboratories (Division 11, National Bureau of Standards), and is concerned with the development and application of modern statistical methods in the physical sciences and engineering.

J. H. Curtiss
Chief, National Applied
Mathematics Laboratories

A. V. Astin
Director
National Bureau of Standards

EFFECT OF TOTAL SAMPLE SIZE ON LEVEL OF SIGNIFICANCE AT THE START OF A CONTROL CHART

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F. Proschan and I. R. Savage

Introduction: In starting a control chart to detect a shift in a normal population mean, it is generally assumed that one knows μ, the population mean, and σ, the population standard deviation, or that one has accumulated enough past data to estimate them with negligible error. If these assumptions are correct, the level of significance turns out to be .3 per cent (more accurately, .26 per cent) under the usual practice of setting the control limits at $\mu \pm 3\sigma_{\bar{\bar{X}}}$. However, suppose that it is important to start the control chart as soon as possible so that only a relatively small amount of past data is available. How is the level of significance affected by the number, m, and the size, n, of subgroups used to set the limits?

Summary: For various sample sizes n, the largest number of subgroups m for which the significance level α exceeds 1 per cent is given in Table I:

TABLE I

n	m
2	15
3	9
4	7
5	6
6	5
7	5
8	4
9	4
10	4
12	4
14	4
16	3
18	3
20	3

In each case above, $\alpha < 2$ per cent.

Deviation: Let

$\bar{x}_i$ = mean of i^{th} sample

$\bar{\bar{x}}$ = mean of $\bar{x}_i$ $(i = 1, 2, \ldots, m)$

n = common sample size

m = number of samples available at the start of the control chart

$\bar{R}$ = average of the m sample ranges

d_n = expected value of the range in the samples of n, randomly selected from a normal population of unit standard deviation

$$u = \left| \frac{\bar{x}_{m+1} - \bar{\bar{x}}}{\frac{\bar{R}}{d_n}\sqrt{\frac{1}{n} + \frac{1}{mn}}} \right|$$

Control limits are customarily set at $\mu \pm 3\sigma_{\bar{x}}$. Suppose u and σ are unknown so that the control limits are set at

$$\bar{\bar{x}} \pm 3 \frac{\bar{R}}{d_n \sqrt{n}} \qquad (1)$$

The probability, α, of the next sample mean, $\bar{\bar{x}}_{m+1}$, falling outside of the control limits is given by

$$\alpha = \text{Prob.} \left(|\bar{\bar{x}}_{m+1} - \bar{\bar{\bar{x}}}| \geq 3 \frac{\bar{\bar{R}}}{d_n \sqrt{n}} \right) \qquad (2)$$

or

$$\alpha = \text{Prob.} \left(u\sqrt{\frac{m+1}{m}} \geq 3 \right) = \text{Prob.} \left(u \geq 3\sqrt{\frac{m}{m+1}} \right) \quad (3)$$

Values of $3\sqrt{\frac{m}{m+1}}$ are as follows:

m	$3\sqrt{\frac{m}{m+1}}$	m	$3\sqrt{\frac{m}{m+1}}$
1	2.1213	11	2.8723
2	2.4495	12	2.8823
3	2.5981	13	2.8909
4	2.6833	14	2.8983
5	2.7386	15	2.9047
6	2.7775	16	2.9104
7	2.8062	17	2.9155
8	2.8484	18	2.9200
9	2.8460	19	2.9240
10	2.8604	20	2.9277

Bounds for α may be determined by the use of the tables of [1], which give the 90, 95, 98, 99, 99.8, and 99.9 percentiles of u. By using Table 6 of [1], which gives the 99 percentile of u, for a given value of n the largest value of m was determined such that $\alpha \geq 1$ per cent. (See Table I) Six point Lagrangian interpolation was performed to determine the 99 percentile value of u for $n = 2$, $m = 16$ and four point Lagrangian interpolation for the 99 percentile of u for $n = 4$, $m = 7$. In addition, by using Table 5 of [1] it was established that α was actually less than 2 percent for the combinations of m and n given in Table I.

Using Sample Standard Deviation Instead of Range

With only a small amount of data available at the start of a control chart for means, it might be considered advisable to estimate the population variance by pooling the sample variances. The control limits would then be established as

$$\bar{\bar{x}} \pm 3 \frac{s}{\sqrt{n}} \qquad (4)$$

where

$$s = \sqrt{\sum_{i=1}^{m} s_i^2/m}$$

and S_i^2 = variance of i^{th} sample.

If this more efficient procedure for estimating the population variance is followed, will the level of significance of the control chart for means come out closer to the desired value, .3 per cent? Or, putting it another way, will there be any decrease in the maximum number of subgroups, for which α, the level of significance, exceeds 1 per cent?

In general the answer is no, as the following table shows. Table II gives the largest value of m for a given value of n such that $\alpha \geq 1$ per cent, using (4) as the control limits.

TABLE II

n	m
2	16
3	9
4	7
5	6
6	5
7	5
8	4
9	4
10	4
12	4
14	3
16	3
18	3
20	3

In each case, $\alpha < 2$ per cent.

Note that Table II differs from Table I in only 2 cases: $n = 2, 14$. In both cases m differs in the two tables by 1 only. Hence, in general, control limits established by (4) are not noticeably superior to those of (1) (using as the criterion of superiority, the maximum number of subgroups for which $\alpha > 1$ per cent).

REFERENCE

E. Lord, The use of range in place of standard deviation in the t-test, Biometrika, Vol. 34, Parts 1, 2 (1947), pp. 41-67.

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.00). Information on calibration services and fees can be found in NBS Circular 483, Testing by the National Bureau of Standards (25 cents). Both are available from the Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.

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