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Refractive Indices and Densities of Aqueous Solutions of Invert Sugar

U.S. DEPARTMENT OF COMMERCE

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Refractive Indices and Densities of Aqueous Solutions of Invert Sugar

Carl F. Snyder and Albert T. Hattenburg

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Refractive Indices and Densities of Aqueous Solutions of Invert Sugar [1]

Carl F. Snyder and Albert T. Hattenburg

The refractive indices and densities of aqueous solutions of invert sugar have been determined, at temperatures of 15, 20, 25, and 30° C, for concentrations up to about 82 percent of invert sugar (by weight). From the results, equations have been derived which relate the refractive index and percent of invert sugar (weight in air) at each temperature. Other equations relate the absolute density and percent of invert sugar (weight in vacuum) at each temperature. Five-decimal tables giving the refractive indices and densities of invert sugar solutions containing 1 to 85 percent of sugar are given for each percentage of invert sugar at the four temperatures.

1. Refractive Index at Various Temperatures

Carl F. Snyder

1.1. Introduction

The increasing use of refractometric methods for determining the dissolved solids in commercial, liquid sugar-products has made it desirable to obtain more-complete data on the refractive indices of aqueous solutions of pure invert sugar at various temperatures. These refractive indices have, therefore, been determined for concentrations up to about 82 percent (by weight), at 15, 20, 25, and 30° C. From the results, an equation was derived relating refractive index and percent of invert sugar (by weight) at each temperature. From the four equations thus obtained, values of the index for equal increments of concentration were calculated.

1.1.1. Historical Discussion

Although fragmentary data on the refractive indices of solutions of some sugars had previously been reported, Stolle [1] [2] is generally credited with the first systematic study of the refractive indices of sugars other than sucrose. He observed little difference in the indices of the different sugars in solutions of the same concentration. Tolman and Smith [2], using an Abbe refractometer, found that, for a number of sugars and related compounds, a solution of a given percentage by weight has approximately the same refractive index. In neither of these investigations were measurements made on solutions of invert sugar. Staněk and Vondrák [3] showed that the refractive index of an invert sugar solution is lower than that of a sucrose solution of the same concentration. The same observation was made by Schneller [4]. Most of the solutions measured by Schneller were prepared by the acid hydrolysis of sucrose and contained various proportions of invert sugar and sucrose. Subsequently, de Whalley [5] studied the refractometric estimation of dissolved solids in sirups containing invert sugar; he measured 17 solutions of invert sugar prepared by dissolving equal weights of pure D-glucose and D-fructose, for the range of 9.61 to 66.72 percent (by weight). The index was measured on an Abbe refractometer, and the observed refractive index was converted to percent solids by means of the table of the refractive indices of sucrose solutions. Both Schneller and de Whalley found that an appreciable difference exists between the refractive index of a solution of sucrose and one of invert sugar of the same concentration. Thus, estimation of the total solids in a sirup containing invert sugar (with or without sucrose) by conversion of the observed refractive index to the equivalent sucrose (from the sucrose table) gave results which were too low. From his experimental results, de Whalley calculated an average correction of 0.022 for each percent of invert sugar present; this was to be added to the refractometric result for solids expressed as sucrose. Later, Zerban [6] showed that, for the data of both Schneller and de Whalley, the magnitude of the correction varies with the concentration. Zerban and Martin [7] then determined the refractive indices of invert sugar solutions at 20 °C for the range of 0 to 80 percent (by weight), and devised the following equation where p is the percent by weight of invert sugar solids.

$$n_D^{20} \text{ (invert sugar solutions)} = 1.33299 + 1.411210\times10^{-3}p + 0.5198919\times10^{-5}p^2 + 1.56855\times10^{-8}p^3 - 0.3776638\times10^{-10}p^4. \quad (1)$$

Since the above measurements of the refractive index of invert sugar solutions were made only at 20 °C, it was deemed desirable to repeat these measurements and to extend them to other temperatures.

[1] This Monograph is based on work partly supported by the Sugar Research Foundation, Inc.

[2] Figures in brackets indicate the literature references at the end of this Monograph.

1.2. Preparation of Sugars

1.2.1. D-Glucose

The purest form of commercial dextrose hydrate that was readily available was dissolved in sufficient hot, distilled water to make a solution of approximately 60 °Brix; the solution was treated with vegetable charcoal and filtered. The filtrate was warmed and refiltered through hardened filter-paper until all traces of charcoal had been removed. The final filtrate was concentrated under diminished pressure to a sirup containing about 80 percent of dextrose (by weight); this was heated to 55 °C, nucleated with anhydrous D-glucose, and placed in a rotating crystallizer enclosed in an insulated cabinet thermostated at 55 °C. The crystallization was usually complete within a few hours. The temperature within the cabinet was then lowered until the mass had reached a temperature of about 30 to 35 °C, and the crystals were separated from the mother liquor by centrifuging and thoroughly washed with absolute ethanol. Recrystallization was repeated until analysis of the product showed that the sugar was of the required purity. The final crystals were air-dried and then contained less than 0.1 percent of moisture. The sugar was finely powdered, dried at 60 °C, and stored in vacuum desiccators over suitable drying agents.

1.2.2. D-Fructose

Relatively pure D-fructose (levulose) was available from the stock prepared at the National Bureau of Standards from tubers of the Jerusalem artichoke [8]. The sugar had already been recrystallized from water and it consisted of large crystals having a purity of over 99.9 percent. For this investigation, the sugar was further purified by the procedure of Jackson and Mathews [9]. A solution of the sugar in about an equal weight of water was treated with vegetable charcoal and the suspension was warmed and filtered. The filtrate was refiltered through hardened filter-paper to remove all traces of the charcoal, and concentrated at diminished pressure to a sirup containing 82 to 85 percent of D-fructose. The thick sirup was mixed with absolute ethanol and nucleated with crystals of D-fructose, and the suspension was rotated in a motor-driven apparatus at room temperature until crystallization was complete. The crystals were removed by centrifuging, thoroughly washed with absolute ethanol, air-dried, finely powdered, dried in a vacuum oven at 50 °C for about 1 hour, and stored in vacuum desiccators containing suitable desiccants.

1.3. Preparation of Solutions

The measurements of the refractive indices and densities were made on solutions prepared by dissolving a mixture of equal weights of pure D-glucose (dextrose) and pure D-fructose (levulose) in distilled water.

A suitable quantity of dry D-fructose was placed in a tared glass-stoppered flask, and the flask and contents were weighed and then dried to constant weight in a vacuum oven at 50 °C. An equal weight of dry D-glucose was transferred to the flask, and the flask and contents were weighed and then dried in a vacuum oven at 50 °C to constant weight. The final weight of the contents represented the weight of dry invert sugar. Enough water to yield a solution of the desired concentration was added. The flask was mechanically rotated in an inclined position on an apparatus so constructed that its rotating shaft extended into an air bath fitted with heaters and a thermostat. For the solutions in the lower range of concentrations, elevated temperatures were not necessary, but, in the higher range, the air bath was heated to the lowest temperature that would ensure complete solution. When all of the sugar had dissolved, the flask was cooled to room temperature (if necessary) and weighed. From the weight of the solution and the weight of invert sugar taken, the composition of each solution was calculated in percent by weight (in air).

1.4. Determination of Refractive Indices

The solution of invert sugar was kept at room temperature for 24 hours to establish mutarotational equilibrium. Determinations of the refractive index were made with a Bausch and Lomb precision refractometer (Serial 6326), using a sodium lamp. For all readings, the instrument was adjusted to read (approximately) correctly for distilled water and the scale was calibrated by means of test plates of known refractive index. The instrument was kept in a room whose temperature was regulated to within about 1 °C. Water from a large water-thermostat was continuously circulated through the prism jackets to maintain the same temperature of the solution and prisms to within 0.01 or 0.02 °C.

For each series of measurements, the temperature of the room and water bath was adjusted to the desired value. A series of readings was taken with distilled water, the prism faces were thoroughly dried, and the instrument was filled with the solution of invert sugar. After a series of observations had been made on the solution, the prism faces were cleaned and a series of readings was taken with distilled water These operations were repeated until the required data had been obtained.

On completion of the series of measurements at one temperature, the temperature of the room and water bath was adjusted to a different temperature and a set of measurements was made at the new temperature. A random order was followed with respect to the measurements at different temperatures.

The observed readings on the instrument scale were converted to their equivalents in refractive index by means of the tables furnished with the instrument.

1.4.1. Results

Measurements of the refractive indices were made on 39 solutions of invert sugar of concentrations ranging from about 5 to 82 percent by weight (in air). Each solution was measured at two or more of the selected temperatures (15, 20, 25, and 30 °C). From the data obtained at each temperature, an equation relating refractive index and percent of invert sugar by weight (in air) was derived (by the method of averages)

The four equations are as follows:

$$n_D^{15}=1.3333872+1.4292\times10^{-3}p+0.515855\times10^{-5}p^2+0.1153\times10^{-7}p^3 \quad (2)$$

$$n_D^{20}=1.3329877+1.4149\times10^{-3}p+0.52729\times10^{-5}p^2+0.1105\times10^{-7}p^3 \quad (3)$$

$$n_D^{25}=1.3325026+1.4114\times10^{-3}p+0.51088\times10^{-5}p^2+0.1244\times10^{-7}p^3 \quad (4)$$

$$n_D^{30}=1.3319403+1.4016\times10^{-3}p+0.5134\times10^{-5}p^2+0.1249\times10^{-7}p^3 \quad (5)$$

The experimental values obtained at each temperature for the invert sugar solutions of known composition and the values calculated from the corresponding equation are given in table 1.

By employing equations (2) to (5), the refractive indices at equal increments of concentration were calculated; they are given in table 2. These values (table 2) for the refractive indices at 20 °C are slightly higher than those reported by Zerban and Martin [7].

TABLE 1. *Refractive indices of invert sugar solutions*

Invert sugar by wt. (in air)	n_D at 15 °C			n_D at 20 °C			n_D at 25 °C			n_D at 30 °C		
	Obs.	Calc.	Calc.−Obs.	Obs.	Calc.	Calc.−Obs.	Obs.	Calc.	Calc.−Obs.	Obs.	Calc.	Calc.−Obs.
%												
4.9106	1.34049	1.34053	+0.00004	1.34001	1.34006	+0.00005	1.33956	1.33956	0.00000	1.33888	1,33895	+0.00007
5.0244	1.34076	1.34070	−.00006	1.34024	1.34023	−.00001	1.33973	1.33972	−.00001	1.33913	1.33911	−.00002
9.6687	1.34768	1.34770	+.00002	1.34712	1.34717	+.00005	1.34665	1.34664	−.00001	1.34593	1.34598	+.00005
10.0548				1.34779	1.34776	−.00003	1.34724	1.34722	−.00002			
11.4836				1.34997	1.34995	−.00002				1.34871	1.34873	+.00002
11.4961	1.35055	1.35052	−.00003	1.34994	1.34997	+.00003	1.34936	1.34942	+.00006	1.34871	1.34875	+.00004
12.5735	1.35225	1.35220	−.00005	1.35171	1.35163	−.00008	1.35109	1.35108	−.00001	1.35051	1.35040	−.00011
15.0320	1.35615	1.35608	−.00007	1.35542	1.35549	+.00007	1.35499	1.35492	−.00007	1.35420	1.35421	+.00001
15.1660	1.35627	1.35629	+.00002	1.35570	1.35570	.00000	1.35504	1.35513	+.00009	1.35433	1.35442	+.00009
16.3219	1.35805	1.35814	+.00009	1.35753	1.35754	+.00001	1.35698	1.35696	−.00002	1.35629	1.35624	−.00005
16.5251	1.35855	1.35847	−.00008	1.35786	1.35786	.00000	1.35729	1.35728	−.00001	1.35656	1.35656	.00000
18.7642	1.36203	1.36210	+.00007	1.36143	1.36147	+.00004	1.36085	1.36087	+.00002	1.36010	1.36013	+.00003
20.0077	1.36407	1.36414	+.00007	1.36360	1.36350	−.00010	1.36290	1.36289	−.00001	1.36228	1.36214	−.00014
20.0818				1.36365	1.36362	−.00003	1.36305	1.36301	−.00004			
25.0411	1.37265	1.37259	−.00006	1.37197	1.37190	−.00007	1.37128	1.37125	−.00003			
25.0691	1.37268	1.37264	−.00004				1.37132	1.37129	−.00003			
26.9445	1.37581	1.37587	+.00006	1.37509	1.37516	+.00007	1.37434	1.37449	+.00015	1.37360	1.37368	+.00008
30.0336	1.38138	1.38128	−.00010	1.38060	1.38054	−.00006	1.37991	1.37984	−.00007			
30.9705	1.38283	1.38294	+.00011	1.38213	1.38220	+.00007	1.38134	1.38149	+.00015	1.38073	1.38064	−.00009
33.2872				1.38639	1.38634	−.00005	1.38566	1.38561	−.00005			
34.9011	1.38994	1.39004	+.00010	1.38929	1.38926	−.00003	1.38864	1.38852	−.00012	1.38761	1.38764	+.00003
39.3212				1.39753	1.39745	−.00008	1.39678	1.39666	−.00012			
42.2839	1.40390	1.40391	+.00001	1.40299	1.40308	+.00009	1.40223	1.40226	+.00003	1.40124	1.40133	+.00009
45.0871	1.40946	1.40937	−.00009									
50.1960	1.41958	1.41958	.00000	1.41869	1.41870	+.00001	1.41774	1.41780	+.00006	1.41680	1.41681	+.00001
52.4589	1.42420	1.42422	+.00002	1.42325	1.42332	+.00007	1.42230	1.42240	+.00010	1.42134	1.42140	+.00006
55.2430	1.43005	1.43003	−.00002	1.42909	1.42911	+.00002	1.42810	1.42816	+.00006	1.42709	1.42714	+.00005
58.8565	1.43771	1.43773	+.00002	1.43677	1.43679	+.00002	1.43583	1.43581	−.00002	1.43484	1.43477	−.00007
60.3033	1.44090	1.44086	−.00004	1.43994	1.43991	−.00003	1.43900	1.43892	−.00008	1.43804	1.43787	−.00017
64.9237	1.45114	1.45108	−.00006	1.45017	1.45010	−.00007	1.44923	1.44908	−.00015	1.44814	1.44800	−.00014
65.1626	1.45166	1.45161	−.00005	1.45070	1.45064	−.00006				1.44859	1.44853	−.00006
68.6833	1.45974	1.45962	−.00012	1.45880	1.45863	−.00017	1.45772	1.45758	−.00014	1.45656	1.45648	−.00008
70.7986	1.46454	1.46452	−.00002	1.46359	1.46352	−.00007	1.46249	1.46245	−.00004	1.46137	1.46134	−.00003
73.1294	1.47010	1.47000	−.00010	1.46898	1.46899	+.00001	1.46786	1.46791	+.00005	1.46685	1.46678	−.00007
75.5289	1.47563	1.47573	+.00010	1.47482	1.47470	−.00012	1.46365	1.46361	−.00004			
75.5960	1.47584	1.47589	+.00005	1.47494	1.47486	−.00008	1.47385	1.47377	−.00008	1.47268	1.47263	−.00005
78.1373	1.48203	1.48206	+.00003	1.48096	1.48102	+.00006	1.47984	1.47992	+.00008	1.47867	1.47877	+.00010
80.1473	1.48687	1.48701	+.00014	1.48568	1.48596	+.00028	1.48469	1.48485	+.00016	1.48365	1.48369	+.00004
82.7504	1.49347	1.49351	+.00004	1.49221	1.49245	+.00024	1.49114	1.49133	+.00019	1.48987	1.49016	+.00029

TABLE 2. *Refractive indices of invert sugar solutions**

Invert sugar by weight (in air)	n_D at			
	15 °C	20 °C	25 °C	30 °C
%				
0	1.33339	1.33299	1.33250	1.33194
1	1.33482	1.33441	1.33392	1.33335
2	1.33627	1.33584	1.33535	1.33476
3	1.33772	1.33728	1.33678	1.33619
4	1.33919	1.33873	1.33823	1.33763
5	1.34066	1.34020	1.33969	1.33908
6	1.34215	1.34167	1.34116	1.34054
7	1.34364	1.34315	1.34264	1.34201
8	1.34516	1.34465	1.34413	1.34349
9	1.34668	1.34616	1.34563	1.34498
10	1.34821	1.34768	1.34714	1.34648
11	1.34975	1.34920	1.34866	1.34800
12	1.35130	1.35075	1.35020	1.34952
13	1.35286	1.35230	1.35174	1.35106
14	1.35444	1.35386	1.35330	1.35260
15	1.35603	1.35544	1.35487	1.35416
16	1.35762	1.35702	1.35644	1.35573
17	1.35923	1.35862	1.35803	1.35731
18	1.36085	1.36023	1.35964	1.35891
19	1.36248	1.36185	1.36125	1.36051
20	1.36413	1.36348	1.36287	1.36213
21	1.36578	1.36513	1.36451	1.36375
22	1.36745	1.36679	1.36616	1.36539
23	1.36913	1.36846	1.36782	1.36705
24	1.37082	1.37014	1.36949	1.36871
25	1.37252	1.37183	1.37118	1.37038
26	1.37424	1.37354	1.37287	1.37207
27	1.37596	1.37525	1.37458	1.37377
28	1.37770	1.37698	1.37630	1.37548
29	1.37945	1.37873	1.37803	1.37721
30	1.38122	1.38048	1.37978	1.37895
31	1.38299	1.38225	1.38154	1.38070
32	1.38478	1.38403	1.38331	1.38246
33	1.38658	1.38582	1.38509	1.38423
34	1.38840	1.38763	1.38689	1.38602
35	1.39022	1.38944	1.38869	1.38782
36	1.39206	1.39128	1.39052	1.38964
37	1.39391	1.39312	1.39235	1.39146
38	1.39578	1.39498	1.39420	1.39330
39	1.39766	1.39685	1.39606	1.39515
40	1.39955	1.39873	1.39793	1.39702
41	1.40145	1.40063	1.39982	1.39890
42	1.40337	1.40254	1.40172	1.40079
43	1.40530	1.40446	1.40363	1.40270
44	1.40724	1.40640	1.40556	1.40462
45	1.40920	1.40835	1.40750	1.40655
46	1.41117	1.41031	1.40945	1.40849
47	1.41315	1.41229	1.41142	1.41045
48	1.41515	1.41428	1.41340	1.41243
49	1.41716	1.41628	1.41539	1.41442
50	1.41919	1.41830	1.41740	1.41642
51	1.42122	1.42033	1.41942	1.41843
52	1.42328	1.42238	1.42146	1.42046
53	1.42534	1.42444	1.42351	1.42251
54	1.42742	1.42651	1.42558	1.42457
55	1.42952	1.42860	1.42766	1.42664
56	1.43163	1.43070	1.42975	1.42873
57	1.43375	1.43282	1.43186	1.43083
58	1.43589	1.43495	1.43398	1.43294
59	1.43804	1.43710	1.43612	1.43507
60	1.44020	1.43926	1.43827	1.43722
61	1.44238	1.44143	1.44043	1.43938
62	1.44458	1.44362	1.44262	1.44155
63	1.44679	1.44582	1.44481	1.44374
64	1.44901	1.44804	1.44702	1.44595
65	1.45125	1.45027	1.44925	1.44817
66	1.45350	1.45252	1.45149	1.45040
67	1.45577	1.45479	1.45375	1.45265
68	1.45805	1.45706	1.45602	1.45492
69	1.46035	1.45936	1.45830	1.45720
70	1.46266	1.46166	1.46060	1.45950
71	1.46499	1.46399	1.46292	1.46181
72	1.46734	1.46633	1.46525	1.46414
73	1.46970	1.46868	1.46760	1.46648
74	1.47207	1.47105	1.46997	1.46884
75	1.47446	1.47343	1.47235	1.47121
76	1.47687	1.47583	1.47474	1.47360
77	1.47929	1.47825	1.47715	1.47601
78	1.48172	1.48068	1.47958	1.47843
79	1.48418	1.48313	1.48203	1.48087
80	1.48664	1.48559	1.48449	1.48333
81	1.48913	1.48807	1.48696	1.48580
82	1.49163	1.49057	1.48945	1.48828
83	1.49414	1.49308	1.49196	1.49079
84	1.49668	1.49560	1.49449	1.49331
85	1.49922	1.49815	1.49703	1.49584

*A condensed form of this table was published in the Proceedings of the Twelfth Session of the International Commission for Uniform Methods of Sugar Analysis, 1958, page 26.

2. Density at Various Temperatures

Albert T. Hattenburg and Carl F. Snyder

2.1. Introduction

Densimetric methods are applicable to the determination of dissolved solids in commercial liquid-sugar products. The true densities of 15 solutions of invert sugar of various concentrations were determined at 15, 20, 25, and 30 °C. By using the experimental data at each of these temperatures, an equation was derived relating true density and percent of invert sugar by weight (in vacuum). From the four equations thus obtained, the values of the density at equal increments of concentration (percent of invert sugar by weight) were calculated.

2.1.1. Historical Discussion

The determination of the densities of sugar solutions of various concentrations and temperatures has been the subject of extensive investigation. Because of the commercial importance of sucrose, the densities of its aqueous solutions have received the most attention. Since their publication in 1900, Plato, Domke, and Harting's tables of densities of sucrose solutions [10] have been generally accepted as being the most accurate that were available. Although, strictly, these tables are applicable to solutions of pure sucrose only, they are frequently employed for expressing the total content of dissolved solids in a sugar solution containing other dissolved solids besides sucrose. For such materials, the values have been expressed as apparent percent of sucrose or as "sucrose equivalent."

By basing calculations on the values of Plato and co-workers [10] for the densities of sucrose solutions, many other tables have been prepared. Among these is the expanded table of Sidersky [11] which gives the number of grams of sucrose per 100 g and per 100 ml of solution for concentrations up to 30 percent by weight at t/4 °C and t/15 °C (between 10 and 30°). Domke [12] published a table of apparent specific gravities at

20 °C/20 °C for aqueous solutions containing 0 to 90 percent of sucrose by weight. Bates and Bearce [13] calculated a table giving the Baume equivalents in specific gravities at 20 °C/4 °C and 20 °C/20 °C. A table of weights of sucrose per gallon and per cubic foot of aqueous solutions was computed by Snyder and Hammond [14].

For use in the tropics, a table of specific gravities of sucrose solutions at 27.5 °C/4 °C was calculated by Dowes-Dekkar and Erlee [15] and at 28 °C/4 °C by Sidersky [11]. For use in Hawaii, a table giving apparent specific gravities at 27.5 °C/27.5 °C was given in the Methods of Chemical Control of the Association of Hawaiian Sugar Technologists.

The densities of aqueous solutions containing up to 30 percent of D-glucose were determined by Jackson [16] and by Riiber [17]. Subsequently, Jackson and Matthews [9] determined the densities of aqueous solutions containing up to 70 percent of D-fructose.

The lack of published data on the densities of aqueous solutions of invert sugar led to the experimental determinations now reported.

2.2. Determination of Density

Data on the densities of invert sugar solutions were obtained from a series of determinations made at 15, 20, 25, and 30 °C, for solutions of 15 different concentrations prepared as described in section 1.3. The weight of invert sugar and the weight of the solution (in air) of each of the solutions were converted to weights in vacuum, and the percent (by weight) of invert sugar (in vacuum) was calculated. The measurements were made by the use of a pycnometer which consisted of a long cylindrical glass body of about 100-ml capacity. Access to the body was provided by two capillary tubes whose upper ends protruded into a small bowl fitted with a ground-glass cover. The small bowl served to contain the solution which spilled out of the tubes upon removal from a high-temperature bath.

2.2.1. Results

From the data at each temperature, an equation relating absolute density (in g/ml) and percent of invert sugar by weight (in vacuum) was derived by the method of averages.

The four equations are as follows:

$$d^{15°}=0.9991266+0.3895362\times10^{-2}q+0.124621\times10^{-4}q^2+0.4632\times10^{-7}q^3-0.21854\times10^{-9}q^4 \quad (6)$$

$$d^{20°}=0.9982343+0.3863641\times10^{-2}q+0.1214466\times10^{-4}q^2+0.5417\times10^{-7}q^3-0.25882\times10^{-9}q^4 \quad (7)$$

$$d^{25°}=0.9970770+0.3827780\times10^{-2}q+0.1261903\times10^{-4}q^2+0.4235\times10^{-7}q^3-0.1661\times10^{-9}q^4 \quad (8)$$

$$d^{30°}=0.9956780+0.3798374\times10^{-2}q+0.1264516\times10^{-4}q^2+0.4180\times10^{-7}q^3-0.1533\times10^{-9}q^4 \quad (9)$$

Table 3 shows the observed densities for each concentration at each temperature and the values calculated from the corresponding equation.

By means of equations (6) to (9), the absolute densities at equal increments of concentration were calculated; they are given in table 4.

TABLE 3. *Densities of invert sugar solutions*

Invert sugar by wt. (in vacuo)	Density (g/ml) at											
	15 ° C			20 ° C			25 ° C			30 ° C		
	Obs.	Calc.	Calc.−Obs.	Obs.	Calc.	Calc.−Obs.	Obs.	Calc.	Calc.−Obs.	Obs.	Calc.	Calc.−Obs.
%												
4.9086	1.01857	1.01855	−0.00002	1.01754	1.01750	−0.00004	1.01619	1.01617	−0.00002	1.01462	1.01463	+0.00001
9.6649	1.03798	1.03798	.00000	1.03674	1.03676	+.00002	1.03527	1.03529	+.00002	1.03360	1.03361	+.00001
11.4917	1.04563	1.04560	−.00003	1.04436	1.04432	−.00004	1.04282	1.04279	−.00003	1.04110	1.04106	−.00004
15.1604	1.06116	1.06120	+.00004	1.05972	1.05978	+.00006	1.05812	1.05815	+.00003	1.05629	1.05631	+.00002
18.7575	1.07685	1.07686	+.00001	1.07532	1.07531	−.00001	1.07358	1.07358	.00000	1.07165	1.07163	−.00002
26.9360	1.11394	1.11388	−.00006	1.11204	1.11204	.00000	1.11010	1.11008	−.00002	1.10790	1.10790	.00000
30.9614	1.13282	1.13285	+.00003	1.13085	1.13087	+.00002	1.12875	1.12879	+.00004	1.12650	1.12650	.00000
34.8913	1.15184	1.15186	+.00002	1.14975	1.14974	−.00001	1.14757	1.14755	−.00002	1.14514	1.14515	+.00001
42.2733	1.18875	1.18887	+.00012	1.18653	1.18653	.00000	1.18408	1.18411	+.00003	1.18150	1.18151	+.00001
52.4482	1.24276	1.24274	−.00002	1.24023	1.24014	−.00009	1.23743	1.23740	−.00003	1.23458	1.23455	−.00003
58.8461	1.27846	1.27833	−.00013	1.27564	1.27559	−.00005	1.27276	1.27266	−.00010	1.26971	1.26967	−.00004
64.9138	1.31327	1.31329	+.00002	1.31030	1.31044	+.00014	1.30728	1.30736	+.00008	1.30418	1.30424	+.00006
70.7899	1.34828	1.34827	−.00001	1.34531	1.34532	+.00001	1.34216	1.34214	−.00002	1.33890	1.33891	+.00001
75.5211	1.37725	1.37723	−.00002	1.37418	1.37420	+.00002	1.37094	1.37097	+.00003	1.36764	1.36768	+.00004
82.7439	1.42273	1.42276	+.00003	1.41967	1.41964	−.00003	1.41642	1.41641	−.00001	1.41309	1.41304	−.00005

Table 4. *Densities of invert sugar solutions*

Invert sugar by wt. (in vaccum)	Density (g/ml) at			
	15 °C	20 °C	25 °C	30 °C
%				
0	0.99913	0.99823	0.99708	0.99568
1	1.00303	1.00211	1.00092	.99949
2	1.00697	1.00601	1.00478	1.00333
3	1.01093	1.00994	1.00868	1.00719
4	1.01491	1.01389	1.01259	1.01108
5	1.01892	1.01786	1.01654	1.01499
6	1.02296	1.02186	1.02051	1.01893
7	1.02702	1.02589	1.02450	1.02290
8	1.03111	1.02995	1.02853	1.02690
9	1.03523	1.03403	1.03258	1.03092
10	1.03937	1.03814	1.03666	1.03497
11	1.04354	1.04227	1.04076	1.03904
12	1.04774	1.04644	1.04490	1.04315
13	1.05197	1.05063	1.04906	1.04728
14	1.05622	1.05484	1.05325	1.05144
15	1.06051	1.05909	1.05747	1.05563
16	1.06482	1.06337	1.06171	1.05985
17	1.06916	1.06767	1.06599	1.06410
18	1.07353	1.07200	1.07030	1.06837
19	1.07793	1.07637	1.07463	1.07268
20	1.08235	1.08076	1.07899	1.07701
21	1.08681	1.08518	1.08339	1.08138
22	1.09130	1.08963	1.08781	1.08577
23	1.09581	1.09411	1.09226	1.09020
24	1.10036	1.09862	1.09674	1.09465
25	1.10494	1.10316	1.10126	1.09913
26	1.10954	1.10773	1.10580	1.10365
27	1.11418	1.11233	1.11037	1.10819
28	1.11885	1.11697	1.11498	1.11277
29	1.12355	1.12163	1.11961	1.11738
30	1.12828	1.12633	1.12428	1.12201
31	1.13304	1.13105	1.12897	1.12668
32	1.13783	1.13581	1.13370	1.13138
33	1.14265	1.14060	1.13846	1.13612
34	1.14750	1.14542	1.14325	1.14088
35	1.15239	1.15027	1.14807	1.14567
36	1.15730	1.15516	1.15293	1.15050
37	1.16225	1.16007	1.15781	1.15536
38	1.16723	1.16502	1.16273	1.16025
39	1.17224	1.17000	1.16768	1.16517
40	1.17729	1.17502	1.17266	1.17013
41	1.18236	1.18006	1.17768	1.17512
42	1.18747	1.18514	1.18273	1.18014
43	1.19261	1.19025	1.18780	1.18519
44	1.19778	1.19539	1.19292	1.19027
45	1.20298	1.20057	1.19806	1.19539
46	1.20821	1.20577	1.20324	1.20054
47	1.21348	1.21101	1.20845	1.20573
48	1.21878	1.21629	1.21369	1.21094
49	1.22411	1.22159	1.21896	1.21619
50	1.22947	1.22693	1.22427	1.22148
51	1.23487	1.23230	1.22961	1.22679
52	1.24030	1.23771	1.23498	1.23214
53	1.24576	1.24315	1.24039	1.23753
54	1.25125	1.24861	1.24583	1.24294
55	1.25678	1.25412	1.25130	1.24839
56	1.26233	1.25965	1.25681	1.25388
57	1.26792	1.26522	1.26235	1.25939
58	1.27354	1.27082	1.26792	1.26494
59	1.27920	1.27646	1.27353	1.27053
60	1.28488	1.28112	1.27917	1.27615
61	1.29060	1.28782	1.28484	1.28180
62	1.29635	1.29355	1.29055	1.28748
63	1.30214	1.29932	1.29629	1.29320
64	1.30795	1.30511	1.30206	1.29896
65	1.31380	1.31094	1.30787	1.30474
66	1.31968	1.31680	1.31371	1.31056
67	1.32559	1.32269	1.31958	1.31642
68	1.33153	1.32862	1.32548	1.32231
69	1.33750	1.33458	1.33142	1.32823
70	1.34351	1.34057	1.33740	1.33418
71	1.34954	1.34659	1.34340	1.34017
72	1.35561	1.35264	1.34944	1.34620
73	1.36171	1.35872	1.35551	1.35226
74	1.36784	1.36484	1.36162	1.35835
75	1.37400	1.37099	1.36776	1.36447
76	1.38020	1.37717	1.37393	1.37063
77	1.38642	1.38337	1.38013	1.37682
78	1.39268	1.38962	1.38637	1.38305
79	1.39896	1.39589	1.39264	1.38931
80	1.40528	1.40219	1.39895	1.39560
81	1.41162	1.40852	1.40528	1.40193
82	1.41800	1.41488	1.41165	1.40829
83	1.42441	1.42128	1.41805	1.41469
84	1.43084	1.42770	1.42449	1.42111
85	1.43731	1.43415	1.43095	1.42757

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