NATIONAL BUREAU OF STANDARDS REPORT

10 833

FIRE SERVICE LOCATION–ALLOCATION MODELS

Prepared for

The Fire Research Program
National Bureau of Standards

U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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FIRE SERVICE LOCATION-ALLOCATION MODELS

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IMPORTANT NOTICE

NBS

U.S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

ABSTRACT

This paper compiles the various types of location-allocation models which analyze the impact of varying the number and location of fire stations. The assumptions of each model, the relationships between models, and possible heuristics and algorithms are discussed. In addition, a methodology of spatial concepts analogous to those used in transportation planning is presented.

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1.0 INTRODUCTION

1.1 Purpose of Report

The purpose of this report is to provide such technical people as operations researchers, mathematicians, and planners with a perspective on analytical approaches to the fire station location problem.

The objective is to present generic classes of location-allocation models which use different objective functions as indicators of the level of service provided. This paper does not provide a detailed state-of-the-art review on location theory, since there are adequate reviews in the literature. In particular, the paper by Revelle, Marks, and Liebman[1/] presents a general survey of the literature through 1969. To detail the various facets of fire service activities goes beyond the scope of this report, but the interested reader can gain some appreciation of the extensive and involved functions performed by fire departments by consulting their training manuals.[2/]

1/ C. Revelle, D. Marks, and J. Liebman, "An Analysis of Private and Public Sector Location Models," Management Science, Vol. 16, No. 11, 1970.

2/ See, for example:

a. J. F. Casey, (Ed.), The Fire Chief's Handbook, Third Edition, New York: The Reuben H. Donnelley Corp, 1968.

b. L. W. Erven, Fire Company Apparatus and Procedures, Beverly Hills, California: Glencoe Press, 1969.

1.2 The Problem

City governments are being faced with new and greater demands for public services at the same time as the cost of providing such services is steadily increasing. There are some indications, as noted in Table 1, that the fire service function is appropriating an increasingly larger portion of the overall city budget. Consequently, there is a pressing need for tools to analyze the delivery of fire services and find means to increase their efficiency and effectiveness.

As cities grew from small communities, new facilities for fire services were gradually added at heavy demand points. Consequently, the fire stations often are not optimally located for the city as it currently exists. Similarly, as different parts of the city decay and suffer a negative growth rate, there is a need to re-examine the locational pattern of facilities.

Some method, more objective than the widely accepted usage of insurance ratings, by which a city can assess the adequacy of its level of fire protection is urgently needed. A necessary part of an objective approach would be the consideration of where facilities should be located and a measurement of the level of service provided, as a function of the number of the facilities and their current locations. The models which address the above problem are called location-allocation models. The difficulties of location-allocation analysis are dual. First, what is meant by "level of service" or "effectiveness" of the fire department must be defined. Second, analytical means must be found to assess the benefits of different locational patterns.

Table 1

Fire Department Budgets[3/]

1967, 1970

(Selected Cities of 100,000 or over Population)

City	1967 F.D. Budget % of General Revenue	1970 F.D. Budget % of General Revenue	1970 F.D. Expenditure (in thousands)	1970 Salaries % of F.D. Budget
Chicago, Ill.	10	12	$ 68,478	98
Los Angeles, Cal.	12	12	62,348	98
New York City, N.Y.	3	4	233,296	100
Atlanta, Ga.	8	10	8,544	89
Pittsburgh, Pa.	10	13	11,181	98
St. Louis, Mo.	8	9	12,211	98
Washington, D. C.	3	4	22,683	94
Cincinnati, Ohio	5	5	11,215	96
Long Beach, Cal.	7	10	8,137	96
Miami, Fla.	13	19	7,619	100
Portland, Ore.	16	14	8,857	95
San Jose, Cal.	10	12	6,855	87
Wichita, Kan.	12	12	4,445	91
Baton Rouge, La.	12	10	3,246	89
Columbia, S.C.	15	18	1,391	94
Lansing, Mich.	27	13	3,325	91
Montgomery, Ala.	21	19	2,503	96
Richmond, Va.	5	7	5,620	77
Wichita Falls, Tex.	7	8	962	92

[3/] International City Management Association, The Municipal Year Book, Washington, D. C., 1967, 1971.

1.3 Report Content

This report consists of six sections including the introduction. Section 2 gives brief summaries of three historic location-allocation studies. These studies demonstrate that the difficult problem of how many fire stations a city should have and where they should be located has received attention from the OR profession for a long time.

Section 3 determines what is to be located, (facilities, companies, equipment, or men), and provides a terminology for spatial characteristics to be used in the models. Section 4 describes several different types of location models; Section 5 discusses methods of providing solutions to these models. Section 6 summarizes the contributions of this paper.

2.0 MAJOR LOCATION-ALLOCATION STUDIES

This section describes three location-allocation studies which represent milestones in the application of location theory to the fire services. The Valinsky study was done in the early 1950's for the city of New York. It used the A.I.A.[4/] schedule as a constraint to determine the number and location of fire stations, supplemented by: (a) a crude hazard analysis, (b) availability analysis, and (c) a historical analysis of extreme situations of resource utilization to determine the risk of all units being used simultaneously. Jane Hogg's contribution was the use of an analytical model based upon network travel times and weighted demand points to locate stations. The Berlin-Santone study developed methodologies for (a) evaluating different location patterns, (b) hazard assessment, and (c) service districting.

2.1 Valinsky's Study

In 1951 David Valinsky was assigned to New York City's Mayor's Committee on Management Survey to examine the efficiency of fire company locations and determine whether or not statistical methods could usefully be applied to the entire problem of location.

[4/] The American Insurance Association Standard Grading Schedule for determining the level of fire protection in a city. See: "Standard Schedule for Grading Cities and Towns of the United States with Reference to their Fire Defenses and Physical Conditions," National Board of Fire Underwriters, New York, Chicago, San Francisco, 1956, with 1963 and 1964 Amendments.

The Standard Schedule is also reproduced in the 1969 Municipal Yearbook, op. cit., page 1.

The study consisted of four major tasks. First, Valinsky referenced the A.I.A. Standard Grading specifications for the location of apparatus and constructed an initial set of locations for the distribution of engine and ladder companies in New York that met the A.I.A. specifications. Since these standards were based primarily on the size of the areas to be protected with little consideration of such characteristics as population density and high-rise structures, the general A.I.A. specifications had to be reconsidered in the light of local conditions of special concern from a fire protection point of view.

The second task in the study was an analysis of burnable material concerning the physical characteristics of structures and the special fire hazards of their occupancies. A block-by-block study of the entire area was required in order to determine the special characteristics which differentiate areas of similar geographic proportions as to their inherent risk potential. This phase of the study demonstrated that the distribution of companies as initially determined by the A.I.A. specifications was not totally adequate to meet the requirements of certain high hazard areas. Additional companies were located in such areas.

The third part of the study statistically analyzed the work load of the companies in order to determine the expected availability of the companies initially placed in the first two parts of the study. The geographic distributions of work performed were determined by studying the average number of runs, sizeable fires, and amount of time spent at work per year. Next, the amount of time at work and out-of-quarters was calculated, and the distribution of fire incidence by time and borough of

New York was determined. Averages and absolute ranges of indicators of work performance such as the relation of unnecessary* alarms to working alarms by area were determined, as was the time consumed in runs. The study resulted in more companies being added to the ones located in the first two phases.

The final phase of the study was an analysis of the problems of mass response. In particular, consideration was given to probabilistic questions of the form "What might happen?". These questions led to studying:

1. The distribution of multiple alarms
2. Fire losses
3. Fire fatalities
4. Probabilities of extreme fire situation.

The study determined the density of alarms area-by-area and hour-by-hour, and evaluated the historical and anticipated incidence of multiple alarms with high loss potential. The probabilities inherent in these high loss situations were computed and resulted in the repositioning of some companies.

2.2 Jane Hogg's Study

In 1958, the Home Office, London, England, received a request from Glasgow, Scotland, to help plan fire station locations. Since central Glasgow was to be virtually flattened and rebuilt by the 1980's, nearly

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*"Unnecessary alarms" include malicious false alarms, good intent alarms, and accidental alarms.

all of the stations were to be relocated. Therefore, there was a wide choice of possible station locations.

Jane Hogg, of the Scientific Advisor's Branch, Home Office, chose the objective function of minimizing the total-journey-time of all the engines traveling to fires, including both first responding engines and any reinforcements.

Jane Hogg points out[5/] that the response time to a fire depends on at least three factors:

1. Where the fire occurs; i.e., the pattern of fire incidence
2. Where the nearest station is located
3. The type of road network, and possible travel speed along the network.

The study first designated a large number of possible (initial) sites for stations, determined in such a way that political constraints were satisfied. The sites were fairly evenly scattered over Glasgow. Hogg chose to locate only one engine per location in order to obtain a lower total-journey-time.

[5/] Jane M. Hogg, "The Deployment of the Fire Services in Glasgow in 1980," Needs of the Fire Services, Proceedings of a Symposium Oct. 1968, The Committee on Fire Research, Div. of Eng., National Research Council, National Academy of Sciences, Washington, D. C., (1969).

Jane M. Hogg, "The Siting of Fire Stations," Operational Research Quarterly, Vol. 19, No. 3, 1968, pp. 275-287.

Jane M. Hogg, "A Distribution Model for an Emergency Service: Masters of Science Thesis," University of London, Imperial College of Science and Technology, Sept. 1970.

Next, she created a map depicting the projected incidence pattern in 1980, obtained through a regression analysis associating fire incidence with residential and working class populations. The map of the city was then divided into subareas composed of several one square kilometer cells of the map grid. She formed as many subareas as were feasible, yet large enough so that there would be a sufficient number of fires to enable the frequency distribution of engines called for service to be estimated.

Topographical features such as rivers, canals, and railway lines were considered in the creation of the subareas wherever they appeared to be barriers to passage. These boundary lines represented lines of low fire incidence. Furthermore, each of the possible station sites was identified with a particular subarea.

Once the subareas had been delineated, the center of gravity of calls for each of the subareas was determined, and the travel (or journey) time between the subareas and all possible station locations was computed.

Jane Hogg determined those locations from the whole set of possible sites which were best for the total number of engines (in this case 41). She then eliminated the one site which increased the total response time by the least amount. There were now 40 engines. This procedure was repeated. The previously rejected site (number 40) was compared with each of the retained sites. If an exchange would minimize the total-journey-time, then one was made. The same analysis was repeated for station number 39 and the procedure continued until no sites were rejected.

2.3 East Lansing Study

Early in 1968, the International City Managers' Association, the American Society of Planning Officials, the Fels Institute at the University of Pennsylvania, and the Technical Analysis Division of the National Bureau of Standards initiated a Housing and Urban Development Department sponsored project to consider the applicability of systems analysis to the resolution of urban problems. The objective of the project was to demonstrate how city staffs, given adequate technical assistance and guidance, could use the methods of systems analysis to solve their particular problems. A study was conducted in East Lansing, Michigan, to show how systems analysis could be applied to solving planning problems in fire departments. In particular, a computer model was developed by the Technical Analysis Division and the city's staff[6/] as a tool for the city to use in planning the number and locations of fire stations.

The city staff and NBS analysts agreed that response time was an important factor to consider in planning the location of fire stations. It was determined that response time could be reduced both by increasing the number of fire stations and by strategically locating individual fire stations within their districts of responsibility.

The model, as developed, did not rigorously determine the number of fire stations required; however, it provided a means to evaluate the effecttiveness of a particular configuration of fire district boundaries. The

[6/] Louis Santone and Geoffrey Berlin, "Location of Fire Stations," Systems Analysis for Social Problems, Washington Operations Research Council, 1970, pp. 81-91.

effort was essentially a districting procedure,[7/] rather than strictly a location procedure. The model outputs give information helpful in determining boundary changes which might lead to a better balanced coverage by the fire stations.

The city of East Lansing was described by a highway network consisting of nodes and links, and by an indexed listing of all structures. Each structure was associated with one of the nodes of the network and represented a potential source of demand. The candidate fire station locations to be evaluated were located at a subset of the network nodes.

The study team developed an approach to explicitly rank alternative fire station locations. Once the specific locations had been chosen, the model would evaluate the objective function and determine service districts. The objective function was based on a combination of travel times from the proposed fire station locations to the node of demand and the demand for fire protection at these nodes.

The travel times were estimated on the basis of shortest path travel throughout the street network. The demand for service at each node was defined as the sum of the demands at each individual structure assigned to a node. The demand for service at each structure was defined as the product

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[7/] The methodology of districting has been used in political nonpartisan voter redistricting. See, for example:

a. S. W. Hess, J. B. Weaver, H. J. Siegfeldt, J. N. Whelan and P. A. Zitlan, "Nonpartisan Political Redistricting by Computer," Operations Research, Vol. 13, No. 6, 1965, pp. 998-1006.

b. Crond, Inc., REDIST, Version 3.3, Program Description and User Manual, National Municipal League, New York, N.Y., 1967.

of the probability that a fire would occur in that structure during a given interval of time (for example, a year), and the expected loss that would result from the fire. The probability of a fire occurring was estimated from fire history data and a regression model that included such variables as age, size, and construction type of buildings. The number of people at risk and the value of the building and contents was implicit in these variables. A similar approach was used to estimate the expected losses. Since the available data concerning losses to property referred to losses that occurred after a certain response time, travel time from the known fire station was incorporated as a variable in the function predicting the amount of loss.

The project team hypothesized a linear form for the demand for service at individual buildings, but made no attempt to define expected damage. Instead, the function was intended to reflect the relative importance of providing rapid response to a fire in a particular building. The variables in the linear function were determined by members of the city staff and analysts from the National Bureau of Standards. The values of the coefficients were determined, at a two-day session attended by city and fire department personnel, by ranking the importance of protecting typical structures such as schools, churches, and single family dwellings on a value scale. A set of linear equations was formulated relating the average judgement value to the characteristics of each structure.

3.0 LOCATIONAL CONCEPTS

In order to develop location-allocation models, it is necessary to discuss what is to be located and what spatial characteristics need to be considered.

3.1 Concept of Unit

Some writers assume that it is the fire station that must be located[8/], some assume that it is the fire company, and still others the engine companies.[9/] These may be entirely different problems to fire departments. One fire station, for instance, may contain several pumper and ladder companies.

Using a functional breakdown of fire department services, the following three categories of basic units are defined:

1. Fire Suppression Unit (FSU) - That unit of apparatus, usually called an engine or pumper, complete with assigned manpower, however configured (i.e., numbers of men assigned), in a particular jurisdictional area, whose primary function is the control and extinguishment of fires.

[8/] L. C. Santone, G. Berlin, Location of Fire Stations, Presented at WORC Symposium on "Systems Analysis for Social Problems," May 26-28, 1969. Also see National Bureau of Standards Report #10 093.

[9/] D. Valinsky, "A Determination of the Optimum Location of Fire Fighting Units In New York City," Operations Research, Vol. 3, 1955, pp. 494-512.

E. Nilsson, J. Swartz, "Applications of Systems Analysis to the Alexandria, Va. Fire Department," NBS Report 10 454, January, 1972.

2. Fire Rescue Unit (FRU) - That unit of apparatus, usually called a ladder or a truck, whose primary function is the rescue of persons from sites above the first floor level.

3. Special Support Unit (SSU) - That unit of personnel or special apparatus and equipment used to support the FSU and FRU in conducting their missions.

Pumper companies, however, often conduct salvage, overhaul, and rescue activities in addition to fire suppression. Similarly, ladder companies often do ventilation, salvage, and overhaul.[10/] The motivation for formulating the definitions was to abstract the basic functional units from the various particular apparatus and manpower configurations.

3.2 Concept of Jurisdiction

It is assumed that a fire department serves a bounded geographical jurisdiction made up of contiguous subareas separated by natural or man-made boundaries. A jurisdiction is defined as a bounded geographic area that may or may not be internally separated by such things as mountain ranges, rivers, or railroad tracks.

[10/] Ventilation, salvage, overhaul, and rescue are technical fire department terminology. Ventilation is the planned and systematic removal of smoke, gases, and heat from a structure. Salvage refers to the covering or removal of goods which may be damaged by fire or water. Overhaul refers to the practice of searching for any sparks, embers, or fire that may remain in a building, or in any structure, place, or thing that may have been subject to a fire. Rescue refers to extracting and caring for persons trapped and/or injured in structures, vehicles, traffic accidents, train wrecks, airplane crashes, floods, wind storms, and earthquakes.

Natural barriers may divide a geo-political region into areas which the fire department views as unconnected. To illustrate this point, a hypothetical city and county will be described. Figure 1 shows Phoenix County (hypothetical) divided by a river and a mountain range. Frequent flooding in the spring and summer prevents fire apparatus from using the bridges often enough that the fire department will not consider an engine company on one side of the river as offering protection to the other side. The mountain range cannot be crossed in less than fifteen minutes. Therefore, the two sides of the river may be considered as disconnected for the purpose of locational analysis, and a separate location study performed for each side of the river. Finally, suppose the region of the county below the mountain range is essentially divided into two areas because of a lack of connecting roads. Each of the above considerations makes it either necessary (as in the case of the Smoky River), or desirable (from the computational point of view), to analyze each of these areas separately. The geographical divisions described above might be appropriate in rural county jurisdictions which do not have mutual aid agreements with surrounding jurisdictions.

A region can also be divided for analysis into areas with qualitatively different fire problems. Figure 2 illustrates the situation in Sparks City (hypothetical). The downtown part of the city contains many high-rise buildings which necessitate ladders and rescue equipment, as well as a large capacity for water delivery.

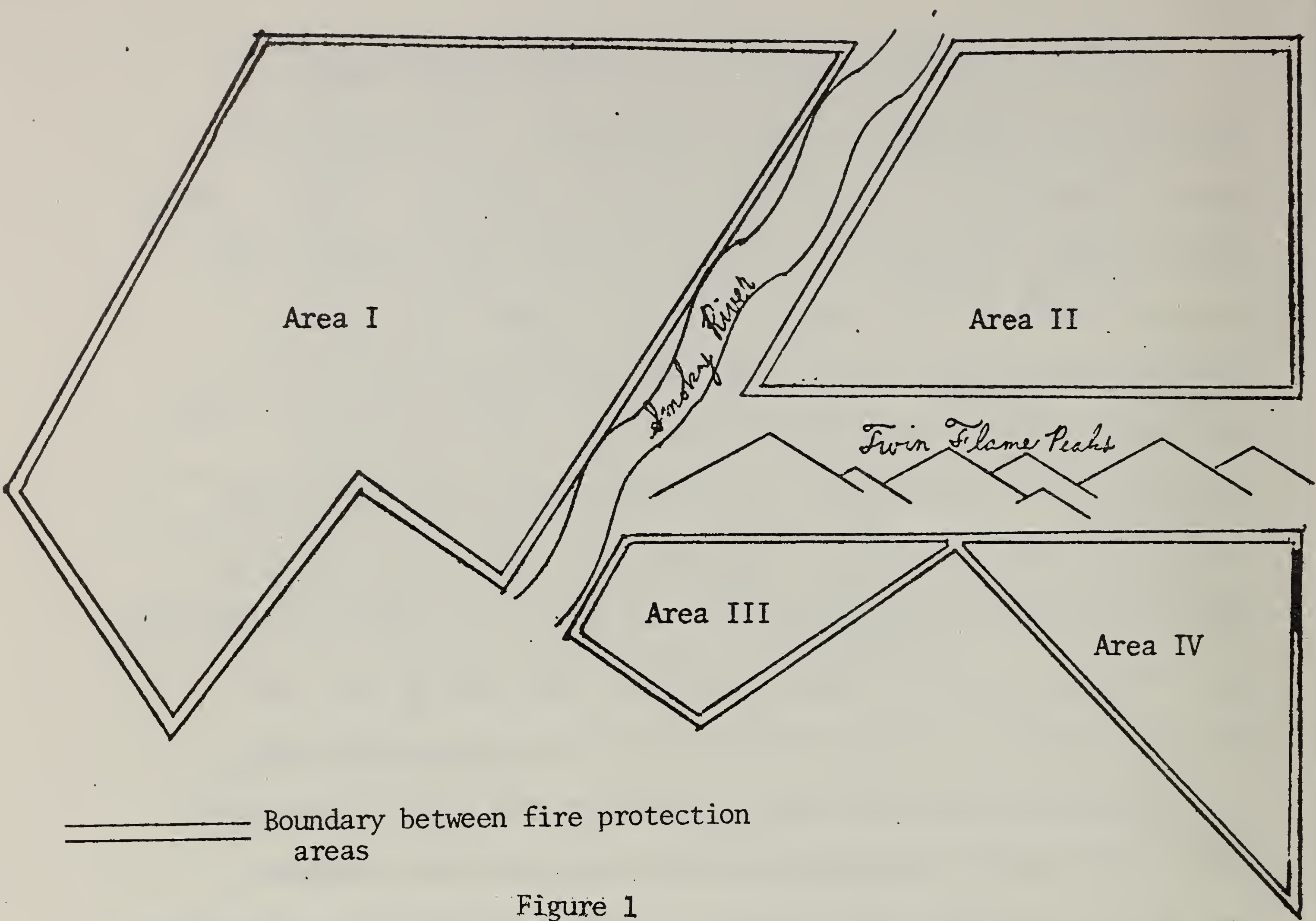


Figure 1

Phoenix County

Division into Areas on the Basis of Political and Natural Boundaries

Without Mutual Aid

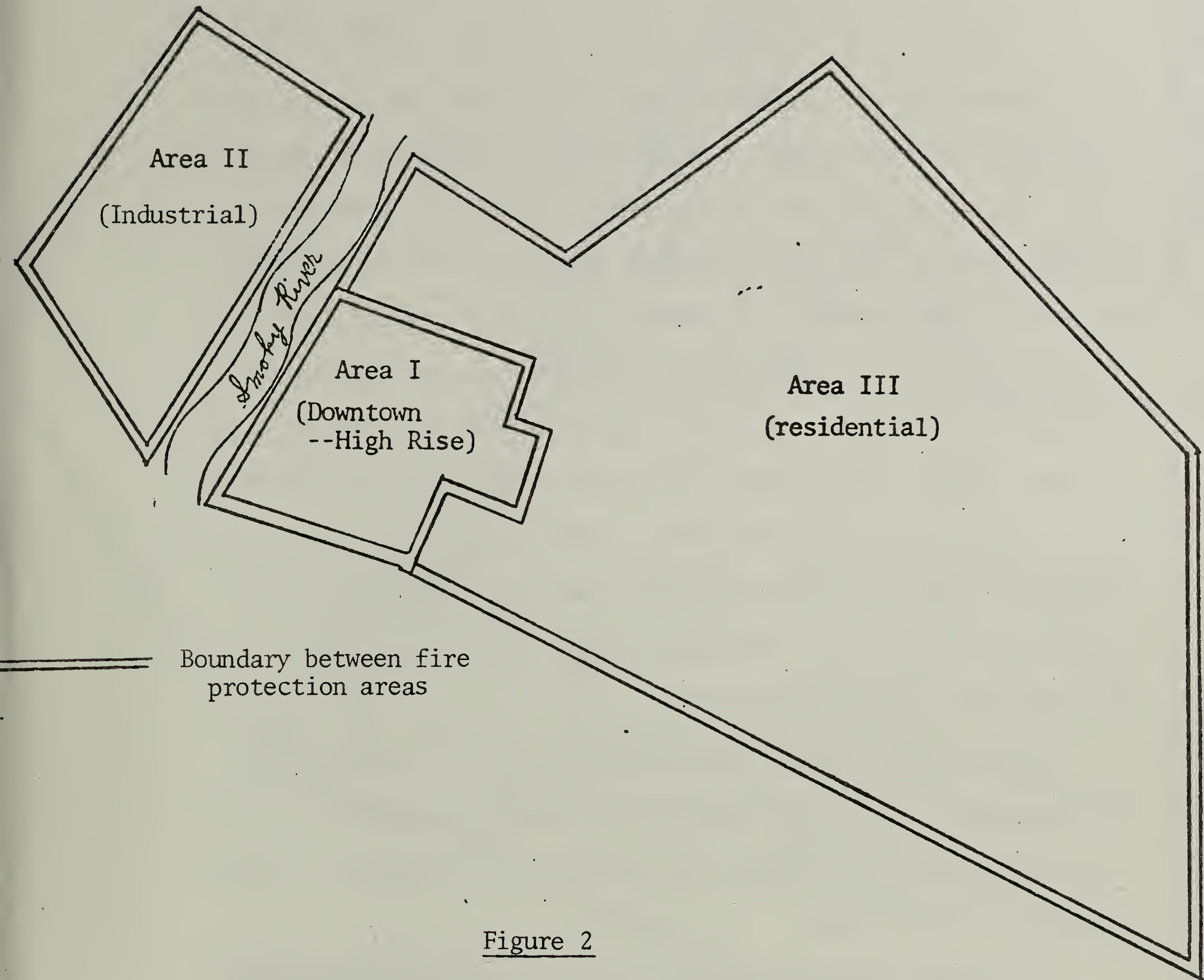


Figure 2

Sparks City

Division into Areas on the Basis of Qualitatively Distinct Firefighting Problems
Without Mutual Aid

The companies in the industrial area, on the other hand, need foam capability and larger water supplies than the residential part of the city. The area divisions in this example affect the locational analysis in the sense that a greater importance will be assigned to one area than another. As Figure 2 indicates, Sparks City is affected by both a geographical division, and the qualitative fire problem division. An analyst might perform two location analyses in this example, one for each side of the river. On the side of the river containing Areas I and III, he would perform a single location analysis, weighting the demands from Area I in a different manner from those of Area III. (For the example, that such weightings can be determined will be assumed.)

The previous examples have indicated two ways (quantitative or geographic) of partitioning jurisdictions for locational analysis. A jurisdiction, as in Sparks City's case, may require both divisions applied to its problems.

If Phoenix County and Sparks City have a mutual aid agreement which makes the nearest engine company responsible for responding to alarms without regard to geo-political boundaries, analysis areas which cross political boundaries are feasible. This situation might be partitioned as in Figure 3.

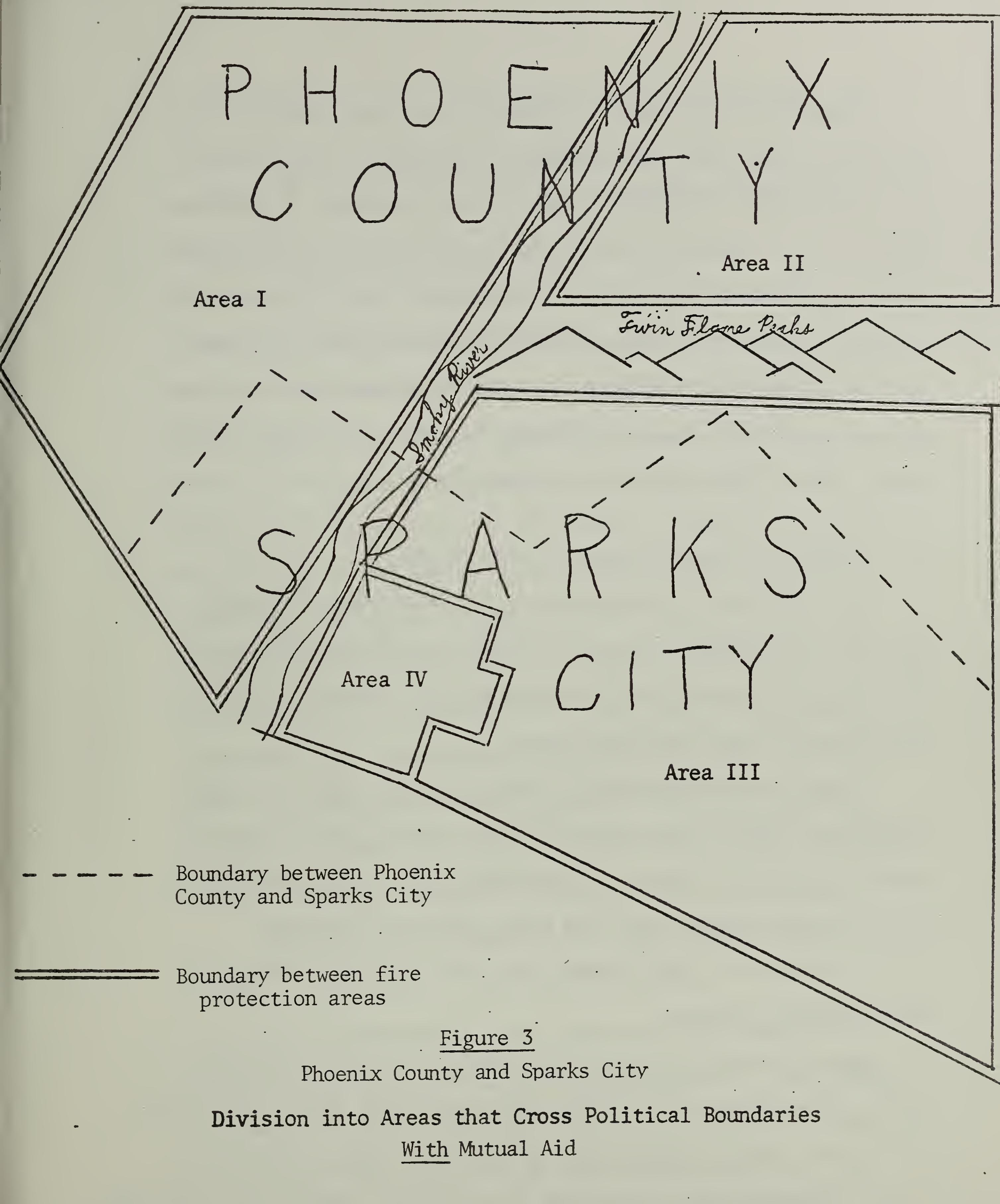


Figure 3

Phoenix County and Sparks City

Division into Areas that Cross Political Boundaries

With Mutual Aid

Although the references to Phoenix County and Sparks City are entirely hypothetical, such instances are typical. For example, Dade County, Florida, exhibits some of these problems. Dade County consists of 27 jurisdictions without central dispatch to all areas, so that individual jurisdictions cannot always rely on mutual aid. A further complication arises from the causeways across to Miami Beach which have draw bridges. The fire departments must consider the possibility of a draw bridge being raised at a critical time; namely, when a fire engine is responding to an alarm.

3.3 Concepts of Demand Zones and Focal Points

It is necessary to develop means for specifying the spatial distribution of demand for service within a given jurisdiction. The models in this paper are based on the two concepts of fire service demand zones and focal points.

A demand zone represents an area of the city with relatively homogeneous land use. The demand for fire service for the zone is assumed to occur at one point called the focal point. The concepts of fire service demand zones and focal points are analogous to the concepts of traffic demand zones amd centroids used in the transportation sciences.

These concepts can be made operational in the following manner:

1. The ultimate size of the fire demand zone should be related to a non-critical travel time. For example, if the city considers that 30 seconds is a critical response time, the fire demand zone should not be larger than 30 seconds driving time.

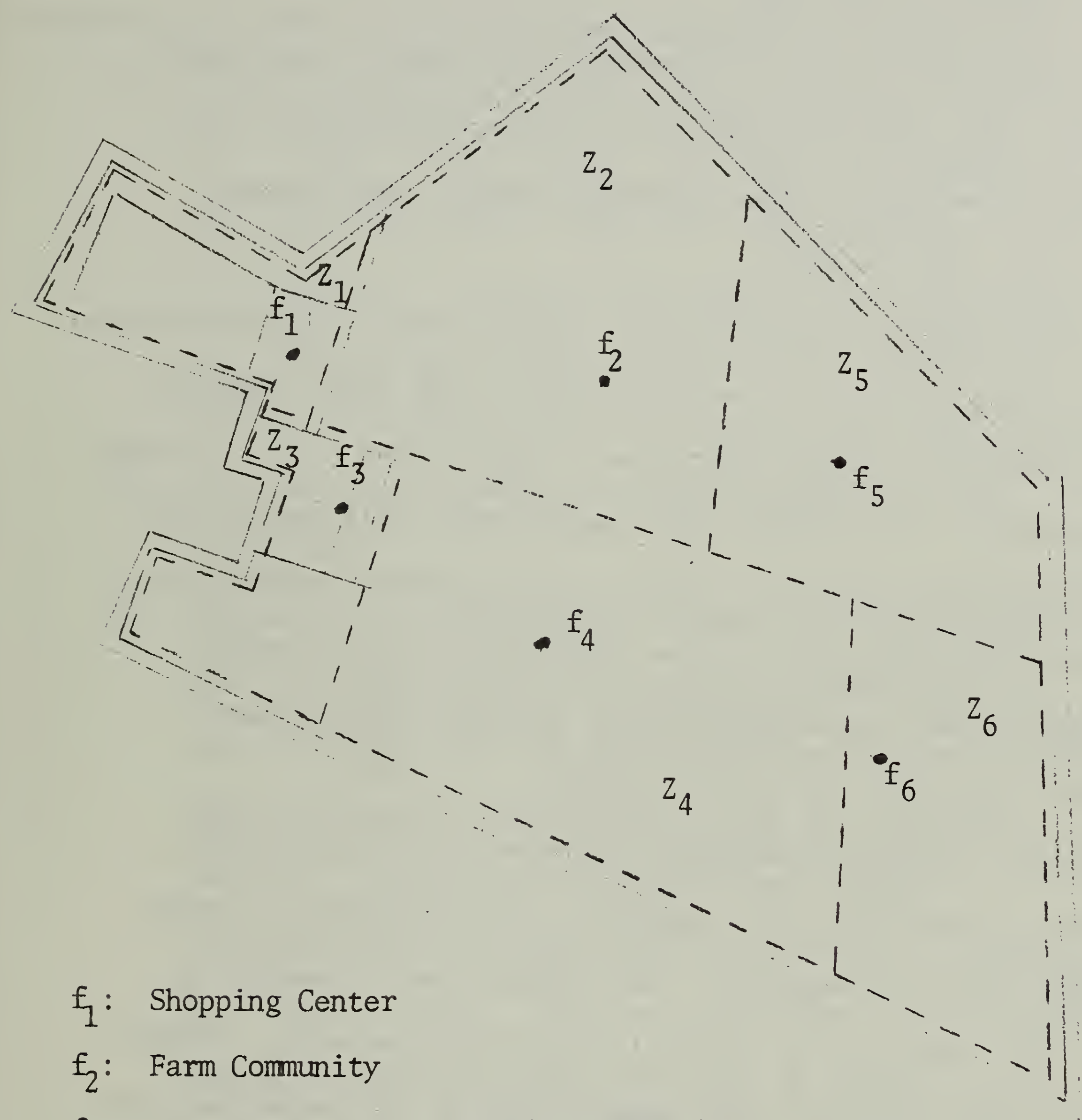
2. A fire demand zone can be a single complex of buildings; e.g., a factory producing or using hazardous materials, a church or a hospital, or an area of relatively homogeneous structures.
3. The focal points are chosen to be points within the fire demand zone representing the principal hazard for that zone, or the centroid of the zone computed by weighting all hazards in the zone. A hospital, as a significant entity within a residential area, could be treated in two ways. First, an area of the residential community containing the hospital satisfying the travel time criteria for that particular city could be marked off, and the focal point for that zone placed at the hospital. Second, the hospital may be treated as a separate fire demand zone within another fire demand zone representing the residential area. In this case, two focal points would be placed, one at the hospital and another, perhaps at the centroid, for the residential area.
4. A street network is assumed to exist for the jurisdiction, and a focal point will generally be a node of the street network.
5. A measure of importance is associated with each focal point. Different measures are needed for various models. For example, the measure could be the number of calls for service originating at the focal point, or a sum of the different types of calls weighted by the hazard to life or property represented by that type.

The resource and time constraint models will require a measure which gives a maximum travel time from the nearest facility to the focal point, or a measure requiring K basic suppression units to respond within L minutes.

Figure 4 illustrates how fire demand zones and focal points might appear in the residential area of Sparks City without mutual aid (see Figure 2). The fire demand zones, Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, each have a focal point, f_1, f_2, f_3, f_4, f_5, f_6. This is only an illustration of how one area could be partitioned. For example, the high value area of Sparks City may have much smaller fire demand zones, possibly on the order of a block in size, indicating the importance of a response to the associated focal points.

Figure 4

Fire Demand Zones and Focal Points in Sparks City



f_1: Shopping Center

f_2: Farm Community

f_3: Gas Stations in a Suburban Community

f_4: Church in Farm Community

f_5: Grain Storage Elevator

f_6: Farm Community

4.0 LOCATIONAL MODELS

This section describes the major variations of location models proposed for fire suppression unit location and provides a discussion of how they relate to each other.

First, an initial list of assumptions to be used in the models is presented. The remaining subsections discuss variations of the basic weighted-time model, time constrained models, and balanced workload models, respectively.

All of the models considered share several general characteristics. All of the models rely on the response times of the units involved, rather than on their response distances as used in the A.I.A. Standard Grading approach. The response time in the models will notationally be identified by T_{ij}, where T_{ij} refers to the shortest average travel time from facility location j to the focal point of fire demand zone i. These times can be generated by applying shortest path algorithms to specific city street networks.

The models evaluate alternative locations of a finite number of existing and potential fire suppression unit locations in terms of the given objective function. The locations are assumed to be coincident with nodes in the city's transportation network. Finally, all of the models weight the fire demand zones by the degree and type of hazards represented by the land use pattern of the zones. Some of the models employ the weights directly in the function to be optimized, while others employ weights implicitly as time constraints.

4.1 Initial Assumptions

This section furnishes a fundamental list of the assumptions made in applying the location models. Additional assumptions necessary for some of the models are discussed in context.[11/]

(a) Each fire suppression unit is assigned to one station at a fixed location (node), and that unit responds to all calls for service from its assigned station. Consequently, the models considered in this report do not apply to units which respond to an alarm while on patrol, or from any other place than their fixed location.

(b) The units are assumed to be indistinguishable and equivalent.

(c) The units are indivisible. In many fire departments, the fire suppression units consist of more than one piece of apparatus. In some situations, the unit can be split and each of the sub-units used at a different location. This tactic is particularly useful in fighting brush fires and fires where there is exposure to other buildings. The models described in this paper, however, assume that each unit will be engaged in its entirety, at a single location.

[11/] The assumptions will be lettered (a), (b), etc., so that the required assumptions for a particular model can be referred to by letter and need not be repeated.

(d) A given fire demand zone focal point is served from the closest unit. This assumption represents the usual practice of the fire services.

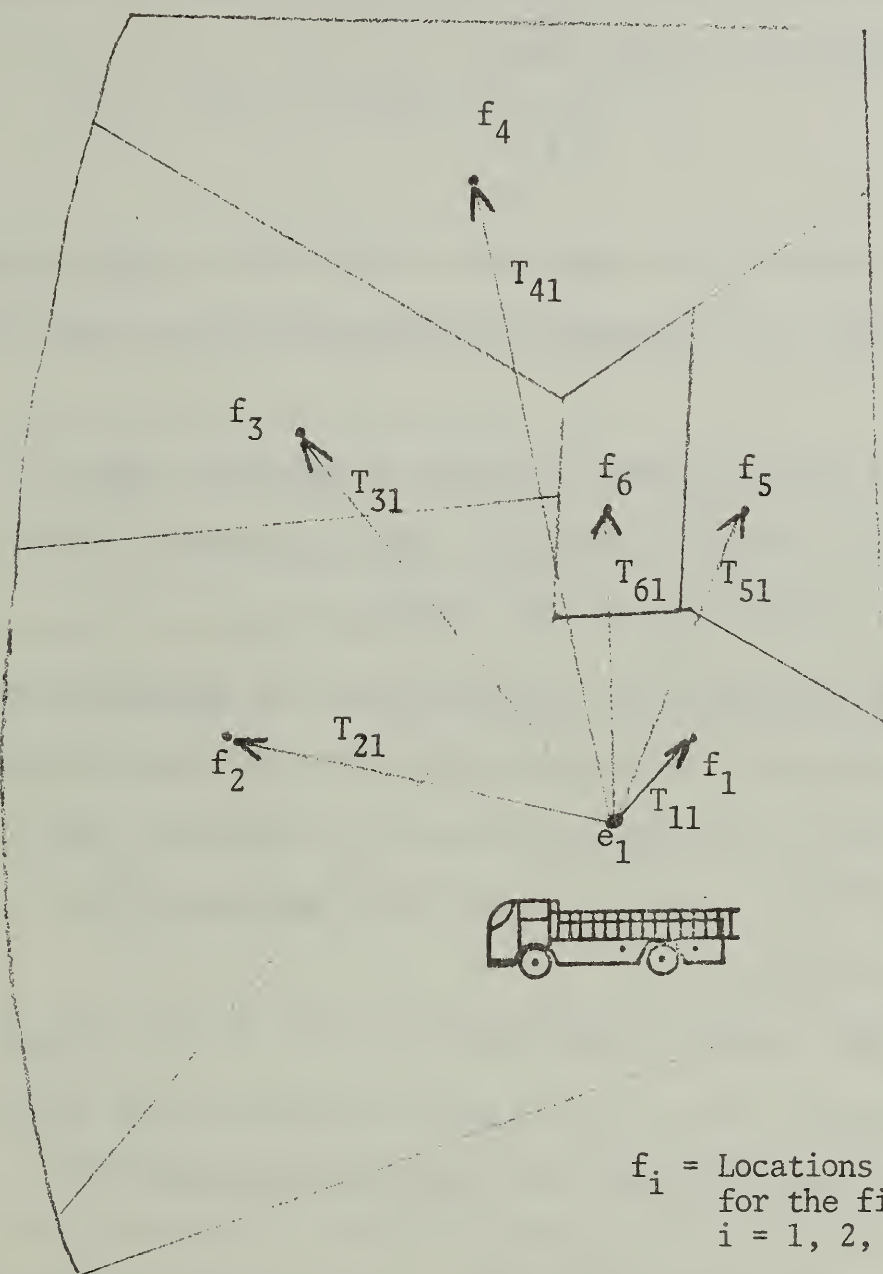
(e) Alarms, or calls for service, will originate from a finite collection of focal points, f_i, i=1, 2, ..., n, (n being the total number of focal points chosen to represent the fire demand zones).

(f) Potential locations for the basic fire suppression units are restricted to a finite set of points in the network denoted by e_j, j = 1, 2, ..., m, called <u>stations</u> (or more precisely, "potential fire station locations"). Generally, $m \leq n$.

(g) The travel-time $T_{ij} \geq 0$, required for a unit at e_j to respond to an alarm at the focal point f_i, is known for all ij. These travel times are illustrated in Figure 5.

Figure 5

Basic Fire Suppression Unit Response Times



f_i = Locations of the focal points for the fire demand zones, i = 1, 2, 3, 4, 5, 6

e_1 = Proposed fire station location

T_{i1} = Time from e_1 to f_i

4.2 Weighted Timed Models

All of the models in this section attempt to minimize the disutility associated with selection of stations for a given number of serving units. If f_i is to be served by a unit at e_j, the disutility associated with this assignment is assumed to be of the form:

$$W_i T_{ij}$$

where W_i is some measure of the importance of providing a rapid response at f_i. In particular, it is assumed for those models with a disutility function that:

(h) W_i = the expected number of alarms at the focal point f_i over a specified length of time, for example, a year. The "disutility" is also linear in T_{ij}.

To facilitate the exposition, it is necessary to introduce some notations. In particular, for the location of M units (where M is the actual number of basic fire suppression units to be located) from m possible locations, e_j, $j = 1, 2, \ldots, m$, $M < m$, and one unit is to be assigned to each location.

There are $\binom{m}{M}$ 12/ subsets of the set $\{e_j: j = 1, 2, \ldots, m\}$ consisting of M distinct elements, that is, the number of different ways of choosing M locations from m possible sites. For notational purposes let

$$M_m = \binom{m}{M}.$$

12/ $\binom{m}{M}$ is the binomial coefficient: $\binom{m}{M} = \frac{m!}{(m-M)!M!}$

Let E_k represent the k^{th} subset of the M_m subsets (i.e., k indexes a particular subset, called E_k, of the set $\{e_j: j = 1, 2, \ldots, m\}$, where $k = 1, 2, \ldots, M_m$). The subsets E_k are each distinct. For notational purposes let

$$e_{k_j} = \text{the } j^{th} \text{ element in subset } E_k.$$

4.2.1 Basic Model

First consider a very special case in which a fire department jurisdiction, represented by n focal points, $\{f_i; i = 1, 2, \ldots, n\}$ is to be served by a single FSU stationed at one of m possible locations, $\{e_j; j = 1, 2, \ldots, m\}$. If it is assumed that alarms occur at such times that the fire suppression unit is always available when required, then the unit can be located by determining that location among the m possible choices which will satisfy

$$\min_{1 \leq j \leq m} \sum_{i=1}^{n} W_i T_{ij}. \qquad \text{[Model I]}$$

It is clear that in this special approach (Model I) it is assumed that as well as (a-h),

(i) A fire suppression unit is available whenever one is required.

(j) Only one unit is required to respond to each alarm.

This model may seem naive, but it is appropriate for many small communities in the United States.

A logical extension of Model I is the location of several fire suppression units. For this model, assumptions (a-j) are used and the total travel time for the M basic fire suppression units is minimized. Thus, the objective function of Model II is:

$$\underset{1 \le k \le M_m}{\text{Min}} \sum_{i=1}^{n} W_i \ \text{Min} \ (T_{ik_j} \mid e_{k_j} \in E_k). \qquad \text{[Model II]}$$

This model is relevant to residential communities with a high resource availability, low structure density, and low life and property hazards.

Given the restricting assumptions of Model II, two constraints on this model are obvious; namely, the assumptions that the closest unit is available, and that only one unit responds to each alarm. The next subsection discusses a procedure for relaxing the first assumption.

4.2.2 Availability Model

This formulation uses the locations determined by the basic model and divides the region to be served into districts, each served exclusively by one station. The districts are determined so as to minimize the total expected travel time.

Carter, Chaiken, and Ignall have constructed a simple model in which the nearest unit is not assumed to be always available. Their model describes a region served by two units at fixed locations 1 and 2 as illustracted in Figure 6. In addition to the previous assumptions (a-h), (j), they assume that:

(k) The arrival of alarms is a Poisson process.

(l) The mean service time is independent of the location of the alarm and the basic fire suppression unit servicing the alarm.

(m) A part of the region R (Figure 6) has been identified as the district assigned to a unit located at 1. The district assigned to unit 1 will be called A, the district assigned to unit 2 will be referred to as B. Furthermore, these two units are assumed to be dispatched according to the following rules:

(1) The two units will respond only to alarms in the region.

(2) A unit, if available, will respond to all alarms in its own district.

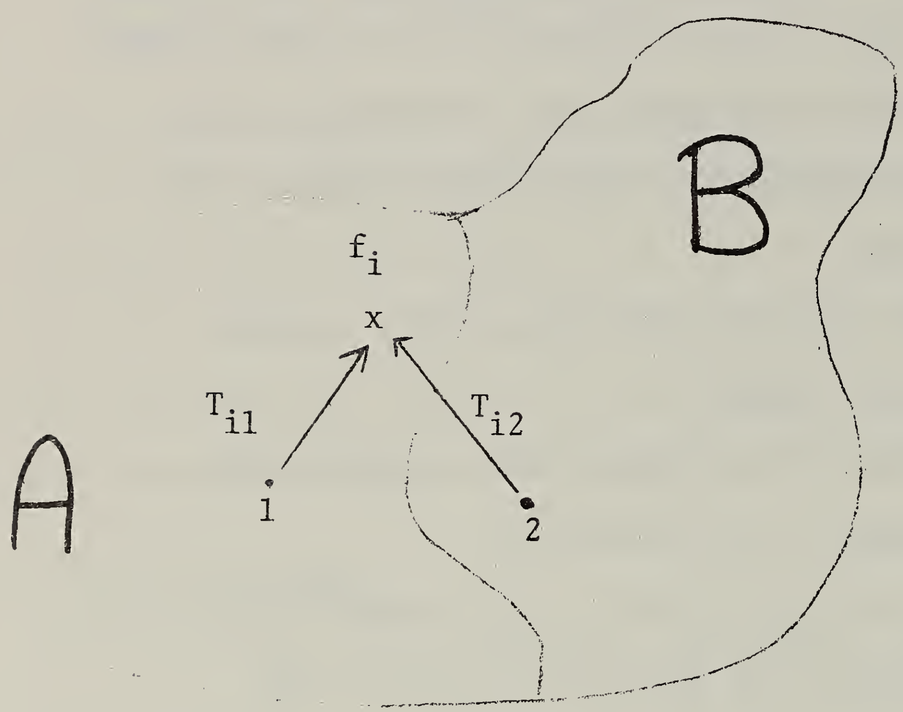
(3) A unit, if available, will respond to an alarm in the other unit's district whenever that unit is unavailable.

(4) When both units are unavailable, alarms will be served by units outside of B.

Let there be no focal points in region B. Then, under the previous assumptions, Carter, et al., showed that the area (or focal points) to be included in district A to minimize the total expected travel time is:

Figure 6

Response Region for Two Units



$$A = \{f_i \in R \mid T_{i1} - T_{i2} \leq \frac{\lambda}{\lambda-\mu} \cdot \frac{n}{\sum_{i=1}^{n} W_i} \sum_{i=1}^{n} (T_{i1} - T_{i2}) W_i\}$$

[Model III]

where

λ = mean arrival rate of alarms

μ = mean service time.

This model is not directly applicable to most resource location problems because of the limitation on the number of units considered.[13]

4.2.3 Multiple Dispatch Model

In this section, a model will be discussed which relaxes the assumption that only one unit is required on all alarms. If the assumptions of (II) are modified to allow for a probability q_i a second unit will be required at f_i, and the unit will be available when required, the total travel time for a given number of units can be minimized by stationing these units at the locations which satisfy:

$$\min_{1 \leq k \leq M_m} \sum_{i=1}^{n} W_i \min \{T_{ik_r} + q_i T_{ik_s} \mid e_{k_r}, e_{k_s} \in E_k, r \neq s\}.$$

[Model IV]

[13] New results relaxing this constraint have been announced by Chaiken and Larson at the 40th National ORSA meeting, October 27-29, 1971, ORSA Bulletin, p. B238. However, these results are not yet published.

Model IV includes assumptions (a-j),[14/] and the new assumption:

(n) A second basic fire suppression unit may be required on an alarm at f_i. The probability of requiring the second unit on an alarm at f_i is designated by q_i.

Model IV could clearly be extended to include probabilities that three or more units are required for each alarm.

4.3 Time Constraint Models

The conventional approach to the fire station location problem is given by the A.I.A. standards which require that a station be located within a given number of miles of each focal point. (The required distances may vary with the "value" of the fire demand zone.) Therefore, models which constrain the maximum allowable distance between a fire demand zone and its serving unit(s) are of interest. Mitchell[15/] suggests one feasible approach to modeling these constraints taking time rather than distance into consideration.

Assume that there is a maximum allowable travel time associated with each focal point:

(o) T_i = Maximum time constraint for a response to f_i, i = 1, 2, ..., n.

In order to insure compliance with these constraints, define a penalty function, G, in the following manner:

$$G(i, t) = \begin{cases} 0 \text{ if } t \leq T_i \\ \alpha \text{ if } t > T_i \end{cases} ; \text{ where } \alpha \text{ is a large value.}$$

[14/] Under assumption (i), the assumption is implicitly included that the first and second nearest units are available whenever required.

[15/] P. S. Mitchell, "Efficient Allocation of Fire Department Resources," Fire Technology, (1971), pp. 327-242.

This penalty function can be appended to the objective function in Models I, II, or IV, in order to create time constrained versions of these models as follows:

$$\text{(IC)} \qquad \underset{1 \leq j \leq m}{\text{Min}} \sum_{i=1}^{n} (W_i T_{ij} + G(i, T_{ij})).$$

Assumptions for (IC): (a-j), (o).

$$\text{(IIC)} \qquad \underset{1 \leq k \leq M_m}{\text{Min}} \sum_{i=1}^{n} W_i \ \text{Min}\{T_{ik_j} + G(i, T_{ik_j}) \mid e_{k_j} \in E_k\}.$$

Assumptions for (IIC): (a-k), (o).

$$\text{(IVC)} \qquad \underset{1 \leq k \leq M_m}{\text{Min}} \sum_{i=1}^{n} W_i \ \text{Min}\{T_{ik_r} + q_i T_{ik_s} + G(i, T_{ik_r}) \mid e_{k_r}, e_{k_s} \in E_k, r \neq s\}.$$

Assumptions for (IVC): (a-k), (n,o).

Model IVC could be extended by appending a second penalty function associated with the travel time of the second closest basic fire suppression unit, T_{ik_s}. Furthermore, alternative functional forms for the penalty function G could be analyzed in place of the $\{o, \alpha\}$ function.

A different approach to the time constraint model has been suggested by Toregas, et al.[16/]

[16/] Toregas, ReVelle, Swain, and Bergman, "The Location of Emergency Service Facilities," Operations Research, Sept.-Oct., (1971).

They formulate the problem as locating the minimum number of fire suppression units in order to insure that each focal point, f_i, lies within a prespecified service time b_i. In particular, define:

b_i = the upper bound on the initial response time of a first due unit to focal point f_i, where $i = 1, 2, \ldots, n$.

The objective of this approach is to minimize the number of units required to satisfy the time constraints.

In order to formulate the model, additional definitions are required:

U = the set of possible unit locations

Y_j = 1 if a unit is located at e_j and 0 otherwise, where $e_j \in U$.

$N_i = \{j \in U \mid T_{ij} \leq b_i\}$, $i = 1, 2, \ldots, n$, and T_{ij} is the shortest path time from focal point f_i to unit location e_j. N is not an empty set for all i.

The problem of identifying the minimum number of unit locations which can provide the desired level of unit service can be formulated as:[17/]

$$\text{Min} \sum_{j=1}^{n} Y_j$$

$$\text{subject to} \sum_{j \in N_i} Y_j \geq 1, \quad i = 1, 2, \ldots, n \qquad \text{[Model V]}$$

where Y_j = 1 or 0, for $j = 1, 2, \ldots, n$.

The assumptions for Model V are (a-g), (i), and (o).

17/ A similar formulation has been suggested to the authors by Dr. William Horn, Applied Mathematics Division, National Bureau of Standards.

4.4 Balanced Workload Model

The balanced workload model attempts to balance the workloads while minimizing the total travel time. The general problem (of selecting M locations from a set, E, of m possible locations) is usually unsolvable. The first model formulated below is a feasibly solvable problem.[18/] The general idea is to begin with a fixed set of M locations. For the purposes of notation, let E represent the fixed set of locations. The problem to be solved is to determine the fraction of the workload at each focal point, f_i, to be assigned to unit j. In particular, this model makes assumptions (a-l) and in addition it assumes:

(p) workload at f_i is proportional to W_i.

The model requires the following definitions:

W_i = weight associated with the workload at focal point f_i, $i = 1, 2, \ldots, n$,

T_{ij} = shortest average route time between focal point f_i and unit j in E_{01},

Y_{ij} = the fraction of the workload at focal point f_i assigned to unit j in E_0.

The problem can be formulated:

$$\text{Min} \sum_{i=1}^{n} \sum_{j=1}^{M} T_{ij} W_i Y_{ij}, \qquad \text{[Model VI]}$$

18/ V. Srinivasan and G. L. Thompson, "A Fortran V Code for Transportation Algorithms," Graduate School of Industrial Administration, Carnegie-Mellon University, Pittsburgh, Pennsylvania, June 1971.

subject to

$$\sum_{j=1}^{M} Y_{ij} = 1 \quad , \quad i = 1, 2, \ldots, n,$$

$$\sum_{i=1}^{n} W_i Y_{ij} = \frac{1}{M} \sum_{i=1}^{n} W_i \quad , \quad j = 1, 2, \ldots, M.$$

The problem that is usually unsolvable but which describes the general locational problem can be formulated in the notation of Model VI where:

T_{ij} = shortest average route time between focal point f_i and unit j in E,

Y_{ij} = the fraction of the workload at focal point f_i assigned to unit j in E,

X_j = 1 if a unit is located at j, and 0 otherwise.

The new problem can be formulated as a quadratic program:

$$\text{Min} \sum_{i=1}^{n} \sum_{j=1}^{m} T_{ij} W_i Y_{ij} X_j \qquad \text{[Model VII]}$$

subject to

$$\sum_{j=1}^{m} Y_{ij} X_j = 1 \quad , \quad i = 1, 2, \ldots, n,$$

$$\sum_{j=1}^{m} X_j \leq M,$$

$$\sum_{i=1}^{n} W_i Y_{ij} X_j = \frac{1}{M} \sum_{i=1}^{n} W_i \quad , \quad j = 1, 2, \ldots, m.$$

5.0 MODEL HEURISTICS AND ALGORITHMS

Each of the models discussed above can be associated with an algorithm or a heuristic which will select locations for the units according to their respective criteria. In spite of the large number of models presented, a small number of solution techniques can accommodate all of them.

These techniques are:

1. Complete enumeration
2. Maranzana heuristic
3. Integer Programming
4. Transportation Algorithm.

5.1 Complete Enumeration

When the number of choices is small, it is feasible to calculate the value of the objective function for each of the alternatives. This approach has been used by Berlin and Santone[19/] in applying Model I. Note that in Model I the number of alternatives is m; i.e., the number of possible locations at which to locate the one unit.

Complete enumeration, where feasible, has an important advantage over more sophisticated computational procedures for identifying the optimum locations. If the value of the objective function is computed for each of

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19/ L. C. Santone, G. Berlin, Location of Fire Stations, Presented at WORC Symposium on "Systems Analysis for Social Problems," May 26-28, 1969. See also National Bureau of Standards Report #10 093.

the alternatives, the alternatives can be ranked. In doing this not only the best choice is identified, but also the second, third, and fourth best choices and differences in the value of the objective function among these choices.

It should be noted that Models II, IIC, IV, AND IVC may also be solved by complete enumeration as long as M_m, the number of alternatives, does not get so large as to make the required number of computations too time consuming.

5.2 The Maranzana Heuristic

5.2.1 Application to Models II, IIC

Several authors have suggested the use of a heuristic developed by Maranzana[20/] for models such as Model II.[21/] The heuristic consists of locating basic fire suppression units at an arbitrary initial selection of M of the m stations, then partitioning the focal points into districts such that all points in a district are served from the same location.

[20/] F. E. Maranzana, "On the Location of Supply Points to Minimize Transport Costs," Operations Research Quarterly, 15, (1964), pp. 261-270.

[21/] Also see Richard Jordan, et al., Systems Analysis of Inland Consolidation Centers for Marine Cargo, NBS Technical Note #530, Nov. 1970.

This report uses a Maranzana type approach and presents an extensive analysis of using different starting centers and different types of constraints. The running times on a UNIVAC 1108 computer were of the order of 1-3 minutes per iteration for a problem of 1400 shippers, 12 inland centers, and 3 ports.

Also see Arnold Weber, Documentation of LOC Models, (NBS Report to be published), for experience in using these approaches.

Next, for each district the set of possible station locations is examined to determine if the value of the objective function in that district can be reduced by selecting one of the alternative locations for the unit serving the district, (Model I). Finally, if new locations are chosen for some of the units, the redistricting is repeated and the process is continued until it fails to recommend new locations for the units. More precisely, with respect to Model II, the heuristic consists of the following steps:

1. Initialize

Make an initial arbitrary selection of M of the possible station locations $\{e_j\}_{j=1}^{m}$, say $E_1 = \{e_{1_j} \mid j = 1, 2, \ldots, M\}$.

2. District

Assign each f_i to one and only one of the M serving basic fire suppression units in order to form districts. Let p be an index representing one of the units. Form the p-th district as follows: Define

$$D_p = \{f_i \mid T_{il_p} \leq T_{il_j}, \; j = 1, 2, \ldots, M\},$$

where p = 1, 2, ..., M. That is, assign each focal point to the nearest serving unit. Futhermore, suppose that unit location e_1 is associated with district D_1, e_{1_2}, is associated with D_2, etc. If some focal point should be equidistant from two or more units, arbitrarily assign this point to the district of the unit that appears first in the list.

3. Move

Examine the set of possible unit locations $\{e_j\}_{j=1}^{m}$ for a new set of M locations which best serve each district D_p, p = 1, 2, ..., M. This is done by choosing the locations in order to satisfy:

$$(*) \qquad \underset{1 \le j \le m}{\text{Min}} \sum_{f_i \varepsilon D_p} W_i T_{ij}.$$

This procedure may be accomplished by a complete enumeration. Let E_2 be the set of M new unit locations which satisfy condition (*), where

$$E_2 = \{e_{2_j} \mid j = 1, 2, \ldots, M\}.$$

4. Terminate

If the unit location has been changed in any of the districts, return to step 2, otherwise stop.

5.2.2 Application to Models IV and IVC

In this section, a variation on the Maranzana heuristic is presented which is applicable to Models IV and IVC discussed in Section 4.4. In order to make this extension of the heuristic, it is necessary to supplement the concept of district D_p, defined above. In this section, D_p will be referred to as the "first-due district," and the "second-due district," is defined as D_p^2. In an intuitive manner, a second-due district can be thought of as a set of next nearest focal points to a basic suppression unit, say unit 1. Thus, they would be a set of focal points falling in the first-due district of some other fire suppression unit, say unit 2, but closer to unit 1 than to units 3, 4, ..., M. Formally, define D_p^2, p = 1, 2, ..., M, as follows:

$$D_p^2 = \{f_i \mid f_i \notin D_p \text{ and for some } p', (p' \neq p, 1 \leq p' \leq M), f_i \varepsilon D_{p'} \text{ and}$$

$$T_{ip} \leq T_{ir}, (r \neq p', 1 \leq r \leq M)\}.$$

Although this is a formal mathematical definition, it can be given an operational interpretation. A focal point f_i belongs to the second due district D_p^2 of some unit indexed p, provided:

1. f_i is not in the first due district D_p of the unit p, but falls in the first due district of some other unit p´, $D_{p'}$, where $p' \neq p$,
2. the following condition on the response times to focal point f_i is satisfied:

$$T_{ip} \leq T_{ip'} \leq T_{ij},$$

where $j \varepsilon \{1, 2, \ldots, M\}, \ \{p, p'\}$.

Figure 7 displays an intuitive pictorial view. The figure depicts three units located at points 1, 2, 3, in a circular jurisdiction with distances measured by straight lines. The associated first-due districts, D_1, D_2, D_3, are delineated by the solid lines. The second due district for unit 1, D_1^2, is represented by the shaded area. Thus, the shaded area in D_2 is nearer to unit 1 than to unit 3, and the shaded area in D_3 is nearer to unit 1 than to unit 2. Similar shadings could be constructed for units 2 and 3.

Using this definition the Maranzana heuristic can be reformulated to apply to Model IV:

Step 1. Initialize

Make an arbitrary selection of M possible unit locations from the possible locations $\{e_j\}_{j=1}^{m}$. As before, designate this set as $E_1 = \{e_{1j} \mid j = 1, 2, \ldots, M\}$. These will be the initial locations for the M units.

Step 2. District

For each unit p, p = 1, 2, ..., M, partition the focal points into first-due districts

$$D_p = \{f_i \mid T_{il_p} \leq T_{il_j},\ j = 1, 2, \ldots, M\},$$

where D_p is the first-due district for unit p, p = 1, 2, ..., M.

Also, partition the focal points into second-due districts[22/]

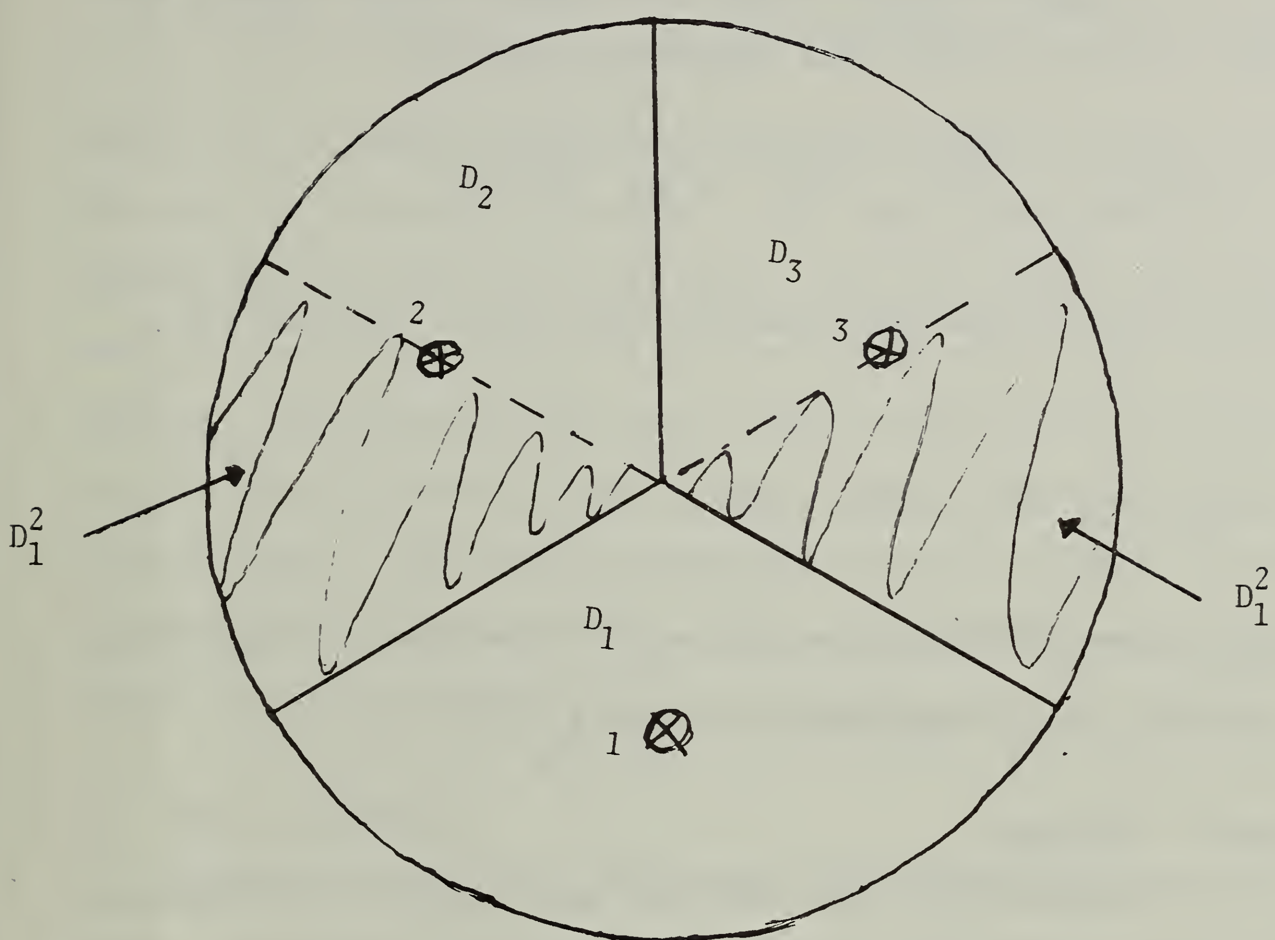
$$D_p^2 = \{f_i \mid f_i \notin D_p \text{ and for some } p',\ (p' \neq p,\ 1 \leq p' \leq M),\ f_i \in D_{p'}$$

$$\text{and } T_{ip} \leq T_{ir},\ (r \neq p',\ 1 \leq r \leq M)\}.$$

[22/] Again, it is necessary to consider the case in which a focal point is equidistant from two or more units in order to ensure that the D_p (also D_p^2) sets partition the focal points. When this is the case, this focal point is arbitrarily assigned to the unit with the smallest unit number p.

Figure 7

First and Second-Due Districts



1, 2, 3: Unit Locations

D_1, D_2, D_3: First-Due Districts for Units 1, 2, 3, respectively

D_1^2 : Second-Due District - Unit 1

Step 3. Move

As in Section 4.2.1, look for an alternative location for each unit. Unit p will be moved to a new location if there is some station in the list $\{e_j\}_{j=1}^{m}$ which reduces the value of the objective function for the points served by unit p. This objective is given by

$$(**) \qquad \underset{1 \leq j \leq m}{\text{Min}} \quad \Big(\sum_{f_i \varepsilon D_p} W_i T_{ij} + \sum_{f_i \varepsilon D_p^2} W_i q_i T_{ij} \Big).$$

Let E_2 be the set of M new unit locations which satisfy condition (**), where

$$E_2 = \{e_{2_j} \mid j = 1, 2, \ldots, M\}.$$

Again e_{2_1} is associated with D_1 and D_1^2, e_{2_2}, is associated with D_2 and D_2^2, etc., and if $e_{2_p} \neq e_{1_p}$, then move unit e_{1_p} to e_{2_p}.[23/]

Step 4. Reiterate

If none of the units have been moved in step 3, the heuristic terminates, otherwise, return to step 2 and continue.

As with the Maranzana algorithm this heuristic does not necessarily achieve an absolute minimum for the objective function. It does achieve some form of a "local" minimum.

[23/] Again, care must be taken that the objective function e_{2_p} is less than the value of the objective function at e_{1_p} and not equal. Otherwise the heuristic may cycle and not converge.

This heuristic may also be applicable to Model IV with condition (**) replaced by the new objective

$$(***) \quad \min_{1 \le j \le m} \left(\sum_{f_i \varepsilon D_p} [W_i T_{ij} + G(i, T_{ij})] + \sum_{f_i \varepsilon D_p^2} W_i q_i T_{ij} \right).$$

However, it is possible that this heuristic will fail in step 2. In fact, the value of the objective function in (***) may be infinity for all possible locations for one of the units. This may occur in cases where there is a solution for Model IVC with no finite value of the objective function. This difficulty may be overcome by replacing the arbitrary selection of initial locations in step 1 with an initial selection of locations based on Model V. This can be accomplished to insure a finite value for each of the objective functions (***) for some values of M. A computational procedure for accomplishing this will be explained in the next section.

5.3 Model V Algorithm

Toregas, ReVelle, Swain, and Bergman (see footnote 16) describe a simple algorithm for solving Model V in some of the cases where the number of units required to serve each focal point is one. Their algorithm consists of applying a linear program to Model V, with additional cuts if necessary:

Step 1: Let the problem be described by:

$$\min \sum_{j=1}^{n} y_j,$$

subject to $\sum_{j \in N_i} j \geq 1$, $i = 1, 2, \ldots, n$,

where $y_j = 1$ or 0 for $j = 1, 2, \ldots, n$, and

N_i, $i = 1, 2, \ldots, n$ have been defined as in section 4.3

Step 2: Apply a linear program to the problem.[24/]

Step 3: If the solutions y_j are all integers, the problem is solved. If one or more of y_j are not integers, define M_0, the number of units required, by

$$M_0 = \sum_{j=1}^{n} y_j.$$

If M_0 is an integer while one or more of the y_j are non-integral, the algorithm fails to produce a solution to Model V. (Toregas, et al., report never encountering this situation in their experience with the algorithm.)

If M_0 is not an integer, add an integral cut to the problem. The new problem is:

$$\text{Minimize } \sum_{j=1}^{n} y_j$$

$$\text{subject to } \sum_{j \in N_i} y_j \geq 1, \quad y_i \geq 0 \text{ for each } i = 1, 2, \ldots, n,$$

$$\text{and } \sum_{j=1}^{n} y_j \geq [M_0] + 1,$$

where $[M_0]$ is the largest integer which is smaller than M_0. Continue the algorithm by repeating steps 2 and 3.

This algorithm cannot continue indefinitely since on each iteration $\sum_{j=1}^{n} y_j$ must increase beyond the next integer greater than $\sum_{j=1}^{n} y_j$, but cannot increase beyond m.

24/ Toregas, et al., report that a mathematical programming code is available for an IBM S/360.

5.4 Model VII Heuristic

One approach to Model VII has been suggested [24/] which makes direct use of the transportation problem. In this formulation, begin by assigning a portion W_{ij} of the workload (W_i) of each focal point to each of the possible fire station locations, e_j, which constrain the utilization of each location. This can be accomplished by solving the transportation problem for the values of W_{ij} which:

$$\text{minimize} \sum_{i=1}^{n} \sum_{j=1}^{m} T_{ij} W_{ij}$$

$$\text{subject to} \sum_{j=1}^{n} W_{ij} = W_i \text{ for each } i = 1, 2, \ldots, n$$

$$\text{and} \sum_{i=1}^{n} W_{ij} \leq \frac{\sum_{i=1}^{n} W_i}{M} .$$

Define the cost of this system of m fire stations to be

$$D_m = \sum_{i=1}^{n} \sum_{j=1}^{m} T_{ij} W_{ij} ,$$

where the values of the W_{ij} have been determined by solving the transportation problem.

24/ By Dr. George Suzuki, Technical Analysis Division, National Bureau of Standards.

One method of solving Model VII would be to compute the costs, D_m^k, $k = 1, 2, \ldots, \binom{m}{M}$, associated with each subset containing M of the possible fire station locations using the method described by Srinivasan and Thompson[25/]. The set of M locations of least cost solves Model VII. However, this approach may prove impractical because of the computation times required. For example, if a problem requiring the selection of the best 10 out of 20 possible locations takes ten seconds to evaluate each possibility, it would take 21 days to evaluate all of the possibilities. As a result, it is necessary to investigate heuristics for solving Model VII.

One method for arriving at a set of M stations consists of first solving the transportation problem for the m stations as described above, and defining the utilization

$$U_j = \sum_{i=1}^{n} W_{ij}$$

associated with each location. The set of possible locations is reduced by eliminating the least utilized location. This procedure is repeated on the remaining m-1 locations until only M stations remain.

Another heuristic for Model VII was programmed by Crond, Inc. This program is referred to as REDIST,[26/] and the general steps of the heuristic are given as follows:

[25/] See footnote 18.

[26/] CROND, Inc. REDIST, Version 3.3, Program Description and User Manual. Copies can be obtained from National Municipal League, 47 E. 68th St., New York, N.Y., 10021.

1. Estimate the initial locations of the units to be placed.
2. Use a transportation algorithm to assign focal points to fire suppression units in order to minimize the sum of the weighted response times. This is the districting step.
3. Adjust these assignments so that each focal point lies entirely within one unit district. This process usually destroys the equal distribution workload, but uniquely defines districts.
4. Reassign focal points between districts in order to improve workload equality.
5. Compute new unit locations within each district as new trial locations. The program returns to step 2 until the solution converges.

6.0 SUMMARY

The objective of this paper has been to compile and review tools that can be used in locating fire suppression units. The specific contributions of this paper are:

1. A methodology of spatial concepts to use in locational analysis.
2. A specification of different types of location-allocation models and their assumptions.
3. A discussion of how each type of model relates to the others.
4. Discussions of heuristics and algorithms to be used with the models.

An analyst concerned with fire suppression unit locations should be able to select the type of model most appropriate to his particular problem. In addition, it is hoped that the paper will stimulate work on more efficient algorithms and on relating the locational problems to overall resource allocation problems.

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