STUDY OF PRESTRESSED COMPOSITE CONCRETE

TEE BEAMS UNDER FLEXURAL LOADING
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STUDY OF PRESTRESSED COMPOSITE CONCRETE TEE BEAMS UNDER FLEXURAL LOADING

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IMPORTANT NOTICE

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Abstract

Prestressed Tee-beams constructed by the split-beam method were tested to failure in flexure to study the behavior and ultimate strength of these beams and to compare their flexural characteristics with those of prestressed beams of conventional construction.

The variables in the study included the percentage of prestressing steel, strength of concrete in the compressive element of the composite split-beams, manner of prestressing and web reinforcement.

Results showed that no significant distinction can be drawn between the composite split-beams and monolithically constructed beams on the basis of flexural response and ultimate load. The flange portion of the cross-section of the split-beam is cast after the stem of the beam has been formed and prestressed. The strength of the concrete for the flange section, which is called the compressive element, can be reduced within limits from that required for the prestressed stem (tensile element) without sacrificing ultimate load capacity. The required percentage of reinforcing steel is less for the split-beam compared with conventional beams.
A notable departure from the usual concept of composite prestressed concrete design was developed several years ago by A. Amirikian\(^\frac{1}{*}\) of the U. S. Department of the Navy. The principal objective of this new concept is to optimize the application of prestressing for flexural concrete members by prestressing only the area of the cross-section normally subjected to tension under bending. This requires that the tensile and compressive areas of the beam's cross-section be constructed separately in order to restrict the pre-compression of the concrete to the tensile section. Therefore, this procedure can be considered as a special case of composite construction in which the interface of the two elements is set at the neutral axis of the composite section. Beams constructed by this procedure are called "split-beams" and feature reduced prestressing forces for the same working load capacity compared with similar beams of conventional design.

\(^*\)Numbers in brackets indicate the literature reference at the end of this paper.
A series of prestressed Tee-beams constructed by the split-beam technique were tested to failure to study the behavior and ultimate strength of these beams and to compare their performances with those of conventional prestressed beams. The variables in the study included the percentage of prestressing steel, strength of concrete in the compressive element of the split-beam, manner of prestressing, and web reinforcement.

The work reported here is an extension of an earlier study with split-beams of rectangular cross-section. With the rectangular beams the principle difference between the split-beam and conventional beam was in the required prestressing force and location of the prestressing tendon in the cross-section. The cross-sectional properties of Tee-beams lend themselves to an additional advantageous feature in the split-beam technique since in the flexural response of the section the strain on the compressive surface (top of the flange) is usually considerably less than that on the tensile surface (bottom of the stem) due to the position of the elastic neutral axis. This means that the strength of the concrete in the flange section provided to resist compressive stresses need not be as high as that required in the tensile section which is initially cast and prestressed. This allows for possible savings in materials.
Beams

The concept of split-beam design takes advantage of the technique of composite construction for minimization of the prestress in the cross-section. The design procedure is to determine the overall cross-section for the beam as would be done for a conventional monolithic prestressed beam. From the properties of the full cross-section, the area that will experience tension under loading is defined by the location of the elastic neutral axis. This area will be cast separately in the split-beam construction and is termed the tensile element. After prestressing the tensile element, the zone of the split-beam that will resist compression is cast-on and is stress free prior to the application of live load.

The specimens in this investigation included beams of conventional monolithic construction as well as the split-beams of composite construction. They were all Tee shaped in cross-section with a 3 in. by 15 in. flange, an overall depth of 18 in., and were 19 ft. long.

Figure 1 shows the nominal dimensions of the beams with the location of the prestressing tendon in the
cross-section given for both the monolithic beam and the split-beam. Also, the positions of support and points of loading for tests are indicated.

The channel for the reinforcing tendons in each post-tensioned beam was formed by placing a length of thin-wall steel tubing, 1-in. O.D., in the form in the position specified for the tendon. The tubing was fixed in position at the ends of the form and at the third-point and midspan locations.

A single steel bar, threaded on both ends, was used as the prestressing tendon in the post-tensioned beams. In each case, the tendon was straight and located at a constant depth in the cross-section throughout the length of the beam.

Figure 2 shows a tensile element setup for post-tensioning. The tensioning force in the tendon was measured with a steel dynamometer attached to the tendon at the end of the beam opposite to the jacking end. This force was distributed over the ends of the prestressed element with 1 in. thick bearing plates. Heavy duty steel nuts bore against the dynamometer on one end and the bearing plate on the other end to maintain the prestressing force in the element.

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The pretensioned beams were each prestressed with two strands of steel cable. In profile, the strands were straight and parallel and spaced approximately 1-in. center to center in a vertical plane throughout the central 12 ft. section of the span of the beam (6 ft. to each side of the midspan section). From these points they tapered apart at equal angles to about 3-in. center to center at the ends of the beam. Steel washers with 1 1/2-in. I.D. were used to accomplish the angle change in the profile of the strands. The washers encircled the strands and were located at the points along the length of the strands where it was desired to change the angles. The two strands were tensioned simultaneously against anchors which spaced them about 3 1/2-in. apart. With the washers encircling the strands and located 4 ft. away from the pretensioning anchors, the strands, under tension, tapered down to the location of the washers which restrained the spread from these points inward to the 1-in. spacing described above.

A view of the jacking arrangement for pretensioning is presented in Figure 3. The pretensioning wires were stressed between two stub columns which were set 25 ft. apart and fixed to a tie-down floor. A steel dynamometer was placed between standard chuck type strand grips and the anchoring column at the end opposite to the jacking end to reflect the prestressing force
in the strands.

Stirrups were fabricated from mild steel No. 3 reinforcing bars having a yield strength of 50,000 psi. They were U shaped in the stem of the beam and placed throughout the span length spaced 1 ft. apart. The stirrups looped under the prestressing tendon extended up to the mid-height of the flange and out 6-in. horizontally on both sides.

The tendon in the monolithic beam was located 6.1 in. below the center of gravity for the Tee section and the prestress force was 45,000 lb. The location of the tendon in the split-beam was 8.7 in. below the center of gravity of its composite Tee section and the prestress force was 27,000 lb. The essential difference here is that the location of the tendon and the prestress for the monolithic beam is determined from the properties of the full Tee section while, due to the nature of the construction of the split-beam, the location of the tendon and the prestress is determined by the properties of the tensile element of the composite Tee section.

Figure 4 shows the idealized stress conditions for both types of beams for two stages of loading. The stress conditions (a) show that for the monolithic
beam the stress block tapers from the maximum value at the bottom fiber to zero at the top fiber of the beam. The stress block for the split-beam, for this stage, tapers from the maximum value at the bottom fiber to zero at the neutral axis of the composite section. This difference in the stress blocks reflects the same proportional difference in the required prestressing force for the two types of beams. The value of applied load that will cause zero stress in the bottom fiber is represented by the stress condition (b) and is the same for both the monolithic beam and the split-beam. The stress condition (c) results from the combination of (a) and (b).

Concrete

The concrete used in the first 12 specimens was composed of Type III Portland cement, siliceous sand and pea gravel. This concrete was mixed in the laboratory in a turbine-type mixer of 1/2 cu. yd. capacity. Concrete for the subsequent beams was obtained from a local ready-mix company using the same materials except for the course aggregate which was a Maryland No. 7 crushed stone (size No. 8, ASTM C 33-67). The mix proportions were varied around a design mix for 5000 psi concrete 1:3.2:2.6 by weight of cement, sand, and gravel. The
water content varied from 6 to 11 gal. per sack. The concrete strengths at the time of beam testing are given in Table 1. These strengths represent the average values determined from compressive tests of three 6-by-12-in. control cylinders.

Prestressing Steel

Two types of steel were used as prestressing tendons. The stress-strain curves for the steels are shown in Fig. 5. For the post-tensioned beams, high strength heat-treated, stress-relieved bars were used. Tensile tests of these bars indicated a stress-strain relationship that is essentially linear up to a stress of 108,000 psi, and an initial tangent modulus of approximately $29.8 \times 10^6$ psi. The yield strength of the steel was 170,000 psi as determined by the 0.2 percent offset method. The tensile strength was 190,000 psi. The prestressing tendons for the pretensioned beams were 7 wire strands of high strength steel. Tensile test of the strands showed a linear stress-strain relationship up to a stress of approximately 162,000 psi. The tangent modulus for the strand was $28.1 \times 10^6$ psi. The yield strength was defined at 0.2 percent offset and was 221,000 psi. The ultimate strength of the strands, as could be developed in the beams, was not determined
precisely since in all cases the strands fractured in the grips in the tensile test. However, indications from the tensile tests, and the beam tests suggest that the manufacturers rating of 250,000 psi is valid.

Test Procedure

Beam specimens were tested to failure with equal loads applied at the quarter points. In general, load increments of 2000 lb. were applied except in the range of cracking where increments of 500 lb. were used. After the application of each load increment, the deflection of the beam, the force in the reinforcing tendon for unbonded beams, the strain on the concrete surface, and the extent of cracking were recorded. For the pretensioned beams, electrical resistance gages were applied to individual wires of the strands for strain measurements. However, readings from these gages beyond the cracking loads were erratic in all cases and were discarded.

Test Results and Analysis

Values of observed and computed characteristics of the beams are given in Table 1. In general, the notations used herein are those proposed by ACI-ASCE Joint Committee 323, otherwise, they are defined as they occur. The
first order grouping of the specimens is by method of prestressing and attachment of tendons. There are three classifications: (1) Post-tensioned, unbonded; (2) Post-tensioned, grouted; (3) Pretensioned, bonded. The two beams with grouted tendons, SG-1 and SG-2, experienced bond failures beyond the cracking loads. Since there are no appreciable differences in the performance of beams with bonded and unbonded tendons prior to the onset of cracking, these beams were classed as unbonded for the purpose of comparison.

The beams in this investigation fall into one of five different steel ratio (p) groups. However, test results show that a better ordering of groups can be made in terms of a moment index, $A_s f_{sy} d$. All beams, except SU-11, SU-14, and SG-2, failed after the yield strength of the reinforcement had been reached. Beams SU-11 and SU-14 had 1900 psi and 2000 psi concretes in the compressive zones, respectively, and failed by compression of concrete with the reinforcement in the elastic range. Beam SG-2 failed by interface separation in the shear span.

The performances of the beams are compared in terms of load-deflection characteristics, ultimate strengths, and crack patterns. The moment index $A_s f_{sy} d$ shows
a direct correlation with both the load-deflection relationship and ultimate strength. However, the fact should not be overlooked that all beams in this study were of the same shape and size and were tested in the same manner.

Load Response

A flexural load applied on a reinforced concrete beam of a given cross-section will require a specific force to act at the level of the steel for equilibrium. In a prestressed beam the strains at the level of the steel are a function of the moment of inertia of the full transformed cross-section up to the cracking load. Within this range of loading, the effect of large differences in steel areas on the straining rate at the level of the steel is relatively small. However, once the beam has cracked, the amount of strain in the steel for a given increment of loading will vary inversely with different size tendons.

Typical load-deflection relationships for post-tensioned beams with unbonded solid bar tendons are shown in Figure 6 and for pretensioned beams with bonded strand tendons in Figure 7. The three curves representing the post-tensioned beams in Figure 6 clearly show the
effect of the moment index \( A_{sf_{syd}} \) on the performance of the beams. The same is true for the two curves representing the pretensioned beams in Figure 7. The initial portion of the load deflection curves in all cases was a straight line with practically the same slope, irrespective of the moment index. This portion of the curve reflects the response of the beam to loading prior to the onset of cracking. Subsequent to cracking, however, the curves are ordered in accordance with the moment index.

In Figure 8 a basic difference is seen in the overall characteristics of the load-deflection curves between the post-tensioned and the pretensioned beam groups. The pretensioned beams (SB-1, SB-3) showed considerably more ductility in their response to loading than the post-tensioned group (SU-1, RU-2, SU-13). Distinctively different crack patterns were observed for the two groups of beams. Typical crack patterns for the beams are shown in Figure 9. For the post-tensioned beams, a single crack first appeared at or very near midspan and was followed shortly, as loading proceeded, by the development of two or three additional cracks on both sides of the crack at midspan. In the beams without stirrups, the midspan crack developed into a distinctive
Y pattern with horizontal extensions just under the flange covering a large section of the constant moment zone. When stirrups were used in the post-tensioned beams, the horizontal extensions of the central crack were eliminated. Views of a post-tensioned and a pretensioned beam at ultimate load are presented in Figures 10 and 11, respectively, to illustrate the difference in crack distributions and their effect on deflection. In all post-tensioned unbonded beams the central crack dominated the failure mode causing the maximum compressive strain in the concrete, and consequently the maximum curvature of the beam, to concentrate at the midspan. For the pretensioned bonded beams, ten to twelve equally spaced cracks developed in the constant moment zone. These cracks propagated and opened with equal magnitude as load increments were added and until failure occurred. This caused a more uniform and greater overall curvature in the pretensioned beams than for the post-tensioned ones.

**Ultimate Strength**

Final failure in flexure of a reinforced concrete beam may be initiated by excessive elongation of the reinforcement, in which case it is called a tension failure, or crushing of the concrete may occur before yield of the reinforcement.
which is termed a compression failure. Other types of failures were of secondary concern in this study. In general, the beams in this investigation failed in tension. Two beams, RU-1 and SG-2, failed prematurely and their results are not used for comparisons in the study. Beam RU-1 failed when the threads in an anchor nut on one end of its post-tensioned bar were sheared off while the beam was being loaded. Beam SG-2 failed by complete separation of the interface of the tensile and compressive elements in the shear zone under load.

The expression $A_s f_{sy} d$ has been shown to be a valid index for an ordering of the overall load response for the beams in this investigation. Therefore, the moment index is a means by which a comparison can be made of the tendon sizes required to produce equal load response and capacity for split-beams and monolithic beams. The moment index for the monolithic beams (RU-1, RU-2) is 755 in.-kips and the tendon size is 0.37 sq. in. With these values as references, the required tendon size for equal performance by the split-beam is,

$$A_s = \frac{755 \text{ in.-kips}}{f_{sy} d}$$
With the type of steel used for the post-tensioned beams, the tendon size for the split-beam is found to be 0.30 sq. in. This is a reduction of approximately 20 percent below the size for the monolithic beam.

An analysis of the principle properties of the stress block was conducted to evaluate the performance of the concrete at ultimate load. It has been demonstrated in laboratory tests that the shape of the stress block at the ultimate capacity of a beam varies with the strength of the concrete. The shape of the stress block varies from nearly trapezoidal for low strength concretes in the 2000 psi class to practically triangular for high strength concretes in the 7000 psi class. The stress block has been found to be nearly parabolic for 5000 psi concrete. However, in determining the ultimate strength of beams, the exact shape of the stress block is incidental to the magnitude and location of the internal compressive force. This force can be defined and located in terms of three parameters, \( k_1, k_2, \) and \( k_3. \) The parameter \( k_1 \) is defined as the ratio of the average compressive stress to the maximum compressive stress of the concrete in the compression zone of the beam at ultimate. The parameter \( k_2 \) is defined as the ratio of the depth to the line of action.
of the resultant compressive force to the depth to the neutral axis. Parameter $k_3$ is defined as the ratio of the maximum compressive strength of the concrete in flexure to the cylinder strength.

The assumed stress conditions at ultimate load are shown in Fig. 12. The expression for the ultimate resisting moment is:

$$ M_u = A_s f_{su} (d - k_2 k_u d) = f_{sup} b d^2 (1 - k_2 k_u) $$  \hspace{1cm} (1)

An expression for $k_u$ is obtained from the equilibrium of forces.

$$ k_u = \frac{p f_{su}}{k_1 k_3 f_c} $$  \hspace{1cm} (2)

Substituting Eq. (2) into Eq. (1) and dividing both sides by $f_{c} b d^2$ gives:

$$ \frac{M_u}{f_{c} bd^2} = \frac{p f_{su}}{f_c} [1 - \left(\frac{k_2}{k_1 k_3}\right)\left(\frac{p f_{su}}{f_c}\right)] $$  \hspace{1cm} (3)

Eq. (3) is a convenient relationship for evaluating the expression $\frac{k_2}{k_1 k_3}$ with the measured properties of a beam. This relationship was studied using the measured ultimate moments and measured steel stresses at failure. The results are the plotted points shown in Figure 13. The curve shown by dashed line in this figure was developed from the results of a study by Janny, Hognestad, and McHenry with rectangular beams.
covering five types of reinforcement. This curve represents the relationship in Eq. 3 for a value of \( \frac{k_2}{k_1 k_3} \) equal to 0.52. The plotted points in Fig. 13 are obviously in close agreement with the curve. This indicates that the basic characteristics of the stress block at ultimate load for the Tee beams in this investigation are similar to those for the rectangular beams studied by Janny, et al. It also indicates that the unusually low steel ratios, tendon location, and initial stress gradient discontinuity in the split-beams had no adverse effect on the ultimate load performances of these beams over a wide range of concrete strengths in the compressive elements (flange sections).

As would be expected for beams failing in tension, the strength of the concrete in the compression zone had little if any effect on the ultimate capacity of the beams. In this study only the most general trend might be noted in a comparison of concrete strengths and ultimate moments for beams in the 925 in.-kip moment index group (SU-5 thru SU-14). However, the results for the beams in the 650 in.-kip index group (SU-1 thru SU-4) dispels any argument for a direct linear correlation of concrete strength and ultimate moment.
Discussion

In this study no clear distinction can be drawn between the performance of the beams on the basis of construction method (monolithic or split-beam), nor can it be said that the use of stirrups significantly affected the load response characteristics of the beams. The index $A_g f_{sy} d$, which was used to categorize the beams in relation to a scale of load response and capacity, is essentially a measure of the internal resistant moment capacity for under-reinforced beams. The very close agreement between the index values and the respective measured ultimate moments for the Tee beams can be explained by considering the factors in the moment index expression. The factor $d$ is approached within 10 percent by the actual moment arm at ultimate and the actual stress in the steel is somewhat greater than the yield strength of the steel, $f_{sy}$, by a similar difference but opposite in direction to that for the $d$ factor. Consequently, the two departures from actual conditions compensate for each other quite conveniently.

Due to the manner of construction and the design of the pre-compressed section, the split-beam enjoys the advantage of a reduced amount of reinforcing steel for the same overall flexural characteristics as
compared with the conventional monolithic beam. However, it should be emphasized that the comparison here is between a composite beam and a monolithic beam. Also, no tensile stresses were allowed in the stages of construction. It may be better to evaluate the split-beams in relation to other composite beams. For example, it was stated earlier that the split-beam is a special case of composite construction where the construction joint is designed to coincide with the neutral axis of the composite section. Figure 14 shows the cross section of a composite beam of conventional makeup where the construction joint is located at the intersection of flange and web. The dimensions of the cross section are the same as for the other beams in this investigation. To include this section in a comparison with the other beam sections in this study, the basic prestressing denominators for the three types of beam construction features were computed with respect to the common moment index $A_s f_{sy} d = 755$ in.-kips. These values are presented in Table 2. From a purely performance standpoint, it is apparent that no improvement would be gained in comparison with the split-beam design by locating the construction joint above the neutral axis of the cross section. In fact, for the same flexural characteristics, the required area of the reinforcing steel will increase with the distance of the construction joint above the
neutral axis. Conversely, when the construction joint is located below the neutral axis the required area of the reinforcement will decrease as the distance of the construction joint to the neutral axis increases. The limiting distance of the construction joint below the neutral axis will be affected by several practical considerations. Among these considerations are: (1) The minimum cross-section needed for prestressing to a desired value; (2) The degree to which tensile cracks will be tolerated in the zone between the construction joint and the neutral axis within the working load range.
Conclusions

The construction joint located at the neutral axis of the composite section had no adverse effect on the performance of the split beams when dowels or stirrups were provided.

Stirrups should be provided throughout the span length for these beams to prevent the development of extensive horizontal cracking just above the neutral axis in the region of maximum moment and to serve as reinforcement against possible interface separation.

The product of the factors $A_s f_{sy} d$ was found to be a satisfactory index and very close indication of the ultimate moment for the beams in this investigation. However, the test data agreed extremely well with the more refined relationship

$$\frac{M_{ult}}{f_c^b d^2} = q_u (1 - 0.52 q_u).$$

Concrete strengths in the compression zone can be markedly reduced below that required for the prestressed element without significantly affecting the flexural characteristics of under-reinforced members. A practical lower limit would appear to be 3000 psi. The use of lightweight concrete in the compressive element should not be overlooked as an additional benefit.
The required amount of prestressing steel for the split-beam in this study was approximately 20% less than that for a monolithic beam. However, when compared with a conventional composite beam only a 9% reduction in steel was found in favor of the split beam.

Although a strict economic evaluation for the practical use of split-beams was not within the scope of this study, the experience gained in preparing these specimens raises a serious question as to the balance between materials savings and the added cost of framework and construction handling.
List of References


3) ACI-ASCE Joint Committee 232, "Tentative Recommendations for Prestressed Concrete," Journal of the American Concrete Institute, Vol. 29, No. 7 (January 1958), pp. 545-578.


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<td>4400</td>
<td>5000</td>
<td>0.37</td>
<td>14.7</td>
<td>0.0017</td>
<td>73</td>
<td>190</td>
<td>1.3</td>
<td>437</td>
</tr>
<tr>
<td>SU-7 Composite</td>
<td>None</td>
<td>3800</td>
<td>5400</td>
<td>0.37</td>
<td>14.7</td>
<td>0.0017</td>
<td>73</td>
<td>198</td>
<td>1.5</td>
<td>437</td>
</tr>
<tr>
<td>SU-8 Composite</td>
<td>None</td>
<td>2800</td>
<td>4800</td>
<td>0.37</td>
<td>14.7</td>
<td>0.0017</td>
<td>73</td>
<td>186</td>
<td>1.9</td>
<td>383</td>
</tr>
<tr>
<td>SU-9 Composite</td>
<td>None</td>
<td>6600</td>
<td>4700</td>
<td>0.37</td>
<td>14.7</td>
<td>0.0017</td>
<td>73</td>
<td>198</td>
<td>0.9</td>
<td>377</td>
</tr>
<tr>
<td>SU-10 Composite</td>
<td>dowels@12&quot;c/c</td>
<td>3000</td>
<td>4700</td>
<td>0.37</td>
<td>14.7</td>
<td>0.0017</td>
<td>73</td>
<td>184</td>
<td>1.8</td>
<td>392</td>
</tr>
<tr>
<td>SU-11 Composite</td>
<td>stirrups@12&quot;c/c</td>
<td>1900</td>
<td>4700</td>
<td>0.37</td>
<td>14.7</td>
<td>0.0017</td>
<td>73</td>
<td>162</td>
<td>2.5</td>
<td>346</td>
</tr>
<tr>
<td>SU-12 Composite</td>
<td>stirrups@12&quot;c/c</td>
<td>4400</td>
<td>5100</td>
<td>0.37</td>
<td>14.7</td>
<td>0.0017</td>
<td>73</td>
<td>181</td>
<td>1.2</td>
<td>378</td>
</tr>
<tr>
<td>SU-13 Composite</td>
<td>stirrups@12&quot;c/c</td>
<td>5600</td>
<td>4800</td>
<td>0.37</td>
<td>14.7</td>
<td>0.0017</td>
<td>73</td>
<td>190</td>
<td>1.0</td>
<td>432</td>
</tr>
<tr>
<td>SU-14 Composite</td>
<td>stirrups@12&quot;c/c</td>
<td>2900</td>
<td>5300</td>
<td>0.37</td>
<td>14.7</td>
<td>0.0017</td>
<td>73</td>
<td>149</td>
<td>2.1</td>
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<td>Post-Tensioned, Grouted</td>
<td></td>
<td>None</td>
<td>2100</td>
<td>5100</td>
<td>0.26</td>
<td>14.7</td>
<td>0.0012</td>
<td>104</td>
<td>181</td>
<td>1.8</td>
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<td>SG-1 Composite</td>
<td>None</td>
<td>2400</td>
<td>4800</td>
<td>0.37</td>
<td>14.7</td>
<td>0.0017</td>
<td>73</td>
<td>127</td>
<td>1.5</td>
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<td>Pretensioned, Bonded</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>SB-1 Composite</td>
<td>stirrups@12&quot;c/c</td>
<td>3600</td>
<td>4600</td>
<td>0.16</td>
<td>14.4</td>
<td>0.0007</td>
<td>124</td>
<td>---</td>
<td>---</td>
<td>338</td>
</tr>
<tr>
<td>SB-2 Composite</td>
<td>stirrups@12&quot;c/c</td>
<td>5300</td>
<td>5400</td>
<td>0.16</td>
<td>14.4</td>
<td>0.0007</td>
<td>138</td>
<td>---</td>
<td>---</td>
<td>378</td>
</tr>
<tr>
<td>SB-3 Composite</td>
<td>stirrups@12&quot;c/c</td>
<td>2500</td>
<td>4300</td>
<td>0.22</td>
<td>14.5</td>
<td>0.0010</td>
<td>85</td>
<td>---</td>
<td>---</td>
<td>311</td>
</tr>
<tr>
<td>SB-4 Composite</td>
<td>stirrups@12&quot;c/c</td>
<td>6600</td>
<td>5500</td>
<td>0.22</td>
<td>14.5</td>
<td>0.0010</td>
<td>75</td>
<td>---</td>
<td>---</td>
<td>284</td>
</tr>
</tbody>
</table>

1/ Where bars were used, the area reported is the effective area in the threaded sections near the ends of the beam.

2/ \( a_1 \) = computed depth of stress block at ultimate load using measured force in tendon, \((T_u/0.85 \text{ bfc})\)
Table 2
Computed Prestressing Denominators\(^{(a)}\) for Post-Tensioned Tee-Beams of Different Construction with a Common Moment Index of 755 in-kips

<table>
<thead>
<tr>
<th>Construction Method</th>
<th>$F_o \frac{h}{d}$</th>
<th>$d$</th>
<th>$A_s \text{ in.}^2$</th>
<th>$f_{se}/f_{sy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monolithic</td>
<td>45,000 lbs.</td>
<td>12.1 in.</td>
<td>0.37 in.²</td>
<td>0.71</td>
</tr>
<tr>
<td>Conventional Composite</td>
<td>33,750 lbs.</td>
<td>13.6 in.</td>
<td>0.33 in.²</td>
<td>0.61</td>
</tr>
<tr>
<td>Split Beam</td>
<td>27,000 lbs.</td>
<td>14.7 in.</td>
<td>0.30 in.²</td>
<td>0.53</td>
</tr>
</tbody>
</table>

(a) For the same cross section, initial conditions, and steel bars for this study.

(b) Initial prestressing force.
Figure 2 Tensile element with jacking arrangement setup for post-tensioning.
Figure 3. Jacking arrangement for pretensioning.
\[ f_{ct} = \frac{f_{cb}}{2} \]

Figure 4 Stress conditions at midspan idealized, (a) after prestress, (b) gradient produced by applied load causing stress at bottom fiber equal to prestress, (c) resultant from combining (a) and (b).
Figure 5  Stress-strain curves for prestressing steel tendons.
Figure 6 Load-deflection relationship for post-tensioned beams with unbonded solid bar tendons.
Figure 7  Load-deflection relationship for pretensioned beams with bonded strand tendons.
Figure 8 Combined curves of load-deflection relationship for all methods of prestressing and types and sizes of tendons.
Fig. 9 Typical crack patterns for beams: (a) Post-tensioned unbonded without stirrups; (b) Post-tensioned unbonded with stirrups; (c) Pretensioned bonded.
Figure 10 Post-tensioned beam SU-13 at ultimate load.
Figure 11 Pretensioned beam SB-4 at ultimate load.
Figure 12 Stress conditions at ultimate load.
Fig. 13  Relationship between ultimate moment and $\varphi_u$. 

\[ \frac{M_{ult}}{f_c bd^2} = \varphi_u (1 - 0.52 \varphi_u) \]

- ○ - RU - 1, 2
- △ - SU - 1 - 4
- ○ - SU - 5 - 10
- □ - SU - 11 - 14
- X - SG - 1, 2
- + - SB - 1, 2
Figure 14  Cross-section for conventional composite beam.