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Supplement 1

U.S. DEPARTMENT OF COMMERCE / National Bureau of Standards

Relativistic Many-Body Bound Systems: Electromagnetic Properties

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Relativistic Many-Body Bound Systems: Electromagnetic Properties

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CONTENTS

ABSTRACT

The formulae for the calculation of the electron scattering form factors, and of the static magnetic dipole and electric quadrupole moments, of relativistic many-body bound systems are derived. The framework, given in NBS Monograph 147, is relativistic quantum field theory in the Schrödinger picture ; the physical particles, i.e. the solutions of the interacting fields, are given as linear combinations of the solutions of the free fields, called the parton fields. The parton-photon interaction is taken as given by minimal coupling, $\vec{p} \to \vec{p} - e\vec{A}$; in addition the contribution of the photon-vector meson vertex of the vector dominance model is derived.

Key words: Electromagnetic properties; form factors; magnetic moments; quadropole moments; quantum field theory; relativistic many body systems; vector dominance.

CHAPTER I

ELECTROMAGNETIC INTERACTION OPERATORS

I.1 - INTRODUCTION

In a previous publication[1], NBS Monograph 147, the description of a nucleus has been formulated as a stationary problem of relativistic quantum field theory. In that report, only the problem of the stationary state energies and wave functions has been considered.

In the present Monograph, we shall derive the formulae needed for computation of the electromagnetic properties of these states, i.e. the electron scattering form factors, the magnetic and quadrupole static and transition moments. To this end we shall give the explicit form of the one-body matrix elements in terms of the single particle discretized basis states used in NBS Monograph 147. These matrix elements can then be used in a well-known fashion to calculate the properties of the many-body solutions.

From now on, we shall refer to NBS Monograph 147 as I. In this report, we shall use all the definitions and notations introduced in I. We shall refer to a formula from that book as for example I(4.123) and to a page number as for example I p.105. In particular, we refer to Chapter II for the phase conventions and the angular momentum diagramatic coupling techniques and to Chapter III for the definition of the metric. As in I, we shall treat successively the cases of spin 0, spin 1/2, and spin 1 fields. For the spin 1 field, we employ the formulation of R. Hayward, ref. [2].

I.2 - FORM OF THE INTERACTIONS

As interaction with the electromagnetic field we consider only the minimal coupling, i.e. the forms obtained by the replacement

$$\vec{p} \to \vec{p} - e\vec{A}$$

in the Lagrangian. In other words no anomalous moment is ascribed to the spin 1/2 and the spin 1 fields. However, we shall add explicitly the photon-vector meson interaction of the vector dominance model. These two prescriptions are introduced since in the present framework on the one hand a certain part of the mesonic contributions to the magnetic moments is treated explicitly while on the other hand the configuration spaces contemplated in I-Chapter V are truncated at rather low energies (about 1 GeV). In particular, they do not contain any baryon-antibaryon components which would presumably contribute the bulk of the $\rho\gamma$ vertex.

Since we consider photon emission or absorption processes, we need only terms linear in the photon field A_μ. Thus we can write in general

$$\mathscr{H}_{\text{interaction}} = - \int d^3r \; J_\mu A_\mu \; . \tag{1.1}$$

The currents J_μ are of the form, for spin 0 fields

$$J_\mu = ie(\phi^+ \frac{\partial}{\partial x_\mu} \tau_z \, \phi - \frac{\partial}{\partial x_\mu} \phi^+ \, \tau_z \, \phi) \; , \tag{1.2}$$

for spin 1/2 fields

$$J_\mu = ie \, \overline{\psi} \, \gamma_\mu \; (1/2 + \tau_z/2)\psi \; , \tag{1.3}$$

and for spin 1 fields [2]

$$J_\mu = ie(\overline{\rho} \; \gamma_\mu \; \gamma_4 \; \tau_z \; \overline{\pi} + \pi \; \gamma_4 \; \gamma_\mu \; \tau_z \; \rho) \; . \tag{1.4}$$

The electron scattering form factors, after factorizing out the photon propagator, are given by (see for example refs. [3] and [4])

$$F_\mu(\vec{q}) = \int d^3r \; J_\mu(\vec{r}) \; e^{i\vec{q}\vec{r}} \; . \tag{1.5}$$

The quadrupole moment is obtained by evaluating the quadrupole operator

$$\underset{\sim}{q}(\vec{r}) = 2z^2 - x^2 - y^2 \; , \tag{1.6}$$

with the charge density $\rho_o = - i \; J_4$

$$Q = - i \int d^3r \; J_4(\vec{r}) \; \underset{\sim}{q}(\vec{r}) \; . \tag{1.7}$$

This form follows from the application of Siegert's theorem, which we briefly recall here. Let us note first that in the limit $q \to 0$ the electric field of the magnetic multipolarities goes to zero faster than that of the electric and longitudinal multipolarities. Furthermore, for real photons the longitudinal multipolarities are absent. Then, one can apply Siegert's theorem which is based on the identity for the electric field

$$\vec{\mathcal{E}}_\ell = - \vec{\nabla} V_\ell + (\vec{qr}) f_\ell(\vec{qr}) , \qquad (1.8)$$

where V_ℓ denotes the scalar multipole

$$V_\ell(\vec{r}) = a_\ell Y_\ell(\hat{r}) j_\ell(qr) . \qquad (1.9)$$

Thus for $q \to 0$ and combining the proper time dependence in the real fields A_μ and J_μ in order to obtain a real energy

$$- \int d^3r\, J_\mu A_\mu = \frac{i}{\omega} \int d^3r\, \vec{J}.\vec{\mathcal{E}} = - \frac{i}{\omega} \int d^3r\, \vec{J}.\vec{\nabla}V$$

$$= \frac{i}{\omega} \int d^3r\, \nabla J.V = - \frac{i}{\omega} \int d^3r\, \dot{\rho}_o V = \int d^3r\, \rho_o V ,$$

which yields the form (1.7) for the quadrupole part of V.

The magnetic moment operator is in general of the form, ref. [2]

$$\vec{\underset{\sim}{\mu}} = \mu_o(\vec{\ell} + \vec{\sigma}) \qquad (1.10)$$

where $\vec{\sigma}$ is the spin operator of the field. Therefore we have for spin 0

$$\vec{\underset{\sim}{\mu}} = - i\mu_o \int d^3r\, J_4 \vec{\ell} , \qquad (1.11)$$

for spin 1/2

$$\vec{\underset{\sim}{\mu}} = - i\mu_o \int d^3r\, J_4(\vec{j} + \vec{s}) , \qquad (1.12)$$

and for spin 1

$$\vec{\underset{\sim}{\mu}} = - i\mu_o \int d^3r\, J_4 \vec{j} . \qquad (1.13)$$

Note that these expressions contain no anomalous part in agreement with the introductory remarks. Note also that the form (1.10) is valid for arbitrary spin in the formulation of R. Hayward, ref. [2]. In the cases s = 0 and s = 1, eqs. (1.11) and (1.12) thus yield a particularly simple result since the magnetic moment is proportional to a conserved quantity, namely the total angular momentum of the particle. Finally, the use of the fourth component of the conserved electromagnetic current in eqs. (1.11) to (1.13), as is well-known, arises from a quasi-Siegert theorem. Namely from the relation

$$\vec{B} = \text{rot } \vec{A} ,$$

we get the identity in the limit $q \to 0$

$$\vec{A} = \frac{1}{2} \vec{r} \times \vec{B} + \vec{q} \times \vec{r} f(qr) , \qquad (1.14)$$

where $\vec{B}$ is a constant (independent of space coordinates). Herewith we have for the interaction

$$- \int d^3r\, \vec{J}.\vec{A} = - \frac{e}{m} \int d^3r\, \vec{p}.\vec{A} = \frac{-e}{2m} \int d^3r\, \vec{p}.(\vec{r} \times \vec{B})$$

$$= \frac{e}{2m} \int d^3r\, (\vec{r} \times \vec{p}).\vec{B} = \frac{e\vec{B}}{2m} . \int d^3r\, \vec{\ell} \ , \qquad (1.15)$$

where the integral over $\vec{\ell}$ means the expectation value of the orbital angular momentum of the system. The replacement of the current $\vec{J}$ by the velocity $\vec{v}$ must be carried out in a formulation of the Hamiltonian in which the spin-orbit coupling has been eliminated, e.g. for the spin 1/2 particle by going from the first order to the second order Dirac equations, see ref.[5], eq. (12.11), or more generally for any spin, ref.[2], section 4. In these formulations the interaction of the spin with the magnetic field is already in the form $\mu_o\, \vec{s}.\vec{B}$. The expectation value of (1.15) is evaluated by integration over a normalized quantity which has the sign of the charge, i.e. the charge density

$$\rho_o(\vec{r}) = - i\, J_4(\vec{r}).$$

I.3 - VECTOR DOMINANCE

In the vector dominance model, one assumes that the most important interaction of hadrons and photons arises by the process in which a photon is converted into a neutral vector meson which then interacts with the hadronic system via the strong interactions. Consequently the direct interaction of the photons with the hadronic current via the term $J_\mu\, A_\mu$ is assumed to be negligible and is frequently dropped. We shall not make here this assumption. We shall however add a term to the interaction Lagrangian describing the photon-vector meson interaction and call it the vector dominance term.

The simplest gauge invariant Lorentz scalar which is linear in both the photon and vector meson fields is

$$\mathscr{L}^{VD}_{int} = g \int d^4x\, F_{\mu\nu}(x)\, \Phi_{\mu\nu}(x) \ , \qquad (1.16)$$

where

$$F_{\mu\nu}(x) = \partial_\mu\, A_\nu(x) - \partial_\nu\, A_\mu(x) \ , \qquad (1.17)$$

and

$$\Phi_{\mu\nu}(x) = \partial_\mu\, \omega_\nu(x) - \partial_\nu\, \omega_\mu(x) \ , \qquad (1.18)$$

in terms of the photon vector potential $A_\mu(x)$ and the field of the ω-meson $\omega_\mu(x)$. A similar expression holds for the neutral ρ-meson. Since we shall use the Lorentz gauge

$$\partial_\mu \, \omega_\mu = 0, \tag{1.19}$$

for the vector mesons, eq. (1.16) can be simplified by integrating by parts

$$\begin{aligned}\mathcal{L}^{VD}_{int} &= - g \int d^4x (A_\nu \, \partial_\mu \, \Phi_{\mu\nu} - A_\mu \, \partial_\nu \, \Phi_{\mu\nu}) \\ &= - 2g \int d^4x \, A_\nu (\partial_\mu \, \partial_\mu \, \omega_\nu - \partial_\mu \, \partial_\nu \, \omega_\mu) \\ &= - 2g \, m_\omega^2 \int d^4x \, A_\nu(x) \, \omega_\nu(x) \; .\end{aligned} \tag{1.20}$$

In the first term we have used the Klein-Gordon equation, see I(3.149), to replace the d'Alembertian by the meson mass, while the second term vanishes owing to the Lorentz condition (1.19).

Thus the vector dominance interaction for the ω and the (neutral) ρ^o fields are of the form

$$\mathcal{L}^{VD}_{int} = - G_{\gamma\omega} \int d^4x \, A_\mu(x) \, \omega_\mu(x) - G_{\gamma\rho} \int d^4x \, A_\mu(x) \, \rho^o_\mu(x) \; . \tag{1.21}$$

Note that the replacement of the d'Alambertian by the mass in (1.20) is correct since (1.21) will be used to evaluate matrix elements and our basis functions for the vector mesons indeed obey the Klein-Gordon equation. Of course, it is not implied that the Klein-Gordon equation comprises the complete equations of motion. Also recall that (1.16) is only an ad hoc effective interaction which is not contained in the original Lagrangian.

CHAPTER II

ELASTIC ELECTRON SCATTERING FORM FACTOR

II.1 - SPIN 0 FIELD

II.1.1 - Scattering terms

In the case of pions, we must consider separately the photon absorption terms, figure 2.1.a (which we shall call "scattering terms") and the pair creation and annihilation terms, figures 2.1.b and 2.1.c. They differ by the form of the isospin operator and the coupling adopted between the particle states and the transferred multipoles.

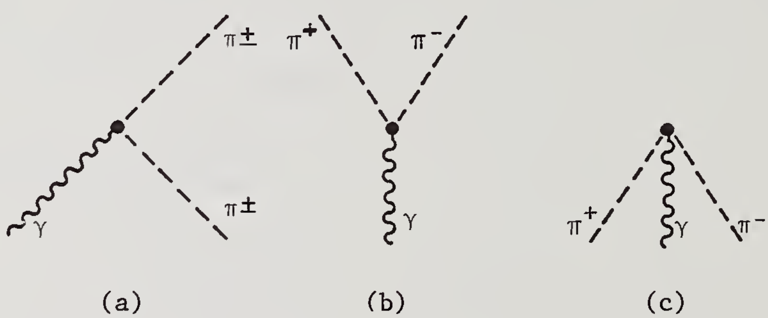


Fig. 2.1

We treat first the scattering terms figure 2.1.a. Only the charged pions contribute, thus we introduce the charge operator

$$\tau_z = \hat{1}[c^{[1]}\ \tau^{[1]}]^{[0]}, \qquad (2.1)$$

with

$$c_m^{[1]} = - i\delta_{m0} \ . \tag{2.2}$$

Polarization in ordinary space is added to polarization in isospin space with an overall space and isospin amplitude denoted by $c^{[11]}$ and $c^{[01]}$ for the space vector and space scalar parts respectively :

$$\vec{J} = ie\ \hat{1}^2[c^{[11]}(\phi'^+(\nabla^{[1]}\ \tau^{[1]}\ \phi') - (\nabla^{[1]}\ \phi'^+)\tau^{[1]}\phi')]^{[00]} \ , \tag{2.3}$$

$$J_4 = ie\ \hat{1}[c^{[01]}(\phi'^+(\frac{\partial}{\partial x_4}\ \tau^{[1]}\ \phi') - (\frac{\partial}{\partial x_4}\ \phi'^+)\tau^{[1]}\phi')]^{[00]} \ . \tag{2.4}$$

These forms are constructed to express the fact that the current is an invariant, see chapter I of ref. [5] and chapter II of ref. [1].

The complex fields ϕ'^+ and ϕ' are obtained in their discretized forms from the real fields, expressions I(3.24), by selecting the proper creation and annihilation parts. In the case of the scalar current one may introduce the π' fields of I(3.25) with the relations I(3.2a) and I(3.2b),

$$\frac{\partial}{\partial x_4}\ \phi' = - i\ \pi'^+ \quad \text{and} \quad \frac{\partial}{\partial x_4}\ \phi'^+ = - i\ \pi' \ . \tag{2.5}$$

Finally in eq. (1.5) we shall make use of the multipole expansion in tensorial form of the plane wave

$$e^{i\vec{q}\vec{r}} = 4\pi \sum_{\ell} (i)^{\ell}\ \hat{\ell}[\hat{q}^{[\ell]}\ \hat{r}^{[\ell]}]^{[0]}\ j_{\ell}(qr) \ . \tag{2.6}$$

In all the terms of the expressions (2.3) and (2.4), the isospin parts yield the same contribution according to the recoupling diagram of figure 2.2, namely

$$\hat{1}^3[\tilde{A}^{[1]}\ \tilde{\eta}^{[1]}]^{[0]}\ [c^{[1]}\ \tau^{[1]}]^{[0]}\ [A^{[1]}\ \eta^{[1]}]^{[0]}$$

$$= - [1|\tau|1][\tilde{A}^{[1]}\ A^{[1]}\ c^{[1]}]^{[0]} \ . \tag{2.7}$$

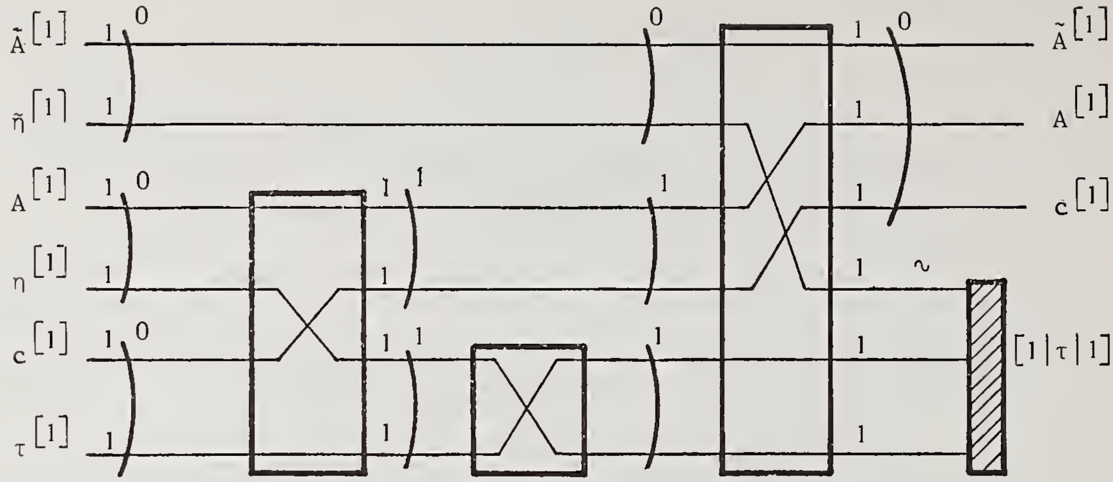


Fig. 2.2

The first term of the vector current contribution, eq. (2.3), is calculated according to figure 2.3. The adopted coupling corresponds to the change of the initial angular momentum ℓ_2 by one unit, $L = \ell_2 \pm 1$, after application of the gradient operator. The second term of eq. (2.3) yields the recoupling diagram of figure 2.4. The final result for the vector form factor for the spin 0 field is thus

$$\vec{F}(\vec{q}) = \frac{ie}{2} 4\pi \sum_{\substack{\nu_1 \ell_1 \\ \ell \\ \nu_2 \ell_2}} (i)^{\ell_2-\ell_1+\ell} (-[1|\tau|1])$$

$$\times \sum_L \frac{1}{\hat{L}} \Big\{ (-)^{\ell_2+1+L} \alpha_{L\ell_2} [\ell_1|L|\ell] \mathcal{F}^{\nu_1\nu_2}_{\ell_1 L \ell \ell_2}(q)$$

$$\times [\tilde{A}^{[\ell_1 1]} [A^{[\ell_2 1]} c^{[11]}]^{[L1]} \hat{q}^{[\ell 0]}]^{[00]}$$

$$- (-)^{\ell_1+1+L} \alpha_{L\ell_1} [\ell_2|L|\ell] \mathcal{F}^{\nu_2\nu_1}_{\ell_2 L \ell \ell_1}(q)$$

$$\times [[\tilde{A}^{[\ell_1 1]} c^{[11]}]^{[L1]} A^{[\ell_2 1]} q^{[\ell 0]}]^{[00]} \Big\} , \qquad (2.8)$$

where we have defined

$$\mathscr{F}^{\nu_1\nu_2}_{\ell_1 L \ell \ell_2}(q) = \sqrt{\frac{\pi}{2}} \int r^2 dr \int \frac{p^3 dp}{\sqrt{E}}\, g_{\nu_1\ell_1}(r)\, j_L(qr)\, j_\lambda(pr)\, f_{\nu_2\ell_2}(p) \;. \qquad (2.9)$$

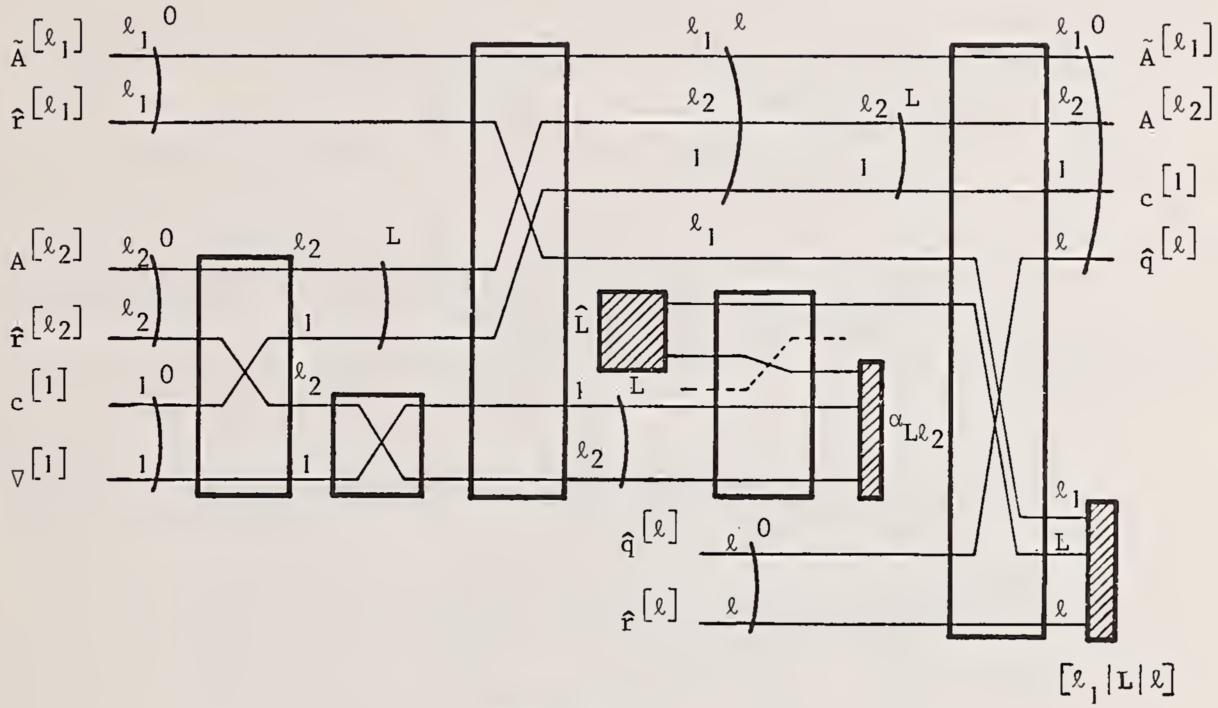


Fig. 2.3

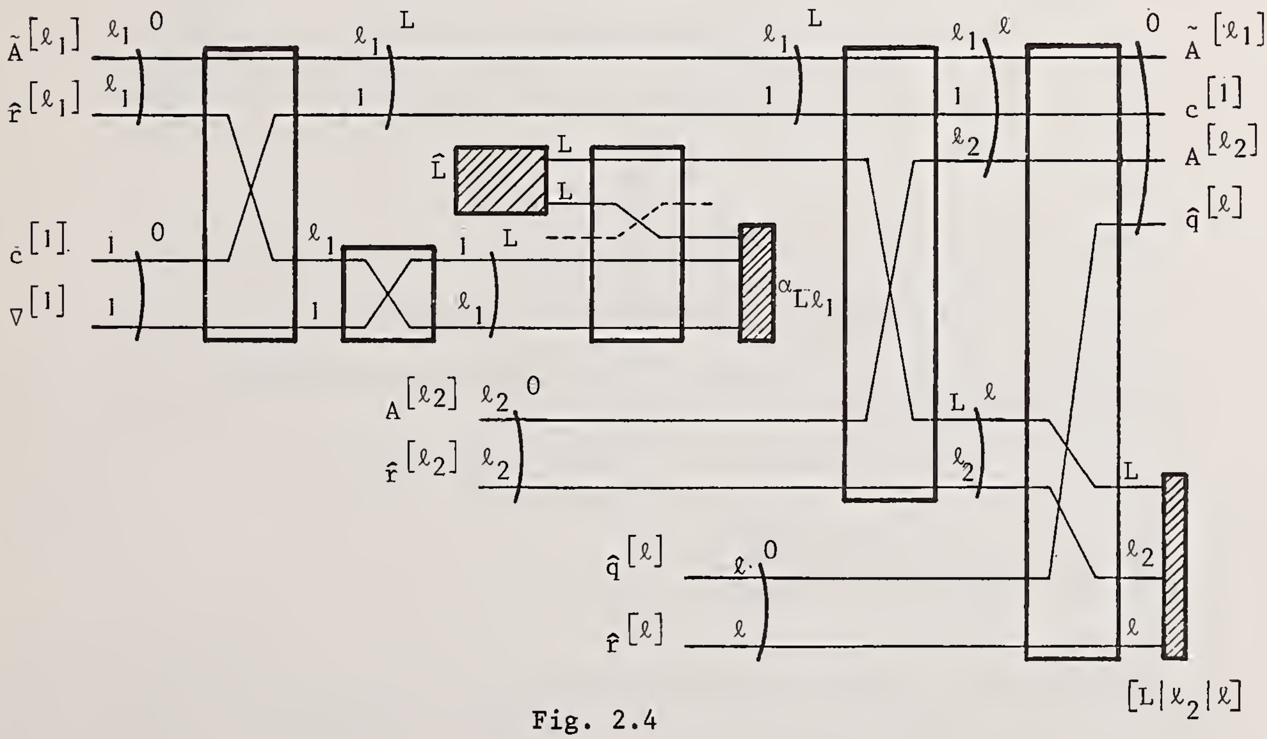


Fig. 2.4

Next we evaluate the scalar form factor due to the scalar current, eq. (2.4). Owing to eq. (2.5), the two terms give equal contributions, the space part of which is evaluated in figure 2.5. Together with the isospin contribution of eq. (2.7) we get

$$F_4(\vec{q}) = -\,ie\,4\pi \sum_{\substack{\nu_1\ell_1 \\ \ell \\ \nu_2\ell_2}} (i)^{\ell_2-\ell_1+\ell} (-[1|\tau|1])[\ell_1|\ell_2|\ell] \tag{2.10}$$

$$\times \int r^2 dr\, g_{\nu_1\ell_1}(r)\, j_\ell(qr)\, h_{\nu_2\ell_2}(r) [\tilde{A}^{[\ell_1 1]} A^{[\ell_2 1]} c^{[01]} \hat{q}^{[\ell 0]}]^{[00]}.$$

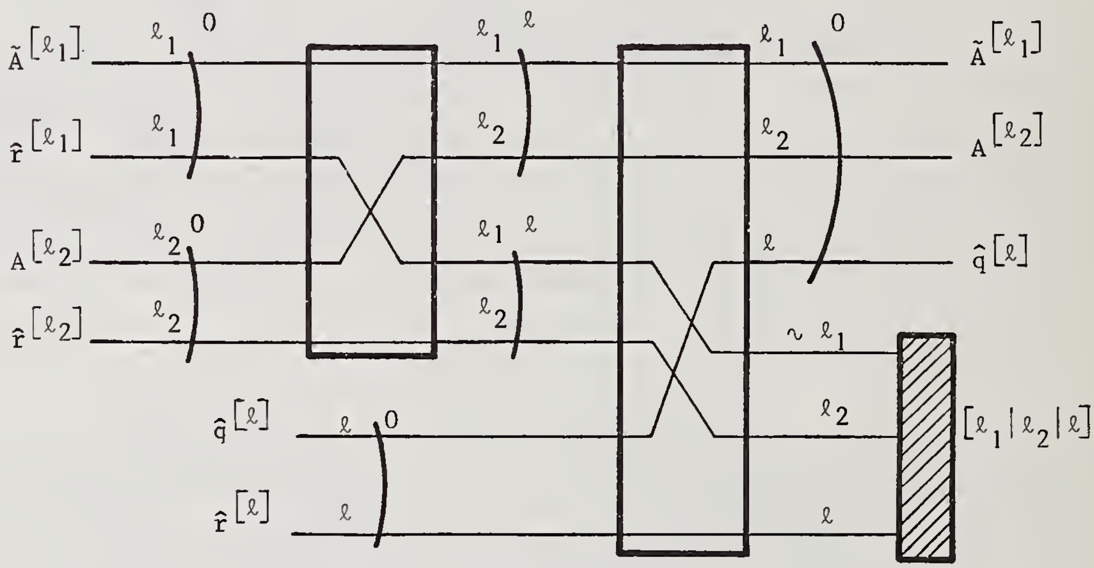


Fig. 2.5

II.1.2 - Pair creation and pair annihilation terms

In the case of pair production or annihilation, figures 2.1.b and 2.1.c, we must supply in isospin space the operator

$$\Delta T_z = \sum_T \hat{L}\, [C^{[T]}\, \tau^{[1]}\, \tau^{[1]}]^{[0]} \tag{2.11}$$

with

$$C_M^{[T]} = \delta_{M\,0}\, (1\;1\quad 1 -1\;|T\;0)$$

since only charged pions contribute. Thus, with polarization amplitudes $c^{[xT]}$ which include both ordinary space and isospin space, the invariant form of the current for the pair diagram is (we write $\nabla^{[10]}$ to indicate that it is a unit scalar in isospin space ; conversely for $\tau^{[01]}$)

$$\vec{J} = ie\ \hat{1} \sum_T \hat{T}[c^{[1T]}(\phi'\ (\nabla^{[10]}\ \tau^{[01]}\ \tau^{[01]}\ \phi') - (\nabla^{[10]}\ \phi'\ )\tau^{[01]}\ \tau^{[01]}\ \phi')]^{[00]}$$

$$+ \text{ same term with } \phi'^{+} \text{ in place of } \phi', \tag{2.12}$$

$$J_4 = ie \sum_T \hat{T}[c^{[0T]}(\phi'\ (\frac{\partial}{\partial x_4}\ \tau^{[01]}\ \tau^{[01]}\ \phi') - (\frac{\partial}{\partial x_4}\ \phi'\ )\tau^{[01]}\ \tau^{[01]}\ \phi')]^{[00]}$$

$$+ \text{ same term with } \phi'^{+} \text{ in place of } \phi'. \tag{2.13}$$

We consider first the pair creation term of figure 2.1.b.

The isospin part yields according to figure 2.6 the following contribution, both for the scalar and vector form factors

$$\hat{1}^2\ \hat{T}[\tilde{A}^{[1]}\ \tilde{\eta}^{[1]}]^{[0]}\ [c^{[T]}\ \tau^{[1]}\ \tau^{[1]}]^{[0]}\ [\tilde{A}^{[1]}\ \tilde{\eta}^{[1]}]^{[0]}$$

$$= \frac{1}{\hat{1}} \begin{bmatrix} 1 & 1 & T \\ 1 & 1 & T \\ 1 & 1 & 0 \end{bmatrix}\ [1|\tau|1]^2\ [\tilde{A}^{[1]}\ \tilde{A}^{[1]}\ c^{[T]}]^{[0]}\ . \tag{2.14}$$

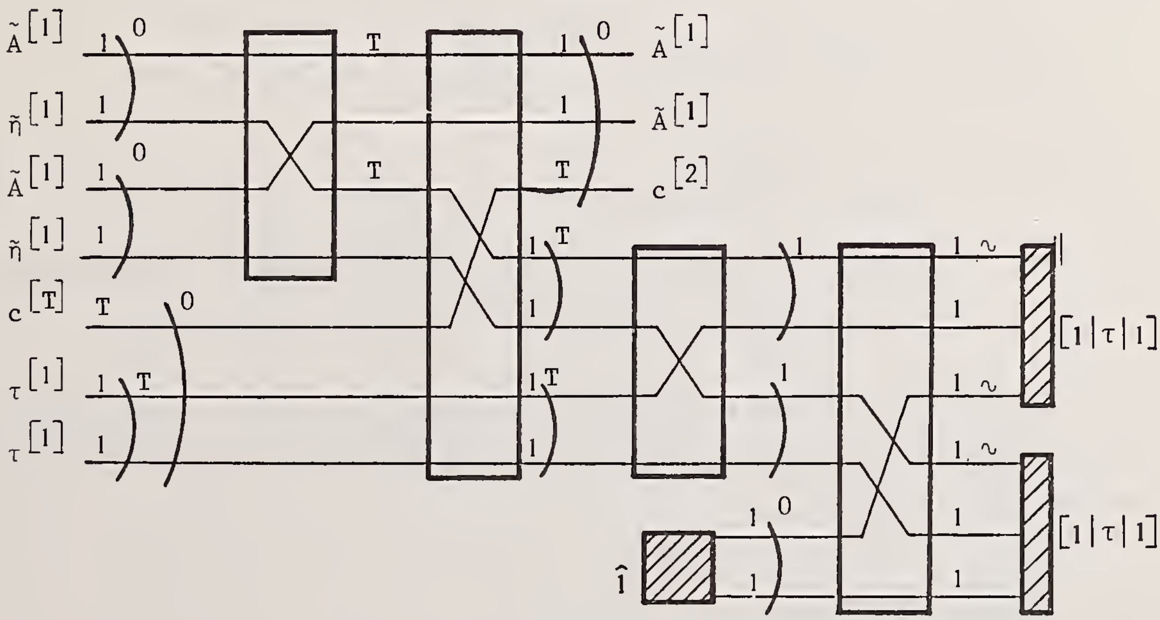


Fig. 2.6

We turn to the space part. We consider first the vector form factor. The recoupling geometry is the same as in figures 2.3 and 2.4, except for a phase $(-)^{\ell_2}$. The final expression is thus

$$\vec{F}(\vec{q}) = \frac{ie}{2}\, 4\pi \sum_{\substack{\nu_1\ell_1 \\ \ell \\ \nu_2\ell_2}} \sum_{T} (i)^{-\ell_2-\ell_1+\ell} \left(\frac{1}{\hat{1}} \begin{bmatrix} 1 & 1 & T \\ 1 & 1 & T \\ 1 & 1 & 0 \end{bmatrix} \quad [1|\tau|1]^2\right)$$

$$\times \sum_L \frac{1}{\hat{1}} \Big\{ (-)^{\ell_2+1+L} \quad \alpha_{L\ell_2} [\ell_1|L|\ell] \mathcal{F}^{\nu_1\nu_2}_{\ell_1 L\ell\ell_2}(q)$$

$$\times [\tilde{A}^{[\ell_1 1]} [\tilde{A}^{[\ell_2 1]} \; c^{[1T]}]^{[L1]} \; \hat{q}^{[\ell 0]}]^{[00]}$$

$$- (-)^{\ell_1+1+L} \quad \alpha_{L\ell_1} [\ell_2|L|\ell] \mathcal{F}^{\nu_2\nu_1}_{\ell_2 L\ell\ell_1}(q)$$

$$\times [[\tilde{A}^{[\ell_1 1]} \; c^{[1T]}]^{[L1]} \; \tilde{A}^{[\ell_2 1]} \; \hat{q}^{[\ell 0]}]^{[00]} \Big\} \; . \tag{2.15}$$

Likewise for the scalar form factor, see figure 2.5 together with figure 2.6

$$F_4(\vec{q}) = -\, ie\, 4\pi \sum_{\substack{\nu_1\ell_1 \\ \ell \\ \nu_2\ell_2}} \sum_{T} (i)^{-\ell_2-\ell_1+\ell} \left(\frac{1}{\hat{1}} \begin{bmatrix} 1 & 1 & T \\ 1 & 1 & T \\ 1 & 1 & 0 \end{bmatrix} \quad [1|\tau|1]^2\right) \tag{2.16}$$

$$\times \int r^2 dr\, g_{\nu_1\ell_1}(r)\; j_\ell(qr)\; h_{\nu_2\ell_2}(r) [\tilde{A}^{[\ell_1 1]} \; \tilde{A}^{[\ell_2 1]} \; c^{[0T]} \; \hat{q}^{[\ell 0]}]^{[00]} \; .$$

For the pair annihilation terms these expressions hold upon replacement of the creation operators $\tilde{A}$ by the annihilation operators A and upon multiplication by the phase $(-1)^{\ell_1+\ell_2}$.

II.2 - SPIN 1/2 FIELD

Owing to the limitation of our configuration space, see Chapter VI ref. [1] we here have to consider only the scattering terms. Furthermore, since we assume the neutron to have no electromagnetic interactions we introduce the charge projection operator

$$P = \frac{1}{2}\left(1 + \hat{1}\,[c^{[1]}\ \tau^{[1]}]^{[0]}\right), \tag{2.17}$$

with, as in eq. (2.2)

$$c_m^{[1]} = -\,i\ \delta_{m0}\ . \tag{2.18}$$

Introducing again polarization amplitudes $c^{[xy]}$ in ordinary (index x) and isospin (index y) spaces the currents are

$$\vec{J} = \frac{ie}{2}\ \hat{1}\,[\bar{\psi}\ c^{[10]}\ \gamma^{[10]}\ \psi + \hat{1}\ \bar{\psi}\ c^{[11]}\ \gamma^{[10]}\ \tau^{[01]}\ \psi]^{[00]}\ , \tag{2.19}$$

$$J_4 = \frac{ie}{2}\ [\bar{\psi}\ c^{[00]}\ \gamma_4\ \psi + \hat{1}\ \bar{\psi}\ c^{[01]}\ \gamma_4\ \tau^{[01]}\ \psi]^{[00]}\ . \tag{2.20}$$

The isospin part is treated in figure 2.7 and yields

$$\left(\frac{\hat{1}}{2}\right)\ [\tilde{B}^{[1/2]}\ \tilde{\eta}^{[1/2]}]^{[0]}\ \frac{1}{2}\ (1 + \hat{1}[c^{[1]}\ \tau^{[1]}]^{[0]})\left(\frac{\hat{1}}{2}\right)\ [B^{[1/2]}\ \eta^{[1/2]}]^{[0]}$$

$$= \frac{1}{2}\sum_{y=0,1}\ (-)^y[1/2|\tau^{[y]}|1/2]\,[\tilde{B}^{[1/2]}\ B^{[1/2]}\ c^{[y]}]^{[0]}\ , \tag{2.21}$$

with

$$c^{[0]} = 1\ . \tag{2.22}$$

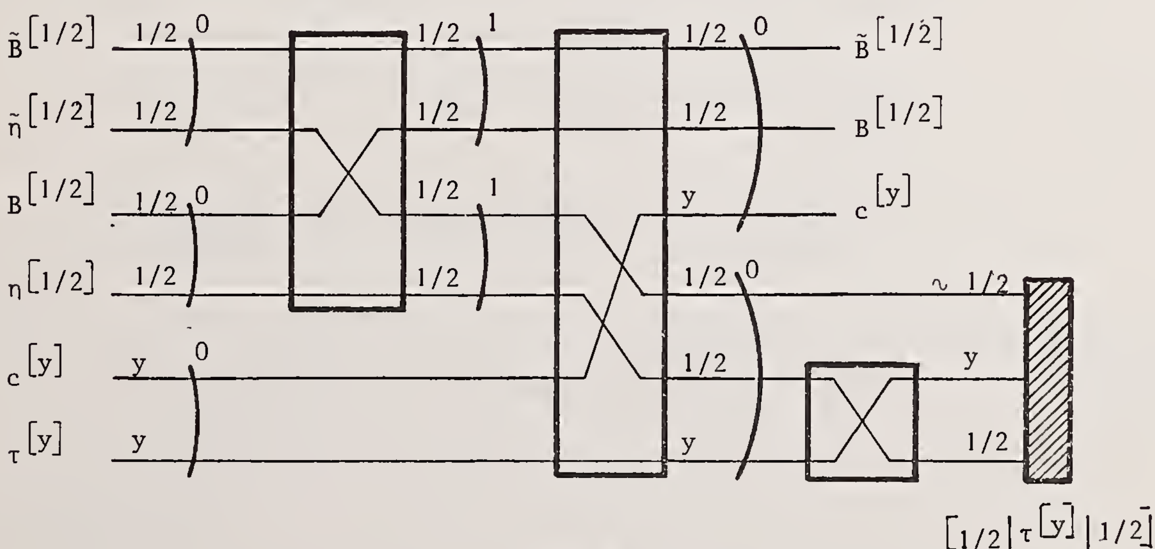


Figure 2.7

We shall evaluate first the vector current contribution to the form factor. The spinor field with its large and small components

$$\psi = \begin{pmatrix} L \\ S \end{pmatrix} , \tag{2.23}$$

is given in its discretized form in I(3.98). Expressing the γ matrices in terms of the Pauli matrices σ, the expression (2.19) for the vector current becomes

$$\vec{J} = \frac{ie}{2} \sum_{y=1,0} \hat{1}\,\hat{y}(-)^y [c^{[1y]} (L^+ \sigma^{[10]} \tau^{[0y]} S + S^+ \sigma^{[10]} \tau^{[0y]} L)]^{[00]} . \tag{2.24}$$

The vector part of the form factor has thus a geometry given by figure 2.8 together with figure 2.7 for the isospin

$$\vec{F}(\vec{q}) = - ie\, 2\pi \Bigg\{ \frac{1}{2} \sum_{y=1,0} (-)^y [1/2|\tau^{[y]}|1/2][1/2|\sigma^{[1]}|1/2]$$

$$\times \sum_{\substack{\nu_1\ell_1 j_1(\lambda_1) \\ \ell \\ \nu_2\ell_2 j_2(\lambda_2)}} \sum_L (-)^{\ell_2-\ell_1+\ell} \frac{\hat{L}}{\hat{1}\,\hat{\ell}} \left(\begin{bmatrix} 1/2 & \ell_1 & j_1 \\ 1/2 & \lambda_2 & j_2 \\ 1 & \ell & L \end{bmatrix} [\ell_1|\lambda_2|\ell] \mathscr{G}^{\nu_1\nu_2}_{\ell_1\ell_2\lambda_2\ell}(q) \right.$$

$$\left. + \begin{bmatrix} 1/2 & \lambda_1 & j_1 \\ 1/2 & \ell_2 & j_2 \\ 1 & \ell & L \end{bmatrix} [\lambda_1|\ell_2|\ell] \mathscr{G}^{\nu_2\nu_1}_{\ell_2\ell_1\lambda_1\ell}(q) \right) \Bigg\}$$

$$\times [\tilde{B}^{[j_1 1/2]}_{\nu_1\ell_1} B^{[j_2 1/2]}_{\nu_2\ell_2} [\hat{q}^{[\ell 0]} c^{[1y]}]^{[Ly]}]^{[00]} , \tag{2.25}$$

where

$$\mathscr{G}^{\nu_1\nu_2}_{\ell_1\ell_2\lambda_2\ell}(q) = \int r^2 dr\, u_{\nu_1\ell_1}(r)\, j_\ell(qr)\, v_{\nu_2\ell_2\lambda_2}(r) . \tag{2.26}$$

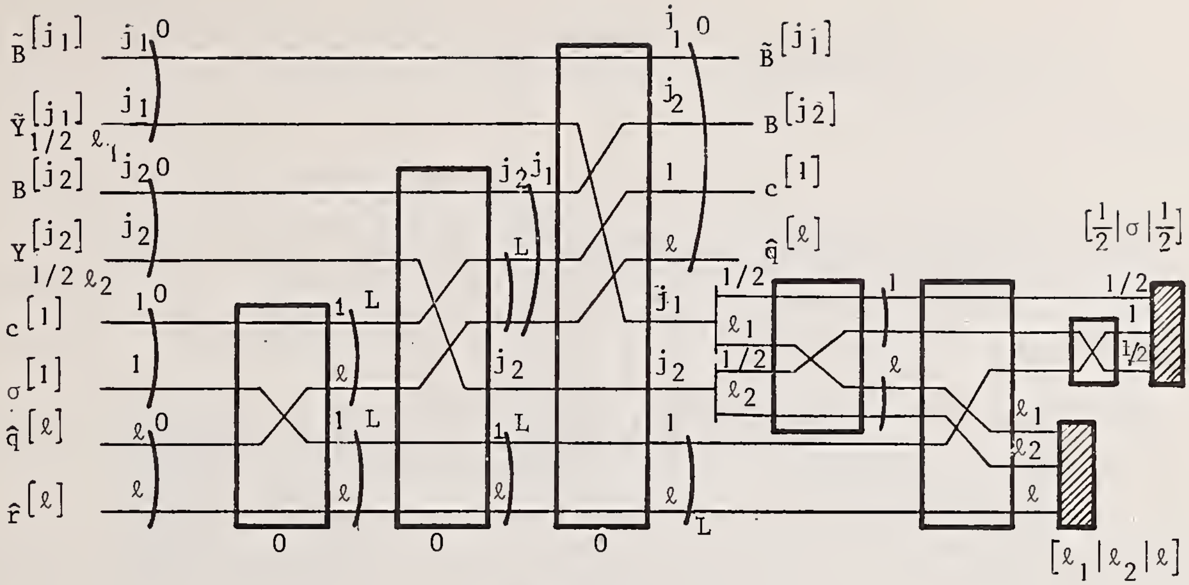


Fig. 2.8

The scalar current (2.20) is in terms of the large and small components eq. (2.23),

$$J_4 = \frac{ie}{2}\sum_{y=0,1} \hat{y}\left[c^{[0y]}\left(L^+ \tau^{[y]} L + S^+ \tau^{[y]} S\right)\right]^{[00]} . \tag{2.27}$$

The evaluation of the scalar form factor for the space geometry is given in figure 2.9 and the final result together with the isospin part (2.21) is

$$F_4(q) = ie\,2\pi \left\{ \frac{1}{2}\sum_{y=1,0} (-)^y [1/2|\tau^{[y]}|1/2]\,\widehat{1/2} \times \sum_{\substack{\nu_1\ell_1 j_1(\lambda_1)\\ \ell \\ \nu_2\ell_2 j_2(\lambda_2)}} (i)^{\ell_2-\ell_1+\ell} \left(\begin{bmatrix}1/2 & \ell_1 & j_1\\ 1/2 & \ell_2 & j_2\\ 0 & \ell & \ell\end{bmatrix} [\ell_1|\ell_2|\ell]\,\mathcal{U}^{\nu_1\nu_2}_{\ell_1\ell_2\ell}(q) + \begin{bmatrix}1/2 & \lambda_1 & j_1\\ 1/2 & \lambda_2 & j_2\\ 0 & \ell & \ell\end{bmatrix} [\lambda_1|\lambda_2|\ell]\,\mathcal{V}^{\nu_1\nu_2}_{\ell_1\lambda_1\ell_2\lambda_2\ell}(q)\right) \left[\tilde{B}^{[j_1 1/2]}_{\nu_1\ell_1}\, B^{[j_2 1/2]}_{\nu_2\ell_2}\, c^{[0y]}\, \hat{q}^{[\ell 0]}\right]^{[00]} \right\}, \tag{2.28}$$

with the definitions

$$\mathcal{U}^{\nu_1\nu_2}_{\ell_1\ell_2\ell}(q) = \int r^2 dr\, u_{\nu_1\ell_1}(r)\, u_{\nu_2\ell_2}(r)\, j_\ell(qr) , \tag{2.29}$$

$$U^{\nu_1\nu_2}_{\ell_1\lambda_1\ell_2\lambda_2\ell}(q) = \int r^2 dr\; v_{\nu_1\ell_1\lambda_1}(r)\; v_{\nu_2\ell_2\lambda_2}(r)\; j_\ell(qr) \;. \qquad (2.30)$$

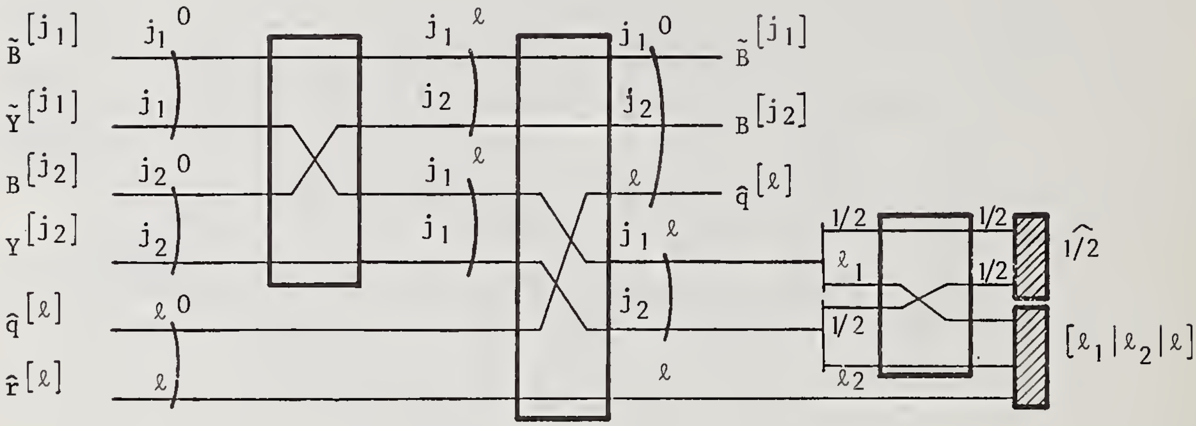


Fig. 2.9

II.3 - SPIN 1 FIELD

II.3.1 - The current operators

The invariant form of the spin 1 current operator (1.4) is obtained by introducing explicitly the γ matrices for spin 1, I(3.118), and the ρ' fields of I(3.131), I(3.132) and I(3.133). These fields are already broken into their scalar and vector components. Thus, noting that in the Lorentz gauge, see I(3.134)

$$\pi'_4 = \bar{\pi}'_4 = 0 \;,$$

we get for the vector current

$$\vec{J} = ie\; \hat{1}^2 (\bar{\rho}' [c^{[11]}\; \gamma^{[10]}\; \tau^{[01]}]^{[00]}\; \gamma_4\; \bar{\pi}'$$
$$+ \pi'\; \gamma_4 [c^{[11]}\; \gamma^{[10]}\; \tau^{[01]}]^{[00]}\; \rho')$$

$$\equiv ie\; \hat{1}^2 (\rho'_4 [c^{[11]}\; \gamma^{[10]}\; \tau^{[01]}]^{[00]}\; \vec{\pi}'$$
$$- \vec{\pi}'^{+} [c^{[11]}\; \gamma^{[10]}\; \tau^{[01]}]^{[00]}\; \rho_4'^{+}) \;, \qquad (2.31)$$

where the charge projection operator of eq. (2.1) has been introduced and where $c^{[11]}$ denotes both space and isospin polarization amplitudes.

Likewise the scalar current is given by

$$J_4 = ie\ \hat{1}(\vec{\rho}\,'[c^{[01]}\ \tau^{[01]}]^{[00]}\ \vec{\pi}\,' + \vec{\pi}\,'^{+}[c^{[01]}\ \tau^{[01]}]^{[00]}\ \vec{\rho}\,'^{+})\ . \tag{2.32}$$

II.3.2 - Form factor matrix elements

We have to consider only scattering terms of the type shown in figures 2.1.a owing to the limitation of our configuration space to at most one spin 1 vector meson, see Chapter VI ref. [1].

The evaluation of the isospin matrix elements in (2.31) and (2.32) is identical to the spin 0 case and is given in eq. (2.7), figure 2.2.

For the vector part of the current and from the explicit form of the γ matrices, one sees that after acting on the fields, i.e. in the step from the first to the second form of equation (2.31), the remaining parts of $\gamma^{[10]}$ are simply unit vectors $e^{[10]}$. We therefore will have to evaluate terms of the form

$$[d^{[1]}\ \gamma^{[1]}]^{[0]}[A^{[J]}\ Y^{[J]}_{1L}]^{[0]} = \frac{1}{\hat{1}}\ [A^{[J]}\ d^{[1]}\ \hat{r}^{[L]}]^{[0]}\ , \tag{2.33}$$

according to figure 2.10, where the symmetry properties of the invariant triple product have been used. We also have introduced the space polarization amplitudes $d^{[1]}$.

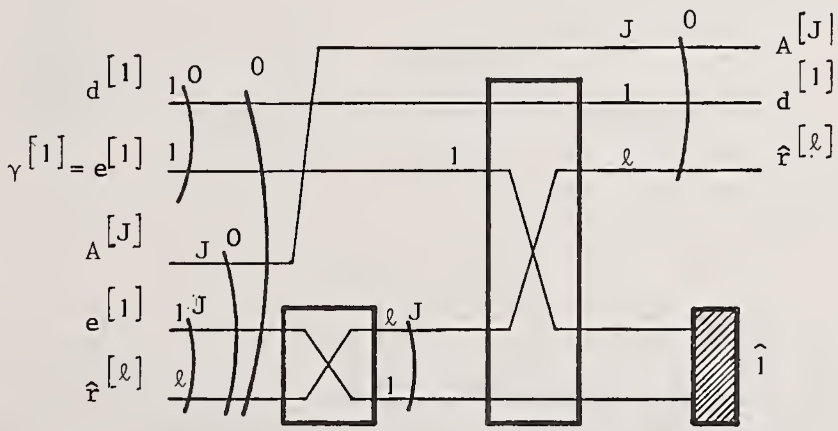


Fig. 2.10

When evaluating the several contributions to the vector current (2.31) from the multipolarities $\kappa = \mathscr{M}$, $\mathscr{E}$ and $\mathscr{L}$ we note that for the ρ_4' field only the longitudinal multipole $\mathscr{L}$ exists. Furthermore, we note that the second term in eq. (2.31) is the hermitian conjugate of the first one. We therefore calculate explicitly its contribution only to the vector form factor, the space geometry of which is given in figure 2.11 and yields

$$[\tilde{A}^{[J_1]}_{\mathscr{L}\nu_1}\, \hat{r}^{[J_1]}]^{[0]}\, [\hat{q}^{[\ell]}\, \hat{r}^{[\ell]}]^{[0]}\, [A^{[J_2]}_{\kappa\nu_2}\, d^{[1]}\, \hat{r}^{[\ell_2]}]^{[0]}$$

$$= \frac{1}{\hat{J}_1\hat{\ell}\hat{\ell}_2}\, [J_1|\ell|\ell_2]\, [\tilde{A}^{[J_1]}_{\mathscr{L}\nu_1}\, [A^{[J_2]}_{\kappa\nu}\, d^{[1]}]^{[\ell_2]}\, \hat{q}^{[\ell]}]^{[0]} \quad . \qquad (2.34)$$

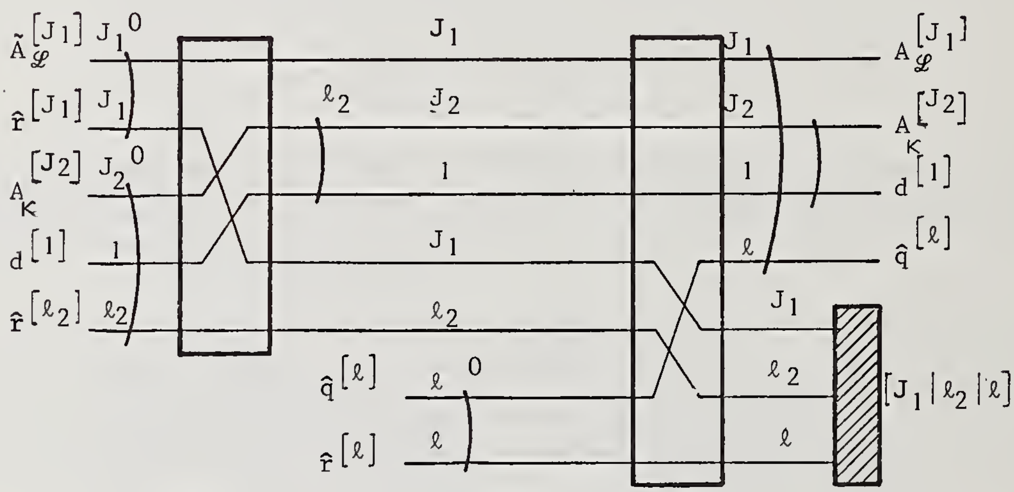


Fig. 2.11

The complete vector form factor with the isospin geometry of figure 2.2 thus is, denoting again by $c^{[11]}$ both the space and isospin amplitudes

$$\vec{F}(\vec{q}) = ie\; 4\pi(-\; [1|\tau^{[1]}|1]) \sum_{\substack{\nu_1\ell_1\\ \ell\\ \nu_2\ell_2}} (i)^{J_2-J_1-1+\ell}$$

$$\times \Big\{ -\; i\, \mathscr{K}^{\mathscr{M}\nu_1\nu_2}_{J_1J_2J_2\ell}(q) [\tilde{A}^{[J_1 1]}_{\mathscr{L}\nu_1} [A^{[J_2 1]}_{\mathscr{M}\nu_2}\; c^{[11]}]^{[J_2 1]}\; \hat{q}^{[\ell 0]}]^{[00]}$$

$$+ \frac{\sqrt{J_2}}{\widehat{(J_2+1)}} \mathscr{K}^{\mathscr{E}\nu_1\nu_2}_{J_1J_2J_2+1\ell}(q) [\tilde{A}^{[J_1 1]}_{\mathscr{L}\nu_1} [A^{[J_2 1]}_{\mathscr{E}\nu_2}\; c^{[11]}]^{[J_2+1\; 1]}\; \hat{q}^{[\ell 0]}]^{[00]}$$

$$+ \frac{\sqrt{J_2+1}}{\widehat{(J_2-1)}} \mathscr{K}^{\mathscr{E}\nu_1\nu_2}_{J_1J_2J_2-1\ell}(q) [\tilde{A}^{[J_1 1]}_{\mathscr{L}\nu_1} [A^{[J_2 1]}_{\mathscr{E}\nu_2}\; c^{[11]}]^{[J_2-1\; 1]}\; \hat{q}^{[\ell 0]}]^{[00]}$$

$$- \frac{\sqrt{J_2+1}}{\widehat{(J_2+1)}} \mathscr{L}^{\nu_1\nu_2}_{J_1J_2J_2+1\ell}(q) [\tilde{A}^{[J_1 1]}_{\mathscr{L}\nu_1} [A^{[J_2 1]}_{\mathscr{L}\nu_2}\; c^{[11]}]^{[J_2+1\; 1]}\; \hat{q}^{[\ell 0]}]^{[00]}$$

$$+ \frac{\sqrt{J_2}}{\widehat{(J_2-1)}} \mathscr{L}^{\nu_1\nu_2}_{J_1J_2J_2-1\ell}(q) [\tilde{A}^{[J_1 1]}_{\mathscr{L}\nu_1} [A^{[J_2 1]}_{\mathscr{L}\nu_2}\; c^{[11]}]^{[J_2-1\; 1]}\; \hat{q}^{[\ell 0]}]^{[00]}$$

$$+\; h.c. \;\ldots \Big\} \quad , \tag{2.35}$$

with the definitions

$$\mathscr{K}^{\kappa\nu_1\nu_2}_{J_1J_2\ell_2\ell}(q) = [J_1|\ell_2|\ell] \int r^2 dr\, \mathcal{V}^{\mathscr{L}}_{\nu_1J_1}(r)\; j_\ell(qr)\, \mathfrak{X}^{\kappa}_{\nu_2J_2\ell_2}(r) \;, \tag{2.36}$$

$$\mathscr{L}^{\nu_1\nu_2}_{J_1J_2\ell_2\ell}(q) = [J_1|\ell_2|\ell] \int r^2 dr\, \mathcal{V}^{\mathscr{L}}_{\nu_1J_1}(r)\; j_\ell(qr)\, \mathcal{Z}^{\mathscr{L}}_{\nu_2J_2\ell_2}(r) \;, \tag{2.37}$$

where the radial functions $\mathcal{V}^{\mathscr{L}}$, $\mathfrak{X}^{\kappa}$ and $\mathcal{Z}^{\mathscr{L}}$ are given in eqs. I(3.236), I(3.233) and I(3.235) respectively. As an example of the hermitian conjugate contributions, we obtain for the second term of (2.35) (recall eq. I(2.28)),

$$- ie\; 4\pi(+\; [1|\tau^{[1]}|1]) \sum (-i)^{J_1-J_2-1+\ell} \frac{\sqrt{J_1}}{\widehat{(J_1+1)}}$$
$$\times \mathscr{K}^{\mathscr{E}\nu_2\nu_1}_{J_2J_1J_1+1\ell}(q) [\hat{q}^{[\ell 0]} [c^{[11]}\; \tilde{A}^{[J_1 1]}_{\mathscr{E}\nu_1}]^{[J_1+1\; 1]}\; A^{[J_2 1]}_{\mathscr{L}\nu_2}]^{[00]} \;. \tag{2.38}$$

We now turn to the scalar current, eq. (2.32). Here the two terms give the same contribution and we have

$$J_4(q) = 2ie\ \hat{1}\ \vec{\rho}\,'\left[c^{[01]}\ \tau^{[01]}\right]^{[00]}\ \vec{\pi}\,' \quad . \tag{2.39}$$

The scalar form factor thus is, see figure 2.12

$$F_4(\vec{q}) = 2ie\ \left(-\ [1|\tau^{[1]}|1]\right)\ 4\pi \sum_{\substack{\nu_1 J_1 \\ \ell \\ \nu_2 J_2}} (i)^{J_2-J_1+\ell}\ \frac{\hat{1}}{\hat{J}_1\hat{J}_2}$$

$$\times \sum_{\substack{\kappa_1\lambda_1 \\ \kappa_2\lambda_2}} \begin{bmatrix} 1 & \lambda_1 & J_1 \\ 1 & \lambda_2 & J_2 \\ 0 & \ell & \ell \end{bmatrix} [\lambda_1|\lambda_2|\ell] \int r^2 dr\ K^{\kappa_1\nu_1}_{J_1\lambda_1}(r)\ L^{\kappa_2\nu_2}_{J_2\lambda_2}(r)\ j_\ell(qr)$$

$$\times \left[\tilde{A}^{[J_1 1]}_{\kappa_1\nu_1}\ A^{[J_2 1]}_{\kappa_2\nu_2}\ c^{[01]}\ \hat{q}^{[\ell 0]}\right]^{[00]} \quad , \tag{2.40}$$

with (see eqs. I(3.232) through I(3.236))

$$K^{\mathcal{M}\nu}_{J\lambda}(r) = \delta_{\lambda J}\ \hat{J}\mathcal{W}^{\mathcal{M}}_{\nu JJ}(r) \quad , \tag{2.41}$$

$$K^{\mathcal{E}\nu}_{J\lambda}(r) = \delta_{\lambda J-1}\ i\sqrt{J+1}\,\mathcal{W}^{\mathcal{E}}_{\nu JJ-1}(r) + \delta_{\lambda J+1}\ i\sqrt{J}\,\mathcal{W}^{\mathcal{E}}_{\nu JJ+1}(r) \quad , \tag{2.42}$$

$$K^{\mathcal{L}\nu}_{J\lambda}(r) = \delta_{\lambda J-1}\ i\sqrt{J}\,\mathcal{W}^{\mathcal{L}}_{\nu JJ-1}(r) - \delta_{\lambda J+1}\ i\sqrt{J+1}\,\mathcal{W}^{\mathcal{L}}_{\nu JJ+1}(r) \quad , \tag{2.43}$$

and

$$L^{\mathcal{M}\nu}_{J\lambda}(r) = -\ \delta_{\lambda J}\ i\ \hat{J}\mathcal{X}^{\mathcal{M}}_{\nu JJ}(r) \quad , \tag{2.44}$$

$$L^{\mathcal{E}\nu}_{J\lambda}(r) = \delta_{\lambda J+1}\sqrt{J}\,\mathcal{X}^{\mathcal{E}}_{\nu JJ+1}(r) + \delta_{\lambda J-1}\sqrt{J+1}\,\mathcal{X}^{\mathcal{E}}_{\nu JJ-1}(r) \quad , \tag{2.45}$$

$$L^{\mathcal{L}\nu}_{J\lambda}(r) = -\ \delta_{\lambda J+1}\sqrt{J+1}\ \mathcal{Z}^{\mathcal{L}}_{\nu JJ+1}(r) + \delta_{\lambda J-1}\sqrt{J}\ \mathcal{Z}^{\mathcal{L}}_{\nu JJ-1}(r). \tag{2.46}$$

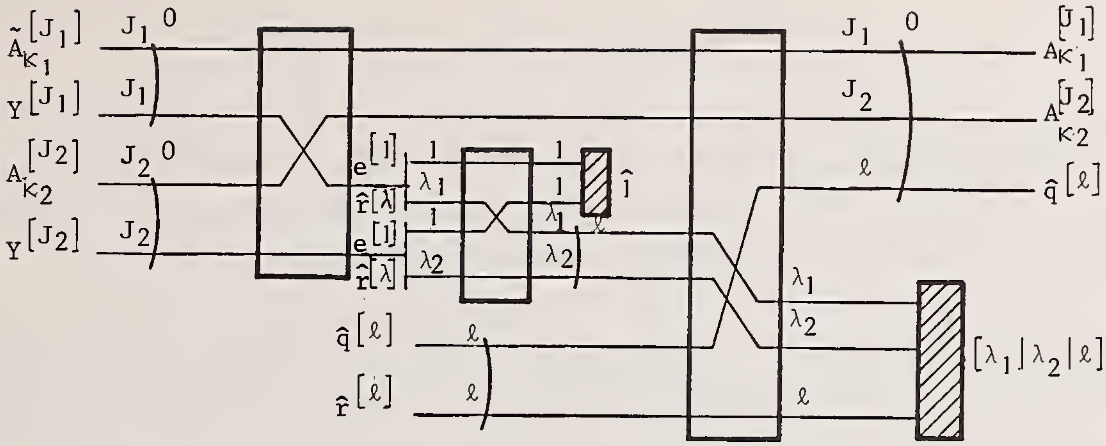


Fig. 2.12

II.4 - CENTER OF MASS AND RECOIL CORRECTIONS

We must distinguish the pseudo-center of mass motion introduced into the solutions by the use of the center of mass pseudo-Hamiltonian, eq. I(1.41), and by the use of a truncated basis, from the relativistic recoil of the target nucleus in the electron scattering process. We shall discuss successively the extraction of the center of mass motion and the corrections required to account for the relativistic recoil.

We assume that the pseudo-Hamiltonian, I(1.50) and I(1.41)

$$\mathcal{H} = H + \frac{1}{2}\,\xi(\vec{P}^2 + \Omega^2\vec{R}^2), \tag{2.47}$$

has solutions of the form I(1.55) for the ground state

$$\psi_{g.s.} = \phi_o(\vec{R})\ \chi_{g.s.}(\vec{\xi}_i), \tag{2.48}$$

where $\phi_o(R)$ is the normalized 0s center of mass wave function and $\chi(\vec{\xi}_i)$ is the physical ground state wave function of the relative intrinsic particle coordinates $\vec{\xi}_i$. From the relation

$$\langle\psi_{g.s.}|e^{i\vec{q}\cdot\vec{r}}|\psi_{g.s.}\rangle = \langle\phi_o|e^{i\vec{q}\cdot\vec{R}}|\phi_o\rangle\langle\chi_{g.s.}|e^{i\vec{q}\cdot\vec{\xi}}|\chi_{g.s.}\rangle, \tag{2.49}$$

we see that the physical elastic form factor is of the form

$$F_\mu(\vec{q})_{\text{corrected}} = e^{q^2/4\Omega^2}\, F_\mu(\vec{q})_{\text{calculated}} \tag{2.50}$$

where $F_\mu(\vec{q})$ calculated is the form factor calculated from the solution $\psi_{g.s}$ with the one-body matrix elements given in the previous sections. Note that the correction resulting from eq. (2.50) can be important even for small values of the transferred momentum $\vec{q}$ depending on the magnitude of the parameter Ω.

We now evaluate the effect of the relativistic recoil of the target nucleus at large momentum transfer. The following remarks are limited to the case of elastic scattering. It is best to use the brick-wall coordinate system in which the recoiling composite object moves with momentum $-\vec{q}/2$ before collision and $+\vec{q}/2$ after collision. The velocity is

$$\vec{v} = \pm(\vec{q}/2)(M^2 + (q^2/4))^{-1/2}, \tag{2.51}$$

and the Lorentz factor

$$\gamma = (1 + (\tfrac{q}{2M})^2)^{1/2}, \tag{2.52}$$

where M is the mass of the composite particle. In the brick-wall system the Lorentz contraction is the same in the initial and final states. Thus, in the many-body matrix elements, the overlaps associated with the constituents which do not interact with the electromagnetic field are unity. We must however evaluate the effect of the boost on the value of the one-body matrix element involving the interacting constituent particle.

For example, for a spin 0 constituent in the basis state $\phi_{\nu\ell}(x)$ of the solution ψ calculated in the laboratory system, we get in the brick-wall system

$$F(q) = \langle \phi'_{\nu\ell}(\vec{q}/2)\,|\,e^{iqz}\,|\,\phi'_{\nu\ell}(-\vec{q}/2)\rangle \tag{2.53}$$

where we have assumed the transferred momentum to be along the z direction. In eq. (2.53), $\phi'_{\nu\ell}(\pm\vec{q}/2)$ is the state boosted with the velocity $\vec{v}$, eq. (2.51). For the spin 0 case, the boosted functions $\phi'_{\nu\ell}$ differ from the laboratory functions $\phi_{\nu\ell}$ firstly by a Lorentz contraction, secondly by a change of normalization. The Lorentz contraction is of course the same for the initial and final states. The change of normalization is given by the Lorentz factor γ. Therefore we have

$$\phi'_{\nu\ell}(x, y, z) = \sqrt{\gamma}\,\phi(x, y, \gamma z), \tag{2.54}$$

and eq. (2.53) becomes

$$F(q) = \int dx\, dy\, \gamma dz\, \phi^*(x, y, \gamma z)\, e^{iqz}\, \phi(x, y, \gamma z) \quad . \qquad (2.55)$$

A change of variables yields finally

$$F(q) = \int dx\, dy\, d\xi\, \phi^*(x, y, \xi)\, e^{i(q/\gamma)\xi}\, \phi(x, y, \xi) \quad . \qquad (2.56)$$

Thus the effect of the relativistic recoil for a spin 0 constituent is fully taken into account by scaling of the transfer momentum

$$F(q)_{physical} = F_{corrected}(q/\gamma) \quad . \qquad (2.57)$$

However, for a constituent of non-zero spin, in addition to compensating for the Lorentz contraction as in eq. (2.52), we must also evaluate the effect of the Lorentz transformation on the spinors. This way eq. (2.56) becomes

$$F(q) = \frac{1}{\mathcal{N}} \int dx\, dy\, d\xi (\overline{\psi}(x, y, \xi) + \delta\overline{\psi}(x, y, \xi))\, e^{i(q/\gamma)\xi}$$

$$\times\ (\psi(x, y, \xi) + \delta\psi(x, y, \xi)) \quad , \qquad (2.58)$$

$$\mathcal{N} = \langle \psi + \delta\psi | \psi + \delta\psi \rangle \quad . \qquad (2.59)$$

We outline the calculation of the change $\delta\psi$ in the wave function of the constituent, using as an example the case of a spin 1/2 particle. There only the small components are modified by the boost which entails the substitution

$$\vec{\sigma}.\vec{\nabla} \rightarrow \sigma_x \partial_x + \sigma_y \partial_y + \gamma\sigma_\xi\, \partial_\xi = \vec{\sigma}.\vec{\nabla}_\xi + (\gamma - 1)\sigma_\xi \partial_\xi \quad . \qquad (2.60)$$

The term $\vec{\sigma}.\vec{\nabla}_\xi$ contributes to ψ as in eqs. I(3.72) and I(3.73) which have been evaluated to yield eqs. I(3.88) and I(3.89). In order to evaluate the correction term we introduce

$$(\gamma - 1)\sigma_\xi \partial_\xi = \hat{1}^2 [h^{[1]}\ \sigma^{[1]}]^{[0]}\ [h^{[1]}\ \nabla^{[1]}]^{[0]}, \qquad (2.61)$$

with

$$h_m^{[1]} = -\, i(\gamma - 1)^{1/2}\, \delta_{m0} \quad . \qquad (2.62)$$

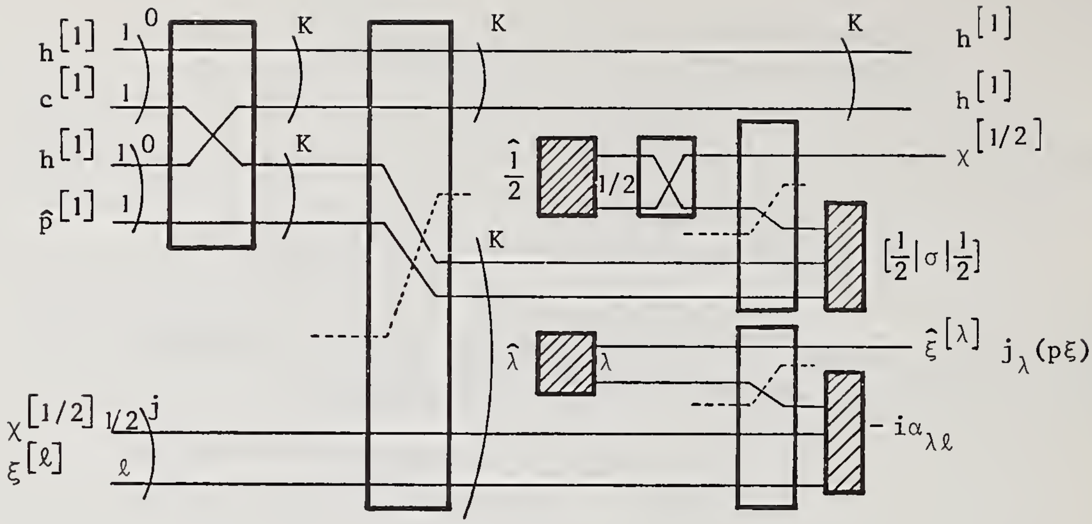


Fig. 2.13

From the recouplings of figure 2.13 we obtain the field expansion

$$\delta\psi(\xi) = \frac{1}{\sqrt{2}} \sum_{\substack{\nu\ell j \\ N\lambda K}} (i)^\ell \, (-)^{j-N} \, \frac{\hat{N}}{\hat{\lambda}} \begin{bmatrix} 1 & 1 & K \\ 1/2 & \ell & j \\ 1/2 & \lambda & N \end{bmatrix} \sqrt{6}\, \alpha_{\lambda\ell}$$

$$\left\{ \left[B_{\nu\ell}^{[j\ 1/2]} \, \eta^{[1/2]} \begin{pmatrix} 0 \\ [[h^{[1]}h^{[1]}]^{[K]} \, Y_{1/2\lambda}^{[N]}(\hat{\xi})\delta v_{\nu\ell\lambda}(\xi)]^{[j]} \end{pmatrix} \right]^{[00]} \right.$$

$$\left. + (-)^\ell \left[\tilde{C}_{\nu\ell}^{[j\ 1/2]} \, \tilde{\eta}^{[1/2]} \begin{pmatrix} [[h^{[1]}h^{[1]}]^{[K]} \tilde{Y}_{1/2\lambda}^{[N]}(\hat{\xi})\delta v_{\nu\ell\lambda}(\xi)]^{[j]} \\ 0 \end{pmatrix} \right]^{[00]} \right\} \tag{2.63}$$

where

$$\delta v_{\nu\ell\lambda}(\xi) = \sqrt{\frac{2}{\pi}} \int p^2 \, dp (2E(E+m))^{-1/2} \, f_{\nu\ell}(p) \, j_\lambda(p\xi) \ . \tag{2.64}$$

We note however that for the relatively low transfer momentum region covered by the truncated configuration space of Chapter VI, ref. [1], we expect the recoil corrections to be small, in contrast to the center of mass correction discussed above, since $\delta\psi$ is of the order of

$$\gamma - 1 \sim \frac{q^2}{8M^2} \ . \tag{2.65}$$

We finally note for completeness that for the case of inelastic scattering the calculation is most easily performed in the generalized brick wall system in which the γ-factors are the same in the initial and the final states. Then, considering the excitation of the system of rest-mass M by the energy B, so that the rest-mass of the final state is $M_2 = M + B$, we have $\vec{P}_1 = - M\gamma\vec{v}$, $\vec{P}_2=(M+B)\gamma\vec{v}$, $E_1=M\gamma$, $E_2=(M+B)\gamma$ and the momentum transfer four-vector is $q = \left((2M+B)\gamma\vec{v},\ iB\gamma\right)$. With obvious modifications the formulae of this Chapter then hold also for inelastic form factors. We shall not go into detail here.

CHAPTER III

QUADRUPOLE AND MAGNETIC STATIC MOMENTS

III.1 - QUADRUPOLE MOMENTS

The quadrupole moment operator the form of which is derived in Chapter I section 2

$$Q(\vec{r}) = 2z^2 - x^2 - y^2 = \sqrt{\frac{16\pi}{5}}\, r^2\, Y_{20}(\hat{r}) \quad , \tag{3.1}$$

reads in tensorial form

$$Q(\vec{r}) = \hat{2}\, \sqrt{\frac{16\pi}{5}}\, \left[b^{[2]}\, \hat{r}^{[2]}\right]^{[0]}\, r^2 \, , \tag{3.2}$$

where the amplitudes $b_m^{[2]}$ corresponding to the definition (3.1) are

$$b_m^{[2]} = - \delta_{m0} \quad . \tag{3.3}$$

Furthermore, the complete quadrupole operator must of course include the charge projection operators defined in eqs. (1.2), (1.3) and (1.4) for the spin 0, 1/2 and 1 cases respectively.

III.1.1 - Spin 0 field

The scattering terms, figure 2.1.a give the contribution to the quadrupole moment

$$Q = - ie \sum_{\substack{\nu_1\ell_1 \\ \nu_2\ell_2}} (i)^{\ell_2-\ell_1} [1|\tau^{[1]}|1][\ell_1|2|\ell_2] \tag{3.4}$$

$$\times \int r^2 dr \sqrt{\frac{16\pi}{5}} g_{\nu_1\ell_1}(r)\, r^2 h_{\nu_2\ell_2}(r) [\tilde{A}^{[\ell_1 1]} A^{[\ell_2 1]}{}_c{}^{[01]}{}_b{}^{[20]}]^{[00]} ,$$

where the recoupling is similar to that used in figure 2.5 for eq. 2.10. The definitions of the amplitudes are given by eqs. (3.3) and (2.2). Likewise for the pair creation contribution, figure 2.1.b, we get from figures 2.5 and 2.6 and from eq. (2.16)

$$Q = - ie \sum_{\substack{\nu_1\ell_1 \\ \nu_2\ell_2}} \sum_{T} (i)^{-\ell_2-\ell_1} \left(\frac{1}{\hat{1}} \begin{bmatrix} 1 & 1 & T \\ 1 & 1 & T \\ 1 & 1 & 0 \end{bmatrix} [1|\tau^{[1]}|1]^2\right) \tag{3.5}$$

$$\times \int r^2 dr \sqrt{\frac{16\pi}{5}} g_{\nu_1\ell_1}(r)\, r^2 h_{\nu_2\ell_2}(r) [\tilde{A}^{[\ell_1 1]} \tilde{A}^{[\ell_2 1]}{}_c{}^{[0T]}{}_b{}^{[20]}]^{[00]} ,$$

with the definition of eq. (2.11). The pair annihilation term, figure 2.1.c, gives the same contribution as (3.5) except for the additional phase $(-)^{\ell_1+\ell_2}$.

III.1.2 - Spin 1/2 field

The quadrupole matrix elements for the spin 1/2 scattering term is derived according to the recoupling of figure 2.9 and, in similar fashion as in eq. (2.28), we get

$$Q = \frac{ie}{2} \Bigg\{ \frac{1}{2} \sum_{y=1,0} (-)^y [1/2|\tau^{[y]}|1/2] \left(\frac{\hat{1}}{2}\right) \sum_{\substack{\nu_1\ell_1\lambda_1 j_1 \\ \nu_2\ell_2\lambda_2 j_2}} (i)^{\ell_2-\ell_1}$$

$$\times \Bigg(\begin{bmatrix} 1/2 & \ell_1 & j_1 \\ 1/2 & \ell_2 & j_2 \\ 0 & 2 & 2 \end{bmatrix} [\ell_1|\ell_2|2] \int r^2 dr \sqrt{\frac{16\pi}{5}}\, r^2\, u_{\nu_1\ell_1}(r)\, u_{\nu_2\ell_2}(r)$$

$$+ \begin{bmatrix} 1/2 & \lambda_1 & j_1 \\ 1/2 & \lambda_2 & j_2 \\ 0 & 2 & 2 \end{bmatrix} [\lambda_1|\lambda_2|2] \int r^2 dr \sqrt{\frac{16\pi}{5}}\, r^2\, v_{\nu_1\ell_1\lambda_1}(r)\, v_{\nu_2\ell_2\lambda_2}(r) \Bigg)$$

$$\times [\tilde{B}_{\nu_1\ell_1}^{[j_1 1/2]} B_{\nu_2\ell_2}^{[j_2 1/2]}{}_c{}^{[0y]}{}_b{}^{[20]}]^{[00]} \Bigg\} . \tag{3.6}$$

III.1.3 - Spin 1 field

For the spin 1 field figure 2.12 and eq. (2.40) yield

$$Q = -\ 2ie[1|\tau^{[1]}|1] \sum_{\substack{\nu_1 J_1\\ \nu_2 J_2}} (i)^{J_2-J_1} \frac{\hat{1}}{\hat{J}_1\hat{J}_2} \sum_{\substack{\kappa_1\lambda_1\\ \kappa_2\lambda_2}} \begin{bmatrix} 1 & \lambda_1 & J_1 \\ 1 & \lambda_2 & J_2 \\ 0 & 2 & 2 \end{bmatrix} [\lambda_1|\lambda_2|2]$$

$$\times \int r^2 dr \sqrt{\frac{16\pi}{5}}\, r^2\, K^{\kappa_1\nu_1}_{J_1\lambda_1}(r)\, L^{\kappa_2\nu_2}_{J_2\lambda_2}(r) \times [\tilde{A}^{[J_1 1]}_{\kappa_1\nu_1}\ A^{[J_2 1]}_{\kappa_2\nu_2}\ c^{[01]}\ b^{[20]}]^{[00]}\ , \tag{3.7}$$

where the functions K and L are given in eqs. (2.41) through (2.46). The summations κ_1 and κ_2 in (3.7) extend of course over all the multipolarities of the particle.

III.2 - MAGNETIC MOMENTS

III.2.1 - Spin 0 field

From the remarks of Chapter I section 2 we get immediately for the spin 0 "scatering" term,

$$\mu = ie[1|\tau^{[1]}|1] \sum_{\substack{\nu_1\ell_1\\ \nu_2\ell_2}} \delta_{\nu_1\nu_2}\, \delta_{\ell_1\ell_2} [\ell_1|\ell^{[1]}|\ell_2]$$

$$\times [\tilde{A}^{[\ell_1 1]}_{\nu_1}\ A^{[\ell_2 1]}_{\nu_2}\ c^{[01]}\ b^{[10]}]^{[00]}\ , \tag{3.8}$$

and similar terms for pair creation and annihilation contributions with the added phase $(-)^{\ell_1}$ and $(-)^{\ell_1+\ell_2}$ respectively.

III.2.2 - Spin 1/2 field

For spin 1/2 the $\vec{j}$ term of (1.12) is immediate since $\vec{j}$ is a good quantum number, while the spin term $\vec{s}$ requires a simple recoupling,

$$\mu = \frac{ie}{2}\ \frac{1}{2}\sum_{y=1,0} (-)^y [1/2|\tau^{[y]}|1/2] \left(\frac{\hat{1}}{2}\right) \sum_{\substack{\nu_1\ell_1(\lambda_1)j_1 \\ \nu_2\ell_2(\lambda_2)j_2}} (i)^{\ell_2-\ell_1}$$

$$\times \Big\{ \delta_{j_1j_2}\ \delta_{\nu_1\nu_2}\ \delta_{\ell_1\ell_2} (-\ [j_1|j^{[1]}|j_2])$$

$$+ \left(\delta_{\ell_1\ell_2}\ \hat{\ell}_1 \begin{bmatrix} 1/2 & \ell_1 & j_1 \\ 1/2 & \ell_1 & j_2 \\ 1 & 0 & 1 \end{bmatrix} \int r^2 dr\ u_{\nu_1\ell_1}(r)\ u_{\nu_2\ell_1}(r) \right.$$

$$\left. + \delta_{\lambda_1\lambda_2}\ \hat{\lambda}_1 \begin{bmatrix} 1/2 & \lambda_1 & j_1 \\ 1/2 & \lambda_1 & j_2 \\ 1 & 0 & 1 \end{bmatrix} \int r^2 dr\ v_{\nu_1\ell_1\lambda_1}(r)\ v_{\nu_2\ell_2\lambda_1}(r) \right) (-\ [1/2|s^{[1]}|1/2]) \Big\}$$

$$\times [\tilde{B}^{[j_1 1/2]}_{\nu_1\ell_1}\ B^{[j_2 1/2]}_{\nu_2\ell_2}\ c^{[0y]}\ b^{[10]}]^{[00]} \quad . \tag{3.9}$$

III.2.3 - Spin 1 field

For spin 1 the expression is as simple as in the spin 0 case since $\vec{J}$ is a good quantum number

$$\mu = ie[1|\tau^{[1]}|1] \sum_{\substack{\nu_1 J_1 \kappa_1 \\ \nu_2 J_2 \kappa_2}} \delta_{\nu_1\nu_2}\ \delta_{J_1J_2}\ \delta_{\kappa_1\kappa_2} [J_1|J^{[1]}|J_2]$$

$$\times [\tilde{A}^{[J_1 1]}_{\kappa_1\nu_1}\ A^{[J_2 1]}_{\kappa_2\nu_2}\ c^{[01]}\ b^{[10]}]^{[00]} \quad , \tag{3.10}$$

where the summation extends over all multipolarities κ_1 and κ_2 of the particle.

Note that in expressions (3.8), (3.9) and (3.10) we have set $\mu_0 = 1$, cf. eqs. (1.11), (1.12) and (1.13) in conformity with our basic assumption of no anomalous moment. Furthermore, the truncated space of ref. [1], Chapter VI, does not allow for pair creation or annihilation terms in the static moments of spin 1/2 and spin 1 fields.

CHAPTER IV

VECTOR DOMINANCE

IV.1 - FORM FACTORS

The contribution to the elastic scattering form factor of the ω meson through the vector dominance graphs of figure 4.1.a and 4.1.b is simply of the form, according to Chapter I section 3

$$F_{\mu}^{(\omega)}(\vec{q}) = G_{\gamma\omega} \int d^3r\, e^{i\vec{q}\vec{r}}\, \omega_{\mu}(\vec{r}) \,, \tag{4.1}$$

where $\omega_{\mu}(\vec{r})$ is the field, the expansion of which is given in eqs. I(3.222) through I(3.225).

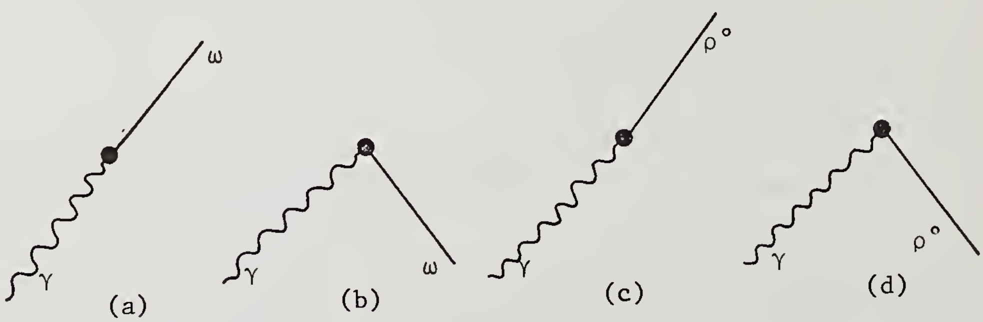


Fig. 4.1

The analogous expression for the ρ meson must include a projector on the neutral component ρ^0

$$F_\mu^{(\rho)}(\vec{q}) = G_{\gamma\rho} \int d^3r\, e^{i\vec{q}\vec{r}} (P_0\, \vec{\rho})$$

$$= G_{\gamma\rho} \int d^3r\, e^{i\vec{q}\vec{r}}\, \hat{1}[c^{[1]}\ \eta^{[1]}]^{[0]} . \vec{\rho} \quad , \qquad (4.2)$$

with the isospin amplitudes $c_m^{[1]}$ given by eq. (2.2) and where in (4.2) the dot implies a scalar product in isospin space. Thus the isospin wave function in each term of the ρ expansion is replaced as follows

$$\hat{1}[c^{[1]}\ \eta^{[1]}] . \hat{1}[\tilde{A}^{[1]}\ \tilde{\eta}^{[1]}] = \hat{1}[\tilde{A}^{[1]}\ c^{[1]}]^{[0]} \quad . \qquad (4.3)$$

Apart from this isospin coupling of the creation (or annihilation) operators to the amplitude $c^{[1]}$, the ω and ρ mesons yield identical expressions.

Furthermore, the Fourier-Bessel transforms of the radial part of the field which appear in the matrix elements are, for example, of the general form (see expressions I(3.232) through I(3.236))

$$\int r^2 dr\, j_\ell(qr)\, \mathcal{W}^\kappa_{\nu J\lambda}(r)\, \delta_{\ell\lambda} = \int r^2 dr \int p^2 dp\, j_\ell(qr)\, j_\ell(pr) \sqrt{\frac{N_\kappa}{2E}} \sqrt{\frac{2}{\pi}}\, f_{\nu J}(p)$$

$$= \int p^2 dp\ \frac{\pi}{2} \frac{\delta(q-p)}{p^2} \left(\sqrt{\frac{N_\kappa}{2E}} \sqrt{\frac{2}{\pi}}\, f_{\nu J}(p)\right)$$

$$= \sqrt{\frac{\pi}{2}} \sqrt{\frac{N_\kappa}{2E}}\, f_{\nu J}(q) \ , \qquad (4.4)$$

while the space geometry is obtained, introducing space polarization amplitudes $d^{[1]}$ as in eq. (2.33)

$$\int d\hat{r} [\hat{q}^{[\ell]}\ \hat{r}^{[\ell]}]^{[0]}\ [A^{[J]}_{\kappa\nu}\ d^{[1]}\ \hat{r}^{[\lambda]}]^{[0]}$$

$$= \delta_{\ell\lambda}\ \hat{\ell} [A^{[J]}_{\kappa\nu}\ d^{[1]}\ \hat{q}^{[\ell]}]^{[0]} \quad . \qquad (4.5)$$

The vector form factor for the ω-meson dominance term is thus given by

$$\vec{F}(\vec{q}) = G_{\gamma\omega}\ 4\pi \sqrt{\frac{\pi}{2}} \sum_{\kappa\nu J\ell} (i)^{\ell+J} \sqrt{\frac{N_\kappa}{2E}}\, \mathcal{N}^\kappa_{NJ\ell}$$

$$\times f_{\nu J}(q) ([A^{[J]}_{\kappa\nu}\ d^{[1]}\ \hat{q}^{[\ell]}]^{[0]} + (-)^{J+M} [\tilde{A}^{[J]}_{\kappa\nu}\ d^{[1]}\ \hat{q}^{[\ell]}]^{[0]}) \quad , \qquad (4.6)$$

where

$$\mathcal{N}^{\mathcal{M}}_{\nu J\ell} = \delta_{\ell J}\ \hat{J} , \qquad (4.7)$$

$$\mathcal{N}^{\mathcal{E}}_{\nu J\ell} = - i(\delta_{\ell,J-1} \sqrt{J+1} + \delta_{\ell,J+1} \sqrt{J}), \tag{4.8}$$

$$\mathcal{N}^{\mathcal{L}}_{\nu J\ell} = - i(\delta_{\ell,J-1} \sqrt{J} - \delta_{\ell,J+1} \sqrt{J+1}) \quad , \tag{4.9}$$

and

$$\begin{aligned} M &= 0 \text{ if } \kappa = \mathcal{M} \quad , \\ M &= 1 \text{ if } \kappa = \mathcal{E}\, , \ \mathcal{L} \quad . \end{aligned} \tag{4.10}$$

The scalar form factor is

$$\begin{aligned} F_4(\vec{q}) = iG_{\gamma\omega}\, 4\pi \ \sqrt{\frac{\pi}{2}} \sum_{\nu J} \ (-)^J\, \hat{J} \ \sqrt{\frac{1}{2E}}\, \frac{q}{m}\, f_{\nu J}(q) \\ \times \left[(A^{[J]}_{\mathcal{L}\nu} + (-)^{J+1}\, \tilde{A}^{[J]}_{\mathcal{L}\nu})\hat{q}^{[J]} \right]^{[0]} \quad . \end{aligned} \tag{4.11}$$

For the ρ field, these expressions hold with a replacement according to 4.3, for example

$$\left[A^{[J]}_{\kappa\nu}\ d^{[1]}\ \hat{q}^{[\ell]} \right]^{[0]} \rightarrow \hat{1}\left[A^{[J1]}_{\kappa\nu}\ d^{[10]}\ c^{[01]}\ \hat{q}^{[\ell 0]} \right]^{[00]} \quad . \tag{4.12}$$

IV.2 - QUADRUPOLE MOMENT

The evaluation of the quadrupole moment requires a special treatment, since the usual procedure, followed in Chapter III, involves an interaction of the electromagnetic field with a conserved vector current. In contrast, here the interaction energy arises from the emission or absorption of a neutral particle. Thus we shall start from the defining equation for the energy of a quadrupole moment (rank two tensor denoted by $\underset{\approx}{Q}$) interacting with an electric field gradient

$$E = \underset{\approx}{Q} \otimes \underset{\approx}{\nabla\mathcal{E}} \equiv Q\, \underset{\approx}{\hat{Q}} \otimes \underset{\approx}{\nabla\mathcal{E}} \ , \tag{4.13}$$

where the tensor product implies contraction of the tensor indices.

We have here introduced the unit tensor of rank 2, $\underset{\approx}{\hat{Q}}$. Eq. (4.13) is valid as long as the gradient of the field is constant over the volume of the system. On the other hand the vector dominance interaction energy is given by (1.21). Hence we shall recognize the vector dominance quadrupole moment Q by equating these energies ,

$$G_{\gamma\omega} \int d^3r\ A_\mu\ \omega_\mu = Q\, \underset{\approx}{\hat{Q}} \otimes \underset{\approx}{\nabla\mathcal{E}} \quad \text{for} \quad q \rightarrow 0 \quad . \tag{4.14}$$

For the photon vector potential A_μ we use a plane wave in the Lorentz gauge

$$\vec{A} = \vec{a}\, e^{i\vec{q}\vec{r}} = \hat{1}[a^{[1]}\ e^{[1]}]^{[0]}\ e^{i\vec{q}\vec{r}} \quad . \qquad (4.15)$$

The electric field $\vec{\mathscr{E}}$ thus is

$$\vec{\mathscr{E}} = -\dot{\vec{A}} = iq\vec{a}\ e^{i\vec{q}\vec{r}} \quad , \qquad (4.16)$$

and the electric field gradient

$$\underset{\sim}{\nabla\vec{\mathscr{E}}} = -q\ \underset{\sim}{\vec{q}\otimes\vec{a}}\ e^{i\vec{q}\vec{r}} \quad , \qquad (4.17)$$

where the wavy line denotes the construction of rank two tensors according to

$$\underset{\sim}{\nabla\vec{\mathscr{E}}} = -q\ \hat{1}[q^{[1]}\ e^{[1]}]^{[0]}\ \hat{1}[a^{[1]}\ e^{[1]}]^{[0]} e^{i\vec{q}\vec{r}}$$

$$\equiv -q^2\ \hat{2}[[\hat{q}^{[1]}\ \vec{a}^{[1]}]^{[2]}\ [e^{[1]}\ e^{[1]}]^{[2]}]^{[0]}\ e^{i\vec{q}\vec{r}} \ . \qquad (4.18)$$

Let us write eq. (3.2) as

$$\underset{\sim}{Q} = \hat{2}\ Q[b^{[2]}\ \hat{Q}^{[2]}]^{[0]} \quad , \qquad (4.19)$$

with $\hat{Q}^{[2]}$ the unit spherical tensor of rank 2. We get with a simple recoupling

$$E_{\lim\quad q\to 0} = -Q\ \hat{2}\ q^2[b^{[2]}\ \hat{q}^{[1]}\ \vec{a}^{[1]}]^{[0]}\ e^{i\vec{q}\vec{r}} \ , \qquad (4.20)$$

where we have made use of the defining relation

$$[\hat{Q}^{[2]}\ e^{[1]}\ e^{[1]}]^{[0]} = \hat{2} \ . \qquad (4.21)$$

On the other hand the left hand term of eq. (4.14) is calculated with the multipole J = 2 of the ω field which in the limit $q \to 0$ has the dominant components (of space character), see eqs. I(3.222) through I(3.225)

$$\vec{\omega} \Rightarrow \sum_\nu \left\{ (i)^{2-1} \left\{ [A^{[2]}_{\mathscr{E}\nu}[e^{[1]}\ \hat{r}^{[1]}]^{[2]}]^{[0]}\ \sqrt{2+1}\, \mathscr{W}^{\mathscr{E}}_{\nu 21}(qr) \right.\right.$$

$$+ [A^{[2]}_{\mathscr{L}\nu}[e^{[1]}\ r^{[1]}]^{[2]}]^{[0]}\ \sqrt{2}\, \mathscr{W}^{\mathscr{L}}_{\nu 21}(qr)$$

$$\left. + \text{h.c.} \ldots \right\} \quad . \qquad (4.22)$$

We substitute the plane wave expansion (2.6) for the vector potential $\vec{A}$ in eq. (4.14) together with the field expansion (4.22). The recoupling geometry of figure 4.2 yields in the limit $q \Rightarrow 0$

$$E = -4\pi \sqrt{\frac{\pi}{2}}\, f_{\nu 2}(q \Rightarrow 0)\, G_{\gamma\omega} \left\{ \sqrt{\frac{2+1}{2E_\omega}} \left[(A^{[2]}_{\mathcal{E}\nu} - \tilde{A}^{[2]}_{\mathcal{E}\nu})\, \hat{q}^{[1]}\, \vec{a}^{[1]} \right]^{[0]} + \sqrt{\frac{E_\omega}{m_\omega}} \left[(A^{[2]}_{\mathcal{L}\nu} - \tilde{A}^{[2]}_{\mathcal{L}\nu})\, \hat{q}^{[1]}\, \vec{a}^{[1]} \right]^{[0]} \right\} , \tag{4.23}$$

where we have used eqs. I(3.232), I(3.215) and, for the integration over r the relation

$$\int r^2 dr\, j_1(pr)\, j_1(qr) = \frac{\pi}{2} \frac{\delta(p-q)}{p^2} \; . \tag{4.24}$$

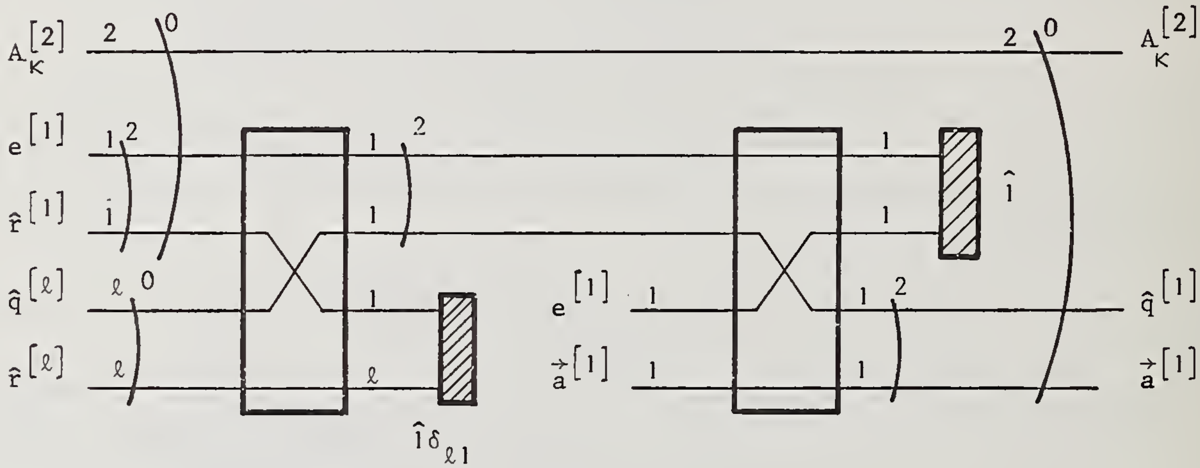


Fig. 4.2

We finally compare the expressions (4.23) and (4.20) noting that (see I(3.31))

$$f_{\nu 2}(q) \xrightarrow[\lim\ q\to 0]{} \alpha^{7/2}\, C_{\nu 2}\, L_\nu^{5/2}(0)\, q^2 \quad , \tag{4.25}$$

which yields the complete contribution of the vector dominance term to the static quadrupole moment from the ω field

$$Q = \sum_\nu Q_\nu^{\mathcal{E}}\, \hat{2} \left[(A^{[2]}_{\mathcal{E}\nu} - \tilde{A}^{[2]}_{\mathcal{E}\nu})\, d^{[2]} \right]^{[0]} + \sum_\nu Q_\nu^{\mathcal{L}}\, \hat{2} \left[(A^{[2]}_{\mathcal{L}\nu} - \tilde{A}^{[2]}_{\mathcal{L}\nu})\, d^{[2]} \right]^{[0]} \quad , \tag{4.26}$$

with

$$Q_\nu^{\mathcal{E}} = \frac{2}{2}\, \pi^{3/2}\, \alpha^{7/2}\, C_{\nu 2}\, L_\nu^{5/2}(0) \sqrt{\frac{3}{m_\omega}}\, G_{\gamma\omega} \quad , \tag{4.27}$$

$$Q_\nu^{\mathcal{L}} = \sqrt{\frac{2}{3}}\, Q_\nu^{\mathcal{E}} \quad , \tag{4.28}$$

since in (4.22) the field energy E_ω equals the ω mass m_ω.

In eq. (4.26) the quadrupole polarization tensor $d^{[2]}$ has been substituted according to

$$d^{[2]} = [\hat{q}^{[1]} \vec{a}^{[1]}]^{[2]} \quad , \tag{4.29}$$

where $\vec{a}^{[1]}$ is the polarization vector of the real photon field; it is coupled with the momentum direction amplitude $\hat{q}^{[1]}$ of the real photon. The values of $d_m^{[2]}$ are associated with the preparation of the system according to well known rules, for example see ref. [5].

As in the case of section IV.1, one goes from the contribution of the ω meson to the contribution of the ρ^0 meson by a substitution analogous to eq. (4.12), namely

$$[(A_{\kappa\nu}^{[2]} - \tilde{A}_{\kappa\nu}^{[2]})\ d^{[2]}]^{[0]} \to \hat{1}[(A_{\kappa\nu}^{[21]} - \tilde{A}_{\kappa\nu}^{[21]})\ d^{[20]}\ c^{[01]}]^{[00]} \quad . \tag{4.30}$$

In order to calculate eq. (4.27) the following values are useful

$$L_\nu^{5/2}(0) \begin{cases} = 1 \text{ if } \nu = 0 \quad , \\ = 5/2 \text{ if } \nu = 1 \quad , \\ = 63/8 \text{ if } \nu = 2 \quad . \end{cases} \tag{4.31}$$

IV.3 - MAGNETIC MOMENT

We shall follow a similar path as in the previous section. On the one hand the magnetic coupling energy is of the form

$$E = \vec{\mu}\,\vec{B} \quad . \tag{4.32}$$

With a plane wave potential $\vec{A}$, eq. (4.15)

$$\begin{aligned} \vec{B} = \text{rot}\,\vec{A} &= \hat{1}\sqrt{2}[e^{[1]}\ \nabla^{[1]}\ \vec{a}^{[1]}]^{[0]}\ e^{i\vec{q}\vec{r}} \\ &= \hat{1}\sqrt{2}\ iq[e^{[1]}\ \hat{q}^{[1]}\ \vec{a}^{[1]}]^{[0]}\ e^{i\vec{q}\vec{r}} \quad , \end{aligned} \tag{4.33}$$

see I(2.38). Hence with a simple recoupling

$$E = iq\ \hat{1}\sqrt{2}\ \mu\ \hat{1}[\hat{\mu}^{[1]}\ \hat{q}^{[1]}\ \vec{a}^{[1]}]^{[0]}\ e^{i\vec{q}\vec{r}} \quad , \tag{4.34}$$

where μ is the unknown magnetic moment and $\hat{\mu}^{[1]}$ a unit tensor of rank 1.

On the other hand we shall use the expression of the energy in terms of the interacting fields, limiting the expansion of ω_μ, I(3.222), to the magnetic dipole and taking the limit q =>0. Thus

$$\vec{\omega} \Rightarrow \sum_\nu i^1 \mathcal{W}^{\mathcal{M}}_{\nu 1 1}(qr)\ \hat{1}[[e^{[1]}\ \hat{r}^{[1]}]^{[1]}\ (A^{[1]}_{\mathcal{M}\nu} - \tilde{A}^{[1]}_{\mathcal{M}\nu})]^{[0]}\quad . \tag{4.35}$$

Utilising the recoupling of figure 4.3 corresponding to the substitution of its expansion for the plane wave real photon potential $\vec{A}$

$$E = G_{\gamma\omega}\ 4\pi \sum_\nu \int r^2 dr\ j_1(qr)\mathcal{W}^{\mathcal{M}}_{\nu 1 1}(r)\ \hat{1}[(A^{[1]}_{\mathcal{M}\nu} - \tilde{A}^{[1]}_{\mathcal{M}\nu})\ \hat{q}^{[1]}\ \vec{a}^{[1]}]^{[0]}\ . \tag{4.36}$$

We note that

$$\int r^2 dr\ j_1(qr)\mathcal{W}^{\mathcal{M}}_{\nu 1 1}(qr) = \sqrt{\frac{\pi}{2}}\ \sqrt{\frac{1}{2m_\omega}}\ f_{\nu 1}(q)\quad , \tag{4.37}$$

$$f_{\nu 1}(q)_{q \to 0} = \alpha^{5/2}\ C_{\nu 1}\ q\ L^{3/2}_\nu(0)\quad , \tag{4.38}$$

$$L^{3/2}_\nu(0) \begin{cases} = 1. \text{ if } \nu = 0\ , \\ = 3/2 \text{ if } \nu = 1\ , \\ = 35/8 \text{ if } \nu = 2\ . \end{cases} \tag{4.39}$$

Thus the contribution of the ω field dominance term to the magnetic moment is equal to

$$\mu = \sum_\nu \mu_\nu\ \hat{1}[(A^{[1]}_{\mathcal{M}\nu} - \tilde{A}^{[1]}_{\mathcal{M}\nu})\ d^{[1]}]^{[0]}\quad , \tag{4.40}$$

with from comparing eq. (4.34) to eqs. (4.36), (4.37) and (4.38)

$$\mu_\nu = -\ i\sqrt{2}\ \pi^{3/2}\ \alpha^{5/2}\ C_{\nu 1}\ L^{3/2}_\nu(0)\ \sqrt{\frac{1}{m_\omega}}\ G_{\gamma\omega}\quad . \tag{4.41}$$

As before we have introduced the system polarization amplitudes

$$d^{[1]} = [\hat{q}^{[1]}\ \vec{a}^{[1]}]^{[1]}\quad .$$

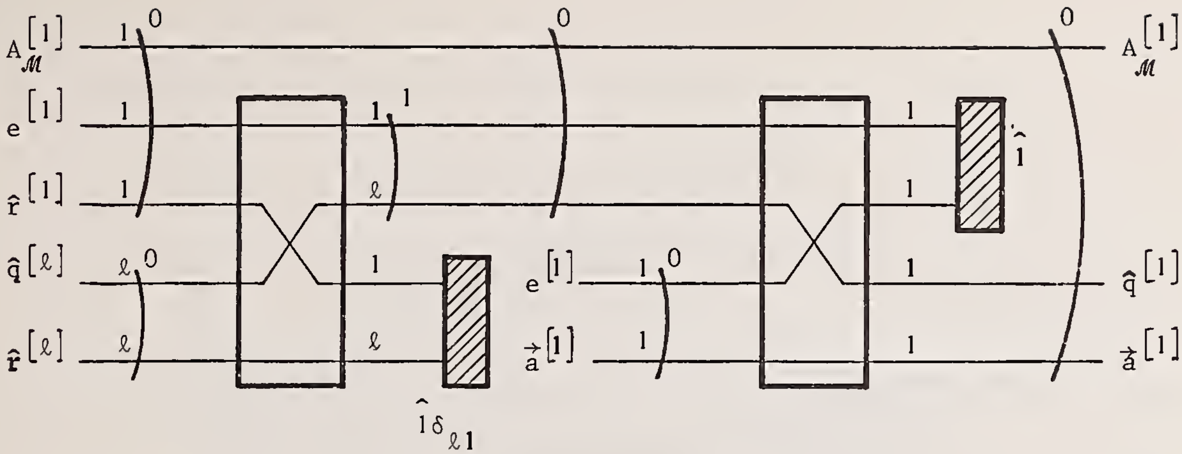


Fig. 4.3

Finally, the ρ^0 field contributions are similar to eq. (4.39) with the substitution of eq. (4.12), namely

$$[(A_{\mathcal{M}\nu}^{[1]} - \tilde{A}_{\mathcal{M}\nu}^{[1]})\, d^{[1]}]^{[0]} \to \hat{1}[(A_{\mathcal{M}\nu}^{[11]} - \tilde{A}_{\mathcal{M}\nu}^{[11]})\, d^{[10]}\, c^{[01]}]^{[00]} \ . \tag{4.42}$$

APPENDIX

ADDENDUM AND ERRATA TO N.B.S. MONOGRAPH 147

A - DIRECT CALCULATION OF THE C.M. QUADRATIC COORDINATE MATRIX ELEMENTS

In N.B.S. Monograph 147, ref. [1], the matrix elements of the operator

$$\frac{1}{2}\left(\vec{x}_i^{\,2}\; e^{-zE}\; E^2 + e^{-zE}\; E^2\; \vec{x}_i^{\,2}\right) \quad , \tag{A.1}$$

were calculated by expanding twice on complete basis vector sets. This leads to complicated numerical steps. Furthermore, since by necessity any functional space must be truncated, the expressions I(4.43) and I(4.44) may yield poor accuracy. This is of course not the case for the bi-linear terms I(4.45) which involve no intermediate sums.

We shall give here an alternate way for calculating the matrix elements (A.1), which is direct and free from the limitations of intermediate expansions. From the result

$$\int x^4\, dx\; j_\ell(p_1 x)\; j_\ell(p_2 x) = \frac{\pi}{2}\left\{\frac{-\delta''(p_1-p_2)}{p_1 p_2} + \frac{\ell(\ell+1)}{2}\left(\frac{\delta(p_1-p_2)}{p_1^4}\right.\right.$$

$$\left.\left. + \;\delta'(p_1-p_2)\,\frac{1}{p_1 p_2}\left(\frac{1}{p_1} - \frac{1}{p_2}\right)\right)\right\} \quad , \tag{A.2}$$

we get the general expression

$$[\nu_1\ J_1\ \kappa\ |\tfrac{1}{2}\ (\vec{x}^2\ e^{-zE}\ E^2 + e^{-zE}\ E^2\ \vec{x}^2)|\nu_2\ J_2\ \kappa]$$

$$= \delta_{J_1J_2}\ \hat{J}_1\ \hat{1}\ \frac{2}{\pi}\int p_1^2\ dp_1 \int p_2^2\ dp_2\ F_\kappa(E_1,E_2)$$

$$\times\ f_{\nu_1\ell_1}(p_1)\ f_{\nu_2\ell_2}(p_2)\ \frac{1}{2}\ (E_1^2\ e^{-zE_1} + E_2^2\ e^{-zE_2})$$

$$\times\ \frac{\pi}{2}\left\{\frac{-\delta''(p_1-p_2)}{p_1p_2} + A_\kappa^J\left(\frac{\delta(p_1-p_2)}{p_1^4} + \delta'(p_1-p_2)\ \frac{(p_2-p_1)}{p_1^2\ p_2^2}\right)\right\} \quad , \qquad (A.3)$$

where $F_\kappa(E_1,E_2)$ is the norm factor for the various cases κ and A_κ^J is a weight. Namely for the π-meson, $\kappa = 1$, $J = \ell$

$$F_1(E_1,E_2) = \frac{1}{2}\ \left(\sqrt{\frac{E_1}{E_2}} + \sqrt{\frac{E_2}{E_1}}\right) \quad , \qquad (A.4)$$

$$A_1^J = \frac{\ell(\ell+1)}{2} \quad ; \qquad (A.5)$$

for the ω-and ρ-mesons, magnetic multipolarities, $\kappa = 2$, $J = \ell$

$$F_2 = F_1 \quad \text{and} \quad A_2^J = A_1^J \quad ; \qquad (A.6)$$

for the ω-and ρ-mesons, electric multipolarities, $\kappa = 3$, $J = \ell \pm 1$

$$F_3 = \left(\sqrt{\frac{E_1}{E_2}} + \sqrt{\frac{E_2}{E_1}}\right) \quad , \qquad (A.7)$$

$$A_3^J = \frac{J}{\hat{J}^2}\ \frac{J(J-1)}{2} + \frac{J+1}{\hat{J}^2}\ \frac{(J+1)(J+2)}{2} = J(J+1) + 2 \quad ; \qquad (A.8)$$

for the ω-and ρ-mesons, longitudinal multipolarities, $\kappa = 4$, $J = \ell \pm 1$

$$F_4 = \frac{m}{\sqrt{E_1E_2}} \quad , \qquad (A.9)$$

$$A_4^J = A_3^J \quad ; \qquad (A.10)$$

for the nucleon, $\kappa = 5$, $J = \ell \pm 1/2$, $\lambda = J \pm 1/2$

$$F_5 = \sqrt{\frac{E_1+m}{E_1}}\ \sqrt{\frac{E_2+m}{E_2}} \quad , \tag{A.11}$$

$$A_5^J = \frac{E+m}{E}\ \ell(\ell+1) + \frac{E-m}{E}\ \lambda(\lambda+1) \quad , \tag{A.12}$$

where in A_5^J we have already taken the limit $E_1 = E_2$.

The evaluation of eq. (A.3) is carried out with the change of variables,

$$p_1-p_2 = p \qquad p_1+p_2 = P \qquad dp_1\ dp_2 = \frac{1}{2}\ dP\ dp \quad , \tag{A.13}$$

setting in the integrand

$$\delta'(p)\ F(p) = -\ F'(p)/_{p=0} \quad , \tag{A.14}$$

$$\delta''(p)\ F(p) = F''(p)/_{p=0} \quad , \tag{A.15}$$

the limit $p = 0$ being taken after differentiation.

We thus obtain the general result

$$\begin{aligned}
&\left[\nu_1\ J_1\ \kappa\ \left|\tfrac{1}{2}(\vec{x}^2\ e^{-zE}\ E^2 + e^{-zE}\ E^2\ \vec{x}^2)\right|\nu_2\ J_2\ \kappa\right] \\
&\quad = \delta_{J_1J_2}\ \delta_{\ell_1\ell_2}\ \hat{l}\ \hat{J}_1 \left\{ \int dP\ f_{\nu_1\ell}(P)\ f_{\nu_2\ell}(P)\ E^2\ e^{-zE} \right. \\
&\quad \times \left(N_\kappa + \frac{1}{2} - \frac{1}{2}\frac{P^2}{E^2} + \frac{zP^2}{4E} + \frac{3}{4}\frac{zP^4}{E^3} - \frac{z^2P^4}{4E^2}\right) \\
&\quad \left. - \int P^2 DPE^2\ e^{-zE}\left(f''_{\nu_1\ell}(P)f_{\nu_2\ell}(P)+2f'_{\nu_1\ell}(P)f'_{\nu_2\ell}(P)+f_{\nu_1\ell}(P)f''_{\nu_2\ell}(P)\right)\right\}
\end{aligned} \tag{A.16}$$

In this expression $\ell = J_1$ for the meson cases and $\ell = \ell_1$ for the nucleon case. The factor $(\delta_{\ell_1\ell_2})$ exists only for the nucleon case. The coefficient N_κ takes the following values

$\kappa = 1$, π-meson

$$N_1 = J_1(J_1+1) - \frac{P^4}{4E^4} \quad ; \tag{A.17}$$

$\kappa = 2$, ω- and ρ-mesons, magnetic multipolarities

$$N_2 = N_1 \quad ; \tag{A.18}$$

$\kappa = 3$, ω- and ρ-mesons, electric multipolarities

$$N_3 = J_1(J_1+1) + 2 - \frac{P^4}{4E^4} \quad ; \tag{A.19}$$

$\kappa = 4$, ω- and ρ-mesons, longitudinal multipolarities

$$N_4 = J_1(J_1+1) + 2 + \left(\frac{P^2}{4E^3} - \frac{P^4}{2E^5}\right)m \quad ; \tag{A.20}$$

$\kappa = 5$, the nucleon

$$N_5 = \frac{1}{2}\left(\frac{\ell(\ell+1)(E+m)}{E} + \frac{\lambda(\lambda+1)(E-m)}{E}\right) - \frac{P^4}{4E^4} + \frac{P^2}{4E^2} \quad . \tag{A.21}$$

Finally the factor $\hat{1}$ in eq. (A.16) comes from the expression of $\vec{X}^2$ in tensorial form. No isospin coefficient (which would be $\hat{t}$) is included.

B - DIRECT CALCULATION OF THE C.M. LINEAR COORDINATE MATRIX ELEMENTS

When computing the matrix elements of the operator $\mathcal{E}\vec{x}$, I(4.31), an expansion on a complete set has been used, see I(4.32), with the above inconveniences. A more direct expression can be obtained starting from I(4.31) and carrying out the steps I(4.35) through I(4.37) with the full integrand $\mathcal{E}\vec{x}$. One notes that $\mathcal{E} = e^{-zE}\,E^n$ is a symmetric function of p_1 and p_2. When integrating by part the $\delta'(p_1-p_2)$ term, see I(4.36), the $\mathcal{E}'(p)$ factor goes to zero when $p = p_1-p_2$ goes to zero. Consequently the expression for I(4.32) is directly (see I(4.37))

$$[\nu_1\ell_1|\mathcal{E}\vec{x}^{[1]}|\nu_2\ell_2] = i\int P^2\,dP\;\mathcal{E}(P)\left[\frac{(\alpha_{\ell_1\ell_2})^3}{P}\,f_{\nu_1\ell_1}(P)\,f_{\nu_2\ell_2}(P) + \frac{\alpha_{\ell_1\ell_2}}{2}\left(f'_{\nu_1\ell_1}(P)\,f_{\nu_2\ell_2}(P) - f_{\nu_1\ell_1}(P)\,f'_{\nu_2\ell_2}(P)\right)\right] \quad . \tag{B.1}$$

The same applies to I(4.38), I(4.39), I(4.41) and I(4.42).

C - NEW PHASE DEFINITION FOR THE PION FIELD

Finally note that the way the pion field is defined p. 42, eq. (3.8), the pion nucleon matrix elements p. 98, eq. (5.16), are purely imaginary. It is therefore worthwhile, for computational purposes, to redefine the pion field as

$$\Phi(\vec{r},t) = \left(\frac{1}{2\pi}\right)^{3/2} \sum_{\kappa} \int d^3p \frac{i}{\sqrt{2E}} \left\{ e^{i(\vec{p}\,\vec{r} - E\,t)} a^{(1)}_{\kappa\vec{p}} \; \eta^{[1]}_{\kappa} - e^{-i(\vec{p}\,\vec{r} - E\,t)} a^{(1)+}_{\kappa\vec{p}} \; \eta^{[1]+}_{\kappa} \right\} .$$

Using that definition the complex phase of (5.16) becomes the real number $(i)^{-\ell_3+\ell_2+\ell_1+1}$.

D - ERRATA TO N.B.S. MONOGRAPH 147

The errata are given in the form of the complete corrected formula or of a fraction thereof such that they may be directly pasted over the wrong parts in the Monograph 147

p. 7, eq. (1.8)

$$\psi^{(S)}(x) = \sum_{j} (u_j(x) b_j + v_j(x) c_j^{+})$$

p. 7, eq. (1.12)

$$\psi^{(H)}(x,t) = \sum_{j} \left(e^{-i\,\varepsilon_j t} u_j(x) b_j + e^{i\,\varepsilon_j t} v_j(x) c_j^{+} \right)$$

p. 19, eq. (2.4)

$$Y^{(\ell)}_{m}(\theta\varphi) = \hat{r}^{(\ell)}_{m} = (-)^{\ell+m}\, Y^{[\ell]}_{-m}(\theta\varphi)$$

p. 37, eq. (2.79)

$$\alpha_{n+1}(T)\ \beta_n(T-1) \begin{bmatrix} T & 1 & T-1 \\ 0 & 1 & 1 \\ T & t & T \end{bmatrix} + \beta_{n+1}(T)\ \alpha_n(T+1) \begin{bmatrix} T & 1 & T+1 \\ 0 & 1 & 1 \\ T & t & T \end{bmatrix} = 0 \ , \qquad t = 1 \ .$$

p. 46, eq. (3.37)

$$\mathcal{L} = - \sum_{\kappa} (\bar{\psi}_{\kappa}\, \gamma_{\mu}\, \partial_{\mu}\, \psi_{\kappa}) + m(\bar{\psi}_{\kappa}\, \psi_{\kappa}) \ ,$$

p. 73

Utilizing the formulas (2.60) and (2.61), it can be checked that

p. 75, second line of eq. (3.238)

$$+\frac{J+1}{\hat{J}^2}\left(\mathcal{W}^{\mathscr{E}}_{\nu\,J\,J+1}(r)\,\mathfrak{X}^{\mathscr{E}}_{\nu\,J\,J+1}(r') + \mathcal{W}^{\mathscr{L}}_{\nu\,J\,J+1}(r)\,\mathcal{Z}^{\mathscr{L}}_{\nu\,J\,J+1}(r')\right) = 3\,\frac{\delta(r-r')}{r^2}\,.$$

p. 76, add to the end of the section the comment :

We list the parities and the number ν of quantas for the lowest energy discretized multipole solutions of the spin 1 fields

J	ν	$\mathscr{E}$	$\mathscr{M}$	$\mathscr{L}$
0	0			+
1	1	−	+	−
2	2	+	−	+
3	3	−	+	−

p. 80, eq. (4.13)

$$[\nu_1\ell_1|\vec{p}^{\,[1]}|\nu_2\ell_2] = \int p^2 dp\; f_{\nu_1\ell_1}(p)\; f_{\nu_2\ell_2}(p)\; p \times \alpha_{\ell_1\ell_2}$$

p. 83, last line of eq. (4.18)

$$+\sqrt{(J_1+1)J_2}\begin{bmatrix} 1 & J_1-1 & J_1 \\ 1 & J_2+1 & J_2 \\ 0 & 1 & 1 \end{bmatrix} \alpha_{J_1-1,J_2+1}\; f_{\nu_1 J_1-1}\; f_{\nu_2 J_2+1}\Bigg\} \qquad ,$$

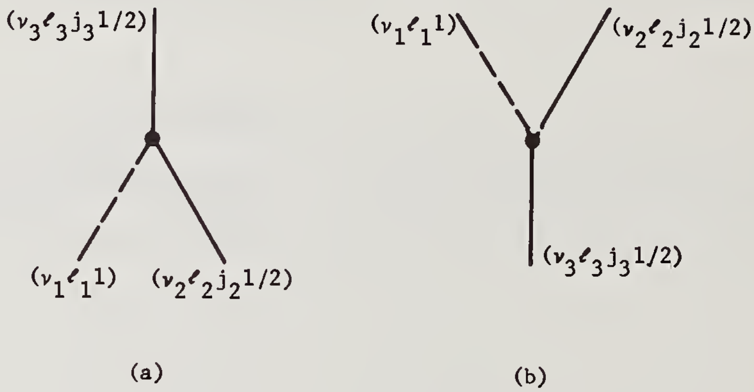
p. 90, second line of eq. (4.41)

$$\times\left[\sqrt{J_1(J_2+1)}\begin{bmatrix} 1 & J_1+1 & J_1 \\ 1 & J_2-1 & J_2 \\ 0 & 1 & 1 \end{bmatrix} \mathscr{F}^{\nu_1\nu_2}_{J_1+1,J_2-1}(p) + \sqrt{J_1J_2}\begin{bmatrix} 1 & J_1+1 & J_1 \\ 1 & J_2+1 & J_2 \\ 0 & 1 & 1 \end{bmatrix} \mathscr{F}^{\nu_1\nu_2}_{J_1+1,J_2+1}(p)\right.$$

p. 94, first line of eq. (5.8)

$$\mathscr{H}_{PV(\pi NN)}(\text{time-like}) = \frac{\sqrt{3}}{2} G_{PV(\pi NN)} \sum_{\substack{\nu\nu'\nu'' \\ \ell\ell'\ell'' \\ j'j''}} i^{-\ell''+\ell'+\ell}\, \hat{j}'\, \hat{j}''\, (-)^{j'+\lambda'+1/2}$$

p. 95, Exchange in figure 5.3.b the boson and fermion lines



p. 96, first line of eq. (5.15)

$$_f\langle(\nu_1\nu_2\ell_1\ell_2 j_2) j_3 1/2 \,|\, [\tilde{B}^{[j''1/2]}_{\nu''\ell''} (A^{[\ell\ 1]}_{\nu} \pm (-)^{\ell} \tilde{A}^{[\ell\ 1]}_{\nu}) B^{[j'1/2]}_{\nu'\ell'}]^{[00]} \,|\, \nu_3\ell_3 j_3 1/2\rangle_i$$

p. 98, fourth line of eq. (5.16)

$$\times\left(\begin{Bmatrix} \ell_3 & j_3 & 1/2 \\ j_2 & \lambda_2 & \ell_1 \end{Bmatrix} [\ell_3|\lambda_2|\ell_1] \int r^2 dr\, h_{\nu_1\ell_1}\, u_{\nu_3\ell_3}\, v_{\nu_2\ell_2\lambda_2} - \begin{Bmatrix} \lambda_3 & j_3 & 1/2 \\ j_2 & \ell_2 & \ell_1 \end{Bmatrix} [\lambda_3|\ell_2|\ell_1] \right.$$

p.100, first line of eq. (5.26)

$$\mathscr{H}_{(4\pi)} = \frac{G_{(4\pi)}}{4} \sum_{R=0,2} I(\alpha\beta\gamma\delta)\, i^{\alpha+\beta+\gamma+\delta}\, Q^{L}_{\alpha\beta}\, Q^{\delta}_{L\gamma}\, Q^{0}_{\delta\delta} \sqrt{4\pi}$$

p.102, first line of eq. (5.27)

$$\mathscr{H}_{(4\pi)} = \frac{G_{(4\pi)}}{4} \sum I(\alpha\beta\gamma\delta)\, i^{-(\alpha+\beta+\gamma+\delta)}\, Q^{L}_{\alpha\beta}\, Q^{\delta}_{L\gamma}$$

p.102, first line of eq. (5.28)

$$\mathscr{H}_{(4\pi)} = \frac{G_{(4\pi)}}{4} \sum \quad I(J\beta\gamma\delta)\; i^{(I-\beta-\gamma-\delta)}\; Q^{L}_{\beta\gamma}\; Q^{J}_{L\delta}$$

p.102, first line of eq. (5.29)

$$\mathscr{H}_{(4\pi)} = \frac{G_{(4\pi)}}{4} \sum \quad I(J\beta\gamma\delta)\; i^{(-J+\beta+\gamma+\delta)}\; Q^{L}_{\beta\gamma}\; Q^{J}_{L\delta}$$

p.102, first line of eq. (5.30)

$$\mathscr{H}_{(4\pi)} = \frac{G_{(4\pi)}}{4} \sum \quad I(\alpha\beta\gamma\delta)\; i^{(-\alpha-\beta+\gamma+\delta)}\; Q^{J}_{\alpha\beta}\; Q^{J}_{\gamma\delta}$$

p.125, eq. (A.5)

$$x_1 = x_o + v_1(t-t_o) \quad ,$$

p.125, eq. (A.6)

$$x_2 = x_o + v_2(t-t_o) \quad ,$$

p.135, add at the bottom of the page the missing equation :

$$\begin{aligned} \psi = {} & 4\pi \sum_{\lambda} i^{\lambda}\, \hat{\lambda}\, F_{\lambda j}(pr) \left[\hat{p}^{[\lambda]}\; \hat{r}^{[\lambda]}\right]^{[0]} \\ & \times \sum_{k\ell} \hat{k}\, \hat{\ell} \left[b_N^{[k]}(\vec{p})\; \chi_{\nu N}^{[k]}(\vec{\xi}_i)\right]^{[0]} \left[b_\pi^{[\ell]}(-\vec{p})\; \chi_{\mu\pi}^{[\ell]}(\vec{\xi}_j)\right]^{[0]} \end{aligned} \qquad \text{(C.4)}$$

p.136, eq. (C.7)

$$\chi_{\lambda j}^{(\nu,\mu)} = \mathcal{N}\, \hat{j}\, F_{\lambda j}(pr) \sum_{k\ell} \left[B_{k\ell}^{[j]}(p) \left[\chi_{\nu N}^{[k]}(\vec{\xi}_i)\, \chi_{\mu\pi}^{[\ell]}(\vec{\xi}'_j)\right]^{[S]} \hat{r}^{[\lambda]} \right]^{[0]}$$

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