NATIONAL BUREAU OF STANDARDS REPORT

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APPROACHES TO EVALUATING THE EFFECTS OF VFR TOWERS ON FLOW AND SAFETY AT AIRPORTS

Interagency Agreement No. DOT-FA70 WAI-188

NBS

U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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APPROACHES TO EVALUATING THE EFFECTS OF VFR TOWERS ON FLOW AND SAFETY AT AIRPORTS

L. S. Joel
W. A. Steele
J. J. Filliben
G. B. Hare

Interagency Agreement No. DOT-FA70 WAI-188

NBS

U.S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

ABSTRACT

The Federal Aviation Administration is reviewing its criteria (now based on annual traffic levels) for airports to be eligible for the installation of VFR towers. In support of that review, the study documented here was commissioned to seek mathematical methods and models for measuring the effects of a VFR tower on flow and safety at airports with different traffic volumes. This report discusses a variety of methodologies, viewpoints, and concepts, with the following highlights: (1) The first version of a mathematical model, intended to portray a tower's ability to expedite flow by abridging the full operation sequence, has been formulated and exercised in illustrative calculations. (2) Available aggregated data are not adequate for identifying functional relations between collision rates and activity levels at tower and non-tower airports; however a novel statistical approach has established the association between tower-presence and lower collision rates on a firmer basis than before. (3) More knowledge is needed concerning pilots' information needs, relative to potential hazards, which go unmet in the absence of a VFR tower; as an initial contribution along this line, a computerized model has been developed to aid in studying the pilot's time-varying field of vision as limited by cockpit geometry and other structural obstructions.

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1. INTRODUCTION

This report describes work, performed by the Technical Analysis and Applied Mathematics Divisions of the National Bureau of Standards, for the Economic Analysis Branch of the Federal Aviation Administration's Office of Aviation Economics, in support of the FAA's review of criteria for airports to qualify for the installation of VFR towers. Our task was to attempt to find methods and mathematical models to measure the benefits of VFR tower installation at medium-activity airports, specifically the contribution of towers to safety and to the improvement of traffic flow. By "measurement", we mean the determination of numbers describing the difference between the presence and absence of a VFR tower in terms of readily identifiable relevant units; for instance, in the case of safety, the expected number of fatalities per million operations.

The FAA has already experienced the conceptual and technical difficulties of this problem area, so that it will come as no surprise that our actual accomplishments (apart from orientation, and from barking up the inevitable false trails before identifying them as such), are all of a somewhat preliminary nature. It became apparent quite early that, while no simple solution is at hand, practicable models could indeed be devised albeit at greater cost than admissable in our own study effort. The work statement gave us the option of recommending discontinuation of the research under these circumstances (or if we deemed valid models to be totally inaccessible). We did not so recommend, because we believe that the models whose first stages we actually have realized as computer programs have utility for the indirect measurement of safety and flow benefits of towers, as well as collateral applications of great interest within FAA.

The major problems in constructing measurement (really prediction) models fall naturally into four classes: (1) finding suitable units of measurement, (2) identifying relevant parameters, (3) formalizing the model structure (i.e., writing equations), and (4) assessing sources of data and developing a "strategy" for acquiring and assimilating these data. We devoted attention to each of these subjects, and they are addressed, where appropriate, in the various sections of the report.

The report is divided into two major sections: Chapter 2, devoted to the question of traffic flow (primarily as related to tower control) considered independently from questions on safety, and Chapter 3, on safety. There is a fourth chapter, outlining some thoughts on promising directions for making the models more comprehensive, valid and refined.

Chapter 2, "Flow Effects", after describing the kinds of traffic movements to be analyzed, soon becomes substantially a discussion of the applicability of queuing models and Markov chains. It contains the formulation and analysis for what we call the

"String of Slots" model, a representation of arrival traffic in which the intervals along a generalized landing path are assigned probabilities as desired entry points to the landing pattern. These probabilities are "inputs" to the analysis, and we would expect to base them on a distribution of directions of approach to the airport as influenced by the information available to the pilots. A general version of the model would measure average delays under various rules concerning what are the alternative entry points if the desired position is occupied by another aircraft. The present primitive version does not consider a mix of different aircraft types, nor the dovetailing of VFR traffic with IFR traffic, nor departures; it has however been brought to the point of illustrative calculations estimating the differences expected in landing time, under control environments intended (respectively) to represent the presence and absence of a VFR tower at airports with various levels of annual activity.

Chapter 3, "Safety", begins with an informal discussion of the operational meaning of safety in the context of measurement of tower effects. We then attempt to deal with safety benefits using the probability of mid-air collision as "cost" criterion. The inadequacy of available data for implementing the conceptually simplest approach to measurement, that of direct regression on collisions by volume class at tower and non-tower airports, is regretfully noted. The next best alternative, significance testing on the data, is described. This refers to methods of determining whether a meaningful correlation between tower presence and collision rates exists, without reference to the degree of the dependence. The verification of such a correlation by modern so-called "distribution free" tests is described in Appendix C.

The thrust of the measurement effort is then shifted to considering the VFR tower as an information source and studying the importance of information to safety. The chapter closes with the details of a specific computerized model which measures "cockpit visibility", i.e., the way that the critical portion of a pilot's field of visibility varies during a landing or departure (or, indeed any flight path) as a function of the structural configuration of the aircraft itself. This chapter also includes mention of models occurring in four of our principal background source documents.

The staff orientation for this project required studying a large number of reports. They are cited wherever a specific point of clear pertinence occurs, and the bibliography[1] contains these as well as all others which contained useful (relevant) background reading. The reports we read, the two field trips to towers made

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1 In Chapter 5; references to its listing are shown as numbers in square braces.

by project staff members, discussions with FAA people and with active licensed pilots at NBS, and the polemical working sessions of the project staff, while hardly constituting a program of data gathering, interviews and formal analysis in depth, afforded us (in our view at least) valuable insights into the problem under study and informed our models. The general project activity generated a large number of qualitative conjectures, and we have recorded them in the report wherever they contain ideas which we have not encountered in our readings, even though they do not contribute directly to the establishment of quantitative standards. Many of them relate to intangibles such as pilot confidence; in particular, here is where the relationship of control and orderly flow to safety is exposited.

Appendix A presents the theoretical underpinning for the treatment of the string-of-slots model in Chapter 2. In particular, it discusses a key assumption on which our current solution method for that model is based, showing why this assumption is believed to yield reasonable approximations to the exact solution.

Appendix B reproduces a mathematical study, motivated only in part by the present project, of alternative approaches to controlling the "right of way" in the presence of two competing traffic streams --- e.g., the liftoff queues on each of a pair of intersecting runways, or perhaps a landing stream and a stream of departures. The approaches considered were: (a) periodic shifting of the "green light" from one stream to the other,

(b) "adaptive" or "feedback" control based on which stream's waiting-line is longer, and (c) as a benchmark, a hypothetical control system which could correctly "prophesy" the pattern of future arrivals in both streams. It is shown that under the study's assumptions, most of the benefit in passing from periodic to "prophesying" control is already attained in the passage to feedback control.

Appendix C, the report of tests of correlation between (a) collision frequencies at or around airports (of various volume classes) and (b) the presence or absence of a VFR tower, has been mentioned above. It is a self-contained study by a staff member of the Statistical Engineering Laboratory of the Bureau's Applied Mathematics Division. This appendix contains also some revisions and corrections to previously tabulated collision data.

During the course of the study, a member of the Technical Analysis Division's Behavioral Sciences Group conducted an examination of the literature on human factors in accident evasion, in order to furnish a basis for refining the visibility models. This work was initiated only "late in the game", i.e. after we found ourselves obliged to abandon the possibility of establishing simple formulae by statistical analysis of available collision data, and so there was not time to integrate its findings into the formal "structure" of the total research effort. In consequence, this material has been set apart and appears as Appendix D to this report. The appendix is self-contained and includes its own bibliographical list of references.

A program listing of the "cockpit visibility" model described in Chapter 3 as well as sample output from the model constitutes Appendix E.

Appendix F consists of the interagency agreement DOT-FA70WAI-188 between the FAA and NBS.

This project was a joint effort by the National Bureau of Standards' Technical Analysis Division and Applied Mathematics Division. It was conducted under the administrative supervision of R. T. Penn (manager of the first-named division's Mathematical Modeling Group), and the technical supervision of L. S. Joel (of the second division's Operations Research Section). Other members of the study group are listed below:

Technical Analysis	Applied Mathematics
G. Hare	J. Filliben
E. H. Short	A. J. Goldman
W. A. Steele	W. A. Horn
	J. Levy
	M. H. Pearl
	R. Traub

Particular responsibility for the present volume rests with Joel and Steele. Appendices B, C, and D are due to Levy and Pearl, Filliben and Hare respectively. We wish also to acknowledge many helpful consultations with W. F. Druckenbrod of the Technical Analysis Division.

2. FLOW CONSIDERATIONS

2.1 Problems in Analyzing Flow at Small Airports

In this chapter, the analysis of flow which was developed during our study will be described. The current section (2.1) notes some problems encountered in analyzing operations at smaller airports, and gives "schematic" accounts of these operations. Section 2.2 discusses several attempts made to formulate models of such traffic flow, and points out features of each formulation which mar its applicability. Then in Section 2.3, the model which we feel best balances the "costs" of increased solution efforts against the "benefits" of increased detail is described; solution efforts and numerical demonstrations are postponed until 2.4.

This chapter, then, concerns itself with measuring the performance of traffic handling procedures (with and without a VFR tower) in moving traffic-- and <u>not</u> their implications for safety, a subject which will be discussed in Chapter 3. Such a discussion must logically begin by considering what performance measure (or set of measures) is appropriate.

Conceptually, at least, we are talking about smoothness of operation, i.e., orderly flow of traffic. The lay notion of order is that everything is and remains in "its proper place". Thus the governing idea is <u>predictability</u>: the absence of surprises. This idea, though attractive because of its simplicity and perspicuity, is of limited use to us except as a mise-en-scène because of the extreme difficulty one encounters in attempting quantification. We pass therefore to explicitly numerical measures.

One measure which has been used [1] in the study of larger airports is "capacity" or "maximum throughput rate," which is defined by assuming a continuous demand for the use of facilities, and then counting the number of operations which can be handled per unit time. Obviously this measure can be useful only when the assumption of "continuous demand" holds over extended durations -- a situation which occurs at most a few times each year at the types of airports under consideration here (30,000-200,000 annual total operations).

A more meaningful measure, which is probably at least as descriptive is the length of time an aircraft expects to "wait" for service, i.e., the "average delay." The word "wait" is emphasized because unlike an aircraft ready to take-off which sits at the end of a runway doing what is unquestionably "waiting", a landing aircraft will go into one of several maneuvers designed to stretch out its flight until the facility is ready to receive it. Since some of these maneuvers closely resemble unaltered flight paths, it may at times not be apparent whether or not delay is being experienced.

Because path-stretching delay does not admit direct clear-cut identification and measurement, one must instead decide on some ideal or "no-delay" time as a benchmark to which to compare the actual elapsed time for the operation; the difference between the two is taken as a measure of the aircraft's delay. This ideal time might, for example, be the time which would be required for the aircraft to be processed if no other traffic were in the system and no ATC rules need be obeyed; alternatively, it might be the processing time with no other traffic around but the current ATC rules still in force. More generally, it could be the time to be processed when (a) the traffic is in some arbitrary but fixed state at the time of entry and (b) some stipulated ATC rules (not necessarily the current ones) must be adhered to. Different ideal scenarios might be useful for different analysis purposes; it is not obvious which (if any) deserves to be called "the correct" no-delay case.

For this reason, we have tried to model alternative traffic handling procedures (i.e. tower and non-tower), to test their abilities to handle identical traffic loads, and then to compare the relationships between the resultant average service times. This approach evades the need to decide arbitrarily what is and what is not delay.

To create a basis for discussing the relative merits of different models for evaluating aircraft-handling procedures, we will next describe these procedures themselves. More precisely, what will be described are "idealized" or "stylized" or "formalized" versions of these procedures. In practice there is considerably more in the way of local differences and individual pilot options than is indicated by the rigid step-by-step accounts given below; the FAA, recognizing the great diversity among airport situations and other factors such as the degree of pilot familiarity with the site, has chosen <u>not</u> to develop and enforce a set of stringent confining rules for the operation of aircraft in the vicinity of small airports. For analysis purposes, however, one must select <u>some</u> definite picture of the course of events to work with; one which is reasonably representative and realistic, at least to the point where results obtained with its aid are clearly <u>relevant</u> (even if not perfectly applicable) for understanding the somewhat fuzzily-defined mass of real-world alternatives. The formulation described below was chosen with care; it is based primarily on [2], [3], and conversations with both pilots and controllers.

<u>First</u>, the operating procedures for airports with FAA operated VFR towers:

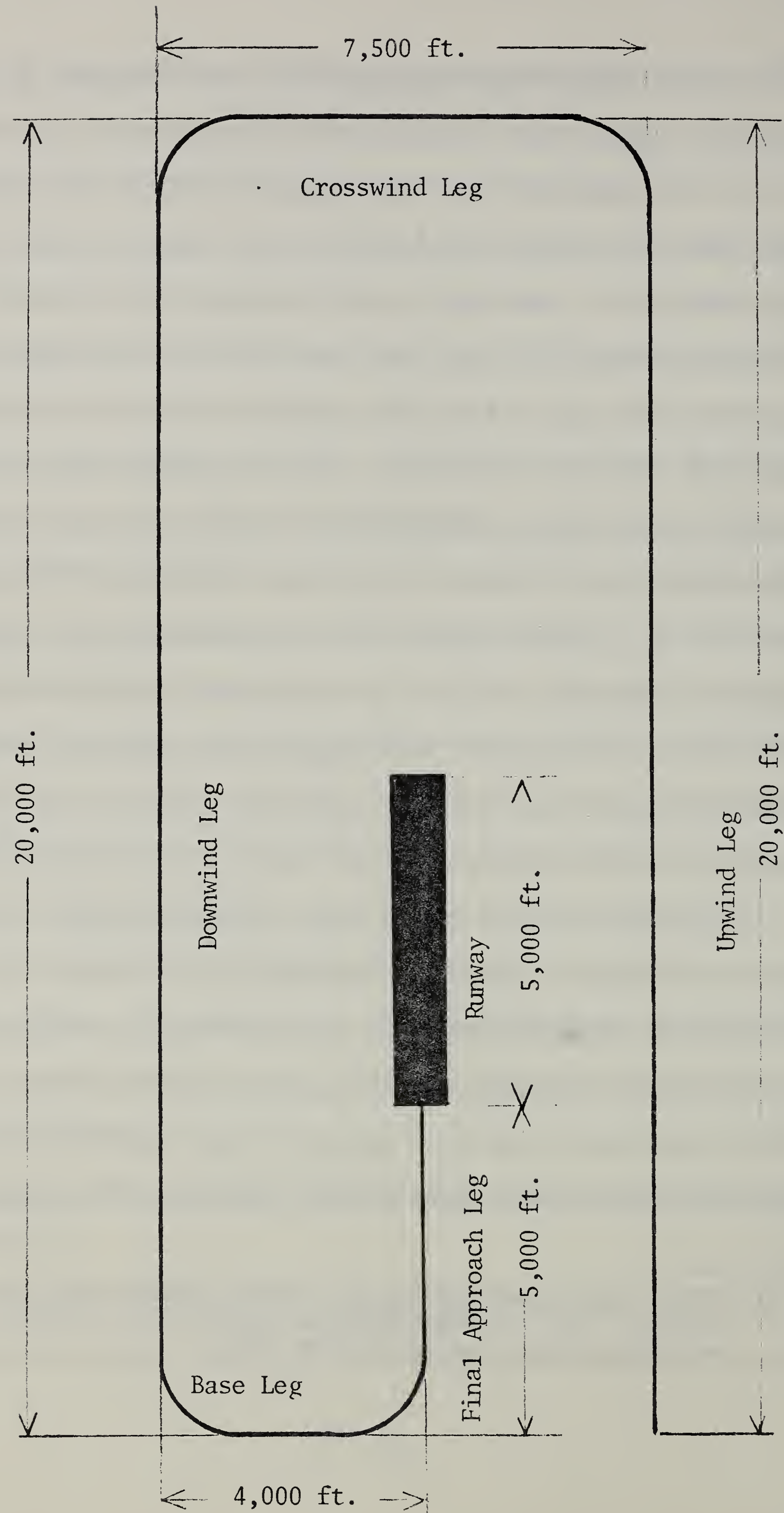
A pilot approaching an airport operating with a VFR tower is required to convey, by radio (a radio is required for aircraft desiring to use FAA-operated airports), his intentions (e.g. fly-over, land, touch and go) before he penetrates a cylinder 5 miles in radius and 2,000 feet high surrounding the airport. The controller, after considering (i) the request of the pilot, (ii) the pilot's position, bearing and speed, and (iii) the activity of other aircraft under the control of the tower, will provide directions to the pilot, which if followed, will provide the "best service" to him (i.e., fastest service subject to safety considerations and priority of earlier pilots). Such directions take the form of an approach heading to the airport, an entry to a flight pattern, or perhaps an alerting to geographical landmarks which aid in navigation. Since the controller theoretically has perfect information on all other aircraft in the control zone, he can clear the pilot for approaches which would normally seem precarious, such as "straight in on final approach".(1)

A pilot wishing to depart from a tower-controlled airport, after starting his engine and radioing the controller with a request for takeoff permission, will be directed to that point on the runway configuration which permits earliest departure. (This point may be nearer to the pilot than the head of the runway he would normally have used, thereby speeding up his departure.)

---

(1) Terms used in describing the traffic pattern are those defined in [3]. They are shown here in Figure 2.1.1, which has been reproduced from page 69 of that work.

Figure 2.1.1: A Standard Lefthand Pattern



After taxiing to this point, he waits for the controller to make sure that no other aircraft will interfere with his departure, and then takes off and follows the directions of the controller for clearing the control zone.

Second, for airports without FAA-operated VFR towers: (For the purposes of this study we ignore the few VFR towers which are not operated by the FAA; they do not have standard operating procedures, and also their small number could not significantly affect total national delay. Therefore, for us the only alternative to an FAA-operated VFR tower is no tower at all.)

A pilot performs a landing by (i) "passing over" the airport to get an idea of the runway in use and the current traffic pattern, (ii) "merging into" the traffic pattern and (iii) maintaining "safe separation" from other aircraft while flying the remainder of the traffic pattern and landing. Although this description appears quite ambiguous, it is probably more precise than operating procedures as the FAA defines them. As stated above, the reason for this vagueness is the great variation in pilot familiarity and usage at airports across the country (e.g., a stringent set of rules applied to a remote, little-used airport might cause great inconvenience and delay to users without appreciably increasing their safety).

A departing aircraft taxies to the end of the runway in use, waits until the pilot feels he can take off "without interfering with" either the next arriving aircraft or the last departing aircraft, takes off, joins the traffic pattern in the cross-wind leg, and stays in the pattern until the pilot feels he can safely leave it.

Before moving on to explore the models which might be helpful in analyzing these procedures, it seems advisable (for clarity) to elaborate on the four terms which were enclosed in quotation marks, in the last two paragraphs.

"Passing over" refers to the procedure pilots use to gain knowledge of operating procedures at the particular airport. For a pilot unfamiliar with the installation, this procedure might involve circling the field several times above the traffic pattern looking for a windsock, traffic pattern descriptors, the runway in use, and geographical landmarks which might aid in following the traffic pattern. An "old hand" at this airport, who had been flying touch and goes all afternoon, would, on the other hand, probably be able to enter safely on the downwind leg.

"Merging into" simply refers to finding two successive aircraft separated by a distance large enough to accommodate oneself, and then entering this gap.

"Safe separation," as its use implies, is a distance which the pilot of a following aircraft feels allows sufficient maneuverabilit to avoid an accident (it is analogous to the much-talked-about, but little-adhered-to, rule of one car length for each ten miles per hour in automobile driving).

Finally, "without interfering with," like "safe separation," requires a subjective decision about a pilot's skills, and therefore varies widely from one pilot to the next.

2.2 Candidate Models[2]

To the best of our knowledge, the problem of quantifying the effects of a VFR control tower on delay at small airports has received almost no study in the past. Our only record of a completed effort is that made by the Bureau of the Budget in [4]. Unfortunately, the technique used was the direct application of results obtained by Airborne Instruments Laboratory (AIL) in previous work [5] performed for the FAA, work based on a simple, Poisson-fed, constant service-time queueing model, which does not apply to the airports in question. (This will be explained further in our discussion of queueing models.)

The first candidate considered was a technique which has been employed in other studies of the ATC system (see [6] or [7]), that of an appropriately detailed Monte-Carlo simulation of the operations to be analyzed. The main advantage of this approach is its flexibility; the situation studied need not be distorted in order to fit it into a particular mathematical framework, since unreasonable idealizations of the real problem can usually be avoided by passing to a finer level of detail in the model (at a corresponding cost in complexity and data-requirements).

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(2) Some readers may prefer to skip over this section, which requires some knowledge of the mathematical techniques of operations research for full comprehension.

The three major drawbacks of this approach, however, were in the present case sufficient reasons for eliminating it from our candidate set. These drawbacks are: (i) the high cost of data-assimilation and model development (even the smallest of simulation models usually must be computerized to handle the complex interactions of system elements and to be in a form which permits easy repetitive use to obtain statistically valid results), (ii) the lack of a "closed-form" solution, which in turn requires (iii) for each numerical case of interest, a substantial and costly number of runs of the model to obtain a "sample size" adequate for statistical significance. Due to the resource constraints we faced, it was felt that an analysis technique with more modest requirements was indicated.

The second approach, that of [4], uses queueing formulas which have been transformed by AIL into a set of graphs, which can easily be consulted to obtain a delay figure for a given operation rate, runway configuration, mix of aircraft type, mode of operation (here VFR), physical properties of runways and landing-to-takeoff ratio.

However, several features of the AIL work rule out straightforward application to the current study. Most important of these is the fact that [5] does not distinguish between VFR operations at a tower-equipped airport and those at a non-tower airport, a distinction which is the critical one for our purposes. Also, the AIL material measures only delay caused by a facility's congestion, and so would not credit the tower with being able to use "short cuts" to speed up service when traffic conditions permit.

Despite these shortcomings (for our study) of the particular work in [5], it still appeared feasible that an analytical queueing model suitable for this problem could be found or developed. The basic scenario of queueing theory, that of "customers" arriving for, waiting for, and finally receiving "service," seemed a very natural one for representing operations at a small airport. We were encouraged in particular, by the recent work along these lines by Arthur D. Little, Inc., though this was aimed primarily at the larger "IFR airports." In particular, a rather innovative model(3) was developed which relaxed three of the usual restrictions limiting the applicability of previous work:

a) The requirement that mean arrival and service rates be constant over extended periods of time was relaxed to assuming that they vary with time but exhibit some periodicity (i.e., the pattern repeats itself every day).

b) The assumption that arrival and service rates be independent of the number of aircraft currently in the system was relaxed to allow for the fact that under congestion a controller may tend to speed up service (sometimes by "bending" ATC rules),

(3) See [8], Chapter 6. (Only earlier versions were available during our study.)

while pilots tend to "go away and come back later" for service.

c) Finally, the possibility of an unlimited queue of waiting aircraft can be avoided; a "largest reasonable number of waiting aircraft" can be specified, and arrivals which normally would add to the line are assumed to go elsewhere or return later.

Unfortunately this model (like AIL's and all other queueing formulations, to our knowledge) is incapable of letting a single server simultaneously handle several customers, each in a different phase of service. It is precisely this phenomenon -- that while one aircraft is flying the traffic pattern, preparing to land, several others can be involved in other stages of the same traffic pattern -- that caused us finally to abandon queueing theory as a preferred approach.(4)

A final technique which was considered at length was that of a Markov-chain model based on the concept of a "string of slots". This concept will be elaborated in the next section, but the general approach can be sketched as follows:

1) The traffic pattern (See Figure 2.1.1) is divided into a number of segments, the length of each being equal to a "standard" safe separation distance between aircraft.

2) Since each segment can contain at most one aircraft

(4) One _might_ succeed in portraying such situations using something like series-parallel networks of queues, but then the advantageous simplicity of "ordinary" queueing theory is lost.

there are 2^N possible arrangements of aircraft and empty slots within the string, where N denotes the number of slots.

3) Assume a time unit chosen so that an aircraft in the string would move up exactly one slot during each stage of the process. Suppose we know the probability of a new (aircraft) arrival to the system per stage, and also the probabilities that each of the slots is the desired entry point into the string for such a new arrival.

4) Then it is possible in principle, by solving a suitable set of simultaneous linear equiations, to compute the steady-state probability of each particular arrangement of occupied and empty slots, and from these to calculate the expected service time for an aircraft.

Unfortunately, even a string which contains only 20 slots would lead to a system of over one million simultaneous equations (with an equal number of unknowns) to be set up and solved. The programming effort and computer time required for such a gigantic calculation (one per case of interest) are orders of magnitude beyond the scale of this study. Thus the simultaneous-equations approach, the standard one for "solving" Markov-chain models, is hopelessly impractical in this case. There remained the possibility of developing an alternative solution technique for this particular mathematical model. Our work in this direction is presented in Section 2.4, but first the model itself must be described more precisely.

2.3 The String of Slots Model

This section treats a mathematical model, already sketched above, which appears most applicable to the study of delays at small airports with light traffic. As described here, the model deals with the landing process only. We cannot, without further testing, estimate the adequacy of its realism for delay analyses, but may note (a) that the existence of a computationally simple solution process (described below, and justified in Appendix A) is relevant in the typical modeling compromise between realism and tractability, and (b) that the type of approach presented here may prove extendable to more complicated and realistic versions of the model. The same or similar techniques may also prove useful in studies of moving sidewalks or other conveyor-belt-like systems.

The model focuses upon the part of the landing pattern prior to the final descent-to-runway. This portion, essentially a helical arc, is visualized as being unwound, "straightened out", and segmented into a string of slots as shown in Figure 2.3.1. Slot length (and hence the number of slots) is assumed chosen consistent with the requirement that each slot can contain at most one aircraft at a time. (In Figure 2.3.1, the "occupied" slots are designated by asterisks.) Since aircraft in fact enter the landing pattern at various points, the model permits entry to the string to occur at any slot, and not only at the last slot. Once in the string, a plane moves progressively from left to right, finally "vanishing" on the right (i.e., entering its final descent). Thus the pattern is a kind of fixed highway in the sky, but the point at which a given aircraft will attempt to enter it will depend on various factors, for instance the direction of approach to this airfield and the altitude and exit of preliminary surveillance pass over the airfield.

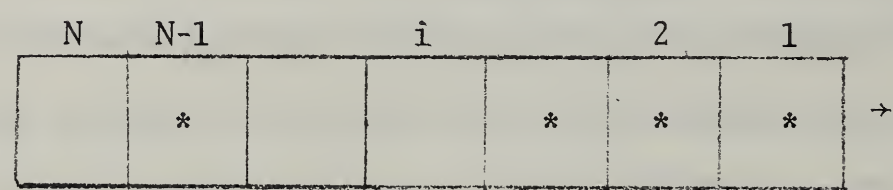


Figure 2.3.1 The "String of Slots" picture

Time is treated as divided into short discrete periods, denoted $t = 1, 2, \ldots$ The assumed sequence of events within any one period is as follows:

(a) Each aircraft in the string moves one slot to the right.

(b) Either a new aircraft arrives (with known probability α), or none does (with probability $1-\alpha$). The desired point of entry into the string, for a new arrival, is slot 1 with known probability p_1, slot 2 with known probability p_2, etc.

(c) If the new arrival's desired entry point is empty, entry is made there. Otherwise the new aircraft enters the first empty slot to the left of the desired one.

Several points in this description require some elaboration. First, note the "light-traffic" assumption that at most one new aircraft arises during a single time period. Note here, by the way, that α is constant arrival probability, not a constant arrival rate, i.e. in each time interval a coin is tossed to determine whether there is an arrival. If there is one, a "roulette wheel" is spun to determine where the entry will be attempted. Second, there is no danger that a new arrival will find no empty slot to receive "him"; after step (a) above, the left-most slot (slot N) is certainly free. Third, we emphasize that whether or not a new aircraft appears --- and the identity of its desired entry slot, if one does appear --- is independent of the current pattern of occupancy and non-occupancy of slots.

The fourth point concerns the interpretation of the particular sequence (a), (b), (c) chosen above. This description is equivalent to and easier to work with than the more complicated sequence we actually have in mind. In the latter, the new aircraft (if any) arrives at some intermediate stage in the forward shift of the aircraft already in the string, and regards a slot as "filled" if he projects that it would be occupied by the time he would enter it should he try to do so.

Apart from the number (N) of slots, there are two <u>input data</u> characterizing any particular instance of the model. One is the probability α of a new arrival, which may be interpreted as a <u>traffic intensity parameter</u>; we require for the mathematics only that $0 < \alpha < 1$, but in view of the underlying "light traffic" supposition, α-values in excess of (say) 1/2 would be unnatural. The <u>second</u> input is the probability distribution

$$\{p_i\}_{i=1}^{N} = \{p_1, p_2, \ldots, p_N\}$$

of desires for entry slots. These are non-negative numbers summing to 1; we assume $p_N > 0$ for technical convenience. In the illustrative calculations presented later, we shall vary this distribution in a manner intended to represent the potential of a VFR tower for "aiming" new arrivals toward entry points farther up toward final descent, with consequent reduction in delay.

The <u>output</u> sought from the model is the ensemble $\{e_i\}_1^N$, where

e_i = steady-state probability that a new arrival (if there is one) will enter the string at slot i.

These are of importance because entry <u>via</u> slot i implies a wait of i time periods before leaving the string of slots (i.e., before beginning the final descent), so that the expected value (mean) of the wait-in-string can be calculated from the e_i's as

$$E(W) = \sum_{i=1}^{N} ie_i, \tag{2.1}$$

a quantity which can perhaps be best interpreted relative to the corresponding value

$$E(W)_{min} = \sum_{i=1}^{N} ip_i \tag{2.2}$$

for a new arrival in the absence of other traffic. In addition to the mean (2.1), the associated standard deviation

$$\sigma(W) = [\sum_{i=1}^{N} i^2 e_i - \{E(W)\}^2]^{1/2} \tag{2.3}$$

is also of interest.

2.4 Analysis of the Model

The solution method to be developed involves the probabilities

$f_i(t)$ = probability that slot i is full (occupied) at the end of time period t,

$e_i(t)$ = probability that a new arrival, if one occurred in time period t, would enter slot i.

More precisely, it involves the associated steady-state probabilities,

$$f_i = \lim_{t\to\infty} f_i(t), \qquad e_i = \lim_{t\to\infty} e_i(t).$$

The existence of these limits, independent of the pattern of slot-occupancy at the beginning of the initial time period, will be verified in Appendix A.

The relationship between the f_i's and e_i's is determined as follows. After time period t+1, slot i (where i < N) is full if and only if one of the following two mutually exclusive events occurred:

(a) slot i was entered during time period t + 1, or

(b) slot i + 1 was full after time period t.

It follows that

$$f_i\,(t+1) = \alpha\, e_i\,(t+1) + f_{i+1}(t).$$

Transposing terms and letting $t\to\infty$, we obtain

$$e_i = \alpha^{-1}\,(f_i - f_{i+1}) \qquad (1 \le i < N). \tag{2.4}$$

Since slot N is full at the end of a time period if and only if it was entered during that period, (2.4) is supplemented by

$$e_N = \alpha^{-1} f_N \tag{2.5}$$

Combining the last two equations with (2.1), we find that the expected waiting time can be expressed in terms of the f_i's as

$$E(W) = \alpha^{-1} (f_1 + f_2 + \ldots. + f_N). \tag{2.6}$$

In other words, $\alpha E(W)$ is equal to the expected number of aircraft in the string.

To determine f_1, observe that every arriving aircraft does enter one and only one slot, so that

$$e_1 + e_2 + \ldots + e_N = 1.$$

Using (2.4) and (2.5) for substitutions into this equation, we find after multiplying by α that

$$(f_1 - f_2) + (f_2 - f_3) + \ldots + (f_{N-1} - f_N) + f_N = \alpha$$

and so

$$f_1 = \alpha, \tag{2.7}$$

a result easily checked by observing that the average rate of entry to the string of aircraft, all ultimately passing through slot 1, must equal the mean rate of movement through that slot.

We turn now to describing a method for determining the f_i's. If N=1, the full solution is supplied by (2.7). Let us assume that $N > 1$.

To determine f_2, we note that slot 1 is entered in time period t+1 if and only if (i) there is in that time period a new arrival desiring to enter slot 1, and (ii) slot 1 is free at the moment - i.e., slot 2 was free at the end of time period t. This leads to

$$e_1 = (1 - f_2)\, p_1. \tag{2.8}$$

On the other hand, (2.4) with i = 1 yields

$$e_1 = \alpha^{-1} (f_1 - f_2). \tag{2.9}$$

Equating the right-hand sides of (2.8) and (2.9) yields

$$f_2 = (\alpha^{-1} f_1 - p_1)/(\alpha^{-1} - p_1). \tag{2.10}$$

If N > 2, we would like to continue the solution process "in the same manner". In order to describe what this means, it is convenient to think of a new arrival which desires to enter slot j as <u>considering</u> slots j, j+1, j+2, etc. in succession until the first empty one is found. Let

$c_i(t)$ = probability that a new arrival during stage t (if there is one) will <u>c</u>onsider slot i,

and let c_i denote the corresponding steady-state probability. Then since slot i will be entered if and only if it is both considered and empty, we write

$$e_i(t) = c_i(t)\ [1-f_{i+1}\ (t-1)]; \tag{2.11}$$

use of the simple product form involves an <u>independence assumption</u> whose discussion is deferred to Appendix A. From (2.11) it follows that

$$e_i = c_i\ (1-f_{i+1}) \qquad (1 \leq i < N). \tag{2.12}$$

Equating the right-hand side of (2.12) to that of (2.4) yields an equation which can be solved for f_{i+1}; the result is

$$f_{i+1} = (\alpha^{-1} f_i - c_i)/(\alpha^{-1} - c_i). \tag{2.13}$$

Using the starting value $f_1 = \alpha$, this last relation permits f_2, f_3, ... to be calculated one by one, <u>if</u> the c_i's are known. Since

$c_1 = p_1$, the case i = 1 of (2.13) checks with (2.10).

It is still necessary to specify how the c_i's can be calculated. As just noted, the first of them is clearly given by

$$c_1 = p_1. \tag{2.14}$$

Next, slot i+1 is considered if and only if one of the following two mutually exclusive events occurs: (i) the new arrival desires to enter this slot, or (ii) slot i was considered but found occupied. On this basis we write

$$c_{i+1}(t) = p_{i+1} + c_i(t)\, f_{i+1}\,(t-1); \tag{2.15}$$

the product formulation on the right-hand side involves the same <u>independence</u> <u>assumption</u> noted earlier. From (2.15) it follows that

$$c_{i+1} = p_{i+1} + c_i\, f_{i+1}. \tag{2.16}$$

The solution process, (2.13) and (2.16), can be described as a step-by-step algorithm in a form suitable for computer programming (or desk calculation):

STEP 1: Set i = 1, $f_1 = \alpha$, $c_1 = p_1$.

STEP 2: Increment i by 1. If i > N, STOP.

STEP 3: Set $f_i = (\alpha^{-1} f_{i-1} - c_{i-1})/(\alpha^{-1} - c_{i-1})$. Also set $c_i = p_i + c_{i-1} f_i$. Return to Step 2.

In the remainder of this section, we seek to illustrate the usefulness of the string-of-slots model in the analysis of delay at airports. For this it is necessary (a) to select a set of alternative operating procedures to be investigated, (b) to select values for the model parameters which will represent each of these procedures, and (c) to apply the model and observe the differences in average service time (for a variety of traffic loads) for the different procedures.

The choice of operating procedures to be compared was obvious -- namely, tower and non-tower -- but the selection of model parameters to represent these procedures was not. We chose N, the number of slots, to be the same for both cases; because of the "constant time to traverse a slot" assumption, this implies traffic patterns of equal length. While this was true in a formal sense, the values of the p_i's were (as will be detailed below) chosen for the "tower" case so that the pattern's "effective length" was shorter in this case, i.e. the tail end of the string was almost never used, corresponding to the notion that a pilot requires less information-gathering circling of the airport when aided by a VFR tower.

The particular numerical value chosen for N was arrived at by using a traffic pattern similar to that described in Section 3.5. This is a standard left-hand traffic pattern with total length of about 58,000 feet. Assuming a minimum separation of 3000 feet, 20 slots would be able to cover the traffic pattern (plus an additional 2,000 feet on the end). Hence our choice of

$$N = 20.$$

The general idea guiding the selection of the p_i's for the two cases was that the tower's presence resulted in aircraft being able (safely) to enter further "up" in the string. Of course this is too indefinite to single out a specific set of p_i's. Another difficulty is that the p_i's refer to desired (rather than actual) entry points, and so would not be directly available from recorded data, if they do exist.

It was decided that the "general idea" just mentioned could be conveniently expressed by choosing the p_i's in geometric progression, i.e.

$$p_i = kr^{i-1} \qquad (0 < r \leq 1) \tag{2.17}$$

where k is chosen so that the p_i's will sum to 1, i.e.

$$k = (1-r)/(1-r^N) = 1/(1+r+r^2+\ldots+r^{N-1}). \tag{2.18}$$

To represent non-tower airports (or more generally, the absence of a "helping hand" in gaining entry to "early" slots), we chose r = 1, yielding the uniform distribution

$$p_i = 1/N \qquad (1 \leq i \leq N). \tag{2.19}$$

The smaller the value of r, the stronger the "helping hand"; to represent the "tower" case we chose the illustrative value r = 1/2, approximating (2.17) and (2.18) by

$$p_i = 1/2^i \qquad (1 \leq i < N), \tag{2.20}$$

$$p_N = 1/2^{N-1}. \tag{2.21}$$

At this point, it is convenient to interject (for later reference) the following approximations. Call a value of α <u>effectively small</u> if, when that value holds, an arriving aircraft will with very high probability be able to enter the slot it desires, and so will enter slot i with probability p_i. In such cases

$$E(W) \approx \sum_1^N i p_i, \tag{2.22}$$

the right-hand side being the minimum possible value of E(W) for the given p_i's (it is the limiting value as $\alpha \to 0$). For the uniform distribution ($p_i = 1/N$), it follows that

$$E(W) \approx (N + 1)/2, \tag{2.23}$$

while if the ratio r in (2.17)'s geometric progression is not too close to 1, then (2.22) yields

$$\begin{aligned} E(W) &\approx \textstyle\sum_1^N i\, k\, r^{i-1} \\ &\approx k \textstyle\sum_1^\infty i\, r^{i-1} \\ &\approx (1 - r)(1 - r)^{-2} = (1 - r)^{-1}. \end{aligned} \tag{2.24}$$

The only model parameter still to be prescribed is the traffic intensity parameter α. Values of α can be calculated from the relation

$$\alpha = (\Delta t).\lambda \tag{2.25}$$

where

$$\lambda = \text{arrival rate at the airport,}$$

$$\Delta t = \text{length of time an A/C occupies a slot}$$

$$= \text{"stage duration" of the model.}$$

The first factor, Δt, is determined by the separation distance and the velocity of the aircraft. Assuming a nominal speed of 100 knots (which we feel is representative of the aircraft under consideration), and again using 3,000 feet as the minimum separation distance, we obtain

$$\Delta t = 0.296 \text{ min.}$$

To illustrate the sizes of the mean service times: over the range[5]

$$0 \leq \lambda \leq 50 \qquad \text{(A/C per hour)} \qquad (2.26)$$

we find, using (2.6) and the f_i's calculated by the method described above, that for (2.17) with r=1/2, E(W) varies over the range (in min., obtained by multiplying by Δt),

$$.59 \leq E(W) \leq .62 \qquad \text{("tower"),} \qquad (2.27)$$

while for (2.19) it varies over the range

$$3.11 \leq E(W) \leq 3.15 \qquad \text{("non-tower").} \qquad (2.28)$$

Thus we see an average difference of roughly 2.5 min. The "effectively small" approximations (2.24) and (2.23) yield

$$E(W) \approx 0.59 \text{ min.,} \qquad E(W) \approx 3.11 \text{ min.,}$$

---

(5) The value of 50 was obtained from Appendix C of [9].

in good agreement with (2.27) and (2.28) respectively.

A more meaningful pair of results, unfortunately somewhat more difficult to compute, are the total annual service times (mean values) with and without a tower, for arrivals at an "average airport", in terms of the number of annual arrivals at the airport. A mathematical formula for this quantity is found as follows. Let

L = annual number of arrivals at the airport,

$\lambda(L)$ = maximum hourly arrival rate for "average" airport of activity-level L,

$H(\lambda,L)$ = number of hours during year when "average" airport of activity-level L has hourly arrival rate λ,

so that

$$\int_0^{\lambda(L)} H(\lambda,L)\lambda d\lambda = L. \tag{2.29}$$

Furthermore, let

$\overline{W}(\lambda)$ = value of average service time $E(W)$ for α as given by (2.25),

$T(L)$ = mean value of total annual service time for arrivals at "average" airport with activity-level L.

Then $T(L)$ is the quantity we are trying to evaluate, and it is given by the formula

$$T(L)= \int_0^{\lambda(L)} H(\lambda,L)\ \lambda\ \overline{W}(\lambda)\ d\lambda. \tag{2.30}$$

Because $\overline{W}(\lambda)$ is an increasing function of λ, we have [6]

(6) Here $\overline{W}(0)$ denotes the limit of $\overline{W}(\lambda)$ as $\lambda \to 0$.

$$\overline{W}(0) \leq \overline{W}(\lambda) \leq \overline{W}(\lambda(L))$$

in (2.30), and thus

$$\overline{W}(0) \int_0^{\lambda(L)} H(\lambda,L)\lambda d\lambda \leq T(L) \leq \overline{W}(\lambda(L) \int_0^{\lambda(L)} H(\lambda,L)\lambda d\lambda,$$

which by (2.29) gives

$$L\overline{W}(0) \leq T(L) \leq L\overline{W}(\lambda(L)), \qquad (2.31)$$

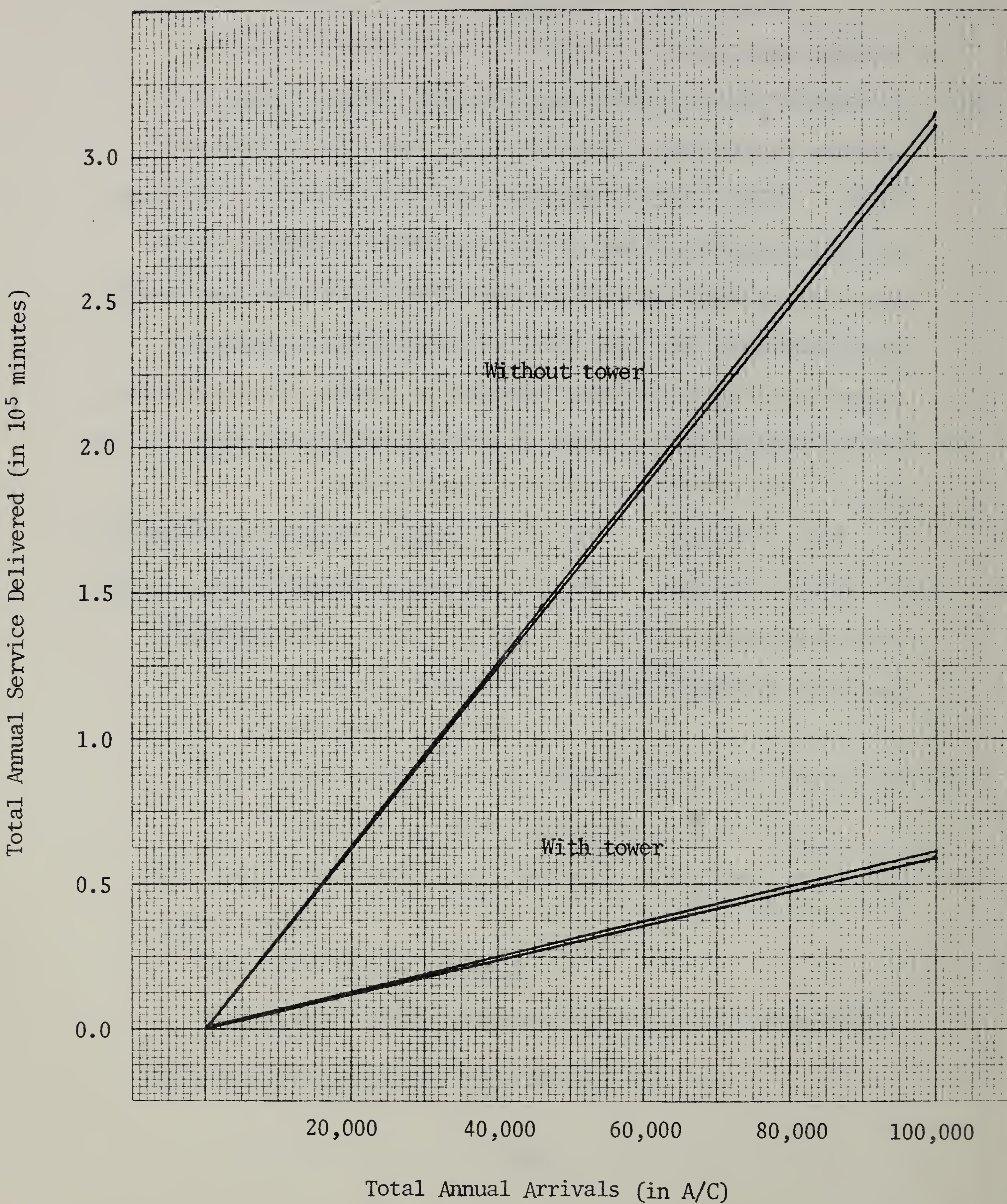
thus bracketing the value of T(L). Appendix C of [9] shows that for the range of activity-levels (i.e., L-values) under discussion here, the "peak hour of a sunny August afternoon" figure $\lambda(L)$, for an "average" airport, is at most 50 arrivals/hour, so that (2.26) applies. Thus (2.27) and (2.28) also apply, i.e. we have

$$\overline{W}(0) = .59 \text{ min.} \quad , \quad \overline{W}(\lambda(L)) - .62 \text{ min.} \quad (\text{"tower"}), \qquad (2.32)$$

$$\overline{W}(0) = 3.11 \text{ min.} \quad , \quad \overline{W}(\lambda(L)) - 3.15 \text{ min} \quad (\text{"non-tower"}), \qquad (2.33)$$

so that the "bracketing" in (2.31) is tight. The situation is portrayed in Figure 2.4.1.

Figure 2.1.1: Total Annual Time in Service



3. SAFETY

3.1 The Direct Approach

The objective definition of Safety in the context of airport operations is not simple. Naively we may identify this notion with the absence of "accidents", but this last term is less clear than it may at first appear; indeed some writers [10,11] recommend that the term be abandoned.

Because of our specific interest in the role (or roles) of VFR towers relative to safety, we may follow Gansle [12] in partially circumventing this definitional problem by restricting attention to "tower-preventable accidents." This category too, is somewhat fuzzy without further specification; e.g. a tower cannot prevent a mechanical failure on board an aircraft, but may be able to minimize its harmful effects by "stand clear" advisories to nearby planes or by early visual detection of the malfunction (as in the failure of landing gear to descend). But _if_ the category can be defined in a suitably explicit way, then the _direct approach_ to the task addressed in this chapter would be to develop and apply yardsticks of an actuarial variety to measure the objective probabilities of tower-preventable incidents (collisions, casualties, etc.), both with and without a tower, at given levels of airport activity. _If_ this measurement could be accomplished satisfactorily, we would have succeeded in ascertaining quantitatively most of the local safety benefits of VFR towers in normal airport operation. As noted in [4, 12], there would remain a residue of tower activities, such as aid to lost pilots and emergency traffic control after disasters, which are clearly relevant to safety though lying outside the scope of the indicated measurement scheme.

Let us examine this direct measurement approach, and past experience with it, more specifically. The most optimistic procedure would involve two regressions of (tower-preventable) incident levels versus airport activity levels: one regression for airports with VFR towers, the other for non-tower airports. Examination of the two resultant functional relationships, if they are amenable to validation, could obviously provide information relevant to determining the appropriate activity-level criteria for an airport to be eligible for tower installation.

The dependent variable for such analyses, i.e., the index of accident-level, has generally been taken to be the level of mid-air collisions (MAC's) in the vicinity of the airport. This restriction is easy to explain. Incidents on the ground such as collisions between taxiing aircraft or air-ground collisions, while putatively as tower-preventable as MAC, have consequences in terms of injury or damage likely to be small relative to those of MAC's. We feel, incidentally, that because these accidents occur with frequencies of the same order of magnitude as MAC's, are frequently explained in the same conventional terms (i.e., "failure to see and aovid"), and seem subject to amelioration by the same procedures which are generally mentioned as MAC remedies, they should be considered in defining nominal safety levels. Enroute collisions occurring remote from airports cannot be regarded as "tower-preventable"; similarly, major incidents during landings or takeoffs which result from mechanical malfunctions cannot be construed as related to VFR control procedures or other VFR tower functions.

The independent variables, i.e. the indices of airport activites, are generally limited by data avilability to 2 figures: the approximate annual numbers of itinerant and total operations. For non-tower airports, even these gross statistics are regarded as subject to fairly wide uncertainty. And as noted in [12], these particular variables may be quite inadequate in "explanatory value" without more information on the "peaking" of activity.

The three most common reasons for failure of this approach are:

(1) insufficiency of data to overcome the distorting effect of chance "outliers" among the observations;

(2) an actual functional dependence more complicated than those suspected and tested, or

(3) the actual lack of existence of a valid functional dependence of the "dependent variable" (e.g., mid-air collision rates) upon the proposed set of "independent variables" (here, airport activity level, and tower presence or absence).

Regarding (3), note that even if traffic volume and tower-or-not status are assumed major determinants of the accident rate, a number of auxiliary "explanatory" parameters may also be required to secure a successful formal relationship. Possible prime candidates include: proximity to cities; proximity to large airports; degree of scatter in the azimuth-distribution of ambient air traffic.

Gansle [12] attempted such a regression, using collision data edited to omit incidents readily identifiable as atypical. He tried 6 different functional forms, with not completely satisfactory results in all cases.

In such cases --- i.e., when regression fails to reveal a mathematical relationship through which the influences of the independent variables on the dependent one can be measured -- fallback to a less ambitious goal may be in order. Namely, by applying appropriate statistical tests to the data it may at least be possible to say with some assurance that the independent variables (in particular, the presence of a VFR tower) do influence the dependent variable in a statistically significant way.

Using 1961-1968 statistics on collisions and traffic levels, Wirt [4] and Gansle [12] have carried out such analyses, arriving at differing conclusions. Wirt asserts that the level of collision risk is not affected by the presence or absence of a VFR tower; Gansle reaches the "Scotch verdict" that such an effect is not proven in a statistical sense from the available data (in particular, in view of the small number of "observations" due to the rarity of MAC's near airports). It is our impression that Wirt's actual analysis, as distinguished from his formal conclusions, is in fact quite consistent with Gansle's findings.

During the present study, we have attempted additional analyses employing both standard and distribution-free statistical techniques. As detailed in Appendix C, the latter succeeded in rigorously demonstrating an association between lower collision rates and "tower airports" (as distinguished from those without a VFR tower). A causal interpretation of this finding (i.e., "towers reduce risk") can of course be advanced only with caution, because of such possible complications as the following: The mix of pilots using non-tower airports may differ systematically, in the "accident-proneness" of their flying styles and flight purposes, from that of tower-airport users; installation of a tower at a given airport may induce part of the riskier element in the mix to transfer their activities to some other (non-tower) field, thus intensifying the above-mentioned difference even farther. Thus the data comparisons showing lower collision rates at VFR-tower airports would be harder to interpret, since they would no longer correspond simply to introducing a tower into a "homogeneous" population of pilots.

As noted above, at least part of the problem in reaching conclusions that will withstand the statistician's crucible is the "small-sample" implication of the (fortunate) scarcity of relevant MAC's. Because "hazardous near mid-air collisions" (NMAC's) are considerably more frequent, it is natural to consider the possibility of (a) establishing a functional relationship between hazardous NMAC rates and MAC rates, (b) replacing the meager MAC data by the more ample NMAC figures as dependent variable in the above-mentioned analyses, and (c) combining (a) with (b) to obtain the desired inferences about the variation of collision rates with traffic level at tower and non-tower airports. In principle, this is merely equivalent to "rounding to zero" the closest-approach distance of two aircraft, so long as it lies below the threshold which qualifies the encounter as a near miss.

Part (a) of this tactic is in fact treated in [14], where a theoretical ratio of MAC and NMAC rates is deduced, based on the fraction of a sphere of "NMAC-distance" radius which would be occupied by an aircraft of typical dimensions concentered with the sphere. The results appear consistent with 1968 MAC and NMAC totals. (A very similar analysis appears in Section 5.4 of [8].) Moreover, the accuracy of the annual NMAC compilations should be enhanced by their inclusion of an ingenious method (described in the 1968 NMAC Report [15]) for correcting the estimate of the actual number of incidents to allow for unreported ones, given the number reported by only one pilot and the number reported by two or more.

While this line of attack (use of NMAC data) appears promising, its limitations should be recognized.(1) Collection of NMAC data was

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(1) One would encounter additional difficulties here if our previous suggestion, that encounters of interest be extended to include ground-ground and air-ground collisions, were adopted.

initiated only 2 years ago, so that establishing the stability over time of postulated relationships will not be possible for a while. Cool accurate estimating of miss distances,(2) by the pilots involved in such potentially critical situations, seems rather much to expect(3) (cf. [15]). The hypothesized ratio of MAC to NMAC rates may not be valid in the somewhat more special environment near an airport.(4) And of course, the problems of better identifying tower-preventable incidents, and of securing more reliable and meaningful information on airport activity levels, would remain.

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(2) The NMAC report separates hazardous NMAC's into two classes: critical, in which avoidance of actual collision is due to chance; and potential, in which a collision might have occurred except for evasive action by the pilots. In either case the "miss distance" is a major determinant.

(3) For incidents near VFR-tower airports, the tower observers may have a useful role to play in this regard.

(4) In fact [16] in contrast with [8] and [15], states that in general there is <u>not</u> a significant correlation between MAC's and NMAC's.

3.2 The Tower as Sentry and Information Source

As the considerations presented in the last section became clear, they led to the decision that it would be unprofitable for the present study to place major emphasis on further efforts along the lines of the "direct approach". The search for "mere" statistical correlations, without investigating the causal chains involved, appears unpromising at least for the present; it seems necessary to consider more specifically what the VFR tower has to offer in particular situations of potential hazard. The answer in most cases is information, and our main thrust in the balance of this chapter will be on the role of the tower as an information source. It will be convenient, however, to approach this subject from the flank, digressing first in order to dispose of another topic, namely the role of the VFR tower as police sentry.[5]

Although recreational aviation is in principle accessible to a large fraction of our affluent society, there remains a mystique about flying as a dangerous and exhilarating kind of activity, reserved for a "special breed". It is not for the timorous, and pilots are likely to be men of strong self-confidence. It is to be expected that more than an infinitesimal fraction of such men would tend

(5) The reader will recognize that the following discussion is well spiced with conjecture and speculation.

occasionally to violate procedural safeguards, on the supposition that these rules cause excessive delay and are "really" intended to be binding only on fliers of less skill than they. A VFR tower, even without any authority of enforcement, stands as conspicuous reminder that a pilot's judgments are subject to observation and ratification. It has an effect on most pilots' actions analogous to that of the "radar in use" road sign (which slows down even local motorists who know that the implied threat is, in fact, vacuous). Of course for some fraction of pilots the absence of a tower will serve to inhibit reckless behavior.

A corollary is that the installation of a tower should (as one effect) increase both the subjective and actual security levels of users, pilots being confident that it will inhibit "some nut from cutting in on me in the pattern", or will at worst provide a warning if the nut is not deterred. In addition, beyond the admonitory effect of the tower, the instinct of unruly pilots to conform to majority behavior patterns will be reinforced by the changed deportment of other former mavericks.

The tower's sentry role, resulting as it does in a more orderly flow of traffic, is alter ego to its role as information source. We can distinguish two senses in which this is true. First, the presence of the sentry conveys to a pilot the assurance of a higher probability that the aircraft nearby will conform to the "rules of the road"; this is in itself valuable (implicit) information, influencing how the pilot need divide his attention between surveillance and other tasks.

Second, increased orderliness may be essential to reduce to a reasonable level the load of explicit information-handling needed to provide adequate protection; the amount of information needed to describe the exceptions and deviations arising in a basically regular flow pattern is enormously less than that required to characterize a less systematic set of trajectories.

The relationship between "orderliness" and "safety" is a fascinating and difficult one. The correlation between them is so obvious that there is a temptation to equate the two, but this is simply not correct. Suppose for example that every second plane in a landing pattern proceeded full throttle toward the plane ahead, while the target aircraft continued serenely along their course. This situation is highly "orderly", in that its further evolution is entirely predictable, but one would scarcely term it "safe".

As a next attempt at elucidating the relationship, it can be pointed out that mass madness (as in the preceding example) is really not the danger. The critical point is that if a localized difficulty occurs in an orderly pattern, then corrective procedures involving just a few aircraft can be formulated on the basis that the remaining planes' movements can be both "counted on" and readily extrapolated.

This explanation, too, cannot be the whole story; if the original "orderly" pattern were a crowded one, a "safe" set of corrective maneuvers in response to a local difficulty would still be hard to find. By way of analogy, we may observe that even if the police presence imposed compliance with the laws, that would not assure the public safety unless the laws themselves were well-designed to achieve such a goal. Without attempting to probe this complex topic any further, we can at least say that the "sentry's" role in promoting compliance with an orderly flow pattern characterized by adequate separations is a positive contribution to safety.

It is a contribution, however, which is hard to quantify. A very rough start can be made by comparing recorded MAC and NMAC rates with those that would be expected from very "disorderly" flows, i.e. patterns of movement that are "random" in some appropriate sense. Numerous studies of this type have been made. Borrowed in concept or detail from the physicist's kinetic theory of gasses, they range from analyses of the expected number of collisions per unit time of randomly moving particles in an enclosed volume as it depends on their number and velocities [8, 14], through simple planar gas models based on the idea of altitude separation [17, 18] to fairly sophisticated ones involving random motions interspersed with controlled ones in various specifically-designated paths or otherwise restricted portions of airspace simulating enroute flight [8, 18-20], [9] or terminal operations [8, 21]. Such models at the very least can be useful in determining bounds on the expected number of MAC's under simple assumptions.

We now return to a point briefly mentioned earlier, namely the assurance provided a pilot by the tower's presence, and its consequences as to how he can allot his attention [22]. Tension under stress is universally acknowledged to be a causative agent in collision, so that reducing this stress (when unnecessary) is also a contribution toward collision prevention. Quantifying this contribution, too, is difficult, but it should be possible to measure fairly precisely the increments in time made available to the pilot for other chores because the tower permits a lessening of his own scanning activities; moreover, the associated diminution of tension and increase in confidence may be susceptible to estimation through appropriate physiometric and psychometric techniques.

There is, however, another facet to the question of the relationship between order and safety. It stems from the general notion that "order" and "flexibility" are somewhat antithetical principles; e.g. the more efficiency in its "normal" enviroment of "well-behaved" random perturbations is achieved for a system by clever design, the more disruptive may be large disturbances of unexpected kinds.In the present instance, the assurance engendered by the tower's presence and the availability of its warning services (perhaps "beefed up" by more observers and improved equipment) might induce in some pilots a reduction in scanning so great that should a warning of emergency come, it would arrive as a bewildering "total surprise" which could not be reacted to promptly.

We may ask, then, at what level (if any) is subjective security too <u>high</u>? That is, is there a risk of surrounding pilots with an amniotic-like situation to the extent that they are lulled into carelessness about events outside their own craft? At least one study (on radar operators) [23] which has come to our attention indicates that vigilance is seriously affected by long periods when the task appears unnecessary. This may not be a factor in a pilot's performance -- landings and departures may be intrinsically stressful enough to keep adrenal secretions and general alertness up to snuff -- but it is worth noting as a possible problem area.

A second aspect of the confrontation between flexibility and order appears to explain the phenomenon, noted in [4] and [12], of the temporary decrease in local air traffic following installation of a VFR tower. Some pilots tend to avoid tower airports because orderliness, although conducive to safety, does not necessarily promote maximum expeditiousness of flow, the latter requiring freedom to exploit temporary features of the "tactical" situation. Thus, while our earlier language may have suggested a picture of the tower as benignly inhibiting the rash behavior of arrogant pilots, we must in fairness present a second picture as it may be perceived: that of possibly unnecessary constraints placed upon experienced and generally prudent pilots (e.g., flight instructors), by the introduction of monitors at least implicitly enforcing formal separation rules and substituting an imposed structure of flight operations for the pilots' own smoothly-functioning (though informal) system.

3.3 Contributions of Tower Observations to Risk Reduction

At this point we set aside the VFR tower functions of traffic control and the relaying of general information. The sole concern here will be the tower's responsibility to observe its surroundings and to report the relevant findings to nearby aircraft. In this capacity, the tower provides a recognizably substantial augmentation of the pilot's own visual resources. For specificity we sketch three distinguishable elements of this augmentation:

(a) Firstly, the mere presence of a supplementary pair of eyes clearly increases the critical information available to the pilot. Such an increase is evident even in the two extreme cases which might appear "least favorable" for such a contribution. The first of these extreme cases is the situation in which a tower observer is continuously scanning precisely the same points in space as a pilot and hence, apparently, not adding anything to the pilot's "information system." But in this case the tower can obviously provide confirmation of hazards detected by the pilot, in addition to a possibility of earlier detection. Moreover, the tower can reduce the effect of panic action due to false alarms (although the tower controller may not know the cause of the pilot's apparently irrational maneuver). The second extreme case is that in which the tower observer is totally absorbed in scanning a sector diametrically opposite in direction to that from which an aircraft is approaching the airport. While this would usually not add to the pilot's set of relevant data, there is certainly a positive probability (however small) that some crisis presenting a sudden hazard to the aircraft -- say, a helicopter out of control -- will in fact originate in the airspace not normally of interest to the pilot [24, 25].

(b) The "eyes" provided by the tower are not merely additional to those in the aircraft, but also are intrinsically or potentially superior in a number of ways: The tower normally is or can be manned by several operators, so that irrespective of other tower duties surveillance can be maintained continuously and attentively. The tower's field of vision is panoramic compared to that of an aircraft's pilot or pilot-copilot pair. The tower is not in the disadvantageous situation of observing from a changing frame of reference; it is not subject to involuntary motions due to various kinds of turbulence [26]; it provides a stationary observation platform from which objects can be located much more precisely relative to terrain features. The tower observer is in a better position to persevere when subject to sun glare, since he can move about in the tower room to secure a better vantage point. He can magnify his vision by the use of binoculars(6), in particular high-powered ones in firmly anchored pivoting mounts; he can improve his estimation of aircraft position and/or direction by using an angular measuring device (e.g., transit or sextant)(7) [27-29]

(c) Lastly, the tower as a communication hub for local traffic can receive continuous reports of pilots' intentions and can transmit them to other pilots when the advisability of doing so is indicated by the tower's observations. (The basis for selective dissemination of information, indicated by the underlined clause, distinguishes the

(6) A copilot, when there is one, can of course also use binoculars, but then the disadvantages of motion are intensified.

(7) [27] mentions a contrary point: altitude estimation is somewhat more difficult from the ground than from the air. This may not be germane in considering elevated towers.

VFR tower's role in this respect from that of an FSS, which after all is also a communications center and information source for air traffic.)

These contributions to risk reduction are not easy to quantify. There is experimental verification [27, 29, 30] of the relatively greater difficulty, in a moving aircraft, of identifying objects as "threats" by their direction of motion. Other controlled experiments and analyses [27, 31, 32] have shown that a subject will frequently fail to achieve sufficiently prompt detection (8) of an aircraft on a collision course with him, even when the subject has been forewarned of such threats. And our background reading disclosed studies which claim that unless a pilot is kept continuously apprised of the locations and velocities of potential threats, his visual surveillance is not likely to provide adequate protection. [25, 28, 31, 33, 24].

In what follows, we shall be concerned with assessing how much pertinent information, in excess of what the pilot can acquire unaided, can be made available to local traffic through the resources of a VFR tower.

The proposed indices of tower benefit in the present context are then, roughly speaking, the ratio of pilot-developed relevant data to total relevant data in each of the situations of interest. This requires assumptions as to the degrees of pertinence of various categories of data. It requires investigating the extent to which the pilot's own efforts leave deficiencies in critical information (the first phase of one such assessment is reported later). And, as stated, it

(8) Early enough for unequivocally effective evasive maneuver.

involves the implicit assumption that a tower would be capable of furnishing the full balance of the desired data. This last assumption, though perhaps not strictly true, appears warranted as an approximation except in quite unusual cases (e.g., a sudden northward-moving small rain squall enveloping a tower and interfering with observation while not affecting mutual observation of craft in traffic two miles to the south); in fact, the assumption appears to weaken roughly as the difficulty of measuring pilot-generated information increases.

The main conceptual problem in evaluating such an index is that of how to measure "amounts of information", not in the antiseptic context of mathematical information theory (which abstracts from the content and significance of the messages involved), but rather in the setting of pertinence to the avoidance of aircraft hazard. It is natural, therefore, to seek to classify or rank types of information according to the anticipated difficulty of this measurement problem. Our own efforts along this line led to recognizing a fundamental two-way classification which overshadows and logically precedes any finer distinctions. The two broad categories of data thus identified are:

(1) "Visibility-related" information, reflecting the capability of a tower observer to detect aircraft and other potential hazards which are obscured from a pilot's vision. [28, 30].

(2) "Identification and tracking" information, corresponding to the greater impediments to acquiring and correctly processing visual data, which face a moving observer subject to distractions, as opposed to a stationary viewer with such scanning as his major responsibility.

The contrast between (1) and (2) lies in the simple fact that what is "seeable" (i.e., within the physical field of vision of the observer) can nevertheless be overlooked ("missed") or misinterpreted. For example, a pilot in a moving plane, distracted by the necessity to fly the airlane, may not be in a position to assess the directions and altitudes of other aircraft (or even to note their presence) as easily as can a man in a tower; moreover his knowledge even of his own position and attitude can be impaired by faulty instruments or by misleading referents such as a tilted cloud bank, whereas the tower has supplementary visual clues [24, 27-30].

As indicated above, these two classes differ radically in the difficulty of their associated measurement-of-information problems. A comprehensive assessment of benefits from VFR towers (and other elements of the Air Traffic Control System) must ultimately come to grips with the second category, i.e., must attain a reliable quantitative understanding of the factors and processes which preclude a pilot's achieving early recognition and continued alert "tracking" of all threats in his environment. For the moment, however, we can do no more than observe that a serious effort toward this goal would involve integrated experimental and data-gathering programs far transcending the scope of our present study. Such efforts, aimed at predicting the probability and rapidity of threat detection and establishing its precise relationship to collision avoidance(9), would entail extensive investigations of psychophysiological questions, of

(9) We are aware of a substantial body of literature on the analysis of warning times, etc., in connection with research on detection and warning devices. We have scanned but not read some of these papers. Some of the references we have cited contain some material in this area.

equipment-reliability questions, and no doubt of questions in a variety of other fields as well.

On the other hand, "visibility" considerations permit relatively rapid and inexpensive study. They clearly form an important, indeed an essential building-block for the broader type of program mentioned above, and so constitute a natural point of penetration into the larger analysis area. As will be seen, meaningful numerical measures of the physical impediments to "see and be seen" can be formulated, and can be estimated with the aid of fairly straightforward computer models.

We feel that a visibility model, besides being a refining component of a general probability-of-collision model such as Graham's [14] provides the following collateral benefits:

(a) Analysis of the performance of piloted aircraft, given measures of available visibility, can be used to evaluate standards for instrument-provided information.

(b) The cockpit geometry model of Section 3.4, in particular, can be used to develop a standard by which to evaluate specific aircraft designs

(c) On the surface at least, radar visibility is very closely analogous to ocular visibility, being strongly limited by line of sight considerations, and operating in such a way that all the varieties of interference that we have identified have electromagnetic couterparts. Thus the model configuration when completed may be easily adaptable to determining measures of radar detectability equivalent to those for visual detectability.

Note however that we share the reservations of Ernst, that a very extensive model is almost never worth its total cost.

We therefore proceed to enumerate some readily identifiable sources of visual obstruction in flight. Many have been cited as causal factors in the accident descriptions listed in the survey reports, or in articles contained in the flight periodicals, which comprised the bulk of background readings in this study. The enumeration is without explanation except where the entries are not self-evident, and includes terms from both categories of data defined previously:

Cockpit geometry.

Structural elements of the aircraft (e.g. wingstruts, wings).

The sun, particularly in early morning or late afternoon.

Sun reflections, particularly glare on snow, sand or water.

Windshield refraction and parallax at "steep" angles.

Dirt on aircraft window surfaces.

Mountains.

Elevated structures (including, ironically, airport towers).

Heat shimmer.

Smoke.

Meteorological phenomena such as mist, rain, clouds, lightning, snow, haze, hail; electromagnetic anomalies such as fireballs, St. Elmo's fire and the like. These phenomena are not customarily associated with VFR conditions, but all of them may be found in circumstances which qualify as VFR.

Other aircraft.

Groundcover variegation (particularly piebald effects).

Shadows.

Birds, singly or in flocks.

Overcast, which decreases contrast.

Some of these act as distractions as well as, or rather than, direct obstructions. Moreover, the shielding effects of some frequently exceed their dimensions because of edge diffraction. To appreciate the significance of seemingly inconsequential obstructions, say dirt specks on cockpit windows, recall that at a distance of two statute miles a sphere of 50 foot diameter (a conservative approximation of the mean dimensions of typical general aviation craft) subtends a visual angle of less than (1/4°). Certain factors omitted from the catalog above but mentioned previously have the effect of misdirection rather than concealment (occasionally those on the list have this effect as well): Jitter resulting from turbulence, altitude and orientation apperceptive errors because of cloudbanks (or canted terrain), and barometrically induced altimeter error. [15, 24, 27, 29, 36, 37].

A related failure of perspective, which is widespread even among experienced pilots [27], is the inability to judge relative altitudes in an uncluttered sky. Its cause is unknown but conjectured to be our conditioned dependence on visual frames of reference. This phenomenon is not mentioned as a cause in any of the accident reports we have available, but a plausible scenario is easy to construct:

In a close approach situation in which the standard rules for evasion are ambiguous (and they are alleged to be so [27]), there seems to be an instinctive tendency to nose up to avoid hazard. A pilot, then, who judges another aircraft to be at or below his own altitude when it is in fact 500 ft. above, could "evade" himself directly into a collision.[11]

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[11] For additional and related material, see Appendix D and its bibliography.

3.4 The Visibility Model

This section takes up the model developed as a tool for analyzing a pilot's field of vision. The need for, development of, data requirements for and means of exercising this model will be addressed here at the conceptual level; the details of its (limited) computer-program implementation, the results obtained from its experimental use to date, and recommendations for further uses, are all deferred to Section 3.5.

In past discussions of the contributions of VFR towers to risk reduction, it has generally been assumed (and we have found no reason to dispute) that during VFR weather conditions an observer in the control tower has at least 3 miles of visibility in all directions[(12)], whereas a pilot's vision during flight is restricted in certain directions due to a limited windshield and various other obstructions. Unfortunately, little has been written about the view available to the pilot. It has been noted that he definitely cannot see straight up, straight back, or straight down; but the effects of even these simple restrictions on the ability to detect other planes in flight near the airport are dependent on the direction of the pilot's aircraft, its bank angle, and its position relative to the airport.

Any evaluation of the extra information provided by a tower must necessarily be based on knowledge of what information pilots could

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[(12)] Upward vision is of course restricted if (as is generally the case) the tower's cab is covered with an opaque roof. However, it appears reasonable to assume general cognizance of an A/C maintained as it flies through this zone.

acquire even in the tower's absence. Therefore, a model was formulated to aid in analyzing the information available to the pilot through his own powers of sight. The main characteristics of this model will now be described; they involve the particular representations chosen for the three key elements of the situation being modeled: pilot's field of vision relative to his aircraft; points to be viewed from; points to be viewed.

(a) Field of Vision. Central to the function of the model is its means of describing the effects of cockpit geometry and other obstructions on the available field of vision. (The word "available" is inserted to emphasize that the present model does not take into consideration the probability that a pilot actually utilizes his field of vision or any particular subfield, a probability which depends on the press of other cockpit duties, the expectation of gaining critical information by scanning just then , etc.). Because these geometry factors vary greatly among individual aircraft, we have provided a parametric representation for them --- with parameter-values to be supplied by the model's user --- rather than building in a few "standard" configurations as constants of the model.

The present parametric representation can be described in three steps:

(1) The field of vision is taken to consist of a finite number of "cones of vision". Each cone emanates from the pilot's eye position, which is determined by his stature as well as the positioning of his seat within the cockpit; for uniformity we may consider the pilot to

be of average height (say, 5' 10"). These cones utilize all windows available to the pilot, but exclude all struts, wings, or windshield posts through which he cannot see. Each cone is assumed to extend out to the mean detection range for small aircraft in clear weather[13], and to be convex (i.e., to have no "holes" in it). Note that this version of the model ignores the possibility of certain bodily movements to secure visual data not available in the pilot's normal position; he may turn his head to take advantage of panoramic window arrangements, but is not "permitted" to crane his neck or bend over or reposition his seat, so that his eyes are constrained like a movie camera on a "pan" mount. Areas for model generalization --- in relaxing this last restriction, in permitting random deviations from average pilot height and/or mean detection range, in including the observational capabilities of a co-pilot --- are apparent.

(2) Each cone is approximated (if necessary) by one which is polyhedral, i.e. is bounded by a finite set of flat "walls" (planar faces). Such a cone is specified by listing its "edges" as a finite set of geometrical rays emanating from the pilot's eye position; each pair of "neighboring" rays has passed through it a unique plane forming one of the faces on the cone.

(3) Each ray, in turn, is specified by giving its direction as angular components in a spherical coordinate system which measures angular displacements from a level reference axis along the craft's fuselage.

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[13] About 8 statute miles, as determined by various studies [27, 29]. In any event this number would be an input parameter to the model.

Thus the field of vision is described only relative to the position and orientation of the pilot's aircraft; the latter must also be specified (see below) before the visibility or invisibility of particular points in space can be determined.

(b) Observer Points. As noted above, it is necessary to specify the points "from which an aircraft desires to look," as well as its orientation at each such point. Two options for entering this information are provided for the model's user:

(1) The first option is most suitable for describing a set of viewing points which are unsystematically located relative to the airport. Each point's position is given by (x, y, z) components --- in feet --- in a coordinate system centered at one of the ends of the "landing direction" of the runway. Aircraft orientation at this point is given by the 3 components (in the same coordinate system) of a vector pointing along the fuselage, and by a bank angle, θ.

(2) The second option is designed for the case in which the viewing points (observer points) are naturally regarded as successive positions along a flight path of the viewing aircraft. A description of the path --- or of each of several paths to be considered consecutively --- must be supplied by the user in terms of parametric equations

$$x = x(p),\ y = y(p),\ z = z(p)\ \ \theta=\theta(p),$$

where the "running" parameter (p) along the curve might represent either time, or distance traversed along the path. It is actually more natural to represent θ as a function of the components (v_x, v_y, v_z)

of the associated velocity vector $\vec{v}$, whose direction is also needed to describe the aircraft's orientation. If p represents time, then the components of $\vec{v}$ can be obtained from mathematical (user-supplied) or numerical (computer-performed) differentiation of the first three parametric equations; if p represents distance-along-path, the user would also have to describe how speed (the length of vector $\vec{v}$) varies with time, or some equivalent information.

The basic idea underlying this option is that visibility will change relatively little as the scanning aircraft moves from one point to a neighboring one along the smooth portions of a flight curve; thus an entirely adequate picture, of fluctuations in the visibility pattern as the viewing aircraft traverses the path, is obtained by choosing as observer points a discrete set of suitably spaced locations along the path. The user is required to specify the number of discrete points by which a flight path is to be represented; the model will then approximate the path by the indicated number of points, equally spaced with respect to time or path-distance, according to the significance assigned the parameter p.

(c) Target Points. The third main ingredient of the model is the set of points to be viewed, i.e., whose visibility from the scanning aircraft is to be ascertained. This set of points can be described by the user *via* either of the options described under (b) above; when option (2) is used, the points to be viewed are naturally regarded as the flight paths of one or more other planes whose continuing visibility to the scanning aircraft is to be analyzed. In case the set of points to be viewed coincides with the set of scanning positions, the user need not repeat their input. Information on θ and $\vec{v}$ is not required for the target points.

The options are, of course, independently chosen for the observing points, and the target points, since we have described what amount to subprograms for insertion in a computer program implementing the model.

A third option for target points combines (1), and (2). Here, a large number of points would be stored, and divided into subsets corresponding to different observer points. The options can be chosen independently for observers and for targets, since they are represented by subprograms in the model computer program.

Specification of the three model elements just described (field of vision, observer points, and target points) is all that is required to exercise the visibility model. The model operates by shifting the scanning aircraft from each viewing point to the next, making tests at each such point to determine which of the points to be viewed fall within the pilot's available field of vision. This information is recorded by filling in a table V, whose entries are either blank (for "not visible") or the symbol "v" (for "Visible"). Thus the entry in the i-th row and j-th column of V is

$$v_{ij} = \begin{cases} v & \text{if j-th target point is visible from i-th observer point,} \\ \text{(blank)} & \text{otherwise.} \end{cases}$$

Various aggregates could be calculated, e.g.: The mean fraction of target points visible, the largest interval of visibility for each target point, the mean fraction of flight time visibility for target points (the "dual" of the first aggergate named), the size (number of points or diameter) of the largest contiguous target set continuously visible.

3.5 Computer Implementation of the Visibility Model

3.5.1 Scanning Points and Scanned Points. In our first realization of the visibility model described in the preceding section, the scanner path employed is a simplified representative left-hand approach pattern at constant height plus a ground-level runway, with the normal 45° approach to the downwind leg replaced by additional segments consisting of an "upwind leg" and a "crosswind leg" turning into the downwind leg. The height h of the pattern and the lengths of its legs (including the runway) are adjustable inputs to the computer program, but in fact we chose

$$h = 800 \text{ ft.}$$

and chose the dimensions of the pattern as indicated in Figure 2.1.1 This pattern, chosen as a fair compromise between realism and convenience of calculation, is the one that appears in the Pilot's Handbook [2] for the purpose of establishing nomenclature; it is also the one employed in Chapter 2 of this report.

All turns are taken as "1-minute turns" , their radius R being determined by the adjustable computer-code input

$$v = \text{(constant) airspeed of scanning aircraft,}$$

which we took to be

$$v = 90 \text{ knots.}$$

This value yields

$$2\pi R = 90 \text{ (n.mi./hr.)} \times 6076 \text{ (ft./n.mi.)}/60\text{(min./hr.)},$$

so that the program calculates

$$R = 1451.5 \text{ ft.}$$

Scanner attitude is level except during turns, where the (constant) bank angle is given by

$$\theta = \tan^{-1} (v^2/Rg)$$

where g is the gravitational acceleration constant (32. ft./sec.^{2}); thus the code computes

$$\theta = 26.3°.$$

Descent is uniform from the end of the final turn to the touchdown point on the runway.

The formation of the parametric equations of motion

$$x = x(p),\ y=y(p),\ z = z(p),\ \theta = \theta(p)$$

($p = t$) on the basis of the above information is a routine matter; the equations can be read from the program listing of the subroutine PNTCAL in Appendix E, and will not be repeated here.

Observations are made at n equally spaced points along the scanning path. The parameter n is also an adjustable input to the code; we chose

$$n = 100,$$

so that the visibility pattern was recorded every 4 sec., or equivalently every 591 ft.

The target (scanned) points for this prototype of the model were selected as identical to the observation points. This rather obvious initial choice was made because:

(1) It simplified the program and reduced computation time.

(2) It leads to outputs which are especially easy to read and interpret.

(3) Points on the approach path forward of the scanner, especially those within 15 - 20 sec., are obviously of critical importance. Points to the rear in the flight path are plausible candidates to represent that often-cited villain in collision dramas, "the overtaking aircraft." Thus the selected points were clearly of interest; what other points are critical for observation during approach-and-landing is a difficult-sounding question which we have not as yet paused to consider systematically.

3.5.2 Aircraft Types. Three aircraft types, one conceptual and two real, were represented in our experimental runs. The first was concocted from the specifications of a visibility standard [39] for commercial transport aircraft proposed in 1960 by the CAA (the predecessor of the FAA). These specifications are:

> It is recommended that the cockpit visibility be such that the pilot when utilizing binocular vision and head and eye rotation and measured from a point 41" above the pilot's heel rest and 5" aft of the rearmost control wheel position with the aircraft in level flight attitude shall command an area of clear vision as follows:
>
> a. 20° forward above the horizon, unbroken
> b. 12° forward below the horizon, unbroken
> c. 40° above the horizon 90° to the left
> d. 30° below the horizon 90° to the left
> e. 135° to the left
> f. 100° to the right

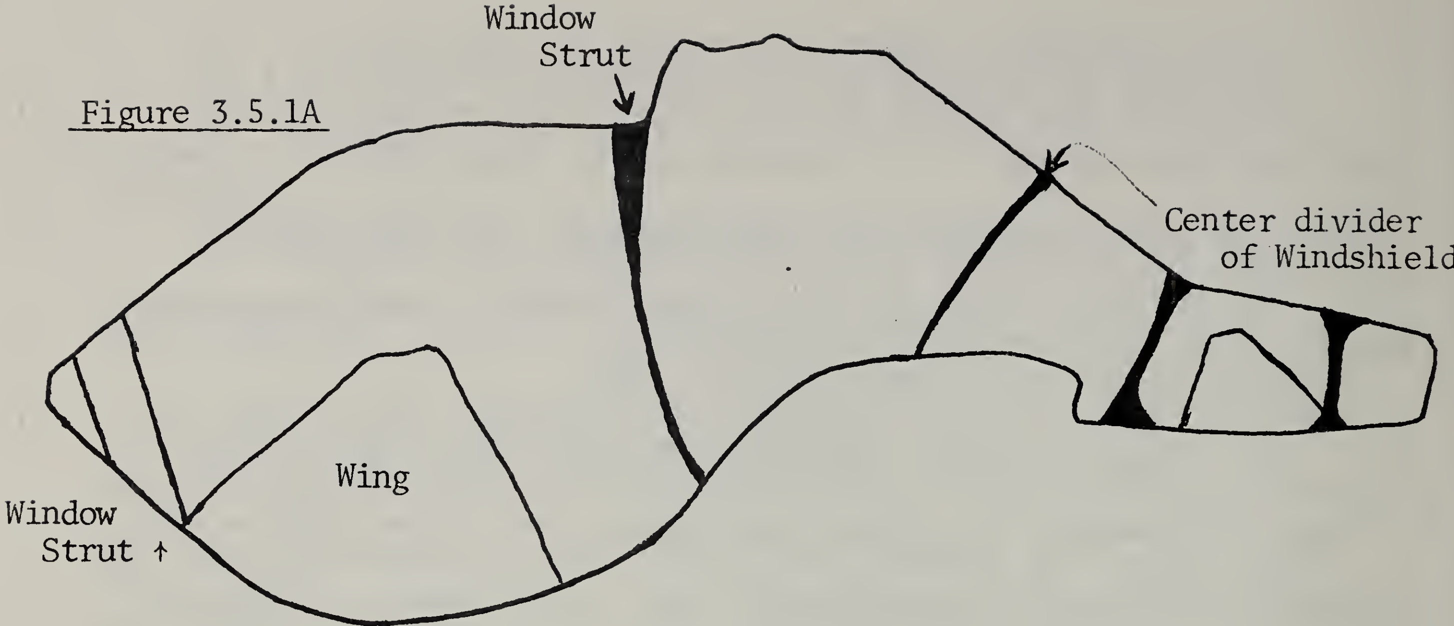
The other two aircraft are the CESSNA 210 (a high-wing type) and the PIPER Cherokee 6 (a low-wing type). The visibility "cones" of these two were represented only approximately, for three reasons:

(1) It seems unlikely that aggregate results would be extremely sensitive to the exact shape of these cones.

(2) A more accurate representation would have required more cones, thus perhaps increasing computation times beyond what we considered appropriate in experimental runs. (The computer times would still have been very small, however.) This point is illustrated in Figure 3.5.1; panel A is typical of the kinds of shapes encountered, panel B shows a rather accurate coverage employing 18 polygons (which project into 18 cones), while panel C presents a moderately good approximation using only 8 cones. (There is no difficulty introduced by the overlapping of cones, as in panel C.)

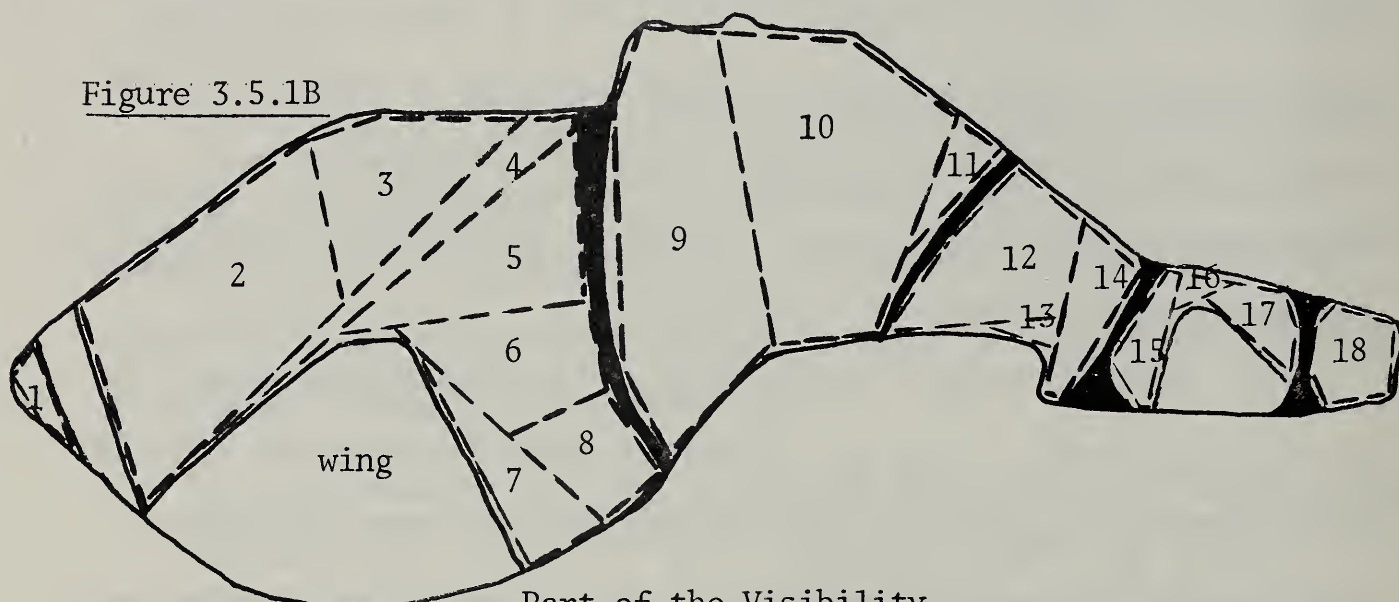
(3) What the "exact" cones should be is a bit indeterminate, for the following reason. Our data came from gridded photographs taken at FAA's NAFEC in response to a request from the Air Force. The photos were taken by twin cameras positioned 30.5" above the mid depression of the pilot's seat, which is in "center" position in its tracks. The lenses of the twin cameras were arranged to simulate an interocular spacing of 2.5", leading to <u>two</u> neighboring patterns juxtaposed on the photographs; this introduces a "fuzziness" for the purposes of our model's present version, which corresponds to monocular vision.

Figure 3.5.1A



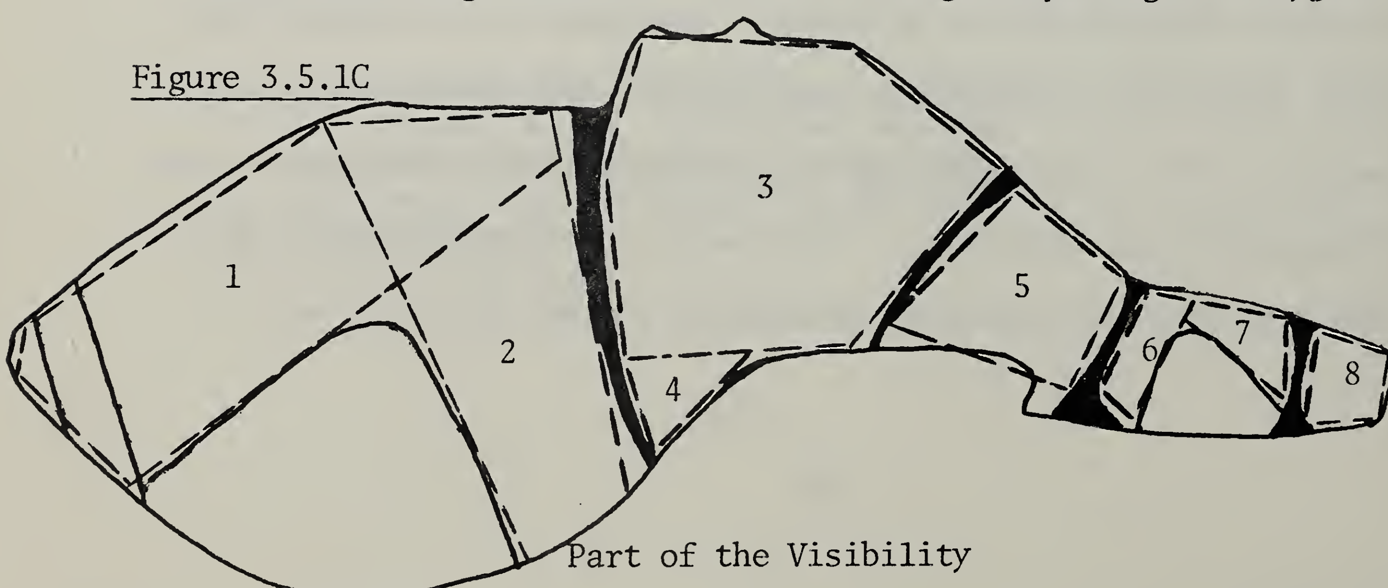
Part of the Visibility Field of a Low Wing Aircraft

Figure 3.5.1B



Part of the Visibility Field of a Low Wing Aircraft Accurate Coverage Requiring 18 Polygons

Figure 3.5.1C



Part of the Visibility Field of a Low Wing Aircraft Approximate Coverage by 8 Convex Polygons

Note that the stipulations of the cones of visibility for these three aircraft types are not "built into" the computer model; they represent three different specifications of the appropriate set of adjustable input parameters.

3.5.3 Model Outputs. The following list specifies the information printed out at the end of each run of our present computerized version of the visibility model. Some of these have been suppressed, for brevity, in Appendix E's presentation of sample output.

(1) Input data, labelled, to identify the run.

(2) Coordinates of the scan points; direction vector and bank angle of the observing aircraft at each of these points. (The coordinate system was described in Section 3.4)

(3) The algebraic equations of each face plane of each visibility cone. (The derivation of these equations is a routine exercise in 3-dimensional analytical geometry which will not be detailed here; the procedure can be read off from the appropriate parts of the program listing in Appendix E.)

(4) The dimensions of the scanner's path (i.e., the landing pattern).

(5) A computerized picture of a plane projection of the flight path, with scan points indicated by "v".

(6) For each scan point: a line of print indicating each visible target point with a "V" and each invisible one with a blank. Inspection of the line-to-line shifts in the pattern of "V's" displays how the pattern of visibility changes as the scanner traverses its flight path.

(7) For each scan point, the fraction of target points which are visible, and also the fraction of those target points "forward" of the scan point which are visible. The average values of these statistics are given at run termination.

(8) The average over the scanning points of : the number of visible target points, and the number of such points forward of the scanner along its path.

3.5.4 Some Natural Extensions. We have three items to propose here:

(1) The scanner aircraft would not in reality follow its nominal path exactly; even if it attempted to do so, there would be small random perturbations to its trajectory due to wind gusts, clear-air turbulences etc. A representation of these effects can readily be incorporated in the model, and the sensitivity of its outputs to such random "noise" can be ascertained. A related possibility is to incorporate visibility-increasing maneuvers such as deliberate rolls, in order to study their effectiveness.

(2) For arriving aircraft, incorporate a slightly more complex flight path involving a circular pass over the airport at 1000 ft. above pattern height, followed by a standard 45° approach with descent into the downwind leg; as a first cut at a path for departures, a rising helical curve.

(3) Pairs of flight trajectories can be run, where in each pass one aircraft is the "scanner" and the other is the "intruder", to

determine for instance at what points an overtaking intruder can be detected. The runs can be repeated with roles reversed, yielding (with slight computation time since at each scanning position only a single target point must be scanned) a fairly complete visibility analysis of a potential collision. The "heavy" part of the labor is producing the trajectory subroutines.

(4) Calculation of ranges of target points from scan points. Also, recording of intervals during which each target point is uninterruptedly visible. This information should prove useful in subsequent estimations of probabilities of detecting an intruder aircraft and of accurately appraising its speed and direction of motion. Such estimations will require considerations of the size and relative brightness of the intruder. (In a nutshell, the conclusions documented by Cornell Aero Lab, Applied Psychology Corp. and other studies in the past 15 years, are that the threshold of detectability depends on size, light contrast and duration of scan, while estimation of speed and direction depends on these along with range and shape. [25, 27, 31, 36].

4. NEXT STEPS

As stated in the Introduction, our objective in this study was to attempt to develop mathematical models and methods which would aid in measuring the benefits of VFR towers --- especially as regards flow and safety considerations --- for airports of various activity levels. The preceding text has discussed a number of methodologies, viewpoints, and concepts relevant to this goal. In this concluding chapter we wish to focus briefly on the three areas in which we have achieved concrete progress, and to sketch what appear to be the natural next steps toward enlarging and exploiting this progress.

(1) String-of-Slots Model. This model, whose formulation, analysis and illustrative "application" are given in Chapter 2, appears to provide a promising and convenient analytical tool for estimating time savings due to a tower's ability to advise pilots when "short-cuts" can safely be taken. It is still, however, in a relatively unevaluated prototype stage. Procedures for estimating its parameters from field observations need to be thought through and tested. Further study is required to see whether its basic sleight-of-hand --- replacing a time-consuming maneuver to allow a desired "slot" to be freed by an instantaneous entry to a slot behind the desired one --- needs to be supplanted, and if so, by what. Extension of the model to encompass takeoffs as well as landings seems essential, if the full range of this species of tower benefits is to be considered. Another important step is to represent the (itinerant, non-itinerant) mix within the model.

(2) Visibility Model. We have come to feel that real insight into how a VFR tower promotes safety, requires detailed consideration of pilots' information needs (and how the tower can help meet them). The visibility model described in Chapter 3 illustrates both the feasibility and the potential value of such investigations, and may be of interest to the FAA in a broader context than the "tower criteria" review. Straightforward next steps, listed in Section 3.5, include (a) studying the effects, on the "domain of visibility", of the observing aircraft's fluctuations from its nominal flight path, and (b) examining the visibility relations between two successive planes traversing a landing pattern. Extension of the computer implementation to handle takeoffs is a must. Accomplishment of these tooling-up tasks will provide the basis from which to proceed in identifying the explicit scenarios to which this model's application might be most useful and timely.

(3) Analysis of Collision Data. In Appendix C, a more positive statistical methodology than had previously been applied was used to establish the existence of a significant association between the presence of VFR towers, at airports, and lower collision rates there. The "lower" was not quantified in this analysis. It is possible to adapt the methodology to test quantitative versions of the assertion, and this should be attempted. The nature of the data employed, however, makes uphill climbing of all efforts to exhibit and interpret correlations. Addressing this point, Section C.4 offers a number of suggestions concerning more extensive articulation of more detailed data, so as to make more

definitive analyses possible. We urge that these suggestions be considered in the planning of future reporting procedures. If some of them can be presently realized by extracting more material from available report forms, then additional statistical analyses could be undertaken promptly.

The tripartite set of tasks described above has deliberately been restricted to relatively concrete and explicit items, natural continuations of the major elements of the work documented in this report. We have therefore omitted such fascinating but more speculative areas as (a) efforts to provide quantitative elucidation of the "orderly flow" concept and its relationship with safety, and (b) more systematic study of what constitutes "critical information" (and "information glut") for a pilot relative to his various duties in the situations he encounters. These subjects would be most welcome areas for further exploratory research, but we felt that most (not all!) of our explorer's license expired with the completion of the current trek, and that our obligation in this chapter was not to propose plans for a second expedition, but rather to recommend how the newly penetrated territories might best be assimilated and developed.

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No. 5 Pilot Judgments of Simulated Collisions and Near Misses: A Comparison of Performance With Uncoded and Two-Tone Coded Models

No. 6 9 & 14 Effects of Backscattered Light on Target Light Detectability in a Ground Test Environment

No. 7 Outdoor Test Range Evaluation of Aircraft Paint Patterns

No. 8 Flight Simulator Tests of Altitude-Coded Lights

No. 10 & 11 Pilot Judgments of Aircraft Range and Relative Altitude: Ground-to-Air and Air-to-Air Observations

No. 12 Distance Estimation of Frequency-Coded and Uniformly Flashing Lights

No. 13 Conspicuity of Selected Signal Lights Against City-Light Backgrounds

No. 15 Altitude Evasion in Visual Collision Avoidance

No. 16 Flight Test of an Altitude-Coded Aircraft Light

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The Role of Range and Altitude Judgment in Mid-Air Collision Prevention

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APPENDIX A: TECHNICAL ASPECTS OF STRING-OF-SLOTS MODEL

In this appendix we present the technical arguments needed to complete the discussion of the solution method set forth in Section 2.4 for the string-of-slots model.

Define a state of the string of slots to be a specification of exactly which slots are occupied; thus there are 2^N possible states. For any states τ, σ (not necessarily distinct), let $S(\tau, \sigma)$ denote the set of integers k between 1 and N inclusive with the following property: if the string begins a time period in state τ, and a new aircraft desiring entry to slot k arrives, then the string will end the period in state σ. Then if the string ends one time period in state τ, the probability that it will end the next period in state σ is

$$P(\tau \rightarrow \sigma) = \Sigma \; \{d_k: k \text{ in } S(\tau,\sigma)\} \qquad \text{(A.1)}$$

where we have set

$d_k = \alpha p_k$ = probability that an aircraft desiring entry to slot k arrives.

Because the probability of achieving a given state σ is fully determined by the last previous state, independent of other past history or of the current time (t), the random process under study is an ordinary Markov chain, M. Consider the state ε in which all slots are empty. Clearly ε can be reached from any state in at most N time periods, each with no new aircraft arriving; this sequence of events has probability at least $(1-\alpha)^N > 0$. Moreover any other state σ can be reached from ε in N time periods; if σ has j slots occupied, this involves j strategically-timed arrivals of new aircraft desiring entry to slot N, a sequence with probability $(\alpha p_N)^j (1-\alpha)^{N-j} > 0$

It follows that in the language of Feller[1], the Markov chain M is <u>irreducible</u>. Also, the string can move from ε to ε in <u>any</u> prescribed number m of time periods with positive probability (e.g., by having no arrival in any period); thus the chain M is not <u>periodic</u> (<u>op cit</u>, pp. 321-322). It follows (<u>op cit</u>, p. 329) that the quantities

$p(\sigma)$ = steady-state probability of ending a time period in state σ

do exist.

Now let S_i be the set of states in which slot i is occupied. If $p(\sigma,t)$ denotes the time-varying analog of $p(\sigma)$, then by definition

$$f_i(t) = \Sigma\ \{p(\sigma,t):\ \sigma \text{ in } S_i\},$$

and so f_i exists and is given by

$$f_i = \Sigma\ \{p(\sigma):\quad \sigma \text{ in } S_i\}. \qquad \text{(A.2)}$$

The equation prior to (2.4) then implies that e_i also exists. The existence of the c_i's is proven similarly.

The <u>usual procedure</u> for "solving" such a Markov-chain model consists of first determining the $p(\sigma)$'s, and then calculating the quantities of interest from them; in our case the second step would be the computation of the f_i's from (A.2). As for the first step, the $p(\sigma)$'s are found from the system of "balance equations"

$$\Sigma_\tau\ p(\tau)\ P(\tau\to\sigma) = p(\sigma) \qquad \text{(all } \sigma) \qquad \text{(A.3)}$$

these equations are not independent, but adjunction of the additional equation

$$\Sigma_\sigma\ p(\sigma) = 1 \qquad \text{(A.4)}$$

makes them determinate.

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[1] [44, p. 318].

In the present situation, however, (A.3) is a system of 2^N equations in 2^N variables $p(\sigma)$; since $N\approx 20$ in our applications, this system is much too large to set up and solve without undue effort. Thus we sought alternative methods for determining the N f_i's without having first to determine the much more numerous quantities $p(\sigma)$. This attack led to the formulas

$$f_1 = \alpha, \quad f_2 = (1-p_1)/(\alpha^{-1}-p_1) \tag{A.5}$$

derived earlier as Eqs. (2.7) and (2.10); with the notation

$$d_o = 1-\alpha \quad (\text{so that } \Sigma_o^N d_i = 1), \tag{A.6}$$

these relations read

$$f_1 = 1-d_o, \quad f_2 = (\alpha-d_1)/(1-d_1). \tag{A.7}$$

The attack also led to the material in the next paragraph, which (as will be seen) permits the calculation of f_N.

Let E_o be the event that all slots are occupied, and for $1\leq i\leq N$ let E_i be the event that slot i is empty but all subsequent slots (i+1, i+2,...,N) are occupied. We will use the notation

ε_i = steady-state probability of E_i,

and will use the notation $E_i(t)$ and $\varepsilon_i(t)$ for the associated time-dependent events and probabilities. Since the events $\{E_i\}_{i=o}^N$ are exhaustive and exclusive,

$$\Sigma_{i=o}^N \varepsilon_i = 1. \tag{A.8}$$

For $1\leq i<N$, it can be checked (we omit the details of the reasoning) that $E_i(t)$ occurs at the end of stage t if and only if (a) $E_{i+1}(t-1)$ was the case, and also (b) stage t brought a new arrival desiring to enter one of the slots i+1 through N inclusive. This relation gives an equation involving $\varepsilon_i(t)$ and $\varepsilon_{i+1}(t-1)$, which on letting $t\to\infty$ yields

$$\varepsilon_i = (\Sigma_{j=i+1}^N d_j)\, \varepsilon_{i+1} \qquad (1\leq i<N);$$

the analog for i=0 is found to be

$$\varepsilon_o = \alpha(\varepsilon_o + \varepsilon_1).$$

These equations can be rewritten

$$\varepsilon_1 = (d_o/\Sigma_1^N d_j)\ \varepsilon_o, \qquad (A.9)$$

$$\varepsilon_{i+1} = (1/\Sigma_{j=i+1}^N d_j)\ \varepsilon_i \qquad (1 \le i < N). \qquad (A.10)$$

Eqs. (A.8), (A.9), (A.10) form a set of N+1 equations for the N+1 probabilities $\{\varepsilon_i\}_{i=o}^N$, and because of its special form this system can be solved quite readily; one sets $\varepsilon_o' = 1$, applies the relations (A.9) and (A.10) to calculate in turn quantities $\varepsilon_1', \varepsilon_2', \ldots, \varepsilon_N'$, then computes

$$K = \Sigma_{i=o}^N \varepsilon_i'$$

and sets

$$\varepsilon_i = \varepsilon_i'/K. \qquad (A.11)$$

A final observation here is that E_N is precisely the event that slot N is empty, so that this analysis permits the calculation of f_N by

$$f_N = 1 - \varepsilon_N. \qquad (A.12)$$

We did not succeed in finding a simple rigorous way to determine the remaining f_i's, namely f_3 through f_{N-1}, though the conjecture remains that this is possible. The solution method employed in Section 2.4, and characterized by Eqs. (2.13) and (2.16), which we rewrite (with $C_i = \alpha c_i$) as

$$f_1 = \alpha,\ C_1 = d_1, \qquad (A.13)$$

$$f_{i+1} = (f_i - C_i)/(1 - C_i), \qquad (A.14)$$

$$C_{i+1} = d_{i+1} + C_i\ f_{i+1}, \qquad (A.15)$$

is less-than-rigorous in its dependence on an extra independence assumption. This assumption concerns the N events defined (for $1 \le i \le N$) by

F_i: slot i is full (i.e., occupied),

and asserts, for $2 \leq j \leq N$, that $(F_1, F_2 \ldots, F_j)$ are independent in the steady-state, i.e. that

$$\text{Prob}\ \{F_1 \text{ and } F_2 \text{ and} \ldots \text{and } F_j\} = \Pi_{i=1}^{j} f_i. \tag{A.16}$$

It would of course be preferable if this were true exactly. However, a quite tolerable fall-back situation for our purposes is that this assumption holds as a good enough approximation (i.e., interdependencies are weak enough) that results calculated on its basis are also good approximations to the rigorously correct answers. The remainder of this Appendix presents the evidence we have amassed to date bearing on this point; it appears that the "good approximation" presumption is applicable for most cases of interest.

(a) For N=2, the independence assumption is exactly true. This is proved by solving the equations (A.3) and (A.4), yielding

$$p(EE) = d_o^2/(d_o+d_2), \quad p(EF) = d_o(d_1+d_2),$$

$$p(FE) = d_o d_2/(d_o+d_2), \quad p(FF) = d_2(d_1+d_2)/(d_o+d_2),$$

where we have used the notation $\sigma = (\sigma_N, \sigma_{N-1}, \ldots, \sigma_1)$ with σ_i = F (or E) if slot i is full (or empty) in configuration σ. Here (A.16) takes the form

$$p(FF) = f_1\ f_2,$$

which is easily checked using (A.7) and the above formula for p(FF).

(b) For N=3, events F_2 and F_3 are independent. This is proved (we omit details) by solving equations (A.3) and (A.4), or rather a compactified version which focuses only on slots 2 and 3. The results of this are

$$p(EEE) + p(EEF) = d_o^2(d_o+d_1)/\Delta,$$

$$p(EFE) + p(EFF) = d_o\ (d_o+d_1)(d_2+d_3)/\Delta,$$

$$p(FEE) + p(FEF) = d_o\ d_3\ (1-d_1)/\Delta,$$

$$p(FFE) + p(FFF) = d_3\ (1-d_1)(d_2+d_3)/\Delta,$$

where the normalizing factor Δ is given by

$$\Delta = (1-d_1)\ [d_o\ (d_o+d_1) + d_3(1-d_1)].$$

The assertion is that

$$\text{Prob}\ \{F_2 \text{ and } F_3\} = f_2\ f_3,$$

which is equivalent to

$$p(FFE) + p(FFF) = f_2\ \{[p(FEE) + p(FEF)] + [p(FFE) + p(FFF)]\},$$

and is readily checked using (A.7) and the formulas above.

(c) We might intuitively expect that fairly low traffic levels, the situation of interest in our study, would be conducive to the presence of only weak interactions among the events F_i. This supposition is borne out by calculations for N=4, in which the system (A.3-4) was solved numerically for various values of the d_k's. Respectable-size violations of consequences of the independence assumption were found only for values of α considerably higher than those of interest here.

(d) The preceding evidence dealt with small values of N, whereas N>10 is the range of actual concern. Since no analytical assurance of (approximate) independence for such large N was available, we developed a simple Monte-Carlo fast-time simulation of the string-of-slots, coded it in FORTRAN (Figure A.1 shows a listing), and exercised it for several scenarios.

The aim of this exercise was to estimate the values of the f_i's by the corresponding simulation relative frequencies, to estimate the mean waiting time E(W) by Eq. (2.6), and to see how well these values agreed with those obtained by the solution method of Section 2.4, which is based on the independence assumption. The results for E(W) are given in Table A.1, while those for the f_i's are illustrated in Table A.2. The agreement is quite good, indicating that the solution method in Section 2.4 indeed yields satisfactory approximations.

Table A.1: Mean Waiting Times(*) from Simulation and Solution Method

N	α	$p_i^{(**)}$	E(W) from simul.	E(W) from sol. method
3	1/3	1/N	2.09	2.14
4	1/3	1/N	2.64	2.66
5	1/3	1/N	3.21	3.17
15	1/3	1/N	8.16	8.20
15	1/3	$1/2^i$	2.17	2.13
20	1/3	1/N	10.68	10.70

(*) In slot-times; multiply by $\Delta t=0.296$ to convert to minutes.
(**) In next-to-last entry, $p_{15}=1/2^{14}$.

Table A.2: f_i - Values from Simulation and Solution Method ($N=15$, $\alpha=1/3$, $p_i=1/2^i$ except that $p_{15}=1/2^{14}$)

i	f_i(sim.)	f_i(sol.meth.)	i	f_i(sim.)	f_i(sol.meth.)
1	.338	.333	9	.001	.001
2	.202	.200	10	.001	.000
3	.096	.094	11	.000	.000
4	.046	.044	12	.000	.000
5	.021	.021	13	.000	.000
6	.011	.010	14	.000	.000
7	.005	.005	15	.000	.000
8	.002	.002			

Figure A.1 Listing of Simulation Program

```
'IT FOR CGEOMT,CGEOMT
      SUBROUTINE CGEOMT(P,CUMP,NTOT)
      DIMENSION CUMP(50)
      Q=1.-P
      PI=P
      NLS1=NTOT-1
      CUMP(1)=P
      DO 10 I=2,NLS1
      PI=PI*Q
 10   CUMP(I)=CUMP(I-1)+PI
      CUMP(NTOT)=1.0
      RETURN
      END
'I ASM RANDNO,RANDNO
RANDNO*  L,017   12,0        .  CLEAR 12          RANDOM NUMBER GENERATOR
         L       13,I+1      .  LOAD LOW HALF     FEBRUARY 1967.
         MF      13,M        .  MULT, SHIFT       THIS IS A FORTRAN CALLABLE
         SSC     14,1        .  SET IN SIGN       FUNCTION.. RANDNO( J )
         L       15,I        .  LOAD UPPER HALF   IF J = 0, RESULT IS GE 0.
         MI      15,M        .  MULT              AND LT 1.   IF J NE 0,
         DA      12,15       .  COMBINE HALVES    RESULT IS GT -1. AND LT 1.
         LSSL    13,1        .  SET SIGN-BIT=0
         SSL     13,1        .                       GENERATOR
         DS      13,I        .  SAVE PROD.        I = MOD( 5**13*I, 2**70 )
         L,017   12,0170     .  SETUP EXP         RANDNO = FLOAT( I )/2.**70
         LCF     12,13       .  FLOAT ANS.
         L       12,15       .  ANS. TO 12        WALTER GILBERT, NBS
         TNZ     *0,11       .  TEST ARG=0
         J       2,11        .  =0, RETURN.       16.625 MICROSEC TO HERE.
         A       12,BIT      .  MULT BY 2.
         FAN     12,ONE      .  SUB 1.
         J       2,11        .  RETURN            19.875 MICROSEC TO HERE.
$(1),I   +0154447730601      .  RANDOM INITIAL VALUE
         +0255751305264      .  (DOUBLE PRECISION INTEGER)
M        +0011060471625      .  5**13, MULTIPLIER.
BIT      +000170000000       .
ONE      +0200777777777      .
         END                 .
```

```
'IT FOR  SLOTS,SLOTS
      DIMENSION PROBIN(50),IATIME(50),TIMDST(50)
      INTEGER FRSTPL,SLOT(20000),FULL(50),DESIRE(50),ENTER(50),SNPSHT
C
C
C  ***************************DESCRIPTION OF INPUT REQUIRED   *******************
C
C  DATA DESCRIPTION FOR CARD NUMBER 1.
C
C  ------------------------------------------------------------------------------
C   COLUMNS . JUSTIFY . VARIABLE . MEANING
C           .         . NAME     .
C  ------------------------------------------------------------------------------
C   1-10    .      10 .   IFLAG  . =1 - DATA CARDS 2,3 AND 4 FOLLOW.
C           .         .          .-----------------------------------------------
C           .         .          . =2 - ONLY DATA CARDS 3 AND 4 FOLLOW.
C           .         .          .-----------------------------------------------
C           .         .          . =3 - ONLY DATA CARD 4 FOLLOWS.
C           .         .          .-----------------------------------------------
C           .         .          . =4 - NO MORE DATA, TERMINATE RUN.
C  ------------------------------------------------------------------------------
C
C  DATA DESCRIPTION FOR CARD NUMBER 2.
C
C  ------------------------------------------------------------------------------
C   COLUMNS . JUSTIFY . VARIABLE . MEANING
C           .         . NAME     .
C  ------------------------------------------------------------------------------
C   1-10    .      10 .  NSLOTS  . TOTAL NUMBER OF SLOTS IN TRAFFIC PATTERN..
C  ------------------------------------------------------------------------------
C  11-20    .      20 . NINSLT   . FIRST SLOT INTO WHICH AN ENTRY MAY BE MADE ,
C           .         .          .   I.E., THE FIRST SLOT LEFT OF FINAL APPROACH
C  ------------------------------------------------------------------------------
C  21-30    .      30 .  LAMBDA  . RATE OF ARRIVALS. (IN A/C PER HOUR)
C  ------------------------------------------------------------------------------
C   31-40   .      40 . SLTTIM   . TIME REQUIRED FOR AIRCRAFT TO FLY FROM
C           .         .          . ONE SLOT TO THE NEXT.
C  ------------------------------------------------------------------------------
```

```
C
C  DATA DESCRIPTION FOR CARD NUMBER 3.
C
C  ----------------------------------------------------------------------------
C   COLUMNS . JUSTIFY . VARIABLE . MEANING
C           .         . NAME     .
C  ----------------------------------------------------------------------------
C   1-10    .      10 . PROBIN(I). PROBIN(I) FOR I GOING FROM NINSLT TO NSLOTS,
C  11-20    .      20 .          . EQUALS THE PROBABILITY OF WANTING TO ENTER
C    .      .       . .          . SLOT I, GIVEN AN OPERATION OCCURS AT THE
C    .      .       . .          . MOMENT.
C    .      .       . .          . (NOTE' THIS FORMAT APPLIES TO ALL CARDS
C  71-80    .      80 .          . FOLLOWING, WHICH ARE REQUIRED TO READ IN
C           .         .          . PROBIN.
C  ----------------------------------------------------------------------------
C
C  DATA DESCRIPTION FOR CARD NUMBER 4.
C
C  ----------------------------------------------------------------------------
C   COLUMNS . JUSTIFY . VARIABLE . MEANING
C           .         . NAME     .
C  ----------------------------------------------------------------------------
C   1-10    .      10 . IRESET   . =0 WIPE-OUT ALL PLANES THAT ARE CURRENTLY IN
C           .         .          .    THE STRING.
C           .         .          .---------------------------------------------
C           .         .          . =1 BEGIN COLLECTING DATA WITH CURRENT PLANES
C           .         .          . IN SLOTS.
C  ----------------------------------------------------------------------------
C  11-20    .      20 . NTOTAC   . = THE NUMBER OF AIRCRAFT TO BE RUN THRU MODEL
C           .         .          .---------------------- OR -------------------
C           .         .          . =0,IF THE USER DESIRES TO STOP AFTER A FIXED
C           .         .          . PERIOD OF TIME RATHER THAN A FIXED NO. OF A/C
C  ----------------------------------------------------------------------------
C  21-30    .      30 . NDTIME   . THE TIME TO STOP THE SIMULATION.
C           .         .          . NOTE' ONLY MEANINGFUL WHEN NTOTAC=0.
C  ----------------------------------------------------------------------------
C  31-40    .      40 . SNPSHT   . NUMBER OF AIRCRAFT TO BE PROCESSED BEFORE
C           .         .          . PRINTING OUT INTERMEDIATE STATISTICS.
C  ----------------------------------------------------------------------------
C  ----------------------------------------------------------------------------
C  ****************************************************************************
   1 READ(5,9000)IFLAG
     GOTO(2,3,4,9999),IFLAG
   2 READ(5,9002)NSLOTS,NINSLT,LAMBDA,SLTTIM
 9002 FORMAT(3I10,F10.0)
```

```
9000 FORMAT(6I10)
   3 READ(5,9001)(PROBIN(I),I=NINSLT,NSLOTS)
9001 FORMAT(8F10.0)
     NINPL1=NINSLT+1
     DO5I=NINPL1,NSLOTS
   5 PROBIN(I)=PROBIN(I-1)+PROBIN(I)
     WRITE(6,9012)(N,PROBIN(N),N=NINSLT,NSLOTS)
9012 FORMAT(I10,F6.3)
   4 READ(5,9000)IRESET,NTOTAC,NDTIME,SNPSHT
     IRESET=IRESET+1
     IF(NTOTAC.EQ.0)GOTO30
     NDTIME=34359738367
     GOTO40
  30 NTOTAC=34359738367
  40 GOTO(41,42),IRESET
  41 FRSTPL=0
     LASTPL=0
  42 DO50I=1,NSLOTS
     IATIME(I)=0
     DESIRE(I)=0
     ENTER(I)=0
  50 FULL(I)=0
     KRNTIM=0
     LIMTIM=50
     P=(LAMBDA*SLTTIM)/60.
     CALL CGEOMT(P,TIMDST,LIMTIM)
     WRITE(6,9012)(I,TIMDST(I),I=1,LIMTIM)
     DO800 NOAC=1,NTOTAC
     NUMBAC=NOAC
C  THE TIME UNTIL THE NEXT ARRIVAL IS CALCULATED HERE     ----------------------
     X=RANDNO(0)
     DO 55 I=1,LIMTIM
     IF(X.GT.TIMDST(I))GOTO55
     NEXTOP=I
     IATIME(NEXTOP)=IATIME(NEXTOP)+1
     GOTO59
 55  CONTINUE
 59  IF(KRNTIM+NEXTOP.GT.NDTIME)GOTO10[illegible]
C   THE DESIRED SLOT FOR ENTRY IS FOUND HERE     ---------------------------------
     X=RANDNO(0)
     DO60I=NINSLT,NSLOTS
     IF(X.GT.PROBIN(I))GOTO60
     INSLOT=I
     DESIRE(I)=DESIRE(I)+1
     GOTO65
  60 CONTINUE
```

```
   65 IF(FRSTPL.EQ.0)GOTO300
      IF(NEXTOP.EQ.0)GOTO105
      NEW1ST=FRSTPL
C  UPDATING OF STRING TO TIME OF NEXT ARRIVAL IS DONE HERE   ------------------
      DO100I=FRSTPL,LASTPL
      NXTSLT=SLOT(I)-NEXTOP
      IF(NXTSLT.GT.0)GOTO70
      NEW1ST=I+1
      IF(SLOT(I).EQ.1)GOTO100
      NXTSLT=1
   70 LOSLOT=SLOT(I)-1
      DO80J=NXTSLT,LOSLOT
   80 FULL(J)=FULL(J)+1
      SLOT(I)=NXTSLT
  100 CONTINUE
      FRSTPL=NEW1ST
C  THE SLOT WHICH IS ACTUALLY ENTERED IS CALCULATED HERE  ---------------------
      IF(FRSTPL.GT.LASTPL)GOTO155
  105 DO150I=FRSTPL,LASTPL
      ITEMP=I
      IF(SLOT(I)-INSLOT)150,110,160
  110 INSLOT=INSLOT+1
  150 CONTINUE
  155 SLOT(LASTPL+1)=INSLOT
      LASTPL=LASTPL+1
      FULL(INSLOT)=FULL(INSLOT)+1
      GOTO200
  160 LASTPL=LASTPL+1
      NTEMP=LASTPL-ITEMP
      DO170I=1,NTEMP
  170 SLOT(LASTPL-I+1)=SLOT(LASTPL-I)
      SLOT(ITEMP)=INSLOT
      FULL(INSLOT)=FULL(INSLOT)+1
      GOTO200
  300 FRSTPL=1
      LASTPL=1
      SLOT(1)=INSLOT
      FULL(INSLOT)=FULL(INSLOT)+1
  200 KRNTIM=KRNTIM+NEXTOP
      ENTER(INSLOT)=ENTER(INSLOT)+1
      IF(MOD(NOAC,SNPSHT).NE.0)GOTO800
      IF(NUMBAC.EQ.NTOTAC)GOTO800
      XTIME=FLOAT(KRNTIM)
      DO790I=1,NSLOTS
      XFULL=FLOAT(FULL(I))/XTIME
      XDESIR=FLOAT(DESIRE(I))/XTIME
```

```
      XENTER=FLOAT(ENTER(I))/XTIME
      WRITE(6,9011)I,XFULL,XDESIR,XENTER
  790 CONTINUE
  800 CONTINUE
 1000 WRITE(6,9010)NUMBAC,KRNTIM
 9010 FORMAT('1'///'    THE SIMULATION HAS PROCESSED ',I5,' OPERATIONS IN
     1',I5,' CLOCK UNITS'//20X,'THE F(I) FOLLOW.'//9X,'I',5X,'F(I)',6X,'
     2P(I)',6X,'E(I)'/)
      XTIME=FLOAT(KRNTIM)
      ALPHA=FLOAT(FULL(1))
      DO1010I=1,NSLOTS
      XFULL=FLOAT(FULL(I))/XTIME
      XDESIR=FLOAT(DESIRE(I))/ALPHA
      XENTER=FLOAT(ENTER(I))/ALPHA
      WRITE(6,9011)I,XFULL,XDESIR,XENTER,IATIME(I)
 9011 FORMAT(I10,3F10.5,I20)
 1010 CONTINUE
      GOTO1
 9999 CALLEXIT
      END
' XQT SLOTS
```

APPENDIX B: A COMPARISON OF CONTROL POLICIES FOR AN INTERSECTION OF TWO TRAFFIC STREAMS[1]

The paper which follows, considers the optimal (total delay minimizing strategy in the operation of a traffic signal at an intersection of two one-way streams of vehicular traffic. The work was substantially supported by an activity other than the project covered in this report, and is included primarily as a matter of record because of discernable relevance to air traffic control. At the level of abstraction represented (i.e. roughly the same as that of the string-of-slots model of Chapter 2), the model could be interpreted in terms of ATC in several ways:

(1) Assigning landing priorities to elements in "streams" of aircraft arriving at an airfield at which there are two principal directions of approach. (The main thrust of the work undertaken by the authors for the current project was to attempt to generalize the analytic result to many streams, to make the model applicable to omnidirectional approach to airports. At the time of preparation of this report the mathematical obstacles had been discovered to be formidable.)

---

(1) In reproducing this document, we have not revised its internal structure (i.e. the numbering of its sections, and the presence of what now becomes an appendix-to-an-appendix) to conform with the over-all format of the report.

(2) Assigning departure priorities (weather permitting) at airports with intersecting runways in periods when there are few arrivals.

(3) Investigating the consequences of substituting tower controlled priorities between departures and arrivals for the current inflexible precedence fo arriving aircraft over departure.

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A COMPARISON OF CONTROL POLICIES

FOR AN INTERSECTION OF TWO TRAFFIC STREAMS

ABSTRACT

Numerical comparisons are carried out for the "traffic loss rates" resulting from three approaches to regulating the flow at an intersection of two traffic streams. The studies involve Markov chain models, of traffic generation and intersection operation, yielding fixed vectors from which the average loss rates can be calculated. The control systems treated are of non-feedback, feedback, and "prophesying" natures respectively.

1. INTRODUCTION

In a recent pair of papers ([1] and [2])(1), the authors treated the "traffic loss rate" at the intersection of several streams of traffic. Various assumptions were made concerning the pattern of control and the amount of information available to the traffic control mechanism. In each case, the system was modeled as a Markov chain, yielding a fixed vector ("steady-state probabilities") from which the mean traffic loss rate could be calculated.

In this paper we introduce a third system in addition to those investigated in [1] and [2], and present numerical values for the expected traffic loss rate when the intersection is controlled by each of the three systems. Specifically, this permits us not only to compare the performance of an optimal feedback (or "adaptive") control system(2) with that of a non-feedback system, but also to place the resultant improvement in perspective by comparing it with the performance of a hypothetical system with "prophesying" capability. The problem of the relative effectiveness of traffic control systems under varying amounts of information about the state of the system is as significant for air traffic control as it is for surface travel. In particular this would seem to be one aspect of the question to be examined in a comparison of tower controlled landings at an airfield versus landing at an airfield where there is no tower. We therefore also report these results under a program investigating the latter subject.

---

(1) [1] J. Levy and M. Pearl, The traffic loss at a merge-point controlled by non-feedback regulation. NBS Report 9989, 2/69.

[2] J. Levy and M. Pearl, Feedback regulation of traffic at an intersection of two streams. Working Paper No. 11, Sept. 1969.

(2) "Optimal" in the sense of minimizing expected traffic loss rate.

The remainder of this introductory section contains a description of the common features of the three control policies to be studied. Section 2 then takes up the individual aspects of the three mathematical models. Finally, Section 3 presents our numerical results together with some commentary.

For all three models, time is treated as discrete and is divided into periods sufficiently short that (i) at most one unit of traffic can pass through the intersection in one time period, and (ii) at most one new unit of traffic can be generated in each stream during one time period. For each stream, the probability (p) that a unit of traffic is generated in that stream during a time period is one of the parameters of the analysis; another parameter is the capacity (c) of the stream, i.e. the maximum number of waiting traffic units it can hold. In the case of 2 traffic streams, S_0 and S_1, the corresponding parameters are denoted

$$p_0 \; , \; c(0) \; , \; p_1 \; , \; c(1) \; ,$$

and we adopt the convention and abbreviations

$$p_0 \leq p_1 \; ; \; q_0 = 1-p_0 \; ; \; q_1 = 1-p_1 \; . \qquad (1.1)$$

A control policy is a set of rules for determining, for each time period, the traffic stream (if any) to which the intersection is "open" during that period(3). Some of our earlier work involved systems with an "orange light"; i.e., each change in the identity of the stream to which the intersection was open had to be preceded by a time period ("dead" or "switchover" time) in which the intersection was closed to all traffic. Such systems are not considered in the present paper; it is assumed that the intersection is always open to some one of the streams.

---

(3) One might define a "control system" to be a physical embodiment of a "control policy", but we shall treat the two phrases as synonymous.

Suppose a unit of traffic is generated in a stream to which the intersection is currently closed. Ordinarily, the unit joins that stream's queue of units awaiting access to the intersection. However, the queue may already exhaust the capacity of the stream. In this case the new traffic unit is regarded as lost. The (average) traffic loss rate (L) of a control system is the expected number of units of traffic lost per time period. Interpretable as a mean "turn-away" or "disappointment" rate, it is the only index of system (mis) performance treated in this paper apart from the closely associated relative traffic loss rate (L_r), the probability that a generated unit of traffic will be lost.

2. THE THREE CONTROL POLICIES

2.1 Non-Feedback Control

A non-feedback control policy is one which does not use information concerning either (i) the traffic currently waiting at the intersection, or (ii) the pattern of traffic to be generated in future time periods. In [1], a natural "cyclic" non-feedback system of control at the intersection of two or more streams of traffic was studied, and an explicit expression for the traffic loss rate L was derived.

However, that system was assumed to involve an "orange light" in any transition from allowing traffic thru from one source to allowing it thru from the second source. This is not so in the work reported here. See next to last paragraph of the previous section.

It will be convenient to focus attention on a single one of the traffic streams, with parameters p and c. (Let $q=1-p$.) Define the state of the stream to be the number of traffic units waiting in its queue at the beginning of a time period; this will be an integer between 0 and c inclusive. The matrix of state-to-state transition probabilities, over a time period in which the stream does have access to the intersection, is the $(c+1)\times(c+1)$ array

$$G = \begin{vmatrix} 1 & 0 & 0 & \ldots\ldots & 0 & 0 \\ q & p & 0 & \ldots\ldots & 0 & 0 \\ 0 & q & p & \ldots\ldots & 0 & 0 \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & 0 & & q & p \end{vmatrix} ;$$

the corresponding matrix, if the intersection is closed to the stream, is

$$R = \begin{vmatrix} q & p & 0 & . & . & . & . & 0 & 0 \\ 0 & q & p & . & . & . & . & 0 & 0 \\ 0 & 0 & q & . & . & . & . & 0 & 0 \\ . & . & . & & & & & . & . \\ . & . & . & & & & & . & . \\ . & . & . & & & & & . & . \\ 0 & 0 & 0 & & & & & q & p \\ 0 & 0 & 0 & & & & & 0 & 1 \end{vmatrix} .$$

The non-feedback systems studied in [1] involved a repetitive pattern in each cycle of which the intersection is open to the stream for k consecutive time periods and then closed to the stream for ℓ consecutive periods. Thus the transition matrix for a full cycle is

$$X = R^{\ell} G^{k} .$$

If the stationary vector of X is $Y = [u_o, u_1, \ldots, u_{c-1}, u_c]$, then the traffic loss rate for the (single) stream, L_s, is ([1], p.12)

$$L_s = \frac{1}{k+\ell} [p\ell - \tilde{Y} (I-V^{\ell}) C] , \qquad (2.1)$$

where

$$\tilde{Y} = [u_o, u_1, \ldots, u_{c-1}] ,$$

$$V = q I + p N ,$$

$$N = [n_{st}] , \; n_{st} = \begin{cases} 1 & \text{if } s = t-1 \\ o & \text{otherwise} \end{cases}$$

$$C = \begin{vmatrix} c \\ c-1 \\ . \\ . \\ . \\ 1 \end{vmatrix}$$

When the probabilities of a unit of traffic being generated are equal for the two streams, then the number of consecutive time periods during which the intersection is kept open are equal for the two streams. Thus we set

(i) $k = \ell$, when there is no orange light

(ii) $k + 2 = \ell$, when there is an orange light.

In this paper we do not consider systems with an orange light and hence we set

$$k = \ell .$$

Previously unreported computations which have been made, for non-feedback control of two streams of traffic in which the probabilities of a unit of traffic being generated are equal $(p_o = p_1)$, indicate that the minimal traffic loss occurs when

$$k = \ell = 1 .$$

In this case (2.1) reduces to

$$L_s = pu_c 1/2 , \tag{2.2}$$

and the total traffic loss rate L for the system is

$$L = pu_c . \tag{2.3}$$

Thus

$$L_r = u_c/2 .$$

2.2 Feedback Control

A system to regulate the flow of traffic at the intersection will be called a feedback system if it takes into account the traffic currently waiting at the intersection, but does not consider the pattern of traffic to be generated in future time periods. In [2], such a system was studied for the case of two intersecting streams, S_0 and S_1.

The results of [2] show that the class of optimal feedback control policies for the 2-stream case are precisely those consistent with the following four rules:

(a) If both streams' queues are filled to capacity, open the intersection to S_1. (Recall that $p_0 \leq p_1$.)

(b) If both streams' queues are empty, open the intersection to S_1.

(c) If one queue is filled to capacity and the other is not, open the intersection to the stream with filled queue.

(d) If one queue is empty and the other is not, open the intersection to the stream which is not empty.

Define the state of the system to be the total number of traffic units waiting (in both queues), so that $C = c(0) + c(1)$ is the maximum possible value of this "state variable". Then under any optimal feedback control policy, the matrix of period-to-period transition probabilities is given [2] by the $(C+1) \times (C+1)$ array

$$\begin{vmatrix} q_o & p_o & 0 & \cdots & 0 & 0 \\ q_oq_1 & p_oq_1+p_1q_o & p_op_1 & \cdots & 0 & 0 \\ 0 & q_oq_1 & p_oq_1+p_1q_o & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & & & \\ 0 & 0 & 0 & & p_oq_1+p_1q_o & p_op_1 \\ 0 & 0 & 0 & & q_1 & p_1 \end{vmatrix}$$

If $U = [u_o, u_1, \ldots, u_C]$ is the fixed vector of X, then the system traffic loss rate is given by

$$L = u_C p_o \, , \tag{2.4}$$

and

$$L_r = u_C p_o / (p_o + p_1) \, . \tag{2.5}$$

2.3 Prophesying Control

Let us now consider what information, in addition to that employed by a feedback control system, might be useful in reducing traffic loss rates. One such item of information, clearly, is (a) knowledge of which streams, during the current time period, will have new units of traffic generated. A second such item would be (b) knowledge of the traffic generation patterns for future time periods. Note that (a), and some version of (b), are physically attainable through the use of detection devices placed "upstream" of the intersection.

Ordinarily, the availability of both (a) and (b) rather than (a) alone would confer some advantage. However, this is not the case under our present assumptions (no orange light, only 2 traffic streams). To see why, define "state" as in the preceding subsection. One can demonstrate the optimality, among all control policies, of a class of policies which make no use of knowledge concerning future time periods. Specifically, these policies are those obtained from the optimal feedback rules (a) - (d) of the last subsection by the following two modifications:

(a') If in state C (i.e., both queues filled to capacity), and only one of the streams is to have a new traffic unit generated during the current time period, then open the intersection to that stream.

(b') If in state 0 (i.e., both queues are empty), and only one of the streams is to have a new traffic unit generated during the current time period, then open the intersection to that stream.

The proof of optimality follows the same logic used in [2], and so will not be written out.

For any of these optimal policies, the matrix of period-to-period transition probabilities is given by the (C+1) x (C+1) array

$$X = \begin{vmatrix} 1-p_o p_1 & p_o p_1 & 0 & \cdots & 0 & 0 \\ q_o q_1 & p_o q_1 + p_1 q_o & p_o p_1 & \cdots & 0 & 0 \\ 0 & q_o q_1 & p_o q_1 + p_1 q_o & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & & & \\ 0 & 0 & 0 & & p_o q_1 + p_1 q_o & p_o p_1 \\ 0 & 0 & 0 & & q_o q_1 & 1-q_c q_1 \end{vmatrix}$$

which differs only in its top and bottom rows from the matrix given in the previous subsection. If $U = [u_o, u_1, \ldots, u_C]$ is the fixed vector of X, then the system traffic loss rate is given by

$$L = u_C p_o p_1 \qquad (2.6)$$

and

$$L_r = u_C p_o p_1/(p_o + p_1) \; . \qquad (2.7)$$

3. NUMERICAL RESULTS

Attached is a tabulation of the expected traffic loss at an intersection under the three types of control described earlier. The computations were carried thru for $p = .05(.05).95$ and capacities 1(1)4 at each of the two intersecting roads. The improvement resulting in each case from the use of feedback control over the non-feedback control, and that (over simple feedback) from knowing whether a vehicle will appear in the current period, have also been tabulated.

One aspect of the data is that for each capacity each of the improvements mentioned above is the same for p as for $1-p$. This tends to indicate that the functions I(feedback, non-feedback) (improvement resulting from the use of feedback control over the non-feedback control) and I(known traffic, feedback) (improvement obtained from forecast of traffic over that of simple feedback control) are related to the entropy of the system.

The following example shows that the exact nature of the relationship would have to be formulated with care. Suppose we are given a system with states $\{S_1, S_2\}$ and actions $\{a_1, a_2\}$. The cost is: $C(S_1,a_1) = C(S_2,a_2) = 0$ $C(S_1,a_2) = 1$ and $C(S_2,a_1) = 100$.

Independent of the current state and action the transition probabilities are: $Pr. (S_1) = p$ $Pr (S_2) = 1-p$. For $p > 1/2$, the entropy of the system increases as p decreases while the function I(feedback, non-feedback) decreases with decreasing p.

Aside from the above qualitative interpretation of the data, the information, accompanied by cost functions, could be used for optimization decisions in the usual manner. For example, in a situation in which feedback control is more expensive than non-feedback, and the cost per unit overflow can be expressed in units comparable with those stating the cost of control, a direct cost-benefit comparison is available.

Even in the absence of so much explicit additional price information, if it is known that some one of several intersections can be converted from a non-feedback to a feedback control scheme, then, where the parameters fall in the range carried by the computation, the improvement in each case can be read off and presumably the change effected at that intersection where the improvement will be largest.

The computations carried out are examples, that can be extended to ranges of the parameters not included in the tabulation here presented. They demonstrate the feasibility of the optimization techniques described above.

In our tabulation of the data we have used the symbols I(feedback, non-feedback) and I(know traffic, feedback) to denote respectively the improvement obtained in the objective function when using feedback control as against non-feedback control, and the improvement obtained from knowing the current traffic generated as against simple feedback.

For the system under study, in which knowledge of which periods in the future will yield units of traffic along the individual road, does not enable us to improve the system performance over that attained from knowing current traffic before deciding to which intersection the light should be green, our data also show that I (feedback, non-feedback) $\geq$ I(know traffic, feedback). When the left side of the inequality is positive the inequality is strict.

TABLE 1. Expected overflow under different control policies

	Nonfeedback	Feedback	Known Traffic	I(feedback, non-feedback)	I (known tr… feedback)
CAPACITY = 2					
.05	.000131	.000007	.000000	.000124	.000007
.10	.001099	.000122	.000002	.000977	.000120
.15	.003868	.000680	.000021	.003189	.000658
.20	.009524	.002353	.000147	.007171	.002206
.25	.019231	.006250	.000687	.012981	.005563
.30	.034177	.013966	.002494	.020212	.011472
.35	.055502	.027534	.007495	.027967	.020039
.40	.084211	.049231	.019248	.034980	.029983
.45	.121096	.081200	.042854	.039896	.038346
.50	.166667	.125000	.083333	.041667	.041667
.55	.221096	.181200	.142854	.039896	.038346
.60	.284211	.249231	.219248	.034980	.029983
.65	.355502	.327534	.307495	.027967	.020040
.70	.434177	.413966	.402494	.020212	.011472
.75	.519231	.506250	.500687	.012981	.005563
.80	.609524	.602353	.600147	.007171	.002206
.85	.703868	.700680	.700021	.003189	.000658
.90	.801099	.800122	.800001	.000977	.000120
.95	.900131	.900007	.900000	.000124	.000007
CAPACITY = 4					
.05	.000000	.000000	.000000	.000000	.000000
.10	.000014	.000000	.000000	.000014	.000000
.15	.000120	.000001	.000000	.000119	.000001
.20	.000587	.000009	.000001	.000577	.000009
.25	.002066	.000076	.000008	.001990	.000068
.30	.005868	.000456	.000084	.005412	.000372
.35	.014224	.002135	.000616	.012088	.001519
.40	.030332	.008121	.003530	.022211	.004591
.45	.057890	.025128	.015531	.032762	.009597
.50	.100000	.062500	.050000	.037500	.012500
.55	.157890	.125128	.115531	.032762	.009597
.60	.230332	.208121	.203530	.022211	.004591
.65	.314224	.302135	.300616	.012088	.001519
.70	.405868	.400456	.400084	.005412	.000372
.75	.502066	.500076	.500008	.001990	.000068
.80	.600587	.600009	.600001	.000577	.000009
.85	.700120	.700001	.700000	.000119	.000001
.90	.800014	.800000	.800000	.000014	.000000
.95	.900000	.900000	.900000	.000000	.000000

	Nonfeedback	Feedback	Known Traffic	I(feedback, non-feedback)	I(known traffic, feedback
CAPACITY = 6					
.05	.000000	.000000	.000000	.000000	.000000
.10	.000000	.000000	.000000	.000000	.000000
.15	.000004	.000000	.000000	.000004	.000000
.20	.000037	.000000	.000000	.000037	.000000
.25	.000229	.000001	.000000	.000228	.000001
.30	.001065	.000015	.000003	.001050	.000013
.35	.003990	.000178	.000052	.003811	.000127
.40	.012433	.001553	.000687	.010880	.000866
.45	.032528	.007889	.006410	.022639	.003479
.50	.071429	.041667	.035714	.029762	.005952
.55	.132528	.109889	.106410	.022639	.003479
.60	.212433	.201553	.200687	.010880	.000866
.65	.303990	.300178	.300052	.003811	.000127
.70	.401065	.400015	.400003	.001050	.000013
.75	.500229	.500001	.500000	.000228	.000001
.80	.600037	.600000	.600000	.000037	.000000
.85	.700004	.700000	.700000	.000004	.000000
.90	.800000	.800000	.800000	.000000	.000000
.95	.900000	.900000	.900000	.000000	.000000

	Nonfeedback	Feedback	Known Traffic	I(feedback, non-feedback)	I(known traffic, feedback
CAPACITY = 8					
.05	.000000	.000000	-.000000	.000000	.000000
.10	.000000	.000000	.000000	.000000	.000000
.15	.000000	.000000	.000000	.000000	.000000
.20	.000002	.000000	.000000	.000002	.000000
.25	.000025	.000000	.000000	.000025	.000000
.30	.000195	.000001	.000000	.000195	.000000
.35	.001146	.000015	.000004	.001131	.000011
.40	.005341	.000305	.000135	.005036	.000170
.45	.019661	.004202	.002774	.015459	.001428
.50	.055556	.031250	.027778	.024306	.003472
.55	.119661	.104202	.102774	.015459	.001428
.60	.205341	.200305	.200135	.005036	.000170
.65	.301146	.300015	.300004	.001131	.000011
.70	.400195	.400001	.400000	.000195	.000000
.75	.500025	.500000	.500000	.000025	.000000
.80	.600002	.600000	.600000	.000002	.000000
.85	.700000	.700000	.700000	.000000	.000000
.90	.800000	.800000	.800000	.000000	.000000
.95	.900000	.900000	.900000	.000000	.000000

The following is a listing of the program used to generate the tabulation given above. The listing for the subroutine FIXV is reproduced in Reference [1], "The Traffic Loss at a Merge-Point Controlled by Non-feedback Regulation".

```
      DIMENSION XR(9,9),XG(9,9),X(9,9),U(9),XF(9,9),UF(9)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DO 99 I1=2,5
      I2=I1-1
      I3=I2-1
      J1=2*I1-1
      J2=J1-1
      J3=J2-1
      WRITE (6,300)  J2
      DO 10 I = 1,I1
      DO 10 J = 1,I1
      XR(I,J)=0.0
      XG(I,J)=0.0
   10 X(I,J)=0.0
      DO 15 I=1,J1
      DO 15 J=1,J1
   15 XF(I,J)=0.0
      XR(I1,I1)=1.0
      XG(1,1)=1.0
      DO 99 M=1,19
      T=FLOAT(M)
      P=.05*T
      Q=1.0-P
      DO 20 I=1,I2
      XR(I,I)=Q
      XG(I+1,I+1)=P
      XR(I,I+1)=P
   20 XG(I+1,I)=Q
      DO 25 I=1,I1
      DO 25 J=1,I1
   25 X(I,J)=0.0
      DO 30 I=1,I1
      DO 30 J=1,I1
      DO 30 K=1,I1
   30 X(I,J)=X(I,J)+XR(I,K)*XG(K,J)
```

```
      CALL FIXV(X,U,I1,I2)
      EB=U(I1)*P
      XF(1,1)=Q
      XF(1,2)=P
      XF(J1,J2)=Q
      XF(J1,J1)=P
      DO 40 I=1,J3
      XF(I+1,I)=Q*Q
      XF(I+1,I+1)=2.*Q*P
   40 XF(I+1,I+2)=P*P
      CALL FIXV(XF,UF,J1,J2)
      EF=UF(J1)*P
      XF(1,1)=1.-P*P
      XF(1,2)=P*P
      XF(J1,J2)=Q*Q
      XF(J1,J1)=1.-Q*Q
      CALL FIXV(XF,UF,J1,J2)
      EFI=UF(J1)*P*P
      EI1=EB-EF
      EI2=EF-EFI
      WRITE (6,100) P,EB,EF,EFI,EI1,EI2
   99 CONTINUE
  100 FORMAT (2X,F6.2,2X,F9.6,2X,F9.6,2X,F9.6,9X,F9.6,2X,F9.6)
  300 FORMAT (/'     CAPACITY    =     ',I2)
      END
```

APPENDIX C: ANALYSIS OF COLLISION DATA FOR TOWER AND NON-TOWER AIRPORTS

This appendix, in essence, answers affirmatively the following question: "Is there a valid criterion by which the observed differences in the frequencies of mid-air collisions in the tower and non-tower environments have statistical significance?"

The scarcity of MAC's has thwarted all attempts to characterize any relevant underlying probability distributions (frequency of collision per 1,000,000 operations, etc.) in order to apply conventional statistical methods to estimate the possible effectiveness of towers.

Thus, with available data we cannot answer the question using standard techniques of analysis, and could merely infer subjectively that it "looks as if towers can effect the collision rate". On the other hand, with the method developed here we can state that the differences in the collision rates are not merely adventitious, i.e., that the subjective inference is in fact formally justified. We cannot, however, make any more precise statement such as "The chance of collision at such and such a traffic level is x percent smaller in the presence of a tower". This latter kind of result might be inferred from standard techniques (assuming e.g., normal or Poisson occurrence of collision) if the data were sufficiently detailed.

The following quotations [47, 48] may put this matter in proper perspective. (We have added emphasis by underlining)

(1) "... the important place ascribed to the normal distribution in statistical theory was justified on the basis that any known

---

(1) [47, p. 385].

continuous distribution could be transformed to the normal distribution. But, of course, experimenters quite frequently have no knowledge of the form of the distribution with which they are dealing, or at least so little information that they cannot prescribe a normalizing transformation. Until recently there was not much to be done in this situation, and experimenters were more or less forced to make wholesale assumptions of normality. During the past few years, however, techniques have been developed for estimating parameters and testing hypotheses which require no assumption about the form of the distribution function. These techniques are called non-parametric methods, or better, distribution-free methods. While the collection of distribution-free methods is not nearly so comprehensive as that based in normal theory, a good beginning has been made."

(2) Most of the methods of estimation and of testing hypotheses we have studied so far are based on the assumption that the observations are taken from normal populations. These methods extract all the information that is available in a sample, and they usually attain the best possible precision, that is, the most reliable results. In spite of this, there are several reasons why we may wish to use other, less precise methods - the assumption of normality may be grossly incorrect, the labor involved in carrying out the more precise methods may be excessive, or a short-cut method may be desired to determine in advance whether it is worthwhile to carry out the more detailed calculations.

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(2) [48, pp. 207-209].

Certain methods of inference have the important advantage that they do not require the more stringent assumptions of the methods based on the normal distribution. These methods, which usually have the additional advantage that they require less burdensome calculations, are known as nonparametric (or distribution-free) methods, since they are generally not related specifically to the parameters of given distributions. The main advantage of nonparametric methods is that exact tests can be performed when the assumptions underlying so-called "standard" methods cannot all be met; essentially, these methods do not depend on the distribution of the population (or populations) from which the samples are obtained. The major disadvantage of nonparametric methods is that they may be wasteful of information, and usually they have a smaller efficiency than the corresponding parametric methods, provided that the assumptions of the standard (parametric) methods can be met.

C.1. Preparation of the Data

This appendix presents a statistical analysis of the data in [12] on the frequencies of collisions at tower and non-tower airports, respectively. Using a "modified signs test," it is shown that the presence of a VFR tower is associated with lower collision rates. This is a very weak-sounding statement, but apparently had not previously been solidly established due to the difficulty of rigorously drawing inferences from the small number of collisions reported. Appropriate cautions are noted, and suggestions for facilitating further analyses are offered.

The balance of the present Section C.1 describes the basic data and how they are brought into the form to which the analysis method is applied. This method is presented at the conceptual level in Section C.2, with technical details deferred. Section C.3 sets forth our findings, with some associated caveats. Better-articulated and more detailed data would of course promote the possibility of more incisive analyses leading to sharper conclusions; particular recommendations along this line are given in C.4. Finally, Section C.5 reports the details of the methodology and its application in the present instance.

In keeping with Tables 13 and 14 of [12], our Tables C.1.1 through C.1.8 appear as four matched pairs; the first table in each pair (e.g., C.1.3) refers to airports with an FAA tower, while the second (e.g., C.1.4) refers to non-tower airports. The columns of these tables correspond to the successive years 1961-1968.

Each table is in turn divided into two panels. The rows of the upper panel correspond to airport activity levels in terms of itinerant flights, while those of the lower panel correspond to levels of total (itinerant and local) traffic activity.

Tables C.1.1 and C.1.2 present simple counts of the number of airports which fell into each of the activity-level groups during each of the study years. For 1965-1968 the "tower airport" numbers are taken[1] from Table 8 in [12];the data for 1961-1964 were obtained from the author of [12]. For non-tower airports the 1961-1967 data were obtained from that author, and the 1968 figures from Table 9 in [12].

Tables C.1.3 and C.1.4 present estimates of the number of operations of each kind (itinerant in the upper panel, total in the lower) occurring at all airports of each category during each year. Each entry in Table C.1.3 is found by multiplying the corresponding entry of Table C.1.1 by the midpoint of the interval of activity-levels that labels its row; Table C.1.4 is obtained from Table C.1.2 by the same procedure.

Tables C.1.5 and C.1.6 give the number of midair collisions reported each year for each category of airport. These are taken from Tables 13 and 14 (left-hand sides) in [12].[2] Blank cells indicate zero entries. The collisions in question must occur within 5 miles of the airport, and must satisfy certain additional "relevance" conditions indicated in Appendix II of [12].

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[1]The last 1964 entry in the upper panel of Table C.1.1 is a correction to Table 8 of [12].

[2]The upper panel of Table C.1.6 amplifies (and differs from) Table 14 of [12] in containing an additional 1968 incident at an airport in the 30-40,000 itinerant-operations category. The first incident in the 1965 column apparently occurred at a field with less than 40,000 total operations, hence is not reflected in the lower panel.

NUMBER OF AIRPORTS

TOWER

TABLE C.1.1

Annual Itinerant Operations (thousands)

	1961	1962	1963	1964	1965	1966	1967	1968
20-25	8	13	11	14	9	6	2	4
25-30	15	18	17	10	16	15	14	9
30-35	16	18	25	23	19	11	13	14
35-40	14	17	20	15	22	23	18	13
40-45	11	11	17	21	13	17	10	14
45-50	17	13	14	11	19	18	22	9
50-55	14	11	13	19	15	17	14	19
55-60	17	19	13	14	14	14	21	21

Annual Total Operations (thousands)

	1961	1962	1963	1964	1965	1966	1967	1968
40-50	22	19	18	18	17	17	11	12
50	20	20	22	23	20	16	16	8
60	11	18	21	16	17	20	16	15
70	16	10	16	17	20	18	15	9
80	23	25	26	13	15	11	14	13
90	22	24	16	17	15	16	13	23
100	20	19	22	17	13	11	24	19
110	11	13	15	23	19	16	14	17
120	9	9	9	16	19	20	9	15
130	7	8	8	10	14	14	14	12
140-150	10	13	9	4	9	9	20	8

NUMBER OF AIRPORTS

NON-TOWER

TABLE C.1.2

Annual Itinerant Operations (thousands)	1961	1962	1963	1964	1965	1966	1967	1968
20-25	52	54	59	64	64	65	68	70
25	25	26	29	31	31	31	33	34
30	22	23	25	27	28	28	29	30
35	15	16	18	19	19	19	20	21
40	4	4	4	4	4	4	5	5
45	4	5	5	5	5	5	5	6
50	3	3	3	3	4	4	4	4
55-60	2	2	3	3	3	3	3	3
Annual Total Operations (thousands)								
40-50	60	63	68	74	74	75	79	81
50	43	45	49	53	53	54	56	58
60	30	32	35	37	38	38	40	41
70	15	16	18	19	19	19	20	21
80	15	16	18	19	19	19	21	21
90	7	8	8	9	9	9	10	10
100	5	5	6	6	6	6	6	7
110	5	5	6	6	6	6	7	7
120	5	5	6	6	6	7	7	7
130	3	3	3	3	3	3	3	4
140-150	3	3	3	3	3	3	4	4

TABLE C.1.3

TOTAL NUMBER OF OPERATIONS = # Airports x # Operations/Airports

TOWER

multiply entry by 10^3

Annual Itinerant Operations (thousands)

	1961	1962	1963	1964	1965	1966	1967	1968	TOTAL
20-25	180	292.5	247.5	315	202.5	135	45	90	1507.5
25-30	412.5	495	467.5	275	440	412.5	385	247.5	3135
30-35	520	585	812.5	747.5	617.5	357.5	422.5	455	4517.5
35-40	525	637.5	750	562.5	825	862.5	675	487.5	5325
40-45	467.5	467.5	722.5	892.5	552.5	722.5	425	595	4845
45-50	807.5	617.5	665	522.5	902.5	855	1045	427.5	5842.5
50-55	735	577.5	682.5	997.5	787.5	892.5	735	997.5	6405
55-60	977.5	1092.5	747.5	805	805	805	1207.5	1207.5	7647.5
Total	4625	4765	5095	5117.5	5132.5	5042.5	4940	4507.5	39,225.0

Annual Total Operations (thousands)

	1961	1962	1963	1964	1965	1966	1967	1968	TOTAL
40-50	990	855	810	810	765	765	495	540	6030
50-60	1100	1100	1210	1265	1100	880	880	440	7975
60-70	715	1170	1365	1040	1105	1300	1040	975	8710
70-80	1200	750	1200	1275	1500	1350	1125	675	9075
80-90	1955	2125	2210	1105	1275	935	1190	1105	11900
90-100	2090	2280	1520	1615	1425	1520	1235	2185	13870
100-110	2100	1995	2310	1785	1365	1155	2520	1995	15225
110-120	1265	1495	1725	2645	2185	1840	1610	1955	14720
120-130	1125	1125	1125	2000	2375	2500	1125	1875	13250
130-140	945	1080	1080	1350	1890	1890	1890	1620	11745
140-150	1450	1885	1305	580	1305	1305	2900	1160	11890
Total	14935	15860	15860	15470	16290	15440	16010	14525	124390

TABLE C.1.4

TOTAL NUMBER OF OPERATIONS = # Airports x Operations/Airport

NON-TOWER

	1961	1962	1963	1964	1965	1966	1967	1968	TOTAL	multiply entry by 10^3
20-25	1170	1215	1327.5	1440	1440	1462.5	1530	1575	11160	
25	687.5	715	797.5	852.5	852.5	852.5	907.5	935	6600	
30	715	747.5	812.5	877.5	910	910	942.5	975	6890	
35	562.5	600	675	712.5	712.5	712.5	750	787.5	5512.5	
40	170	170	170	170	170	170	212.5	212.5	1445	
45	190	237.5	237.5	237.5	237.5	237.5	237.5	285	1900	
50	157.5	157.5	157.5	157.5	210	210	210	210	1470	
55-60	115	115	172.5	172.5	172.5	172.5	172.5	172.5	1265	
Total	3767.5	3957.5	4350	4620	4705	4727.5	4962.5	5152.5	36242.5	
40-50	2700	2835	3060	3330	3330	3375	3555	3645	25830	
50	2365	2475	2695	2915	2915	2970	3080	3190	22605	
60	1950	2080	2275	2405	2470	2470	2600	2665	18915	
70	1125	1200	1350	1425	1425	1425	1500	1575	11025	
80	1275	1360	1530	1615	1615	1615	1785	1785	12580	
90	665	760	760	855	855	855	950	950	6650	
100	525	525	630	630	630	630	630	735	4935	
110	575	575	690	690	690	690	805	805	5520	
120	625	625	750	750	750	875	875	875	6125	
130	405	405	405	405	405	405	405	540	3375	
140-150	435	435	435	435	435	435	580	580	3770	
Total	12645	13275	14580	15455	15520	15745	16765	17345	121330	

TABLE C.1.5

TOWER

Annual Itinerant Operations (thousands)

	1961	1962	1963	1964	1965	1966	1967	1968	TOTAL
20-25									0
25									0
30			1						1
35									0
40									0
45									0
50									0
55-60									0
Total	0	0	1	0	0	0	0	0	1

Annual Total Operations (thousands)

	1961	1962	1963	1964	1965	1966	1967	1968	TOTAL
40-50			1						1
50									0
60									0
70									0
80									0
90									0
100									0
110		1				1			2
120									0
130									0
140-150									0
Total	0	1	1	0	0	1	0	0	3

Table C.1.6

TOTAL NUMBER OF COLLISIONS

NON-TOWER

	1961	1962	1963	1964	1965	1966	1967	1968	TOTAL
20-25				2	1	4			7
25			3				1	1	5
30	1		1	1				1	4
35	1								1
40				2		1			3
45					1	1			2
50									0
55-60							1		1
Total	2	0	4	5	2	6	2	2	23
40-50		1	1	2		4			8
50			1	1			2		4
60	1	1	1		1			1	5
70									0
80								1	1
90			1			1	1		3
100									0
110	1							1	2
120						1			1
130	1								1
140-150				2					2
	3	2	4	5	1	6	3	3	27

Tables C.1.7 and C.1.8 give the estimated probability of collision each year in each airport category. The probabilities are of course estimated by the associated relative frequencies. That is, an entry in Table C.1.7 is obtained (apart from a final multiplication by 10^6) as the quotient of the corresponding datum in Table C.1.5 by that in Table C.1.3; Table C.1.8 is obtained in the same fashion from Tables C.1.6 and C.1.4. This pair of tables corresponds to, but shows deviations from, the right-hand panels of Tables 13-14 in [12].

Finally, the entries of the signs table (Table C.1.9) are obtained in two steps. First, each entry in Table C.1.7 (for tower airports) is subtracted from the corresponding entry of Table C.1.8 (for non-tower airports). This yields a difference table. Then positive entries of the difference table are replaced by plus signs, negative entries by minus signs, and zero entries by zeros (designated, for simplicity, by blanks).

TABLE C.1.7

ESTIMATED COLLISION PROBABILITY

TIMES 10^6, TOWER AIRPORTS

Itinerant Operations (1000's)	1961	1962	1963	1964	1965	1966	1967	1968		TOTAL
20-25										
25-30										
30-35			1.23							0.22
35-40										
40-45										
45-50										
50-55										
55-60										
Total			0.20							0.03
Total Operations (1000's)										
40-50			1.23							0.17
50-60										
60-70										
70-80										
80-90										
90-100										
100-110										
110-120		0.67				0.54				0.14
120-130										
130-140										
140-150						0.06				0.02
Total		0.06	0.06							

TABLE C.1.8

TIMES 10^6, NON-TOWER AIRPORTS

Itinerant Operations (1000's)	1961	1962	1963	1964	1965	1966	1967	1968		TOTAL
20-25				1.39	0.69	2.74				0.63
25-30			3.76				1.10	1.07		0.76
30-35	1.40		1.23	1.14				1.03		0.58
35-40	1.78									0.18
40-45				11.77		5.88				2.08
45-50					4.21	4.21				1.05
50-55										
55-60								5.80		0.79
Total	0.53		0.92	1.08	0.43	1.27	0.40	0.40		0.64
40-50		0.35	0.33	0.60		1.19				0.31
50-60			0.37	0.34			0.65			0.18
60-70	0.51	0.48	0.44		0.40			0.38		0.26
70-80										
80-90								0.56		0.08
90-100			1.32			1.17	1.05			0.45
100-110								1.24		
110-120	1.74									0.36
120-130						1.14				0.16
130-140	2.47									0.30
140-150				4.66						0.53
Total	0.24	0.15	0.27	0.32	0.06	0.38	0.18	0.17		0.22

TABLE C.1.9

SIGNS OF DIFFERENCES FOR (C.1.8) - (C.1.7)

Itinerant Operations (1000's)	1961	1962	1963	1964	1965	1966	1967	1968	TOTAL
20-25				+	+	+			+
25-30			+				+	+	+
30-35	+			+				+	+
35-40	+								+
40-45						+			+
45-50				+	+	+			+
50-55									
55-60							+		+
Total	+		+	+	+	+	+	+	+
40-50		+	-	+		+			+
50-60			+	+			+		+
60-70	+	+	+		+			+	+
70-80									
80-90								+	+
90-100			+			+	+		+
100-110								+	
110-120	+	-				-			+
120-130						+			+
130-140	+			+					+
140-150									+
Total	+	+	+	+	+	+	+	+	+

C.2 Analysis Method

In this section we describe the method used to analyze the upper and lower panels of Table C.1.9. The description will be at the conceptual level, mathematical particulars being deferred to Section C.5.

A plus sign indicates a situation (combination of activity-level and year) in which the "observed probability" of collision is greater at non-tower airports than at tower airports. A minus sign has the opposite significance. Each panel displays both a healthy number of plusses, and a decided preponderance of them over minuses. The question is whether this preponderance is great enough to be a reliable indicator of a genuine propensity to plus rather than minus signs -- i.e., of a systematic association of tower-airports with lower collision rates -- or whether it might reasonably have arisen by sheer chance even if a control tower's presence has no implications for an airport's collision rate. For this question to be a fair one, it must of course be assumed that tower and non-tower airports (of the same activity level, in the same year) do not systematically differ in some collision-relevant feature <u>other</u> than the presence or absence of a tower.

For analysis purposes, the situation is complicated by the presence of so many zeros (blanks), which arise by the differencing 0-0=0 from the rarity of collisions as reflected in the many blanks of Tables C.1.5 and C.1.6. Ordinarily coincidence would generate few if any zeros; they could be ignored, and the following <u>sign test</u> [45; p. 280] would be applied.

Regard the "observations" (the entries in the array to be analyzed) as independent drawings in a binomial-probability model governed by the two parameters

p^+ = probability of a plus sign,

p^- = probability of a minus sign.

We will also use the notations

N = number of observations,

P(0) = number of plus signs in given array,

M(0) = number of minus signs in given array.

If P and M denote the respective numbers of plusses and minuses in a generic array of the same dimensions, then

$$P + M = P(0) + M(0) = N. \qquad (C.1)$$

From this it is easy to characterize those arrays which are at least as favorable (in supporting the relevance of VFR towers to lower collision rates) as is the given array; they are precisely the arrays for which the two equivalent statements

$$P \geq P(0), \quad M \leq M(0) \qquad (C.2)$$

both hold.

Recall that our question is whether the observed predominance of plus signs (or an even greater predominance!) would have a reasonable probability of occurrence even if there were no systematic propensity for plusses (i.e., no underlying association of tower-airports with lower collision rates). In terms of the possible event

F: an array at least as favorable as the given one occurs, and the "null hypothesis", that plusses and minuses are equi-likely,

$$H_o : p^+ = p^-, \tag{C.3}$$

the question is whether the conditional probability

$$\text{Prob } \{F, \text{ given } H_o\} \tag{C.4}$$

is large enough that one is willing to accept the occurrence of F (it did occur!) as compatible with hypothesis (C.3). On the basis of the binomial-probability model, plus the observation that since

$$p^+ + p^- = 1, \tag{C.5}$$

the hypothesis H_o is equivalent to $p^+ = p^- = 1/2$, the probability (C.4) can readily be computed; it is just the probability of P(0) "heads" (or more) in N tosses of a fair coin. With the probability known, the judgment can be made as to whether or not it is "large enough"; if not, then H_o is rejected and hence the association of tower-airports with lower collision rates will have been confirmed.

To accommodate the frequency of zeros in Table C.1.9, we have developed a simple modification of the procedure just described, i.e., a <u>modified signs test</u>. It involves a <u>tri</u>nomial rather than a <u>bi</u>nomial probability model; the previous parameters p^+ and p^- are joined by

$$p^o = \text{probability of a zero},$$

and the previous notation is supplemented by

$Z(0)$ = number of zeros in given array,

Z = number of zeros in a generic array with N observations.

The previous relations (C.1) and C.5) become

$$P + M + Z = P(0) + M(0) + Z(0) = N, \tag{C.6}$$

$$p^+ + p^- + p^o = 1. \tag{C.7}$$

The two statements in (C.2) are no longer equivalent, but we continue to use them as the definition of the class F of arrays "at least as favorable" as the given one. Note the implicit assumption that all entries have the same probability (p^o) of yielding a zero.

Complication comes from the fact that the probability (C.4) is not determined; its calculation requires p^+, p^-, and p^o to be known, but (C.3) and (C.7) fail to determine these quantities. To deal with this difficulty, we introduce a parameter u ranging over the interval [0,1], and define the family of hypotheses

$$H_o(u) : p^+ = p^-, \text{ and } p^o = u.$$

In analogy with (C.4), define

$$f(u) = \text{Prob}\ \{F, \text{ given } H_o(u)\}, \tag{C.8}$$

and let

$$f_{max} = \max\ \{f(u) : 0 \leq u \leq 1\}. \tag{C.9}$$

Here each f(u) is determinable, since under $H_o(u)$

$$p^+ = p^- = (1-u)/2, \quad p^o = u. \tag{C.10}$$

Hence f_{max} is in principle determinable. If f_{max} is "too small to be credible," then so is every f(u); hence every hypothesis $H_o(u)$ is rejected, the same is true of H_o which is the union of the $H_o(u)$'s, and so the suspected association will have been confirmed.

The results obtained using this method will be reported in the next section. In concluding our sketch of methodology, it should be noted that the procedure just described can be adapted to deal with quantitative versions of the question at issue. That is, suppose we ask whether there is a sound basis for saying that collision rates at tower airports would be X% lower than those at non-tower airports. Then the ordinary sign test could be applied to a modified version of Table C.1.9, in which a plus sign would appear only if the corresponding entries in Tables C.1.7 and C.1.8 displayed a relative difference of X% or more; otherwise a minus sign would be entered. It seems worthwhile to carry out such an analysis for different levels of X, but this has not yet been done.

C.3 Findings and Cautions

The method just described was applied to the upper panel of Table C.1.9, for which

$$N = 64,\ P(0) = 15,\ M(0) = 0,\ Z(0) = 49. \qquad (C.10)$$

The result was

$$f_{max} < 1 \times 10^{-5}, \qquad (C.11)$$

which seems clearly interpretable as "too small to be credible."

One may certainly question the propriety of treating entries in the same row as independent drawings (from the trinomial model), since it is suspected that airport activity-level (which is the row-heading) significantl influences the safety-related need for a tower.[3] This suggests supplementing the above analysis by one in which all entries (over all years) for the same activity-level are aggregated into a single entry. The resultant data appear in the "Totals" columns in Tables C.1.3 through C.1.9; the upper panel in C.1.9 gives

$$N = 8,\ P(0) = 7,\ M(0) = 0,\ Z(0) = 1. \qquad (C.12)$$

Additionally, one might question the treatment of a single year's (column's) entries, which for example might share exceptional weather conditions relevant to the question being analyzed, as independent. This in turn suggests a different auxiliary analysis, this one utilizing the "Totals" rows. For the upper panel in Table C.1.9, the relevant values are again given by (C.12). For these values, it was found that

[3]This supposition is in fact embodied in the current eligibility criteria.

$$f_{max} < 5 \times 10^{-3}. \qquad (C.13)$$

While much larger than (C.11), this too is small enough to signal "reject H_o" in conventional statistical practice.

The same procedure was applied to the lower panel in Table C.1.9. For the "all entries" test,

$$N = 88,\ P(0) = 20,\ M(0) = 3,\ Z(0) = 65, \qquad (C.14)$$

leading to

$$f_{max} \approx 3 \times 10^{-5}. \qquad (C.15)$$

The test on the "Totals" column has

$$N = 11,\ P(0) = 9,\ M(0) = 0,\ Z(0) = 2, \qquad (C.16)$$

while that on the "Totals" row has

$$N = 8,\ P(0) = 8,\ M(0) = 0,\ Z(0) = 0. \qquad (C.17)$$

These lead, respectively, to

$$f_{max} < 1 \times 10^{-3},\quad f_{max} < 4 \times 10^{-3}. \qquad (C.18)$$

Again, rejection of the "null hypothesis" H_o is indicated.

Several interrelated cautions should be voiced at this point. Their general nature can probably be anticipated from the careful wording of the finding reached above, namely that a statistically demonstrable association exists between tower presence and lower collision rates.

The first caution is that statistical association does not necessarily imply a causal relationship. Lower collision rates might be causally related to some other factor, which has "happened" to be correlated with tower presence in the past, but might not be in the future. (A conjectural candidate for such a factor would be the reduced presence of a more "risk-taking" class of pilots, who might be driven to shift to a non-tower airport by the disciplinary overtones of a tower's installation; such transfers will presumably become more difficult as the density of tower airports rises.) Such possibilities may appear remote, but evidently cannot be ruled out by the limited type of analysis reported here.

The term "limited" leads to our second set of cautions. As noted at the start of Section 3.2, preliminary reconnaissance of the collision-data situation indicated that heavy investment in statistical analysis during the current project would probably not prove worthwhile. Early analysis efforts using more common statistical methods did nothing to dispel this impression. Thus the substudy reported in this appendix was a late and limited step; its expansion, once encouraging results were forthcoming, could no longer be accommodated in the study's time-frame. Specifically, the substudy confined itself to the question "is there a demonstrable association between tower presence and lower collision rates?" as distinguished from the more general question "what factors are most strongly associated with lower collision rates, and how do these factors interact?" Its starting point was, in effect, Tables 13 and 14 of [12], plus the particular structuring and aggregations leading to those tables, whereas a broader investigation to explore the more general question should of course begin much closer to the "raw data." In the next section, we indicate some kinds of data desirable for such an investigation.

Our third caution is somewhat independent of the first two. Even if towers were clearly identified as causing certain desirable effects relative to safety, it would not follow that they were the best means of of attaining those effects, and so there could be no immediate inference of an injunction "build more towers." One would want (a) to gain more insight -- perhaps by such approaches as the "visibility model" of Chapter 3 -- into the physical processes through which towers inhibit collisions, and (b) to consider whether these results might not be more effectively or economically attained by alternative means such as on-board proximity-detection devices. Such considerations, of a "cost-effectiveness" or "engineering economics" nature, were explicitly set outside the boundaries of the study reported in this document.

C.4 Recommendations to Facilitate Further Analysis

The category of mid-air collisions might profitably be expanded to include crashes ("collisions" of an aircraft with the ground), and incidents in which one or both of the planes were located on the ground. The subsequent remarks, however, are largely oriented to the "mid-air" category.

To effect the type of broader statistical investigation proposed above, a more complete and systematic organization of the original data is needed. In particular, for each of the 30 (= 3 + 27)mid-air collisions referred to in Tables 13 and 14 of [12] the following specifics should be available, preferably in one table which can easily be surveyed by eye:

(1) Pilot Data (for each aircraft):

- (a) Age
- (b) Sex
- (c) Type -- commercial, private, student, etc.
- (d) Total no. of hrs. logged
- (e) No. of hrs. logged in this A/C type
- (f) No. of people in cockpit?
- (g) Was there a copilot?
- (h) Passenger or passengers in cockpit?
- (i) Relevant physical condition -- eyesight, fatigue, sobriety, etc.

(2) Aircraft Data (for each plane):

- (a) Age
- (b) Manufacturer
- (c) Engine type -- single, multi, etc.
- (d) Wing type -- high, low, etc.
- (e) Instruments

(3) Pre-Collision Data (for each aircraft):

(a) Trip type -- local or itinerant

(b) Trip purpose -- pleasure, business, etc.

(c) Phase of operation -- landing, takeoff, etc.

(d) Existence of flight-plan?

(4) Collision Data:

(a) Year

(b) Month

(c) Date

(d) Day of week

(e) Time of day

(f) Part of day -- daylight, dusk, etc.

(g) Convergence angle

(h) Convergence direction (above, below, left, right)

(i) Proximity to airport

(j) Altitude

(k) Weather

(l) Visibility

(m) Cause of accident

(5) Post-Collision Data (for each aircraft):

(a) Extent of personal injury

(b) Extent of aircraft damage

(c) Other damage

(6) Airport Data:

(a) Name

(b) State

(c) No. of annual itinerant operations

(d) No. of annual local operations

(e) Total no. of annual operations

(f) No. of runways

(g) Runway length

(h) Annual no. of single-engine operations

(i) Annual no. of multi-engine operations

(j) Tower or non-tower?

(k) Proximity of other medium-activity airports

(l) Proximity of high-activity airports

If much of this information for 1961-1968 is in fact available, then further statistical analysis could be attempted promptly. If not, then we urge that careful consideration be given to the inclusion of these items in planning the future reporting process.

Obviously, the above list reflects in part the preconceptions of the writer (neither a pilot or a specialist in air safety) as to how a relevant accident might be described and which of its attributes are likely to prove significant. The expert reader may well be able to improve the list (through addition, deletion, and/or modification), to provide superior wording for some of the present entries, and to phrase appropriate answer categories for some of the stickier items such as (4m).

Our current notions about (4m), plus some of the"preconceptions" just alluded to, can be articulated by noting our (layman's) general impression that the collision of aircraft is a very special event which almost always occurs in one of the following four mutually exclusive ways:[4]

[4]These are phrased for the non-tower case, and would require modification to allow for the presence of a tower.

(1) A pilot did not see the other plane in time (due to lack of experience, distraction by cockpit duties, distraction by passengers, etc.), OR:

(2) A pilot could not see the other plane in time (due to insufficient window area, faulty cockpit design, low-visibility weather, etc.), OR:

(3) The pilot did see the other plane in time, but did not or could not effect prompt proper evasive action (due to malfunction of his plane, impaired judgment or slowed reflexes attributable to alcohol or sleepiness or fatigue, etc.), OR:

(4) The air traffic congestion near the incident exceeded the limitations of human ability to avoid the contact/collision.

The roles and potentials of VFR towers (or other means of air traffic control) are clearly not identical for all of these "failure modes," so that some degree of "mode-specific" analysis should be required.

C.5 Technical Details

In this concluding section, we describe in detail the mathematical apparatus used in implementing the analysis method sketched in Section C.2. The reader is asked to review the notation introduced at that point; it will not be repeated here.

Elementary combinatorial probability shows that the probability of any particular triple (P, M, Z) under the "trinomial model" is given by

$$(N!)(p^+)^P(p^-)^M(p^o)^Z/(P!)(M!)(Z!).$$

Since $p^+ = p^- = (1 - u)/2$ and $p^o = u$ under the hypothesis $H_o(u)$, we have

$$f(u) = \sum{}^*(N!)[(1-u)/2]^{P+M}u^Z/(P!)!(M!)(Z!),$$

where $\sum{}^*$ denotes a sum over all triples (P,M,Z) for which

$$P \geq P(0),\ 0 \leq M \leq M(0),\ Z \geq 0,\ P + M + Z = N.$$

Using the relation

$$Z = N - P - M,$$

and introducing the auxiliary quantity

$$x = (1-u)/2u, \tag{C.19}$$

we obtain

$$f(u) = u^N \sum_{M=0}^{M(0)} \binom{N}{M} x^M \sum_{P=P(0)}^{N-M} \binom{N-M}{P} x^P. \tag{C.20}$$

To handle the applications (C.10) and C.14), involving large values of N (N = 64 and 88, respectively), we make the further substitution

$$y = (1-u)/(1+u), \tag{C.21}$$

leading from (C.20) to

$$f(u) = \sum_{M=0}^{M(0)} \binom{N}{M} [(1-u)/2]^M [(1+u)/2]^{N-M} \beta(N-M,y,P(0))$$

where $\beta(N-M, y, S)$ is the value of the complementary cumulative binomial distribution which gives the probability of S or more successes in N-M independent trials each with a success probability of y. Since M(0) is small (0 and 3, respectively), there are only a few summands in the last equation, and the β-values can be looked up in the tables of [46]. This procedure was used to evaluate f(u) over a grid of u-values, yielding the approximate f_{max}-values in (C.11) and (C.15).

The remaining applications all had M(0) = 0, so that (C.20) reduces to

$$f(u) = u^N \sum_{P=P(0)}^{N} \binom{N}{P} x^P, \qquad (C.22)$$

and also had P(0) close to N, so that there were few summands in (C.22). Thus analytical treatment of these cases could be carried further before resorting to numerical calculations. Specifically, (C.12), (C.16) and (C.17) yield <u>via</u> (C.22) the respective functions

$$f(u) = [(1-u)/2]^7 \, (15u + 1)/2, \qquad (C.23)$$

$$f(u) = [(1-u)/2]^9 \, (199u^2 + 20u + 1)/4, \qquad (C.24)$$

$$f(u) = [(1-u)/2]^8. \qquad (C.25)$$

For (C.23) it is found that $f'(0) > 0$, while $f'(u)$ vanishes only for u=1 and for the roots

$$u^- = -0.039, \quad u^+ = 0.129$$

of the quadratic equation $198u^2 - 18u - 1 = 0$. Hence

$$f_{max} = f(u^+) = 97 \times 10^{-5},$$

yielding the first part of (C.18). For (C.25), evidently

$$f_{max} = 2^{-8},$$

yielding the second part of (C.18).

There is just one further point to be addressed. Recall that the essence of the analysis is to prove that

$$f(u) = \text{Prob}\ \{F, \text{ given } H_o(u)\}$$

is <u>small</u>. To be "conservative," F should therefore be defined to be as "large" as is consistent with its interpretation as "at least as favorable as (P(0), M(0), Z(0))." We have defined F by the conditions

$$P \geq P(0), \quad M(0) \leq M, \tag{C.26}$$

but perhaps a less stringent interpretation is plausible. For example, one <u>might</u> regard (P, M, Z) = (10, 1, 0) as "at least as favorable" as (9, 0, 2), though this was not done in analyzing (C.16). That is, replacing a pair of zeros by a plus sign and a minus sign <u>might</u> be regarded as yielding a situation "no less favorable" (for establishing our finding) than before. In this case, F would correspond to all triples

$$P = P(0) + z, \quad M = M(0) + z, \quad Z = Z(0) - 2z \tag{C.27}$$

where z runs over all non-negative integers not exceeding Z(0)/2. The preceding analysis was redone using this alternative and more conservative approach, based on (C.27). The resultant values of f_{max} are shown in Table C.1.10; while necessarily higher (or no lower) than before, they still indicate rejection of the "null hypothesis" H_o.

Table C.1.10: Maximum Value (f_{max}) of Prob {F, given $H_o(u)$}

P(0)	M(0)	Z(0)	(C.26)	(C.27)
15	0	49	$5x10^{-6}$	.03304
7	0	1	.00482	.00482
65	3	20	.00003	.03769
9	0	2	.00097	.00586
8	0	0	.00391	.00391

"I always fly VFR but I can't always be looking in all directions. Other aircraft approached head on."

"The Apache overtook me from my left side, descending and crossing from my left rear to my right front. I was in his blind spot and he was in mine."

"Jet climbed up from blind spot under left engine, passed within 15 feet across nose."

"Pilot of a single engine aircraft reported being missed by a jet coming within 100 feet. The jet was making a simulated flameout landing."

"We were distracted momentarily by paper work in cockpit. I had just given a position report. I never did see the plane approaching -- seen by co-pilot."

"Too many cockpit duties plus necessary company work was responsible for this near miss. Simplification is one of the answers to this."

These comments appeared in a report issued by the U. S. Civil Aeronautics Board, "Selected pilot comments taken from near-collision reports submitted during calendar year 1957."[1] Comments like some of these imply that cockpit visibility is not adequate, and that new design criteria are imperative. However, an examination of the various parameters of near-miss air collision (NMAC) data indicates that increased cockpit visibility is no panacea to the problem of collision avoidance. Both changes in aircraft characteristics (e.g., speed), and the increased number of planes in the skies, have compounded this problem to such a degree that no single solution appears adequate.

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[1]U. S. Civil Aeronautics Board,"Selected pilot comments taken from near-collision reports submitted during calendar your 1957".

For many years there has been primary reliance on the "see-and-be-seen" doctrine for the avoidance of mid-air collisions. Visual sighting of the intruding aircraft was the most frequently cited first cause of alert in both day and night near-miss air collision reports submitted in 1968.[2] Yet a quite different impression of the doctrine's adequacy is given by:

(a) the finding that 93% of all reported hazardous incidents which occurred during the day were such that visual sighting alone would not have detected the intruding aircraft in time to avoid an air collision[3], and

(b) the numerous statements that most mid-air collisions are attributable to failure by one or both pilots to see the intruding aircraft in time to maneuver out of the collision path.[4]

In view of these conflicting impressions, an appeal to the basic properties of visual perception seems in order. The absence of "evolution" of these properties is the natural suspect as "limiting factor", i.e. they do not accomodate to the faster aircraft and the crowded airways of today. First, the eye must detect the intruding aircraft. In order to sight a head-on cross section of a 10 foot fuselage at seven miles, for example, an angular size of one minute of arc at the eye is required; at ten degrees from the center of vision, the threshold angle is 10 minutes of arc and the "just visible" distance is reduced to 0.7 miles. Additionally, each eye fixation from the center field of vision requires approximately 0.5 second. Using Chart A.1[5] as an example, the severity of the visual demands involved can

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[2] "Near Midair Collision Report of 1968", Department of Transportation, Federal Aviation Administration, Air Traffic & Flight Standards Technical Report, July 1969.

[3] Ibid.

[4] Graham, Walton and Orr, Robert H. "Separation of Air Traffic By Visual Means: An Estimate of the Effectiveness of the See and Avoid Doctrine", Department of Transportation, Air Traffic Advisory Committee Report, Dec. 1969.

[5] "Synopsis of the Federal Aviation Agency Airborne Collision Prevention Program", December 31, 1960.

be appreciated. Assume that the pilot is scanning from the center of his vision to the left, back through the center to the right and back again. It could take a pilot 6 seconds to perform just one such scan.

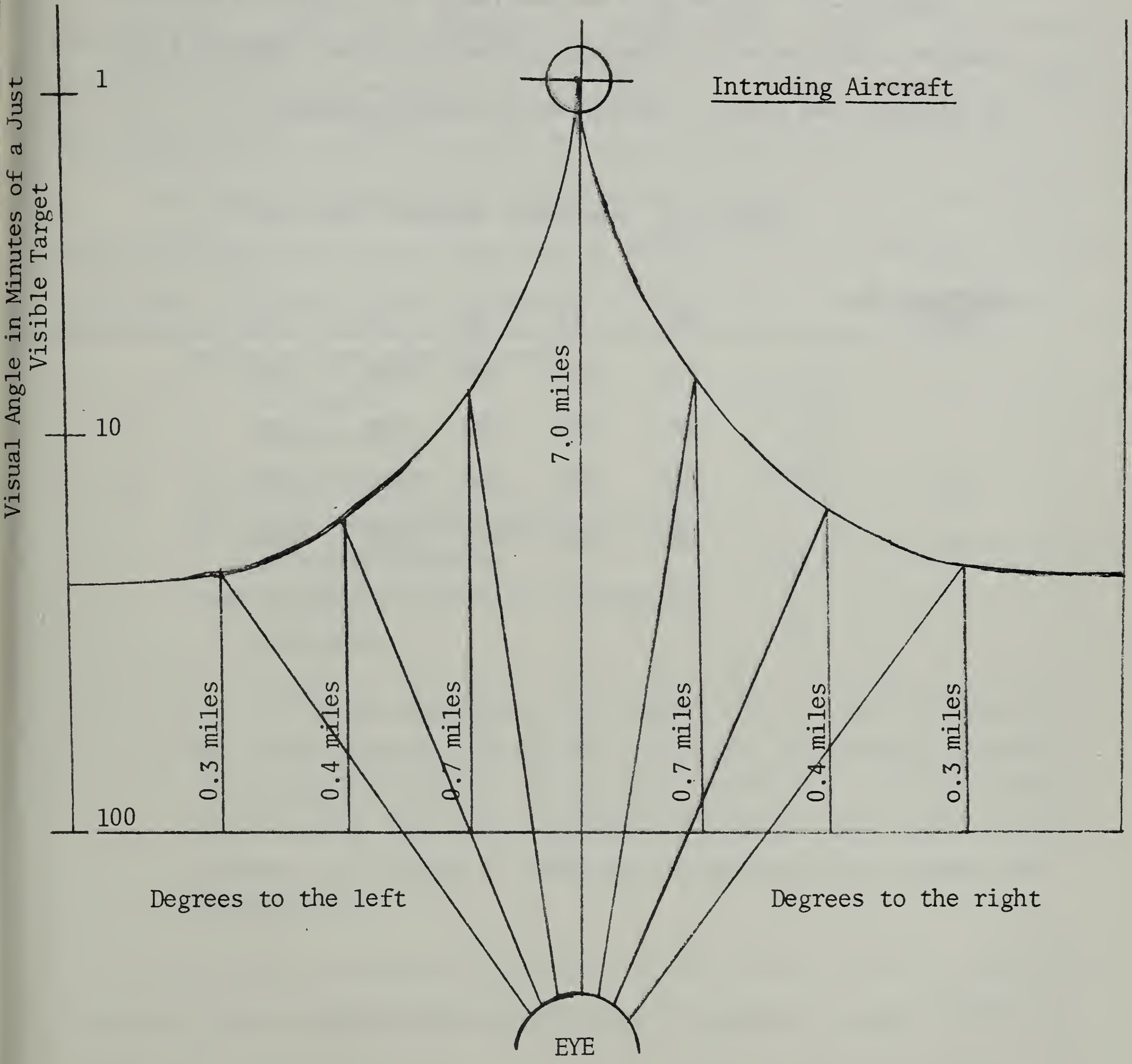
Visual Angle in Minutes of a Just Visible Target
1
10
100
Intruding Aircraft
7.0 miles
0.3 miles
0.4 miles
0.7 miles
0.7 miles
0.4 miles
0.3 miles
Degrees to the left
Degrees to the right
EYE

TABLE D.1

Table D.1[6] lists the time available for a pilot to perform evasive action once the intruding aircraft has been sighted, based on distance and speed. (Velocities are closing speeds.)

Table D.2: Available Response Time (Sec.)

Distance In Miles	180	90	45	30	15
3	60	120	240	360	720
5	100	200	400	600	1200
10	200	400	800	1200	2400
20	400	800	1600	2400	4800

(6) Ibid.

Once the intruder has been detected, five seconds (at the least) are required for positive detection, evasive decision, movement of the controls, and the time lag for the instruments to perform their changes. (Five seconds before collision, for example, two aircraft on a collision course at a rate-of-closure speed of 1200 knots will be approximately only a mile apart.)

A mid-air collision can be avoided only if the pilots are capable of reacting within the critical time and of making the correct decision. Pilots cited the following factors responsible in 75% of all mid-air collisions:[7]

Failed to see other aircraft:	
In flight	46%
In pattern	3%
Misjudged distance	23%
Did not change flight pattern in time to avoid collision	19%
Used improper crossover or crossunder technique	9%

> "It is an interesting coincidence that in order of frequency the errors committed in mid-air collisions fall roughly into the same sequence, that is, the greatest number of errors are related to perception, the second greatest to decision lags and the third greatest are related to inappropriate decision and judgment and responses."[8]

The ability of the pilot to perceive an intruding aircraft is dependent on several factors which bear little if any relation to cockpit visibility.

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[7]Zeller, Anchard F. "Human Aspects of the Mid-Air Collision Prevention Program". Presented at the 30th Annual Meeting of the Aero Medical Association, Los Angeles, California, April 1959.

[8]Ibid.

From many accident reports it is evident that the pilot did not react with an "average perceptual response". The day-to-day psychological and physiological variations in any individual markedly affect his ability to react and hence his performance from day to day and situation to situation. In other works, an "average" perceptual response is not an adequate indicator of behavior, because the day-to-day fluctuations in a pilot's response to a given situation are so great.[9]

Characteristics of the visual "field" to be searched further contribute to the difficulty of detection. Poor brightness contrast between the intruder and its background frequently makes it very difficult to perceive. This factor is dependent on such variables as angle of the sun, directions and speeds of the two aircraft, etc., but in general the plane is flying against a neutral background (the sky), which results in poor brightness contrast. Painting the aircraft with highly visible fluorescent paints increases the probability of detection somewhat. Orange-red fluorescent paints are the most conspicuous and can be detected up to 4 1/2 miles away 90% of the time. This is quite an advantage over non-fluorescent paints whose color can be detected only 2 miles away 90% of the time. Ideally, the sighting of color could be used as a range estimate but apparently pilots are not using these distance cues consciously and are probably ignoring them entirely.[10]

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[9] McIntosh, B. B. "Mid-Air Collisions in the U. S. A. F." Report of Presentations and General Discussions at the Mid-Air Collision Symposium, Indianapolis, Indiana, November 1955.

[10] Applied Psychology Corporation, "The Role of Range and Altitude Judgment in Mid-Air Collision Prevention". Prepared for Federal Aviation Agency, Project No. 110-512R, May 1963.

The pilot relies for perception on his visual system and the cues he receives. This is somewhat of a disadvantage to him in the sky, since he is trying to apply the same visual rules that apply on the ground. For instance, a pilot has difficulty in determining the relative size of another aircraft. Unless the specific type of intruding aircraft has been identified, the speck sighted in the sky could equally well be a Piper Cub at five miles or a trans-continental jet at ten. The pilot cannot carry out the usual process of mentally comparing the size of the "unknown" object (aircraft) with the known size of some other object in the visible environment. Moreover, the pilot has no stationary frame of reference with which to gauge speed and approach-angle. There are no stationary objects in the sky -- everything is in motion and flux, including the pilot himself. He has no physical objects to serve as referents in determining the relative speed and angle of another aircraft. Some approach angles are easier to determine than others, but the most difficult is also the most dangerous: the head-on situation.

The phenomenon of motion parallax additionally distorts perception:

> When the eye moves relative to the environment, or the environment relative to the eye, the different angular velocities of objects at various distances result in differences in perceived motion of these objects. Near objects move more rapidly than far objects and against the direction of the observer's movement, while far objects move with him.[11]

The set of factors already given could be expanded, and discussions given of linear perspective, retinal disparity, accommodation, convergence, binocular and monocular blindness,[11a] but they would point up the same fact: the

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[11] Ibid.

[11a] McFarland, Ross A., Human Factors in Air Transportation, McGraw Hill, 1953.

visual stimuli and reactions that apply on the ground do not function in the same manner in the sky. Conditions and cues in the air are different, and the visual system cannot function 100% using ground rules.

One of the most dangerous phenomena in visual search is "altitude myopia". The term is derived from experiences in high altitude flights where the environment is one of infinite uniformity, i.e., an empty visual field.[12] The human eye is incapable of focusing at infinity if there is no detail at infinity which is capable of being sharply focused.[13] An empty visual field can occur at any altitude. All that is required is a sufficiently uniform surrounding: fog, night, total overcast conditions, etc., and the myopia takes effect, reducing the focal distance to less than 5 meters.[14] There is nothing for the eye to focus on, and it will not detect another aircraft as effectively as when flying in a variegated field.

Boredom and cockpit duties contribute significantly to the problems of visual sighting. The psychological and physiological state of the men in the cockpit affect their efficiency in detection. Preoccupation with personal problems, fatigue and general boredom can result in a type of myopia similar to that in an empty visual field.

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[12] Whiteside, Thomas C. O. The Problems of Vision in Flight at High Altitude, London, Butterworths Scientific Publications, 1957.

[13] Zeller, Anchard F. "Human Aspects of the Mid-Air Collision Prevention Program". Presented at the 30th Annual Meeting of the Aero Medical Association, Los Angeles, California, April 1959.

[14] Ibid.

As the complexity of aircraft increases, the required cockpit activities also grow. An Air Force study[15] developed some of the following statistics:

--The average time to read a standard Air Force altimeter is seven seconds; 1/6 of these readings are in error, 1/10 in error by as much as 1,000 feet.

--If an individual looks into the cockpit, looks outside and then re-focuses on the instrument panel, a minimum of two seconds has elapsed.

--Other activities such as changing radio channels, monitoring the fuel systems, and re-setting the altimeters are even more time consuming.

--When an instrument flight plan must be followed, the time spent monitoring the instruments is much greater, even under VFR conditions.

--Under emergency or anticipated emergency conditions the time spent looking inside the cockpit is also appreciably greater.

Cockpit activities have repeatedly been cited as one of the most time-consuming operations in flight. Instruments have to be monitored, entries into logs made, chart courses checked, etc., all of which reduce the time available for visual search.

Over-crowded airports appear to be one of the major conditions cited for air collisions. There are many reports which state that so many near-misses or collisions occurred within five miles of an airport at 1,000 feet or below, and that take-off and landing operations are particularly susceptible to the dangers of over-crowding. During the taxi phase of take-off, the pilot

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[15] Ibid.

scans horizontally approximately 120° to the left of center and 100° to the right. Pilots rarely look more than 15° above the horizon but look down at the instrument panel quite often. This uniform scanning allows the pilot to sight other planes and objects on the ground during the taxi phase. Once the aircraft leaves the runway and begins to ascend, the pilot spends almost 2/3 of his time monitoring flight and engine instruments. He has only 1/3 of his time available for visual detection of collision paths in the high-density area surrounding the airport. During landing only a small portion of the windshield is used for scanning. The pilot scans about 3/4 of the time during landing, but most of this is directed straight ahead in order to maneuver the aircraft into alignment with the runway.[16,17] Clearly, the pilot is too occupied with this primary task to scan effectively for intruding aircraft. This latter function is the responsibility of the tower. Some visual search can be conducted by the co-pilot if present, but the danger of collision must be risked where there is no second person cockpit. The maintenance-of-scan problem is compounded at smaller air fields where there is often no tower, though the lower traffic densities involved reduce the associated average risk.

The effectiveness of the "see-and-be-seen" principle is susceptible of some improvement. Pilots can be trained to search more effectively and accurately. Use of fluorescent paints, smoke trails, flashing lights (in night flying) and other conspicuity tactics aid in visual sighting. However, the human visual system cannot keep pace with technological advances. Designs for

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[16] Graves, J. A. "Visibility in Modern Transport Aircraft.", The Society of Experimental Test Pilots Quarterly Review, No. 2, Winter, 1959, pp. 77-79.

[17] Baker, Charles A. "Visual Aspects in Collision Avoidance for Air Force Aircraft", Visual Search Techniques, National Academy of Sciences -- National Research Council, Washington, D. C., 1960 , pp. 28-31.

cockpits affording greater visibility too often cannot be incorporated into the sophisticated aerodynamic silhouettes of the present and future. Greater speeds and traffic densities make it more and more difficult to avoid mid-air collisions. The development and usage of Pilot Warning Indicators (PWI) or a sophisticated Collision Avoidance System (CAS) appears to be the logical replacement or supplemental help the pilot needs. On a less costly scale, explicit pilot training in visual search is needed, uniformity and unambiguity of air traffic rules should be improved, and tower personnel should perhaps be expanded to give the information and guidance that is needed for air traffic control in highly congested areas. The see-and-be-seen concept may be a valid method for smaller aircraft which do not have the high speed capabilities of other planes -- perhaps improved cockpit visibility would be of some assistance to this class. However, the more sophisticated aircraft are rapidly rendering the visual sighting concept obsolete, creating a dire need to replace the human visual system with a mechanical detection-decision device.

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APPENDIX E: PROGRAM LISTING AND SAMPLE OUTPUTS FOR CURRENT VERSION OF VISIBILITY MODEL

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00101         DIMENSION THETRA(6,6),PHIRAY(6,6),NRAYS(6),A(3,6,6),OUTPUT(100),
00101        1          XRAY(6,6),YRAY(6,6),ZRAY(6,6),TFP(20),VUPTS(100,3),
00101        2          TANGNT(100,4),TRNSFM(3,3),C(6),TEST(6),E(3,6),
00101        3          TPPLOT(101,51),VIZIBL(3,6)
00103         COMMON LEGCNT(20),T(20),V,W,DRUN,DFIN,NTARGT,DBASE,HPAT,DELT2,
00103        1       TARGTS(100,3),NBREAK
00104         EQUIVALENCE(T(1),TFP(1)),(NVUPTS,NTARGT),(NSEGNT,NFPSGT)
00105         READ(5,9000)DRUN,DFIN,DBASE,RBDRY,HPAT,RTURN1,RTURN2,DUPA,DCROSS
00120    9000 FORMAT(5F10.3/5F10.3)
00121         DDOWN=DUPA
00122         READ(5,9001)SPEED,NCONES,(NRAYS(I),I=1,NCONES)
00132    9001 FORMAT(F10.0,7I10)
00133         DO 10 I=1,NCONES
00136         K=NRAYS(I)
00137         READ(5,9002)(THETRA(I,J),PHIRAY(I,J),J=1,K)
00146    9002 FORMAT(12F6.0)
00147      10 CONTINUE
00147   C--  HERE RAYS ARE CALCULATED FROM THE VISIBILITY ANGLES - - - - - - - - - - -
00151         DO21I=1,NCONES
00154         K=NRAYS(I)
00155         DO20J=1,K
00160         THETRA(I,J)=THETRA(I,J)/57.29583
00161         PHIRAY(I,J)=PHIRAY(I,J)/57.29583
00162         XRAY(I,J)=-COS(PHIRAY(I,J))*SIN(THETRA(I,J))
00163         YRAY(I,J)=COS(PHIRAY(I,J))*COS(THETRA(I,J))
00164         ZRAY(I,J)=SIN(PHIRAY(I,J))
00165      20 CONTINUE
00167         VIZIBL(1,I)=XRAY(I,1)+XRAY(I,2)+XRAY(I,3)
00170         VIZIBL(2,I)=YRAY(I,1)+YRAY(I,2)+YRAY(I,3)
00171         VIZIBL(3,I)=ZRAY(I,1)+ZRAY(I,2)+ZRAY(I,3)
00172      21 CONTINUE
00172   C-- HERE THE FACES OF THE POLYHEDRON ARE CALCULATED FROM THE RAYS - - - - - - -
00174         DO40I=1,NCONES
00177         K=NRAYS(I)
00200         DO 30 J=1,K
00203         IF(J.EQ.K)GOTO31
00205         A(1,I,J)=YRAY(I,J)*ZRAY(I,J+1)-ZRAY(I,J)*YRAY(I,J+1)
00206         A(2,I,J)=XRAY(I,J+1)*ZRAY(I,J)-XRAY(I,J)*ZRAY(I,J+1)
00207         A(3,I,J)=XRAY(I,J)*YRAY(I,J+1)-XRAY(I,J+1)*YRAY(I,J)
00210      30 CONTINUE
00212      31 A(1,I,K)=YRAY(I,K)*ZRAY(I,1)-ZRAY(I,K)*YRAY(I,1)
00213         A(2,I,K)=XRAY(I,1)*ZRAY(I,K)-XRAY(I,K)*ZRAY(I,1)
00214         A(3,I,K)=XRAY(I,K)*YRAY(I,1)-XRAY(I,1)*YRAY(I,K)
00215         WRITE(6,9007)((A(II,I,J),II=1,3),J=1,K)
00226    9007 FORMAT(3F20.3)
00227      40 CONTINUE
00231         READ(5,9003)NSEGNT,NTARGT,(T(I),I=1,NSEGNT)
00241    9003 FORMAT(2I5,7F10.0/8F10.0)
00242         FSPEED=SPEED*6080./60.
```

```
      QTURN=FSPEED*0.25
      RADIUS=FSPEED/(2*3.14159)
      T(1)=DUPW-RADIUS
      T(2)=QTURN
      T(3)=DCROSS-2*RADIUS
      T(4)=QTURN
      T(5)=DDOWN-2*RADIUS
      T(6)=QTURN
      T(7)=DBASE-2*RADIUS
      T(8)=QTURN
      T(9)=SQRT(HPAT**2+DFIN**2)-RADIUS
      T(10)=DRUN
      TOT=0.
      DO50I=1,NSEGMT
   50 TOT=TOT+T(I)
      DELT2=TOT/FLOAT(NVUPTS)
      WRITE(6,9004)DELT2,TOT,(T(I),LEGCNT(I),I=1,NSEGMT)
 9004 FORMAT(2F10.2/10(F6.0,I4))
      CALLPNTCAL(TARGTS,NTARGT,NSEGMT,T,LEGCNT,DELT2,HPAT,DFIN,DRUN,
     1           RADIUS,DBASE,DCROSS,DUPW,DDOWN)
      READ(5,9003)NEPSGT,NVUPTS,(IEP(I),I=1,NEPSGT)
      V=FSPEED/60.
      BNKANG=ATAN2(V*V,32.2*RADIUS)
      WRITE(6,9010)
 9010 FORMAT('1')
      CALLVUCALC(VUPTS,NVUPTS,TARGTS)
      CALLTNGNTS(TANGNT,NVUPTS,VUPTS)
      CALLBANK(TANGNT,NVUPTS,LEGCNT,5,BNKANG)
      WRITE(6,9005)(I,(TARGTS(I,J),J=1,3),I=1,NTARGT)
 9005 FORMAT(I10,5X,3F12.1)
      WRITE(6,9010)
      WRITE(6,9006)(I,(VUPTS(I,J),J=1,3),(TANGNT(I,J),J=1,4),I=1,NVUPTS)
 9006 FORMAT(I10,5X,3F12.1,10X,4F12.3)
      WRITE(6,9020)SPEED,RADIUS,BNKANG
 9020 FORMAT('1'///42X,'THE FOLLOWING IS A RUNDOWN OF THE VIEWING AIRCRA
     1FT'//20X,'SPEED IS',F5.0,' KNOTS'/20X,'MEANING ONE-MINUTE TURNS AR
     2E MADE WITH A RADIUS OF',F8.1,' FEET'/20X,'AND WITH A ',F5.3,' RAD
     3IAN BANK ANGLE')
      WRITE(6,9021)NCONES
 9021 FORMAT(/20X,'THE WINDSHIELD IS MADE UP OF',I2,' CONE(S)')
      DO900I=1,NCONES
      K=NRAYS(I)
      WRITE(6,9022)I,(J,(A(L,I,J),L=1,3),J=1,K)
 9022 FORMAT(/20X,'CONE',I2,':'/(29X,'FACE',I2,' IS  ',F5.3,' X',F5.3,'
     1Y',F5.3,' Z'))
  900 CONTINUE
      WRITE(6,9023)DUPW,DCROSS,DDOWN,DBASE,DFIN,DRUN
 9023 FORMAT(////40X,'THE FOLLOWING IS A RUNDOWN ON THE TRAFFIC PATTERN
     1USED FOR THE RUN'/20X,'IT IS A STANDARD LEFT-HAND PATTERN WITH THESE DIMEN
     2SE DIMENSIONS:'//30X,'UPWIND LEG =',F10.0,/30X,'CROSS WIND LEG =',
     3F10.0,/30X,'DOWN WIND LEG =',F10.0/30X,'BASE LEG =',F10.0/30X,'FIN
     4AL APPROACH LEG =',F10.0/30X,'RUNWAY LENGTH =',F10.0)
      WRITE(6,9010)
      WRITE(6,9008)
 9008 FORMAT(' VIEWED     NO. OF     NO. OF'/
     1       '  FROM      POINTS     PRE-PNTS'/
     2       '  POINT     VISIBLE    VISIBLE'/
     3T35,'1',T45,'2',T55,'3',T65,'4',T75,'5',T85,'6',T95,'7',T105,'8' T
```

```
     4(I5,'9',T125,'*'')
      NCTOT=0
      NARCTO=0
      DO5001/I=1,NVUPTS
      NSEL=1
      NAREUC=0
C--   AT THIS PT THE TRANSFORMATION MATRIX IS DETERMINED - - - - - - - - - - - - -
      DELTA=SQRT(TANGNT(IVIEW,1)**2+TANGNT(IVIEW,2)**2)
      COSTHT=TANGNT(IVIEW,2)/DELTA
      SINTHT=TANGNT(IVIEW,1)/DELTA
      COSPHI=DELTA
      SINPHI=TANGNT(IVIEW,3)
      IF(ABS(TANGNT(IVIEW,4)).GT.1.0E-2)GOTO75
      TRNSFM(1,1)=COSTHT
      TRNSFM(2,1)=-SINTHT
      TRNSFM(3,1)=0.0
      TRNSFM(1,2)= COSPHI*SINTHT
      TRNSFM(2,2)= COSPHI*COSTHT
      TRNSFM(3,2)= SINPHI
      TRNSFM(1,3)=-SINPHI*SINTHT
      TRNSFM(2,3)=-SINPHI*COSTHT
      TRNSFM(3,3)= COSPHI
      GOTO80
   75 COSALF=COS(TANGNT(IVIEW,4))
      SINALF=SIN(TANGNT(IVIEW,4))
      TRNSFM(1,1)= COSALF*COSTHT-SINALF*SINPHI*SINTHT
      TRNSFM(2,1)=-COSALF*SINTHT-SINALF*SINPHI*COSTHT
      TRNSFM(3,1)= SINALF*COSPHI
      TRNSFM(1,2)= COSPHI*SINTHT
      TRNSFM(2,2)= COSPHI*COSTHT
      TRNSFM(3,2)= SINPHI
      TRNSFM(1,3)=-SINALF*COSTHT-COSALF*SINPHI*SINTHT
      TRNSFM(2,3)= SINALF*SINTHT-COSALF*SINPHI*COSTHT
      TRNSFM(3,3)= COSALF*COSPHI
   80 DO78II=1,NTARGT
   78 OUTPUT(II)=' '
C--   NO   TO TRANSFORM EACH FACE OF THE POLYHEDRON     - - - - - - - - - - - - - -
      DO500I=1,NCOLS
      KEY=KAYS(I)
      DO 100 J=1,K
      DO 95 L=1,3
      B(L,J)=TRNSFM(L,1)*A(1,I,J)+TRNSFM(L,2)*A(2,I,J)+TRNSFM(L,3)*A(3,I
     1    ,J)
   95 CONTINUE
  100 C(J)= -B(1,J)*VUPTS(IVIEW,1)-B(2,J)*VUPTS(IVIEW,2)-B(3,J)*VUPTS(IV
     1    IEW,3)
C--   B(L,J) ARE THE FACES EXPRESSED IN THE AIRPORTS COORDINATE SYSTEM - - - - -
C -   NOW TO COMPARE THE NOSE WITH THE TARGETS     - - - - - - - - - - - - - - - -
      XNOSE=TRNSFM(1,1)*VIZIBL(1,I)+TRNSFM(1,2)*VIZIBL(2,I)+TRNSFM(1,3)*
     1    VIZIBL(3,I)+VUPTS(IVIEW,1)
      YNOSE=TRNSFM(2,1)*VIZIBL(1,I)+TRNSFM(2,2)*VIZIBL(2,I)+TRNSFM(2,3)*
     1    VIZIBL(3,I)+VUPTS(IVIEW,2)
      ZNOSE=TRNSFM(3,1)*VIZIBL(1,I)+TRNSFM(3,2)*VIZIBL(2,I)+TRNSFM(3,3)*
     1    VIZIBL(3,I)+VUPTS(IVIEW,3)
      DO105J=1,K
  105 TEST(J)=B(1,J)*XNOSE+B(2,J)*YNOSE+B(3,J)*ZNOSE+C(J)
      DO130ITARGT=1,NTARGT
      DO115J=1,K
```

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00534          TEMP=B(1,J)*TARGTS(ITARGT,1)+B(2,J)*TARGTS(ITARGT,2)+B(3,J)*TARGTS
00534        1      (ITARGT,3)+C(J)
00535          IF(TEMP*TEST(J))130,115,115
00540      115 CONTINUE
00542          IF(OUTPUT(ITARGT).EQ.'V')GOTO130
00544          OUTPUT(ITARGT)='V'
00545          NSEE=NSEE+1
00546          IF(ITARGT.GT.IVIEW)NAHEDC=NAHEDC+1
00550      130 CONTINUE
00552      150 CONTINUE
00554          DO140KK=1,NSEGMT
00557          ITEMP=LEGCNT(KK)
00560      140 OUTPUT(ITEMP)='*'
00562          WRITE(6,9009)IVIEW,NSEE,NAHEDC,(OUTPUT(ITARGT),ITARGT=1,NTARGT)
00573     9009 FORMAT(I5,2I10,100A1)
00574          NCTOT=NCTOT+NSEE
00575          NAHCTO=NAHCTO+NAHEDC
00576      500 CONTINUE
00600          AVGSEE=FLOAT(NCTOT)/FLOAT(NVUPTS)
00601          AVAHDC=FLOAT(NAHCTO)/FLOAT(NVUPTS)
00602          WRITE(6,9012)AVGSEE,AVAHDC
00606     9012 FORMAT('1    AVERAGE NO. VISIBLE',F6.2,'AVERAGE NO. VISIBLE AHEAD O
00606        1F A/C',F6.2)
00607          WRITE(6,9011)NSEGMT,(LEGCNT(K),K=1,NSEGMT)
00616     9011 FORMAT(I5,10I10)
00617          DO8010I=1,101
00622          DO8011J=1,51
00625     8011 TPPLOT(I,J)=' '
00627     8010 CONTINUE
00631          YSCALE=DUPA/100.
00632          XSCALE=YSCALE*1.6666667
00633          DO8020I=1,NVUPTS
00636          IX=IFIX(VUPTS(I,1)/XSCALE)+26
00637          IY=IFIX(VUPTS(I,2)/YSCALE)+51
00640          TPPLOT(IY,IX)='V'
00641     8020 CONTINUE
00643          WRITE(6,9010)
00645          WRITE(6,9013)TPPLOT
00653     9013 FORMAT(10X,101A1)
00654          CALLEXIT
00655          END
```

```
WIT FOR PNTCAL,PNTCAL                                                          24 AUG 70            18:21:21.929
UNIVAC 1108 FORTRAN V LEVEL   2206 0018 F5018P
THIS COMPILATION WAS DONE ON 24 AUG 70 AT 18:21:22

   SUBROUTINE PNTCAL    ENTRY POINT 000477

   STORAGE USED (BLOCK, NAME, LENGTH)

        0001   *CODE   000541
        0000   *DATA   000065
        0002   *BLANK  000000

   EXTERNAL REFERENCES (BLOCK, NAME)

        0003   ATAN2
        0004   SIN
        0005   COS
        0006   NERR2$
        0007   NWDU$
        0010   NIO1$
        0011   NIO2$
        0012   NERR3$

   STORAGE ASSIGNMENT FOR VARIABLES (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001     000440 10L      0001     000124 100L     0001     000113 110L     0001     000403 130L     0001     000365 20L
    0001     000452 216G     0001     000330 30L      0001     000312 40L      0001     000260 50L      0001     000243 60L
    0001     000211 70L      0001     000067 8L       0001     000175 80L      0001     000072 9L       0001     000141 90L
    0000     000014 5000F    0000 R 000007 ARCLTH    0000 R 000002 COSRHO    0000 R 000005 FINAL     0000 I 000013 I
    0000 I 000006 LEG       0000 I 000010 N         0000 I 000011 NEG       0000 R 000000 RHO       0000 R 000001 SINRHO
    0000 R 000012 THETA     0000 R 000003 YT        0000 R 000004 ZT

00101      1*          SUBROUTINE PNTCAL(POINTS      ,NPTS,NOLEGS,END,LEGCNT,DELS,HPAT,
00101      2*         1           DFIN,DRUN,RADIUS,DBASE,DCROSS,DUPW,DDOWN)
00103      3*          DIMENSION POINTS(100,3),END(20),LEGCNT(20)
00104      4*          POINTS(NPTS,1)=0.0
00105      5*          POINTS(NPTS,2)=0.0
00106      6*          POINTS(NPTS,3)=0.0
00107      7*          RHO=ATAN2(HPAT,DFIN)
00110      8*          SINRHO=SIN(RHO)
00111      9*          COSRHO=COS(RHO)
00112     10*          YT=-DRUN-DFIN+RADIUS*COSRHO
00113     11*          ZT=HPAT-RADIUS*SINRHO
00114     12*          FINAL=DFIN+DRUN
00115     13*          LEG=NOLEGS
00116     14*          ARCLTH=0.0+DELS
00117     15*          N=NPTS
00120     16*          LEGCNT(LEG)=N
00121     17*          N=N-1
00122     18*      8   NEG=LEG+1
```

```
9     GOTO(10,20,30,40,50,60,70,80,90,100,110),NEG
110   POINTS(N,1)=0.0
      POINTS(N,2)=-ARCLTH
      POINTS(N,3)=0.0
      GOTO130
100   POINTS(N,1)=0.0
      POINTS(N,2)=-DRUN-COSRHO*ARCLTH
      POINTS(N,3)=SINRHO*ARCLTH
      GOTO130
90    THETA=ARCLTH/RADIUS
      POINTS(N,1)=RADIUS*(COS(THETA)-1.)
      POINTS(N,2)=YT-RADIUS*SIN(THETA)*COSRHO
      POINTS(N,3)=ZT+RADIUS*SIN(THETA)*SINRHO
      GOTO130
   80 POINTS(N,1)=-RADIUS-ARCLTH
      POINTS(N,2)=-FINAL
      POINTS(N,3)=HPAT
      GOTO130
70    THETA=ARCLTH/RADIUS
      POINTS(N,1)=(RADIUS-DBASE)-RADIUS*SIN(THETA)
      POINTS(N,2)=-FINAL+RADIUS*(1.-COS(THETA))
      POINTS(N,3)=HPAT
      GOTO130
60    POINTS(N,1)=-DBASE
      POINTS(N,2)=ARCLTH+RADIUS-FINAL
      POINTS(N,3)=HPAT
      GOTO130
50    THETA=ARCLTH/RADIUS
      POINTS(N,1)=RADIUS*(1.-COS(THETA))-DBASE
      POINTS(N,2)=(DDOWN-FINAL -RADIUS)+RADIUS*SIN(THETA)
      POINTS(N,3)=HPAT
      GOTO130
   40 POINTS(N,1)=ARCLTH-DBASE+RADIUS
      POINTS(N,2)=DDOWN-FINAL
      POINTS(N,3)=HPAT
      GOTO130
30    THETA=ARCLTH/RADIUS
      POINTS(N,1)=DCROSS-DBASE+RADIUS*(SIN(THETA)-1.)
      POINTS(N,2)=DDOWN-FINAL-RADIUS*(1.-COS(THETA))
      POINTS(N,3)=HPAT
      GOTO130
20    POINTS(N,1)=DCROSS-DBASE
      POINTS(N,2)=DDOWN-FINAL-RADIUS-ARCLTH
      POINTS(N,3)=HPAT
130   N=N-1
      ARCLTH=ARCLTH+DELS
      IF(N.LE.0)GOTO10
      IF(ARCLTH.LE.END(LEG))GOTO9
      ARCLTH=ARCLTH-END(LEG)
      LEG=LEG-1
      LEGCNT(LEG)=N
      GOTO8
10    WRITE(6,9000)N,ARCLTH,DELS,(LEGCNT(I),I=1,NOLEGS),LEG
9000  FORMAT(I5,2F10.2,/20I5)
      RETURN
      END
```

```
                    THE FOLLOWING IS A RUNDOWN OF THE VIEWING AIRCRAFT

SPEED IS  95. KNOTS
MEANING ONE-MINUTE TURNS ARE MADE WITH A RADIUS OF   1451.5 FEET
AND WITH A  .458 RADIAN BANK ANGLE

THE WINDSHIELD IS MADE UP OF 6 CONE(S)

CONE 1:
          FACE 1 IS  -.197 X .416 Y-.298 Z
          FACE 2 IS  -.168 X .492 Y-.339 Z
          FACE 3 IS  -.513 X .116 Y .234 Z
          FACE 4 IS   .578 X .430 Y .105 Z
          FACE 5 IS   .412 X .419 Y-.097 Z

CONE 2:
          FACE 1 IS  -.471 X .117 Y-.592 Z
          FACE 2 IS  -.692 X-.634 Y-.358 Z
          FACE 3 IS  -.312 X-.022 Y .262 Z
          FACE 4 IS  -.029 X .934 Y .352 Z

CONE 3:
          FACE 1 IS  -.611 X .162 Y-.756 Z
          FACE 2 IS  -.438 X .025 Y-.567 Z
          FACE 3 IS  -.241 X-.598 Y .681 Z
          FACE 4 IS  -.279 X-.036 Y .251 Z
          FACE 5 IS  -.260 X .014 Y .127 Z

CONE 4:
          FACE 1 IS  -.021 X .455 Y-.463 Z
          FACE 2 IS  -.111 X .322 Y .202 Z
          FACE 3 IS   .150 X .168 Y .935 Z
          FACE 4 IS   .6   X .141 Y-.617 Z

CONE 5:
          FACE 1 IS   .110 X .191 Y-.206 Z
          FACE 2 IS  -.003 X .405 Y .196 Z
          FACE 3 IS   .26  X-.066 Y .143 Z
          FACE 4 IS   .372 X-.334 Y-.151 Z

CONE 6:
          FACE 1 IS   .944 X .161 Y-.841 Z
          FACE 2 IS   .159 X .139 Y .066 Z
          FACE 3 IS   .364 X-.107 Y .155 Z
          FACE 4 IS   .013 X-.403 Y .650 Z


                    THE FOLLOWING IS A RUNDOWN ON THE TRAFFIC PATTERN USED FOR THE RUN
 IT IS A STANDARD LEFT-HAND PATTERN WITH THESE DIMENSIONS:

          UPWIND LEG =       20000.
          CROSS WIND LEG =       7500.
          DOWN WIND LEG =      20000.
```

PIPER CH... KEF 6(97-32-2AP) VIB.POINT 42 IN ABOVE SEAT   DATE 081070 PAGE

```
BASE LEG =        4000.
FINAL APPROACH LEG =      5000.
RUNWAY LENGTH =      5000.
```

VIEWED FROM POINT	NO. OF POINTS VISIBLE	NO. OF PAIRS VISIBLE
1	97	96
2	96	95
3	94	93
4	93	92
5	92	91
6	91	90
7	90	89
8	89	88
9	88	87
10	86	85
11	86	85
12	85	84
13	84	83
14	83	82
15	82	81
16	81	80
17	80	79
18	81	80
19	81	80
20	79	78
21	79	78
22	78	77
23	77	76
24	74	73
25	72	71
26	70	69
27	67	66
28	66	65
29	64	63
30	35	34
31	34	[illegible]
32	64	63
33	97	[illegible]
34	97	[illegible]
35	97	[illegible]
36	97	[illegible]
37	95	[illegible]
38	94	61
39	91	58
40	91	59
41	91	59
42	83	57
43	85	57
44	87	54
45	95	52
46	97	54
47	97	53
48	95	52
49	94	51
50	92	50
51	91	49
52	90	48
53	87	46

PIPER (CHEROKEE [illegible]) VIEWPOINT 42 IN ABOVE SEAT DATE 081074 PAGE

```
                    THE FOLLOWING IS A RUNDOWN OF THE VIEWING AIRCRAFT

SPEED IS  90. KNOTS
HEADING ONE-MINUTE TURNS ARE MADE WITH A RADIUS OF  1451.5 FEET
AND WITH A  .459 RADIAN BANK ANGLE

THE WINDSHIELD IS MADE UP OF 6 CONE(S)

CONE 1:
          FACE 1 IS  -.065 X .471 Y-.512 Z
          FACE 2 IS  -.169 X .237 Y-.209 Z
          FACE 3 IS  -.670 X .048 Y .739 Z
          FACE 4 IS   .713 X .621 Y-.012 Z

CONE 2:
          FACE 1 IS  -.056 X .544 Y-.627 Z
          FACE 2 IS  -.215 X .467 Y .081 Z
          FACE 3 IS   .245 X .059 Y .917 Z
          FACE 4 IS   .634 X .122 Y-.275 Z

CONE 3:
          FACE 1 IS   .075 X .005 Y-.337 Z
          FACE 2 IS  -.286 X-.447 Y-.076 Z
          FACE 3 IS  -.084 X-.003 Y .069 Z
          FACE 4 IS   .082 X .549 Y .225 Z

CONE 4:
          FACE 1 IS   .218 X .121 Y-.867 Z
          FACE 2 IS  -.042 X-.606 Y-.162 Z
          FACE 3 IS  -.339 X-.036 Y .251 Z
          FACE 4 IS  -.417 X .012 Y .296 Z
          FACE 5 IS  -.340 X .193 Y .041 Z

CONE 5:
          FACE 1 IS  -.074 X .043 Y-.711 Z
          FACE 2 IS   .159 X .265 Y-.001 Z
          FACE 3 IS   .299 X-.074 Y .632 Z
          FACE 4 IS   .07. X-.292 Y-.048 Z

CONE 6:
          FACE 1 IS  -.132 X .016 Y-.287 Z
          FACE 2 IS   .195 X .127 Y .016 Z
          FACE 3 IS   .123 X-.127 Y .258 Z
          FACE 4 IS  -.172 X-.215 Y-.024 Z


                    THE FOLLOWING IS A RUNDOWN OF THE TRAFFIC PATTERN USED FOR THE RUN
IT IS A STANDARD LEFT-HAND PATTERN WITH THESE DIMENSIONS:

          UPWIND LEG =        2?000.
          CROSS WIND LEG =     7500.
          DOWN WIND LEG =     20000.
          BASE LEG =           4000.
```

```
PIPER CH[illegible] REF [illegible]-32-2[illegible] VIEWPOINT 42 IN ABOVE SEAT      DATE 081070  PAGE

         BASE LEG =        4000.
         FINAL APPROACH LEG =        5000.
         [illegible] LENGTH =        5000.
```

CESSNA 21[illegible] VIEWPOINT 42.5 INCH ABOVE FLR,5INCH AFT CONTROLSDATE 0819[illegible] PAGE

VIEWED FROM POINT	NO. OF POINTS VISIBLE	NO. OF PRE-PNTS VISIBLE
1	72	71
2	69	68
3	66	65
4	64	63
5	61	60
6	59	58
7	57	56
8	54	53
9	51	50
10	48	47
11	45	44
12	43	42
13	41	40
14	38	37
15	35	34
16	32	31
17	30	29
18	28	27
19	27	26
20	25	24
21	23	22
22	22	21
23	20	19
24	18	17
25	16	15
26	14	13
27	12	11
28	10	9
29	9	8
30	7	6
31	5	4
32	6	5
33	14	13
34	17	16
35	19	18
36	17	16
37	15	14
38	13	12
39	12	11
40	10	9
41	8	7
42	6	5
43	5	4
44	8	7
45	41	40
46	69	54
47	70	52
48	74	52
49	66	51
50	64	[illegible]
51	62	49
52	60	44
53	56	47

CESSNA 210 VIEWPOINT 42.5 INCH ABOVE FLR,5INCH AFT CONTROLSDATE 0812[illegible] PAGE

APPENDIX F: INTERAGENCY AGREEMENT BETWEEN FAA AND NBS

DEPARTMENT OF TRANSPORTATION
FEDERAL AVIATION ADMINISTRATION

WASHINGTON, D.C. 20590

INTERAGENCY AGREEMENT DOT-FA70WAI-188

BETWEEN

DEPARTMENT OF TRANSPORTATION, FEDERAL AVIATION ADMINISTRATION

AND

NATIONAL BUREAU OF STANDARDS

A. PURPOSE:

This agreement between the Department of Transportation, Federal Aviation Administration (hereinafter referred to as FAA), and the National Bureau of Standards (hereinafter referred to as NBS), covers an analysis of effectiveness of VFR Air Traffic Control Towers.

B. BACKGROUND:

The FAA is reviewing the criteria for installing new VFR towers at airports across the country. A VFR tower is the raised structure enabling a controller to maintain separation between aircraft in an airport area by visual and radio contact, and without the benefit of radar or other advanced traffic control facilities.

The current criteria for an airport to become a candidate to receive a VFR control tower are: 1) public ownership 2) 24,000 annual itinerant operations for airports with air carrier service, and 50,000 annual itinerant operations for airports serving civil aviation but not air carriers ("general aviation" airports).

In view of the funds necessary to implement the 167 tower installations planned for the next 10 years -- about $320,000 each as well as yearly operating and maintenance costs of about $100,000 -- the present criteria must be reviewed and, if warranted, new criteria selected. This analysis of effectiveness will provide an integral part of the information needed to determine appropriate criteria. The analysis of dollar costs and their comparison to benefits will be done by the FAA.

C. STATEMENT OF WORK:

The NBS shall provide the necessary qualified personnel, facilities, equipment, materials and services and perform the following work:

Task I. Determine what methodology is best suited in terms of adequacy, feasibility and efficiency to estimate VFR tower effectiveness (as identified in the discussion of the analytical problem).

Task II. Specify the model devised for estimating VFR tower effectiveness. If a stochastic model, the method for estimating probabilities should be made explicit. (Will they be judgmental or determined by simulation?) If deterministic, the techniques which will be used to describe the flow of airport traffic should be clearly described. Explicit treatment of any and all assumptions, hypotheses, parameters, variables, statements of operating characteristics, and necessary data will be required.

Task III. Obtain and process sample selected data required as input to the model to estimate the VFR tower effectiveness.

Task IV. Estimate the amounts of effectiveness (as identified under the heading "Analytical Problem") for representative parameter levels, using the model specified in Task II and the data collected in Task III.

Task V. To the extent practicable, verify the model and results of the analysis by means of statistical measures, comparisons to known data, the ability of the model to predict the effectiveness of towers actually in operation, or other means. Provide sensitivity analyses where appropriate.

D. THE ANALYTICAL PROBLEM:

The analytical effort should be directed to answering these questions:

1) How does the probability of aircraft collisions vary with traffic volumes and mix of aircraft types, under conditions of an airport with and without a VFR control tower? Aircraft collisions should include midair, air-ground, and (least important) ground-ground collisions (but these kinds of collisions need not be combined for the analysis). The probability of collisions, should, of course, be an output of the estimation model, rather than a parameter written into the model.

2) What are the mean and range of changes in flight time and track distance due to the controllers in the tower maintaining separation between aircraft, organizing traffic flows, and providing information to the pilot? (These are termed "traffic flow" effects.) How do these vary with traffic volume and aircraft mix?

E. REVIEW OF FAA MIDAIR COLLISION ANALYSIS:

An FAA statistical analysis of observed data on midair collisions and their relationship to annual traffic should be reviewed to determine whether further inferences can be made by use of these data. (See attached interim report, An Analysis of the Cost and Effectiveness of Air Traffic Control Towers.) (No analysis of traffic flow effects was attempted in the report.)

F. KINDS OF TRAFFIC FLOW EFFECTS:

The effect of VFR air traffic tower control on traffic flows, it must be recognized, may be the net effect of any adverse effects on traffic flow rates (especially, increases in aircraft service times caused by spacing or vectoring), and any beneficial effects, such as the controller's allowing the saving of aircraft operating time by aiding the pilot to make an intersection takeoff. (It is not intended here to state that either adverse or beneficial traffic flow effects exist -- this must be determined by the analysis. The NBS should estimate and report on any such negative and positive traffic flow effects, as well as the net difference.)

G. VARIABLES INFLUENCING VFR TOWER EFFECTIVENESS:

The units of effectiveness shall be measured as a function of at least the level of traffic and types of aircraft using the airport.

1) In estimating VFR tower effectiveness, tower and non-tower airports with up to 100,000 itinerant and 200,000 total operations should be compared, beginning with at least 10,000 and 30,000, respectively. The mixture of itinerant and local traffic should also be considered as a factor affecting collision probability rates and traffic flow effects.

The analysis itself may be done in terms of traffic in specified periods (e.g., hours) characteristic of airports with the above amounts of annual traffic. The effectiveness units should then be related to annual traffic, given the relationship between annual traffic and traffic in the period used in the analysis.

2) The aircraft mixes must include aircraft having significant variations in size and operating speeds. (Other aircraft characteristics which are found to significantly affect the collision and traffic flow

rates (e.g., time to maneuver) should also be included as variables in the analysis).

Traffic mixes can also be measured in terms of pilot proficiency or ratings -- perhaps student pilots compared to all others, but this measure of mix has a lower priority as the proficiency of the pilot is probably closely related to the type of aircraft and operations (local vs. itinerant).

3) The specification of these two variables is not intended to exclude other variables which may, during the course of the analysis, be found to significantly effect VFR tower effectiveness (e.g., number of configuration of runways at the airport).

In determining the effectiveness in terms of saved time (i.e., aircraft operating time), a distribution of aircraft by time saved (either in absolute or proportional terms) should be made. The frequency classes should be 0 to 5, 5 to 10, 10 to 15, 15 to 30 and 30+ minutes.

H. SCOPE OF WORK:

The work shall be limited to estimating the physical units (or probability of their occurrence) of VFR tower effectiveness. The following analytical work shall not be included in this effort: estimating the dollar values of VFR tower effectiveness; estimating the costs of installing and operating VFR towers; comparing the estimated dollar benefits and costs of VFR towers; analyzing the structure of or forecasting the demand for VFR tower services.

I. REPORTS:

(a) NBS shall meet with the FAA's Contracting Officer's Representative every two weeks to review study progress.

(b) NBS shall submit letter type technical progress reports, five (5) copies. The reports shall be submitted every four (4) weeks and shall detail results of work performed to date and methodological, analytical or data related problems.

(c) NBS shall submit a final report, 200 copies, summarizing results of all work performed.

Ten (10) copies of the final report draft shall be submitted for FAA review. The FAA will require thirty (30) days to review and approve, disapprove or request changes to the draft report.

(d) The NBS staff will present two briefings to FAA personnel, one at the end of three months and the second on completion of the study.

J. COMPLETION OF WORK:

Draft copies of the final report shall be submitted within six (6) months after date of execution of this agreement by both parties. The estimated time for completion of the work, including submission of completion of final report, is nine (9) months from the date of execution of this agreement by both parties.

Because the work requires the determination and use of new approaches to a problem which has long resisted satisfactory analysis, it is undetstood that NBS will devote its best effort to accomplish the tasks listed in this schedule, with no guarantee of positive results.

K. TERMINATION:

This agreement may be terminated at any time by either party giving to the other thirty (30) days notice thereof in writing.

L. DISSEMINATION OF INFORMATION:

During performance of this agreement, any information, oral or written, concerning the results or conclusions made pursuant to performance of this agreement shall not be published or permission granted for publication, or distributed for public consumption without approval of FAA.

M. FUNDING:

The estimated cost for performance of the work is $60,000.00. However, the agreement is funded in the amount of $30,000.00 for the work performed during FY-1970. No FY-1971 expenditures are herein authorized. Upon availability of FY-1971 funds, the agreement will be amended to provide for additional funds in the amount of $30,000.00.

FAA shall reimburse NBS in an amount not to exceed $30,000.00 for actual costs incurred during FY-1970 in the performance of work herein. The FAA shall be advised in writing and the written consent of the FAA Contracting Officer obtained prior to undertaking additional obligations.

An advance of funds in the amount of $30,000.00 may be requested via Standard Form 1080 citing Appropriation 001.0/1220/000/2591 and Inter-Agency Agreement No. DOT-FA70WAI-188. These references shall be cited on all correspondence relating to fiscal and contractual matters. Standard Form 1080 shall be submitted to the Chief, Fiscal Accounting Branch, HQ-220, Federal Aviation Administration, 800 Independence Avenue, S.W., Washington, D.C. 20590, who will make payment thereon.

NBS shall furnish an informal report of transferred funds reflecting expenditures every other pay period; to be followed by a formal report to be furnished by the Accounting Division of NBS at a later date.

At the conclusion of FY-1970, and upon expiration of this agreement, or upon completion of the work called for hereunder, whichever first occurs, NBS shall refund any portion of the transferred funds which has not been expended under this agreement.

N. CONTRACTING OFFICER'S REPRESENTATIVE:

The Contracting Officer's Representative for technical matters on this agreement is Mr. James J. Gansle, EC-100, Office of Aviation Economics, FAA, Washington, D.C. 20590. Telephone 202/962-5163, or such other person as may subsequently be designated in writing by the Contracting Officer.

AGREED:

DEPARTMENT OF TRANSPORTATION, FEDERAL AVIATION ADMINISTRATION	NATIONAL BUREAU OF STANDARDS
BY Dorothy Grapp	BY
TITLE Contracting Officer	TITLE
DATE 4/1/70	DATE

PR WA5I-0-0683
PL-70-37

DEPARTMENT OF COMMERCE ★ NATIONAL BUREAU OF STANDARDS ★