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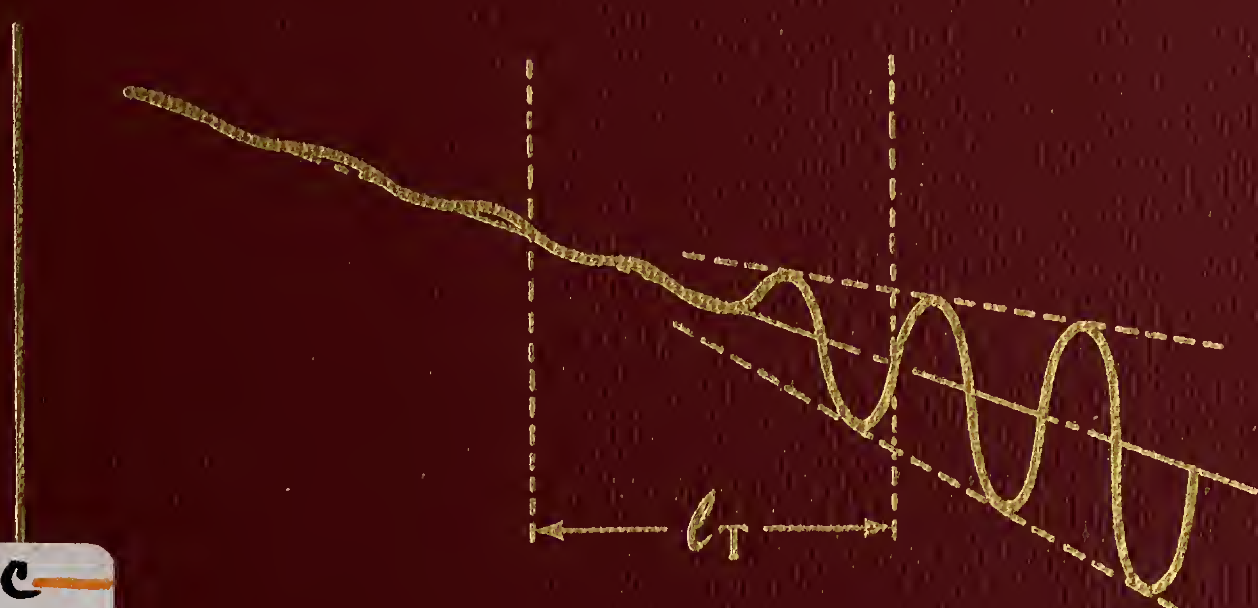
A METROLOGY GUIDE



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THE MEASUREMENT OF NOISE PERFORMANCE FACTORS: A METROLOGY GUIDE

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ABSTRACT

This metrology guide provides the basis for critical comparisons among seven measurement techniques for average noise factor and effective input noise temperature. The techniques that are described, discussed, and analyzed include the (1) Y-Factor, (2) 3-dB, (3) Automatic, (4) Gain Control, (5) CW, (6) Tangential, and (7) Comparison Techniques. The analyses yield working equations and error equations by which accuracy capabilities are compared. Each technique is also analyzed for (a) frequency range for best measurement results, (b) special instrumentation requirements, (c) speed and convenience, (d) operator skill required, and (e) special measurement problems. General instrumentation requirements and practical measurement problems are discussed for the benefit of the non-expert metrologist. Worked examples illustrate the principles involved in applying the working and error equations. An extensive bibliography and suggested reading list aid the metrologist to locate additional material on these measurements. Taken altogether, this guide will be helpful in selecting the best measurement technique for any of a wide range of operational requirements and, once the technique is selected, it will be of further benefit in helping the metrologist identify where his efforts should be placed to derive the greatest efficiency and accuracy from his measurement system.

KEY WORDS

Effective input noise temperature; measurement errors; noise factor; noise measurements; noise performance factors; noise temperature; Y-factor measurements.

EDITOR'S NOTE

This Guide is one of a series of Metrology Guides sponsored by The Electromagnetic Metrology Information Center of the National Bureau of Standards Electromagnetics Division designed to be critical comparisons of measurement methods for a variety of electromagnetic quantities. The objective is to provide guidance in the selection, use and evaluation of methods for a particular application. These Guides, written by measurement specialists and based on extensive literature searches, are tailored to the needs of technical people who may not possess specialized training in the measurement of the quantity that is the subject of a particular guide. Therefore, these Guides will be useful to teachers, design engineers, contract monitors, and practicing metrologists, as well as general engineers and scientists who need specific measurement know-how but do not have the time or facilities to do their own research and study on the complexities involved. With the above objective in mind, each guide includes the following:

1. A description of the physical principles underlying the measurement technique.

2. An indication of the accuracy obtainable with each method whether by discussion of typical ranges and accuracies or through discussion of the error equations.

3. A discussion of the technical strengths and weaknesses of each technique. This includes a discussion of the sources of error or, wherever possible, the error equations of specific operating systems.

4. A discussion of the instrumentation requirements (including standards) for each technique.

5. Operational problems, suggestions, or examples.

6. An extensive bibliography to assist the reader in pursuing the details of methods beyond the depth of the guide.

If the reader wishes to receive information about any future Guides or wishes to comment on this Guide, please use the form at the back of the book.

W. J. Anson
Editor -
Metrology Guide Series

ACKNOWLEDGMENT

My sincere appreciation goes to the following persons for their contributions to this Guide. Wilbur J. Anson provided much guidance, assistance, and encouragement throughout the long process from conception to birth of this Guide. Margaret Woolley typed the manuscript with much dedication to the task. Illustrations were drawn by Nick Sanchez. The manuscript was read by Drs. William D. McCaa, Robert A. Kamper and Charles H. Manney for technical accuracy. Finally, this Guide could not have been produced without the commitment and support of Dr. R. C. Sangster, Chief of the Electromagnetics Division, NBS.

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The Measurement of Noise Performance Factors;
A Metrology Guide

1. INTRODUCTION

The purpose of this guide is to describe, discuss, and analyze methods of measuring the average noise factor and average effective input noise temperature of an electronic transducer[1]. Noise factor is a measure of the degree to which a transducer degrades the signal-to-noise ratio of an incoming waveform. Every signal source supplies both signal power and noise power, and therefore its output waveform has a signal-to-noise ratio (see figure 1). Real-world transducers add noise, thus producing an output waveform having a poorer signal-to-noise ratio. Effective input noise temperature is a measure of how much noise power the transducer adds to its input signal (see figure 2). Further discussion of, and the relationship between, noise factor and effective input noise temperature are found in Sections 2.1 and 2.2.

Strictly speaking, noise factor, $F(f)$, and effective input noise temperature, $T_e(f)$, are functions of frequency, and are defined in terms of the noise power in a one hertz bandwidth at a specified frequency, f. However, the manner in which transducers are used and the practical limitations of instrumentation usually require measurements to be made over a band of frequencies greater than one hertz in width. Thus the quantities actually measured are AVERAGE NOISE FACTOR, $\overline{F}$, and AVERAGE EFFECTIVE INPUT NOISE TEMPERATURE, $\overline{T_e}$, the averages being taken over the frequency band.

This guide is not primarily concerned with the validity of the definitions of $\overline{F}$ and $\overline{T_e}$, nor with questions pertaining to their applicability. Discussions of those matters can be found in the technical literature [1] - [9].

$\overline{F}$ and $\overline{T_e}$ are widely measured by a large number of people for a variety of purposes. The cost, quality, and sometimes even the feasibility of an electronics system are often significantly influenced by noise performance. For example, the operating costs of commercial satellite communications systems can increase by many hundreds of thousands of dollars per decibel of noise factor increase. The effective range of radar is inversely proportional to noise factor. The sensitivity of nuclear magnetic resonance spectrometers; the ability to detect, measure, and study heavy organic molecules in deep space with radio telescopes; the radiometric mapping of Earth for natural resources, ecological purposes, etc.; all these improve with reduced noise factor.

[1]The term "transducer" as used in this guide is a generalized concept as defined in Appendix A, Definition of Terms. Refer to Appendix A for definitions of terms important to the proper understanding of this guide.

Indeed, the noise performance of systems is a fundamental limiting factor to their ultimate sensitivity, capacity, and operational efficiency. The measurement of noise performance factors therefore often serves an important role in the design and operation of systems.

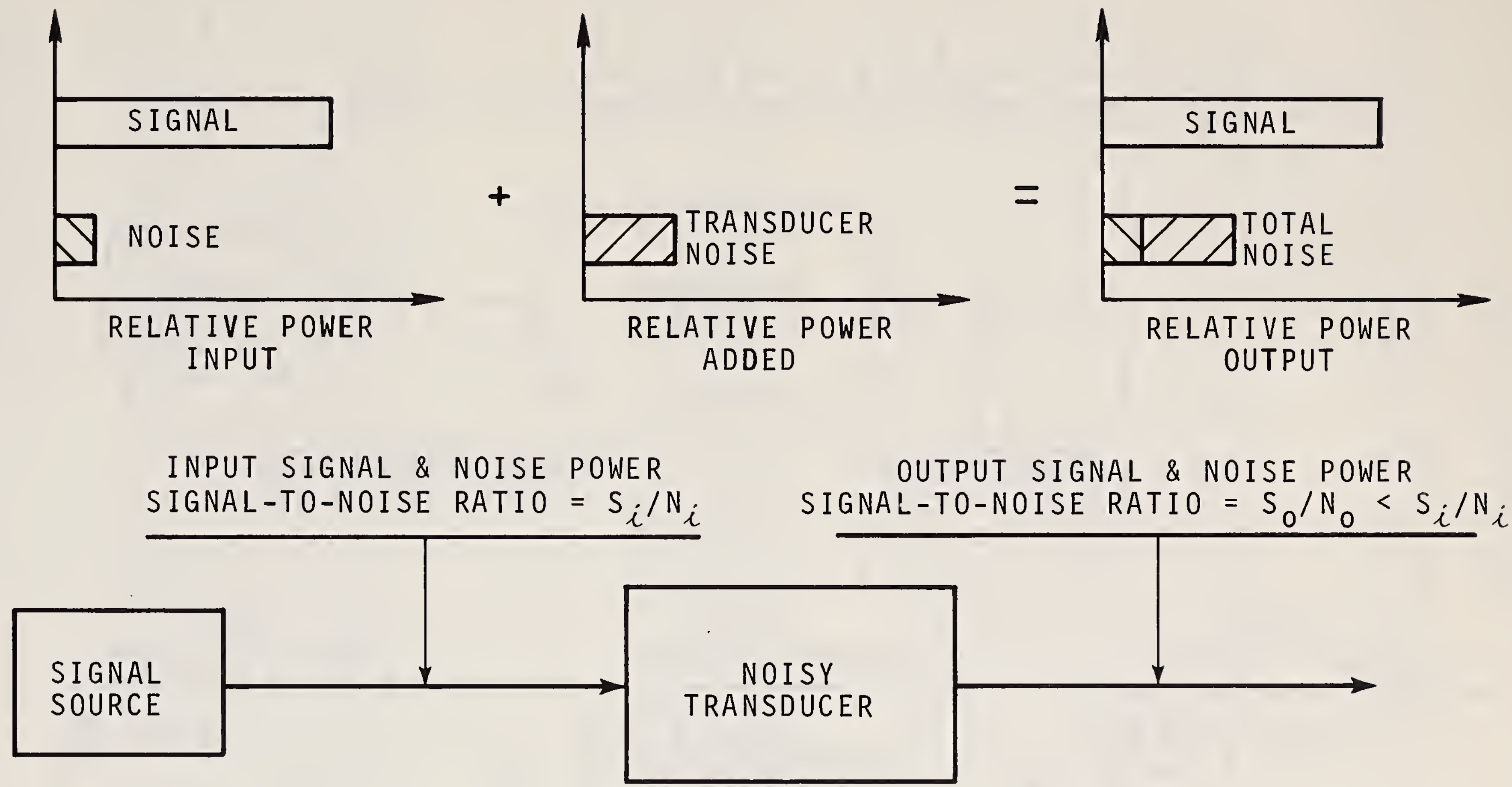


Figure 1. Effect of Transducer Noise on Signal-to-noise Ratio. Basis for Noise Factor, F.

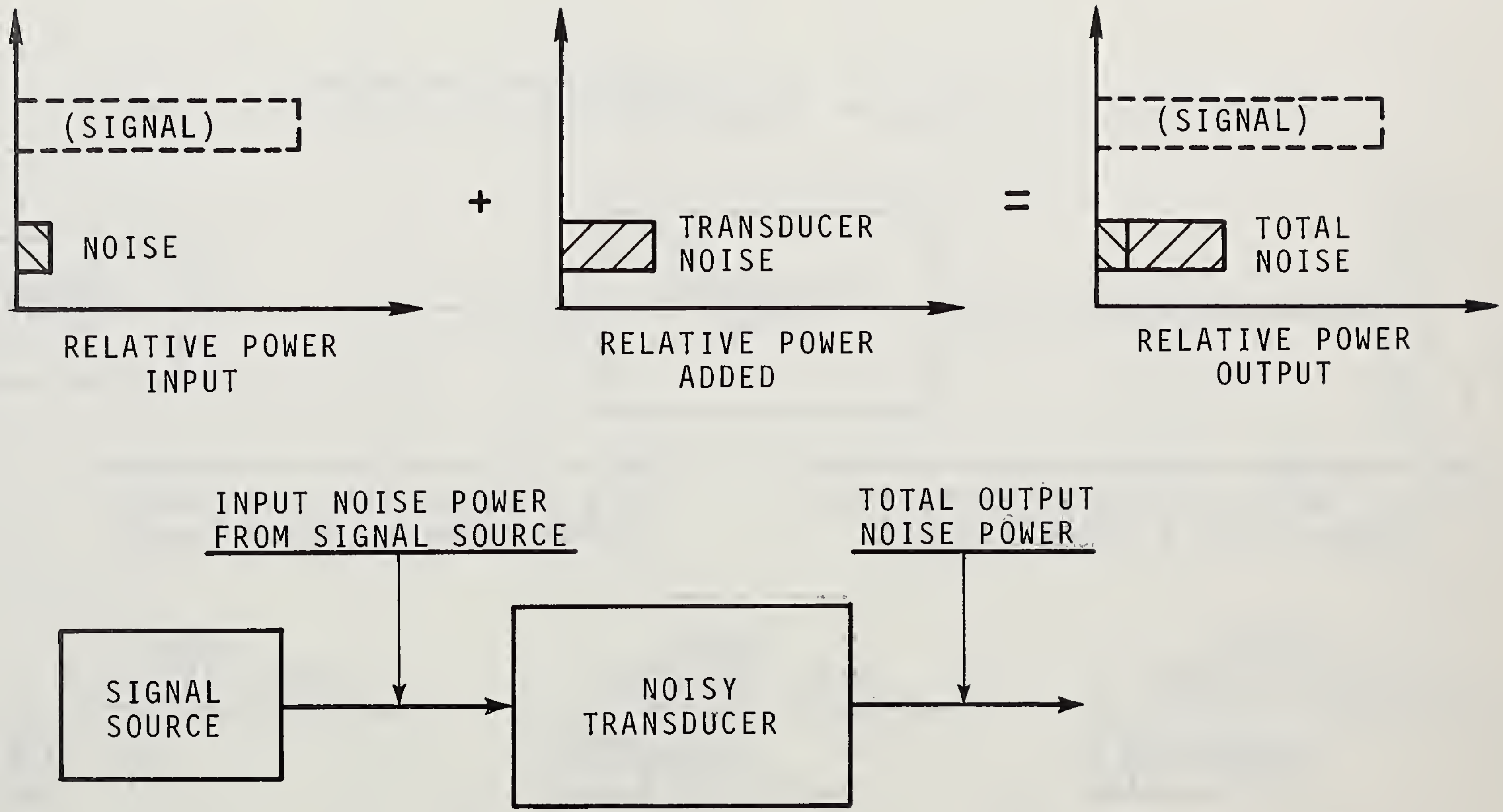


Figure 2. Effect of Transducer Noise on Total Output Noise Power. Basis for Effective Input Noise Temperature, T_e.

2. GENERAL CONSIDERATIONS

To establish the basis for the measurement methods discussed in this guide, the definitions and certain fundamental features of $\overline{F}$ and $\overline{T_e}$ are given below.

2.1. Average Noise Factor

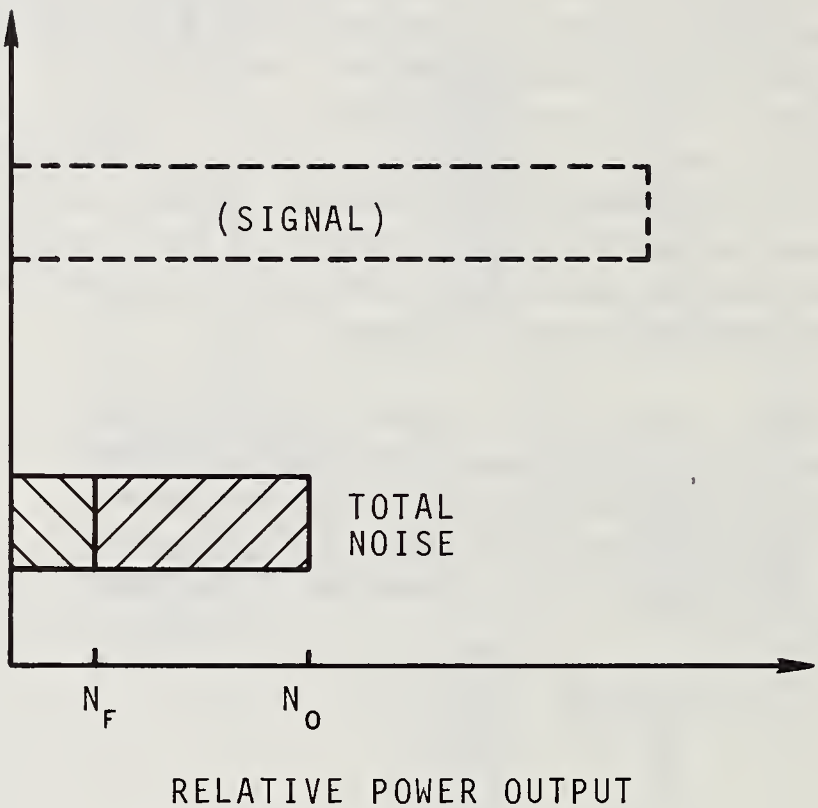
Average noise factor, $\overline{F}$, is a dimensionless ratio that was originated to indicate the noisiness of a two-port transducer. There are two points of view that lead to $\overline{F}$. Historically, the first, introduced by North [10], is that $\overline{F}$ is the ratio of (1) the output noise power from the transducer to (2) the output noise power from an equivalent noise-free transducer (see figure 3). Thus, it is the ratio of two powers. The second, introduced by Friis [11], is that $\overline{F}$ is the ratio of (1) the input signal-to-noise ratio, S_i/N_i, to (2) the output signal-to-noise ratio, S_o/N_o, of the transducer (see figure 4). Thus, it is the ratio of two power ratios. These two points of view lead to the same result, which is expressed formally by the IEEE definition [12] (see figure 5) given in Appendix A. The essential part of this definition is as follows:

> The average noise factor (average noise figure) of a two-port transducer is the ratio of N_o, the total noise power delivered by the transducer into its output termination when the noise temperature of its input termination is standard (290 K) at all frequencies, to N_s, that portion of N_o engendered by the input termination. Thus,

$$\overline{F} = \frac{N_o}{N_s} . \tag{1}$$

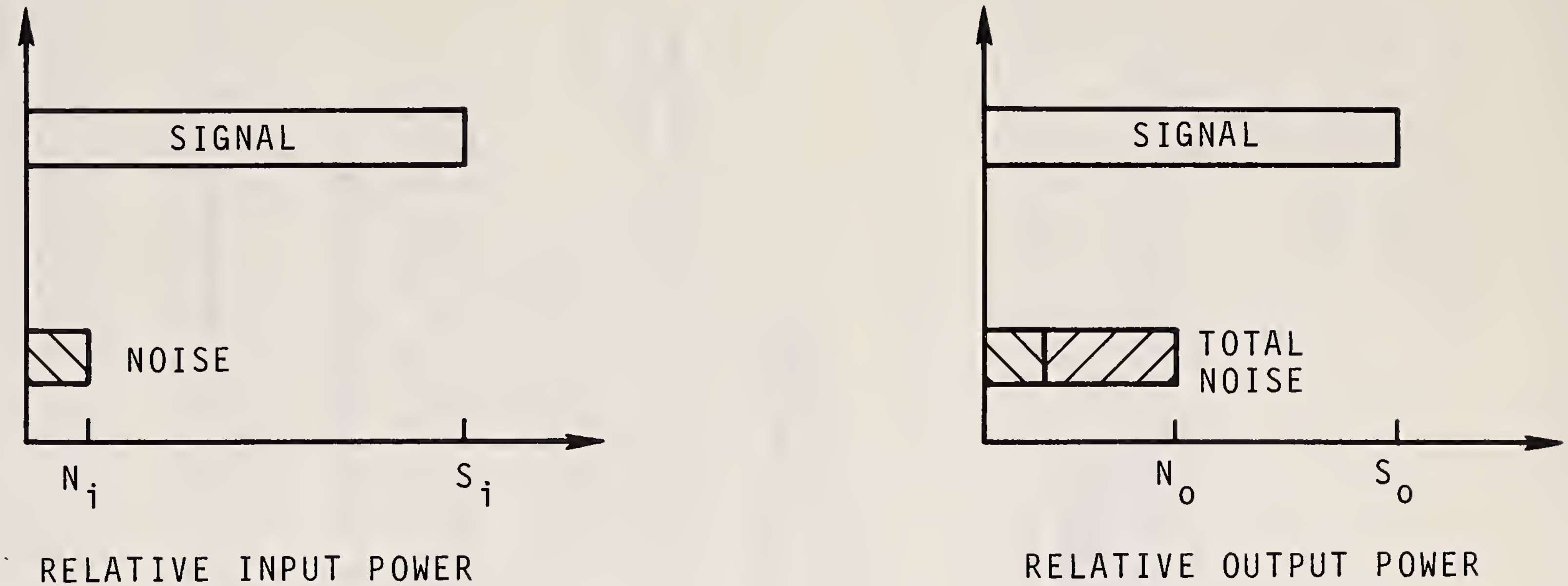
For a transducer that has gain in more than one frequency band, such as a heterodyne or other multiple-response system, the denominator, N_s, includes only that noise power from the input termination that lies in the same frequency interval(s) as an information-bearing system signal. N_s does not include noise contributions, such as those from an unused image-frequency or unused idler-frequency band, where no system signal exists. Because of this interpretation of the definition, $\overline{F}$ of a given transducer may depend upon the signal characteristics of the system in which it is used. This situation has given rise to terms such as

a. single channel noise figure
b. narrow band noise figure
c. radar noise figure
d. single sideband noise figure



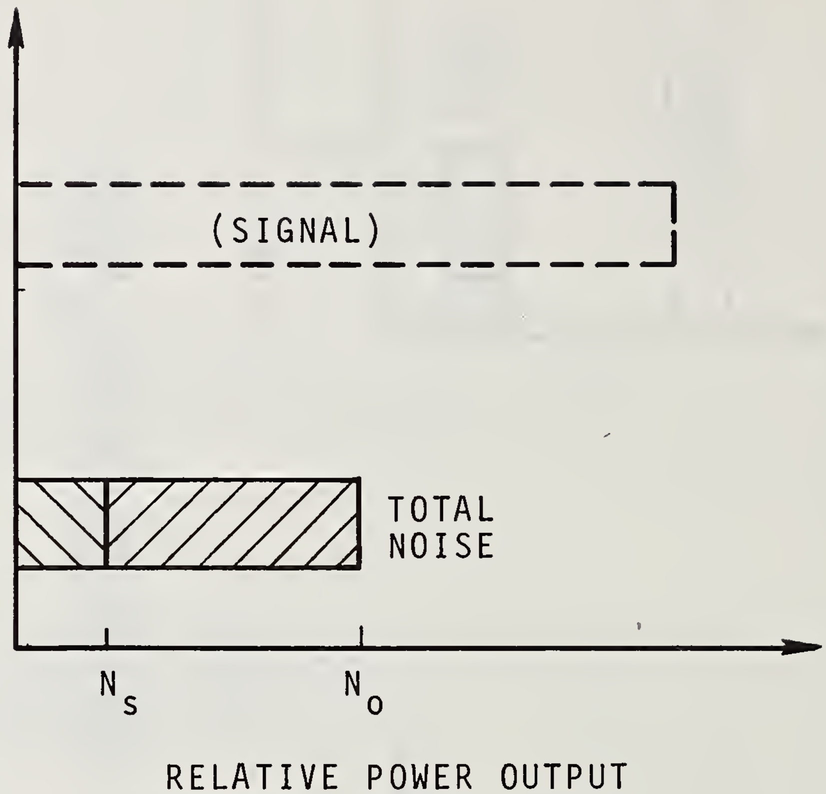
$$\overline{F} = \frac{N_O}{N_F} = \frac{\text{TOTAL OUTPUT NOISE POWER}}{\text{OUTPUT NOISE POWER FROM EQUIVALENT NOISE-FREE TRANSDUCER}}$$

Figure 3. D. O. North's Concept of $\overline{F}$.



$$\overline{F} = \frac{S_i/N_i}{S_o/N_o} = \frac{\text{INPUT SIGNAL-TO-NOISE RATIO}}{\text{OUTPUT SIGNAL-TO-NOISE RATIO}}$$

Figure 4. H. T. Friis' Concept of $\overline{F}$.



$$\overline{F} = \frac{N_o}{N_s} = \frac{\text{TOTAL OUTPUT NOISE POWER}}{\text{PORTION OF } N_o \text{ ORIGINATING IN INPUT TERMINATION}}$$

Figure 5. IEEE's Concept of $\overline{F}$.

which pertain to transducers in which the system signal lies in only one frequency band, and

e. double channel noise figure

f. broad band noise figure

g. astronomy noise figure

h. double sideband noise figure

which pertain to transducers in which the system signal lies in two or more frequency bands. In the special case of a transducer that has gain in only one frequency interval, $\overline{F}$ is by definition a "single channel" type of noise factor, regardless of the bandwidth of the transducer. In general, for a given multiple-response transducer, the single channel noise factor, $\overline{F_s}$, is greater than the broad band noise factor, $\overline{F_b}$ (see Section 5.6.)

The relationship between $\overline{F_s}$ and $\overline{F_b}$ is given by the equation

$$\overline{F_s} = \overline{F_b} \; \frac{(GB)_b}{(GB)_s} \qquad (2)$$

where (GB) is the area under that part of the transducer gain function, G(f), that includes the frequency band(s) in which the system signal lies. That is,

$$(GB)_s = \int_{fs} G(f)\,df \qquad (3)$$

and

$$(GB)_b = \int_{fb} G(f)\,df, \qquad (4)$$

where the integral in (3) is over only the frequency range, f_s, that includes the narrow band signal, and the integral in (4) is over the frequency ranges, f_b, that includes the broad band signal.

Measurement techniques that use a broad band test source, such as a wideband noise generator, will normally yield $\overline{F_b}$ unless the transducer has gain in only a single band of frequencies, or unless special measures are taken to limit the measurement frequency range by filters (see Section 4.7.1). Conversely, measurement techniques that use a CW signal generator as a test source will yield either $\overline{F_s}$ or $\overline{F_b}$ depending upon whether (GB) is evaluated over one or more bands of G(f). Thus measurements of $\overline{F_b}$ are readily made by dispersed-signal techniques, but the $\overline{F_s}$ of a multiple-response transducer must be made either by a CW technique or by using filters in a dispersed-signal technique. The measured value of $\overline{F}$ (i.e., either $\overline{F_s}$ or $\overline{F_b}$) has meaning only when the measurement frequency interval, f_s or f_b, is specified.

Average noise factor depends also upon the impedance (admittance) of the transducer's input termination; therefore, it is meaningful only when the impedance (or admittance) of the input termination is specified (see Section 5.1).

Some engineers have adopted the practice of using the term "noise factor" to indicate the numerical power ratio and the term "noise figure" to indicate the power ratio expressed in decibels. This practice is not universally followed and, since it is contrary to the intent of the committee who formulated the IEEE definition (see p. 42 of reference [13]), this practice is discouraged. These terms may be used interchangeably.

2.2. Average Effective Input Noise Temperature

Average effective input noise temperature, $\overline{T_e}$, is another widely used measure of transducer noisiness, but the basic concept is different from that of noise factor. $\overline{T_e}$ is a fictitious TEMPERATURE, having the dimensions of kelvins (K), that is assigned to a real transducer to represent how much noise power the transducer adds to an input signal. The IEEE definition of $\overline{T_e}$ [14] is given in Appendix A; the essential part of this definition is as follows:

> The average effective input noise temperature, $\overline{T_e}$, of a multi-port transducer, with one port designated as the output port, is the noise temperature, in kelvins, which, when assigned simultaneously to the specified impedance terminations at all frequencies at all accessible ports except the designated output port of a NOISEFREE EQUIVALENT of the transducer, would yield the same total noise power in a specified output band delivered to the output termination as that of the ACTUAL TRANSDUCER connected to noisefree equivalents of the terminations at all ports except the output port.

To help understand this definition, two block diagrams are shown in figure 6. In the upper diagram, the input ports of the NOISE-FREE EQUIVALENT of the multiport transducer are terminated in source impedances, each at temperature $\overline{T_e}$. The total noise power available from the transducer is P_L. In the lower diagram, the input ports of the ACTUAL (noisy) multiport transducer are terminated in source impedances of the same impedance values as before, but each such input termination is NOISE-FREE (noise temperature of 0 K). The total noise power available from this transducer is also P_L. The value of $\overline{T_e}$ necessary to

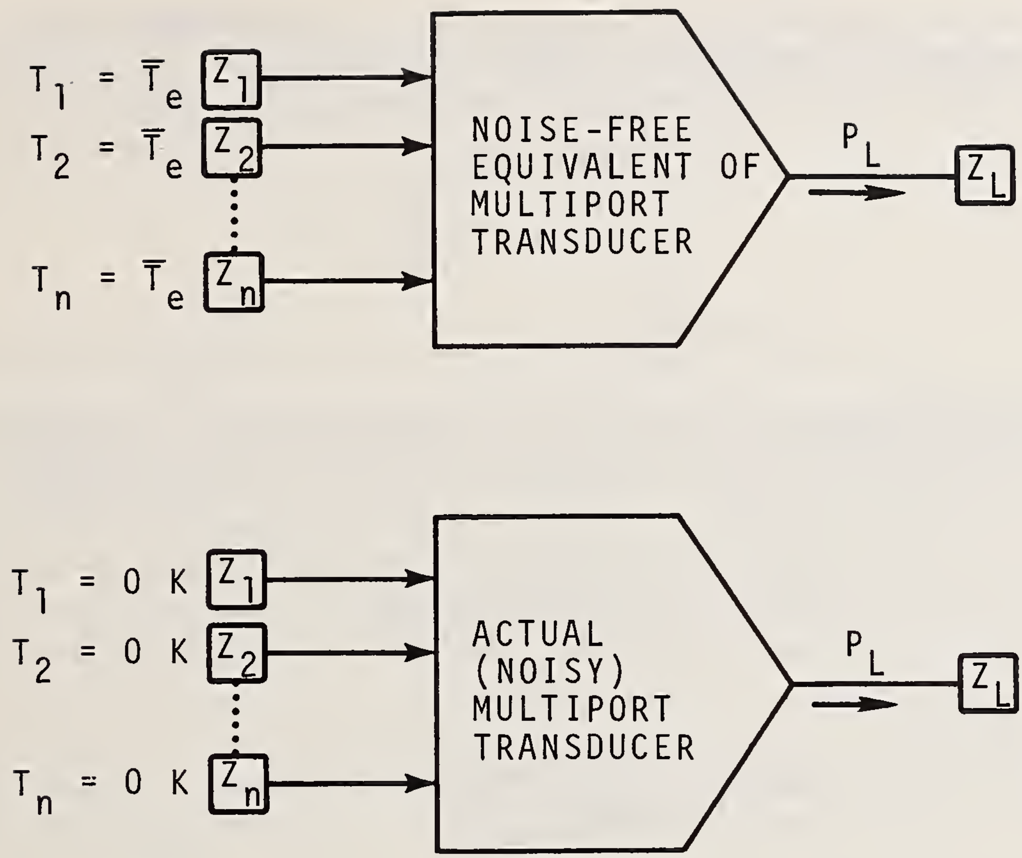


Figure 6. IEEE's Concept of $\overline{T_e}$.

produce the same value of P_L in both diagrams is equal to the value of the average effective input noise temperature of the actual multiport transducer.

Note that each of the diagrams of figure 6 requires the use of fiction (e.g., noise-free equivalent components). However, the real quantity, $\overline{T_e}$ of the real, noisy multiport transducer is established through the use of this fiction.

For a two-port transducer with a single input and a single output frequency band, $\overline{T_e}$ is related to $\overline{F}$ by the equations

$$\overline{T_e} = 290(\overline{F} - 1) , \tag{5}$$

and

$$\overline{F} = \frac{\overline{T_e}}{290} + 1. \tag{6}$$

For transducers with gain in more than one frequency band, $\overline{T_e}$ is related to $\overline{F_b}$ as given in (5) and (6), above, and to $\overline{F_s}$ according to the relationships [13]

$$\overline{T_e} = 290 \left(\overline{F_s} \frac{(GB)_s}{(GB)_b} - 1 \right), \tag{7}$$

and

$$\overline{F_s} = \left(1 + \frac{\overline{T_e}}{290} \right) \frac{(GB)_b}{(GB)_s} . \tag{8}$$

$\overline{T_e}$ differs from $\overline{F}$ in the following ways:

a. $\overline{F}$ pertains to a two-port transducer;
   $\overline{T_e}$ pertains to multi-port transducers.

b. $\overline{F}$ is defined <u>only</u> when the input termination to the transducer is at a temperature of 290 K;
   $\overline{T_e}$ is defined independently of the temperature of the input termination.

c. $\overline{F}$ is a function of the frequency distribution of the system signal;
   $\overline{T_e}$ is independent of the frequency distribution of the system signal.

d. $\overline{F}$ has a minimum limiting value of unity;

$\overline{T_e}$ has a minimum limiting value of zero.

$\overline{T_e}$ is most readily measured by wide band dispersed-signal techniques, although it can be measured by CW techniques. In the latter case, the transducer's noise bandwidth must also be measured to obtain $\overline{T_e}$ (see Appendix C).

2.3. Noise Bandwidth

Noise bandwidth, B, is the width of an ideal rectangular gain function whose area, G_oB, is the same as the transducer's gain function area, $(GB)_s$ (see Appendix C). That is,

$$B = \frac{(GB)_s}{G_o} \tag{9}$$

where G_o is the transducer gain at some reference frequency, f_o.

It is important to recognize that B is a variable parameter of the transducer. Its value depends upon the choice of f_o because G_o depends upon frequency. A given transducer does not have a unique value of B because there are no adequate standards that specify the choice of f_o. For a transducer with a single maximum in its gain function, f_o is commonly chosen as the frequency of maximum gain, and B has a well-defined value. However, for transducers having gain functions of more complicated shape or for those having gain in more than one frequency band, the choice of f_o may not be so obvious; f_o may not be selected the same by all metrologists. Hence the value of B may differ from case to case. The effect of arbitrarily choosing f_o is to make B large if G_o is small, or to make B small if G_o is large.

A brief analysis of the CW techniques would show that the actual value of B, per se, is not important (see Appendix C). The important transducer parameter, for the purpose of measuring $\overline{F}$, is not its noise bandwidth, B, but rather its gain-bandwidth product, $(GB)_s$. Indeed, B is "measured" by measuring $(GB)_s$ and dividing it by the measured value of G_o [15]. $(GB)_s$ is a constant for a given transducer. Therefore, f_o can be selected, in principle, at any frequency within the interval of f_s without affecting the value of $\overline{F}$. In practice however, the measurement is normally most accurate if f_o is chosen where G(f) is large (at or near a maximum) rather than out on the "skirts" of the gain function.

The need to evaluate either B or $(GB)_s$ does not arise with dispersed-signal measurement techniques. These techniques are distinctly advantageous for this reason; the measurement of $(GB)_s$ in the absence of competent equipment of advanced design can be tedious, difficult, and inaccurate.

This discussion of noise bandwidth is given to place B into proper perspective in the MEASURERMENT of $\overline{F}$. It must not be inferred that the noise PERFORMANCE of a transducer is independent of its noise bandwidth. On the contrary, if a transducer's noise bandwidth is poorly chosen with respect to the system signal's bandwidth, the transducer's performance may be quite poor.

2.4. Transducer Linearity

The definitions of $\overline{F}$ and $\overline{T_e}$ do not require that the transducer be linear. In principle, every transducer has an $\overline{F}$ and $\overline{T_e}$ regardless of the signal level. However, if the transducer is non-linear, $\overline{F}$ and $\overline{T_e}$ may vary with signal level, depending upon the nature of the non-linearity and other transducer characteristics. A single value of $\overline{F}$ (or $\overline{T_e}$) then does not describe the noisiness of the transducer, and its meaningfulness is reduced.

On the other hand, each of the known techniques for measuring $\overline{F}$ and $\overline{T_e}$ is tacitly based on the assumption that either (1) the transducer is linear or (2) $\overline{F}$ and $\overline{T_e}$ are independent of signal level. When either condition is not the case, measurement results on a given transducer may be variable. This is the reason for the practically sound but technically incorrect belief that $\overline{F}$ and $\overline{T_e}$ apply only to linear transducers.

3. MEASUREMENT TECHNIQUES

Techniques for measuring $\overline{F}$ and $\overline{T_e}$ are broadly divided between broad band and narrow band techniques. Broad band techniques typically use noise generators or other dispersed-signal sources as a measurement signal, whereas narrow band techniques use CW signal generators, either unmodulated or modulated.

The most generally accurate broad band measurement technique is the Y-factor technique. Two variations of this technique are described below. The most widely used broadband techniques are the 3-dB and automatic noise figure meter (ANFM) techniques. Another broad band technique that has at times been used is the gain control technique.

The most widely used narrow band measurement technique is the CW technique using an unmodulated signal. Other narrow band techniques include the tangential and comparison techniques.

All of these techniques are described in this section, along with their relative merits and weaknesses. The purpose of these discussions is to provide an understanding of the basic procedures involved in each technique. Therefore, certain ideal conditions are assumed, viz., (a) the two-port transducer is linear, stable, well shielded and has gain in only one frequency band, (b) the measurement instrumentation is properly selected for the intended purpose, (c) the transducer has no peculiar electrical or mechanical characteristics that prevent the measurement from being made as indicated, and (d) the noise temperature of generator source impedances is standard (290 K) unless otherwise stated. Deviations from these ideal conditions, including their effects and how to cope with them, are discussed in later sections of this Guide , particularly Section 5, Practical Measurement Problems.

Although the equations included in this section are for the assumed ideal conditions, they are often adequate for many real-world situations where, e.g., the measurement accuracy requirements are modest. For more complete equations, and for an understanding of the derivation of these equations, refer to Section 6, Working Equations.

Some idea of test equipment characteristics and availability as required by these techniques can be obtained from Section 4, Instrumentation. The accuracy that can be expected under a given set of conditions can be estimated by the methods given in Section 7, Measurement Errors, and examples of hypothetical measurement results are given in Section 8, Measurement Examples.

These measurement techniques apply generally across the full frequency spectrum, from very low frequencies through the millimeterwave frequencies. The particular choice of technique for a given situation will depend upon many factors including (a) desired accuracy, (b) instrumentation required, (c) equipment availability, (d) equipment costs, (e) frequency range, (f) transducer type, (g) convenience of technique, (h) measurement speed, and (i) operator experience. Most of these factors are discussed briefly in this section. Costs are not discussed in detail since they will vary from time to time and from one situation to another. However, the information contained in this Guide should be helpful in estimating comparative costs when used in conjunction with current manufacturer's price information.

A summary of the more important characteristics of these measurement techniques is given in Table I.

3.1. Y-Factor Technique

The Y-factor technique uses dispersed-signal sources of power applied to the transducer under test. The most commonly used sources generate random noise power that is distributed over a wide frequency range. The Y-factor technique derives its name from the quantity

$$Y = \frac{P_1}{P_2} \tag{10}$$

where P_1 and P_2 are two noise power levels at the OUTPUT PORT of the transducer that result from two different levels of noise power applied to its input port.

Two Y-factor methods are commonly used, viz., (a) the power meter method and (b) the variable attenuator method. They differ principally in the way the Y-factor is measured.

3.1.1. Power Meter Method

<u>Basic Method</u>

This method uses a pair of random noise generators and a power meter [13] as shown in figure 7. One of the noise generators (designated "hot") has a higher noise temperature, T_h, than the other (designated "cold"), T_c. The hot and cold noise generators provide known available powers to the transducer. The power meter measures the output power levels from the transducer. The ratio of the two output power levels corresponding to the two input powers

TABLE I. Summary of Characteristics of Measurement Techniques

Technique	Accuracy		Frequency Range	Instrumentation		Speed
	Best	Typical		Availability	Cost	
1. Y-Factor						
a. Power Meter Method	1%	2% to 10%	<10kHz to >60 GHz	Very good	Medium to high	Medium
b. Attenuation Method	1%	2% to 10%	<10kHz to >60 GHz	Very good	Medium to high	Medium
2. 3-dB						
a. Variable Source Method	5%	10% to 60%	1 MHz to 3 GHz	Good	Low to medium	Fast
b. Fixed Source Method	2%	5% to 25%	<10kHz to >60 GHz	Very good	Medium to high	Medium
3. Automatic	5%	7% to 25%	10 MHz to >60 GHz	Good	Medium	Fast
4. Gain Control						
a. Variable Source Method	5%	10% to 60%	1 MHz to 3 GHz	Good	Low to medium	Fast
b. Fixed Source Method	2%	5% to 40%	<10kHz to >60 GHz	Very good	Medium to high	Medium
5. CW	1%	5% to 25%	<1kHz to >100 GHz	Excellent	Low to medium	Slow
6. Tangential	~30%	30% to 200%	<1MHz to >10 GHz	Very good	Medium	Fast
7. Comparison	~25%	30% to 100%	<1kHz to >10 GHz	Very good	Low to medium	Fast

TABLE I (continued)

Convenience	Operator Skill Required	Source of Principal Error(s)	Special Problems*
Low to medium	Moderately high	Power Meter	None
Low to medium	Moderately high	Attenuator	None
Medium to high	Medium	TLD generator	Frequency range is limited by TLD generator
Low to medium	Moderately high	Attenuators	None
High	Medium	Hot noise generator	Instrumentation calibration
Medium to high	Medium	TLD generator	Frequency range is limited by TLD generator
Low to medium	Moderately high	Attenuators	None
Very Low	Medium	Measurement of noise bandwidth	Noise bandwidth measurement is tedious. Equipment drift with time.
High	High	Measurement of output S/N	Measurement is highly subjective
High	Medium	Many, about equal	Prototype transducer is required

*General problems are discussed in Section 5

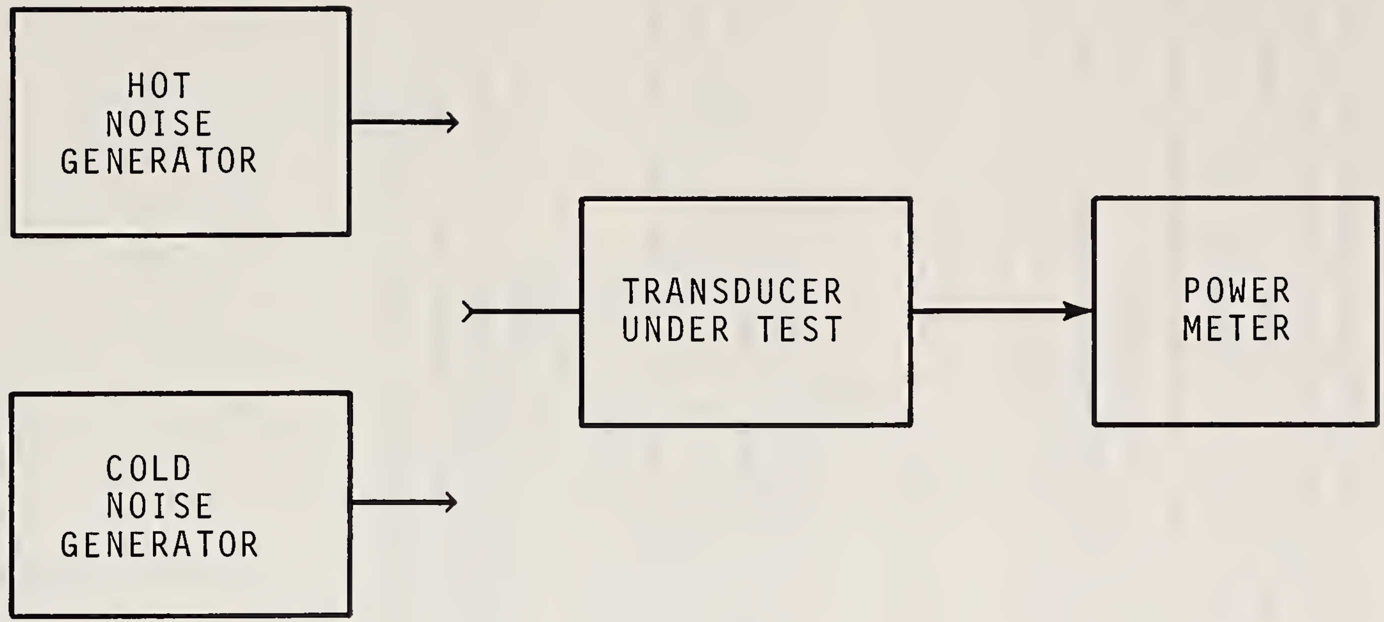


Figure 7. Y-Factor/Power Meter Method.

is the Y-factor. $\overline{T_e}$ and $\overline{F}$ are computed from the measured Y-factor and the known noise temperatures of the two noise sources.

Operational Characteristics

This method is capable of very high accuracy; measurement uncertainties as small as 1% (0.04 dB) are possible under best conditions, and typical uncertainties lie between 2% (0.1 dB) and 10% (0.4 dB). It is capable of high precision, particularly when the measurement system is automated [16]. For these reasons, this method often is chosen over other methods (except for the Y-factor/attenuator method) when high accuracy and precision are desired. It is used when an accurate power meter is available and an accurate variable attenuator is not. The principal measurement error in most typical situations comes from the power meter rather than from the noise sources.

The method is applicable over a very wide range of frequencies, e.g., from below 10 kHz to beyond 60 GHz.

Measurement apparatus for this method is commercially available for many, but not all, of the measurement situations encountered. When using precision noise generators, instrumentation tends to be expensive; this is also true if the measurement system is automated and if the highest accuracy is desired. Otherwise, instrumentation costs are modest.

When performed manually, the power meter method is not rapid, and is not as convenient as the ANFM technique for displaying the effects of adjustments on the transducer. However, it is straight-forward and comparatively trouble-free. It requires a moderately high level of operator skill.

Test Equipment Required

a. Hot noise generator
b. Cold noise generator
c. Power meter

Basic Procedure

1. Arrange the equipment as shown in figure 7.

2. Connect the Hot Noise Generator to the transducer input port.

3. Record the reading of the Power Meter, P_h.

4. Disconnect the Hot Noise Generator and connect the Cold Noise Generator to the transducer input port.

5. Record the reading of the Power Meter, P_c.

6. Compute the Y-factor from the equation

$$Y = \frac{P_h}{P_c} . \tag{11}$$

7. Compute $\overline{T_e}$ from the equation

$$\overline{T_e} = \frac{T_h - YT_c}{Y - 1} . \tag{12}$$

8. Compute $\overline{F}$ from the equation

$$\overline{F} = \frac{T_h - YT_c}{290(Y - 1)} + 1, \tag{13}$$

or from (6).

$$\overline{F} = \frac{\overline{T_e}}{290} + 1 \tag{6}$$

Also,

$$\overline{F}\ (\mathrm{dB}) = 10 \log_{10} \overline{F}. \tag{14}$$

3.1.2. Attenuator Method

Basic Method

This method is similar to the Power Meter method, except that it uses an accurate variable attenuator and a signal level indicator to measure Y [17]. See figure 8.

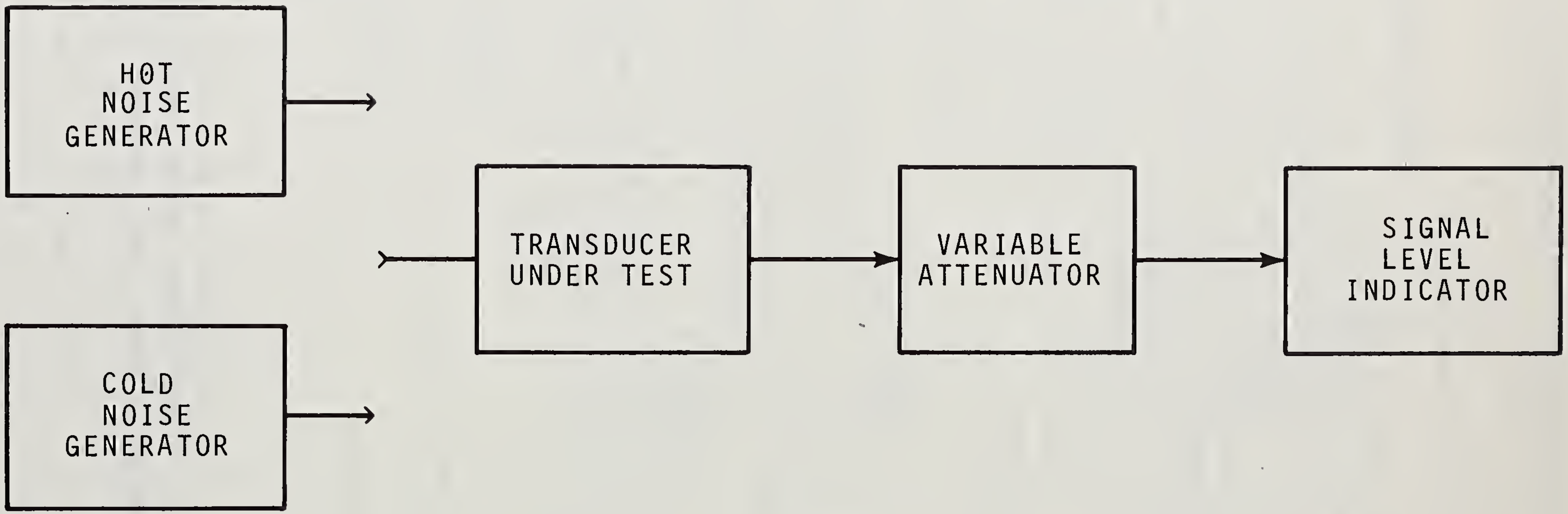


Figure 8. Y-Factor/Attenuator Method.

Operational Characteristics

The operational characteristics of this method are generally the same as for the Power Meter method. This method is used when an accurate variable attenuator is available and an accurate power meter is not. The principal measurement error comes typically from the attenuator error rather than from the noise sources or the signal level indicator.

Test Equipment Required

a. Hot noise generator
b. Cold noise generator
c. Variable attenuator
d. Signal level indicator

Basic Procedure

1. Arrange the apparatus as shown in figure 8.

2. Connect the Cold Noise Generator to the transducer input port.

3. Adjust the Variable Attenuator to produce an Indicator reading near full scale.

4. Record the Indicator reading I_c.

5. Record the Attenuator setting, A_c(dB), in decibels.

6. Disconnect the Cold Noise Generator and connect the Hot Noise Generator to the transducer input port.

7. Adjust the Attenuator to produce the same Indicator reading I_c.

8. Record the new Attenuator setting, A_h(dB), in decibels.

9. Compute the Y-factor in decibels from the equation

$$Y\ (\mathrm{dB}) = A_c\ (\mathrm{dB}) - A_h\ (\mathrm{dB}). \qquad (15)$$

10. Convert the Y-factor in decibels to Y-factor by the equation

$$Y = \text{antilog}_{10} \left[\frac{Y\ (\text{dB})}{10} \right] \quad . \tag{15}$$

11. Compute $\overline{T_e}$ from equation (12).

$$\overline{T_e} = \frac{T_h - YT_c}{Y - 1} \tag{12}$$

12. Compute $\overline{F}$ and $\overline{F}$(dB) from equations (6) or (13), and (14).

$$\overline{F} = \frac{\overline{T_e}}{290} + 1 \tag{6}$$

$$\overline{F} = \frac{T_h - YT_c}{290(Y - 1)} + 1 \tag{13}$$

$$\overline{F}\ (\text{dB}) = 10 \log_{10} \overline{F} \tag{14}$$

3.2. 3-dB Technique

This technique is similar to the Y-Factor/Attenuator Method except that an accurate fixed attenuator of 3 dB insertion loss is inserted in place of the variable attenuator, and the Y-factor is forced to have the fixed value of 2 (i.e., 3 dB). Because of this, the noise source must have a continuously adjustable output level.

Two methods are possible. The most common method uses a temperature-limited diode (TLD) of the thermionic vacuum type as a source of shot noise. Such a generator has an easily adjustable output level. A less common method uses a fixed-level noise generator and a variable attenuator to adjust its output level. The choice of method depends primarily upon the equipment available and the transducer frequency range.

3.2.1. Variable Source Method

Basic Method

This method uses a TLD noise generator, a 3 dB attenuator, and a signal level indicator [18] as shown in figure 9. The TLD noise generator supplies an adjustable but known input power to the transducer over a range of frequencies that normally exceeds that passed by the transducer. The signal level indicator is used to indicate a reference level of output power, which needs not be measured.

With the generator output control adjusted to produce zero shot noise (output is only residual thermal noise), and with the 3 dB attenuator out of the system, the transducer output power produces an indication on the signal level indicator. Then the 3 dB attenuator is inserted into the system, and the noise generator output is increased until the same indication is obtained. $\overline{F}$ is read directly from the calibrated emission current meter for the TLD generator, or is computed from the measured TLD emission current.

> Note: In some situations, inserting the 3 dB attenuator into the system may be difficult. An alternate scheme is to use a power meter instead of the 3 dB attenuator and signal level indicator, and adjust the noise generator for twice the initial output power level from the transducer.

Operating Characteristics

Typically, the accuracy of available TLD noise generators is not as good as for fixed level generators, and the practical accuracy of this method suffers. Measurement uncertainties as small as 5% (0.2 dB) are possible under best conditions, and typical uncertainties lie between 10% (0.4 dB) and 60% (2 db).

This method is commonly used over a moderately wide range of frequencies, e.g., from 1 MHz to 3 GHz, although in principle it can be used outside this range. The frequency range is limited primarily by the range of shot noise generators (see Section 4.1.2.3.).

Measurement apparatus is commercially available for use in the frequency range just cited, and is typically less expensive than apparatus required for other broad band methods (except the Gain Control/Variable Source Method).

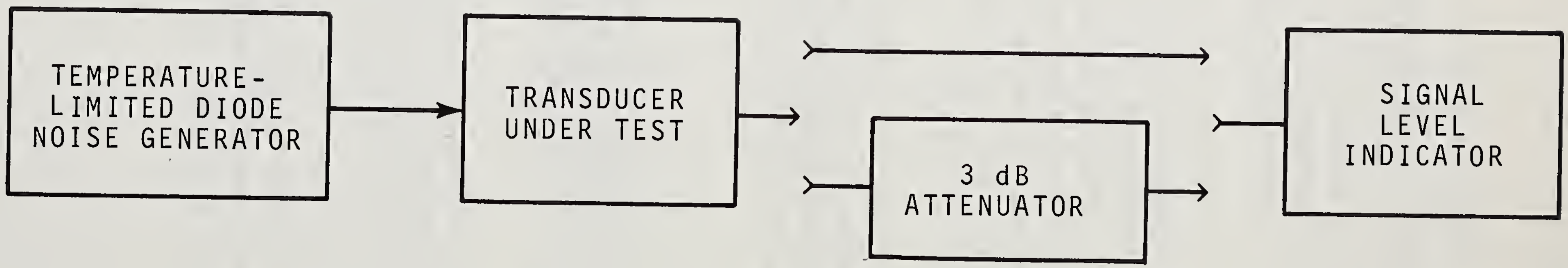


Figure 9. 3-dB/Variable Source Method.

This 3-dB method is somewhat faster to perform than most other methods, but not as convenient as the ANFM technique for displaying the effects of adjustments on the transducer. However, it is comparatively simple to perform and requires a moderate level of operator skill.

Test Equipment Required

a. Temperature-limited diode noise generator
b. 3 dB fixed attenuator
c. Signal level indicator

Basic Procedure

1. Arrange the apparatus as shown in figure 9.

2. Connect the output port of the transducer directly to the Signal Level Indicator.

3. Turn the emission current of the TLD Noise Generator to zero (leave the generator connected to the transducer under test).

4. Adjust the sensitivity of the Signal Level Indicator to produce a reading near full scale.

5. Record the Indicator reading, I.

6. Disconnect the transducer from the Signal Level Indicator.

7. Insert the 3 dB Attenuator between the transducer output port and the Signal Level Indicator.

8. Increase the emission current of the TLD Noise Generator to produce the Indicator reading, I.

9. Record the indicated value of $\overline{F}$(dB), in decibels, as read from the TLD Generator meter.

10. If the emission current meter is not calibrated in terms of $\overline{F}$(dB), compute $\overline{F}$ from the equation

$$\overline{F} = 20\ I_d R, \tag{17}$$

where

I_d = emission current, in amperes, obtained in Step 8,

R = TLD source resistance, in ohms.

11. Compute, if desired, the value of $\overline{F}$ from the equation

$$\overline{F} = \text{antilog}_{10}\left[\frac{\overline{F}\ (\text{dB})}{10}\right] . \tag{18}$$

12. Compute the value of $\overline{T_e}$ from (5).

$$\overline{T_e} = 290\ (\overline{F} - 1) \tag{5}$$

3.2.2. Fixed Source Method

Basic Method

This method uses a fixed-level noise generator, a variable attenuator, a 3 dB attenuator, and a signal level indicator [19] as shown in figure 10. The noise generator supplies a known input power to the variable attenuator whose attenuation, α, is known. Thus, the generator-attenuator pair serves the same function as the TLD noise generator described above. Except for the noise source, this method is closely similar to the Variable Source Method (see Section 3.2.1.). However, instead of $\overline{F}$, $\overline{T_e}$ is computed from the known values of noise temperature and α.

Operating Characteristics

Because high precision noise generators and attenuators can be used, this method is capable of very high accuracy; measurement uncertainties as small as 2% (0.1 dB) are possible under best conditions, and typical uncertainties lie between 5% (0.2 dB) and 25% (1 dB).

This method is applicable over a very wide range of frequencies, e.g., from below 10 kHz to beyond 60 GHz.

Measurement apparatus for this method is commercially available for many, but not all, of the measurement situations encountered. Instrumentation costs range from modest to high, depending primarily upon the desired measurement accuracy.

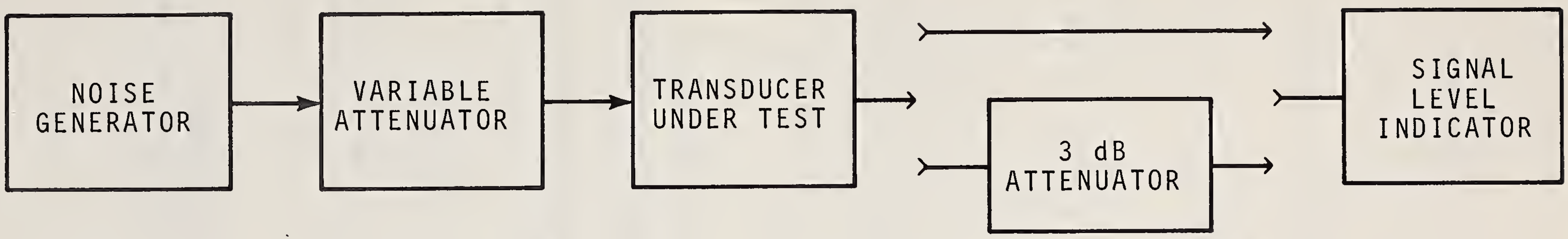


Figure 10. 3-dB/Fixed Source Method.

The method is moderately fast, but not as convenient as the ANFM Technique. It has little to offer over the Y-factor/Attenuator Method, except in special cases where the Y-factor must be fixed at 3 dB. It requires a moderately high level of operator skill, but is comparatively straight-forward and trouble-free.

Test Equipment Required

a. Random noise generator
b. Variable attenuator
c. 3 dB fixed attenuator
d. Signal level indicator

Basic Procedure

1. Arrange the apparatus as shown in figure 10.

2. Connect the output port of the transducer directly to the Signal Level Indicator.

3. Turn the Noise Generator OFF (leave the generator connected to the transducer under test).

4. Adjust the sensitivity of the Signal Level Indicator to produce a reading near full scale.

5. Record the Indicator reading, I.

6. Disconnect the transducer from the Signal Level Indicator.

7. Insert the 3 dB Attenuator between the transducer output port and the Signal Level Indicator.

8. Turn the Noise Generator ON and adjust the Variable Attenuator to again produce the Indicator reading, I.

9. Record the attenuation, A (dB), in decibels, of the Variable Attenuator.

10. Convert the attenuation, A (dB), in decibels, to transmittance, α, by the equation

$$\alpha = \text{antilog}_{10}\left[\frac{-A\ (\text{dB})}{10}\right] . \tag{19}$$

11. Compute $\overline{T_e}$ from the equation

$$\overline{T_e} = (T_n - T_a)\alpha - T_a, \tag{20}$$

where T_n is the noise temperature of the Noise Generator, and T_a is the temperature of both the generator source impedance and the Variable Attenuator.

12. Compute $\overline{F}$ and $\overline{F}$(dB) from equations (6) and (14).

$$\overline{F} = \frac{\overline{T_e}}{290} + 1 \tag{6}$$

$$\overline{F}(\text{dB}) = 10 \log_{10} \overline{F} \tag{14}$$

3.3. Automatic (ANFM) Technique

Basic Method

This technique uses an automatic noise figure measurement system [20], [21], [22], which includes a switched random noise generator and an automatic noise figure meter (ANFM) as shown in figure 11. The ANFM cyclicly switches the noise generator between two output power levels, and automatically computes $\overline{F}$ from the transducer output power. The measured value of $\overline{F}$(dB) is displayed by the ANFM's panel meter.

Operating Characteristics

This technique, as typically performed with commercial equipment, has medium accuracy because of difficulties in calibrating the switched noise generator and the analog circuitry in the ANFM. It is, however, in principle capable of high accuracy, but usually at the sacrifice of speed and convenience. Measurement uncertainties as small as 5% (0.2 dB) are possible under best conditions with commercial equipment, and typical uncertainties lie between 7% (0.3 dB) and 25% (1 dB).

Commercially available apparatus is available only for frequencies between 10 MHz and 40 GHz, although the principle of the method can be applied over the range from below 10 kHz to over 60 GHz. Apparatus is modest in cost and not difficult to use.

The method is fast, and is probably the most convenient of all methods of medium to high accuracy for quickly displaying the effects of adjustments on the transducer under test. It requires only a moderate level of operator skill. Regular calibration of the automatic display apparatus may be necessary for best results.

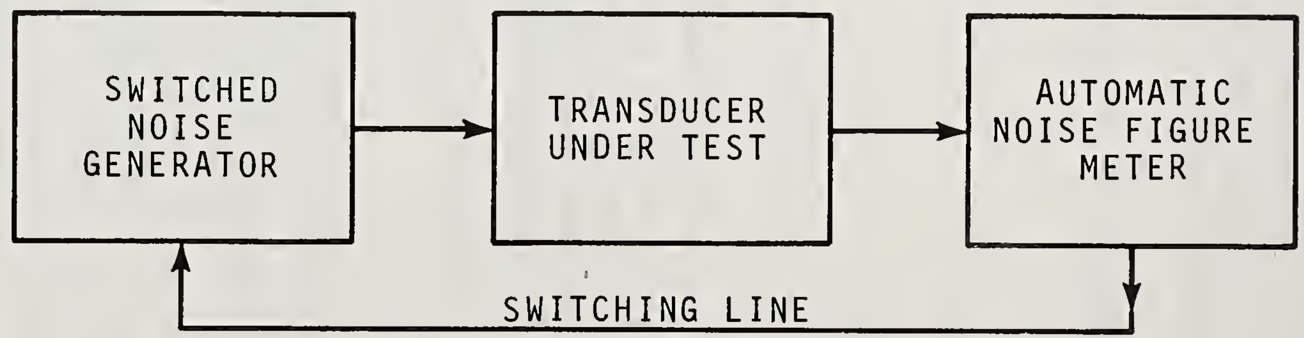


Figure 11. Automatic (ANFM) Technique.

Test Equipment Required

a. Automatic noise figure meter (ANFM)
b. Noise generator for use with the ANFM

Basic Procedure

1. Arrange the apparatus as shown in figure 11.

2. Adjust the Noise Generator current and other controls according to manufacturer's instructions.

3. Set Operate switch to read Noise Figure on panel meter.

4. Record the indicated value of $\overline{F}$(dB).

5. Compute, if desired, the value of $\overline{F}$ from equation (18).

$$\overline{F} = \text{antilog}_{10} \left[\frac{\overline{F}(\text{dB})}{10} \right] \tag{18}$$

6. Compute the value of $\overline{T_e}$ from equation (5).

$$\overline{T_e} = 290\ (\overline{F} - 1) \tag{5}$$

3.4. Gain Control Technique

The gain control technique [23] uses a random noise generator having an adjustable output level, such as is used in the 3-dB technique. It requires that the gain of the transducer be adjustable over a limited range, and this gain change must not significantly change the $\overline{F}$ or $\overline{T_e}$ of the transducer.

The gain control technique is useful in cases where the transducer tends to saturate or clip in its later stages. If the transducer includes either a linear or a power detector, this detector can often be used in the measurement system.

As with the 3-dB technique, two methods are possible; one uses a temperature-limited diode noise generator and the other uses a fixed-level noise generator and a variable attenuator. The choice of method depends primarily upon the equipment availability and the transducer frequency range.

3.4.1. Variable Source Method

Basic Method

This method uses a TLD noise generator and a signal level indicator as shown in figure 12. The TLD noise generator supplies an adjustable but known input power to the transducer over a range of frequencies that normally exceeds that passed by the transducer. The signal level indicator is used to indicate two different reference levels of output power, neither of which needs to be known.

With the generator output control adjusted to produce zero shot noise (output is only residual thermal noise), and with maximum transducer gain, the transducer output power produces an indication, I_1, on the signal level indicator. Then the noise generator output is increased to produce a larger indication, I_2. The transducer gain is reduced to again produce I_1, and the noise generator output is increased to again produce I_2. $\overline{F}$ is computed from the measured values of TLD emission current by means of (21) which is given below.

Operating Characteristics

Typically, the accuracy of available TLD noise generators is not as good as for fixed level generators, and the practical accuracy of this method suffers. Measurement uncertainties as small as 5% (0.2 dB) are possible under best conditions, and typical uncertainties lie between 10% (0.4 dB) and 60% (2 dB). dB).

The method is commonly used over a moderately wide range of frequencies, e.g., from 1 MHz to 3 GHz, although in principle it can be used outside this range. The frequency range is limited primarily by the range of shot noise generators (see Section 4.1.2.3.).

Measurement apparatus is commercially available for use in the frequency range just cited, and is typically less expensive than apparatus required for other broad band methods (except the 3-dB/Variable Source method).

This method is moderately fast, but not as convenient as the ANFM technique. It requires a moderate level of operator skill, and is comparatively straightforward to perform.

Test Equipment Required

a. Temperature-limited diode noise generator
b. Signal level indicator

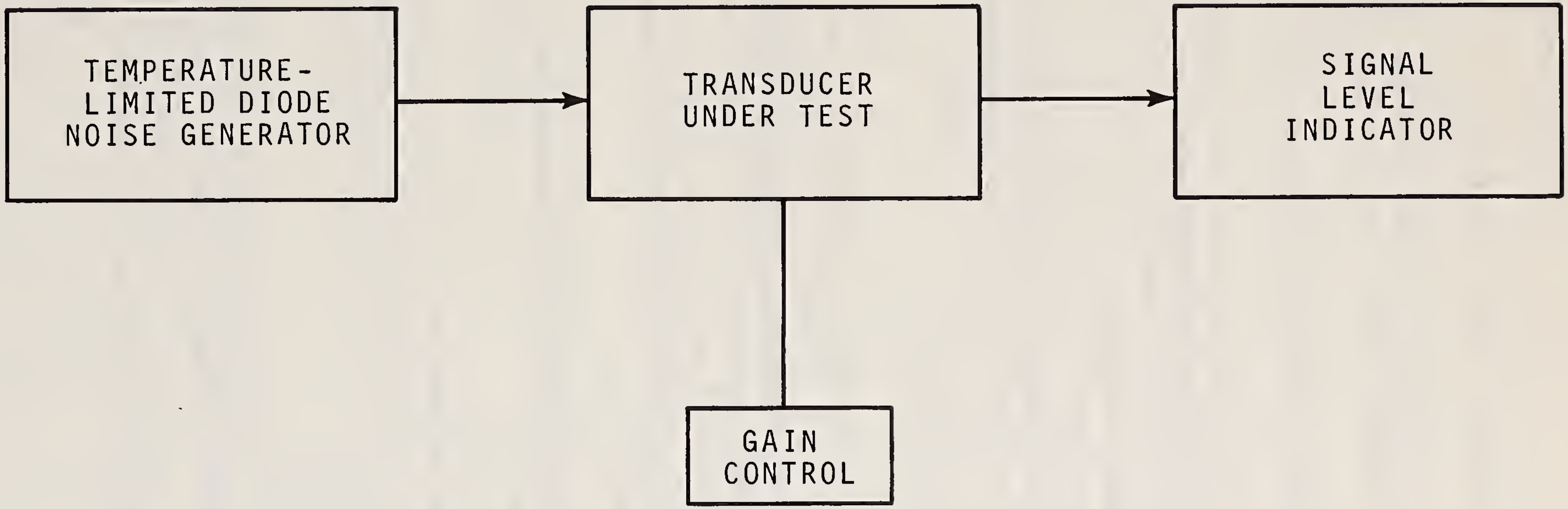


Figure 12. Gain Control/Variable Source Method.

Basic Procedure

1. Arrange the apparatus as shown in figure 12.

2. Turn the emission current of the TLD Noise Generator to zero (leave the generator connected to the transducer under test).

3. Adjust the transducer gain to its maximum value.

4. Adjust the sensitivity of the Signal Level Indicator to produce a reading at least 10 dB above its own noise level.

5. Record the Indicator reading I_1.

6. Increase the emission current of the TLD Noise Generator to produce a new Indicator reading I_2 that is 3 dB or more above I_2.

7. Record the Indicator reading I_2.

8. Record the emission current, I_{d1}, in amperes, that produces I_2.

9. Reduce the gain of the transducer to produce the Indicator reading I_1 with emission current I_{d1}.

10. Increase the emission current of the TLD Noise Generator to again produce the Indicator reading I_2.

11. Record the emission current, I_{d2}, in amperes, that produces I_2 with reduced transducer gain.

12. Compute $\overline{F}$ from the equation

$$\overline{F} = \frac{20R(I_{d1})^2}{I_{d2}-2I_{d1}}, \tag{21}$$

where R = TLD source resistance, in ohms.

13. Compute $\overline{F}$(dB) and $\overline{T_e}$ from equations (14) and (5).

$$\overline{F}(\mathrm{dB}) = 10 \log_{10} \overline{F} \tag{14}$$

$$\overline{T_e} = 290(\overline{F} - 1) \tag{5}$$

3.4.2. Fixed Source Method

Basic Method

This method uses a fixed-level noise generator, a variable attenuator, and a signal level indicator as shown in figure 13. The noise generator supplies a known input power to the variable attenuator whose attenuation, α, is known. Thus the generator-attenuator pair serves the same function as the TLD noise generator described above. Except for the noise source, this method is closely similar to the Variable Source Method (see Section 3.4.1.). However, instead of $\overline{F}$, $\overline{T_e}$ is computed from the known values of noise temperature and α.

Operating Characteristics

Because high precision generators and attenuators can be used, this method is capable of very high accuracy. However, since changing the transducer gain may change some of the other transducer properties, this accuracy potential is not always obtained. Measurement uncertainties as small as 2% (0.1 dB) are possible under best conditions, and typical uncertainties lie between 5% (0.2 dB) and 40% (1.5 dB).

This method is applicable over a very wide range of frequencies, e.g., from below 10 kHz to beyond 60 GHz.

Measurement apparatus for this method is commercially available for many, but not all, of the measurement situations encountered. Instrumentation costs range from modest to high, depending primarily upon the desired measurement accuracy.

The method is moderately fast, but not as convenient as the ANFM technique. It requires a moderately high level of operator skill, but is comparatively straight-forward and trouble-free.

Test Equipment Required

a. Random noise generator

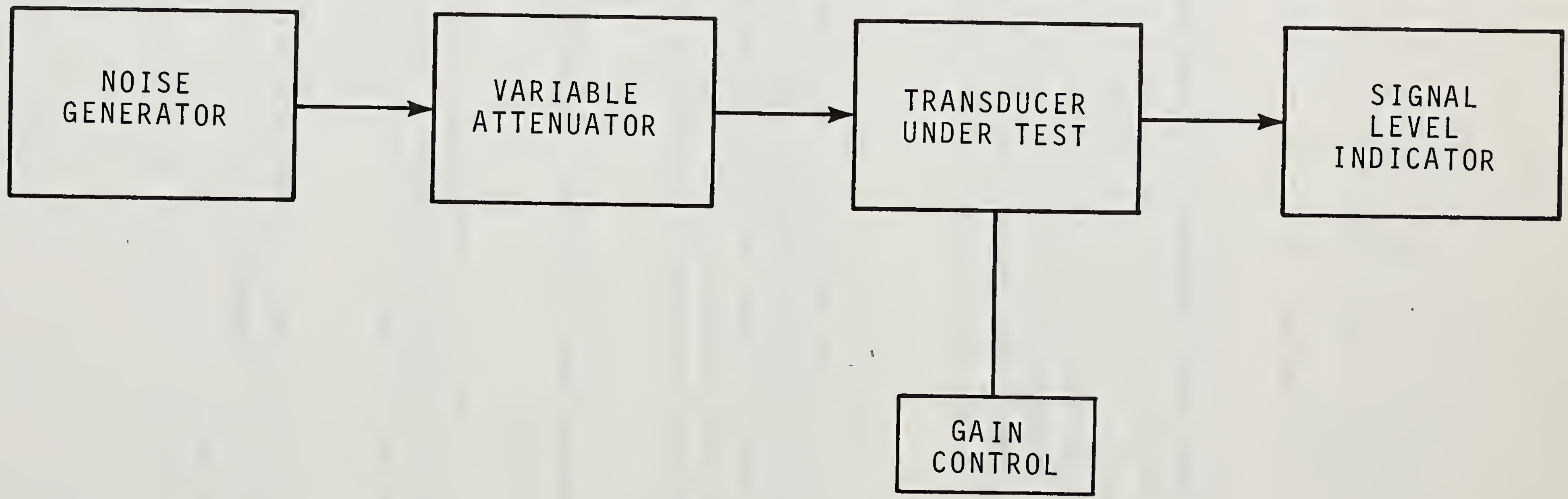


Figure 13. Gain Control/Fixed Source Method.

b. Variable attenuator

c. Signal level indicator

Basic Procedure

1. Arrange the apparatus as shown in figure 13.

2. Turn the Noise Generator OFF (leave the generator connected to the transducer under test).

3. Adjust the transducer gain to its maximum value.

4. Adjust the sensitivity of the Signal Level Indicator to produce a reading at least 10 dB above its own noise level.

5. Record the Indicator reading, I_1.

6. Turn the Noise Generator ON and adjust the Variable Attenuator to produce a new Indicator reading I_2 that is from 3 dB to 6 dB above I_1.

7. Record the Indicator reading I_2.

8. Record the attenuation, A_1 (dB), in decibels, of the Variable Attenuator, that produces I_2.

9. Reduce the gain of the transducer to again produce the Indicator reading I_1 with the Attenuator set to A_1 (dB).

10. Decrease the attenuation of the Variable Attenuator to again produce the Indicator reading I_2.

11. Record the attenuation, A_2 (dB), in decibels, that produces I_2 with the reduced transducer gain.

12. Convert the attenuations, A_1 (dB) and A_2 (dB), in decibels, to attenuations, α_1 and α_2, by the equations

$$\alpha_1 = \text{antilog}_{10} \left[\frac{-A_1 \text{ (dB)}}{10} \right] \tag{22}$$

$$\alpha_2 = \text{antilog}_{10}\left[\frac{-A_1\ (\text{dB})}{10}\right]. \tag{23}$$

13. Compute $\overline{T_e}$ from the equation

$$\overline{T_e} = (T_n - T_a)\frac{\alpha_1{}^2}{\alpha_2 - 2\alpha_1} - T_a, \tag{24}$$

where T_n is the noise temperature of the Noise Generator, and T_a is the temperature of both the Generator source impedance and the Variable Attenuator.

14. Compute $\overline{F}$ and $\overline{F}$ (dB) from equations (6) and (14).

$$\overline{F} = \frac{\overline{T_e}}{290} + 1 \tag{6}$$

$$\overline{F}(\text{dB}) = 10\ \log_{10}\ \overline{F} \tag{14}$$

3.5. CW Technique

The CW technique of measuring $\overline{F}$ is a narrow band measurement technique that uses an adjustable frequency sine-wave signal generator [15]. Thus it has application when a dispersed-signal (noise) generator is unavailable.

Basic Method

This technique uses a CW signal generator and a power meter as shown in figure 14. A frequency meter is also required if the generator's frequency calibration is inadequate. The CW generator supplies a known input power, P_s, to the transducer at the measurement frequency, f_o. The power meter measures the transducer output power under two conditions: (a) with the CW generator OFF, in which case the output power P_1, is entirely noise, and (b) with the CW generator ON, in which case the output power, P_2, is a mixture of noise and CW. The signal generator and power meter are also used to measure the noise bandwidth of the transducer (see Appendix C). $\overline{F}$ is computed from the known generator output, the measured output powers from the transducer, and the measured noise bandwidth.

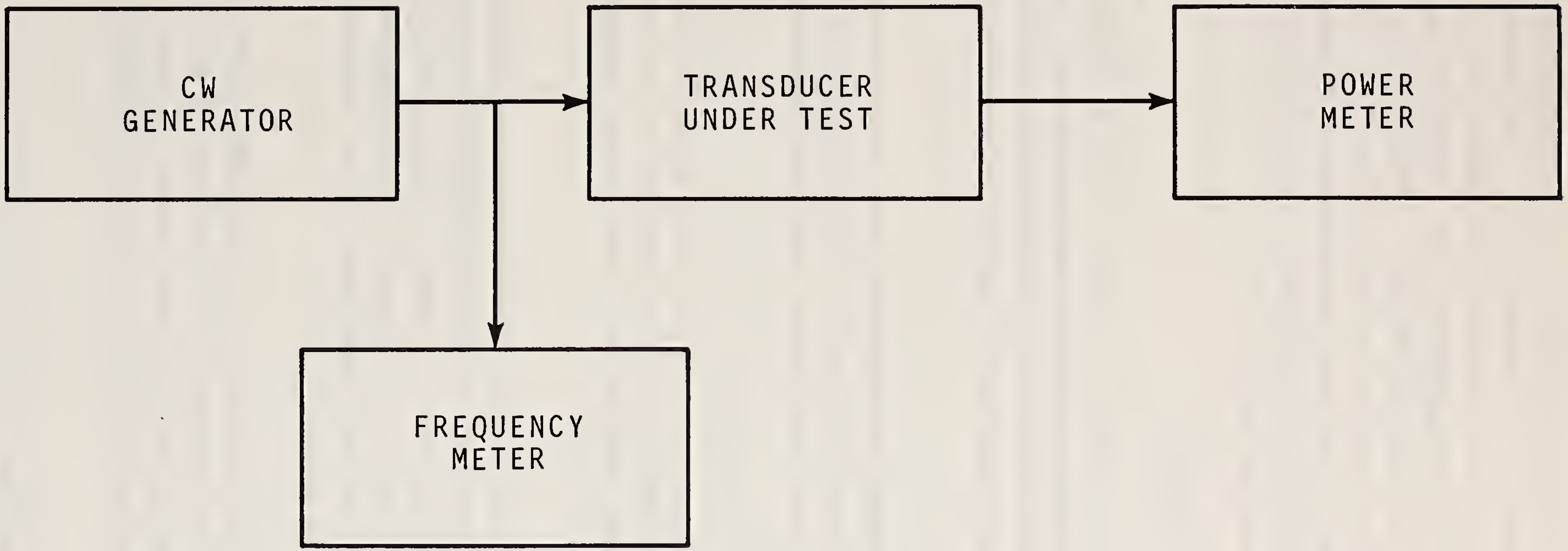


Figure 14. CW Technique.

Note: For convenience in practice, P_s is often adjusted to make $P_2 = 2P_1$. This results in a second "3-dB" technique that is similar in concept to that described in Section 3.2.1. However, note that in the CW 3-dB technique, the transducer output power, P_2, is a mixture of noise and CW power. Therefore care must be taken that the meter used to indicate the output level is a true power detector and is not one that responds differently to noise and CW power. Otherwise, an indicated 3 dB change in output level may be in error (see Section 4.3.1.).

Operating Characteristics

In principle, this technique is capable of high accuracy. However, primarily because of uncertainties in measuring noise bandwidth, the potential accuracy is not always obtained. Measurement uncertainties in $\overline{F}$ as small as 1% (0.04 dB) are possible under best conditions, but typical uncertainties lie between 5% (0.2 dB) and 25% (1 dB).

This technique is applicable over an extremely wide range of frequencies, e.g., from near zero hertz to over 300 GHz. It probably finds its greatest application below 10 MHz, where other techniques are less applicable for one reason or another.

Measurement apparatus for this technique is commercially available for virtually every measurement situation encountered. No special instruments are needed beyond those required for other common measurements such as gain, bandwidth, sensitivity, etc.

The CW technique is somewhat tedious and time-consuming, especially if high accuracy is desired and if ordinary laboratory equipment is used. However, when automated generators and detectors are available, relatively rapid measurements are possible. Even so, its speed does not approach that of other techniques with presently available equipment. The technique requires a moderately high level of operator skill.

Because of the time required by the technique, drifts in both the transducer and the measurement apparatus can be a problem. Other troublesome problems include (a) CW signal purity, (b) power meter response to the noise/CW mixture, and (c) properly accounting for the multiple responses of the transducer (see Sections 4 and 5).

Test Equipment Required

a. CW signal generator

b. Power Meter

c. Frequency Meter

Basic Procedure

1. Arrange the equipment as shown in figure 14.

2. Adjust the CW Generator output to zero (leave the generator connected to the transducer under test).

3. Record the transducer output power, P_1.

4. Adjust the CW Generator output to a level that the transducer can amplify linearily, and which produces at least a 20 dB greater output power level than P_1 when the frequency of the CW signal is tuned to the measurement frequency, f_o.

5. Record the available power level, P_s, from the Signal Generator.

6. Record the transducer output power, P_2.

7. Determine the noise bandwidth, B, of the transducer by standard means (see Appendix C).

8. Compute $\overline{F}$ from the equation

$$\overline{F} = \frac{P_s}{4\text{x}10^{-21}\ B\left(\frac{P_2}{P_1} - 1\right)} . \qquad (25)$$

9. Compute $\overline{F}$(dB) and $\overline{T_e}$ from equations (14) and (5).

$$\overline{F}(\text{dB}) = 10\ \log_{10} \overline{F} \qquad (14)$$

$$\overline{T_e} = 290\ (\overline{F} - 1) \qquad (5)$$

3.6. Tangential Technique

The tangential technique is a visual estimation technique that uses an oscilloscope to display information about the transducer noise [24]. It is a narrow band technique in that it uses a CW signal generator, but it may also be classified a broad band technique in that the CW signal generator is 100% modulated with a square-wave signal, thus producing sidebands that extend over a wide range of frequencies.

Basic Method

This technique uses a pulsed signal generator and an oscilloscope as shown in figure 15. If the transducer under test does not contain a suitable detector (see Section 5.10), an external detector must be supplied. The frequency and power meters are used with the signal generator to measure the transducer's noise bandwidth, B (see Section 2.3). When pulsed ON, the signal generator supplies a known CW power, P_s, to the transducer at the measurement frequency, f_o. When pulsed OFF, the transducer input is thermal noise power. The oscilloscope displays the detected output from the transducer as a square-wave modulated noise signal. When P_s is adjusted to produce a "tangential pattern," as shown in figure 16, a known output signal-to-noise ratio, $(S/N)_o$ is assumed. This has been emperically found to be approximately 11 dB for linear detectors and approximately 8 dB for square-law detectors. $\overline{F}$ is computed from the known value of P_s, the measured value of B, and the assumed value of $(S/N)_o$.

Operating Characteristics

The tangential technique is a low-accuracy technique because of the subjective nature of visually interpreting what one sees on the oscilloscope screen. Measurement uncertainties are probably never smaller than 30% (1.2 dB) even under best conditions. The precision of measurement by a single, well-trained operator is typically 10% (0.4 dB) to 25% (1 dB), but the spread of measurement results among different well-trained operators is typically no smaller than 30% (1.2 dB).

The technique can be applied, in principal, over a very wide range of frequencies, but best results are obtained with wide-bandwidth systems. It is commonly used only with radar and similar equipment, although it applies to other VHF and UHF transducers as well.

This technique is rapid and simple, and uses commonly-available test equipment. The measurement data are displayed instantly and continuously

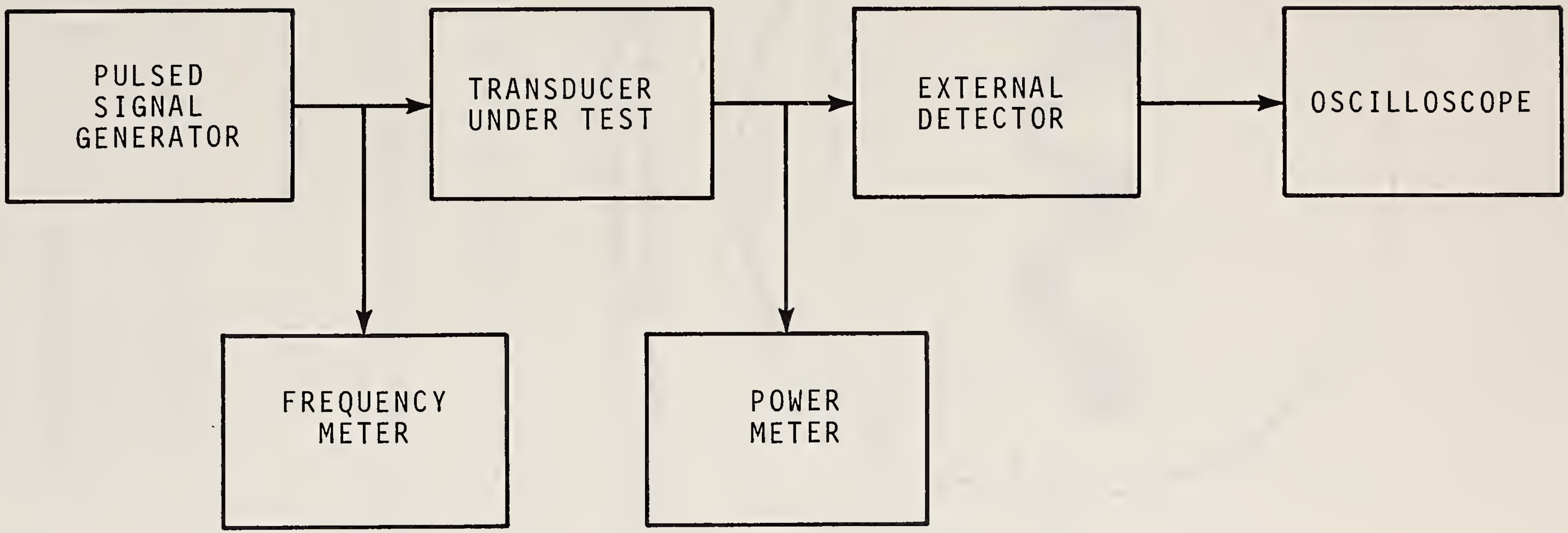


Figure 15. Tangential Technique.

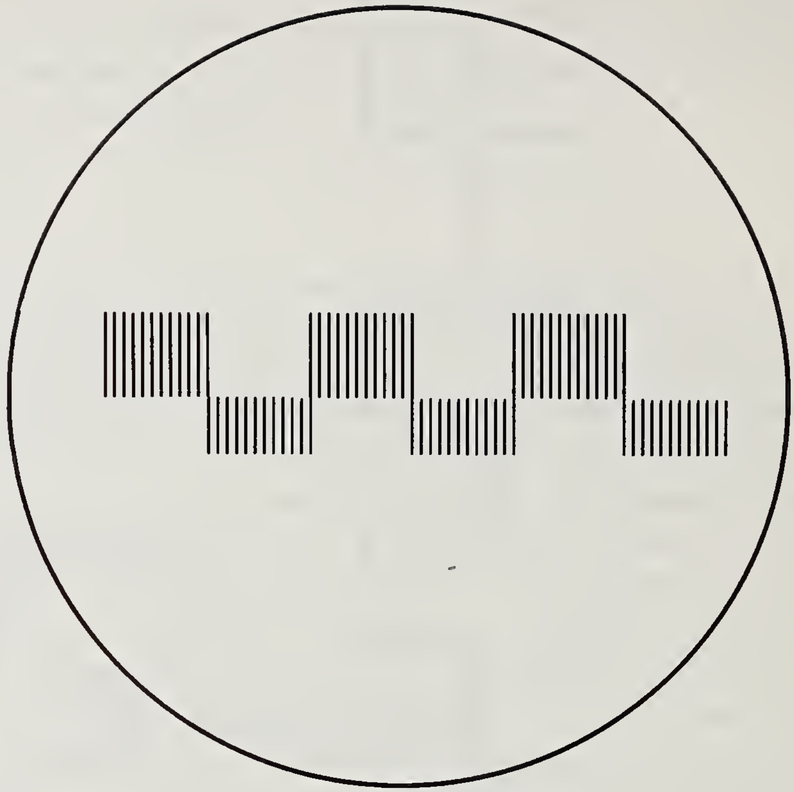


Figure 16. Oscilloscope Display of the "Tangential Pattern."

on the oscilloscope screen, an advantage when making adjustment on the transducer. The technique requires a high degree of operator skill to reliably interpret when the desired "tangential pattern" is obtained.

Test Equipment Required

a. Pulsed signal generator
b. Oscilloscope
c. External detector
d. Frequency meter
e. Power meter

Basic Procedure

1. Arrange the equipment as shown in figure 15. The Detector may be either a linear (envelope) or square-law (power) detector.

2. Adjust the output power of the Signal Generator to zero.

3. Adjust the gain of the Oscilloscope to produce a noise trace of 1/4 to 2 centimeters height.

4. Square-wave modulate the Signal Generator 100% with a modulation frequency between 1 kHz and 5 kHz.

5. Adjust the output level of the Signal Generator so that the upper segment of the trace is just completely above the bottom segment of the trace (see figure 16).

6. Determine the power, P_s, in the CW signal during the ON half-cycle of modulation.

7. Determine the noise bandwidth, B, of the transducer by standard means (see Appendix C).

8. Compute $\overline{F}$ by either equation (26) or (27).

$$\overline{F} = \frac{2P_s}{B} \times 10^{19} \text{ (linear detector)} \tag{26}$$

$$\overline{F} = \frac{4P_s}{B} \times 10^{19} \text{ (square-law detector)} \tag{27}$$

9. Compute $\overline{F}$(dB) and $\overline{T_e}$ from equations (14) and (5).

$$\overline{F}(\text{dB}) = 10 \log_{10} \overline{F} \tag{14}$$

$$\overline{T_e} = 290(\overline{F} - 1) \tag{5}$$

3.7. Comparison Technique

The comparison technique requires a "master" transducer of known noise factor against which transducers of like kind are compared [15]. It is normally classified a narrow band technique in that it uses a CW generator as a reference source.

Basic Method

This technique uses a CW signal generator, a master transducer, and a power meter as shown in figure 17. The master transducer is a prototype of the transducer under test, and its average noise factor, $\overline{F_m}$, is assumed to be known. The CW generator supplies an adjustable input power, P_s, to the transducer at the measurement frequency, f_o. The value of P_s need not be known. The power meter measures each transducer's output under two conditions: (a) with the CW generator OFF, in which case the output power, P_1, is entirely noise, and (b) with the CW generator ON, in which case the output power, P_2 is a mixture of noise and CW. $\overline{F_x}$ for the transducer under test is computed from the known value of $\overline{F_m}$ and the measured output powers P_{1m}, P_{2m}, P_{1x} and P_{2x}.

Note 1: Since the measured output powers are used in ratio to each other to compute $\overline{F_x}$, the ratios (P_{m2}/P_{m1}) and (P_{x2}/P_{x1}) may be determined directly by ratio techniques (e.g., with an attenuator) if desired.

Note 2: This technique may also be performed with an amplitude modulated signal generator, in which case P_m and P_x are the audio or video powers of the demodulated signals.

Operating Characteristics

The comparison technique is typically a low-accuracy technique because of the inherent differences between the master transducer and the transducer under test. Measurement uncertainties are probably never smaller than 25% (1 dB), and typically lie between 30% (1.2 dB) and 100% (3 dB).

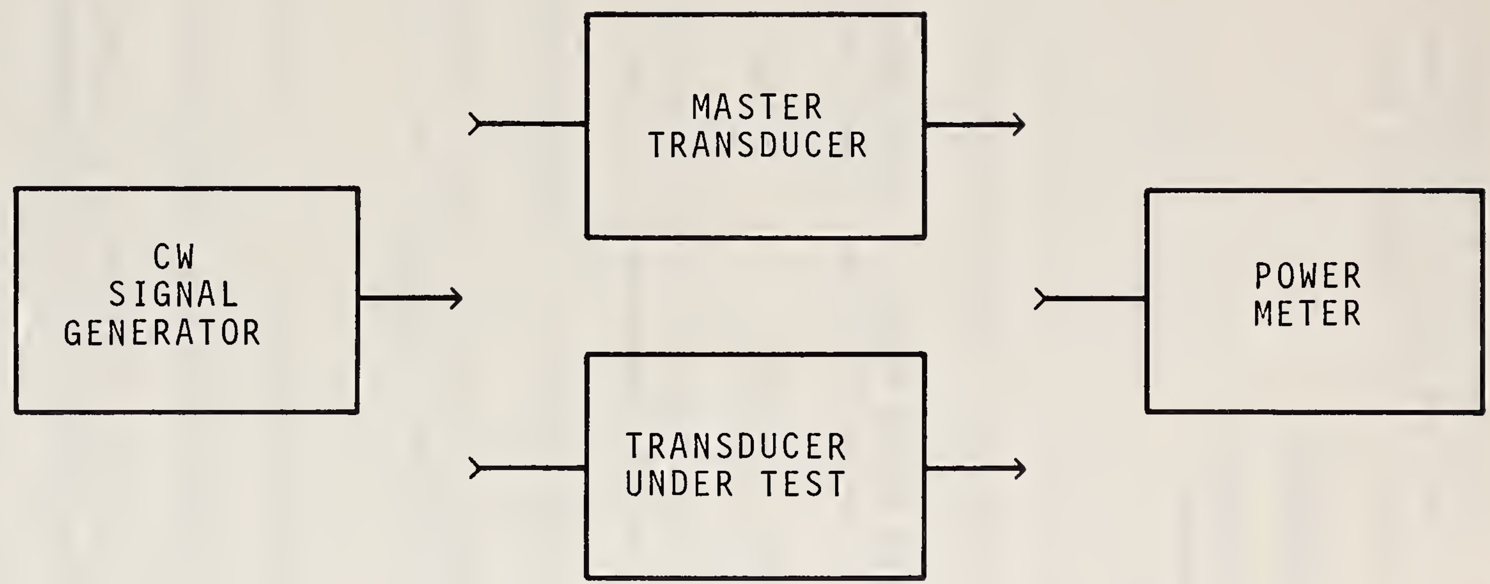


Figure 17. Comparison Technique.

This technique is applicable over a very wide range of frequencies and noise factor. It is rapid and simple, and requires no special equipment. It requires only a moderate level of operator skill.

Test Equipment Required

a. CW signal generator
b. Master transducer
c. Power meter

Basic Procedure

1. Arrange the equipment as shown in figure 17.

2. Connect the CW Signal Generator to the Master Transducer's input port, and the Master Transducer's output port to the Power Meter.

3. Tune the Signal Generator to the measurement frequency, f_o.

4. Adjust the output level of the Signal Generator to zero (leave the Signal Generator connected to the Master Transducer).

5. Measure the power output, P_{m1}, from the Master Transducer.

6. Adjust the output level of the Signal Generator to a value that produces a transducer output power that is approximately 20 dB above P_{m1}.

7. Record the output power, P_s, of the Signal Generator.

8. Measure the power output, P_{m2}, from the Master Transducer.

9. Disconnect the Signal Generator and the Power Meter from the Master Transducer, and connect them to the Transducer Under Test.

10. Repeat steps 4 and 5 to obtain P_{x1}.

11. Repeat steps 6, 7, and 8 to obtain P_{x2}. Be certain the output power from the signal generator is P_s, as in step 6.

12. Compute $\overline{F_x}$ for the Transducer Under Test, in terms $\overline{F_m}$ for the Master Transducer, by the equation

$$\overline{F_x} = \overline{F_m} \; \frac{(P_{m2} - P_{m1})\; P_{x1}}{(P_{x2} - P_{x1})\; P_{m1}} \, , \tag{28}$$

or

$$\overline{F_x} \doteq \overline{F_m} \left(\frac{P_{m2}}{P_{m1}}\right) \left(\frac{P_{x1}}{P_{x2}}\right) . \tag{29}$$

13. Compute $\overline{F_x}$ (dB) and $\overline{T_{ex}}$ from equations (14) and (5).

$$\overline{F_x}\ (\mathrm{dB}) = 10 \log_{10} \overline{F_x} \tag{14}$$

$$\overline{T_{ex}} = 290(\overline{F_x} - 1) \tag{5}$$

4. INSTRUMENTATION

In this section we will discuss the important characteristics of the instruments that are required by the various measurement methods. The availability or cost of such instruments often determine which method is used.

4.1. Noise Generators

Random noise power is generated by a variety of means, but the three basic types of generators that are commonly used for these measurements are thermal, shot, and plasma (gas discharge) sources. Each of these sources uses a different fundamental physical phenomenon, and their operational characteristics are different. However, the random signals that are produced by each are all nearly gaussian and therefore equivalent for the purposes of this measurement, at least in the frequency ranges of current engineering use. A brief summary of commercial noise generator characteristics is given in Table II.

4.1.1. General Features

4.1.1.1. Thermal Noise Generators

Operationally, a thermal noise generator uses a resistive termination, in either hollow [25] or coaxial [26] waveguide, at a controlled temperature, T, to produce Johnson noise [27] according to the relationship

$$S(f) = hf\ [\exp\left(\frac{hf}{kT}\right) - 1]^{-1} \tag{30}$$

where

$S(f)$ = available noise power per unit bandwidth centered at frequency f

h = Planck's constant, 6.626×10^{-34} joule-second

k = Boltzmann's constant, 1.381×10^{-23} joule per kelvin.

For frequency-to-temperature ratios, f/T, of less then 10^9 hertz per kelvin, S(f) is approximated by the equation

$$S(f) \doteq kT\ . \tag{31}$$

The noise power, P_a, available in a frequency interval (f_1, f_2) is therefore

TABLE II. Summary of Noise Generator Characteristics

Type	Typical Accuracy	Noise Temperature	Frequency Range	Waveguide	Cost
1. Thermal	1% to 3%	77k, 300K, 373K (fixed)	0 to 9 GHz	Coaxial and Rectangular	Medium to High
2. Shot					
a. Thermionic	5% to 15%	300K to 29,000K (variable)	100kHz to 3 GHz	Coaxial	Low
b. Solid State	5% to 10%	1,000K to 600,000K (fixed)	100kHz to 35 GHz	Coaxial and Rectangular	Medium
3. Plasma	3% to 6%	10,000K, 18,000K (fixed)	200MHz to 75 GHz	Coaxial and Rectangular	Medium

$$P_a = \int_{f_1}^{f_2} S(f)\, df \tag{32}$$

or

$$P_a \doteq kTB, \tag{33}$$

where

$$B = \int_{f_1}^{f_2} df = f_2 - f_1 \tag{34}$$

if S(f) is constant over the interval (f_1, f_2).

The noise temperature, T_n, of a thermal noise generator differs from the thermodynamic temperature, T, of the termination because of losses in the waveguide that conveys the noise power from the termination to the output port of the generator. A first-order approximation of this effect is given by the equation

$$T_n = \alpha T + (1-\alpha) T_a \tag{35}$$

where

α = power efficiency of the waveguide

and

T_a = temperature of the waveguide.

The value of T_n is obtained either by calibration [28], [29] or by calculation from noise theory, using the physical parameters (temperature, resistivity, linear dimensions, etc.) of the generator apparatus.

4.1.1.2. Shot Noise Generators

Operationally, a shot noise generator consists of a resistor, R, through which current from a shot noise source is passed. The shot noise source is typically a biased diode, either thermionic [30] or semiconductor [31], through which electrons flow in a random manner [32].

In a thermionic diode operating in the temperature-limited mode, the mean-square fluctuations, $\overline{i^2}$, in the average (d.c.) emission current, I, is given by the equation

$$\overline{i^2} = 2\,\varepsilon\,I \int_0^\infty \Phi(f)\,df \qquad (36)$$

where

ε = electron charge, 1.602×10^{-19} coulomb

and $\phi(f)$ = correction factor.

The correction factor $\phi(f)$ accounts for the effects of transit time, operating frequency, cathode temperature, and cathode-anode potential gradient. It typically has a value near unity at low frequencies, and decreases towards zero as frequency increases (see figure 18).

The available power per unit bandwidth, S(f), from a temperature-limited diode (TLD) generator is given by the equation

$$S(f) = \tfrac{1}{2}\varepsilon IR\phi + k\,T_a \qquad (37)$$

where T_a is the thermodynamic temperature of R. The first term on the right-hand side of equation (37) is shot noise, and the second term is thermal noise from R. From the definition of noise temperature, T_n, and (37),

$$T_n = \frac{\varepsilon IR\phi}{2k} + T_a \qquad (38)$$

for a TLD noise generator.

The TLD noise generator can be made very stable by stabilizing I [33]. Also it is particularly easy to vary T_n by adjusting the value of I. However, because of the great difficulty of evaluating $\phi(f)$ from the physical parameters of the generator circuit [34], the value of T_n must be obtained by calibration at frequencies where $\phi(f)$ deviates significantly from unity. Depending upon the accuracy desired, this may occur as low as 100 MHz. Special thermionic diodes have been designed to provide useful output up to 3 GHz.

At low operating frequencies, (e.g., $\leq 10^4$ hertz), a TLD generator produces flicker noise in addition to shot and thermal noise. The theory of flicker or (1/f) noise is not yet well established, so no equations to account for this effect are given here. For this reason, TLD generators are not normally used below 100 kilohertz.

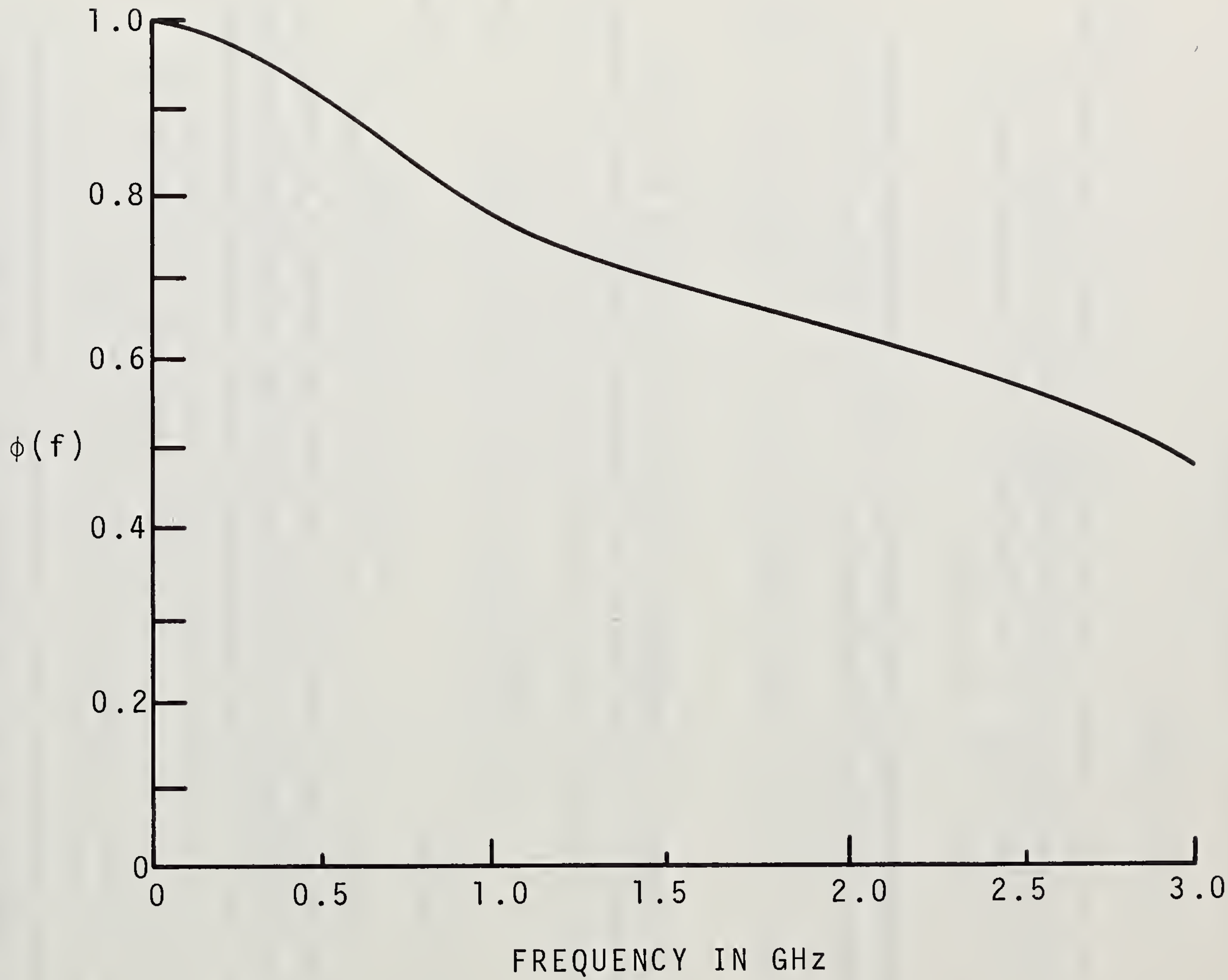


Figure 18. Variation of Correction Factor, ϕ, With Frequency For a Plane-Parallel Diode.

As indicated by equation (38), the smallest noise temperature from a TLD generator is T_a (I = 0), which is typically between 300 and 310 kelvins. Common thermionic diodes generate shot noise power that is approximately 16 to 20 dB above this level (in 50-ohm systems); i.e., T_n (due to both shot and thermal noise) reaches approximately 12,000 to 29,300 kelvins. The latter level is in excess of that produced by either thermal or plasma (gas discharge) sources.

The theory of shot noise in a semiconductor diode is still in the process of development [35], and no equations for this source are given here. The most common source of this noise is from microplasmas that occur when current is passed through the diode operating in the avalanche mode. Random fluctuations of this current are passed through a resistor as in the TLD noise generator to produce a random noise voltage. Semiconductor diodes generate additional noise by other mechanisms (thermal, recombination, etc.) which adds to the shot noise power available from these devices.

Because of the low junction voltages involved, a large average current can be passed through the semiconductor diode with the result that a large value of T_n can be produced. Values between 10^5 and 10^7 kelvins have been reported [36]. However, because of the temperature sensitivity of the semiconductor junction, T_n may be unstable unless special means are used to stabilize this effect.

The transit time in a semiconductor diode is quite small with the result that the noise spectrum extends into the UHF and SHF ranges ($3\text{x}10^9$ to $3\text{x}10^{11}$Hz). However, the impedance of the mounted diode is usually quite variable with frequency so that the VSWR of the generator deviates from unity to a significant amount. To overcome this problem, common practice is to "pad" the output of the source to stabilize the VSWR. Since the noise temperature of the source is large to begin with, the loss of power due to padding presents no particular concern, and typical generators provide noise temperatures from 1000 to 30,000 kelvins with VSWR's of 1.02 or less.

Flicker noise is also present in semiconductor noise sources, thus limiting the lower frequency at which they are used to approximately 100 kHz.

At the present state-of-the-art, it is not usual to vary T_n by varying the current through the semiconductor diode. T_n is determined by calibration at a specified current and frequency.

4.1.1.3. Plasma(Gas Discharge) Noise Generators

This type of noise generator uses the random fluctuations in the electromagnetic fields produced by the ionization of a noble gas confined in a suitable container. The configuration commonly used is a special form of the fluorescent lamp [37] filled with either neon or argon at pressures that are chosen according to the dimensions of the lamp tube. The fluorescent coating is, of course, omitted.

The noise temperature of a plasma noise generator is approximately the electron temperature in the ionized gas. However, a definite difference exists between these two temperatures, and attempts to calculate T_n from the electron temperature and other physical parameters of the source have been only partially successful [38]. Thus, T_n of a plasma generator must be obtained by calibration. Noise temperatures range between approximately 10,000 and 18,000 kelvins, depending upon the gas, pressure, and dimensional configuration. When operated from a stabilized excitation power supply, plasma sources are very stable and repeatable.

The noise generating mechanism for plasma noise is a very wideband phenomenon. However, the frequency range over which gas-discharge sources can be used depends principally upon the method used to couple the electromagnetic fluctuations into a waveguide system.

4.1.2. Important Characteristics

The important characteristics of noise sources for these measurement techniques include the following considerations:

a. Accuracy
b. Output level
c. Frequency range
d. Stability

4.1.2.1. Accuracy

The uncertainty in the generator's available noise power may cause a large measurement error, depending upon the transducer's $\overline{F}$ or $\overline{T_e}$. This can be seen by examining the uncertainty coefficients in the error equations given in Section 7, Measurement Errors. Therefore, for high accuracy results, it is necessary to use a noise source of certified and adequate accuracy. Accuracies for typical commercial noise generators range from 1% to 15% (see Table II). The effect of a given generator accuracy may be estimated by means of the equations given in Section 7. In some techniques the generator inaccuracy may be the principal source of measurement error (see Table I).

4.1.2.2. Output Level

The output power level available from the generator may affect the measurement accuracy, and may even determine whether or not a particular measurement technique can be used.

Generally speaking, best results are obtained with the Y-factor technique when the source noise temperatures bracket $\overline{T_e}$ according to the relation [25]

$$\overline{T_e} = \sqrt{T_c\, T_h}\ . \tag{39}$$

Thus the measurement system for a transducer having a large $\overline{T_e}$ should use a hot noise generator having a large T_h. Similarly, if $\overline{T_e}$ is small, T_c should be low. Otherwise, the measurement system may lack sufficient sensitivity for an accurate measurement. The same applies to the 3-dB and Gain Control Techniques using a fixed-level source. All three types of noise sources (thermal, shot, and plasma) are suitable for use in these methods. However, thermal noise sources are limited in their maximum noise power, and both shot and plasma noise sources are limited in their minimum power levels (see Table II). Some selection can be made, therefore, to obtain the best source temperature for a particular measurement situation.

The 3-dB and Gain Control variable source methods require noise generators having an adjustable output level. At the present time, only temperature-limited diode shot noise generators are generally available with this feature. Further, the output level must be sufficient for the value of $\overline{F}$ being measured. Commercial units are suitable for measuring $\overline{F}$(dB) up to 16 dB or 20 dB, depending upon the type of noise diode that is used.

The maximum $\overline{F}$ that can be measured by the automatic noise figure meter technique increases as the noise temperature of the higher temperature source increases. On the other hand, in some ANFM designs the resolution and accuracy for the lower values of $\overline{F}$ improve as this noise source temperature decreases. Therefore, when a choice is available, choose the proper noise source according to the value of $\overline{F}$ that is expected.

In any of the techniques using fixed-level noise generators, there will be an optimum pair of noise source temperatures for a given value of $\overline{F}$ or $\overline{T_e}$. Situations where the highest accuracy and precision are required may warrant the extra effort required to determine those temperatures, and to acquire sources that generate them accurately.

4.1.2.3. Frequency Range

For best measurement accuracy, the noise power should be constant with frequency over the ranges of frequencies passed by the transducer. All noise sources exhibit some variation in either their available or delivered power, and for high accuracy measurements this must be taken into account.

The frequency range of thermal noise sources is generally very wide. It is normally limited by the transmission system that connects the noise generating element to the generator output port. Corrections for this transmission effect are given by the manufacturer of the thermal noise generator. Although thermal noise changes with frequency due to the quantum effect according to (30), this effect is not great enough to be seen by present-day equipment.

Plasma noise sources are believed to have a constant frequency spectrum over a large range, but the method of mounting noise tubes in helixes or in hollow waveguide causes their frequency range to be restricted to a moderately narrow band. This problem is met by building sources for individual segments of the frequency spectrum, and such sources are commercially available for a wide range of frequencies. Accurate measurements do require corrections within a given band segment, however.

The upper frequency range of shot noise generators is significantly limited by the transit time effect in TLD noise sources (less so in solid state sources), and is further limited by the parasitic reactances in the noise generator circuit package. Therefore, exercise care in choosing a shot noise source for use at high frequencies.

A further precaution must be exercised when choosing a noise source for measurements at very low frequencies. Thermal noise sources are generally satisfactory, but some shot noise generators contain a large amount of flicker (1/f) noise. Select a generator that has been designed for use at low frequencies.

4.1.2.4. Stability

Noise sources must be stable for best measurement results. Thermal noise sources must be adequately designed to assure a constant noise temperature that is insensitive to normal variations in the measurement environment (temperature, atmospheric pressure, power supply fluctuations, etc.). Corrections may be required as indicated by the manufacturer. As an example, the noise temperature of a liquid nitrogen cooled source will fluctuate with changes in atmospheric pressure. This may amount to one or two kelvins for elevation changes between sea level and 5000 feet.

Shot and plasma noise sources can be stabilized by stabilizing the emission, junction, or discharge current. Some commercial sources incorporate this feature; others do not. Stabilization factors greater than 1000:1 are possible, and are adequate for most purposes. Thermal stabilization may also be required, particularly with solid state and plasma sources.

The noise power from plasma sources may not be stable under pulsed conditions. Specially designed noise tubes are available for this type of service.

Several measurement techniques require a shot or plasma noise source to supply thermal noise in an "OFF" condition. Such a source should be designed so that the thermal noise power is stable and predictable, as discussed in Section 5.2.

The source impedance (or reflection coefficient) of a source should be constant under conditions that require it to be pulsed or tuned on and off. This is particularly important for a transducer whose $\overline{F}$ or $\overline{T_e}$ is sensitive to its input termination impedance.

Finally, noise sources must be adequately shielded and filtered to prevent extraneous signals from entering the measurement system and altering the source's apparent noise temperature. See also Section 5.5.

4.2. CW Signal Generators

In noise performance measurements, CW signal generators are used for two purposes: (a) as a source of reference power, and (b) to determine noise bandwidth. The first application requires a known output level of CW power; therefore, the generator must either have a calibrated output indicator, or the output must be measured by some means. The second application requires the output level to be adjustable in known amounts; this also requires either a calibrated output indicator, or a calibrated attenuator. Laboratory CW signal generators usually have these features, and are generally applicable to the CW technique.

A further requirement is accurate frequency calibration. If the chosen generator does not provide sufficient accuracy, a precision frequency meter, either digital or cavity type, can be used to measure the signal frequency.

Adequate shielding and signal purity are essential. These should be checked by conventional means to be certain that harmonics or spurious signals do not degrade the measurement results.

The Tangential technique requires that the CW signal be 100% square-wave modulated. The duty factor of the square wave is not highly important; however, it should be near 50% and must be stable for best results. An adjustable modulation frequency is a convenience in the measurement, but the exact frequency is not so important as long as a crisp, stable display can be obtained on the oscilloscope.

4.3. Power Meters

Several of these techniques require a power meter to measure the noise and/or CW power delivered by the transducer. Because of the characteristics of noise signals, the power meter must meet the requirements given below.

4.3.1. True Power

The power meter must measure the true power in the signal regardless of waveform. This requires (a) a true power response, (b) a bandwidth sufficient to pass all frequency components delivered by the transducer, and (c) a dynamic range sufficient to respond to noise peaks without significant distortion. The bolometer-type power meter meets these requirements for most frequency ranges from below 10 MHz to over 60 GHz. Rectifier-type power meters, that are intended to measure CW power, are unsuitable for this application except when crude measurements of low accuracy are acceptable.

4.3.2. Accuracy

As shown in Section 8, the uncertainty in the power meter reading contributes a large amount of the total measurement uncertainty for certain techniques. Therefore, for high accuracy results, it is necessary to use a power meter of certified and adequate accuracy. Unless it has been calibrated recently, the meter's accuracy may be as poor as 10%, whereas a 2% to 5% accuracy is typical of calibrated meters.

4.3.3. Sensitivity

The power meter must be capable of giving a reliable reading of transducer output power for the lowest levels involved, e.g., when a cryogenically-cooled noise source is connected to a low-noise amplifier of low gain. If the power meter has perceptible internal noise, correction of the reading may be desirable depending upon the desired measurement accuracy. See Section 5.7 for methods of making this correction. Further, if the power meter sensitivity is insufficient to provide a reliable reading, an amplifier must be used between it and the transducer. In this case, the measurement results may be corrected (as explained in Section 5.7) for the noise contribution of the amplifier, when significant.

Sensitivities in the range of 0.01 to 10 milliwatts full scale are commonly suitable for these measurements.

4.3.4. Bandwidth

The CW bandwidth of the power meter should normally be greater than that of the transducer under test. However, if it is desired to make the measurement within a bandwidth that is smaller than that of the transducer, the power meter may be selected with this narrower bandwidth, or a filter may be used as described below in Section 4.7.1. In the former case, the frequency response of the power meter must be known in order to properly interpret the measurement results (see Section 5.7.4.).

The bandwidth of bolometer-type power meters is normally very wide, e.g., from 10 MHz to over 10 GHz, depending upon the bolometer mount. Other types, such as the calorimeter type, extend to d.c.

4.4. Signal Level Indicators

In several of these measurement techniques, an uncalibrated output level indicator may be used because a measurement of output power is not required. Suitable instruments for this purpose include radio receivers, broad band voltmeters, and other sensitive detectors of various types.

Inasmuch as the signal level indicator serves only to indicate a fixed level of output noise power, its detector law is not important. However, it must be stable for the time required to make a measurement, and it must have sufficient sensitivity and bandwidth. These requirements are generally the same as for power meters (see Sections 4.3.3. and 4.3.4.).

4.5. Attenuators

4.5.1. Precision Variable Attenuators

For use in the Y-factor/attenuation and the 3-dB and gain control fixed source methods, choose a variable attenuator with the following characteristics:

4.5.1.1. Accuracy

High attenuator accuracy is important because the attenuator error can be a large part of the total measurement error (see Section 8). Attenuator uncertainties of 0.5 dB to 2 dB per 10 dB are typical, and uncertainties as small as 0.1 dB per 10 dB are possible in highly stable, recently calibrated attenuators. The contribution of a given attenuator uncertainty to the overall measurement uncertainty are discussed in Sections 7 and 8.

4.5.1.2. Range

A change in attenuation of 20 dB is sufficient for the transducers normally encountered and the noise sources normally used. This can be seen from equation (40); the minimum change will occur when $\overline{T_e}$ is very large, and the maximum will occur when $\overline{T_e}$ is very small. In the latter case, Y approaches the ratio T_{hot}/T_{cold}, which is normally no greater than 100 (20dB).

$$Y = \frac{T_{hot} + \overline{T_e}}{T_{cold} + \overline{T_e}} \tag{40}$$

When measuring large values of Y, accuracy can be improved by using a fixed precision attenuator in series with a variable attenuator. The fixed attenuator is chosen such that the variable attenuator is used over only a small portion of its range. In this way, the variable attenuator error is kept small and hence also the total attenuator error.

4.5.1.3. Bandwidth

Bandwidth requirements are similar to those discussed for a power meter (see Section 4.3.4.). If the bandwidth is so narrow as to affect the measurement bandwidth of the system (rather than the transducer bandwidth being the limiting factor) the frequency response of the attenuator must be known in order to properly interpret the measurement results (see Section 5.7.4.).

The bandwidth of resistive-type variable attenuators is normally very wide, e.g., dc to over 1 GHz. Other types, such as directional coupler and waveguide-below-cutoff types, have narrower bandwidths that typically range from 5% to over 100% of their design frequency.

4.5.2. Precision Fixed Attenuators

For use in the 3-dB technique, the requirements of accuracy and bandwidth are generally the same as given for variable attenuators. The potential accuracy of a fixed attenuator is normally greater than for a variable attenuator, with uncertainties as small as 0.05 dB in 3 dB attainable. To get high accuracy, take care that the termination impedances match the characteristic impedance of the attenuator. Otherwise the insertion attenuation will not be equal to the expected value.

4.6. Amplifiers

When the gain of the transducer under test is insufficient to provide proper operation of the indicator in a given measurement set-up, additional gain must be supplied by auxiliary amplifiers. The important considerations are as follows:

a. Gain
b. Dynamic range
c. Noise factor
d. Bandwidth

4.6.1. Gain

The gain of the auxiliary amplifier must of course be sufficient to provide proper operation of the measurement system. However, when a gain control is provided, make certain that it does not degrade the noise figure of the amplifier, as might occur if the control were an input attenuator or a bias setting on an input tube or transistor. Also, make certain that no automatic gain control (AGC, AVC) circuits are in operation; otherwise, the amplifier gain is not constant with signal level, and the measurement results are meaningless.

Gain requirements for auxiliary amplifiers range from 10 dB to 120 dB. These requirements can often be met with generally available amplifiers. However, in special cases a custom amplifier may be required, or a duplicate of the transducer under test can be pressed into service.

4.6.2. Dynamic Range

Due care must be given to the choice of an auxiliary amplifier to make certain that it does not overload or significantly clip the noise peaks that it is required to pass [39], [40]. The measurement error caused by clipping generally increases as the clipping threshold level decreases. A normally safe practice is to select an auxiliary amplifier with a distortion-free CW power handling capability that is 10 dB greater than the average noise power that is to be delivered by the amplifier.

4.6.3. Noise Factor

The auxiliary amplifier should have a small value of $\overline{F}$ or $\overline{T_e}$ for the reasons discussed in Section 5.7.3. Generally speaking, noise factors from 3 dB to 20 dB greater than the transducer under test are permissible, depending upon the transducer's gain and noise factor. If the latter parameters are large, the auxiliary amplifier's noise factor need not be small.

4.6.4. Bandwidth

As discussed in Section 5.7.4., it is generally best to select an auxiliary amplifier with a bandwidth that is comparable to (e.g., from one to three times) the bandwidth of the transducer under test. When the transducer noise figure is fairly constant with frequency over its operating range, the auxiliary amplifier bandwidth may be narrower without serious effect. When the transducer noise figure and/or gain are large, the auxiliary amplifier bandwidth may be larger without serious effect.

4.7. Filters

Filters for noise measurements fall into two classes: "pre-detection" and "post-detection" filters. Pre-detection filters are used to establish the location and bandwidth of the measured noise power in the frequency domain. Post-detection filters are used to establish the time-constant of the measurement system indicator. Each class of filters is discussed below.

4.7.1. Pre-Detection Filters

Pre-detection filters may be bandpass, band stop, high pass or low pass filters, depending on the required application. They may be located at a variety of places in the measurement system as shown in figure 19.

A bandpass, high pass, or low pass filter would be located at position A to limit the noise power entering a superheterodyne transducer via the image or principal response frequencies, depending upon which noise factor is to be measured, so as to obtain a "single-sideband" noise factor rather than a "double-sideband" noise factor. A band pass filter would be located at positions B and/or C to restrict the measurement bandwidth to less than the transducer bandwidth if, for example, it were desired to measure a more nearly "spot" noise factor (see Appendix A) at one or more frequencies within the pass band of the transducer. A band pass filter would be located at position C to restrict the bandwidth of a very wide band auxiliary amplifier when it is greater than the transducer bandwidth or the desired measurement bandwidth (see Section 5.7.4.).

A band stop filter would be located at any of the three positions to reject a narrow-band signal, such as a CW signal, which might be present, as, for example, when a measurement is to be made with the transducer in an operating system (on-line testing).

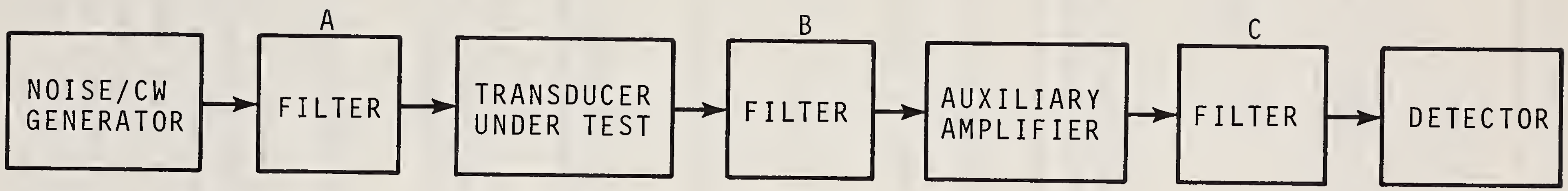


Figure 19. Possible Locations For Pre-Detection Filters.

Other applications of pre-detection filters will become apparent upon study of the system conditions and measurement requirements.

4.7.2. Post-Detection Filters

Post-detection filters are used primarily to obtain maximum measurement accuracy. In most measurement systems, the relative error, $\delta P/P$, in the measurement or indication of noise power is given by the general relationship

$$\frac{\delta P}{P} \propto \frac{1}{\sqrt{B\tau}} \tag{41}$$

where B is the pre-detection measurement bandwidth and τ is the post-detection time constant. Thus an increase in post-detection system time constant will produce a decrease in the measurement uncertainty.

Time constant τ can be increased in a variety of ways; for example:

1) Choose a power meter having a mechanical movement with a long time constant.

2) Insert on large time constant RC network between detector and indicator.

3) Use an integrator, either electrical or mechanical, in the detector output.

4) Use a strip chart recorder to display the detector output, and average the output graphically.

Improved accuracy by these means is obtained at the sacrifice of measurement speed. Given a pre-detection bandwidth, the metrologist must balance the trade-off of accuracy versus the economics of speed for his particular measurement situation.

4.8. Impedance Bridge/Reflectometer

To assure an accurate measurement of $\overline{F}$ or $\overline{T_e}$, the terminal impedances as a function of frequency should be measured for both the signal sources and the transducer input. This accomplishes two things: (1) It reveals any discrepancies between the desired source impedance and the actual source impedance(s) of the generators. (2) It reveals any deviations of input impedance from the design specification of the transducer. With this information, corrections can be applied to account for the actual impedance conditions prevailing during the measurement.

Measurement of source impedance can usually be made by common techniques with no need to observe special precautions. However, measuring the input impedance of an active transducer often requires a system which applies only a very small signal to the transducer input port in order to insure that the impedance remains unchanged from its normal operating value during the measurement. This requires either a very sensitive bridge/reflectometer detector or a sensitive vector impedance meter.

5. PRACTICAL MEASUREMENT PROBLEMS

In actual practice, many circumstances complicate the idealized situations described in Section 3. We now discuss the nature and effects of these complications.

5.1. Source Impedance

Noise factor is not a constant of the transducer; it changes with the source impedance, Z_s, of the signal source. For example, a given transducer will not have the same $\overline{F}$ when fed from a 45-ohm resistive source as when fed from a 50-ohm resistive source, all other conditions being kept the same. Nor will the transducer have the same $\overline{F}$ when fed from a 50 + j0-ohm source as when fed from a 50 + j5-ohm source. Therefore, source impedance must be established and known before a meaningful measurement of $\overline{F}$ can be made. Further, if the source impedances of the noise or CW sources are not the same as the required source impedance, the measurement result will be in error [41].

Experimental evidence and approximation models [42] indicate that $\overline{F}$ is a hyperbolic function of Z_s, and that a minimum $\overline{F}$ exists at some particular value of Z_s, viz., Z_s(opt).

The amount of measurement error, $\delta\overline{F}$, in $\overline{F}$ due to the source impedance effect depends upon the individual transducer. Typically, $\delta\overline{F}$ ranges from 0% to 10% of $\overline{F}$ as Z_s varies from Z_s(opt).

A measurement error occurs in the Y-factor methods when the source impedances of T_h and T_c are equal to each other but different from the required source impedance [43]. Another possibility is for the source impedances of T_h and T_c to be different from each other, with either one or the other, or neither, being equal to the required source impedance [44].

Another aspect of the problem has to do with the variation of source impedance with frequency. Suppose we require the transducer to operate from a given source, such as an antenna, whose source impedance varies over the range of frequencies passed by the transducer. We desire the $\overline{F}$ of the transducer for this condition. To measure $\overline{F}$ with maximum accuracy the source impedances of both T_h and T_c must duplicate that of the antenna over the frequency range of the transducer. Measurement error occurs when either or both impedance functions deviate from that of the antenna.

5.2. Input Termination Temperature

Because $\overline{F}$ is defined only when the noise temperature of the transducer's input termination is standard (290 K), many commercial noise figure meters are calibrated for this condition. Also, the working equations found in the literature are often written for this condition. However, this condition seldom exists in practice, and the measurement results are therefore in error. For non-critical situations, this error is often ignored, but high-accuracy measurements require correcting for this effect. The measurement techniques affected by this condition include the following:

a. 3-dB Technique (Variable Source Method)
b. Automatic Noise Figure Meter Technique
c. Gain Control Technique (Variable Source Method)
d. CW Technique
e. Tangential Technique

Not all manufacturers of TLD noise generators provide guidance in their instruction manuals to correct for this effect. On the other hand, the manufacturers of high quality automatic noise figure meters usually provide such information. In the CW and Tangential techniques, the metrologist is on his own. Normally, this effect is ignored in the Tangential technique, which is inherently a low-accuracy technique.

The relationship between the measured value, $\overline{F_m}$, of noise factor and the true value, $\overline{F}$, is given by the equation

$$\overline{F} = \overline{F_m} + 1 - \frac{T_a}{290}, \tag{42}$$

where T_a is the noise temperature of the input termination connected to the transducer under test. For example, if a commercial TLD noise source is used in the 3-dB technique, the meter reading will be $\overline{F_m}$ (dB). The true noise factor in decibels, is found by converting $\overline{F_m}$ (dB) to a numeric ratio by equation (18), adding the quantity

$$1 - \frac{T_a}{290},$$

and converting this result to true noise factor (in decibels) by equation (14). In the case of commercial ANFM's (42) is incorporated into the manufacturer's instructions for finding the true noise factor. The working equations of Section 6 include the correction given by (42).

T_a is frequently difficult to evaluate. Suggested techniques include the following: (a) Estimate T_a from a temperature measurement of the interior of the noise or CW generator. (b) When accessible, directly measure the physical temperature(s) of the components comprising the source impedance with a thermocouple or other suitable sensor. (c) Directly measure the noise temperature at the generator's output port with a noise radiometer [45]. This method requires equipment not normally available. (d) Use a WAG.

5.3. TLD Frequency Correction

When a temperature-limited diode (TLD) is used for measurements at very high frequencies, the correction factor, $\phi(f)$, (see Section 4.1.1.2.) may have a significant effect upon the measurement results. Manufacturers of TLD noise generators usually treat this effect in one of two ways: (a) some place an upper frequency limit on their instruments, above which the specified accuracy no longer holds; (b) others supply a correction curve for correcting the measured value of $\overline{F}$ as a function of frequency. Either procedure is satisfactory as long as the instrument is within specifications. However, for critical work at the higher frequencies, the noise generator should be calibrated [28], [29] or otherwise compared against a known source [45].

With diodes mounted in the conventional T-5½ glass envelope or similar construction (e.g., Type 5722), TLD noise generators can be designed to have a constant noise temperature with frequency up to 100 or 200 MHz, and to be within 10% of their low frequency value up to 400 or 500 MHz but falling off rapidly thereafter. Using the special TT-1 coaxial construction (Type 6144), the useful range is extended, and the noise temperature may be within 50% of its low frequency value up to 3 GHz. Solid state noise diodes maintain their low frequency output level to even higher frequencies (see Section 4.1.1.2.).

5.4. Non-Linearity

Non-linearity in the transducer may cause either a positive or a negative error in the measured value of $\overline{F}$ or $\overline{T}_e$, depending upon the cause of the non-linearity. Typical causes of non-linearity are (a) voltage amplitude limiting, (b) square-law behavior of circuit elements, and (c) feedback. When amplitude limiting occurs, the Y-factor may be smaller than for a linear transducer of the same $\overline{T}_e$ because of the relatively greater clipping of the noise peaks from T_h than from T_c (see equation (40)). In the case of positive feedback where transducer gain increases with signal amplitude, the opposite may be true.

In addition to the above non-linear effects which are manifest as variations in the effective power gain of the transducer, non-linear phenomena may also change the actual value of $\overline{T}_e$. That is, $\overline{T}_e$ may be a function of signal level. An example is a transistor or mixer diode whose noisiness varies with signal amplitude.

The amount of error introduced into the measured value of $\overline{T}_e$ by transducer non-linearity cannot be readily predicted without detailed knowledge of the transducer. However, for typical well-designed transducers, the error is generally less than 5%.

An estimate of this error can be made by measuring $\overline{T}_e$ with several differing values of T_h and plotting $\overline{T}_e$ on a chart. A trend away from a constant $\overline{T}_e$ may indicate a significant non-linear effect.

5.5. Leakage and Shielding

Extraneous electromagnetic energy may cause error in the measured value of $\overline{T}_e$ if it is able to enter the measurement system because of inadequate shielding or filtering. If the same amount of extraneous energy enters when either T_h or T_c are connected to the transducer the measured value of Y will be smaller than if no extraneous energy is present. This is because

$$\frac{[k\ (T_h + \overline{T}_e)\ B + P_x]G}{[k\ (T_c + \overline{T}_e)\ B + P_x]G} < \frac{k\ (T_h + \overline{T}_e)\ BG}{k\ (T_c + \overline{T}_e)\ BG} = Y \tag{43}$$

where P_x is the extraneous power referred to the input port of the transducer. On the other hand, if the leakage through T_h is different from that through T_c, the measured value of Y may fall either less than or greater than the value obtained in the absence of the extraneous energy.

To be certain that extraneous signals are not a source of error, follow one or more of the following procedures: (1] Perform the measurement in a shielded enclosure of known integrity. (2) Test for leakage into the measurement system by deliberately attempting to introduce energy from a medium power signal source. (3) Display the output signal from the transducer on an oscilloscope and inspect for any periodic waveforms. (4) Check for interference produced within the measurement apparatus itself, e.g., timing circuits in digital power meter.

5.6. Spurious Responses

The ideal two-port transducer power gain function, G(f), has a form that is optimum for the particular transmitted signal. G(f) is zero at frequencies where the signal is zero, and its value at signal frequencies is shaped for best signal-to-noise ratio. However, most real-world transducers have gain at other than signal frequencies which may give rise to undesired or spurious responses. The most common such response for narrow-band communications systems is the image response of a superheterodyne receiver. Responses can be caused by energy at other than the image frequency band because of transducer non-linearity, instability, leakage, or impure heterodyning signals.

For transducers having spurious responses, the total output noise power, N_{on}, contains energy originating at frequencies other than within the principal-frequency transformation(s) of the system. Therefore, N_{on} may be larger than for a similar transducer having no spurious responses, and the measured value of $\overline{F}$ may differ from the true value of $\overline{F}$, depending upon whether a broad band or narrow band technique is used. N_{on} is given by the equation

$$N_{on} = k\int_0^{\infty}(T_n + \overline{T_e})\ G(f)\ df, \tag{44}$$

where T_n is the noise temperature of the signal source. Assuming T_n and $\overline{T_e}$ are constant with frequency (44) can be approximated by the equation

$$N_{on} = k(T_n + \overline{T_e})\ \left[(GB)_p + (GB)_i + (GB)_1 + (GB)_2 + \ldots\right], \tag{45}$$

where

$$(GB)_p = \int_{fp} G(f)\ df \tag{46}$$

is the area under the gain function over the band of frequencies that is the transducer's principal-frequency transformation,

$$(GB)_i = \int_{fi} G(f)\ df \tag{47}$$

is the area under the gain function over the transducer's image-frequency band, and

$$(GB)_1 = \int_{f1} G(f)\ df \tag{48}$$

$$(GB)_2 = \int_{f2} G(f)\ df \tag{49}$$

etc.,

are the areas under the gain function over the transducer's spurious-frequency bands. From the definition of average noise factor, the transducer with spurious responses has a larger $\overline{F}$ than an equivalent spurious-free transducer because

$$N_{on} > k(T_n + \overline{T_e})\ (GB)_p . \tag{50}$$

This also follows from Note 1 of the IEEE definition, (see Appendix A).

If the measurement technique treats the spurious response frequencies as signal channels, as might be the case with broad band techniques, the measured noise factor will in general be smaller than that obtained by a narrow band technique. For example, consider a transducer having gain in only the principal- and image-frequency bands and at no other spurious response frequencies. The single channel noise factor, $\overline{F_s}$, is given by

$$\overline{F_s} = \frac{k(T_o + \overline{T_e})\ \left[(GB)_p + (GB)_i\right]}{k\ T_o (GB)_p} . \tag{51}$$

If

$$(GB)_p = (GB)_i\ , \tag{52}$$

then

$$\overline{F_s} = \frac{2\ k(T_o + \overline{T_e})\ (GB)_p}{kT_o (GB)_p} = 2\left(1 + \frac{\overline{T_e}}{T_o}\right) . \tag{53}$$

A broad band measurement technique would measure the double channel noise factor, $\overline{F_d}$, where

$$\overline{F_d} = \frac{k(T_o + \overline{T_e})\left[(GB)_p + (GB)_i\right]}{kT_o\left[(GB)_p + (GB)_i\right]} = 1 + \frac{\overline{T_e}}{T_o} . \tag{54}$$

From (53) and (54),

$$\overline{F_s} = 2\ \overline{F_d} . \tag{55}$$

If the desired noise factor is $\overline{F_s}$, the measurement result, $\overline{F_d}$, will be in error by a factor of two. Note that, in general, $\overline{F_s}$, may range in value from $\overline{F_d}$ to $2\overline{F_d}$ (or greater) depending upon the relative transducer gain in the image-frequency band. Thus the popular concept that $\overline{F_s}$ (dB) is always 3 dB greater than $\overline{F_d}$(dB) is correct only when $(GB)_i$ equals $(GB)_p$.

5.7. System Gain

It is not uncommon to proceed to measure the $\overline{F}$ of an amplifier only to find that there is insufficient gain in the measurement system to give an adequate display on the output indicator. For example, commercial automatic noise figure meters typically require a minimum signal level of -60 to -70 dBm for proper operation. Or your particular power meter may require several milliwatts to obtain a reliable reading. In such cases, auxiliary amplifiers must be used.

Several factors must be borne in mind when selecting an auxiliary amplifier:

a. Gain; gain stability
b. Dynamic range
c. Noise factor
d. Frequency response
e. Input impedance
f. Spurious generation of signals

5.7.1. Gain

Select an auxiliary amplifier to have adequate gain stability as well as adequate gain. Short-term stability is adequate for the automatic, tangential, and comparison techniques, but longer-term stability may be required for the other non-automatic techniques [46]. In general, the measurement uncertainty, in percent, due to gain fluctuations, will be approximately equal to twice the gain fluctuation (gain uncertainty), in percent.

5.7.2. Dynamic Range

The comments of Section 4.6.2. apply also to auxiliary amplifiers. Clipping is often a greater problem here because of the larger signals passed by the auxiliary amplifier.

5.7.3. Noise Factor

When two twoport transducers are connected in cascade (see figure 20), the overall noise factor, F_{12}, is given by the equation [47]

$$\overline{F_{12}} = \overline{F_1} \frac{(GB)_{12t}}{(GB)_{12s}} + (\overline{F_2} - 1) \frac{(GB)_{2t}}{(GB)_{12s}}, \tag{56}$$

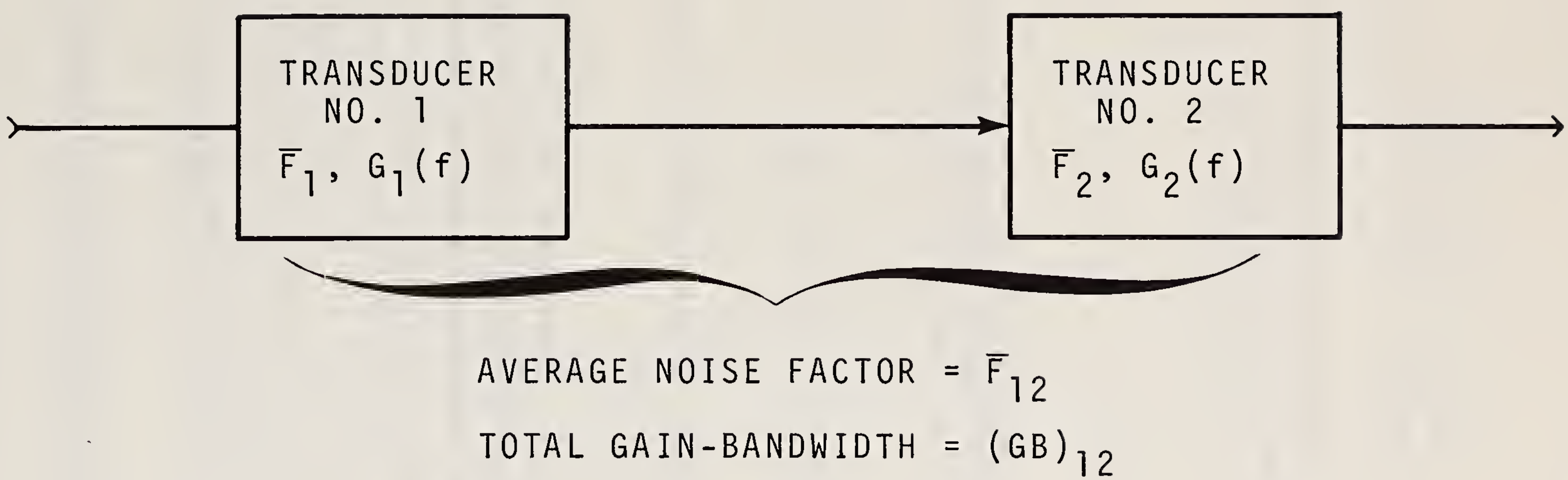


Figure 20. Two Cascaded Twoport Transducers.

where $\overline{F_1}$ and $\overline{F_2}$ are the narrow band average noise factors for the first and second transducers, respectively. The gain-bandwidth products, $(GB)_{12t}$, $(GB)_{12s}$, and $(GB)_{2t}$, are as follows:

$$(GB)_{12t} = \int_0^\infty G_{12}(f)\, df, \tag{57}$$

where $G_{12}(f)$ is the overall gain function of the two cascaded transducers, and $(GB)_{12t}$ is the total area under this gain function.

$$(GB)_{12s} = \int_{fs} G_{12}(f)\, df, \tag{58}$$

where $(GB)_{12s}$ is the area under $G_{12}(f)$ for only the frequency range(s), f_s, that includes the system signal (see Section 2.1).

$$(GB)_{2t} = \int_0^\infty G_2(f)\, df, \tag{59}$$

where $G_2(f)$ is the gain function of the second transducer, and $(GB)_{2t}$ is the total area under this gain function.

Equation (56) shows that the contribution of $\overline{F_1}$ and $\overline{F_2}$ to the measured noise factor, $\overline{F_{12}}$, is affected by the gain-bandwidth products, GB, of the individual transducers. For transducers each having gain in only one frequency band, the ratio $(GB)_{12t}/GB_{12s}$, by which $\overline{F_1}$ is multiplied, reduces to unity. That is,

$$(GB)_{12t} = (GB)_{12s}. \tag{60}$$

In general, however, for multiple-response transducers, the ratio is greater than unity, i.e.,

$$(GB)_{12t} \geq (GB)_{12s}. \tag{61}$$

For transducers with gain in only one and exactly the same frequency range, the ratio $(GB)_{2t}/(GB)_{12s}$, by which $(\overline{F_2} - 1)$ is multiplied, reduces to $1/\overline{G_1}$, that is,

$$(GB)_{12s} = \overline{G_1}\,(GB)_{2t}, \tag{62}$$

where G_1 is the average gain of the first transducer in the frequency interval f_s; viz.,

$$\overline{G_1} = \frac{\int_{fs} G_1(f)\,df}{\int_{fs} df} . \tag{63}$$

In general, however, this ratio may range in value from a lower limit of $1/\overline{G_1}$ to a large value without limit (infinity), depending upon the shapes of $G_1(f)$ and $G_2(f)$ (see Section 5.7.4. below).

In the simple cases where (60) and (62) apply, (56) becomes

$$\overline{F_{12}} = \overline{F_1} + \frac{\overline{F_2} - 1}{\overline{G_1}} , \tag{64}$$

which is the relationship commonly found in the literature [11], [13], [18].

From the above discussion, it can be readily seen that $\overline{F_{12}}$ is usually greater than F_1, and that the difference between these two quantities is determined by the relative sizes of $\overline{F_1}$ and $\overline{F_2}$ and by the characteristics of $G_1(f)$ and $G_2(f)$. If $\overline{F_2}$ and $G_2(f)$ are small, or if G_1 is large, the measured value of $\overline{F_{12}}$ may be very nearly equal to $\overline{F_1}$, which is sought. It is therefore **prudent to select an auxiliary amplifier with as small an $\overline{F}$ as possible. When G_1** is small, or when $\overline{F_2}$ is large, the value of $\overline{F_1}$ **for the transducer under test** can be computed from the measured values of $\overline{F_{12}}$, $\overline{F_2}$ and $\overline{G_1}$ by the relationship

$$\overline{F_1} = \overline{F_{12}} - \frac{\overline{F_2} - 1}{\overline{G_1}} , \tag{65}$$

if the conditions for (64) prevail. Otherwise use (56).

An analysis of two cascaded twoport transducers in terms of effective input noise temperature would show that, for the simple case where (64) applies, the overall average effective input noise temperature, $\overline{T_{e_{12}}}$, is given by

$$\overline{T_{e_{12}}} = \overline{T_{e_1}} + \frac{\overline{T_{e_2}}}{\overline{G_1}} . \qquad (66)$$

The extension of the cascading equations, (64) and (66), to more than two transducers is, for the simple case, straight forward. It is seldom necessary to go beyond the two transducer case when measuring $\overline{F}$ or $\overline{T_e}$, however. For further information about multiple cascading, the reader is referred to references [13] or [18].

5.7.4. Frequency Response

The frequency response of an auxiliary amplifier can alter the measurement results according as it includes a narrower or wider range of frequencies than that of the transducer under test. The shape of the amplifier's gain function, $G_2(f)$, in conjunction with the transducer's gain function, $G_1(f)$, determines the frequencies that are involved in the measurement of $\overline{F_1}$ and $\overline{T_{e_1}}$.

It goes without saying that the auxiliary amplifier must have gain in the frequency range of the output power from the transducer under test. If such is not the case because of an oversight or misunderstanding, the gain-bandwidth product $(GB)_{12S}$ of (56) will be very small or zero and $\overline{F_{12}}$ will be a very large value. An apparent measurement can be obtained, but the computed value of $\overline{F_1}$ may be completely unreliable.

If the auxiliary amplifier has gain in only one frequency range whose width is narrower than that of the transducer, $\overline{F_{12}}$ becomes a narrow band noise factor whose value pertains to the frequency range passed by the measurement system. In this case, the measurement frequency range is determined primarily by the amplifier rather than by the transducer (assuming of course, that other parts of the system, such as the power meter and attenuators, do not narrow the frequency range). In general, the measured value of $\overline{F_1}$ for the transducer may then vary according to the center frequency and width of the auxiliary amplifier's frequency response, depending upon how $\overline{F_1}$ varies with frequency. If $\overline{F_1}$ is constant with frequency over the transducer's frequency range, it may be measured anywhere within this range and with any practical measurement bandwidth without obtaining a variation in its measured value.

If the auxiliary amplifier has gain in frequency ranges in addition to those passed by the transducer, $\overline{F_1}$ may be difficult to determine because, depending upon $\overline{F_2}$ and $G_2(f)$, the output power from the amplifier may consist largely of noise originating within the auxiliary amplifier. In this case, the measured value of $\overline{F_{12}}$ is comparatively insensitive to the value of $\overline{F_1}$. This can be seen from (56); the gain-bandwidth product $(GB)_{2t}$ may be very large, and it would dominate the right-hand term rather than the transducer gain $G_1(f)$, which enters via the gain-bandwidth product $(GB)_{12s}$ (compare equation (63)). The result is a potentially large error in $\overline{F_1}$, as can be seen by rearranging (56) as follows:

$$\overline{F_1} = \overline{F_{12}} \frac{(GB)_{12s}}{(GB)_{12t}} - (\overline{F_2} - 1) \frac{(GB)_{2t}}{(GB)_{12t}} . \tag{67}$$

Both terms on the right-hand side of (67) will be large, and they will be nearly equal. Therefore, a small relative error in either term will produce a large relative error in $\overline{F_1}$.

From the above discussion, it can be appreciated that the auxiliary amplifier selected should have a frequency response that is similar to that of the transducer under test. In general, it should not be narrower unless a specific narrow band noise factor measurement is desired, and it should not be a great amount wider unless $G_1(f)$ is large and both $\overline{F_2}$ and $G_2(f)$ are small. A frequency response that is from one to three times wider than that of the transducer under test is usually satisfactory for the normal measurement situation.

5.7.5. Input Impedance

The input impedance of the auxiliary amplifier affects the measured value of $\overline{F_1}$ inasmuch as it affects $G_1(f)$ [48]. Normally, measurement difficulties are minimized when this impedance matches the output impedance of the transducer under test, and a transmission line of the same impedance is used to connect the two units.

5.7.6. Spurious Generation of Signals

The auxiliary amplifier must be stable and free of spurious signal sources. Check out the amplifier before using it.

5.8. Gain Stability

Short-term gain variations in the transducer under test can cause measurement errors in all of the techniques described in this Guide. The automatic Y-factor technique, and automated power or attenuator methods tend to reduce this effect when the switching rate is fast compared with the rate of gain fluctuations. Very high-gain transducers can be particularly difficult to measure. For example, in one case of a low noise 30 MHz IF amplifier having 78 dB gain, repeated measurement resulted in data spread over a range of 2 to 1. This was traced to gain changes caused by (1) unstable feedback due to leakage between the input, output, and power cables, and (2) variations in source impedance due to faulty connectors. Had only a single measurement been made, this phenomenon would not have been apparent.

5.9. Connectors and Cables

A prevalent problem source in noise measurements is the means by which networks are connected together in the measurement system. Connectors, cables and waveguide can contribute to leakage, mismatch, losses, and change of frequency response. Accurate measurements cannot be made unless proper selection of these components is made.

5.9.1. Connectors

When the transducer under test is fitted at both its input and output ports with good connectors, problems generally arise only when making precision measurements. Otherwise, other sources of error usually predominate. In precision measurements, connectors must be carefully selected and installed.

When the transducer under test is not fitted with connectors, new problems arise which often require ingenuity to solve. The most common problem occurs when the signal to be measured must be extracted from the transducer at a point where no connector has been provided, such as the input to the detector in a receiver. When such a point is actually inaccessible, either (1) an alternate point must be chosen, (2) an alternate measurement technique must be used which allows the detector and post-detector circuits to be included in the measurement system, or (3) give up. If the desired extraction point is accessible, a connection technique must be used that either (1) insignificantly alters the circuit conditions in the transducer, or (2) alters conditions in a known way so that corrections can be made in the measurement results. Successful connection methods include the following:

(1) High impedance circuit: - Capacitively couple from the point of measurement. Use a matching network if necessary. In tube circuits using glass tubes, fit a connector to a tube shield that is modified so as not to contact chassis ground. Disconnect input to receiver detector, or otherwise isolate the measured signal from being affected by the detector.

(2) Low impedance circuit: - Inductively couple or use a resistance matching network. Locate the ground return with care to minimize undesired ground loop interactions.

(3) Miscellaneous: - (a) Devise a connector to plug in place of the tube or transistor at the stage where the measurement is made. (b) Install a connector on the chassis near the point of measurement, and connect to the circuit through a suitable coupling/matching network.

5.9.2. Cables and Waveguide

An ideal measurement system has no interconnecting cables or waveguide. Practical systems, however, usually require a line from the signal source to the transducer input port, and from the transducer output port to the remainder of the measurement system. Serious measurement errors can be caused by improper choice and installation of these lines. Many of the measurement problems discussed in this section can be minimized by (1) using the shortest length practicable; (2) using the correct characteristic impedance; (3) using high-quality, low-loss, lines; (4) properly installing the correct connectors on the lines.

5.10. Detection

Proper attention must be given to detection in the measurement system if the desired measurement results are to be obtained. Detector requirements are not the same for all of the techniques discussed above. Some transducers, such as voltage and power amplifiers, do not have built-in detectors; others, such as receivers, include detectors of which there is a variety of types (envelope, square-law, phase, video, etc.). In this section we will discuss some of the more important considerations that influence the choice and treatment of detectors.

5.10.1. Built-in Detectors

The detector that is built into the transducer under test, may or may not be used in the measurement, depending upon the following factors:

(1) If it is an envelope detector (linear rectifier, video detector), it can be used without calibration in the 3-dB Gain Control, Tangential, and Comparison Techniques, and in the Attenuator Method of the Y-Factor Technique. If the envelope detector is calibrated so that its output signal voltage is known as a function of its input noise power, it can also be used in the Power Meter Method of the Y-Factor Technique. This calibration is not ordinarily known, however. An envelope detector cannot be used in the CW Technique to

yield good accuracy because its response depends upon the particular mixture of noise and CW powers present [49].

(2) If it is a square-law detector (power detector, quadratic detector), it can be used in any of the techniques described except the Automatic Y-Factor Technique, which uses its own external detector.

(3) If it is a phase detector (multiplier, product detector), it behaves simply as a frequency translator and the transducer is equivalent to one having no detector (see below).

When the built-in detector is used in the measurement, the measurement technique is chosen as indicated in (1) or (2) above. The instrumentation required to follow the detector is chosen according to the needs for additional gain and filtering as required by the selected output indicator.

When the built-in detector is not used in the measurement, proceed as discussed below under External Detectors. The principal problem in this case is usually that of tapping into the receiver ahead of the detector to extract the signal without introducing changes in the system that cause the measurement results to be inaccurate.

5.10.2. External Detectors

When the transducer under test has no built-in detector, the detector network of the measurement system is chosen according to the desired measurement technique and/or the instrumentation at hand.

The Y-Factor/Power Meter Method and the CW Technique require a power measuring device. A square-law, power or quadratic detector works well for both techniques, and a commercial power meter can be used for convenience. A calibrated linear detector (calibrated in terms of noise power), can also be used in the Y-Factor method but not in the CW Technique, for reasons given above.

All other techniques described in this Guide except the Automatic Technique can use either a linear or a power-type external detector in the measurement system. The reason for this is that in those techniques the detector and post-detector networks serve only to indicate one or more reference levels of power. These levels need not be known. And, since these levels remain constant for a given measurement of $\overline{F}$ or $\overline{T_e}$, the law of the detector is unimportant as long as it is stable during the measurement.

The Automatic Technique utilizes a detector that is internal to the noise factor indicator system. This detector is usually a square-law device, which simplifies the meter scale calibration on the indicator.

5.11. Cooled Networks

When a noise source and a transducer under test are at different thermodynamic temperatures, the heat flow between them can give rise to measurement errors. An illustration of this condition is the case of a cooled parametric amplifier to which is connected a hot thermal noise generator for making a Y-Factor measurement. Alternatively, one might have a very warm tube-type receiver to which is connected a cryogenically cooled thermal noise generator. In each case, heat flow could change the noise temperatures of either device, depending upon the heat capacities and/or temperature regulation that might prevail.

For precision measurements, some form of thermal isolation should be used between source and transducer. For example, a length of stainless steel waveguide with a thin plating of silver on its inner surfaces will provide good electrical conductivity and poor thermal conductivity. With careful design, the thermal gradients and electrical losses can be determined so as to provide information for computation of corrections for the effects of heat transfer. Without such precautions, the noise temperature of a thermal noise generator cooled with a liquid cryogen might be raised to several degrees above the cryogen temperature, thus producing a significant error when measuring small values of $\overline{T_e}$.

5.12. Multiport Transducers

The noise factor of a multiport transducer is undefined. The $\overline{T_e}$ of a transducer having one output port and more than one input port can be measured by the Y-Factor technique in the following way (basic procedure):

1. Arrange the equipment as shown in figure 21.

2. Connect Hot Noise Generators to the transducer input ports.

3. Record the reading of the Power Meter, P_h.

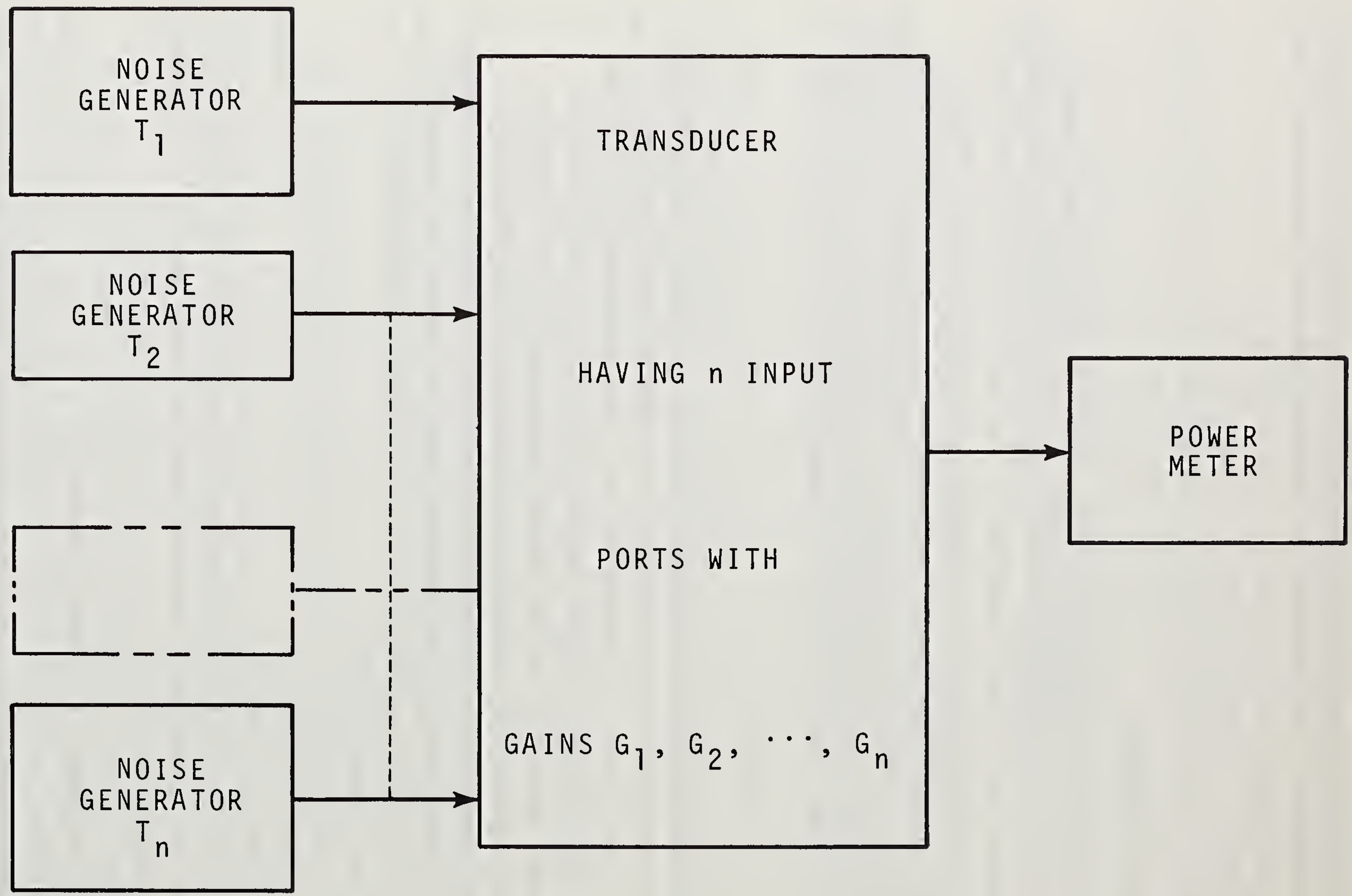


Figure 21. Y-Factor/Power Meter Method for a Multiport Transducer.

4. Disconnect the Hot Noise Generators and connect Cold Noise Generators to the transducer input ports.

5. Record the reading of the Power Meter, P_c.

6. Calculate the Y-Factor from the equation

$$Y = \frac{P_h}{P_c} . \tag{68}$$

7. Calculate $\overline{T_e}$ from the equation

$$T_e = \frac{1}{Y-1} \cdot \frac{\int_o^\infty T_{h1} G_1 \, df + \int_o^\infty T_{h2} G_2 \, df + \ldots + \int_o^\infty T_{hn} G_n \, df}{\int_o^\infty G_1 \, df + \int_o^\infty G_2 \, df + \ldots + \int_o^\infty G_n \, df} + $$

$$- \frac{Y}{Y-1} \cdot \frac{\int_o^\infty T_{c1} G_1 \, df + \int_o^\infty T_{c2} G_2 \, df + \ldots + \int_o^\infty T_{cn} G_n \, df}{\int_o^\infty G_1 \, df + \int_o^\infty G_2 \, df + \ldots + \int_o^\infty G_n \, df} , \tag{69}$$

where the subscript "n" refers to the nth port.

8. If noise sources are chosen such that

$$T_{h1} = T_{h2} = \ldots = T_{hn} = T_h \tag{70}$$

and

$$T_{c1} = T_{c2} = \ldots = T_{cn} = T_c \tag{71}$$

at all frequencies, equation (69) becomes

$$\overline{T_e} = \frac{T_h - YT_c}{Y-1} . \tag{72}$$

It is evident from the above brief procedure outline that the measurement of $\overline{T_e}$ for this particular example would be difficult and probably unsatisfactory. However, it illustrates the nature of the problem. There are other possible multi-port configurations which pose even more difficult challenges. These multi-port situations are rare, and a discussion of them is not considered to be within the scope of this guide.

6. WORKING EQUATIONS

The derivations in this section are based on idealized models of the ten measurement methods in order that the reader can clearly and quickly see what is basically taking place in each technique. Actual measurement set-ups using these methods will introduce additional factors that complicate their analysis. Many of these factors have been discussed in Section 5, Practical Measurement Problems. A thorough rigorous analysis of these real-world situations is beyond the scope of this guide.

A summary of working equations for these techniques is given in Table III.

6.1. Y-Factor Technique (Idealized Conditions)

6.1.1. Power Meter Method

The three parts of the measurement system are the following (see figure 22):

a. Transducer under test

b. Random noise generator

c. Power Meter

The transducer has two ports, an in-port and an out-port, and is linear. It has a gain G that is constant from frequency f_1 to f_2 and is zero outside of this interval. It has a $\overline{T_e}$ that is constant in the interval (f_1, f_2).

The random noise generator provides two levels of available noise power per unit bandwidth, viz., kT_h and kT_c where

$$T_h > T_c. \tag{73}$$

Both T_h and T_c are constant from frequency f_1 to f_2. The generator source impedance is the value for which the transducer has an average effective input noise temperature $\overline{T_e}$.

The power meter is noise-free, linear, and sensitive enough to measure the noise power levels available from the transducer out-port. Its sensitivity is constant over the frequency interval (f_1, f_2). No reflection of power occurs at its in-port.

The measurement system is assumed to be perfectly shielded against all extraneous interference.

TABLE III. Summary of Working Equations

(See key at end for definitions of symbols)

Technique	$\overline{F}$	$\overline{T_e}$
1. Y-Factor a. Power Meter Method	$\overline{F} = \dfrac{\dfrac{T_h}{T_o} - Y\dfrac{T_c}{T_o}}{Y-1} + 1$ (80)	$\overline{T_e} = \dfrac{T_h - YT_c}{Y-1}$ (79)
b. Attenuator Method	$\overline{F} = \dfrac{\dfrac{T_h}{T_o} - Y\dfrac{T_c}{T_o}}{Y-1} + 1$ (80)	$\overline{T_e} = \dfrac{T_h - YT_c}{Y-1}$ (79)
2. 3-dB a. Variable Source	$\overline{F} = \dfrac{\varepsilon\, I_d\, R\, \phi\, A}{2kT_o(1-A)} - \dfrac{T_a}{T_o} + 1$ (92)	$\overline{T_e} = \dfrac{\varepsilon\, I_d\, R\, \phi\, A}{2k(1-A)} - T_a$ (93)
b. Fixed Source	$\overline{F} = \alpha\left(\dfrac{T_n - T_a}{T_o}\right)\dfrac{A}{1-A} - \dfrac{T_a}{T_o} + 1$ (99)	$\overline{T_e} = \alpha\left(T_n - T_a\right)\dfrac{A}{1-A} - T_a$ (96)
3. Automatic	$\overline{F} = \dfrac{\dfrac{T_2}{T_o} - \dfrac{T_1}{T_o}\dfrac{gP_2}{gP_1}}{\dfrac{gP_2}{gP_1} - 1} + 1$ (106)	$\overline{T_e} = \dfrac{T_2 - T_1\left(\dfrac{gP_2}{gP_1}\right)}{\dfrac{gP_2}{gP_1} - 1}$ (105)
4. Gain Control a. Variable Source	$\overline{F} = \dfrac{\varepsilon(I_{d1})^2\, R\, \phi}{2kT_o(I_{d2}-2I_{d1})} - \dfrac{T_a}{T_o} + 1$ (117)	$\overline{T_e} = \dfrac{\varepsilon(I_{d1})^2\, R\, \phi}{2k(I_{d2} - 2I_{d1})} - T_a$ (116)
b. Fixed Source	$\overline{F} = \left(\dfrac{T_n - T_a}{T_o}\right)\dfrac{(\alpha_1)^2}{\alpha_2 - 2\alpha_1} - \dfrac{T_e}{T_o} + 1$ (125)	$\overline{T_e} = \left(T_n - T_a\right)\dfrac{(\alpha_1)^2}{\alpha_2 - 2\alpha_1} - T_a$ (123)

TABLE III. (continued)

5. CW	$\overline{F} = \dfrac{P_s}{k\ T_o\ B\left(\dfrac{P_2}{P_1} - 1\right)} - \dfrac{T_a}{T_o} + 1$	(129)	$\overline{T_e} = \dfrac{P_s}{k\ B\left(\dfrac{P_2}{P_1} - 1\right)} - T_a$	(128)
6. Tangential a. General	$\overline{F} = \dfrac{P_s}{k\ T_o\ B(S/N)_o} - \dfrac{T_a}{T_o} + 1$	(139)	$\overline{T_e} = \dfrac{P_s}{k\ B(S/N)_o} - T_a$	(142)
b. Linear Detector	$\overline{F} \doteq \dfrac{2P_s}{B} \times 10^{19}$	(140)	-	
c. Square-law Detector	$\overline{F} \doteq \dfrac{4P_s}{B} \times 10^{19}$	(141)	-	
7. Comparison	$\overline{F_x} \doteq \overline{F_m}\ \dfrac{P_{m2}\ P_{x1}}{P_{m1}\ P_{x2}}$	(154)	$\overline{T_{ex}} \doteq \overline{T_{em}}\ \dfrac{P_{m2}\ P_{x1}}{P_{m1}\ P_{x2}}$	(156)

Key to Symbols:

$\overline{F}$ - Average noise factor

$\overline{T_e}$ - Average effective input noise temperature

T_h - Noise temperature of hot noise generator

T_c - Noise temperature of cold noise generator

T_o - Standard noise temperature, 290 K

Y - Y-factor

T_a - Ambient temperature

A - 3 dB attenuator transmittivity

ε - Electron charge, 1.602×10^{-19} coulomb

I_d - TLD emission current in amperes

R - Source resistance in ohms

ϕ - TLD correction factor

k - Boltzmann's constant, 1.381×10^{-23} joule/kelvin

α - Variable attenuator transmittivity

T_n - Noise temperature of source

P_1 - Transducer output power, input condition No. 1

P_2 - Transducer output power, input condition No. 2

g - Automatic noise figure meter gain

P_s - Available power from CW generator

$(S/N)_o$ - Output signal-to-noise ratio

B - Noise bandwidth

P_m - Master transducer output power

P_x - Transducer under test output power

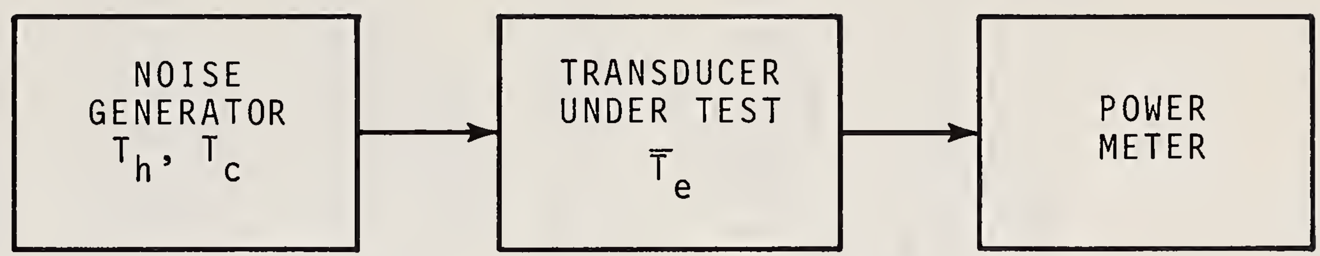


Figure 22. Y-Factor/Power Meter Method.

The generator, transducer and meter are connected together as shown in figure 22. When the generator noise temperature is T_h, the power meter reading is P_h, where

$$P_h = k(T_h + \overline{T_e})BG \tag{74}$$

and

$$B = f_2 - f_1. \tag{75}$$

When the generator noise temperature is T_c, the power meter reading is P_c, where

$$P_c = k(T_c + \overline{T_e})BG. \tag{76}$$

The Y-Factor is

$$Y = \frac{P_h}{P_c}. \tag{77}$$

From (74), (76), and (77),

$$Y = \frac{T_h + \overline{T_e}}{T_c + \overline{T_e}}. \tag{78}$$

Solving (78) for $\overline{T_e}$ gives

$$\overline{T_e} = \frac{T_h - YT_c}{Y - 1}. \tag{79}$$

From (6) and (79)

$$\overline{F} = \frac{\frac{T_h}{290} - Y\frac{T_c}{290}}{Y - 1} + 1. \tag{80}$$

6.1.2. Attenuator Method

The four parts of the measurement system are the following (see figure 23):

a. Transducer under test

b. Random noise generator

c. Variable attenuator

d. Signal level indicator

The transducer has two ports, an in-port and an out-port, and is linear. It has a gain G that is constant from frequency f_1 to f_2 and is zero outside of this interval. It has a $\overline{T_e}$ that is constant in the interval (f_1, f_2).

The random noise generator provides two levels of available noise power per unit bandwidth, viz., kT_h and kT_c where

$$T_h > T_c. \tag{81}$$

Both T_h and T_c are constant from frequency f_1 to f_2. The generator source impedance is the value for which the transducer has an average effective input noise temperature $\overline{T_e}$.

The variable attenuator is a precision unit with enough range to meet the measurement requirements stated below. Its attenuation at a given setting is constant over the frequency interval (f_1, f_2). Its terminal impedances match those of the transducer output impedance and signal level indicator input impedance, respectively.

The signal level indicator is noise-free, stable, and sensitive enough to provide a precise indication of the signal level applied to it. It responds equally to all frequencies in the interval (f_1, f_2).

The measurement system is assumed to be perfectly shielded against all extraneous interference.

The generator, transducer, attenuator, and indicator are connected together as shown in figure 23. When the generator noise temperature is T_c and the attenuator transmittance is α_c, the indicator reading is I_c, where

$$I_c = k(T_c + \overline{T_e})BG\ \alpha_c \tag{82}$$

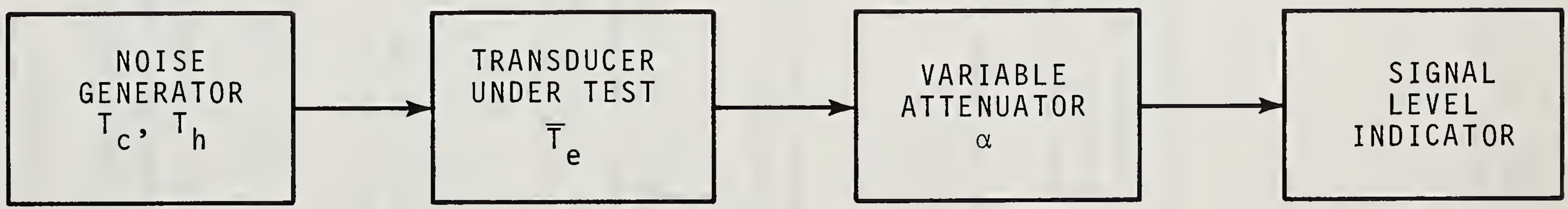


Figure 23. Y-Factor/Attenuator Method.

and

$$B = f_2 - f_1. \tag{75}$$

When the generator noise temperature is T_h, the attenuator transmittance is changed to α_h, where

$$\alpha_h < \alpha_c, \tag{83}$$

to produce the same indicator reading I_c. Then

$$I_c = k(T_h + \overline{T_e})BG\,\alpha_h. \tag{84}$$

The Y-Factor is

$$Y = \frac{\alpha_c}{\alpha_h}. \tag{85}$$

from (82), (84), and (85),

$$Y = \frac{T_h + \overline{T_e}}{T_c + \overline{T_e}}. \tag{78}$$

Solving (78) for $\overline{T_e}$ gives

$$\overline{T_e} = \frac{T_h - YT_c}{Y - 1}. \tag{79}$$

From (6) and (79),

$$\overline{F} = \frac{\frac{T_h}{290} - Y\frac{T_c}{290}}{Y - 1} + 1. \tag{80}$$

6.2. 3-dB Technique (Idealized Conditions)

6.2.1. Variable Source Method

The four parts of the measurement system are the following (see figure 24):

a. Transducer under test

b. Temperature-limited diode (TLD) noise generator

c. 3-dB fixed attenuator

d. Signal level indicator

The transducer has two ports, an in-port and an out-port, and is linear. It has a gain G that is constant from frequency f_1 to f_2 and is zero outside of this interval. It has an $\overline{F}$ that is constant in the interval (f_1, f_2).

The TLD noise generator has a continuously adjustable available noise power with a range sufficient for the measurement. The available noise power at any given setting is constant from frequency f_1 to f_2. The meter scale of the generator is calibrated directly in $\overline{F}$(dB). The generator source impedance is the value, R, for which the transducer has an average effective input noise temperature $\overline{T_e}$. The temperature of the source impedance is 290K.

The fixed attenuator is a precision unit with a transmittance, A, of 0.5 (3dB). Its attenuation is constant over the frequency interval (f_1, f_2). Its terminal impedances match those of the transducer output impedance and signal level indicator impedance, respectively.

The signal level indicator is noise-free, stable, and sensitive enough to provide a precise indication of the signal level applied to it. It responds equally to all frequencies in the interval (f_1, f_2). Its input impedance matches the output impedance of the transducer.

The measurement system is assumed to be perfectly shielded against all extraneous interference.

The generator, transducer, attenuator, and indicator are connected together as shown in figure 24. When the TLD emission current is zero and the transducer is connected directly to the indicator, the indicator reading is I, where

$$I = kT_o \overline{F} BG, \tag{86}$$

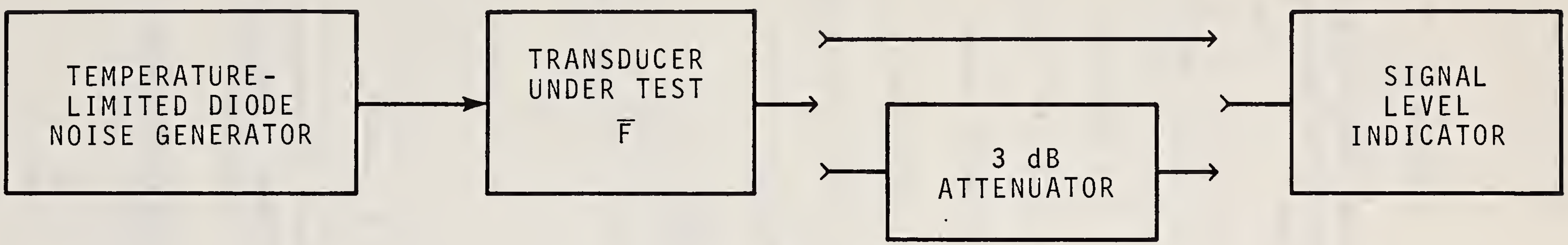


Figure 24. 3-dB/Variable Source Method.

$$T_o = 290\ K, \tag{87}$$

and

$$B = f_2 - f_1. \tag{75}$$

When the attenuator is inserted between the transducer and the indicator, the emission current is increased to the value I_d, in amperes, that produces the same indicator reading I. Then

$$I = A\ k\ (T_o\ \overline{F} + \frac{\varepsilon}{2k}\ I_d\ R\ \phi)BG \tag{88}$$

where A = transmittance of the 3-dB attenuator, 0.5

k = Boltzmann's constant, 1.381×10^{-23} joule per kelvin

ε = electron charge, 1.602×10^{-19} coulomb

R = source resistance of noise generator, in ohms

and ϕ = frequency correction factor (see Section 5.3.).

Solving (86) and (88) for $\overline{F}$ gives

$$\overline{F} = \frac{\varepsilon\ I_d\ R\ \ \phi\ A}{2\ k\ T_o\ (1-A)}\ . \tag{89}$$

Substituting into (89) the values for A, ε, k, and T_o gives

$$\overline{F} = 20\ I_d\ R. \tag{90}$$

For R = 50 ohms, $\overline{F}$ becomes

$$\overline{F} = 1000\ I_d. \tag{91}$$

As stated above, the generator's milliammeter, which measures I_d, is calibrated directly in units of $\overline{F}$(dB) so that the measurement result is read directly without need of any computation for a 50-ohm source impedance.

When the source impedance R has a temperature T_a that is different from T_o = 290K, $\overline{F}$ is given by

$$\overline{F} = \frac{\varepsilon\ I_d\ R\ \phi\ A}{2\ k\ T_o\ (1-A)} - \frac{T_a}{T_o} + 1, \qquad (92)$$

instead of by (89). From (5) and (92),

$$\overline{T_e} = \frac{\varepsilon\ I_d\ R\ \phi\ A}{2\ k(1-A)} - T_a \qquad (93)$$

6.2.2. Fixed Source Method

The five parts of the measurement system are the following (see figure 25):

a. Transducer under test

b. Random noise generator

c. Variable attenuator

d. 3-dB fixed attenuator

e. Signal level indicator

The transducer, 3-dB fixed attenuator and signal level indicator are as described in Section 6.2.1.

The random noise generator provides two levels of available noise power per unit bandwidth, viz., kT_a when it is turned OFF, and kT_n when it is turned ON. T_a and T_n are constant from frequency f_1 to f_2. The generator source impedance is also constant in the interval (f_1, f_2).

The variable attenuator is a precision unit with enough range to meet the measurement requirements stated below. Its attenuation at a given setting is constant over the frequency interval (f_1, f_2). Its input impedance matches the output impedance of the noise generator. Its output impedance is the value for which the transducer has an average effective input noise temperature $\overline{T_e}$. The thermodynamic temperature of the attenuator is T_a.

The measurement system is assumed to be perfectly shielded against all extraneous interference.

The generator, variable attenuator, transducer, fixed attenuator, and indicator are connected together as shown in figure 25. When the generator

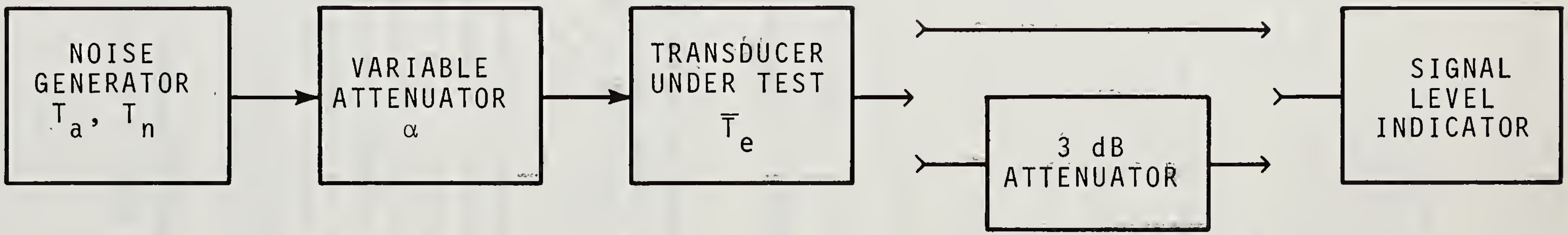


Figure 25. 3-dB/Fixed Source Method.

is turned OFF and the transducer is connected directly to the indicator, the indicator reading is I, where

$$I = k(T_a + \overline{T_e})BG \tag{94}$$

and

$$B = f_2 - f_1. \tag{75}$$

When the 3-dB fixed attenuator is inserted between the transducer and the indicator, the generator is turned ON and and the variable attenuator is adjusted to a transmittance of α that produces the same indicator reading. Then, using (35),

$$I = A\,k\left[T_n\,\alpha + T_a\,(1 - \alpha) + \overline{T_e}\right]\,BG, \tag{95}$$

where A is the transmittance of the 3-dB attenuator.

Solving (94) and (95) for $\overline{T_e}$ gives

$$\overline{T_e} = \alpha(T_n - T_a)\,\frac{A}{1-A} - T_a. \tag{96}$$

If A = 0.5, $\overline{T_e}$ becomes

$$\overline{T_e} = (T_n - T_a)\,\alpha - T_a. \tag{97}$$

If T_a = 290K, $\overline{F}$ is given by

$$\overline{F} = \alpha\left(\frac{T_n}{290} - 1\right)\frac{A}{1-A}\,. \tag{98}$$

If T_a is different from T_o = 290K, $\overline{F}$ is given by

$$\overline{F} = \alpha\left(\frac{T_n - T_a}{T_o}\right)\,\frac{A}{1-A} - \frac{T_a}{T_o} + 1 \tag{99}$$

instead of by (98).

6.3. Automatic Technique (Idealized Conditions)

The three parts of the measurement system are the following (see figure 26):

a. Transducer under test

b. Switched noise generator

c. Automatic noise figure meter (ANFM)

The transducer has two ports, an in-port and an out-port, and is linear. It has a gain G that is constant from frequency f_1 to f_2 and is zero outside of this interval. It has an $\overline{F}$ that is constant in the interval (f_1, f_2).

The noise generator is capable of being switched rapidly between two noise power levels, kT_1B and kT_2B, where

$$k\, T_1\, B < k\, T_2\, B. \tag{100}$$

T_1 and T_2 are constant over the frequency interval, (f_1, f_2). The generator source impedance is the same at both noise power levels, and is the value for which the transducer has an effective input noise temperature $\overline{T_e}$. Switching between kT_1B and kT_2B is automatically controlled by a signal from the ANFM.

The ANFM is a signal processing/display instrument that provides sufficient gain and low noise to properly process its input signals. Its circuits are designed to electronically compute and display $\overline{F}$ by one of the several methods described below. The measurement range of the instrument is adequate for the measurement.

The measurement system is assumed to be perfectly shielded against all extraneous interference.

The generator, transducer, and ANFM are connected together as shown in figure 26. When the noise generator is switched to the kT_1B level, the output power, P_1, from the transducer is given by

$$P_1 = k\,(T_1 + \overline{T_e})\,BG. \tag{101}$$

When the generator is switched to the kT_2B level, the output power is

$$P_2 = k\,(T_2 + \overline{T_e})\,BG. \tag{102}$$

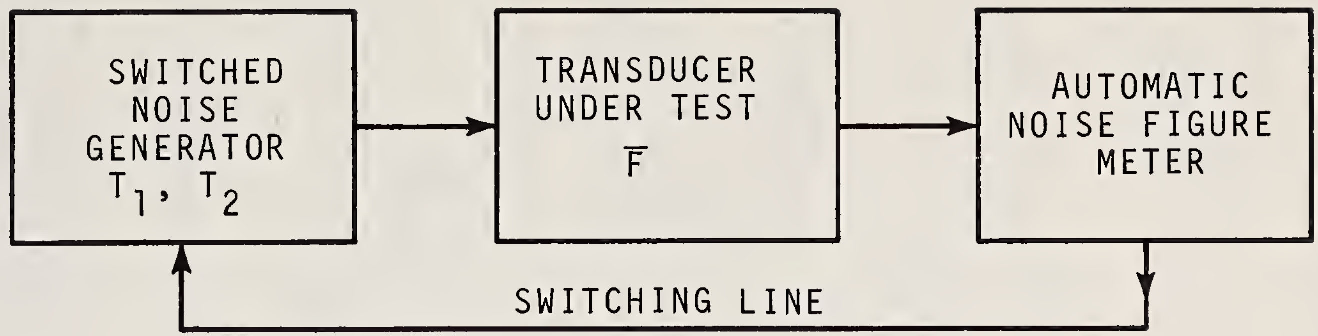


Figure 26. Automatic (ANFM) Method.

Let the gain of the ANFM be g, so that the power detected in the ANFM is gP_1 and gP_2, respectively, for the two conditions given above. Further, let the automatic gain control (AGC) system in the ANFM operate so as to hold gain g at a constant value that will produce a full scale deflection on the ANFM display meter when the noise source is generating kT_2B. From (101) and (102),

$$gP_1 = g\,k\,(T_1 + \overline{T_e})\,BG \tag{103}$$

and

$$gP_2 = g\,k\,(T_2 + \overline{T_e})\,BG. \tag{104}$$

Solving (103) and (104) for $\overline{T_e}$ gives

$$\overline{T_e} = \frac{T_2 - T_1\left(\dfrac{gP_2}{gP_1}\right)}{\dfrac{gP_2}{gP_1} - 1}. \tag{105}$$

Using (6), $\overline{F}$ is given by

$$\overline{F} = \frac{\dfrac{T_2}{290} - \dfrac{T_1}{290}\,\dfrac{gP_2}{gP_1}}{\dfrac{gP_2}{gP_1} - 1} + 1. \tag{106}$$

Note that the ratio gP_2/gP_1 is the Y-Factor.

The quantity gP_2 is fixed by circuit design as stated above. The display meter circuit is designed to deflect as a function of gP_1. Therefore, since all the factors on the right-hand side of (106) are constant except gP_1, the meter scale can be printed to indicate $\overline{F}$ or $\overline{F}$(dB) directly.

In the above description, the automatic gain control operates so as to maintain a constant value of gP_2. Note that it is also possible to design an ANFM in which the value of gP_1 is held constant, or in which the average of gP_1 and gP_2 (i.e., $g(P_1 + P_2)/2$) is held constant. Such variations as these are the basis for different designs used in commercial ANFM's.

6.4. Gain Control Technique (Idealized Conditions)

6.4.1. Variable Source Method

The three parts of the measurement system are the following (see figure 27):

a. Transducer under test

b. Temperature-limited diode (TLD) noise generator

c. Signal level indicator

The transducer has two ports, an in-port and an out-port, and is linear. It has a gain G that is adjustable over a range sufficient to meet the measurement requirements stated below. At any given setting, G is constant within the frequency interval (f_1, f_2), and is zero outside of this interval. The value of $\overline{F}$ is constant in the interval (f_1, f_2), and is independent of the gain settings required in the measurement.

The TLD noise generator has a continuously adjustable available noise power with a range sufficient for the measurement. The available noise power at any given setting is constant from frequency f_1 to f_2. The meter scale of the generator displays the TLD emission current, I_d. The generator source impedance is the value, R, for which the transducer has an average effective input noise temperature $\overline{T_e}$. The temperature of the source impedance is 290K.

The signal level indicator is noise-free, stable, and sensitive enough to provide a precise indication of the signal level applied to it. It responds equally to all frequencies in the interval (f_1, f_2). Its input impedance matches the output impedance of the transducer.

The measurement system is assumed to be perfectly shielded against all extraneous interference.

The generator, transducer, and indicator are connected together as shown in figure 27. When the TLD emission current is zero and the transducer gain is maximum, G_1, the indicator reading is I_1, where

$$I_1 = k\ (T_o + \overline{T_e})\ BG_1, \tag{107}$$

$$T_o = 290\ \text{K}, \tag{87}$$

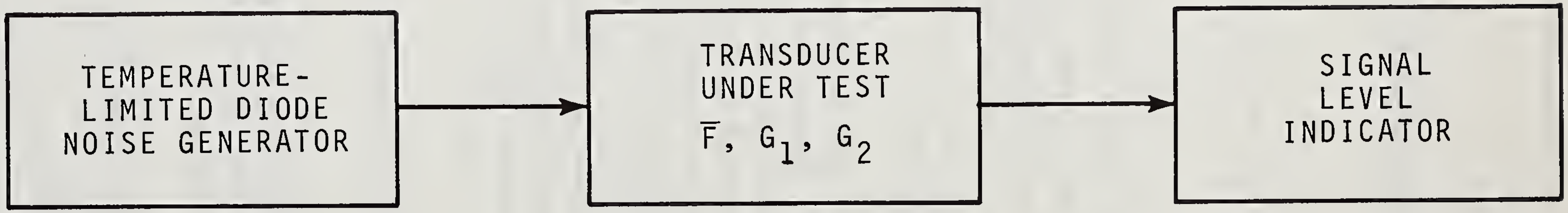


Figure 27. Gain Control/Variable Source Method.

and

$$B = f_2 - f_1. \tag{75}$$

When the TLD emission current is adjusted to a value I_{d1}, the new indicator reading is I_2, where

$$I_2 = \left[k \ (T_o + \overline{T_e}) + \tfrac{1}{2} \ \varepsilon \ I_{d1} \ R \ \phi \right] G_1 \ B \tag{108}$$

and

$$I_2 > I_1 \ . \tag{109}$$

Now, reducing G to a value G_2 to again produce the former indicator reading I_1 gives

$$I_1 = \left[k \ (T_o + \overline{T_e}) + \tfrac{1}{2} \ \varepsilon \ I_{d1} \ R \ \phi \right] G_2 \ B \tag{110}$$

where

$$G_2 < G_1. \tag{111}$$

Finally, increasing the TLD emission current to a value I_{d2} that again produces the larger indicator reading I_2 gives

$$I_2 = \left[k \ (T_o + \overline{T_e}) + \tfrac{1}{2} \ \varepsilon \ I_{d2} \ R \ \phi \right] G_2 \ B. \tag{112}$$

Solving (107), (108), (110), and (112) for $\overline{T_e}$ gives

$$\overline{T_e} = \frac{\varepsilon \ (I_{d1})^2 \ R \ \phi}{2 \ k \ (I_{d2} - 2 \ I_{d1})} - T_o. \tag{113}$$

Using (6), $\overline{F}$ is given by

$$\overline{F} = \frac{\varepsilon \ (I_{d1})^2 \ R \ \phi}{2 \ k \ T_o \ (I_{d2} - 2 \ I_{d1})} \tag{114}$$

or,

$$\overline{F} = \frac{20\ (I_{d1})^2\ R\ \phi}{I_{d2} - 2\ I_{d1}}\ . \tag{115}$$

When the source impedance R has a temperature T_a that is different from T_o = 290K, $\overline{T_e}$ and $\overline{F}$ are given by

$$\overline{T_e} = \frac{\varepsilon\ (I_{d1})^2\ R\ \phi}{2\ k\ (I_{d2} - 2\ I_{d1})} - T_a \tag{116}$$

and

$$\overline{F} = \frac{\varepsilon\ (I_{d1})^2\ R\ \phi}{2\ k\ T_o\ (I_{d2} - 2\ I_{d1})} - \frac{T_a}{T_o} + 1, \tag{117}$$

instead of by (113) and (114).

6.4.2. Fixed Source Method

The four parts of the measurement system are the following (see figure 28):

a. Transducer under test

b. Random noise generator

c. Variable attenuator

d. Signal level indicator

The noise generator and variable attenuator have the characteristics described in 6.2.2., and the transducer and signal level indicator have the characteristics described in 6.4.1. When the generator noise temperature is T_a, the power meter reading is P_{o1}, where

$$P_{o1} = k\ (T_a + \overline{T_e})\ G_1\ B. \tag{118}$$

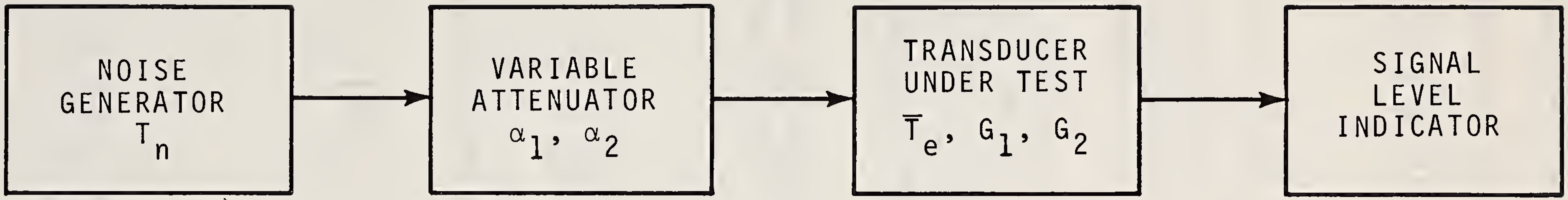


Figure 28. Gain Control/Fixed Source Method.

As in 6.4.1., we can continue by writing

$$P_{o2} = k \left[T_n \ \alpha_1 + T_a(1-\alpha_1) + \overline{T_e} \right] G_1 \ B \tag{119}$$

$$P_{o1} = k \left[T_n \ \alpha_1 + T_a(1-\alpha_1) + \overline{T_e} \right] G_2 \ B \tag{120}$$

and

$$P_{o2} = k \left[T_n \ \alpha_2 + T_a(1-\alpha_2) + \overline{T_e} \right] G_2 \ B, \tag{121}$$

where

$$\alpha_1 < \alpha_2 < 1 \tag{122}$$

and

$$G_2 < G_1 \ . \tag{111}$$

Solving (118), (119), (120), and (121) for $\overline{T_e}$ gives

$$\overline{T_e} = (T_n - T_a) \ \frac{(\alpha_1)^2}{\alpha_2 - 2\alpha_1} - T_a. \tag{123}$$

If T_a = 290K, $\overline{F}$ is given by

$$\overline{F} = \left(\frac{T_n}{T_o} - 1 \right) \frac{(\alpha_1)^2}{\alpha_2 - 2\alpha_1} \quad . \tag{124}$$

If T_a is different from T_o = 290K, $\overline{F}$ is given by

$$\overline{F} = \left(\frac{T_n - T_a}{T_o} \right) \frac{(\alpha_1)^2}{\alpha_2 - 2\alpha_1} - \frac{T_a}{T_o} + 1, \tag{125}$$

instead of by (124).

6.5. CW Technique (Idealized Conditions)

The three parts of the measurement system are the following (see figure 29):

a. Transducer under test

b. CW signal generator

c. Power meter

The transducer has two ports, an in-port and an out-port, and is linear. It has a gain G that is constant from frequency f_1 to f_2 and is zero outside of this interval. It has an $\overline{F}$ that is constant in the interval (f_1, f_2).

The CW generator has both adjustable frequency and adjustable output power level, the ranges of which are adequate for the measurement requirements stated below. The CW signal is a pure sinusoid with no modulation or spurious components. The generator source impedance is the value for which the transducer has an average effective input noise temperature of $\overline{T_e}$. The thermodynamic temperature of the source impedance is T_a.

The power meter is noise free, linear, and sensitive enough to measure the noise power levels available from the transducer out-port. Its sensitivity is constant over the frequency interval (f_1, f_2). No reflection of power occurs at its in-port.

The measurement system is assumed to be perfectly shielded against all extraneous interference.

The generator, transducer, and power meter are connected together as shown in figure 29. When the CW signal level is zero, the power meter reading is P_1, where

$$P_1 = k\ (T_a + \overline{T_e})\ GB, \tag{126}$$

and

$$B = f_2 - f_1. \tag{75}$$

When the available CW power from the generator is set to P_s, and its frequency is set to the measurement frequency, f_o, the power meter reading is P_2, where

$$P_2 = k\ (T_a + \overline{T_e})\ GB + P_s\ G. \tag{127}$$

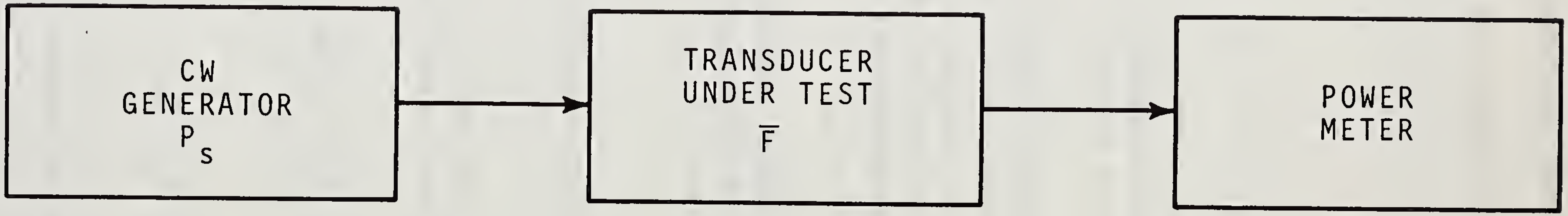


Figure 29. CW Technique.

Solving (126) and (127) for $\overline{T_e}$ gives

$$\overline{T_e} = \frac{P_s}{k\ B\left(\frac{P_2}{P_1} - 1\right)} - T_a. \tag{128}$$

Using (6), $\overline{F}$ is given by

$$\overline{F} = \frac{P_s}{k\ T_o\ B\left(\frac{P_2}{P_1} - 1\right)} - \frac{T_a}{T_o} + 1. \tag{129}$$

If $T_a = T_o = 290\mathrm{K}$, $\overline{F}$ is given by

$$\overline{F} = \frac{P_s}{4\mathrm{x}10^{-21}\ B\left(\frac{P_2}{P_1} - 1\right)}. \tag{130}$$

6.6. Tangential Technique (Idealized Conditions)

The four parts of the measuring system are the following (see figure 30):

a. Transducer under test

b. Pulsed CW generator

c. Linear detector or square-law detector

d. Oscilloscope

The transducer has two ports, an in-port and an out-port, and is linear. It has a gain G that is constant from frequency f_1 to f_2 and is zero outside of this interval. It has an $\overline{F}$ that is constant in the interval (f_1, f_2).

The CW generator has both adjustable frequency and adjustable output power level, the ranges of which are adequate for the measurement requirements stated below. The CW signal is 100% modulated with a square wave voltage. No other modulation or spurious components exist on the signal. The generator source impedance is the value for which the transducer has an average effective input noise temperature $\overline{T_e}$. The temperature of the source impedance is 290K.

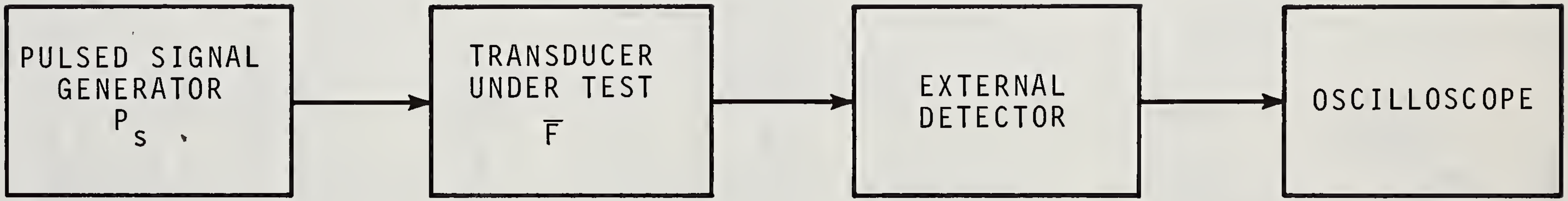


Figure 30. Tangential Technique.

The linear detector follows the law

$$v_o = a\ v_i \quad \text{for } v_i \geqslant o \tag{131}$$

$$= o \quad \text{for } v_i \leqslant o \tag{132}$$

where v_i and v_o are the input and output voltages, respectively, and a is a characteristic parameter of the detector. The value of a is constant over the frequency interval (f_1, f_2).

The square-law detector follows the law

$$v_o = b\ v_i^2 \tag{133}$$

where b is a characteristic parameter of the detector. The value of b is constant over the frequency interval (f_1, f_2).

The oscilloscope is noise-free, stable, and sensitive enough to provide a clear display of the signal levels applied to it. It responds equally to all frequencies in the output voltages from both detectors.

The measurement system is assumed to be perfectly shielded against all extraneous interference.

The generator, transducer, detector, and oscilloscope are connected together as shown in figure 30. When the CW signal level is zero, the power delivered to the detector is P_1, where

$$P_1 = k\ (T_o + \overline{T_e})\ GB \tag{134}$$

and

$$B = f_2 - f_1. \tag{75}$$

When the available CW power from the modulated signal generator is set to P_s during the ON period of modulation, and the CW frequency is set to the measurement frequency, f_o, the power delivered to the detector is P_2, where

$$P_2 = k\ (T_o + \overline{T_e})\ GB + P_sG. \tag{135}$$

P_1 is entirely noise power. P_2 is a mixture of noise power, $k(T_o + \overline{T_e})GB$, and CW power, P_sG. The output signal-to-noise power ratio, $(S/N)_o$, when the CW generator is ON, is

$$(S/N)_o = \frac{P_s}{k\ (T_o + \overline{T}_e)\ B}\ , \tag{136}$$

and the input signal-to-noise power ratio, $(S/N)_i$, is

$$(S/N)_i = \frac{P_s}{k\ T_o\ B}\ . \tag{137}$$

From one definition of $\overline{F}$, Section 2.1., and using (137),

$$\overline{F} = \frac{(S/N)_i}{(S/N)_o} = \frac{P_s}{k\ T_o\ B\ (S/N)_o}\ . \tag{138}$$

When the generator source impedance has a temperature, T_a, that is different from T_o = 290K, $\overline{F}$ is given by

$$\overline{F} = \frac{P_s}{k\ T_o\ B\ (S/N)_o} - \frac{T_a}{T_o} + 1. \tag{139}$$

$(S/N)_o$ is determined empirically for a "tangential pattern" display on the oscilloscope. The "tangential pattern" occurs when P_s is set to a value that causes the upper segment of the oscilloscope trace to be just completely above the lower segment of the trace. The value of P_s that produces the tangential pattern is not specific because the determination of when the tangential pattern is obtained is a subjective determination. However, experiments indicate that $(S/N)_o$ is within ±1 dB of 11 dB for most "linear" detectors, and within ±1 dB of 8 dB for most "square-law" detectors.

Using these values of $(S/N)_o$, $\overline{F}$ is given very approximately by

$$\overline{F} \doteq \frac{2P_s}{B} \times 10^{19} \quad \text{for linear detectors} \tag{140}$$

and

$$\overline{F} \doteq \frac{4P_s}{B} \times 10^{19} \quad \text{for square-law detectors.} \tag{141}$$

From (5) and (139),

$$\overline{T_e} = \frac{P_s}{k\ B\ (S/N)_o} - T_a. \tag{142}$$

6.7. Comparison Technique (Idealized Conditions)

The four parts of the measurement system are the following (see figure 31):

a. Transducer under test

b. Master transducer

c. CW signal generator

d. Power meter

The transducer under test has two ports, an in-port and an out-port, and is linear. It has a gain G_x that is constant from frequency f_1 to f_2, and is zero outside this interval. It has an $\overline{F} = \overline{F_x}$ that is constant in the interval (f_1, f_2), and which obtains when its in-port is terminated by an impedance Z_s.

The master transducer is the same as the transducer under test except that it has a gain G_m and $\overline{F} = \overline{F_m}$.

The CW generator has both adjustable frequency and adjustable output power level, the ranges of which are adequate for the measurement requirements stated below. The CW signal is a pure sinusoid with no modulation or spurious components. The generator source impedance, Z_s, is the value for which the transducers have average effective input noise temperatures of $\overline{T_{ex}}$ and $\overline{T_{em}}$. The thermodynamic temperature of the source impedance is T_a.

The power meter is noise-free, linear, and sensitive enough to measure the noise power levels available from the transducer out-port. Its sensitivity is constant over the frequency interval (f_1, f_2). No reflection of power occurs at its in-port.

The measurement system is assumed to be perfectly shielded against all extraneous interference.

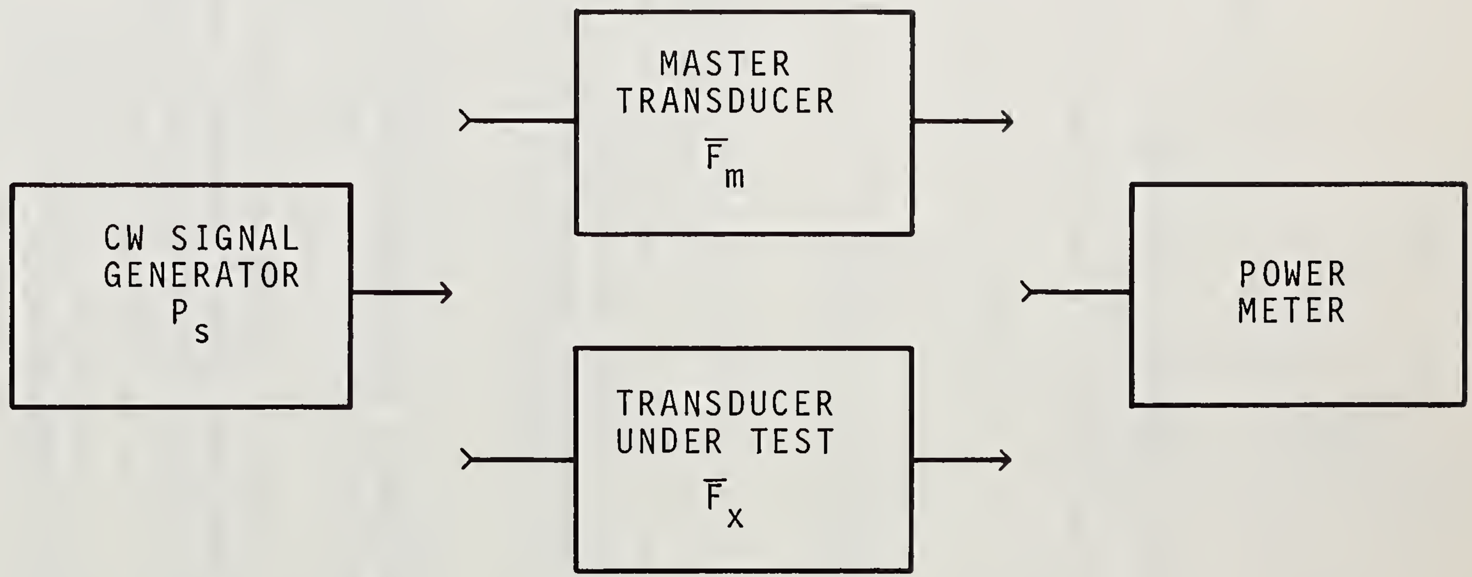


Figure 31. Comparison Technique.

The generator, transducers, and power meter are connected together as shown in figure 31. When the generator is connected to the master transducer and the CW signal level is zero, the power meter reading is P_{m1}, where

$$P_{m1} = k\ (T_a + \overline{T_{em}})\ B\ G_m \tag{143}$$

and

$$B = f_2 - f_1. \tag{75}$$

When the available CW power from the generator is set to P_s, and its frequency is set to the measurement frequency, f_o, the power meter reading is P_{m2}, where

$$P_{m2} = k\ (T_a + \overline{T_{em}})\ B\ G_m + P_s\ G_m. \tag{144}$$

When the generator is connected to the transducer under test and the CW signal level is zero, the power meter reading is P_{x1}, where

$$P_{x1} = k\ (T_a + \overline{T_{ex}})\ B\ G_x. \tag{145}$$

Similarly for a CW power level of P_s,

$$P_{x2} = k\ (T_a + \overline{T_{ex}})\ B\ G_x + P_s\ G_x. \tag{146}$$

Solving (143) and (144) for $\overline{T_{em}}$ gives

$$\overline{T_{em}} = \frac{P_s}{k\ B\left(\dfrac{P_{m2}}{P_{m1}} - 1\right)} - T_a, \tag{147}$$

so that, from (6),

$$\overline{F_m} = \frac{P_s}{k\ T_o\ B\left(\dfrac{P_{m2}}{P_{m1}} - 1\right)} - \frac{T_a}{T_o} + 1. \tag{148}$$

If $T_a = T_o = 290\text{K}$, $\overline{F_m}$ is given by

$$\overline{F_m} = \frac{P_s}{k\ T_o\ B\left(\frac{P_{m2}}{P_{m1}} - 1\right)} . \tag{149}$$

Similarly, from (145) and (146),

$$\overline{T_{ex}} = \frac{P_s}{k\ B\left(\frac{P_{x2}}{P_{x1}} - 1\right)} - T_a, \tag{150}$$

so that, from (6),

$$\overline{F_x} = \frac{P_s}{k\ T_o\ B\left(\frac{P_{x2}}{P_{x1}} - 1\right)} - \frac{T_a}{T_o} + 1, \tag{151}$$

and, if $T_a = T_o = 290K$, $\overline{F_x}$ is given by

$$\overline{F_x} = \frac{P_s}{k\ T_o\ B\left(\frac{P_{x2}}{P_{x1}} - 1\right)} . \tag{152}$$

From (149) and (152), $\overline{F_x}$ is given in terms of $\overline{F_m}$ by the equation

$$\overline{F_x} = \overline{F_m}\ \frac{(P_{m2} - P_{m1})\ P_{x1}}{(P_{x2} - P_{x1})\ P_{m1}} . \tag{153}$$

If $P_{m2} >> P_{m1}$ and $P_{x2} >> P_{x1}$, $\overline{F}_x$ is approximately

$$\overline{F_x} \doteq \overline{F_m}\ \frac{P_{m2}\ P_{x1}}{P_{m1}\ P_{x2}} . \tag{154}$$

From (147) and (150), $\overline{T_{ex}}$ is given in terms of $\overline{T_{em}}$ by the equation

$$\overline{T_{ex}} = \overline{T_{em}} \; \frac{\dfrac{P_s \; P_{x1}}{P_{x2} - P_{x1}} - k \; T_a \; B}{\dfrac{P_s \; P_{m1}}{P_{m2} - P_{m1}} - k \; T_a \; B} \; . \tag{155}$$

If $P_{x2} >> P_{x1}$, $P_{m2} >> P_{m1}$, and if $P_s \; P_{x1}$ and $P_s \; P_{m1}$ are much greater than $k \; T_a \; B$, $\overline{T_{ex}}$ is approximately

$$\overline{T_{ex}} \doteq \overline{T_{em}} \; \frac{P_{m2} \; P_{x1}}{P_{m1} \; P_{x2}} \; . \tag{156}$$

The uncertainty in the measured value of $\overline{F}$ or $\overline{T_e}$ is found by considering all of the significant sources of uncertainty and combining their individual contributions by means of an error equation. In this section we will discuss the error equation for each of the measurement techniques, and also we will briefly discuss the various sources of measurement uncertainty. General comments on the treatment of errors are found in Appendix B.

The basic error equations are derived from the working equations given in Section 6. The first-order (linear) error equation usually provides sufficient accuracy for most measurement requirements. Worked examples that illustrate the use of the working and error equations are found in Section 8.

7.1. Y-Factor Technique

7.1.1. Power Meter Method

The working equation for the Y-Factor/power meter method is the following:

$$\overline{T_e} = \frac{T_h - YT_c}{Y-1} . \tag{79}$$

The uncertainty, $\delta\overline{T_e}$, in $\overline{T_e}$ comes from uncertainties in T_h, T_c, and Y. The uncertainty in Y comes from the uncertainty in measuring P_h and P_c. The first-order approximation of $\delta\overline{T_e}$ is

$$\delta\overline{T_e} = \delta T_h \frac{\partial\overline{T_e}}{\partial T_h} + \delta T_c \frac{\partial\overline{T_e}}{\partial T_c} + \delta Y \frac{\partial\overline{T_e}}{\partial Y} , \tag{157}$$

where δT_h, δT_c, and δY are the uncertainties in the values of T_h, T_c, and Y, respectively, and the partial derivatives are "uncertainty coefficents" corresponding to each of these quantities (more on this below). $\delta\overline{T_e}$ is evaluated by evaluating each uncertainty and uncertainty coefficient on the right-hand side of (157).

Uncertainties δT_h and δT_c are obtained from the accuracy specification of the noise generators. This may be obtained from the manufacturer, from a calibration of the generators, or by computing the generator noise temperature uncertainty from its physical characteristics.

Uncertainty δY is comprised of several parts as follows:

a. Power meter inaccuracy

b. Source impedance differences

c. Non-linearity

d. Leakage

e. Spurious response

f. Gain fluctuations

g. Connector and line imperfections

The part due to the power meter inaccuracy is obtained from the manufacturer's specifications on the meter. The remaining parts of δY are discussed in Section 5 and are often very difficult to determine. With care, in some measurement situations these parts can be made less significant than the power meter inaccuracy, in which case they are ignored in the determination of δY. Since Y is the ratio of two power measurements, the uncertainties in these two measurements must be pooled properly to give δY (see Appendix B).

The uncertainty coefficients are obtained by differentiating (79) thus:

$$\frac{\partial \overline{T_e}}{\partial T_h} = \frac{1}{Y-1} \tag{158}$$

$$\frac{\partial \overline{T_e}}{\partial \overline{T_c}} = -\frac{Y}{Y-1} \tag{159}$$

$$\frac{\partial \overline{T_e}}{\partial Y} = -\frac{T_h - T_c}{(Y-1)^2} \tag{160}$$

7.1.2. Attenuator Method

The working equation, error equation, and uncertainty coefficients for the Y-factor/attenuator method are the same as for the Y-factor/power meter method (Section 7.1.1). The only difference is that the variable attenuator inaccuracy is used where the power meter inaccuracy was used in the previous discussion.

7.2. 3-dB Technique

7.2.1. Variable Source Method

The working equation for this 3-dB method is the following:

$$\overline{F} = \frac{\varepsilon\ I_d\ R\ \phi\ A}{2k\ T_o(1-A)} - \frac{T_a}{T_o} + 1. \tag{92}$$

The uncertainty, $\delta\overline{F}$, in $\overline{F}$ comes from uncertainties δI_d, δR, $\delta\phi$, δA, and δT_a in I_d, R, $\phi(f)$, A, and T_a, respectively. The first-order approximation of $\delta\overline{F}$ is

$$\delta\overline{F} = \delta I_d \frac{\partial\overline{F}}{\partial I_d} + \delta R \frac{\partial\overline{F}}{\partial R} + \delta\phi \frac{\partial\overline{F}}{\partial\phi} + \delta A \frac{\partial\overline{F}}{\partial A} + \delta T_a \frac{\partial\overline{F}}{\partial T_a} . \tag{161}$$

The uncertainty δI_d represents the uncertainty in the calibration of the emission current milliammeter, whereas $\delta\phi$ represents the uncertainty in the correction to be applied to I_d because of the effects of the various phenomena cited in 4.1.1.2. The uncertainty δR is obtained from the tolerances on the components that comprise the source impedance of the generator. δR may be reduced by measurement of the source impedance with accurate instruments. δA is obtained from the manufacturer's specifications on the 3 dB attenuator. The uncertainty δT_a is obtained by estimation or by measurement.

Measurement uncertainties resulting from system imperfections discussed in Section 5 are often difficult to determine. Their effect enters the working and error equations through the factor A which then takes on an expanded meaning to include these system parameters as well as the 3 dB attenuator transmittance. With care, the effect of these imperfections can be made insignificant in comparison with other uncertainties.

The uncertainty coefficients are obtained by differentiating (92) thus:

$$\frac{\partial\overline{F}}{\partial I_d} = \frac{\varepsilon}{2kT_o} R\ \phi\ \frac{A}{(1-A)} \tag{162}$$

$$\frac{\partial\overline{F}}{\partial R} = \frac{\varepsilon}{2kT_o} I_d\ \phi\ \frac{A}{(1-A)} \tag{163}$$

$$\frac{\partial \overline{F}}{\partial \phi} = \frac{\varepsilon}{2kT_o} I_d R \frac{A}{(1-A)} \tag{164}$$

$$\frac{\partial \overline{F}}{\partial A} = \frac{\varepsilon I_d R \phi}{2k T_o (1-A)^2} \tag{165}$$

$$\frac{\partial \overline{F}}{\partial T_a} = - \frac{1}{T_o} \tag{166}$$

7.2.2. Fixed Source Method

The working equation for this 3-dB method is the following:

$$\overline{T_e} = \alpha (T_n - T_a) \frac{A}{1-A} - T_a. \tag{96}$$

The uncertainty, $\delta\overline{T_e}$, in $\overline{T_e}$ comes from uncertainties $\delta\alpha, \delta T_n$, δT_a, and δA in α, T_n, T_a, and A, respectively. The first-order approximation of $\delta\overline{T_e}$ is

$$\delta\overline{T_e} = \delta\alpha \frac{\partial \overline{T_e}}{\partial \alpha} + \delta T_n \frac{\partial \overline{T_e}}{\partial T_n} + \delta T_a \frac{\partial \overline{T_e}}{\partial T_a} + \delta A \frac{\partial \overline{T_e}}{\partial A} . \tag{167}$$

Uncertainty δT_n is obtained from the specifications of the noise generator. δT_a is obtained by estimating or otherwise determining the uncertainty in the value of T_a. $\delta\alpha$ and δA are obtained from the specifications of the variable and the 3 dB fixed attenuators, respectively. Refer to Section 7.2.1. for a brief discussion of how to determine uncertainties resulting from system imperfections.

The uncertainty coefficients are obtained by differentiating (96) thus:

$$\frac{\partial \overline{T_e}}{\partial \alpha} = (T_n - T_a) \frac{A}{1-A} \tag{168}$$

$$\frac{\partial \overline{T_e}}{\partial T_n} = \frac{\alpha A}{1-A} \tag{169}$$

$$\frac{\partial \overline{T_e}}{\partial T_a} = -\left(\frac{\alpha A}{1-A} + 1\right) \tag{170}$$

$$\frac{\partial \overline{T_e}}{\partial A} = \frac{\alpha(T_n - T_a)}{(1-A)^2} \tag{171}$$

7.3. Automatic Technique

The working equation for the automatic technique is the following:

$$\overline{F} = \frac{\frac{T_2}{290} - \frac{T_1}{290}\frac{gP_2}{gP_1}}{\frac{gP_2}{gP_1} - 1} + 1. \tag{106}$$

The uncertainty, $\delta\overline{F}$, in $\overline{F}$ comes from uncertainties δT_1, δT_2, δgP_1, and δgP_2 in T_1, T_2, gP_1, and gP_2, respectively. The uncertainty δgP_1 includes the uncertainty, $\delta(gP_1)_d$, of detecting the power in gP_1 and the uncertainty, $\delta(gP_1)_i$, of displaying this detected power on the panel meter of the ANFM. The uncertainty $\delta(gP_2)$ includes the uncertainty, $\delta(gP_1)_d$, of detecting the power in gP_2, and the uncertainty, $\delta(gP_2)_c$, of holding gP_2 constant with the AGC system. The first-order approximation of $\delta\overline{F}$ is

$$\delta\overline{F} = \delta T_1 \frac{\partial\overline{F}}{\partial T_1} + \delta T_2 \frac{\partial\overline{F}}{\partial T_2} + \delta gP_1 \frac{\partial\overline{F}}{\partial gP_1} + \delta gP_2 \frac{\partial\overline{F}}{\partial gP_2}\ . \tag{172}$$

Uncertainties δT_1 and δT_2 are obtained from the specified accuracy of the noise generator for its two output power levels. Uncertainties δgP_1 and δgP_2 are obtained from the manufacturer of the ANFM, or from measurements made on the ANFM circuits.

In addition to these uncertainties, other measurement errors occur that do not appear explicitly in the error equation. They depend upon the particular ANFM design and upon the measurement set-up. Examples include errors due to cable loss, VSWR, frequency, termination temperature, insertion loss (gas-discharge sources), and image response. A thorough analysis of all of these errors has not been made, and is beyond the scope of this guide. The manufacturer's instruction book on a given automatic noise figure meter usually has an adequate treatment of the more important of these errors.

The uncertainty coefficients are obtained by differentiating (106) thus:

$$\frac{\partial \bar{F}}{\partial T_1} = \frac{gP_2}{290(gP_2 - gP_1)} \tag{173}$$

$$\frac{\partial \bar{F}}{\partial T_2} = \frac{gP_1}{290(gP_2 - gP_1)} \tag{174}$$

$$\frac{\partial \bar{F}}{\partial gP_1} = \frac{(T_2 - T_1)\ gP_2}{290(gP_2 - gP_1)^2} \tag{175}$$

$$\frac{\partial \bar{F}}{\partial gP_2} = \frac{(T_1 - T_2)\ gP_1}{290(gP_2 - gP_1)^2} \tag{176}$$

7.4. Gain Control Technique

7.4.1. Variable Source Method

The working equation for the gain control technique, using a TLD noise generator, is the following:

$$\bar{F} = \frac{\varepsilon (I_{d1})^2\ R\ \phi}{2kT_o\ (I_{d2} - 2I_{d1})} - \frac{T_a}{T_o} + 1. \tag{117}$$

The uncertainty, $\delta\bar{F}$, in $\bar{F}$ comes from uncertainties, δI_{d1}, δI_{d2}, δR, $\delta\phi$, and δT_a in I_{d1}, I_{d2}, R, ϕ, and T_a, respectively. The first-order approximation of $\delta\bar{F}$ is

$$\delta\bar{F} = \delta I_{d1} \frac{\partial \bar{F}}{\partial I_{d1}} + \delta I_{d2} \frac{\partial \bar{F}}{\partial I_{d2}} + \delta R \frac{\partial \bar{F}}{\partial R} + \delta\phi \frac{\partial \bar{F}}{\partial \phi} + \delta T_a \frac{\partial \bar{F}}{\partial T_a} . \tag{177}$$

The uncertainties δI_{d1} and δI_{d2} represent the uncertainty in the calibration of the emission current milliammeter, whereas $\delta\phi$ represents the uncertainty in the correction to be applied to I_d because of the effects of the various phenomena cited in 4.1.1.2. The uncertainty δR is obtained from the tolerances on the components that comprise the source impedance of the generator. δR may be reduced by measurement of the source impedance with accurate instruments. The uncertainty δT_a is obtained by estimation or by measurement.

The uncertainty coefficients are obtained by differentiating (117) thus:

$$\frac{\partial \overline{F}}{\partial I_{d1}} = \frac{\varepsilon\ R\ \phi\ I_{d1}\ (I_{d2} - I_{d1})}{k\ T_o\ (I_{d2} - 2I_{d1})^2} \tag{178}$$

$$\frac{\partial \overline{F}}{\partial I_{d2}} = - \frac{\varepsilon\ R\ \phi\ (I_{d1})^2}{2\ k\ T_o (I_{d2} - 2I_{d1})^2} \tag{179}$$

$$\frac{\partial \overline{F}}{\partial R} = \frac{\varepsilon (I_{d1})^2\ \phi}{2\ k\ T_o\ (I_{d2} - 2I_{d1})} \tag{180}$$

$$\frac{\partial \overline{F}}{\partial \phi} = \frac{\varepsilon (I_{d1})^2\ R}{2\ k\ T_o\ (I_{d2} - 2I_{d1})} \tag{181}$$

$$\frac{\partial \overline{F}}{\partial T_a} = - \frac{1}{T_o} \tag{182}$$

7.4.2. Fixed Source Method

The working equation for this gain control technique is the following:

$$\overline{T_e} = (T_n - T_a) \frac{(\alpha_1)^2}{\alpha_2 - 2\alpha_1} - T_a. \qquad (123)$$

The uncertainty, $\delta\overline{T_e}$, in $\overline{T_e}$ comes from the uncertainties, δT_n, δT_a, $\delta\alpha_1$, and $\delta\alpha_2$, in T_n, T_a, α_1, and α_2, respectively. The first-order approximation of $\delta\overline{T_e}$ is

$$\delta\overline{T_e} = \delta T_n \frac{\partial\overline{T_e}}{\partial T_n} + \delta T_a \frac{\partial\overline{T_e}}{\partial T_a} + \delta\alpha_1 \frac{\partial\overline{T_e}}{\partial\alpha_1} + \delta\alpha_2 \frac{\partial\overline{T_e}}{\partial\alpha_2} . \qquad (183)$$

Uncertainty δT_n is obtained from the specifications of the noise generator. δT_a is obtained by estimating or otherwise determining the uncertainty in the value of T_a. $\delta\alpha_1$ and $\delta\alpha_2$ are obtained from the specifications of the variable attenuator. Refer to Section 7.2.1. for a brief discussion of how to treat uncertainties resulting from system imperfections.

The uncertainty coefficients are obtained by differentiating (123) thus:

$$\frac{\partial\overline{T_e}}{\partial T_n} = \frac{(\alpha_1)^2}{\alpha_2 - 2\alpha_1} \qquad (184)$$

$$\frac{\partial\overline{T_e}}{\partial T_a} = - \frac{(\alpha_1)^2}{\alpha_2 - 2\alpha_1} - 1 \qquad (185)$$

$$\frac{\partial\overline{T_e}}{\partial\alpha_1} = \frac{2(T_n - T_a)(\alpha_1 \alpha_2 - (\alpha_1)^2)}{(\alpha_2 - 2\alpha_1)^2} \qquad (186)$$

$$\frac{\partial\overline{T_e}}{\partial\alpha_2} = - \frac{(\alpha_1)^2 (T_n - T_a)}{(\alpha_2 - 2\alpha_1)^2} \qquad (187)$$

7.5. CW Technique

The working equation for the CW technique is the following:

$$\overline{F} = \frac{P_s}{kT_oB\left(\frac{P_2}{P_1} - 1\right)} - \frac{T_a}{T_o} + 1. \qquad (129)$$

The uncertainty, $\delta\overline{F}$, in $\overline{F}$, comes from uncertainties δP_s, δT_a, δB, δF_1, and δP_2, in P_s, T_a, B, P_1, and P_2, respectively. The first-order approximation of $\delta\overline{F}$ is

$$\delta\overline{F} = \delta P_s \frac{\partial\overline{F}}{\partial P_s} + \delta T_a \frac{\partial\overline{F}}{\partial T_a} + \delta B \frac{\partial\overline{F}}{\partial B} + \partial P_1 \frac{\partial\overline{F}}{\partial P_1} + \partial P_2 \frac{\partial\overline{F}}{\partial P_2} . \tag{188}$$

Uncertainty δP_s is obtained from the specifications of the CW generator. δT_a is obtained by estimating or otherwise determining the uncertainty in the value of T_a. δB is obtained as described in Appendix C. δP_1 and δP_2 are obtained from the specifications of the power meter.

The uncertainty coefficients are obtained by differentiating (129) thus:

$$\frac{\partial\overline{F}}{\partial P_s} = \frac{1}{kT_oB\left(\frac{P_2}{P_1} - 1\right)} \tag{189}$$

$$\frac{\partial\overline{F}}{\partial T_a} = - \frac{1}{T_o} \tag{190}$$

$$\frac{\partial\overline{F}}{\partial B} = - \frac{P_s}{kT_oB^2\left(\frac{P_2}{P_1} - 1\right)} \tag{191}$$

$$\frac{\partial\overline{F}}{\partial P_1} = \frac{P_s P_2}{kT_oB\ (P_2 - P_1)^2} \tag{192}$$

$$\frac{\partial\overline{F}}{\partial P_2} = - \frac{P_s P_1}{kT_oB\ (P_2 - P_1)^2} \tag{193}$$

7.6. Tangential Technique

The working equation for the tangential technique is the following:

$$\overline{F} = \frac{P_s}{kT_oB(S/N)_o} - \frac{T_a}{T_o} + 1. \tag{139}$$

The uncertainty, $\delta\overline{F}$, in $\overline{F}$ comes from uncertainties δP_s, δT_a, δB, and $\delta(S/N)_o$ in P_s, T_a, B, and $(S/N)_o$, respectively. The first-order approximation of $\delta\overline{F}$ is

$$\delta\overline{F} = \delta P_s \frac{\partial\overline{F}}{\partial P_s} + \delta T_a \frac{\partial\overline{F}}{\partial T_a} + \delta B \frac{\partial\overline{F}}{\partial B} + \delta(S/N)_o \frac{\partial\overline{F}}{\partial(S/N)_o} \; . \tag{194}$$

Uncertainty δP_s is obtained from the specifications of the CW generator. δT_a is obtained by estimating or otherwise determining the uncertainty in the value of T_a. δB is obtained as described in Appendix C. $\delta(S/N)_o$ is obtained either from the values given in Section 6.6., or by measurement as indicated therein.

The uncertainty coefficients are obtained by differentiating (139) thus:

$$\frac{\partial\overline{F}}{\partial P_s} = \frac{1}{kT_oB(S/N)_o} \tag{195}$$

$$\frac{\partial\overline{F}}{\partial T_a} = -\frac{1}{T_o} \tag{196}$$

$$\frac{\partial\overline{F}}{\partial B} = -\frac{P_s}{kT\ B^2(S/N)_o} \tag{197}$$

$$\frac{\partial\overline{F}}{\partial(S/N)_o} = -\frac{P_s}{kT_oB(S/N)_o^{\ 2}} \tag{198}$$

7.7. Comparison Technique

The working equation for the comparison technique is the following:

$$\overline{F}_x \doteq \overline{F}_m \frac{P_{m2}\ P_{x1}}{P_{m1}\ P_{x2}} \; . \tag{154}$$

The uncertainty, $\delta\overline{F}_x$, in $\overline{F}_x$ comes from uncertainties $\delta\overline{F}_m$, δP_{m1}, δP_{m2}, δP_{x1}, and δP_{x2} in $\overline{F}_m$, P_{m1}, P_{m2}, P_{x1}, and P_{x2}, respectively. The first-order approximation of $\delta\overline{F}_x$ is

$$\delta\bar{F}_x = \delta\bar{F}_m \frac{\partial\bar{F}_x}{\partial\bar{F}_m} + \delta P_{m1} \frac{\partial\bar{F}_x}{\partial P_{m1}} + \delta P_{m2} \frac{\partial\bar{F}_x}{\partial P_{m2}} + \delta P_{x1} \frac{\partial\bar{F}_x}{\partial P_{x1}} + \delta P_{x2} \frac{\partial\bar{F}_x}{\partial P_{x2}} \quad . \tag{199}$$

Uncertainty $\delta\bar{F}_m$ is obtained from the specifications of the master transducer. δP_{m1}, δP_{m2}, δP_{x1}, and δP_{x2}, are obtained from the specifications of the power meter.

The uncertainty coefficients are obtained by differentiating (154) thus:

$$\frac{\partial\bar{F}_x}{\partial\bar{F}_m} = \frac{P_{m2}\, P_{x1}}{P_{m1}\, P_{x2}} \tag{200}$$

$$\frac{\partial\bar{F}_x}{\partial P_{m1}} = - \frac{\bar{F}_m\, P_{m2}\, P_{x1}}{(P_{m1})^2\, P_{x2}} \tag{201}$$

$$\frac{\partial\bar{F}_x}{\partial P_{m2}} = \frac{\bar{F}_m\, P_{x1}}{P_{m1}\, P_{x2}} \tag{202}$$

$$\frac{\partial F_x}{\partial P_{x1}} = \frac{\bar{F}_m\, P_{m2}}{P_{m1}\, P_{x2}} \tag{203}$$

$$\frac{\partial\bar{F}_x}{\partial P_{x2}} = - \frac{\bar{F}_m\, P_{m2}\, P_{x1}}{P_{m1}\, (P_{x2})^2} \tag{204}$$

8. MEASUREMENT EXAMPLES

This section is provided to illustrate how $\overline{F}$, $\overline{T_e}$, and their uncertainties are computed from measurement data. To do this, a hypothetical but typical two-port amplifier has been "measured" and an error analysis performed for each of the ten methods described in this Guide. These examples show not only the comparative accuracies obtained with the various methods for the assumed conditions, but they also show the relative contributions of the individual sources of error to the total measurement error. A summary of these results is given in Table IV.

It must be emphasized that these results apply only to the hypothetical conditions that are stated. In actual practice, individual circumstances may cause the accuracies that may be attained by a given measurement method to differ from those given in this section. The hypothetical amplifier has the following specifications:

f_o = 2 GHz

B = 60 MHz (no image response)

$\overline{F}$ = 6 dB

$\overline{T_e}$ = 870 K

$G(f)_o$ = 10^5 (50 dB)

Z_i = 50 Ω

Z_o = 50 Ω

Other Characteristics:

- Linear amplitude response
- No spurious responses
- No spurious signals generated
- Perfect shielding
- Matched output termination
- Proper input termination for desired $\overline{F}$, $\overline{T_e}$

TABLE IV. Summary of Measurement Examples for Ten Measurement Techniques

Transducer Characteristics	
Measurement frequency, f_o	2 GHz
Bandwidth, B	60 MHz
Average noise factor, $\overline{F}$	6 dB
Average effective input noise temperature, $\overline{T_e}$	870 K
Gain, $G(f_o)$	50 dB

1. Y-Factor/Power Meter

Error Source	Amount
Total Error in $\overline{T_e}$	9.0%
From noise source, T_h	2.6%
From noise source, T_c	0.4%
From power meter	6.0%
Total error in $\overline{F}$	6.7%

2. Y-Factor/Attenuator

Error Source	Amount
Total Error in $\overline{T_e}$	10.0%
From noise source, T_h	2.6%
From noise source, T_c	0.4%
From attenuator	7.0%
Total error in $\overline{F}$	7.5%

3. 3-dB/Variable Source

Error Source	Amount
Total Error in $\overline{F}$	17.5%
From noise source, I_d	2.0%
From noise source, R	0.5%
From noise source, ϕ	10.1%
From 3 dB attenuator	4.6%
From ambient temperature, T_a	0.3%
Total error in $\overline{T_e}$	23.3%

4. 3-dB/Fixed Source

Error Source	Amount
Total Error in $\overline{T_e}$	15.2%
From variable attenuator	6.0%
From noise source, T_n	1.6%
From ambient temperature, T_a	0.4%
From 3 dB attenuator	6.2%
Total error in $\overline{F}$	11.4%

5. Automatic Technique

Error Source	Amount
Total error in $\overline{F}$	13.2%
From noise source, T_1	0.29%
From noise source, T_2	3.9%
From measured power, gP_1	5.6%
From measured power, gP_2	3.4%
Total error in $\overline{T_e}$	17.6%

6. Gain Control/Var. Source

Error Source	Amount
Total error in $\overline{F}$	19.6%
From noise source, I_{d1}	5.4%
From noise source, I_{d2}	3.4%
From noise source, R	0.5%
From noise source, ϕ	10.1%
From ambient temperature, T_a	0.2%
Total error in $\overline{T_e}$	26.1%

7. Gain Control/Fixed Source

Error Source	Amount
Total error in $\overline{T_e}$	15.6%
From noise source, T_n	2.6%
From noise source, T_a	0.4%
From variable attenuator, α_1	12.1%
From variable attenuator, α_2	0.5%
Total error in $\overline{F}$	11.7%

8. CW Technique

Error Source	Amount
Total error in $\overline{F}$	11.5%
From signal source, P_s	2.0%
From ambient temperature, T_a	0.3%
From bandwidth, B	5.1%
From power meter, P_1	2.0%
From power meter, P_2	2.1%
Total error in $\overline{T_e}$	15.3%

9. Tangential Technique

Error Source	Amount
Total error in $\overline{F}$	33.5%
From signal source, P_s	2.0%
From ambient temperature, T_a	0.3%
From bandwidth, B	5.1%
From oscilloscope, (S/N_o)	26.1%
Total error in $\overline{T_e}$	44.7%

10. Comparison Technique

Error Source	Amount
Total error in $\overline{F}$	30.0%
From noise factor, $\overline{F}_m$	10.0%
From power meter, P_{m1}	5.0%
From power meter, P_{m2}	5.0%
From power meter, P_{x1}	5.0%
From power meter, P_{x2}	5.0%
Total error in $\overline{T_e}$	39.0%

8.1. Y-Factor/Power Meter Method

a. Given parameters:

T_h = 10,580 K (ENR*= 15.5 dB)

T_c = 300 K

δT_h = 200 K

δT_c = 3 K

δP = 2% of P

b. Let the calculated value of Y, from (77), be

Y = $\underline{9.79}$; Y(dB) = $\underline{9.98 \text{ dB}}$

c. From (79),

$$\overline{T_e} = \frac{10{,}580 - (9.79 \times 300)}{9.79 - 1} = \underline{870\text{K}}$$

d. From (158), (159), and (160),

$$\frac{\partial T_e}{\partial T_h} = \frac{1}{9.79 - 1} = \underline{0.114}$$

$$\frac{\partial \overline{T_e}}{\partial T_c} = - \frac{9.79}{9.79 - 1} = \underline{-1.114}$$

$$\frac{\partial \overline{T_e}}{\partial Y} = - \frac{10{,}580 - 300}{(9.79 - 1)^2} = \underline{-133 \text{ K}}$$

e. The uncertainty δY is taken to be the worst-case value, viz.,

$\delta Y = (2\% + 2\%) Y = 0.040 \times 9.79 = \underline{0.392}$.

f. From (157),

$\delta \overline{T_e}$ = $(200 \times 0.114) + (3 \times 1.11) + (0.392 \times 133)$

* ENR = Excess Noise Ratio

$$= 22.8 + 3.33 + 52.1 = \underline{78.2\ K}$$

g. Summary:

$Y = 9.79$ (9.98 dB)

$\overline{T_e} = 870$ K

$\delta\overline{T_e} = 78.2$ K (9.0% of $\overline{T_e}$)

Components of $\delta\overline{T_e}$:	From T_h:	22.8 K	(2.6%)
	From T_c:	3.3 K	(0.4%)
	From P :	52.1 K	(6.0%)
		78.2 K	(9.0%)

h. $\overline{F} = \dfrac{870}{290} + 1 = \underline{4.00}$ $(\underline{6.0\text{ dB}})$

$$\delta\overline{F} = \frac{\delta\overline{T_e}}{290} = \frac{78.2}{290} = \underline{0.270}$$

$$\frac{\delta\overline{F}}{\overline{F}} = \frac{0.27}{4.00} = \underline{0.067} \quad (\underline{6.7\%}\ ;\ \underline{0.28\text{ dB}})$$

8.2. Y-Factor/Attenuator Method

a. Given parameters:

$T_h = 10{,}580$ K (ENR = 15.5 dB)

$T_c = 300$ K

$\delta T_h = 200$ K

$\delta T_c = 3$ K

$\delta A(\text{dB}) = 2\%$ of $A(\text{dB})$

b. Let the calculated value of Y(dB), from (85), be

$Y(dB) = \underline{9.98\ dB}$; $Y = \underline{9.79}$

c. From (79)

$$\overline{T_e} = \frac{10{,}580 - (9.79 \times 300)}{9.79 - 1} = \underline{870\ K}$$

d. From (158), (159), and (160),

$$\frac{\partial \overline{T_e}}{\partial T_h} = \frac{1}{9.79 - 1} = \underline{0.114}$$

$$\frac{\partial \overline{T_e}}{\partial T_c} = - \frac{9.79}{9.79 - 1} = - \underline{1.114}$$

$$\frac{\partial \overline{T_e}}{\partial Y} = - \frac{10{,}580 - 300}{(9.79 - 1)^2} = \underline{-133\ K}$$

e. The uncertainty δY is given by

$\delta Y(dB) = 0.02 \times 9.98\ dB = 0.20\ dB$

$\therefore\ \delta Y = 0.047\ Y = 0.047 \times 9.79 = \underline{0.460}$

f. From (157),

$$\delta \overline{T_e} = (200 \times 0.114) + (3 \times 1.11) + (0.460 \times 133)$$

$$= 22.8 + 3.33 + 61.2 = \underline{87.3\ K}$$

g. Summary:

$Y = 9.79$ (9.98 dB)

$\overline{T_e} = 870$ K

$\delta \overline{T_e} = 87.3$ K (10.0% of $\overline{T_e}$)

Components of $\delta\overline{T_e}$:	From T_h:	22.8 K	(2.6%)
	From T_c:	3.3 K	(0.4%)
	From A :	<u>61.2 K</u>	<u>(7.0%)</u>
		87.3 K	(10.0%)

h. $\overline{F} = \frac{870}{290} + 1 = \underline{4.00} \quad \underline{(6.0\ \text{dB})}$

$$\delta\overline{F} = \frac{\delta\overline{T_e}}{290} = \frac{87.3}{290} \doteq \underline{0.301}$$

$$\frac{\delta\overline{F}}{\overline{F}} = \frac{0.301}{4.00} = \underline{0.075} \quad (\underline{7.5\%}\ ;\ \underline{0.31\ \text{dB}})$$

8.3. 3-dB/Variable Source Method

a. Given parameters:

$\phi(f) = 0.62$

$R = 50\ \Omega$

$T_a = 300$ K

$A = 0.5$

$\delta I_d = 2\%$ of I_d

$\delta\phi = 10\%$ of $\phi(f)$

$\delta R = 5\%$ of R

$\delta T_a = 3$ K

$\delta A = 0.1$ dB (2.3%)

b. Let the measured value of I_d be

$I_d = \underline{6.50\ \text{ma}}.$

Note: On an uncorrected TLD noise generator, this would indicate an $\overline{F}$ of

$\overline{F}$ = 6.50 (8.1 dB).

c. From (92),

$$\overline{F} = \frac{1.602\text{x}10^{-19} \text{ x } 50 \text{ x } 6.50 \text{ x } 10^{-3} \text{ x } 0.62}{2 \text{ x } 1.381 \text{ x } 10^{-23} \text{ x } 290} + (1 - \frac{300}{290})$$

$$= 4.033 - 0.0344 = \underline{4.00}$$

d. From (162), (163), (164), (165) and (166),

$$\frac{\partial \overline{F}}{\partial I_d} = \frac{1.602\text{x}10^{-19} \text{ x}50\text{x}0.62}{2\text{x}1.381\text{x}10^{-23} \text{ x } 290} = \underline{620 \text{ ampere}^{-1}}$$

$$\frac{\partial \overline{F}}{\partial R} = \frac{1.602\text{x}10^{-19} \text{ x}6.50 \text{ x } 10^{-3} \text{ x}0.62}{2\text{x}1.381\text{x}10^{-23} \text{ x } 290} = \underline{0.0807 \text{ ohm}^{-1}}$$

$$\frac{\partial \overline{F}}{\partial \phi} = \frac{1.602\text{x}10^{-19}\text{x}6.50\text{x}10^{-3}\text{x}50}{2\text{x}1.381\text{x}10^{-23}\text{x } 290} = \underline{6.50}$$

$$\frac{\partial \overline{F}}{\partial A} = \frac{1.602\text{x}10^{-19} \text{ x}6.50\text{x}10^{-3}\text{x}50\text{x}0.62}{2\text{x}1.381\text{x}10^{-23}\text{x}290\text{x}0.25} = \underline{16.1}$$

$$\frac{\partial \overline{F}}{\partial T_a} = - \frac{1}{290} = \underline{-0.00345 \text{ K}^{-1}}$$

e. The uncertainty δA is given by

$\delta A = 0.023 \text{ x } 0.5 = \underline{0.0115}$.

f. From (161),

$\delta \overline{F}$ = (0.02x6.50x10^{-3} x 620) + (0.05x50x0.0807) + (0.10x0.62x6.50) +

$+ (0.0115 \times 16.1) + (3 \times 0.00345)$

$= 0.0806 + 0.0202 + 0.404 + 0.185 + 0.0104 = \underline{0.700}$

$$\frac{\delta\overline{F}}{\overline{F}} = \frac{0.700}{4.00} = \underline{0.175} \rightarrow \underline{17.5\%}$$

g. Summary:

I_d = 6.50 ma

$\overline{F}$ = 4.00 (6.0 dB)

$\delta\overline{F}$ = 0.700 (17.5% of $\overline{F}$; 0.70 dB)

Components of $\delta\overline{F}$:	From I_d:	0.0806	(2.0%)
	From R :	0.0202	(0.5%)
	From $\phi(f)$:	0.404	(10.1%)
	From A :	0.185	(4.6%)
	From T_a:	0.0104	(0.3%)
		0.700	(17.5%)

h. $\overline{T_e} = 290\ (4.00 - 1) = \underline{870\ K}$

$\delta\overline{T_e} = 290 \times 0.700 = \underline{203\ K}$

$$\frac{\delta\overline{T_e}}{\overline{T_e}} = \frac{203}{870} = 0.233 \rightarrow \underline{23.3\%}$$

8.4. 3-dB/Fixed Source Method

a. Given parameters:

T_n = 10,580 K (ENR = 15.5 dB)

T_a = 300 K

A = 0.5

δT_n = 200 K

δT_a = 3 K

$\delta\alpha$ = 2% of α(dB)

δA = 0.1 dB (2.3%)

b. Let the measured value of α(dB) be

$\alpha(\text{dB}) = \underline{-\ 9.44\ \text{dB}}\ ;\ \alpha = \underline{0.1138}$

c. From (96),

$$\overline{T_e} = 0.1138 \times (10{,}580 - 300) - 300 = \underline{870\ \text{K}}$$

d. From (168), (169), (170) and (171),

$$\frac{\partial \overline{T_e}}{\partial \alpha} = (10{,}580 - 300)\ \frac{0.5}{1 - 0.5} = \underline{10{,}280\ \text{K}}$$

$$\frac{\partial \overline{T_e}}{\partial T_n} = \frac{0.1138 \times 0.5}{1 - 0.5} = \underline{0.1138}$$

$$\frac{\partial \overline{T_e}}{\partial T_a} = -\left(\frac{0.1138 \times 0.5}{1 - 0.5} + 1\right) = \underline{-\ 1.1138}$$

$$\frac{\partial \overline{T_e}}{\partial A} = \frac{0.1138(10{,}580 - 300)}{(1 - 0.5)^2} = \underline{4680\ K}$$

e. The uncertainty $\delta\alpha$ is given by

$$\delta\alpha(dB) = 0.02 \times 9.44\ dB = 0.189\ dB$$

$$\therefore\ \delta\alpha = 0.045\alpha = 0.045 \times 0.1138 = \underline{0.00506}$$

f. The uncertainty δA is given by

$$\delta A = 0.023A = 0.023 \times 0.500 = \underline{0.0115}$$

g. From (167),

$$\delta\overline{T_e} = (0.00506 \times 10{,}280) + (200 \times 0.1138) + (3 \times 1.1138) + (0.0115 \times 4680)$$

$$= 52.0 + 22.8 + 3.34 + 53.8 = \underline{131.9\ K}$$

h. Summary:

α = - 9.44 dB

$\overline{T_e}$ = 870 K

$\delta\overline{T_e}$ = 131.9K (15.2% of $\overline{T_e}$)

Components of $\delta\overline{T_e}$:			
	From α :	52.0 K	(6.0%)
	From T_n:	22.8 K	(1.6%)
	From T_a:	3.34 K	(0.4%)
	From A :	53.8 K	(6.2%)
		131.9 K	(15.2%)

i. $\overline{F} = \dfrac{870}{290} + 1 = \underline{4.00}$ $\underline{(6.0\text{ dB})}$

$$\delta\overline{F} = \frac{\delta\overline{T_e}}{290} = \frac{131.9}{290} = \underline{0.455}$$

$$\frac{\delta\overline{F}}{\overline{F}} = \frac{0.455}{4.00} = \underline{.114} \quad \underline{(11.4\%;} \quad \underline{0.47\text{ dB}})$$

8.5. Automatic Technique

a. Given parameters:

$T_1 = 300$ K

$T_2 = 10{,}580$ K (ENR = 15.5 dB)

$g = 10^4$ (40 dB)

$\delta T_1 = 3$ K

$\delta T_2 = 400$ K

$(\delta gP_1)_d = 2\%$ of gP_1

$(\delta gP_1)_i = 3\%$ of gP_1

$(\delta gP_2)_d = 2\%$ of gP_2

$(\delta gP_2)_c = 1\%$ of gP_2

(See the NOTE at the end of this example.)

b. Let the value of gP_2 be

$gP_2 = \underline{9.48\text{ mw}}.$

c. Let the indicated value of gP_1 be

$gP_1 = \underline{0.969\text{ mw}}.$

d. From (106),

$$\overline{F} = \frac{\frac{10{,}580}{290} - \frac{300}{290} \cdot \frac{9.48}{0.969}}{\frac{9.48}{0.969} - 1} + 1 = \underline{4.00}$$

e. From (173), (174), (175), and (176)

$$\frac{\partial \overline{\mathbf{F}}}{\partial T_1} = \frac{9.48 \mathrm{x} 10^{-3}}{290(9.48 - 0.969)\mathrm{x}10^{-3}} = \underline{0.00384 \ \mathrm{K}^{-1}}$$

$$\frac{\partial \overline{F}}{\partial T_2} = \frac{0.969 \mathrm{x} 10^{-3}}{290(9.48 - 0.969)\mathrm{x}10^{-3}} = \underline{0.000393 \ \mathrm{K}^{-3}}$$

$$\frac{\partial \overline{F}}{\partial gP_1} = \frac{(10{,}580 - 300)9.48\mathrm{x}10^{-3}}{290(9.48 - 0.969)^2\mathrm{x}10^{-6}} = \underline{4640 \ \mathrm{watt}^{-1}}$$

$$\frac{\partial \overline{F}}{\partial gP_2} = \frac{(300 - 10{,}580)0.969\mathrm{x}10^{-3}}{290(9.48 - 0.969)^2\mathrm{x}10^{-6}} = -\underline{474 \ \mathrm{watt}^{-1}}$$

f. From (172),

$$\delta \overline{F} = (3\mathrm{x}0.00384) + (400\mathrm{x}0.000393) +$$

$$+ (0.05\mathrm{x}0.969\mathrm{x}10^{-3}\mathrm{x}4640) + (0.03\mathrm{x}9.48\mathrm{x}10^{-3}\mathrm{x}474)$$

$$= 0.0115 + 0.157 + 0.225 + 0.135 = \underline{0.529}$$

$$\frac{\delta \overline{F}}{\overline{F}} = \frac{0.529}{4.00} = \underline{0.132} \rightarrow \underline{13.2\%}$$

g. Summary:

$$\frac{gP_1}{gP_2} = 9.71 \ (9.87 \ \mathrm{dB})$$

$\overline{F} = 4.00$ (6.0 dB)

$\delta\overline{F} = 0.529$ (13.2% of $\overline{F}$; 0.54 dB)

Components of δF:	From	T_1:	0.0115	(0.29%)
	From	T_2:	0.157	(3.9%)
	From	gP_1:	0.225	(5.6%)
	From	gP_2:	<u>0.135</u>	<u>(3.4%)</u>
			0.529	(13.2%)

h. $\overline{T_e} = 290(4.00 - 1) = \underline{870\ K}$

$\delta\overline{T_e} = 290 \times 0.529 = \underline{153\ K}$

$$\frac{\delta\overline{T_e}}{\overline{T_e}} = \frac{153}{870} = \underline{0.176} \rightarrow \underline{17.6\%}$$

NOTE: Other measurement uncertainties, discussed in Section 7.3, have not been taken into account in this example, thus making the results optimistic.

8.6. Gain Control/Variable Source Method

a. Given parameters:

$$\phi(f) = 0.62$$

$$R = 50\ \Omega$$

$$T_a = 300\ K$$

$$\delta I_d = 2\%\ \text{of}\ I_d$$

$$\delta\phi = 10\%\ \text{of}\ \phi(f)$$

$$\delta R = 5\%\ \text{of}\ R$$

$$\delta T_a = 3\ K$$

b. Let the measured values of I_d be

$I_{d1} = \underline{19.5\ ma}.$

$I_{d2} = \underline{97.6\ ma}$

c. From (117),

$$\overline{F} = \frac{1.602x10^{-19}\ x\ (19.52x10^{-3})^2\ x50x0.62}{2x1.381x10^{-23}\ x\ 290(97.58-2x19.52)x10^{-3}} - \frac{300}{290} + 1$$

$$= 4.034 - 1.034 + 1 = \underline{4.00}$$

d. From (178), (179), (180), (181), and (182),

$$\frac{\partial\overline{F}}{\partial I_{d1}} = \frac{1.602x10^{-19}\ x50x0.62x19.5\ x10^{-3}x(97.6-19.5)x10^{-3}}{1.381x10^{-23}x290(97.6-2x19.5)^2\ x10^{-6}}$$

$$= \underline{551.4\ \text{ampere}^{-1}}$$

$$\frac{\partial\overline{F}}{\partial I_{d2}} = -\frac{1.602x10^{-19}\ x\ 50\ x\ 0.62\ x\ (19.5)^2\ x\ 10^{-6}}{2x1.381x10^{-23}\ x290(97.6\ -\ 2\ x19.5)^2\ x\ 10^{-6}}$$

$$= -\underline{68.94\ \text{ampere}^{-1}}$$

$$\frac{\partial\overline{F}}{\partial R} = \frac{1.602\text{x}10^{-19} \text{ x } (19.5)^2 \text{ x } 10^{-6} \text{ x } 0.62}{2\text{x}1.381\text{x}10^{-23}\text{x}290(97.6 - 2\text{x}19.5)\text{x}10^{-3}} = \underline{0.0807 \text{ ohm}^{-1}}$$

$$\frac{\partial\overline{F}}{\partial \phi} = \frac{1.602\text{x}10^{-19} \text{ x}50 \text{ x } (19.5)^2 \text{ x}10^{-6}}{2\text{x}1.381\text{x}10^{-23} \text{ x}290(97.6 - 2 \text{ x } 19.5)\text{x}10^{-3}} = \underline{6.509}$$

$$\frac{\partial\overline{F}}{\partial T_a} = -\frac{1}{290} = \underline{-0.00345 \text{ K}^{-1}}$$

e. From (177)

$$\delta\overline{F} = (0.02\text{x}19.5\text{x}10^{-3} \text{ x } 551.4) + (0.02\text{x}97.6\text{x}10^{-3} \text{ x}68.9) +$$

$$+ (0.05\text{x}50\text{x}0.0807) + (0.1\text{x}0.62\text{x}6.509) + (3\text{x}0.00345)$$

$$= 0.215 + 0.135 + 0.0202 + 0.4036 + 0.0103 = \underline{0.784}$$

$$\frac{\delta\overline{F}}{\overline{F}} = \frac{0.784}{4.00} = \underline{0.196} \rightarrow \underline{19.6\%}$$

f. Summary:

I_{d1} = 19.5 ma

I_{d2} = 97.6 ma

$\overline{F}$ = 4.00 (6.0 dB)

$\delta\overline{F}$ = 0.784 (19.6% of $\overline{F}$; 0.78 dB)

Components of $\delta\overline{F}$:	From I_{d1}:	0.215	(5.4%)
	From I_{d2}:	0.135	(3.4%)
	From R :	0.0202	(0.5%)
	From ϕ :	0.404	(10.1%)
	From T_a :	0.0103	(0.2%)
		0.784	(19.6%)

g. $\overline{T_e} = 290\ (4.00 - 1) = \underline{870\ K}$

$\delta\overline{T_e} = 290 \times 0.784 = \underline{227\ K}$

$$\frac{\overline{\delta T_e}}{\overline{T_e}} = \frac{227}{870} = 0.261 \rightarrow \underline{26.1\%}$$

8.7. Gain Control/Fixed Source Method

a. Given parameters:

$T_n = 10{,}580$ K (ENR = 15.5 dB)

$T_a = 300$ K

$\delta T_n = 200$ K

$\delta T_a = 3$ K

$\delta\alpha = 2\%$ of α(dB)

b. Let the measured values of α be

$\alpha_1 = \underline{0.228}$ (-6.43 dB)

$\alpha_2 = \underline{0.910}$ (-0.408 dB)

c. From (123),

$$\overline{T_e} = (10{,}580 - 300)\ \frac{0.228^2}{0.910 - 2 \times 0.228} - 300 = \underline{870\ K}$$

d. From (184), (185), (186), and (187),

$$\frac{\partial \overline{T_e}}{\partial T_n} = \frac{0.228^2}{0.910 - 2 \times 0.228} = \underline{0.1138}$$

$$\frac{\partial \overline{T_e}}{\partial T_a} = -\left(\frac{0.228^2}{(0.910 - 2 \times 0.228)} + 1\right) = \underline{-1.1138}$$

$$\frac{\partial \overline{T_e}}{\partial \alpha_2} = \frac{2(10{,}580 - 300)(0.228 \times 0.910 - 0.228^2)}{(0.910 - 2 \times 0.228)^2} = \underline{15420\ \mathrm{K}}$$

$$\frac{\partial \overline{T_e}}{\partial \alpha_2} = -\frac{0.228^2\ (10{,}580 - 300)}{(0.910 - 2 \times 0.228)^2} = -\ \underline{2570\ \mathrm{K}}$$

e. The uncertainties, $\delta\alpha$, are given by

$\delta\alpha_1$ (dB) = 0.02 x 6.43 dB = 0.129 dB

$\therefore \delta\alpha_1 = 0.030\alpha_1 = 0.030 \times 0.228 = \underline{0.00683}$

$\delta\alpha_2$ (dB) = 0.02 x 0.408 dB = 0.00816 dB

$\therefore \delta\alpha_2 = 0.0019\alpha_2 = 0.0019 \times 0.910 = \underline{0.00173}$

f. From (183),

$$\delta\overline{T_e} = (200 \times 0.1138) + (3 \times 1.1138) + (0.00683 \times 15420) + $$

$$+ (0.00173 \times 2570)$$

$$= 22.8 + 3.34 + 105.3 + 4.45 = \underline{135.9\ \mathrm{K}}$$

g. Summary:

$\alpha_1 = 0.228 \quad (-6.43 \text{ dB})$

$\alpha_2 = 0.910 \quad (-0.408 \text{ dB})$

$\overline{T_e} = 870 \text{ K}$

$\delta\overline{T_e} = 135.9 \text{ K } (15.6\% \text{ of } \overline{T_e})$

Components of $\delta\overline{T_e}$:			
	From T_n:	22.8 K	(2.6%)
	From T_a:	3.34 K	(0.4%)
	From α_1:	105.3 K	(12.1%)
	From α_2:	4.45 K	(0.5%)
		135.9K	(15.6%)

h. $\overline{F} = \frac{870}{290} + 1 = \underline{4.00} \quad \underline{(6.0 \text{ dB})}$

$$\delta\overline{F} = \frac{\delta\overline{T_e}}{290} = \frac{135.9}{290} = \underline{0.469}$$

$$\frac{\delta\overline{F}}{\overline{F}} = \frac{0.469}{4.00} = \underline{0.117} \quad \underline{(11.7\% \text{ ; } (0.48 \text{ dB}))}$$

8.8. CW Technique

a. Given parameters:

$P_s = 10^{-10}$ watt

$T_a = 300$ K

$\delta P = 2\%$ of P

$\delta T_a = 3$ K

$\delta B = 5\%$ of B

b. Let the measured values of P_1 and P_2 be

P_1 = $\underline{9.69 \times 10^{-8} \text{ watt}}$ (0.097 μw)

P_2 = $\underline{10.1 \times 10^{-6} \text{ watt}}$ (10.1 μw)

c. From (129)

$$\overline{F} = \frac{10^{-10}}{1.381 \times 10^{-23} \times 290 \times 6 \times 10^{7}} \frac{}{\left(\dfrac{10.1 \times 10^{-6}}{9.69 \times 10^{-8}} - 1\right)} - \frac{300}{290} + 1$$

$$= 4.035 - 1.034 + 1 = \underline{4.00}$$

d. From (189), (190), (191), (192), and (193),

$$\frac{\partial \overline{F}}{\partial P_s} = \frac{1}{1.381 \times 10^{-23} \times 290 \times 6 \times 10^{7} \left(\dfrac{10.1 \times 10^{-6}}{9.69 \times 10^{-8}} - 1\right)} = \underline{4.035 \times 10^{-10} \text{ watt}^{-1}}$$

$$\frac{\partial \overline{F}}{\partial T_a} = -\frac{1}{290} = \underline{-\ 0.00345 \ \ K^{-1}}$$

$$\frac{\partial \overline{F}}{\partial B} = -\frac{10^{-10}}{1.381 \times 10^{-23} \times 290 \times 6^{2} \times 10^{14} \left(\dfrac{10.1 \times 10^{-6}}{9.69 \times 10^{-8}} - 1\right)} = \underline{-\ 6.726 \times 10^{-8} \text{ Hz}^{-1}}$$

$$\frac{\partial \overline{F}}{\partial P_1} = \frac{10^{-10} \times 10.1 \times 10^{-6}}{1.381 \times 10^{-23} \times 290 \times 6 \times 10^{7} \ (10.1 \times 10^{-6} - 9.69 \times 10^{-8})^{2}} = \underline{4.205 \times 10^{7} \text{ watt}^{-1}}$$

$$\frac{\partial \overline{F}}{\partial P_2} = -\frac{10^{-10} \times 9.69 \times 10^{-8}}{1.381 \times 10^{-23} \times 290 \times 6 \times 10^{7} \ (10.1 \times 10^{-6} - 9.69 \times 10^{-8})^{2}} = \underline{-\ 4.035 \times 10^{5} \text{ watt}^{-1}}$$

e. From (188),

$$\delta \overline{F} = (0.02 \times 10^{-10} \times 4.035 \times 10^{10}) + (3 \times 0.00345) +$$

$$+ \quad (0.05 \times 6 \times 10^{7} \times 6.726 \times 10^{-8}) +$$

$$+ \quad (0.02 \times 9.69 \times 10^{-8} \times 4.205 \times 10^{7}) +$$

$$+ \quad (0.02 \times 10.1 \times 10^{-6} \times 4.035 \times 10^{5})$$

$$= \quad 0.0807 + 0.0103 + 0.2018 + 0.0815 + 0.0848 = \underline{0.459}$$

$$\frac{\delta\overline{F}}{F} = \frac{0.459}{4.00} = \underline{0.115} \rightarrow \underline{11.5\%}$$

f. Summary:

$P_1 = 9.69 \times 10^{-8}$ watt

$P_2 = 10.1 \times 10^{-6}$ watt

$P_s = 10^{-10}$ watt

$\overline{F} = 4.00$ (6.0 dB)

$\delta\overline{F} = 0.459$ (11.5% of $\overline{F}$; 0.47 dB)

Components of $\delta\overline{F}$:			
	From P_s:	0.0807	(2.0%)
	From T_a:	0.0103	(0.3%)
	From B :	0.202	(5.1%)
	From P_1:	0.0815	(2.0%)
	From P_2:	0.0848	(2.1%)
		0.459	(11.5%)

g. $\overline{T_e} = 290(4.00 - 1) = \underline{870\ K}$

$\delta\overline{T_e} = 290 \times 0.459 = \underline{133\ K}$

$$\frac{\delta\overline{T_e}}{\overline{T_e}} = \frac{133}{870} = 0.153 \rightarrow \underline{15.3\%}$$

8.9. Tangential Technique

a. Given parameters:

T_a = 300 K

$(S/N)_o$ = 11 dB (12.59)

δP_s = 2% of P_s

δT_a = 3 K

$\delta (S/N)_o$ = 1 dB (25.9%)

δB = 5% of B

b. Let the measured value of P_s be

$P_s = \underline{12.2 \times 10^{-12} \text{ watts}}$

c. From (139)

$$\overline{F} = \frac{12.2 \times 10^{-12}}{1.381\text{x}10^{-23}\text{x}290\text{x}6\text{x}10^{7}\text{x}12.59} - \frac{300}{290} + 1$$

$$= 4.034 - 1.034 + 1 = \underline{4.00}$$

d. From (195), (196), (197), and (198),

$$\frac{\partial \overline{F}}{\partial P_s} = \frac{1}{1.381\text{x}10^{-23}\text{x}290\text{x}6\text{x}10^{7}\text{x}12.59} = \underline{0.3305\text{x}10^{12}\text{watt}^{-1}}$$

$$\frac{\partial \overline{F}}{\partial T_a} = -\frac{1}{290} = -\ \underline{0.00345\ K^{-1}}$$

$$\frac{\partial \overline{F}}{\partial B} = -\frac{12.2 \times 10^{-12}}{1.381\text{x}10^{-23}\text{x}290\text{x}36\text{x}10^{14}\text{x}12.59} = -\ \underline{6.72\text{x}10^{-8}\text{Hz}^{-1}}$$

$$\frac{\partial \overline{F}}{\partial (S/N)_o} = -\frac{12.2 \times 10^{-12}}{1.381\text{x}10^{-23}\text{x}290\text{x}6\text{x}10^{7}\text{x}(12.59)^{2}} = -\ \underline{0.3205}$$

e. The uncertainty $\delta(S/N)_o$ is given by

$$\delta(S/N)_o = 0.259 \times 12.59 = \underline{3.261}$$

f. From (194),

$$\delta\overline{F} = (0.02 \times 12.2x10^{-12}x0.3305x10^{12}) +$$

$$+ (3x0.00345) + (0.05x6x10^{7}x6.724x10^{-8}) +$$

$$+ (3.261x0.3205)$$

$$= 0.0807 + 0.0134 + 0.2017 + 1.045 = \underline{1.34}$$

$$\frac{\delta\overline{F}}{\overline{F}} = \frac{1.34}{4.00} = \underline{0.335} \rightarrow \underline{33.5\%}$$

g. Summary:

P_s = 12.2×10^{-12} watts

$\overline{F}$ = 4.00 (6.0 dB)

$\delta\overline{F}$ = 1.34 (33.5% of $\overline{F}$; 1.25 dB)

Components of $\delta\overline{F}$:	From P_s:	0.081	(2.0%)
	From T_a:	0.013	(0.3%)
	From B :	0.202	(5.1%)
	From $(S/N)_o$:	1.045	(26.1%)
		1.34	(33.5%)

h. $\overline{T_e} = 290(4.00 - 1) = \underline{870\ K}$

$$\delta\overline{T_e} = 290 \times 1.34 = \underline{389\ K}$$

$$\frac{\delta\overline{T_e}}{\overline{T_e}} = \frac{389}{870} = 0.447 \rightarrow \underline{44.7\%}$$

8.10. Comparison Technique

a. Given parameters:

$\bar{F}_m$ = 4.00 (6.0 dB)

T_a = 300 K

$\delta\bar{F}_m$ = 0.40 (0.41 dB)

$\delta P \doteq 5\%$ of P

(See the NOTE at the end of this example).

b. Let the measured values of P be

$P_{m1} = \underline{9.7\text{x}10^{-8}\text{ watt}}$

$P_{m2} = \underline{9.7\text{x}10^{-6}\text{ watt}}$

$P_s = \underline{9.6\text{x}10^{-11}\text{ watt}}$

$P_{x1} = \underline{10.5\text{x}10^{-8}\text{ watt}}$

$P_{x2} = \underline{9.7\text{x}10^{-6}\text{ watt}}$

c. From (154),

$$\bar{F}_x \doteq 4.00\ \frac{9.7\text{x}10^{-6}\text{x}10.5\text{x}10^{-8}}{9.7\text{x}10^{-8}\text{x}9.7\text{x}10^{-6}} = \underline{4.33}\quad (6.4\text{ dB})$$

d. From (200), (201), (202), (203), and (204),

$$\frac{\partial\bar{F}_x}{\partial\bar{F}_m} = \frac{9.7\text{x}10^{-6}\text{x}10.5\text{x}10^{-8}}{9.7\text{x}10^{-8}\text{x}9.7\text{x}10^{-6}} = \underline{1.08}$$

$$\frac{\partial\bar{F}_x}{\partial P_{m1}} = -\ \frac{4.00\text{x}9.7\text{x}10^{-6}\text{x}10.5\text{x}10^{-8}}{(9.7)^2\text{x}10^{-16}\text{x}9.7\text{x}10^{-6}} = \underline{-\ 0.448\text{x}10^{8}\text{ watt}^{-1}}$$

$$\frac{\partial\bar{F}_x}{\partial P_{m2}} = \frac{4.00\text{x}10.5\text{x}10^{-8}}{9.7\text{x}10^{-8}\text{x}9.7\text{x}10^{-6}} = \underline{0.448\text{x}10^{6}\text{ watt}^{-1}}$$

$$\frac{\partial \overline{F_x}}{\partial P_{x1}} = \frac{4.00 x 9.7 x 10^{-6}}{9.7 x 10^{-8} x 9.7 x 10^{-6}} = \underline{0.412 x 10^{8} \text{ watt}^{-1}}$$

$$\frac{\partial \overline{F_x}}{\partial P_{x2}} = - \frac{4.00 x 9.7 x 10^{-6} x 10.5 x 10^{-8}}{9.7 x 10^{-8} x (9.7)^{2} x 10^{-12}} = - \underline{0.447 x 10^{6} \text{ watt}^{-1}}$$

e. From (199),

$$\delta\overline{F_x} = (0.40 \text{ x } 1.08) + (0.05 \text{ x } 9.7 x 10^{-8} x 0.448 x 10^{8}) +$$

$$+ \quad (0.05 x 9.7 x 10^{-6} x 0.448 x 10^{6}) + (0.05 x 10.5 x 10^{-8} x 0.412 x 10^{8}) +$$

$$+ \quad (0.05 x 9.7 x 10^{-6} x 0.447 x 10^{6})$$

$$= \quad 0.432 + 0.217 + 0.217 + 0.217 + 0.217 = \underline{1.30}$$

$$\frac{\delta\overline{F_x}}{\overline{F_x}} = \frac{1.30}{4.33} = \underline{0.300} \rightarrow \underline{30.0\%}$$

f. Summary:

$F_m = 4.00$ (6.0 dB)

$\delta\overline{F_m} = 0.40$ (10.0% of $\overline{F_m}$; 0.41 dB)

$\overline{F_x} = 4.33$ (6.4 dB)

$\delta\overline{F_x} = 1.30$ (30.0% of $\overline{F_x}$; 1.1 dB)

Components of $\delta\overline{F_x}$:	From $\overline{F_m}$:	0.432	(10.0%)
	From P_{m1}:	0.217	(5.0%)
	From P_{m2}:	0.217	(5.0%)
	From P_{x1}:	0.217	(5.0%)
	From P_{x2}:	0.217	(5.0%)
		1.30	(30.0%)

g. $\overline{T_e} = 290\ (4.33 - 1) = \underline{966\ K}$

$\delta\overline{T_e} = 290 \times 1.30 = \underline{377\ K}$

$$\frac{\delta\overline{T_e}}{\overline{T_e}} = \frac{377}{966} = 0.390 \rightarrow \underline{39.0\%}$$

NOTE: This example assumes there are no differences between the transducers. In practice, the transducers will have somewhat different transfer functions, terminal impedances, and $\overline{F}$'s. Such differences may increase the measurement uncertainty.

9. APPENDICES

APPENDIX A

Definitions of Terms

The terms defined in this Appendix have been selected to aid in understanding this Guide. They by no means represent all the terms pertaining to the subject. For terms not included, the reader is referred to standard sources and to the literature. Definitions taken from the IEEE standards are identified in parentheses at the end of the definition.

1. Accuracy - The degree to which a stated or measured value of a quantity agrees with its true value. Correctly expressed in qualitative terms (high, low, good, poor), it is often wrongly used to express a quantitative concept which is properly expressed by the term "error" or "uncertainty." For a discussion of this and related terms, see reference [50].

2. Bandwidth, effective (bandpass filter in a signal transmission system). - The width of an assumed rectangular bandpass filter having the same transfer ratio at a reference frequency and passing the same mean square of a hypothetical current and voltage having even distribution of energy over all frequencies. (IEEE Std 100-1972) [51]

   Note: In this Guide, effective bandwidth of a filter and noise bandwidth of a transducer are considered to be identical.

3. Dispersed-Signal Source - A signal source whose available power is distributed over a range of frequencies. Examples include noise generators, impulse generators, comb generators, noise-modulated sinusoids, etc.

4. Effective Input Noise Temperature T_e (of a multiport tranducer with one port designated as the output port). - The noise temperature in kelvins which, assigned simultaneously to the specified impedance terminations at the set of frequencies contributing to the output, at all accessible ports except the designated output port of a noisefree equivalent of the transducer, would yield the same available power per unit bandwidth at a specified output frequency at the output port as that of the actual transducer connected to noisefree equivalents of the terminations at all ports except the output port. (62 IRE 7.S2)

   Note 1: A noisefree equivalent of the transducer is one for which all internal (inaccessible) noise sources are removed, and for which the termination at the output port is noisefree.

   Note 2: The effective input noise temperature may depend significantly on the impedance and the noise temperature of the termination at the output port at frequencies other than the specified output frequency.

   Note 3: For a two-port transducer with a single input and a single output frequency the effective input noise temperature T_e is related to the noise factor F by the equation

$$T_e = 290(F - 1).$$

5. Effective Input Noise Temperature, Average $\overline{T_e}$ (of a multiport transducer with one port designated as the output port). - The noise temperature in kelvins which, assigned simultaneously to the specified impedance terminations at all frequencies at all accessible ports except the designated output port of a noisefree equivalent of the transducer, would yield the same total noise power in a specified output band delivered to the output termination as that of the actual transducer connected to noisefree equivalents of the terminations at all ports except the output port. (62 IRE 7.S2)

Note 1: A noisefree equivalent of the transducer is one for which all internal (inaccessible) noise sources are removed and for which the termination at the output port is noisefree.

Note 2: The load is to be taken as noisefree over the output frequency band when determining the delivered output power.

Note 3: $\overline{T_e}$ may depend significantly on the impedances and averaged noise temperatures of the termination at the output port within frequency bands other than the specified output frequency band.

Note 4: For a two-port transducer with a single input and a single output frequency band, the average effective input noise temperature $\overline{T_e}$ is related to the average noise factor $\overline{F}$ by the equation

$$\overline{T_e} = 290(\overline{F} - 1).$$

6. Error - The difference between the stated or measured value and the true value of a quantity. Usually expressed in the same units as the quantity, or in per cent.

7. Linear Transducer - A transducer for which the pertinent measures of all the waves concerned are linearly related. (IEEE Std 100-1972)

Note 1: By linearly related is meant any relation of linear character whether by linear algebraic equation, by linear differential equation, by linear integral equation, or by other linear connection.

Note 2: The term WAVES CONCERNED connotes actuating waves and related output waves, the relation of which is of primary interest in the problem at hand.

8. Noise - Unwanted disturbances superposed upon a useful signal that tend to obscure its information content. (IEEE Std 100-1972)

Note: When applied to the measurement of noise factor and effective input noise temperature, a randomly fluctuating signal which would be classified as noise in a communications system becomes a useful signal for the purposes of the measurement.

9. Noise Bandwidth - See bandwidth, effective.

10. Noise Factor (Noise Figure) (of a Two-Port Transducer). - At a specified input frequency the ratio of 1) the total noise power per unit bandwidth at a corresponding output frequency available at the output port when the Noise Temperature of its input termination is standard (290 K) at all frequencies to 2) that portion of 1) engendered at the input frequency by the input termination at the Standard Noise Temperature (290 K). (57 IRE 7.S2)

Note 1: For heterodyne systems there will be, in principle, more than one output frequency corresponding to a single input frequency, and vice versa; for each pair of corresponding frequencies a Noise Factor is defined. 2) includes only that noise from the input termination which appears in the output via the principal-frequency transformation of the system, i.e., via the signal-frequency transformation(s), and does not include spurious contributions such as those from an unused image-frequency or an unused idler-frequency transformation.

Note 2: The phrase "available at the output port" may be replaced by "delivered by system into an output termination."

Note 3: To characterize a system by a Noise Factor is meaningful only when the admittance (or impedance) of the input termination is specified.

11. Noise Factor (Noise Figure), Average (of a Two-Port Transducer). - The ratio of 1) the total noise power delivered by the transducer into its output termination when the Noise Temperature of its input termination is standard (290 K) at all frequencies, to 2) that portion of 1) engendered by the input termination. (57 IRE 7.S2)

Note 1: For heterodyne systems, 2) includes only that noise from the input termination which appears in the output via the principal-frequency transformation of the system, i.e., via the signal-frequency transformation(s), and does not include spurious contributions such as those from an unused image-frequency or an unused idler-frequency transformation.

Note 2: A quantitative relation between the Average Noise Factor $\overline{F}$ and the Spot Noise Factor F(f) is

$$\overline{F} = \frac{\int_0^\infty F(f)G(f)df}{\int_0^\infty G(f)df}$$

where f is the input frequency, and G(f) is the transducer gain, i.e., the ratio of 1) the signal power delivered by the transducer into its output termination, to 2) the corresponding signal power available from the input termination at the input frequency. For heterodyne systems, 1) comprises only power appearing in the output via the principal-frequency transformation i.e., via the signal-frequency transformation(s) of the system; for example, power via unused image-frequency or unused idler-frequency transformation is excluded.

Note 3: To characterize a system by an Average Noise Factor is meaningful only when the admittance (or impedance) of the input termination is specified.

12. Noise Factor (Noise Figure), Spot. See: Noise Factor (Noise Figure)(of a Two-Port Transducer). (57 IRE 7.S2)

Note: This term is used where it is desired to emphasize that the Noise Factor is a point function of input frequency.

13. Noise Temperature (general)(at a pair of terminals and at a specific frequency). - The temperature of a passive system having an available noise power per unit bandwidth equal to that from the actual terminals. (IEEE Std 100-1972)

Note: Thus, the noise temperature of a simple resistor is the actual temperature of the resistor, while the noise temperature of a diode through which current is passing may be many times the observed absolute temperature.

14. Noise Temperature (standard). - The standard reference temperature T_o for noise measurements is 290 kelvins. (IEEE Std 100-1972)

Note: kT_o/e = 0.0250 volt, where e is the electron charge and k is Boltzmann's constant.

15. Plasma Noise - Random noise generated by non-equilibrium processes in a gas discharge device or its solid-state equivalent. See reference [52].

16. Port (electronic devices or networks) - A place of access to a device or network where energy may be supplied or withdrawn or where the device or network variables may be observed or measured. (IEEE Std 100-1972)

Notes: (A) In any particular case, the ports are determined by the way the device is used and not by its structure alone. (B) The terminal pair is a special case of a port. (C) In the case of a waveguide or transmission line, a port is characterized by a specified mode of propagation and a specified reference plane. (D) At each place of access, a separate port is assigned to each significant independent mode of propagation. (E) In frequency changing systems, a separate port is also assigned to each significant independent frequency response.

17. Precision - The degree to which a collection of measurement results cluster together. Usually expressed in qualitative terms (e.g., high, low, moderate, good, poor). For a discussion of this and related terms, see reference [50].

18. Random Noise - Noise that comprises transient disturbances occurring at random. (IEEE Std 100-1972) Examples include thermal noise, shot noise, and plasma noise. Examples may not include impulsive noise, ignition noise, and interference of a non-random nature.

19. Signal-to-noise ratio - The ratio of the value of the signal to that of the noise. (IEEE Std 100-1972)

Note: When dealing with random noise, this ratio is a power ratio, usually expressed in decibels.

20. Shot Noise - Random noise generated by a non-equilibrium process in which fluctuations occur on an otherwise steady-state flow of current. See references [35] and [53].

21. Source - That which supplies signal power to a transducer or system. (IEEE Std 100-1972)

22. Termination - A one-port load that terminates a section of a transmission system in a specified manner. (IEEE Std 100-1972)

23. Thermal Noise - Random noise generated by thermal processes in equilibrium with their environment. See reference [27].

24. Thermionic Diode, temperature-limited - A vacuum diode tube operating with a cathode temperature and a cathode-anode voltage such that all electrons emitted from the cathode are collected at the anode.

25. Transducer - A device by means of which energy can flow from one or more transmission systems or media to one or more other transmission systems or media. (IEEE Std 100-1972) Examples include amplifiers, receivers, mixers, detectors, attenuators, couplers, transformers, filters, etc.

26. Transducer gain - The ratio of the power that the transducer delivers to the specified load under specified operating conditions to the available power of the specified source. (IEEE Std 100-1972)

Notes: (A) If the input and/or output power consists of more than one component, such as multifrequency signals or noise, then the particular components used and their weighting must be specified. (B) This gain is usually expressed in decibels.

27. Uncertainty - The amount of lack of absolute certainty in the value of a quantity or measurement result. Usually expressed either in the same units as the quantity, or in percent. Example: "The measured value of $\overline{T_e}$ is 400 K; the uncertainty in this value is 20 K or 5%."

APPENDIX B

Treatment of Errors

This Appendix deals very briefly with the treatment of measurement errors. A thorough treatment of this subject is beyond the scope of this Guide, and may be found by reference to the literature [50], [54], [55].

Measurement Errors

A measurement error is the difference between the true value and the measured value of a quantity. Measurement errors are of two types: systematic and random. A systematic error is an error caused by a bias in some feature of the measurement system, and is the difference between the true value and the limiting mean of a set of measured values of a quantity. A random error is an error caused by a random process, and is often expressed in terms of the standard deviation of the measurement data.

Measurement errors result primarily from uncertainties in the data. They can also result from procedural mistakes, but this source of error can be removed by using care, repeating measurements, cross-checking, and a variety of other procedural methods. Uncertainties in the data come from such sources as (a) uncertainties in the true value of a meter or dial reading, (b) uncertainties in the calibration of an instrument, (c) imprecision in the indicated meter reading due to random fluctuations or poor resolution, (d) variations in the parameter being measured during the measurement interval, and (e) variations caused by human frailties in performing the measurement.

The total measurement error is obtained by means of an error equation. The error equation comes as a result of an error analysis of the measurement process, and contains terms which represent the individual sources of error.

The total error of a measurement normally consists of both systematic and random errors. In general, the error equation applies separately to both types of errors. Thus, the error equation is used twice, once for systematic errors and once for random errors. The total error is then expressed in two parts; viz., a systematic part and a random part. In some cases, one type may predominate over the other, in which case the total error may be substantially systematic or random. For proper use of error equations, refer to the section on Pooling Errors, below.

Repeating Measurements

Measurements should be made more than once for the following reasons:

a. Gross errors and mistakes may be revealed.

b. The average result usually has a smaller error than each individual result.

c. The spread of the individual results provides an estimate of the measurement error.

Repeating the measurement many times can reduce the random error, but it will not reduce the systematic error. Systematic error is reduced by using test equipment having greater inherent accuracy, and by eliminating any erroneous adjustments or procedures.

Random error is expressed in terms of the standard deviation, s, of the measurement data. It is common practice to take 3s (three standard deviations) as the random error of the measured value of the parameter.

Standard deviation decreases with the square root of the number of measurement results, n. Significant improvements accrue as n increases from one to two to three, but note that the relative improvement decreases with increasing n. Seldom is it worth while to repeat measurements of the type given in this Guide more than ten times.

Pooling Errors

Independent systematic errors are combined algebraically. That is, if one error is positive (+) and the other negative (-), the total of these two errors is their algebraic sum. If a systematic error can be either positive or negative (±) then judgement must be excercised when combing it with other errors. Unless knowledge about an error directs otherwise, the worst-case combination should be used. This normally means that the errors are pooled by adding their absolute values, thus disregarding their algebraic signs. If there is some interdependence (correlation) between individual errors, it may be taken into account to obtain a pooled error that is smaller than their worst-case sum.

Random errors are combined on a root-mean-square basis. That is, the two errors, Δa and Δb, produce a total error, Δc, given by the equation

$$\Delta c = [(\Delta a)^2 + (\Delta b)^2]^{1/2}. \qquad (205)$$

When there is a question as to whether an error is systematic or random, usual practice is to pool it with other errors on a worst-case basis.

Obtaining Measurement Uncertainties

In order to estimate a measurement error, it is necessary to obtain quantitative values for the uncertainties in the data. The usual source of information for systematic measurement uncertainties is the manufacturer's accuracy specification for the particular test equipment involved. An alternative and possibly superior source is a recent calibration certificate for the instrument, which should include the limits of error on the calibration results. Lacking either of these sources, measurement uncertainties can be obtained by measuring a known quantity with the instrument, and comparing the measurement results with the known value.

Random uncertainties are obtained from a statistical analysis of the data. Refer to standard textbooks on the subject for procedures [56], [57].

Measurement of Noise Bandwidth

Certain of the measurement techniques discussed in this Guide (see Sections 3.5 and 3.6) require a value for the noise bandwidth, B, of the transducer. In other techniques, a knowledge of B is useful to interpret the measurement results, as, for example, to distinguish between "broad band noise factor" and "narrow band noise factor." This Appendix is a brief discussion of techniques for measuring B.

Noise Bandwidth

Noise bandwidth, B, is defined in terms of a fictional gain function having the shape of a rectangle (see figure 32). The area, A, enclosed by this rectangle is, by definition, the same as the area beneath the actual gain function, G(f), of the transducer or system involved (see figure 33), where

$$A = \int_{fs} G(f)\,df \;. \tag{206}$$

The height (gain), G_o, of the fictional gain function is chosen as discussed below. The width, B, of the fictional gain function is then determined by the area under the actual gain function, and is, by definition, this area divided by the height of the fictional gain function.

Therefore, to determine the noise bandwidth of a transducer or system, two quantities must be known; viz., (1) the area under the actual gain function, and (2) the value of gain for which noise bandwidth is computed. These two quantities are discussed below.

Measurement of Area A

The first step to obtain A is to measure the gain function, G(f), of the transducer. Techniques for doing this are well established and referenced in the literature [58], [59]. They include point-by-point techniques and swept-frequency techniques. Variations on the several techniques have been devised to meet particular measurement problems imposed by frequency range, dynamic range, terminal impedances, feedback, leakage, interference susceptibility, and noise factor.

The next step is to produce a plot of G(f) on linear graph paper. If G(f) is known analytically, it can be plotted from its equation. If data on G(f) is in tabular numerical form, G(f) is plotted manually. Some measurement techniques for G(f) produce data directly as a curve, either on an X-Y plotter, or on an

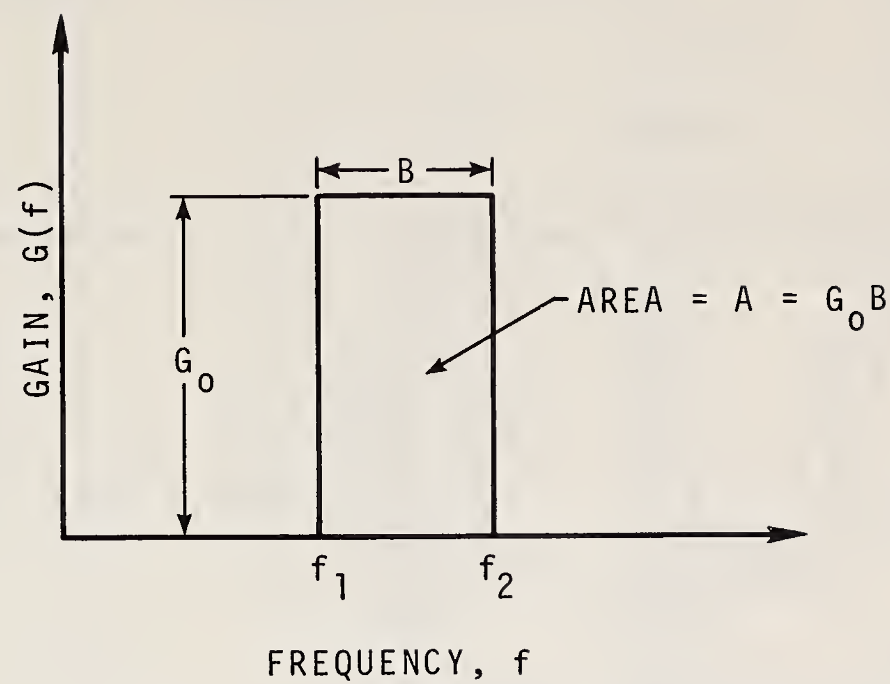


Figure 32. Ideal (fictional) Gain Function.

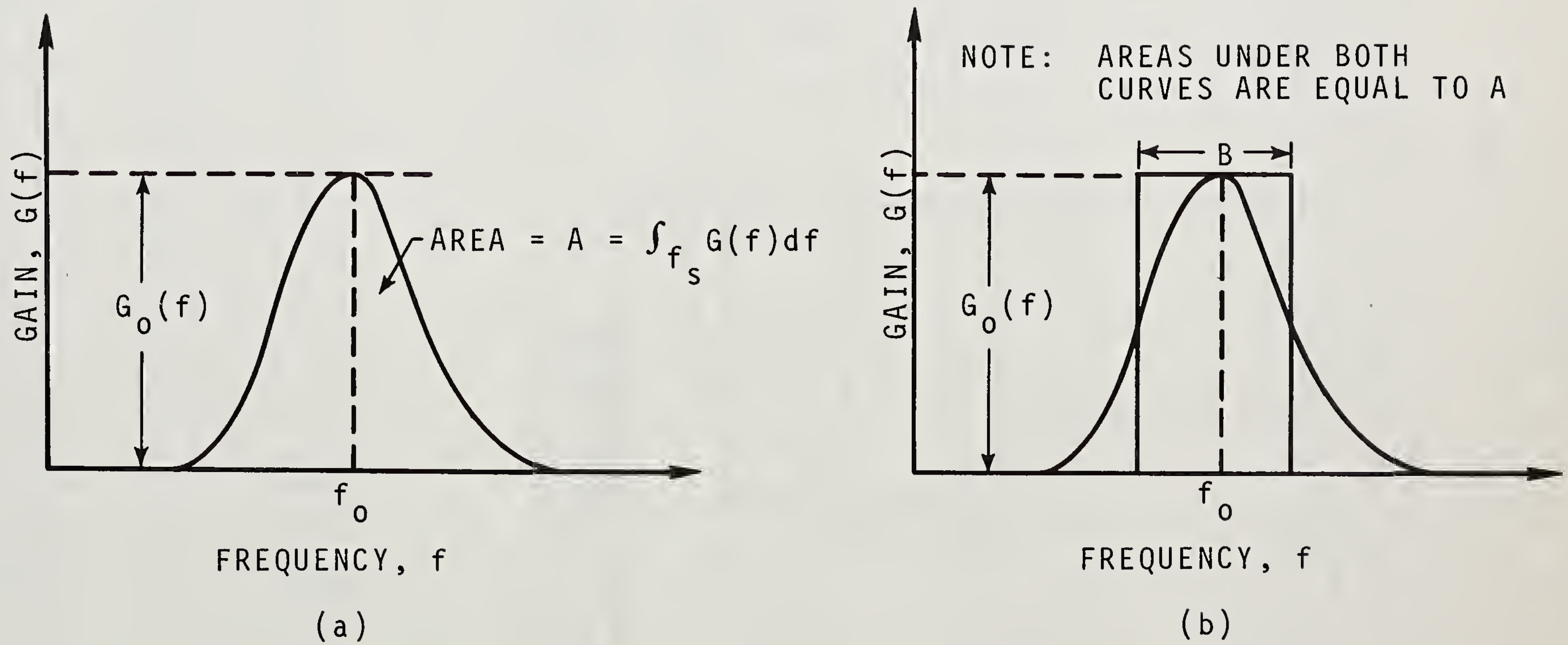


Figure 33. Typical Gain Function.

oscilloscope screen which must then be photographed or the curve can be traced. In any event, the area may then be determined by any of the well-known techniques such as (a) counting squares, (b) use of a planimeter, or (c) linear line-segment approximation [60]. If the plot is on semi-log or log-log coordinates, or if the data are in decibels rather than as a simple ratio of powers, perform whatever steps are necessary to obtain the equivalent results as from a linear plot of power ratio versus frequency.

Measurement of Gain G_o

Gain G_o is measured by any of several well established techniques as mentioned in the preceeding section. G_o may be assigned the arbitary value of unity "1", and the values of G(f) are then measured relative to G_o.

The choice of measurement frequency, f_o, at which G_o is evaluated, is, in principle, arbitrary. That is, B can be determined from any value of G(f), and the computed value of $\overline{F}$ will be independent of the choice of f_o as explained below. However, f_o should be chosen such that G_o may be measured accurately; therefore f_o is normally chosen in a range where G(f) is large. In the case of a simple gain function having a shape similar to that shown in figure 33, f_o is commonly chosen as the frequency of maximum gain.

Computation of B

Transducer noise bandwidth, B, is computed simply by dividing the value of A by the value of G_o.

$$B = \frac{A}{G_o} = \frac{\int_{fs} G(f)df}{G_o} \tag{207}$$

In cases where the transducer has gain in more than one frequency interval (multiple-response systems) the interval(s) for which A is evaluated must be stated. Also, the frequency f_o at which G_o is evaluated should be stated to complete the information concerning B.

The uncertainty, δB, in the value of B comes from uncertainties δA and δG_o in A and G_o, respectively. The first-order approximation of δB is

$$\delta B = \delta A \frac{\partial B}{\partial A} + \delta G_o \frac{\partial B}{\partial G_o} . \tag{208}$$

Uncertainty δA is obtained by evaluating the uncertainties in (a) gain measurement (power meter error), (b) frequency measurement (frequency meter error), and (c) the technique used to determine A (planimeter error, etc.). δG_o is obtained from the power meter specifications.

The uncertainty coefficients are obtained by differentiating (208) thus:

$$\frac{\partial B}{\partial A} = \frac{1}{G_o} \tag{209}$$

$$\frac{\partial B}{\partial G_o} = - \frac{A}{G_o^{\,2}} \tag{210}$$

Choice of f_o

The reason why f_o is arbitrary is explained as follows: Note that in the CW technique, $\overline{F}$ is given by the equation

$$\overline{F} = \frac{P_s}{k\ T_o\ B\left(\frac{P_2}{P_1} - 1\right)} - \frac{T_a}{T_o} + 1 \tag{129}$$

where

$$P_1 = k(T_a + \overline{T_e}) \int_o^{\infty} G(f)df \tag{126}$$

and

$$P_2 = k(T_a + \overline{T_e}) \int_o^{\infty} G(f)df + P_s\ G_o. \tag{127}$$

By definition,

$$B = \frac{\int_{fs} G(f)df}{G_o}\ . \tag{9}$$

From (126) and (127),

$$\frac{P_2}{P_1} - 1 = \frac{P_s\ G_o}{P_1}\ . \tag{211}$$

Substituting (9) and (211) into (129) gives

$$\overline{F} = \frac{P_s}{k\ T_o\ \frac{\int_{fs} G(f)df}{G_o}\ \frac{P_s G_o}{P_1}} - \frac{T_a}{T_o} + 1 \tag{212}$$

For a given measurement set-up, all of the quantities in (212) EXCEPT G_o, are independent of the choice of f_o. But G_o has no effect on the computed value of $\overline{F}$ since its effect on B is exactly cancelled by its effect on the quantity $(P_2/P_1) - 1$. Therefore, f_o may be chosen at any frequency except where G(f) is zero (B is then undefined).

A similar argument can be presented based upon the Tangential technique.

10. SUGGESTED READING

The published literature on noise factor and noise factor measurement is quite extensive. It represents a large fraction of the total literature on noise, which itself is vast. To read it all is for most people an impractical task. However, from this huge resource may be selected certain papers which have various valuable features that recommend themselves.

Basic to the understanding of noise factor and effective input noise temperature are the tutorial papers and standards written by the IEEE Subcommittee 7.9 on Noise [12], [15], [42]. A very helpful recent book on this subject is a small one by Mumford and Scheibe [13], which, although succinct, covers the subject clearly and in a way that is adequate for many purposes.

To obtain a historical perspective of the subject, one should read the original papers by North [10] and Friis [11] who presented the first discussions of noise factor in the open literature. Goldberg's paper [18] is a good but dated review of Friis' work. As the subject has matured, various circuit models have been proposed as bases for interpreting observations, and the one by Rothe and Dahlke [B87] has received good acceptance.

The practice of measuring and specifying noise factor has not been without its trials and tribulations. It is helpful to read the frustrations and travesties described by Cohn [8] and Greene [9] in order to be alerted to common and sometimes subtle difficulties that continue to plague the unwary.

The techniques of noise factor measurement are described in a large number of books and papers. Unfortunately, some of these are misleading or too simplistic, and in other cases, incorrect. Although old, Lebenbaum's chapter [B73] in the Handbook of Electronic Measurements, Vol. II, is still an excellent treatment of the subject. The IEEE paper on measurement methods [15] should also be read.

The literature contains hundreds of articles pertaining to noise generation and noise sources of various types. The classic papers by Johnson [27], Nyquist [B33], Schottky [B40] and Mumford [37] should be read for historical perspective. The paper by Hart [B24] is an excellent review of the basic principles of thermal, diode, and discharge gas sources suitable for noise factor measurements. Gupta [35] provides an up-to-date review of solid-state noise sources, including an extensive list of references. Potential accuracy capabilities are discussed in IEEE Standards Report No. 294 [45].

Finally, for help in the treatment of measurement errors, Zapf's chapter [50] on "Accuracy and Precision", in the Basic Electronic Instrument Handbook, will be found useful.

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16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

This metrology guide provides the basis for critical comparisons among seven measurement techniques for average noise factor and effective input noise temperature. The techniques that are described, discussed, and analyzed include the (1) Y-Factor, (2) 3-dB, (3) Automatic, (4) Gain Control, (5) CW, (6) Tangential, and (7) Comparison Techniques. The analyses yield working equations and error equations by which accuracy capabilities are compared. Each technique is also analyzed for (a) frequency range for best measurement results, (b) special instrumentation requirements, (c) speed and convenience, (d) operator skill required, and (e) special measurement problems. General instrumentation requirements and practical measurement problems are discussed for the benefit of the non-expert metrologist. Worked examples illustrate the principles involved in applying the working and error equations. An extensive bibliography and suggested reading list aid the metrologist to locate additional material on these measurements. Taken altogether, this guide will be helpful in selecting the best measurement technique for any of a wide range of operational requirements and, once the technique is selected, it will be of further benefit in helping the metrologist identify where his efforts should be placed to derive the greatest efficiency and accuracy from his measurement system.

17. KEY WORDS (Alphabetical order, separated by semicolons)

Effective input noise temperature; measurement errors; noise factor; noise measurements; noise performance factors; noise temperature; Y-factor measurements.

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