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MISCELLANEOUS PUBLICATION OF THE BUREAU OF STANDARDS No. 56
SHEET 1
TEXT

TABLES AND GRAPHS FOR FACILITATING THE COMPUTATION OF SPECTRAL ENERGY DISTRIBUTION BY PLANCK'S FORMULA

CONSISTING OF SEVEN SHEETS (FIVE CHARTS) AS FOLLOWS: TEXT (SHEET I). GRAPHS: 1,000 TO 5,000° K. (SHEETS 2, 3, 4; CHARTS 1, 2, 3); 5,000 TO 8,000° K. (SHEET 5; CHART 4); 8,000 TO 24,000° K. (SHEET 6; CHART 5). TABLES (SHEET 7).

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ABSTRACT

Very frequently it has been necessary to compute the energy distribution of a "black body" in the visible spectral region, at some particular temperature, from Planck's formula:

$$E_\lambda = \frac{C_1\lambda^{-5}}{e^{\frac{C_2}{\lambda\theta}}-1}$$

Such computation consumes much time and labor, in consequence, the following short cuts in the way of tables and graphs have been devised.

Using the equation in the form

$$\frac{E_\lambda}{E_m} = \left(\frac{A}{\lambda\theta}\right)^5 \left(e^{\frac{C_2}{A}}-1\right)\left(e^{\frac{C_2}{\lambda\theta}}-1\right)^{-1}$$

a table has been made with values of $\lambda\theta$ and corresponding values of $\frac{E_\lambda}{E_m}$. For any given temperature the wave lengths in the visible, and partly in the ultra-violet and infra-red regions, corresponding to these values of relative energy, can readily be obtained from the products $\lambda\theta$. Hence the spectral energy curve can be plotted at once.

Two sets of tables have also been made up, each for 38 temperatures, in which the relative energy values computed directly from Planck's formula are recorded for the wave lengths 400 to 720 mμ at intervals of 10 millimicrons—one set having the energy value equal to 100 at 590 mμ, the other equal to 100 at 560 mμ. The temperature range is from 1,000 to 28,000° K. (accuracy, within 0.15 per cent).

From one of these tables a series of isochromatic curves has been constructed. The relative energy has been plotted (accuracy, within 0.33 per cent) against the absolute temperature, and one complete curve drawn for each of the 33 wave lengths. From these graphs, the energy distribution for the visible region can be obtained in a few minutes for any temperature whatever in the range above specified.

I. INTRODUCTION

In connection with the work of the color laboratory, very frequent use is made of the spectral energy distribution at various temperatures of "black body" radiation, as represented by Planck's formula:

$$E_\lambda = \frac{C_1\lambda^{-5}}{e^{\frac{C_2}{\lambda\theta}}-1} \qquad (1)$$

E_λ is the radiant energy for any wave length λ, at any temperature θ (degrees absolute); C_1 and C_2 are constants, and e is the base of the natural logarithms. As a rule it is desired to know the relative values of E at a given temperature for a range of 320 mμ throughout the visible spectrum; that is, 400 to 720 mμ, at intervals of 10 mμ. Moreover, these values must conform to a scale such that at some particular wave length E will always be the same for all temperatures, for instance, equal to 100 at wave length 560 mμ.[2] When computed directly from the formula, this proves to be a long and tedious process; therefore, certain short cuts have been devised. Tables have been made and curves plotted which greatly facilitate this work. It was thought worth while to publish these for the benefit of others who make similar use of this formula. The following reference tables have been found of great assistance: F. W. Newman and J. W. D. Glaisher, Tables of Exponential Functions, C. E. VanOrstrand, Tables of the Exponential Function.[3]

II. EXPLANATION OF TABLES

1. TABLE 1

Instead of using Planck's equation in the form given above, it was combined[4] with Wien's displacement law connecting the temperature with the wave length at which the maximum energy occurs; that is,

$$\lambda_m\theta = A, \text{ a constant} \qquad (2)$$

This maximum energy is given by

$$E_m = \frac{C_1\lambda_m^{-5}}{e^{\frac{C_2}{\lambda_m\theta}}-1} \qquad (3)$$

Substituting from (2) $\lambda_m = \frac{A}{\theta}$, equation (3) takes the form

$$E_m = \frac{C_1\left(\frac{A}{\theta}\right)^{-5}}{e^{\frac{C_2}{A}}-1} \qquad (4)$$

Dividing (1) by (4) we obtain

$$\frac{E_\lambda}{E_m} = \left(\frac{A}{\lambda\theta}\right)^5 \left(e^{\frac{C_2}{A}}-1\right)\left(e^{\frac{C_2}{\lambda\theta}}-1\right)^{-1} \qquad (5)$$

An inspection of this equation will show that the right-hand side is a function of $\lambda\theta$ alone. The values of the constants used in the computations are:

$A = 2{,}890$ micron degrees
$C_2 = 14.350$ micron degrees.[5]

Table 1 has been arranged with a wide range of values of $\lambda\theta$ and corresponding values of $\frac{E_\lambda}{E_m}$, computed to four significant figures.

Use of Table 1.—From the products $\lambda\theta$ the wave lengths can be readily computed for any temperature θ. (These wave lengths are in microns, and must be multiplied by 1,000 to be expressed in millimicrons.) The values of relative energy at these wave lengths are already known from the table; hence the distribution curve for this temperature can at once be plotted. The next step is to transform the values $\frac{E_\lambda}{E_m}$ so that at 560 mμ the energy will be equal to 100. This is accomplished by reading from the curve the energy coordinate for 560 mμ, taking its reciprocal (times 100) and multiplying the other coordinates by this factor. The spectral energy curve is thus well defined and will pass through the required point at 560 mμ.

It is frequently necessary to know the energy at every 10 mμ beginning at 400 mμ. These values, while not obtainable directly from the $\lambda\theta$ computations, are readily obtained from the distribution curve mentioned above, and can be read with accuracy (about 0.2 to 0.6 per cent), provided the scale has been suitably chosen.

In order, therefore, to obtain energy values for any temperature θ at every 10 mμ, with a given value at 560 mμ, the use of Table 1 involves four distinct steps: (1) Computation of λ from the products $\lambda\theta$; (2) plotting of λ against $\frac{E_\lambda}{E_m}$ given in the table; (3) reading from this curve the values of $\frac{E_\lambda}{E_m}$ at intervals of 10 mμ; and (4) multiplying these values by the reciprocal of the value at 560 mμ, and the product by 100.

This process of computation, while more easily and somewhat more rapidly performed than direct calculation from Planck's formula, still consumes a considerable amount of time. Moreover, without extreme care in the plotting of the curves, the accuracy obtainable may not be greater than 0.6 per cent. The use of Table 1 can not be recommended for great speed or great accuracy, under the conditions imposed in the introduction; but it is very useful to give readily the shape of the energy distribution curve and the relative values of the energy when the above conditions need not be met. Moreover, with the use of Table 1, the curve can be extended into the ultra-violet and infra-red regions of the spectrum.

2. TABLES 2A AND 2B

These tables have been prepared directly from Wien's and Planck's formulas, with the exception only of the values for 1,000° K., which were calculated directly from Table 1. Wien's formula was used for temperatures 1,100 to 2,000° K., Planck's formula for 2,200 to 28,000° K. Since the energy values given in these tables are all relative to the value chosen for wave length 560 (or 590) mμ the constant C_1 does not enter into consideration. C_2 is taken equal to 14,350 micron degrees. The values in these tables are considered correct to within 0.15 per cent.[6]

The temperatures for which the energy values are given in these tables are as follows:

1,000 to 2,000, by steps of 100°.
2,000 to 3,000, by steps of 200°.
3,000 to 4,000, by steps of 250°.
4,000 to 7,000, by steps of 500°.
7,000 to 10,000, by steps of 1,000°.
10,000 to 28,000, by steps of 2,000°.

In Table 2A the energy has been made equal to 100 at 590 mμ for each temperature; in Table 2B the value 100 has been chosen at 560 mμ.

III. EXPLANATION OF GRAPHS

The plotting of the graphs was suggested by the existence of Tables 2A and 2B. The energy distribution is often required to be known for some temperature intermediate to those given, and a series of curves based on these tables should be very suitable to yield these data. This led to a series of isochromatic curves, one for each wave length at intervals of 10 mμ, from 400 to 720 mμ. E_λ can be read directly for any temperature desired within the range 1,000 to 28,000° K.

The plotting of these curves met with some difficulties for the following reasons: (1) It was desired to obtain an accuracy of plotting and of reading to 0.33 per cent. (2) The energy values vary from 0.007 (approximately) to 3,000; in other words, the largest value is about 400,000 times the smallest value. This necessitates several changes of scale for the energy coordinates.[7] (3) The isochromatics crowd very closely together between temperatures 4,000 to 7,000°.

In view of these considerations, it has been found necessary to plot the curves in sections on 40 by 50 cm sheets. Only that series is given for which E_λ has the value of 100 at 560 mμ. The other scale of values (100 at 590 mμ) can readily be obtained from these by the use of a suitable factor. The accuracy mentioned above (0.33 per cent) should apply except for E_λ less than 3.5 and possibly also on very steep parts or near the line $\theta = 1{,}000°$ K. The temperature can be estimated to within 1° for the range 1,000 to 8,000°, and to within 5° for the range 8,000 to 28,000°. The curves have been tested by comparing values furnished by the graphs (for given temperatures other than those recorded in Table 2B) with the values computed for these temperatures.

IV. PRACTICAL APPLICATION

It has been shown[8] that a number of the ordinary illuminants have a spectral energy distribution that approximates closely that of a Planckian radiator. This property is valuable in that the unknown spectral energy distribution of an illuminant can very readily be determined if it can be matched in color with a so-called "black body." The temperature to which the "black body" is brought in order to produce color match with the illuminant is defined as the color temperature of the latter. Since this quantity is not difficult to determine in laboratories having suitable equipment, the energy distribution in the visible region of such illuminant can at once be recorded by reading from the charts the energy values for the given temperature.

V. SUMMARY

If it is required to know the spectral distribution of a Planckian radiator at any temperature θ throughout the visible and into the ultra-violet and infra-red regions, Table 1 affords a ready means of computing it with very little labor. But if the distribution curve is required to pass through a certain point (for instance, 100 at 560 mμ), and if the values of energy are desired for 10 mμ intervals, further computation is necessary. Tables 2A and 2B have been computed giving these values for 38 temperatures in the interval 1,000 to 28,000° K.

In order to obtain directly the spectral distribution for any temperature whatever in this interval, at wave lengths 10 mμ apart, and having the value 100 at 560 mμ, a series of isochromatic curves has been constructed. From these curves, which are plotted to a large scale, the energy values can be read to an accuracy usually within 0.33 per cent, the temperature values within 0.1 per cent.

VI. APPENDIX[9]

The question may arise as to what variation will be caused in the relative values of E_λ by the adoption of a value of C_2 different from 14,350; for example, Coblentz's most recent value[10] for this constant is 14,320; the International Critical Tables has adopted 14,330; and further research may eventually cause further, though probably very small, revision of these latest values.

An inspection of the equation, $E_\lambda = \frac{C_1\lambda^{-5}}{e^{\frac{C_2}{\lambda\theta}}-1}$ shows that the effect of a variation of C_2 with θ constant is the same as, but opposite in sense to, a variation of θ with C_2 constant.

So far as the graphs are concerned, in which values of E_λ are plotted against values of θ for the various wave lengths, we have, therefore, a ready means of obtaining true values of E_λ (within the precision already noted in the main part of the paper) for any value of C_2 which it may be desired to use. The method is simply this: Compute the percentage difference between 14,350 and the value to be used. Call this percentage difference p. Multiply the temperature by $1 \pm p/100$ and read values of E_λ from the graphs at this corrected temperature. If C_2 is decreased, θ must be increased, and vice versa.

While true values of E_λ may thus be obtained for any value of C_2, and as readily as for the value 14,350, the difference in E_λ obtained by using 14,320 or 14,330 is so small as to be negligible for most work for which these tables and graphs will be used. In the first place, the difference in temperature necessary to make the correction on the graphs is about 0.2 per cent, the order of magnitude of differences in color temperature determinations by experienced observers. In the second place, the differences in values of E_λ relative to that at 560 mμ caused by this 0.2 per cent change in C_2 is but a few tenths of 1 per cent, except at the extreme wave lengths for temperatures below about 1,500°, where they may become greater than 1 per cent. Spectral energy measurements can scarcely be guaranteed within 1 per cent, while the discrepancies between spectral energy measurements and values of energy computed for the corresponding color temperature may be several per cent in the end regions of the visible spectrum.

While, therefore, true values of E_λ may be readily obtained from the graphs for any value of C_2, it is probable that for most work the error involved in using values of E_λ based on $C_2 = 14{,}350$—that is, those given by the tables or read from the graphs without the temperature correction—will be negligible.

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[1] The authors wish to acknowledge their indebtedness to Harry J. Keegan for his able assistance with the computations.

[2] Cf. Lockiesh, The equality-point in spectral energy and luminosity distribution curves of illuminants, J. Frank. Inst., 183, pp. 633–634; 1917.

[3] Published in the Jour. Wash. Acad. Sci., 3, No. 11, June 19, 1913.

[4] This arrangement was suggested by I. G. Priest.

[5] Coblentz, B. S. Sci. Paper No. 348, p. 470, 1919. See also appendix.

[6] Cf. Table given in Jour. Op. Soc. Am., 4, p. 331, 1920; W. E. Forsythe. This table is convertible into Table 2A or 2B by the use of suitable factors. Some of the values, however, have been found to be slightly in error, and it is understood are in process of revision.

[7] The logarithmic scale was discarded as unsuitable for certain regions. Nomograms proved rather cumbersome because of the form the equation takes for transforming the energy values to 100 at 560 mμ, etc.; that is, $\frac{E_\lambda}{E_m} = f(\theta)\,\phi(\lambda\theta)$.

[8] Hyde and Forsythe, The quality of light from an illuminant as indicated by its color temperature, J. Frank. Inst., 183, pp. 353–354; 1917; Priest, Measurement of the Color Temperature of the More Efficient Artificial Light Sources by the Method of Rotatory Dispersion, B. S. Sci. Paper No. 443, 1922.

[9] The authors wish to acknowledge the suggestions of Dr. K. S. Gibson in connection with the appendix.

[10] B. S. Sci. Paper No. 406, December, 1920.

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SHEET 2
CHART 1

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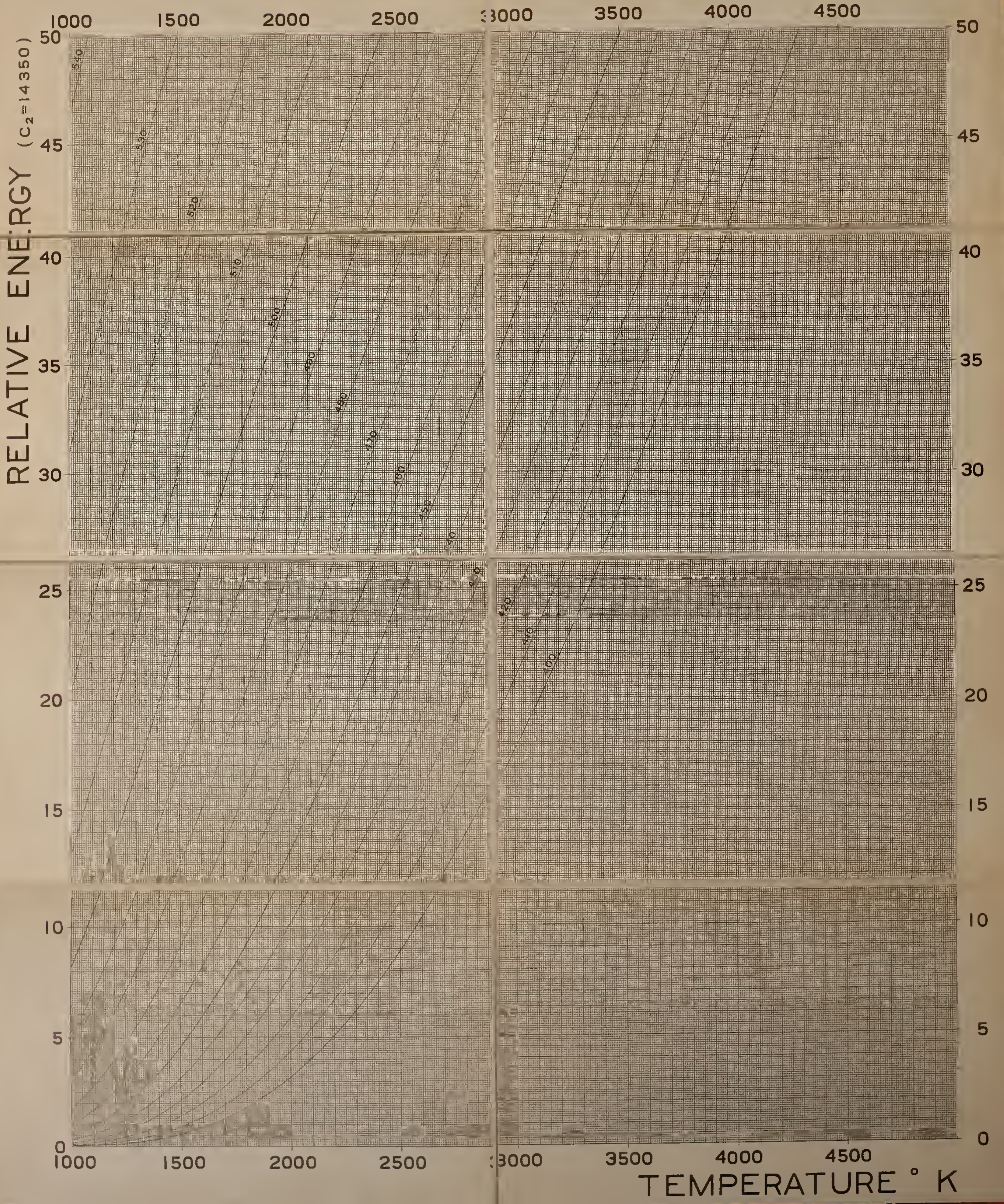
ENERGY DISTRIBUTION BY PLANCK'S FORMULA

100 AT WAVE LENGTH 560 mμ



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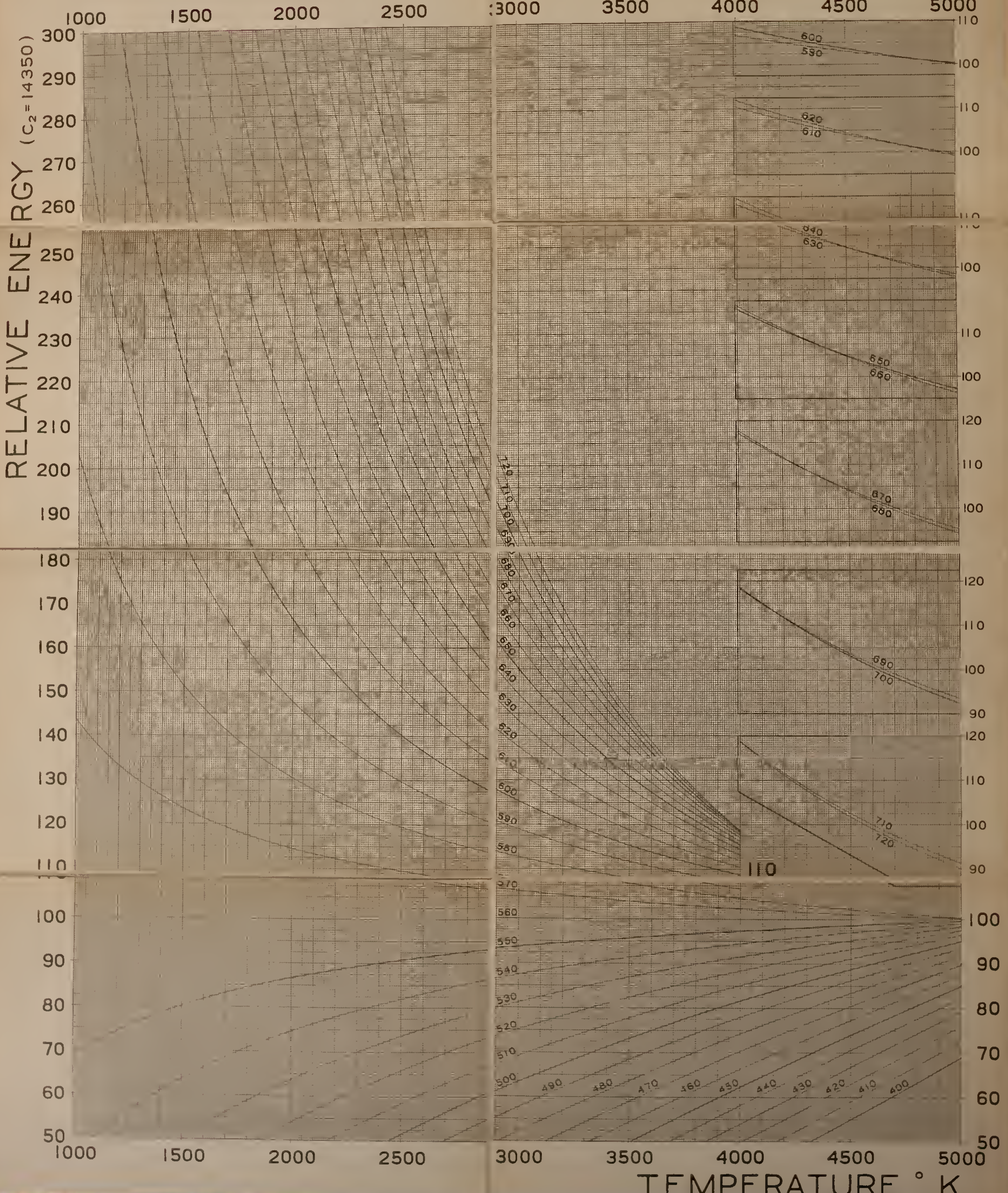
ENERGY DISTRIBUTION BY PLANCK'S FORMULA

100 AT WAVE LENGTH 560 mμ



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SHEET 4
CHART 3

DEPARTMENT OF COMMERCE
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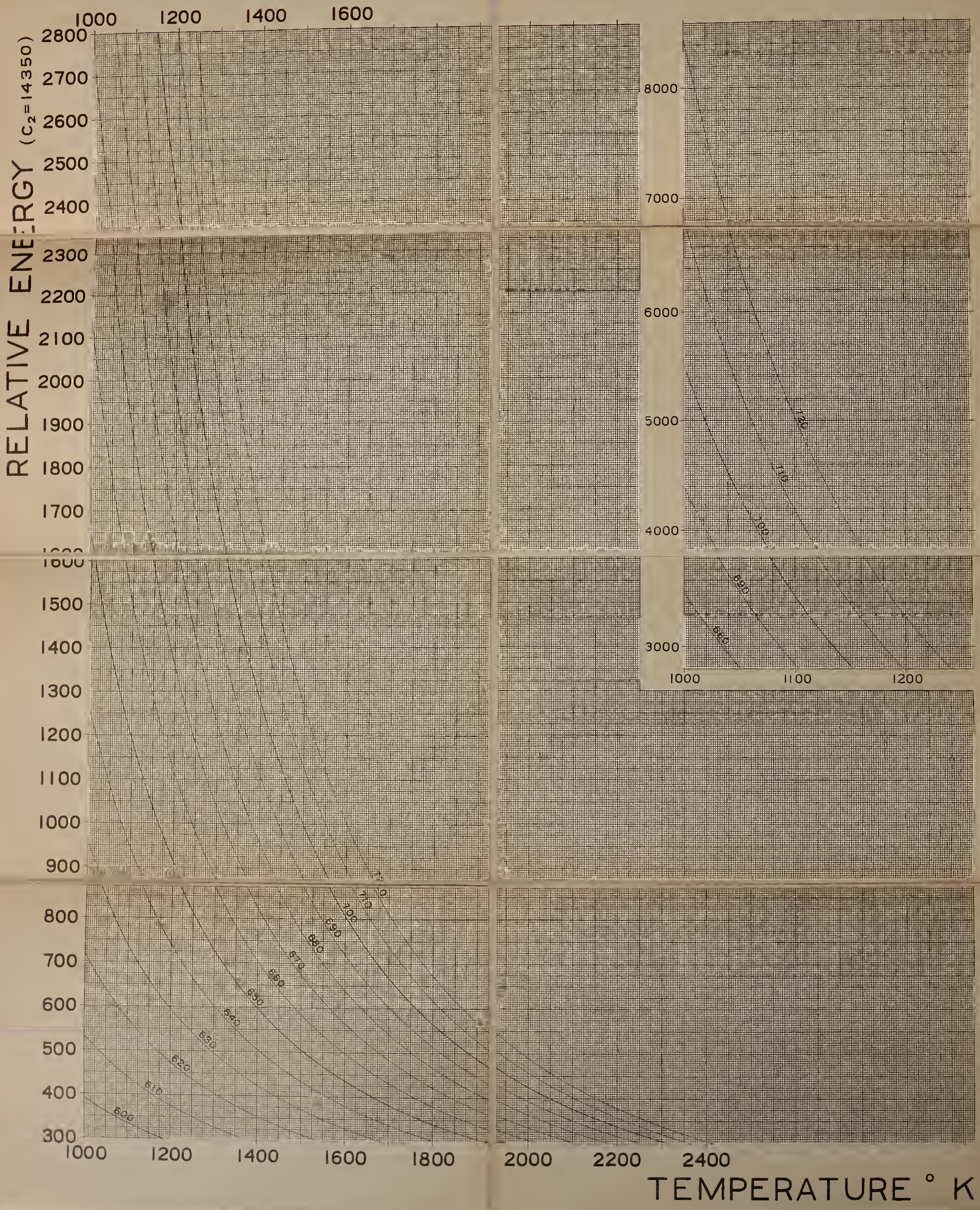
ENERGY DISTRIBUTION BY PLANCK'S FORMULA

100 AT WAVE LENGTH 560 mμ



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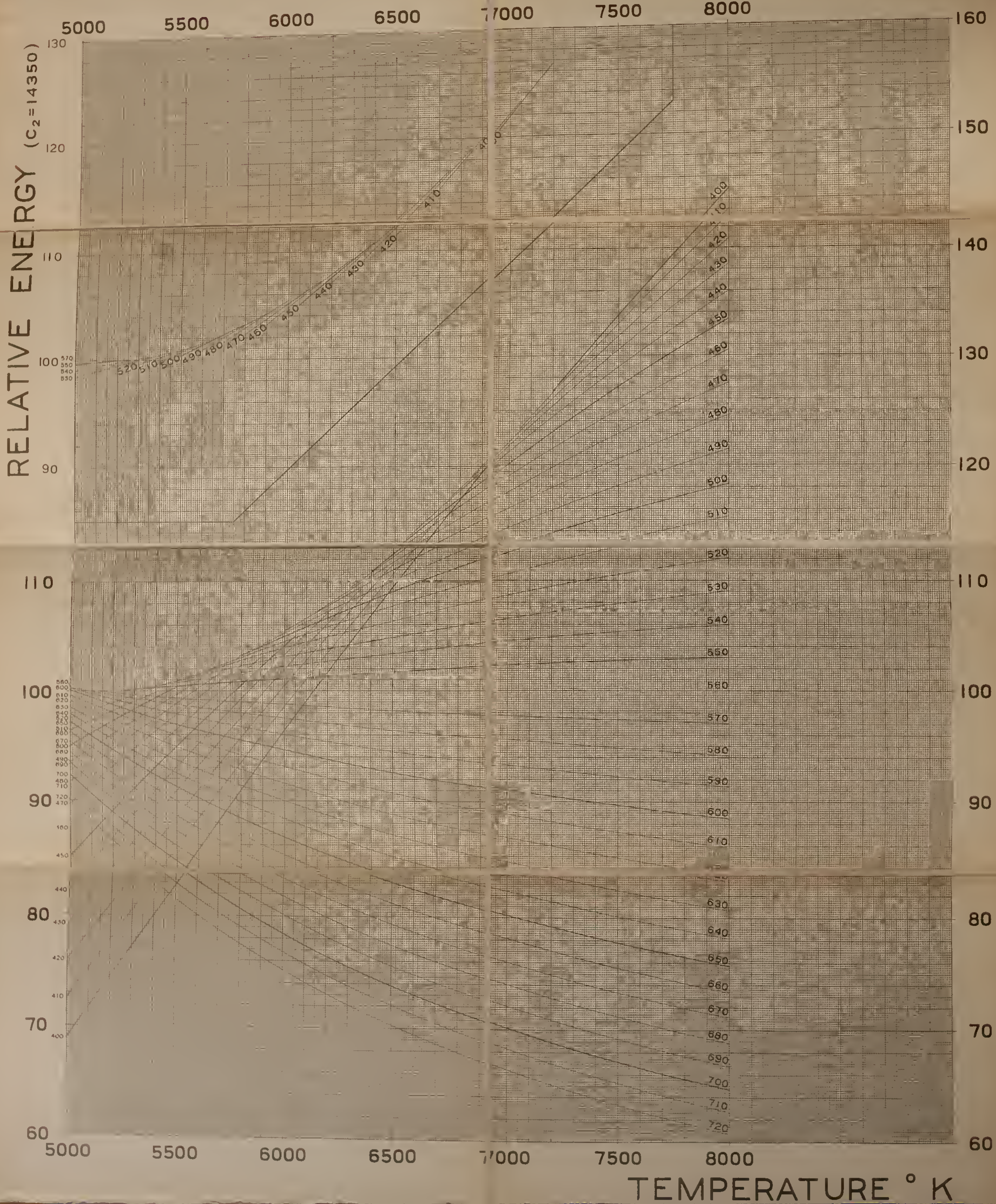
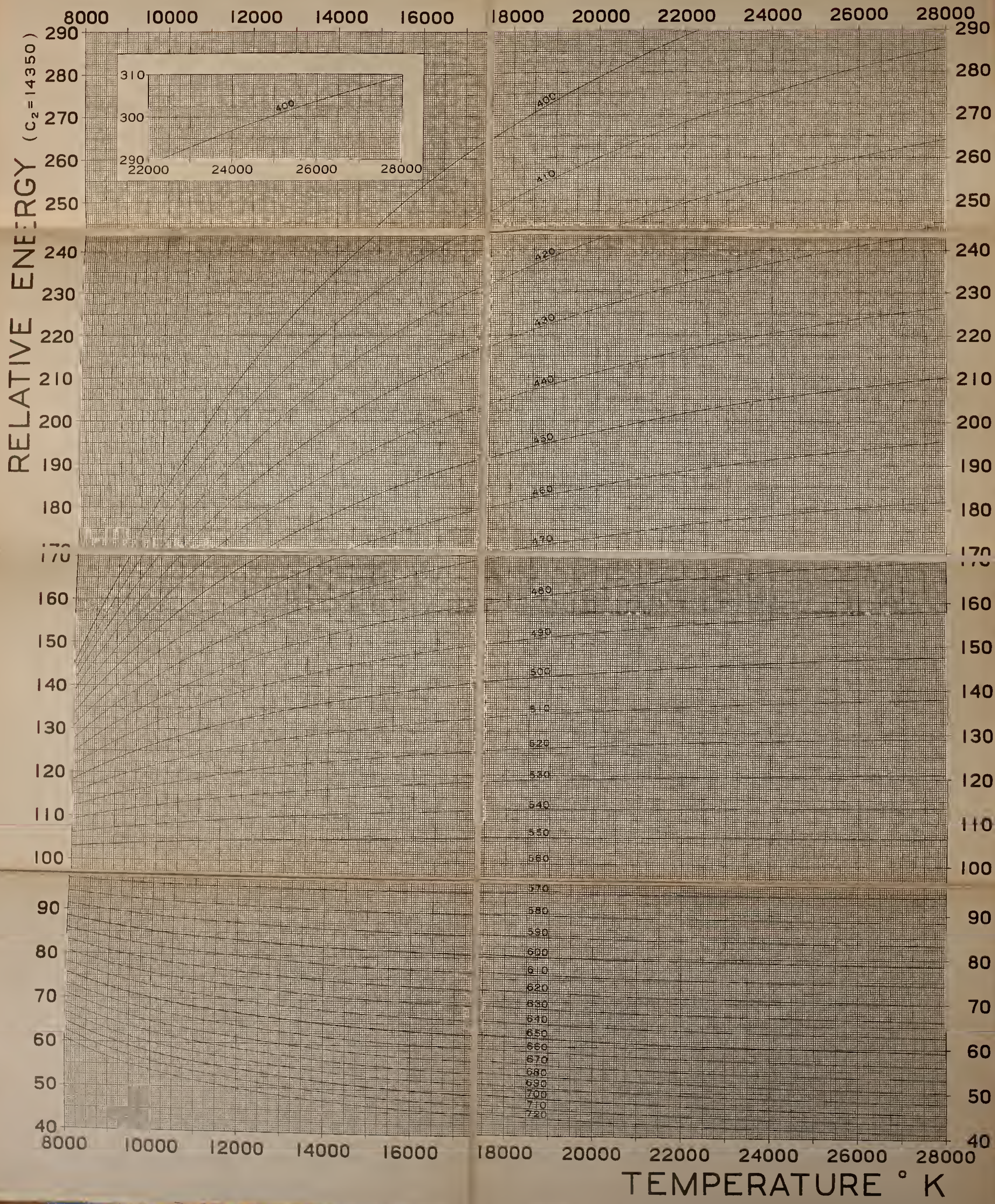
ENERGY DISTRIBUTION BY PLANCK'S FORMULA

100 AT WAVE LENGTH 560 mμ



ENERGY DISTRIBUTION BY PLANCK'S FORMULA

100 AT WAVE LENGTH 560 mμ



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TABLES AND GRAPHS FOR FACILITATING THE COMPUTATION OF SPECTRAL ENERGY DISTRIBUTION BY PLANCK'S FORMULA

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SHEET 7
TABLES

TABLE 1.—$\frac{E_\lambda}{E_m}=\left(\frac{A}{\lambda\theta}\right)^5\left(e^{\frac{C_2}{A}}-1\right)\left(e^{\frac{C_2}{\lambda\theta}}-1\right)^{-1}$

[A=2,890 micron degrees, C_2=14,350 micron degrees, $e^{\frac{C_2}{A}}-1$=142.37]

λθ in micron degrees	$\frac{E_\lambda}{E_m}$	λθ in micron degrees	$\frac{E_\lambda}{E_m}$	λθ in micron degrees	$\frac{E_\lambda}{E_m}$
400	0.0000000007367	1,160	0.05562	4,700	0.6209
410	.0000000015563	1,200	.07291	4,800	.5907
420	.000000003163	1,250	.05278	4,900	.5740
430	.000000006270	1,300	.09720	5,000	.5511
440	.00000001193	1,300	.1343	5,200	.5103
450	.0000000002201	1,350	.1568	5,400	.4715
460	.0000000003943	1,400	.1887	5,600	.4355
470	.0000000006879	1,450	.2254	5,800	.4023
480	.000000001171	1,500	.2647	6,000	.3715
490	.000000001942	1,550	.3059	6,200	.3435
500	.000000003154	1,600	.3486	6,400	.3177
510	.000000005015	1,650	.3922	6,600	.2940
520	.000000007818	1,700	.4363	6,800	.2723
530	.00000001186	1,750	.4804	7,000	.2523
540	.00000001759	1,800	.5241	7,200	.2340
550	.00000002560	1,850	.5669	7,400	.2173
560	.00000003674	1,900	.6086	7,600	.2019
570	.00000005161	1,950	.6487	7,800	.1878
580	.00000007169	2,000	.6871	8,000	.1748
590	.00000009818	2,050	.7236	8,400	.1519
600	.0000001331	2,100	.7579	8,800	.1324
610	.0000001786	2,150	.7900	9,200	.1159
620	.0000002373	2,200	.8196	9,600	.1018
630	.0000003124	2,250	.8470	10,000	.08970
640	.0000004069	2,300	.8719	10,400	.07932
650	.0000006389	2,350	.8945	10,800	.07036
660	.0000008275	2,400	.9147	11,200	.06291
670	.000001061	2,450	.9325	11,600	.05588
680	.000001351	2,500	.9479	12,000	.05001
690	.000001705	2,550	.9612	12,400	.04458
700	.000002135	2,600	.9722	12,800	.04009
710	.000002655	2,650	.9815	13,200	.03644
720	.000003276	2,700	.9888	13,600	.03293
740	.000004897	2,750	.9940	14,000	.02986
760	.000007136	2,800	.9974	14,400	.02713
780	.00001018	2,850	.9995	14,800	.02469
800	.00001419	2,900	1.0000	15,200	.02252
820	.00001944	2,950	.9990	15,600	.02059
840	.00002615	3,000	.9967	16,000	.01886
860	.00003458	3,100	.9886	16,600	.01693
880	.00004503	3,200	.9762	17,000	.01525
900	.00005110	3,300	.9603	17,500	.01379
920	.00005782	3,400	.9418	18,000	.01246
940	.00007326	3,500	.9209	18,500	.01130
960	.00009169	3,600	.8982		
980	.0001134	3,700	.8712	19,000	.01027
1,000	.0001388	3,800	.8492	19,500	.009362
1,030	.0001652	3,900	.8256	20,000	.008548
1,060	.0002206	4,000	.7975	20,500	.007820
1,090	.0002609	4,100	.7714		
1,100	.0003314	4,200	.7453		
1,120	.0003852	4,300	.7193		
1,140	.0004441	4,400	.6938		
1,150	.0005436	4,500	.6686		
1,160	.0005794	4,600	.6440		

TABLE 2A.—Energy Distribution of Planckian Radiator

[Computed from formula $E_\lambda=\frac{\lambda^{-5}}{e^{\frac{C_2}{\lambda\theta}}-1}$ in which C_2=14,350]

[A factor has been used for each temperature that makes E_λ=100 at wave length 590 mμ]

λ in millimicrons	Relative values of E_λ for the following temperatures, θ in degrees absolute						
	1,000°[1]	1,100°	1,200°	1,300°	1,400°	1,500°	1,600°
400	0.006703	0.01917	0.04599	0.09646	0.1820	0.3155	0.5104
410	.01421	.03753	.08424	.1671	.3006	.5000	.7795
420	.02898	.07097	.1495	.2813	.4831	.7722	1.153
430	.05705	.1300	.2579	.4506	.7579	1.166	1.669
440	.1086	.2309	.4325	.7063	1.152	1.724	2.415
450	.2003	.3987	.7072	1.119	1.741	2.498	3.420
460	.3558	.6708	1.128	1.754	2.559	3.551	4.724
470	.6259	1.102	1.763	2.625	3.692	4.963	6.424
480	1.066	1.768	2.696	3.853	5.217	6.877	8.609
490	1.767	2.775	4.045	5.535	7.299	9.331	11.37
500	2.870	4.274	5.952	7.883	10.03	12.36	14.82
510	4.564	6.460	8.616	11.01	13.58	16.29	19.08
520	7.114	9.589	12.28	15.14	18.15	21.21	24.28
530	10.88	13.98	17.22	20.54	23.91	27.26	30.55
540	16.37	20.10	23.83	27.52	31.18	34.70	38.09
550	24.20	28.45	32.50	36.42	40.17	43.69	47.09
560	35.25	38.73	43.82	47.61	51.16	54.49	57.45
570	50.60	54.68	58.30	61.60	64.56	67.26	69.66
580	71.60	74.41	76.75	78.86	80.77	82.40	83.82
590	100.0	100.0	100.0	100.0	100.0	100.0	100.0
600	137.8	132.8	128.8	125.5	122.7	120.5	118.4
610	187.7	174.9	164.6	156.4	149.7	144.1	139.3
620	253.1	227.5	207.9	192.9	180.9	171.1	162.7
630	337.2	292.3	260.7	236.1	217.1	201.8	189.1
640	444.9	370.4	324.2	287.0	258.7	236.4	218.2
650	581.2	474.3	390.7	348.5	306.1	275.3	250.8
660	753.0	596.2	490.1	415.1	360.5	318.8	286.2
670	965.4	742.0	595.5	494.4	421.3	367.0	325.2
680	1,229	917.2	718.6	584.8	490.0	420.5	367.7
690	1,531	1,127	862.0	687.7	567.0	479.3	413.8
700	1,943	1,374	1,028	804.6	652.2	543.7	463.2
710	2,416	1,663	1,217	935.8	747.3	614.3	517.2
720	2,981	2,001	1,434	1,082	851.1	690.6	574.5

λ in millimicrons	Relative values of E_λ for the following temperatures, θ in degrees absolute						
	1,700°	1,800°	1,900°	2,000°	2,200°	2,400°	2,600°
400	0.7807	1.138	1.596	2.163	3.660	5.663	8.210
410	1.153	1.637	2.233	2.962	4.816	7.206	10.15
420	1.671	2.304	3.072	3.990	6.231	9.045	12.41
430	2.370	3.186	4.153	5.268	7.958	11.20	14.96
440	3.303	4.331	5.517	6.867	10.01	13.69	17.86
450	4.520	5.786	7.218	8.808	12.44	16.55	21.10
460	6.084	7.619	9.324	11.18	15.27	19.80	24.67
470	8.077	9.895	11.87	13.97	18.54	23.43	28.62
480	10.57	12.66	14.93	17.32	22.28	27.49	32.87
490	13.63	16.08	18.55	21.15	26.51	31.97	37.51
500	17.42	20.10	22.84	25.63	31.27	36.92	42.46
510	21.97	24.87	27.82	30.74	36.59	42.28	47.74
520	27.40	30.50	33.57	36.58	42.45	48.02	53.39
530	33.83	37.02	40.10	43.13	48.88	54.24	59.28
540	41.38	44.56	47.56	50.44	55.77	60.83	65.48
550	50.16	53.15	55.98	58.62	63.57	67.91	71.96
560	60.28	62.93	65.37	67.71	71.82	75.42	78.65
570	71.92	73.89	75.76	77.52	80.61	83.23	85.55
580	85.08	86.29	87.31	88.37	90.07	91.43	92.72
590	100.0	100.0	100.0	100.0	100.0	100.0	100.0
600	116.6	115.1	113.8	112.6	110.5	108.8	107.5
610	133.3	131.8	128.8	126.1	123.7	116.0	115.1
620	155.9	150.1	144.9	140.6	133.2	127.4	122.8
630	178.6	169.9	162.4	155.9	145.4	137.0	130.5
640	203.7	191.2	181.0	172.2	158.0	146.9	138.3
650	230.9	214.4	200.9	189.2	171.0	157.0	146.2
660	260.5	239.3	221.8	207.3	184.5	167.3	154.0
670	292.2	265.7	244.1	226.2	198.2	177.6	161.8
680	328.5	293.9	267.6	245.7	212.4	188.0	169.6
690	363.3	323.9	292.2	266.2	227.1	198.5	177.4
700	402.8	355.4	318.1	287.6	241.9	209.0	185.0
710	444.7	389.0	345.0	309.3	256.8	219.6	192.6
720	489.1	423.5	374.6	331.8	271.9	230.2	200.1

[1] Computed from Table 1.

TABLE 2A—Continued

λ in millimicrons	Relative values of E_λ for the following temperatures, θ in degrees absolute							
	2,800°	3,000°	3,250°	3,500°	3,750°	4,000°	4,500°	5,000°
400	11.27	14.84	19.97	25.70	32.01	38.75	53.37	68.74
410	13.82	17.54	23.09	29.17	35.75	42.64	57.27	72.37
420	16.25	20.54	26.44	32.60	39.57	46.53	61.10	75.83
430	19.19	23.80	30.03	36.59	43.49	50.48	64.84	79.04
440	22.44	27.32	33.80	40.52	47.44	54.43	68.44	81.97
450	25.98	31.07	37.75	44.53	51.44	58.30	71.87	84.70
460	29.80	35.11	41.85	48.65	55.45	62.09	75.12	87.21
470	33.92	39.32	46.09	52.83	59.44	65.84	78.15	89.50
480	38.33	43.73	50.47	56.99	63.42	69.47	81.01	91.50
490	42.97	48.35	54.94	61.25	67.27	72.97	83.74	93.27
500	47.92	53.12	59.46	65.43	71.15	76.43	86.22	94.79
510	53.05	58.06	64.03	69.61	74.97	79.69	88.51	96.15
520	58.41	63.08	68.67	73.70	78.48	82.79	90.67	97.27
530	63.96	68.25	73.26	77.78	81.92	85.72	92.51	98.17
540	69.67	73.45	77.84	81.73	85.40	88.48	94.26	98.83
550	75.57	78.73	82.43	85.60	88.56	91.13	95.77	99.41
560	81.53	84.02	86.92	89.46	91.72	93.63	97.12	99.76
570	87.58	89.34	91.33	93.07	94.58	95.89	98.17	99.98
580	93.84	94.68	95.74	96.59	97.38	98.03	99.21	100.0
590	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
600	106.3	105.2	104.2	103.3	102.5	101.7	100.6	99.83
610	112.6	110.5	108.3	106.6	104.8	103.3	101.1	99.33
620	118.9	115.5	112.1	109.2	106.9	104.8	101.5	98.89
630	125.1	120.3	115.9	112.0	108.9	106.0	101.7	98.26
640	131.3	125.4	119.6	114.7	110.7	107.2	101.8	97.60
650	137.4	130.2	123.0	117.1	112.3	108.2	101.8	96.93
660	143.6	134.8	126.4	119.4	113.8	109.0	101.6	96.01
670	149.4	139.5	129.6	121.6	115.0	109.6	101.3	95.12
680	155.3	143.9	132.6	123.6	116.2	110.2	101.0	94.16
690	161.1	148.1	135.4	125.3	117.3	110.7	100.6	93.22
700	166.6	152.3	138.1	127.0	118.2	110.9	100.0	92.31
710	172.2	156.1	140.6	128.5	118.9	111.1	99.50	91.02
720	177.5	159.7	142.9	129.8	119.6	111.2	98.78	89.85

λ in millimicrons	Relative values of E_λ for the following temperatures, θ in degrees absolute							
	5,500°	6,000°	6,500°	7,000°	8,000°	9,000°	10,000°	12,000°
400	84.51	100.4	115.8	130.7	158.7	183.8	206.1	243.7
410	87.59	102.7	117.0	131.0	155.5	176.9	199.4	233.6
420	90.41	104.6	118.0	130.9	154.5	174.8	192.6	222.1
430	92.86	106.2	118.7	130.5	151.6	170.1	186.0	211.8
440	95.05	107.5	119.0	129.9	148.9	165.3	179.4	201.9
450	96.98	108.4	119.0	128.7	146.0	160.5	172.9	192.7
460	98.59	109.1	118.7	127.6	142.9	155.7	166.4	183.6
470	99.97	109.5	118.2	126.6	139.6	150.9	160.4	175.1
480	101.0	109.9	117.4	124.4	136.3	146.1	154.2	167.0
490	101.9	109.8	116.4	123.6	133.0	141.4	148.4	159.3
500	103.5	109.3	115.3	120.7	129.5	136.8	142.7	151.8
510	102.9	108.8	114.0	118.8	126.2	132.3	137.2	144.8
520	103.1	108.2	112.5	116.5	122.7	127.9	131.9	138.3
530	103.1	107.3	110.9	114.3	119.3	123.3	126.8	131.9
540	102.9	106.4	109.4	112.0	116.0	119.2	121.8	125.7
550	102.6	105.3	107.6	109.7	112.8	115.2	117.2	120.1
560	102.2	104.1	105.8	107.2	109.5	111.2	112.7	114.7
570	101.5	102.9	103.9	104.8	106.3	107.4	108.2	109.5
580	100.9	101.5	102.0	102.5	103.1	103.6	104.0	104.6
590	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
600	98.99	98.53	97.96	97.60	96.87	96.49	96.01	95.62
610	98.04	96.93	96.02	95.25	93.99	93.05	92.42	91.47
620	96.94	95.41	93.96	92.97	91.13	89.76	88.92	87.42
630	95.70	93.72	91.99	90.90	88.32	86.64	85.39	83.70
640	94.49	92.05	89.90	88.30	85.57	83.57	82.18	80.08
650	93.23	90.29	87.87	85.98	82.99	80.68	78.99	76.58
660	91.92	88.60	85.92	83.71	80.20	77.84	76.07	73.45
670	90.55	86.88	83.94	81.53	77.85	75.12	73.13	70.37
680	89.12	85.12	81.94	79.37	75.34	72.51	70.41	67.43
690	87.74	83.43	79.98	77.30	73.04	69.99	67.74	64.68
700	86.28	81.71	78.09	75.21	70.80	67.56	65.21	62.05
710	84.78	80.00	76.26	73.22	68.60	65.23	62.83	59.53
720	83.30	78.32	74.38	71.21	66.52	62.98	60.48	57.10

λ in millimicrons	Relative values of E_λ for the following temperatures, θ in degrees absolute							
	14,000°	16,000°	18,000°	20,000°	22,000°	24,000°	26,000°	28,000°
400	272.9	295.5	315.2	330.5	343.6	354.3	363.4	371.7
410	258.3	278.4	294.9	308.4	319.2	328.3	336.2	343.0
420	244.2	262.1	275.9	287.4	296.9	304.6	311.3	317.3
430	231.1	245.3	258.4	268.0	276.4	282.9	288.4	293.5
440	218.7	[illegible]	242.1	250.4	257.7	263.1	268.0	272.1
450	207.0	218.4	227.0	234.1	240.1	244.9	248.6	252.6
460	196.2	205.7	213.2	219.2	224.2	228.3	231.7	234.8
470	185.9	194.0	200.3	205.3	209.6	213.0	216.0	218.3
480	176.1	182.9	188.3	192.5	196.2	198.9	201.3	203.4
490	166.8	172.7	177.1	180.8	183.8	186.0	188.1	189.8
500	158.2	163.0	166.8	169.7	172.1	174.1	175.7	177.2
510	150.0	154.0	157.1	159.5	161.6	163.1	164.5	165.6
520	142.5	145.6	148.1	150.0	151.7	152.9	154.1	155.1
530	135.2	137.8	139.8	141.3	142.6	143.5	144.5	145.2
540	128.5	130.5	132.0	133.1	134.4	134.9	135.6	136.2
550	122.0	123.5	124.6	125.5	126.3	127.2	127.7	127.7
560	116.1	117.0	117.8	118.5	119.0	119.4	119.7	120.1
570	110.4	110.9	111.4	111.8	112.3	112.4	112.7	112.9
580	105.0	105.3	105.5	105.7	105.9	106.0	106.2	106.2
590	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
600	95.24	94.94	94.72	94.60	94.60	94.40	94.32	94.36
610	90.74	90.28	89.96	89.59	89.50	89.22	89.05	89.02
620	86.48	85.83	85.37	84.93	84.68	84.37	84.20	84.00
630	82.47	81.60	81.05	80.53	80.26	79.85	79.61	79.35
640	78.63	77.75	76.93	76.63	76.05	75.61	75.37	75.08
650	75.08	73.99	73.12	72.54	72.05	71.65	71.34	71.02
660	71.71	70.51	69.60	68.95	68.48	67.97	67.63	67.27
670	68.49	67.19	66.25	65.52	64.98	64.49	64.00	63.81
680	65.50	64.12	63.14	62.35	61.80	61.24	60.90	60.49
690	62.57	61.17	60.16	59.31	58.77	58.18	57.76	57.41
700	59.90	58.44	57.31	56.50	55.87	55.32	54.83	54.57
710	57.28	55.72	54.66	53.80	53.21	52.62	52.22	51.83
720	54.83	53.30	52.20	51.28	50.66	50.09	49.61	49.28

TABLE 2B.—Energy Distribution of Planckian Radiator

[Computed from formula $E_\lambda=\frac{\lambda^{-5}}{e^{\frac{C_2}{\lambda\theta}}-1}$ in which C_2=14,350]

[A factor has been used for each temperature that makes E_λ=100 at wave length 560 mμ]

λ in millimicrons	Relative values of E_λ for the following temperatures, θ in degrees absolute						
	1,000°	1,100°	1,200°	1,300°	1,400°	1,500°	1,600°
400	0.01902	0.04826	[illegible]	0.2026	0.3558	0.5790	0.8883
410	.04032	.09454	[illegible]	.3510	.5875	.9176	1.357
420	.08221	.1786	[illegible]	.5907	.9443	1.417	2.025
430	.1618	.3272	[illegible]	.9674	1.481	2.140	2.956
440	.3050	.5815	[illegible]	1.546	2.270	3.164	4.237
450	.5681	1.004	[illegible]	2.412	3.403	4.584	5.951
460	1.018	1.688	[illegible]	3.683	5.001	6.519	8.221
470	1.776	2.774	[illegible]	5.513	7.215	9.108	11.18
480	3.023	4.450	[illegible]	8.092	10.24	12.53	14.98
490	5.013	6.985	[illegible]	11.67	14.26	16.98	19.78
500	8.141	10.76	[illegible]	16.56	19.61	22.69	25.79
510	12.95	16.26	[illegible]	23.12	26.54	29.90	33.21
520	20.18	24.14	[illegible]	31.81	35.47	38.93	42.25
530	30.87	35.18	[illegible]	43.14	46.74	50.02	53.17
540	46.44	50.60	[illegible]	57.80	60.95	63.69	66.29
550	68.66	71.61	74.16	76.49	78.51	80.18	81.60
560	100.0	100.0	100.0	100.0	100.0	100.0	100.0
570	143.6	137.6	133.0	129.4	126.2	123.4	121.2
580	203.1	187.3	175.2	165.6	157.9	151.2	145.9
590	283.7	251.7	228.2	210.0	195.5	183.5	174.0
600	390.8	334.4	293.9	283.5	239.9	221.0	296.1
610	532.5	410.3	375.6	328.5	292.5	264.4	242.4
620	717.9	572.6	474.5	405.1	351.6	313.9	283.2
630	956.6	738.5	594.9	495.9	474.3	370.3	329.1
640	1,262	942.5	739.9	602.8	505.5	433.8	379.8
650	1,640	1,194	912.3	727.8	599.9	506.2	436.2
660	2,136	1,501	1,118	871.9	704.7	585.0	498.0
670	2,739	1,868	1,359	1,038	823.5	673.5	565.9
680	3,437	2,309	1,640	1,228	957.6	771.5	640.0
690	4,401	2,837	1,967	1,444	1,108	879.5	720.1
700	5,511	3,458	2,346	1,690	1,275	997.7	806.3
710	6,853	4,166	2,778	1,966	1,461	1,127	900.1
720	8,456	5,037	3,273	2,272	1,663	1,287	999.8

TABLE 2B—Continued

λ in millimicrons	Relative values of E_λ for the following temperatures, θ in degrees absolute						
	1,700°	1,800°	1,900°	2,000°	2,200°	2,400°	2,600°
400	1.295	1.808	2.441	3.194	5.096	7.512	10.44
410	1.916	2.601	3.417	4.374	6.705	9.531	12.91
420	2.772	3.660	4.700	5.878	8.676	12.00	15.78
430	3.931	5.063	6.353	7.781	11.08	14.85	19.03
440	5.479	6.882	8.440	10.14	13.94	18.16	22.71
450	7.493	9.194	11.04	13.01	17.31	21.95	26.83
460	10.09	12.11	14.29	16.49	21.26	26.25	31.36
470	13.40	15.72	18.16	20.63	25.82	31.07	36.38
480	17.53	20.14	22.85	25.52	31.03	36.45	41.80
490	22.64	25.50	28.38	31.24	36.92	42.39	47.69
500	28.90	31.94	34.95	37.85	43.55	46.95	53.99
510	36.44	39.52	42.55	45.40	50.95	56.03	60.75
520	45.46	48.46	51.35	54.03	59.13	63.67	67.88
530	56.12	58.82	61.34	63.70	66.05	71.92	75.37
540	[illegible]	[illegible]	72.75	[illegible]	[illegible]	[illegible]	[illegible]
550	83.20	84.45	85.85	86.57	88.52	90.04	91.49
560	100.0	100.0	100.0	100.0	100.0	100.0	100.0
570	119.3	117.4	115.9	114.5	112.3	110.4	108.8
580	141.1	137.1	133.6	130.5	125.4	121.2	117.9
590	165.9	158.9	153.0	147.7	139.3	132.6	127.1
600	193.5	182.9	174.0	160.3	153.9	144.2	136.6
610	224.4	209.5	197.1	156.3	169.5	156.4	146.3
620	258.6	238.5	221.7	207.7	185.5	164.9	156.1
630	296.3	270.0	248.5	258.2	202.4	181.7	163.9
640	337.8	303.9	276.9	254.3	220.0	194.7	175.9
650	383.0	340.7	307.4	279.4	258.2	208.1	185.9
660	432.1	380.2	339.2	306.0	250.9	221.8	195.7
670	484.7	422.2	373.4	334.0	276.0	235.5	205.8
680	541.5	467.0	409.3	362.9	295.7	249.3	213.6
690	602.7	514.6	446.9	393.1	318.2	263.1	225.6
700	668.1	564.8	486.6	424.7	336.8	277.2	235.3
710	737.7	618.0	527.7	456.9	357.6	291.1	244.9
720	811.3	672.9	570.0	490.1	378.6	505.2	254.4

λ in millimicrons	Relative values of E_λ for the following temperatures, θ in degrees absolute							
	2,800°	3,000°	3,250°	3,500°	3,750°	4,000°	4,500°	5,000°
400	13.82	17.66	22.97	28.73	34.90	41.39	54.95	64.91
410	16.71	20.87	26.57	32.60	58.98	45.54	38.97	72.54
420	19.94	24.44	30.42	38.67	43.14	49.70	62.91	76.00
430	23.54	28.33	34.56	40.90	47.41	53.92	60.76	79.23
440	27.53	32.52	38.88	45.30	51.72	58.13	70.48	82.17
450	31.86	36.99	43.43	49.79	56.08	62.26	74.00	64.90
460	36.55	41.78	48.15	54.38	60.45	66.31	77.35	87.42
470	41.60	46.81	53.03	59.06	64.80	70.32	60.47	69.71
480	47.03	52.08	58.07	63.70	69.14	74.20	63.42	91.72
490	52.71	57.54	63.21	64.47	73.34	77.93	86.22	83.49
500	58.78	63.22	68.41	73.14	77.57	81.63	88.78	95.02
510	65.08	69.11	73.67	77.82	81.63	63.11	91.13	96.41
520	71.64	75.08	79.01	82.39	85.56	88.42	93.36	97.50
530	78.45	91.23	84.29	60.94	89.40	91.55	95.25	98.41
540	63.45	87.43	89.56	91.36	93.10	94.50	97.05	99.07
550	92.68	93.71	94.84	95.69	96.55	97.33	98.58	99.65
560	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
570	107.4	106.3	100.1	104.0	103.1	102.4	101.1	100.2
580	115.1	112.7	110.2	108.0	100.2	104.7	102.2	100.3
590	122.7	119.0	115.1	111.8	109.0	106.8	100.0	100.2
600	130.4	125.3	119.9	115.4	111.7	108.7	103.6	100.1
610	138.1	131.5	124.6	118.9	114.3	110.4	104.1	99.57
620	145.8	137.5	129.0	122.1	116.5	111.9	104.6	99.13
630	153.5	143.4	133.3	125.2	118.7	113.2	104.7	98.50
640	161.1	149.3	137.6	128.2	120.7	114.5	104.8	97.92
650	186.6	155.0	141.6	130.9	122.4	115.5	104.8	97.16
660	176.1	160.5	145.4	133.5	124.0	116.3	104.6	96.24
670	183.2	160.0	149.1	135.9	125.4	117.1	104.3	95.35
680	190.5	171.2	152.6	138.1	126.7	117.7	104.0	94.39
690	197.5	176.3	153.8	140.1	127.9	118.2	103.5	93.44
700	204.4	181.2	158.8	142.0	128.8	118.5	100.0	92.33
710	211.3	185.8	161.8	143.6	139.7	118.7	102.5	91.24
720	217.7	190.1	164.4	145.1	130.4	118.7	101.7	90.07

λ in millimicrons	Relative values of E_λ for the following temperatures, θ in degrees absolute							
	5,500°	6,000°	6,500°	7,000°	8,000°	9,000°	10,000°	12,000°
400	82.72	96.37	109.4	121.9	145.0	165.3	182.9	212.4
410	85.73	98.58	110.6	122.2	143.1	161.2	177.0	202.7
420	86.50	100.5	111.5	122.1	140.9	157.2	170.9	193.6
430	90.90	102.0	112.2	121.8	138.5	152.9	163.1	184.6
440	93.05	103.3	112.5	121.2	136.0	148.6	159.2	176.0
450	94.93	104.1	112.4	120.1	133.4	144.3	153.4	167.9
460	96.51	104.8	112.2	119.0	130.5	140.0	147.7	160.0
470	97.86	105.1	111.7	117.6	127.5	135.7	142.3	152.7
480	98.87	105.5	111.0	116.1	124.5	131.4	136.8	145.6
490	99.70	105.3	110.0	114.3	121.5	127.2	131.7	158.7
500	100.4	105.0	109.0	112.6	118.3	123.0	126.7	132.3
510	100.7	104.5	107.7	110.6	115.3	119.0	121.8	126.2
520	100.9	100.9	100.3	108.7	112.1	115.0	117.0	120.4
530	100.9	100.1	104.8	106.5	109.0	111.1	112.5	114.9
540	100.7	102.2	100.4	104.5	106.0	107.2	106.1	109.6
550	100.4	101.1	101.7	102.3	103.0	103.6	104.0	104.7
560	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
570	99.33	98.77	98.20	97.75	97.05	96.53	95.98	95.46
580	98.73	97.51	96.41	95.60	94.14	93.14	92.31	91.19
590	97.89	96.03	94.51	93.28	91.36	89.92	86.75	87.16
600	96.90	94.62	92.58	91.05	88.50	86.76	85.21	83.34
610	95.97	93.09	90.75	88.84	85.87	83.66	82.02	79.73
620	94.89	91.63	88.80	86.73	85.24	80.71	78.91	76.20
630	93.68	90.02	60.94	84.50	80.68	77.90	75.78	72.97
640	92.49	88.40	84.96	83.37	78.18	75.21	72.93	69.78
650	91.26	86.71	83.05	80.21	75.73	72.54	70.10	86.83
660	69.98	63.09	81.20	78.15	73.38	69.99	67.51	64.02
670	64.63	83.43	79.33	76.05	71.12	67.35	64.90	61.33
680	87.24	81.83	77.44	74.04	68.83	65.20	62.49	36.77
690	85.86	60.17	75.59	72.11	86.73	62.93	60.12	36.38
700	84.46	78.47	73.80	70.16	64.68	60.75	57.87	54.09
710	82.98	76.82	72.07	60.30	62.67	58.65	55.76	51.89
720	81.54	75.20	70.29	66.43	60.70	56.62	53.70	49.77

λ in millimicrons	Relative values of E_λ for the following temperatures, θ in degrees absolute							
	14,000°	16,000°	18,000°	20,000°	22,000°	24,000°	26,000°	28,000°
400	235.1	253.4	267.6	278.9	288.7	296.8	303.6	309.6
410	222.5	237.9	250.4	260.0	268.3	275.0	280.9	285.7
420	210.4	224.0	234.3	242.5	249.5	255.2	260.1	264.2
430	199.1	210.5	219.4	226.2	232.2	237.0	241.0	244.5
440	188.4	198.3	205.6	211.3	216.5	220.4	223.9	226.6
450	178.4	186.6	192.8	197.5	201.8	205.2	207.9	210.4
460	169.0	175.8	181.1	155.0	186.5	191.3	193.5	195.6
470	160.0	165.8	170.1	173.2	176.2	178.4	180.4	181.8
480	151.7	156.3	159.8	162.5	164.9	186.7	160.2	169.4
490	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	155.8	157.2	158.1
500	136.3	139.3	141.7	143.2	144.7	145.9	116.8	147.8
510	129.3	131.6	133.4	134.6	135.8	136.6	137.4	138.0
520	122.7	124.5	125.8	126.6	127.6	128.1	128.8	129.0
530	116.5	117.8	118.7	119.2	119.8	120.3	120.7	120.9
540	110.7	111.5	112.1	112.3	112.7	113.0	113.2	113.4
550	105.1	105.5	105.8	105.9	106.2	100.2	106.3	106.4
560	100.0	100.0	100.0	180.0	100.0	100.0	100.0	100.0
570	95.10	94.81	94.60	94.37	94.38	94.19	94.15	94.02
580	90.49	90.05	89.55	89.22	88.98	88.80	68.55	88.42
590	60.15	85.46	84.91	84.38	84.04	83.78	83.55	83.28
600	82.05	81.14	80.42	79.83	79.50	79.09	78.81	78.58
610	78.17	77.16	76.30	75.64	75.21	74.75	74.40	74.14
620	74.51	73.29	72.40	71.67	71.16	70.68	70.35	69.96
630	71.05	69.74	68.82	67.95	67.45	66.89	60.51	60.09
640	67.74	60.44	65.32	64.49	63.91	63.55	62.97	62.53
650	64.69	63.23	62.09	61.21	60.55	60.03	59.61	59.14
660	61.78	60.26	59.10	58.18	57.54	56.94	56.50	56.02
670	59.01	57.42	56.25	55.39	54.59	54.03	53.54	53.14
680	56.43	54.79	53.61	52.61	51.93	51.31	50.79	50.38
690	53.90	52.28	51.08	50.05	49.39	48.75	48.26	47.83
700	51.61	49.94	48.66	47.68	46.95	46.35	45.81	45.43
710	49.35	47.70	46.64	45.40	44.40	44.03	43.03	43.16
720	47.24	45.55	45.04	43.27	42.59	41.97	41.43	41.04