

NATIONAL BUREAU OF STANDARDS REPORT

10003

PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION
A Semi-Annual Report
July through December 1968



U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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NBS PROJECT

205.0

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U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION FUNCTIONS

Conduct research and provide consulting services to the Bureau and other Federal agencies in various fields of mathematics important to physical and engineering sciences, automatic data processing, and operations research with emphasis on statistical, numerical and combinatorial analysis, and mathematical physics. Develop tools for mathematical work such as mathematical tables, handbooks, manuals, mathematical models and computational methods, and advise on their use. Provide training in disciplines related to these functions.

NUMERICAL ANALYSIS SECTION: The advancement of computation and the theory of numerical analysis, particularly in the development of computing algorithms, approximations to functions, and methods to facilitate the use of high speed electronic computers by subject matter specialists. Design of mathematical tables; exploratory calculations on automatic machines. Consulting services and training, and preparation of manuals in these fields. Research in underlying branches of pure and applied mathematics, such as matrix algebra, combinatorial analysis, and number theory.

OPERATIONS RESEARCH SECTION: Development and application of mathematical and computational techniques for the analysis, improvement or optimization of complex systems or activity-patterns. This includes (1) research in specific relevant areas of mathematics, such as linear programming, the theory of linear graphs, and the theory of strategic contests, (2) investigations in the art of constructing useful mathematical models of complex systems, and of obtaining information about the system by applying analytic or simulation methods, and (3) application of these techniques to selected problems, of general methodological significance, arising in the work of the Bureau or of other Government agencies lacking specialized personnel in this field.

STATISTICAL ENGINEERING SECTION: Consulting services in the application of mathematical statistics to physical science experiments and engineering tests, particularly in the design of experiments and in the analysis and interpretation of data. Research on pertinent topics in probability and mathematical statistics. Preparation of reports, manuals, tables, studies of computational methods and other aids to the application of modern statistical methods.

SYSTEMS DYNAMICS SECTION: Research and consulting in applied mathematics basic to physics and engineering, with emphasis on analysis of the dynamic behavior of complex physical systems. This involves, primarily, the development and application of techniques for solving linear and non-linear systems of differential equations and integral equations, or combinations of both. Of concern also is simulation of the behavior of physical systems by means of electronic computers using approximation techniques and semi-analytic methods. Attention is given to problems in plasma dynamics and the behavior of solid matter and multicomponent liquid systems, with emphasis on developing mathematical methods of wide range of applicability beyond the scope of the immediate problem. Investigations are carried out on the special functions encountered in the analysis and algorithms for their evaluation are prepared.

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^o Only unclassified material is included in this report.

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July 1 through December 31, 1968

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Chan Mou Tchen, Ph.D.^o

*Part-time

***Guest Worker

**Postdoctoral Resident Research Associate

^oOn Leave of Absence

¹Joseph M. Cameron detailed to the Office of the Director, Institute for Basic Standards;
Dr. Joan R. Rosenblatt detailed as Acting Chief, Statistical Engineering Laboratory, effective
August 22, 1968.

Status of Projects

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS

Task 20501-12-2050110/55-55

11540

Origin: NBS

Authorized 8/29/54

Manager: Morris Newman

Full task description: July - September 1954 issue, p. 1

Status: CONTINUED. M. Newman and M. Marcus have proved that an irreducible matrix group with uniformly bounded trace is bounded, and have applied this to the study of certain matrix groups arising in investigations of singular values.

M. Newman and L. Greenberg have proved theorems on the structure of solvable groups generated by two elements of relatively prime order, and have used them to prove that the density of such groups (in a certain well-defined sense) is 0.

M. Newman has shown that if $t \geq 3$ then no subgroup of finite index of $SL(t, \mathbb{Z})$ is the free product of cyclic groups.

M. Newman has derived a number of miscellaneous results on class numbers of quadratic fields. For example if k satisfies $k > 2$, $k \equiv 2 \pmod{4}$, k square-free, and $k = y^p - x^2$, where p is a prime $\equiv 1 \pmod{4}$, then the class number of the field $R(\sqrt{-k})$ is divisible by p .

J. Lehner and A.O.L. Atkin have made a thorough study of the Hecke operators on the groups $\Gamma_0(m)$, where m is a positive integer not restricted to primes. The results for $m \neq$ prime are believed to be new. Spaces of simultaneous eigenforms are developed and are used to build up the space of cusp modular forms. Arithmetic properties of the Fourier coefficients are studied.

K. Goldberg continued his investigation of the coefficients of the powers of the iterates of formal power series.

S. Haber continued a program of calculations designed to determine parameters for, and convergence properties of, Korobov quadrature formulas. He also found some new classes of stochastic quadrature formulas, and began a comparison of the use of various sampling schemes in connection with these formulas.

M. Newman and S. Pierce have determined necessary and sufficient conditions that a certain class of (not necessarily irreducible) matrix groups be equivalent to unitary groups.

F.W.J. Olver has completed an error analysis for the method of stationary phase. Applications are being made to asymptotic expansions of Bessel and other functions.

Publications:

- (1) Semi-groups with zeroids. K. Goldberg. To appear in the Journal of Research NBS.
- (2) A recurrence related to monotone subsequences in permutations. K. Goldberg. To appear in the Journal of Research NBS.
- (3) The l.u.b. of a set of determinants of order 3. K. Goldberg. To appear in the Journal of Research NBS.
- (4) On a theorem of Piatetsky-Shapiro and evaluation of multiple integrals. S. Haber and C.F. Osgood. To appear in Math. Comp.
- (5) A combination of classical and Monte Carlo methods for evaluating multiple integrals. S. Haber. Bull. A.M.S. 74, 683-686 (1968).
- (6) On the sum $\sum < \alpha_n >^{-t}$. S. Haber and C.F. Osgood. Submitted to a technical journal.
- (7) Maximal normal subgroups of the modular group. M. Newman. Proc. Amer. Math. Soc. 19, 1138-1144 (1968).
- (8) Solving equations exactly. M. Newman. J. Res. Nat. Bur. Standards 71B, 171-179 (1967).
- (9) Pairs of matrices generating discrete free groups and free products. M. Newman. Michigan Math. J. 15, 155-160 (1968).
- (10) A bound for the number of conjugacy classes of a group. M. Newman. J. London Math. Soc. 43, 108-110 (1968).
- (11) Bounds for the number of generators of a finite group. M. Newman. J. Res. Nat. Bur. Standards 71B, 247-248 (1967).
- (12) A diophantine equation. M. Newman. J. London Math. Soc. 43, 105-107 (1968).
- (13) Matrix representations of groups. M. Newman. AMS-60, of the NBS.
- (14) An extension of Miller's algorithm. F.W.J. Olver. Aplikace Matematiky, vol. 13, pp. 174-176 (1968).
- (15) Error bounds for the Laplace approximation for definite integrals. F.W.J. Olver. J. of Approx. Theory (to appear).
- (16) Subgroups of $SL(t, z)$. M. Newman. To appear in J. Res. Nat. Bur. Standards.

2. OPERATIONS RESEARCH

CONSULTATION IN MATHEMATICAL OPERATIONS RESEARCH

Task 205-12-2050151
11570

Authorized 12/30/60

Origin and Sponsor: NBS
Manager: Alan J. Goldman

Full task description: October-December 1960 issue, p.3

Status: CONTINUED.

(1) Demand for miscellaneous consulting and advisory services continued heavy. Section staff provided such services in 104 recorded instances; of these 63 involved assistance to NBS staff. The 104 instances totalled to 575 recorded man-hours. Other agencies assisted included National Institutes of Health, Coast Guard, National Highway Safety Council, AID, U.S. Travel Service, Office of Emergency Planning, Bureau of Public Roads and Bureau of Fish and Wildlife. Requests from universities, industry, professional groups and journals were also met.

(2) J. Gilsinn, A.J. Goldman and W. Horn completed assistance to the Maritime Administration in a study (with Division 431) to determine locations and sizings for inland cargo consolidation centers. A mathematical model was developed, programmed, and given illustrative application to the extent that data permitted. The work was documented for the sponsor. (Reported here for convenience; supported under Project 4314422). Further efforts under NBS auspices are aimed at improving this tool to aid in locating collection-distribution facilities.

(3) W.G. Hall continued assistance in developing a simulation model for a highway maintenance depot. He also continued helping the Army Data Field Systems Command in planning, design and numerical-analysis aspects of a proposed new tactical artillery-fire control system. He continued assistance to the National Highway Safety Bureau in planning and evaluating the data-systems elements of its activities. (Reported here for convenience; supported under Projects 4565425 and 2050625 respectively.)

(4) A.J. Goldman, W.G. Hall, W.A. Horn, J. Levy and M.H. Pearl began assistance to the Post Office Department's Bureau of Research and Engineering on a major systems engineering project. (Reported here for convenience; supported under Projects 4314442, 4314444, 4314448, 4314449.)

(5) A.J. Goldman and P.R. Meyers continued the study of "contractification" of self-mappings of metric spaces by remetrization, extending previous results for the commutative case to obtain conditions for the simultaneous contractification of a finite family of maps. W.A. Horn initiated a study of "convex homotopies" between convex real-valued functions, examining existence, monotonicity and interpolation properties of such maps.

Publications

- (1) W.F. Druckenbrod (Div. 431), W.G. Hall, et al. A simulation of a highway maintenance operational unit. NBS Report 9938 (October, 1968).
- (2) A.J. Goldman. Fractional container-loads and topological groups. To appear in Operations Research.
- (3) W.A. Horn. Some fixed point theorems for compact maps and flows in Banach spaces. Submitted to a technical journal.
- (4) W.A. Horn. Determining optimal container inventory and routing. NBS Report 9936 (October, 1968).
- (5) R.H. Jordan (Div. 431), J. Gilsinn, A.J. Goldman, W.A. Horn et al. Systems analysis of inland consolidation centers for the U.S. Maritime Administration. NBS Report 9892 (August, 1968).

COMBINATORIAL METHODS

Task 205-12-2050152
11540

Origin and Sponsor: NBS

Manager: Jack Edmonds

Authorized 12/30/60

Full task description: October-December 1964 issue, p. 3; April-June 1962, p. 15

Status: CONTINUED.

(1) J. Edmonds continued work on various aspects of polyhedral combinatorics, theory of submodular functions, matching theory, and combinatorial algorithms. Edmonds and Oved Shisha prepared a paper titled "Acute Bijections".

(2) A.J. Goldman considered the problem of locating n "centers" in a network so as to minimize transportation costs. He showed S. Hakimi's result, that only vertex-locations for the centers need be considered, remains valid when shipped material is to pass through one or two centers in its movement from source to destination, the applicable unit transport cost changing with each such passage.

(3) W.A. Horn established the extendibility, of a geodesic metric on the 1-skeleton of a simplicial complex (e.g., a graph imbedded in the plane), to the whole complex.

Publications

- (1) J. Edmonds and D.R. Fulkerson (RAND Corporation). Bottleneck extrema. To appear in J. Combinatorial Theory.

LINEAR AND NON-LINEAR PROGRAMMING

Task 205-12-2050153
11540

Origin and Sponsor: NBS

Manager: Jack Edmonds

Authorized 12/30/60

Full task description: October-December 1960 issue, p. 3

Status: CONTINUED.

A.J. Goldman and P.R. Meyers continued studies on the minimax error selection of incompletely specified discrete probability distributions, with error given by the L_∞ norm. Attention focussed on the bivariate case with given margins; this situation was shown to admit a linear programming formulation which may permit a "tailored" solution algorithm.

Publications

- (1) A.J. Goldman. The minimax transportation problem. To appear in Transportation Science (Letters to Editor).
- (2) A.J. Goldman. Minimax error selection of a discrete univariate distribution with prescribed componentwise ranking. Journal of Research NBS, 72B (1968), pp. 273-277.
- (3) A.J. Goldman and P.R. Meyers. Minimax error selection of a discrete univariate distribution with prescribed componentwise bounds. Journal of Research NBS 72B (1968), pp. 263-271.

MATHEMATICAL METHODS FOR HIGH SPEED GROUND TRANSPORTATION STUDY

Task 205-12-2058456
51550

Origin: Technical Analysis Division, NBS

Sponsor: Northeast Corridor Transportation Project, Dept. of Transportation

Manager: A.J. Goldman

Full task description: January-June 1966 issue, p. 14

(1) J. Gilsinn, A.J. Goldman, W.A. Horn and P. Saunders prepared memoranda on various phases of the Northeast Corridor Transportation Project, including reviews of several contractors' reports.

(2) P. Saunders continued work on improving and implementing mathematical scheduling methods. Several versions for high-speed trains are now available on the NBS computer. An adaptation to a VTOL/STOL context was made, and is now in routine use by MITRE in its study of this mode for the Corridor Project.

(3) J. Gilsinn, A.J. Goldman and L.S. Joel (with members of Division 431) are involved in the development of improved models for predicting demand by mode for intercity passenger movement. An important aspect of this work is the identification of appropriate criteria for testing such models. Estimation of parameters is being carried out using (a) OMNITAB for linearized models, (b) a program written by W.G. Hall and P. Saunders for non-linear least-squares estimation in log-linear models, and (c) a general non-linear regression procedure (Marquardt) made operational at NBS by B.L. Joiner (205.03).

(4) J. Levy and M.H. Pearl continued the development of methods for evaluating feedback vs. non-feedback methods for regulating flow at a merge point in a transport network. Several working papers on this subject have been prepared.

(5) J. Gilsinn continued computational experiments for the comparative evaluation of shortest-path algorithms. Three of the methods tested, on both real and synthesized graphs have performed significantly better than the others.

(6) W.A. Horn studied the problem of optimally providing cross-ties between roughly parallel routes to achieve a connected network.

Publications

- (1) W.A. Horn. Optimal networks connecting N points in the plane. To appear in Proceedings of Fourth International Symposium (Karlsruhe, Germany; June, 1968) on Traffic Flow.
- (2) W.A. Horn. Minimum-length covering by intersecting intervals. To appear in Journal of Research NBS, 73B (1969).

3. PROBABILITY AND MATHEMATICAL STATISTICS

RESEARCH IN PROBABILITY AND MATHEMATICAL STATISTICS

Task 20503-12-2050131/63-1259
11550

Origin: NBS
Manager: Joan Raup Rosenblatt
Full task description: July - December 1962

Authorized 10/1/62

Status: CONTINUED. Two linear least squares test problems, both fifth degree polynomials, have been run by Roy H. Wampler on more than twenty different computer programs in order to assess their numerical accuracy. Among the programs tested were representatives from various widely used statistical packages; some of these were highly inaccurate in comparison to good programs (such as NBS's ORTHO) that have been available for many years. Mr. Wampler's paper reports a number of programs that yielded results (coefficients) that were completely erroneous, containing not even one correct significant digit.

Essentially five different algorithms were used in the various programs to obtain the coefficients of the least squares fits. The tests were run on several different computers, in double precision as well as single precision. By comparing the coefficients reported, it was found that those programs using orthogonal Householder transformations or Gram-Schmidt orthonormalization were much more accurate than those using elimination algorithms. Programs using orthogonal polynomials (suitable only for polynomial fits) also proved to be superior to those using elimination algorithms. Morris Newman's program, using congruential methods and integer arithmetic, was used to obtain exact solutions to approximate problems with rounded input. The paper includes detailed documentation of the results, including the sources of programs and the computers used.

H. H. Ku has served as editor of Volume I, Statistical Concepts and Procedures, now in press as part of the twelve-volume NBS Special Publication 300, Precision Measurement and Calibration. This 441-page volume collects, for easy reference, reprints of 40 selected articles, mainly by Bureau authors, on topics of special importance to metrologists. Each of six sections of the volume is preceded by a foreword pointing up the purpose for which the papers are included. Section titles are: (1) The measurement process, precision, systematic error, and accuracy; (2) Design of experiments in calibration; (3) Interlaboratory test; (4) Functional relationships; (5) Statistical treatment of measurement data; (6) Miscellaneous topics. A list of references and an extensive subject index are provided.

Ruth Varner prepared two FORTRAN V programs, KKV68-A and KKV68-B, for the analysis of four-way contingency tables using the information approach (Ku and Kullback 1968). The dimension limitations of the table and memory location requirements of the computer are as follows:

		<u>R</u>	<u>C</u>	<u>D</u>	<u>T</u>	<u>Memory location</u>
KKV68	A	≤ 9	19	9	4	48818
KKV68	B	≤ 7	9	4	3	21257

Publications:

- (1) The distribution of the sample variance from a two-point binomial population. D. Hogben. American Statistician, Vol. 22, No. 5, December 1968.
- (2) The mean deviation and range for $n = 3$. B. L. Joiner and J. R. Rosenblatt. American Statistician, Vol. 22, No. 4, October 1968.
- (3) Expressions of imprecision, systematic error, and uncertainty associated with a reported value. H. H. Ku. M and D Measurements and Data, Vol. 2, No. 4, July-August 1968, pp. 72-77.
- (4) Interaction in multidimensional contingency tables: An information theoretic approach. H. H. Ku. and S. Kullback (G.W.U.). J. Res. NBS - Math. Sciences, Vol. 72B, No. 3, July-Dec. 1968, pp. 159-199.

- (5) The median significance level and other small sample measures of test efficacy. B. L. Joiner. Submitted to a technical journal.
- (6) Precision Measurement and Calibration, Vol. 1 - Statistical Concepts and Procedures. H. H. Ku, ed. NBS Spec. Pub. 300, in press.
- (7) An application of minimum discrimination information estimation to a problem of Grizzle and Berkson on the test of "no interaction" hypothesis. H. H. Ku and S. Kullback (G.W.U.). Submitted to a technical journal.
- (8) Approximating discrete probability distributions. H. H. Ku and S. Kullback (G.W.U.). Submitted to a technical journal.

DEVELOPMENT OF "OMNITAB"

Task 20503-12-2050131
11550

Origin and Sponsor: NBS
Managers: David Hogben, Sally T. Peavy

Authorized 11/1/68

Objective: To develop and conduct research on, techniques of improving OMNITAB, for the purpose of adding efficiency to computing effort of scientists at NBS and elsewhere. This includes, in particular, development and addition of new instructions; action on user's comments; expanding the size of the worksheet; adding capability for handling tape, double precision and user's own sub-routines; revision and refinement of existing instructions.

Background: There is a great need for easy, fast and effective access to the computer by scientists who are not programmers. OMNITAB is a general-purpose computer program for data, statistical, and numerical analysis which is designed to meet this need. The value of OMNITAB has been demonstrated by its adoption at various government agencies, universities and private concerns, but further development is highly desirable. Standards ("tentative") of computing practice are needed and OMNITAB has attempted to set these standards.

The first version of OMNITAB was written specifically for the IBM 7090/7094, partly in machine language. The guiding philosophy was conceived and developed by Joseph Hilsenrath of the NBS Equation of State Section, and most of the programming was done by members of the NBS Computation Laboratory. The operation of the program is described in NBS Handbook 101, OMNITAB: A Computer Program for Statistical and Numerical Analysis (1966) prepared by Hilsenrath together with Guy G. Ziegler, Carla G. Messina, Philip J. Walsh, and Robert J. Herbold.

Major innovations and improvements were developed by Walter J. Gilbert, NBS Computer Services Division, who initiated the conversion of OMNITAB to ASA FORTRAN for the UNIVAC 1108. The present relatively machine-independent version is largely the work of Sally T. Peavy and Ruth N. Varner of the Statistical Engineering Laboratory.

Status: NEW. Primary responsibility for development and maintenance of the OMNITAB program now resides in the Statistical Engineering Laboratory. Earlier work on this task has been reported under Task 63-1259, Research in Probability and Mathematical Statistics.

Sally T. Peavy and Ruth N. Varner have finished rewriting OMNITAB in ASA standard FORTRAN to make it virtually machine independent. The NBS version was put in the absolute mode giving a substantial reduction in computing cost. Several instructions have been added to the program. To increase the size of the worksheet from 5,000 to 12,500 and to make possible the addition of more instructions, overlay and segmentation features have been developed. The number of instructions that can now be added is essentially unlimited.

J. M. Cameron, S. T. Peavy and R. N. Varner made major revisions in the instructions FIT and POLYFIT. Several additional statistics can now be stored automatically and the amount of storage can be controlled. Items added to the automatic printout include, in particular, the standard deviation of predicted values, analysis of variance and an assessment of computing accuracy. The last item represents an important step forward in computer practice.

Also, the set of matrix instructions was substantially improved and increased in number. In particular, a new instruction was written which gives about twenty properties of a matrix (or array) including the trace, sum of squares and orthogonality.

To keep users informed of latest developments, notices are now printed at the end of each OMNITAB computer run giving corrections, additions and improvements incorporated in the current version.

Thirty-five user's comments were carefully studied and many of them resulted in improvements which were incorporated in the OMNITAB program.

David Hogben wrote two subroutines which enable the printing of numbers in a form more readable than heretofore available. Unlike existing methods, these subroutines make it possible for the data to determine the "best" format. An initial version of a new instruction, RPRINT, with several options, incorporates these subroutines. In addition, a new instruction, TWOWAY, was constructed which easily gives an analysis of variance for a balanced or unbalanced two-way table, including Tukey's test for non-additivity in the balanced case.

4. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF

NBS RESEARCH AND TESTING

Task 13911-61-2050950/51-1

99500

Origin: NBS

Authorized 7/1/50

Managers: J. M. Cameron, H. H. Ku, J. R. Rosenblatt

Full task description: July - September 1950 issue, p. 60

Status: CONTINUED. This is a continuing project involving cooperation with other Bureau scientists on the statistical aspects of their investigations. These services vary from short (one-hour) sessions to extended collaborations involving several man-months; and are concerned primarily with statistical design of experiments, analysis and interpretation of data, and the use of computers in statistical analysis of data. Typical examples of the services performed are the following.

Janace A. Speckman is collaborating with Thomas E. Wells of the Resistance and Reactance Section on studies leading to identification of components of variance in the data generated by intercomparisons of standard resistors. Estimates of these variance components are needed for comparative evaluation of alternative experiment designs that have been proposed for use in routine intercomparisons.

Brian L. Joiner participated in an international Workshop on Mass Spectrometric Analysis of Solids, sponsored by the NBS Spectrochemical Analysis Section, on November 18 and 19, 1968. Topic 1 (of 5 discussed) was "The Accuracy and Precision Problem," and the discussion of this topic was opened by an informal talk on "The role of statistics," presented by Dr. Joiner. There were about 100 participants in the workshop, including 10 from 6 foreign countries.

Joan R. Rosenblatt is collaborating with H. S. Lew and E. O. Pfrang of the Structures Section (NBS Building Research Division) on development of methods for analysis of data from fatigue-test experiments on composite steel-and-concrete beams.

The NBS Office of Standard Reference Materials, and a number of sections in the Analytical Chemistry Division, are being assisted by Brian L. Joiner in the development of experimental designs and statistical analyses appropriate for evaluating the extent of any inhomogeneity in the material. Similar statistical problems arise in a number of areas where a nominal value and some uncertainty limits must be reported for the material, e.g. on the certificate that accompanies a Standard Reference Material issued by the Bureau.

Publications:

- (1) The statistical consultant in a scientific laboratory. J. M. Cameron. To appear in Technometrics.
- (2) Interlaboratory comparison of the potential heat test method. D. Gross (421.02) and M. G. Natrella. Submitted to a technical journal.
- (3) A survey of blemishes on government microfilm. C. S. McCamy (212.13), R. S. Wiley (Army Map Service), and J. A. Speckman. To appear in J. Res. NBS - Phys. and Chem., Jan.-Feb. 1969.

STATISTICAL SERVICES

Task 20503-40-2050132/58-346

11550

Origin and Sponsors: Various Agencies
Manager: J. M. Cameron, J. R. Rosenblatt
Full task description: January - March 1958 issue, p. 45

Authorized 3/31/58

Status: CONTINUED. This is a continuing project which involves providing, upon request, statistical services to other governmental agencies, universities, industrial organizations, and other non-governmental agencies. Approximately 30 such requests are handled per month ranging from short conferences to collaboration involving several days work.

A three-day Seminar on Precision and Accuracy in Measurement and Calibration was presented at the West Coast University in Orange, California, to 37 metrologists from industrial and government laboratories. The seminar had been presented four times in Washington (to a total of 136 participants) and its presentation on the West Coast was in response to requests for a presentation there in the interest of savings in travel expense. The presentation was sponsored by the National Conference of Standards Laboratories with the cooperation of the West Coast University which made facilities available at no charge.

The seminar dealt with the concept of a National Measurement System as developed by R. D. Huntcon and the role of the National Bureau of Standards in the system (presented by J. M. Cameron) and with the approach to measurement as a production process and the description of precision and accuracy in terms of parameters of that process (R. C. Raybold).

Lectures were presented on Statistics of Control of a Measurement Process (B. L. Joiner), Problems of Interlaboratory Measurement Agreement (J. M. Cameron), Propagation of Error and Representation of Precision and Accuracy (B. L. Joiner), and Design of Experiments (J. M. Cameron).

The participation of personnel from Bureau laboratories apart from the Statistical Engineering Laboratory added considerably to the value of the program. Woodward G. Eicke of the Electrochemistry section presented some new methods for intercomparing groups of voltage standards and maintaining surveillance over the measurement process involved. Robert C. Raybold of the Office of Measurement Services presented an account of measurement problems in mass and in photometry and steps in the evaluation of parameters of those processes. Paul E. Pontius of the Mass and Volume Section presented a paper on Measurement from a Systems Viewpoint and the concluding lecture of the seminar was his account of the Pilot Program in mass and the demonstration of compatible processes in 17 major laboratories.

Joan R. Rosenblatt has completed a three-year term of office as Program Secretary of the Institute of Mathematical Statistics.

5. SYSTEMS DYNAMICS SECTION
RESEARCH IN MATHEMATICAL PHYSICS
AND RELATED FIELDS

Task 20540-12-2050141/55-57
11540

Origin: NBS

Manager: H. J. Oser

Authorized 9/1/54

Full Task description: July-September 1954 issue, p. 27

Research has been conducted¹ in the area of multicomponent diffusion and heat transfer. Analytical solutions have been formulated¹ for the system of equations describing these phenomena that include i) the coupled flux expressions postulated in irreversible thermodynamics, ii) chemical reactions described by linear rate laws, and iii) time dependent diffusion and kinetic parameters. A stable numerical technique for the transport contribution has been developed that steps each equation independently through time according to the associated eigenvalue in the transport coefficient matrix.

The role of thermodynamic coupling in gas absorption systems has been investigated² by studying the characteristics of a physical model of a quinary system that include i) the coupled flux expressions, ii) heats of absorption released at the gas-liquid interface, and iii) an equilibrium relation between the surface concentrations and temperature.

Multicomponent friction coefficients have been completely defined on the basis of the entropy generation due to diffusion³. Symmetry in these coefficients has been shown to be both necessary and sufficient for the Onsager reciprocal relation to be valid. A method for calculating the coefficients from diffusion and thermodynamic data obtained in arbitrary composition scales and velocity frames has been derived.

Two methods of analyzing diaphragm cell data have been devised that do not impose additional mathematical or theoretical restrictions on the experimental technique for measuring multicomponent diffusion coefficients. The methods are not restricted by the number of species in the mixture and a general analysis of the experimental error has been developed.

A discussion of thermodynamic coupling was presented at the National Meeting of the American Institute of Chemical Engineers, Tampa, Florida, May 19-22, 1968.

In collaboration with Dr. R. deWit of the Lattice Defects and Microstructure Section (312.03), J. T. Fong continued to study the geometric foundation of a class of continuum theories of dislocations. In particular, a manuscript entitled "On existence of shifters in a general geometric manifold and their role in a class of continuum theory of dislocations" is being prepared for submittal to a technical journal.

In collaboration with L. Zapas of the Rheology Section (213.05), J. T. Fong began a study of the propagation of viscoelastic waves in a semi-infinite medium obeying the constitutive equations of a Bernstein-Kearsley-Zapas fluid. The possibility of deriving the elastic potential for a real material from a molecular theory of polymer solution with some measure of polymer interactions is being explored.

A paper by J. C. Smith of the Polymer Interface Section (311.07) and J. T. Fong entitled "On the coupling of longitudinal and transverse waves in a linear three-element viscoelastic string subjected to transverse impact" appears in the July-September issue of the NBS J. Research, Vol. 72B, No. 3 (1968).

R. S. Kraft directed his research towards the following three areas: (a) A technique due to Garabedian of investigating elliptic systems of partial differential equations by converting them into equations of hyperbolic type has been simplified and generalized. The generalized technique was used to obtain analyticity and reflectivity properties of first order elliptic systems. A paper containing these results is being submitted for publication. (b) A formal procedure for constructing Riemann functions by the method of generalized progressing wave expansions has been obtained. The convergence properties of the wave expansions are under investigation. (c) Numerical solution of the Nernst-Planck equations are being studied in collaboration with Professor David Goldman of Womens Medical College, Philadelphia, Pennsylvania.

W. L. Sadowski and Z. Ruthberg continued the analysis of Hermite polynomial series for 1) the two-stream initial condition and 2) a temperature rise in a plasma. A computer program was written to determine the number of terms needed in each of these cases to express the function in the form of a truncated series within a specified error.

An asymptotic expansion for the Hermite polynomials yielded closed forms for the expansion coefficients, and an estimate for the neglected terms as a function of N in each case. A spot check of the computer results indicated that the error bounds found analytically are consistent with the numerical results.

Routine consultations by other members of the staff involved the computation of special functions, the solution of a specified problem in the theory of helium lasers and continuation of the cooperation with the Rheology Section on a problem in propagation of acoustic waves.

Publications

(1) An Analysis of Nonisothermal Multicomponent Diffusion with Chemical Reaction. George B. DeLancey (with S. H. Chiang, University of Pittsburgh). Submitted to Chem. Eng. Sci.

(2) Analysis of Multicomponent Diaphragm Cell Data. George B. DeLancey. Submitted to J. Phys. Chem.

(3) Friction Coefficients in Multicomponent Systems. George B. DeLancey. Submitted to Ind. Eng. Chem. Fundam.

(4) On the Coupling of Longitudinal and Transverse Waves in a Linear Three-Element Viscoelastic String Subjected to Transverse Impact. J. C. Smith (311.07, NBS) and J. T. Fong. July-Sept. 1968, NBS J. Res., Vol. 72B, No. 3.

5. SYSTEMS DYNAMICS SECTION

RESEARCH IN MATHEMATICAL PHYSICS

AND RELATED FIELDS

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11540

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Manager: H. J. Oser

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PLASMA PHYSICS

Task 20504-12-2050140/59-442

11540

Origin: NBS

Authorized 10/13/61

Manager: H. J. Oser

Full task description: October-December 1961 issue, p. 12

The work on the combinatorial approach to the solution of nonlinear partial differential equations was continued. A manuscript has been completed on general combinatorial properties of binomial nonlinear differential operators. Determination of the coefficients with the aid of the LISP language (a list processor) indicated that this language is too cumbersome to handle a general combinatorial problem of this kind. An analytic approach permits calculating the combinatorial weights starting from a much higher level than was possible with a straightforward repetitive LISP program.

The combinatorial method was specifically applied to the nonlinear Vlasov equation and the problem is now in process of being programmed for the computer. The advantage of the special method is that only a few physically significant vectors are calculated as functions of time in the eigenfunction solution.

A manuscript has been completed on the possibility of selective excitation of harmonics by counter-streaming beams of electrons travelling through a plasma. The necessary collimation in velocity space can be obtained by a truncated set of Hermite polynomials. The coefficients of this expansion were obtained in closed form by the use of transformation properties of Hermite polynomials. Computer solutions of the expansion are in good agreement with the analytical result. The analytically obtained truncation error is a realistic upper bound to the one obtained numerically. The possibility of continuous transformation of the base function in velocity space was examined and the transformation coefficients derived. The results show, however, that truncation errors in the expansion would not be significantly reduced for the values of parameters used in the study.

BIOMEDICAL IMAGE AND LANGUAGE PATTERN PROCESSING

Task 20500-12-2050404

51560

Origin: NBS
Sponsor: National Institutes of Health
Manager: Russell A. Kirsch
Full task description: January-June 1964 issue, p. 19

Authorized 1/21/64

Status: CONTINUED. A general procedure was developed on the Q32 computer for automatic segmentation of quantized images. This procedure yields as output a partial ordering (induced by the inclusion relation) of all morphological objects in an image. The elements of this partial ordering serve as inputs for a higher level syntactic analysis. This procedure was applied to a cortex section photograph and to a sequence of optical serial section photographs of a single neuron. A serial section reconstruction was also made for the single neuron. Considerable expansion of the LISPAX image processing system took place during this period. The programmable microscope stage and focus control was connected to the LINC-PDP-8 computer and used in conjunction with a spectrophotometer to control the stage and focus while doing photometric measurements.

SCANNING MICROSCOPE PROCESSING

Task 20500-2050408

51560

Origin: NBS
Sponsor: National Institutes of Health
Manager: Russell A. Kirsch

Authorized 6/18/68

Objective: To study ways of using a programmable scanning microscope with a general purpose digital computer. The computer is to have facilities for image processing and for scanner control. The use of the scanner-computer complex is in automating auto-radiographic analysis of tritium-labelled blood cells.

Status. NEW. Five kinds of leucocytes were scanned from photographs made at several optical wavelengths. These quantized images were analyzed on the Q32 computer. The optical frequency selection was made on the programmable microscope using computer control of a monochromator light source. Some modifications were made to the electronics of the microscope scanner controls.

CHEMICAL BIOLOGICAL INFORMATION PROCESSING

Task 20500-2050410

51560

Origin: NBS
Sponsor: National Institutes of Health
Manager: Russell A. Kirsch

Authorized 6/27/68

Objective: To consult on the use of remote computers in communicating pharmacological information regarding chemical structure and biological activity among a group of collaborating pharmacologists.

Status: NEW. Advisory work continued on Chemical Biological Information Handling Techniques. The use of the Q32 facility was provided to the sponsor for file handling experiments.

MATHEMATICAL TABLES

Task 20500-40-2050121/57-216

11550

Origin and Sponsor: NBS

Manager: I. A. Stegun

Full task description: July-December 1964 issue, p. 4.

Status. CONTINUED. The updating of bibliographic material and correction lists continued for all published volumes. A seventh printing of AMS 55 - the Handbook of Mathematical Functions was released in May 1968.

Assistance at the rate of ten consultations per month to NBS, other government agencies, industry and universities was offered in the evaluation or modification of computing techniques, identification of functions and their tabulation.

In response to a request for obtaining eigenvalues-eigenvectors of a non-symmetric matrix a FORTRAN program published in the Communications of the Association for Computing Machinery was explored. It was found satisfactory for the given cases but failed for other test cases. I. A. Stegun and R. Zucker have continued updating a file of test matrices. Routines recommended for general library usage must supply satisfactory results for the present file.

I. A. Stegun and R. Zucker have started on a more systematic evaluation of methods of computing special functions. Concentration at present is centered on accuracy, and feasible range of power series and asymptotic expansions.

Lectures and Technical Meetings

Mathematical Statistics Seminar

HOADLEY, A. B. (Bell Telephone Laboratories) A Bayesian approach to the inverse regression problem. September 30.

Mathematics Division Expository Seminar

EDMONDS, J. Optimum branchings. July 24.
ROSENBLATT, J. R. Distribution-free statistical tests for comparative experiments. August 14.
HABER, S. Multiple integration using classical and Monte Carlo approaches. September 25.
CLATWORTHY, W. H. (SUNY Buffalo and Univ. of No. Carolina) Isomorphisms and non-isomorphisms in balanced incomplete block designs. November 13.

Systems Dynamics Section Seminar

BERMAN, Mones (National Institutes of Health) SAAM program and its uses in the study of metabolic and kinetic systems. October 9.
PLOTKIN, A. (University of Maryland) A numerical solution for the laminar wake behind a finite flat plate. November 13.
HEARON, John Z. (National Institutes of Health) Generalized inverses and linear systems of equations. December 11.

Colloquia - Special Topics in Mathematics

OLVER, F. W. J. Asymptotic Expansions.

In-Hours Courses, Fall Semester

GOLDBERG, Karl Introduction to Abstract Algebra
HALL, William G. Analysis II
NATRELLA, Mary G. Statistics of Measurement
NEWMAN, Morris Selected Topics in Group Theory and Group Representations
VARNER, Ruth N. FORTRAN Programming for Beginners

Papers and Invited Talks

Presented by Members of the Staff

At Meetings of Outside Organizations

CAMERON, J. M. The statistical consultant in a scientific laboratory. American Statistical Assoc., Pittsburgh, August 22.
EDMONDS, J. R. Matroid intersection. Cornell University, November 26.
EDMONDS, J. R. Matroid intersection. University of Michigan, December 9.

GOLDMAN, A. J. Minimaximax selection of incompletely specified probability distributions. Graduate School of Business, Information, Decision and Control Workshop. University of Chicago, Chicago, Ill., October 24.

The minimax transportation problem. Operations Research Society of America. Philadelphia, Pennsylvania, November 6.

GOLDMAN, A. J. Minimax error selection of a discrete univariate distribution.
MEYERS, P. R. Operations Research of America. Philadelphia, Pennsylvania, November 6.

HOGBEN, D. Approximations to control chart constants for electronic computers. Fourteenth Conference on the Design of Experiments in Army Research, Development and Testing, Edgewood Arsenal, Md., October 23.

JOINER, B. L. Some properties of the range in samples from Tukey's lambda-distributions. American Statistical Association, Pittsburgh, August 22.

JOINER, B. L. OMNITAB in 3 lectures:
1. Introduction, Sept. 16.
2. Intermediate, Sept. 26.
3. Advanced, October 4.

The role of statistics. Workshop on Mass Spectrometric Analysis of Solids, NBS, November 18.

KU, H. H. Minimum discrimination information estimation of cell probabilities in a contingency table - applications. Institute of Mathematical Statistics, Madison, August 29.

Analysis of multidimensional contingency tables. Fourteenth Conference on the Design of Experiments in Army Research, Development and Testing, Edgewood Arsenal, Md., October 23.

WAMPLER, R. H. An evaluation of linear least squares computer codes. Fourteenth Conference on the Design of Experiments in Army Research, Development and Testing, Edgewood Arsenal, Md., October 23.

PUBLICATIONS ACTIVITIES

1.0 PUBLICATIONS THAT APPEARED DURING THIS PERIOD

1.1 Mathematical Tables

Handbook of Mathematical Functions, AMS 55, Seventh Printing (with corrections), May 1968.

1.2 Monograph

Matrix representations of groups. M. Newman. AMS 60, July 1968.

1.3 Technical Papers

On the coupling of longitudinal and transverse waves in a linear three-element viscoelastic string subjected to transverse impact. J. C. Smith (Polymer interface Section, 311.07) and J. T. Fong. J. of Research NBS, 72B, No. 3 (1968).

Minimax error selection of a discrete univariate distribution with prescribed componentwise ranking. A. J. Goldman. J. of Research NBS, 72B, pp. 273-277 (1968).

Minimax error selection of a discrete univariate distribution with prescribed componentwise bounds. A. J. Goldman and P. R. Meyers. J. of Research NBS, 72B, pp. 263-271 (1968).

A combination of classical and Monte Carlo methods for evaluating multiple integrals. S. Haber. Bull. A.M.S. 74, 683-686 (1968).

The distribution of the sample variance from a two-point binomial population. D. Hogben. American Statistician, Vol. 22, No. 5, December 1968.

The mean deviation and range for $n = 3$. B. L. Joiner and J. R. Rosenblatt. American Statistician, Vol. 22, No. 4, October 1968.

Citation searching and bibliographic coupling with remote on-line computer access. F. L. Alt (AIP) and R. A. Kirsch. J. of Research NBS, 72B, pp. 61-78 (1968).

Expressions of imprecision, systematic error, and uncertainty associated with a reported value. H. H. Ku. M and D Measurements and Data, Vol. 2, No. 4, July-August 1968, pp. 72-77.

Interaction in multidimensional contingency tables: An information theoretic approach. H. H. Ku and S. Kullback (G.W.U.). J. of Research NBS, 72B, No. 3, pp. 159-199, (December 1968).

Maximal normal subgroups of the modular group. M. Newman. Proc. Amer. Math. Soc. 19, 1138-1144 (1968).

2.0 MANUSCRIPTS IN THE PROCESS OF PUBLICATION

2.3 Technical Papers

The statistical consultant in a scientific laboratory. J. M. Cameron. To appear in Technometrics.

Friction coefficients in multicomponent systems. George B. Delancey. Submitted to Industrial and Engineering Chemistry, Fundamentals.

Role of coupling in nonisothermal diffusion: Gas absorption. George B. Delancey and S. H. Chiang (Univ. of Pittsburgh.) To be submitted to a technical journal.

An analysis of nonisothermal multicomponent diffusion with chemical reaction. George B. Delancey and S. H. Chiang (Univ. of Pittsburgh). Submitted to a technical journal.

Bottleneck extrema. J. Edmonds and D. R. Fulkerson (RAND Corporation). To appear in J. Combinatorial Theory.

Stochastic quadrature formulas. S. Haber. To appear in Math. Comp.

On a theorem of Piatetsky-Shapiro and evaluation of multiple integrals. S. Haber and C. F. Osgood. To appear in Math. Comp.

On the sum $\sum \langle \alpha_n \rangle^{-t}$. S. Haber and C. F. Osgood. Submitted to a technical journal.

Optimal networks connecting N points in the plane. W. A. Horn. To appear in Proceedings of Fourth International Symposium on Traffic Flow (Karlsruhe, Germany; June, 1968).

Minimum-length covering by intersecting intervals. W. A. Horn. To appear in J. of Research NBS, 73B (1969).

The median significance level and other small sample measures of test efficacy. B. L. Joiner. Submitted to a technical journal.

An unusual mixed initial boundary problem in two independent variables. Richard Kraft. Submitted to J. Math. Analysis and Applications, Academic Press, Inc.

Precision Measurement and Calibration, Vol. 1 - Statistical Concepts and Procedures. H. H. Ku, ed. NBS Spec. Pub. 300, in press.

An application of minimum discrimination information estimation to a problem of Grizzle and Berkson on the test of "no interaction" hypothesis. H. H. Ku and S. Kullback (G.W.U.). Submitted to a technical journal.

Approximating discrete probability distributions. H. H. Ku and S. Kullback (G.W.U.). Submitted to a technical journal.

On the multipliers of the Dedekind modular function. J. Lehner. To appear in J. of Research NBS.

Interlaboratory comparison of the potential heat test method. D. Gross (421.02) and M. G. Natrella. Submitted to a technical journal.

Some results on unitary matrix groups. M. Newman and M. Marcus. To appear in J. of Linear Algebra.

Subgroups of $SL(t, z)$. M. Newman. To appear in J. of Research NBS.

Error bounds for the Laplace approximation for definite integrals. F. W. J. Olver. To appear in J. of Approx. Theory.

A method in diophantine approximation IV. C. F. Osgood. To appear in Acta Mathematica (Warsaw, Poland).

A survey of blemishes on government microfilm. C. S. McCamy (212.13), R. S. Wiley (Army Map Service), and J. A. Speckman. To appear in J. of Research NBS - Phys. and Chem., Jan.-Feb. 1969.

2.4 Notes

Analysis of multicomponent diaphragm cell data. George B. Delancey. Submitted to Journal of Physical Chemistry.

The minimax transportation problem. A. J. Goldman. To appear in Transportation Science (Letters to Editor).

Fractional container-loads and topological groups. A. J. Goldman. To appear in Operations Research.

2.5 Books

Boundary Layers of Flow and Temperature, by Alfred Walz, Translated by Hans J. Oser. M.I.T. Press, (in press).

NBS TECHNICAL PUBLICATIONS

PERIODICALS

JOURNAL OF RESEARCH reports National Bureau of Standards research and development in physics, mathematics, chemistry, and engineering. Comprehensive scientific papers give complete details of the work, including laboratory data, experimental procedures, and theoretical and mathematical analyses. Illustrated with photographs, drawings, and charts.

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Federal Information Processing Standards Publications. This series is the official publication within the Federal Government for information on standards adopted and promulgated under the Public Law 89-306, and Bureau of the Budget Circular A-86 entitled, Standardization of Data Elements and Codes in Data Systems.

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