NATIONAL BUREAU OF STANDARDS REPORT

1834

HOW MANY GUARDS?

By

W. R. Knight
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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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NATIONAL BUREAU OF STANDARDS
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THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section is engaged in specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside of the back cover of this report.

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Statistical Engineering Laboratory

To

Administrative Services
Division 42
National Bureau of Standards

NBS

FOREWORD

This report was prepared in the Statistical Engineering Laboratory of the National Bureau of Standards for use by NBS Division 42, Administrative Services. The Statistical Engineering Laboratory is Section 11.3 of the National Applied Mathematics Laboratories (Division 11, National Bureau of Standards), and is concerned with the development and application of modern statistical methods in the physical sciences and engineering.

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Summary: In preparing work programs for the guard force at the National Bureau of Standards one should take into account the fact that guards take sick leave and some of their annual leave unexpectedly, which gives rise to the questions of whether and by what amount the number of guards should exceed the number of posts to be filled, and thus the theory of probability is called into play. This report gives an outline of a method for preparing a work program for a guard force, the outline taking into account the element of probability that is present. It is further shown that several different work programs can be devised that cost essentially the same amount, and consequently the director of these programs has some flexibility in choosing the method for running the force.

It should be emphasized that this report only gives an outline of a method for devising an economic method for running the guard force since not all of the fiscal complexities have been taken into account. In practice the detailed analysis and design of work programs can only be done with the intimate cooperation between the analyst and the accounting sections of the guard system.

Introduction: Mr. Dalzell, the chief of the Administrative Services Division, National Bureau of Standards, has noticed that probability comes into the planning of the guard force since guards take some of their leave (annual and sick) unexpectedly and consequently he cannot always plan on having as many guards actually coming to work as were asked to come to work. When guards are unexpectedly absent it is often necessary to hire extra men at overtime rates in order to make sure that all the guards posts are filled. It is the purpose of this report to show how one can take the above into account in order to devise work programs that are economical.

The Problem: Mr. Dalzell must hire by the year enough guards to take care of all his posts. The bookkeeping year is 364 days of exactly 52 weeks; this gives 260 weekdays and 104 Saturdays and Sundays, some of which are the 8 holidays. On weekdays 13 posts must be filled; on Saturdays, Sundays, and holidays it is only necessary to fill 12. However, if a holiday falls on Sunday, only 12 posts need be filled the following Monday.*

* This is because the Bureau will be closed the following Monday as the other employees get that day off to make up for the holiday.

If we assume that of the 8 holidays, one will fall on Saturday, one on Sunday, and the rest on weekdays, then there are 253 days upon which 13 posts must be filled, and 103 days (in addition to the 8 holidays which are treated separately for reasons which will shortly become clear.) on which 12 posts must be filled. The days upon which 13 posts must be filled will be hereafter referred to as weekdays, and the days upon which 12 posts must be filled other than holidays will, be referred to as having fallen on a weekend, even though this may not be strictly the case.

The absence rate among the guards is not exactly known, but it is somewhere between four and six percent. In numerical illustrations it will, for purposes of this paper, be taken as five percent.

There are 364 days in the year. After deducting for weekends, holidays, annual and sick leave, etc., there are 212 days on which a man must work. Of this, 15 days are sick leave; we suppose that the guards are sick unexpectedly, hence they will be <u>asked</u> to work during these 15 days, but are sick and therefore do not come. Also there will be a few days on which a guard is asked to work, but does not come because he has taken unexpected annual leave. Thus while a guard works 212 days plus whatever sick leave he does not use, he is <u>asked</u> to work on more than 212 days by the amount of unused sick leave on which he must work, plus the amount of used sick leave

on which he was asked to work but didn't, plus whatever unexpected annual leave he has taken. In short: Each guard is asked to work 227 days plus unexpected annual leave. For purposes of numerical illustration 5 days of unexpected annual leaver per man will be assumed although no figures are available on this.

The guards are paid at an annual rate on the basis of 260 days per year (all weekdays). In addition to this basic annual pay, they get time and a half for any overtime except on holidays. When they are called upon to work overtime an a holiday, they receive double the regular daily pay. When a Holiday happens to fall on a scheduled work day, guards receive their regular daily pay plus a bonus consisting of a full day's pay. It will be seen that regular and overtime pay is thus the same on holidays, the only difference, being that on regular time, half of the holidays pay is included in the regular pay check, while on overtime all of it is separate.

There are two choices that must be made in this situation. First, how many men should be asked to work on a given day? Secondly, how many men should be hired to cover the posts for the year? There will be any number of programs that can be used, depending on the number of men asked to work each day of the year and the total number of men hired. The cost of any program will be the sum of the following items:

(a) The basic annual salaries of the men hired.

(b) Additional holiday pay.

(c) Overtime pay at the end of the year to fill vacant posts in the event that not enough men were hired to fill all of the posts that had been planned upon.

This item comes into the bookkeeping since we cannot hire fractions of men.

(d) The amount of overtime that must be paid to have the posts filled when the men are absent. This will be equal to the amount of overtime that would be paid in one day (average) summed over all the days of the year. Thus in a simple case, it would be 253 times the amount of overtime that would, on the average, be paid on a day when T men were asked to come to work to fill 13 posts,* plus 103 times the overtime that would be paid on a day when T' men were asked to come to fill 12 posts.

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* The formula determining this quantity is derived in Appendix I, and a table for a few selected values is included as Appendix II. The average amount of overtime that will be paid during one day is given by the expression:

$$\text{overtime} = c' \left[TA \sum_{Y=(T-1)-P}^{T-1} B(A, T-1, Y) - (T-P) \sum_{Y=T-P}^{T} B(A, T, Y) \right]$$

Where: P = the number of posts that must be filled

T = the number of men asked to come to work

Y = the number of men asked who don't come

A = the average absence rate

c' = one man's daily pay at overtime rates

(continued on bottom of next page)

Thus, given a prospective program, the sum of these four items will give the cost of such a program. The problem is to find the program with the least possible cost. Unfortunately there is no direct method for doing this. However, common sense will eliminate all but a few of all the possible programs. For example, it is clear that at least as many men must be asked to come to work as there are posts to be filled, and that, given any reasonable absence rate, it should not be necessary to ask very many more men to come to work than there are posts to be filled. Also if D_T equals the number of days per year on which T men are asked to come to work, and W equals the number of days per year that one man can be asked to come to work, then the total number of men to hire to cover the year's posts would be:

$$\Sigma T D_T / W = \text{number of men to hire.}$$

Unfortunately this will not usually be an integer, but it will be seen that the number of men to hire must be an integer in the neighbor of this quantity. Thus in practice it will suffice to simply calculate the cost of a few programs which common sense will point out. The one of these resulting programs

(Footnote continuing from page 5)

$$B(A,T,Y) = \frac{T!}{(T-Y)!Y!} A^Y (1-A)^{T-Y}$$

This function and its partial summations over Y are tabulated in the National Bureau of Standards Applied Mathematics Series No. 6.

which proves cheapest is, of course, the one to select.

The following programs will be figured in units of one man's pay per day. That is, the number "one" refers to an amount of money equal to the amount one man would receive in one day, the number "one and one half" refers to an amount of money equal to that which one man would receive in one day at overtime rates, etc. Thus one man's basic yearly pay would be represented as "260."

Program I.

Let us examine the following program: 13 men will be asked to work on weekdays, 12 on weekends. It has been pointed out that the overtime rate for holidays is the same as the regular rate for holidays, therefore let us always have men work holidays on an overtime basis; this will enable fewer men to be hired for the year. Since the holidays are now all overtime, they will not be considered in determining how many men we need to fill our regular posts this number being:

$$\text{number of men} = \frac{253\text{x}13 + 103\text{x}12}{212 + 15 + 5} = 19.5$$

according to the formula given on page 6. As was remarked at the time, the result is not an integer. It will be necessary to examine both a program hiring 20 men, and one hiring 19; taking 20 first:

Since we "really" need 19.5 men, let us consider the hiring of 20 which will give us a few man days "leftover" at the end of the year; however, we can always absorb some of this surplus by asking some men to work holidays on regular time instead of on overtime as was planned. The exact amount of the surplus will be 115 man-days:

13x253 + 12x103	=	4525	= total number of man days needed.
232x20	=	4640	= total number of man days available if 20 men are hired.
		115	= difference between man days needed and man days available = surplus (or shortage if too few men are hired)

it now only remains to determine the cost of program I:

(a)		5200	Basic annual pay for 260 men.
(b)		96	Additional holiday pay. By asking some of the surplus of 115 man days to work holidays, on regular time, half of this has been brought under the regular basic annual pay; if we had had to hire all of them at overtime, it would have been 2x12x8 = 192.
(c)		0	If less man days are available than are needed, men must be hired at overtime rates at the end of the year to make up the shortage; however, in this case there is a surplus rather than a shortage.
(d)	253 times the overtime that would be paid on one day if 13 men were asked to come to fill 13 posts, plus 103 times the overtime that would be paid	339.4	Overtime due to absence. The overtime that would be, on the average, paid in one day if T men are asked to come to fill P posts can be figured by the

in one day if 12 men were asked to come to fill 12 posts		formula given on page 5, or can be looked up in Appendix II for these particular cases.
	5635.4	Total cost of program I for one year in terms of one man's daily pay.

Program I'.

Now let us see what will be the cost if 13 men are asked to work weekdays, and 12 asked to work weekends, and 19 men are hired instead of 20. This will give a shortage of men at the end of the year, the exact number being:

13x253 + 12x103 = 4525 = Total man days needed

19x232 = 4408 = " " " available

117 = Man days short; these will have to be made up at overtime rates.

The cost of the program:

(a)	19x260	4940	Basic annual pay for 19 men
(b)	2x12x8	192	Additional holiday pay (all at overtime rates.)
(c)	(3/2)x117	175.5	Overtime to make up for the shortage of 117 man days.
(d)		330.9	Overtime due to absence. This is figured in the same way as in the previous example. However, the last 117 man days of the year will be hired at overtime rates, hence absence does not cost anything extra on those days. This means that instead of 253 days during which overtime must be paid for absence there are only 253-(117/13) such days.

	(We arrange to take up the shortages on weekdays only as this proves a trifle less expensive.)
5638.4	Total cost of program I.

Program II.

Let us now see what will happen if more men are asked to come to work than there are posts to be filled. Let 14 men be asked to come to work on weekdays, and 13 on weekends. (12 on holidays, of course.) This requires hiring:

$$\frac{14\text{x}253 + 13\text{x}103}{212 + 15 + 5} = 21.04 \text{ men}$$

In this case we will simply round to 21 men. This will result in a small shortage.

14x253 + 13x103 =	4881	Man days needed
21x232 =	4872	Man days available
	9	Man days short.

The cost of program II:

(a)	21x260	5460	Basic annual pay
(b)	2 x12x8	192	Additional holiday pay
(c)	(3/2)x8	12	Overtime due to shortage of men. (reason that 8 rather than 9 is used is that on the last day 13 rather than 14 men are asked to come to work.)
(d)	252x(overtime on one day if 14 men are asked to fill 13 posts) + 103x (overtime if 13 men are asked to fill 12) +	96.6	Overtime due to absence. On the last day only 4 out of 13 men are on regular time; the rest get overtime anyway.

(4/13) times (overtime if
13 men are asked to come to
fill 13 posts.)

$\overline{5760.6}$ Total cost of program II

In this particular case it seems that it is better not to ask more men to come to work than there are posts to fill. It also appears that it doesn't make much difference whether 20 or 19 men are hired, thus giving some freedom of choice to the administrator of the guard force. It should be remembered that these examples are merely a rough treatment to illustrate the general method; not only have fiscal details been omitted for the sake of simplicity, but also two possible programs [(1) asking 14 men to work weekdays and 12 on weekends, and (2) asking 13 men to work weekdays and 13 on weekends] have not been treated, because it seems probable that they would have proved to be more expensive than programs I and I'.

Appendix I.

Derivation of Overtime Pay Due to Absence

The average overtime that will be paid on a day when T men are asked to come to work to fill P posts hereafter denoted F(A,T,P) where A is the absence rate, will be:

$$F(A,T,P) = \sum_{X=0}^{T} f(X,P)\binom{T}{X} A^{T-X}(1-A)^{X}$$

Where: X = The number of men who actually appear for work on that day when asked (= T-Y).

F(X,P) = The amount of overtime that must be paid if X men appear to fill P posts.

$\binom{T}{X}A^{T-X}(1-A)^{X}$ is the probability that, given an absence rate of A, X men will appear when T men were asked to come to work.

$$F(A,T,P) = c' \sum_{X=0}^{P} (P-X)\binom{T}{X} A^{T-X}(1-A)^{X}$$

Making the substitution Y = T-X.

$$= c' \sum_{Y=T-P}^{T} \left[(P-T)\binom{T}{Y} A^{Y}(1-A)^{T-Y} + Y\binom{T}{Y}A^{Y}(1-A)^{T-Y}\right]$$

$$= c'(P-T)\sum_{Y=T-P}^{T}\binom{T}{Y} A^{Y}(1-A)^{T-Y}$$

$$+ c'TA \sum_{Y=(T-1)-P}^{T-1}\binom{T-1}{Y}A^{Y}(1-A)^{(T-1)-Y}$$

Now, for simplicity of notation, define:

$$B(A,T,Y) = \binom{T}{Y} A^{Y}(1-A)^{T-Y}$$

Thus giving:

$$F = c' \left\{ TA \sum_{Y=(T-1)-P}^{T-1} B(A,T-1,Y) - (T-P) \sum_{Y=T-P}^{T} B(A,T,Y) \right\}$$

Appendix II

Average overtime that will be paid in one day to make up for absences. In terms of one man's daily pay; overtime at one and a half time.

		A = absence rate		
		4%	5%	6%
P=13	T=14	.187	.282	.391
	13	.780	.975	1.170
P=12	T=13	.162	.245	.341
	12	.720	.900	1.080

$$F = \frac{3}{2} \left\{ TA \sum_{Y=(T-1)-P}^{T-1} B(A,T-1,Y) - (T-P) \sum_{Y=T-P}^{T} B(A,T,Y) \right\}$$

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.00). Information on calibration services and fees can be found in NBS Circular 483, Testing by the National Bureau of Standards (25 cents). Both are available from the Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.

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