NBS MONOGRAPH 2

Temperature-Induced Stresses in Solids of Elementary Shape

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Temperature-Induced Stresses in Solids of Elementary Shape

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In general, a solid subjected to non-uniform temperature change develops internal stresses. These are determined by, (1) the temperature distribution within the solid, and (2) certain physical constants of the material. Although the necessary fundamental relations are known, the computation of stresses in even simple solids has heretofore been a tedious operation. For two varieties of heating, the equations determining stress have now been put in convenient form for practical use, and tables of certain temperature functions are offered as a means of quickly determining stresses in a slab, in a cylinder, or in a sphere subjected to either of two modes of heating. The temperature-distribution tables independently will provide a useful means for the ready estimation of temperature gradients.

1. Introduction

Stresses in an elastic solid caused by variations in temperature may be evaluated from the temperature change, the coefficient of thermal expansion, and the elastic constants. In principle, this calculation of stress, at least for elementary solids, is quite simple, but in actual practice it may involve laborious and time-consuming computations. There are given here some equations in convenient form for calculating the stresses in solids of certain shapes when subjected (a) to a sudden change of surface temperature, or (b) to a surface temperature that increases or decreases at a constant rate. (It is assumed that there is no creep or yielding in the material.)

2. Sudden Change of Surface Temperature

Slab.—For the case of a slab that is assumed to be of large enough extent so that end effects can be neglected, the stress is in a direction parallel to the face of slab, and it varies, in both magnitude and sign, from surface to the center of the slab. In the following treatment, average values of the elastic constants and the coefficient of thermal expansion over the temperature range are assumed to be adequate. A basis for calculation is found in the equations given by Hopkinson [1],[1] Williamson [2], Williamson and Adams [3], and Adams and Williamson [4].

In order to visualize the stress relations in a slab, let us consider first the case of a slab consisting of a number of detached layers. Upon being heated uniformly on both faces, the outer layers will expand more than the inner layers, as in figure 1a, but if the layers are not separated, the tendency of the outer layers to expand relatively to the inner layers will be partially offset by the constraint imposed by adjacent layers and the block will retain its original shape, as in figure 1b. It is as if in a solid block the final effect were the sum of two effects, namely, (1) the lengthening of each layer by an amount determined by the temperature change, and (2) a shortening or lengthening of the layer owing to the influence of other layers.

The stress distribution is found to be simply related to the thermal expansion and the elastic constants. Let F be the stress (positive for a tension and negative for a compression), θ the temperature at a distance x from the center of the slab, and α the thermal expansibility. Furthermore, let e and f be, respectively, the relative longitudinal lengthening and transverse shorten-

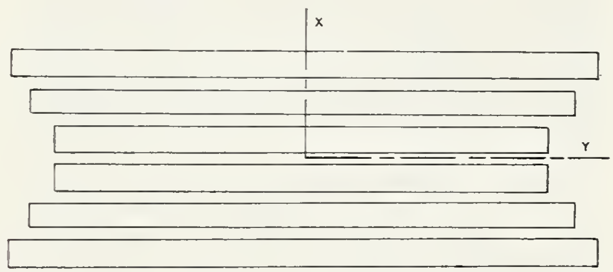

Figure 1a. *The relative changes in dimension in a slab which is being heated when the slab consists of detached layers.*

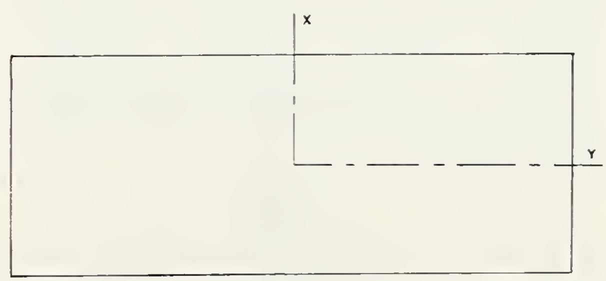

Figure 1b. *There are no relative changes in dimension in a slab which is being heated if the slab is one solid piece.*

[1] Figures in brackets indicate the literature references at the end of this Monograph.

ing produced by unit stress. From symmetry considerations it is evident that in an infinite slab the stresses are the same in all directions parallel to the faces of the slab. It is to be noted that these stresses are purely tensile or compressive and there is no shearing force in planes parallel to the faces of the slab.

Considering a small rectangular element in any plane parallel to the face of the slab, a stress in a given direction will produce a lengthening equal to Fe, while the equal stress in a transverse direction will produce a shortening (in the initial direction) equal to Ff, the aggregate effect being $F(e-f)$. The change in length of any element is also dependent on the temperature; and the thermal and the elastic effects are found to be related in such a way that the sum of the two effects is a constant independent of x. This is the fundamental basis of the present treatment, and may be illustrated as follows.

Suppose that, similar to figure 1, we have a slab consisting of two layers on each side of central plane, and suppose that the layers, originally identical in size, are caused to expand symmetrically by applying a temperature gradient such that $\theta=gx$, the distance from the central plane to the middle of the slab being x and the temperature gradient being g. Then, if we take x as equal to 1 and 2, respectively, for the inside and the outside layers, and if the thermal expansibility is α, the relative lengthening in each transverse direction will be αg and $2\alpha g$. Now suppose that we bring the inner and the outer layers to the same length by stretching the former by $\frac{1}{2}\alpha g$ and compressing the letter by $\frac{1}{2}\alpha g$. The corresponding stresses F will be $\frac{1}{2}\alpha g/(e-f)$ and $-\frac{1}{2}\alpha g/(e-f)$. Finally, we fasten the adjacent surfaces together and remove the external restraints, noting that by reasons of symmetry there will be no overall warping or bending. Simple addition (algebraically) of the effects (neglecting those of the second order) shows that for each of the layers $F(e-f)+\alpha\theta$ is equal to $3/2\alpha g$.

Applying the same procedure to a slab with four layers on each side, first dealing with each outer and each inner pair as in the above example, and then treating the groups of two pairs in the same fashion, we find that in this case $F(e-f)+\alpha\theta$ is equal to a constant, $5/2\alpha g$, for all layers; and it is clear that by further subdivision of the slab into thinner and thinner layers the constancy of the sum of the thermal and mechanical effects will continue to exist, and furthermore that in the limit $F(e-f)+\alpha\theta$ is accurately a constant, which in the simple cases considered here will always be α times the average temperature from center to surface. We have, then,

$$F=\frac{C-\alpha\theta}{(e-f)}$$

or

$$F=C_1-\frac{\alpha\theta}{(e-f)} \tag{1}$$

C_1 being a constant. It may be noted that our fundamental relation is equivalent to the statement that by the aggregate effect of temperature and stress the plane surfaces in the slab remain plane.

It may be noted that $1/e$ is Young's modulus, f/e is Poisson's ratio, and $e-f=1/6G+2/9K$, G and K being, respectively, the rigidity modulus and the bulk modulus, and C_1 is a constant to be evaluated from the condition of mechanical equilibrium to the effect that the algebraic summation of the forces in any direction parallel to the faces of the slab is zero. That is,

$$\int_0^a Fdx=0$$

so that

$$C_1=\frac{\alpha}{(e-f)a}\int_0^a \theta dx. \tag{2}$$

For convenience we employ instead of θ, a temperature function ϕ_1 which may be called the fractional temperature excess for a slab, and is defined by

$$\phi_1=\frac{\theta-\theta_f}{\theta_i-\theta_f} \tag{3}$$

in which θ_i is the initial uniform temperature of the material, and θ_f is the final constant temperature. Inserting in eq (1) the value of C_1 from eq (2) and transforming, we have

$$F=H(\psi_1'-\phi_1), \tag{4}$$

in which

$$H=\frac{\alpha(\theta_i-\theta_f)}{e-f}, \tag{5}$$

and ψ' is the definite integral

$$\psi_1'=\int_0^1 \phi_1 d\left(\frac{x}{a}\right). \tag{6}$$

The function ψ_1' is merely the average fractional temperature excess at any given time (averaged from center to surface). It may be noted that in applying eq (1) the stress, F, is in the same units as the elastic moduli, G and K.

To calculate the stress, then, we need to determine (1) the temperature at a specified time and location, and (2) the average temperature from center to surface at the specified time.

There are two types of equations for calculating the temperature distribution in a slab the temperature of which is changing, a cosine-exponential form and a probability-integral form. The former, for sudden change of surface temperature from an originally uniform temperature, is as follows [3].

$$\phi_1=2\sum_1^m e^{-\frac{Qm^2}{4q^2}}\frac{(-1)^{m+1}}{Qm}\cos\left(Qm\frac{x}{a}\right), \tag{7}$$

in which m means the succession of integers beginning with unity; and $Qm \equiv (2m-1)(\pi/2)$. Of the two dimensionless parameters, x/a and q, the latter is defined by:

$$q \equiv \frac{a}{2\sqrt{\kappa t}}, \tag{8}$$

κ being the thermal diffusivity which is the thermal conductivity divided by the product of the specific heat and density. It may be noted that at the niddle of the slab eq (7) simplifies to:

$$\phi_{1(x=0)} = 2\sum_{1}^{m} e^{\frac{-Qm^2}{4q^2}} \frac{(-1)^{m+1}}{Qm}. \tag{9}$$

The cosine-exponential series converges rapidly for large values of the time, or small values of a (or both). Indeed, if an accuracy of 0.0001 in ϕ_1 is sufficient, only one term of the series is necessary whenever q is less than 0.8 (e.g., when t is greater than 100 sec, κ being 0.004 cm^2/sec and a being 1 cm). On the other hand, for short times or large dimensions many terms of the series may be required.

An alternate equation, [5, 6], for temperatures in a slab is the infinite series of probability integrals:

$$\phi_1 = \text{erf}\, q(1-x/a) + \sum_{0}^{m} [\text{erf}\, q(2m+3-x/a) \; . \; -\text{erf}\, q(2m+1+x/a)](-1)^{m+1}, \tag{10}$$

in which erf (A) is written for $\frac{2}{\sqrt{\pi}}\int_0^A e^{-Z^2}\,dZ$, Z being merely an integration variable.

In contrast with the cos—exp equation, the series (10) converges rapidly for short times, or large dimensions (or both). For an accuracy of 0.0001 in ϕ_1 only one term of the series is required whenever q is greater than 2.7 (e.g., when t is less than 8 sec, if $\kappa = 0.004$ and $a = 1$). In general the convergence is more rapid near the surface than near the center. The same is true for the cos—exp formula.

As an example, for $t=120$ sec and $a=1$ cm, ($q=0.72$), one term of the cos—exp formula is sufficient for obtaining an accuracy of 0.0001 in the fractional temperature excess, while three terms of the prob—int series are required for the same accuracy with a given q. The latter series at longer times requires still more terms—as many as six for $t=700$, $a=1(q=0.3)$. By contrast, for the relatively short time, $t=8$, $a=1(q=2.8)$ the cos—exp formula requires five terms and the prob—int only one.

There is an intermediate range of times (in relation to slab dimensions) for which neither formula can be depended on to converge sufficiently well with one term.

The average temperature, ψ_1', one of the basic factors for determining the temperature-induced stresses, can be computed by applying (6) to either (7) or (10). Thus from the cos—exp formula we have

$$\psi_1' = 2\sum_{1}^{m} e^{-Qm^2/4q^2}/Qm^2. \tag{11}$$

Here, we have been able to integrate analytically, but when it is advantageous to use the *prob—int* formula (as, for example, when t is relatively small) it is necessary to have recourse to numerical integration. In this case, a table of values of ψ_1' can be prepared by calculating ϕ_1 for each desired value of q at several values of x/a, and then applying eq (6) by numerical integration. Alternatively, as a more convenient method for a certain range in t (or more properly in q), the procedure may be as follows: First, for the various values of q, we compute ψ_{11}', which is here defined as the average value of the first term of the summation in eq (11) as x/a varies from 0 to 1. Then from the ψ_{11}''s it turns out that we may readily compute ψ_1' for the given q by the expression

$$\begin{aligned}\psi_1'(q) = \psi_{11}'(q) - [&(3\psi_{11}'(3q) - 2\psi_{11}'(2q)) - (2\psi_{11}'(2q) - \psi_{11}'(q))] \\ +[&(5\psi_{11}'(5q) - 4\psi_{11}'(4q)) - (4\psi_{11}'(4q) - 3\psi_{11}'(3q))] \\ -[&(7\psi_{11}'(7q) - 6\psi_{11}'(6q)) - (6\psi_{11}'(6q) - 5\psi_{11}'(5q))] + \;.\;.\;.\end{aligned} \tag{12}$$

This procedure will permit an appreciable saving in time for calculating ψ_1' when high-speed computing devices are not available. As will appear below, the principal shortcut, for this and other shapes, is achieved through the tabulation of certain temperature-distribution functions. The series given in eq (12) converges rapidly except for relatively small values of q. Thus for $q=0.6$ only two of the terms in square brackets of eq (12) are required for an accuracy of 0.0001 in ψ_1'; for $q=1.0$ only one term (in square brackets); and for $q=2.5$ ψ_1' equals the simply calculated ψ_{11}' within the above specified precision.

Cylinder.—The stresses which develop in a cylinder having a radial temperature gradient may be visualized if the cylinder is first thought of as consisting of a number of separate, concentric, cylindrical shells. These shells would separate if a temperature gradient were established, and there would be radial, tangential and axial strains in each shell. However, since a solid cylinder does not consist of separate shells, internal stresses are created, and the combination of the thermal and elastic effects gives the total strain. The stresses in the cylinder may be evaluated from these equations for strain and additional equations which express the mechanical equilibrium which must exist [2].

For the radial stress, F_r, in a cylinder, we have an extension and modification of the relations given by Williamson [2].

$$F_r = C_2 - \frac{\alpha}{(e-f)x^2}\int_0^x x\theta\, dx, \tag{13}$$

in which C_2 is a constant to be evaluated from the condition that $F_r=0$ when $x=a$. Thus we have

$$C_2 = \frac{\alpha}{(e-f)a^2}\int_0^a x\theta dx. \tag{14}$$

Making the transformations similar to those employed for the slab, we find that the convenient working equation for the cylinder is

$$F_r = \mathrm{H}(\psi_2' - \psi_2) \tag{15}$$

in which

$$\psi_2 = \frac{1}{(x/a)^2}\int_0^{x/a} \frac{x}{a}\phi_2 d\left(\frac{x}{a}\right), \tag{16}$$

ϕ_2 being the fractional temperature excess for a cylinder; and ψ_2' is the value of ψ_2 at $x/a=1$. It may be noted that ψ_2 is one-half the average value of ϕ_2 when averaged with respect to $(x/a)^2$. End effects being neglected, we have for a cylindrical rod a radius a, the surface temperature of which has been suddenly changed, the following equation for ϕ_2, at the time t and the fractional distance x/a from the center:

$$\phi_2 = 2\sum_1^m e^{-\frac{R_m^2}{4q^2}} \frac{1}{R_m J_1(R_m)} J_0(R_m x/a), \tag{17}$$

in which J_0 and J_1, respectively, stand for Bessel functions of the zero'th and first order, and the R_m's are the successive roots of $J_0=0$.

The tangential stress F_t is related to the radial stress as follows,

$$F_t = F_r + x\frac{dF_r}{dx} \tag{18}$$

and is readily found to be

$$F_t = H(\psi_2' + \psi_2 - \phi_2). \tag{19}$$

The function ψ_2 can be calculated by numerical integration from previously determined ϕ_2, or by substituting (17) into (16) and integrating. Thus

$$\psi_2 = \frac{2}{(x/a)^2}\sum_1^m e^{-\frac{R_m^2}{4q^2}} \frac{1}{R_m^2 J_1(R_m)} \frac{x}{a} J_1(R_m x/a). \tag{20}$$

In determining ψ_2', the value of ψ_2 at $x=a$, it is to be noted that when $x=a$, $J_1(R_m x/a) = J_1(R_m)$.

Sphere.—In a manner analogous to that used to analyze the thermal stresses which develop in a slab and in a cylinder, a sphere may be imagined which consists of a number of detached, concentric, spherical shells. These shells will separate when a temperature gradient is established and show radial and tangential strains. These thermal effects may also be seen in a sphere which is not segmented, and, in addition, there are elastic effects in this case, because the solid sphere acts as an entity, and it may be imagined that the shells are bound together. The equations for total strain, together with those for the mechanical equilibrium which must exist in the sphere, suffice to calculate the internal stresses. The following equations are extensions of those given by Williamson [2].

In a sphere the radial stress is determined by

$$F_r = C_3 - \frac{2\alpha}{(e-f)}\frac{1}{x^3}\int_0^x x^2\theta dx. \tag{21}$$

The constant C_3, as in the case of the cylinder, is evaluated by the condition that $F_r=0$ where $x=a$. Thus

$$C_3 = \frac{2\alpha}{e-f}\frac{1}{a^3}\int_0^a x^2\phi dx, \tag{22}$$

and we therefore have

$$F_r = 2H(\psi_3' - \psi_3). \tag{23}$$

The function ψ_3 is defined by

$$\psi_3 = \frac{1}{(x/a)^3}\int_0^{x/a} (x/a)^2\phi_3 d\left(\frac{x}{a}\right) \tag{24}$$

in terms of ϕ_3 the fractional temperature excess for a sphere; and ψ_3' is the value of ψ_3 for $x/a=1$. It may be noted that ψ_3 is one-third of the average value of ϕ_3 when averaged with respect to (x/a).[3]

For the tangential stress

$$F_t = F_r + \frac{x}{2}\frac{dFr}{dx}, \tag{25}$$

and the working equation is, thus

$$F_t = H(2\psi_3' + \psi_3 - \theta_3). \tag{26}$$

The fractional temperature excess ϕ_3 in the sphere, as in the case of the slab, can be calculated from either of two equations. The former is

$$\phi_3 = 2\sum_1^m e^{-\frac{S_m^2}{4q^2}} \frac{(-1)^{m+1}}{S_m}\frac{a}{x}\sin(S_m x/a) \tag{27}$$

in which S_m is written for $m\pi$. Alternately, we may use the probability-integral equation [5,6]:

$$\phi_3 = \frac{x-a}{x} + \frac{a}{x}\left\{\operatorname{erf} q(1-x/a) + \sum_0^m [\operatorname{erf} q(3m+3-x/a) - \operatorname{erf} q(2m+1+x/a)]\right\}. \tag{28}$$

Similarly to the cases for the slab and the cylinder the function ψ_3 can be computed by numerical integration; or for the sphere it can be determined by substituting (27) into (24) and integrating. Thus

$$\psi_3=\frac{2}{(x/a)^3}\sum_1^m e^{-\frac{S_m^2}{4q^2}}\frac{1}{S_m{}^3}(\sin(S_m x/a)-S_m x/a \cos(S_m x/a))(-1)^{m+1}. \quad (29a)$$

It is readily found that

$$\psi_3'=2\sum_1^m e^{-\frac{S_m^2}{4q^2}}/S_m^2. \quad (29b)$$

3. Linear Heating

For the important case in which the surface temperature is changed at a constant linear rate, the internal temperature, as shown by Williamson and Adams [3], can be calculated by equations consisting of three terms, a linear one, a parabolic one, and a third one with an exponential involving the time t. When t becomes large with respect to the size a, the exponential term becomes vanishingly small, the temperature distribution is essentially parabolic and the equations for θ take the simple forms for the "steady state":

Slab $$\theta-\theta_i=ht-\frac{h}{2\kappa}(a^2-x^2) \quad (30)$$

Cylinder $$\theta-\theta_i=ht-\frac{h}{4\kappa}(a^2-x^2) \quad (31)$$

Sphere $$\theta-\theta_i=ht-\frac{h}{6\kappa}(a^2-x^2) \quad (32)$$

in which h is the rate at which the surface is being heated. Having these temperatures we may apply eqs (1), (2), (13), (14), (21), and (22) to (30), (31), and (32), and we find that the stresses are

Slab $$F=\frac{L}{6}(a^2-3x^2) \quad (33)$$

Cylinder $$F_r=\frac{L}{16}(a^2-x^2) \quad (34)$$

$$F_t=\frac{L}{16}(a^2-3x^2) \quad (35)$$

Sphere $$F_r=\frac{L}{15}(a^2-x^2) \quad (36)$$

$$F_t=\frac{L}{15}(a^2-2x^2). \quad (37)$$

In these equations

$$L=\frac{\alpha h}{\kappa(e-f)}.$$

Especially for the case of sudden change of surface temperature, the complexity of the equations for temperature and stress have made the calculation of numerical values a time-consuming process. Extended tables of various temperature functions have now been prepared by the Computation Section of the Applied Mathematics Division of the National Bureau of Standards. By the use of these tables the calculation of stresses in many important instances becomes a matter of simple addition and multiplication. At present there are five tables, namely for ϕ_1 (together with ψ_1'), ϕ_2, ψ_2, ϕ_3, and ψ_3, and these are designated by numbers 5 through 9, respectively.

Quite apart from the utility of the tables in connection with stress calculation, the tables for the ϕ's may prove to have wide application as a labor-saving device in problems involving the estimation of temperature distribution. In this regard it is important to note that for the case of sudden change of surface temperature the tables may be used for calculating temperatures in certain other shapes. Thus, from the complete equation for temperatures in a parallelepiped, it turns out that such temperatures may be obtained by multiplying together 3 ϕ_1's each taken from the table with its appropriate parameter. Furthermore, temperatures in a finite cylinder, for this species of temperature change, may be obtained by multiplying the appropriate ϕ_1 and ϕ_2.

A numerical example of the use of the tables for determining stresses in simple solids subjected to a sudden change of surface temperature is as follows:

The example chosen is for the case of a slab of Pyrex glass 7740, 2 cm in thickness (a=1), the surface temperature having been changed suddenly from 100° to 0° C. The appropriate parameters are found in tables 1 and 2. Since $q=a/2\sqrt{\kappa t}$, after a time lapse of 60 sec q will be 0.786.

TABLE 1. *Parameters for the calculation of stress*

Material	Coefficient of thermal expansion, α (range 0°–100° C)	Diffusivity, κ (at 100° C)	Parameter for elastic effects $(e-f)$ (at 100° C)
	$°C^{-1}$	cm^2sec^{-1}	cm^2kg^{-1}
Plate glass	7.80×10^{-6}	5.49×10^{3}	1.18×10^{-6}
Pyrex 7740	3.46	6.74	1.27
Vycor	0.83	8.82	1.27
Fused silica	.48	9.80	1.16
Pure aluminum	23.5	964	0.951
Pure iron	12.1	198	.359

For q=0.786 it is necessary to interpolate between the values of fractional temperature excess given in table 5. To find the distribution of stress we apply the working equation for the slab,

$$F=H(\psi_1'-\phi_1) \quad (4)$$

TABLE 2. *Calculated thermal stresses in a slab of Pyrex 7740*

x/a*	$q=0.7800$**	$q=0.8000$	$=0.786$	$(\psi_1'-\phi_1)$***	F, kg/cm^2	Temp. °C
0.0	0.4619	0.4856	0.4690	−0.1704	−46.3	46.9
.1	.4562	.4796	.4632	−.1646	−44.8	46.3
.2	.4393	.4618	.4461	−.1475	−40.1	44.6
.3	.4116	.4327	.4179	−.1193	−32.4	41.8
.4	.3737	.3929	.3795	−.0809	−22.0	38.0
.5	.3267	.3435	.3317	−.0331	−9.0	33.2
.6	.2716	.2855	.2758	.0228	6.2	27.6
.7	.2098	.2206	.2130	.0856	23.3	21.3
.8	.4128	.1501	.1450	.1536	41.8	14.5
.9	.0723	.0760	.0734	.2252	61.3	7.3
1.0	.0000	.0000	.0000	.2986	81.2	0
ψ_1':	.2941	.3092	.2986	----------	----------	-------

*Distance from center plane/slab semi-thickness.

**$q=\frac{a}{2\sqrt{\kappa t}}$ where t is the elapsed time and κ is the thermal diffusivity.

***ϕ_1 is the fractional temperature excess and ψ_1' is the average fractional temperature excess (see p. 2).

where

$$H=\frac{\alpha(\theta_i-\theta_f)}{e-f}, \tag{5}$$

and, substituting numerical values, we have $H=272$. With appropriate interpolations, values of fractional temperature excess and of the stress are found as in table 2.

A graphical representation of this variation of stress with position in the slab is shown in figure 2, together with curves for other values of elapsed time.

To find the temperature at various points in the slab for the 100 deg temperature change it is necessary merely to multiply by a factor of 100 the values of fractional temperature excess given in the fourth column of table 2. A plot of this variation of temperature with position in the slab is given in figure 3, together with curves for other values of elapsed time.

In order to gain a better conception of these effects in different materials and with different slab thicknesses and elapsed times, tables 3 and 4 have been prepared, temperature distribution being shown in table 3 and stress distribution in table 4. For illustrative purposes it was felt to be of interest to consider the case of pure aluminum and of pure iron, although the parameters for these materials are not as well known as the corresponding ones for glasses.

Figure 4 shows the distribution of stress in a 2-cm thick slab of plate glass at selected time intervals after the surface temperature has been changed suddenly from 100° to 0° C. Figure 5 affords a comparison of the distribution of stress in four commercial glasses which have undergone a sudden change in surface temperature under the same boundary conditions.

The parameters which were needed for these calculations were obtained from the literature [7, 8, 9], and are given in table 1. The values for plate glass, iron and aluminum are only approximate.

For further illustration an example of stress in a cylinder of plate glass with linear surface heating is given in figure 6.

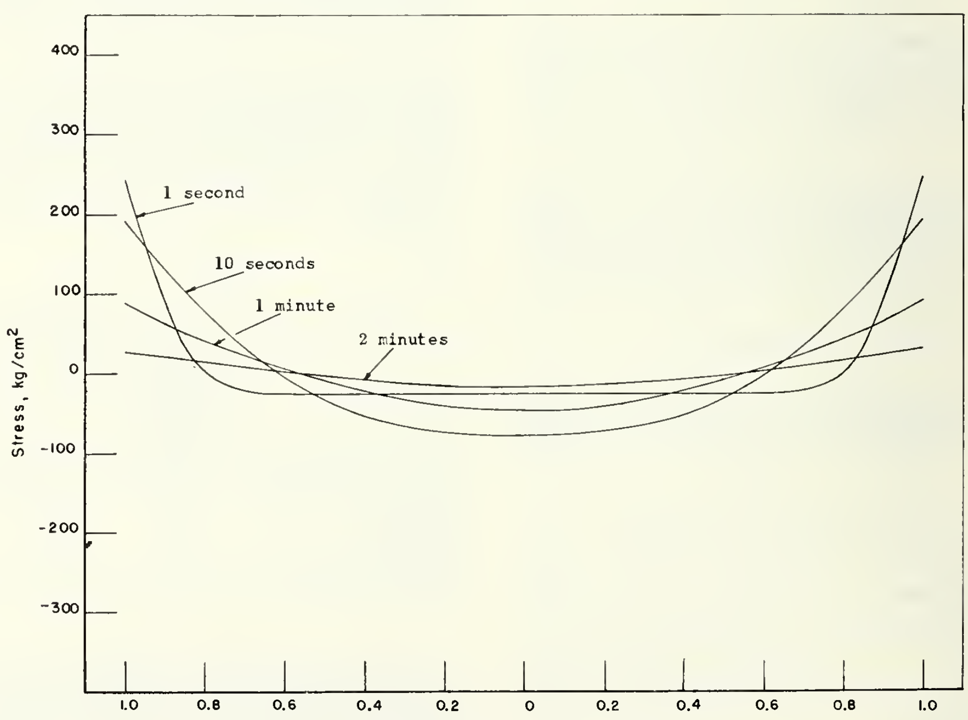

FIGURE 2. *Curves showing the calculated distribution of stress in a slab of Pyrex 7740 2-cm thick, when the surface temperature has been changed suddenly from 100° to 0° C.*

The curves show the distribution of stress which exists after a time lapse of 1 sec, 10 sec, 1 min, and 2 min.

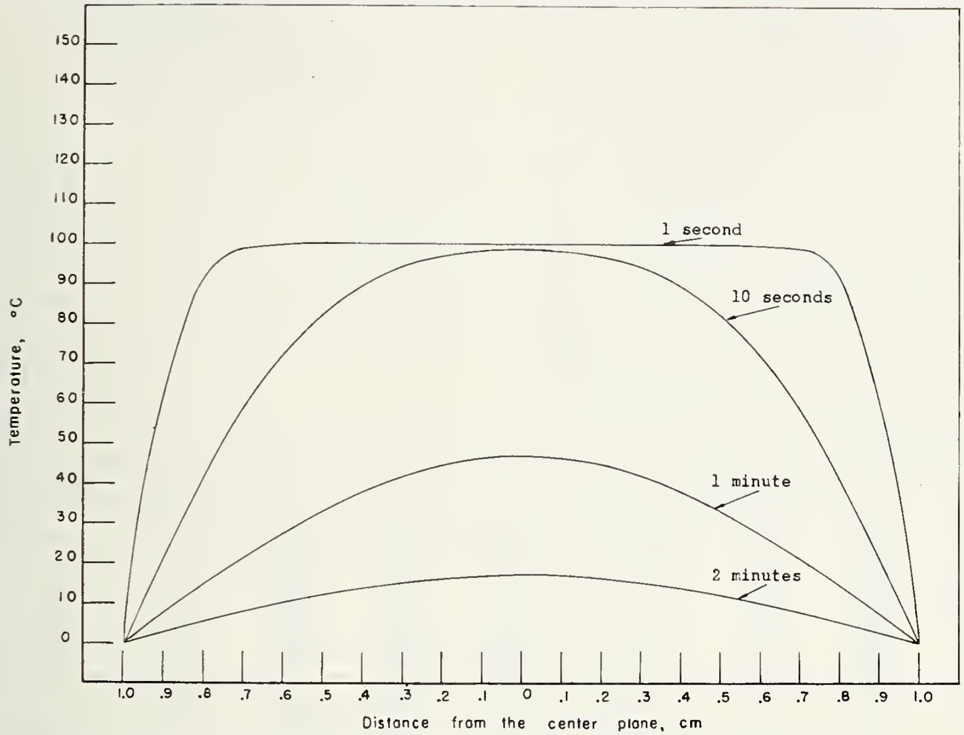

FIGURE 3. *Curves showing the calculated distribution of temperature in a slab of Pyrex 7740 2-cm thick, when the surface temperature has been changed suddenly from 100° to 0° C.*

The curves show the distribution of temperature which exists after a time lapse of 1 sec, 10 sec, 1 min, and 2 min.

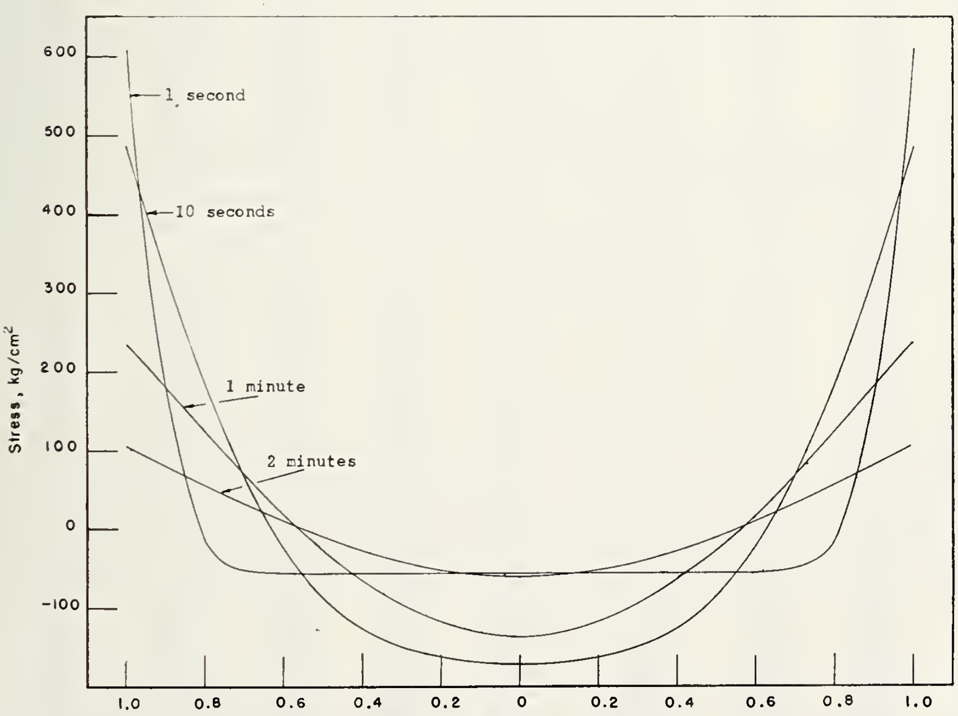

FIGURE 4. *Curves showing the calculated distribution of stress in a slab of plate glass 2-cm thick, when the surface temperature has been changed suddenly from 100° to 0° C.*

The curves show the distribution of stress which exists after a time lapse of 1 sec, 10 sec, 1 min, and 2 min.

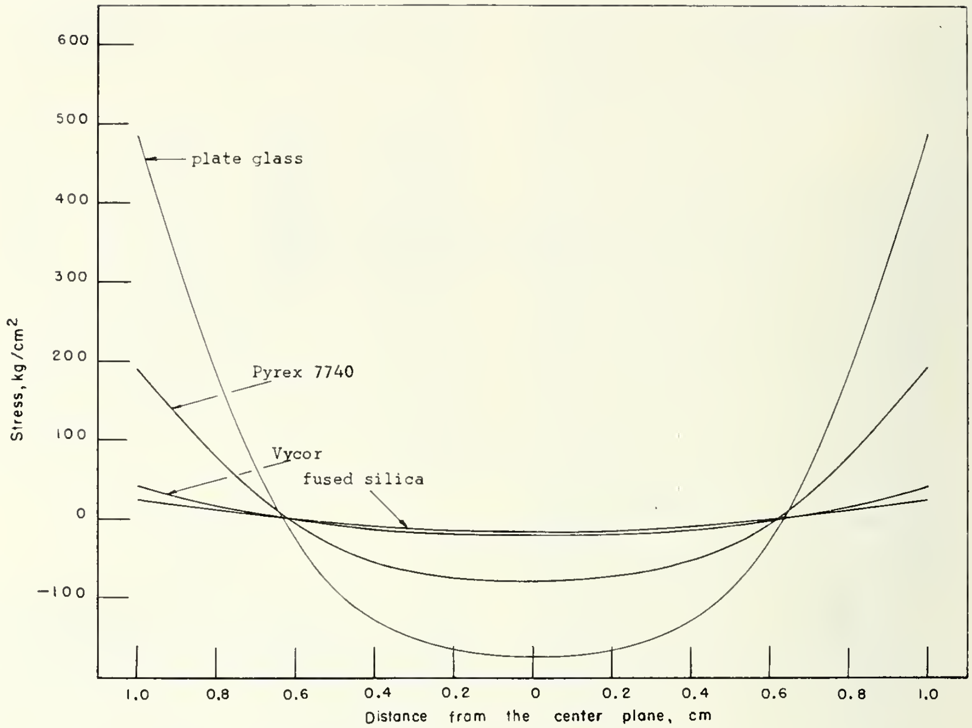

FIGURE 5. *Curves drawn to compare the stresses which develop in slabs of glass 10 sec after the surface temperature has been changed suddenly from 100° to 0° C.*

The values are calculated for slabs 2-cm thick.

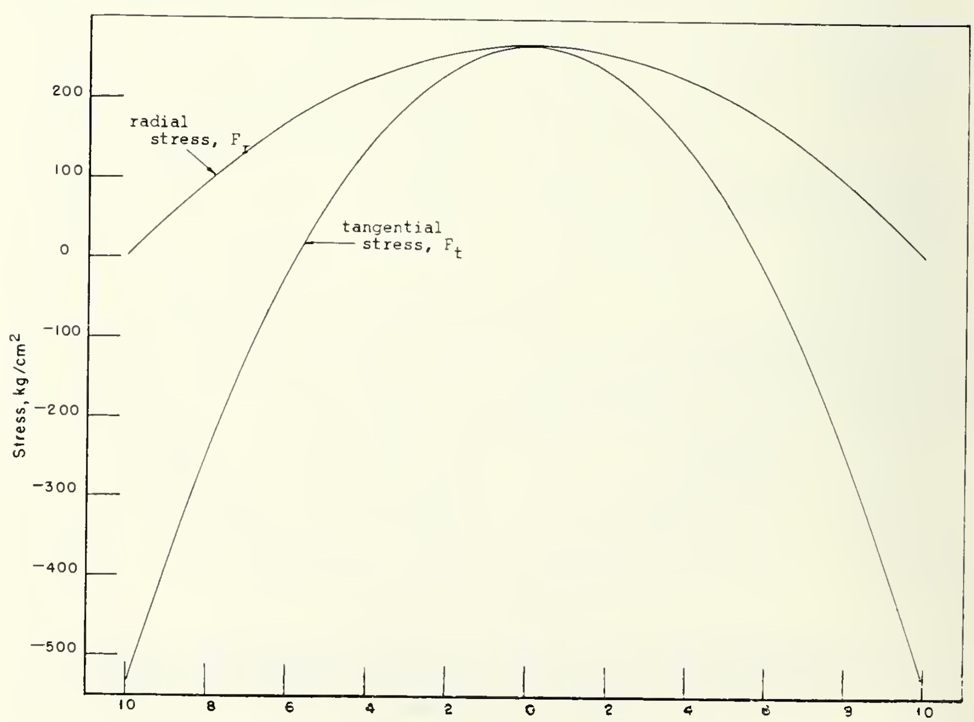

FIGURE 6. *Curves showing the calculated distribution of radial and tangential stresses in a cylinder of plate glass 20 cm in diameter which is being heated linearly at a rate of 2° C/min.*

TABLE 3. *Temperature distribution, ° C in various materials suddenly cooled**

x/a**	Pyrex $a=1$ $t=60$***	Pyrex $a=1$ $t=2$	Aluminum $a=1$ $t=2$	Aluminum $a=1$ $t=1$	Aluminum $a=10$ $t=1$	Iron $a=1$ $t=2$	Iron $a=1$ $t=1$
0.0	47	100	1	12	100	48	77
.1	46	100	1	12	100	47	76
.2	45	100	1	11	100	46	74
.3	42	100	1	11	100	43	69
.4	38	100	1	10	100	39	63
.5	33	100	1	8	100	34	55
.6	28	99	1	7	100	28	46
.7	21	97	1	5	97	22	36
.8	15	84	0	4	85	15	24
.9	7	52	0	2	53	8	12
1.0	0	0	0	0	0	0	0

*See p. 6.
**Distance from center plane/slab semithickness.
***Time in seconds.

TABLE 4. *Thermal stress, kg/cm² in various materials suddenly cooled**

x/a**	Pyrex $a=1$ $t=60$***	Pyrex $a=1$ $t=2$	Aluminum $a=1$ $t=2$	Aluminum $a=1$ $t=1$	Aluminum $a=10$ $t=1$	Iron $a=1$ $t=2$	Iron $a=1$ $t=1$
0.0	−46	−31	−10	−106	−274	−587	−933
.1	−45	−31	−10	−103	−274	−568	−902
.2	−40	−31	−9	−92	−274	−508	−811
.3	−32	−31	−7	−74	−274	−411	−612
.4	−22	−31	−5	−50	−274	−279	−454
.5	−9	−31	−2	−21	−273	−114	−193
.6	6	−30	1	14	−264	79	−116
.7	23	−22	5	53	−198	295	466
.8	42	13	9	95	97	529	850
.9	61	100	13	140	892	776	1,256
1.0	81	241	17	185	2,197	1,029	1,674

*See p. 6.
**Distance from center plane/slab semithickness.
***Time in seconds.

TABLE 5. *Fractional temperature excess, ϕ_1, and generalized temperature averages, ψ'_1, for a slab*

x/a \ q	0.3000	0.3200	0.3400	0.3600	0.3800	0.4000	0.4200	0.4400	0.4600	0.4800
0.	0.0013	0.0031	0.0061	0.0109	0.0178	0.0270	0.0386	0.0526	0.0690	0.0875
0.10	.0013	.0030	.0061	.0108	.0176	.0266	.0381	.0520	.0682	.0865
0.20	.0013	.0029	.0058	.0104	.0169	.0256	.0367	.0500	.0656	.0832
0.30	.0012	.0027	.0055	.0097	.0158	.0240	.0344	.0469	.0615	.0780
0.40	.0011	.0025	.0050	.0088	.0144	.0218	.0312	.0426	.0558	.0708
0.50	.0010	.0022	.0043	.0077	.0126	.0191	.0273	.0372	.0488	.0619
0.60	.0008	.0018	.0036	.0064	.0104	.0158	.0227	.0309	.0406	.0515
0.70	.0006	.0014	.0028	.0050	.0081	.0122	.0175	.0239	.0313	.0397
0.80	.0004	.0010	.0019	.0034	.0055	.0083	.0119	.0163	.0213	.0270
0.90	.0002	.0005	.0010	.0017	.0028	.0042	.0060	.0082	.0108	.0137
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ_1'	0.0009	0.0020	0.0039	0.0069	0.0113	0.0172	0.0246	0.0335	0.0439	0.0557

x/a \ q	0.5000	0.5200	0.5400	0.5600	0.5800	0.6000	0.6200	0.6400	0.6600	0.6800
0.	0.1080	0.1301	0.1535	0.1781	0.2035	0.2295	0.2559	0.2824	0.3090	0 3354
0.10	.1066	.1285	.1516	.1759	.2010	.2267	.2527	.2789	.3052	.3313
0.20	.1027	.1237	.1460	1694	1935	2183	2433	2686	2938	3190
0.30	.0962	.1159	.1368	.1587	.1813	.2045	.2280	.2516	.2753	.2988
0.40	.0874	1052	1242	1441	.1646	1857	2070	2285	2500	2713
0.50	0764	.0920	.1086	.1259	.1439	.1623	.1809	.1997	.2185	.2372
0.60	.0635	.0765	.0902	.1047	.1196	.1349	.1504	.1660	.1816	.1971
0.70	.0490	.0591	.0697	.0809	.0924	.1042	.1162	.1282	.1403	.1523
0.80	.0334	.0402	.0474	.0550	.0629	.0709	.0791	.0873	.0955	.1036
0.90	.0169	.0203	.0240	.0279	.0318	.0359	.0400	.0442	.0483	.0525
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ_1'	0.0687	0.0828	0.0977	0.1134	0.1295	0.1461	0.1629	0.1798	0.1967	0.2135

x/a \ q	0.7000	0.7200	0.7400	0.7600	0.7800	0.8000	0.8200	0.8400	0.8600	0.8800
0.	0.3616	0.3874	0.4127	0.4376	0.4619	0.4856	0.5086	0.5310	0.5527	0.5738
0.10	.3571	.3826	.4077	.4322	.4562	.4796	.5024	.5245	.5459	.5667
0.20	.3439	.3684	.3925	.4162	.4393	.4618	.4838	.5051	.5258	.5458
0.30	.3222	.3452	.3678	.3899	.4116	.4327	.4533	.4733	.4927	.5115
0.40	.2925	.3134	.3339	.3541	.3737	.3929	.4116	.4298	.4474	.4645
0.50	.2557	.2739	.2919	.3095	.3267	.3435	.3598	.3757	.3912	.4062
0.60	.2125	.2277	.2426	.2573	.2716	.2855	.2991	.3124	.3252	.3377
0.70	.1642	.1759	.1874	.1987	.2098	.2206	.2311	.2413	.2513	.2609
0.80	.1117	.1197	.1276	.1353	.1428	.1501	.1573	.1643	.1711	.1777
0.90	.0566	.0606	.0646	.0685	.0723	.0760	.0796	.0832	.0866	.0900
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ_1'	0.2302	0.2466	0.2628	0.2786	0.2941	0.3092	0.3239	0.3382	0.3521	0.3655

x/a \ q	0.9000	0.9200	0.9400	0.9600	0.9800	1.0000	1.0200	1.0400	1.0600	1.0800
0.	0.5941	0.6137	0.6327	0.6509	0.6685	0.6854	0.7017	0.7173	0.7323	0.7467
0.10	.5868	.6062	.6250	.6430	.6604	.6772	.6933	.7087	.7236	.7378
0.20	.5652	.5839	.6020	.6195	.6363	.6525	.6681	.6831	.6976	.7114
0.30	.5297	.5473	.5643	.5807	.5966	.6119	.6267	.6410	.6547	.6680
0.40	.4811	.4972	.5127	.5278	.5423	.5564	.5700	.5831	.5958	.6081
0.50	.4207	.4348	.4485	.4617	.4746	.4870	.4991	.5108	.5221	.5331
0.60	.3499	.3617	.3731	.3842	.3950	.4054	.4156	.4255	.4351	.4445
0.70	.2704	.2795	.2884	.2970	.3054	.3136	.3215	.3293	.3368	.3442
0.80	.1841	.1903	.1964	.2023	.2081	.2137	.2191	.2245	.2297	.2348
0.90	.0932	.0964	.0995	.1025	.1054	.1082	.1110	.1137	.1164	.1190
1.00	0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ_1'	0.3786	0.3912	0.4034	0.4153	0.4267	0.4378	0.4485	0.4588	0.4688	0.4784

TABLE 5. *Fractional temperature excess, ϕ_1, and generalized temperature averages, ψ'_1, for a slab*—Continued

x/a \ q	1.1000	1.1200	1.1400	1.1600	1.1800	1.2000	1.2200	1.2400	1.2600	1.2800
0.	0.7604	0.7736	0.7862	0.7982	0.8097	0.8206	0.8311	0.8410	0.8505	0.8595
0.10	.7515	.7645	.7771	.7890	.8005	.8114	.8218	.8318	.8412	.8503
0.20	.7248	.7376	.7498	.7616	.7729	.7837	.7941	.8040	.8135	.8226
0.30	.6807	.6930	.7048	.7162	.7272	.7378	.7480	.7578	.7672	.7763
0.40	.6200	.6315	.6426	.6534	.6638	.6739	.6837	.6932	.7024	.7113
0.50	.5438	.5542	.5643	.5741	.5837	.5930	.6020	.6109	.6195	.6280
0.60	.4536	.4625	.4712	.4797	.4880	.4962	.5042	.5120	.5197	.5272
0.70	.3514	.3585	.3655	.3723	.3789	.3855	.3920	.3984	.4046	.4109
0.80	.2398	.2447	.2496	.2543	.2590	.2636	.2682	.2727	.2772	.2816
0.90	.1216	.1241	.1266	.1291	.1315	.1339	.1362	.1386	.1409	.1432
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.4878	0.4968	0.5055	0.5140	0.5221	0.5301	0.5377	0.5451	0.5523	0.5593

x/a \ q	1.3000	1.3200	1.3400	1.3600	1.3800	1.4000	1.4200	1.4400	1.4600	1.4800
0.	0.8680	0.8761	0.8838	0.8911	0.8980	0.9046	0.9108	0.9166	0.9221	0.9273
0.10	.8589	.8670	.8748	.8822	.8892	.8958	.9021	.9081	.9137	.9191
0.20	.8313	.8396	.8475	.8551	.8624	.8693	.8759	.8822	.8882	.8939
0.30	.7850	.7935	.8016	.8094	.8169	.8242	.8312	.8379	.8444	.8506
0.40	.7200	.7283	.7365	.7444	.7521	.7596	.7668	.7739	.7808	.7874
0.50	.6362	.6443	.6522	.6599	.6674	.6748	.6821	.6892	.6962	.7030
0.60	.5346	.5419	.5491	.5562	.5632	.5701	.5769	.5836	.5902	.5967
0.70	.4170	.4231	.4291	.4350	.4409	.4467	.4525	.4582	.4639	.4696
0.80	.2860	.2904	.2947	.2990	.3033	.3075	.3118	.3160	.3202	.3243
0.90	.1455	.1478	.1501	.1523	.1546	.1568	.1590	.1613	.1635	.1657
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.5661	0.5726	0.5790	0.5852	0.5912	0.5970	0.6027	0.6082	0.6136	0.6188

x/a \ q	1.5000	1.5200	1.5400	1.5600	1.5800	1.6000	1.6200	1.6400	1.6600	1.6800
0.	0.9322	0.9368	0.9412	0.9453	0.9491	0.9527	0.9561	0.9592	0.9622	0.9650
0.10	.9241	.9289	.9334	.9377	.9417	.9455	.9491	.9524	.9556	.9585
0.20	.8994	.9046	.9096	.9143	.9188	.9231	.9272	.9311	.9348	.9383
0.30	.8566	.8624	.8680	.8734	.8785	.8835	.8883	.8930	.8974	.9017
0.40	.7939	.8002	.8064	.8124	.8182	.8239	.8294	.8348	.8400	.8451
0.50	.7097	.7163	.7227	.7291	.7353	.7414	.7474	.7533	.7591	.7648
0.60	.6032	.6095	.6158	.6221	.6282	.6343	.6403	.6462	.6521	.6579
0.70	.4752	.4807	.4863	.4918	.4972	.5026	.5080	.5134	.5187	.5240
0.80	.3285	.3326	.3368	.3409	.3450	.3491	.3532	.3572	.3613	.3653
0.90	.1679	.1702	.1724	.1746	.1768	.1790	.1812	.1834	.1856	.1878
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.6239	0.6288	0.6336	0.6383	0.6429	0.6474	0.6517	0.6560	0.6601	0.6642

x/a \ q	1.7000	1.7200	1.7400	1.7600	1.7800	1.8000	1.8200	1.8400	1.8600	1.8800
0.	0.9676	0.9700	0.9723	0.9744	0.9763	0.9782	0.9799	0.9815	0.9829	0.9843
0.10	.9613	.9640	.9664	.9687	.9709	.9729	.9748	.9766	.9783	.9798
0.20	.9416	.9448	.9479	.9507	.9535	.9560	.9585	.9608	.9631	.9652
0.30	.9058	.9098	.9136	.9173	.9209	.9243	.9276	.9308	.9338	.9367
0.40	.8501	.8549	.8596	.8642	.8686	.8730	.8772	.8813	.8853	.8891
0.50	.7704	.7758	.7812	.7865	.7917	.7968	.8018	.8067	.8115	.8162
0.60	.6637	.6693	.6749	.6805	.6860	.6914	.6967	.7020	.7073	.7124
0.70	.5292	.5344	.5396	.5447	.5498	.5549	.5600	.5650	.5700	.5749
0.80	.3693	.3734	.3774	.3814	.3854	.3893	.3933	.3972	.4012	.4051
0.90	.1900	.1922	.1944	.1966	.1987	.2009	.2031	.2053	.2075	.2097
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.6681	0.6720	0.6758	0.6794	0.6830	0.6866	0.6900	0.6934	0.6967	0.6999

x/a \ q	1.9000	1.9200	1.9400	1.9600	1.9800	2.0000	2.0200	2.0400	2.0600	2.0800
0.	0.9856	0.9868	0.9878	0.9889	0.9898	0.9906	0.9914	0.9922	0.9928	0.9935
0.10	.9813	.9826	.9839	.9851	.9862	.9872	.9882	.9891	.9899	.9907
0.20	.9672	.9690	.9708	.9725	.9741	.9757	.9771	.9785	.9798	.9810
0.30	.9395	.9422	.9448	.9473	.9497	.9520	.9543	.9564	.9584	.9604
0.40	.8929	.8966	.9001	.9036	.9070	.9102	.9134	.9165	.9195	.9224
0.50	.8208	.8254	.8298	.8342	.8385	.8427	.8468	.8508	.8548	.8586
0.60	.7175	.7226	.7275	.7324	.7373	.7421	.7468	.7515	.7561	.7607
0.70	.5798	.5847	.5895	.5943	.5991	.6039	.6086	.6132	.6179	.6225
0.80	.4090	.4129	.4168	.4207	.4245	.4284	.4322	.4361	.4399	.4437
0.90	.2118	.2140	.2162	.2184	.2205	.2227	.2249	.2270	.2292	.2314
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.7031	0.7062	0.7092	0.7121	0.7151	0.7179	0.7207	0.7234	0.7261	0.7288

TABLE 5. *Fractional temperature excess, ϕ_1, and generalized temperature averages, ψ'_1, for a slab*—Continued

x/a \ q	2.1000	2.1200	2.1400	2.1600	2.1800	2.2000	2.2200	2.2400	2.2600	2.2800
0.	0.9940	0.9946	0.9951	0.9955	0.9959	0.9963	0.9966	0.9969	0.9972	0.9975
0.10	.9914	.9921	.9927	.9932	.9938	.9943	.9947	.9951	.9955	.9959
0.20	.9821	.9832	.9842	.9852	.9861	.9870	.9878	.9886	.9893	.9900
0.30	.9623	.9641	.9658	.9674	.9690	.9705	.9720	.9734	.9747	.9760
0.40	.9252	.9279	.9306	.9332	.9356	.9381	.9404	.9426	.9448	.9470
0.50	.8624	.8661	.8698	.8733	.8768	.8802	.8835	.8868	.8900	.8931
0.60	.7651	.7696	.7739	.7782	.7825	.7867	.7908	.7949	.7989	.8029
0.70	.6270	.6316	.6361	.6405	.6450	.6494	.6537	.6581	.6624	.6666
0.80	.4475	.4512	.4550	.4588	.4625	.4662	.4699	.4736	.4773	.4810
0.90	.2335	.2357	.2378	.2400	.2421	.2443	.2464	.2486	.2507	.2529
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.7313	0.7339	0.7364	0.7388	0.7412	0.7436	0.7459	0.7481	0.7504	0.7525

x/a \ q	2.3000	2.3200	2.3400	2.3600	2.3800	2.4000	2.4200	2.4400	2.4600	2.4800
0.	0.9977	0.9979	0.9981	0.9983	0.9985	0.9986	0.9988	0.9989	0.9990	0.9991
0.10	.9962	.9965	.9968	.9971	.9973	.9976	.9978	.9980	.9981	.9983
0.20	.9906	.9912	.9918	.9924	.9929	.9933	.9938	.9942	.9946	.9950
0.30	.9772	.9783	.9795	.9805	.9815	.9825	.9834	.9843	.9851	.9859
0.40	.9490	.9510	.9529	.9548	.9566	.9583	.9600	.9616	.9631	.9647
0.50	.8961	.8991	.9020	.9048	.9076	.9103	.9130	.9155	.9180	.9205
0.60	.8068	.8106	.8144	.8181	.8218	.8254	.8290	.8325	.8360	.8394
0.70	.6708	.6750	.6792	.6833	.6874	.6914	.6954	.6994	.7034	.7073
0.80	.4847	.4883	.4919	.4956	.4992	.5027	.5063	.5099	.5134	.5170
0.90	.2550	.2572	.2593	.2614	.2636	.2657	.2678	.2700	.2721	.2742
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.7547	0.7568	0.7589	0.7609	0.7629	0.7649	0.7669	0.7688	0.7707	0.7725

x/a \ q	2.5000	2.5200	2.5400	2.5600	2.5800	2.6000	2.6200	2.6400	2.6600	2.6800
0.	0.9992	0.9993	0.9993	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997
0.10	.9984	.9986	.9987	.9988	.9989	.9990	.9991	.9992	.9993	.9993
0.20	.9953	.9956	.9959	.9962	.9965	.9967	.9970	.9972	.9974	.9976
0.30	.9867	.9874	.9881	.9887	.9894	.9899	.9905	.9910	.9915	.9920
0.40	.9661	.9675	.9689	.9702	.9714	.9726	.9738	.9749	.9760	.9770
0.50	.9229	.9252	.9275	.9297	.9319	.9340	.9361	.9381	.9400	.9419
0.60	.8427	.8460	.8492	.8524	.8556	.8586	.8617	.8647	.8676	.8705
0.70	.7112	.7150	.7188	.7226	.7263	.7300	.7337	.7373	.7409	.7445
0.80	.5205	.5240	.5275	.5310	.5344	.5379	.5413	.5448	.5482	.5516
0.90	.2763	.2784	.2806	.2827	.2848	.2869	.2890	.2911	.2932	.2953
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.7743	0.7761	0.7779	0.7796	0.7813	0.7830	0.7847	0.7863	0.7879	0.7895

x/a \ q	2.7000	2.7200	2.7400	2.7600	2.7800	2.8000	2.8200	2.8400	2.8600	2.8800
0.	0.9997	0.9998	0.9998	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999
0.10	.9994	.9994	.9995	.9995	.9996	.9996	.9997	.9997	.9997	.9997
0.20	.9977	.9979	.9981	.9982	.9983	.9985	.9986	.9987	.9988	.9989
0.30	.9925	.9929	.9933	.9937	.9941	.9944	.9948	.9951	.9954	.9956
0.40	.9780	.9790	.9799	.9808	.9817	.9825	.9833	.9840	.9848	.9855
0.50	.9438	.9456	.9473	.9490	.9507	.9523	.9539	.9554	.9569	.9583
0.60	.8733	.8761	.8789	.8815	.8842	.8868	.8893	.8918	.8943	.8967
0.70	.7480	.7515	.7550	.7584	.7618	.7651	.7685	.7718	.7750	.7782
0.80	.5549	.5583	.5617	.5650	.5683	.5716	.5749	.5782	.5814	.5847
0.90	.2974	.2995	.3016	.3037	.3058	.3079	.3100	.3120	.3141	.3162
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.7910	0.7926	0.7941	0.7956	0.7971	0.7985	0.7999	0.8013	0.8027	0.8041

x/a \ q	2.9000	2.9200	2.9400	2.9600	2.9800	3.0000	3.0200	3.0400	3.0600	3.0800
0.	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	.9998	.9998	.9998	.9998	.9998	0.9999	0.9999	0.9999	0.9999	0.9999
0.20	.9990	.9990	.9991	.9992	.9993	.9993	.9994	.9994	.9995	.9995
0.30	.9959	.9962	.9964	.9966	.9968	.9970	.9972	.9974	.9975	.9977
0.40	.9861	.9868	.9874	.9880	.9885	.9891	.9896	.9901	.9906	.9910
0.50	.9597	.9611	.9624	.9637	.9649	.9661	.9673	.9684	.9695	.9706
0.60	.8991	.9014	.9037	.9060	.9082	.9103	.9124	.9145	.9165	.9185
0.70	.7814	.7846	.7877	.7908	.7939	.7969	.7999	.8029	.8058	.8087
0.80	.5879	.5911	.5943	.5975	.6007	.6039	.6070	.6101	.6132	.6163
0.90	.3183	.3204	.3224	.3245	.3266	.3286	.3307	.3327	.3348	.3369
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.8055	0.8068	0.8081	0.8094	0.8107	0.8119	0.8132	0.8144	0.8156	0.8168

TABLE 5. *Fractional temperature excess, ϕ_1, and generalized temperature averages, ψ'_1, for a slab*—Continued

x/a \ q	3.1000	3.2000	3.3000	3.4000	3.5000	3.6000	3.7000	3.8000	3.9000	4.0000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	.9995	0.9997	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	.9979	.9985	.9989	.9992	.9995	0.9996	0.9998	0.9998	0.9999	0.9999
0.40	.9915	.9934	.9949	.9961	.9970	.9977	.9983	.9987	.9991	.9993
0.50	.9716	.9763	.9804	.9838	.9867	.9891	.9911	.9928	.9942	.9953
0.60	.9205	.9297	.9381	.9456	.9523	.9583	.9637	.9684	.9726	.9763
0.70	.8116	.8254	.8385	.8508	.8624	.8733	.8835	.8931	.9020	.9103
0.80	.6194	.6346	.6494	.6638	.6778	.6914	.7047	.7175	.7300	.7421
0.90	.3389	.3491	.3593	.3694	.3794	.3893	.3992	.4090	.4187	.4284
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.8180	0.8237	0.8290	0.8341	0.8388	0.8433	0.8475	0.8515	0.8553	0.8590

x/a \ q	4.1000	4.2000	4.3000	4.4000	4.5000	4.6000	4.7000	4.8000	4.9000	5.0000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000
0.50	.9963	.9970	.9976	.9981	.9985	.9989	.9991	0.9993	0.9995	0.9996
0.60	.9796	.9825	.9850	.9872	.9891	.9907	.9922	.9934	.9944	.9953
0.70	.9181	.9252	.9319	.9381	.9438	.9490	.9539	.9583	.9624	.9661
0.80	.7538	.7651	.7761	.7867	.7969	.8068	.8163	.8254	.8342	.8427
0.90	.4380	.4475	.4569	.4662	.4755	.4847	.4937	.5028	.5117	.5205
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.8624	0.8657	0.8688	0.8718	0.8746	0.8774	0.8800	0.8825	0.8849	0.8872

x/a \ q	5.1000	5.2000	5.3000	5.4000	5.5000	5.6000	5.7000	5.8000	5.9000	6.0000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000
0.60	.9961	.9967	.9973	.9977	.9981	.9985	.9987	0.9990	0.9992	0.9993
0.70	.9695	.9726	.9755	.9780	.9804	.9825	.9844	.9861	.9877	.9891
0.80	.8508	.8587	.8661	.8733	.8802	.8868	.8931	.8991	.9048	.9103
0.90	.5292	.5379	.5465	.5549	.5633	.5716	.5978	.5879	.5959	.6039
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.8894	0.8915	0.8935	0.8955	0.8974	0.8993	0.9010	0.9027	0.9044	0.9060

x/a \ q	6.1000	6.2000	6.3000	6.4000	6.5000	6.6000	6.7000	6.8000	6.9000	7.0000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	0.9994	0.9995	0.9996	0.9997	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999
0.70	.9903	.9915	.9925	.9934	.9942	.9949	.9955	.9961	.9966	.9970
0.80	.9155	.9205	.9252	.9297	.9340	.9381	.9419	.9456	.9490	.9523
0.90	.6117	.6194	.6270	.6346	.6420	.6494	.6566	.6638	.6708	.6778
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.9075	0.9090	0.9104	0.9118	0.9132	0.9145	0.9158	0.9170	0.9182	0.9194

x/a \ q	7.1000	7.2000	7.3000	7.4000	7.5000	7.6000	7.7000	7.8000	7.9000	8.0000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.70	.9974	0.9977	0.9980	0.9983	0.9985	0.9987	0.9989	0.9991	0.9992	0.9993
0.80	.9554	.9583	.9611	.9637	.9661	.9684	.9706	.9726	.9745	.9763
0.90	.6847	.6914	.6981	.7047	.7112	.7175	.7238	.7300	.7361	.7421
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.9205	0.9216	0.9227	0.9238	0.9248	0.9258	0.9267	0.9277	0.9286	0.9295

TABLE 5. *Fractional temperature excess, ϕ_1, and generalized temperature averages, ψ'_1, for a slab*—Continued

x/a \ q	8.1000	8.2000	8.3000	8.4000	8.5000	8.6000	8.7000	8.8000	8.9000	9.0000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1 0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.70	0.9994	0.9995	0.9996	0.9996	0.9997	0.9997	0.9998	0.9998	0.9998	0.9999
0.80	.9780	.9796	.9811	.9825	.9838	.9850	.9861	.9872	.9882	.9891
0.90	.7480	.7538	.7595	.7651	.7707	.7761	.7814	.7867	.7918	.7969
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.9303	0.9312	0.9320	0.9328	0.9336	0.9344	0.9351	0.9359	0.9366	0.9373

x/a \ q	9.1000	9.2000	9.3000	9.4000	9.5000	9.6000	9.7000	9.8000	9.9000	10.0000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.70	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
0.80	.9899	.9907	.9915	.9922	.9928	0.9934	0.9939	0.9944	0.9949	0.9953
0.90	.8019	.8068	.8116	.8163	.8209	.8254	.8299	.8342	.8385	.8427
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.9380	0.9387	0.9393	0.9400	0.9406	0.9412	0.9418	0.9424	0.9430	0.9436

x/a \ q	10.1000	10.2000	10.3000	10.4000	10.5000	10.6000	10.7000	10.8000	10.9000	11.0000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.70	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.80	0.9957	0.9961	0.9964	0.9967	0.9970	0.9973	0.9975	0.9977	0.9979	0.9981
0.90	.8468	.8508	.8548	.8586	.8624	.8661	.8698	.8733	.8768	.8802
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Average ψ'_1	0.9441	0.9447	0.9452	0.9457	0.9463	0.9468	0.9473	0.9478	0.9482	0.9487

TABLE 6. *Fractional temperature excess, ϕ_2, for a cylinder*

x/a \ q	0.4000	0.4200	0.4400	0.4600	0.4800	0.5000	0.5200	0.5400	0.5600	0.5800
0.	0.0002	0.0004	0.0009	0.0017	0.0030	0.0049	0.0076	0.0113	0.0159	0.0218
0.10	.0002	.0004	.0009	.0017	.0030	.0049	.0075	.0111	.0157	.0215
0.20	.0002	.0004	.0009	.0016	.0028	.0047	.0072	.0106	.0150	.0205
0.30	.0002	.0004	.0008	.0015	.0026	.0043	.0067	.0098	.0139	.0190
0.40	.0001	.0003	.0007	.0013	.0024	.0039	.0060	.0088	.0125	.0170
0.50	.0001	.0003	.0006	.0012	.0020	.0033	.0051	.0075	.0107	.0146
0.60	.0001	.0002	.0005	.0009	.0016	.0027	.0041	.0061	.0087	.0118
0.70	.0001	.0002	.0004	.0007	.0012	.0020	.0031	.0046	.0065	.0089
0.80	.0001	.0001	.0002	.0005	.0008	.0013	.0020	.0030	.0043	.0058
0.90	.0000	.0001	.0001	.0002	.0004	.0006	.0010	.0015	.0021	.0028
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	0.6000	0.6200	0.6400	0.6600	0.6800	0.7000	0.7200	0.7400	0.7600	0.7800
0.	0.0289	0.0373	0.0470	0.0580	0.0703	0.0838	0.0985	0.1143	0.1311	0.1488
0.10	.0285	.0367	.0463	.0571	.0693	.0826	.0971	.1126	.1292	.1467
0.20	.0272	.0351	.0443	.0547	.0663	.0790	.0929	.1078	.1236	.1403
0.30	.0252	.0326	.0410	.0507	.0614	.0732	.0861	.0999	.1146	.1301
0.40	.0226	.0291	.0367	.0453	.0549	.0655	.0770	.0893	.1025	.1163
0.50	.0193	.0250	.0315	.0388	.0471	.0561	.0660	.0766	.0878	.0997
0.60	.0157	.0202	.0255	0315	.0382	.0455	.0535	.0621	.0712	.0809
0.70	.0118	.0152	.0191	.0236	.0286	.0342	.0401	.0466	.0534	.0606
0.80	.0077	.0100	.0126	.0155	.0188	.0225	.0264	.0306	.0351	.0399
0.90	.0038	.0049	.0061	.0076	.0092	.0109	.0128	.0149	.0171	.0194
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

TABLE 6. *Fractional temperature excess, ϕ_2, for a cylinder*—Continued

x/a \ q	0.8000	0.8200	0.8400	0.8600	0.8800	0.9000	0.9200	0.9400	0.9600	0.9800
0.	0.1673	0.1866	0.2064	0.2268	0.2476	0.2687	0.2901	0.3117	0.3334	0.3551
0.10	.1649	.1839	.2304	.2235	.2440	.2649	.2860	.3072	.3286	.3500
0.20	.1578	.1759	.1947	.2139	.2335	.2534	.2736	.2940	.3145	.3350
0.30	.1463	.1631	.1804	.1983	.2164	.2349	.2537	.2725	.2915	.3106
0.40	.1308	.1458	.1614	.1773	.1936	.2101	.2269	.2438	.2608	.2779
0.50	.1121	.1250	.1383	.1520	.1659	.1801	.1945	.2090	.2236	.2382
0.60	.0909	.1014	.1122	.1233	.1346	.1461	.1578	.1696	.1814	.1933
0.70	.0682	.0760	.0841	.0924	.1010	.1096	.1184	.1272	.1361	.1451
0.80	.0448	.0500	.0553	.0608	.0664	.0721	.0778	.0836	.0895	.0954
0.90	.0218	.0243	.0269	.0295	.0323	.0350	.0378	.0407	.0435	.0464
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	1.0000	1.0200	1.0400	1.0600	1.0800	1.1000	1.1200	1.1400	1.1600	1.1800
0.	0.3768	0.3984	0.4199	0.4412	0.4622	0.4830	0.5035	0.5236	0.5433	0.5627
0.10	.3714	.3927	.4139	.4349	.4557	.4762	.4964	.5162	.5357	.5548
0.20	.3555	.3759	.3962	.4163	.4362	.4559	.4753	.4944	.5132	.5316
0.30	.3296	.3486	.3674	.3862	.4047	.4231	.4412	.4590	.4766	.4938
0.40	.2949	.3119	.3289	.3457	.3624	.3789	.3952	.4114	.4272	.4429
0.50	.2529	.2675	.2821	.2966	.3110	.3252	.3394	.3533	.3671	.3807
0.60	.2053	.2172	.2290	.2408	.2526	.2642	.2758	.2872	.2986	.3098
0.70	.1540	.1630	.1719	.1808	.1897	.1985	.2072	.2159	.2245	.2330
0.80	.1013	.1072	.1131	.1190	.1248	.1306	.1364	.1421	.1478	.1535
0.90	.0493	.0521	.0550	.0579	.0607	.0636	.0664	.0692	.0720	.0747
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	1.2000	1.2200	1.2400	1.2600	1.2800	1.3000	1.3200	1.3400	1.3600	1.3800
0.	0.5816	0.6001	0.6181	0.6356	0.6527	0.6692	0.6853	0.7008	0.7158	0.7303
0.10	.5735	.5918	.6096	.6270	.6439	.6603	.6762	.6916	.7066	.7210
0.20	.5497	.5673	.5845	.6014	.6178	.6337	.6492	.6643	.6789	.6931
0.30	.5108	.5274	.5436	.5595	.5751	.5902	.6050	.6194	.6335	.6471
0.40	.4583	.4734	.4882	.5028	.5171	.5311	.5448	.5582	.5713	.5841
0.50	.3941	.4073	.4204	.4332	.4458	.4582	.4703	.4823	.4941	.5056
0.60	.3208	.3318	.3426	.3532	.3638	.3741	.3844	.3945	.4044	.4142
0.70	.2414	.2497	.2580	.2662	.2743	.2823	.2902	.2980	.3058	.3135
0.80	.1591	.1647	.1702	.1756	.1811	.1864	1918	1971	2023	.2075
0.90	.0775	.0802	.0829	.0856	.0883	.0909	.0935	.0962	.0988	.1013
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	1.4000	1.4200	1.4400	1.4600	1.4800	1.5000	1.5200	1.5400	1.5600	1.5800
0.	0.7443	0.7578	0.7708	0.7833	0.7952	0.8067	0.8177	0.8282	0.8382	0.847
0.10	.7349	.7484	.7613	.7738	.7857	.7972	.8082	.8188	.8289	.83868
0.20	.7068	.7201	.7329	.7453	.7572	.7687	.7798	.7905	.8007	.8106
0.30	.6604	.6733	.6858	.6980	.7097	.7212	.7322	.7429	.7533	.7633
0.40	.5966	.6088	.6207	.6323	.6437	.6547	.6655	.6760	.6863	.6962
0.50	.5169	.5280	.5389	.5496	.5601	.5704	.5805	.5905	.6002	.6097
0.60	.4239	.4335	.4429	.4522	.4613	.4704	.4793	.4881	.4967	.5053
0.70	.3210	.3286	.3360	.3434	.3507	.3579	.3651	.3722	.3793	.3862
0.80	.2127	.2178	.2229	.2280	.2330	.2380	.2430	.2480	.2529	.2578
0.90	.1039	.1065	.1090	.1116	.1141	.1166	.1191	.1216	.1240	.1265
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	1.6000	1.6200	1.6400	1.6600	1.6800	1.7000	1.7200	1.7400	1.7600	1.7800
0.	0.8570	0.8657	0.8740	0.8818	0.8893	0.8964	0.9031	0.9095	0.9155	0.9212
0.10	.8478	.8566	.8650	.8730	.8806	.8879	.8947	.9013	.9075	.9133
0.20	.8200	.8291	.8378	.8461	.8541	.8617	.8690	.8760	.8827	.8890
0.30	.7730	.7823	.7914	.8001	.8085	.8166	.8244	.8320	.8393	.8463
0.40	.7060	.7154	.7246	.7336	.7423	.7508	.7590	.7670	.7748	.7824
0.50	.6191	.6282	.6372	.6460	.6547	.6631	.6714	.6796	.6876	.6954
0.60	.5137	.5220	.5302	.5383	.5463	.5542	.5620	.5697	.5772	.5847
0.70	.3931	.4000	.4068	.4135	.4202	.4268	.4334	.4399	.4464	.4528
0.80	.2626	.2674	.2723	.2770	.2818	.2865	.2913	.2960	.3006	.3053
0.90	.1290	.1314	.1339	.1363	.1387	.1412	.1436	.1460	.1484	.1508
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	1.8000	1.8200	1.8400	1.8600	1.8800	1.9000	1.9200	1.9400	1.9600	1.9800
0.	0.9265	0.9316	0.9363	0.9408	0.9450	0.9489	0.9527	0.9561	0.9594	0.9624
0.10	.9189	.9241	.9291	.9337	.9381	.9423	.9462	.9499	.9534	.9566
0.20	.8951	.9008	.9063	.9116	.9166	.9213	.9258	.9301	.9341	.9380
0.30	.8530	.8595	.8657	.8717	.8775	.8830	.8884	.8935	.8984	.9031
0.40	.7898	.7969	.8039	.8106	.8172	.8236	.8297	.8357	.8416	.8472
0.50	.7030	.7105	.7179	.7251	.7322	.7391	.7458	.7525	.7590	.7653
0.60	.5921	.5994	.6066	.6136	.6206	.6275	.6343	.6410	.6476	.6542
0.70	.4592	.4655	.4717	.4779	.4841	.4902	.4963	.5023	.5082	5141
0.80	.3099	.3145	.3191	.3237	.3282	.3328	.3373	.3418	.3463	.3507
0.90	.1532	.1556	.1580	.1604	.1628	.1651	1675	1699	.1722	.1746
1.00	0.	0.	0.	0.	0	0.	0.	0.	0.	0.

TABLE 6. *Fractional temperature excess, ϕ_2, for a cylinder*—Continued

x/a \ q	2.0000	2.0200	2.0400	2.0600	2.0800	2.1000	2.1200	2.1400	2.1600	2.1800
0.	0.9653	0.9679	0.9704	0.9727	0.9748	0.9768	0.9787	0.9804	0.9820	0.9835
0.10	.9597	.9625	.9652	.9677	.9701	.9723	.9743	.9762	.9780	.9797
0.20	.9416	.9451	.9484	.9515	.9544	.9572	.9598	.9623	.9646	.9669
0.30	.9077	.9120	.9162	.9202	.9240	.9276	.9311	.9345	.9377	.9408
0.40	.8527	.8580	.8632	.8682	.8730	.8777	.8822	.8866	.8909	.8950
0.50	.7716	.7776	.7836	.7894	.7951	.8007	.8062	.8115	.8167	.8218
0.60	.6606	.6669	.6732	.6794	.6854	.6914	.6973	.7032	.7089	.7146
0.70	.5200	.5258	.5316	.5373	.5430	.5486	.5542	.5597	.5652	.5707
0.80	.3552	.3596	.3640	.3684	.3728	.3771	.3815	.3858	.3901	.3944
0.90	.1769	.1793	.1816	.1840	.1863	.1887	.1910	.1933	.1956	.1980
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	2.2000	2.2200	2.2400	2.2600	2.2800	2.3000	2.3200	2.3400	2.3600	2.3800
0.	0.9849	0.9861	0.9873	0.9884	0.9894	0.9903	0.9912	0.9920	0.9927	0.9933
0.10	.9812	.9827	.9840	.9853	.9864	.9875	.9885	.9894	.9903	.9911
0.20	.9690	.9709	.9728	.9745	.9762	.9777	.9792	.9805	.9818	.9830
0.30	.9437	.9466	.9492	.9518	.9543	.9566	.9588	.9610	9630	.9650
0.40	.8990	.9029	.9066	.9102	.9137	.9171	.9204	.9235	.9266	.9295
0.50	.8268	.8317	.8365	.8411	.8457	.8501	.8544	.8587	.8628	.8669
0.60	.7202	.7256	.7311	.7364	.7416	.7468	.7519	.7569	.7619	.7667
0.70	.5760	.5814	.5867	.5920	.5972	.6023	.6075	.6125	.6176	.6226
0.80	.3986	.4029	.4071	.4113	.4155	.4197	.4239	.4280	.4321	.4362
0.90	.2003	.2026	.2049	.2072	.2095	.2118	.2141	.2164	.2187	.2210
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	2.4000	2.4200	2.4400	2.4600	2.4800	2.5000	2.5200	2.5400	2.5600	2.5800
0.	0.9939	0.9945	0.9950	0.9955	0.9959	0.9963	0.9966	0.9970	0.9972	0.9975
0.10	.9918	.9925	.9931	.9937	.9942	.9947	9952	.9956	.9960	.9963
0.20	.9842	.9853	.9863	.9872	.9881	.9889	.9897	.9904	.9911	.9917
0.30	.9668	.9686	.9703	.9718	.9734	.9748	.9762	.9775	.9787	.9799
0.40	.9324	.9351	.9377	.9403	.9428	.9451	.9474	.9496	.9517	.9538
0.50	.8708	.8746	.8784	.8820	.8856	.8891	.8925	.8958	.8990	.9021
0.60	.7715	.7762	.7809	.7854	.7899	.7943	.7987	.8029	.8072	.8113
0.70	.6275	.6324	.6373	.6421	.6468	.6515	.6562	.6608	.6654	.6700
0.80	.4403	.4444	.4485	.4525	.4565	.4605	.4645	.4685	.4724	.4764
0.90	.2233	.2256	.2279	.2301	.2324	.2347	.2370	.2392	.2415	.2437
1.00	0.	0.	0.	0.	0.	0.	0.	0	0.	0.

x/a \ q	2.6000	2.6200	2.6400	2.6600	2.6800	2.7000	2.7200	2.7400	2.7600	2.7800
0.	0.9978	0.9980	0.9982	0.9984	0.9985	0.9987	0.9988	0.9989	0.9991	0.9992
0.10	.9967	.9970	.9972	.9975	.9977	.9979	.9981	.9983	.9984	.9986
0.20	.9923	.9928	.9934	.9938	.9943	.9947	.9951	.9955	.9958	.9961
0.30	.9810	.9821	.9831	.9841	.9850	.9859	.9867	.9875	.9882	.9889
0.40	.9558	.9577	.9595	.9613	.9629	.9646	.9661	.9676	.9691	.9705
0.50	.9052	.9082	.9111	.9139	.9166	.9193	.9219	.9244	.9269	.9293
0.60	.8154	.8193	.8233	.8271	.8309	.8346	.8383	.8419	.8454	.8489
0.70	.6745	.6789	.6833	.6877	.6920	.6963	.7006	.7048	.7089	.7131
0.80	.4803	.4842	.4881	.4919	.4958	.4996	.5034	.5072	.5110	.5148
0.90	.2460	.2483	.2505	.2528	.2550	.2572	.2595	.2617	.2639	.2662
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	2.8000	2.8200	2.8400	2.8600	2.8800	2.9000	2.9200	2.9400	2.9600	2.9800
0.	0.9993	0.9993	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997
0.10	.9987	.9988	.9990	.9991	.9991	.9992	.9993	.9994	.9994	.9995
0.20	.9964	.9967	.9969	.9972	.9974	.9976	.9978	.9979	.9981	.9983
0.30	.9895	.9902	.9908	.9913	.9918	.9923	.9928	.9932	.9937	.9941
0.40	.9718	.9731	.9743	.9755	.9766	.9777	.9787	.9797	.9807	.9816
0.50	.9316	.9338	.9360	.9382	.9402	.9423	.9442	.9461	.9480	.9497
0.60	.8523	.8557	.8589	.8622	.8653	.8684	.8715	.8745	.8774	.8803
0.70	.7171	.7212	.7251	.7291	.7330	.7369	.7407	.7445	.7482	.7519
0.80	.5185	.5223	.5260	.5297	.5333	.5370	.5406	.5442	.5478	.5514
0.90	.2684	.2706	.2728	.2751	.2773	.2795	.2817	.2839	.2861	.2883
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	3.0000	3.1000	3.2000	3.3000	3.4000	3.5000	3.6000	3.7000	3.8000	3.9000
0.	0.9998	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	.9995	.9997	.9998	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	.9984	.9990	.9993	.9996	.9997	.9998	0.9999	0.9999	1.0000	1.0000
0.30	.9944	.9960	.9971	.9980	.9986	.9990	.9993	.9995	0.9997	0.9998
0.40	.9825	.9863	.9894	.9918	.9937	.9952	.9964	.9973	.9980	.9985
0.50	.9515	.9594	.9662	.9720	.9769	.9810	.9844	.9873	.9897	.9917
0.60	.8831	.8965	.9085	.9194	.9292	.9380	.9458	.9528.	.9590	.9645
0.70	.7556	.7733	.7901	.8058	.8207	.8347	.8478	.8601	.8716	.8823
0.80	.5550	.5726	.5897	.6064	.6226	.6384	.6538	.6687	.6832	.6972
0.90	.2905	.3015	.3123	.3231	.3338	.3445	.3550	.3655	.3759	.3862
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	4.0000	4.1000	4.2000	4.3000	4.4000	4.5000	4.6000	4.7000	4.8000	4.9000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	.9989	.9992	.9994	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	1.0000
0.50	.9933	.9947	.9958	.9967	.9973	.9979	.9984	.9987	.9990	0.9992
0.60	.9693	.9736	.9773	.9806	.9834	.9859	.9880	.9899	.9914	.9928
0.70	.8923	.9016	.9103	.9183	.9257	.9325	.9388	.9446	.9500	.9549
0.80	.7108	.7240	.7367	.7490	.7609	.7724	.7834	.7941	.8044	.8143
0.90	.3964	.4066	.4166	.4266	.4365	.4463	.4560	.4656	.4751	.4846
1.00	0	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	5.0000	5.1000	5.2000	5.3000	5.4000	5.5000	5.6000	5.7000	5.8000	5.9000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	.9994	0.9996	0.9997	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000	1.0000
0.60	.9939	.9949	.9958	.9965	.9971	.9976	.9980	.9984	0.9986	0.9989
0.70	.9594	.9635	.9672	.9706	.9737	.9765	.9790	.9813	.9834	.9853
0.80	.8238	.8329	.8416	.8501	.8581	.8658	.8732	.8803	.8870	.8934
0.90	.4939	.5032	.5123	.5214	.5303	.5392	.5480	.5566	.5652	.5737
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	6.0000	6.1000	6.2000	6.3000	6.4000	6.5000	6.6000	6.7000	6.8000	6.9000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	0.9991	0.9993	0.9994	0.9995	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999
0.70	.9870	.9884	.9898	.9910	.9921	.9930	.9939	.9946	.9953	.9959
0.80	.8996	.9054	.9110	.9163	.9213	.9261	.9307	.9350	.9390	.9429
0.90	.5820	.5903	.5985	.6065	.6145	.6224	.6301	.6378	.6453	.6528
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	7.0000	7.1000	7.2000	7.3000	7.4000	7.5000	7.6000	7.7000	7.8000	7.9000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.70	.9964	.9969	0.9973	0.9977	0.9980	0.9982	0.9985	0.9987	0.9989	0.9990
0.80	.9466	.9500	.9533	.9564	.9593	.9621	.9646	.9671	.9693	.9715
0.90	.6601	.6674	.6745	.6816	.6885	.6953	.7021	.7087	.7152	.7217
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	8.0000	8.1000	8.2000	8.3000	8.4000	8.5000	8.6000	8.7000	8.8000	8.9000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.70	0.9991	0.9993	0.9994	0.9995	0.9996	0.9996	0.9997	0.9997	0.9998	0.9998
0.80	.9735	.9754	.9772	.9789	.9804	.9819	.9832	.9845	.9857	.9868
0.90	.7280	.7342	.7404	.7464	.7523	.7581	.7639	.7695	.7750	.7805
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	9.0000	9.1000	9.2000	9.3000	9.4000	9.5000	9.6000	9.7000	9.8000	9.9000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.70	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000
0.80	.9878	.9888	.9897	.9905	.9912	.9920	0.9926	0.9932	0.9938	0.9943
0.90	.7858	.7911	.7962	.8013	.8063	.8111	.8159	.8206	.8252	.8297
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

TABLE 6. *Fractional temperature excess, ϕ_2, for a cylinder*—Continued

x/a \ q	10.0000	10.1000	10.2000	10.3000	10.4000	10.5000	10.6000	10.7000	10.8000	10.9000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.70	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.80	0.9948	0.9952	0.9956	0.9960	0.9964	0.9967	0.9970	0.9972	0.9975	0.9977
0.90	.8341	.8385	.8427	.8469	.8510	.8549	.8588	.8627	.8664	.8701
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

TABLE 7. *Generalized temperature averages, ψ_2, for a cylinder*

x/a \ q	0.4000	0.4200	0.4400	0.4600	0.4800	0.5000	0.5200	0.5400	0.5600	0.5800
0.	0.0001	0.0002	0.0005	0.0009	0.0015	0.0025	0.0038	0.0056	0.0080	0.0109
0.10	.0001	.0002	.0005	.0009	.0015	.0024	.0038	.0056	.0079	.0108
0.20	.0001	.0002	.0004	.0008	.0015	.0024	.0037	.0055	.0077	.0106
0.30	.0001	.0002	.0004	.0008	.0014	.0023	.0036	.0053	.0075	.0102
0.40	.0001	.0002	0004	.0008	.0013	.0022	0034	.0050	.0071	.0097
0.50	.0001	.0002	.0004	.0007	.0013	.0020	.0032	.0047	.0066	.0090
0.60	.0001	.0002	.0003	.0007	.0011	.0019	.0029	.0043	.0061	.0083
0.70	.0001	.0002	.0003	.0006	.0010	.0017	.0026	.0039	.0055	.0075
0.80	.0001	.0001	.0003	.0005	.0009	.0015	.0023	.0034	.0048	.0066
0.90	.0000	.0001	.0002	.0004	.0008	.0013	.0020	.0029	.0041	.0056
1.00	.0000	.0001	.0002	.0004	.0007	.0011	.0016	.0024	.0034	.0047

x/a \ q	0.6000	0.6200	0.6400	0.6600	0.6800	0.7000	0.7200	0.7400	0.7600	0.7800
0.	0.0144	0.0186	0.0235	0.0290	0.0351	0.0419	0.0492	0.0571	0.0655	0.0744
0.10	.0143	.0185	.0233	.0288	.0349	.0416	.0489	.0567	.0651	.0739
0.20	.0140	.0181	.0228	.0282	.0341	.0407	.0478	.0555	.0637	.0723
0.30	.0135	.0174	.0220	.0271	.0329	.0392	.0461	.0535	.0614	.0697
0.40	.0128	.0166	.0209	.0258	.0312	.0372	.0438	.0508	.0582	.0661
0.50	.0120	.0155	.0195	.0241	.0292	.0348	.0409	.0474	0544	.0617
0.60	.0110	.0142	.0179	.0221	.0267	.0319	.0375	.0435	.0499	.0566
0.70	.0099	.0128	.0161	.0199	.0241	.0287	.0337	.0392	.0449	.0510
0.80	.0087	.0112	.0142	.0175	.0212	.0253	.0297	.0345	.0396	.0449
0.90	.0075	.0097	.0122	.0150	.0182	.0217	.0255	.0296	.0340	.0386
1.00	.0062	.0080	.0101	.0125	.0152	.0181	.0213	.0247	.0283	.0321

x/a \ q	0.8000	0.8200	0.8400	0.8600	0.8800	0.9000	0.9200	0.9400	0.9600	0.9800
0.	0.0837	0.0933	0.1032	0.1134	0.1238	0.1344	0.1451	0.1559	0.1667	0.1776
0.10	.0831	.0926	.1025	.1126	.1229	.1334	.1440	.1547	.1655	.1763
0.20	.0813	.0906	.1002	.1101	.1203	.1305	.1409	.1514	.1619	.1725
0.30	.0783	.0873	.0966	.1062	.1159	.1258	.1359	.1460	.1561	.1663
0.40	.0743	.0829	.0917	.1008	.1100	.1194	.1290	.1386	.1482	.1579
0.50	.0694	.0774	.0856	.0941	.1027	.1115	.1204	.1294	.1384	.1475
0.60	.0637	.0710	.0786	.0863	.0943	.1023	.1105	.1187	.1270	.1353
0.70	.0573	.0639	.0707	.0777	.0848	.0921	.0995	.1069	.1143	.1218
0.80	.0505	.0563	.0623	.0684	.0747	.0811	.0876	.0941	.1007	.1073
0.90	.0434	.0483	.0535	.0588	.0642	.0697	.0752	.0808	.0865	.0921
1.00	.0361	.0403	.0446	.0490	.0535	.0580	.0627	.0673	.0721	.0768

x/a \ q	1.0000	1.0200	1.0400	1.0600	1.0800	1.1000	1.1200	1.1400	1.1600	1.1800
0.	0.1884	0.1992	0.2100	0.2206	0.2311	0.2415	0.2517	0.2618	0.2717	0.2813
0.10	.1871	.1978	.2085	.2190	.2295	.2398	.2500	.2600	.2698	.2794
0.20	.1831	.1936	.2040	.2144	.2246	.2347	.2447	.2545	.2641	.2735
0.30	.1765	.1866	.1967	.2067	.2166	.2264	.2360	.2455	.2548	.2640
0.40	.1675	.1772	.1868	.1963	.2057	.2150	.2242	.2332	.2422	.2509
0.50	.1565	.1655	.1745	.1834	.1922	.2009	.2095	.2180	.2264	.2346
0.60	.1436	.1519	.1602	.1683	.1765	.1845	.1924	.2003	.2080	.2156
0.70	.1293	.1368	.1442	.1516	.1589	.1662	.1733	.1804	.1874	.1943
0.80	.1139	.1205	.1270	.1335	.1400	.1464	.1528	.1590	.1652	.1713
0.90	.0978	.1035	.1091	.1147	1203	.1258	.1313	.1367	.1420	.1472
1.00	.0815	.0862	.0909	.0956	.1002	.1048	.1094	.1139	.1183	.1227

x/a \ q	1.2000	1.2200	1.2400	1.2600	1.2800	1.3000	1.3200	1.3400	1.3600	1.3800
0.	0.2908	0.3000	0.3090	0.3178	0.3263	0.3346	0.3426	0.3504	0.3579	0.3652
0.10	.2888	.2980	.3069	.3157	.3241	.3324	.3404	.3481	.3556	.3628
0.20	.2828	.2918	.3006	.3092	.3176	.3257	.3336	.3412	.3487	.3558
0.30	.2729	.2817	.2903	.2986	.3068	.3147	.3224	.3299	.3372	.3443
0.40	.2595	.2679	.2761	.2841	.2920	.2996	.3071	.3144	.3214	.3283
0.50	.2427	.2507	.2584	.2660	.2735	.2807	.2878	.2948	.3015	.3081
0.60	.2231	.2305	.2377	.2447	.2517	.2585	.2651	.2716	.2780	.2842
0.70	.2011	.2078	.2144	.2208	.2271	.2333	.2394	.2454	.2512	.2570
0.80	.1774	.1833	.1891	.1948	.2005	.2060	.2114	.2168	.2220	.2271
0.90	.1524	.1576	.1626	.1675	.1724	.1772	.1819	.1865	.1911	.1955
1.00	.1270	.1313	.1355	1397	.1437	.1477	.1517	.1555	.1593	.1631

TABLE 7. *Generalized temperature averages, ψ_2, for a cylinder*—Continued

x/a \ q	1.4000	1.4200	1.4400	1.4600	1.4800	1.5000	1.5200	1.5400	1.5600	1.5800
0.	0.3722	0.3789	0.3854	0.3916	0.3976	0.4033	0.4088	0.4141	0.4191	0.4239
0.10	.3698	.3765	.3830	.3893	.3952	.4010	.4065	.4117	.4168	.4216
0.20	.3628	.3695	.3759	.3821	.3881	.3939	.3994	.4047	.4098	.4146
0.30	.3511	.3577	.3641	.3703	.3762	.3820	.3875	.3928	.3980	.4029
0.40	.3349	.3414	.3477	.3538	.3596	.3653	.3708	.3762	.3813	.3863
0.50	.3145	.3208	.3268	.3328	.3385	.3441	.3495	.3547	.3598	.3648
0.60	.2902	.2961	.3019	.3076	.3130	.3184	.3236	.3287	.3336	.3384
0.70	.2626	.2680	.2734	.2786	.2838	.2888	.2937	.2985	.3032	.3077
0.80	.2322	.2371	.2420	.2467	.2513	.2559	.2604	.2647	.2690	.2732
0.90	.1999	.2042	.2085	.2126	.2167	.2207	.2246	.2284	.2322	.2359
1.00	.1667	.1703	.1739	.1773	.1808	.1841	.1874	.1906	.1938	.1969

x/a \ q	1.6000	1.6200	1.6400	1.6600	1.6800	1.7000	1.7200	1.7400	1.7600	1.7800
0.	0.4285	0.4328	0.4370	0.4409	0.4446	0.4482	0.4516	0.4547	0.4577	0.4606
0.10	.4262	.4306	.4347	.4387	.4425	.4461	.4495	.4527	.4557	.4586
0.20	.4193	.4237	.4280	.4320	.4359	.4396	.4431	.4464	.4496	.4526
0.30	.4076	.4122	.4165	.4207	.4247	.4285	.4322	.4357	.4390	.4422
0.40	.3911	.3957	.4001	.4044	.4085	.4125	.4164	.4200	.4236	.4270
0.50	.3695	.3742	.3787	.3830	.3872	.3913	.3953	.3991	.4028	.4063
0.60	.3431	.3477	.3521	.3564	.3606	.3647	.3687	.3726	.3763	.3800
0.70	.3122	.3165	.3208	.3250	.3290	.3330	.3369	.3407	.3444	.3480
0.80	.2773	.2814	.2853	.2892	.2930	.2967	.3003	.3039	.3074	.3108
0.90	.2395	.2431	.2466	.2500	.2534	.2567	.2599	.2631	.2663	.2693
1.00	.2000	.2030	.2059	.2088	.2117	.2144	.2172	.2199	.2225	.2251

x/a \ q	1.8000	1.8200	1.8400	1.8600	1.8800	1.9000	1.9200	1.9400	1.9600	1.9800
0.	0.4633	0.4658	0.4682	0.4704	0.4725	0.4745	0.4763	0.4781	0.4797	0.4812
0.10	.4613	.4639	.4663	.4686	.4708	.4728	.4747	.4765	.4782	.4798
0.20	.4555	.4582	.4608	.4632	.4655	.4677	.4697	.4717	.4735	.4752
0.30	.4453	.4482	.4510	.4536	.4561	.4585	.4608	.4630	.4651	.4670
0.40	.4302	.4334	.4364	.4393	.4421	.4447	.4473	.4497	.4520	.4543
0.50	.4098	.4131	.4163	.4195	.4225	.4254	.4282	.4309	.4335	.4360
0.60	.3835	.3870	.3904	.3936	.3968	.3999	.4029	.4058	.4086	.4113
0.70	.3515	.3550	.3583	.3616	.3648	.3679	.3710	.3740	.3769	.3798
0.80	.3141	.3174	.3206	.3238	.3269	.3299	.3328	.3358	.3386	.3414
0.90	.2723	.2753	.2782	.2811	.2839	.2866	.2893	.2920	.2946	.2972
1.00	.2277	.2302	.2327	.2351	.2375	.2398	.2421	.2444	.2466	.2488

x/a \ q	2.0000	2.0200	2.0400	2.0600	2.0800	2.1000	2.1200	2.1400	2.1600	2.1800
0.	0.4826	0.4840	0.4852	0.4864	0.4874	0.4884	0.4894	0.4902	0.4910	0.4918
0.10	.4812	.4826	.4839	.4851	.4862	.4873	.4883	.4892	.4900	.4908
0.20	.4769	.4784	.4798	.4812	.4825	.4837	.4848	.4858	.4868	.4877
0.30	.4689	.4707	.4723	.4739	.4754	.4769	.4782	.4795	.4807	.4819
0.40	.4564	.4585	.4605	.4623	.4641	.4659	.4675	.4691	.4706	.4720
0.50	.4385	.4408	.4431	.4453	.4474	.4494	.4514	.4533	.4551	.4568
0.60	.4140	.4166	.4191	.4216	.4239	.4263	.4285	.4307	.4328	.4348
0.70	.3825	.3853	.3879	.3905	.3931	.3955	.3979	.4003	.4026	.4049
0.80	.3441	.3468	.3495	.3521	.3546	.3571	.3595	.3619	.3643	.3666
0.90	.2997	.3022	.3046	.3070	.3094	.3117	.3140	.3162	.3184	.3206
1.00	.2510	.2531	.2552	.2572	.2592	.2612	.2632	.2651	.2670	.2689

x/a \ q	2.2000	2.2200	2.2400	2.2600	2.2800	2.3000	2.3200	2.3400	2.3600	2.3800
0.	0.4924	0.4931	0.4937	0.4942	0.4947	0.4952	0.4956	0.4960	0.4963	0.4967
0.10	.4915	.4922	.4928	.4934	.4940	.4945	.4949	.4954	.4957	.4961
0.20	.4886	.4894	.4902	.4909	.4915	.4922	.4927	.4933	.4938	.4942
0.30	.4829	.4840	.4849	.4859	.4867	.4875	.4883	.4890	.4897	.4904
0.40	.4734	.4747	.4760	.4772	.4783	.4794	.4804	.4814	.4824	.4833
0.50	.4585	.4602	.4617	.4633	.4647	.4661	.4675	.4688	.4700	.4712
0.60	.4368	.4388	.4406	.4425	.4442	.4459	.4476	.4492	.4508	.4523
0.70	.4071	.4092	.4113	.4134	.4154	.4173	.4192	.4211	.4229	.4247
0.80	.3688	.3710	.3732	.3753	.3774	.3795	.3815	.3835	.3854	.3873
0.90	.3227	.3248	.3269	.3289	.3309	.3329	.3348	.3368	.3386	.3405
1.00	.2707	.2725	.2743	.2760	.2778	.2795	.2812	.2828	.2844	.2860

x/a \ q	2.4000	2.4200	2.4400	2.4600	2.4800	2.5000	2.5200	2.5400	2.5600	2.5800
0.	0.4970	0.4972	0.4975	0.4977	0.4979	0.4981	0.4983	0.4985	0.4986	0.4988
0.10	.4964	.4967	.4970	.4973	.4975	.4978	.4980	.4981	.4983	.4985
0.20	.4947	.4951	.4954	.4958	.4961	.4964	.4967	.4970	.4972	.4974
0.30	.4910	.4915	.4921	.4926	.4931	.4935	.4939	.4943	.4947	.4950
0.40	.4841	.4849	.4857	.4864	.4871	.4878	.4884	.4891	.4896	.4902
0.50	.4724	.4735	.4746	.4756	.4766	.4776	.4785	.4794	.4802	.4811
0.60	.4538	.4553	.4566	.4580	.4593	.4606	.4618	.4630	.4642	.4653
0.70	.4265	.4282	.4298	.4315	.4331	.4346	.4361	.4376	.4391	.4405
0.80	.3892	.3911	.3929	.3946	.3964	.3981	.3998	.4014	.4030	.4046
0.90	.3423	.3441	.3459	.3476	.3493	.3510	.3527	.3543	.3559	.3575
1.00	.2876	.2892	.2907	.2922	.2937	.2952	.2967	.2981	.2995	.3009

TABLE 7. *Generalized temperature averages, ψ_2, for a cylinder*—Continued

x/a \ q	2.6000	2.6200	2.6400	2.6600	2.6800	2.7000	2.7200	2.7400	2.7600	2.7800
0.	0.4989	0.4990	0.4991	0.4992	0.4993	0.4993	0.4994	0.4995	0.4995	0.4996
0.10	.4986	.4987	.4989	.4990	.4991	.4992	.4992	.4993	.4994	.4994
0.20	.4976	.4978	.4980	.4982	.4983	.4984	.4986	.4987	.4988	.4989
0.30	.4954	.4957	.4960	.4962	.4965	.4967	.4970	.4972	.4973	.4975
0.40	.4907	.4912	.4917	.4921	.4925	.4930	.4933	.4937	.4940	.4944
0.50	.4818	.4826	.4833	.4840	.4847	.4853	.4860	.4866	.4871	.4877
0.60	.4664	.4675	.4685	.4695	.4705	.4714	.4723	.4732	.4741	.4749
0.70	.4419	.4432	.4445	.4458	.4471	.4483	.4495	.4507	.4519	.4530
0.80	.4062	.4077	.4092	.4107	.4122	.4136	.4150	.4164	.4178	.4191
0.90	.3591	.3606	.3621	.3636	.3651	.3666	.3680	.3694	.3708	.3722
1.00	.3023	.3036	.3050	.3063	.3076	.3089	.3101	.3114	.3126	.3138

x/a \ q	2.8000	2.8200	2.8400	2.8600	2.8800	2.9000	2.9200	2.9400	2.9600	2.9800
0.	0.4996	0.4997	0.4997	0.4997	0.4998	0.4998	0.4998	0.4998	0.4998	0.4999
0.10	.4995	.4995	.4996	.4996	.4997	.4997	.4997	.4998	.4998	.4998
0.20	.4990	.4991	.4992	.4992	.4993	.4994	.4994	.4995	.4995	.4996
0.30	.4977	.4979	.4980	.4981	.4983	.4984	.4985	.4986	.4987	.4988
0.40	.4947	.4950	.4952	.4955	.4958	.4960	.4962	.4964	.4966	.4968
0.50	.4882	.4887	.4892	.4897	.4901	.4905	.4909	.4913	.4917	.4921
0.60	.4757	.4765	.4773	.4780	.4787	.4794	.4801	.4807	.4814	.4820
0.70	.4541	.4552	.4562	.4572	.4582	.4592	.4602	.4611	.4620	.4629
0.80	.4204	.4217	.4230	.4242	.4254	.4266	.4278	.4290	.4301	.4312
0.90	.3735	.3749	.3762	.3775	.3788	.3800	.3813	.3825	.3837	.3849
1.00	.3151	.3162	.3174	.3186	.3197	.3209	.3220	.3231	.3242	.3252

x/a \ q	3.0000	3.1000	3.2000	3.3000	3.4000	3.5000	3.6000	3.7000	3.8000	3.9000
0.	0.4999	0.4999	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.10	.4998	.4999	.4999	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.20	.4996	.4997	.4998	.4999	.4999	.5000	.5000	.5000	.5000	.5000
0.30	.4989	.4992	.4995	.4997	.4998	.4998	.4999	.4999	.5000	.5000
0.40	.4970	.4978	.4984	.4988	.4991	.4994	.4995	.4997	.4998	.4998
0.50	.4924	.4940	.4952	.4962	.4970	.4977	.4982	.4986	.4989	.4991
0.60	.4826	.4853	.4876	.4895	.4912	.4926	.4938	.4948	.4957	.4964
0.70	.4638	.4679	.4715	.4747	.4776	.4802	.4825	.4845	.4863	.4879
0.80	.4323	.4376	.4424	.4469	.4510	.4547	.4582	.4614	.4644	.4671
0.90	.3861	.3918	.3971	.4022	.4069	.4113	.4154	.4194	.4231	.4266
1.00	.3263	.3315	.3363	.3409	.3452	.3493	.3532	.3569	.3604	.3638

x/a \ q	4.0000	4.1000	4.2000	4.3000	4.4000	4.5000	4.6000	4.7000	4.8000	4.9000
0.	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.10	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.20	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.30	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.40	.4999	.4999	.4999	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.50	.4993	.4995	.4996	.4997	.4998	.4998	.4999	.4999	.4999	.4999
0.60	.4970	.4975	.4980	.4983	.4986	.4989	.4991	.4992	.4994	.4995
0.70	.4894	.4906	.4917	.4927	.4936	.4944	.4951	.4957	.4963	.4967
0.80	.4696	.4720	.4741	.4761	.4780	.4797	.4812	.4827	.4840	.4853
0.90	.4299	.4331	.4361	.4389	.4416	.4441	.4465	.4488	.4510	.4531
1.00	.3670	.3700	.3729	.3757	.3784	.3809	.3834	.3857	.3880	.3902

x/a \ q	5.0000	5.1000	5.2000	5.3000	5.4000	5.5000	5.6000	5.7000	5.8000	5.9000
0.	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.10	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.20	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.30	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.40	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.50	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.60	.4996	.4997	.4997	.4998	.4998	.4999	.4999	.4999	.4999	.4999
0.70	.4971	.4975	.4978	.4981	.4984	.4986	.4988	.4989	.4991	.4992
0.80	.4864	.4875	.4885	.4894	.4902	.4910	.4917	.4924	.4930	.4936
0.90	.4550	.4569	.4587	.4605	.4621	.4637	.4652	.4666	.4680	.4693
1.00	.3923	.3943	.3962	.3981	.3999	.4016	.4033	.4049	.4065	.4080

x/a \ q	6.0000	6.1000	6.2000	6.3000	6.4000	6.5000	6.6000	6.7000	6.8000	6.9000
0.	0.5000	0.5000	0.5000	0.5000	0.5000	.05000	0.5000	0.5000	0.5000	0.5000
0.10	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.20	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.30	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.40	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.50	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.60	.4999	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.70	.4993	.4994	.4995	.4996	.4997	.4997	.4997	.4998	.4998	.4998
0.80	.4941	.4946	.4950	.4954	.4958	.4962	.4965	.4968	.4970	.4973
0.90	.4705	.4717	.4728	.4739	.4750	.4760	.4769	.4779	.4788	.4796
1.00	.4095	.4109	.4123	.4136	.4149	.4162	.4174	.4186	.4198	.4209

TABLE 7. *Generalized temperature averages, ψ_2, for a cylinder*—Continued

x/a \ q	7.0000	7.1000	7.2000	7.3000	7.4000	7.5000	7.6000	7.7000	7.8000	7.9000
0.	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.10	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.20	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.30	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.40	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.50	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.60	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.70	.4999	.4999	.4999	.4999	.4999	.4999	.5000	.5000	.5000	.5000
0.80	.4975	.4977	.4979	.4981	.4983	.4984	.4985	.4987	.4988	.4989
0.90	.4804	.4812	.4819	.4827	.4834	.4840	.4847	.4853	.4859	.4864
1.00	.4220	.4230	.4241	.4251	.4261	.4270	.4279	.4288	.4297	.4306

x/a \ q	.80000	8.1000	8.2000	8.3000	8.4000	8.5000	8.6000	8.7000	8.8000	8.9000
0.	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.10	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.20	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.30	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.40	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.50	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.60	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.70	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.80	.4990	.4991	.4992	.4993	.4993	.4994	.4994	.4995	.4995	.4996
0.90	.4869	.4875	.4880	.4884	.4889	.4893	.4898	.4902	.4906	.4909
1.00	.4315	.4323	.4331	.4339	.4346	.4354	.4361	.4368	.4375	.4382

x/a \ q	9.0000	9.1000	9.2000	9.3000	9.4000	9.5000	9.6000	9.7000	9.8000	9.9000
0.	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.10	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.20	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.30	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.40	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.50	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.60	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.70	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.80	.4996	.4997	.4997	.4997	.4998	.4998	.4998	.4998	.4999	.4999
0.90	.4913	.4916	.4920	.4923	.4926	.4929	.4932	.4935	.4937	.4940
1.00	.4389	.4395	.4401	.4408	.4414	.4420	4426	.4431	.4437	.4443

x/a \ q	10.0000	10.1000	10.2000	10.3000	10.4000	10.5000	10.6000	10.7000	10.8000	10.9000
0.	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
0.10	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.20	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.30	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.40	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.50	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.60	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.70	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
0.80	.4999	.4999	.4999	.4999	.4999	.5000	.5000	.5000	.5000	.5000
0.90	.4942	.4945	.4947	.4949	.4951	.4953	.4955	.4957	.4958	.4960
1.00	.4448	.4454	.4459	.4464	.4469	.4474	.4479	.4484	.4488	.4493

TABLE 8. *Fractional temperature excess, ϕ_3, for a sphere*

x/a \ q	0.5000	0.5200	0.5400	0.5600	0.5800	0.6000	0.6200	0.6400	0.6600	0.6800
0.	0.0001	0.0002	0.0004	0.0008	0.0013	0.0021	0.0033	0.0048	0.0069	0.0096
0.10	.0001	.0002	.0004	.0008	.0013	.0021	.0032	.0048	.0068	.0095
0.20	.0001	.0002	.0004	.0007	.0012	.0020	.0031	.0045	.0065	.0090
0.30	.0001	.0002	.0004	.0007	.0011	.0018	.0028	.0042	.0060	.0083
0.40	.0001	.0002	.0003	.0006	.0010	.0016	.0025	.0037	.0052	.0073
0.50	.0001	.0001	.0003	.0005	.0008	.0013	.0021	.0031	.0044	.0061
0.60	.0001	.0001	.0002	.0004	.0007	.0011	.0016	.0024	.0035	.0049
0.70	.0000	.0001	.0002	.0003	.0005	.0008	.0012	.0018	.0026	.0035
0.80	.0000	.0001	.0001	.0002	.0003	.0005	.0008	.0011	.0016	.0023
0.90	.0000	.0000	.0000	.0001	.0001	.0002	.0004	.0005	.0008	.0011
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	0.7000	0.7200	0.7400	0.7600	0.7800	0.8000	0.8200	0.8400	0.8600	0.8800
0.	0.0130	0.0171	0.0221	0.0279	0.0347	0.0423	0.0510	0.0606	0.0711	0.0827
0.10	.0128	.0169	.0217	.0275	.0341	.0416	.0501	.0596	.0700	.0813
0.20	.0122	.0160	.0207	.0261	.0324	.0396	.0477	.0567	.0666	.0773
0.30	.0112	.0147	.0190	.0240	.0297	.0363	.0438	.0520	.0611	.0709
0.40	.0098	.0130	.0167	.0211	.0262	.0320	.0386	.0459	.0538	.0626
0.50	.0083	.0109	.0141	.0178	.0221	.0270	.0325	.0386	.0453	.0526
0.60	.0066	.0086	.0111	.0141	.0175	.0214	.0257	.0306	.0359	.0417
0.70	.0048	.0063	.0081	.0103	.0127	.0156	.0188	.0223	.0262	.0304
0.80	.0030	.0040	.0052	.0065	.0081	.0099	.0119	.0142	.0166	.0193
0.90	.0014	.0019	.0024	.0031	.0038	.0046	.0056	.0066	.0078	.0090
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

Table 8. *Fractional temperature excess, ϕ_3, for a sphere*—Continued

x/a \ q	0.9000	0.9200	0.9400	0.9600	0.9800	1.0000	1.0200	1.0400	1.0600	1.0800
0.	0.0951	0.1084	0.1225	0.1375	0.1531	0.1695	0.1865	0.2041	0.2222	0.2407
0.10	.0935	.1066	.1205	.1352	.1506	.1667	.1835	.2008	.2186	.2368
0.20	.0889	.1014	.1146	.1286	.1433	.1586	.1745	.1910	.2079	.2253
0.30	.0816	.0930	.1052	.1180	.1315	.1455	.1602	.1753	.1908	.2068
0.40	.0720	.0820	.0927	.1041	.1159	.1283	.1412	.1546	.1683	.1824
0.50	.0605	.0690	.0780	.0875	.0975	.1080	.1188	.1301	.1416	.1535
0.60	.0480	.0547	.0618	.0694	.0773	.0856	.0942	.1031	.1123	.1217
0.70	.0350	.0399	.0451	.0506	.0564	.0624	.0687	.0752	.0819	.0888
0.80	.0222	.0254	.0287	.0322	.0358	.0397	.0437	.0478	.0521	.0565
0.90	.0104	.0118	.0134	.0150	.0168	.0185	.0204	.0224	.0243	.0264
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	1.1000	1.1200	1.1400	1.1600	1.1800	1.2000	1.2200	1.2400	1.2600	1.2800
0.	0.2597	0.2790	0.2986	0.3183	0.3383	0.3584	0.3785	0.3986	0.4187	0.4388
0.10	.2555	.2745	.2937	.3132	.3328	.3526	.3724	.3923	.4121	.4318
0.20	.2430	.2611	.2795	.2980	.3168	.3356	.3545	.3735	.3924	.4113
0.30	.2231	.2398	.2566	.2737	.2910	.3084	.3258	.3433	.3609	.3783
0.40	.1968	.2115	.2265	.2416	.2569	.2723	.2878	.3034	.3190	.3346
0.50	.1657	.1781	.1907	.2035	.2164	.2295	.2426	.2559	.2691	.2824
0.60	.1314	.1413	.1513	.1615	.1718	.1822	.1927	.2033	.2139	.2246
0.70	.0959	.1031	.1104	.1179	.1254	.1331	.1408	.1486	.1564	.1642
0.80	.0610	.0656	.0702	.0750	.0798	.0847	.0896	.0946	.0996	.1047
0.90	.0285	.0307	.0328	.0351	.0373	.0396	.0419	.0443	.0466	.0490
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	1.3000	1.3200	1.3400	1.3600	1.3800	1.4000	1.4200	1.4400	1.4600	1.4800
0.	0.4587	0.4784	0.4979	0.5172	0.5362	0.5550	0.5734	0.5914	0.6091	0.6263
0.10	.4514	.4709	.4902	.5092	.5280	.5465	.5647	.5825	.6000	.6171
0.20	.4301	.4488	.4673	.4856	.5036	.5215	.5390	.5563	.5732	.5899
0.30	.3958	.4131	.4303	.4474	.4643	.4810	.4974	.5137	.5297	.5454
0.40	.3502	.3657	.3811	.3965	.4117	.4268	.4417	.4565	.4711	.4855
0.50	.2957	.3090	.3222	.3354	.3485	.3616	.3745	.3874	.4001	.4128
0.60	.2353	.2460	.2566	.2673	.2780	.2886	.2992	.3097	.3202	.3306
0.70	.1721	.1800	.1880	.1959	.2038	.2117	.2197	.2276	.2354	.2433
0.80	.1097	.1148	.1199	.1250	.1302	.1353	.1404	.1456	.1507	.1558
0.90	.0514	.0538	.0562	.0586	.0610	.0634	.0659	.0683	.0707	.0732
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	1.5000	1.5200	1.5400	1.5600	1.5800	1.6000	1.6200	1.6400	1.6600	1.6800
0.	0.6432	0.6596	0.6756	0.6912	0.7063	0.7209	0.7350	0.7487	0.7618	0.7746
0.10	.6339	.6502	.6661	.6815	.6965	.7111	.7252	.7388	.7520	.7647
0.20	.6061	.6220	.6375	.6527	.6674	.6818	.6957	.7093	.7224	.7351
0.30	.5608	.5760	.5908	.6054	.6196	.6335	.6471	.6603	.6732	.6857
0.40	.4997	.5137	.5275	.5410	.5543	.5674	.5802	.5928	.6051	.6172
0.50	.4252	.4376	.4498	.4619	.4738	.4856	.4972	.5086	.5199	.5310
0.60	.3409	.3512	.3614	.3715	.3815	.3915	.4013	.4111	.4208	.4304
0.70	.2511	.2589	.2667	.2744	.2821	.2898	.2974	.3050	.3126	.3201
0.80	.1609	.1661	.1712	.1763	.1814	.1865	.1915	.1966	.2017	.2067
0.90	.0756	.0780	.0805	.0829	.0854	.0878	.0902	.0927	.0951	.0976
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	1.7000	1.7200	1.7400	1.7600	1.7800	1.8000	1.8200	1.8400	1.8600	1.8800
0.	0.7868	0.7985	0.8098	0.8206	0.8310	0.8409	0.8504	0.8594	0.8680	0.8762
0.10	.7770	.7888	.8001	.8110	.8214	.8315	.8410	.8502	.8590	.8673
0.20	.7474	.7593	.7707	.7818	.7925	.8027	.8126	.8221	.8312	.8400
0.30	.6979	.7098	.7213	.7325	.7434	.7539	.7641	.7740	.7835	.7927
0.40	.6290	.6406	.6519	.6629	.6737	.6842	.6945	.7045	.7143	.7238
0.50	.5420	.5527	.5633	.5738	.5840	.5941	.6040	.6137	.6233	.6327
0.60	.4398	.4492	.4585	.4677	.4768	.4858	.4947	.5035	.5122	.5208
0.70	.3276	.3350	.3424	.3497	.3570	.3642	.3714	.3786	.3857	.3927
0.80	.2117	.2167	.2217	.2267	.2317	.2367	.2416	.2465	.2515	.2564
0.90	.1000	.1024	.1049	.1073	.1097	.1122	.1146	.1170	.1194	.1218
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	1.9000	1.9200	1.9400	1.9600	1.9800	2.0000	2.0200	2.0400	2.0600	2.0800
0.	0.8840	0.8914	0.8984	0.9051	0.9114	0.9173	0.9230	0.9283	0.9333	0.9380
0.10	.8753	.8828	.8900	.8969	.9034	.9095	.9154	.9209	.9261	.9310
0.20	.8484	.8564	.8641	.8715	.8785	.8852	.8916	.8977	.9035	.9090
0.30	.8016	.8102	.8186	.8266	.8343	.8417	.8489	.8558	.8624	.8688
0.40	.7331	.7422	.7510	.7595	.7679	.7760	.7838	.7915	.7989	.8062
0.50	.6419	.6509	.6598	.6685	.6771	.6854	.6937	.7017	.7096	.7173
0.60	.5293	.5376	.5459	.5541	.5622	.5702	.5781	.5858	.5935	.6011
0.70	.3997	.4067	.4136	.4205	.4273	.4341	.4408	.4475	.4541	.4607
0.80	.2613	.2661	.2710	.2758	.2807	.2855	.2903	.2951	.2998	.3046
0.90	.1243	.1267	.1291	.1315	.1339	.1363	.1387	.1412	.1436	.1460
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

TABLE 8. *Fractional temperature excess, ϕ_3, for a sphere*—Continued

x/a \ q	2.1000	2.1200	2.1400	2.1600	2.1800	2.2000	2.2200	2.2400	2.2600	2.2800
0.	0.9424	0.9466	0.9505	0.9541	0.9575	0.9607	0.9637	0.9665	0.9691	0.9716
0.10	.9357	.9400	.9442	.9481	.9517	.9551	.9583	.9613	.9642	.9668
0.20	.9143	.9193	.9240	.9286	.9328	.9369	.9407	.9444	.9478	.9511
0.30	.8750	.8808	.8865	.8919	.8971	.9021	.9069	.9115	.9159	.9201
0.40	.8132	.8200	.8266	.8330	.8392	.8452	.8510	.8567	.8621	.8674
0.50	.7249	.7323	.7396	.7467	.7536	.7604	.7671	.7736	.7799	.7862
0.60	.6086	.6160	.6232	.6304	.6375	.6445	.6514	.6582	.6648	.6714
0.70	.4672	.4737	.4801	.4865	.4928	.4991	.5053	.5115	.5177	.5237
0.80	.3093	.3141	.3188	.3235	.3281	.3328	.3374	.3420	.3467	.3512
0.90	.1484	.1508	.1532	.1555	.1579	.1603	.1627	.1651	.1675	.1699
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	2.3000	2.3200	2.3400	2.3600	2.3800	2.4000	2.4200	2.4400	2.4600	2.4800
0.	0.9738	0.9759	0.9779	0.9797	0.9814	0.9829	0.9844	0.9857	0.9869	0.9881
0.10	.9693	.9716	.9737	.9757	.9776	.9794	.9810	.9825	.9839	.9852
0.20	.9542	.9571	.9598	.9624	.9648	.9671	.9693	.9713	.9732	.9750
0.30	.9241	.9279	.9316	.9351	.9385	.9417	.9447	.9477	.9504	.9531
0.40	.8726	.8775	.8823	.8869	.8914	.8957	.8999	.9040	.9079	.9116
0.50	.7922	.7982	.8040	.8097	.8152	.8206	.8259	.8311	.8361	.8410
0.60	.6779	.6844	.6907	.6969	.7030	.7090	.7150	.7208	.7266	.7323
0.70	.5298	.5358	.5417	.5476	.5534	.5592	.5649	.5706	.5763	.5818
0.80	.3558	.3604	.3649	.3694	.3739	.3784	.3829	.3874	.3918	.3962
0.90	.1722	.1746	.1770	.1794	.1817	.1841	.1865	.1888	.1912	.1936
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	2.5000	2.5200	2.5400	2.5600	2.5800	2.6000	2.6200	2.6400	2.6600	2.6800
0.	0.9891	0.9901	0.9910	0.9918	0.9925	0.9932	0.9938	0.9944	0.9949	0.9954
0.10	.9864	.9875	.9885	.9895	.9904	.9912	.9919	.9926	.9932	.9938
0.20	.9767	.9783	.9798	.9812	.9825	.9837	.9849	.9859	.9869	.9879
0.30	.9556	.9580	.9603	.9624	.9645	.9665	.9684	.9701	.9718	.9734
0.40	.9153	.9188	.9221	.9254	.9285	.9316	.9345	.9373	.9400	.9426
0.50	.8458	.8505	.8550	.8595	.8638	.8680	.8721	.8761	.8800	.8838
0.60	.7378	.7433	.7487	.7540	.7593	.7644	.7695	.7744	.7793	.7841
0.70	.5874	.5929	.5983	.6037	.6090	.6143	.6195	.6247	.6299	.6350
0.80	.4006	.4050	.4094	.4137	.4181	.4224	.4267	.4309	.4352	.4395
0.90	.1959	.1983	.2006	.2030	.2053	.2077	.2100	.2123	.2147	.2170
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	2.7000	2.7200	2.7400	2.7600	2.7800	2.8000	2.8200	2.8400	2.8600	2.8800
0.	0.9958	0.9962	0.9966	0.9969	0.9972	0.9975	0.9978	0.9980	0.9982	0.9984
0.10	.9944	.9949	.9953	.9957	.9961	.9965	.9968	.9971	.9974	.9976
0.20	.9888	.9896	.9903	.9910	.9917	.9923	.9929	.9934	.9939	.9944
0.30	.9749	.9764	.9777	.9790	.9803	.9814	.9825	.9836	.9845	.9855
0.40	.9451	.9475	.9498	.9520	.9542	.9562	.9582	.9601	.9619	.9637
0.50	.8875	.8911	.8946	.8980	.9013	.9046	.9077	.9108	.9137	.9166
0.60	.7889	.7935	.7981	.8026	.8070	.8113	.8156	.8197	.8238	.8279
0.70	.6400	.6450	.6499	.6548	.6597	.6645	.6692	.6739	.6786	.6832
0.80	.4437	.4479	.4521	.4562	.4604	.4645	.4686	.4727	.4768	.4809
0.90	.2194	.2217	.2240	.2263	.2287	.2310	.2333	.2356	.2379	.2402
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	2.9000	2.9200	2.9400	2.9600	2.9800	3.0000	3.0200	3.0400	3.0600	3.0800
0.	0.9985	0.9987	0.9988	0.9990	0.9991	0.9992	0.9993	0.9993	0.9994	0.9995
0.10	.9978	.9980	.9982	.9984	.9985	.9987	.9988	.9989	.9990	.9991
0.20	.9948	.9952	.9956	.9959	.9963	.9965	.9968	.9971	.9973	.9975
0.30	.9864	.9872	.9880	.9887	.9894	.9901	.9907	.9913	.9918	.9923
0.40	.9653	.9669	.9685	.9700	.9714	.9727	.9740	.9753	.9765	.9776
0.50	.9194	.9221	.9247	.9273	.9298	.9322	.9346	.9368	.9390	.9412
0.60	.8318	.8357	.8395	.8432	.8469	.8505	.8540	.8575	.8609	.8642
0.70	.6878	.6923	.6968	.7012	.7055	.7099	.7141	.7184	.7226	.7267
0.80	.4849	.4889	.4929	.4969	.5009	.5048	.5088	.5127	.5165	.5204
0.90	.2425	.2448	.2471	.2494	.2517	.2540	.2563	.2586	.2609	.2632
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	3.0000	3.1000	3.2000	3.3000	3.4000	3.5000	3.6000	3.7000	3.8000	3.9000
0.	0.9992	0.9995	0.9997	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	.9987	.9992	.9995	.9997	.9999	.9999	1.0000	1.0000	1.0000	1.0000
0.20	.9965	.9977	.9985	.9991	.9994	.9996	0.9998	0.9999	0.9999	0.9999
0.30	.9901	.9928	.9949	.9964	.9975	.9982	.9988	.9992	.9994	.9996
0.40	.9727	.9787	.9834	.9872	.9902	.9925	.9944	.9957	.9968	.9977
0.50	.9322	.9432	.9527	.9608	.9676	.9734	.9782	.9822	.9856	.9884
0.60	.8505	.8675	.8829	.8968	.9093	.9205	.9305	.9394	.9474	.9544
0.70	.7099	.7308	.7506	.7693	.7869	.8035	.8190	.8336	.8472	.8600
0.80	.5048	.5243	.5432	.5617	.5797	.5973	.6143	.6309	.6469	.6625
0.90	.2540	.2655	.2768	.2881	.2993	.3104	.3215	.3324	.3433	.3542
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	4.0000	4.1000	4.2000	4.3000	4.4000	4.5000	4.6000	4.7000	4.8000	4.9000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	0.9997	0.9998	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	.9983	.9987	.9991	.9993	.9995	0.9997	0.9998	0.9999	0.9999	0.9999
0.50	.9906	.9925	.9940	.9953	.9963	.9971	.9977	.9982	.9986	.9989
0.60	.9606	.9660	.9708	.9750	.9787	.9818	.9846	.9869	.9890	.9907
0.70	.8719	.8829	.8932	.9027	.9115	.9197	.9272	.9341	.9404	.9462
0.80	.6776	.6923	.7064	.7201	.7334	.7461	.7585	.7703	.7818	.7928
0.90	.3649	.3755	.3861	.3965	.4069	.4172	.4274	.4375	.4475	.4574
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	5.0000	5.1000	5.2000	5.3000	5.4000	5.5000	5.6000	5.7000	5.8000	5.9000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	.9992	0.9994	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999
0.60	.9922	.9935	.9946	.9955	.9962	.9969	.9974	.9979	.9983	.9986
0.70	.9516	.9565	.9609	.9649	.9686	.9720	.9750	.9777	.9802	.9824
0.80	.8034	.8135	.8233	.8327	.8417	.8503	.8585	.8663	.8739	.8810
0.90	.4672	.4769	.4865	.4961	.5055	.5148	.5240	.5331	.5421	.5510
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	6.0000	6.1000	6.2000	6.3000	6.4000	6.5000	6.6000	6.7000	6.8000	6.9000
0.	1,0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	0.9989	0.9991	0.9992	0.9994	0.9995	0.9996	0.9997	0.9998	0.9998	0.9999
0.70	.9844	.9862	.9878	.9893	.9905	.9917	.9927	.9936	.9944	.9951
0.80	.8879	.8944	.9006	.9065	.9122	.9175	.9226	.9274	.9319	.9363
0.90	.5598	.5685	.5771	.5856	.5940	.6023	.6104	.6185	.6264	.6343
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	7.0000	7.1000	7.2000	7.3000	7.4000	7.5000	7.6000	7.7000	7.8000	7.9000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.70	.9957	.9963	.9968	.9972	0.9976	0.9979	0.9982	0.9984	0.9987	0.9988
0.80	.9403	.9442	.9478	.9513	.9546	.9576	.9605	.9632	.9658	.9682
0.90	.6420	.6496	.6572	.6646	.6719	.6791	.6862	.6931	.7000	.7068
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	8.0000	8.1000	8.2000	8.3000	8.4000	8.5000	8.6000	8.7000	8.8000	8.900
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8.70	0.9990	0.9991	0.9993	0.9994	0.9995	0.9996	0.9996	0.9997	0.9997	0.9998
0.80	.9704	.9726	.9745	.9764	.9781	.9797	.9813	.9827	.9840	.9852
0.90	.7135	.7200	.7265	.7328	.7391	.7452	.7512	.7572	.7630	.7687
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

x/a \ q	9.0000	9.1000	9.2000	9.3000	9.4000	9.5000	9.6000	9.7000	9.8000	9.9000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.70	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000
0.85	.9864	.9875	.9885	.9894	.9902	.9910	.9918	.9924	.9930	.9936
0.90	.7743	.7799	.7853	.7906	.7959	.8010	.8060	.8110	.8158	.8206
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

TABLE 8. *Fractional temperature excess, ϕ_3, for a sphere*—Continued

x/a \ q	10.0009	10.1000	10.2000	10.3000	10.4000	10.5000	10.6000	10.7000	10.8000	10.9000
0.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.74	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.80	0.9942	0.9947	0.9951	0.9955	0.9959	0.9963	0.9966	0.9969	0.9972	0.9974
0.90	.8252	.8298	.8343	.8387	.8429	.8472	.8513	.8553	.8593	.8631
1.00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

TABLE 9. *Generalized temperature averages, ψ_3, for a sphere*

x/a \ q.	0.5000	0.5200	0.54000	0.5600	0.5800	0.6000	0.6200	0.6400	0.6600	0.6800
0.	0.0000	0.0001	0.0001	0.0003	0.0004	0.0007	0.0011	0.0016	0.0023	0.0032
0.10	.0000	.0001	.0001	.0003	.0004	.0007	.0011	.0016	.0023	.0032
0.20	.0000	.0001	.0001	.0002	.0004	.0007	.0010	.0016	.0022	.0031
0.30	.0000	.0001	.0001	.0002	.0004	.0006	.0010	.0015	.0021	.0029
0.40	.0000	.0001	.0001	.0002	.0004	.0006	.0009	.0014	.0020	.0027
0.50	.0000	.0001	.0001	.0002	.0003	.0005	.0008	.0012	.0018	.0025
0.60	.0000	.0000	.0001	.0002	.0003	.0005	.0007	.0011	.0016	.0022
0.70	.0000	.0000	.0001	.0002	.0003	.0004	.0006	.0010	.0014	.0019
0.80	.0000	.0000	.0001	.0001	.0002	.0003	.0005	.0008	.0011	.0016
0.90	.0000	.0000	.0001	.0001	.0002	.0003	.0004	.0006	.0009	.0013
1.00	.0000	.0000	.0000	.0001	.0001	.0002	.0003	.0005	.0007	.0010

x/a \ q.	0.7000	0.7200	0.7400	0.7600	0.7800	0.8000	0.8200	0.8400	0.8600	0.8800
0.	0.0043	0.0057	0.0074	0.0093	0.0116	0.0141	0.0170	0.0202	0.0237	0.0276
0.10	.0043	.0057	.0073	.0092	.0114	.0140	.0168	.0200	.0235	.0273
0.20	.0042	.0055	.0071	.0089	.0111	.0136	.0163	.0194	.0228	.0265
0.30	.0040	.0052	.0067	.0085	.0106	.0129	.0155	.0185	.0217	.0252
0.40	.0037	.0049	.0063	.0079	.0098	.0120	.0145	.0172	.0202	.0234
0.50	.0034	.0044	.0057	.0072	.0089	.0109	.0132	.0156	.0184	.0213
0.60	.0030	.0039	.0051	.0064	.0079	.0097	.0117	.0139	.0163	.0189
0.70	.0026	.0034	.0044	.0055	.0068	.0084	.0101	.0120	.0141	.0163
0.80	.0021	.0028	.0036	.0046	.0057	.0070	.0084	.0100	.0117	.0136
0.90	.0017	.0023	.0029	.0037	.0046	.0056	.0068	.0080	.0094	.0110
1.00	.0013	.0017	.0022	.0028	.0035	.0043	.0052	.0061	.0072	.0084

x/a \ q.	0.9000	0.9200	0.9400	0.9600	0.9800	1.0000	1.0200	1.0400	1.0600	1.0800
0.	0.0317	0.0361	0.0408	0.0458	0.0510	0.0565	0.0622	0.0680	0.0741	0.0802
0.10	.0314	.0358	.0404	.0454	.0505	.0559	.0616	.0674	.0733	.0795
0.20	.0305	.0347	.0392	.0440	.0491	.0543	.0598	.0654	.0712	.0771
0.30	.0290	.0330	.0373	.0419	.0467	.0516	.0568	.0622	.0677	.0734
0.40	.0270	.0307	.0347	.0390	.0434	.0481	.0529	.0579	.0630	.0683
0.50	.0245	.0280	.0316	.0355	.0395	.0438	.0481	.0527	.0574	.0622
0.60	.0218	.024[illegible]	.0281	.0315	.0351	.0388	.0427	.0468	.0509	.0552
0.70	.0188	.0214	.0242	.0272	.0303	.0335	.0369	.0404	.0440	.0477
0.80	.0157	.0179	.0202	.0227	.0253	.0280	.0308	.0337	.0367	.0398
0.90	.0126	.0144	.0163	.0182	.0203	.0225	.0248	.0271	.0295	.0320
1.00	.0096	.0110	.0124	.0139	.0155	.0172	.0189	.0207	.0226	.0244

x/a \ q.	1.1000	1.1200	1.1400	1.1600	1.1800	1.2000	1.2200	1.2400	1.2600	1.2800
0.	0.0866	0.0930	0.0995	0.1061	0.1128	0.1195	0.1262	0.1329	0.1396	0.1463
0.10	.0857	.0921	.0985	.1051	.1117	.1183	.1249	.1316	.1382	.1449
0.20	.0832	.0894	.0957	.1020	.1084	.1149	.1214	.1278	.1343	.1407
0.30	.0792	.0851	.0910	.0971	.1032	.1093	.1155	.1217	.1279	.1340
0.40	.0737	.0792	.0848	.0904	.0961	.1019	.1076	.1134	.1192	.1250
0.50	.0671	.0721	.0772	.0823	.0875	.0928	.0981	.1034	.1087	.1140
0.60	.0596	.0640	.0685	.0731	.0778	.0824	.0871	.0919	.0966	.1014
0.70	.0514	.0553	.0592	.0632	.0672	.0712	.0753	.0794	.0835	.0876
0.80	.0430	.0462	.0495	.0528	.0562	.0596	.0630	.0664	.0699	.0733
0.90	.0345	.0371	.0398	.0424	.0451	.0479	.0506	.0534	.0562	.0590
1.00	.0264	.0284	.0304	.0324	.0345	.0366	.0387	.0408	.0429	.0451

x/a \ q	1.3000	1.3200	1.3400	1.3600	1.3800	1.4000	1.4200	1.4400	1.4600	1.4800
0.	0.1529	0.1595	0.1660	0.1724	0.1787	0.1850	0.1911	0.1971	0.2030	0.2088
0.10	.1514	.1580	.1644	.1708	.1771	.1833	.1894	.1953	.2012	.2069
0.20	.1471	.1535	.1598	.1660	.1722	.1783	.1842	.1901	.1958	.2014
0.30	.1402	.1463	.1523	.1583	.1642	.1700	.1758	.1815	.1870	.1925
0.40	.1308	.1365	.1422	.1478	.1534	.1589	.1644	.1697	.1750	.1802
0.50	.1193	.1245	.1298	.1350	.1401	.1452	.1503	.1553	.1602	.1650
0.60	.1061	.1108	.1155	.1202	.1248	.1294	.1340	.1385	.1430	.1474
0.70	.0918	.0959	.1000	.1041	.1081	.1122	.1162	.1202	.1241	.1280
0.80	.0768	.0803	.0837	.0872	.0906	.0940	.0974	.1008	.1042	.1075
0.90	.0618	.0646	.0674	.0702	.0729	.0757	.0785	.0812	.0839	.0866
1.00	.0472	.0493	.0515	.0536	.0558	.0579	.0600	.0621	.0642	.0663

TABLE 9. *Generalized temperature averages, ψ_3, for a sphere*—Continued

x/a \ q	1.5000	1.5200	1.5400	1.5600	1.5800	1.6000	1.6200	1.6400	1.6600	1.6800
0.	0.2144	0.2199	0.2252	0.2304	0.2354	0.2403	0.2450	0.2496	0.2539	0.2582
0.10	.2125	.2180	.2233	.2285	.2335	.2383	.2430	.2476	.2520	.2562
0.20	.2070	.2123	.2176	.2227	.2276	.2325	.2371	.2417	.2461	.2503
0.30	.1978	.2030	.2082	.2132	.2180	.2228	.2274	.2318	.2362	.2404
0.40	.1853	.1903	.1952	.2000	.2047	.2093	.2138	.2182	.2225	.2266
0.50	.1698	.1745	.1792	.1837	.1882	.1925	.1968	.2010	.2051	.2091
0.60	.1518	.1561	.1604	.1646	.1687	.1727	.1767	.1806	.1845	.1882
0.70	.1319	.1357	.1395	.1432	.1469	.1505	.1541	.1577	.1612	.1646
0.80	.1108	.1140	.1173	.1205	.1237	.1268	.1299	.1329	.1360	.1390
0.90	.0893	.0920	.0946	.0972	.0998	.1024	.1049	.1074	.1099	.1124
1.00	.0683	.0704	.0724	.0744	.0764	.0784	.0803	.0823	.0842	.0861

x/a \ q	1.7000	1.7200	1.7400	1.7600	1.7800	1.8000	1.8200	1.8400	1.8600	1.8800
0.	0.2623	0.2662	0.2699	0.2735	0.2770	0.2803	0.2835	0.2865	0.2893	0.2921
0.10	.2603	.2642	.2680	.2716	.2751	.2784	.2816	.2846	.2875	.2903
0.20	.2544	.2583	.2621	.2658	.2693	.2727	.2759	.2790	.2820	.2849
0.30	.2445	.2485	.2523	.2560	.2596	.2630	.2663	.2695	.2726	.2756
0.40	.2307	.2346	.2384	.2421	.2457	.2492	.2526	.2559	.2591	.2621
0.50	.2131	.2169	.2206	.2243	.2279	.2313	.2347	.2380	.2412	.2443
0.60	.1920	.1956	.1991	.2026	.2061	.2094	.2127	.2159	.2190	.2221
0.70	.1680	.1713	.1746	.1778	.1809	.1840	.1871	.1901	.1931	.1960
0.80	.1419	.1448	.1477	.1505	.1533	.1561	.1588	.1615	.1641	.1667
0.90	.1148	.1172	.1196	.1220	.1243	.1266	.1289	.1311	.1333	.1355
1.00	.0880	.0898	.0916	.0935	.0953	.0971	.0988	.1006	.1023	.1040

x/a \ q	1.9000	1.9200	1.9400	1.9600	1.9800	2.0000	2.0200	2.0400	2.0600	2.0800
0.	0.2947	0.2971	0.2995	0.3017	0.3038	0.3058	0.3077	0.3094	0.3111	0.3127
0.10	.2929	.2954	.2978	.3001	.3022	.3042	.3061	.3079	.3097	.3113
0.20	.2876	.2902	.2927	.2950	.2973	.2994	.3015	.3034	.3052	.3070
0.30	.2784	.2811	.2838	.2863	.2887	.2910	.2932	.2953	.2973	.2992
0.40	.2651	.2679	.2707	.2734	.2759	.2784	.2808	.2831	.2853	.2875
0.50	.2473	.2503	.2531	.2559	.2586	.2612	.2638	.2662	.2686	.2709
0.60	.2251	.2280	.2309	.2337	.2364	.2391	.2417	.2443	.2468	.2492
0.70	.1988	.2016	.2044	.2071	.2097	.2123	.2149	.2174	.2198	.2222
0.80	.1693	.1718	.1743	..1767	.1791	.1815	.1839	.1862	.1884	.1907
0.90	.1377	.1398	.1419	.1440	.1460	.1481	.1501	.1520	.1540	.1559
1.00	.1056	.1073	.1089	.1106	.1122	.1137	.1153	.1168	.1184	.1199

x/a \ q	2.1000	2.1200	2.1400	2.1600	2.1800	2.2000	2.2200	2.2400	2.2600	2.2800
0.	0.3141	0.3155	0.3168	0.3180	0.3192	0.3202	0.3212	0.3222	0.3230	0.3239
0.10	.3128	.3142	.3156	.3168	.3180	.3191	.3202	.3211	.3221	.3229
0.20	.3086	.3102	.3116	.3130	.3143	.3156	.3168	.3179	.3189	.3199
0.30	.3011	.3028	.3045	.3061	.3076	.3090	.3104	.3117	.3129	.3141
0.40	.2895	.2915	.2934	.2952	.2969	.2986	.3002	.3018	.3033	.3047
0.50	.2732	.2753	.2775	.2795	.2815	.2834	.2852	.2870	.2888	.2904
0.60	.2515	.2539	.2561	.2583	.2604	.2625	.2645	.2665	.2685	.2703
0.70	.2246	.2269	.2292	.2314	.2336	.2357	.2378	.2399	.2419	.2438
0.80	.1929	.1950	.1972	.1993	.2014	.2034	.2054	.2074	.2093	.2112
0.90	.1578	.1597	.1615	.1634	.1652	.1670	.1687	.1704	.1722	.1738
1.00	.1214	.1228	.1243	.1257	.1271	.1285	.1299	.1313	.1326	.1340

x/a \ q	2.3000	2.3200	2.3400	2.3600	2.3800	2.4000	2.4200	2.4400	2.4600	2.4800
0.	0.3246	0.3253	0.3260	0.3266	0.3271	0.3276	0.3281	0.3286	0.3290	0.3294
0.10	.3237	.3244	.3251	.3258	.3264	.3269	.3274	.3279	.3284	.3288
0.20	.3208	.3216	.3225	.3232	.3239	.3246	.3252	.3258	.3263	.3269
0.30	.3152	.3163	.3173	.3182	.3191	.3200	.3208	.3215	.3223	.3229
0.40	.3060	.3073	.3086	.3098	.3109	.3120	.3131	.3141	.3150	.3159
0.50	.2920	.2936	.2951	.2966	.2980	.2994	.3007	.3019	.3032	.3043
0.60	.2722	.2739	.2757	.2774	.2790	.2806	.2822	.2837	.2852	.2866
0.70	.2458	.2477	.2495	.2514	.2531	.2549	.2566	.2583	.2599	.2615
0.80	.2131	.2150	.2168	.2186	.2204	.2221	.2238	.2255	.2272	.2288
0.90	.1755	.1772	.1788	.1804	.1820	.1835	.1851	.1866	.1881	.1896
1.00	.1353	.1366	.1379	.1392	.1404	.1417	.1429	.1441	.1453	.1465

x/a \ q	2.5000	2.5200	2.5400	2.5600	2.5800	2.6000	2.6200	2.6400	2.6600	2.6800
0.	0.3297	0.3300	0.3303	0.3306	0.3308	0.3311	0.3313	0.3315	0.3316	0.3318
0.10	.3292	.3295	.3298	.3301	.3304	.3307	.3309	.3311	.3313	.3315
0.20	.3273	.3278	.3282	.3286	.3289	.3293	.3296	.3299	.3301	.3304
0.30	.3236	.3242	.3248	.3253	.3258	.3263	.3267	.3272	.3276	.3279
0.40	.3168	.3176	.3184	.3192	.3199	.3206	.3212	.3219	.3225	.3230
0.50	.3055	.3066	.3076	.3087	.3096	.3106	.3115	.3124	.3132	.3141
0.60	.2880	.2894	.2907	.2920	.2932	.2944	.2956	.2968	.2979	.2990
0.70	.2631	.2646	.2662	.2676	.2691	.2705	.2719	.2733	.2746	.2759
0.80	.2304	.2320	.2336	.2351	2366	.2381	.2396	.2410	.2424	.2438
0.90	.1911	.1925	.1940	.1954	.1968	.1981	.1995	.2008	.2022	.2035
1.00	.1477	.1488	.1500	.1511	.1522	.1533	.1544	.1555	.1566	.1576

TABLE 9. *Generalized temperature averages, ψ_3, for a sphere*—Continued

x/a \ q	2.7000	2.7200	2.7400	2.7600	2.7800	2.8000	2.8200	2.8400	2.8600	2.8800
0.	0.3319	0.3321	0.3322	0.3323	0.3324	0.3325	0.3326	0.3327	0.3327	0.3328
0.10	.3317	.3318	.3320	.3321	.3322	.3323	.3324	.3325	.3326	.3326
0.20	.3306	.3308	.3310	.3312	.3314	.3315	.3317	.3318	.3320	.3321
0.30	.3283	.3286	.3289	.3292	.3295	.3298	.3300	.3302	.3304	.3306
0.40	.3236	.3241	.3246	.3250	.3255	.3259	.3263	.3267	.3270	.3274
0.50	.3148	.3156	.3163	.3170	.3177	.3184	.3190	.3196	.3202	.3207
0.60	.3000	.3011	.3021	.3030	.3040	.3049	.3058	.3066	.3075	.3083
0.70	.2772	.2784	.2797	.2809	.2820	.2832	.2843	.2854	.2865	.2876
0.80	.2452	.2466	.2479	.2492	.2505	.2518	.2530	.2543	.2555	.2567
0.90	.2048	.2061	.2073	.2086	.2098	.2110	.2122	.2134	.2146	.2157
1.00	.1587	.1597	.1607	.1617	.1627	.1637	.1647	.1657	.1666	.1676

x/a \ q	2.9000	2.9200	2.9400	2.9600	2.9800	3.0000	3.0200	3.0400	3.0600	3.0800
0.	0.3328	0.3329	0.3329	0.3330	0.3330	0.3331	0.3331	0.3331	0.3331	0.3332
0.10	.3327	.3328	.3328	.3329	.3329	.3330	.3330	.3330	.3331	.3331
0.20	.3322	.3323	.3324	.3324	.3325	.3326	.3327	.3327	.3328	.3328
0.30	.3308	.3310	.3312	.3313	.3315	.3316	.3317	.3318	.3319	.3320
0.40	.3277	.3280	.3283	.3286	.3289	.3291	.3293	.3296	.3298	.3300
0.50	.3212	.3218	.3223	.3227	.3232	.3236	.3240	.3244	.3248	.3252
0.60	.3091	.3098	.3106	.3113	.3120	.3127	.3134	.3140	.3146	.3152
0.70	.2886	.2896	.2906	.2916	.2925	.2934	.2944	.2952	.2961	.2970
0.80	.2578	.2590	.2602	.2613	.2624	.2635	.2645	.2656	.2666	.2677
0.90	.2169	.2180	.2191	.2203	.2213	.2224	.2235	.2245	.2256	.2266
1.00	.1685	.1694	.1703	.1713	.1722	.1730	.1739	.1748	.1756	.1765

x/a \ q	3.0000	3.1000	3.2000	3.3000	3.4000	3.5000	3.6000	3.7000	3.8000	3.9000
0.	0.3331	0.3332	0.3332	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
0.10	.3330	.3331	.3332	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.20	.3326	.3329	.3330	.3332	.3332	.3333	.3333	.3333	.3333	.3333
0.30	.3316	.3321	.3325	.3328	.3330	.3331	.3332	.3332	.3333	.3333
0.40	.3291	.3302	.3310	.3316	.3321	.3324	.3327	.3328	.3330	.3331
0.50	.3236	.3256	.3271	.3284	.3294	.3303	.3309	.3314	.3319	.3322
0.60	.3127	.3158	.3185	.3208	.3228	.3244	.3259	.3271	.3281	.3290
0.70	.2934	.2978	.3017	.3052	.3083	.3112	.3137	.3159	.3179	.3197
0.80	.2635	.2687	.2735	.2779	.2820	.2859	.2894	.2927	.2957	.2985
0.90	.2224	.2276	.2326	.2372	.2416	.2457	.2497	.2534	.2569	.2603
1.00	.1730	.1773	.1814	.1853	.1890	.1925	.1959	.1991	.2022	.2051

x/a \ q	4.0000	4.1000	4.2000	4.3000	4.4000	4.5000	4.6000	4.7000	4.8000	4.9000
0.	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
0.10	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.20	.3333	.3333	.3333	.3333	.3333	.3333	.3833	.3333	.3333	.3333
0.30	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.40	.3331	.3332	.3332	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.50	.3325	.3327	.3328	.3329	.3330	.3331	.3332	.3332	.3332	.3333
0.60	.3297	.3303	.3308	.3313	.3316	.3319	.3332	.3324	.3326	.3327
0.70	.3213	.3227	.3239	.3251	.3261	.3270	.3277	.3284	.3290	.3296
0.80	.3011	.3036	.3058	.3079	.3098	.3116	.3133	.3148	.3162	.3175
0.90	.2634	.2665	.2693	.2721	.2747	.2771	.2795	.2818	.2839	.2859
1.00	.2079	.2106	.2132	.2156	.2180	.2203	.2225	.2246	.2266	.2286

x/a \ q	5.0000	5.1000	5.2000	5.3000	5.4000	5.5000	5.6000	5.7000	5.8000	5.9000
0.	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
0.10	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.20	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.30	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.40	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
9.50	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.60	.3328	.3329	.3330	.3331	.3331	.3332	.3332	.3332	.3332	.3333
0.70	.3300	.3305	.3308	.3312	.3314	.3317	.3319	.3321	.3323	.3324
0.80	.3188	.3199	.3209	.3219	.3228	.3236	.3244	.3251	.3257	.3263
0.90	.2879	.2897	.2915	.2932	.2949	.2964	.2979	.2993	.3007	.3020
1.00	.2305	.2323	.2341	.2358	.2374	.2390	.2406	.2420	.2435	.2449

x/a \ q	6.0000	6.1000	6.2000	6.3000	6.4000	6.5000	6.6000	6.7000	6.8000	6.9000
0.	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
0.10	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.20	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.30	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.40	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.50	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.60	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.70	.3326	.3327	.3328	.3329	.3329	.3330	.3330	.3331	.3331	.3331
0.80	.3269	.3274	.3279	.3283	.3288	.3291	.3295	.3298	.3301	.3304
0.90	.3033	.3045	.3056	.3067	.3077	.3088	.3097	.3107	.3116	.3124
1.00	.2462	.2475	.2488	.2501	.2513	.2525	.2536	.2547	.2558	.2568

TABLE 9. *Generalized temperature averages, ψ_3, for a sphere*—Continued

x/a \ q	7.0000	7.1000	7.2000	7.3000	7.4000	7.5000	7.6000	7.7000	7.8000	7.9000
0.	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
0.10	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.20	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.30	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.40	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.50	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.60	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.70	.3332	.3332	.3332	.3332	.3333	.3333	.3333	.3333	.3333	.3333
0.80	.3306	.3309	.3311	.3313	.3314	.3316	.3317	.3319	.3320	.3321
0.90	.3132	.3140	.3148	.3155	.3162	.3169	.3175	.3182	.3188	.3193
1.00	.2578	.2588	.2598	.2607	.2616	.2625	.2634	.2643	.2651	.2659

x/a \ q	8.0000	8.1000	8.2000	8.3000	8.4000	8.5000	8.6000	8.7000	8.8000	8.9000
0.	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
0.10	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.20	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.30	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.40	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.50	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.60	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.70	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.80	.3322	.3323	.3324	.3325	.3326	.3327	.3327	.3328	.3328	.3329
0.90	.3199	.3204	.3209	.3214	.3219	.3223	.3227	.3232	.3236	.3240
1.00	.2667	.2675	.2682	.2690	.2697	.2704	.2711	.2718	.2724	.2731

x/a \ q	9.0000	9.1000	9.2000	9.3000	9.4000	9.5000	9.6000	9.7000	9.8000	9.9000
0.	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
0.10	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.20	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.30	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.40	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.50	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.60	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.70	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.80	.3329	.3330	.3330	.3330	.3331	.3331	.3331	.3331	.3332	.3332
0.90	.3243	.3247	.3250	.3254	.3257	.3260	.3263	.3266	.3268	.3271
1.00	.2737	.2743	.2749	.2755	.2761	.2767	.2773	.2778	.2783	.2789

x/a \ q	10.0000	10.1000	10.2000	10.3000	10.4000	10.5000	10.6000	10.7000	10.8000	10.9000
0.	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
0.10	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.20	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.30	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.40	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.50	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.60	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.70	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.80	.3332	.3332	.3332	.3333	.3333	.3333	.3333	.3333	.3333	.3333
0.90	.3273	.3276	.3278	.3280	.3283	.3284	.3286	.3288	.3290	.3292
1.00	.2794	.2799	.2804	.2809	.2814	.2819	.2823	.2828	.2832	.2837

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