NATIONAL BUREAU OF STANDARDS REPORT

NBS Project		NBS Report
1103-21-5119	16 April 1952	1598

PROGRESS REPORT FOR JANUARY - MARCH 1952

on

APPLICATION OF THE THEORY OF STOCHASTIC PROCESSES TO THE STUDY OF TRAJECTORIES

(NBS Project 1103-21-5119)

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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I. SUMMARY

This report contains a summary of the background of the above project and a statement of the problems involved. The work done during the quarter is discussed and the results are stated. A technical report entitled "A property of strongly continuous processes" by Eugene Lukacs will be submitted separately.

II. DISCUSSION OF PROBLEM AND OF WORK DONE DURING THE QUARTER.

In this new project an attempt is made to investigate whether mathematical-statistical tools associated with the theory of stochastic processes can be profitably applied to the analysis of trajectory data gathered at the Naval Ordnance Test Station, Inyokern, California. The theory of stochastic processes deals with time-dependent phenomena in which there is a probability relationship berween the state at a given instant and one or more states at preceeding instants. Some of the difficulties encountered in analysing ordnance data can be overcome by considering a trajectory to be a stochastic process. This approach eliminates the difficulties created by the fact that only one observation is available for the position of the missile at each instant.

The first step in this work is the adoption of a stochastic model. As always in such a situation one has to find a compromise between the wish to establish a realistic model and the necessity to use a model which does not lead to a forbiddingly complicated mathematical analysis. The fundamental random process (Wiener process) is a very simple process and methods

for the estimation of its parameters were developed under a previous Bureau project (Elementary Theory of Stochastic Processes, Task 1103-11-1107/49-3). This is the main reason why it seems desirable to attempt to use in this study the fundamental random process as the stochastic model. Clearly, this involves a certain amount of idealization. It is therefore desirable to safeguard against too radical departure from reality by testing first whether the observed trajectory data can be assumed to come from a fundamental random process.

Since no such test is known at present the first task under this project is the development of a test for the hypothesis that a sequence of observations comes from a fundamental random process.

This necessitates a study of the various properties of the fundamental random process in order to see which of these properties should be emphasized in the test to be developed.

The following theorem was derived in this connection mainly by using a result of Khinchine:

Let $y(t)$ be a stochastic process and assume that

(i) $y(t)$ is a process with independent increments.
(ii) $y(t)$ is strongly continuous in the interval $[a, b]$.

Then $y(b) - y(a)$ is normally distributed.

This theorem shows that the normality of the increments follows from a certain continuity property of the process. In view of the physical situation encountered in the study of trajectories a continuity assumption is certainly reasonable. Therefore the test to be developed should not emphasize the normality of the increments; the independence of the increments seems to be the property deserving greater attention.

A technical report containing the proof of this theorem was completed and will be issued at the beginning of the next quarter.

E. Lukacs visited Professor H. B. Mann in February 1952, and discussed with him various aspects of this project.