NATIONAL BUREAU OF STANDARDS REPORT

1154

REPORT FOR FISCAL YEAR 1951

on

APPLICATION OF STATISTICAL THEORY OF EXTREME VALUES

TO GUST-LOAD PROBLEMS

(NACA Project S50-39; NBS Project 1103-21-5106)

NBS

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

U. S. DEPARTMENT OF COMMERCE
Charles Sawyer, Secretary

NATIONAL BUREAU OF STANDARDS
E. U. Condon, Director

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section is engaged in specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside of the back cover of this report.

1. ELECTRICITY. Resistance Measurements. Inductance and Capacitance. Electrical Instruments. Magnetic Measurements. Electrochemistry.

2. OPTICS AND METROLOGY. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Gage.

3. HEAT AND POWER. Temperature Measurements. Thermodynamics. Cryogenics. Engines and Lubrication. Engine Fuels. Combustion.

4. ATOMIC AND RADIATION PHYSICS. Spectroscopy. Radiometry. Mass Spectrometry. Physical Electronics. Electron Physics. Atomic Physics. Neutron Measurements. Nuclear Physics. Radioactivity. X-Rays. Betatron. Nucleonic Instrumentation. Radiological Equipment. Atomic Energy Commission Instruments Branch.

5. CHEMISTRY. Organic Coatings. Surrace Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Gas Chemistry. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

6. MECHANICS. Sound. Mechanical Instruments. Aerodynamics. Engineering Mechanics. Hydraulics. Mass. Capacity, Density, and Fluid Meters.

7. ORGANIC AND FIBROUS MATERIALS. Rubber. Textiles. Paper. Leather. Testing and Specifications. Organic Plastics. Dental Research.

8. METALLURGY. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion.

9. MINERAL PRODUCTS. Porcelain and Pottery. Glass. Refractories. Enameled Metals. Building Stone. Concreting Materials. Constitution and Microstructure. Chemistry of Mineral Products.

10. BUILDING TECHNOLOGY. Structural Engineering. Fire Protection. Heating and Air Conditioning. Exterior and Interior Coverings. Codes and Specifications.

11. APPLIED MATHEMATICS. Numerical Analysis. Computation. Statistical Engineering. Machine Development.

12. ELECTRONICS. Engineering Electronics. Electron Tubes. Electronic Computers.

13. ORDNANCE DEVELOPMENT. Mechanical Research and Development. Electromechanical Fuzes. Technical Services. Missile Fuzing Research. Missile Fuzing Development. Projectile Fuzes. Ordnance Components. Ordnance Tests. Ordnance Research.

14. RADIO PROPAGATION. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Frequency Utilization Research. Tropospheric Propagation Research. High Frequency Standards. Microwave Standards.

15. MISSILE DEVELOPMENT. Missile Engineering. Missile Dynamics. Missile Intelligence. Missile Instrumentation. Technical Services.

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SUMMARY

Work carried out under the above project[1] for the period July 1, 1950 to March 31, 1951 was covered by Progress Report submitted under date of May 28, 1951 (reference 1). The present report includes work done during the final quarter of FY 1951, April 1 to June 30, which terminates the contract year. This work was largely in connection with a paper entitled "On certain estimators based on large samples of extremes," by Julius Lieblein, submitted separately.

ACTIVITIES FROM JULY 1, 1950 TO MARCH 31, 1951

As stated above, activities during this period were described in the previous progress report (reference 1). In recapitulation, they were, briefly: [2]

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[1] Covered by Contract W-4712 (S50-39) — Services for conducting a special investigation of the application of the Statistical Theory of Extreme Values to gust-load problems — between the National Advisory Committee for Aeronautics and the National Bureau of Standards.

[2] Item (1) was sponsored in part by the NACA and in part by the NBS. Item (2) was sponsored in part by the Office of Air Research, Air Materiel Command, USAF, and in part by the NACA. Publication in each case is being undertaken by the NBS.

(1) "Lectures on the Statistical Theory of Extreme Values and their Practical Applications," by E. J. Gumbel, prepared for publication in the Applied Mathematics Series of the NBS (reference 2).

(2) "Probability Tables for the Analysis of Extreme-Value Data," with an introduction by E. J. Gumbel, accepted for publication in the Applied Mathematics Series of the NBS (reference 3).

(3) "Table of the First Moment of Ranked Extremes," for sample sizes n = 1(1)10(5)60(10)100 and ranks 1(1)25 (in descending order), computed in preliminary form for use in studying B. F. Kimball's graphical techniques. (Submitted as a separate report.)

(4) Generalized Poisson theory applied to analysis of VGH records, and found to be unsuitable.

(5) Various estimators of the parameters of the extreme-value distribution studied for bias and efficiency.

As stated in the previous report, the aim of the NACA-sponsored research at the National Bureau of Standards has been the improved application of the statistical theory of extreme values to gust-load problems. At the Bureau this research has been carried out in the Statistical Engineering Laboratory (Section 11.3) of the Bureau's National Applied Mathematics Laboratories (Division 11, NBS) as Project 1103-21-5106, with Mr. Julius Lieblein as Project Leader. The previous report also discussed the background and purpose of the project, and in particular, the relation of the lectures by Gumbel and the extreme-value probability tables (items (1) and (2) above, respectively) to the aims of the project.

WORK FROM APRIL 1, 1951 TO JUNE 30, 1951

The final quarter of the contract year was devoted largely to preparation of a report under item (5) above, and also included some work under items (1), (2), (3), as follows:

(1) "Lectures on the Statistical Theory of Extreme Values and their Practical Applications," by E. J. Gumbel. The manuscript was reviewed by a technical reader for the National Bureau of Standards Research Editorial Committee. Discussions with the reader were held and a start was made toward incorporating the numerous suggestions into the manuscript.

(2) "Probability Tables for the Analysis of Extreme-Value Data." The manuscript was reviewed by a technical reader, several minor changes were made, and the manuscript was accepted for publication in the Applied Mathematics Series of the NBS.

(3) "Table of the First Moment of Ranked Extremes." Several entries were corrected and an explanatory section added to make the table suitable for submitting as a separate report.

(5) Report on large-sample estimators. As above indicated, the chief outcome of the work of the final quarter is a report entitled "On Certain Estimators Based on Large Samples of Extremes," by Julius Lieblein, which is submitted separately. This paper provides several useful and efficient methods for analyzing large samples of extreme-value data that have advantages over certain methods suggested by other workers in the

field and previously used in an NACA research report (reference 4). Certain technical aspects of the forthcoming paper are indicated in the abstract comprising the Appendix to the present report. It is planned to present this paper at the Eastern Sectional Meeting of the Institute of Mathematical Statistics in Washington, D. C., October 26-27, 1951, subject to the necessary clearance by the NACA.

INDICATED FURTHER RESEARCH

The above-mentioned progress report for the first three quarters of FY 1951 was accompanied by a discussion of proposals for further research (reference 5) that concerned five items warranting attention if the project is continued through a second year. These are briefly, (i) estimators, (ii) prediction, (iii) graphical methods, (iv) testing a single sample, and (v) testing two samples.

Of the above items, one (estimators) depends upon the extension of previous work under the project. The work on estimators is intended to concentrate on methods of analyzing very small samples of data. The availability of item (3) previously mentioned, "Table of the First Moment of Ranked Extremes," makes it possible for the first time to find methods applicable to very small samples which are unbiased, that is, the methods will give answers which will tend to cluster about

the correct value. But to find among these the most efficient methods, i.e. those whose "clustering" is the most concentrated about the true value, so that fewer observations are necessary for the same precision, it is necessary to have available tables of the second moments (variances and covariances) of ranked extremes in small samples. Computation of these tables is expected to absorb about one-third of the estimated total project cost of $15,000.

REFERENCES

1. National Bureau of Standards: Progress Report on Application of Theory of Extreme Values to Gust-Load Problems, for the period July 1, 1950 - March 31, 1951 (NACA Project S50-39; NBS Project 1103-21-5106). Enclosure to letter from E. U. Condon to H. L. Dryden dated 28 May 1951.

2. Gumbel, E. J.: Lectures on the Statistical Theory of Extreme Values and their Practical Applications. To be submitted for publication in the Applied Mathematics Series of the NBS.

3. Probability Tables for the Analysis of Extreme-Value Data. To be published in the Applied Mathematics Series of the NBS.

4. Press, Harry: The Application of the Statistical Theory of Extreme Values to Gust-Load Problems. NACA TN 1926, November 1949, pp. 10-11.

5. National Bureau of Standards: Proposal for Renewal of NACA Contract with NBS on Application of Extreme Values to Gust-Load Problems. Enclosure to letter from E. U. Condon to H. L. Dryden dated 28 May 1951.

APPENDIX

(Abstract of paper for presentation at the Eastern Sectional Meeting of the Institute of Mathematical Statistics in Washington, D.C., October 26-27, 1951.)

On Certain Estimators Based on Large Samples of Extremes.
Julius Lieblein, Statistical Engineering Laboratory, National Bureau of Standards.

E. J. Gumbel and B. F. Kimball have given estimators for the parameters of the asymptotic distribution of largest values, Prob $\{X \leq x\} = \exp(-e^{-\alpha(x-u)})$, which are especially adapted to the analysis of data in large samples. The present paper applies the theory of order statistics to the problem of seeking more efficient estimators which are at the same time simpler to compute. Several large-sample estimators are found which, with one exception, appear to have greater efficiency than those derived from the methods of Gumbel and Kimball, yet require much less effort in computation. If punch cards are used the work can be handled by a mechanical sorter which ranks the observations in order of size and then selects a small number of them with predetermined ranks.

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The National Bureau of Standards is the principal agency of the Federal Government for fundamental and applied research in physics, mathematics, chemistry, and engineering. Its activities range from the determination of physical constants and properties of materials, the development and maintenance of the national standards of measurement in the physical sciences, and the development of methods and instruments of measurement, to the development of special devices for the military and civilian agencies of the Government. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various scientific and technical advisory services. A major portion of the NBS work is performed for other government agencies, particularly the Department of Defense and the Atomic Energy Commission. The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. The scope of activities is suggested in the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: the Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: the Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards (75 cents). Information on calibration services and fees can be found in NBS Circular 483, Testing by the National Bureau of Standards (25 cents). Both are available from the Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.


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