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Progress Report for April - June 1952

on

Applications of the Theory of Stochastic Processes to the Study of Trajectories

(NBS Project 1103-21-5119)

NBS

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

This is a special progress report to the U.S. Naval Ordnance Test Station which sponsors NBS Project 1103-21-5119, Application of the theory of stochastic processes to the study of trajectories.

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PROGRESS REPORT FOR APRIL-JUNE 1952

ON

APPLICATION OF THE THEORY OF STOCHASTIC PROCESSES TO THE STUDY OF TRAJECTORIES

(NBS Project 1103-21-5119)

I. Summary

This report contains a summary of the work done during the quarter. Results of this work are briefly stated Technical reports written in connection with this project are mentioned but are transmitted separately.

II. Discussion of work done during the quarter.

The first task under the project is the development of a test for the hypothesis that a sequence of observations comes from a fundamental random process (Wiener process). In this connection a theorem was derived during the preceding quarter. A statement of this theorem was already given in the progress report for the quarter January - March 1952. [NBS report 1598]. A detailed treatment may be found in a paper by E. Lukacs entitled "A property of strongly continuous processes" (NBS report 1665) which was transmitted to the U.S. Naval Ordnance Test Station at the close of the quarter.

This work provides an analysis of the various assumptions underlying the proposed stochastic model of a fundamental random process (F.R.P.). On the basis of this analysis one can conclude that standard statistical techniques can be used to test the hypothesis that observed trajectory data come from a F.R.P. One has to form the deviations $x(t)$ from the mean value curve. If N equidistant observations are taken during the interval $(0,T)$ one has to form the increments $\Delta_n = x(n\tau) - x(\overline{n-1}\,\tau)$ for $\tau = T/N$ and $n = 1,2,\ldots N$. and test the hypothesis that the Δ_n are independently distributed. A rather wide variety of such tests is known, rejection of the hypothesis of independence of the Δ_n would imply rejection of the hypothesis of a F.R.P.

Another alternative approach was also studied. This approach does not make use of the fact that the normality of the process is — under certain assumptions — equivalent to a continuity property of the process. If one wants to test the hypothesis that a sequence of observations comes from a Wiener process, one might be willing to assume a priori

the continuity of the variance function but should like to test whether the increments are normally and independently distributed. One has therefore to test simultaneously two hypotheses: the hypothesis of normality and the hypothesis of randomness (identical and independent distribution of the random variables). This establishes a connection between this project and a study currently carried under Task 1103-11-1107/52-2. Mr. I. R. Savage proved the following theorem:

Let $x_1 \ldots x_n$ be a sample of n identically and independently distributed random variables. Then any rank order statistic and any symmetric statistic are independently distributed.

This theorem can be applied to construct a test for the hypothesis mentioned above in which the evidence from a χ^2 test of normality and from a rank-order test of randomness can be combined. Mr. Savage's results are summarized in NBS Report No. 1694 entitled "Simultaneous Tests of Randomness and other Hypotheses" which will be issued at the beginning of the next quarter.

E. Lukacs and Professor H. B. Mann visited the Naval Ordnance Test Station in June 1952, and discussed possible applications and problems to be solved under this project. Professor Mann also gave lectures on his recent work on the estimation of mean value functions of stochastic processes.