NATIONAL BUREAU OF STANDARDS REPORT

1661

Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

A QUARTERLY REPORT

January through March 1952

NBS

U. S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

U. S. DEPARTMENT OF COMMERCE
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The scope of activities of the National Bureau of Standards is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section is engaged in specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside of the back cover of this report.

1. **ELECTRICITY.** Resistance Measurements. Inductance and Capacitance. Electrical Instruments. Magnetic Measurements. Electrochemistry.
2. **OPTICS AND METROLOGY.** Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Gage.
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10. **BUILDING TECHNOLOGY.** Structural Engineering. Fire Protection. Heating and Air Conditioning. Exterior and Interior Coverings. Codes and Specifications.
11. **APPLIED MATHEMATICS.** Numerical Analysis. Computation. Statistical Engineering. Machine Development.
12. **ELECTRONICS.** Engineering Electronics. Electron Tubes. Electronic Computers. Electronic Instrumentation.
13. **ORDNANCE DEVELOPMENT.** Mechanical Research and Development. Electromechanical Fuzes. Technical Services. Missile Fuzing Research. Missile Fuzing Development. Projectile Fuzes. Ordnance Components. Ordnance Tests. Ordnance Research.
14. **RADIO PROPAGATION.** Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Frequency Utilization Research. Tropospheric Propagation Research. High Frequency Standards. Microwave Standards.
15. **MISSILE DEVELOPMENT.** Missile Engineering. Missile Dynamics. Missile Intelligence. Missile Instrumentation. Technical Services. Combustion.

NBS PROJECT

NBS REPORT
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Projects and Publications

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NATIONAL APPLIED MATHEMATICS LABORATORIES

January through March 1952

NATIONAL APPLIED MATHEMATICS LABORATORIES
January 1 through March 31, 1952

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Contents

Index of Active Research and Development Projects

NOTE: This index is not intended to cover the numerous special problem solutions, statistical analyses, and other ad hoc services to Government agencies which form an important part of the work of the National Applied Mathematics Laboratories. These services are, however, fully represented in the body of the report.

A. Research: Pure Mathematics

B. Research: Numerical Analysis

C. Research: Applied Mathematics, Physics, Astronomy, and Automatic Translation

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F. Manuals, Bibliographies, Indices, and Technical Information

Index of Active Research and Development Projects

G. Computing Machine Development

Status of Projects

March 31, 1952

I. Institute for Numerical Analysis

(Section 11.1)

1. Fundamental Research

SOLUTION OF SETS OF SIMULTANEOUS ALGEBRAIC EQUATIONS AND TECHNIQUES FOR THE INVERSION AND ITERATION OF MATRICES
Task 1101-11-5100/49-AE2
(formerly 11.1/1-49-AE2)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full task description appears in July-Sept 1949 issue.

Status: CONTINUED. The finite iterative procedure described in publication (6) has been tried on the SWAC and IBM equipment. These experiments have been carried out with good success by R. M. Hayes, U. Hochstrasser, and L. Wilson for certain small test matrices.

Publication (8) presents a summary of card-programmed calculator experiments with the accelerated gradient method performed last summer. The results are compared with those obtained with the "almost-optimum" methods reported in publication (4).

In publication (7) D. H. Lehmer presents his investigations of certain character matrices. Let p be an odd prime, α an integer, and $\chi(a)$ Legendre's symbol for the quadratic character of a(mod p). Simple explicit expressions are derived for the inverse, the characteristic equation, and any positive power of matrices M_α of the form

$$M_\alpha = \{a_{ij}\} = \{\chi(\alpha + i + j)\} .$$

These results facilitate the construction of matrices with simple rational elements that can be used for experimentation in numerical analysis. See also the investigation of C. Lanczos on the evaluation of the smallest root of certain characteristic equations, publication (8).

Publications: (1) "The extent of n random unit vectors," by G. E. Forsythe and J. W. Tukey; IN MANUSCRIPT. (2) "A method of computing exact inverses of matrices with integer coefficients," by J. B. Rosser; accepted by the NBS Journal of Research. (3) "An extension of Gauss' transformation for improving the condition of systems of linear equations," by G. E. Forsythe and T. S. Motzkin; accepted by Mathematical Tables and Other Aids to Computation. (4) "Gradient methods in the solution of systems of linear equations," by M. L. Stein; accepted by the NBS Journal of Research. (5) "Tentative classification of methods and bibliography on solving systems of linear equations," by G. E. Forsythe; to appear in Simultaneous Equations and the Determination of Eigenvalues - Proceedings of an NBS Symposium held in Los Angeles, Aug. 1951. (6) "Method of conjugate gradients for solving linear systems," by E. Stiefel and M. R. Hestenes; IN MANUSCRIPT. (7) "On certain character matrices," by D. H. Lehmer; submitted to a technical journal. (8) "IBM experiments with accelerated gradient methods for linear equations," by A. I. Forsythe and G. E. Forsythe; IN MANUSCRIPT. (9) "Solution of systems of linear equations by minimized iterations," by C. Lanczos; accepted by the NBS Journal of Research.

NUMERICAL METHODS IN CONFORMAL MAPPING
Task 1101-11-5100/49-CM1
(formerly 11.1/1-49-CM1)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The preparation of the report continues. Some further numerical work has been carried out in the case of an ellipse with axis ratio 2:1. The results obtained by the Warschawski method are being checked by differencing and comparison with results obtained on SEAC using the explicit formulae for the mapping given by G. Szego.["Conformal mapping of the interior of an ellipse onto a circle," by G. Szego. Amer. Math. Mo. 57, 474-478 (Aug-Sept 1950)].

CALCULATION OF EIGENVALUES, EIGENVECTORS, AND EIGENFUNCTIONS OF LINEAR OPERATORS
Task 1101-11-5100/50-3
(formerly 11.1/1-50-3)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full task description appears in July-Sept 1949 issue.

Status: CONTINUED. A new publication relevant to this task is publication (2) listed under task 1101-11-5100/52-1. It appeared in 1943 in an inaccessible Russian publication and gives background information about the possible invariant subspaces of the linear transformation defined by a matrix. The paper then goes on to describe specific numerical methods of extending classical iteration methods to obtain these invariant subspaces in a general case. It is a useful reference on such practical problems as separating close eigenvalues, recognizing a defective matrix, and so on. For other work on eigenvalues, see publication (7) under task 1101-11-5100/51-2, p. 5 .

In vibration and flutter problems we are frequently interested in the absolutely smallest root of the characteristic equation. Since the usual iteration processes boost up the large eigenvalues at the cost of the small ones, a preliminary inversion of the matrix is required. The present method dispenses with the preliminary inversion of the matrix. The problem $Ax - \lambda x = 0$ is changed into $\widetilde{A}^*Ax - \lambda\widetilde{A}^*x = 0$, and the smallest principal axis x_0 of the Hermitian problem $\widetilde{A}^*Ax_0 = \xi_0 x_0$ (ξ_0 = positive and the smallest of all possible ξ) is considered as the zeroth approximation of x. In first approximation we obtain

$$x = (1 - \lambda\gamma_0)\, x_0 + \lambda y$$

with the quadratic condition

$$1 - \gamma_0 \lambda + (\gamma_0^2 - \gamma_1) \lambda^2 = 0$$

where

$$Ay = x_0$$

$$\gamma_0 = \frac{1}{\xi} \frac{x_0 \tilde{A} \tilde{x}_0}{x_0 \tilde{x}_0}$$

$$\gamma_1 = \frac{1}{\xi} \frac{y \tilde{A} \tilde{x}_0}{x_0 \tilde{x}_0} .$$

The solution of the quadratic equation for λ approximates the (generally complex) smallest eigenvalue of the generally nonsymmetric and nonreal matrix A. This method, if applied to the characteristic equation associated with a given polynomial, gives an effective method for locating the absolutely smallest root of an algebraic equation, (or the absolutely largest root of the inverted equation) without generating the power sums required in Bernoulli's method. (A talk on this subject is to be presented at the Fresno meeting of the American Mathematical Society, May 3, 1952).

Publications: (1) "Alternative derivations of Fox's escalator formulas for latent roots," by G. E. Forsythe; submitted to a technical journal. (2) "Convergence of a method of solving linear problems," by W. Karush; submitted to a technical journal. (3) "Sufficient conditions for the convergence of Newton's method in complex Banach spaces," by M. L. Stein; submitted to a technical journal. (4) "The determination of latent roots and invariant manifolds of matrices by means of iterations," by K. A. Semendiaev, translated by C. D. Benster, edited by G. E. Forsythe, NBS Report No. 1402.

STUDIES IN THE NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS
Task 1101-11-5100/51-1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. J. W. Green prepared a paper [see publication (8) below] which describes and studies a new method for the solution of linear parabolic differential equations. For the equation

$$L[u] = u_{xx} - u_t - g(x,t)\, u = f(x,t)$$

subject to the initial conditions $u(x,o) = u(o,t) = u(\pi,t) = 0$, the method consists in approximating the solution by the sum

$$u_n(x,t) = \sum_{k=1}^{n} C_{n,k}(t) \sin kt ,$$

the $C_{n,k}(t)$ being determined from the conditions

$$\int_0^{\pi} \left\{ L[u_n] - f \right\} \sin jx \, dx = 0, \quad j = 1,2,\ldots,n.$$

W. Wasow studied the asymptotic behavior, for large λ, of the differential equation

$$u^{(4)} + \sum_{j=1}^{4} a_j(x)\, u^{(4-j)} + \lambda^2 \sum_{k=1}^{4} b_k(x) u^{(2-k)} = 0$$

with $b_0(0) = 0$, $b_0^1(0) \neq 0$. The variable x is complex, and the coefficients are assumed to be analytic. This problem occurs in the theory of the stability of laminar viscous flows. It can be solved completely - as far as the asymptotically leading terms are concerned - by expressing the solutions asymptotically in terms of solutions of the special differential equation

$$y^{(4)} + \lambda^2 (xy'' + y) = 0 .$$

The asymptotic properties of the latter can be derived by means of contour integration [see publication (7) below].

E. Stiefel [see publication (9) below] shows that the numerical work involved in solving a boundary value or eigenvalue problem by finite difference methods in a domain with many symmetries often can be reduced by applying the theory of group characters to the group of symmetries of the domain. He also considers the problem of solving $\Delta u = 0$ in a cube when the prescribed boundary values are invariant under the group of rotations of the cube. In the series representation of the solution in terms of harmonic polynomials only a subset of these polynomials actually occurs, and the theory of group characters facilitates considerably the determination of this subset.

G. Blanch has been studying a class of second order differential operators with constant coefficients. The representation of the solution by an integral equation is well known; but it does not seem to have been observed before that the numerical evaluation of the solution can be so arranged as not to involve directly the upper limit of integration. The troublesome problem of iterating approximate solutions is therefore avoided. Moreover, for certain parameters the solution by means of the integral equation has distinct advantages over the corresponding method of stepwise integration, from the viewpoint of the size of the permissible integration interval, for a given upper bound of error. A paper on the subject is in process of preparation.

Publications: (1) "On integration of parabolic equations by difference methods. I: Linear and quasi-linear equations for the infinite interval," by F. John; accepted by Communications on Pure and Applied Mathematics. (2) "On the numerical solution of parabolic partial differential equations," by G. Blanch; accepted by NBS Journal of Research. (3) "On mildly nonlinear partial difference equations of elliptic type," by L. Bers; submitted to a technical journal. (4) "The expansion theorem for pseudo-analytic functions," by L. Bers and S. Agmon; submitted to a technical journal. (5) "On the truncation error in the solution of Laplace's equation by finite differences," by W. Wasow; accepted by the NBS Journal of Research. (6) "On the approximation of linear elliptic differential equations by difference equations with positive coefficients" by T. S. Motzkin and W. Wasow; submitted to a technical journal. (7) "Asymptotic solution of differential equations of hydrodynamic stability in a domain containing a transition point," by W. Wasow; IN MANUSCRIPT. (8) "An expansion method for parabolic partial differential equations," by J. W. Green; IN MANUSCRIPT. (9) "Two applications of group characters to the solution of boundary-value problems," by E. Stiefel; accepted by the NBS Journal of Research.

PROBABILITY METHODS AND SAMPLING TECHNIQUES
Task 1101-11-5100/51-2

Origin: NBS
Sponsor: Office of Naval Research, USN
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. With the aim of deriving useful new results on error analysis for the Monte Carlo method, within the framework of a unified theory, a study was made by J. H. Curtiss of chain functions of the type

$$Z_N = \psi(S_N)\prod_1^N\left[z(S_{k-1},S_k)\right],\quad \Sigma_N = \psi(S_0) + \sum_1^N Z_k,$$

where S_0, $S_1,\ldots$, is a simple Markov chain. In the discrete case, suppose that the possible states of the process are x_i, $i = \ldots,-2,-1,0,1,2,\ldots$, and that the transition probabilities $Pr(S_{k+1}= y \mid S_k = x) = p(x,y)$ are stationary. Then if we let

$$v_N(x) = E(Z_N \mid S_0 = x),\ w_N(x) = E(\Sigma_N \mid S_0 = x),\ a(x,y) = z(x,y)p(x,y),$$

it is easily shown that

$$(*)\qquad v_{N+1}(x) = \sum_y a(x,y)\, v_N(y),\quad v_0(x) = \psi(x);$$

$$(**)\qquad w_N(x) = \psi(x) + \sum_y a(x,y)\, w_{N-1}(y), w_0(x) = \psi(x).$$

If A denotes the matrix $[a(x_i,x_j)] = [a_{ij}]$, then the formal solutions of these recursion relations are in matrix notation respectively

$$v_N = A^N\psi,\ w_{N-1} = (I-A^N)(I-A)^{-1}\psi$$

If A has the following partitioned form:

$$A = \left[\begin{array}{c|c} A_1 & A_2 \\ \hline 0 & 1 \end{array}\right] = \left[\begin{array}{c|c} \begin{matrix} a_{ij} \\ i=1,\ldots,n \\ j=1,\ldots,n \end{matrix} & \begin{matrix} a_{ij} \\ i=1,\ldots,n \\ j=n+1\ldots \end{matrix} \\ \hline 0 & \delta_{ij} \end{array}\right]$$

where δ_{ij} is Kronecker's symbol, and if

$$\psi = (\psi' \mid \psi'') = (\psi_1, \ldots, \psi_n \mid \psi_{n+1}, \ldots) ,$$

then (*) has the form

(***) $$v_{N+1} = A_1 v_N + A_2 \psi'' ,$$

with formal solution

$$v_N = A_1^N \psi' + (I-A_1^N)(I-A_1)^{-1} A_2 \psi'' .$$

If the same matrix A is used in (**) but the initial-value vector ψ is replaced by $\psi^* = (\psi'' \mid 0)$, then (**) becomes

(****) $$w_N = A_1 w_{N-1} + \psi'' ,$$

and its solution is

$$w_{N-1} = (I-A_1^N)(I-A_1)^{-1} \psi'' ,$$

which is exactly what (***) and its solution become when $\psi' = 0$, $A_2 = I$. The well-known necessary and sufficient condition for $A_1^\infty = 0$ is that all eigenvalues of A_1 be in absolute value less than unity; if this condition is satisfied, then obviously limiting vectors v_∞ and w_∞ exist. If $A_2 = I$, the limiting vector in each case satisfies the same system of linear algebraic equations. This is true whether or not $\psi' = 0$. The coexistence of these two stochastic models for linear equations was pointed out by Wasow in a forthcoming paper in Mathematical Tables and Other Aids to Computation (see publication (5) below). The above discussion, together with its analogue for nondiscrete Markov processes, provides a convenient unified background for most of the applications of the Monte Carlo method which have so far been made, including two summation and quadrature problems (the case N = 1), time-dependent transport and diffusion problems, and boundary value problems associated with elliptic differential equations (the case $N = \infty$). In problems involving differential and difference equations, the mean values $w_N(x)$, $w_\infty(x)$, play the rôle of Green's functions. From the point of view of stochastic processes, the elements of the submatrix A_2 correspond to absorbing or trap states.

The study of error analysis centered about the variances of Z_N and Σ_N and the reduction of these variances by "importance sampling." In what follows, the underlying viewpoint is that the matrix A is given, and the problem is to make a wise choice of z(x,y) and p(x,y) (with $z(x,y)\, p(x,y) = a(x,y)$, $p(x,y) \geq 0$, $\sum_y p(x,y) = 1$). Let $B = [z(x_i,x_j)a(x_i,x_j)]$, and let B_1 and B_2 be the submatrices of B corresponding to A_1 and A_2. Let

$$m_N(x) = E(Z_N^2 \mid S_o = x), \mu_N(x) = E(\Sigma_N^2 \mid S_o = x) .$$

Then

$$m_N = B^N D\psi = B_1^N D'\psi' + (I-B_1^N)(I-B_1)^{-1} B_2 D''\psi'' ;$$

also

$$\mu_N = B^N D \psi + (I-B^N)(I-B)^{-1} \left\{ 2D(I-A)^{-1}\psi - D\psi \right\} - 2H_N (I-A)^{-1} A^2 \psi ,$$

where D is a diagonal matrix whose diagonal elements are the components of ψ, and where

$$H_N = \sum_{0}^{N-1} B^k D A^{N-1-k} .$$

If in Σ_N, the initial vector ψ is replaced by ψ^* as in deriving (****) above, then in μ_N, the B's and A's can be replaced by B_1's and A_1's, and ψ by ψ''. If the eigenvalues of B_1 (as well as those of A_1) are all less than unity in absolute value, then it can be shown that limiting second moments exist for both Z_N and Σ_N and are given respectively by

$$m_\infty = (I-B_1)^{-1} B_2 D'' \psi''$$

$$\mu_\infty = (I-B_1)^{-1} \left\{ 2D(I-A_1)^{-1} \psi'' - D'' \psi'' \right\}$$

(The first of these two formulas was derived in the case $A_2 = I$ by Forsythe and Leibler in Mathematical Tables and Other Aids to Computation IV 127-129 (1950), and the second was studied from a different point of view in the special case in which A is a stochastic matrix and $z(x,y) = 1$ by Wasow in his forthcoming MTAC paper referred to previously. Both papers apply the restriction to A_1 that the eigenvalues of the corresponding matrix of absolute values are less than unity in modulus, but this restriction seems to be academic in view of the fact that Z_∞ and Σ_∞ will always be approximated in practice by Z_N and Σ_N, N large but finite.)

These results permit the discussion of a number of dispersion problems related to the Monte Carlo method. For example, if the duration of the random walk associated with the partitioned matrix A is defined as the number of visits to the states corresponding to the elements in columns numbered $j = 1,\ldots,n$ before reaching the trap states $j = n+1,\ldots,$ and if E_i is the mean duration for initial position x_i, and finally, if $\hat{E}$ is the maximum mean duration for any i, then the variance of the duration for starting at x_i can be shown to be not greater than $2\hat{E}^2 - E_i^2$. (The exact formula for the variance is $2\sum_k G(x_i,x_k)\, E_k - E_i - E_i^2$, where $G(x_i,x_k)$ is the mean number of visits to x_k if the random walk starts at x_i.) Another application is to the comparison of the two methods of matrix inversion mentioned above (one by means of Z_∞ and the other by means of Σ_∞). This application generalizes the comparison of the two methods made by Wasow for stochastic matrices in the forthcoming MTAC paper referred to previously.

Turning to importance sampling, a useful selection of the stochastic model in the case of simple summation or quadrature problems (these correspond to the case $N = 1$, the problem being to evaluate $v_1(x) = \sum a(x,y)\, \psi(y)$ or the analogous integral) seems to be as follows:

Suppose that $a(x,y) \geqq 0$. Choose $\emptyset(y) \geqq 0$ in such a way that $w(x) = \sum a(x,y)\, \emptyset(y)$ can be more easily evaluated than $v_1(x)$, and let $\epsilon(y) = [\psi(y) - \emptyset(y)]/\emptyset(y)$. Let $\epsilon \geqq |\epsilon(y)|$. Now choose

$$p(x,y) = \frac{a(x,y)\emptyset(y)}{w(x)}, \quad z(x,y) = \frac{w(x)}{\emptyset(y)}$$

Then of course

$$E(Z_1 \mid S_o = x) = \sum a(x,y)\psi(y)$$

as was desired, and

$$\mathrm{Var}(Z_1 \mid S_o = x) = w(x)^2 \sum_y \epsilon^2(y)\, p(x,y) - (v_1(x)-w(x))^2$$
$$\leqq \epsilon^2\, w(x)^2 .$$

This simple appraisal involves only the given data, but will generally be rather too high, inasmuch as the neglected term $(v_1(x) - w(x))^2$ is of the order of $\epsilon^2 w(x)^2$ also. If $\epsilon = 0$, Z_1 becomes a zero-variance estimator. For $N > 1$, it turns out that Z_N will be a zero variance estimator if and only if $z(x,y) = K\psi(x)/\psi(y)$. In this case, $Z_N = K^N \psi(x)$; and $E(Z_N)$ can satisfy the recursion relation (*) for $N = 1,2,\ldots$, if and only if K is an eigenvalue of A and ψ is a corresponding eigenvector. In the case of the partitioned matrix A described above, this is not a serious restriction because with $K = 1$, ψ'' can be assigned arbitrarily as before, and although ψ' cannot be assigned arbitrarily if ψ is to be an eigenvector, nevertheless the effect of ψ' fades out as $N \to \infty$. In fact, in the limit the vector $(v_\infty | \psi'')$ is obviously an eigenvector of A corresponding to the eigenvalue $K = 1$, so it is reasonable to suppose that the problem of estimating the solution of (***) in the case $N = \infty$ might lend itself easily to a practical importance sampling technique such as that used in the case $N = 1$ above. The appropriate practical arrangement for the case $N = \infty$, suggested by the case $N = 1$, seems to be as follows: Suppose again that $a(x,y) \geqq 0$. Choose any $\emptyset(y) \geqq 0$ and define

$$\epsilon(x_i) = \emptyset(x_i) - \sum a(x_i,y)\, \emptyset(y_i), \quad i = 1,\ldots,n,$$

$$\epsilon(x_i) = 0 ,$$

$$\emptyset(x_i) = \psi(x_i), \quad i > n .$$

(Thus $\emptyset$ plays the rôle of a trial vector in the deterministic theory of linear equations.) Let $\epsilon \geqq |\epsilon(y)|$. Choose

$$p(x,y) = \frac{a(x,y)\emptyset(y)}{\emptyset(x) - \epsilon(x)}, \quad z(x,y) = \frac{\emptyset(x) - \epsilon(x)}{\emptyset(y)}$$

Set up Z_N with the trial vector $\emptyset$ replacing ψ. Then $E(Z_\infty | S_o = x) = v_\infty(x)$, as desired, and

$$\mathrm{Var}(Z_\infty | S_o = x) = E\,[(Z_\infty - \emptyset(x))^2 \mid S_o = x] - (v_\infty(x) - \emptyset(x))^2$$
$$= E\,[(Z_\infty - \emptyset(x))^2 \mid S_o = x] \leqq \epsilon^2\, C(x) ,$$

where C is a complicated vector whose components involve only the elements of A and components of $\emptyset$. In the (artificial) case $\epsilon = 0$, Z_∞ becomes a zero-variance estimator. The existence of zero-variance chain functions seems to obviate the necessity of going into a more elaborate stochastic process for solving steady-state problems, as advocated by G. E. Albert in a recent series of internal memoranda prepared at the Oak Ridge National Laboratory.

The extensions of the above results to continuous operators instead of matrices are fairly immediate. Similar results have also been obtained for estimating $v_N(x)$ by importance sampling for just one preassigned value of N. The new results contained above are being incorporated in a research paper. The long-range objective of the study is a comprehensive exposition of Monte Carlo techniques for essentially non-stochastic problems, including the estimation of eigenvalues.

In addition to the Monte Carlo work by J. H. Curtiss, M. Cohen and M. Kac completed the manuscript of their report on experiments with the Kac-Donsker method for the determination of the lowest eigenvalue of Schrödinger's equation.

Publications: (1) "Uniformly best constant risk and minimax point estimates," by R. T. Peterson, Jr.; NBS J. Res. 48, 49-53 (Jan. 1952); RP2282. (2) "On the estimation of an eigenvalue by an additive functional of a stochastic process with special reference to the Kac-Donsker method," by R. Fortet; NBS J. Res. 48, 68-75 (Jan. 1952); RP2286. (3) "Additive functionals of a Markoff process," by R. Fortet; submitted to a technical journal. (4) "On some functionals of Laplacian processes," by R. Fortet; NBS J. Res. 48, 32-39 (Jan. 1952); RP2280. (5) "On the inversion of matrices by random walks," by W. Wasow; accepted by Mathematical Tables and Other Aids to Computation. (6) "Metodi probabilistici per la soluzione numerica di alcuni problemi di analyisi", by W. Wasow; to be published in the Proceedings of the Fourth Congress of the Italian Mathematical Union in Messina. (7) "A statistical method for finding the lowest eigenvalue of Schrödinger's equation," by M. Kac and M. Cohen; IN MANUSCRIPT.

VARIATIONAL METHODS
Task 1101-11-5100/51-3

Origin: NBS
Sponsor: Office of Naval Research, USN
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. Publication (4), an internal report, was completed by M. L. Stein. R. M. Hayes is continuing his study of general methods for solving linear boundary value problems involving ordinary and partial differential equations. He has obtained general convergence theorems which are applicable to the Rayleigh-Ritz method and also to a generalization of the method given by E. Stiefel and M. R. Hestenes for the finite case. The proof of these results is based on those given in publication (1).

Publications: (1) "Applications of the theory of quadratic forms in Hilbert space to the calculus of variations," by M. R. Hestenes; Pac. J. Math. I, 525-581 (Dec. 1951). (2) "On methods for obtaining solutions of fixed end point problems in the calculus of variations," by M. L. Stein; accepted by the NBS Journal of Research. (3) "The solution of linear equations by minimization," by M. R. Hestenes and M. L. Stein; NAML Report 52-45. (4) "A Rayleigh-Ritz-like procedure for minimizing integrals," by M. L. Stein; IN MANUSCRIPT.

STUDIES IN APPLIED MATHEMATICS
Task 1101-11-5100/51-4

Origin: NBS
Sponsor: Office of Naval Research, USN
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. The computations for Tables I, II and III of publication (2) were completed as a part of task 1101-53-1101/51-36,p.23.

Publications: (1) An introduction to the tables of the Chebyshev Polynomials has been prepared by C. Lanczos, to be included in the NBS Applied Mathematics Series 9, "Tables of the Chebyshev polynomials $S_n(x)$ and $C_n(x)$"; now in press. (2) "Numerical computation of low moments of order statistics from a normal population," by J. Barkley Rosser; submitted to a technical journal. (3) "Analytical and practical curve fitting of equidistant data," by C. Lanczos; IN MANUSCRIPT.

MISCELLANEOUS STUDIES IN THEORETICAL PHYSICS
Task 1101-11-5100/51-5

Origin: Office of Naval Research, USN
Sponsor: " "
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. The study of the application of variational methods to quantum mechanical scattering problems is being continued (see Oct-Dec 1951 issue, p. 7). Explicit closed form results have been obtained for the differential cross-section for scattering by a Yukawa potential, including exchange, using a plane wave trial function with an adjustable wave number. Computations using these expressions are under way. The results will be compared with "exact" differential cross-sections obtained from a numerical integration of Schrodinger's equation which is now being carried out with the aid of W. Futterman of the University of Southern California. The extension to the case in which tensor forces are included in the interaction potential is being studied.

A. Baños, Jr., University of California at Los Angeles, and R. K. Golden have examined a problem involving electromagnetic effects associated with moving bodies (see July-Sept 1951 issue, p. 9). They considered the electric and magnetic fields which a stationary observer attributes to a rotating uniformly magnetized sphere, first when the sphere is in empty space and then when it is surrounded by a stationary concentric conducting shield. Neglecting terms of order v^2/c^2, they found that in both cases the magnetic field is the same as that for the stationary sphere. In addition both internal and external electric fields are present in the first case while only an internal electric field is present in the second. This study is a first step in the consideration of magneto-hydrodynamical effects in the theory of geomagnetism.

Publications: (1) "Modes of vibration of a suspended chain," by D. S. Saxon and A. S. Cahn; accepted by the Quarterly Journal of Mechanics and Applied Mathematics. (2) "Distribution of electrical conduction currents in the vicinity of thunderstorms," by R. E. Holzer and D. S. Saxon; accepted by the Journal of Geophysical Research. (3) "Radiation characteristics of a turnstile antenna shielded by a section of a metallic tube closed at one end," by A. Baños, Jr., D.S. Saxon, and L. Bailin; accepted by the Journal of Applied Physics. (4) "The torsion of anisotropic elastic cylinders by forces applied on the lateral surface," by H. Luxenberg; accepted by the NBS Journal of Research, (5) "An optical model for nucleon-nuclei scattering," by R. E. LeLevier and D. S. Saxon; accepted by the Physical Review. (6) "The electromagnetic field of a rotating uniformly magnetized sphere," by R. K. Golden and A. Baños, Jr., IN MANUSCRIPT. (7) "Variational calculation of scattering cross-section," by E. Gerjuoy and D. S. Saxon; accepted as a letter to the editor by the Physical Review.

STUDY OF RUSSIAN MATHEMATICAL PROGRESS
Task 1101-11-5100/52-1

Origin: NBS Authorized 3/15/52
Sponsor: Office of Naval Research, USN
Manager: G. E. Forsythe

Objective: To prepare, and keep up to date in readily available form, information regarding a) mathematical monographs in Russian, b) papers in Russian on numerical mathematical analysis. Also, to make this literature accessible to U. S. mathematicians by c) obtaining important monographs and journals for the Institute for Numerical Analysis library, d) publishing bibliographies of Russian material, e) translating selected important material on numerical analysis.

Background: Like other European mathematicians, those of Russia have always devoted considerable attention to publishing good expository monographs on advanced mathematical subjects. The output of such monographs has recently increased to an astounding degree. There have appeared within the past few years a number of first class books, several of which are unparalleled in the Western literature. The existence of these books is not well known in the United States. Obtaining copies of them, publishing lists of them, and translating important excerpts will contribute a great deal towards making them known. It also has the desirable effect of stimulating the study of Russian by mathematicians.

With respect to journal articles the situation is a little different. Formerly a Russian article was ordinarily followed by a summary in French, English, or German. This practice was stopped abruptly about 1947, and moreover, Russian mathematicians then closed the practice of writing some articles in Western languages. Mathematical Reviews is doing excellent work in reviewing Russian articles, and the American Mathematical Society (under an O.N.R. contract) is translating important ones - mostly in pure mathematics. With respect to journal articles, the present task is intended only to supplement these efforts, especially in numerical analysis, which has not benefited much from the translation project.

Comments: The sponsor feels that it is a matter of vital importance to our national self interest that scientists become familiar with the Russian language and with Soviet scientific work. It is perhaps not so important in pure mathematics, where ideas seem to leap freely over barriers of language and censorship. But in applied work it is often essential to follow other men's work and reasoning laboriously from line to line; reading a brief abstract is not sufficient. Unless the reader knows Russian fairly well, he will gain little from the original article.

Status: NEW. The Institute for Numerical Analysis library has acquired copies of over sixty Russian monographs in mathematics (with a few in physics). It subscribes to about six Russian journals. Publication (1) is being typed for distribution. It lists over 350 monographs of interest to the practising mathematician, arranged a) by author and b) by subject. Publication (2) is a translation of an important article on calculating matrix eigenvalues by classical iterative methods. It is available for distribution. Publication (3) is a long and interesting one - over 100 pages in Russian - in which the tools of analysis in Banach spaces are applied to various problems of numerical analysis, - solving differential and integral equations, and determining the inverse and spectrum of a finite matrix.

An informal, but important, result of the task has been the initiation of a Russian class for mathematicians at the University of California at Los Angeles.

Publications: (1) "Bibliographical survey of Russian mathematical monographs 1930-1951," by G. E. Forsythe; IN MANUSCRIPT. (2) "The determination of latent roots and invariant manifolds of matrices by means of iteration," by K. A. Semendiaev translated by C. D. Benster, edited by G. E. Forsythe; NBS Report No. 1402. (3) Translation of "Functional analysis and applied mathematics," by L. V. Kantorovich; IN MANUSCRIPT.

STUDIES IN PURE MATHEMATICS
Task 1101-11-5101/50-4
(formerly 11.1/1-50-4)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full task description appears in July-Sept 1949 issue.

Status: CONTINUED. D. H. Lehmer has continued the investigation into the properties of Kloosterman sums

$$S_p(k) = \sum_{k=1}^{p-1} \exp\left[2\pi i(h + k\bar{h})/p\right] \quad (h\bar{h} \equiv 1 \pmod{p})$$

where p is an odd prime. The inequality

$$|S_p(k)| < 2\sqrt{p}$$

holds uniformly for all $p < 100$. The sums of powers

$$R_r = \sum_{(\frac{k}{p})=+1} S_p^r(k) \text{ and } N_r = \sum_{(\frac{k}{p})=-1} S_p^r(k)$$

extending over the quadratic residues and nonresidues respectively, are integers and have been computed for $r = 0(1)6,8,10,16$ by using the approximate values of $S_p(k)$ obtained previously (see Oct-Dec 1951 issue, p. 8), with the help of certain congruence properties. A further study of these integers is in progress.

A series of tests for primality of Mersenne numbers $2^p - 1$ has been run on the SWAC. The coding for this calculation was sent in by R. M. Robinson of the University of California, Berkeley. The results have been interesting from both a theoretical and a practical viewpoint. The tests have shown a high degree of reliability in SWAC operation, and they have given some unexpected information on the distribution of primes of the form $2^p - 1$. Tests have been run once for $p \leqslant 1867$ and twice for $p \leqslant 1303$ with exact agreement. Only two new primes have been discovered, namely those for p = 521 and p = 607. These are by far the largest known primes. The test consists of computing p-1 terms of the sequence 4, 14, 194,..., $[U_{n+1} = U_n^2 - 2]$ modulo $2^p - 1$. Divisibility of U_{p-1} by 2^p-1 is necessary and sufficient for the primality of 2^p-1. The test requires approximately p^3 microseconds.

L. M. Blumenthal continues his study of the applications of metric methods in abstract algebra (see publication 6). A Boolean metric space $\mathcal{B}$ is obtained by associating with each pair of elements p,q of a set an element d(p,q) of a Boolean algebra as distance, so that d(p,q)=0 if and only if p=q, d(p,q)=d(q,p), and $d(p,q)+d(q,r) \supset d(p,r)$ (+ ,$\supset$ denote Boolean addition and inclusion, in the wide sense, respectively). Boolean geometry studies the invariants of $\mathcal{B}$ under the group of congruences (i.e., distance-preserving mappings). Any Boolean algebra is a space $\mathcal{B}$ with d(p,q)=pq'+p'q (juxtaposition and ' denoting Boolean product and complementation, respectively), and this paper begins the systematic study of these Boolean geometries by developing linearity notions and the theory of metric segments. Defining these in a suitable manner, they are shown to possess many properties of segments in ordinary metric spaces. Thus, (1) the length of a segment S_a^b is the distance d(a,b) of its end-elements, (2) if S_a^b, S_b^c are segments and b is metrically between a,c then their set-union is a segment S_a^c, and (3) for m > 4, any m-tuple of $\mathcal{B}$ is on some segment if and only if each triple of the m-tuple has that property.

E. Stiefel (see publication 7) succeeded in finding a complete answer to the question concerning the possible dimensions of spaces in which generalised systems of Cauchy-Riemann equations can exist. Here n partial differential equations

$$\ell_j = \sum_{i,k}^{n} a_{jk}^{i} \frac{\partial u_k}{\partial x_i} = 0 \qquad (j = 1,2,\ldots,n)$$

with constant complex coefficients are said to form a system of generalised Cauchy-Riemann equations, if there exist constants b_{jk}^h such that

$$\Delta u_j = \sum_{h,k}^{n} b_{jk}^{h} \frac{\partial \ell_k}{\partial x_h}$$

In 1939, O. Taussky-Todd had proved that n must be a power of 2. Using methods from the theory of the representation of algebras, E. Stiefel proved that such systems can exist for n = 1,2,4,8 only and gave an enumeration of all possible cases.

The paper by I. J. Schoenberg and A. Whitney (see publication 8) is based on previous results concerning the behavior of Polya frequency functions. The authors obtain necessary and sufficient conditions for the positivity of any minor of the matrix $\Lambda(x - y)$. The problem separates into a number of subcases, depending on the analytical nature of the Laplace transform of $\Lambda(x)$. Each case is treated exhaustively and the problem solved. The results obtained give a general and elegant discussion of the necessary and sufficient conditions of interpolating uniquely with the help of spline curves of any order.

I. J. Schoenberg discusses a question recently raised by L. M. Blumenthal (see publication 9). It is known that a real inner-product space is ptolemaic, i.e., among the distances between any four points of the space Ptolemy's inequality holds. The question is as follows: Let the real normed linear space S be ptolemaic; does it follow that its norm springs from an inner product? This question is answered in the affirmative. The proof is derived from a new, slightly improved, version of a characterization of inner-product spaces due to M. M. Day.

T. S. Motzkin, in a joint paper with E. G. Straus and F. A. Valentine (see publication 10), studies compact point-sets S in the plane for which, for each x ϵ S, the set of points of S with maximum distance from x always, or never, consists of one single point. In the results special convex curves and various kinds of evolutes of the latter play a role.

Publications: (1) "On subharmonic, harmonic, and linear functions of two variables," by E. F. Beckenbach; Revista de Matematica y Fisica Teorica (Argentina). 8, 7-13 (Nov. 1951). (2) "On relative extrema of Bessel functions," by O. Szász; accepted by the Bolletino della Unione Matematica Italiana (Firenze). (3) "On the relative extrema of the Hermite orthogonal functions," by O. Szász; accepted by the Journal of the Indian Mathematical Society. (4) "On a recursion formula and on some Tauberian theorems," by N. G. de Bruijn and P. Erdös; accepted by the NBS Journal of Research. (5) "Metric methods in integral and differential geometry," by J. W. Gaddum; accepted by the American Journal of Mathematics. (6) "Boolean Geometry I," by L. M. Blumenthal; IN MANUSCRIPT. (7) "On Cauchy-Riemann equations in higher dimensions," by E. Stiefel; accepted by the NBS Journal of Research. (8) "On Polya frequency functions III. The positivity of translation determinants with an application to the interpolation problem by spline curves," by I. J. Schoenberg and A. Whitney; submitted to a technical journal. (9) "A remark on M. M. Day's characterization of inner-product spaces and a conjecture of L. M. Blumenthal," by I. J. Schoenberg; submitted to a technical journal. (10) "The number of farthest points," by T. S. Motzkin, E.G. Straus, and F. A. Valentine; submitted to a technical journal.

COMPUTATION OF THE IMAGINARY ZEROS OF THE RIEMANN ZETA FUNCTION

Task 1101-11-5101/50-13
(formerly 11.1/1-50-13)

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: D. H. Lehmer
Full task description appears in Apr-Jun 1950 issue.

Status: CONTINUED. The calculation of the first 5000 "Gram points" has been run twice on the SWAC. The calculation of the Zeta-function at these points has been coded and is being run on the SWAC. Among the first 1500 Gram points there were found 80 failures of Gram's Law. A special run to take care of these cases is being coded.

2. Applied Research

RESEARCH IN THE MATHEMATICAL THEORY OF PROGRAM PLANNING
Task 1101-21-5102/50-11
(formerly 11.1/1-50-11)

Origin: Office of Air Comptroller, USAF
Sponsor: " "
Full task description appears in Apr-Jun 1950 issue.

Status: CONTINUED. The theory of the transporation problem (F. L. Hitchcock, T. C. Koopmans, G. B. Dantzig, and M. Flood) was furthered by T. S. Motzkin in two ways. A succinct representation of all basic solutions (before minimization) of the system

$$\sum_j x_{ij} = a_i, \ \sum_i x_{ij} = b_j, \ x_{ij} \geqslant 0$$

was found (see publication(1) below). Each basic solution is obtained by superimposing contiguous intervals of lengths b_j on contiguous intervals of lengths a_i and putting x_{ij} equal to the common part of a_i and b_j. Hence (as is known) the coefficients of a_i and b_j in x_{ij} are integers ($\pm$ 1). The same holds for the multi-index problem ($x_{i_1,\ldots,i_n}$, $i_k \leqslant s_k > 1$, r-fold sums) if and only if r=1 and the number of $s_k > 2$ is at most 2 (see publication (2) below). An example with fractional coefficients and an additional behavior discrepancy for the multi-index problem was found by A. J. Hoffman.

Besides further hand computing under the supervision of M. Howard to test relaxation methods, the double description method was coded for SWAC by L. Joel.

Continuing his study of metric methods in linear inequalities, L. M. Blumenthal defined a homogeneous system of inequalities to be <u>irreducibly consistent</u> provided (1) the system has a non-trivial solution, but (2) each subsystem obtained by suppressing a column of the unknowns has only the trivial solution. The subset C of the unit n-sphere S_n associated with the coefficients in such a system is non-global, but the projections of C on each of the great (n-1) spheres in which the coordinate planes intersect S_n are global. It is easily seen that a system is irreducibly consistent if and only if the solution set $\sum(C) \neq 0$ and $\sum(C)$ is contained in the interior of <u>one</u> orthant. This orthant is the one that is in the interior of C*, the convex extension of C.

As a <u>modus operandi</u> the above considerations suggest that a system of inequalities be studied by examining first the one-column subsystems, then the two-column systems, etc., with the knowledge that if none of the k-column subsystems has a nontrivial solution, any consistent (k+1)-column system has all of its solutions in one orthant of the corresponding k-sphere. This method, together with a device for "casting out" orthants was applied to a system of ten inequalities in four unknowns (which turned out to be irreducibly consistent) and the solution orthant, together with a solution, was found.

J. W. Gaddum proved (see publication (4) below) that a convex cone and its polar cone have a nonnull vector in common, unless they are orthogonal linear varieties. This is applied to show, among other things, that if $A x \geqslant 0$ has a solution, it has one which is in the convex cone generated by A, and hence $AA'y \geqslant 0$ has a nonnegative solution.

For other investigations on convex curves compare with task 1101-11-5101/50-4, p. 12.

T. S. Motzkin participated in the Third Annual Meeting of the Logistics Conference, sponsored by The George Washington University and Office of Naval Research, and held on Jan. 2, 3, 7, and 8 at Washington, D.C; in a Round Table Conference on the STATAC-SCOOP machine, held on Jan. 4 at the National Bureau of Standards; and in coordinating conferences with the Linear Programming Group of the Computation Laboratory of the National Applied Mathematics Laboratories.

Publications: (1) "Basic solutions of the transportation problem," by T. S. Motzkin; IN MANUSCRIPT. (2) "The multi-index transportation problem," by T. S. Motzkin; IN MANUSCRIPT. (3) "Two existence theorems for systems of linear inequalities," by L. M. Blumenthal; submitted to a technical journal. (4) "A theorem on convex cones with application to linear inequalities," by J. W. Gaddum; submitted to a technical journal.

LANGUAGE TRANSLATION STUDY
Task 1101-21-5104/52-1

Origin: NBS
Sponsor: The Rockefeller Foundation
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. Work continued on the investigation of word frequency both in Spanish and in German, under the direction of W. E. Bull and V. A. Oswald, respectively. The Spanish project is now in its final stages. The material is being written for publication and charts are being prepared which express the linguistic facts graphically. Present indications are that the data will be significant in many fields of investigation. In German, a microglossary has been constructed for the field of brain surgery by scanning sixteen different articles on the subject. The glossary has been extended well beyond the scope originally planned, since it was found that the recurrence of nontechnical items could be predicated to almost as high a degree as the recurrence of technical items. The statistical import of these findings is now being investigated. The semantic import can be established just as soon as the work of alphabetizing the glossary has been completed.

The Research Laboratory of Electronics of the Massachusetts Institute of Technology, aided by a grant of the Rockefeller Foundation, is planning a Conference on Mechanical Translation to be held June 18-20 at the Massachusetts Institute of Technology, at which conference reports will be presented on the above task.

3. Development

NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER (SWAC)
(previously listed as Air Materiel Command Computing Machine)
Task 1101-34-5104/49-1
(formerly 11.1/22-49-1)

Origin: Flight Research Laboratory, Wright Air Development Center, USAF
Sponsor: " " " "
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. There are three items which have recently produced a substantial increase in operating time on the SWAC. These three items are: 1) decreasing the accelerating voltage to minimize the effect of flaws, 2) installing a delay circuit to minimize the effects of spillover, primarily caused by lowering the accelerating voltage, and 3) installing the motor generator set as of last December to stabilize the voltages and minimize the need for adjustments relative to the Williams' tube memory system. Improvements have been made in the Williams' tube amplifiers and a more thorough routine of systematic maintenance has been established. A new method of input to the SWAC making use of an IBM collator is almost ready to go into operation. The rate of input with this device is approximately 80 times that of the Flexowriter equipment now in use.

LOGICAL NOTATION AND BLOCK DIAGRAM SYMBOLISM FOR A.D.C.M.
Task 1101-34-5103/49-2
(formerly 11.1/22-49-2)

Origin: NBS
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Distribution continued of the preliminary lists of terminology and block diagram symbols to be used in connection with automatic computing machines. The lists are now being revised on the basis of the many comments and suggestions which have been offered in response to our request.

4. Mathematical Services

MATHIEU FUNCTIONS II
Task 1101-53-1101/45-1
(formerly 11.1/2-45-1)

Origin: Applied Mathematics Panel NDRC
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1950 issue.

TABLE OF BESSEL-CLIFFORD FUNCTIONS
(formerly listed as SPECIAL TABLE OF BESSEL FUNCTIONS)
Task 1101-53-1101/48-2
(formerly 11.1/2-48-2)

Origin: NBS
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Jan-Mar 1949 issue.

Status: CONTINUED. The final manuscript has been fully checked.

TABLES FOR ROCKET AND COMET ORBITS
Task 1101-53-1101/48-3
(formerly 11.1/2-48-3)

Origin: NBS
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Jan-Mar 1949 issue.

Status: CONTINUED. The table is being retyped, on account of a flaw in the manuscript.

Publication: Approved for publication as NBS Applied Mathematics Series 20; to be printed by the Government Printing Office.

COMPUTING SERVICES FOR RESEARCH STAFF OF THE INSTITUTE FOR NUMERICAL ANALYSIS
Task 1101-53-1101/49-1a
(formerly 1101-53-1100/49-1)

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: M. Howard
Full task description appears in July-Sept 1949 issue, (see task 11.1/32-49-1).

Status: CONTINUED. Computations on desk calculators were made for various members of the staff. Among such calculations were: (1) For

F. Acton: Computations were made to test approximations for the probability integral useful in numerical analysis. (2) For G. Blanch and D. H. Lehmer: Investigation was made of the zeros of a class of polynomials arising in connection with a certain asymptotic expansion. (3) For C. Lanczos: Computations were made for further investigation of the Chebyshev iteration technique, and the method was extended for finding the smallest eigenvalue of a non-symmetric matrix. (4) For D. H. Lehmer: Brief computations were made involving the evaluation of a matrix and the determination of the roots of several polynomials. (5) For D. Saxon: Further investigation was made involving the evaluation of series and functions for the determination of scattering cross-sections, which are attained by using variational methods.

Problems computed on IBM machines for the research staff included the following: (1) For D. Saxon: Numerical solution was made of a second order differential equation containing two parameters. Solutions were made for thirty sets of parameters.

Problems for the research staff involving the SWAC were: (1) For F. Acton: A code was prepared for the computation of probability distribution leading to confidence limits which will include all of a future sample of m from a normal distribution with probability α. (2) For A. Cahn: The routine for the Goldbach problem was recoded to use Alway's method to test for primality. (3) For M. Cohen: Further work was done on the Monte Carlo solution of Schrödinger's equation. (4) For G.Forsythe: A Monte Carlo solution of a boundary-value problem for Laplace's differential equation was run on SWAC. (5) For M. R. Hestenes: Several sets of 5 linear equations were solved on SWAC, and coding was done to solve 8 equations. (6) For D. H. Lehmer: (a) Mersenne numbers were tested on SWAC for primality, and $(2^{521} - 1)$ and $(2^{607} - 1)$ were found to be prime. See task 1101-11-5101/50-4, p.12, (b) Values of certain character sums were obtained on SWAC, and (c) A routine was coded and checked out for the continued fraction expansions of square roots of integers. (7) For W. E. Milne: A solution obtained on SWAC during the previous quarter for a set of 3 simultaneous differential equations was checked and converted. (8) For J. B. Rosser: Additional coding and computing has been done for task 1101-11-5101/50-13, p. 14

PUNCHED CARD LIBRARY
Task 1101-53-1101/49-2
(formerly 11.1/2-49-2)

Origin: NBS
Full task description appears in Apr-Jun 1949 issue.

Comments: A catalog of tables on punched cards which are on file at the Institute may be obtained by addressing the Institute for Numerical Analysis, 405 Hilgard Avenue, Los Angeles 24, California. Within the limits of the program of the computation unit of the Institute, tables will be duplicated upon request, provided the requester furnishes the blank cards. Request should be addressed directly to the Institute.

Status: CONTINUED. Pearson's table of the Bivariate Normal Distribution Function has been checked by differences up to and including the sixth order. Values of the function for arguments k = 0.1, h = 0,0.1; and k = 0.2, h = 0,0.1, 0.2, for r = $\pm$ .05(.05).95, have been computed on the CPC as a further check. The table of random digits (20,000 cards, 50 digits per card), and the table of random normal deviates (10,000 cards) were duplicated and the copies sent to H. Schutzberger, of Sandia Corporation.

METEOROLOGICAL MEANS
Task 1101-53-1101/50-17
(formerly 11.1/31-50-17)

Origin: Department of Meteorology, UCLA — Authorized 3/31/50
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF — Completed 3/31/52
Manager: E. Yowell

Objective: To calculate the time averages (daily readings for one month) and the space averages (readings, for a given day, around an entire latitude circle) of various meteorological elements (i.e., wind velocity, temperature, height) of the 700 mb, 500 mb, and 300 mb pressure levels.

Background: This computation arises in connection with research in the general circulation of the atmosphere being performed by the U.C.L.A. Meteorology Department for the Watson Laboratories of the AMC.

Comments: Raw data for this computation are furnished to INA on punched cards by the U.C.L.A. Meteorology Department.

Status: COMPLETED. The results were transmitted to the originator.

EARTH TIDES
Task 1101-53-1101/51-1

Origin: Geophysics Department U.C.L.A.
Sponsor: Office of Naval Research, USN
Manager: T. H. Southard
Full task description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1950 issue.

EVAPORATION COMPUTATIONS
Task 1101-53-1101/51-3

Origin: Naval Electronics Laboratory
Sponsor: Bureau of Reclamation, Department of the Interior
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. Data for months of November 1950 through August 1951 have been processed and results transmitted to originator. Daily averages of all data are being prepared. These are 90 percent complete.

RAYDIST DATA ANALYSIS
Task 1101-53-1101/51-10

Origin: Naval Air Missile Test Center (Point Mugu) — Authorized 12/1/50
Sponsor: Bureau of Aeronautics, USN — Terminated 3/31/52
Manager: E. Yowell

Objective: To calculate the coordinates of an object being followed by a Raydist tracking system.

Status: TERMINATED.

COMPUTATIONS IN CONNECTION WITH PROGRAM ANALYSIS
Task 1101-53-1101/51-18

Origin: Operations Analysis Office — Authorized 1/1/51
Sponsor: Office of Air Comptroller, USAF — Terminated 3/31/52
Manager: R. Lipkis

Objective: To perform certain computations, as requested, in connection with this work.

Comments: Most of the computing submitted to be done on this subject will be programmed for the SWAC.

Status: TERMINATED.

STATISTICAL SMOOTHING
Task 1101-53-1101/51-19

Origin: Stanford Research Institute, Stanford University
Sponsor: Office of Research Operations, U. S. Army
Full task description appears in Jan-Mar 1951 issue.

Status: CONTINUED. One additional case was completed.

ROCKET GRAIN BURNING
Task 1101-53-1101/51-21

Origin: Naval Ordnance Test Station (Inyokern), USN — Authorized 2/28/51
Sponsor: Bureau of Ordnance, USN — Completed 3/31/52
Managers: F. John, G. Blanch, R. R. Reynolds

Objective: Part I: To solve the equation

$$u_t = u_{xx} + \exp(-x) + \exp(A - \frac{B}{u+c})$$

subject to the boundary conditions

$$u(x,0) = 0;\ u(\infty,t) = 0;\ u_x(0,t) = 0.$$

Part II: To solve

$$u_t = u_{xx} + \exp(x_b - x) + \exp(A - \frac{B}{u+c}) ,$$

where $x_b(t)$ is that value of x for which $u = u_b$(constant). In part II, x_b defines a moving boundary and replaces the condition $u_x(0,t) = 0$. The other two boundary conditions of Part I hold.

Background: The solution of this equation is desired by the Naval Ordnance Test Station in connection with research being performed at that base.

Comment: A preliminary investigation of this problem has been authorized. Decision as to whether or not a complete study will be made will depend on the preliminary results.

Status: COMPLETED.

COMPUTATION IN CONNECTION WITH A STUDY OF POLARIZATION OF LIGHT
Task 1101-53-1101/51-25

Origin: Department of Meteorology, UCLA
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Jan-Mar 1951 issue.

Status: CONTINUED. Completion of the final report is awaiting introductory material and information concerning format to be furnished by the originator.

CONVERSION OF HEXIDECIMAL NUMBERS
Task 1101-53-1101/51-28

Origin: Naval Air Missile Test Center (Point Mugu)
Sponsor: Bureau of Aeronautics, USN
Full task description appears in Jan-Mar 1951 issue.

Status: CONTINUED. One run of data was reduced this quarter.

TABLES OF THE BIVARIATE NORMAL DISTRIBUTION FUNCTION
Task 1101-53-1101/51-32

Origin: Division 13, NBS
Sponsor: Office of Chief of Ordnance, U. S. Army
Full task description appears in Apr-Jun 1951 issue.

Status: CONTINUED. Differences of Pearson's values and those of E. Fix have been checked. In certain regions entries will have to be recomputed to determine their accuracy. This recomputation will be done on IBM equipment.

PRESSURE FIELDS OF POTENTIAL FLOW PAST A BODY OF REVOLUTION
Task 1101-53-1101/51-33

Origin: Naval Ordnance Test Station (Pasadena)
Sponsor: Bureau of Ordnance, USN
Full task description appears in Apr-Jun 1951 issue.

Status: CONTINUED. An investigation of an integral equation method of solution was started.

SIMPLIFIED ROLLING PULLOUT EQUATIONS
Task 1101-53-1101/51-34

Origin: Cornell Aeronautical Laboratory
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Apr-Jun 1951 issue.

Status: CONTINUED. Experimental data have been received for eight test flights. Coding of these flights for the SWAC is under way.

LOW MOMENTS OF ORDER STATISTICS
Task 1101-53-1101/51-36

Origin: University of Oregon
Sponsor: Office of Naval Research, USN
Full task description appears in Apr-Jun 1951 issue.

Status: CONTINUED. The computations for the first and second moments are complete, as are certain quantities required in the computation of the cross moments. This task, although temporarily halted pending further financial support, is now being continued under a limited budget.

REDUCTION OF HYDROGRAPHIC DATA
Task 1101-53-1101/51-39

Origin: Scripps Oceanographic Institute — Authorized 6/22/51
Sponsor: University of California — Completed 3/31/52
Manager: R. R. Reynolds

Objective: Given sets of observations of salinity S, temperature T, and pressure P, of samples of sea-water:

1. Compute Cl, the chlorinity, by the formula

$$Cl = \frac{S-0.030}{1.8050} ,$$

2. Compute

$$\sigma_{S,0,0} = -0.069 + 1.4708\, Cl - 0.001570\, Cl^2 + .0000398\, Cl^3 .$$

3. Compute the auxiliary quantities A_T, B_T, Σ_T where

$$A_T = 10^{-3}\, T(4.7867 - 0.098185T + .0010843T^2)$$

$$B_T = 10^{-6}\, T(18.030 - 0.8164T + 0.01667T^2)$$

$$\Sigma_T = -\frac{(T - 3.98)^2\,(T + 283)}{503.570\,(T + 67.26)} .$$

4. Compute $\sigma_{S,T,O}$ by the formula

$$\sigma_{S,T,O} = \Sigma_T + (\sigma_{S,O,O} + 0.1324)[1 - A_T + B_T(\sigma_{S,O,O} - 0.1324)] .$$

5. Compute $\alpha_{S,T,P}$ by the formula

$$\alpha_{S,T,P} = \alpha_{S,T,O} - 10^{-9} P \alpha_{S,T,O} \Big\{ \frac{4886}{1 + 0.0000183P} - [227 + 28.33T - 0.551T^2 + 0.004T^3] + 10^{-4} P [105.5 + 9.50T - 0.158T^2] - 10^{-8} (1.5P^2T) - \frac{\sigma_{S,O,O} - 28}{10} [147.3 - 2.72T + 0.04T^2 - 10^{-4}P (32.4 - 0.87T + 0.002T^2)] + \left(\frac{\sigma_{S,O,O} - 28}{10}\right)^2 [4.5 + 0.1T - 10^{-4}P (1.8 - 0.06T)] \Big\}.$$

The quantities σ and α are related by the equations $\alpha = \frac{1}{\rho} \sigma = 10^3(\rho - 1)$.

6. Compute the anomaly in specific volume

$$\delta = \alpha_{S,T,P} - \alpha_{35,0,P}.$$

7. Compute the anomaly in geopotential distance between isobaric surfaces P_1 and P_2

$$\Delta D = \int_{P_1}^{P_2} \delta \, DP.$$

Background: The data consists of measurements of temperature T (in degrees centigrade), and salinity S (in parts per mille), of a sample of water taken at a depth corresponding to a sea-pressure P(in decibars). The density ρ, or the specific volume is a function of all three parameters. It is desired to determine the specific volume for all points of observation by the method given by V. Bjerknes and J. W. Sandstrom, Dynamic Meteorology and Hydrography, part 1, Carnegie Institute of Washington, Pub. No. 88, 1910.

Comments: These data are collected by Scripps Oceanographic Institute in connection with a study of Pacific currents.

Status: COMPLETED. The last phase of this task, the training of Scripps personnel in the methods used to solve the hydrographic data problem, was completed.

AUTOCORRELATION COEFFICIENTS
Task 1101-53-1101/52-2

Origin: Naval Air Missile Test Center (Point Mugu)
Sponsor: Bureau of Aeronautics, USN
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Computations are performed as data are received. One run was computed in connection with phase one during this quarter.

PRESSURE DISTRIBUTION ON A BODY OF REVOLUTION IN SUPERSONIC FLIGHT
Task 1101-53-1101/52-3

Origin: Hughes Aircraft Company — Authorized 8/20/51
Sponsor: Flight Research Laboratory — Completed 3/31/52
Wright Air Development Center, USAF
Manager: R. R. Reynolds

Objective: To devise an I.B.M. procedure for determining the velocity distribution on the surface of a bluff body of revolution moving at supersonic speed in a steady inviscid, rotational, compressible medium.

Background: The coordinates x,r in a meridional plane and the speed q and direction θ of the flow are given along an initial curve, as well as the direction σ of a detached shock front. The equations of the left and right characteristics are

$$dr = \tan(\theta + \alpha)dx \quad \text{and} \quad dr = \tan(\theta - \alpha)dx,$$

respectively, where α is the Mach angle. Tollmien's equations of compatibility

$$d\theta = \frac{\cot\alpha}{q}\,dq - \frac{\sin\theta\,\sin\alpha}{r\,\sin(\theta+\alpha)}\,dr + \frac{\sin\alpha\cos\alpha}{\gamma(\gamma-1)}\,ds, \quad \text{where } \gamma = 1.405$$

$$d\theta = -\frac{\cot\alpha}{q}\,dq + \frac{\sin\theta\,\sin\alpha}{r\,\sin(\theta-\alpha)}\,dr - \frac{\sin\alpha\cos\alpha}{\gamma(\gamma-1)}\,ds$$

also hold on the left and right characteristics, respectively. These four equations are sufficient for the calculation of x,r,q,θ and the entropy s at interior points. On the shock front these quantities are determined from a shock polar diagram.

Status: COMPLETED. Computations were started on a seventh body, but the initial conditions proved to be incompatible. Further work will be done by the originator. The description of the procedure used in this task is reported in the appendix of this issue.

RAMJET CONTROL
Task 1101-53-1101/52-4

Origin: Marquardt Aircraft Company
Sponsor: Air Materiel Command, USAF
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Responses are being determined for another y(t).

MAGNETO-IONIC EQUATIONS
Task 1101-53-1101/52-5

Origin: Air Force Cambridge Research Center, USAF
Sponsor: " " "
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. Possible computation methods were examined further; no computation has been performed as yet.

REDUCTION OF THEODOLITE DATA
Task 1101-53-1101/52-6

Origin: Naval Air Missile Test Center (Point Mugu)
Sponsor: Bureau of Aeronautics, USN
Manager: F. H. Hollander
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. One theodolite run was computed this quarter.

RANGE ERROR COMPUTATION
Task 1101-53-1101/52-7

Origin: Naval Air Missile Test Center (Point Mugu) Authorized 9/19/51
Sponsor: Bureau of Ordnance, USN Completed 3/31/52
Manager: F. H. Hollander

Objective: Given the coordinates of a transmitter and of three receivers, to calculate the following quantities for each of 111 specified points. Let ℓ_0, m_0, n_0 be the direction cosines from the transmitter to a point in space, and ℓ_i, m_i, n_i $(i = 1,2,3)$ be the direction cosines from the receivers to the point. If the matrix (a_{ij}) is the inverse of the matrix

$$\begin{pmatrix} \ell_0 + \ell_1 & m_0 + m_1 & n_0 + n_1 \\ \ell_0 + \ell_2 & m_0 + m_2 & n_0 + n_2 \\ \ell_0 + \ell_3 & m_0 + m_3 & n_0 + n_3 \end{pmatrix}$$

the quantities desired are

$$\frac{\Delta x}{\Delta p} = (a_{11}^2 + a_{12}^2 + a_{13}^2)^{\frac{1}{2}}$$

$$\frac{\Delta v}{\Delta p} = (a_{21}^2 + a_{22}^2 + a_{23}^2)^{\frac{1}{2}}$$

$$\frac{\Delta z}{\Delta p} = (a_{31}^2 + a_{32}^2 + a_{33}^2)^{\frac{1}{2}}$$

The computation is to be made for each of 4 sets of three receivers.

Status: COMPLETED.

THE DETERMINATION OF THE PERIODS AND AMPLITUDES OF THE LIGHT VARIATIONS OF THE STARS δ SCUTI AND 12 LACERTAE

Task 1101-53-1101/52-9
(formerly 1101-53-1100/49-4)

Origin: NBS
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Apr-Jun 1949 issue (see task 11.1/32-49-4).

Status: INACTIVE. For status to date see Oct-Dec 1950 issue.

BUBBLE DISTANCES

Task 1101-53-1101/52-11

Origin: Meteorology Department, University of California at Los Angeles
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. Listings of original data were checked by the Meteorology Department. After corrections were made, the corrected data were listed and differenced. Bubble pairs have been selected on approximately half of the data.

SUBSONIC AIR FORCES

Task 1101-53-1101/52-14

Origin: Flight Research Laboratory, Wright Air Development Center, USAF — Authorized 12/15/51, Terminated 3/31/52
Sponsor: " " "
Manager: R. Reynolds

Objective: To determine lift and moment coefficients for 65 wing-aileron combinations oscillating with reduced frequency $\omega = .04(.04).52$ in a stream at Mach number 0.7. The wing section is assumed to be a straight line extending from $x = -1$ to $x = 1$ with the control surface hinge at $x = -.1(.2).9$ $(x \neq .5)$.

Comments: The formulas used, based principally on the investigation of Poisson's integral equation by H. G. Küssner and L. Schwarz, are derived in H. E. Fettis' "Calculation of non-stationary air forces at subsonic speeds," OAR Technical Report No. 5 (1951), Flight Research Laboratory, Wright Air Development Center, USAF.

Status: TERMINATED. The sponsor withdrew the problem.

EVALUATION OF TRANSCENDENTAL EXPRESSIONS
Task 1101-53-1101/52-17

Origin: North American Aviation, Inc. Authorized 12/15/51
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF Completed 3/31/52
Manager: Roselyn Lipkis

Objective: For each 765 combinations of the parameters A, b, θ, and x, determine the integral values of D and W which produce a minimum of

$$z = \frac{AW + D}{1 - \left[1 - bx^{\frac{\theta}{W+D}}\right]^{W}}$$

subject to the restrictions $0 \leqq D \leqq 30$ and $1 \leqq W \leqq 5$.

Background: These computations arise in connection with the work at North American Aviation, Inc., in connection with an Air Force Contract.

Status: COMPLETED. The computations were performed on the SWAC, and the results were submitted to the contractor.

INHERENT ERROR ANALYSIS FIRE CONTROL EVALUATION PROGRAM
Task 1101-53-1101/52-19

Origin: Naval Ordnance Test Station (Inyokern) Authorized 4/1/52
Sponsor: Bureau of Ordnance, USN
Manager: A. D. Hestenes

Objective: This study is concerned with the inherent error analysis of the air to ground rocket fire control evaluation program of the client and consists of four phases as follows:

(A) Review of evaluation program and determination of the mathematical expressions for the inherent errors.

(B) Recommendation of statistical experiments by which NOTS can determine measurement errors, reading errors, etc., required for a numerical evaluation of the expressions obtained in (A).

(C) Study of the permissable measurement errors for a desired accuracy in the resulting data.

(D) Numerical evaluation of the mathematical expressions obtained in (A) using numerical values of errors obtained as a result of (B).

Background: This study is needed to determine the validity of the results of fire control evaluation programs.

Status: NEW. Phase (A) is proceeding as well as anticipated. No time has been spent on Phases (B), (C), and (D).

TABLE OF BIVARIATE NORMAL DISTRIBUTION FUNCTION FOR SPECIAL VALUE OF THE PARAMETERS
Task 1101-53-1101/52-23

Origin: Engineering Department, Columbia University — Authorized 2/15/52
Sponsor: Office of Naval Research, USN — Completed 3/31/52
Manager: G. Blanch

Objective: To compute

$L(-h, -h, r)$ for $r = \pm \frac{1}{2}$

$L(-h, -2h, r)$ for $r = \frac{1}{2}$

$h = 0(.01)2.6$; 5D

where by definition

$$L(h,k,r) = \int_h^\infty dx \int_k dy\, \phi(x,y,r) ,$$

and

$$\phi(x,y,r) = \frac{1}{2\pi\sqrt{(1-r^2)}} \exp\left[-\tfrac{1}{2}(x^2 + y^2 - 2rxy)/(1-r^2)\right].$$

Background: A table of $L(h,k,r)$ for $h, k \leq 2.6$ has been published by K. Pearson. A supplementary table of the same function for h,k up to 4, computed by E. Fix of the University of California, Berkeley, is available in manuscript form. (The National Bureau of Standards is now collating the tables of Pearson and Fix, to be published in one volume.) Interpolation in the tables of E. Fix and Pearson is not linear, however, and there is a need for the table at the smaller interval, in connection with certain research now in progress.

The computations will involve, essentially, 5-point interpolation in available tables, and use of known relations between $L(h,k,r)$ and $L(-h,-k,r)$. A few values of $L(-h,-2h,r)$ for h beyond 2 will have to be computed especially, since they are outside the range of either table.

Publication: "Tables of $L(-h,-h,r)$ for $r = \pm \frac{1}{2}$ and $L(-h,-2h,r)$ for $r = \frac{1}{2}$ for $h = 0(.01)2.6$; 5D," by G. Blanch; NAML report 52-53.

Status: COMPLETED (NEW).

6th ORDER DIFFERENTIAL SYSTEM
Task 1101-53-1101/52-26

Origin: Hughes Aircraft Co. — Authorized 4/1/52
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Manager: T. H. Southard

Objective: To solve a system of three second order non-linear ordinary differential equations with prescribed initial conditions. The equations involved are similar to those of task 1101-53-1101/51-37 and those of task 1101-53-1101/51-27 from the same originator.

Status: NEW. Approximately half of the task has been computed and turned over to the originator for analysis. Further computations await the originator's decision.

SYSTEMATIC AND RANDOM ERRORS
Task 1101-53-1101/52-29

Origin: North American Aviation Co. Authorized 4/1/52
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Manager: A. D. Hestenes

Objective: Compute the salvo kill probability of a square target of side 2a as a function of the parameters: aiming (systematic) error, ammunition dispersion (random error), and salvo size. The expression to be evaluated is

$$P_{SK} = 2 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} Q(i,j,N)\, P_A(i,j)$$

where

$$Q(i,j,N) = 1 - [1 - P_k\, P_h\, (i,j)]^N$$

$$P_h(i,j) = \left[\varphi\left(\frac{a\sqrt{2}}{\sigma_R}\left(1 - \frac{i+\frac{1}{2}}{n}\right)\right) + \varphi\left(\frac{a\sqrt{2}}{\sigma_R}\left(1 + \frac{i+\frac{1}{2}}{n}\right)\right)\right]$$

$$\left[\varphi\left(\frac{a\sqrt{2}}{\sigma_R}\left(1 - \frac{j+\frac{1}{2}}{n}\right)\right) + \varphi\left(\frac{a\sqrt{2}}{\sigma_R}\left(1 + \frac{j+\frac{1}{2}}{n}\right)\right)\right]$$

$$P_A(i,j) = \left[\varphi\left(\frac{a\sqrt{2}}{\sigma_A}\,\frac{i+1}{n}\right) - \varphi\left(\frac{a\sqrt{2}}{\sigma_A}\,\frac{i}{n}\right)\right]$$

$$\left[\varphi\left(\frac{a\sqrt{2}}{\sigma_A}\left(\frac{j+1}{n} - k\right)\right) - \varphi\left(\frac{a\sqrt{2}}{\sigma_A}\left(\frac{j}{n} - k\right)\right)\right]$$

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{u^2}{2}}\, du.$$

The distributions are assumed to be circular normal distributions with standard deviations σ_R for the random error and σ_A for the aiming error. N is the salvo size and $y_o = ka$, $x_o = 0$ represents the center of the aiming point distribution. P_k is a probability that a target will be destroyed if hit by a single shot independent of the effect of other shots. a/n represents the mesh size. n is to be chosen such as P_{SK} is correct to three decimals.

P_{SK} is to be evaluated for the following values of the parameters

$$P_k = 0.2,\ 0.4,\ 0.7,\ \text{and } 1.0$$
$$N = 1,\ 5,\ 10,\ 25,\ 50,\ 100,\ 150,\ 200$$
$$y_o = 0,\ a,\ 2a,\ 3a,\ 5a,\ 10a,\ 15a,\ 20a$$
$$\sigma_R = a,\ 2a,\ 3a,\ 5a,\ 10a,\ 15a,\ 20a$$
$$\sigma_A = a,\ 2a,\ 3a,\ 5a,\ 10a,\ 15a,\ 20a.$$

Background: These computations are needed to predict performance of new (as well as old) weapons.

Comments: Although much work of this nature has been done piecemeal in the past, the program outlined above goes beyond the previous work and represents a comprehensive program which will be useful to many organizations interested in defense programs.

Status: NEW. A preliminary study of the mesh size has been made. The main portion of the task is now being coded for computation on the SWAC.

CONTROL SYSTEM EQUATIONS
Task 1101-53-1101/52-30

Origin: Summers Gyroscope Company Authorized 4/1/52
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Manager: R. R. Reynolds

Objective: To compute the positive quantities

$$R_\theta = a^3(3a - 2x)/12xz\theta_o,$$

$$R_e = (-3a^4 + 10a^3x - 6a^2x^2 - 6a^2x + 6ax^2$$
$$-12ax^2y + 12a^2xy + 6x^2y - 6x^2y^2)/12xz\theta_o\ ,$$

$$R_S = 2(a - x)/x\ ,$$

$$R_f = x(6y^2 - 2z^2 + 6y + 3z)/3a^2(1 - z)\ ,$$

where

$$x = (1 - c)z\ ,$$

$$\theta_o = R(R + 1)/2\ ,$$

$$R = (6y^2 + 6yz + z^2)/6(1 - z)\ ,$$

a is a root of

$$a^3 - (.75x + \alpha)a^2 + (\alpha x + \beta)a - \gamma = 0 ,$$

$$\alpha = .75(1 - 2y) , \beta = 1.5y(1 - y) , \gamma = .25y^2(3 - 2y) ,$$

and $c_{min} = 1 - [z^3 - \alpha z^2 + \beta z - \gamma)/z^2(.75z - \alpha)]$, for the parameters

$c = c_{min}$, and the values .25(.05).4(.1).9(.025).975 greater than C_{min} ,

z = .005, .01, .02, .05, .1, .2, .5, y = .001,.002, .005, .01, .02, .05.

Background: This problem arises in the roll-axis stability analysis of an airframe using a flicker type control system.

Status: NEW. After some preliminary analysis, the problem is being coded for the SWAC.

SMOG ANALYSIS
Task 1101-53-1101/52-31

Origin: Consolidated Engineering Corporation Authorized 4/1/52
Sponsor: NBS
Manager: E. C. Yowell

Objective: To perform the following computations on 38 sets of mass spectrograph data: let x_{ij} be the mass spectrograph reading of the ith line (counted from an arbitrary m/e value) in the jth sample. Normalize each measurement producing an $\bar{x}_{ij}$ by the equation

$$\bar{x}_{ij} = \frac{x_{ij}}{\sum_i x_{ij}} .$$

Then compute the ratios,

$$k_{i,j,1} = \frac{\bar{x}_{i,j}}{\bar{x}_{i,1}} .$$

Background: The client believes that an examination of the $k_{i,j,1}$ ratios will enable him to identify chemical compounds present in one sample but not in another.

Comments: During the summer of 1951, 38 samples of atmospheric constituents were collected in liquid oxygen traps set up in the Los Angeles area. Many of these samples were taken in contrasting pairs of observing conditions. Each sample was then subjected to a mass spectrograph analysis, and the results of these analyses are being studied for clues as to the primary components contained in the local smog.

Status: NEW. These computations have been 50 percent completed.

CORRELATION COEFFICIENTS
Task 1101-53-1101/52-34

Origin: Sheridan Supply Company Authorized 4/1/52
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Manager: F. H. Hollander

Objective: To compute all two by two tables from a set of 102 scores obtained on an opinion questionnaire given to 600 Basic Airmen. Then 5151 tetrachoric correlations are computed from these tables by the approximate formula given below. The first step involves obtaining the cell frequencies for each possible pair of questions as in the table:

		Variable x: 1	Variable x: 0
Variable y	1	a	b
	0	c	d

The second step involves the computation of the tetrachoric coefficients by the formula

$$r_{xy} = \cos \frac{\pi}{1+\sqrt{\frac{ad}{bc}}}$$

Background: Originally the score for each question could be any integer from zero to eleven, inclusive. These were converted into a score of zero or one by separating the original scale into two parts approximately at the median of the scores of all airmen for that question so that all four marginal totals for each table would be approximately 300. The client feels that this justifies the use of the above approximate formula for tetrachoric correlation.

Status: NEW. These computations have been 90 percent completed.

SIERRA WAVE PROJECT
Task 1101-53-1101/52-36

Origin: Department of Meteorology, U.C.L.A. Authorized 4/1/52
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Manager: T. H. Southard

Objective: To determine the space position of a sailplane as a function of time, being given data as to the difference between the distances of the sailplane from each of several pairs of fixed (Raydist) stations.

Background: The sponsor is interested in investigating the meteorological phenomenon of orographic lee waves in the atmosphere, and is sending sailplanes to probe the wind field existing in the lee of the Sierra Nevada mountains of California.

Status: NEW. First data have been received and key-punched, but no computations have been performed as yet.

CHARACTERISTICS OF MOMENT ESTIMATORS OF EXTREME VALUES
Task 1101-53-1101/52-38

Origin: Section 11.3, NBS — Authorized 4/1/52
Sponsor: Dynamic Loads Division, NACA
Manager: D. Teichroew

Objective: To compute, by empirical sampling, the following moments of the mean, y, and standard deviation, s, in random samples of size n = 10, 20, 30 from the extreme-value distribution with c.d.f. $F(y) = \exp(-e^{-y})$:

$$\sigma^2(\bar{y}) \ , \quad \mathrm{cov}(\bar{y}, s) \ , \quad \mu_1(s) \quad , \quad \sigma^2(s) \ .$$

Background: These computations are needed in the study of the two-parameter asymptotic distribution of extreme values $\phi(x) = \exp(-e^{-y})$, where $y = (x-u)/\beta$, being carried on under task 1103-21-5106/51-1. If a sample of observations is assumed to come from this distribution then the parameter $\xi_P = u + \beta\, y_P$ indicates the (smallest) value which will not be exceeded by the fraction (1-P) of the observations in the long run. y_P is a tabulated value depending only on the probability P. E. J. Gumbel has given an estimator for ξ_P which depends upon the sample mean $\bar{x}$ and standard deviation s_x. This estimator is given by

$$\hat{\xi}_P = \hat{u} + \hat{\beta}\, y_P \ ,$$

where $\hat{u} = \bar{x} - \frac{\sqrt{6}}{\pi} C s_x$, $\hat{\beta} = \frac{\sqrt{6}}{\pi}\, s_x$, and C (=0.5772...) is Euler's constant. The above computations will make it possible for the first time to determine the bias and efficiency of Gumbel's moment-estimator for several samples of moderate size.

Comments: This is a companion project to task 1102-53-1106/52-67, which has for its aim the construction of order-statistics estimators of the parameter ξ_P which are unbiased and have minimum variance. The results of these two tasks will make possible a comparison between the two types of estimators needed in connection with research on estimation of extreme values being carried on under task 1103-21-5106/51-1 (Research on Application of Theory of Extreme Values to Gust-Load Problems).

Status: NEW.

II. Computation Laboratory

(Section 11.2)

1. Research

RESEARCH IN CLASSICAL NUMERICAL ANALYSIS
Task 1102-21-1104/50-1
(formerly 11.2/11-50-1)

Origin: NBS
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. In connection with his study of a new set of orthogonal polynomials H. E. Salzer completed the computation of the zeros and the corresponding Christoffel numbers for the first eight orthogonal polynomials. With the aid of these complex numbers integrals of the type

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{x}\, P\left(\frac{1}{x}\right)\, dx$$

which arise in the inversion of Laplace transforms, can be evaluated exactly for P, any polynomial of degree not exceeding 16 and with no constant term.

An article dealing with the numerical calculation of Laplace transforms of a function given at equal intervals has been completed by Mr. Salzer. Also he has completed an article discussing equally weighted quadrature formulas over infinite intervals.

RESEARCH IN MODERN NUMERICAL ANALYSIS: INVESTIGATION OF BERGMAN'S METHOD FOR THE SOLUTION OF THE DIRICHLET PROBLEM FOR CERTAIN MULTIPLY CONNECTED DOMAINS
Task 1102-21-1104/50-2
(formerly 11.2/11-50-2)

Origin: NBS
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. F. Alt has prepared a paper to be presented to the Association for Computing Machinery describing the results of this investigation. It is expected that this paper and the tabular material will be submitted for publication by the NBS.

MISCELLANEOUS STUDIES IN PURE MATHEMATICS

Task 1102-21-1104/50-4
(formerly 11.2/11-50-4)

Origin: NBS
Manager: O. Taussky-Todd
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. M. Newman is continuing his studies of functions analogous to the Ramanujan function (n) which arise as coefficients in the expansions of certain elliptic modular functions.

Publications: (1) "The coefficients of $\prod_{n=1}^{\infty}(1-x^n)^r$," by M. Newman; IN MANUSCRIPT. (2) "Remarks on some modular identities," by M. Newman; submitted to a technical journal. (3) "On Cauchy-Riemann equations in higher dimensions," by E. Stiefel; accepted by the NBS Journal of Research.

NUMBER THEORETICAL TEST PROBLEMS FOR SEAC

Task 1102-21-1104/50-5a
(formerly 11.2/11-50-5)

Origin: NBS
Managers: O. Taussky-Todd and K. Goldberg
Full task description appears in Apr-Jun 1950 issue.

Status: INACTIVE. For status to date see Oct-Dec 1951 issue.

ANALYSIS OF CRYSTAL STRUCTURE

Task 1102-21-1104/51-3

Origin: NBS
Full task description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1951 issue.

SOLUTION OF LAPLACE EQUATION BY MONTE CARLO METHOD

Task 1102-21-1104/51-6

Origin: NBS
Managers: C. J. Swift and J. Todd
Full task description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see Oct-Dec 1951 issue.

HEAT CONVECTION IN LAMINAR FLOW THROUGH A TUBE
Task 1102-21-1104/51-9

Origin: NBS
Full task description appears in Jan-Mar 1951 issue.

Status: CONTINUED. The report has been written and is now being reviewed.

THREE-BODY PROBLEM
Task 1102-21-1104/52-4

Origin: NBS
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see Oct-Dec 1951 issue.

ANALYSIS OF GEOMAGNETIC FIELD
Task 1102-21-1104/52-8

Origin: NBS
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see Oct-Dec 1951 issue.

INVESTIGATION OF THE APPLICABILITY OF AUTOMATIC DIGITAL ELECTRONIC COMPUTING TO PROBLEMS IN THEORETICAL AND APPLIED STATISTICS
Task 1102-21-1104/52-28

Origin: NBS and Institute of Statistics, University of North Carolina
Managers: J. Levin and R. B. Bryce
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Research was conducted by D. Teichroew on (i) continued fractions, (ii) history of empirical sampling, and (iii) sampling on high-speed machines. The results regarding continued fractions have been set forth in a paper "Use of continued fractions in hi-speed computing," by D. Teichroew, which is to be submitted for publication. Also a report on "importance" sampling has been prepared.

SPECIAL PROBLEMS IN FINITE MATRIX THEORY
Task 1102-21-1104/52-34

Origin: NBS
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. (1) O. Taussky-Todd continued work on matrices with property L and, in particular, on different characterizations of this property. She also worked on inequalities for characteristic roots of products of matrices in connection with recent results of H. Weyl and K. Fan. (2) A. J. Hoffman obtained a new proof of some results of J. von Neumann on the location of the eigenvalues of the second kind of the sum of two matrices. (3) M. Mannos obtained results of "Eigenvectors of matrix polynomials." (4) O.Taussky-Todd and J. Todd completed an extensive manuscript on "Systems of simultaneous equations, matrices and determinants". This was prepared at the request of the editors of a technical journal.

Publications: (1) "Normal matrices with property L," by N.Wiegmann; submitted to a technical journal. (2) "Pairs of normal matrices with property L," by H. Wielandt; submitted to a technical journal. (3) "On the eigenvalues of A + B and AB," by H. Wielandt; submitted to a technical journal.

ROOTS OF POLYNOMIAL EQUATIONS
Task 1102-21-1104/52-51

Origin: NBS Authorized 2/1/52
Manager: I. C. Diehm

Objective: To produce a SEAC routine which will determine the roots, real and complex, of any given polynomial equation, provided its degree is not so large that computation time becomes excessive.

Background: Let C be a closed curve in the complex plane, and f(z) a function which is meromorphic within, and has no singularities on C. Then

$$\frac{1}{2\pi i} \int_C \frac{f'(z)}{f(z)} dz = \sum_j r_j - \sum_k s_k$$

where $r_1, r_2, r_3, \ldots$ are the orders of the zeros and $s_1, s_2, s_3 \ldots$ the orders of the poles of f(z) within C. Since a polynomial has no poles, it can be determined whether the polynomial has zeros in a given region, and, if so, the region can be subdivided and the process repeated until the roots are known to any desired accuracy.

Status: NEW. A code has been completed and is being tested.

DISTRIBUTION OF NORMAL MODES OF VIBRATION OF CUBIC LATTICES
Task 1102-21-1104/52-62

Origin: NBS Authorized 2/25/52
Sponsor: "
Manager: I. C. Diehm

Objective: To compute the distribution of normal modes of vibration in cubic lattices, on which depend the vibrational contribution to the thermodynamic properties of polyatomic molecules and crystals.

Exact distribution functions have been found for two-dimensional lattices, but the amount of computation necessary in the three-dimensional case has hitherto been prohibitive. This program will investigate application of high-speed computing devices to the problem.

Background: The characteristic frequencies of normal modes of vibration of a cubic lattice are roots of cubic equations. Since the number of equations is of the order of the number of particles in the lattice ($O(10^{23})$), the time required for the calculation of the frequencies is tremendous. It is expected that by taking small crystals ($O(10^4)$ lattice points) one would obtain a considerable amount of information concerning the distribution.

Status: NEW. A flow chart has been prepared, and coding is in progress.

RESEARCH IN LINEAR PROGRAMMING
Task 1102-21-5115/50-2
(formerly 1102-53-1106/50-2)

Origin: Air Comptroller's Office, USAF
Sponsor: " "
Managers: J. Todd and A. J. Hoffman
Full task description appears in Jan-Mar 1950 issue, see 11.2/12-50-1.

Status: CONTINUED. Twenty small scale problems have been solved by the Simplex method, the Brown Game method, and the relaxation method. The results which have been obtained are being investigated so as to compare the relative efficiency of these methods in solving problems of linear programming.

The Simplex technique was successfully employed in solving the "Aircraft Deployment" problem, that of finding the most efficient assignment of crews and aircraft to combat and training so that a maximum number of sorties can be flown. The solution time for the handling of a 48 x 71 matrix was 30 hours. The same problem attempted by the Brown Game method showed the latter considerably slower.

Publications: (1) "Preliminary report on the SEAC experience with the relaxation method," by N. Wiegmann; IN MANUSCRIPT. (2) "Preliminary report on the SEAC experience with Brown's method of fictitious play," by M. Mannos; IN MANUSCRIPT. (3) "The order of degeneracy of the solution of the Simplex problem," by A. Hoffman; IN MANUSCRIPT. (4) "The 'closeness' of a vector to a solution of a system of linear inequalities," by A. Hoffman; IN MANUSCRIPT. (5) "Bibliography on bounds for characteristic roots of finite matrices," by O. Taussky-Todd, NBS Report 1162 (Sept 1951); limited number of copies available upon request.

COMPRESSIBLE FLOW-METHOD OF ORTHOGONAL AND KERNEL FUNCTIONS
Task 1102-21-5116/52-16

Origin: Flight Research Laboratory, Wright Air Development Center, USAF and Harvard University
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. A coarse tabulation was made on SEAC of the λ and θ values as functions of M. The codes have been completed and are being checked on SEAC, for: (1) the tabulation of M as a function of λ and θ and (2) the evaluation of F corresponding to the M obtained in (1). Further theoretical investigations are being made to determine the appropriate grid for λ and θ preparatory to the coding of the multiple integrals mentioned in the last issue.

TABLES OF INTEGRALS INVOLVING THE HIGHER TRANSCENDENTAL FUNCTIONS
Task 1102-21-5117/52-33

Origin: NBS
Sponsor: Office of Naval Research
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The preparation of the tables was continued.

NAVIER-STOKES EQUATION
Task 1102-21-5117/52-50

Origin: NBS
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. The numerical evaluation of the Oseen linearization of the Navier-Stokes equations has been continued. The following parts of the problem have been coded: (1) the determination of the Fourier coefficients for the boundary values of the first seven functions of the complete systems and their normal derivatives, and (2) the evaluation of the coefficients of the systems of linear equations which determine approximate solutions for both the classical problems (all velocity components, zero, and the Ackeret problem (normal velocity zero; tangential stress zero). A trial run on SEAC has been made and was successful.

2. Applied Research: Tables and Experimental Computations

BIBLIOGRAPHY OF MATHEMATICAL TABLES AND NUMERICAL ANALYSIS
Task 1102-21-1110/50-5
(formerly 1102-21-1104/50-5)

Origin: NBS
Managers: J. Todd and H. E. Salzer
Full task description appears in Jan-Mar 1950 issue, see 11.2/2-50-5.

Status: CONTINUED. W. H. Durfee has prepared a mimeographed list of standard mathematical periodicals available in the NBS Washington library.

COLLECTED SHORT MATHEMATICAL TABLES OF THE COMPUTATION LABORATORY

Task 1102-21-1110/51-4
(formerly 1102-21-1104/51-4)

Origin: NBS
Manager: H. E. Salzer
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED.

Publication: "Collected Tables of the Computation Laboratory: Volume I. Tables of functions and of zeros of functions"; accepted for publication in the NBS Applied Mathematics Series.

TABLE OF THE BESSEL FUNCTION $Y_n(x)$

Task 1102-21-1110/51-10
(formerly 1102-21-1104/51-10)

Origin: NBS
Full task description appears in Jan-Mar 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1951 issue.

REVISION OF MATHEMATICAL TABLES

Task 1102-21-1110/52-7

Origin: NBS
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The revision of AMS1, "Tables of the Bessel Functions $Y_0(x)$, $Y_1(x)$, $K_0(x)$, $K_1(x)$, $0 \leq x \leq 1$," is completed; this will be reissued as AMS25. Also the revision of the following tables is in progress: (1) "Tables of Circular and Hyperbolic Sines and Cosines for Radian Arguments," (MT3); (2) "Table of Sine and Cosine Integrals for Arguments from 10 to 100," (MT13); (3) "Tables of Probability Function," volume II (MT14); and (4) "Table of Arc Tan x," (MT16). These tables will be reissued in the NBS Applied Mathematics Series.

TABLES OF $E_1(z)$, $(z = x + iy)$

Task 1102-21-5120/43-3)
(formerly 1102-21-1110/43-3)

Origin: Canadian National Research Council
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Apr-Jun 1949 issue, see task 11.2/2-43-3.

Status: CONTINUED. Checking of the final manuscript is underway.

TABLE OF THE GAMMA FUNCTION FOR COMPLEX ARGUMENTS
Task 1102-21-5120/46-1
(formerly 1102-21-1110/46-1)

Origin: NBS
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Managers: H. E. Salzer and M. Stein
Full task description appears in Apr-Jun 1949 issue, see task 11.2/2-46-1.

Status: CONTINUED. The checking and preparation of interpolation schedules was continued. An extensive bibliography of tables in this field and of their applications has been compiled by H. E. Salzer.

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-21-5120/47-2
(formerly 1102-21-1110/47-2)

Origin: NBS
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Apr-Jun 1949 issue, see task 11.2/2-47-2.

Status: CONTINUED. The checking of the manuscript for volume II is being continued.

A program for the computation of the functions on a production basis is now being studied.

Publication: "Tables of Coulomb wave functions, volume I," NBS Applied Mathematics Series 17; in press, Government Printing Office.

TABLE OF ANTILOGARITHMS
Task 1102-21-5120/47-3
(formerly 1102-21-1110/47-3)

Origin: NBS
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Apr-Jun 1949 issue, see 11.2/2-47-3.

Status: CONTINUED. This volume is to be submitted for publication in the NBS Applied Mathematics Series.

TABLE OF LAGRANGIAN COEFFICIENTS FOR SEXAGESIMAL INTERPOLATION
Task 1102-21-5120/48-2
(formerly 1102-21-1110/48-2)

Origin: NBS
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Apr-Jun 1949 issue, see 11.2/2-48-2.

Status: CONTINUED. The manuscript for the three- and four-point interpolants has been typed. Arrangements are being made for the writing of an introduction for the volume.

FERMI FUNCTION, II
Task 1102-21-5120/49-10
(formerly 1102-21-1110/49-10)

Origin: NBS, Section 4.4
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Apr-Jun 1949 issue, see 11.2/33-49-10.

Status: CONTINUED. The second page proofs were corrected.

Publication: The table is being printed by the Government Printing Office and will be issued as "Table for the analysis of β-spectra," NBS Applied Mathematics Series 13.

TABLE OF CHEBYSHEV POLYNOMIALS
Task 1102-21-5120/50-3a
(formerly 1102-21-1110/50-3a)

Origin: NBS
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in July-Sept 1949 issue, see 11.2/2-50-3.

Status: CONTINUED. The final page proofs were corrected.

Publication: "Table of Chebyshev polynomials," NBS Applied Mathematics Series 9; in press, Government Printing Office.

PROBABILITY TABLES FOR EXTREME VALUES
Task 1102-21-5120/50-4a
(formerly 1102-21-1110/50-4a)

Origin: NBS, Section 11.3
Sponsor: Flight Research Laboratory, Wright Air Development Center, USAF
Full task description appears in Oct-Dec 1949 issue, see 11.2/2-50-4.

Status: CONTINUED. Galley proofs have been received and corrected.

Publication: "Probability tables for analysis of extreme-value data", NBS Applied Mathematics Series 22; in press, Government Printing Office.

WAVE FUNCTION FOR LITHIUM
Task 1102-21-5120/50-7
(formerly 1102-21-1104/50-7)

Origin: NBS
Manager: J. Todd
Full task description appears in Apr-Jun 1950 issue.

Status: CONTINUED. A code has been prepared and is being checked.

TABLE OF ARCSIN FOR COMPLEX ARGUMENTS
Task 1102-53-5120/52-14
(formerly 1102-21-1110/52-14)

Origin: NBS
Manager: J. Levin
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. A code has been completed and is being checked.

EXTENSION OF THE TABLE OF HYPERBOLIC SINES AND COSINES
Task 1102-21-5120/52-18
(formerly 1102-21-1110/52-18)

Origin: NBS
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The code has been revised for high-speed output. The completion of the table is pending until machine time becomes available.

TABLE OF THE MODIFIED AIRY INTEGRAL
Task 1102-21-5120/52-23
(formerly 1102-21-1110/52-33)

Origin: NBS
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The truncation error occurring in the numerical integration of the integral is being studied.

TABLE OF ERROR FUNCTION FOR COMPLEX ARGUMENTS
Task 1102-21-5120/52-25
(formerly 1102-21-1110/52-25)

Origin: NBS
Managers: H. E. Salzer and I. C. Diehm
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The code has been completed and checked.

EXTENSION OF TABLES OF THE EXPONENTIAL FUNCTION FOR NEGATIVE ARGUMENTS
Task 1102-21-5120/52-31
(formerly 1102-21-1110/52-31)

Origin: NBS
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The code has been completed and checked and will be run when machine time is available.

WAVE FUNCTION FOR HELIUM
Task 1102-21-5120/52-32
(formerly 1102-21-1104/52-32)

Origin: NBS
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. The difference equation corresponding to the helium wave equation using a coarse mesh has been solved. Plans have been made to solve the equation using a finer mesh.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-21-5120/52-37
(formerly 1102-21-1110/52-37)

Origin: NBS
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. A code has been completed and checked and the first part of the problem solved by its aid. A code for the second part of the problem has been completed and partly checked.

VAN DER POL EQUATION
Task 1102-21-5120/52-43
(formerly 1102-21-1110/52-43)

Origin: NBS
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. A code has been completed and checked and results have been obtained for small values of the parameter.

RADIAL MATHIEU FUNCTIONS
Task 1102-21-5120/52-49

Origin: NBS Authorized 2/1/52
Sponsor: Atomic Energy Commission
Managers: I. Rhodes, J. Todd

Objective: To tabulate the radial solutions of Mathieu's differential equation

$$y'' - (b - s\cosh^2 x)y = 0 . \qquad (1)$$

Range of tabulation: s from 0 to 10, at those points for which the characteristic values and trigonometric coefficients are available; x = 0(.02)2(.01)3; 5D, for orders r = 0,1,...,15. Functions to be tabulated:

$$\left.\begin{matrix} AJe_r(s,x), & AJe'_r(s,x), & Ne_r(s,x)/A, & Ne'_r(s,x)/A \\ AJo_r(s,x), & AJo'_r(s,x), & No_r(s,x)/A, & No'_r(s,x)/A \end{matrix}\right\} \qquad (2)$$

where $A = 4^r \, r! s - \frac{1}{2}r$.

Background: The above functions are defined in the NBS publication Tables Relating to Mathieu Functions (published by Columbia University Press). Associated with equation (1) is the following equation:

$$(3) \qquad y'' + (b - s \cos^2 x)y = 0$$

When the parameter b assumes a value $be_r(s)$ which belongs to a countably infinite set of characteristic values the solutions of (3) are even and periodic, or period π or 2π. Similarly there is another countably infinite set of characteristic values $bo_r(s)$ corresponding to which the periodic solutions (of the same periods) are odd. The characteristic values of orders up to and including 15, together with the trigonometric coefficients associated with them, have been adequately tabulated in the NBS publication mentioned above. A pioneering table of the periodic solutions of the first five or six orders have been published by Ince, and a more extensive table for orders up to 15 is in process of computation at the NBS Computation Laboratory. No tables at all are available for the "radial" solutions--that is, the solutions of (2), although in many problems the periodic solutions cannot be used without the corresponding solutions of (2). The values of the radial solutions at x = 0 can be obtained easily from the "joining factors" tabulated in the NBS publication and the information supplied in the Introduction of the same volume. Moreover, the magnitude of the functions, with varying s, can now be inferred from the properties of the tabulated joining factors, and it is hoped that the scaling factor, A, introduced in (2), is such that the resulting functions will be of order of magnitude of unity throughout their range.

Ideally it would be desirable to tabulate the functions up to a point where the known asymptotic expansions yield approximations to about three significant figures. The range suggested here may not meet this criterion, especially for high orders r. However, the initial tabulation suggested here, or any part of it, will provide some means of judging how much more remains to be done.

The range suggested here comprises about 408,000 entries. If 8 functions are printed across a page, the space needed will be 1,020 pages, or 2 volumes.

Status: NEW. Several programs for the computation of the Mathieu functions have been coded for SEAC. A study of the relative efficiency of these codes is underway.

SIEVERT'S INTEGRAL
Task 1102-21-5120/52-57

Origin: NBS — Authorized 2/12/52
Managers: O. Steiner and W. Soderquist

Objective: To construct a table of values of Sievert's integral S, where

$$S = \int_0^{\phi} e^{-x \sec \theta} \, d\theta$$

for the range of values $\phi = 0(1°)90°$, $x = 0(.01)1(.02)5(.05)10$.

Background: Among the uses of Siebert's integral is the calculation of intensity of radiation from line sources of radioactive material.

Status: NEW. A code is in preparation.

STRUVE FUNCTION OF ORDER 3/2
Task 1102-21-5120/52-60

Origin: NBS
Manager: E. C. Marden
Authorized 2/12/52

Objective: To compute

$$\sqrt{\frac{2\pi}{x}}\, H_{3/2}(x) = 1 - \frac{2 \sin x}{x} + \frac{2}{x^2}(1 - \cos x)$$

for x = 0(.02)15, where $H_{3/2}(x)$ is the Struve Function of order 3/2.

Background: The Struve functions of half integral order appear in radiation problems with spherical symmetry. One important quantity that is proportional to $\sqrt{2\pi/x}\, H_{3/2}(x)$ is the fraction of energy scattered from a plane wave by a spherical obstacle when its wave transmission characteristics are not too different from those of the external medium.

Status: NEW. A code has been completed and checked.

SCATTERING FUNCTIONS
Task 1102-21-5120/52-63

Origin: NBS
Manager : A. N. Gleyzal
Authorized 3/10/52

Objective: To develop practical numerical and analytical methods for determining the scattering of plane waves by obstacles (or force centers in quantum mechanical cases). Mathematically this requires the solution of the wave equation

$$\nabla^2 x + k^2(r)x = 0$$

with proper behavior at large distances from the scatterer. The possibility of using high-speed digital computers for this problem will be investigated.

Background: There is considerable technological interest in the scattering of electromagnetic waves by large molecules, fogs, dust particles, etc. The interpretation of experimental results in the scattering of high energy particles by nuclei is one of the central problems in nuclear physics.

Status: NEW. A study of the literature is being made preparatory to formulating an extensive computation program.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Task 1304-34-6351/51-8

Origin: Section 11.3, NBS
Full task description appears in Apr-Jun 1951 issue.

Status: CONTINUED. A code for the computation of x, where p, q, α are given and $I_x(p,q) = \alpha$, has been completed.

3. Mathematical Services

LINEAR PROGRAMMING ON STANDARD PUNCHED CARD MACHINES
Task 1102-53-1106/49-3
(formerly 11.2/36-49-3)

Origin: Air Comptroller's Office, USAF
Sponsor: " " "
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The computation of Air Force triangular models has been progressing as required by the Office of the Air Comptroller. The 192 x 192 inter-industry matrix has been prepared for inversion on the UNIVAC.

SHOCK WAVE PARAMETERS
Task 1102-53-1106/49-13
(formerly 11.2/33-49-13)

Origin: Bureau of Ordnance, USN
Sponsor: " "
Full task description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Apr-Jun 1951 issue.

WAVE RESISTANCE OF SHIPS, III
Task 1102-53-1106/50-11
(formerly 11.2/33-50-11)

Origin: David Taylor Model Basin, USN
Sponsor: " " "
Full task description appears in Oct-Dec 1949 issue.

Status: CONTINUED. The integral $M_n = \int_0^1 x^n \sin \alpha x \, dx$ has been computed for the new values of α. In addition the integral $R = \int_0^1 x^4 e^{\alpha x} dx$ has been computed for values of α.

MOLECULAR STRUCTURE CALCULATIONS, II
Task 1102-53-1106/50-16
(formerly 11.2/33-50-16)

Origin: Naval Research Laboratory, USN
Sponsor: " "
Full task description appears in Jan-Mar 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1951 issue.

PROGRAM COMPUTATION ON THE SEAC
Task 1102-53-1106/51-7

Origin: Office of the Air Comptroller, USAF
Sponsor: " "
Managers: A. J. Hoffman and F. B. Meek
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. A new code for the solution of aircraft deployment pattern models has been devised. The code has been checked and several test problems have been run off successfully. The new code eliminates the need for hand computation of patterns and in addition, furnishes information which was also previously computed by hand about allocations of planes to combat and training.

FLOW IN SUPERSONIC NOZZLES
Task 1102-53-1106/51-13

Origin: Naval Ordnance Laboratory
Sponsor: " "
Managers: F. Alt and L. Nemerever
Full task description appears in Oct-Dec 1950 issue.

Status: CONTINUED. One of the finite difference schemes investigated has been coded and run on SEAC; it is apparently stable. Results are being transmitted to the sponsor.

INTERNAL CONVERSION COEFFICIENTS FOR L-SHELL
Task 1102-53-1106/51-19

Origin: Atomic Energy Commission, Oak Ridge National Laboratories
Sponsor: " " "
Manager: J. H. Wegstein
Full task description appears in Jan-Mar 1951 issue.

Status: CONTINUED. Coding of the main routine continues.

X-RAY PENETRATION
Task 1102-53-1106/51-20

Origin: Atomic Energy Commission, New York Office
Sponsor: " " "
Full task description appears in Jan-Mar 1951 issue.

Status: CONTINUED. Low energy cases have been run, and several results have been obtained which were transmitted to sponsor. The completion of the problem is pending until machine time becomes available. A code for a point-monodirectional radiation source is in preparation.

LIQUID-VAPOR TRANSITION
Task 1102-53-1106/51-22

Origin: Naval Medical Research Institute
Sponsor: " "
Full task description appears in Jan-Mar 1951 issue.

Status: CONTINUED. The integral equation was solved for additional values of the parameter.

MOLECULAR STRUCTURE, III
Task 1102-53-1106/51-37

Origin: Naval Research Laboratory, USN
Managers: M. Abramowitz and P. J. O'Hara
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Calculation of the most prominent maxima of the series

$$\phi(x,y,z) = \sum_{hkl} C_{h,k,l} \cos 2\pi(hx + ky + lz)$$

was continued.

SHOCK WAVE PARAMETERS
Task 1102-53-1106/51-38

Origin: Bureau of Ordnance, USN
Sponsor: " "
Full task description appears in Apr-Jun 1951 issue.

Status: INACTIVE. For status to date see Oct-Dec 1951 issue.

STANDARD LORAN TABLES
Task 1102-53-1106/51-39: Alaskan Chain
Task 1102-53-1106/52-38: Loran Radux Stations II

Origin: U. S. Navy Hydrographic Office
Sponsor: " "
Full task description appears in July-Sept 1949 issue, see 11.2/34-50-1.

Status: CONTINUED. The manuscript for the Alaskan chain was completed and submitted to the sponsor. The calculations for the first two Radux pairs of Loran Radux Stations II have been completed, and the results have been transmitted to the sponsor.

PRESSURE DISTRIBUTION ON BODIES OF REVOLUTION
Task 1102-53-1106/52-3

Origin: David Taylor Model Basin, USN
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. All cases but one have been completed.

POWDER DIFFRACTION
Task 1102-53-1106/52-6

Origin: NBS, Section 9.7
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1951 issue.

PLASTIC DEFLECTION OF A COLUMN
Task 1102-53-1106/52-11

Origin: NACA, Langley Aeronautical Laboratory
Sponsor: " " "
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The results are being transmitted to the sponsor as completed.

ROUNDING ERRORS IN STEPWISE INTEGRATION OF A PARTIAL DIFFERENTIAL EQUATION
Task 1102-53-1106/52-13

Origin: Naval Ordnance Laboratory, USN
Sponsor: " "
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Results are being transmitted to the sponsor as completed.

NEUTRON DIFFUSION
Task 1102-53-1106/52-15

Origin: Rand Corporation
Sponsor: Air Materiel Command, USAF
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The completion of the computations is pending until data becomes available.

PRECISE DETERMINATION OF THE PARAMETER OF DISPERSION EQUATION FOR SEVERAL TYPES OF OPTICAL GLASS
Task 1102-53-1106/52-17

Origin: NBS, Division 2
Sponsor: "
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The computations are being performed as requested

ROLLING MOMENT DUE TO SIDESLIP
Task 1102-53-1106/52-19

Origin: National Advisory Committee for Aeronautics
Sponsor: " "
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The code is completed and checked. Preparation of data by the IBM unit is almost completed.

SPHERICAL BLAST
Task 1102-53-1106/52-20

Origin: Naval Ordnance Laboratory, USN
Sponsor: " "
Managers: D. H. Jirauch, L. Nemerever

Status: CONTINUED. New equations were submitted by the sponsors after further analysis. A flow chart has been completed.

MAGNETIC FIELD EXTRAPOLATION
Task 1102-53-1106/52-22

Origin: Naval Ordnance Laboratory, USN
Sponsor: " "
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. A code has been completed and checked. The continuation of this task is pending until data for the next problem is available.

CHECKING OF NAVIGATION TABLES
Task 1102-53-1106/52-26

Origin: USN Hydrographic Office
Sponsor: " "
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The checking of these tables is 60 percent completed.

CABLE IN A UNIFORM STREAM
Task 1102-53-1106/52-27

Origin: David Taylor Model Basin
Sponsor: " "
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see Oct-Dec 1951 issue.

LORAN GRID TABLES
Task 1102-53-1106/52-39

Origin: Hydrographic Office, USN
Sponsor: " "
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. Programming for SEAC has been completed.

CALCULATIONS FOR d SPACINGS
Task 1102-53-1106/52-44

Origin: NBS, Division 9
Sponsor: "
Managers: J. A. Jordan and M. Abramowitz
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. The calculations were completed and transmitted to the sponsor.

SMOOTHING AND REDUCTION OF THEODOLITE DATA
Task 1102-53-1106/52-45

Origin: Long Range Proving Grounds, Florida
Sponsor: " "
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. A preliminary study of the form of the approximating polynomial and the number of points to be used is being made.

INTENSITIES OF SPECTRAL LINES II
Task 1102-53-1106/52-46

Origin: NBS, Division 14
Sponsor: "
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. Computations are being performed as requested.

INTER-INDUSTRY ECONOMICS
Task 1102-53-1106/52-47

Origin: The George Washington University
Sponsor: U. S. Navy
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. The calculations are about 60 percent completed.

KINETICS EQUATIONS FOR CONCURRENT CHEMICAL REACTIONS
Task 1102-53-1106/52-48

Origin: NBS, Section 7.3 — Authorized 1/1/52
Sponsor: " — Completed 3/31/52
Manager: A. Gleyzal

Objective: To solve the equations

$$\frac{dx}{dt} = \frac{U_A h}{3}(3A_o - x)(a - x - y)$$

$$\frac{dy}{dt} = K_D h\,(a - x - y)$$

with $x(0) = y(0) = 0$, and to express the solution functions $x(t, K_A, K_D, A_o, h, a)$, $y(t, K_A, K_D, A_o, h, a)$ in terms of functions of two variables.

Background: The kinetics of the oxidation of aldoses of chlorous acid is complicated by the concurrent decomposition of chlorous acid having independent evaluation of orders and rate constants of the decomposition and oxidation reaction; a mathematical solution of the rate equation would establish the stoichiometry of the oxidation of aldoses with chlorous acid.

Status: COMPLETED (NEW). The results were transmitted to the originators.

CONSUMER PRICE INDEX SURVEY
Task 1102-53-1106/52-53

Origin: Bureau of Labor Statistics — Authorized 1/1/52
Sponsor: " " — Completed 3/31/52
Manager: J. A. Jordan

Objective: To obtain the percentage of reported expenditures, and the amount of average expenditure, for each type of commodity, in 91 selected cities. Resulting figures to be used for revision of consumer price index to a 1950 base.

Background: Supplied data consists of numbers of consumer units, amount reporting, and aggregate amount reported within each item number, for each city.

Status: COMPLETED (NEW). The results were transmitted to the sponsor.

BETATRON SCHIFF SPECTRUM CALCULATION
Task 1102-53-1106/52-54

Origin: NBS, Section 4.11 — Authorized 1/1/52
Sponsor: " — Completed 3/31/52
Manager: I. Stegun

Objective: Computation of the quantity $\frac{E^2_e}{k}$, where

$$E_e \text{(electronic energy)} = 0(2)30.51(5)50.51 \text{ mer.},$$
$$k = 0(2)[E_e],$$
$$T = 2(1-\epsilon)(1n\alpha - 1) + \epsilon^2(1n\alpha - \tfrac{1}{2}),$$
$$\epsilon = \frac{k}{E_e}$$
$$\frac{1}{\alpha^2} = \left[\frac{\mu\epsilon}{2E_e(1-\epsilon)}\right]^2 + \left[\frac{z^{1/3}}{111}\right]^2$$
$$= .511 \text{ mer.},$$
$$\text{and } z = 74.$$

Background: The quantity arises in the formula for a spectrum in a straight forward direction. The problem was proposed by Dr. H.W. Koch.

Status: COMPLETED(NEW). The results were transmitted to the sponsor.

HEAT LOSS FROM CAVITIES
Task 1102-53-1106/52-56

Origin: NBS Section 10.3 Authorized 2/12/52
Sponsor: " "
Manager: L. Nemerever

Objective: To obtain an analytical expression for the heat loss from a spherical cavity.

Background: This task arose in connection with research on thermal properties of underground cavities, carried on in the Heating and Air-Conditioning Section of NBS. Dr. Harold Woolley had previously derived a formula applicable to spherical cavities. The present task consisted mainly in verifying Dr. Woolley's work.

Status: NEW. Preliminary checking of material has been done.

WAVE LENGTH OF SPECTRAL LINES
Task 1102-53-1106/52-59

Origin: NBS Section 4.2 Authorized 2/15/52
Sponsor: " Completed 3/31/52
Manager: I. Stegun

Objective: To fit a quadratic polynomial, by the method of least squares, to a set of experimental data.

Background: The computation arose in connection with experimental work of the NBS Radiometry Laboratory. The wave numbers (frequencies) of lines in the rotational-vibrational absorption spectrum of CS_2 are expressed as a function of quantum number. This problem was requested by E. K. Plyer and N. M. Gailar of Section 4.2, NBS.

Status: COMPLETED (NEW). The results were transmitted to the sponsor.

MULTIPLE COMPTON SCATTERING OF LOW ENERGY GAMMA RADIATION
Task 1102-53-1106/52-65

Origin: Naval Research Laboratory Authorized 4/1/52
Sponsor: " "
Manager: I. Stegun

Objective: To evaluate Fourier integrals for a selected set of parameters.

Background: The integrals arise in the theoretical studies being carried out at the Naval Research Laboratory. These computations were specifically requested by Mr. O'Rourke.

Status: NEW.

LONG PATH USABLE FREQUENCY PREDICTIONS
Task 1102-53-1106/52-66

Origin: NBS Section 14.4 Authorized 4/1/52
Sponsor: "
Manager: M. Stein

Objective: To analyze available radio traffic and field strength data to determine observed maximum usable frequencies over long paths. To develop empirical methods of calculating long path maximum usable frequencies to bring them into agreement with observations.

Background: Experience has shown that the maximum usable frequencies predicated by methods currently in use at the Central Radio Propagation Laboratory are discrepant with and in general lower than those actually observed. It is believed that empirical methods can be devised to improve this situation.

Status: NEW.

ORDER-STATISTICS ESTIMATORS OF EXTREME VALUES
Task 1102-53-1106/52-67

Origin: NBS Section 11.3 Authorized 4/1/52
Sponsor: National Advisory Committee for Aeronautics
Manager: I. A. Stegun

Objective: 1. To express the solution $w_i^{(n)*}, \lambda^*, \mu^*$ of the following linear system of n+2 equations explicitly in terms of the two arbitrary quantities a and b:

$$\sum_{j=1}^{n} \sigma_{ij}^{(n)} w_j^{(n)} + \lambda + c_i^{(n)} \mu = 0 \ , \ i = 1,2,\ldots,n.$$

$$\sum_{j=1}^{n} w_j^{(n)} = a$$

$$\sum_{j=1}^{n} c_j^{(n)} w_j^{(n)} = b$$

Here $c_i^{(n)}$ is the first moment of the i-th order statistic (in ascending order) in random samples of size n from the extreme-value distribution with c.d.f. $\exp(-e^{-y})$, and $\sigma_{ij}^{(n)}$ is the covariance between the i-th and j-th order statistics. The coefficients $c_i^{(n)}$, $\sigma_{ij}^{(n)}$ depend upon certain sums to be evaluated and upon the sample size n. The n+2 solutions will all be of the form $r_i a + s_i b$.

2. To evaluate the quadratic form

$$Q_n = Q(w_1^{(n)}, w_2^{(n)}, \ldots, w_n^{(n)}) = \sum_{j=1}^{n} \sum_{i=1}^{n} \sigma_{ij}^{(n)} w_i^{(n)} w_j^{(n)}$$

for $w_i^{(n)} = w_i^{(n)*}$, $i = 1, 2, \ldots, n$. The result will be of the form $Q_n^* = C_n a^2 + D_n ab + E_n b^2$, where C_n, D_n, E_n are the numerical quantities whose values are desired.

Calculations are to be made for at most n = 3(1)7, and accuracy to at least 4D is desired in the coefficients C_n, D_n, E_n, at least for the smaller values of n.

Background: In connection with the study of methods of analyzing extreme-value data under task 1103-21-5106/51-1, it is desired to obtain order-statistics estimators of the parameters of the form $T_n = \sum_{i=1}^{n} w_i^{(n)} x_i$ which are unbiased and have minimum variance. This requires the minimization of the quadratic form Q defined above subject to linear restrictions of the form

$$\sum_{j=1}^{n} w_j^{(n)} = a, \quad \sum_{j=1}^{n} c_j^{(n)} w_j^{(n)} = b$$

The linear equations under (1) express the conditions for minimization.

Comments: See Comments under task 1101-53-1101/52-38, to which this task is a companion project. The results of both these tasks are needed for research in progress under task 1103-21-5106/51-1 (Research in Application of Theory of Extreme Values to Gust Load Problems). Results for each value of n will be transmitted to originator for analysis to determine whether to proceed with calculations for the next succeeding value of n.

Status: NEW.

ATMOSPHERIC REFRACTION
Task 1102-53-1106/52-68

Origin: NBS Section 14.9
Sponsor: "
Manager: R. Brooks

Authorized 4/1/52
Completed 3/31/52

Objective: To obtain the autocorrelation function of time series consisting of measurements of the atmospheric index of refraction for radio waves.

Background: The measurements are made by the NBS Microwave Standards Section (NBS 14.9). The autocorrelation analysis is required in studies of the scattering of microwaves in the atmosphere and has application to investigations of atmospheric turbulence. This analysis was requested specifically.

Status: COMPLETED (NEW). The results have been transmitted by Dr. H. E. Bussey, 14.9 to the sponsor.

GAS ADSORPTION BY HIGH POLYMERS
Task 1102-53-1106/52-70

Origin: Bethesda Naval Medical Center Authorized 4/1/52
Sponsor: " "
Manager: I. A. Stegun

Objective: To obtain numerical solutions of a certain transcendental equation for a large number of values of the parameters occurring in it.

Background: The equation under consideration occurs in theoretical investigations of the adsorption of gases by high-polymer molecules. This was specifically requested by Dr. T. Mill, Bethesda Naval Medical Center.

Status: NEW.

TABLES OF THERMODYNAMIC PROPERTIES OF GASES
Task 0302-51-2606/49-5
(formerly 11.2/33-49-5)

Origin: NBS, Section 3.2
Sponsor: National Advisory Committee on Aeronautics
Managers: I. A. Stegun and M. Stein
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Calculations are being performed as requested. A table for helium (No. 6.10) has been published in the NBS-NACA tables series of thermal properties of gases.

BASIC IONOSPHERIC DATA
Task 1401-34-1473/49-14
(formerly 11.2/33-49-14)

Origin: NBS, Section 14.3
Sponsor: "
Managers: M. Stein and W. Gordon
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The calculations are being performed as requested.

IONOSPHERIC WINDS
Task 1401-11-1401/50-7
(formerly 11.2/33-50-7)

Origin: NBS, Section 14.1 Authorized 12/1/49
Sponsor: " Completed 3/31/52
Manager: B. S. Prusch

Objective: To analyze certain records of fast-fading ionospheric reflections of radio beams by transcribing them from the original paper tape to punched cards and performing correlation analyses by means of punched card machines.

Background: Measurements of intensity of reflection are recorded at three stations located at the vertices of a right triangle. As an ionospheric disturbance travels over the stations, it causes successive fading of the three records. By cross-correlating the records with various time lags and maximizing the correlation coefficients one obtains the components of the average velocity of travel of such disturbances. The problem was suggested by Mr. C. D. Salsberg of the Ionospheric Research Laboratory.

Status: COMPLETED.

RAY TRACING
Task 0202-21-2308/50-13
(formerly 11.2/33-50-13)

Origin: NBS, Section 2.2
Sponsor: "
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Several routine problems on skew ray tracing and numerical analyses of image errors of the third order were done.

COMPUTATION OF SINGLE AND DOUBLE WALL SOUND TRANSMISSION INTEGRALS
Task 0601-31-3527/51-7

Origin: NBS — Authorized 6/25/51
Manager: N. Levine and J. Levin — Terminated 3/31/52

Objective: To compute the integral

$$\tau = 2\int_0^1 \frac{v\,dv}{A^2}$$

where

$$A^2 = 1 + 4\,[R(R+1)+p^2v^2] + 4\,\sin^2 bv\,\left\{[R(R+1)+p^2v^2]^2 - p^2v^2\right\} - 4pv\,\sin 2bv\,[R(R+1)+p^2v^2]\;,$$

R is a damping parameter, b is a parameter which depends on the ratio of an airspace thickness to the wave length of sound, and p is a parameter depending on the mass reactance of the wall and the stiffness reactance resulting from the existence of flexural waves.

Background: This integral represents the ratio of transmitted to incident energy in the case of a random sound field striking a wall consisting of two partitions separated by an airspace. The variable v is the cosine of the angle of incidence.

Comments: This problem was formulated by the Sound Section, National Bureau of Standards. The occurrence of flexural waves in the wall is taken into account.

Status: TERMINATED.

CONDUCTION OF HEAT INWARD FROM A PLANE SURFACE EXPOSED TO HIGH INTENSITY RADIATION

Task 1002-11-4720/51-14

Origin: NBS, 10.0
Sponsor: "
Managers: M. Abramowitz and W. F. Cahill
Full task description appears in Apr-Jun 1951 issue.

Status: CONTINUED. Several new cases have been run on the SEAC.

III. Statistical Engineering Laboratory

(Section 11.3)

1. Fundamental Research in Mathematical Statistics

GLOSSARY OF STATISTICAL ENGINEERING TERMINOLOGY
Task 1103-11-1107/48-3
(formerly 11.3/2-48-3)

Origin: Section 11.0, NBS — Authorized 1/23/48
Manager: I. R. Savage — Terminated 3/31/52

Objective: To prepare a glossary of the statistical terminology associated with acceptance sampling and process control, statistical analysis and interpretation of experimental and test data, and statistical design of experiments and tests.

Background: The application of statistical concepts and techniques to acceptance sampling and process control has given rise to new terms, and many everyday terms are used with very specific connotations. The relatively new art of the statistical design of experiments and tests also has a special vocabulary. Finally, the concepts, principles, and techniques of statistical inference as applied to the analysis and interpretation of experimental and test data have been revised and expanded considerably during the past two decades with consequent changes in the meanings of terms and the introduction of new terms.

It is highly desirable, therefore, that a glossary of statistical engineering terminology be prepared to eliminate some of the present confusion in this field and to facilitate wider understanding of the subject.

Status: TERMINATED. Since the inception of this task, a Committee on Educational Aids (chairman, M. G. Kendall, and manager, W. R. Buckland, both of the University of London) of the International Statistical Institute has come into being. One of its stated purposes is the preparation of a list of statistical terms and symbols in the field of statistical methodology; another is the preparation of a chapter on statistical terminology for inclusion in the handbook of statistics to be issued by the American Statistical Association. The continuation of the present task as such would only rival this similar effort; therefore, it has been decided to terminate this task and to cooperate with and assist the above-mentioned committee as needed. As a start in this direction, at Professor Kimball's request, we are making the glossary in its present form available to the committee and we plan to send additions as these grow out of the work on other tasks. A number of worthwhile additions to our glossary have been made this quarter by members of the technical staff.

Dr. Eisenhart has continued the weekly conferences with G. A. Bicking, Ordnance Research and Development Division, in connection with Task Group 9 (Precision and Accuracy) of Committee E-11 of the American Society for Testing Materials. Continued cooperation in this regard should in no way be affected by the termination of this present task.

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-11-1107/49-1a
(formerly 11.3/2-49-1)

Origin: NBS
Full task description appears in Jan-Mar 1949 issue.

Status: CONTINUED. The present file of abstract cards consists of all abstracts in Mathematical Reviews (1940 through 1950) under the two general headings "Theory of Probability" and "Mathematical Statistics". Classification by a coded subject index of these abstracts has been completed. The emphasis this quarter has been on extending the bibliography to include selected abstracts of statistical significance from other fields, such as actuarial mathematics, astronomy, biological problems, combinatorial analysis, economics, theory of errors, ergodic theory, games, design of experiments, moments, and numerical and graphical methods. Back issues of Mathematical Reviews have been reviewed again to make these further selections. The work of clipping, pasting onto cards, and classifying these additional abstracts continues. Thus, except for this relatively small back-log of abstracts, the preparation of the bibliography is now on a current basis. Decisions about the final mechanical handling of the abstracts and the manner in which information from them shall be made available have yet to be made. This card index has proved useful in connection with the work on non-parametric statistics being carried out under task 1103-11-1107/52-2.

ELEMENTARY THEORY OF STOCHASTIC PROCESSES
Task 1103-11-1107/49-3

Origin: NBS
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. The manuscript of H. B. Mann has been accepted for publication and is in press at the U. S. Government Printing Office.

Publication: "Introduction to the theory of stochastic processes depending on a continuous parameter," by H. B. Mann; to appear as NBS Applied Mathematics Series 24.

ESTIMATION OF LOCATION AND SCALE PARAMETERS
Task 1103-11-1107/50-1
(formerly 11.3/1-50-1)

Origin: NBS
Manager: Churchill Eisenhart
Authorized 3/1/50
Terminated 3/31/52

Objective: To compare statistical properties of alternative estimators of the location and scale parameters of particular probability distributions, from the viewpoints of (1) practical applications and (2) clarification of the aims and principles of statistical estimation.

Background: Statistical estimation is a field of research in which there is considerable activity at the present time. One line of research is directed toward determining the "best" estimators, that is, toward the development of principles and techniques for determining such estimators when they exist; and another line is directed toward the development of easier-to-compute estimators that sacrifice as little as possible of the desirable properties of the "best". Even in the definition of "best" there are some difficulties - thus, for purposes of combination of estimates from several independent samples, unbiased estimators are desired; whereas, for use in single instances, there is much to be said for closest estimators; but unbiasedness and closeness are not always compatible, e.g., in estimating the standard deviation of a normal distribution. Much of the work to date along these lines has concerned itself with the determination of asymptotically "best" estimators, which have certain optimum properties for infinitely large samples. The present project, on the other hand, aims to concentrate primarily on estimators of value in the case of finite samples, particularly "small" samples of ten values or less, such as are frequently met with in physical science and engineering measurement.

Comments: This task, conceived as a continuing study, unifies and broadens the scope of work initiated under projects 11.3/1-47-1, 11.3/1-47-2, and 11.3/1-49-2. The first of these was completed in March 1949 (for a summary of results obtained, see Projects and Publications Jan-Mar 1949). The other two are now terminated.

The numerical results of project 11.2/33-49-18 (full project description appears in Apr-June 1949 issue) bear on the study of the arithmetic mean as an estimator of location being continued under the present task.

Likewise, the numerical results on moments of order statistics in samples from a normal distribution evaluated under project 11.1/32-50-5 (full project description appears in Oct-Dec 1949 issue) promise to be of value in connection with the present task.

Status: TERMINATED. Hereafter work of the type summarized below, and in recent status reports under this task, will be reported under task 1103-11-1107/51-2, Miscellaneous Studies in Probability and Statistics.

C. Eisenhart and L. S. Deming resumed work on the table of probability points of order statistics in small samples previously described under this task (see Jan-Mar 1950 issue, p. 40). For all of the probability levels concerned, the table has now been completed and checked from n = 2 to n = 6; and it has been about 75 percent completed from n = 7 to n = 10. A substantial portion of the entries for the .001 level came from L. Joel, as a by-product of SEAC computation relating to task 1304-34-6351/51-8. The table is being extended to n = 20, and the entries for n = 11 to n = 20 are about 50 percent completed for the probability levels from .005 to .995 inclusive.

In response to a request from industry, E. P. King is preparing a memorandum on the efficiency of the average range as an estimate of the standard deviation of a normal population which partitions the total loss of efficiency into two portions; one is attributable to the grouping into sub-samples, and the other is attributable to the use of a linear estimate rather than a root mean square estimate.

EXTREME-VALUE THEORY AND APPLICATIONS
Task 1103-11-1107/50-1a
(formerly 11.3/2-50-1)

Origin: NBS
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. The review of the draft of the monograph by E. J. Gumbel by a technical reader continued.

MANUAL ON FITTING STRAIGHT LINES
Task 1103-11-1107/50-2
(formerly 11.3/2-50-2)

Origin: NBS
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Several aspects of the two-variables-in-error problems were worked on by F. S. Acton in Princeton in conjunction with J. Tukey, as were also several of the confidence region problems that arise in these chapters. Illustrative examples were collected both in Washington and Princeton.

TABLE TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-11-1107/51-1

Origin: NBS
Full task description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1951 issue.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-11-1107/51-2

Origin: NBS
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. (1) Two short working papers were prepared by I. R. Savage on the limits of ES/σ and on a problem in evaluating definite integrals using various Monte Carlo methods. (2) O. Szász and E. Lukacs continued their study of non-negative trigonometric polynomials and are preparing a second paper on the subject. (For details see Sept-Dec 1951 issue, p. 59) (3) O. Szász and E. Lukacs studied properties of characteristic functions which are analytic in a neighborhood of the origin.

Publications: (1) "On the derivation and accuracy of certain formulas for sample sizes and operating characteristics of non-sequential sampling procedures," by U. Chand; NBS J. Res. 47, 491-502(Dec 1951). (2) "Properties of statistics involving the closest pair in a sample of three observations," by J. Lieblein; accepted by the NBS Journal of Research. (3) "An essential property of the Fourier transforms of distribution functions," by E. Lukacs; accepted for publication in the Proceedings of the American Mathematical Society. (4) "The stochastic independence of symmetric and homogeneous linear and quadratic statistics," by E. Lukacs; submitted to a technical journal. (5) "Some non-negative trigonometric polynomials connected with a problem in probability," by E. Lukacs and O. Szász; accepted by the NBS Journal of Research.

LAW OF PROPAGATION OF ERROR
Task 1103-11-1107/52-1

Origin: NBS
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1951 issue.

PROCEDURES OF NON-PARAMETRIC STATISTICS
Task 1103-11-1107/52-2

Origin: NBS
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. (1) The bibliography of non-parametric methods is nearing completion; a final check for accuracy will be made before its preparation as a report. (2) The final draft of the reports on Tchebycheff inequalities has been prepared. (3) A working paper on the asymptotic power of non-parametric procedures was prepared. (4) A working paper on a useful transformation of multivariate data was prepared. (5) Preliminary work on a paper showing the independence of tests of randomness and on other tests of hypotheses was begun.

2. Applied Research in Mathematical Statistics

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3011-60-0002/51-1
(formerly 3000-21-0002/51-1)

Origin: NBS
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. Activity under this project fell into two main categories:

A. Design of Experiments: A quintuple lattice with 25 treatments, separable into 5 replications, was constructed for comparing 25 enameled metals in 25 tests of 5 specimens each. A position effect in the test was eliminated by running a complete replication at each test position.

B. Development or Selection of the Appropriate Methods for Analysis and Interpretation of Data: A method was devised for testing whether two curves of unknown functional form have the same functional form.

Publications: (1) "Tables for constructing and for computing the operating characteristics of single sampling plans," by J. M. Cameron; accepted by Industrial Quality Control. (2) "The operating characteristic of the control chart for sample means," by E. P. King; submitted to a technical journal. (3) "Confidence and tolerance intervals for the normal distribution," by F. Proschan; submitted to a technical journal. (4) "The use of random numbers," by F. Proschan; accepted by Industrial Quality Control. (5) "Control charts may be all right - but ..., by F. Proschan; accepted by Industrial Quality Control. (6) "Statistics and planning tests at elevated temperatures," by W. J. Youden; accepted by Experimental Stress Analysis. (7) "The interpretation of chemical data," by W.J.Youden; accepted by Industrial Quality Control. (8) "Statistical units of measurements," by W. J. Youden; submitted to a technical journal. (9) "Experimental statistics - a review," by W. J. Youden and R. J. Hader; Anal. Chem. 24, 120-124 (Jan 1952).

STATISTICAL ASPECTS OF NBS ADMINISTRATIVE OPERATIONS
Task 3011-60-0002/52-1

Origin: NBS
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. A study of sick and annual leave practices of NBS employees for the calendar year 1950 was completed. At the request of the Personnel Division, a quick estimate of the percentage of NBS employees in a particular age category was obtained from a small sample of employee records to serve as a basis for determining the feasibility of a periodic health examination of employees in this group. A memorandum was submitted to the Chief of the Administrative Services Division on a combinatorial problem arising in connection with the preparation of keys for tumbler locks.

RESEARCH ON APPLICATION OF THEORY OF EXTREME VALUES TO GUST LOAD PROBLEMS
Task 1103-21-5106/51-1

Origin: NACA, Dynamic Loads Division
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. Work during the present quarter has been marked by a discovery that radically alters the basic computation techniques for this task. The new method involves a feasible means of determining the variance of extremal estimators based on order statistics for samples of any size. With this tool it is expected to obtain unbiased estimators of good efficiency, and the scope of the results can thereby be greatly extended without much, if any, increase in cost over what had been planned. These matters were outlined in an administrative progress report submitted to the National Advisory Committee for Aeronautics (NBS Report 1521, March 14, 1952).

A meeting with NACA representatives was held on March 20, 1952, at which time plans for completion of the work were taken up. Afterwards, two computation tasks were initiated: (a) 1102-53-1106/52-67, (see p.57)

for hand computation of "minimum"-variance order-statistics estimators for small samples; and (b) 1101-53-1101/52-38, (see p.34), for bias and variance of an estimator (based on sample moments) for several moderate sample sizes.

At the close of the quarter, a technical article was in preparation presenting the mathematical basis of the new method.

RESEARCH IN APPLICATIONS OF MATHEMATICAL STATISTICS TO PROBLEMS OF THE CHEMICAL CORPS
Task 1103-21-5118/52-1

Origin: Biological Laboratories, Chemical Corps, Dept. of the Army
Sponsor: " " "
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. An operations analysis approach to extensive bivariate data led to the development of a simple performance criterion. A certain set of data has been resolved into its principal components by the fitting of decay type curves. Estimates of error have been obtained which may be used in evaluating the fitted curves. Special analysis of covariance techniques are being devised to determine the effects of several treatments on some bivariate observational data with an inherently large experimental error.

APPLICATION OF THE THEORY OF STOCHASTIC PROCESSES TO THE STUDY OF TRAJECTORIES
Task 1103-21-5119/52-1

Origin: U. S. Naval Ordnance Test Station, Inyokern Authorized 1/1/52
Sponsor: " " "
Manager: E. Lukacs

Objective: To determine whether mathematical-statistical tools associated with the theory of stochastic processes can be profitably applied to the analysis of trajectory data of the type gathered at the Naval Ordnance Test Station, Inyokern.

Background: The theory of stochastic processes deals with time-dependent phenomena in which there is a probability relationship between a state at a given instant and one or more states at preceding instants. Some of the difficulties encountered in analyzing ordnance data can be overcome by considering a trajectory to be a stochastic process, thus eliminating the difficulties created by the fact that only one observation is available for the position of the missile at each instant.

Status: NEW The first step in this work was the adoption of a stochastic model. As always in such a situation one has to find a compromise between the wish to establish a realistic model and the necessity to use a model which does not lead to a forbiddingly complicated mathematical analysis. A fundamental random process is the Wiener process, and methods for the estimation of its parameters have been developed under a previous Bureau task (Elementary Theory of Stochastic Processes, Task 1103-11-1107/49-3). This is the main reason why it seems desirable to attempt to use in this study the Wiener process as the stochastic model. Clearly, this involves a certain amount of idealization. It is therefore desirable to safeguard against too radical departure from reality by testing first whether the observed trajectory data can be assumed to come from a Wiener random process.

Since no such test is known at present the first problem under this task is the development of a test for the hypothesis that a sequence of observations comes from a Wiener process.

The following theorem was derived in this connection mainly by using a result of Khinchine:

Let y(t) be a stochastic process and assume that

(i) y(t) is a process with independent increments.
(ii) y(t) is strongly continuous in the interval [a,b].

Then y(b) - y(a) is normally distributed.

This theorem shows that the normality of the increments follows from a certain continuity property of the process. In view of the physical situation encountered in the study of trajectories a continuity assumption is certainly reasonable. Therefore the test to be developed should not emphasize the normality of the increments; the independence of the increments seems to be the property deserving greater attention.

A technical report containing the proof of this theorem was completed and will be issued at the beginning of the next quarter.

E. Lukacs visited Professor H. B. Mann in February 1952 and discussed with him various aspects of this task.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-53-5105/52-1

Origin: Ship Structure Committee, NRC
Sponsor: " "
Full task description appears in Oct-Dec 1952 issue.

Status: INACTIVE. For status to date see Oct-Dec 1952 issue.

IV. Machine Development Laboratory

(Section 11.4)

in cooperation with

Electronic Computer Section

(Section 12.3)

1. Development: Design and Construction of Automatic Digital Computing Machines

THE BUREAU OF THE CENSUS COMPUTING MACHINE
Task 1104-34-5107/47-1
(formerly 11.4/21-47-1)

Origin: The Bureau of the Census
Sponsor: " "
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. At the end of the quarter, the first UNIVAC System had been in operation for a full year. This equipment has remained in the Philadelphia plant of the Eckert-Mauchly Division of Remington Rand Inc., and has been maintained by the company under contract with the National Bureau of Standards. The Bureau of the Census has employed the equipment predominantly for the tabulation of a portion of the 17th decennial census. The operating experience obtained during this period has been reported in considerable detail in a recent paper, "Review of electronic digital computers ," by J. L. McPherson (Bureau of the Census) and S. N. Alexander (National Bureau of Standards), presented at the Joint AIEE-IRE Computer Conference, February 1952.

Alterations to improve the reliability of auxiliary equipment are being provided on the second and third UNIVAC Systems and will be incorporated into the Bureau of the Census UNIVAC during the coming quarter. The present plans are to leave the installation in Philadelphia during most of 1952, with the Bureau of the Census assuming complete responsibility for its operation and maintenance.

THE NAVY COMPUTING MACHINE
Task 1104-34-5107/47-2
(formerly 11.4/22-47-2)

Origin: Mathematical Sciences Division, ONR
Sponsor: " " "
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The assembly of the Hurricane Computer, on whose design the ONR-NBS machine is to be patterned, has been completed, and the engineering tests and modifications have been in progress during the entire past quarter. Certain components of the ONR-NBS machine, which had been authorized earlier, have been completed. However, no additional work will be authorized until the Hurricane Computer has passed its acceptance test and a satisfactory understanding is reached with the company regarding both the price and the delivery of the ONR-NBS machine.

THE AIR COMPTROLLER'S COMPUTING MACHINE
Task 1104-34-5107/47-3
(formerly 11.4/24-47-3)

Origin: Office of the Air Comptroller, USAF
Sponsor: " "
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. During February 1952, the fourth part of the revised acceptance test procedure was passed, and the UNIVAC System was accepted for delivery to Washington. This was the second attempt to pass the more rigid requirements that were put on the ability of the central computer to communicate with the magnetic input-output system. In order to meet these requirements, the company found it necessary to introduce several design modifications in the Uniservos and the input-output circuitry. These same modifications will be used in System No. 3, and appropriate modifications of System No. 1 will be made in the near future.

The National Bureau of Standards contracted for the transfer and installation of the UNIVAC System in the Pentagon Building. During the past quarter, the power and control wiring, the power supply, and the central computer has been installed at the site. It is anticipated that all elements of the system will be delivered and put into operation during the coming quarter.

NATIONAL BUREAU OF STANDARDS EASTERN AUTOMATIC COMPUTER (SEAC)
Task 1104-34-5107/49-1
(formerly 11.4/24-49-1)

Origin: NBS
Sponsor: Air Comptroller's Office, USAF
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. During this quarter, the over-all operating reliability of SEAC was 79 percent for a 168-hour week which includes 8 hours of scheduled maintenance. Reliability has been somewhat affected by the recent change to temporary a-c power lines. Since SEAC has thus far operated off unregulated power supplies, a-c stabilizers have been ordered to correct the situation.

Additional circuit refinements have been made in the Williams electrostatic memory, and its use in problem solution has increased steadily during the past quarter. The additional circuitry for the 3-address system has been installed but will not be put into service until the next quarter. Changes and additions to the control circuitry associated with the input-output equipment have also been installed so that the Raytheon tape unit can be utilized along with the other magnetic tape units.

WRIGHT DEVELOPMENT CENTER COMPUTING MACHINE
(formerly AIR MATERIEL COMMAND COMPUTING MACHINE)
Task 1104-34-5107/49-1a
(formerly 11.4/23-49-1)

Origin: Flight Research Laboratory, Wright Air Development Center, USAF
Sponsor: " " " " "
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The assembly of the computer is completed except for the magnetic drum memory. The completed equipment is undergoing engineering tests, and individual operations are being initiated with the aid of a small test drum. The Inscriber and Outscriber equipment are now assembled, and tests on them have been started. The complete assembly and the testing of the entire system will be delayed until next quarter because of mechanical difficulties with the magnetic drum for the main memory. The drum was coated and assembled for trial operation. However, at operating speed, the eccentricity was found to be excessive, and the drum has been returned for adjustment. Present plans are to have all equipment tested and proved in so that, when the drum is installed, complete system tests can be started immediately.

ARMY MAP SERVICE COMPUTING MACHINE
Task 1104-34-5107/49-1b
(formerly 11.4/25-49-1)

Origin: Army Map Service, USA
Sponsor: " "
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. During the past quarter, UNIVAC System No. 3 was completely assembled and has been undergoing engineering tests. The tests on several of the auxiliaries has been completed and accepted. The acceptance test for the central computer and the input-output system has been modified slightly in the light of experience gained in the acceptance tests for the first two UNIVAC Systems. The final tests will require the UNIVAC to utilize a complete set of ten Uniservos. Since it is anticipated that the acceptance test will be concluded early in the coming quarter, arrangements for transferring the system to its site in the Army Map Service Building have been made.

CODING RELATED TO THE RAYTHEON COMPUTER
Task 1104-53-5108/49-3
(formerly 11.4/3-49-3)

Origin: Mathematical Sciences Division, ONR — Authorized 12/1/48
Sponsor: Office of Naval Research, USN — Terminated 3/31/52

Objective: To evaluate the mathematical sufficiency of the proposed Raytheon electronic computer.

Background: The Raytheon Company has submitted to the Bureau a series of design modifications intended to increase the flexibility and power of the proposed computer. By coding basic mathematical routines for solution on the Raytheon computer, in its various stages of design, the Bureau has evaluated successive designs.

Status: TERMINATED.

INVESTIGATION OF THE APPLICABILITY OF AUTOMATIC DIGITAL ELECTRONIC COMPUTING TO PROBLEMS OF THE SOCIAL SECURITY AGENCY
Task 1104-53-5108/51-1

Origin: Social Security Agency
Sponsor: " "
Full task description appears in Oct-Dec 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1951 issue.

Lectures and Symposia

Numerical Analysis Colloquium Series
(Los Angeles, California)

CURTISS, J. H. Some chain functions which are useful in the Monte Carlo method. January 21, 1952.

FRANKEL, S. P. (California Institute of Technology). Accuracy and stability considerations in partial difference equations. February 18, 1952.

SCHOENBERG, I. J. (University of Pennsylvania, University of California, and NBS). Interpolation by spline curves. March 17, 1952.

Statistical Engineering Seminars

ACTON, F. S. Analysis of straight line data. January 16, 1952.

EISENHART, C. On the use of randomization in experimentation. February 8, 1952.

LIEBLEIN, J. Summary of some contributed papers given at the Boston meeting of the Institute of Mathematical Statistics. January 11, 1952.

MANDEL, J. (NBS Section 7.5) Fitting a straight line to cumulative data. March 7, 1952.

SAVAGE, I. R. On some portions of the proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability. January 25, 1952.

Talks Presented by Members of the Staff At Statistical Seminars For NBS Division 13 and Section 12.2

EISENHART, C. The evaluation of the effectiveness of material - statistical aspects. January 31,1952.

KING, E. P. (1) Use of the t-test. March 6, 1952. (2) Use of randomized blocks. March 20, 1952. (3) Concepts of interactions. March 27, 1952.

PROSCHAN, F. (1) Confidence limits for the population proportion. February 21, 1952. (2) Comparison of two percentages. February 28, 1952. (3) Analysis of variance. March 13, 1952.

YOUDEN, W. J. Statistics at work. February 7, 1952.

Papers and Invited Talks Presented by Members of the Staff at Outside Organizations

EISENHART, C. The reliability of measured values--fundamental concepts. Presented to the meeting of the American Society of Photogrammetry Washington, D. C., January 10, 1952.

FORSYTHE, G. E. Automatic digital computations. Presented to the Los Angeles Chapter of the American Statistical Association, Los Angeles, Cal., January 24, 1952.

HESTENES, M. R. (1) Iterative method for solving linear systems. Presented to Laboratory Personnel of the Naval Ordnance Test Station, Inyokern, China Lake, Cal., January 30, 1952. (2) Minimax problems on calculus of variations. Presented to the Peripatetic Seminar at the University of California at Los Angeles, February 4, 1952.

HUSKEY, H. D. (1) Mathematical services available at the Institute for Numerical Analysis. Presented at the U. S. Navy Electronics Laboratory, San Diego, California, March 7, 1952. (2) Recent developments in computer design. Presented at the Industrial Engineering Conference sponsored by the University of California at Los Angeles and the American Society of Mechanical Engineering, Los Angeles, March 26, 1952.

LANCZOS, C. Radiation of cylindrical antenna. Presented at the Physics Seminar, University of California at Los Angeles.

LEHMER, D. H. Computing machine development and research in pure mathematics. Presented to the Southern California Section of the Mathematical Association of America Meeting, Occidental College, Los Angeles, March 8, 1952.

SCHOENBERG, I. J. On the shape of curves. Presented to the Southern California section of the Mathematical Association of America Meeting, Occidental College, Los Angeles, March 8, 1952.

TODD, J. The development of numerical analysis for electronic digital computers. Presented to the meeting of the Columbia Mathematics Club, Washington, D. C., January 7, 1952.

WASOW, W. Approximation of elliptic differential equations by difference equations with positive coefficients. Presented to the Peripatetic Seminar at the University of California at Los Angeles, March 3, 1952.

YOUDEN, W. J. (1) Statistics at work. Presented to the Parkersburg, W. Va. Section of the American Society for Quality Control, January 2, 1952. (2) The control and measurement of experimental error. Presented to the personnel of the Naval Powder Factory, Indian Head, Md., January 28, 1952. (3) Statistics at work. Presented to the personnel of the Naval Powder Factory, Indian Head, Md., February 7, 1952. (4)-(12) Interpretation of chemical data. Presented to the following section meetings of the American Chemical Society; Warren, Pa., February 14; Youngstown, Ohio, February 15; Columbus, Ohio, February 18; Painesville, Ohio, February 19; Cleveland, Ohio, February 20; Erie, Pa., February 21. Also presented to the staff of the Diamond Alkali Co., Painesville, Ohio, February 19. Presented to the research personnel of the Celanese Corporation of America, Bishop, Tex., February 25; and to the Border Section of the American Chemical Society, Corpus Christi, Tex., February 26. (13) Statistical units of measurement. Presented to the Philadelphia Section of the American Society for Metals, March 4, 1952. (14) Principles of experimental design. Presented to the Regional Conference of the American Society for Quality Control, New York, N. Y., March 28-29, 1952.

Publication Activities

1. PUBLICATIONS WHICH APPEARED DURING THE QUARTER

1.2 Manuals, Bibliographies, Indices

(1) A guide to tables on punched cards. G. Blanch and E. C. Yowell. MTAC V, 185-212 (Oct. 1951). Reprints available.

1.3 Technical Papers

(1) Fifth order aberration in an optical system. R. K. Anderson. Proceedings. I.B.M. Computation Seminar. 130-131 (Aug. 1951). (Published by I.B.M. Corp., 590 Madison Ave., New York, 1951).

(2) On subharmonic, harmonic, and linear functions of two variables. E. F. Beckenbach. Revista de Matematica y Fisica Teorica (Argentina) 8, 7-13 (Nov. 1951).

(3) On the derivation and accuracy of certain formulas for sample sizes and operating characteristics of non-sequential sampling procedures. U. Chand. NBS J. Res. 47, 491-501 (Dec. 1951); RP2277. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 10 cents.

(4) On some functionals of Laplacian processes. R. Fortet. NBS J. Res. 48, 32-39 (Jan. 1952); RP2280. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 10 cents.

(5) On the estimation of an eigenvalue by an additive functional of a stochastic process with special reference to the Kac-Donsker method. R. Fortet. NBS J. Res. 48, 68-75 (Jan. 1952); RP2286. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 10 cents.

(6) Random determinants. R. Fortet. NBS J. Res. 47, 465-470 (Dec. 1951); RP2274. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 10 cents.

(7) Applications of the theory of quadratic forms in Hilbert space to the calculus of variations. M. R. Hestenes. Pac. J. Math. I, 525-581 (Dec. 1951). Reprints available.

(8) The solution of $Ax = \lambda Bx$. M. R. Hestenes and W. Karush. NBS J. Res. 47, 471-478 (Dec. 1951); RP2275. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 10 cents.

(9) Semi-automatic instruction on the National Bureau of Standards Western Automatic Computer. H. D. Huskey. Proceedings of a Second Symposium on Large-Scale Digital Computing Machinery, Annals of the Harvard Computation Laboratory xxvi, 83-90 (Dec. 1951). No reprints.

(10) Determination of the extreme values of the spectrum of a bounded self-adjoint operator. W. Karush. Proc. Am. Math. Soc. 2, 980-989 (Dec. 1951). Reprints available.

(11) On the variation of the determinant of a positive definite matrix. A. M. Ostrowski and O. Taussky. Proceedings Royal Netherlands Academy of Sciences at Amsterdam Series A, 54, No. 5, and Indag. Math. 13, No. 5, 383-385 (1951). Reprints available.

(12) Uniformly best constant risk and minimax point estimates. R. T. Peterson, jr. NBS J. Res. 48, 49-53 (Jan. 1952); RP2282. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 10 cents.

(13) Formulas for finding the argument for which a function has a given derivative. H. E. Salzer. MTAC V, 213-215 (Oct. 1951). Reprints available.

(14) Some general theorems in iterants. P. Stein. NBS J. Res. 48, 82-83 (Jan. 1952); RP2288. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 5 cents.

(15) A note on the bounds of multiple characteristic roots of a matrix. P. Stein. NBS J. Res. 48, 59-60 (Jan. 1952); RP2284. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 5 cents.

(16) The convergence of Seidel iterants of nearly symmetric matrices. P. Stein. MTAC V, 236-240 (Oct. 1951). Reprints available.

1.4 Reviews

(1) Experimental statistics--a review. R. J. Hader and W. J. Youden. Anal. Chem. 24, 120-124 (Jan. 1952). Reprints available.

1.5 Miscellaneous Publications

(1) Gauss to Gerling on relaxation. G. E. Forsythe. MTAC V, 255-258 (Oct. 1951). Reprints available.

(2) New frontiers in business management control are being established by electronic computers. H. D. Huskey and V. R. Huskey. Journal of Accountancy 69-75 (Jan. 1952). Reprints available.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION March 31, 1952.

2.1 Mathematical Tables

(1) Tables of the Chebyshev polynomials $S_n(x)$ and $C_n(x)$. NBS Applied Mathematics Series 9. In press, Government Printing Office.

(2) Table for the analysis of β spectra. NBS Applied Mathematics Series 13. In press, Government Printing Office.

(3) Tables of Coulomb wave functions, Vol. I. NBS Applied Mathematics Series 17. In press, Government Printing Office.

(4) Tables for rocket and comet orbits. S. Herrick. NBS Applied Mathematics Series 20. In press, Government Printing Office.

(5) Probability tables for analysis of extreme-value data. NBS Applied Mathematics Series 22. In press, Government Printing Office.

(6) Tables of normal probability functions. NBS Applied Mathematics Series 23. Supercedes NBS Mathematical Table MT14, Tables of probability functions, volume II.) In press, Government Printing Office.

(7) Tables of Bessel functions $Y_0(x)$, $Y_1(x)$, $K_0(x)$, $K_1(x)$. $0 \leq x \leq 1$. NBS Applied Mathematics Series 25. (Reissue of AMS1.) In press, Government Printing Office.

(8) Table of Arctan x. (Formerly NBS Mathematical Tables MT16.) To be issued in the Applied Mathematics Series.

(9) Collected tables of the Computation Laboratory: Volume I. Tables of functions and of zeros of functions. To be issued in the NBS Applied Mathematics Series.

(10) Tables of 10^x. Submitted for publication.

(11) The third Wilson prime and an extended table of Wilson quotients. K. Goldberg. Submitted to a technical journal.

(12) Table of zeros and weight factors of the first twenty Hermite polynomials. H. E. Salzer, R. Zucker, and R. Capuano. Accepted for publication in the NBS Journal of Research.

2.2 Manuals, Bibliographies, Indices

(1) A guide to the tables of the normal probability integral. NBS Applied Mathematics Series 21. In press, Government Printing Office.

(2) Introduction to the theory of stochastic processes depending on a continuous parameter. H. B. Mann. NBS Applied Mathematics Series 24. In press, Government Printing Office.

(3) The hypergeometric and Legendre functions with applications to integral equations of potential theory. C. Snow. NBS Applied Mathematics Series 19. (Supersedes NBS MT15, which is out of print.) In press, Government Printing Office.

2.3 Technical Papers

(1) Analyzing straight line data. F. S. Acton. Submitted to a technical journal for publication.

(2) The relaxation method for linear inequalities. S. Agmon. Submitted to a technical journal for publication.

(3) The expansion theorem for pseudo-analytic functions. S. Agmon and L. Bers. Submitted to a technical journal.

(4) On subordination in complex theory. E. F. Beckenbach and E. W. Graham . To appear in "The construction and applications of conformal maps: Proceedings of a symposium," to be published by the National Bureau of Standards.

(5) On mildly nonlinear partial difference equations of elliptic type. L. Bers. Submitted to a technical journal.

(6) On the numerical solution of parabolic partial differential equations. G. Blanch. Accepted for publication in the NBS Journal of Research.

(7) Two existence theorems for systems of linear inequalities. L. M. Blumenthal. Submitted to a technical journal.

(8) Tables for constructing and computing the operating characteristics of single sampling plans. J. M. Cameron. Accepted for publication in Industrial Quality Control.

(9) Contribution to the theory of Markov chains. Kai Lai Chung. Accepted for publication in the NBS Journal of Research.

(10) On the recursion formula and on some Tauberian theorems. N.G. de Bruijn and P. Erdös. Accepted by the NBS Journal of Research.

(11) Alternative derivations of Fox's escalator formulas for latent roots. G. E. Forsythe. Submitted to a technical journal.

(12) An extension of Gauss' transformation for improving the condition of systems of linear transformations. G. E. Forsythe and T. S. Motzkin. Accepted by Mathematical Tables and Other Aids to Computation.

(13) Additive functionals of a Markoff process. R. Fortet. Submitted to a technical journal.

(14) Practical solution of linear equations and inversion of matrices. L. Fox. Accepted for publication in the NBS Journal of Research.

(15) Metric methods in integral and differential geometry. J. W. Gaddum. Accepted for publication by the American Journal of Mathematics.

(16) The sums of the dihedral and trihedral angles in a tetrahedron. J. W. Gaddum. Submitted to a technical journal.

(17) A theorem on convex cones, with applications to linear inequalities. J. W. Gaddum. Submitted to a technical journal.

(18) A 2-basic set of density zero. K. Goldberg. Submitted to a technical journal.

(19) Bergman's Integraloperator erster Art und Riemannsche Funktion. P. Henrici. Submitted to a technical journal.

(20) On the variation of the spectrum of a normal matrix. A. J. Hoffman and H. W. Wielandt. Submitted to a technical journal.

(21) Distribution of electrical conduction currents in the vicinity of thunderstorms. R. E. Holzer and D. S. Saxon. Accepted for publication in the Journal of Geophysical Research.

(22) On integration of parabolic equations by difference methods. I: Linear and quasi-linear equations for the infinite interval. F. John. Accepted for publication in Communications on Pure and Applied Mathematics.

(23) Convergence of a method of solving linear problems. W. Karush. Submitted to a technical journal.

(24) The operating characteristic of the control chart for sample means. E. P. King. Submitted to a technical journal.

(25) Solution of systems of linear equations by minimized iterations. C. Lanczos. Accepted for publication in the NBS Journal of Research.

(26) On certain character matrices. D. H. Lehmer. Submitted to a technical journal.

(27) Properties of statistics involving the closest pair in a sample of three observations. J. Lieblein. Accepted by the NBS Journal of Research.

(28) A method of summing infinite series. S. Lubkin. Accepted by the NBS Journal of Research.

(29) The stochastic independence of symmetric and homogeneous linear and quadratic statistics. E. Lukacs. Submitted to a technical journal.

(30) An essential property of the Fourier transforms of distribution functions. E. Lukacs. Accepted for publication in the Proceedings of the American Mathematical Society.

(31) Some non-negative trigonometric polynomials connected with a problem in probability. E. Lukacs and O. Szász. Accepted for publication in the NBS Journal of Research.

(32) The torsion of anisotropic elastic cylinders by forces applied on the lateral surface. H. Luxenberg. Accepted for publication in the NBS Journal of Research.

(33) Graphing with the 407 accounting machine. S. Mallos. Submitted to a technical newsletter.

(34) The estimation of parameters in certain stochastic processes. H. B. Mann. To be published in Sankhya: The Indian Journal of Statistics.

(35) On a method for the determination of converging factors, applied to the asymptotic expansions for the parabolic cylinder functions. J.C.P. Miller. Submitted to a technical journal.

(36) The double description method. T. S. Motzkin, H. Raiffa, G. L. Thompson, R. M. Thrall. To appear in the Annals of Mathematical Studies Volume on Contributions to the Theory of Games, II.

(37) The number of farthest points. T. S. Motzkin, E. G. Strauss, F. A. Valentine. Submitted to a technical journal.

(38) On lineal entire functions of n complex variables. T.S.Motzkin and I. Schoenberg. Submitted to a technical journal.

(39) Matrices with property L. T. S. Motzkin and O. Taussky. Accepted for publication in the Transactions of the American Mathematical Society.

(40) On representations of finite groups. T. S. Motzkin and O. Taussky. Submitted to a technical journal.

(41) On the derivative of a polynomial and Chebyshev approximation. T. S. Motzkin and J. L. Walsh. Submitted to a technical journal.

(42) On the approximation of linear elliptic differential equations by difference equations with positive coefficients. T. S. Motzkin and W. Wasow. Submitted to a technical journal.

(43) Remarks on some modular identities. M. Newman. Submitted to a technical journal.

(44) On two problems in abstract algebra connected with Horner's rule. A. M. Ostrowski. Submitted to a technical journal.

(45) On a discontinuous analogue of Theodorsen's and Garrick's method. A. M. Ostrowski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published in the National Bureau of Standards Applied Mathematics Series.

(46) On the convergence of Theodorsen's and Garrick's method of conformal mapping. A. M. Ostrowski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published in the National Bureau of Standards Applied Mathematics Series.

(47) Bounds for the greatest latent root of a positive matrix. A. Ostrowski. Accepted for publication in the Journal of the London Mathematical Society.

(48) Confidence and tolerance intervals for the normal distribution. F. Proschan. Submitted to a technical journal.

(49) A method of computing exact inverses of matrices with integer coefficients. J. B. Rosser. Accepted by the NBS Journal of Research.

(50) Numerical computation of low moments of order statistics from a normal population. J. B. Rosser. Submitted to a technical journal.

(51) Formulas for numerical differentiation in the complex plane. H. E. Salzer. Accepted for publication in the Journal of Mathematics and Physics.

(52) An elementary note on powers of quaternions. H. E. Salzer. Submitted to a technical journal.

(53) On calculating the zeros of polynomials by the method of Lucas. H. E. Salzer. Accepted for publication in the NBS Journal of Research.

(54) An optical model for nucleon-nuclei scattering. D. S. Saxon. and R. E. LeLevier. Accepted for publication in Physical Review.

(55) Modes of vibration of a suspended chain. D. S. Saxon and A.S.Cahn. Accepted for publication in the Quarterly Journal of Mechanics and Applied Mathematics.

(56) Distribution of electrical conduction currents in the vicinity of thunderstorms. R. E. Holzer and D. S. Saxon. Submitted to a technical journal.

(57) A remark on M. M. Day's characterization of inner-product spaces and a conjecture of L. M. Blumenthal. I. J. Schoenberg. Submitted to a technical journal.

(58) On Polya frequency functions III: The positivity of translation determinants with an application to the interpolation problem by spline curves. I. J. Schoenberg and A. Whitney. Submitted to a technical journal.

(59) A bibliography of numerical methods in conformal mapping. W.Seidel. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published in the National Bureau of Standards Applied Mathematics Series.

(60) Gradient methods in the solution of systems of linear equations. M. L. Stein. Accepted for publication in the NBS Journal of Research.

(61) On methods for obtaining solutions of fixed end point problems in the calculus of variations. M. L. Stein. Accepted for publication in the NBS Journal of Research.

(62) Sufficient conditions for the convergence of Newton's method in complex Banach spaces. M. L. Stein. Submitted to a technical Journal.

(63) A note on the bounds of the real parts of the characteristic roots of a matrix. P. Stein. Accepted for publication in the NBS Journal of Research.

(64) On Cauchy-Riemann equations in higher dimensions. E. Stiefel. Accepted for publication in the NBS Journal of Research.

(65) Two applications of group characters to the solution of boundary-value problems. E. Stiefel. Accepted for publication in the NBS Journal of Research.

(66) On the relative extrema of the Hermite orthogonal functions. O. Szász. Accepted for publication in the Journal of the Indian Mathematical Society.

(67) On the relative extrema of Bessel functions. O. Szász. Accepted for publication by the Bolletino della Unione Matematica Italiana (Firenze).

(68) On the Gibb's phenomenon for a class of linear transforms. O. Szász. Accepted by "Publications de l'Inst. Math. de l'Acad. Serbe des Sciences," Vol. IV.

(69) Classes of matrices and quadratic fields, II. O. Taussky-Todd. Accepted for publication in the Journal of the London Mathematical Society.

(70) On conformal mapping of variable regions. S. E. Warschawski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published in the National Bureau of Standards Applied Mathematics Series.

(71) On the truncation error in the solution of Laplace's equation by finite differences. W. Wasow. Accepted for publication in the NBS Journal of Research.

(72) On the inversion of matrices by random walks. W. Wasow. Accepted by Mathematical Tables and Other Aids to Computation.

(73) On singular perturbation problems in the theory of non-linear vibrations. W. Wasow. To be published in the Proceedings of the Symposium on Non-Linear Vibrations, held at Isle de Porquerolles, France, September 18-22, 1951.

(74) Metodi probabilistici per la soluzione numerica di alcuni problemi di analisi. W. Wasow. To be published in the Proceedings of the 4th Congress of the Italian Mathematical Union in Messina.

(75) Normal matrices with property L. N. Wiegmann. Submitted to a technical journal.

(76) Pairs of normal matrices with property L. H. Wielandt. Submitted to a technical journal.

(77) On the eigenvalues of A+B and AB. H. Wielandt. Submitted to a technical journal.

(78) Statistical units of measurement. W. J. Youden. Submitted to a technical journal.

2.5 Miscellaneous Publications

(1) The construction and applications of conformal maps: Proceedings of a symposium held at the NBS Institute for Numerical Analysis Los Angeles, California, June 1949. NBS Applied Mathematics Series 18. In press, U. S. Government Printing Office.

(2) Simultaneous equations and the determination of eigenvalues. Proceedings of an NBS Symposium held in Los Angeles, August 1951. To appear in the NBS Applied Mathematics Series.

(3) Scientific teamwork in a Computation Laboratory. J. H. Curtiss. To appear in the Proceedings of the Third Institute on Administration of Scientific Research and Development, held at American University, October 13, 1951.

(4) The use of random numbers. F. Proschan. Accepted for publication in Industrial Quality Control.

(5) Control charts may be all right, but--. F. Proschan. Accepted for publication in Industrial Quality Control.

(6) Statistics and planning tests at elevated temperatures. W. J. Youden. Accepted for publication in Experimental Stress Analysis

3. NBS Reports

Note: The following reports contain material for which formal publication in scientific journals is not presently planned. A limited number of copies of each was available at the end of the quarter.

(1) Translation by C. D. Benster (UCLA) of K. A. Semendiaev. "The determination of latent roots and invariant manifolds of matrices by means of iterations." Translation edited by G. E. Forsythe. NBS Report 1402.

(2) A statistical method for finding the lowest eigenvalue of Schroedinger's equation. M. Kac and M. Cohen. NBS Report 1553.

(3) Combining tolerances. E. P. King. NBS Report 1313.

(4) Bibliography on bounds for characteristic roots of finite matrices. O. Taussky. NBS Report 1162.

Appendix

Pressure Distribution on a Body of Revolution in Supersonic Flight (Task 1101-53-1101/52-3)

In August,1951, at the request of the Missile Aerodynamics Department of Hughes Aircraft Company, the Institute for Numerical Analysis began a series of computations for the purpose of devising a workable procedure on the Model 1 IBM Card-Programmed Calculator (CPC) for determining the velocity distribution on the surface of the forward portion of an arbitrary body of revolution moving at supersonic speed in the direction of its axis through a steady, inviscid medium. The flow was assumed to be rotational; i.e., the pressure depended on both density and entropy,although the latter remained constant along a given streamline. In addition, the rest enthalpy was assumed constant everywhere. Six bodies were considered, each of them a cylinder with a spherical cap attached by a transition surface concave to the axis.

If such a body is assumed to extend indefinitely rearward, the flow field around it is divided into two regions by a detached shock surface which resembles the body close to the axis and approaches a definite conical shape farther away from the axis. On the side of the shock surface away from the axis the flow is uniform, the velocity being parallel to the axis with constant Mach number $M > 1$. As the flow enters the region between the shock surface and the body, it is deflected away from the axis and reduced in speed. Close to the nose the flow is subsonic ($M < 1$); farther back it becomes supersonic ($M > 1$) and approaches the speed and direction of the uniform flow in the undisturbed stream outside the shock surface.

Take the point of intersection of the body and its axis as the origin of the x-r plane, the positive x-axis pointing along the body axis toward the rear and the r-axis being any normal to the x-axis at the origin. This plane, then, is a meridian section and the non-uniform flow is delimited by the shock curve and the body curve (see insert in Fig. 1). If the velocity and entropy are known accurately on an initial curve extending from the body through an entirely supersonic region to the shock curve, then it is possible to determine these flow variables to the right of the initial curve by the method of characteristics. Initial data for the six problems computed are presented in Table 1. (Values of r, Θ, and s have been smoothed from empirical and theoretical data to a uniform number of decimal places.) The curve is seen to rise from the body with a steep, increasing, positive slope which soon becomes infinite; thereafter, it veers to the left with increasing curvature. The variable Θ is the angle

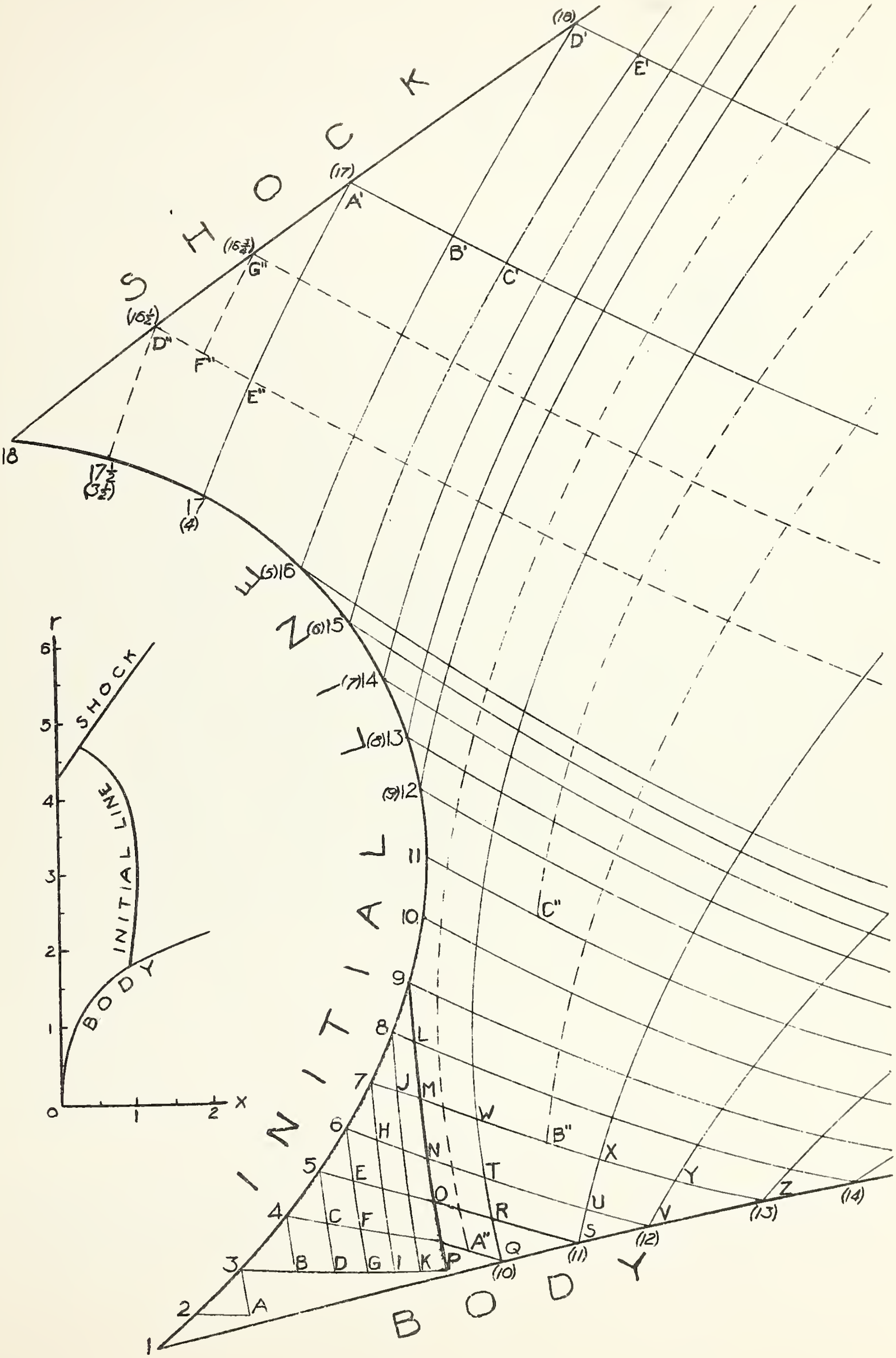

Figure 1

Table 1

Number of Point	x	r	θ	s
(body) 1	.918	1.813	.6335	.0476
2	.9225	1.858	.6085	.0471
3	.927	1.948	.5638	.0462
4	.936	2.037	.5272	.0447
5	.9495	2.216	.4682	.0430
6	.963	2.351	.4343	.0411
7	.9765	2.530	.3985	.0390
8	.99	2.709	.3699	.0364
9	.9945	2.888	.3467	.0340
10	1.008	3.112	.3208	.0314
11	1.0125	3.291	.3038	.0288
12	.999	3.515	.2860	.0264
13	.972	3.695	.2725	.0240
14	.9405	3.874	.2618	.0218
15	.873	4.098	.2493	.0198
16	.7875	4.277	.2413	.0182
17	.6525	4.501	.2306	.0168
(shock) 18	.3375	4.680	.2234	.0142

(in radians) between the velocity vector and the positive x-axis; s is the dimensionless quantity $(\bar{s} - \bar{s}_0)/c_v$, where $\bar{s}$ and $\bar{s}_0$ are the entropies per unit mass at the point and in the undisturbed stream, and c_v is the specific heat at constant volume. Along the initial curve the speed q is assumed constant; q*, the ratio between q and the critical speed of sound, can be defined by

$$(1) \qquad q^* = M \left\{ (\gamma + 1)/[\gamma - 1) M^2 + 2] \right\}^{\frac{1}{2}}, \quad \gamma = 1.405,$$

and is given = 1.048. Moreover, the Mach number of the undisturbed stream is given = 1.62 and the angle between the positive x-axis and the tangent to the shock curve at point 18 on the initial curve is σ = 1.018 radians. The upper meridian section of each of the bodies consists of an arc of a semicircle tangent to the r-axis, a transition curve which is different for each body, and a straight line. It will be seen later that if the velocity distribution on the body between the initial curve and some point P is to be found, it is necessary first to determine x, r, q*, Θ, and s at a sufficient number of points in the region bounded by the body, the initial curve, the shock curve, and a certain curve (the right characteristic) drawn from the shock curve (or the initial curve) through P.

The use of characteristics for solving problems of this type has been systematized by G. Guderley, R. Sauer, and W. Tollmien and is described in detail in Technical Report No. F-TR-1173A-ND (GDAM A-9-M II/1), Air Materiel Command, Wright Field, 1947: The Method of Characteristics in Compressible Flow, Part I (Steady Supersonic Flow), prepared by J. S. Isenberg under the supervision of C. C. Lin in the Graduate Division of Applied Mathematics, Brown University, for the Analysis Division, Intelligence Department, under contract W33-038ac15004 (16351). A description of Tollmien's method adapted to CPC computation follows.

In the region of non-uniform supersonic flow, the stream function $\psi(x,r)$ satisfies a nonlinear, second order, hyperbolic, partial differential equation, in which the coefficients of the second order derivatives are expressible as rational functions of the velocity components $u = q \cos \Theta$, $v = q \sin \Theta$; it is thus possible to compute q(or q^*) and Θ directly without using ψ. The equations of the characteristics of the differential equation are

$$(2_L) \qquad dx = \mu\, dr \;, \qquad \mu = \cot(\Theta + \alpha) \;,$$

$$(2_R) \qquad dr = \lambda\, dx \;, \qquad \lambda = \tan(\Theta - \alpha) \;,$$

where α is the first quadrant angle

$$(3) \qquad \alpha = \sin^{-1} \left\{ [(\gamma + 1) - (\gamma - 1)(q^*)^2]/2(q^*)^2 \right\}^{\frac{1}{2}}$$

At any point in the field, equations (2) define directions making angles $+\alpha$ and $-\alpha$ with the velocity vector; the family of non-intersecting curves representing the solutions of (2_L) are called left characteristics, while the family of non-intersecting curves representing the solutions of (2_R) are called right characteristics. Together they form a network covering the field. Although they cannot be constructed beforehand because of the dependence of μ and λ on q^* and Θ, nevertheless their general behavior can be predicted from the fact that as the flow moves toward the rear and away from the body, Θ decreases from a first quadrant angle to zero, and q^* increases from a little more than 1 to its value in the undisturbed stream. Hence, α decreases to a value depending on M in the free stream, and $\Theta + \alpha$ decreases to zero from an angle

M	1	1.5	2	2.5	3	3.5	4	∞
q^*	1	1.36	1.63	1.82	1.96	2.06	2.13	2.44
α(degrees)	90	42	30	24	19	17	14	0

which may lie in the second quadrant for points near the intersection of the initial curve and the body curve. Moreover, $\theta - \alpha$ is a negative, acute angle which approaches zero toward the rear. Thus, right characteristics slant from the upper left to the lower right, while left ones slant from the lower left to the upper right, except those starting from a point on the body close to the initial curve where $\theta + \alpha > 90°$ – these slant upwards to the left and, if they do not run into the initial curve, they become tangent to a parallel to the r-axis and then slant to the right. The curvilinear quadrilaterals formed by intersecting left and right characteristics increase in size away from the front end of the body. This is partially due to the fact that at points farther away from the initial curve the characteristics have such little curvature that all quantities vary almost linearly and it is accordingly expeditious to drop points so that each mesh becomes almost square. Near the initial and shock curves the meshes are smaller and elongated in the direction of the left characteristics; the actual size of the meshes depends on the distribution of points where data are given on the initial curve.

If μ and λ are known at two points $P_1(x_1, r_1)$ and $P_2(x_2, r_2)$ which occupy such positions that the left characteristic from the former intersects the right characteristic from the latter at a point $P_3(x_3, r_3)$ lying in the interior of the nonuniform flow region to the right of the initial curve, then a first approximation to the coordinates of this point is obtained by replacing the differentials in (2) by differences and solving the system

$$(4_L) \qquad x_3 - x_1 = \mu_1 (r_3 - r_1) \, ,$$

$$(4_R) \qquad r_3 - r_2 = \lambda_2 (x_3 - x_2)$$

simultaneously. The subscripts appended to μ and λ indicate that these coefficients are to be evaluated at P_1 and P_2.

The remaining desired variable q^*, Θ, and s are obtained from the compatibility equations

$$(5_L) \qquad d\Theta = A dq^* - B dr + H ds ,$$

$$(5_R) \qquad d\Theta = -A dq^* + \beta ds - H ds ,$$

where

$$A = \cot \alpha / q^*$$

$$(6) \qquad B = \sin\Theta \sin\alpha / r \sin(\Theta + \alpha) ,$$

$$= \sin\Theta \sin\alpha / r \cos(\Theta - \alpha) ,$$

$$H = \sin\alpha \cos\alpha / \gamma (\gamma - 1) .$$

It is known that (5_L) holds on a left characteristic and (5_R) holds on a right one. The difference approximations for the triad P_1, P_2, P_3 are

$$(7_L) \quad \Theta_3 - \Theta_1 = A_1 (q_3^* - q_1^*) - B_1(r_3 - r_1) + H_1(s_3 - s_1) ,$$

$$(7_R) \quad \Theta_3 - \Theta_2 = -A_2(q_3^* - q_2^*) + \beta_2(x_3 - x_2) - H_2(s_3 - s_2) .$$

These are not sufficient for the evaluation of the three unknowns q_3^*, Θ_3, and s_3; however, substitution of

$$\tau = \ln [\gamma + 1) - (\gamma - 1)(q^*)^2] - \ln 2$$

transforms (5) into

$$(9_L) \qquad d\Theta = -H(\gamma d\tau - ds) - B dr ,$$

$$(9_R) \qquad d\Theta = H(\gamma d\tau - ds) + \beta dx ,$$

with the attendant difference equations

$$(10_L) \quad \Theta_3 - \Theta_1 = -H_1[(\gamma\tau_3 - s_3) - (\gamma\tau_1 - s_1)] - B_1(r_3 - r_1) ,$$

$$(10_R) \quad \Theta_3 - \Theta_2 = H_2[(\gamma\tau_3 - s_3) - (\gamma\tau_2 - s_2)] + \beta_2(x_3 - x_2) ,$$

which can be solved for Θ_3 in terms of previously computed quantities by eliminating $\gamma\tau_3 - s_3$. The variable s is computed as follows: Let

(x_m, r_m) be the intersection point of the streamline through P_3 and the line joining P_1 and P_2; then, since Θ_3 is the direction of the first line,

(11) $$r_3 - r_m = \tan\Theta_3(x_3 - x_m) \ .$$

Also, if it is assumed that s varies linearly on the second line,

(12) $$(x_1 - x_m)/(x_1 - x_2) = (r_1 - r_m)/(r_1 - r_2) = (s_1 - s_m)/(s_1 - s_2) \ .$$

From (11) and (12) it is possible to get s_m, which is the same as s_3, since the entropy remains constant on a streamline. Finally, either of equations (7) yields q_3^*.

In practice the foregoing formulas are reduced to the following five

(13a) $$x_3 = [x_1 + \mu_1(r_2 - r_1 - \lambda_2 x_2)]/[1 - \mu_1 \lambda_2] \ ,$$

(13b) $$r_3 = r_2 + \lambda_2(x_3 - x_2) \ ,$$

(13c) $$\Theta_3 = [H_1\Theta_2 + H_2\Theta_1 + H_1H_2\left\{\gamma(\tau_1 - \tau_2) - (s_1 - s_2)\right\}$$
$$- B_1H_2(r_3 - r_1) + \beta_2 H_1(x_3 - x_2)]/[H_1 + H_2] \ ,$$

(13d) $$s_3 = s_1 + (s_2-s_1)[(x_3-x_1)\tan\Theta_3 - (r_3-r_1)]/[(x_2-x_1)\tan\Theta_3 - (r_2-r_1)] \ ,$$

(13e) $$q_3^* = q_2^* + [\beta_2(x_3-x_2) - H_2(s_3-s_2) - (\Theta_3-\Theta_2)]/A_2 \ ,$$

for the first approximation. A second approximation is obtained by putting these values of x, r, Θ, s, q^* in (2), (3), and (6) to calculate μ_3, λ_3, α_3, H_3, B_3, β_3, and A_3, and then recomputing (13) with the coefficients μ_1, λ_2, H_1, H_2, B_1, β_2, A_2 replaced by the averages $(\mu_1 + \mu_3)/2$, $(\lambda_2 + \lambda_3)/2$, $(H_1 + H_3)/2$, $(H_2 + H_3)/2$, etc. This procedure has the effect of giving the coefficients in (4), (7), and (10) mean values which make the difference equations approximate the differential equations more accurately. Reiteration is possible; however, in the examples investigated even the first iteration changed the value by much less than one percent.

A method for starting the computation is now described. The directions $\theta + \alpha$ and $\theta - \alpha$ of characteristics emanating from points on the initial curve are obtained from Table 1 and equation (3) ($\gamma = 1.405$, $q^* = 1.048$). For any pair of successive initial points below the one numbered 11 the backward left characteristic (at an angle $\theta_1 + \alpha_1 \pm \pi$) from the upper point (x_1, r_1) and the forward right characteristic (at an angle $\theta_2 - \alpha_2$) from the lower point (x_2, r_2) intersect at a point (x_3, r_3) lying to the right of the initial line. The pentad $x_3, r_3, q_3^*, \theta_3, s_3$ is calculated by iterating (13). Call the right characteristic from an initial point numbered n, Rn, and the left characteristic from the same point, Ln; also, let the intersection point of Rm and Ln be designated (m,n). Then the first interior point to be calculated is (2,3), obtained as the intersection R2 drawn from 2 and L3 drawn from 3. This is the point marked A in the lower left corner of Fig. 1. Points described subsequently are also marked by a capital letter in the figure. From 3 and 4 the point B(3,4) is obtained, and from 4 and 5, C(4,5), L5 is extended farther down to intersect with R3 in D(3,5). Along L6 the points E(5,6), F(4,6), and G(3,6) are determined; along L7, H(6,7),... I(3,7); along L8, J(7,8),...K(3,8); and along L9, L(8,9),...P(3,9).

It is not practical to continue in this manner since the left characteristics drawn down from points 10 and 11 almost coincide with L9. The data on L9 are used to construct right characteristics leading to the body curve. The simultaneous solution of (4_R) and the equation

$$(14) \qquad r = f(x) \qquad x = x_3 , \qquad r = r_3 ,$$

of the body curve yields the coordinates of the point (x_3, r_3) on the body from information at the interior point (x_2, r_2). The angle is given by

$$(15) \qquad \theta_3 = \tan^{-1} (df/dx)_3 ;$$

s_3 is equal to the value of s at point 1 (the intersection of the body curve and the initial curve) since the body itself is a streamline; and q_3^* is computed from (13e). The approximations are improved by replacing the coefficients λ_2 in (4_R) and β_2, H_2, and A_2 in (13e) by their averages with λ_3, β_3, H_3, and A_3, just as in computing at an interior point. If r = f(x) is not given analytically, it may be fitted by a formula. In either case, if the function is too complicated, a first approximation to (x_3, r_3) can be got by determining the intersection of the right characteristic from (x_2, r_2) with the tangent drawn from a previously computed point on the body. For the value of x thus calculated, r on the body is found. From this body point another tangent is drawn to intersect with the characteristic and the procedure is repeated as often as desired. Two such iterations were sufficient to give results converging in the sixth decimal for the transition curves occurring in the problems considered.

If the point of intersection of R4 with the body curve is called Q(4, 10), then a left characteristic (which can be designed L10 without ambiguity because no left characteristic was drawn through point 10) is started upward by getting R(5,10) from O(5,9) and Q(4,10); R5 is continued from R(5,10) to S(5,11) on the body by the procedure of the last paragraph, and L11 is started. To complete R6, T(6,10) is computed from N(6,9) and R(5,10), U(6,11) from T(6,10) and S(5,11), and V(6,12) (on the body) from U(6,11) (and possibly S(5,11) if the body curve has a complicated equation). To complete R7, W(7,10) is computed from M(7,9) and T(6,10), X(7,11) from W(7,10) and U(6,11), Y(7,12) from X(7,11) and V(6,12), and Z(7,13) (on the body) from Y(7,12) (and possibly V(6,12)). All the remaining interior points are computed in this same systematic manner. Starting with known values on L9, the initial curve (between points 9 and 16), or the shock curve (see next paragraph), it is possible to determine the point (m,n) from (m,n − 1) on the same right characteristic Rm and (m − 1,n) on the preceding right characteristic

R(m − 1). [This uniform numbering scheme holds provided the left characteristics extending upwards from 12,13,...17 are called L9, L8,...L4.] When the quantities x, r, σ, Θ, s at any point are computed on the CPC, they are held in the memory until values are obtained at the next point and also are punched on a single card. A set of these cards for all the points on a certain right characteristic is combined with the instruction deck containing formulas (13) to calculate points on the next right characteristic. The point being computed is (x_3, r_3), the one on the same characteristic is (x_2, r_2), the one on the preceding characteristic is (x_1, r_1). The instructions are to take $x_1, \ldots s_1$ from the card, take $x_2, \ldots s_2$ from the memory, compute the appropriate coefficients, evaluate the formulas in (13), iterate once, put the results $x_3, \ldots s_3$ in the memory and punch them on a new card. The process requires three minutes for each point and is repeated for each point on Rm without intervention of the operator until the body is reached. Here the instruction deck is replaced by another which determines the point on the body as described in the last paragraph. If f(x) is given by more than one analytic expression, the appropriate portion of the deck must be altered accordingly. Each such point (including two iterations) requires five minutes to compute.

As the flow crosses the shock surface, the uniform velocity and entropy suffer an abrupt change. The following formulas are valid immediately behind the shock:

(16) $$(1 + \tan^2\Theta)(u^*)^3 - [2M^* + (1 + \tan^2\Theta)(M^*)^{-1} + 2M^*\tan^2\Theta/(\gamma + 1)](u^*)^2 + [M^*)^2 + 2]\, u^* - M^* = 0 \ ,$$

(17) $$v^* = u^* \tan\Theta \ ,$$

(18) $$q^* = [u^*)^2 + (v^*)^2]^{\frac{1}{2}} = u^*/\cos\Theta \ ,$$

(19) $$\sigma = \tan^{-1}[(M^* - u^*)/v^*] \ , \quad 0 < \sigma < \pi/2,$$

(20) $$s = -(\gamma + 1)\ln(\gamma + 1) - 2\gamma\ln\bar{M} + \gamma\ln[(\gamma - 1)\bar{M}^2 + 2] + \ln[2\gamma\bar{M}^2 - (\gamma - 1)] ,$$

where $\bar{M} = M \sin\sigma$, M is the Mach number of the undisturbed stream, σ is the angle between the shock curve and the positive x-axis, $u^* = q^* \cos\Theta$, $v^* = q^* \sin\Theta$, and M^* is the constant value of q^* in the undisturbed stream (see (1)). Six quantities (x,r,q,Θ,s,σ) are known at point 18, the intersection of the shock curve and the initial curve. A first approximation to the next point on the shock curve is obtained from the intersection of its tangent at 18 with L4, the left characteristic running up from 17. Better approximations are obtained by the iteration procedure described subsequently. The right characteristic emanating from this point is called R17 (since no right characteristic was drawn through 17) and is extended by getting B'(17,5) from A'(17,4)(the point on the shock curve) and 16, and continuing to the body. The next shock point D'(18,5) is computed as the intersection of the shock curve and L5; R18 is started by getting E'(18,6) from D'(18,5) and C'(17,6). More shock points and net points are calculated in this way until the velocity distribution is known over as much of the body surface as desired.

Let (x_3,r_3) be the shock point being computed, (x_2,r_2) the previously calculated one, and (x_1,r_1) the first interior point of the right characteristic through (x_2,r_2) (or an initial point). The intersection of the left characteristic from (x_1,r_1) with the tangent

(21) $$r_3 - r_2 = \tan\sigma_2(x_3 - x_2)$$

to the shock curve at (x_2,r_2) is given by

(22) $$x_3 = [x_1 + \mu_1)r_2 - r_1 - x_2 \tan\sigma_2)]/[1 - \mu_1 \tan\sigma_2] ,$$

(23) $$r_3 = r_1 + (x_3 - x_1)/\mu_1 \qquad \text{(from } (4_L)) .$$

If q^* and s are assumed constant on the shock curve segment, then (7_L) yields Θ_3, which can be used in (16),(17), and (18) to calculate a second approximation of q_3^*. Finally, σ_3 and an improved s_3 are given by (19) and (20). These values of x_3, r_3, θ_3, q_3^*, σ_3, and s_3 are used to calculate the coefficients μ_3, etc.; then formulas (22), (23), (7_L), (16), (17), (18), (19), and (20) are used with μ_1 replaced by $(\mu_1 + \mu_3)/2$, etc., to recompute the hexad. About six such iterations were necessary to produce results good to five significant figures. Most of the time of computing for a shock point - twenty minutes in all - is consumed in finding the root of the cubic (16). As a first approximation to this root, which is the intermediate one of three real roots, the value of u* at the previous shock point is used; five iterations by Newton's method were programmed in the instructions.

Data on the initial curve had been determined empirically except at the shock curve where the results of theoretical investigation were used. These values must be physically correct to about five per cent and must be compatible with the shape of the body. If, after some computation, everything is known on a certain left characteristic L, then L may be treated as a new initial curve. The values on L depend only on the data on the original initial curve and the shape of the body to the left of L. Consequently, in the six cases considered, L was taken as the rightmost left characteristic that intersected the circular arc common to the meridian sections of all the bodies. The complete network was constructed for only one body; for the other five it was different only to the right of L. Calculation time was cut considerably, for instead of the 16 hours required to compute the complete field (including 200 interior points, 15 shock points, and 25 body points, at respective rates of 3, 20, and 5 minutes per point), the field to the right of L was done in only 4 hours, although it did require about 5 hours to change the coding for each body. A glance at the formulas shows that if the Mach number of the undisturbed stream is altered, the coding of shock points only is affected, while if the body curve is changed, the coding of body points only must be altered.

Interpolation of points in the network becomes necessary when (1) a segment between adjacent net points is too curved to be replaced by a straight line, (2) a pair of adjacent characteristics diverge, (3) the left characteristic rising to the shock curve is too long, or (4) the initial data are too widely spaced. It is more desirable to graph the net points as they are computed and just notice the behavior of the characteristics than to attempt to include any selection principle in the coding; however, an all purpose interpolation deck can be prepared and inserted at will by the operator. This deck is made up to interpolate each variable linearly at a point designated $(m, n + \frac{1}{2}p)$ between (m,n) and $(m, n + p)$, $p < 1$. Examples of such points in the problems solved were A"(4,9 1/2), B"(7,10 1/2) and C"(11, 10 1/4). Actually, only five points were given on the initial curve; the eighteen points in Table 1 were determined by smoothing the data and taken as initial points. Since the left characteristic from point 17 intersected the shock curve at a considerable distance from 17, the point 17 1/2 was interpolated linearly on the initial. The left characteristic from 17 1/2 was called L3 1/2 and intersected the shock curve at D"(16 1/2, 3 1/2), whence R16 1/2 was started. The point F"(16 1/2, 3 3/4) was interpolated between D"(16 1/2, 3 1/2) and E"(16 1/2,4) and a left characteristic run up to the shock curve at G"(16 3/4, 3 3/4). This cumbrous notation may be simplified by relabeling the characteristics L3 1/2, L3, R16 1/2, R16 3/4, R17, and R18 as L3, L2, R17, R18, R19, and R20.

The procedure used for calculating at most of the interior points in the six examples may be used for all of the interior points if the initial curve has such a position that left characteristics run upward from it and right ones run downward from it. Otherwise, as in the examples, a different scheme has to be used in the neighborhood of certain segments of the initial curve.

Although the bodies considered here all have rounded noses, the method is not restricted to such forms but may be used for axially symmetric bodies of more general shape.

The details of computation on the CPC were worked out by E. C. Yowell and carried out by P. Bremer. This report was prepared by R. R. Reynolds.

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